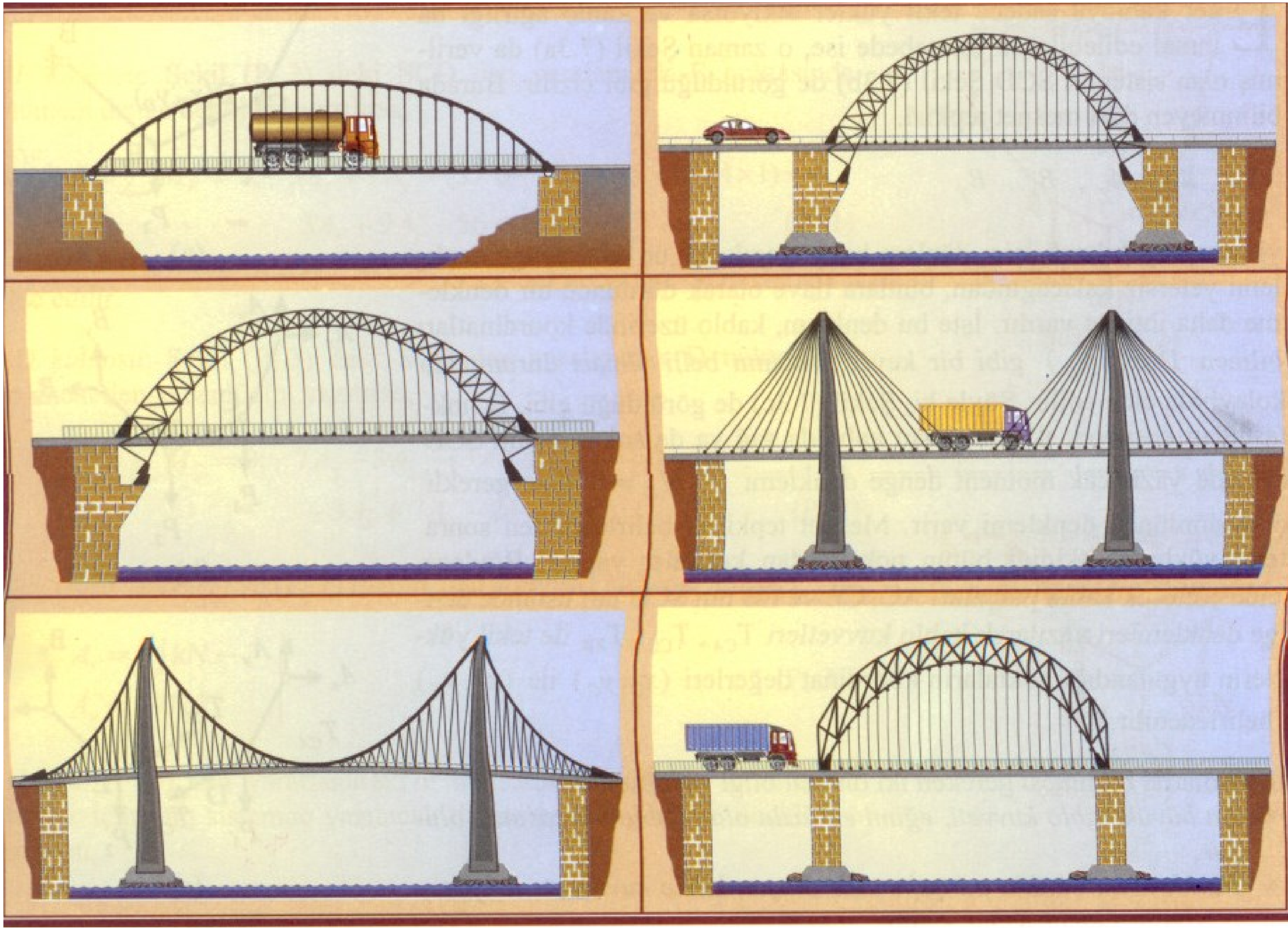
YILDIZ TECHNICAL UNIVERSITY
CIVIL ENGINEERING DEPARTMENT
DIVISION OF MECHANICS

STATICS

CHAPTER EIGHT

CABLES

CABLES



Cables:

- Cables are used in many enginerring applications, such as suspension bridges, transmission lines, aerial transways, guy wires for high towers, etc.

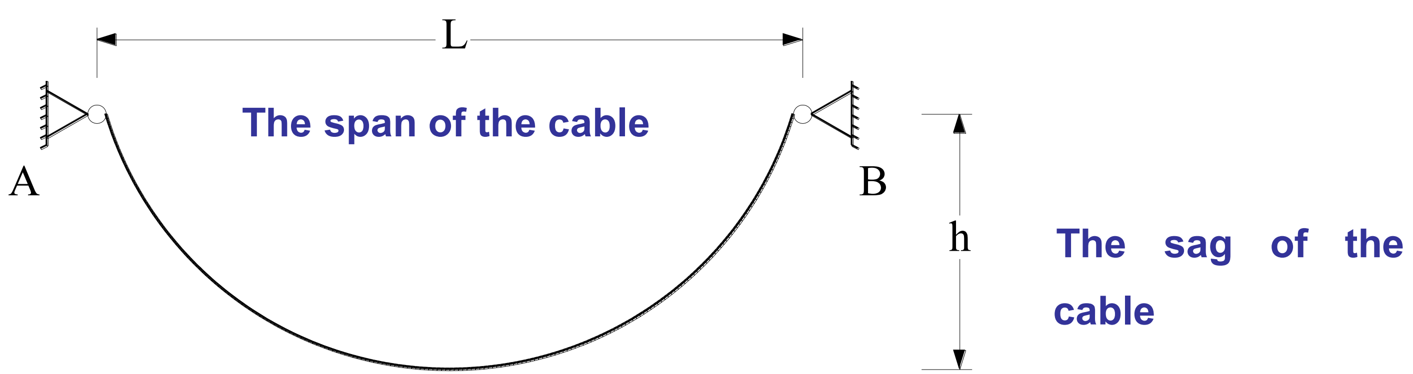
- Due to their flexibilities, the resistance to bending is small and can be neglected. Therefore; cables are considered as a system having infinite numbers of pins joined together.

- According to this assumption, the internal force at any point in the cable reduce to a force of tension directed along the cable.

Cables may be divided into two categories, according to their loading:

1) Cables supporting concentrated loads

2) Cables supporting distributed loads



1. Cables Supporting Concentrated Loads :

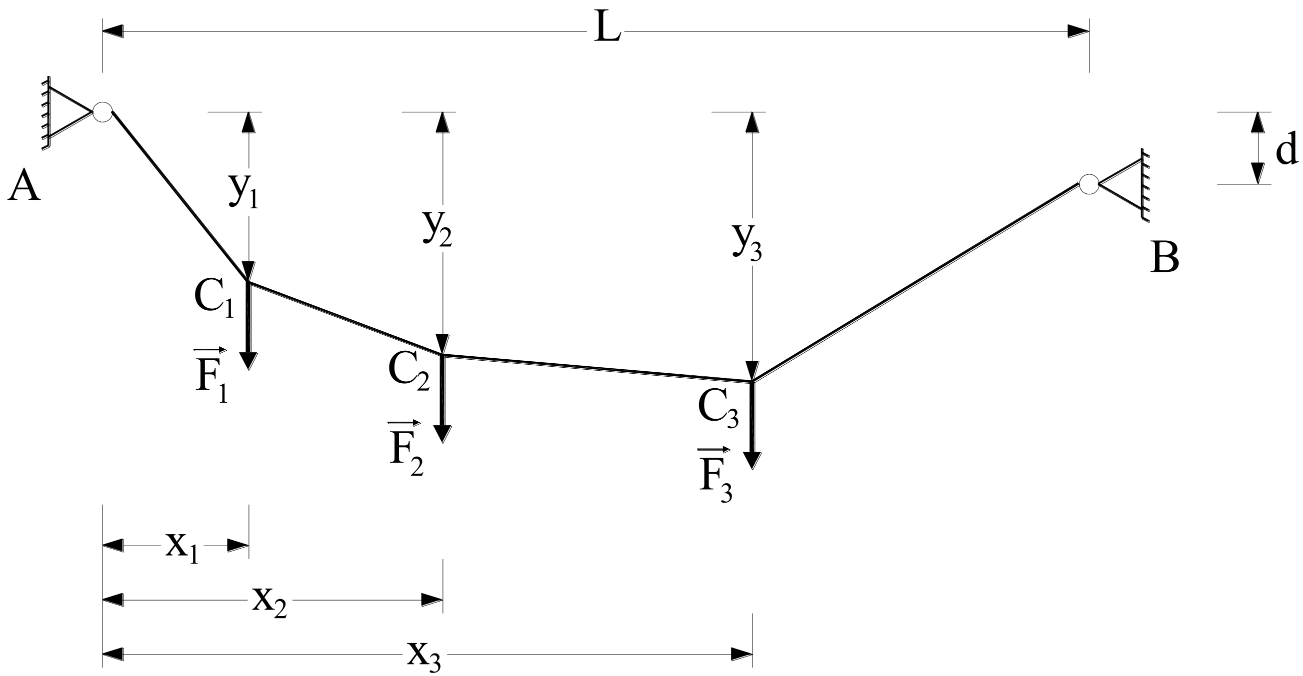


Fig. 8.1 A cable under concentrated loads

The weight of the cable is negligible under the concentrated loads and the loads are applied to the cable in a given vertical line.

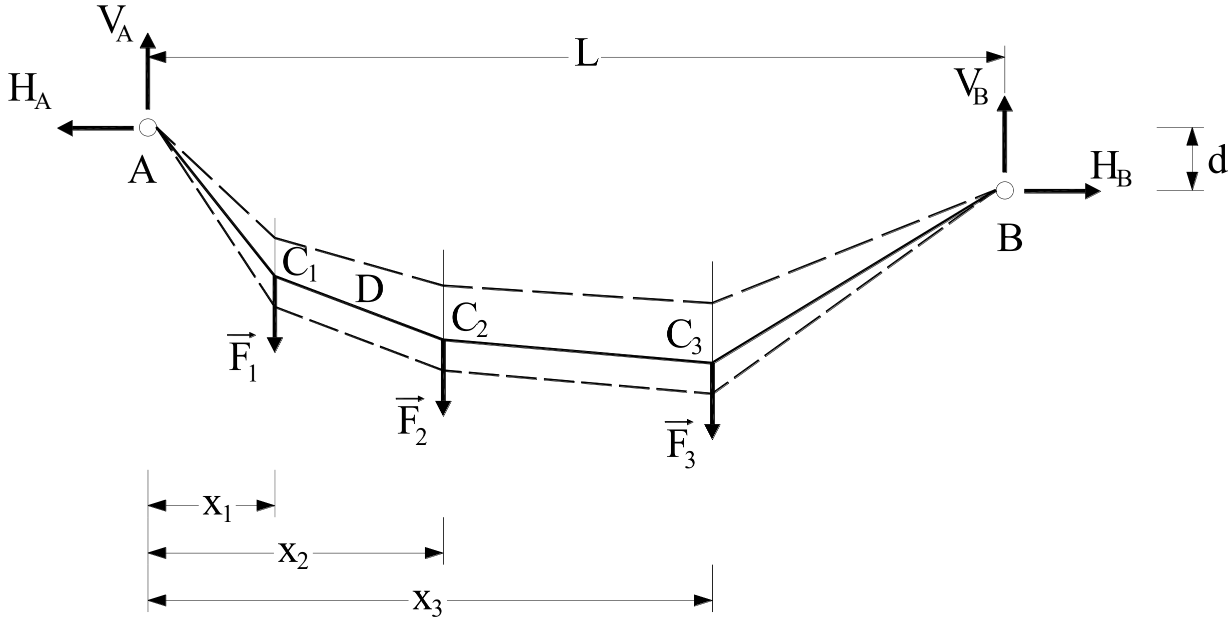


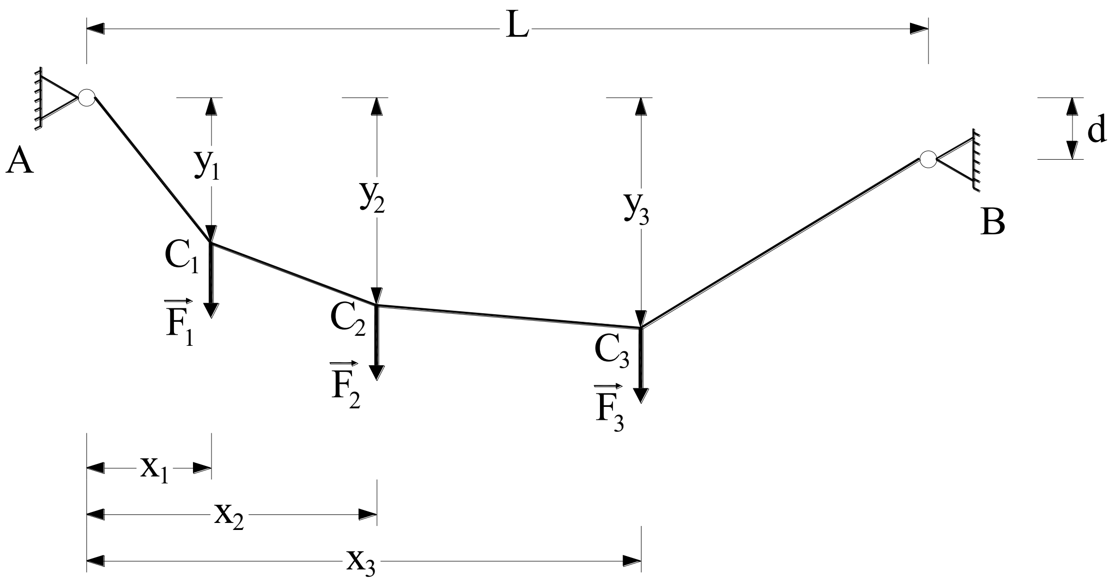
Fig. 8.2 Free-body diagram of a cable under concentrated loads

Support reactions at A and B must be represented by two components each from the free-body diagram of the entire cable.

The three equations of equilibrium are not sufficient to determine the reactions at A and B, thus an additional equation will be required.

If the coordinates of x and y of a point D of the cable is known, the additional equilibrium equation can be written.

The free-body diagram of the AD portion is drawn and the moment equilibirum equation of the left of the right portion of the cable should be equal to zero.



The equilibirum equations for the entire cable:

$$\sum F_x = 0 ,$$

$$\sum F_y = 0 , \qquad \textbf{(1)}$$

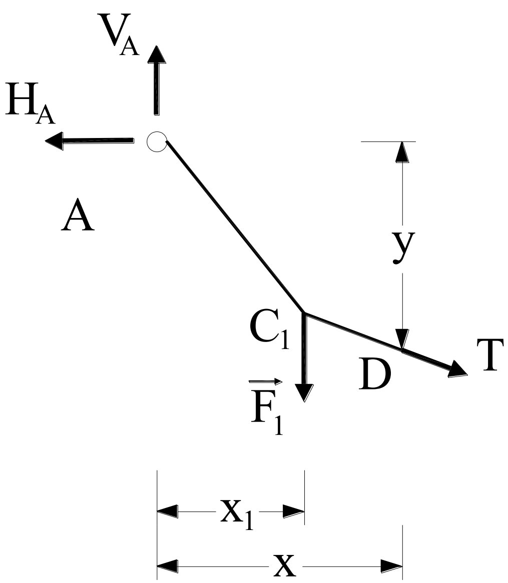
$$\sum M_B = 0 ,$$

Three equations

for the AD portion:

$$\sum M_D = 0 \qquad \textbf{(2)}$$

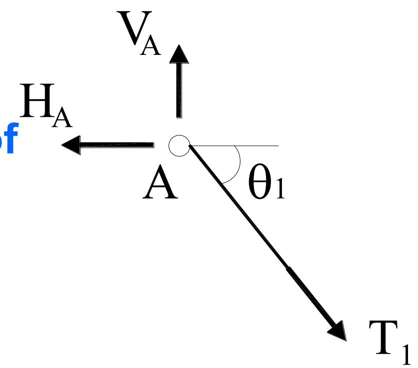
Additional equation



After determining H_A and V_A, the vertical distance from A to a point on the cable can easily be calculated.

The condition that moment about the considered point must be zero is used for this calcualtion.

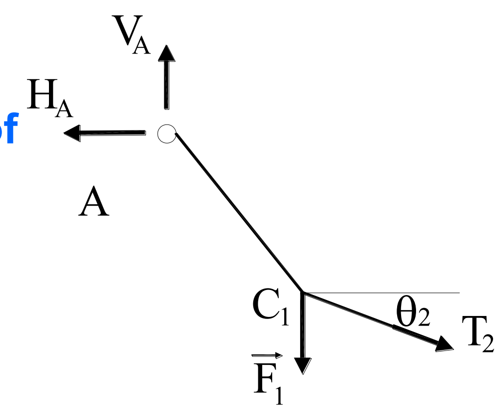
The portion of A-C1 of cable



$$\sum F_x = 0 \rightarrow -H_A + T_1.\cos\theta_1 = 0$$

$$T_1 = \frac{H_A}{\cos\theta_1} \qquad \textbf{(8.3)}$$

The portion of C1-C2 of cable



$$\sum F_x = 0 \rightarrow -H_A + T_2.\cos\theta_2 = 0$$

$$T_2 = \frac{H_A}{\cos\theta_2} \qquad \textbf{(8.4)}$$

From Eqs. (8.3) and (8.4), it is concluded that the cable force at any point of the cable is written as:

$$T_i = \frac{H_A}{\cos\theta_i} \qquad \textbf{(8.5)}$$

subsequently

$$\begin{aligned} \theta_{\max} &\rightarrow T_{\max} \\ \theta_{\min} &\rightarrow T_{\min} \end{aligned} \qquad \textbf{(8.6)}$$

2. Cables Supporting Distributed Loads :

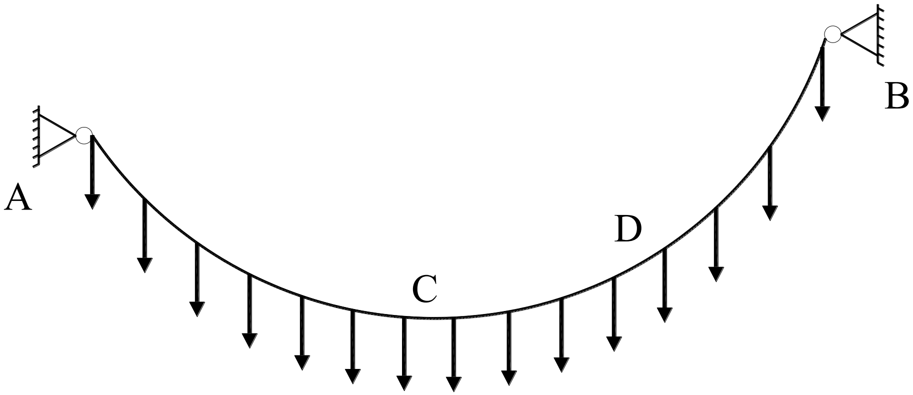
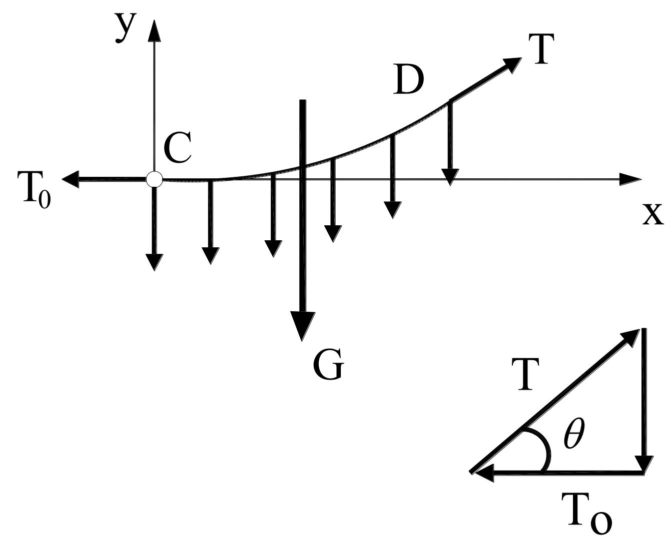


Fig. 8.3 A cable under distributed loads



- Loads are dependent only on the x variable.

- The slope of the cable at the lowest point of it is zero, and the origin of the coordinate axes is chosen at this point.

From the equilibirum of the CD portion of the cable:

$$T = \sqrt{T_o^2 + G^2} \qquad (8.7)$$

Besides;

$$tg\theta = \frac{G}{T_o} \qquad (8.8)$$

By writing the horizontal equilibrium equations for this portion of the cable:

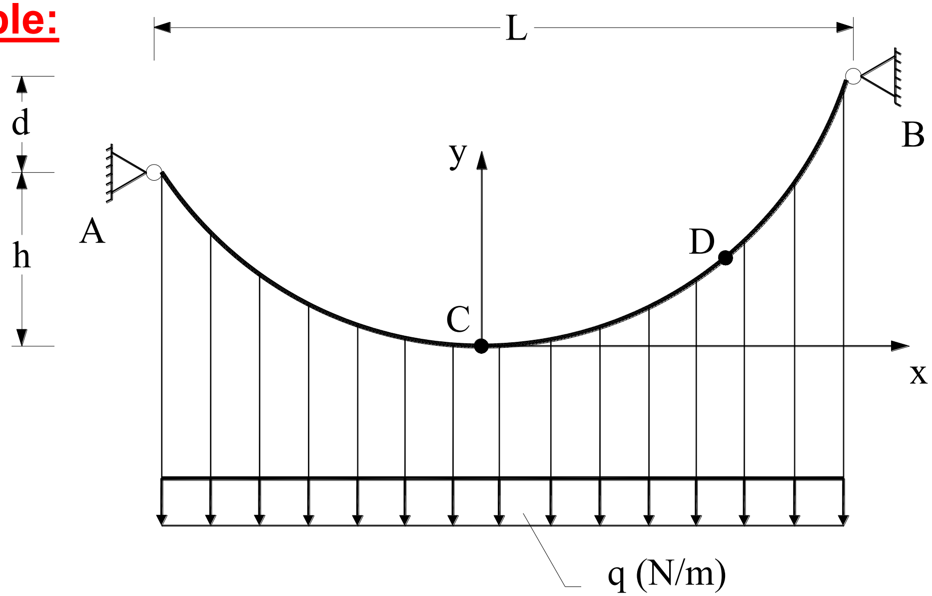
$$\sum F_x = 0 \rightarrow -T_o + T.\cos\theta = 0$$

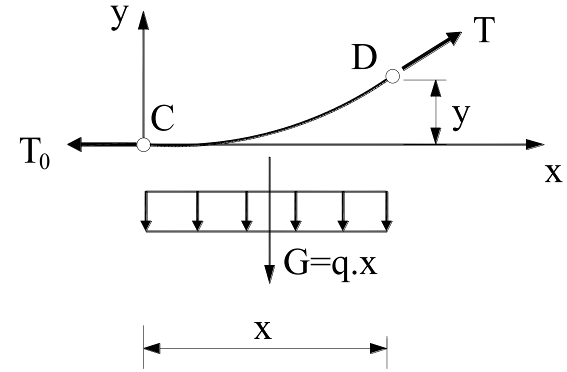
$$T = \frac{T_o}{\cos\theta} \qquad (8.9)$$

By using the vertical equilibrium equation,

$$T = \frac{G}{\sin\theta} \qquad (8.10)$$

Parabolic Cable:





The cable AB carries a load q uniformly distributed along the horizontal line and the weight of the cable is neglected compared with the weight of the roadway. Summing moments about D,

$$\sum M_D = 0 \rightarrow T_o.y - q.x.\frac{x}{2} = 0 \qquad \textbf{(8.11)} \longrightarrow \qquad y = \frac{q.x^2}{2.T_o} \qquad \textbf{(8.12)}$$

From the end conditions:

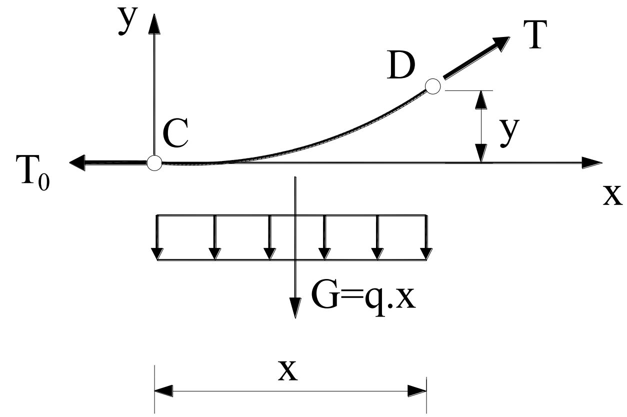
for A(x_A, y_A) $\quad y_A = h = \dfrac{q.x_A^2}{2.T_o}$

for B(x_B, y_B) $\quad y_B = h + d = \dfrac{q.x_B^2}{2.T_o}$

$$\longrightarrow \quad \frac{y_A}{y_B} = \frac{h}{h+d} = \frac{x_A^2}{x_B^2} \qquad L = x_A + x_B$$

$$x_A = x_B\sqrt{\frac{h}{h+d}}$$

The minimum cable force:



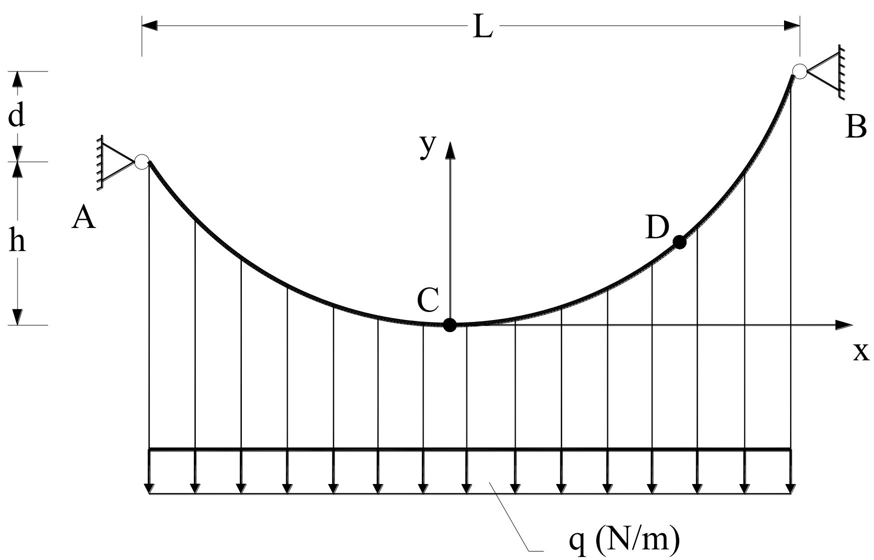
$$T_{\min} = T_o = \frac{q\,x_A^2}{2\,y_A} = \frac{q\,x_B^2}{2\,y_B} \qquad \textbf{(8.13)}$$

For this type of loading:

$$G = q\,x \qquad \textbf{(8.14)}$$

$$T = \sqrt{T_o^2 + q^2\,x^2} \qquad \textbf{(8.15)}$$

$$T_{\max} = \sqrt{T_o^2 + q^2\,x_{\max}^2} \qquad \textbf{(8.16)}$$

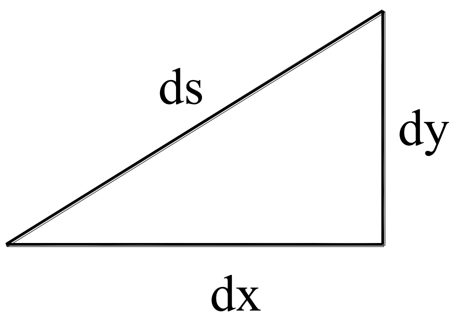


The maximum cable force is occurred at the point that the slope of the cable is the maximum. The maximum slope is where the x coordinate of support is the maximum.

The horizontal component of cable force at any point on the cable is equal to T_0.

Therefore; the horizontal components of the support reactions are equal to T_0.

Finding the Length of a Parabolic Cable:



The length of the arc ds: $$ds^2 = dx^2 + dy^2 \qquad \textbf{(8.17)}$$

$$ds = \sqrt{\frac{dx^2}{dx^2} + \frac{dy^2}{dx^2}}\, dx = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad \textbf{(8.18)}$$

$$y' = \frac{dy}{dx} = \frac{q.x}{T_o} \longrightarrow y = \frac{q.x^2}{2.T_o}$$ **using** $$ds = \sqrt{1+(y')^2}\, dx = \sqrt{1 + \left(\frac{q.x}{T_o}\right)^2}\, dx \qquad \textbf{(8.19)}$$

The length of the cable from its lowest point C to its support A can be obtained from the folllowing formula:

$$\int_0^{S_A} ds = \int_0^{x_A} \sqrt{1 + \left(\frac{q.x}{T_o}\right)^2}\, dx \qquad \textbf{(8.20)}$$

By using the binomial theorem to expand the radical in an infinite series,

$$s_A = \int_0^{x_A} \left(1 + \frac{q^2 x^2}{2.T_o^2} - \frac{q^4 x^4}{8.T_o^4} + \ldots\ldots\right) \mathrm{dx} \qquad \textbf{(8. 21)} \longrightarrow s_A = x_A \left[1 + \frac{q^2 x_A^2}{6T_o^2} - \frac{q^4 x_A^4}{40T_o^4} + \ldots\ldots\ldots\right] \qquad \textbf{(8.22)}$$

from the end conditions: $y_A = \dfrac{q\,x_A^2}{2T_o}$

$$s_A = x_A\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_A}{x_A}\right)^4 + ...\right] \qquad \textbf{(8.23)}$$

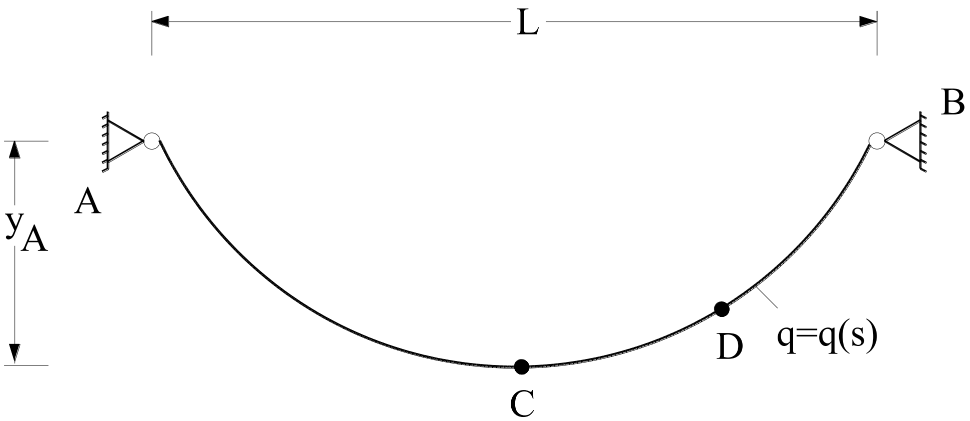
can be found.

In most cases, only the first two terms of the series need to be computed for the total cable length.

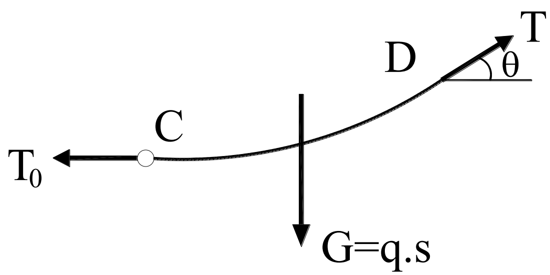
For calculating the total cable length, s_B must be calculated. This length can be found by following the same procedure used for finding s_A. The total cable length is found as

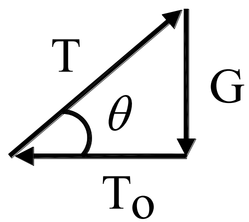
$$s = s_A + s_B \qquad \textbf{(8.24)}$$

Catenary:



Catenary



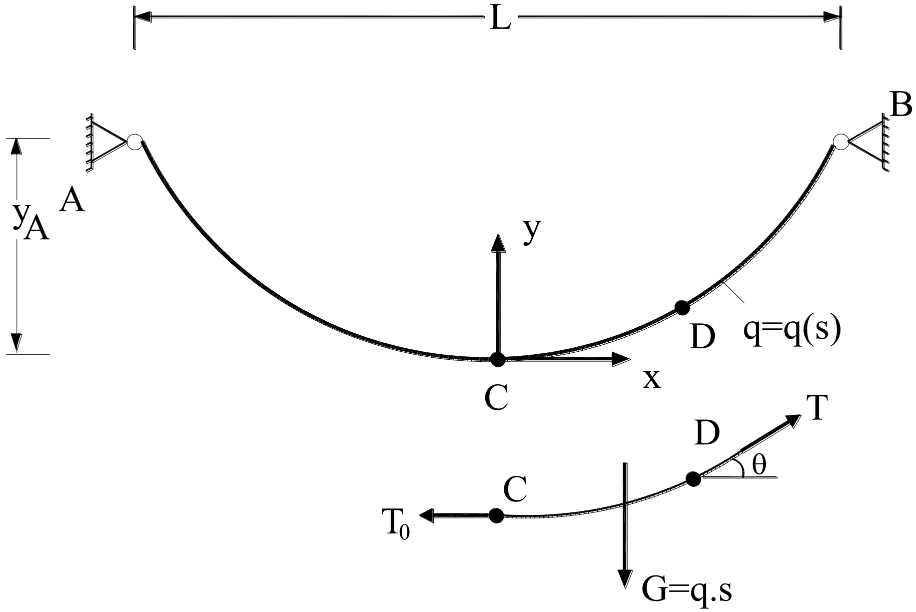


The load of a cable hanging under its own weight is a uniformly distributed load (q) along the cable itself.

a) The support elevations are the same

b) The support elevations are different

a) The support elevations are the same:



The equation of the cable:

$$y = e\left[\cosh\left(\frac{x}{e}\right) - 1\right] \qquad \textbf{(8.25)}$$

$$\frac{y_A}{e} + 1 = \cosh\left(\frac{x_A}{e}\right)$$

e **is found by trial-error method.**

The minimum cable force:

$$\frac{T_o}{q} = e \rightarrow T_o = q\,e \qquad \textbf{(8.26)}$$

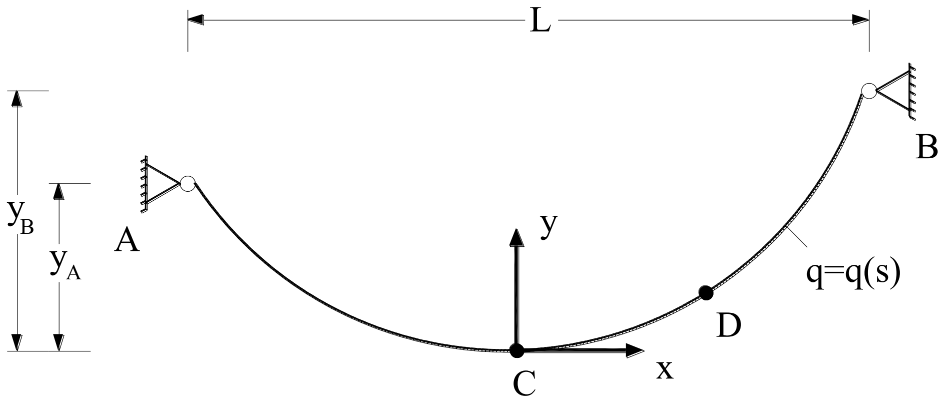
The cable force at any point of the cable starting from the lowest point on the cable (y vertical distance):

$$T = T_o + q\,y \qquad \textbf{(8.27)}$$

The cable length between the lowest point and any point on the cable:

$$s_x = \frac{T_o}{q}\sinh\left(\frac{q\,x}{T_o}\right) \qquad \textbf{(8.28)}$$

b) The support elevations are different:



The equation of the cable:

$$y = e\left[\cosh\left(\frac{x}{e}\right) - 1\right] \qquad \textbf{(8.29)}$$

When the elevations of the supports A and B are different, x_A and x_B are not definite. In this situation, by substituting the coordinates of A and B into Eq. (8.29), the following expressions can be found:

$$\frac{x_A}{e} = \cosh^{-1}\left(\frac{y_A + e}{e}\right) \quad ; \quad \frac{x_B}{e} = \cosh^{-1}\left(\frac{y_B + e}{e}\right) \qquad \textbf{(8.30)}$$

$$L = x_A + x_B \qquad \textbf{(8.31)}$$

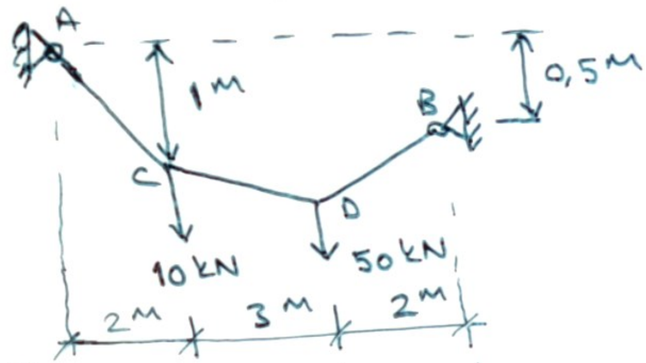
$$\frac{L}{e} = \cosh^{-1}\left(\frac{y_B + e}{e}\right) + \cosh^{-1}\left(\frac{y_A + e}{e}\right) \qquad \textbf{(8.32)} \longrightarrow$$

e **is found by trial-error method.**

$$\frac{T_o}{q} = e \qquad T = T_o + q\,y \qquad s_x = \frac{T_o}{q}\sinh\left(\frac{q\,x}{T_o}\right) \qquad \textbf{(8.33)}$$

Problem 1:

Sample Problem: The cable AB supports two vertical loads from the points indicated. If point C is 1 m below the left support, determine (a) the elevation of point D, (b) the maximum slope and the maximum tension in the cable.



Free Body: Entire Cable

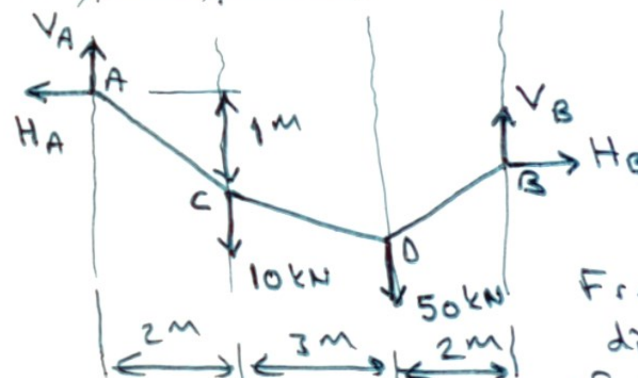
$$\overset{+\curvearrowright}{\sum M_B} = 0 = 7 \cdot V_A - 0{,}5 \cdot H_A - 5 \cdot 10 - 2 \cdot 50 \quad (1)$$

Free Body: Part AC

$$\overset{+\curvearrowright}{\sum M_C} = 0 = 2 \cdot V_A - 1 \cdot H_A \quad (2)$$

Solving the two equations simultaneously, we obtain

$$\boxed{V_A = 25 \text{ kN} \;;\; H_A = 50 \text{ kN}}$$



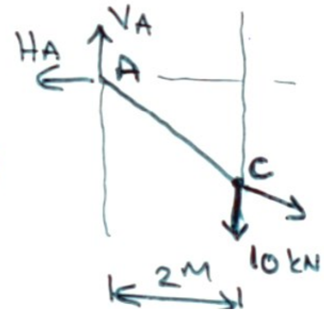
Free body diagram of the entire cable

(a) Elevation of point D:

Free body: Part ACD

$$\overset{+\curvearrowright}{\sum M_D} = 0 = -y_D \cdot 50 + 5 \cdot 25 - 3 \cdot 10 = 0 \quad (3)$$

from (3) $\boxed{y_D = 1{,}9 \text{ m}}$

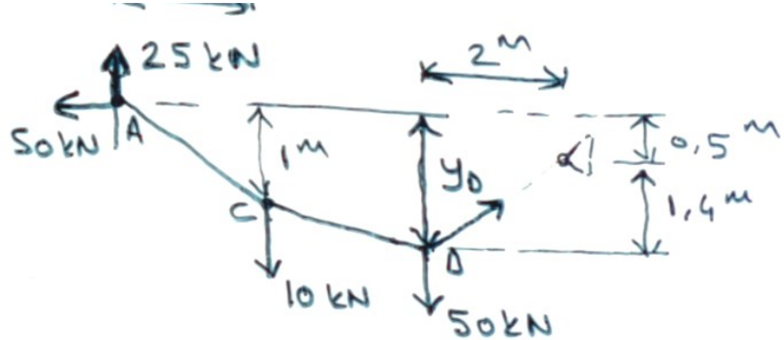


Free body diagram of the cable part AC

(b) Maximum sl

Problem 1:

…aximum slope and maximum tension: We observe that the maximum slope occurs in portion DB. Since the horizontal component of the tension is constant and equal to 50 kN, we write



$$\tan(\theta_{max}) = \frac{1.4}{2} \rightarrow \theta_{max} = 35°$$

$$\boxed{T_{max} = \frac{H_A}{\cos(\theta_{max})} = 61{,}0387 \text{ kN}} = \sqrt{V_B^2 + H_B^2}$$

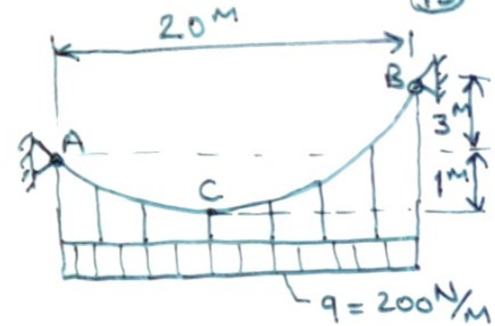
Also, if it is desired V_B and H_B can be obtained:

Free Body: entire cable:

$$\overset{+}{\rightarrow} \Sigma F_x = 0 = -50 + H_B = 0 \rightarrow \boxed{H_B = 50 \text{ kN}} \quad +\uparrow \Sigma F_y = 0 = 25 - 10 - 50 + V_B = 0 \rightarrow \boxed{V_B = 35 \text{ kN}}$$

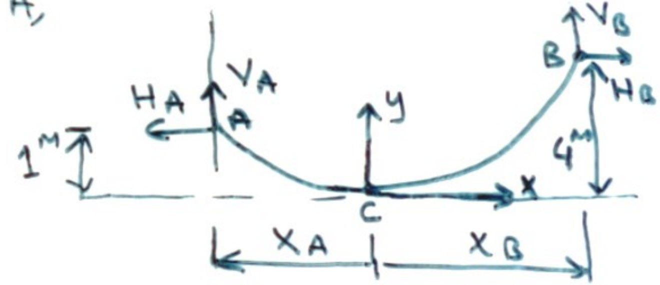
Problem 2:

Sample Problem: Cable AB supports a load uniformly distributed along the horizontal as shown. Knowing that the lowest point of the cable is located at a distance 1 m below A, determine



(a) the support reactions V_A, H_A, V_B, H_B,

(b) the maximum tension in the cable



(c) the angle θ_B that the cable forms with the horizontal at B.

Solution:
$$y = \frac{q x^2}{2 T_0} \qquad (8.11)$$

$$y_A = 1 = \frac{200 \cdot x_A^2}{2 T_0} \quad ① \; ; \qquad y_B = 4 = \frac{200 \cdot x_B^2}{2 T_0} \quad ②$$

from Eqs. ① and ② $\quad \frac{y_A}{y_B} = \frac{1}{4} = \frac{x_A^2}{x_B^2} \rightarrow x_B = \sqrt{4 x_A^2} = 2 x_A \quad ③$

$$\ell = 20^m = x_A + x_B \quad ④ \quad \rightarrow \quad 20 = x_A + 2x_A \rightarrow \boxed{\begin{array}{l} x_A = 6{,}67^m \\ x_B = 13{,}33^m \end{array}}$$

(a) Free bod

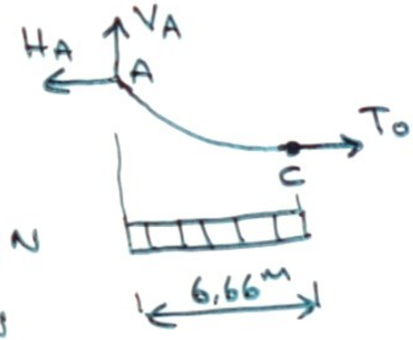
Problem 2:

$\boxed{x_B = 13{,}33^m}$

(a) Free body diagram: portion AC

From (8.11)

$$T_0 = \frac{200 \cdot 6{,}67^2}{2 \cdot 1} = 4444{,}44\ N$$



Free body of the portion AC

$+\uparrow \sum F_y = 0 = V_A - 6{,}667 \cdot 200 = 0 \rightarrow V_A = 1333{,}33\ N$

$\overset{+}{\rightarrow} \sum F_x = 0 = -H_A + T_0 = 0 \rightarrow H_A = 4444{,}44\ N$

From the equilibrium equations for the entire cable

$\overset{+}{\rightarrow} \sum F_x = 0 = -H_A + H_B = 0 \rightarrow H_B = 4444{,}44\ N$

$+\uparrow \sum F_y = 0 = 1333{,}33 - 20 \cdot 200 + V_B = 0 \rightarrow V_B = 2666{,}667\ N$

(b) $T_{max} = \sqrt{T_0^2 + q^2 \cdot x_{max}^2} = \sqrt{4444{,}44^2 + 200^2 \cdot 13{,}333^2} = 5183{,}07\ N$

(c) $tg\,\theta = y' = \dfrac{qx}{T_0}$; $tg\,\theta_B = \dfrac{200 \cdot 13{,}333}{4444{,}44} = 0{,}6 \rightarrow \theta_B = 30{,}96^\circ$

Also, the cable length is

$$S = S_A + S_B = 6{,}667\left[1 + \frac{2}{3}\left(\frac{1}{6{,}667}\right)^2 - \frac{2}{5}\left(\frac{1}{6{,}667}\right)^4\right] + 13{,}33\left[1 + \frac{2}{3}\left(\frac{4}{13{,}33}\right)^2 - \frac{2}{5}\left(\frac{4}{13{,}33}\right)^4\right] = 20{,}8558^m$$

$S = 20{,}8558^m$

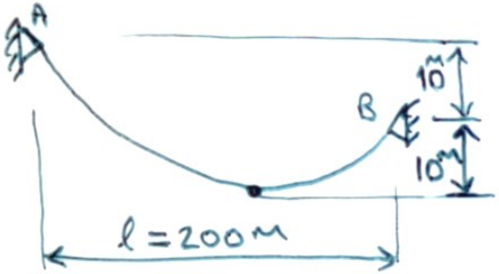
Problem 3:

Sample Problem: The cable AB has a weight per unit length of 0,5 kN/m. Knowing that the lowest point of the cable is located at a distance 2m below the support A, determine

(a) the maximum and minimum tension in the cable,

(b) the location of the lowest point

(c) the cable length s.



Solution: From Eq. (8.53)

$$\frac{l}{e} = \cosh^{-1}\left(\frac{y_B + e}{e}\right) + \cosh^{-1}\left(\frac{y_A + e}{e}\right) \qquad (8.53)$$

Lets assume e as 300 for the first step

$$\frac{200}{300} = \cosh^{-1}\left(\frac{10+300}{300}\right) + \cosh^{-1}\left(\frac{20+300}{300}\right) \rightarrow 0{,}666 \neq 0{,}6206$$

Lets assume e as 350 for the second step

$$\frac{200}{350} = \cosh^{-1}\left(\frac{10+350}{350}\right) + \cosh^{-1}\left(\frac{20+350}{350}\right) \rightarrow 0{,}571 \neq 0{,}575$$

Problem 3:

Lets assume e as 345,8 for the third step

$$\frac{200}{345,8} = \cosh^{-1}\left(\frac{10+345,8}{345,8}\right) + \cosh^{-1}\left(\frac{20+345,8}{345,8}\right) \rightarrow 0,5784 = 0,5784$$ ok.

(a) $T_o = q \cdot e = 0,5 \cdot 345,8 = 172,9 \text{ kN} = T_{min}$

$T_A = T_o + q \cdot y_A = 172,9 + 0,5 \cdot 20 = 182,9 \text{ kN} = T_{max}$

(b) $\frac{x_A}{e} = \cosh^{-1}\left(\frac{20+345,8}{345,8}\right) = 0,33849 \rightarrow \boxed{x_A = 117,05\text{ m};\ x_B = 82,95\text{ m}}$

(c) $S = S_A + S_B = \frac{T_o}{q}\sinh\left(\frac{q x_A}{T_o}\right) + \frac{T_o}{q}\sinh\left(\frac{q x_B}{T_o}\right)$

$$S = \frac{172,9}{0,5} \cdot \sinh\left(\frac{0,5 \cdot 117,05}{172,9}\right) + \frac{172,9}{0,5} \cdot \sinh\left(\frac{0,5 \cdot 82,95}{172,9}\right) = 203,05\text{ m}$$