Physics-1 Recitation-7

Rotation of a Rigid Object About a Fixed Axis

1. The angular position of a point on a wheel is described by $\Theta = 5 + 10t + 2t^2\ (rad)$.

a) Determine angular position, angular speed, and angular acceleration of the point at t=0 and t=3s.

b) If the distance between the point and center of wheel is 0.5 m; for t=3s, determine magnitude of linear speed, linear acceleration, radial acceleration and find magnitude of total acceleration.

1-a) $\theta = 5 + 10t + 2t^2$ (rad)

$t=0 \longrightarrow \theta_1 = 5 + 10(0) + 2(0)^2 = 5$ rad

$t=3s \longrightarrow \theta_2 = 5 + 10(3) + 2(3)^2 = 53$ rad

$$\omega\Big|_{t=0} = \frac{d\theta}{dt}\Big|_{t=0} = 10 + 4t\Big|_{t=0} = 10 \text{ rad/s}$$

$$\omega\Big|_{t=3} = 22 \text{ rad/s}$$

$$\alpha\Big|_{t=0} = \frac{d\omega}{dt}\Big|_{t=0} = 4 \text{ rad/s}^2 \quad ; \quad \alpha\Big|_{t=3} = 4 \text{ rad/s}^2$$

b) $t = 3s$, $r = 0,5m$

$v = r\omega = 0,5 \times 22 = 11$ m/s

$a_r = r\omega^2 = 0,5 \times 22^2 = 242$ m/s^2

$a_t = r\alpha = 0,5 \times 4 = 2$ m/s^2

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{242^2 + 2^2} \cong 242 \text{ m/s}^2$$

2. A small object with mass 4.00 kg moves counterclockwise with constant speed 4.50 m/s in a circle of radius 3.00 m centered at the origin

a) It started at the point with cartesian coordinates (3m, 0). When its angular displacement is 9.00 rad, what is its position vector, int cartesian unit vector notation?

b) In what quadrant is the particle located, and what angle does its position vector make with the positive x axis?

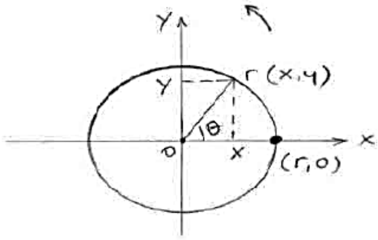
c) What is its velocity vector, in unit vector notation?

d) In what direction is it moving? Make a sketch of the position and velocity vectors

e) What is its acceleration, expressed in unit-vector notation?

f) What total force acts on the object? (Express your answer in unit vector notation)

2.



$V = 4{,}5$ m/s
$m = 4$ kg
$r = 3$ m

$x = r\cos\theta$, $y = r\sin\theta$

$360^\circ \rightarrow 2\pi$ radyan $\rightarrow \theta\,(\text{derece}) = \frac{360}{2\pi} \cdot \theta\,(\text{radyan})$

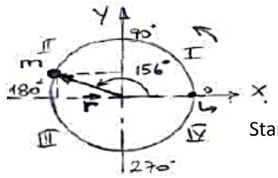
$\theta = 9$ radyan $\Rightarrow \theta\,(\text{derece}) = \frac{360}{2\pi} \cdot 9$ radyan $\cong 516^\circ$

$x = r\cos\theta = 3 \cdot \cos 516 = -2{,}74$ m
$y = r\sin\theta = 3 \cdot \sin 516 = 1{,}22$ m

a) $\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} = (-2{,}74\,\vec{i} + 1{,}22\,\vec{j})$ m

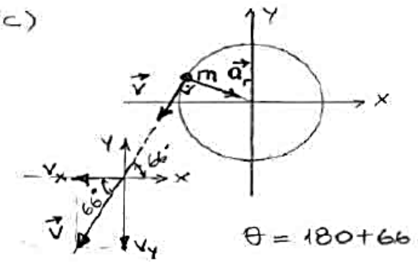
b) Object m makes second tour:

$\theta = (516 - 360) = 156^\circ$, 2^{nd} quarter



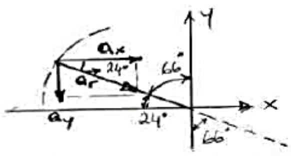
Starting point

c)



$V_x = V \cdot \cos 66 = 4{,}5 \cdot \cos 66$
$V_x = 1{,}83$ m/s
$V_y = V \cdot \sin 66 = 4{,}11$ m/s
$\vec{V} = (-1{,}83\,\vec{i} - 4{,}11\,\vec{j})$ m/s

$\theta = 180 + 66 = \underline{\underline{246^\circ}}$



$a_x = a \cdot \cos 24$
$= 6{,}75 \cdot \cos 24$
$a_x = 6{,}16$ m/s^2

d) $a = \frac{V^2}{r} = \frac{4{,}5^2}{3} = 6{,}75$ m/s^2

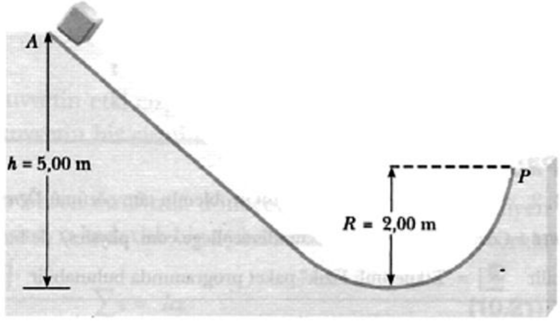
$\vec{a}_r = a_x\vec{i} + a_y\vec{j} = (6{,}16\,\vec{i} - 2{,}74\,\vec{j})$ m/s^2

$\theta = 270 + 66$
$\theta = \underline{\underline{336^\circ}}$

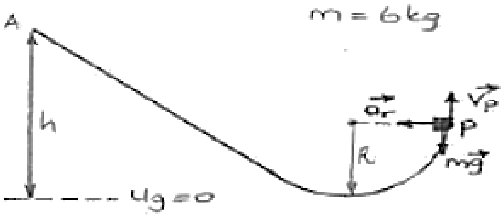
$a_y = a \cdot \sin 24 = 2{,}74$ m/s^2

e) $\Sigma\vec{F} = m \cdot \vec{a} = 4(6{,}16\,\vec{i} - 2{,}74\,\vec{j}) = (24{,}6\,\vec{i} - 11\,\vec{j})$ N

3. A 6.0 kg block is released from A on the frictionless track shown in Figure. Determine the radial and tangential components of acceleration for the block at P.



$h = 5\ m$
$R = 2\ m$
$m = 6\ kg$



$\mu = 0 \Rightarrow \Delta E = 0$

$E_A = E_P$

$\cancel{K_A} + U_A = K_P + U_P$

$mgh = \frac{1}{2} m V_P^2 + mgR$

$V_P^2 = 2g(h-R) = 60\ m^2/s^2$

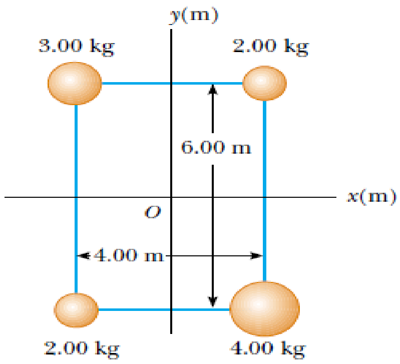
Centripetal acceleration at point P: $a_r = \frac{V_P^2}{R} = \frac{60}{2} = 30\ m/s^2$

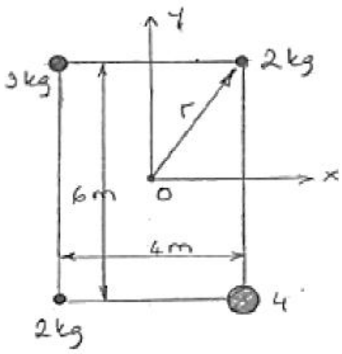
Tangential acceleration at point P: $a_t = g = 10\ m/s^2$

4. The four particles in Figure are connected by rigid rods of neglibigle mass. The origin is at the center of the rectangle. If the system rotates in the xy plane about the z axis with an angular speed of 6.00 rad/s, calculate

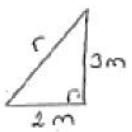
a) The moment of inertia of the system about the z axis and

b) The rotational energy of the system.





a) The distances of the masses to the rotation axis is same. $r_1 = r_2 = r_3 = r_4 = r$



$r = \sqrt{2^2 + 3^2} = \sqrt{13}\ m$

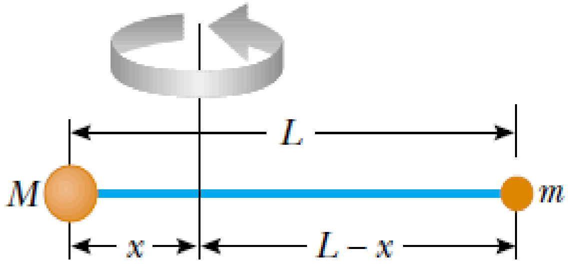
$I_{sistem} = \sum m_i \cdot r_i^2$ (bağlantı çubuklarının kütleleri ihmal ediliyor)

$= r^2 (m_1 + m_2 + m_3 + m_4) = 13(2 + 2 + 3 + 4)$

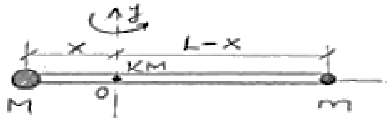
$= \underline{143}\ kg m^2$ The moment of inertia of the system about the z axis.

b) $K_d = \frac{1}{2} I w^2 = \frac{1}{2} I w^2 = \frac{1}{2} (143\ kgm^2)(6\ rad/s^2) = 2574$ Joule

5. Two mases M and m are connected by a rigid rod of length L and of negligible mass, as shown in Figure. For an axis perpendicular to the rod, Show that the system has the minimum moment of inertia when the axis passes through the center of mass. Show that this moment of inertia is $I = \mu L^2$, where $\mu = mM/(m+M)$.



5.



System= (M+m+rod)

$$I_{sistem} = \sum m_i \cdot r_i^2$$

$$I_{km} = M \cdot x^2 + m(L-x)^2 \rightarrow$$ Moment of intertia with respect to the center of mass of the system

Requirement for min. $\Rightarrow \frac{dI}{dx} = 0$ olmalı

$$\Rightarrow \frac{dI}{dx} = \frac{d}{dx}\left[Mx^2 + m(L-x)^2\right] = 0$$

$$= M \cdot 2x + 2m(L-x)(-1) = 0$$

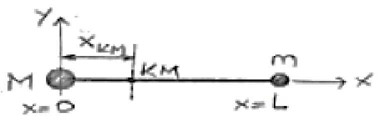
$$2Mx = 2m(L-x) \rightarrow \boxed{x = \frac{mL}{M+m}}$$

* We can find the coordinates center of mass of the system to check our result:

$$y_{km} = 0$$

$$x_{km} = \frac{\sum m_i \cdot x_i}{\sum M_i} = \frac{M \cdot 0 + m \cdot L}{m+M}$$

$$x_{km} = \frac{mL}{m+M}$$



$$KM\left[\frac{mL}{m+M}, 0\right]$$

Böylece $$I_y = M\left(\frac{mL}{M+m}\right)^2 + m\left[L - \frac{mL}{M+m}\right]^2$$

$$I_y = \underbrace{\left(\frac{Mm}{M+m}\right)}_{\mu} L^2$$ elde edilir.

$$I_y = \mu L^2$$

6. A uniform thin metal plate has width a, length b and mass m_1.

a) Find the moment of inertia about the corner of the plate which is perpendicular to the z-axis.

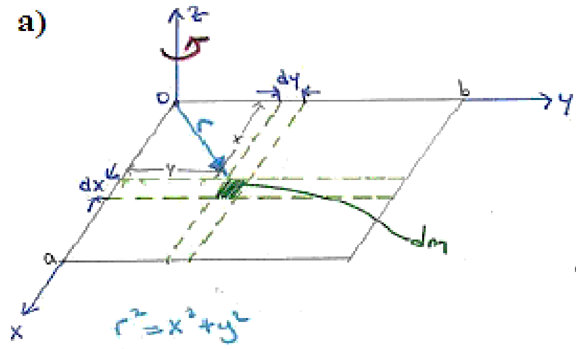
b) Using the results of part a) , Find the moment of inertia about the center of the plate which is perpendicular to the axis.

6. $I = \int r^2 dm$

$\sigma = \frac{M}{A} = \frac{M}{a.b}$

a)

$\sigma = \frac{dm}{dA}$; $dm = \sigma dA = \sigma dx dy$



$r^2 = x^2 + y^2$

$$\boxed{I_o = \int r^2 dm = \iint (x^2+y^2)\sigma dx dy}$$

$$I_o = \sigma \int_0^a \int_0^b (x^2+y^2) dx dy$$

$$I_o = \sigma \int_0^a dx \int_0^b (x^2+y^2) dy$$

$$I_o = \sigma \int_0^a dx \left[\left(x^2 y + \frac{y^3}{3} \right) \Big|_0^b \right]$$

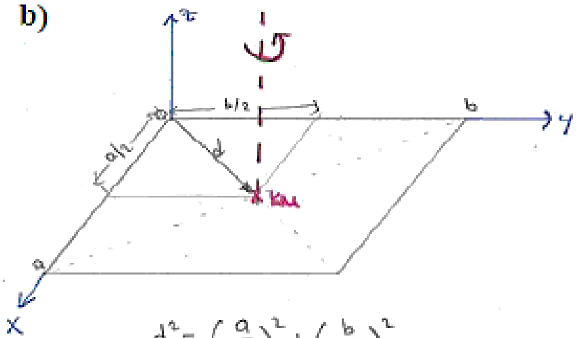
$$I_o = \sigma \int_0^a dx \left(x^2 b + \frac{b^3}{3} \right) = \sigma \left(\frac{x^3}{3} b + x \frac{b^3}{3} \right) \Big|_0^a$$

$$I_o = \underbrace{\sigma}_{\frac{M}{a.b}} \left(\frac{a^3 b}{3} + \frac{a b^3}{3} \right) = \frac{M}{ab} \cdot \frac{ab}{3} (a^2+b^2) = \boxed{\frac{M}{3}(a^2+b^2)}$$

Parallel Axis Theorem:

b)

$$I_o = I_{km} + Md^2$$



$$I_{km} = I_o - Md^2 = \frac{M}{3}(a^2+b^2) - Md^2$$

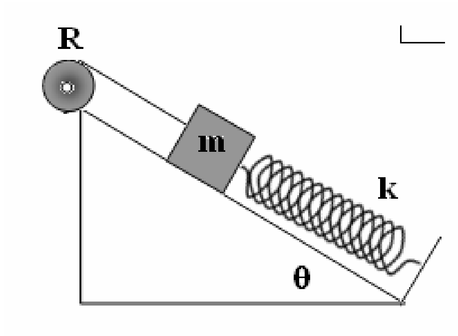
$$I_{KM} = \frac{M}{3}(a^2+b^2) - M\left[\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 \right]$$

$$I_{KM} = \frac{M}{3}(a^2+b^2) - \frac{M}{4}(a^2+b^2)$$

$d^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$

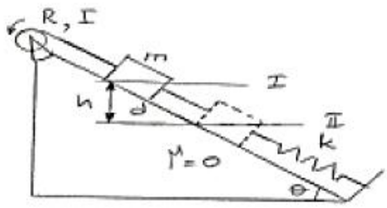
$$\boxed{I_{KM} = \frac{M}{12}(a^2+b^2)}$$

7. The pulley shown in Figure has radius R and moment of inertia I. One end of the mass m is connected to a spring of force constant k, and the other end is fastened to a cord wrapped around the pulley. The pulley axle and the incline are frictionless. If the pulley is wound conterclockwise so that the spring is stretched a distance d from its unstretched position and is then released from rest, find



a) The angular speed of the pulley when the spring is again unstretched and
b) A numerical value for the angular speed at this point if $I=1.00\ kgm^2$, $R=0.300m$, $k=50.0N/m$, $m=0.500kg$, $d=0.200m$, and $\Theta=37^0$.

7.



$\sin\theta = \frac{h}{d}$

$h = d\cdot\sin\theta$

No friction $,\ \mu=0 \Rightarrow \Delta E=0 \Rightarrow E_1=E_2$

a)

$$E_1 = E_2$$
$$\Sigma U_1 + K_1 = \Sigma U_2 + K_2$$
$$(U_{yay}+U_g)_1 + \underbrace{\cancel{K_1}}_{0} = (\underbrace{\cancel{U_{yay}}}_{0} + \underbrace{\cancel{U_g}}_{0})_2 + (\underbrace{K_{d\ddot{o}nme}}_{(makara)} + \underbrace{K_{\ddot{o}teleme}}_{(blok)})_2$$

$$\frac{1}{2}kd^2 + mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \quad , \quad v=\omega\cdot R$$

$$\frac{1}{2}kd^2 + mgd\sin\theta = \frac{1}{2}I\omega^2 + \frac{1}{2}m\omega^2R^2$$

$$\frac{1}{2}kd^2 + mgd\sin\theta = \frac{1}{2}\omega^2(I+mR^2)$$

$$\omega = \sqrt{\frac{kd^2 + 2mgd\sin\theta}{(I+mR^2)}}$$

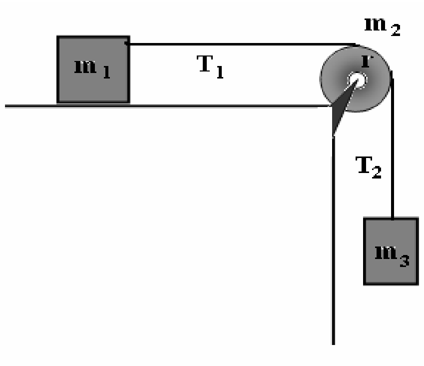
b) $$\omega = \sqrt{\frac{50\cdot(0{,}2)^2 + 2(0{,}5)\cdot 10\cdot(0{,}2)\cdot\sin 37}{(1+0{,}5\cdot 0{,}3^2)}} = \sqrt{3{,}06} = 1{,}75\ rad/s$$

8. A mass m_1 and m_3 are suspended by a string of negligible mass passing over a pulley of Radius r and moment of inertia $I = \frac{1}{2} m_2 r^2$. The pulley and the table are frictionless.

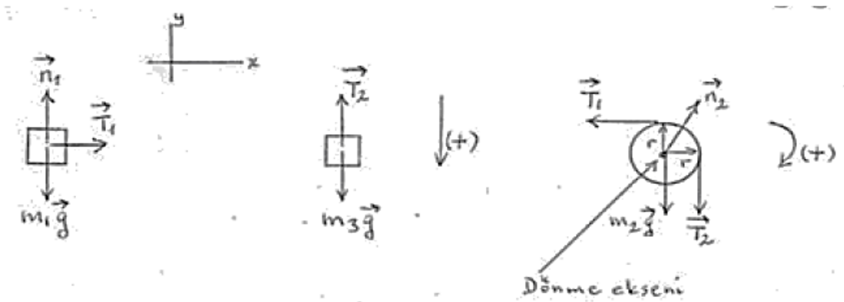
a) Determine the acceleration of the system,

b) The tension T_1 and T_2 in the string.

(m_1=0.15 kg, m_2=0.10 kg, m_3=0.10 kg, r=0.10 kg, g=10 m/s^2)



8. Free body diagrams of two masses and the reel:



Equations of motions for the masses and the reel

$$T_1 = m_1 a \qquad (1)$$

$$m_3 g - T_2 = m_3 a \qquad (2)$$

$$T_2 \cdot r - T_1 r = I\alpha \qquad (3)$$

$$(T_2 - T_1) r = I \frac{a}{r} = \frac{1}{2} m_2 r^2 \left(\frac{a}{r}\right)$$

According to the right hand rule, the net torque is directed into the page. As a result of this, the direction of the angular acceleration is same and into the page.
Using Eqns.(1),(2) and (3)

$$\cancel{T_1} + m_3 g - \cancel{T_2} + \cancel{T_2} - \cancel{T_1} = \left(m_1 + m_3 + \frac{1}{2} m_2\right) a$$

$$a = \frac{m_3 g}{m_1 + m_3 + \frac{1}{2} m_2} = \frac{0{,}10 \times 10}{0{,}15 + 0{,}10 + \frac{1}{2} \times 0{,}10} = \frac{10}{3} \ \frac{m}{s^2}$$

and the Tensions in the strings:

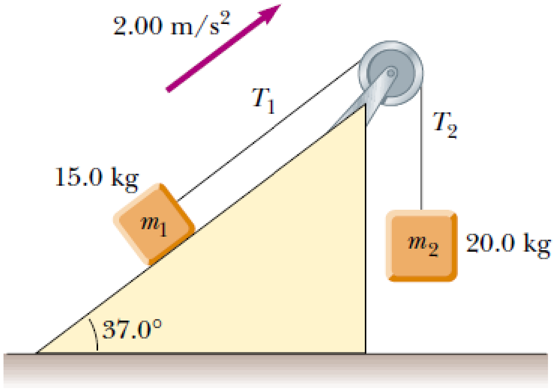
$$T_1 = m_1 a = 0{,}15 \times \frac{10}{3} = 0{,}5 \ N$$

$$T_2 = m_3 (g - a) = 0{,}10 \left(10 - \frac{10}{3}\right) = \frac{2}{3} \ N.$$

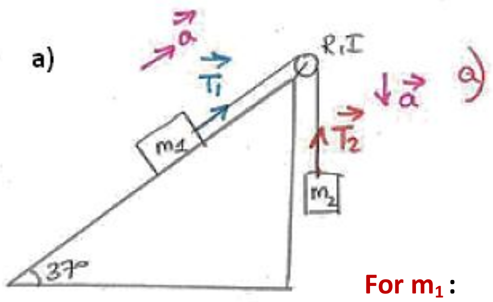
9. Two blocks, as shown in Figure, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia *I*. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s^2.

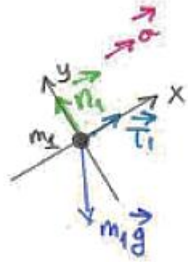
(a) Determine T_1 and T_2, the tensions in the two parts of the string.

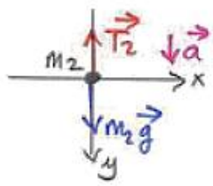
(b) Find the moment of inertia of the pulley.



9. **a)**







$m_1 = 15$ kg

$m_2 = 20$ kg

$a = 2 m/s^2$

$R = 0,25 m$

For m$_1$:

$\Sigma F_x = m_1 . a$

$T_1 - m_1 g \sin 37° = m_1 a$

$T_1 = m_1 (a + g \sin 37°)$

$T_1 = 15 [2 + 10 . \sin 37°]$

$\boxed{T_1 = 120 (N)}$

For m$_2$:

$\Sigma F_y = m_2 a$

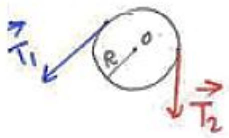
$m_2 g - T_2 = m_2 a$

$T_2 = m_2 (g - a)$

$T_2 = 20 (10 - 2)$

$\boxed{T_2 = 160 (N)}$

b)



$\Sigma \tau_o = I . \alpha$

$T_2 . R - T_1 R = I . \alpha$

$I = \dfrac{(T_2 - T_1) R}{\alpha}$

$I = \dfrac{(160 - 120) . 0,25}{8}$

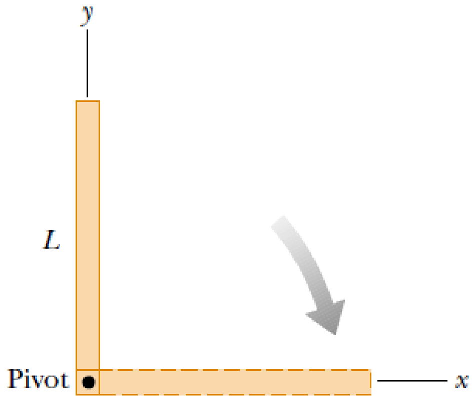
$I = 1,25 \ (kg.m^2)$

$a_t = \alpha R$

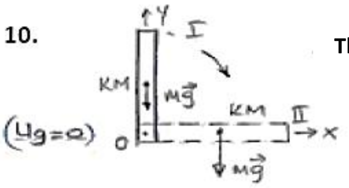
$\alpha = \dfrac{a_t}{R} = \dfrac{2}{0,25} = 8 \ (rad/s^2)$

10. A long, uniform rod of length L and mass M is pivoted about a horizontal, frictionless pin passing through one end. The rod is released from rest in a vertical position, as shown Figure. At the instant the rod is horizontal, find

a) Its angular speed,
b) The magnitude of its angular acceleration,
c) The x and y components of the acceleration of its center of mass, and
d) The components of the reaction force at the pivot.



10.



The moment of inertia of the rod through the center of mass : $I_{KM}=\frac{1}{12}ML^2$

$I_o = I_{km} + Md^2$ **The moment of inertia about "O"**

$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \underline{\underline{\frac{1}{3}ML^2}}$

a) $E_I = E_{II}$

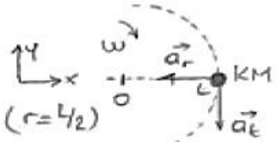
$\cancel{K_I} + U_I = K_{II} + \cancel{U_{II}}$

$mg\left(\frac{L}{2}\right) = \frac{1}{2}I_o\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{L}}$

b) $\Sigma\tau_o = I\cdot\alpha$

$(Mg)\left(\frac{L}{2}\right) = \frac{1}{3}ML^2\cdot\alpha \Rightarrow \alpha = \frac{3g}{2L}$

c)



$a_t = \alpha\cdot r = \left(\frac{3g}{2L}\right)\left(\frac{L}{2}\right) = \frac{3}{4}g$

$a_r = \frac{v^2}{r} = \omega^2 r = \left(\frac{3g}{L}\right)\left(\frac{L}{2}\right) = \frac{3}{2}g$

$\vec{a} = \left[-\frac{3}{2}g\,\vec{i} - \frac{3}{4}g\,\vec{j}\right]\ m/s^2$

d) **According to the Newton's Second Law:** $F = ma$

$\Rightarrow \begin{cases} \Sigma F_x = m\cdot a_x \\ R_x = m\cdot\left(-\frac{3}{2}g\right) = -\frac{3}{2}mg \end{cases}$

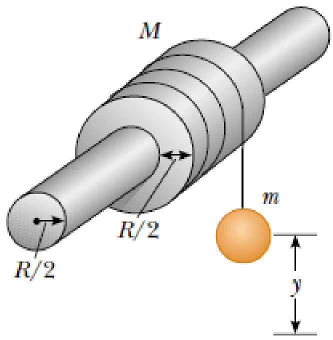
$\Sigma F_y = m\cdot a_y$

$R_y - mg = m\cdot\left(-\frac{3}{4}g\right)$

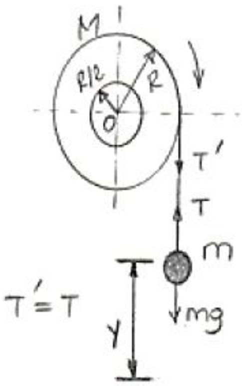
$R_y = \frac{mg}{4}$

11. A uniform, hollow, cylindrical spool has inside Radius R/2, outside radius R, and mass M. It is mounted so that it rotates on a massless horizontal axle. A mass m is connected to the end of a string wound around the spool. The mass m falls from rest through a distance y in time t. Show that the torque due to the frictional forces between spool and acle is

$$\tau_f = R\left[m\left(g-\frac{2y}{t^2}\right)-M\frac{5y}{4t^2}\right]$$



11.



The torque due to the frictional forces prevents motion.

$$\Sigma \tau = I \cdot \alpha$$

$$T \cdot R - \tau_f = I \cdot \alpha \quad \Rightarrow \quad \boxed{\tau_f = TR - I\alpha} \quad (1)$$

Equation of motion for mass m:

$$\Sigma F_y = m \cdot a \, ,$$

$$mg - T = m \cdot a \quad \Rightarrow \quad \boxed{T = m(g-a)} \quad (2)$$

$$y = \cancel{V_0 t}^{\,0} + \frac{1}{2} a t^2 \rightarrow \boxed{a = \frac{2y}{t^2}} \; (3) \quad \Rightarrow \quad \alpha = \frac{a}{R} = \boxed{\frac{2y}{Rt^2}} \; (4)$$

Moment of inertia of a Hollow cylinder:

$$I = \frac{1}{2} M \left(R_{ic}^2 + R_{dis}^2\right) = \frac{1}{2} M \left[\left(\frac{R}{2}\right)^2 + R^2\right] = \boxed{\frac{5}{8} M R^2} \; (5)$$

Using Eqns.(2),(3),(4),(5) and (1);

$$\tau_f = R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4} M \left(\frac{y}{t^2}\right)\right]$$