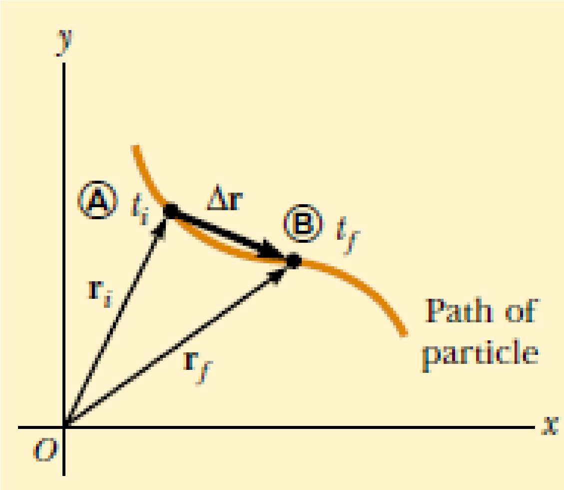
Motion in Two Dimensions

The Position, Velocity and Acceleration Vectors



$$\Delta \mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i$$

displacement vector

$$\overline{\mathbf{v}} \equiv \frac{\Delta \mathbf{r}}{\Delta t}$$

average velocity

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

instantaneous velocity

$$\overline{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

average acceleration

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

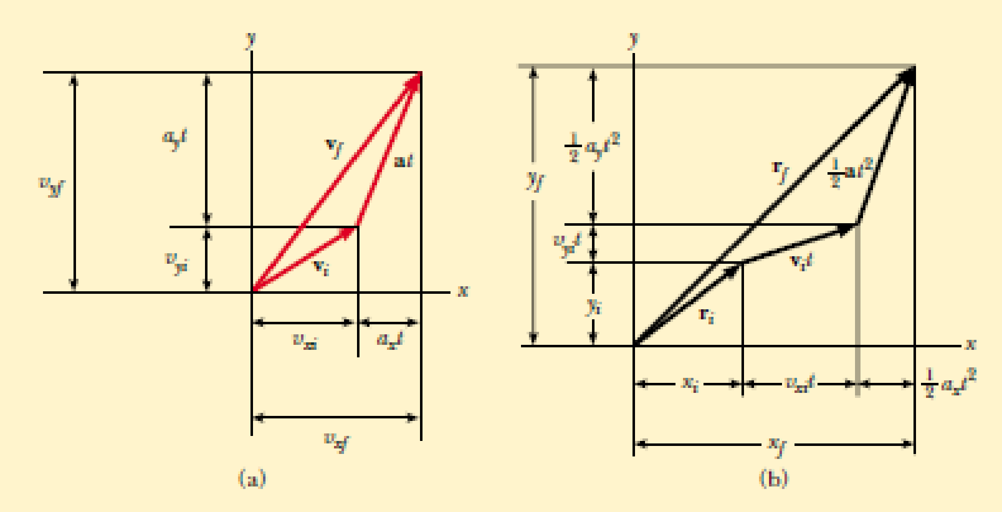
instantaneous acceleration

Two-Dimensional Motion with Constant Acceleration

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

$$\mathbf{v}_f = (v_{xi} + a_x t)\hat{\mathbf{i}} + (v_{yi} + a_y t)\hat{\mathbf{j}}$$
$$= (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}) + (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t$$
$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$$



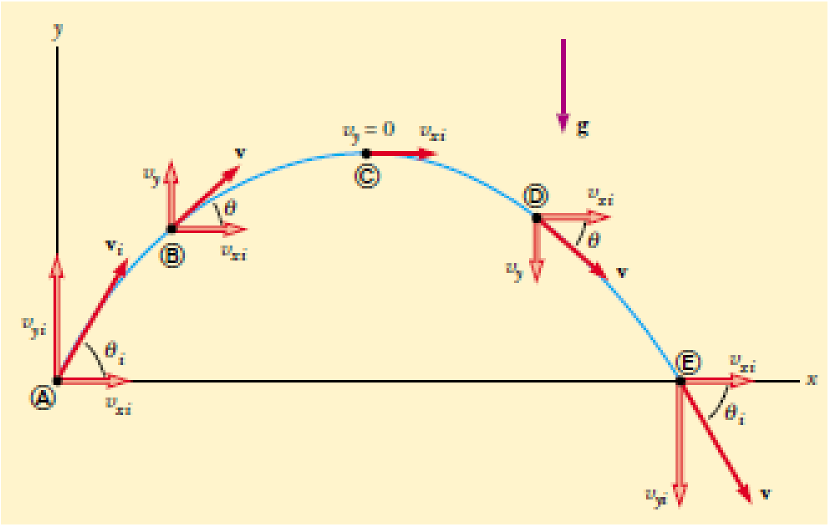
$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2$$

$$\mathbf{r}_f = (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\hat{\mathbf{i}} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\hat{\mathbf{j}}$$
$$= (x_i\hat{\mathbf{i}} + y_i\hat{\mathbf{j}}) + (v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}})t + \tfrac{1}{2}(a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})t^2$$
$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t \quad \begin{cases} v_{xf} = v_{xi} + a_x t \\ v_{yf} = v_{yi} + a_y t \end{cases}$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{a}t^2 \quad \begin{cases} x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \\ y_f = y_i + v_{yi}t + \tfrac{1}{2}a_y t^2 \end{cases}$$

Projectile Motion



$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$

$$x_f = v_{xi}t = (v_i \cos\theta_i)\,t$$

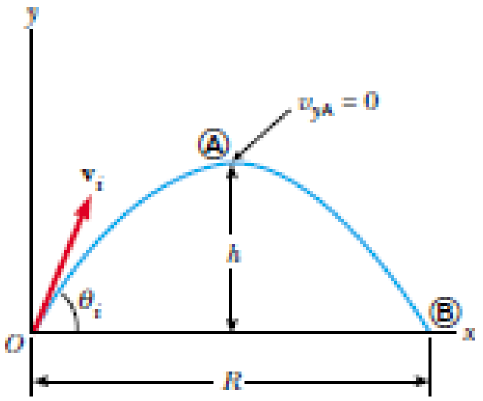
$$y_i = 0 \text{ and } a_y = -g$$

$$y_f = v_{yi}t + \tfrac{1}{2}a_y t^2 = (v_i \sin\theta_i)\,t - \tfrac{1}{2}gt^2$$

$$y = (\tan\theta_i)x - \left(\frac{g}{2v_i^{\,2}\cos^2\theta_i}\right)x^2$$

$$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \tfrac{1}{2}\mathbf{g}t^2$$

Horizontal Range and Maximum Height of a Projectile



$$v_{yA} = 0.$$

$$v_{yf} = v_{yi} + a_y t$$

$$0 = v_i \sin\theta_i - g t_A$$

$$t_A = \frac{v_i \sin\theta_i}{g}$$

$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}$$

$$t_B = 2t_A.$$

$v_{xi} = v_{xB} = v_i \cos\theta_i$ $\quad$ $x_B = R$ at $t = 2t_A$,

$$R = v_{xi} t_B = (v_i \cos\theta_i) 2t_A$$

$$= (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

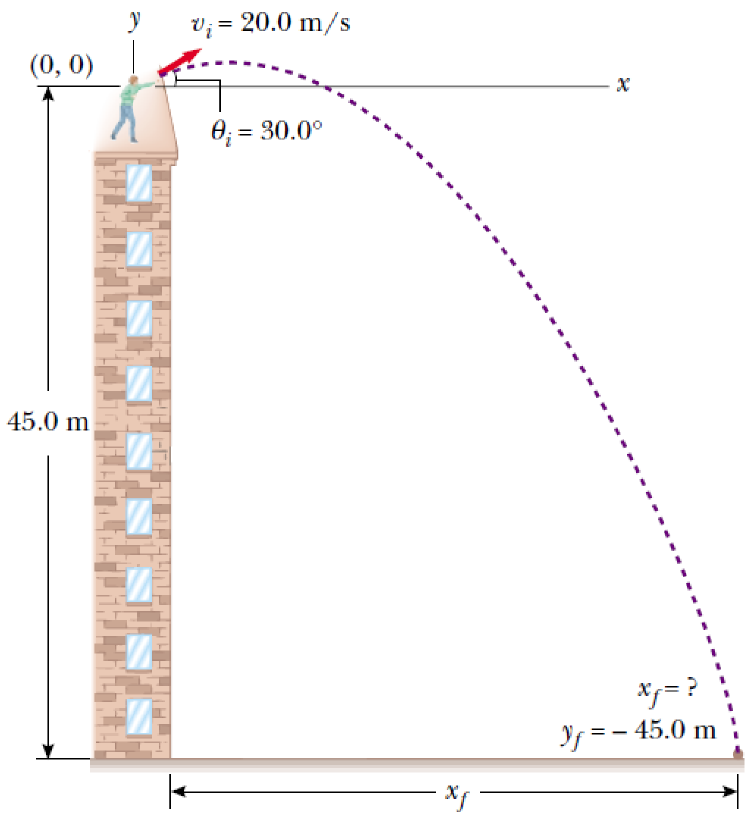
$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$

Therefore, *R* is a maximum when the initial angle is 45°.

Ex/A stone is thrown from the top of a building upward at an angle of 30° to the horizontal with an initial speed of 20.0 m/s, as shown in Figure. If the height of the building is 45 m.

a) How long does it take the stone to reach the ground?

b) What is the speed of the stone just before it strikes the ground?



a) $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$y_i = 0,\ y_f = -45.0 \text{ m},\ a_y = -g,$

$v_{xi} = v_i \cos\theta_i = (20.0 \text{ m/s})\cos 30.0° = 17.3 \text{ m/s}$

$v_{yi} = v_i \sin\theta_i = (20.0 \text{ m/s})\sin 30.0° = 10.0 \text{ m/s}$

$-45.0 \text{ m} = (10.0 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$

t=4.22 s

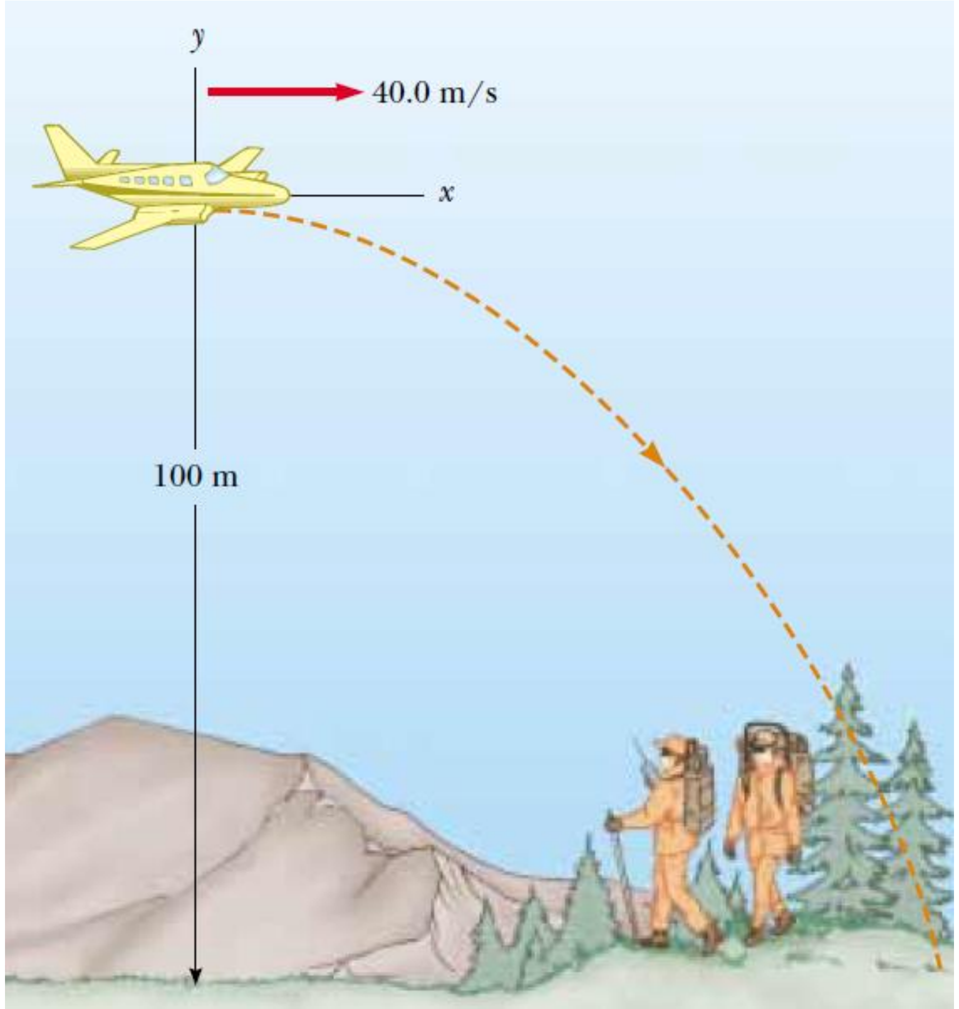
b) $v_{yf} = v_{yi} + a_y t,$

$v_{yf} = 10.0 \text{ m/s} - (9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.4 \text{ m/s}$

$v_{xf} = v_{xi} = 17.3 \text{ m/s}$

$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m/s} = 35.9 \text{ m/s}$

Ex/ A plane drops a package of supplies as shown in Figure. If the plane is traveling horizontally at 40 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it is released?



$$x_f = x_i + v_{xi}t$$

$$x_f = (40.0\ \text{m/s})\,t$$

$$y_f = -\tfrac{1}{2}gt^2$$

$$-100\ \text{m} = -\tfrac{1}{2}(9.80\ \text{m/s}^2)\,t^2$$

$$t = 4.52\ \text{s}$$

$$x_f = (40.0\ \text{m/s})(4.52\ \text{s}) = \boxed{181\ \text{m}}$$

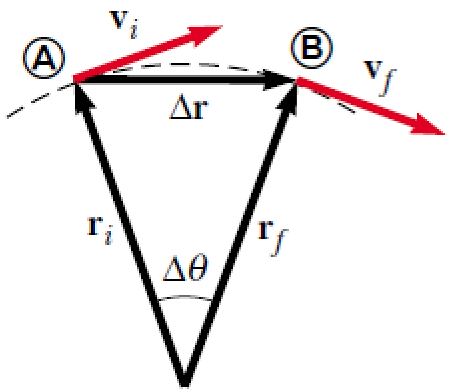
Uniform Circular Motion

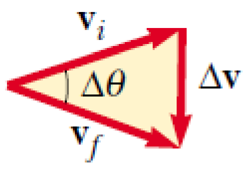
Motion a circular path with *constant speed v is* called uniform circular motion.

Even though an object moves at a constant speed in a circular path, it still has an acceleration.

Acceleration depends on *the change in the velocity vector*. Velocity vector has both *magnitude* and *direction. In uniform circular motion only direction of velocity vector changes.*

The velocity vector is always tangent to the path of the object and perpendicular to the radius of the circular path.





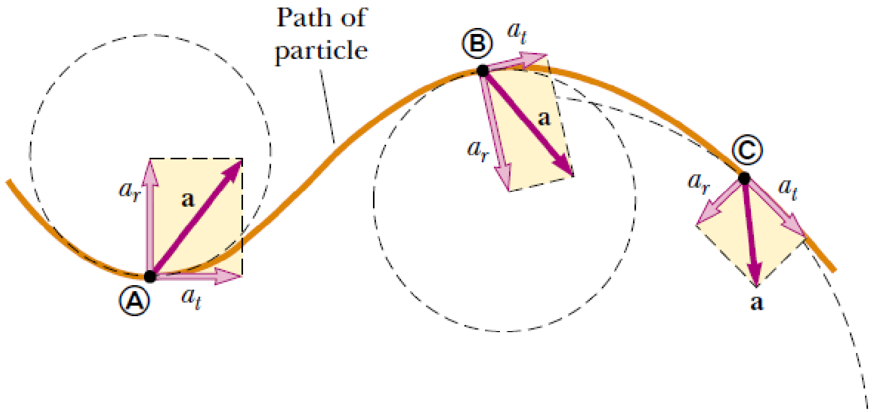
Acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle and *called centripetal acceleration.*

Magnitude of the centripetal acceleration is $a_c = \frac{v^2}{r}$

Period «T»: The time required for one complete revolution.

$$T \equiv \frac{2\pi r}{v}$$

Tangential and Radial Acceleration



The *total* acceleration vector a can be written as the vector sum of the component vectors

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$$

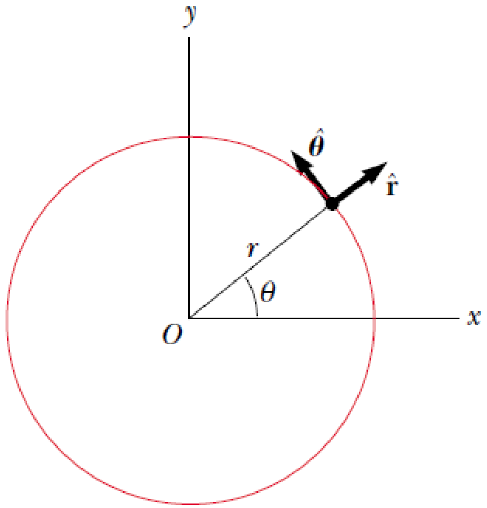
$$a = \sqrt{a_r^2 + a_t^2}.$$

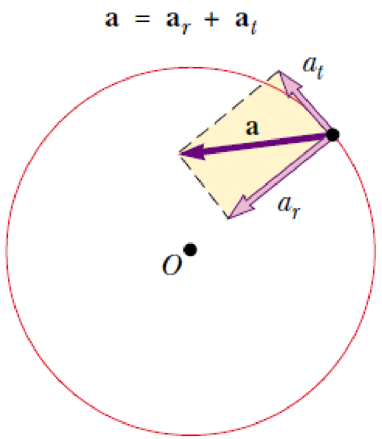
The tangential acceleration component causes the change in the speed of the particle. This component is parallel to the instantaneous velocity

$$a_t = \frac{d|\mathbf{v}|}{dt}$$

The radial acceleration component arises from the change in direction of the velocity

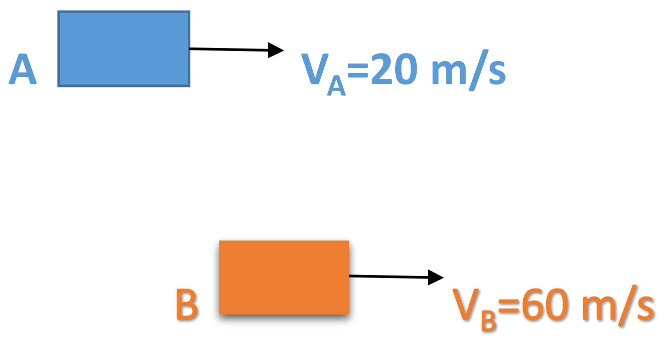
$$a_r = -a_c = -\frac{v^2}{r}$$





$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r = \frac{d|\mathbf{v}|}{dt}\,\hat{\boldsymbol{\theta}} - \frac{v^2}{r}\,\hat{\mathbf{r}}$$

Relative Velocity and Relative Acceleration



Speed of A relative to B

$V_{AB}=V_A-V_B$= 20-60= -40 m/s

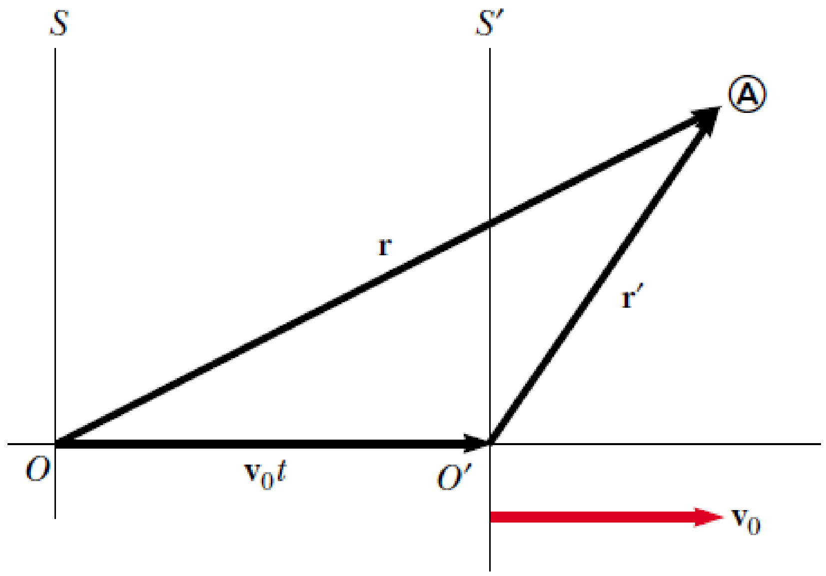
Speed of B relative to A

$V_{BA}=V_B-V_A$= 60-20= 40 m/s

a particle located at point A in the Figure.

Imagine that the motion of this particle is being described by two observers, one in reference frame *S*, fixed relative to Earth, and another in reference frame *S′*, moving to the right relative to *S* (and therefore relative to Earth) with a constant velocity $\mathbf{V}_o$. One observer at «O» and other at «O′»

(Relative to an observer in *S′*, *S* moves to the left with a velocity $-\mathbf{V}_o$.)



At t=0 origins of the two reference frames coincide in space.

$$\mathbf{r} = \mathbf{r}' + \mathbf{v}_0 t,$$

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t$$

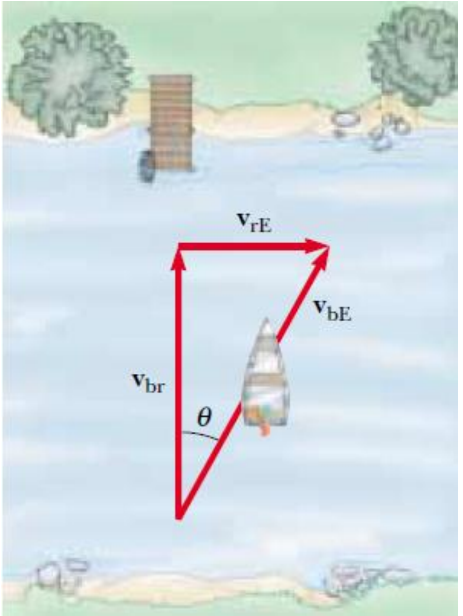
$$\frac{d\mathbf{r}'}{dt} = \frac{d\mathbf{r}}{dt} - \mathbf{v}_0$$

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0$$

$$\frac{d\mathbf{v}'}{dt} = \frac{d\mathbf{v}}{dt} - \frac{d\mathbf{v}_0}{dt}$$

$$\mathbf{a}' = \mathbf{a}$$

Ex/ A boat heading due north crosses a wide river with a speed of 10 km/h relative to the water. The water in the river has a uniform speed of 5 km/h due east relative to the Earth. Determine the velocity of the boat relative to an observer standing on either bank.



$$\mathbf{v}_{bE} = \mathbf{v}_{br} + \mathbf{v}_{rE}$$

$$v_{bE} = \sqrt{v_{br}^2 + v_{rE}^2} = \sqrt{(10.0)^2 + (5.00)^2}\ \text{km/h}$$

$$= 11.2\ \text{km/h}$$

The direction of $\mathbf{v}_{bE}$ is

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.0}\right) = 26.6°$$