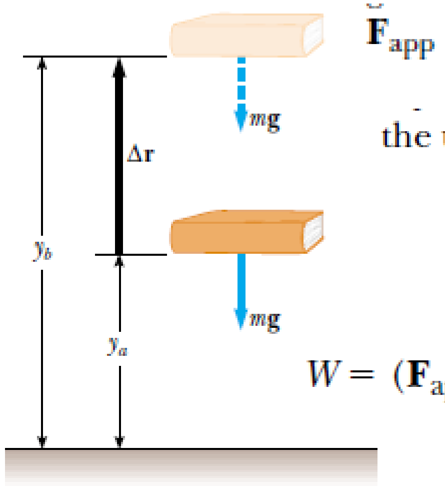
Potential Energy

Potential Energy of a System

gravitational potential energy:

The object is lifting up by constant velocity

So $\mathbf{F}_{app}$= -mg



the upward displacement $\Delta\vec{\mathbf{r}} = \Delta y\hat{\mathbf{j}}$:

$$W = (\vec{\mathbf{F}}_{app}) \cdot \Delta\vec{\mathbf{r}} = (mg\hat{\mathbf{j}}) \cdot [(y_b - y_a)\hat{\mathbf{j}}] = mgy_b - mgy_a$$

Thus, we can identify the quantity mgy as the gravitational potential energy U_g:

$$\boxed{U_g \equiv mgy}$$

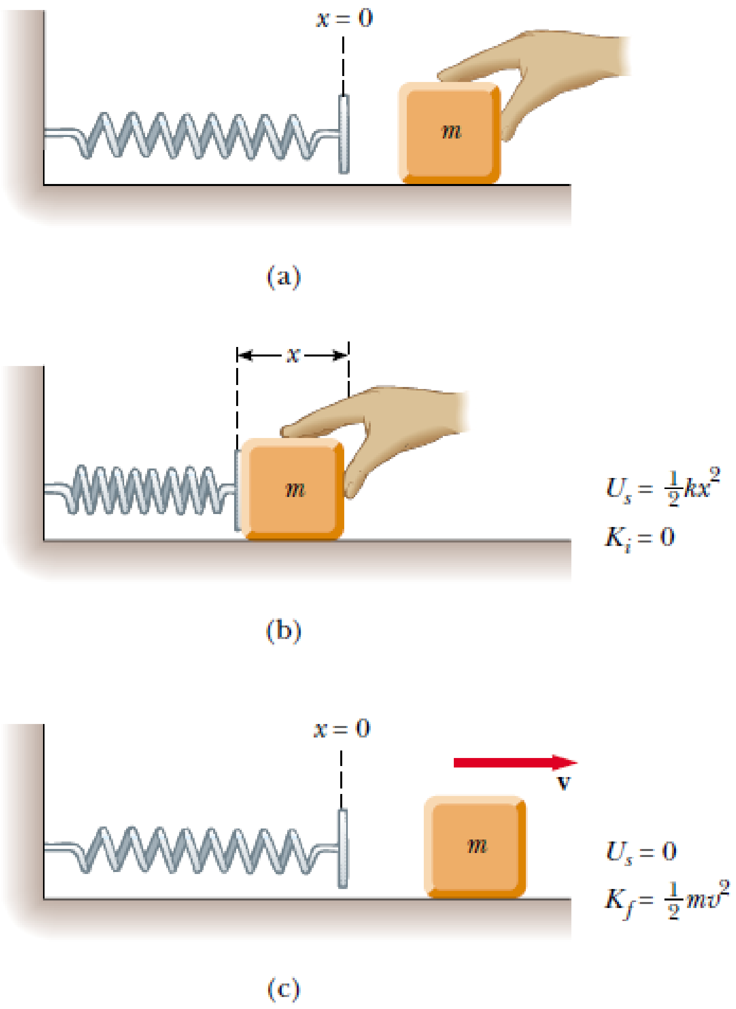
The gravitational potential energy depends only on the vertical height of the object.

Elastic Potential Energy

For the mass - spring system

$$W_{F_{app}} = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2$$

$$\boxed{U_s \equiv \tfrac{1}{2}kx^2}$$



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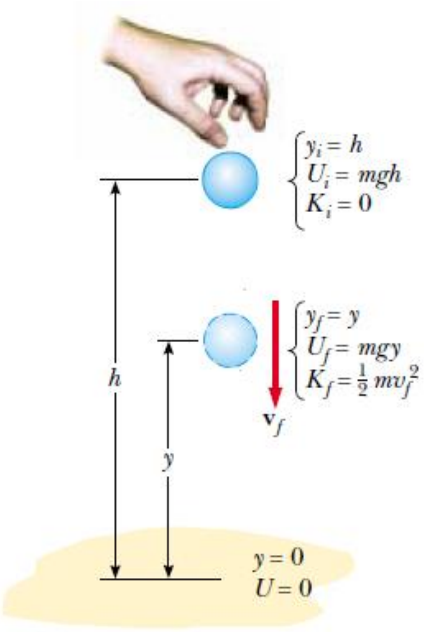
Ex/ A ball of mass *m* is dropped from a height *h* above the ground, as shown in Figure 8.6.

a) Neglecting air resistance, determine the speed of the ball when it is at a height *y* above the ground.

b) Determine the speed of the ball at *y* if at the instant of release it already has an initial upward speed *vi* at the initial altitude *h*.



a)

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} m v_f^2 + mgy = 0 + mgh$$

$$v_f^2 = 2g(h - y)$$

$$v_f = \boxed{\sqrt{2g(h - y)}}$$

b)

$$\frac{1}{2} m v_f^2 + mgy = \frac{1}{2} m v_i^2 + mgh$$

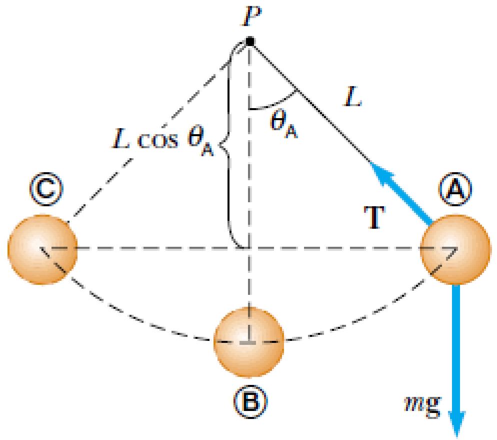
$$v_f^2 = v_i^2 + 2g(h - y)$$

$$v_f = \boxed{\sqrt{v_i^2 + 2g(h - y)}}$$

Ex/ A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure. The sphere is released from rest at point A when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

a) Find the speed of the sphere when it is at the lowest point B.

b) What is the tension T_B in the cord at B?



a) If we measure the y coordinates of the sphere from point P;

$$y_A = -L \cos \theta_A \text{ and } y_B = -L.$$

$$U_A = -mg L \cos \theta_A \text{ and } U_B = -mgL.$$

$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2} m v_B^2 - mgL = 0 - mgL \cos \theta_A$$

$$\boxed{v_B = \sqrt{2gL(1 - \cos \theta_A)}}$$

b) $$\sum F_r = T_B - mg = ma_r = m \frac{v_B^2}{L}$$

$$T_B = mg + 2mg(1 - \cos \theta_A) = \boxed{mg(3 - 2 \cos \theta_A)}$$

Conservative and Nonconservative Forces

Conservative Forces

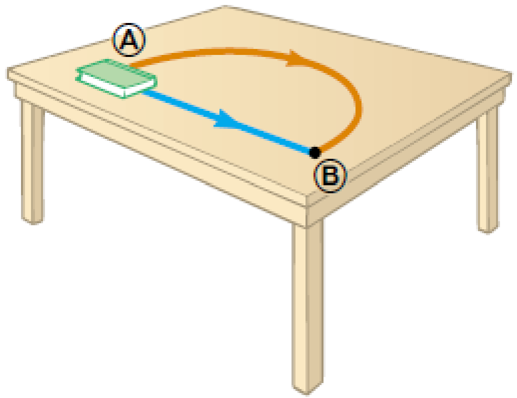
Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

Nonconservative Forces

A force is nonconservative if it does not satisfy properties 1 and 2 for conservative forces. Nonconservative forces acting within a system cause a *change* in the mechanical energy E_{mech} of the system. We have defined mechanical energy as the sum of the kinetic and all potential energies. For example, if a book is sent sliding on a horizontal surface that is not frictionless, the force of kinetic friction reduces the book's kinetic energy. As the book slows down, its kinetic energy decreases.



Changes in Mechanical Energy for Nonconservative Forces

$$\Delta K = -f_k d$$

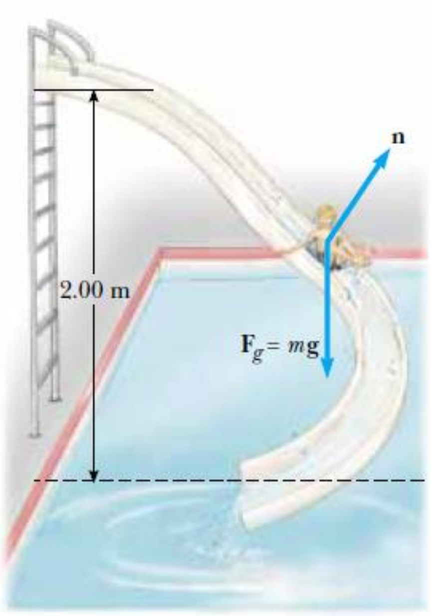
$$\Delta E_{\text{mech}} = \Delta K + \Delta U_g = -f_k d$$

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k d$$

Ex/ A child of mass m rides on an irregularly curved slide of height h = 2 m, as shown in Figure. The child starts from rest at the top.

a) Determine his speed at the bottom, assuming no friction is present.

b) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that v_f = 3.00 m/s and m = 20 kg.



a)

$$y_i = h, \; y_f = 0,$$

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^2 + 0 = 0 + mgh$$

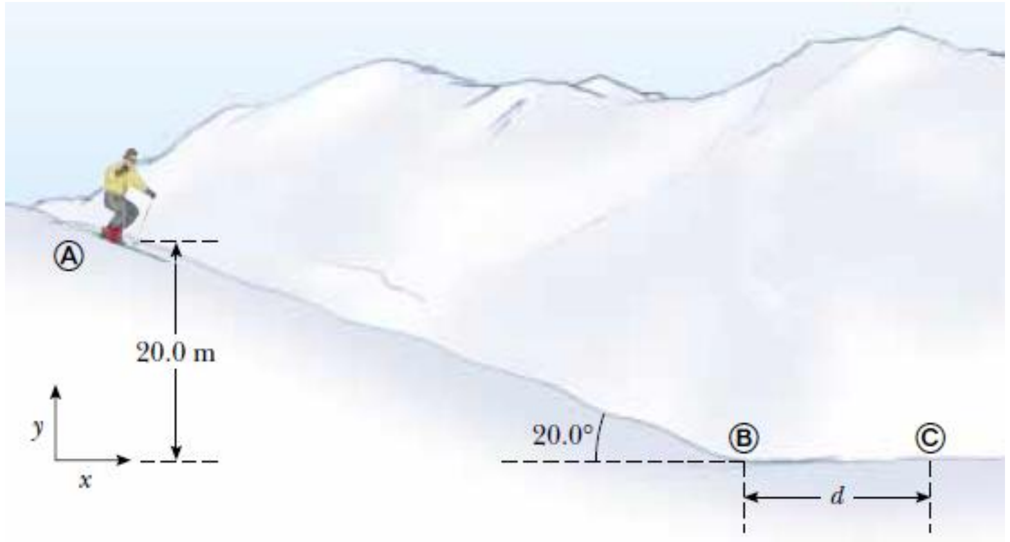
$$v_f = \sqrt{2gh}$$

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{m})} = \boxed{6.26\,\mathrm{m/s}}$$

b)

$$\begin{aligned}\Delta E_{\mathrm{mech}} &= (K_f + U_f) - (K_i + U_i)\\ &= (\tfrac{1}{2}mv_f^2 + 0) - (0 + mgh) = \tfrac{1}{2}mv_f^2 - mgh\\ &= \tfrac{1}{2}(20.0\ \mathrm{kg})(3.00\ \mathrm{m/s})^2\\ &\quad - (20.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{m})\\ &= \boxed{-302\ \mathrm{J}}\end{aligned}$$

Ex/ A skier starts from rest at the top of a frictionless incline of height 20 m, as shown in Figure. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.21. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?



a)

$$v_B = \sqrt{2gh} = \sqrt{2(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m})} = 19.8\ \mathrm{m/s}$$

$$\Delta E_{\mathrm{mech}} = -f_k d,$$

$$K_C = 0$$

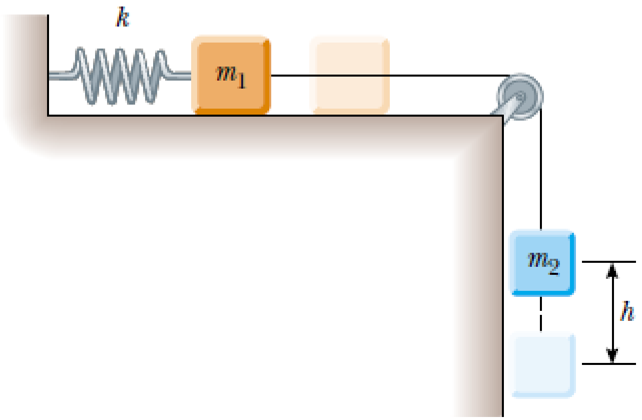
$$\Delta E_{\mathrm{mech}} = E_C - E_B = -\mu_k mgd$$

$$(K_C + U_C) - (K_B + U_B) = (0 + 0) - (\tfrac{1}{2}mv_B^2 + 0)$$

$$= -\mu_k mgd$$

$$d = \frac{v_B^2}{2\mu_k g} = \frac{(19.8\ \mathrm{m/s})^2}{2(0.210)(9.80\ \mathrm{m/s^2})} = 95.2\ \mathrm{m}$$

Ex/ Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



$$\Delta K = 0,$$

$$\Delta E_{\text{mech}} = \Delta U_g + \Delta U_s$$

$$\Delta E_{\text{mech}} = -f_k h = -\mu_k m_1 g h$$

$$\Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

$$\Delta U_s = U_{sf} - U_{si} = \tfrac{1}{2} k h^2 - 0$$

$$-\mu_k m_1 g h = -m_2 g h + \tfrac{1}{2} k h^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2} k h}{m_1 g}$$

Relationship Between Conservative Forces and Potential Energy

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx$$

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i$$

$$dU = -F_x \, dx$$

$$F_x = -\frac{dU}{dx}$$

Example for a spring $U_s = \frac{1}{2}kx^2$

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

in general for a 3-dimensional force;

$$\mathbf{F} = -\frac{\partial U}{\partial x}\hat{\mathbf{i}} - \frac{\partial U}{\partial y}\hat{\mathbf{j}} - \frac{\partial U}{\partial z}\hat{\mathbf{k}}$$