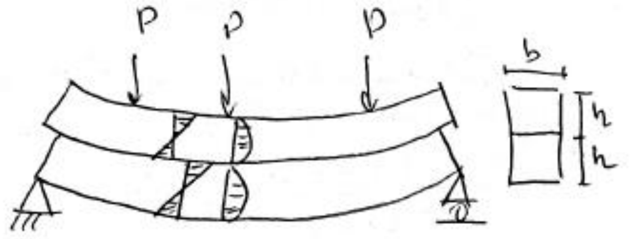
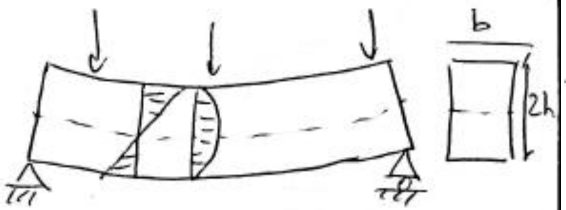
COMPOUND BEAMS

In order to increase the moment resistance under the effect of bending bars can be connected to eachother by wedges, bolts rivets and weld. The resulting beams are called compound beams





no connection between beams
free relative sliding
no shear occurs (It is assumed that friction is too low)

$$M_b = 2\left(\frac{\sigma_{em}\,\frac{1}{12} b h^3}{\frac{1}{2} h}\right) = \frac{1}{3}\sigma_{em} b h^2$$

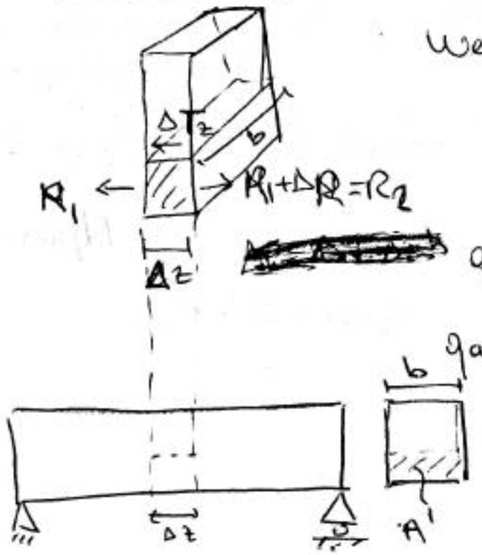
connected beams (work together)
Shear occurs at the interface

$$M_c = \frac{\sigma_{em}\,\frac{1}{12} b (2h)^3}{h} = \frac{2}{3}\sigma_{em} b h^2$$

$$\boxed{M_c = 2 M_b}$$

compound beam can carry two fold load.

Shear Flux



We know that $R_1 + \Delta T_z = R_2$ and $\Delta T_z = \Delta R$

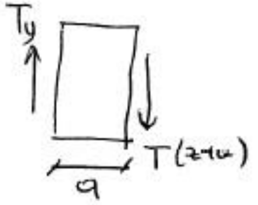
$$\Delta T_z = \tau_{zy}\, b(y)\, \Delta z = \int_{A'} \Delta\sigma_z \, dA'$$

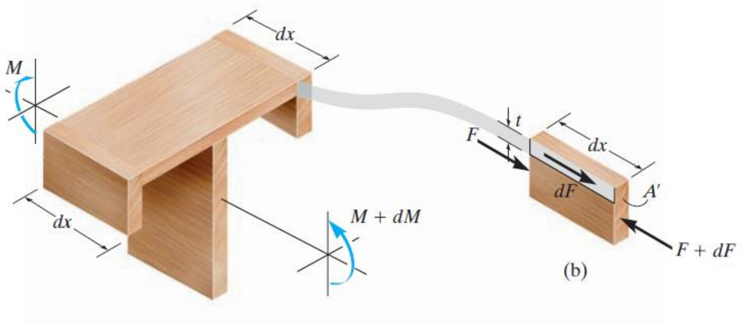
$$q = \frac{\Delta T_z}{\Delta z} = \tau_{zy}\, b(y) = \frac{T_y S_x}{I_x} = \int_{A'} \frac{d\sigma_z}{dz}\, dA$$

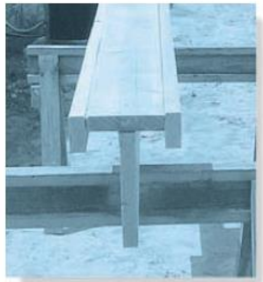
$q \rightarrow$ shear flow : shear force per unit length

$$q_{avg} = -\frac{T_{avg} \cdot S_x}{I_x} \quad \text{(average shear flow)}$$

$$T_{avg} = \frac{\int_z^{z+a} T_y(z)\, dz}{a}$$







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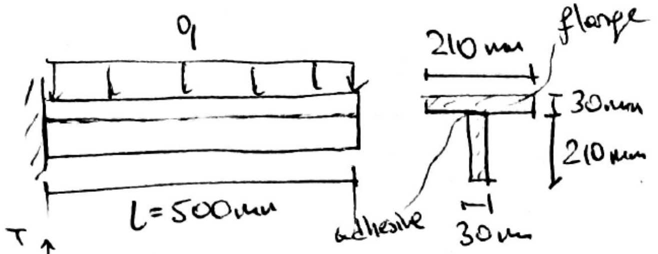
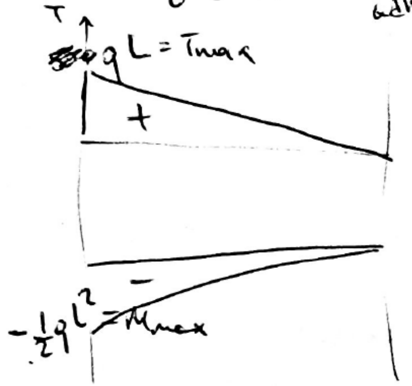


The flange of wooden T section is connected to the web by adhesive.

$(\tau_{em})_{adh} = 2$ MPa

$(\sigma'_{em})_{wood} = 8$ MPa

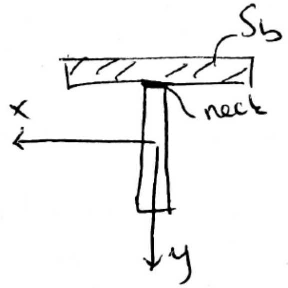
Find the maximum q value that beam can carry safely.



$$y_G = \frac{225(210\times30)+105(210\times30)}{2(210\times30)} = 165 \text{ mm}$$

$$I_x = \frac{1}{12}\,210\,30^3 + 210.30\,(210+15-165)^2$$

$$+ \frac{30\,210^3}{12} + 210.30\,(105-165)^2 = 6900\,10^4 \text{ mm}^4$$



check for shear at transitiong region

$$S_b = -210.30.(210+15-165) = -378\,10^3 \text{ mm}^3$$

$$-\frac{T_{max} S_b}{I_x t} \le (\tau_{em})_{adh} \qquad -\frac{500q\,(-378\,10^3)}{6900\,10^4\,30} \le 2$$

$$q \le 21.9 \text{ N/mm}$$

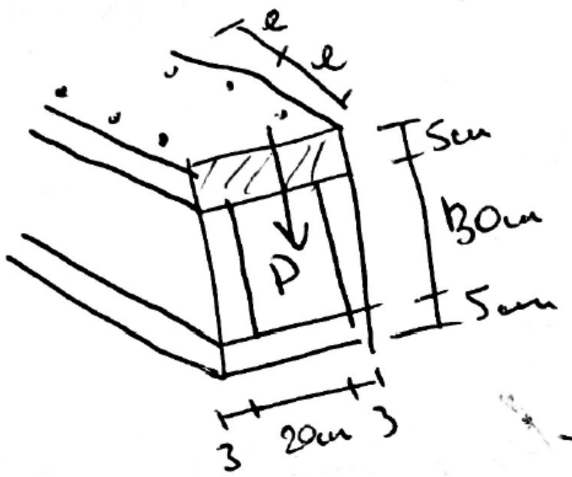
check for bending at the bottom of the section (max. stress-compressive)

$$\frac{M_{max}}{I_x} y_{max} \le (\sigma_{em})_{wood} \qquad \frac{(\frac{1}{2}q\,500^2)165}{6900\,10^4} \le 8$$

$$q = 26.75 \text{ N/mm}$$

$$q_{max} = 21.9 \text{ N/mm}$$

Ex:



Each screws shown in figure can sustain 1 kN shear force. Taking into account bending with shear effect, determine the distance e between to successive screws if a load P=6kN is applied to the cross section.

$$I_x = \frac{26 \cdot 40^3}{12} - \frac{20 \cdot 30^3}{12} = 93666{,}67 \text{ cm}^4$$

$$S_b = -5 \cdot 26 \cdot 17{,}5 = -2275 \text{ cm}^3$$

$$q = -\frac{P \cdot S_b}{I_x} = \frac{-6 \cdot -2275}{93666{,}67} = 0{,}1457$$

$$q \cdot e = \underset{\substack{\downarrow \\ \text{number of screws for}}}{2} \cdot 1$$

$$0{,}1457 \cdot e = 2 \cdot 1$$

$$e = 13{,}72 \text{ cm}$$

The beam is constructed from four boards glued together as shown in Fig. 7–16*a*. If it is subjected to a shear of $V = 850$ kN, determine the shear flow at B and C that must be resisted by the glue.

SOLUTION

Section Properties. The neutral axis (centroid) will be located from the bottom of the beam, Fig. 7–16*a*. Working in units of meters, we have

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.15\text{ m}](0.3\text{ m})(0.01\text{ m}) + [0.205\text{ m}](0.125\text{ m})(0.01\text{ m}) + [0.305\text{ m}](0.250\text{ m})(0.01\text{ m})}{2(0.3\text{ m})(0.01\text{ m}) + 0.125\text{ m}(0.01\text{ m}) + 0.250\text{ m}(0.01\text{ m})}$$

$$= 0.1968\text{ m}$$

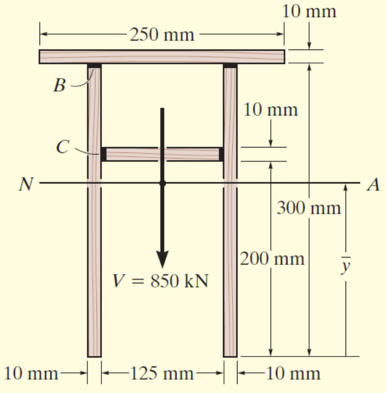
The moment of inertia about the neutral axis is thus

$$I = 2\left[\frac{1}{12}(0.01\text{ m})(0.3\text{ m})^3 + (0.01\text{ m})(0.3\text{ m})(0.1968\text{ m} - 0.150\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.125\text{ m})(0.01\text{ m})^3 + (0.125\text{ m})(0.01\text{ m})(0.205\text{ m} - 0.1968\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.250\text{ m})(0.01\text{ m})^3 + (0.250\text{ m})(0.01\text{ m})(0.305\text{ m} - 0.1968\text{ m})^2\right]$$

$$= 87.52(10^{-6})\text{ m}^4$$



(a)

Since the glue at B and B' in Fig. 7–16*b* "holds" the top board to the beam, we have

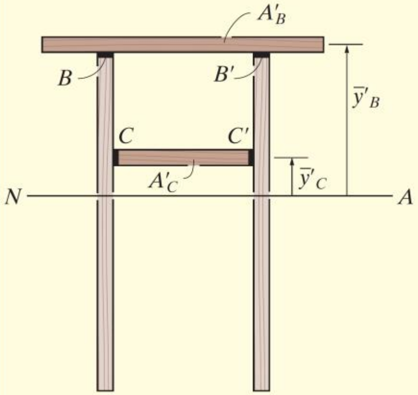
$$Q_B = \bar{y}'_B A'_B = [0.305\text{ m} - 0.1968\text{ m}](0.250\text{ m})(0.01\text{ m})$$

$$= 0.271(10^{-3})\text{ m}^3$$

Likewise, the glue at C and C' "holds" the inner board to the beam, Fig. 7–16*b*, and so

$$Q_C = \bar{y}'_C A'_C = [0.205\text{ m} - 0.1968\text{ m}](0.125\text{ m})(0.01\text{ m})$$

$$= 0.01026(10^{-3})\text{ m}^3$$



(b)

Fig. 7–16

Shear Flow. For B and B' we have

$$q'_B = \frac{VQ_B}{I} = \frac{850(10^3)\text{ N}(0.271(10^{-3})\text{ m}^3)}{87.52(10^{-6})\text{ m}^4} = 2.63\text{ MN/m}$$

And for C and C',

$$q'_C = \frac{VQ_C}{I} = \frac{850(10^3)\text{ N}(0.01026(10^{-3})\text{ m}^3)}{87.52(10^{-6})\text{ m}^4} = 0.0996\text{ MN/m}$$

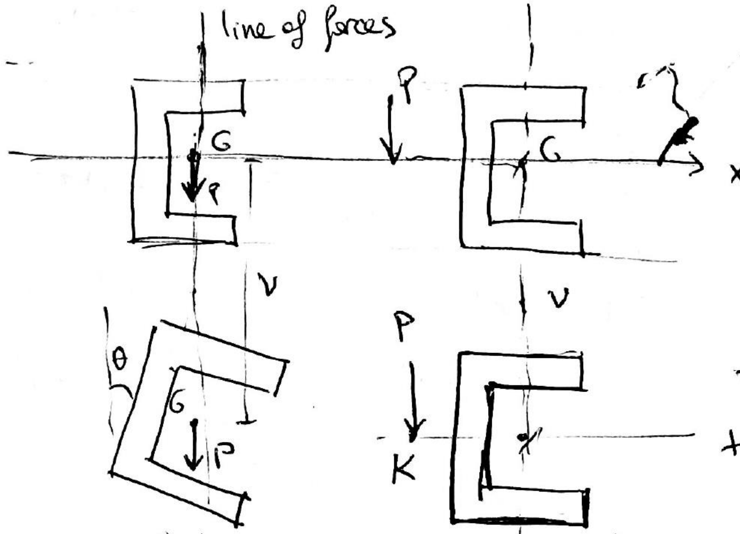
Since *two seams* are used to secure each board, the glue per meter length of beam at each seam must be strong enough to resist *one-half* of each calculated value of q'. Thus,

$$q_B = 1.31\text{ MN/m} \qquad \text{and} \qquad q_C = 0.0498\text{ MN/m} \qquad \textit{Ans.}$$

BENDING WITH AND WITHOUT TORSION

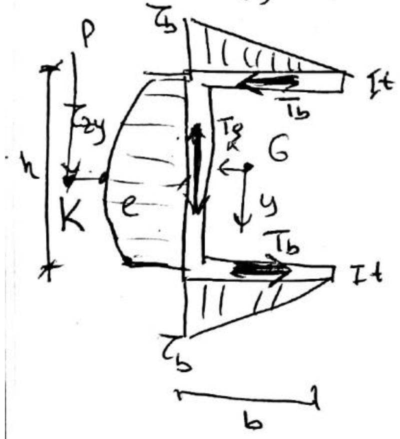
1 week

In the case of bending with shear, if the cross-section of a beam is not symmetric with respect to the line of forces which pass through the center of gravity, bending and torsion occurs together.



Let's find a point such that the shear force acts this point and does not cause torsion in the cross-section. This point is called the "Shear Center".

To clarify the concept, consider the section given below.



The forces which are the resultant of τ_{zx} occur on the horizontal arms are equal and opposite.

$$T_b = \frac{1}{2}\tau_b\, b.t = \frac{1}{2}\left(-\frac{PS_b}{I_x t}\right).b.t \quad \text{where;} \quad S_b = \frac{1}{2}hbt$$

$$= \frac{htb^2}{4I_x t}$$

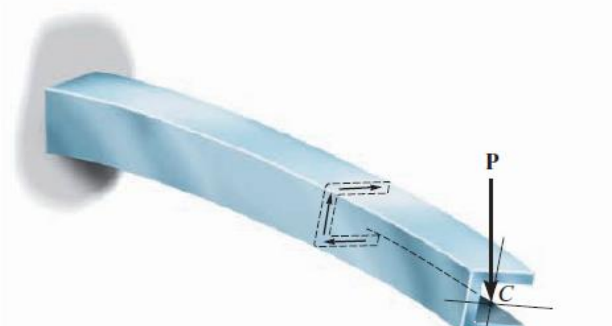
from vertical equilibrium

$$T_G = P$$

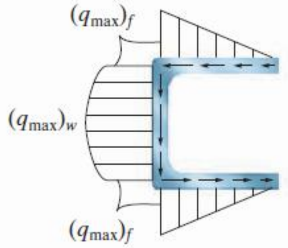
~~The total moment about z axis is~~

$$P.e = T_b.h = 0$$

$$\boxed{e = \frac{h^2b^2t}{4I_x}}$$

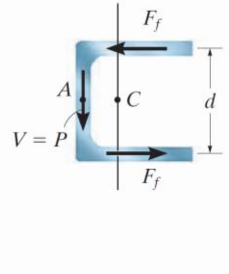


(a)

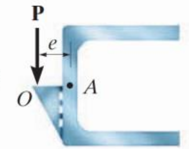


Shear-flow distribution

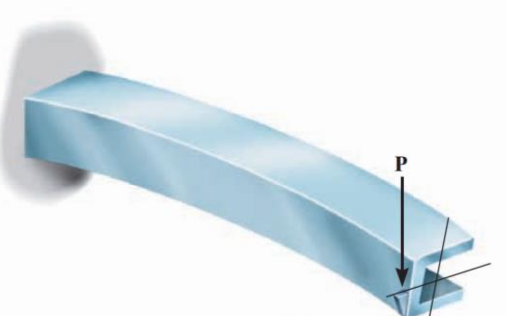
(b)



(c)



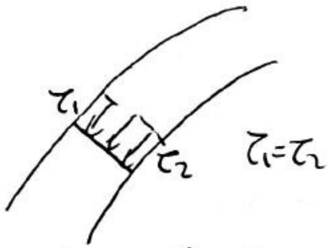
(d)



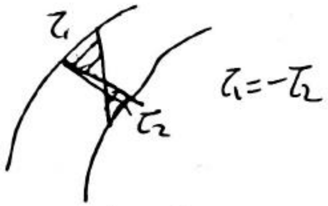
(e)

SHEAR CENTER FOR OPEN SECTIONS

In the case of bending with torsion both shear stress and normal stress distributions are changed. Hence; for unsymmetrical section a way should be found such that no torsion occurs

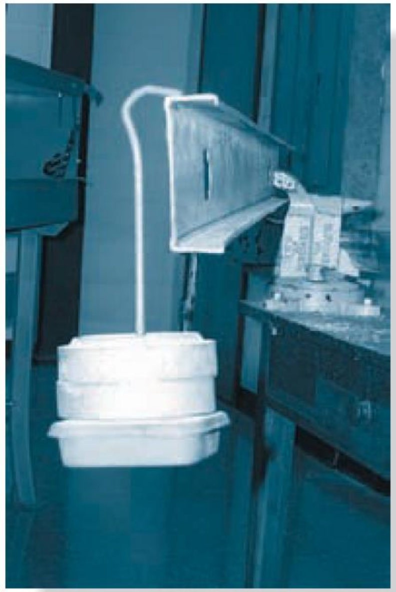


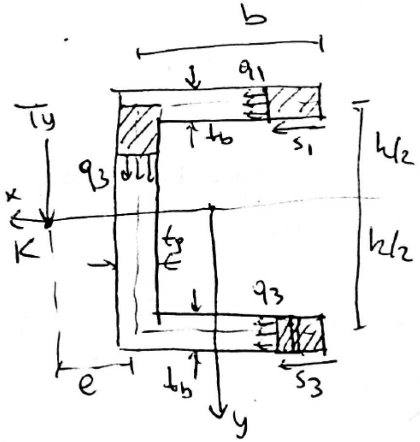
Bending without torsion



Only torsion

torsion + bending





$$q_1 = \tau_{zx} t_b = -\frac{T_y (S_x)_1}{I_x}$$

$$q_2 = \tau_{zy} t_g = -\frac{T_y (S_x)_2}{I_x}$$

$$q_3 = \tau_{zx} t_g = -\frac{T_y (S_x)_3}{I_x}$$

$$(S_x)_1 = s_1 t_b \left(-\frac{h}{2}\right) = -\frac{1}{2} s_1 t_b h$$

$$(S_x)_2 = b t_b \left(-\frac{h}{2}\right) + s_2 t_g \left(-\left(\frac{h}{2} - \frac{s_2}{2}\right)\right)$$

$$= -\frac{1}{2}\left(b t_b h + t_g (h s_2 - s_2^2)\right)$$

$$(S_x)_3 = s_3 t_b \frac{h}{2} = \frac{1}{2} s_3 t_b h$$

Shear forces on the section arms

$$T_1 = \int_0^b q_1 ds_1 = -\frac{T_y}{I_x} \int_0^b \left(-\frac{1}{2} s_1 t_b h\right) ds_1 = \frac{T_y (b^2 t_b h)}{4 I_x}$$

$$T_3 = \int_0^b q_3 ds_3 = -\frac{T_y}{I_x} \int_0^b \frac{1}{2} s_3 t_b h \, ds_3 = -\frac{T_y (b^2 t_b h)}{4 I_x}$$

Shear force on the web

$$I_x = 2\left[\frac{1}{12} b t_b^3 + b t_b \left(\frac{h}{2}\right)^2\right] + \frac{1}{12} t_p h^3$$

$$= \frac{1}{6} b t_b^3 + \frac{1}{2} b t_b h^2 + \frac{1}{12} t_p h^3 \approx \frac{1}{2} b t_b h^2 + \frac{1}{12} t_p h^3$$

$$T_2 = \int_0^h q_2 \, ds_2 = -\frac{T_y}{I_x}\int_0^h -\left[\frac{1}{2} b t_b h + \frac{1}{2} t_p (h s_2 - s_2^2)\right] ds_2$$

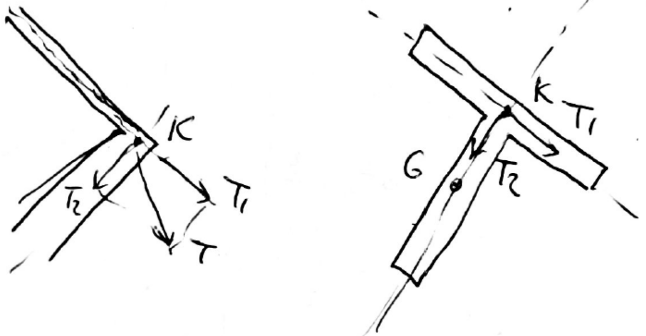
$$= \frac{T_y}{I_x}\left(\frac{1}{2} b t_b h^2 + \frac{1}{12} t_p h^3\right) = T_y$$

Determination of shear center:

$$+\circlearrowleft \sum M_K = 0 \qquad T_2 e - \frac{h}{2} T_1 + \frac{h}{2} T_3 = 0$$

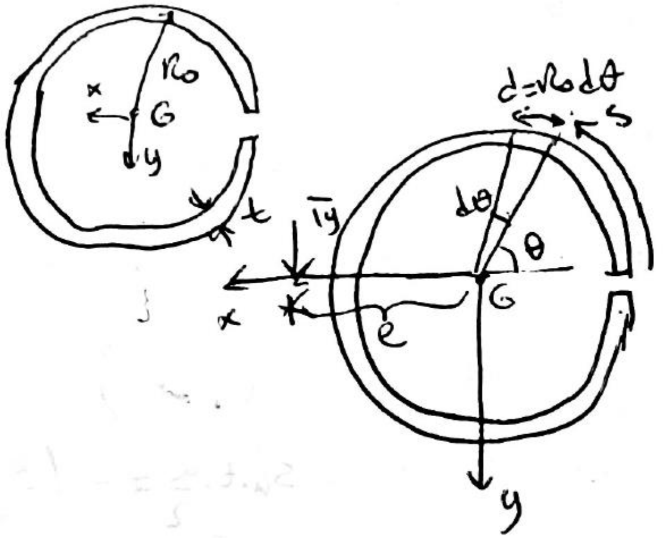
$$e = \frac{h^2 b^2 t_b}{4 I_x}$$

The shear center of the cross sections formed by two thin arms is the point of intersection of the axes of the arms



Ex

Find the shear center?



$d = R_o d\theta$

$dA = t\,ds = t R_o d\theta$

$y = -R_o \sin\theta$

$$S_x = \int_A y\,dA$$

$$= \int_0^{\theta} -R_o \sin\theta \, t R_o \, d\theta$$

$$= -tR_o^2 \int \sin\theta \, d\theta$$

$$= -tR_o^2 (1-\cos\theta)$$

$$I_x = \int_A y^2 dA$$

$$= \int_0^{2\pi} (-R\sin\theta)^2 \, tR_o d\theta$$

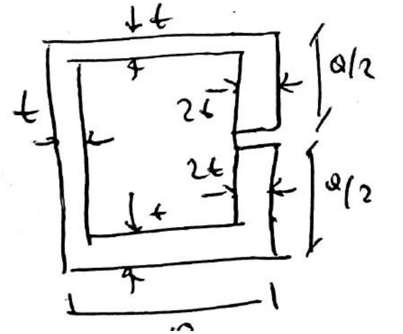
$$= tR_o^3 \int_0^{2\pi} \frac{1}{2}(1-\cos 2\theta) d\theta = \pi t R_o^3$$

Moment about G

$$T_y . e = -\int_0^{2\pi} q . ds . R_o = -\int_0^{2\pi} \frac{T_y S_x}{I_x} R_o \, ds$$

$$e = -\frac{1}{\pi t R_o^3} \int_0^{2\pi} R_o \left(-tR_o^2 (1-\cos\theta)\right) R_o d\theta = 2R_o$$

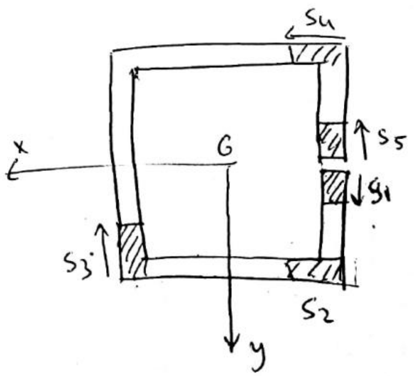
Ex



Find the shear center of open tube cross-section.

$$I_x \cong \frac{1}{12} t a^3 + \frac{1}{12} 2t a^3 + at\left(\frac{a}{2}\right)^2 + at\left(\frac{a}{2}\right)^2$$

$$= \frac{3}{4} a^3 t$$ (the terms includes t^3 are ignored)



$$S_x^1 = s_1 \cdot 2t \cdot \frac{s_1}{2} = s_1^2 t$$

$$S_x^2 = + s_2 t \cdot \frac{a}{2} + \frac{a}{2} \cdot 2t \cdot \frac{a}{4} = \frac{s_2 a t}{2} + \frac{a^2 t}{4}$$

$$S_x^3 = t \cdot s_3\left(\frac{a}{2} - \frac{s_3}{2}\right) + \frac{a}{2} \cdot 2t \cdot \frac{a}{4} + a t \cdot \frac{a}{2}$$

$$= \frac{3}{2} a^2 t + t s_3\left(\frac{a}{2} - \frac{s_3}{2}\right)$$

$$S_x^4 = -\frac{a}{2} \cdot 2t \cdot \frac{a}{4} - s_4 \cdot t \cdot \frac{a}{2} = -\left(\frac{a^2 t}{4} + \frac{a t s_4}{2}\right)$$

$$S_x^5 = -s_5 \cdot 2t \cdot \frac{s_5}{2} = -s_5^2 t$$

shear flux on arms

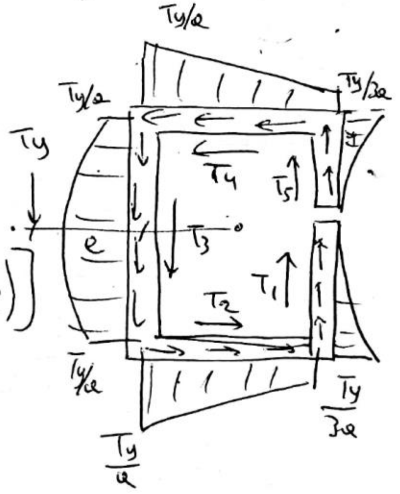
$$q_1 = \frac{-T_y (S_x)^1}{I_x} = \frac{T_y s_1^2 t}{3/4 a^3 t} = \frac{-4}{3} \frac{T_y s_1^2}{a^3}$$

$$q_2 = \frac{-T_y (S_x)^2}{I_x} = \frac{-T_y}{3a}\left(1 + \frac{2 s_2}{a}\right)$$

$$q_3 = \frac{-T_y (S_x)^3}{I_x} = \frac{-T_y}{a}\left[1 + \frac{2}{3}\left(\frac{s_3}{a} - \frac{s_3^2}{a^2}\right)\right]$$

$$q_4 = -\frac{T_y (S_x)^4}{I_x} = \frac{T_y}{3a}\left(1 + \frac{2 s_4}{a}\right)$$

$$q_5 = -\frac{T_y (S_x)^5}{I_x} = \frac{4 T_y s_5^2}{3 a^3}$$



(7)

$$T_1 = \int_0^{a/2} q_1 \, ds_1 = \int_0^{a/2} -\frac{4}{3} \frac{T_y s_1^2}{a^3} \, ds_1 = \frac{-T_y}{18}$$

$$T_2 = \int_0^a q_2 \, ds_2 = \frac{-2}{3} T_y$$

$$T_3 = \int_0^a q_3 \, ds_3 = \frac{10}{9} T_y$$

$$T_4 = -T_2 = \frac{2}{3} T_y$$

$$T_5 = -T_1 = \frac{T_y}{18}$$

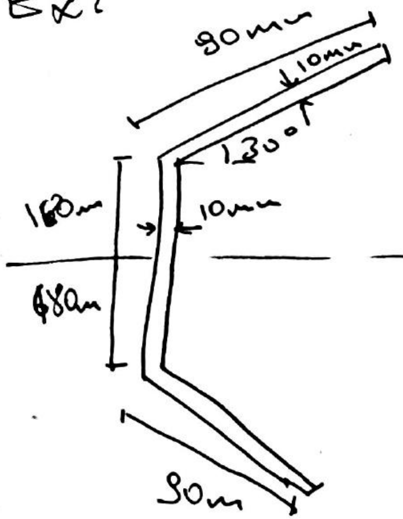
~~Total~~ moment about a point on the vertical arm is zero

$$T_y \cdot e - T_5 \cdot a - T_1 \cdot a - T_4 \frac{a}{2} - T_2 \cdot \frac{a}{2} = 0$$

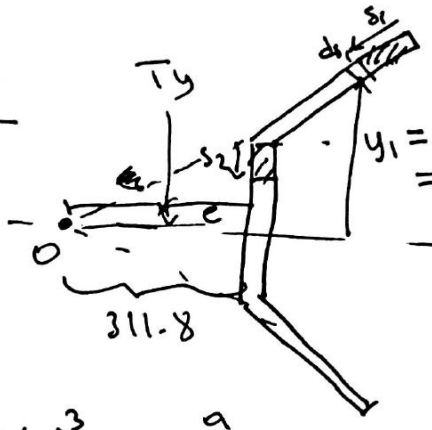
$$e = \frac{7}{9} a$$

(8)

Ex2



Find the location of shear center



$y_1 = 180+(90-s_1)\sin 30$
$= 225-\frac{s_1}{2}$

$$I_x = \frac{t(4a)^3}{12} + 2\int_0^a y_1^2 (t\,ds)$$

$$= 3888\,10^4 + 2\left(506250 s_1 - 1125 s_1^2 + \frac{6}{5} s_1^3\right)\Big|_0^{90}$$

$$= 113\,10^6 \text{ mm}^4$$

$$S_x^1 = -s_1 t\left[180 + \left(90-\frac{s_1}{2}\right)\sin 30\right]$$
$$= -2250 s_1 + 2.5 s_1^2$$

$$S_x^2 = S_x^1\Big|_{s_1=90} + t s_2 \cdot \left\{-\left(180-\frac{s_2}{2}\right)\right\}$$
$$= -182250 - 1800 s_2 + 5 s_2^2$$

$$q_1 = -\frac{T_y S_x^1}{I_x} \qquad T_1 = \int_0^{90} q_1\, ds_1 = -\frac{T_y}{I_x}\int_0^{90} -2250 s_1 + 2{,}5 s_1^2\, ds_1$$

$$= -\frac{T_y}{I_x}\left(\frac{-2250 s_1^2}{2} + \frac{2{,}5 s_1^3}{3}\right)\Bigg|_0^{90} = 0{,}0753\, T_y$$

$$q_2 = -\frac{T_y S_x^2}{I_x} \qquad T_2 = \int_0^{360} q_2\, ds_2 = -\frac{T_y}{I_x}\int_0^{360}\left(-182250 - 1800 s_2 + 5 s_2^2\right) ds_2$$

$$= -\frac{T_y}{I_x}\left(-182250 s_2 - \frac{1800 s_2^2}{2} + \frac{5 s_2^3}{3}\right)\Bigg|_0^{360}$$

$$= 0{,}9247\, T_y$$

Moment about O

$$T_y \cdot (311.8 - e) - 0{,}9247\, T_y \cdot 311.8 = 0$$

$$\boxed{e = 23.5 \text{ mm}}$$

Calculation shear flux when the coordinate axis is not principal

If the coordinat axis of the cross-section is not principal, then the normal stresses can be calculated by

$$\sigma_z = \frac{M_x\,(I_y \cdot y - I_{xy} x) + M_y\,(I_{xy} y - I_x x)}{I_x I_y - I_{xy}^2} \quad (1)$$

The shear flux is given as

$$\tau . t = q = -\int_A \frac{d\sigma_z}{dz}\, dA \quad (2)$$

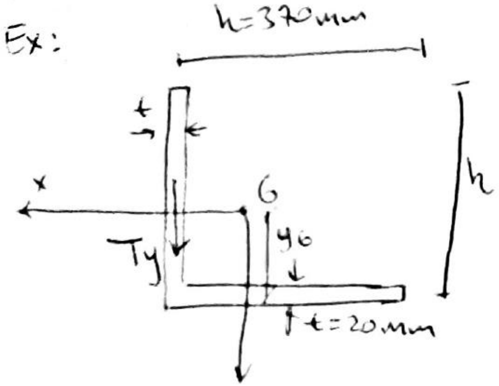
After substituting (1) into (2) and using relations $T_y = \frac{dM_x}{dz}$ one can obtain

$$\tau . t = \frac{-1}{I_x I_y - I_{xy}^2} \int_{A'} \left[T_y\,(I_y \cdot y - I_{xy} x) - T_x\,(I_{xy} y - I_x x) \right] dA$$

Considering $S_x = \int y\, dA$ and $S_y = \int x\, dA$

$$q(s) = -\frac{1}{I_x I_y - I_{xy}^2} \left(T_y \left[I_y S_x(s) - I_{xy} S_y(s) \right] - T_x \left[I_{xy} S_x(s) - I_x S_y(s) \right] \right)$$

Ex:



Find the shear stress distribution on the given cross section

$$y_G = \frac{20 \cdot 370 \cdot (\frac{1}{2} 370)}{2(20 \cdot 370)} = 92{,}5\ mm$$

$$x_G = 92.5\ mm$$

$$I_x = \frac{1}{12}(20 \cdot 370^3) + 20 \cdot 370(-92{,}5)^2 + \frac{1}{12}(370 \cdot 20^3) + 370 \cdot 20 \cdot 92{,}5^2 =$$
$$= 2113 \; 10^5\ mm^4$$

$I_y = 2113 \; 10^5\ mm^4$ (due to

$$I_{xy} = (20 \cdot 370)\ 92{,}5\ (-92{,}5) + 370 \cdot 20\ (-92.5)\ 92.5 = -1266\ 10^5\ mm^4$$

direction of principal axis (x_1, y_1) $\tan 2\alpha_0 = \dfrac{-2 I_{xy}}{I_x - I_y} = \infty$

$2\alpha_0 = 90 \quad \alpha_0 = 45^\circ$

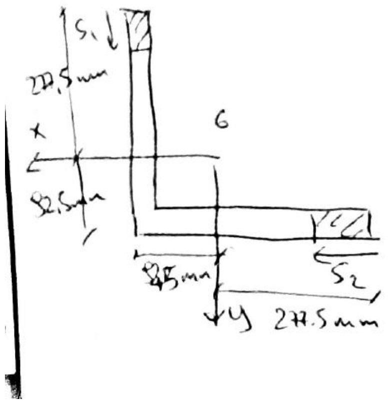
In general form

$$q = \tau t = -\frac{1}{I_x I_y - I_{xy}^2}\Big(T_y\left[I_y S_x(s) - I_{xy} S_y(s)\right] - T_x\left[I_{xy} S_x(s) - I_x S_y(s)\right]\Big)$$

since $T_x = 0$

$$q = \tau t = -\frac{T_y}{I_x I_y - I_{xy}^2}\left(I_y S_x - I_{xy} S_y\right)$$

$$I_x I_y - I_{xy}^2 = 2861 \; 10^{13}\ mm^8$$



$$S_{x_1} = 20 s_1 \left[-\left(277.5 - \tfrac{1}{2} s_1\right)\right] = -5550 s_1 + 10 s_1^2$$

$$S_{y_1} = 20 s_1\ 92{,}5 = 1850 s_1$$

$$S_{x_2} = 20 s_2\ 92.5 = 1850 s_2$$

$$S_{y_2} = 20 s_2 \left[-\left(277.5 - \tfrac{s_2}{2}\right)\right] = -5550 s_2 + 10 s_2^2$$

$$q_1 = -\frac{T_y}{I_x I_y - I_{xy}^2}\left(I_y S_{x_1} - I_{xy} S_{y_1}\right) = T_y\left(3280 s_1 - 7.38 s_1^2\right) 10^{-8}$$

$$q_2 = -\frac{T_y}{I_x I_y - I_{xy}^2}\left(I_y S_{x_2} - I_{xy} S_{y_2}\right) = T_y\left(1090 s_2 - 4.42 s_2^2\right) 10^{-8}$$

The max value of q_1 can be found as

$$\frac{dq_1}{ds_1} = 0 \quad \cancel{\Rightarrow 1090 s_2 - 4.42 s_2^2 = 0}$$

$s_1 = 222$ m

The point in which q_2 is zero

$1090 s_2 - 4.42 s_2^2 = 0 \quad (s_2)_1 = 0 \quad (s_2)_2 = 246$ mm

max value of q_2

$$\frac{\partial q_2}{\partial s_2} = 0 \Rightarrow s_2 = 12 \text{ mm}$$

Finally

$q_1\big|_{s_1=222} = 36.41\, 10^{-4}\, T_y$

$q_1\big|_{s_1=370} = 20.26\, 10^{-4}\, T_y$

$q_2\big|_{s_2=123} = 6.7\, 10^{-4}\, T_y$

$q_2\big|_{s_2=370} = -20.26\, 10^{-4}\, T_y$