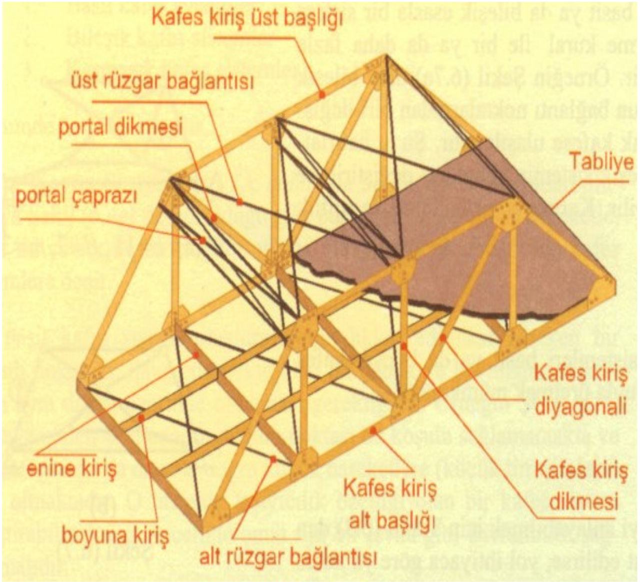
YILDIZ TECHNICAL UNIVERSITY
CIVIL ENGINEERING DEPARTMENT
DIVISION OF MECHANICS

STATICS

CHAPTER SEVEN
TRUSSES

Definition of a truss: A truss consists of straight members connected at joints. Truss members are connected at their extremities only; thus no member is continuous through a joint. The members are actually joined together by means of bolted, riveted or welded connections. It is customary to assume that the members are pinned together, therefore, the forces acting at each end of a member reduce to a single force and no couple, Fig. 2.

The truss is one of the major types of engineering structures. It provides both a practical and an economical solution to many engineering situations, especially in the design of bridges and buildings. The trusses can be classified as follows:



1. Plane Trusses:

a) Simple trusses

b) Trusses made of several simple trusses: Compound Trusses

c) Complex trusses

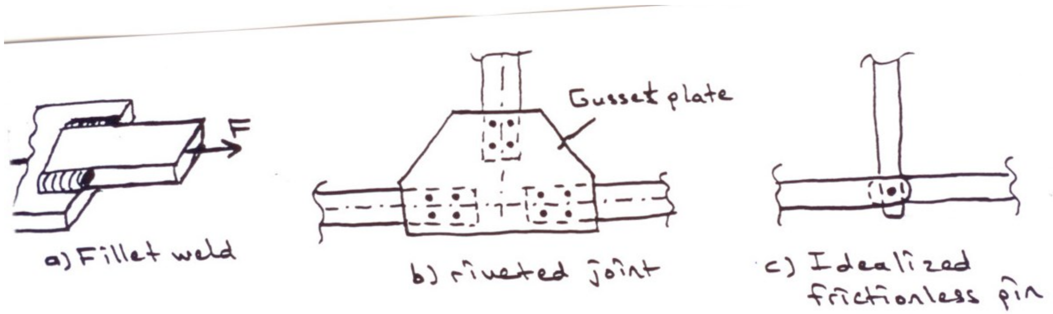


Fig. 2 Joint connections

2. Space Trusses:

Some properties of plane trusses:

❖ Plane trusses are multi-component systems which are formed by straight lines.

❖ A truss consists of straight members connected at joints.

❖ Truss members are connected at their extremities only; thus no member is continuous through a joint.

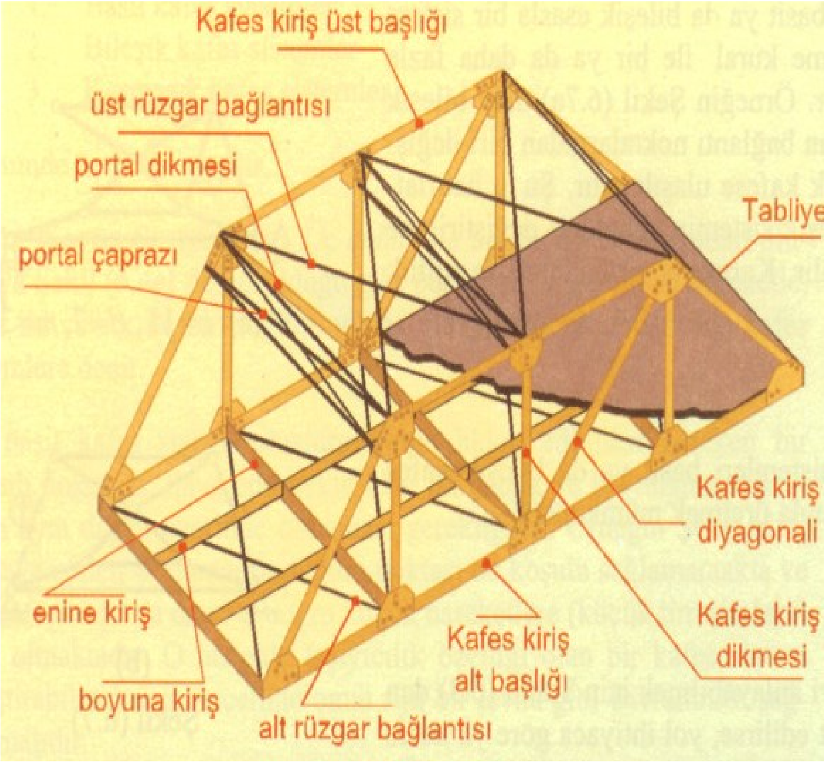
❖ In real trusses, the members are usually bolted, welded or riveted to a gusset plate, as shown in Fig. 7.2.

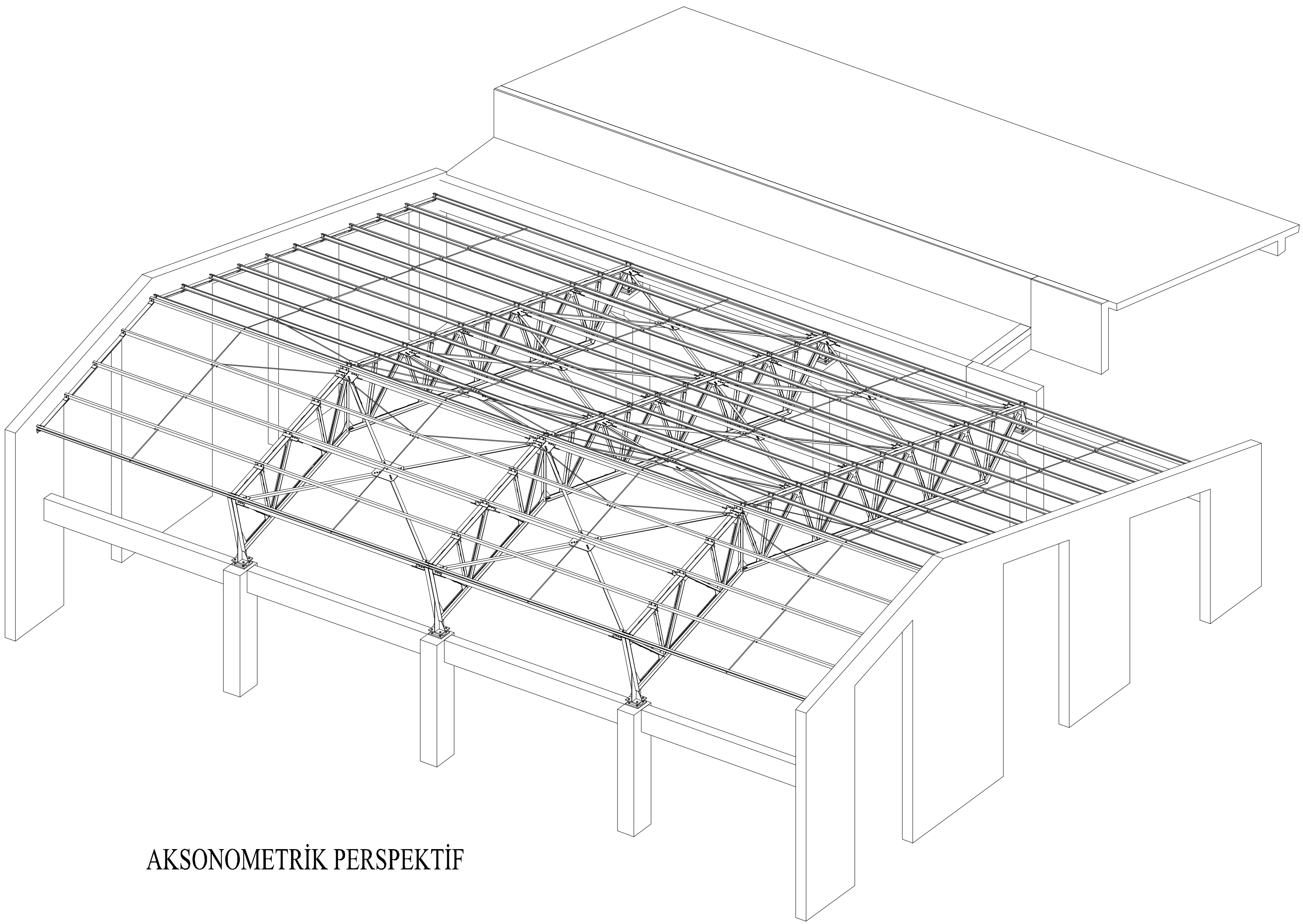
❖ Most actual structures are made of several trussses joined together to form a space framwork, Fig. 7.1.

❖ Each truss is designed to carry those loads which act in its plane and thus may be treated as a two-dimensional structure.

❖When a concentrated load is to be applied between two points, or when a distributed load is to be supported by the truss, as in the case of a bridge truss, a floor system must be provided which, through the use of stringers and floor beams, transmits the load to the joints, Fig. 7.1.

❖The only forces assumed to be applied to a truss member are a single force ar each end of the member, Fig. 7.3.





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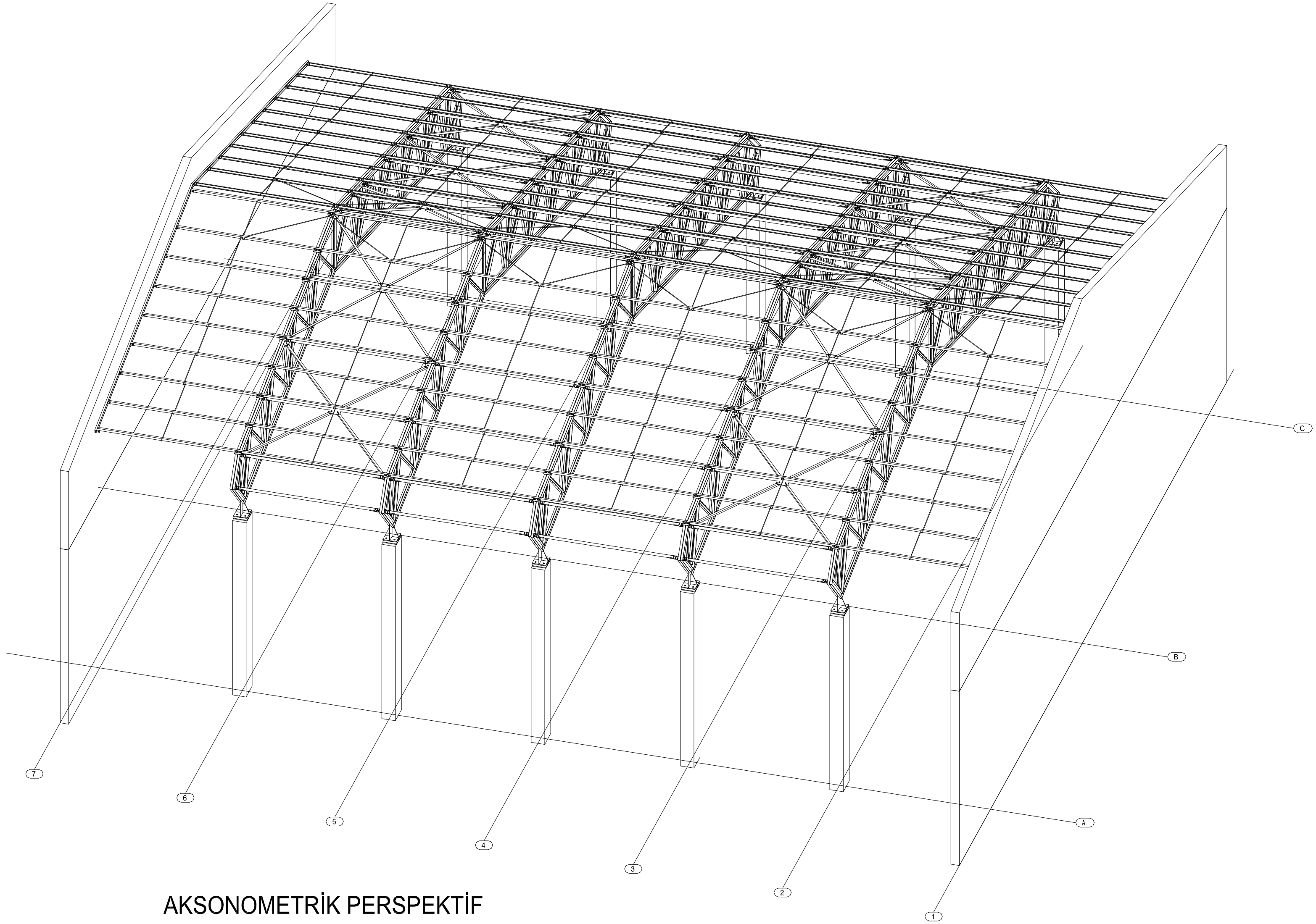

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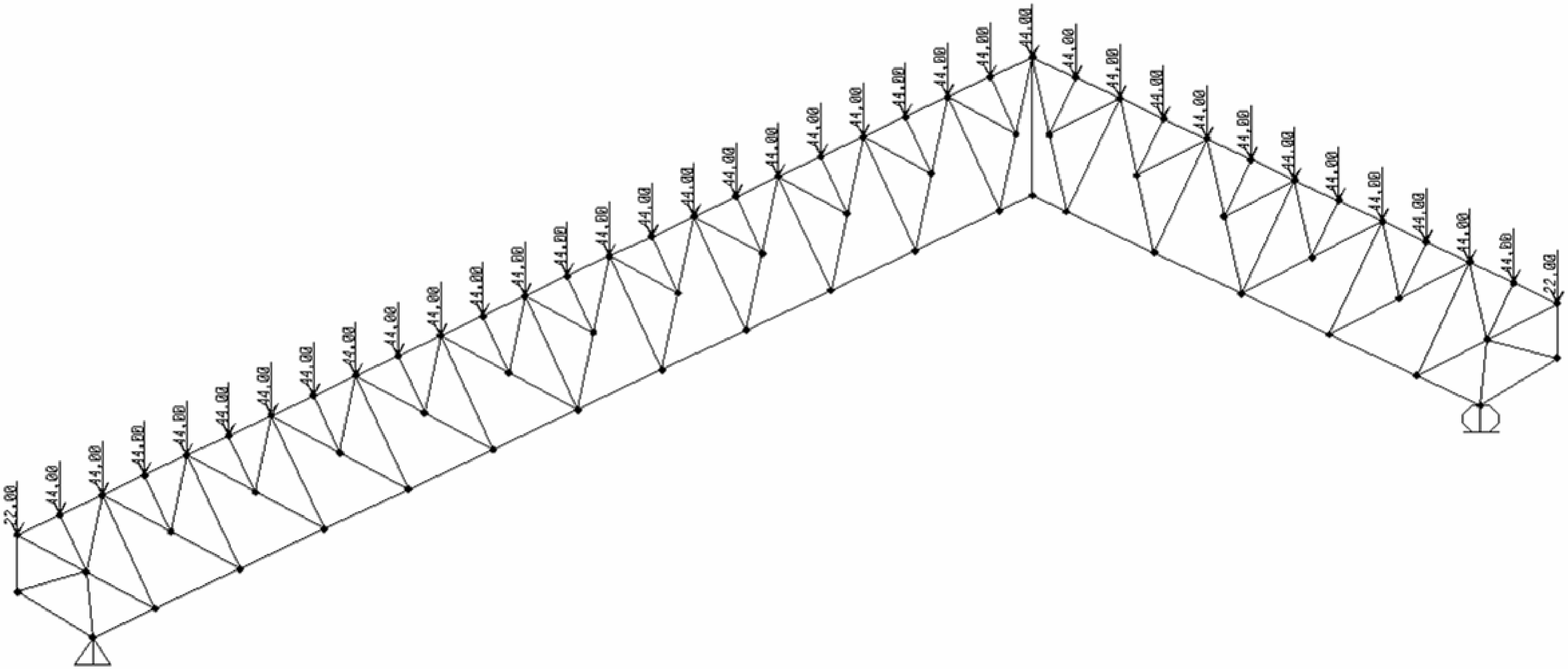

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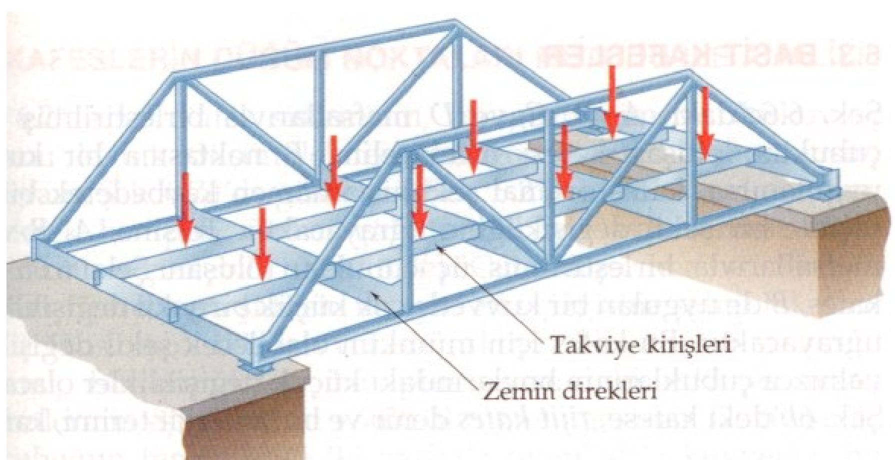
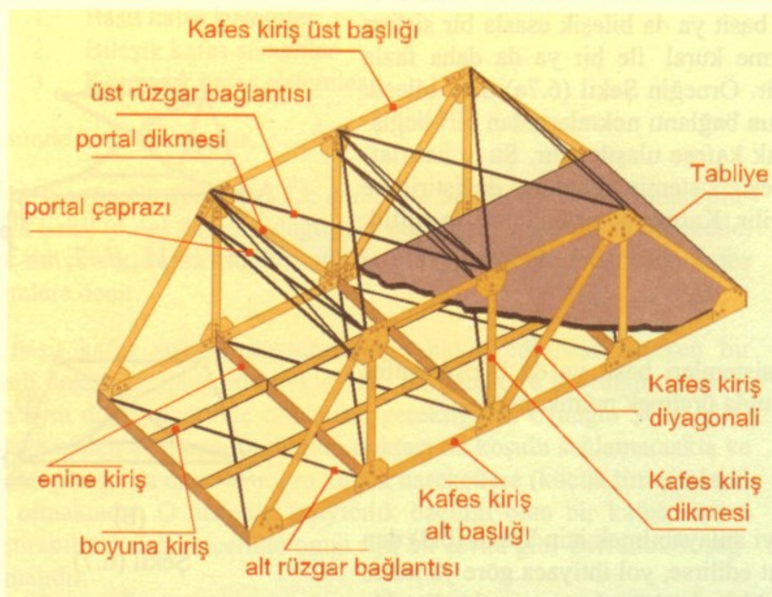
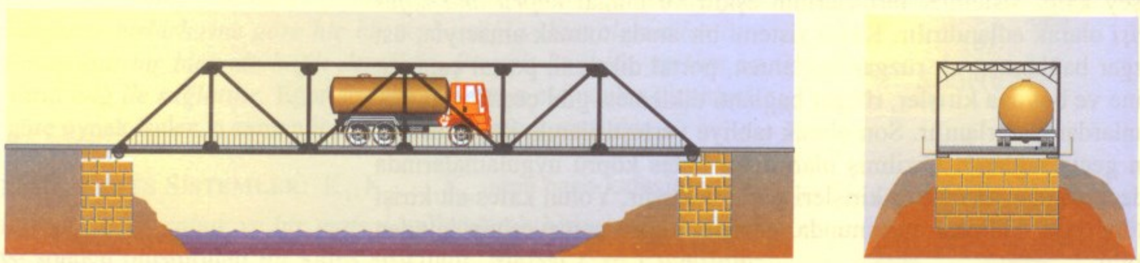

Takviye kirişleri
Zemin direkleri

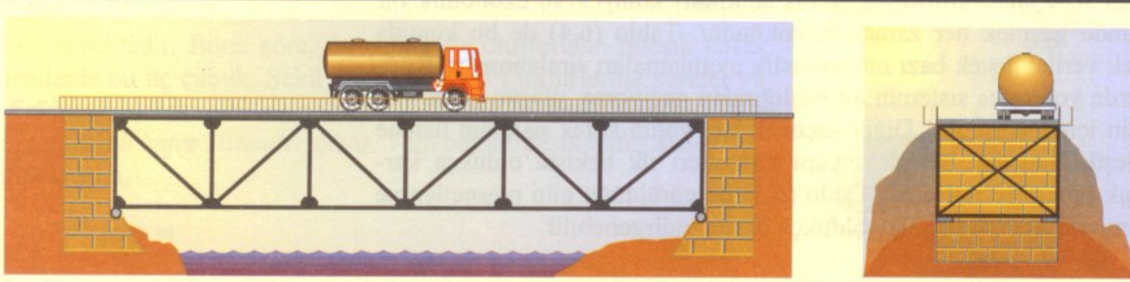


ANA KAFES KİRİŞ

Yol alt başlıkta



Yol üst başlıkta



a) Simple Trusses:

Rigid Truss – basic triangular truss:

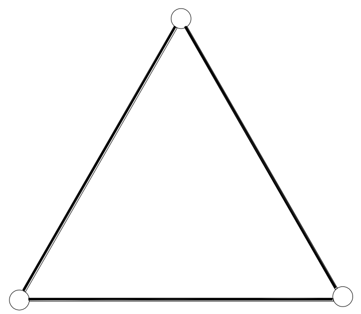


Fig. 7.4 Basic triangular truss

The only possible deformation for this truss is one involving small changes in the length of the members. This truss is called as a rigid truss. The relative positions of A, B, and C according to each other do not change for a rigid truss.

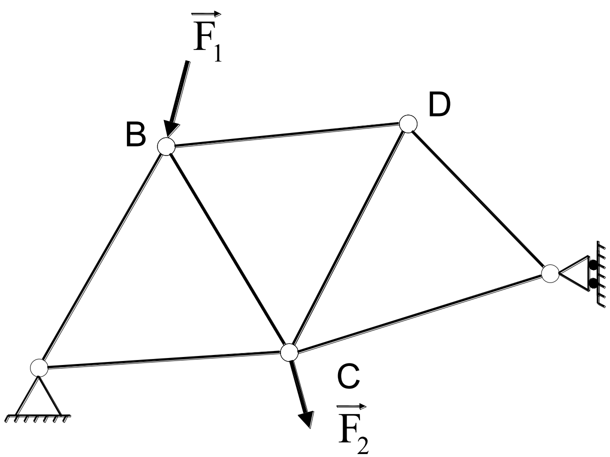


Fig. 7.5 Simple truss

As shown in Fig. 7.5, a larger rigid truss can be obtained by adding two members BD and CD to the base triangular truss of Fig. 7.4.

b) Trusses Made of Several Simple Trusses: Compound Trusses

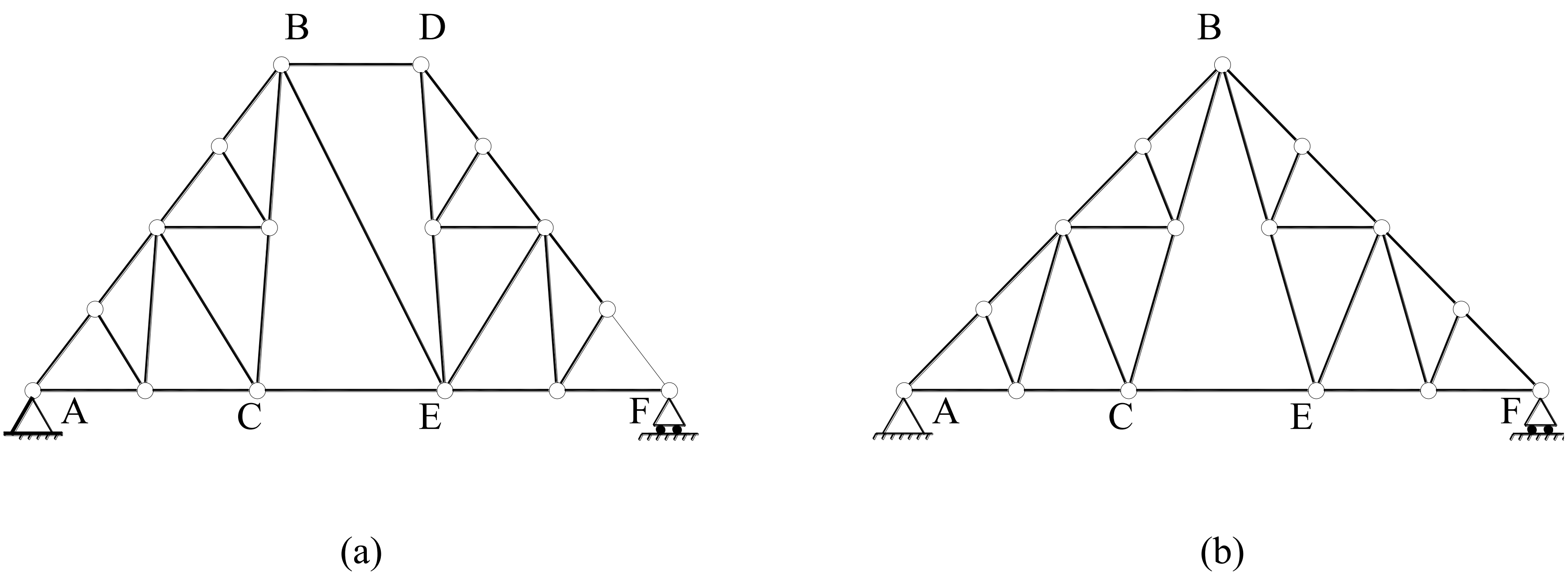
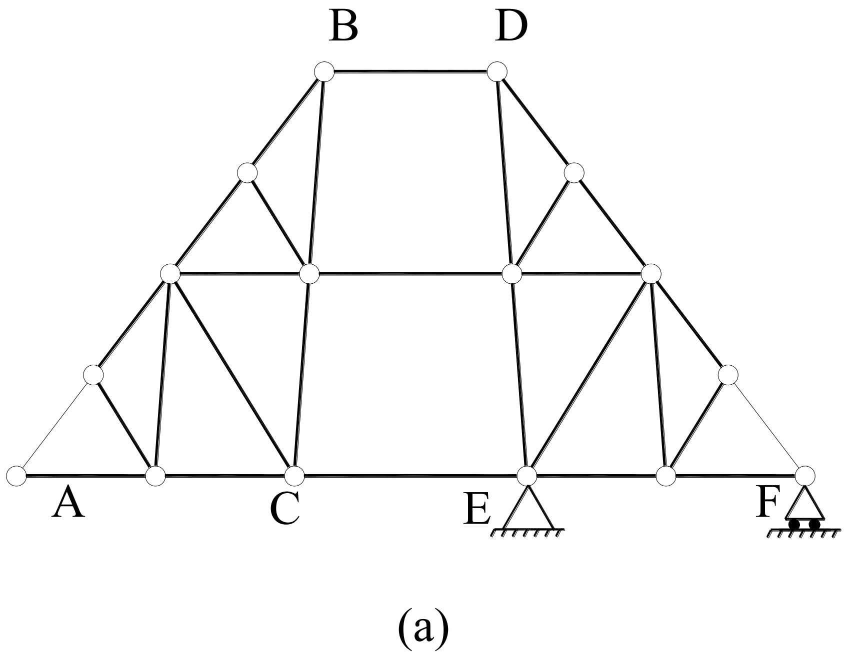


Fig. 7.6 Compound Trusses

Consider two simple trusses ABC and DEF. If they are connected by three bars BD, BE and CE as shown in Fig. 7.6a, they will form together a rigid truss ABDF. The trusses ABC and DEF can also be combined into a single rigid truss by joining joints B and D into a single joint B and by connecting joints C and E by a bar CE, Fig. 7.6b.



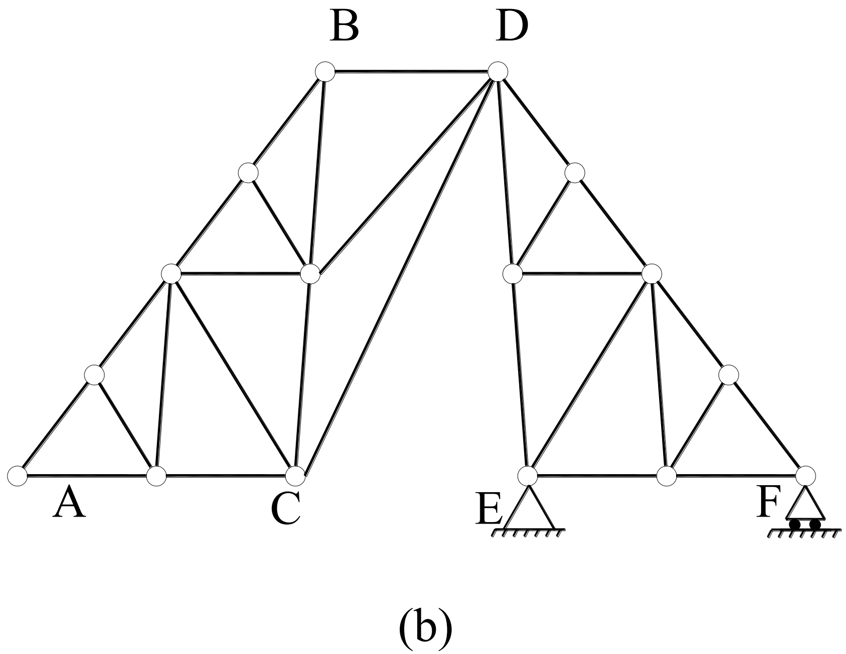


Fig. 7.7. Improperly Connected Trusses

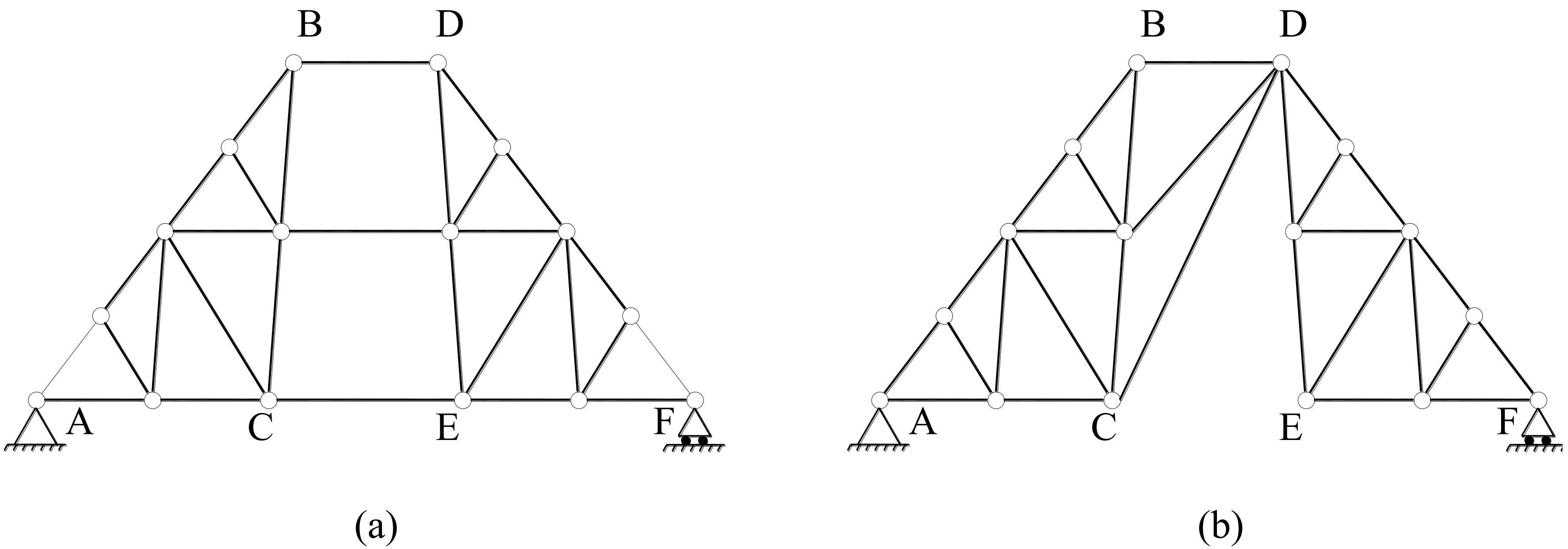


Fig. 7.7. Improperly Connected Trusses

If two simple trusses are connected by threee bars which are parallel to each other or concurrent at a point, then it is said that these trusses are connected improperly. Beacuse this situation arises from an inadequate arrangement or geometry of the bars, it is often referred to as geometric instability.

c) Complex Trusses:

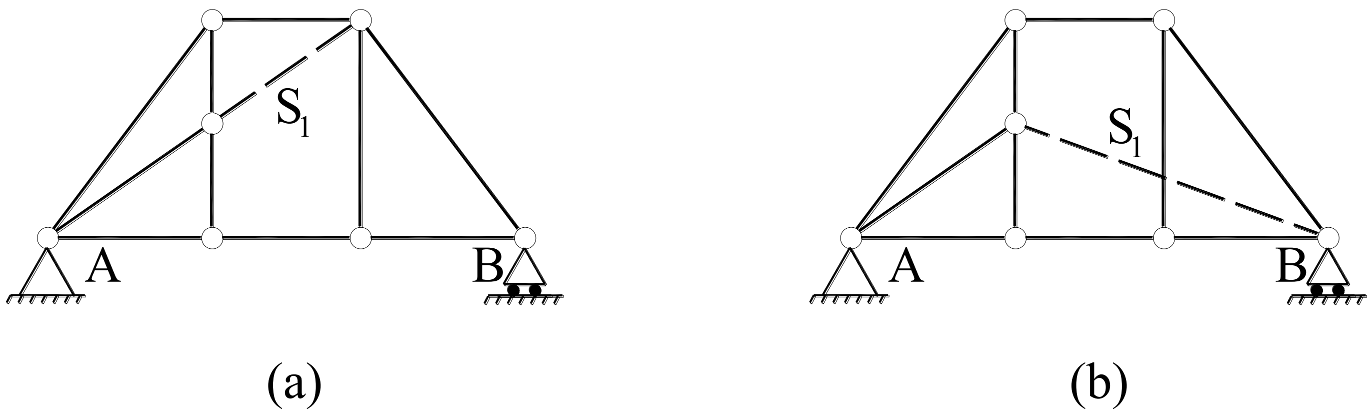


Fig. 7.8 Complex Trusses

❖ **Complex trusses are designed without any connection rules.**

❖ **After designing a simple truss, one or more bars are reconnected to other joints to be able to define a complex truss.**

The degree of freedom in the plane trusses:

$$h = m + r - 2n$$

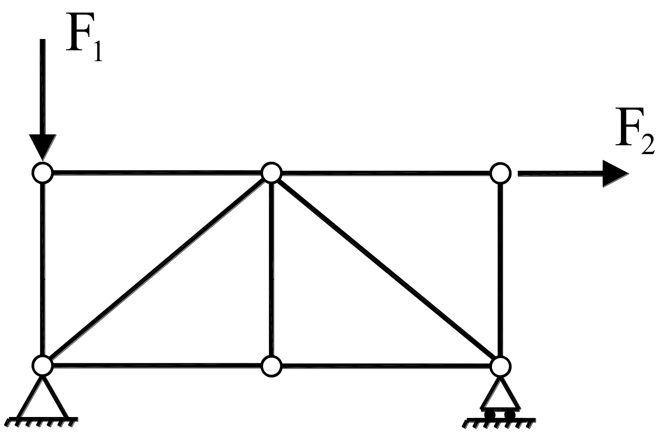
m **is the total number of the members of the truss,** r **is the number of support reactions,** n **is the total number of joints.**

External Indeterminancy:

If the support reactions are the reason to make the truss indeterminate, it is called externally indeterminate.

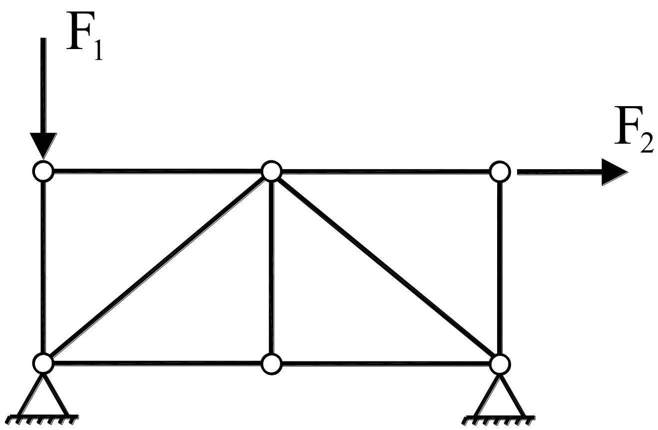
Internal Indeterminancy:

If the bars needs to be altered, then the truss is a internally indeterminate.



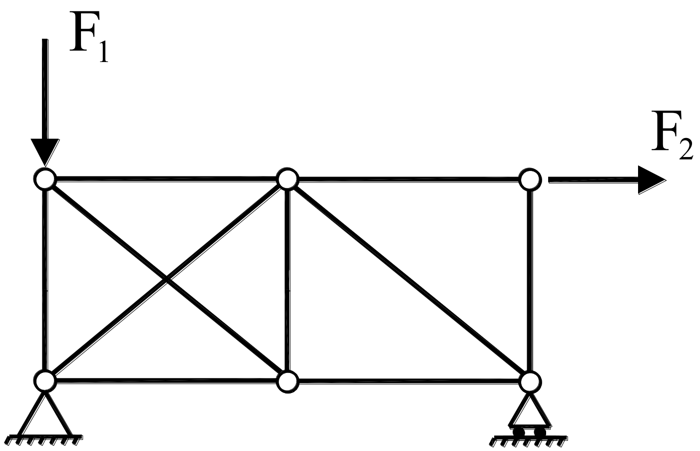
a) Internally and externally determinate truss

$$h = 9 + 3 - 2 * 6 = 0$$



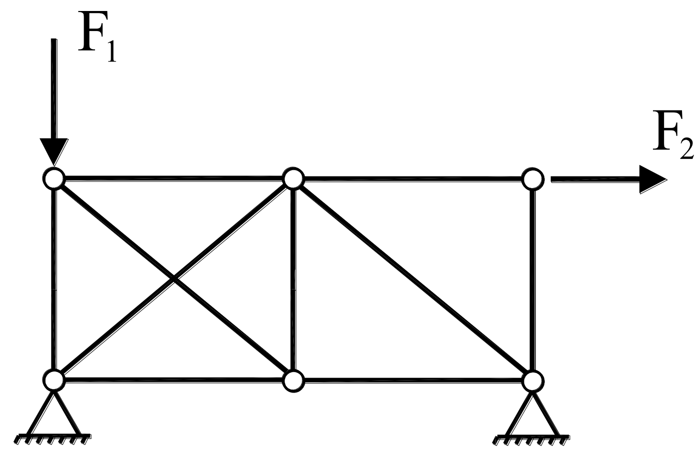
b) Externally 1 degree indeterminate truss

$$h = 9 + 4 - 2 * 6 = 1$$



a) Internally 1 degree indeterminate truss

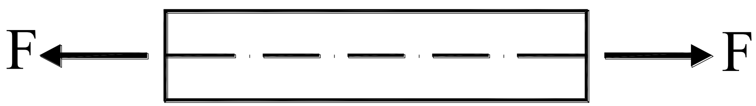
$$h = 10 + 3 - 2 * 6 = 1$$



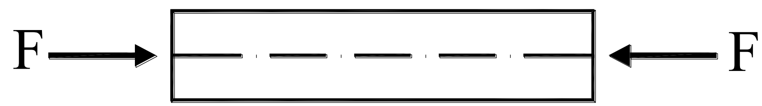
b) Externally and Internally 1 degree indeterminate truss

$$h = 10 + 4 - 2 * 6 = 2$$

Sign convention of bars of a truss:



(+) Çekme kuvveti



(-) Basınç kuvveti

Analysis Methods of Trusses:

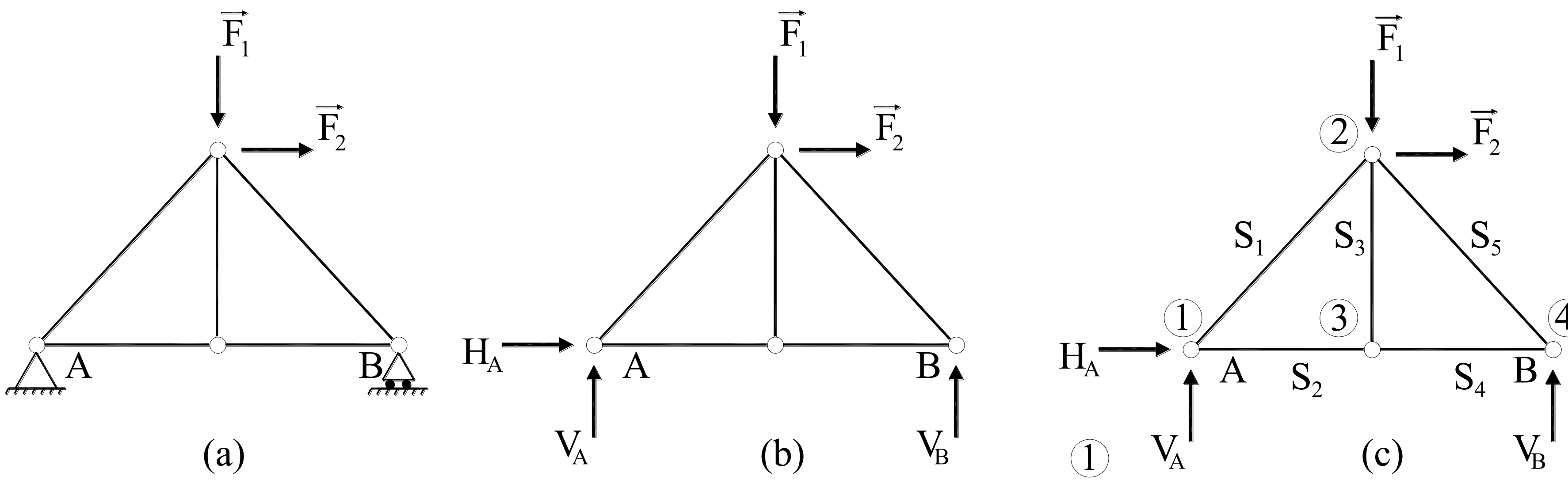
Analytical Methods:

1) **Method of Joints**
2) **Method of Sections (Ritter)**
3) **Method of Changing Bars (Henneberg)**

1. Method of Joints:

In the considered method, because each joint is taken as a pin, and there is no load applied out of the joints, by using two equilibrium equations for each joint, all of the forces of the truss members can be determined.

Solution procedure: 1) **The support reactions are determined.**



2) The truss is dismembered and a free body diagram is drawn for each pin and each member, Figs. 7.9 and 7.10. Bar forces can be determined by using the equilibrium equations.

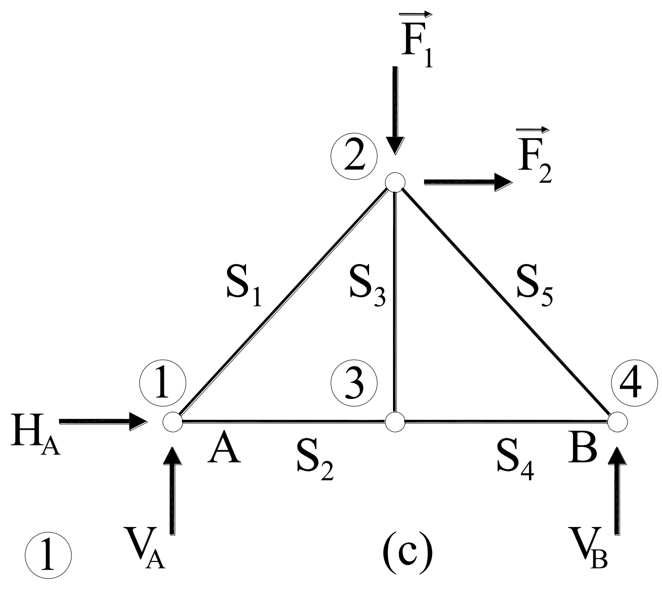


Fig. 7.9.

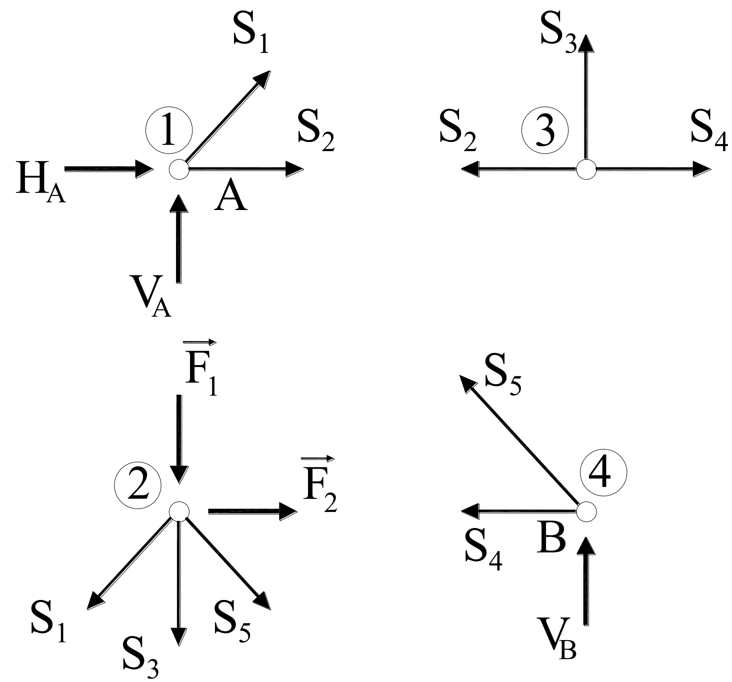
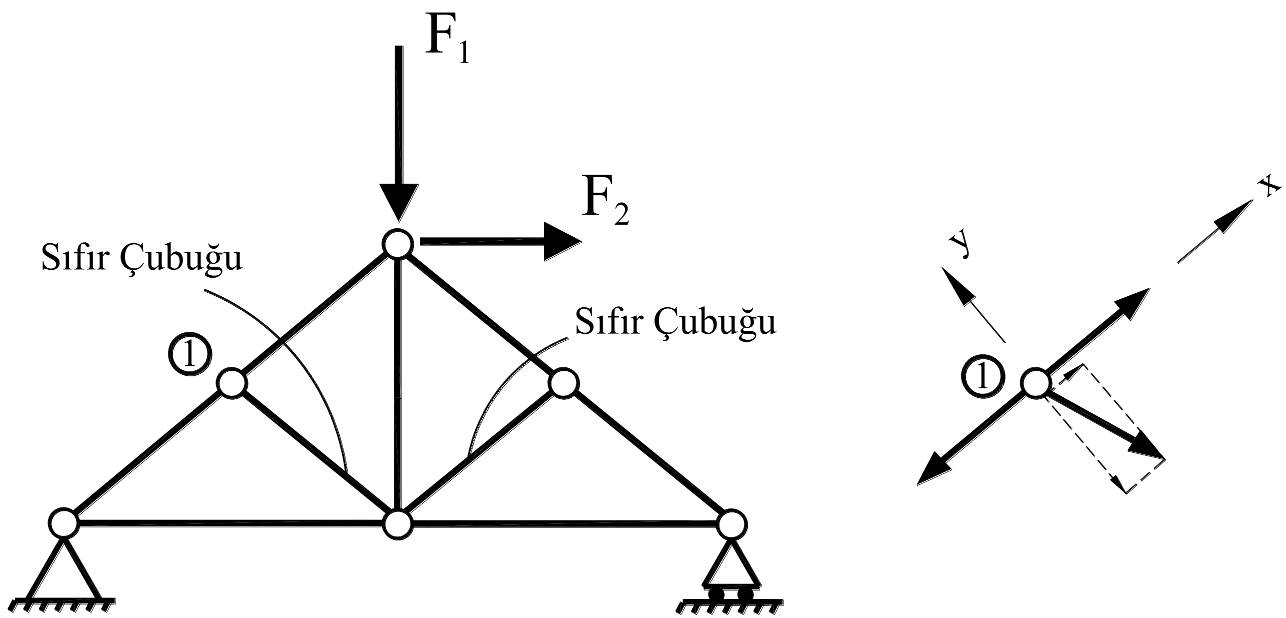


Fig. 7.10

3) At first, the unknown bar forces at each joint are assumed to be tension forces. These forces are drawn towards the outside from the joint. These bar forces are considered to be positive (tension) and if the computation gives negative, then these are compression forces.

4) These values are transferred to the adjacent joints with their true directions and treated as known quantities at these joints. After the first joint, the other joints must be chosen successively as including only two unknown forces.

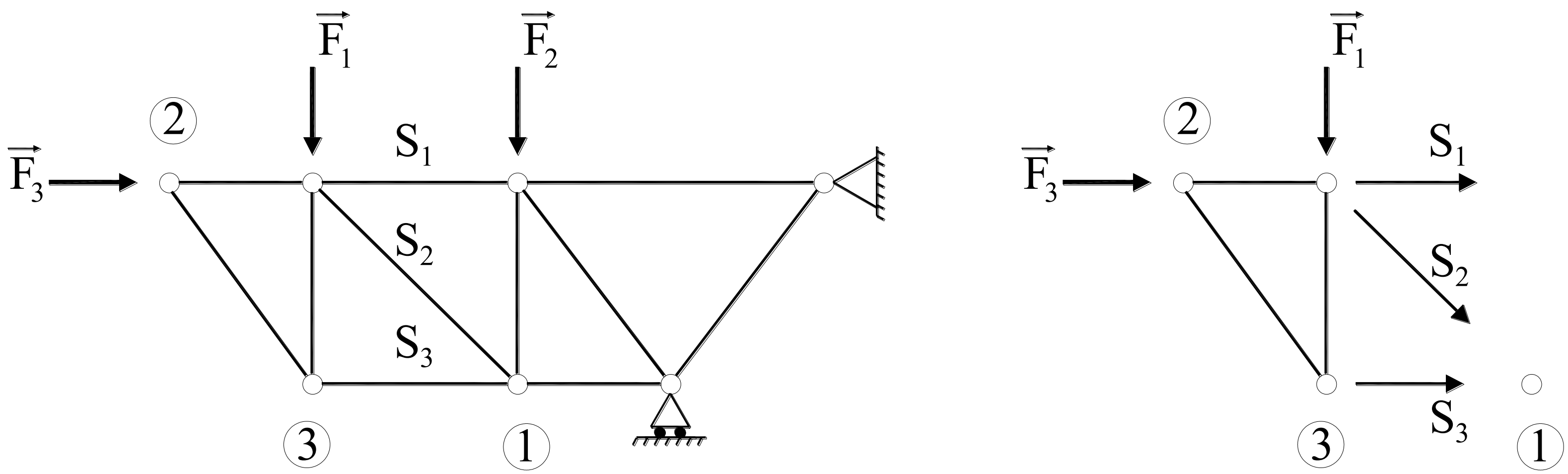
Zero force bars:



These zero force bars are utilized to decrease the buckling length or to take the outside joint forces which are not recommended at the beginning of the design.

2. Method of Sections:

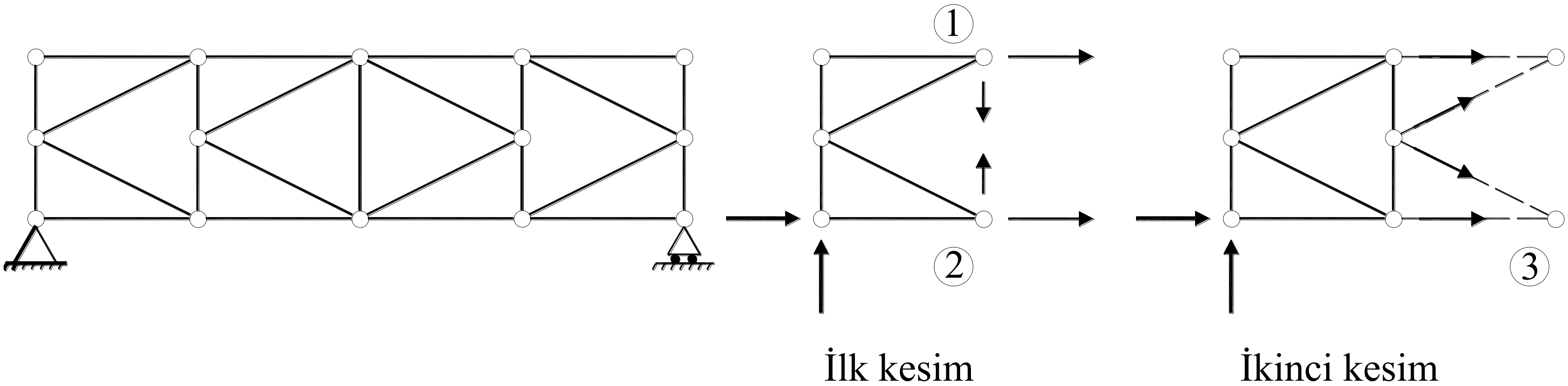
- Generally, the support reactions are found firstly. In some situations, there is no need to find support reactions.
- The considered system is separated into two portions by parsing a section through three members of the truss, one of which is the desired member, i.e., by drawing a line which divides the truss into two completely separate parts but does not intersect more than three members.



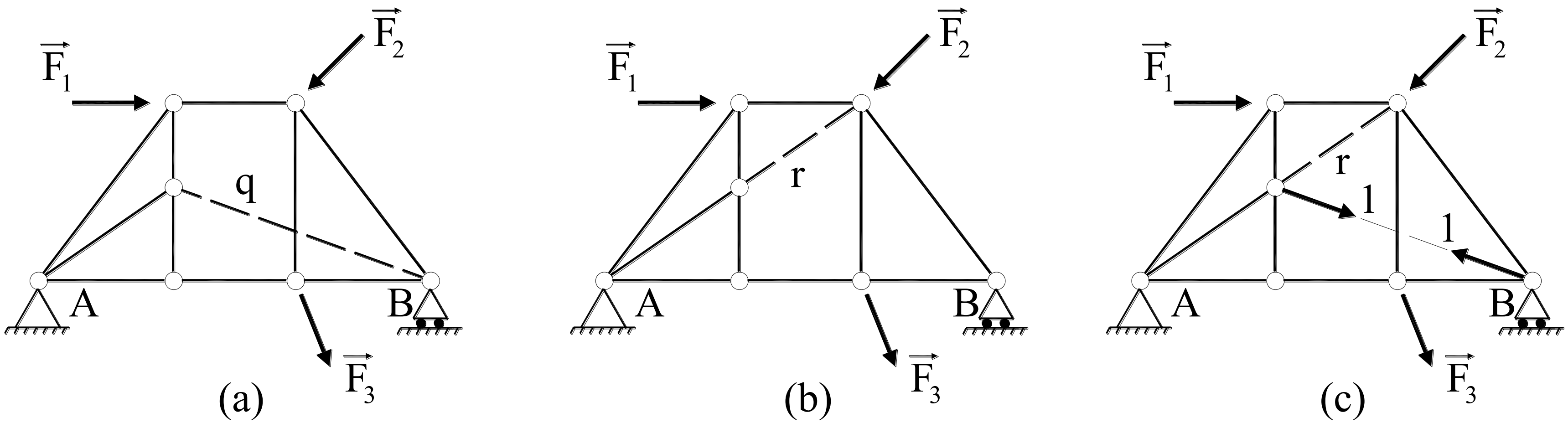
- Either of two portions of the truss obtained after the intersected members have been removed can then be used as a free body and the three equilibrium equations can be applied.

❖ **A positive sign in the answer will indicate that our assumption regarding the sense of force was correct. A negative sign will indicate that our assumption was incorrect and the considered member is in compression.**

K shape truss:



3. Method of Changing Bars (Henneberg):



❖ **If the bar, q, replaces with the bar, r, there will be a simple truss system. Lets assume the bar forces and joints solved by using the two methods as described earlier.**

$$S_1, S_2, ..., S_{10}, S_r$$

❖ **However, a clarification will be used. Unit tension forces will be applied outward the joints that the bar q is connected as shown the figure above.**

$$S_1', S_2', ..., S_{10}', S_r'$$

❖ **if the force P is applied to the joints instead of unit forces, then the bar forces will be**

$$pS_1', pS_2', ..., pS_{10}', pS_r'$$

❖ **when the loads on the truss are also considered, the bar forces will be**

$$(S_1 + pS_1'), (S_2 + pS_2'), \ldots\ldots, (S_{10} + pS_{10}'), (S_r + pS_r')$$

❖ **Finally, because there is no bar r in the real truss, the force P will be determined to make the force of bar r zero. Therefore;**

$$(S_r + pS_r') = 0 \rightarrow p = \frac{-S_r}{S_r'}$$

❖ **The bar forces for the other bars are then calculated using the equations above.**

$$(S_1 - \frac{S_r}{S_r'} \cdot S_1'), (S_2 - \frac{S_r}{S_r'} \cdot S_2') \ldots\ldots, (S_{10} - \frac{S_r}{S_r'} \cdot S_{10}')$$

2) Space Trusses:

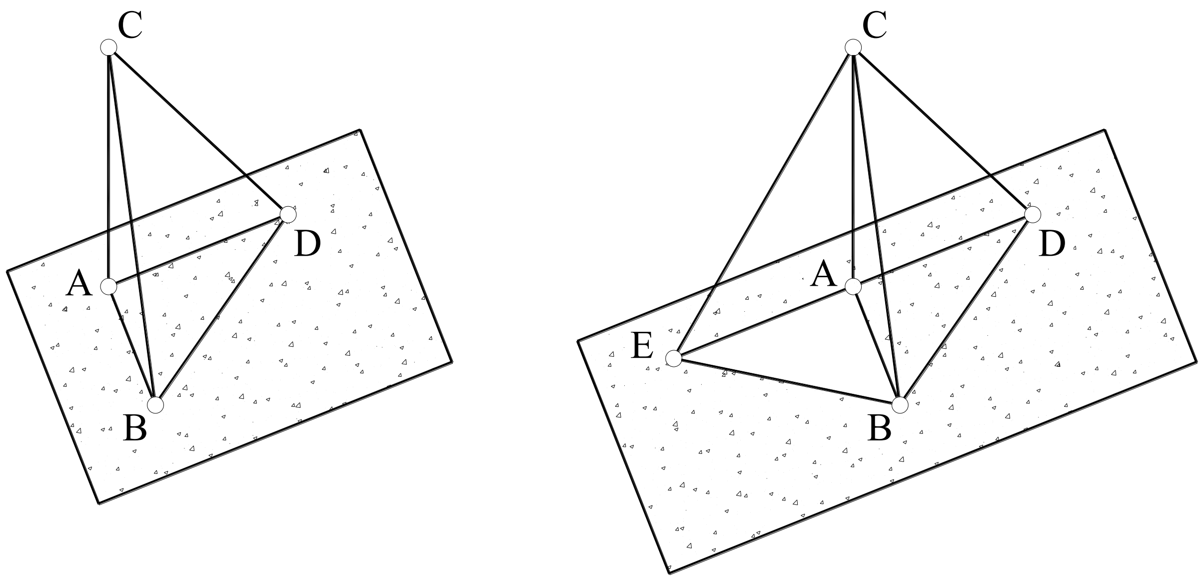


Fig. 7.11 Basic tetrahedron of a space truss

- **When straight members are joined together as their extremities to form a three dimensional configuration, the structure obtained is called a space truss.**

- **The basic tetrahedron has six members and four joints.**

- **In this truss, three force equilibrium equations are used to calculate the bar forces because the forces are intersected at one joint.**

In a simple space truss, the number of degree of freedom is:

$$h = m + r - 3.n$$

The method of sections developed before may also be applied to space trusses. The vector equations must be satisfied for any section of the truss.

$$\sum \vec{F} = 0 \quad , \qquad \sum \vec{M} = 0$$

Since the two vector equations are equivalent to six scalar equations, we conclude that a section should in general not be passed through more than six members whose forces are unknown.

In plane trusses, a moment axis can be found which eliminates all but one unknown.

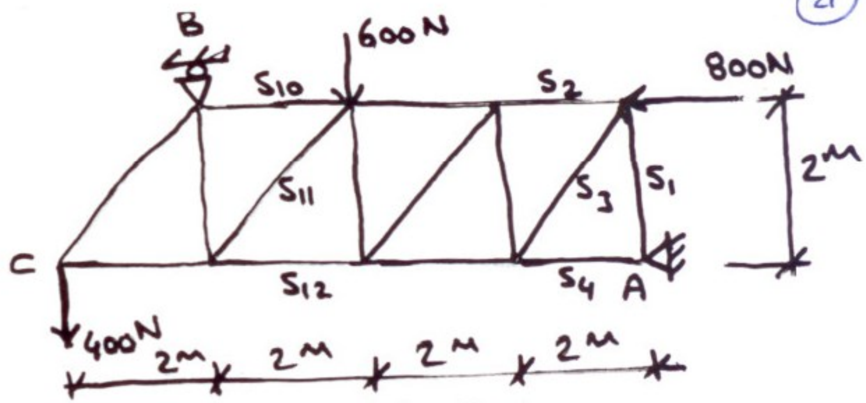
However, the moment axis can seldom be found for space trusses, the method of sections for space trusses is not widely used.

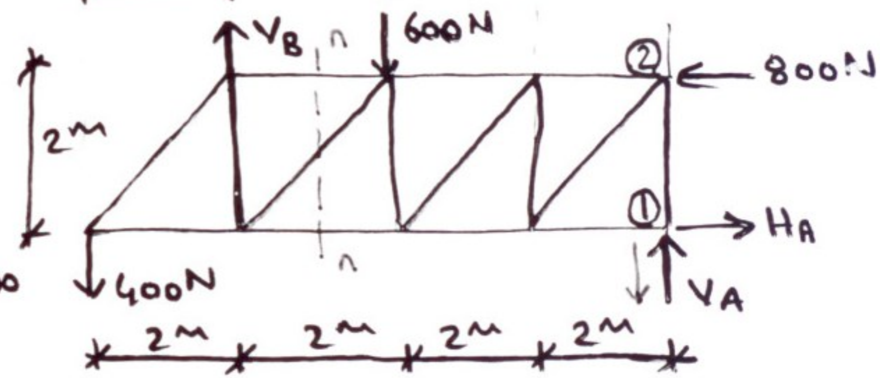
Problem 1:

(21)

Problem 1: a) Using the method of joints, determine the force in the members 1, 2, 3, 4 of the truss shown.

b) Determine the forces in the members 10, 11, 12 of the truss shown by using the method of sections.





Solution: $\overset{+}{\rightarrow}\Sigma F_x = 0: H_A - 800 = 0$

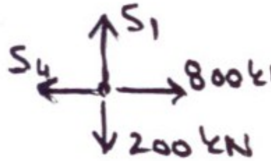
$H_A = 800\ N$

$\overset{+}{\curvearrowright}\Sigma M_A = 0: -400.8 + 6.V_B - 2.800 - 4.600$

$V_B = 1200\ N$

$\uparrow \Sigma F_y = 0: -400 + 1200 - 600 + V_A = 0 \rightarrow V_A = -200\ N$

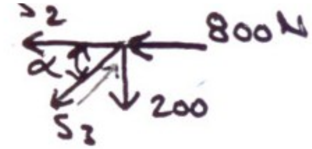
a) joint 1



$\xrightarrow{+}$ $\Sigma F_x = 0: -S_4 + 800 = 0 \rightarrow S_4 = 800N$

$+\uparrow \Sigma F_y = 0: S_1 - 200 = 0 \rightarrow S_1 = 200N$

joint 2:

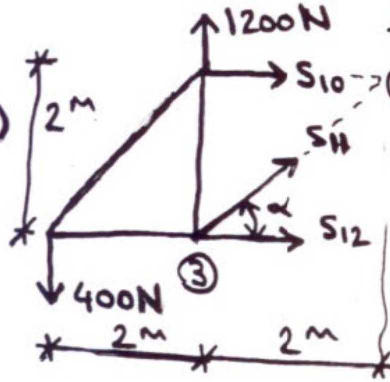


$+\uparrow \Sigma F_y = 0: S_3 . \sin\alpha + 200 = 0$

$S_3 = -282,84N$

$\xrightarrow{+}$ $\Sigma F_x = 0: -S_2 - (-282,84).\cos\alpha - 800 = 0 \rightarrow S_2 = -600N$

b)



$\circlearrowleft + \Sigma M_{③} = 400.2 - 2.S_{10} = 0$

$S_{10} = 400N$

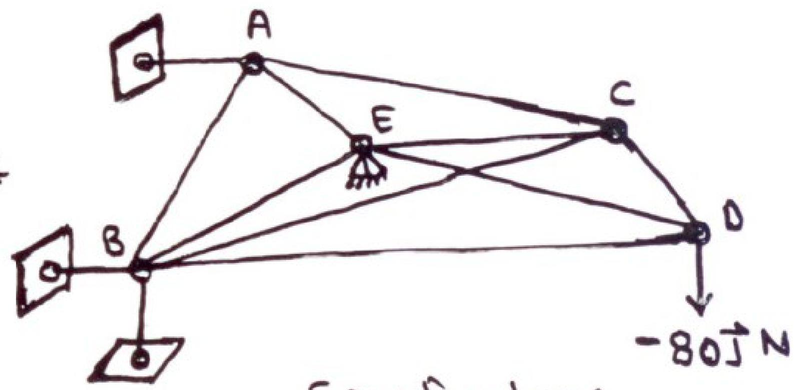
$\circlearrowleft + \Sigma M_{④} = 4.400 - 1200.2 + 2.S_{12} = 0 \rightarrow S_{12} = 400N$

$+\uparrow \Sigma F_y = 0 = -400 + 1200 + S_{11}.\sin\alpha \rightarrow S_{11} = -1131,37N$

Section n-n is passed through the truss so that it intersects the members S_{10}, S_{11}, S_{12}. After the intersected members have been removed, we choose the left-hand portion of the truss as a free body. Three unknown forces are involved; to eliminate the two forces passing through point ③, we write $\Sigma M_{③} = 0$. The sense of S_{10} was chosen assuming this member to be in tension; the positive sign obtained indicates that the member is in tension. For obtaining S_{12}, we write $\Sigma M_{④} = 0$. To determine S_{11}, We write $\Sigma F_y = 0$. The negative sign obtained indicates that the member is in compression.

Problem 2:

Problem 2: The space truss shown consists of 9 members and is supported by a short link at A, two short links at B, and a ball-and-socket at E.



Coordinates:

A(0; 2; -1)^m; B(0; 0; 3)^m

C(3; 0; 0)^m; D(5; -2; 0)

E(0; 0; -4)^m

a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate.

b) Determine the forces in each short link and the reaction at the ball-and-socket joint at E.

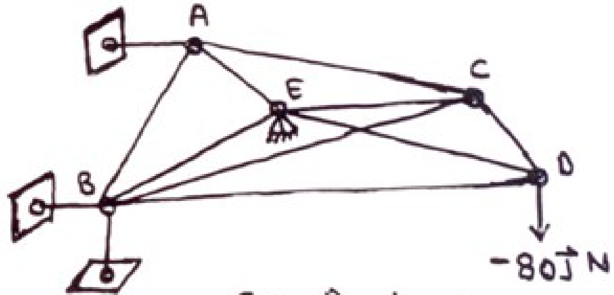
c) Determine the force in the members DC, DE, and DB.

Solution: b) $(2\vec{j}+3\vec{k})\times(-T_A\vec{i})+(7\vec{k})\times(-T_{Bx}\vec{i}-T_{By}\vec{j})+(5\vec{i}-2\vec{j}+4\vec{k})\times(-80\vec{j})=0=\Sigma\vec{M}_E$

$$2T_A\vec{k}-3T_A\vec{j}-7T_{Bx}\vec{j}+7T_{By}\vec{i}-400\vec{k}+320\vec{i}=0$$

$$(2T_A-400)\vec{k}+(-3T_A-7T_{Bx})\vec{j}+(7T_{By}+320)\vec{i}=0$$

$$T_A=200\text{N};\quad T_{Bx}=-85{,}71\text{ N};\quad T_{By}=-45{,}71\text{ N}$$



Coordinates:

A(0 ; 2 ; -1)^m ; B(0 ; 0 ; 3)^m

C(3 ; 0 ; 0)^m ; D(5 ; -2 ; 0)

E(0 ; 0 ; -4)^m

$$\Sigma \vec{F} = 0 = -200\vec{i} + 85{,}71\vec{i} + 45{,}71\vec{j} - 80\vec{j} + E_x\vec{i} + E_y\vec{j} + E_z\vec{k} = 0$$

$E_x = 114{,}29$ N ; $E_y = 34{,}29$ N ; $E_z = 0$

c) $\vec{\lambda}_{DC} = -0{,}71\vec{i} + 0{,}71\vec{j}$; $\vec{\lambda}_{DB} = -0{,}81\vec{i} + 0{,}32\vec{j} + 0{,}49\vec{k}$

$\vec{\lambda}_{DE} = -0{,}75\vec{i} + 0{,}30\vec{j} - 0{,}60\vec{k}$

$$\Sigma \vec{F}_D = 0 = T_{DB}\cdot\vec{\lambda}_{DB} + T_{DC}\cdot\vec{\lambda}_{DC} + T_{DE}\cdot\vec{\lambda}_{DE} - 80\vec{j} = 0$$

$$T_{DB}\cdot(-0{,}81\vec{i} + 0{,}32\vec{j} + 0{,}49\vec{k}) + T_{DC}(-0{,}71\vec{i} + 0{,}71\vec{j}) + T_{DE}(-0{,}75\vec{i} + 0{,}30\vec{j} - 0{,}60\vec{k}) - 80\vec{j} = 0$$

$$(-0{,}71T_{DC} - 0{,}81T_{DB} - 0{,}75T_{DE})\vec{i} + (0{,}71T_{DC} + 0{,}32T_{DB} + 0{,}30T_{DE} - 80)\vec{j} +$$
$$+ (0{,}49T_{DB} - 0{,}60T_{DE})\vec{k} = 0$$

$T_{DC} = 187{,}2$ N ; $T_{DB} = -93{,}6$ N ; $T_{DE} = -76$ N