YILDIZ TECHNICAL UNIVERSITY
CIVIL ENGINEERING DEPARTMENT
DIVISION OF MECHANICS

STATICS

SIXTH CHAPTER

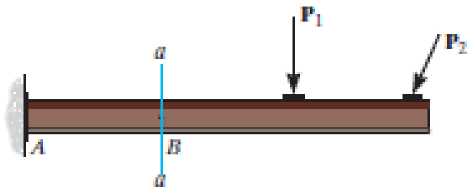
INTERNAL FORCES IN BARS-AXIAL FORCE, SHEAR, BENDING MOMENT DIAGRAMS

6.1 Internal forces in bars-axial force, shear, bending moment diagrams

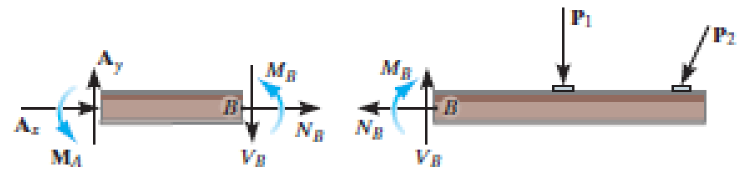
For designing structural systems, i.e. is very important, even inevitable to know the distribution of internal forces of bars which are under external forces.

As it is known, external forces are gravity forces, wind forces, earthquake forces, connection forces etc.

However, internal forces are the actions and reactions between the particles which form the body.

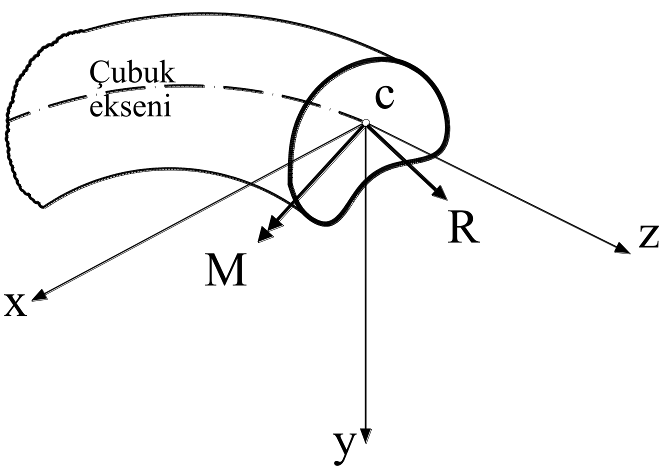


(a)



(b)

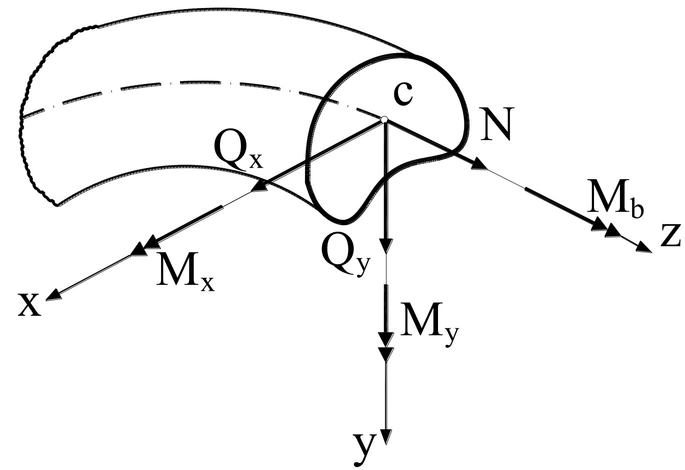
Internal forces in bars



If a bar under external loads are separated into two parts, then for equilibrium of the parts, there must be some forces on the cross-section of these parts. These forces are distributed over the cross-section of the bar.

These distributed forces are reduced to the center of gravity of the cross-section of one part of the bar as a resultant force $\vec{R}$ and a resultant moment $\vec{M}$, Fig. 6.1.

The components of $\vec{R}$ and $\vec{M}$ along the coordinate axes are called as follows:



Components of $\vec{R}$:

- The component perpendicular to the cross-section of the bar, N , is called as axial force.

- The components which lie on the cross-section of the bar is called as shear forces, Q_x and Q_y .

Components of $\vec{M}$:

- The component perpendicular to the cross-section of the bar, M_b , is called as twisting moment.
- The components which lie on the cross-section of the bar is called as bending moments, $M_x \text{ ve } M_y$.

If it is desired to investigate the variation of internal loads with respect to the coordinate axis z and if this variation is shown on a diagram, then the senses of the forces becomes different with respect to the which side of the section is considered.

In this case, there is need to a sign convention to show the internal forces on diagrams which is independent of the considered portion of the bar on one side of the section.

The beam shown in the figure is cut at C which is between A and B. Then the internal loads which are needed to satisfy the equilibrium conditions are desired to be determined. The internal loads on the beam portion AC at the section C is $\vec{V}, \vec{M}, \vec{N}$ while the internal loads on the beam portion BC are $\vec{V}', \vec{M}', \vec{N}'$. From the action-reaction principle, the senses of these loads are opposite to each other but have same magnitudes. The resultant of the flexural stresses on any transverse section has been shown to be a couple (If only transverse loads are considered) and has been designated as $\vec{M}$.

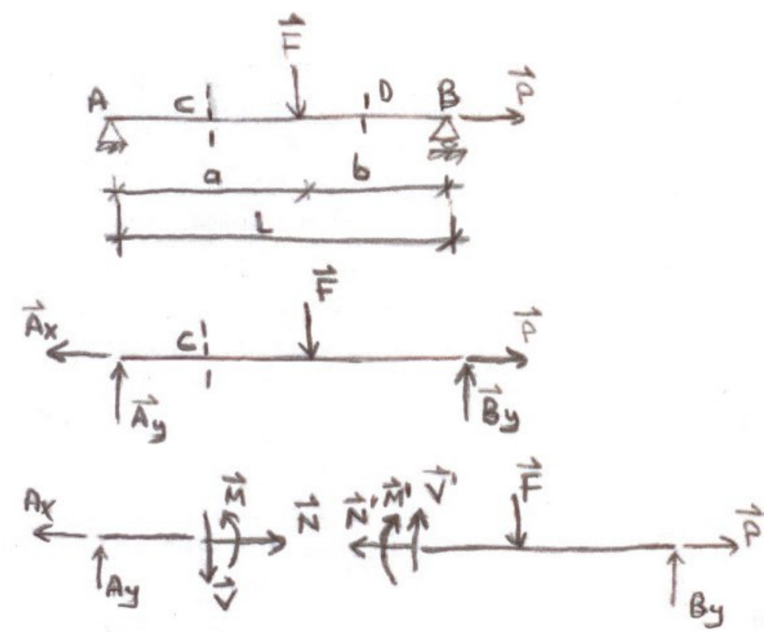
If we calculate the internal forces on the part AC and show them on a diagram, then it is necessary to state on which diagram the calculations are performed.

Therefore, there is need to a new definition of sign for obtaining the same diagrams which are independent of the chosen part and have the same sign for the two portions. Namely, when this sign convention is used, it is not important which part on which side of the section is considered. You can either look at right or left side of the cross-section. It is not important to deal with either side of the cross-section. This is accomplished by choosing

Common magnitude of the vectors $\vec{V}$ and $\vec{V}'$ as V

Common magnitude of the vectors $\vec{M}$ and $\vec{M}'$ as M

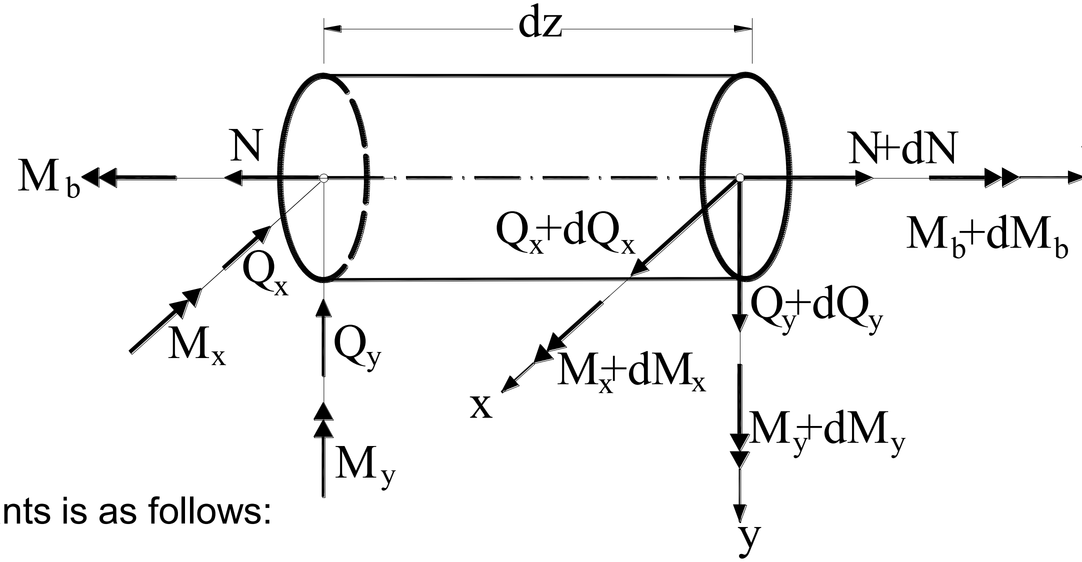
Common magnitude of the vectors $\vec{N}$ and $\vec{N}'$ as N



and then assuming the sign convention special to shear force, axial force and bending moment, the related diagrams can be drawn without mentioning the side of the part considered during the calculations. The following convention will give consistent results regardless of whether one proceeds from left to right or from right to left along the beam. The sign conventions for shear, axial force and bending moment are given below.

Sign convention for resultants

When both the sense of outer normal and the sense of the force is positive or when both the sense of outer normal and the sense of the force are negative, then the resultants are positive. When the sense of the outer normal is positive and the sense of the force is negative or the sense of outer normal is negative and the sense of force is positive, then the resultants are negative.

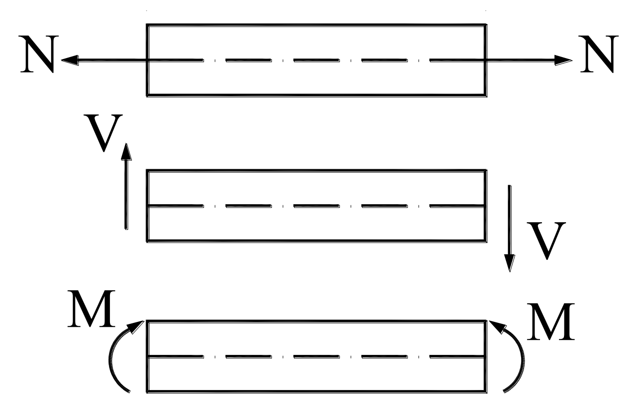


For a beam in plane, sign convention for resultants is as follows:

If axial forces elongates a portion of beam, then these axial forces are positive, otherwise they shorten it, they are negative.

When the shear forces tend to rotate the portion of the beam clockwise, then these shear forces are positive. When they tend to rotate the portion of the beam counterclockwise, they are negative.

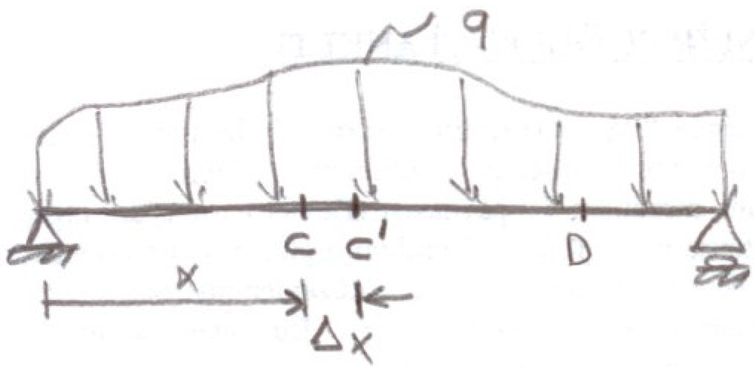
The bending moment in a horizontal beam is positive at sections for which the top of the beam is in compression and the bottom is in tension. In vertical bars, we mark the chosen tension side with a dashed line to define the positive and negative bending moments in advance.

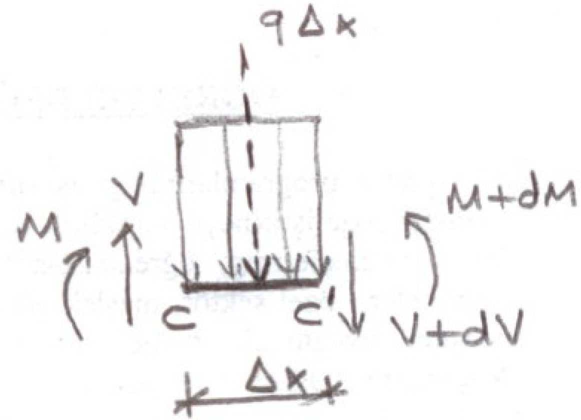


(a)

6.2 Relations between distributed load, shear force and bending moment

In this stage, the relations between distributed load in plane, shear force and bending moment will be obtained for a beam in the same plane with the distributed load.





$$V - (V + \Delta V) - q\,\Delta x = 0 \quad \longrightarrow \quad \frac{dV}{dx} = -q \qquad \textbf{(6.9)}$$

By integrating Equation (6.9) between the points C and D, we obtain

$$V_D - V_C = -\int_{x_C}^{x_D} q\,dx \qquad \textbf{(6.10)} \qquad V_D - V_C = -(\text{the area under the load curve between } C \text{ and } D \text{ points})$$

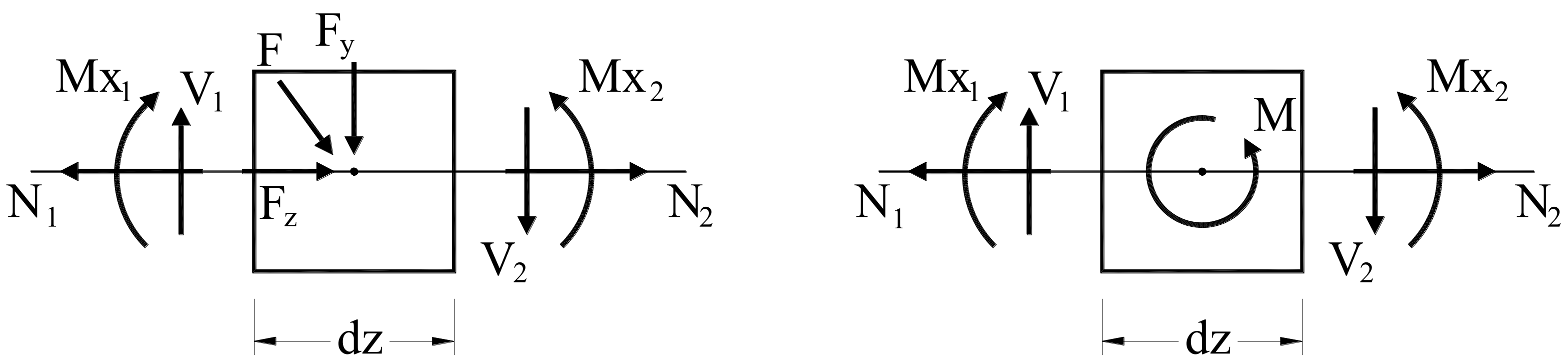
$$(M + \Delta M) - M - V\,\Delta x + q\,\Delta x\frac{\Delta x}{2} = 0$$

$$\Delta M = V\,\Delta x - \frac{1}{2}q(\Delta x)^2 \quad \longrightarrow \quad \frac{dM}{dx} = V \qquad \textbf{(6.11)}$$

By integrating Equation (6.11) between the points C and D, we obtain

$$M_D - M_C = \int_{x_C}^{x_D} V\, d\, x \qquad (6.12) \qquad M_D - M_C = (\textit{the area under the shear force curve between } C \textit{ and } D \textit{ points})$$

If some concentrated forces or some concentrated moments affect on the some points of the beam, then the axial force, the shear force and the bending moment variation just before and just after these points do not remain very small, and there are discontinuities at related quantities at these points. In the figure given below, a beam element of the length dz is considered. The equilibrium equations for this element are as follows:



$$\left.\begin{aligned} N_2 - N_1 + F_z &= 0 \\ V_2 - V_1 + F_y &= 0 \\ M_{x2} - M_{x1} + F_y \cdot dz/2 &= 0 \end{aligned}\right\} \quad \textbf{(6.13)}$$

$$\left.\begin{aligned} N_2 &= N_1 - F_z \\ V_2 &= V_1 - F_y \\ M_{x2} &= M_{x1} \end{aligned}\right\} \quad \textbf{(6.14)}$$

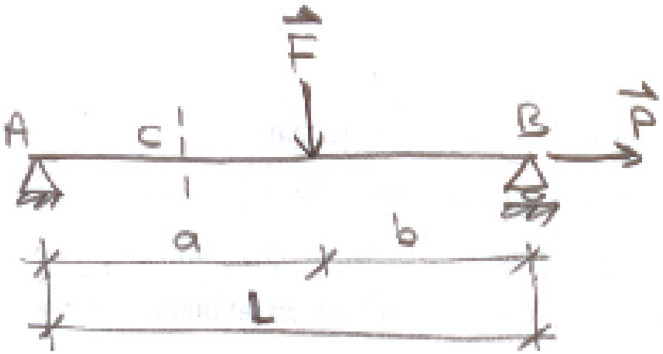
$$\left.\begin{aligned} N_2 &= N_1 \\ V_2 &= V_1 \\ M_{x2} &= M_{x1} - M \end{aligned}\right\} \quad \textbf{(6.15)}$$

6.3 Obtaining the axial force, shear force and bending moment diagrams

a) Section method

In this method, after calculating the reaction forces, the beam is separated into two parts at a section where the shear force and bending moment will be obtained. Using the free body diagram of one of the parts, we can determine the shear force at the considered section by equating to zero the sum of the vertical components of all the forces acting on this part. Similarly, the bending moment at this section can be found by equating to zero the sum of the moments about the considered section.

In this method the beam must be cut at just before and just after the points where there are supports, concentrated forces, concentrated moments, beginning and end of the distributed loads.



First the support reactions must be obtained by using the free body diagram of the whole beam.

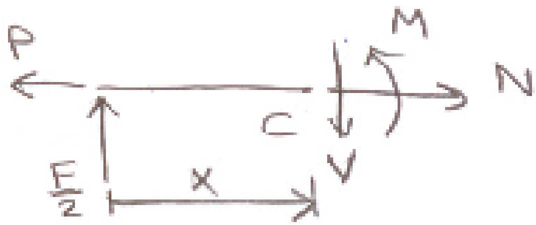
Now the axial force, shear force and bending moment diagrams of the beam shown on the right side will be obtained by cutting the beam at any arbitrary location x and drawing the free body diagram for the portion of the beam to the left (or for the portion to the right) of the transverse cross section.

Then equilibrium equations are written free body diagrams and solved to get the resultants at the location x.

The equations obtained will be valid for a range of x for which the nature of the loading does not change.

The process must be repeated for each different segment of the beam.

From equilibrium of portion A-C

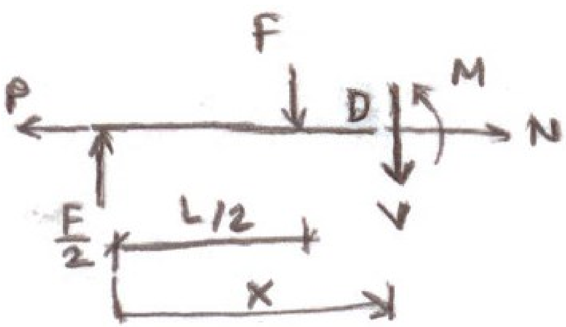


$$+\uparrow\sum F_y = 0 = \frac{F}{2} - V = 0 \rightarrow V = F/2$$

$$\sum M_C = 0 = \frac{F}{2}\times x - M = 0 \rightarrow M = \frac{F\,x}{2}$$

$$\sum \overset{\rightarrow +}{F_x} = 0 = -P + N = 0 \rightarrow N = P$$

From equilibrium of portion A-D



$$+\uparrow\sum F_y = 0 = \frac{F}{2} - F - V = 0 \rightarrow V = -F/2$$

$$\sum M_D = 0 = \frac{F}{2}\times x - F\times\left(x - \frac{L}{2}\right) - M = 0 \rightarrow M = \frac{F\,L}{2} - \frac{F\,x}{2}$$

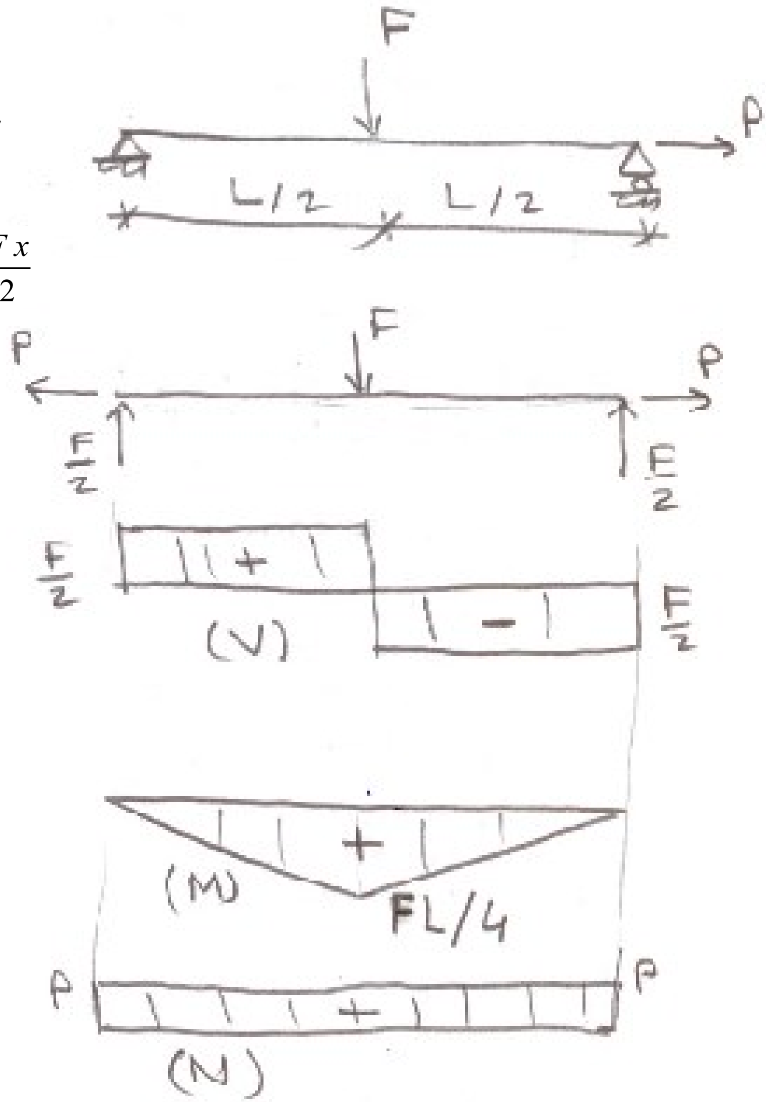
$$\sum \overset{\rightarrow +}{F_x} = 0 = -P + N = 0 \rightarrow N = P$$

Axial force, shear force and bending moment diagrams

$$+\uparrow\sum F_y = 0 = \frac{F}{2} - V = 0 \rightarrow V = F/2$$

$$\sum M_C = 0 = \frac{F}{2}\times x - M = 0 \rightarrow M = \frac{F\,x}{2}$$

$$\sum \overset{\rightarrow +}{F_x} = 0 = -P + N = 0 \rightarrow N = P$$



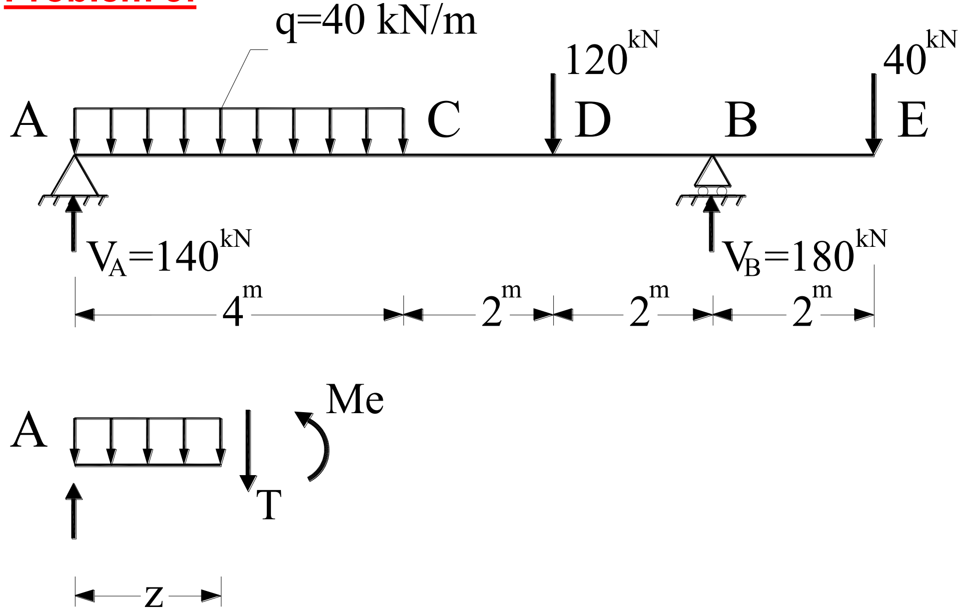
$$+\uparrow\sum F_y = 0 = \frac{F}{2} - F - V = 0 \rightarrow V = -F/2$$

$$\sum M_D = 0 = \frac{F}{2}\times x - F\times\left(x - \frac{L}{2}\right) - M = 0$$

$$\rightarrow M = \frac{F\,L}{2} - \frac{F\,x}{2}$$

$$\sum \overset{\rightarrow +}{F_x} = 0 = -P + N = 0 \rightarrow N = P$$

Problem 3:



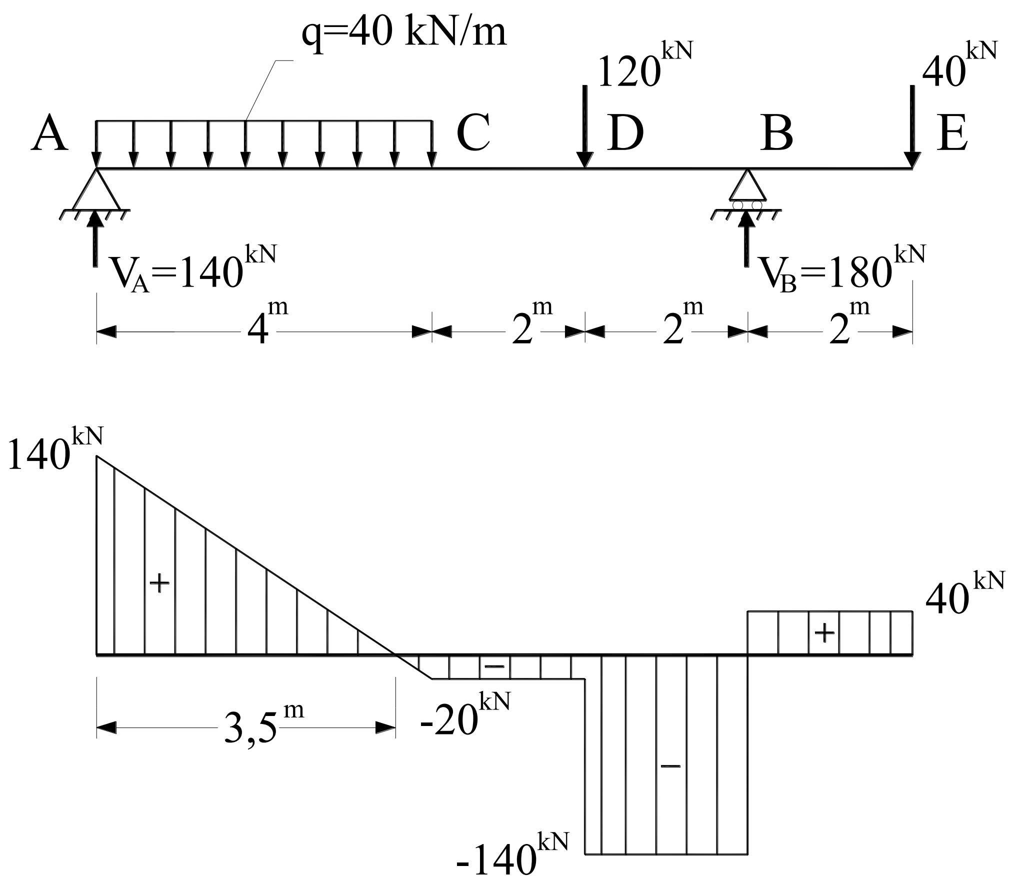
By using equilibrium equations for the whole beam, the reaction forces are obtained. By isolating the beam part with z length, and considering the internal and external forces which affect on this part, the free body diagram, which is shown in figure at the left hand side, is obtained. By using equilibrium equations for the considered beam part, the shear and bending moment expressions are obtained as follows:

$$\sum F_y = 0 \rightarrow 140 - 40z - T = 0 \rightarrow T = 140 - 40z$$

$$\sum M_x = 0 \rightarrow 140z - 40\frac{z^2}{2} - M_x = 0 \rightarrow M_x = 140z - 20z^2$$

These expressions are valid in the region *0<z<4m* (*AC* region). By performing similar procedures for the regions CD, DB, BE, the shear and bending moment diagrams can be found.

Shear force diagram



Shear force at the end A is 140 kN which is equal to the support reaction.

$$Q_A = 140\ kN$$

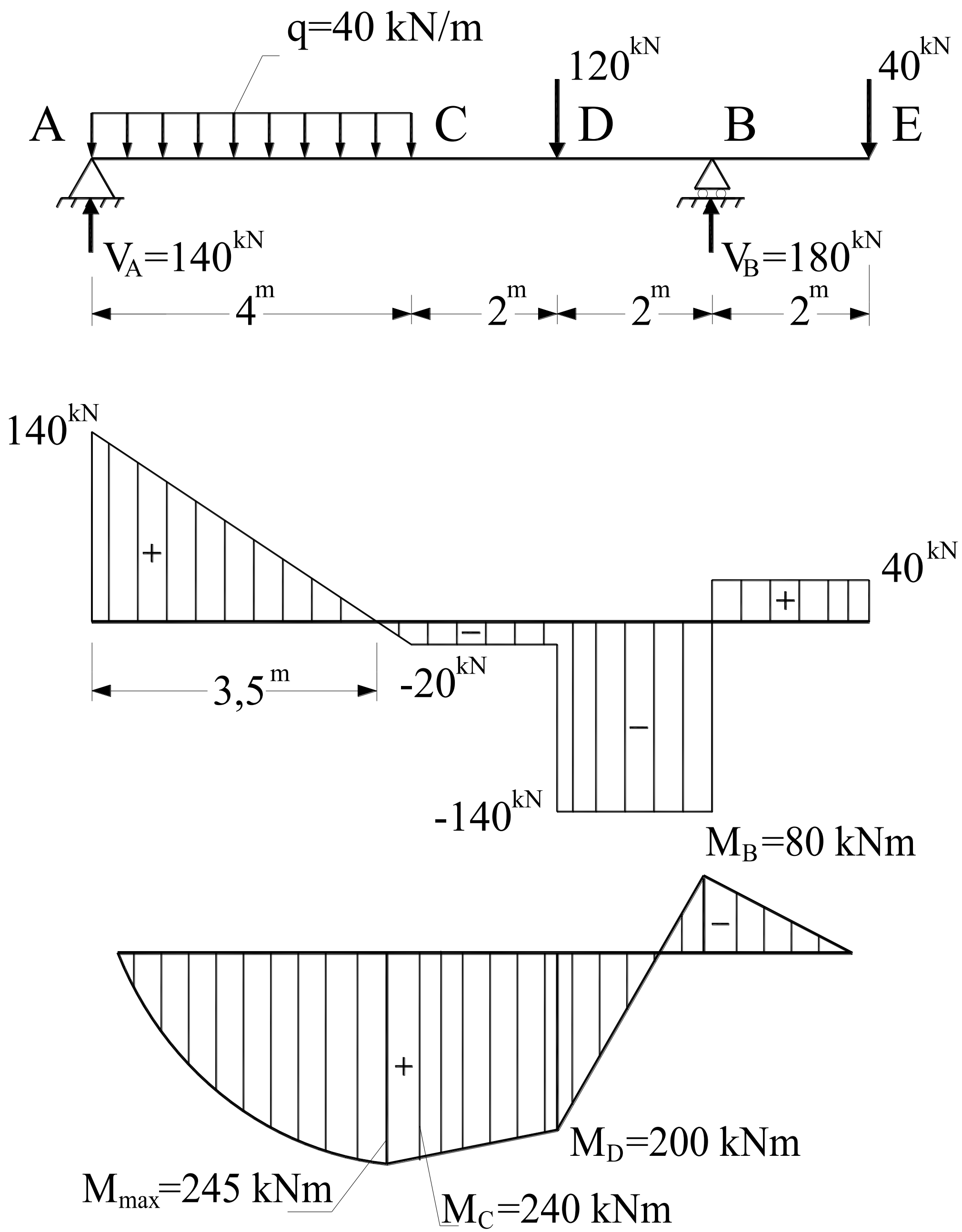
$$Q_c = 140 - 40 \cdot 4 = -20 kN$$

$$Q_{D sağ} = -20 - 120 = -140\ \ kN$$

$$Q_{B-E} = -140 + 180 = 40 kN$$

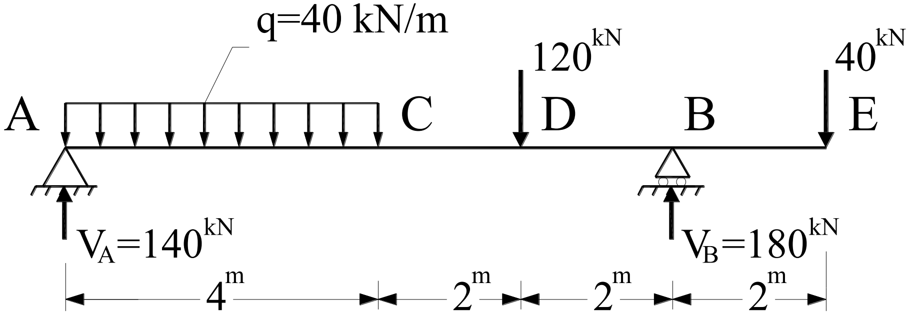
$$Q_E = 40\ kN$$

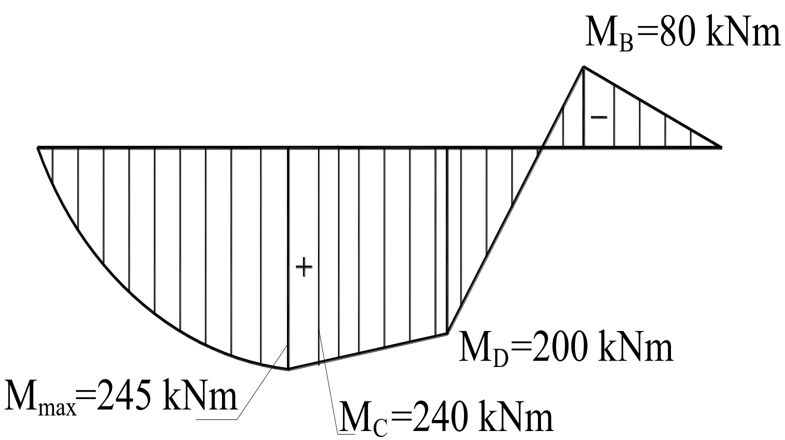
The shear force is zero at z=3.5 m. At this point the bending moment is maximum.



q=40 kN/m
120 kN
40 kN
A
C
D
B
E
V_A=140 kN
V_B=180 kN
4 m
2 m
2 m
2 m
140 kN
+
40 kN
+
−
-20 kN
3,5 m
−
-140 kN
M_B=80 kN
−
+
M_D=200 kNm
M_max=245 kNm
M_C=240 kNm

Bending moment diagram





The shear force is zero at z=3.5 m. At this point the bending moment is maximum.

$$\left(M_x\right)_{z=3.5} = M_{max} = 140 \times 3.5 - 40 \times \frac{3.5^2}{2}$$

$$M_{max} = 245 \ \ kNm$$

$$\left(M_x\right)_C = 140 \times 4 - 40 \times \frac{4^2}{2} = 240 \ \ kNm$$

$$\left(M_x\right)_D = 140 \cdot 6 - 4 \cdot 40\left(\frac{4}{2} + 2\right) = 200 \ kNm$$

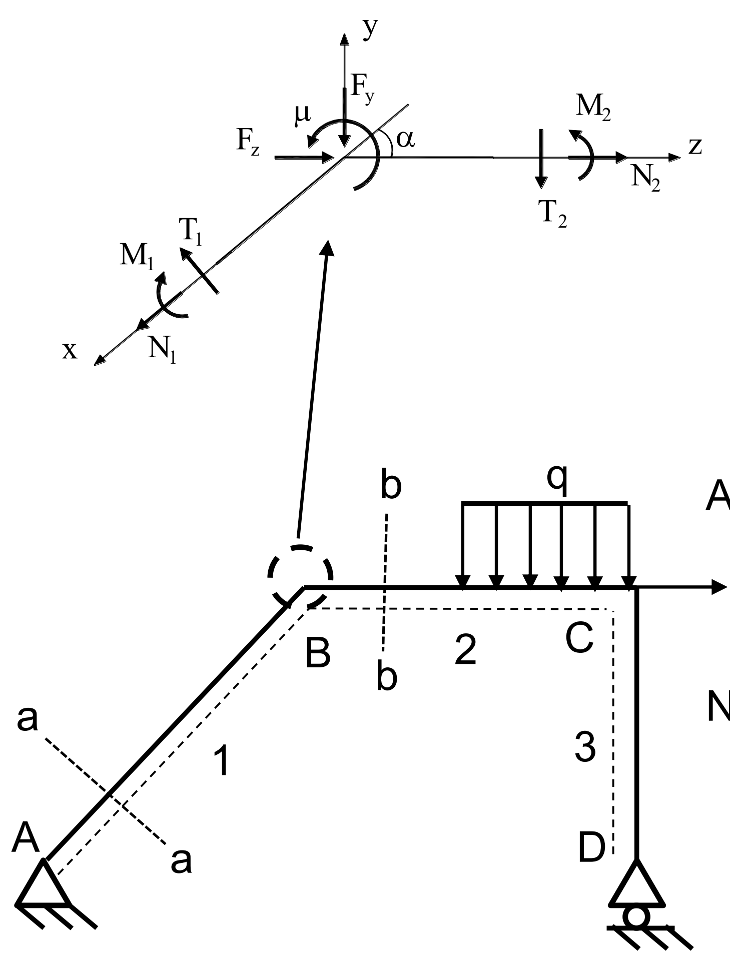
$(M_x)_B$=140.8-40.4.(4/2+4)-120.2=-80 kNm

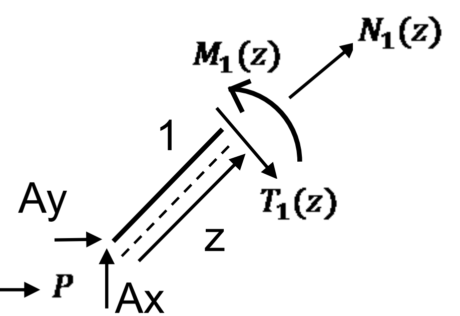
The same results can also be found by using the right hand parts of the beam.

6.4 Axial force shear force and bending moment diagrams of statically determinate frames

For drawing axial force, shear force and bending moment diagrams of this type of systems, the above mentioned rules are valid in just the same way.

In addition to these rules, it is useful to know the relations between the axial force, shear force and bending moment just before and just after the corner points. By considering equilibrium of the corner point the following expressions are obtained.

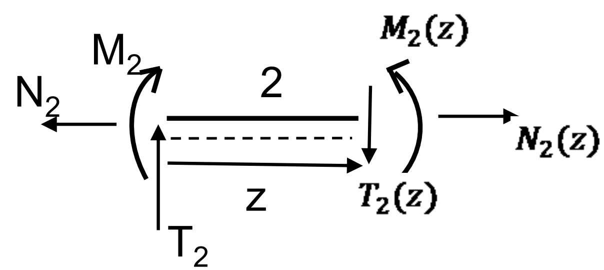




$$
\left.\begin{aligned}
N_2 &= N_1 \cos\alpha + T_1 \sin\alpha - F_z \\
T_2 &= T_1 \cos\alpha - N_1 \sin\alpha - F_y \\
M_2 &= M_1 - \mu
\end{aligned}\right\} \quad (6.16)
$$

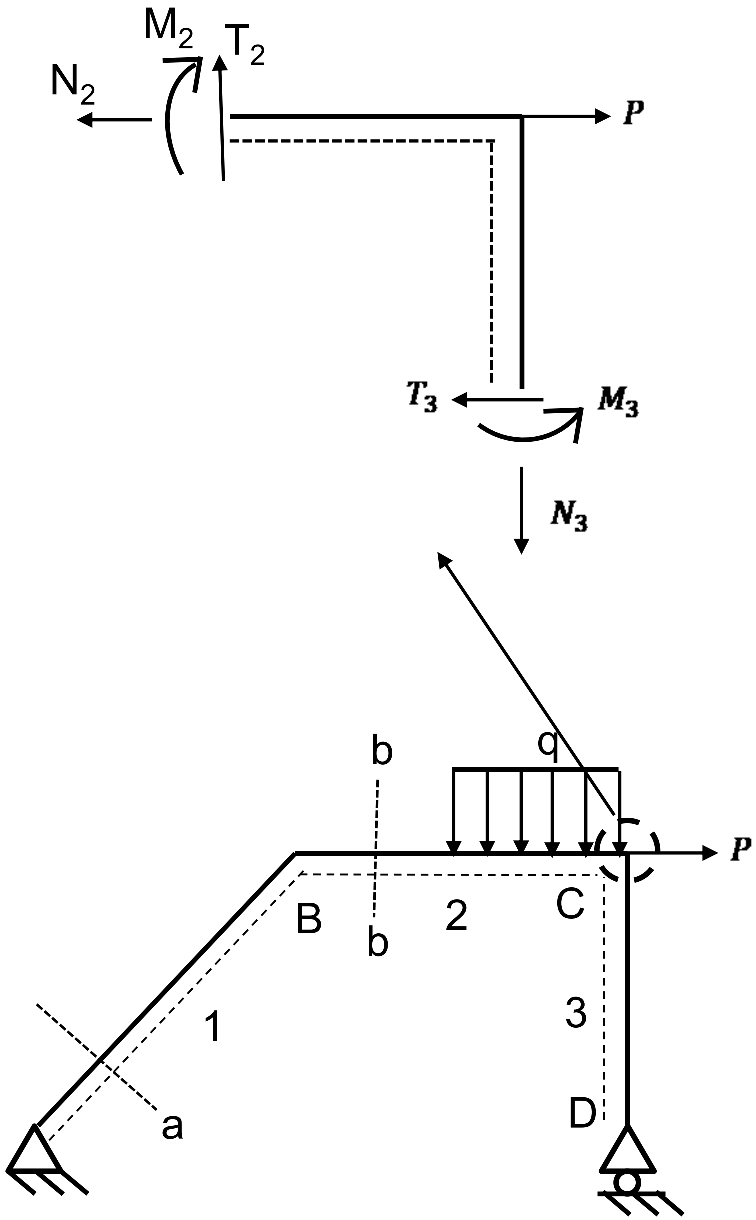
If there are no external concentrated forces and external concentrated moments, then Eq. (6.16) becomes as follows:

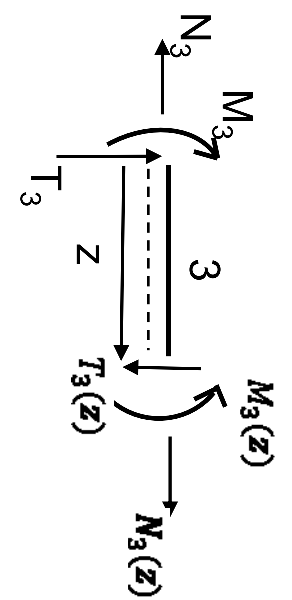
$$
\left.\begin{aligned}
N_2 &= N_1 \cos\alpha + T_1 \sin\alpha \\
T_2 &= T_1 \cos\alpha - N_1 \sin\alpha \\
M_2 &= M_1
\end{aligned}\right\} \quad (6.17)
$$



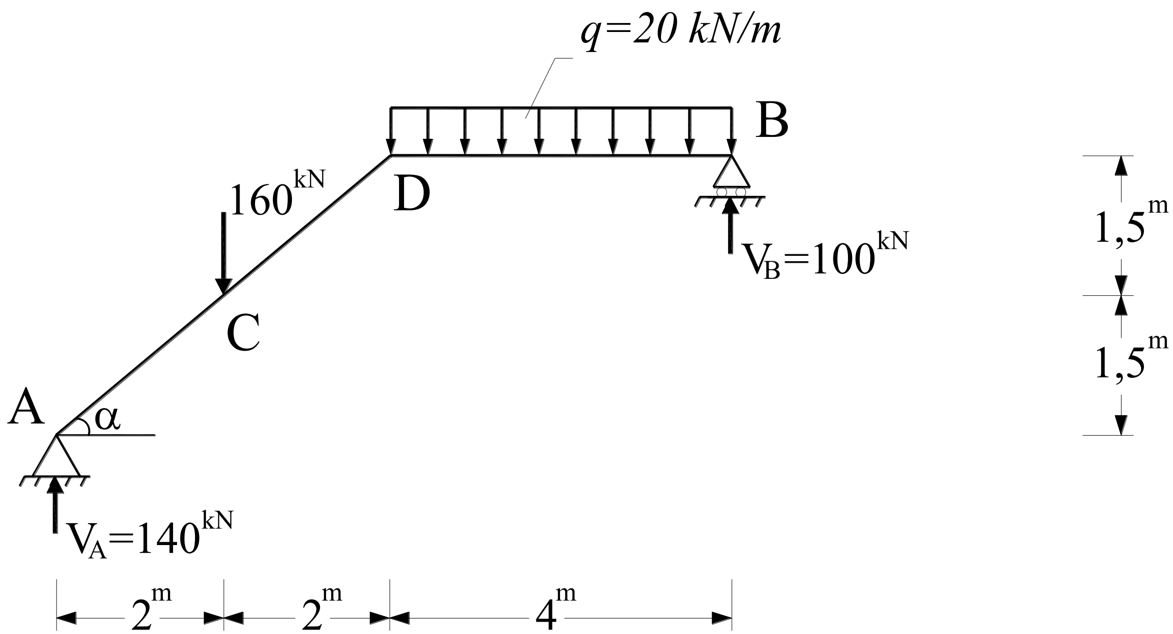
If the corner angle is a right angle, then Eq. (6.16) becomes

$$N_3 = T_2$$
$$T_3 = -N_2 + P \qquad (6.18)$$
$$M_3 = M_2$$



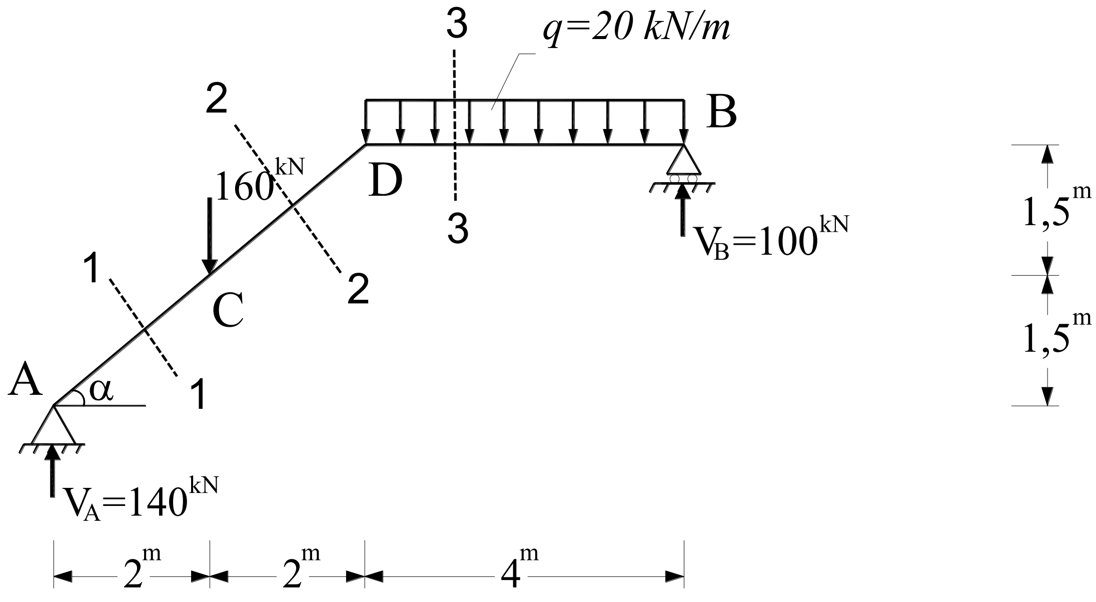


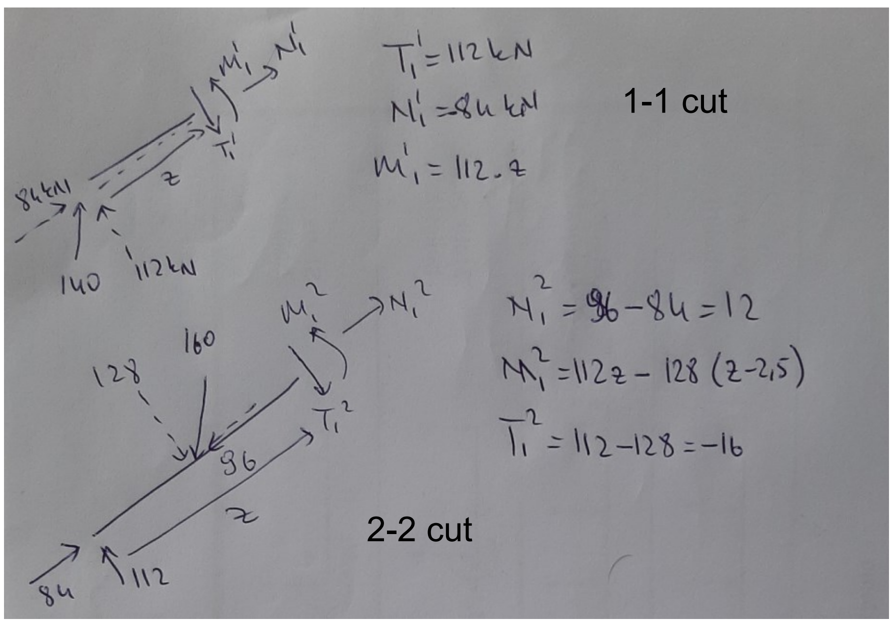
Problem 4:

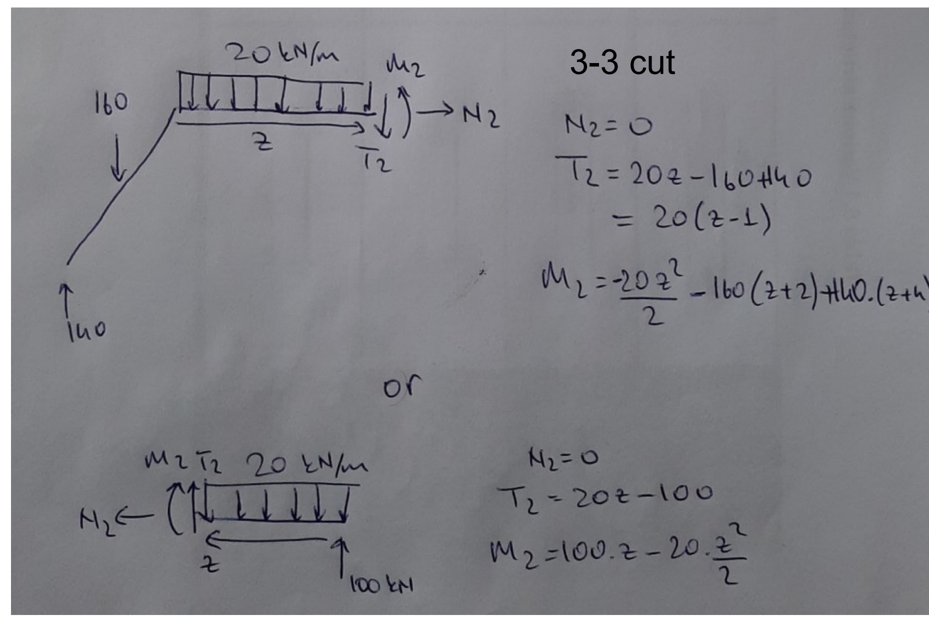


Q_B = -100 kN $\qquad$ N_B = 0 kN

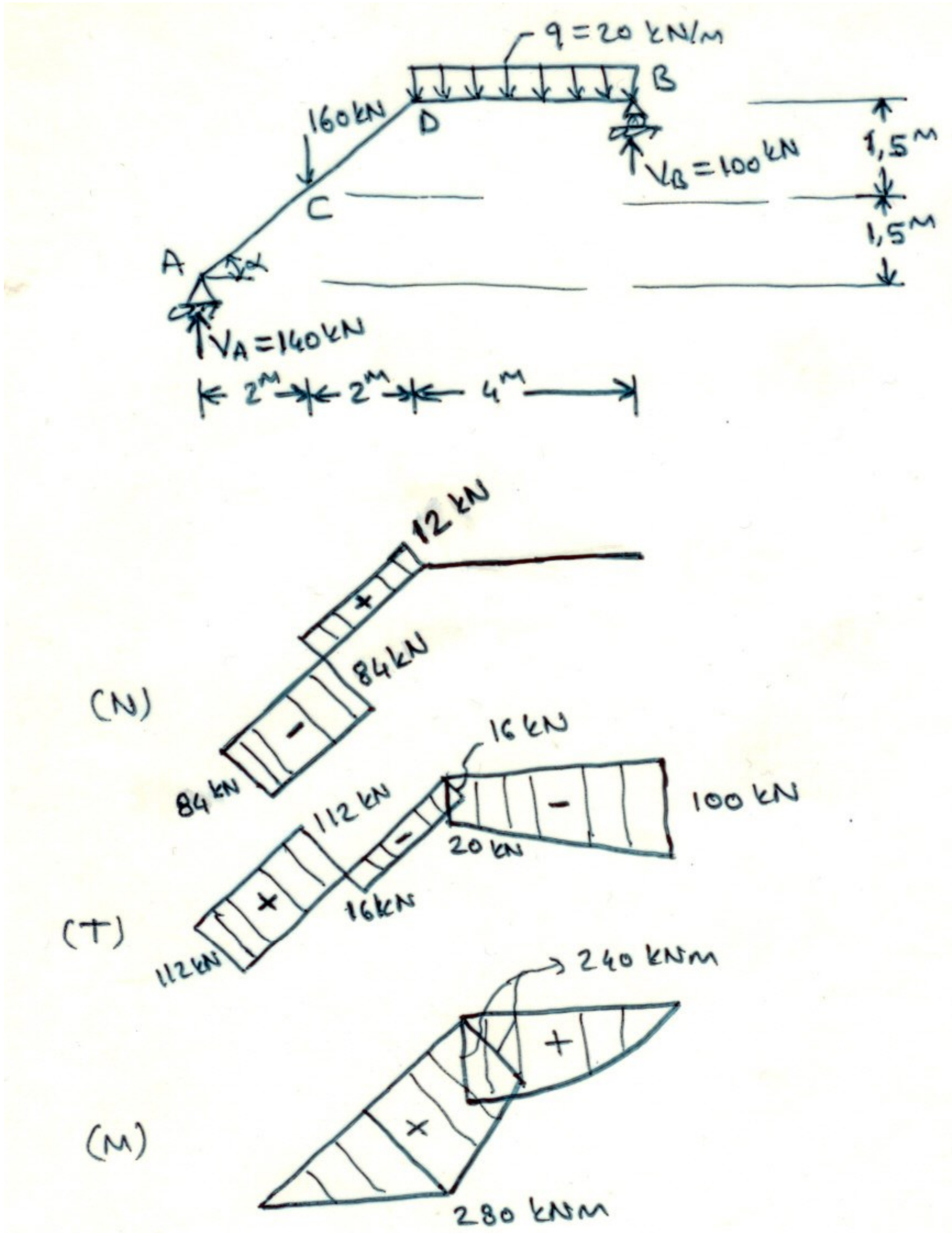
M_A= 0 kNm $\qquad$ M_C= 140.2=280 kNm $\qquad$ M_D=140.4-160.2=240 kNm $\qquad$ M_B= 0 kNm


q=20 kN/m
B
D
C
A
α
160 kN
V_B=100 kN
V_A=140 kN
1 1
2 2
3 3
2 m
2 m
4 m
1,5 m
1,5 m


1-1 cut
$T_1^1 = 112$ kN
$N_1^1 = -84$ kN
$M_1^1 = 112 \cdot z$
84 kN
140
112 kN
z
2-2 cut
$N_1^2 = 96 - 84 = 12$
$M_1^2 = 112z - 128(z-2{,}5)$
$T_1^2 = 112 - 128 = -16$
128
160
96
84
112
z


3-3 cut
20 kN/m
160
140
z
$N_2 = 0$
$T_2 = 20z - 160 + 140$
$= 20(z-1)$
$M_2 = -\frac{20z^2}{2} - 160(z+2) + 140 \cdot (z+4)$
or
20 kN/m
100 kN
z
$N_2 = 0$
$T_2 = 20z - 100$
$M_2 = 100 \cdot z - 20 \cdot \frac{z^2}{2}$

Problem 5:

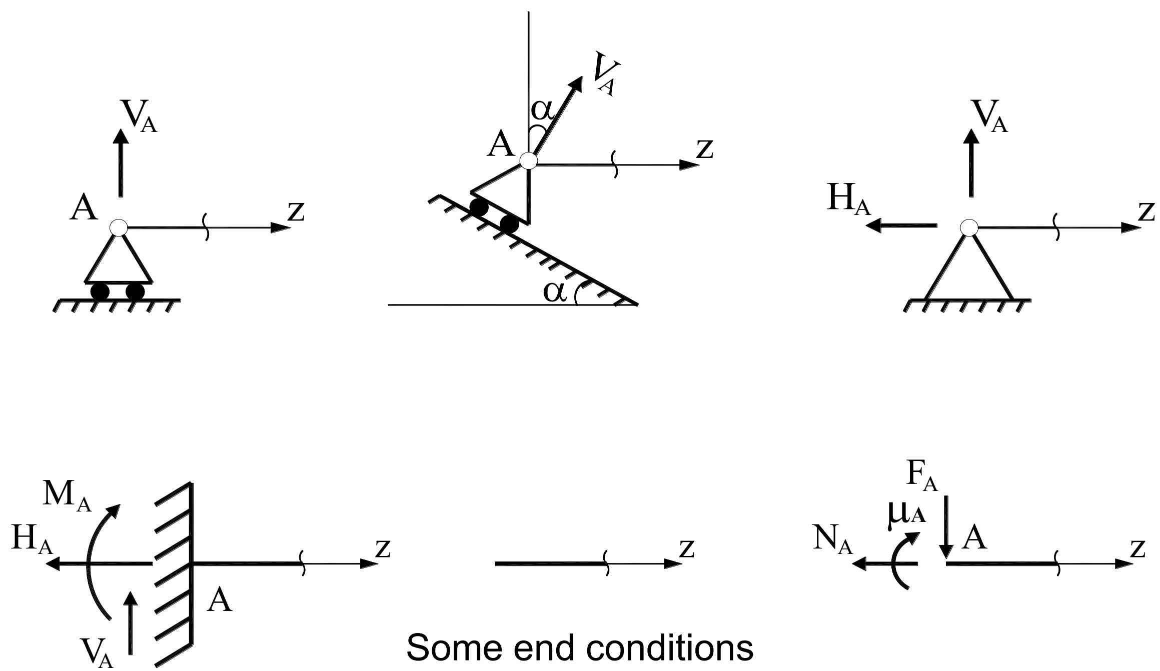


q = 20 kN/m
B
160 kN
D
V_B = 100 kN
1,5 m
C
1,5 m
A
α
V_A = 140 kN
2 m
2 m
4 m
12 kN
+
84 kN
(N)
-
84 kN
16 kN
100 kN
-
112 kN
-
20 kN
+
16 kN
(T)
112 kN
240 kNm
+
+
(M)
280 kNm

b) Integration method

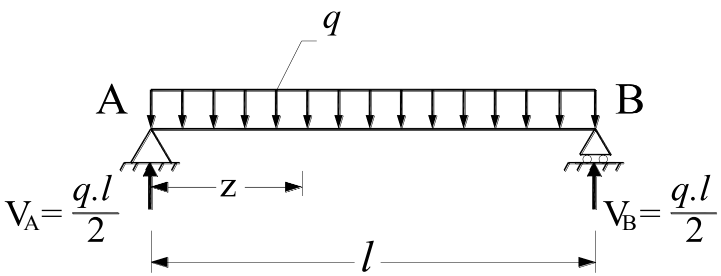
For a statically determinate bar in the yz plane, and under the distributed forces in the yz plane, the shear and the bending moment expressions can be obtained from (6.9) and (6.11) by integration. Integration constants which will be encountered while integrating these equations can be evaluated from the support and end conditions of the statically determinate beam.

$$\frac{dV}{dz} = -q \qquad \textbf{(6.9)} \qquad \frac{dM}{dz} = V \qquad \textbf{(6.11)} \qquad \frac{d^2M}{dz^2} = -q$$



Some end conditions

Example 1:



First step: Considering the entire beam as a free body, we determine the reactions: These reactions are evaluated and shown on the beam in the figure shown at the left side.

Second step: Integrating the differential equations given in Eq. (9, 11):

$$\frac{dN}{dz} = 0 \rightarrow N = C_1$$

; $$\frac{d^2 M_x}{dz^2} = -q_y = -q = sabit \rightarrow \frac{dM_x}{dz} = Q = -qz + C_2 \rightarrow M_x = -q\frac{z^2}{2} + C_2 z + C_3$$

Third step: Evaluating C_1, C_2, C_3 constants by using end conditions:

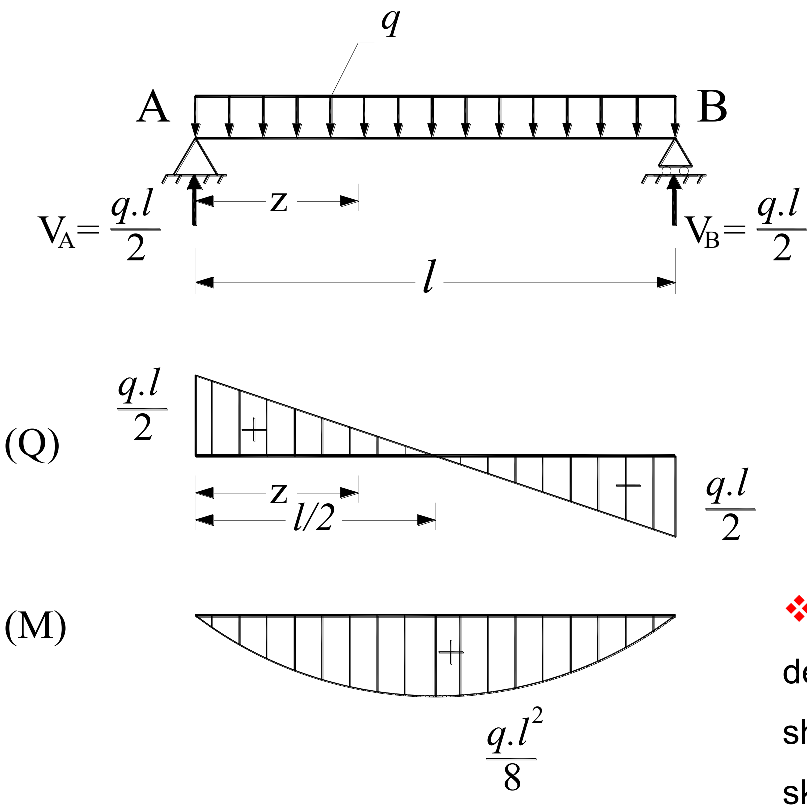
$$z = 0 \rightarrow (N)_{z=0} = N_A = 0 \rightarrow C_1 = 0$$

$$(Q)_{z=0} = Q_A = V_A = q\,l/2 \rightarrow C_2 = q\,l/2$$

$$(M_x)_{z=0} = M_{xA} = 0 \rightarrow C_3 = 0$$

By substituting these values in the related equations above, the following expressions are found:

$$N = 0 \qquad Q = \frac{ql}{2} - qz \qquad M_x = q\frac{l}{2}z - q\frac{z^2}{2}$$



$$N = 0 \qquad Q = \frac{ql}{2} - qz \qquad M_x = q\frac{l}{2}z - q\frac{z^2}{2}$$

❖ Value of the slope of the bending moment curve at any point is equal to the value of the shear force at that point. $\frac{dM}{dx} = V$

❖ Negative value of the slope of the shear force curve at any point is equal to the load per unit length at that point. According to this relation, when the distributed load is uniform, then the shear diagram is an oblique straight line with constant slope. $\frac{dV}{dx} = -q$

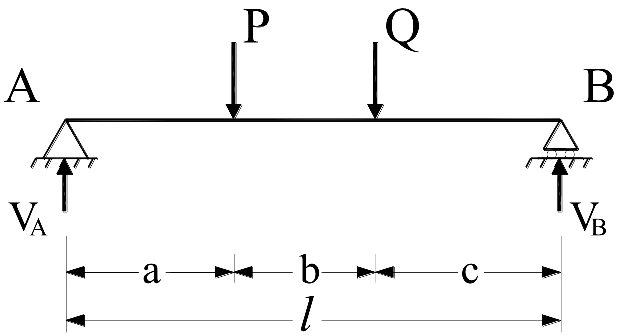
❖ The shear and bending moment curves will always be one and two degrees higher than the load curve. Thus, once a few values of the shear and bending moment have been computed, we should be able to sketch the shear and bending moment diagrams without actually determining the shear and bending moment functions. The bending moment curve is a parabola for this example.

❖ The second derivative of the bending moment is negative. Therefore the moment diagram is concave upward. Otherwise it would be concave downward.

❖ Since shear force and thus the slope of the bending moment is zero in the midpoint of the beam, the bending moment is extremum, in this case maximum, at that point.

$$M_{x\max} = (M_x)_{z=l/2} = \left(q\frac{l}{2}z - q\frac{z^2}{2}\right)_{z=l/2} = \frac{ql^2}{8}$$

Example 2:



There are only concentrated forces on the beam. This type of forces cause discontinuities at shear diagram as it was explained earlier. For this reason, shear and bending moment diagrams will be calculated in three different region on the beam.

First region:

$$0 < z < a \quad b\ddot{o}lgesi: \quad \frac{d^2 M_{x1}}{d z^2} = 0 \rightarrow \frac{d M_{x1}}{d z} = Q_1 = C_1 \rightarrow M_{x1} = C_1 z + C_2$$

End conditions at the point A:

$$z = 0 \rightarrow M_{x1} = 0 \rightarrow C_2 = 0$$

$$z = 0 \rightarrow Q_1 = V_A \rightarrow C_1 = V_A$$

Shear force and bending moment

$$Q_1 = V_A$$

$$M_{x1} = V_A z$$

Second region:

$$a < z < a + b \quad b\ddot{o}lgesi: \quad \frac{d^2 M_{x2}}{d z^2} = 0 \rightarrow \frac{d M_{x2}}{d z} = Q_2 = C_3 \rightarrow M_{x2} = C_3 z + C_4$$

Discontinuity conditions at z=a by using Eq. (6.10)

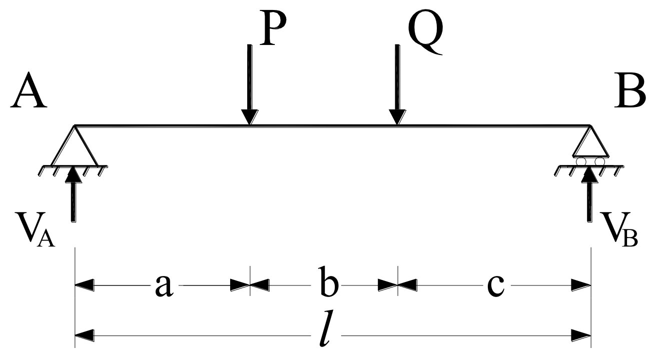
$$z = a \rightarrow Q_2 = Q_1 - P \rightarrow C_3 = V_A - P$$

$$z = a \rightarrow M_{x2} = M_{x1} \rightarrow (V_A - P)a + C_4 = V_A \cdot a \rightarrow C_4 = P \cdot a$$

Shear force and bending moment

$$Q_2 = V_A - P$$

$$M_{x2} = (V_A - P)z + P \cdot a$$



Third section:

$$a+b<z<l \quad bölgesi: \quad \frac{d^2 M_{x3}}{dz^2}=0 \rightarrow \frac{dM_{x3}}{dz}=Q_3=C_5 \rightarrow M_{x3}=C_5 z+C_6$$

End conditions at the point B:

$$z=l \quad Q_3=-V_B \qquad C_5=-V_B$$

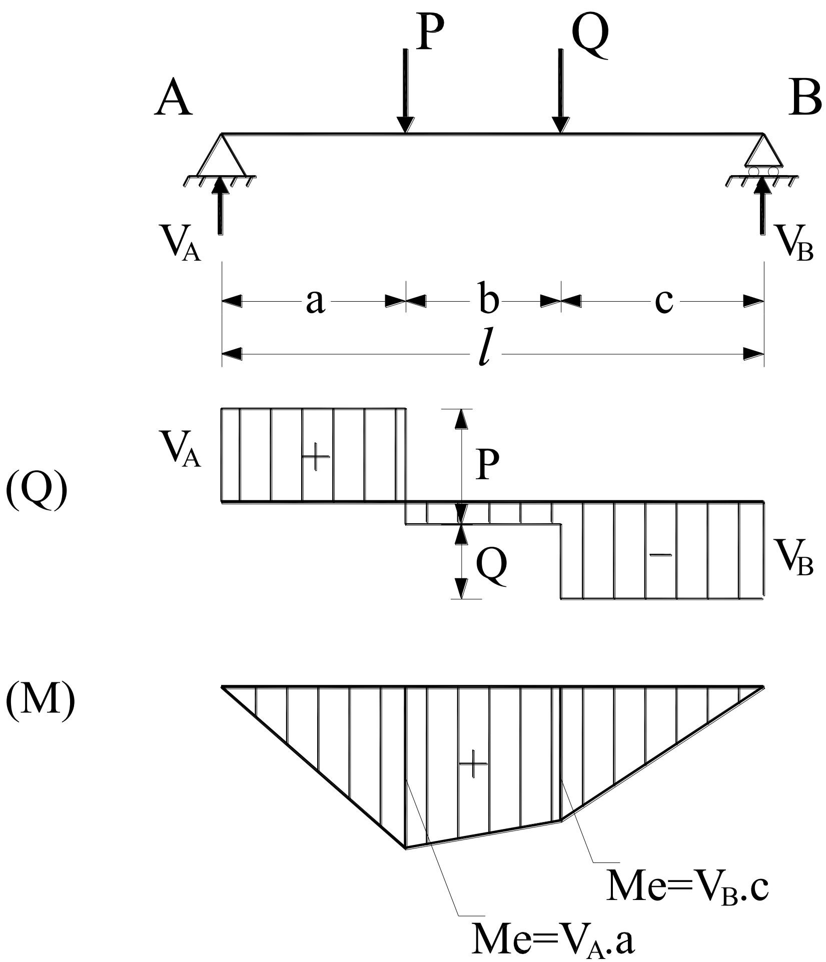
$$z=l \quad M_{x3}=0 \qquad 0=-V_B \cdot l+C_6 \quad C_6=V_B \cdot l$$

In this region, integration constants can also be calculated by using the discontinuity conditions at *z=a+b*

Shear force and bending moment

$$Q_3=-V_B$$

$$M_{x3}=-V_B z+V_B l$$



First region:

$$Q_1 = V_A$$
$$M_{x1} = V_A z$$

Second region:

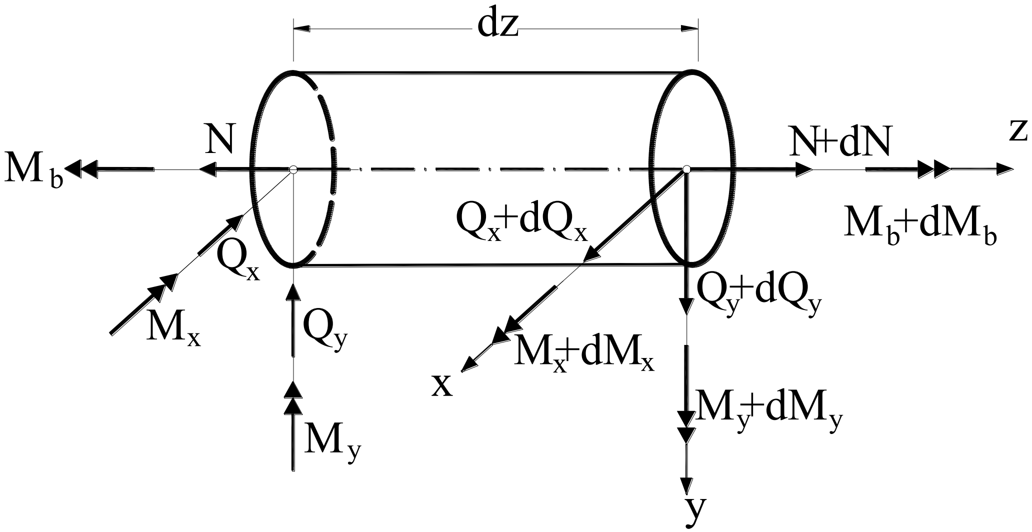
$$Q_2 = V_A - P$$
$$M_{x2} = \left(V_A - P\right) z + P \cdot a$$

Third region:

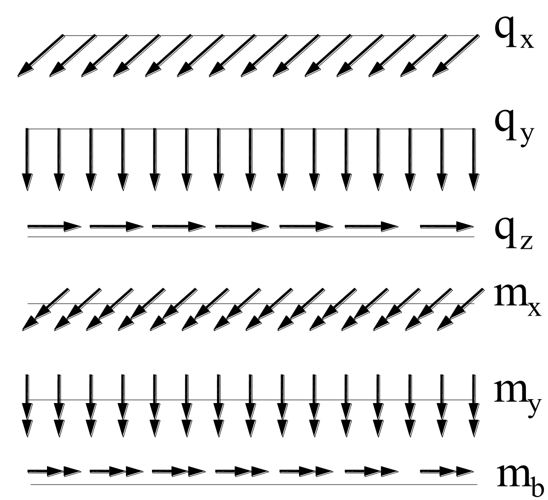
$$Q_3 = -V_B$$
$$M_{x3} = -V_B z + V_B l$$

As it is seen from example problem 6.2, if the load discontinuities increases on the beam, then integration regions increases and as a result of this situation, the number of integration constants which must be calculated increases. Hence, the integration method loses its efficiency for drawing shear and bending moment diagrams.

6.2 Relations between distributed load, shear force and bending moment in the three dimensional loading conditions



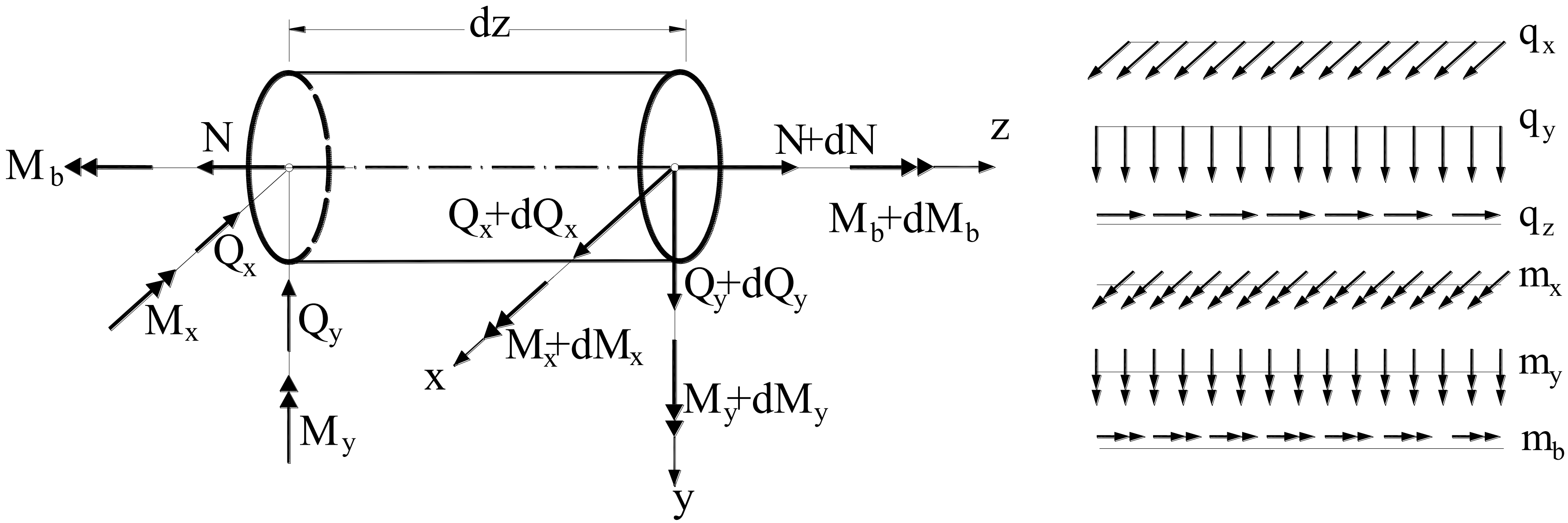
Distributed forces and moments



$$\sum F_x = 0 \rightarrow -Q_x + q_x dz + Q_x + dQ_x = 0 \rightarrow \frac{dQ_x}{dz} = -q_x \tag{6.1}$$

$$\sum F_y = 0 \rightarrow -Q_y + q_y dz + Q_y + dQ_y = 0 \rightarrow \frac{dQ_y}{dz} = -q_y \tag{6.2}$$

$$\sum F_z = 0 \rightarrow -N + q_z dz + N + dN = 0 \rightarrow \frac{dN}{dz} = -q_z \tag{6.3}$$

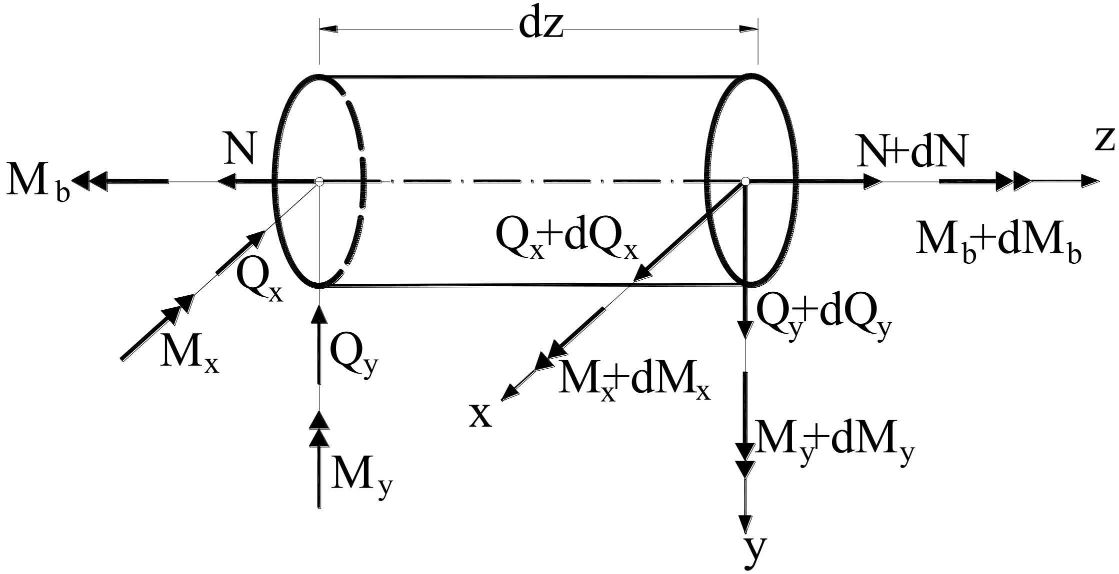


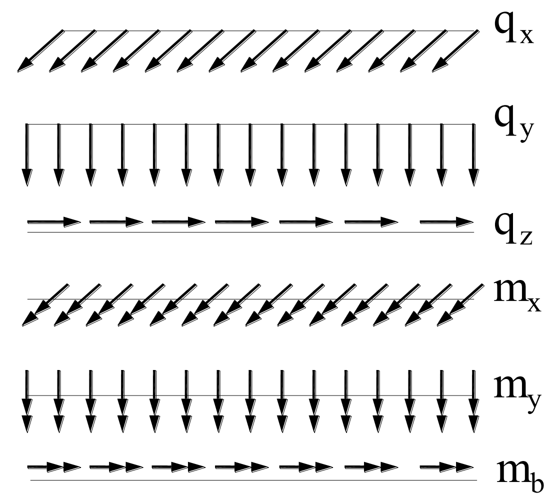
$$\sum M_x = 0 \rightarrow -M_x + m_x dz - Q_y\, dz + M_x + dM_x = 0 \rightarrow \frac{dM_x}{dz} = Q_y - m_x \tag{6.4}$$

$$\sum M_y = 0 \rightarrow -M_y + m_y dz + Q_x\, dz + M_y + dM_y = 0 \rightarrow \frac{dM_y}{dz} = -Q_x - m_y \tag{6.5}$$

$$\sum M_z = 0 \rightarrow -M_b + m_b dz + M_b + dM_b = 0 \rightarrow \frac{dM_b}{dz} = -m_b \tag{6.6}$$

As it is noticed from Eqs. (6.1-6.6), among $Q_x, Q_y, N \; and \; M_x, M_y, M_b$, only $N \; and \; M_b$ are independent of the others.

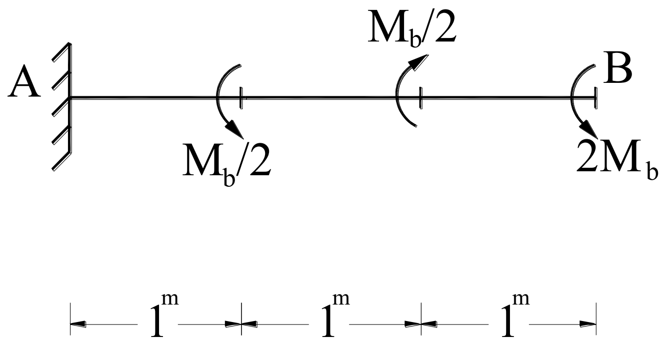


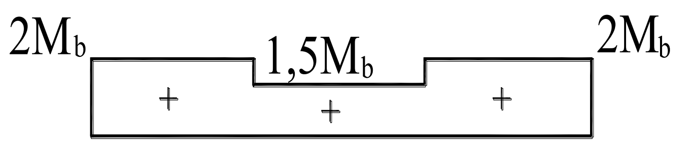


$$\frac{dQ_x}{dz} = -q_x \qquad \frac{dQ_y}{dz} = -q_y \qquad \frac{dN}{dz} = -q_z \qquad \frac{dM_x}{dz} = Q_y - m_x \qquad \frac{dM_y}{dz} = -Q_x - m_y \qquad \frac{dM_b}{dz} = -m_b$$

Let's consider a beam in the yz plane and under the forces in the yz plane again. In this situation, if there are no distributed external moments, namely $m_x = 0$, the relations between the external distributed loads, bending moment and shear force are as follows:

Example Problem: Torsional moments are applied to the axle shown below. Draw the torsional moment diagram for the axle.





Problem 1: Draw the shear and bending moment diagram for the beam and loading shown.

Solution: The reactions are determined by considering the entire beam as a free body

$H_B = 0$, $V_B = 23$ kN, $V_D = 7$ kN

We first determine the internal forces just to the right of the 10 kN load at A. Considering the stub of beam to the left of section 1 as a free body and assuming V and M to be positive (according to the standard convention), we write

$+\uparrow \Sigma F_y = 0: \quad -10 - V_1 = 0 \rightarrow V_1 = -10$ kN

$\circlearrowleft + \Sigma M_1 = 0: \quad 10.0 + M_1 = 0 \rightarrow M_1 = 0$

We next consider as a free body the portion of beam to the left of section 2 and write

$+\uparrow \Sigma F_y = 0: \quad -10 - V_2 = 0 \rightarrow V_2 = -10$ kN

$\circlearrowleft + \Sigma M_2 = 0: \quad 10.5 + M_2 = 0 \rightarrow M_2 = -50$ kNm

The shear and bending moment at sections 3, 4, 5, and 6 are determined in a similar way from the free body diagrams shown. We obtain

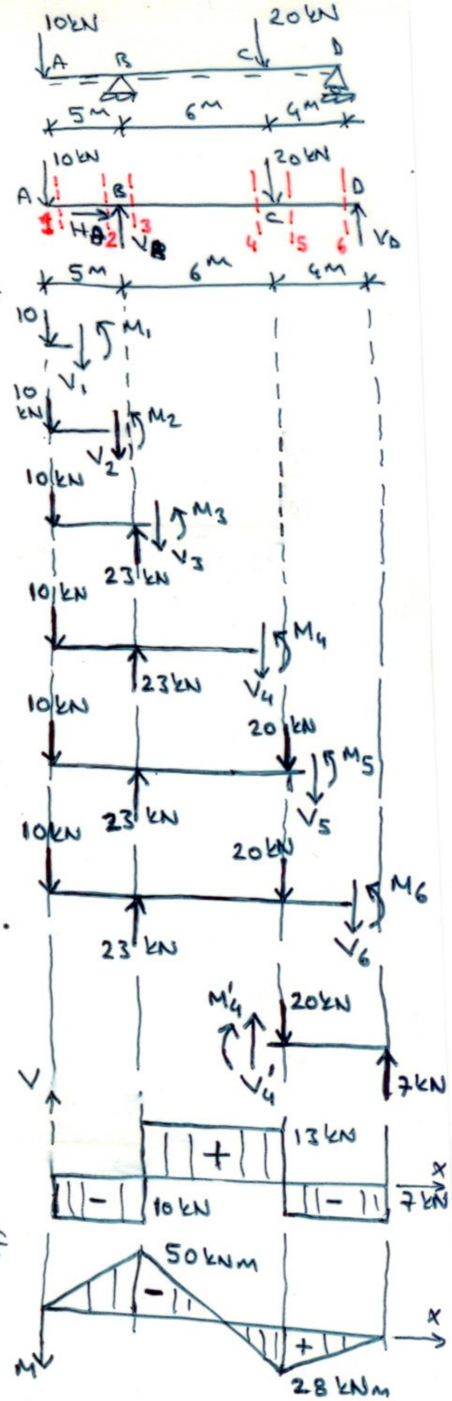
$V_3 = 13$ kN $\qquad M_3 = -50$ kNm

$V_4 = 13$ kN $\qquad M_4 = 28$ kNm

$V_5 = -7$ kN $\qquad M_5 = 28$ kNm

$V_6 = -7$ kN $\qquad M_6 = 0$

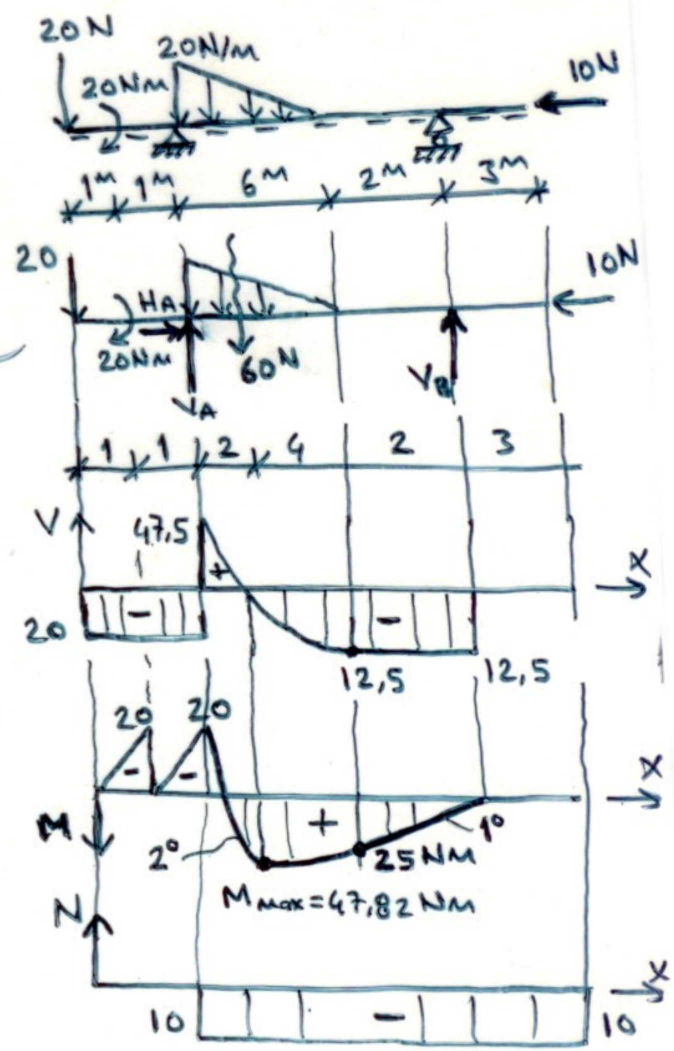
For several of the latter sections, the results may be more easily obtained by considering as a free body the portion of the beam to the right of the section. For example, considering the portion of the beam to the right of section 4, we write



$+\uparrow \Sigma F_y = 0: \quad V_4 - 20 + 7 = 0 \rightarrow V_4 = 13$ kN

$\circlearrowleft + \Sigma M_4 = 0: \quad -M_4 + 7.4 = 0 \rightarrow M_4 = 28$ kNm

Problem 5: Draw the axial force, shear force and bending-moment diagrams for the beam and loading shown.



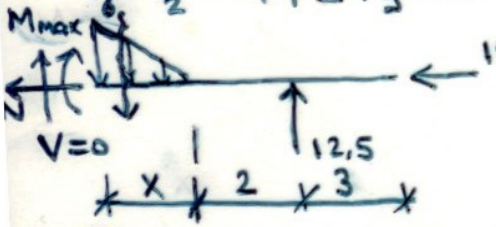
Solution: $\overset{+}{\rightarrow}\ \Sigma F_x = 0 : H_A - 10 = 0 \rightarrow H_A = 10N$

$\Sigma M_A = 0 : -20.2 + 20 + 2.60 - 8V_B = 0 \rightarrow V_B = 12{,}5N$

$\Sigma M_B = 0 : \rightarrow V_A = 67{,}5$

Check: $+\uparrow \Sigma F_y = 0 : -20 + 67{,}5 - 60 + 12{,}5 = 0$ ✓

$\frac{20}{6}\cdot\frac{x^2}{2}$ $\quad +\uparrow \Sigma F_y = 0 : -\frac{20}{6}\cdot x \cdot \frac{x}{2} + 12{,}5 = 0$



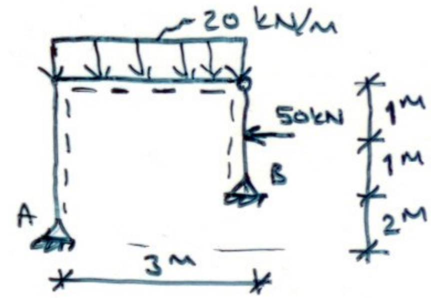
$x = 2{,}74^M$

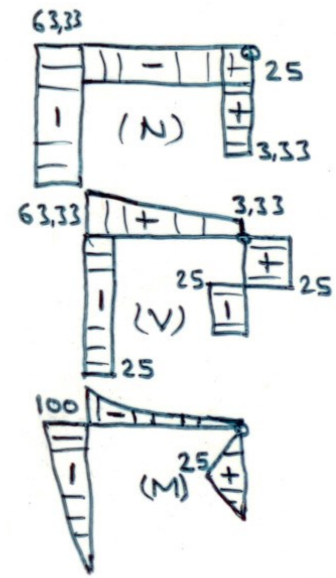
$\Sigma M = 0 : M_{max} + \frac{20}{6}\cdot\frac{2{,}74^2}{2}\cdot\frac{2{,}74}{3} - 4{,}74 \cdot 12{,}5 = 0$

$M_{max} = 47{,}82\ Nm$

Problem 6: Draw the axial force, shear force and bending-moment diagrams for the frame and loading

Problem 6: Draw the axial force, shear force and bending-moment diagrams for the frame and loading shown.





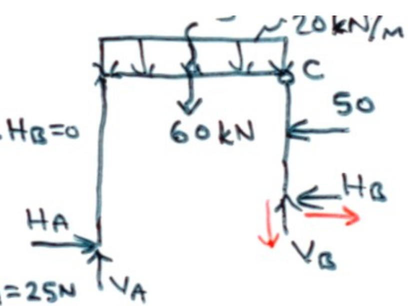
shown.

Solution:

$\sum M_A = 0: 60.1,5 - 50.3 - 3.V_B - 2.H_B = 0$

$\sum M_C = 0: 50.1 + 2.H_B = 0$

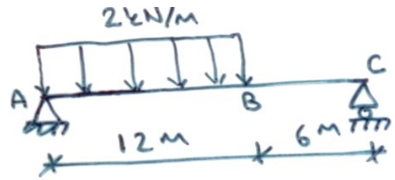
$V_B = -3,33\ N$; $H_B = -25N$



$\sum F_x = 0: H_A - 50 + 25 = 0 \rightarrow H_A = 25N$

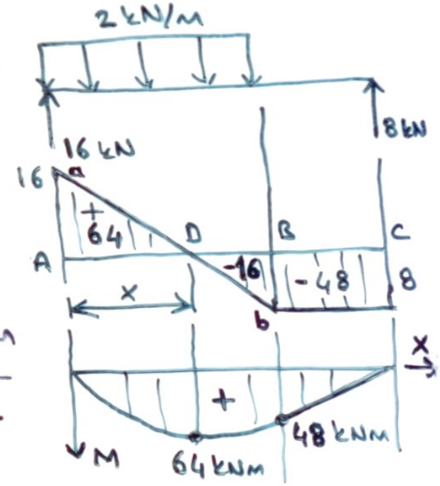
$\uparrow \sum F_y = 0: V_A - 60 - 3,33 = 0 \rightarrow V_A = 63,33\ N$

Problem 4: Draw the shear and bending-moment diagrams for the beam and loading shown.



Solution: Considering the entire beam as a free body, we obtain the reactions

$$V_A = 16 \text{ kN}, \quad V_C = 8 \text{ kN}$$



Shear diagram: The shear just to the right of A is $V_A = 16$ kN. Since the change in shear between two points is equal to minus the area under the load curve between the same two points, we obtain V_B by writing

$$V_B - V_A = -(2)(12) = -24$$

$$V_B = -24 + V_A = -24 + 16 = -8 \text{ kN}$$

The slope $dV/dx = -q$ being constant between A and B, the shear diagram between these two points is represented by a straight line. Between B and C, the area under the load curve is zero; therefore,

$$V_C - V_B = 0 \rightarrow V_C = V_B = -8 \text{ kN}$$ and shear is constant between B and C.

Bending-moment diagram: We note that the bending moment at each end of the beam is zero. In order to determine the maximum bending moment, we locate the section D of the beam where $V = 0$. Considering the portion of the shear diagram between A and B, We note that the triangles DAa and DBb are similar; thus,

$$\frac{x}{16} = \frac{12 - x}{8} \rightarrow x = 8 \text{ m}$$

The maximum bending moment occurs at point D, where we have $dM/dx = 0 = V$. The areas of the various portions of the shear diagram are computed and are given on the diagram. Since the area of the shear diagram between two points is equal to the change in bending moment between the same two points, we write

$$M_D - M_A = 64 \text{ kNm} \rightarrow M_D = 64 \text{ kNm}$$

$$M_B - M_D = -16 \text{ kNm} \rightarrow M_B = 48 \text{ kNm}$$

$$M_C - M_B = -48 \text{ kNm} \rightarrow M_C = 0$$

The bending-moment diagram consists of an arc of parabola followed by a segment of straight line; the slope of the parabola at A is equal to the value of V at that point.