The Laws of Motion

The Concept of Force

Net force causes an object to accelerate.

The object accelerates only if the net force acting on it is not equal to zero.

The net force acting on an object is defined as the vector sum of all forces acting on the object. (We sometimes refer to the net force as the *total force*, the *resultant force*, or the *unbalanced force*.)

If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.

That is, if the net force acting on the object is zero, the object either remains at rest or continues to move with constant velocity.

When the velocity of an object is constant (including when the object is at rest), the object is said to be in ***equilibrium***.

Newton's First Law and Inertial Frames

Newton's first law of motion, sometimes called the *law of inertia.*

Newton's First Law : In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

Simply, when no force acts on an object, the acceleration of the object is zero.

The tendency of an object to resist any attempt to change its velocity is called ***inertia***.

An inertial frame of reference has no acceleration.

Mass

Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity.

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Newton's Second Law

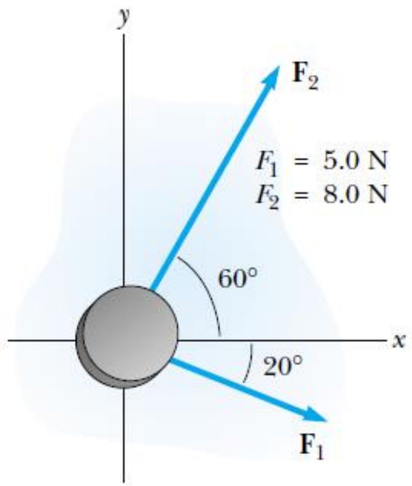
The acceleration of an object is directly proportional to the force acting on it. The magnitude of the acceleration of an object is inversely proportional to its mass.

Newton's second law: When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\boxed{\sum \mathbf{F} = m\mathbf{a}} \qquad 1\,\mathrm{N} \equiv 1\,\mathrm{kg \cdot m/s^2}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

Ex/ A hockey puck having a mass of 0.3 kg slides on the horizontal, frictionless surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure. The force F_1 has a magnitude of 5 N, and the force F_2 has a magnitude of 8 N. Determine both the magnitude and the direction of the puck's acceleration.



$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos(-20°) + F_2 \cos 60°$$
$$= (5.0\text{ N})(0.940) + (8.0\text{ N})(0.500) = 8.7\text{ N}$$

$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin(-20°) + F_2 \sin 60°$$
$$= (5.0\text{ N})(-0.342) + (8.0\text{ N})(0.866) = 5.2\text{ N}$$

$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7\text{ N}}{0.30\text{ kg}} = 29\text{ m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{5.2\text{ N}}{0.30\text{ kg}} = 17\text{ m/s}^2$$

$$a = \sqrt{(29)^2 + (17)^2}\text{ m/s}^2 = 34\text{ m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30°$$

The Gravitational Force and Weight

The attractive force exerted by the Earth on an object is called the gravitational force F_g . This force is directed toward the center of the Earth and its magnitude is called the weight of the object.

$$\Sigma\mathbf{F} = m\mathbf{a}$$

$$\mathbf{a} = \mathbf{g}$$

$$\Sigma\mathbf{F} = \mathbf{F}_g$$

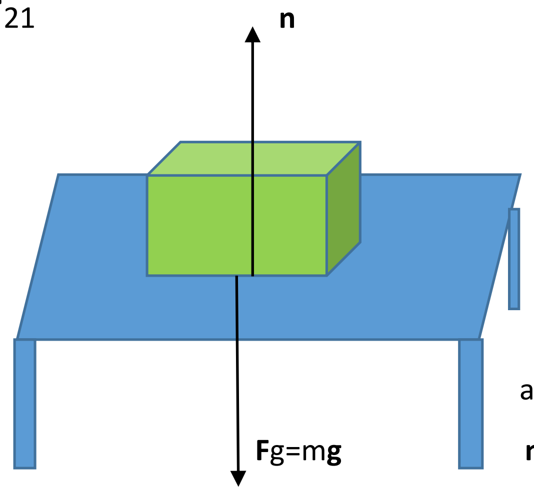
$$\mathbf{F}_g = m\mathbf{g}$$

Newton’s Third Law

The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects and must be of the same type.

If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force F_{21} exerted by object 2 on object 1.

$$\mathbf{F}_{12} = -\,\mathbf{F}_{21}$$



as vectors

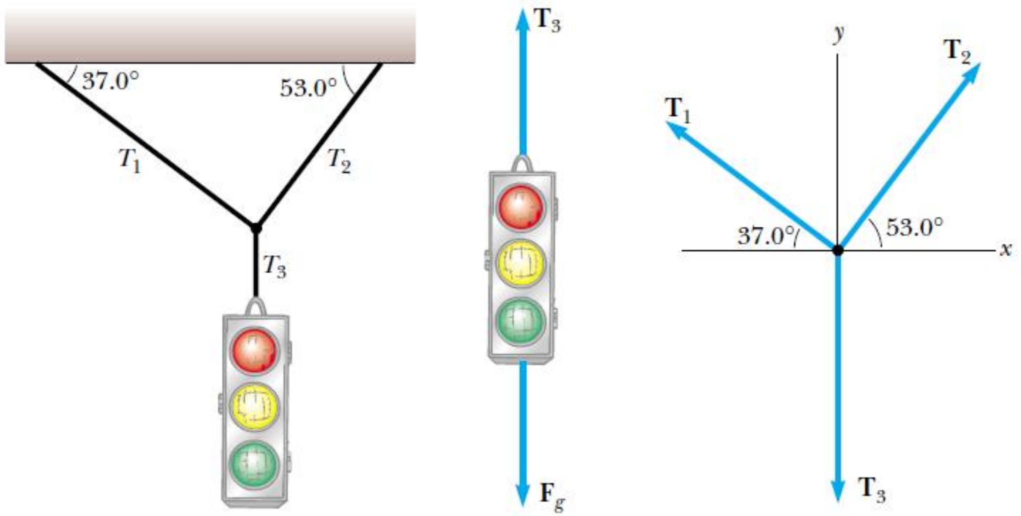
n=-m**g**

n+m**g**=0

in magnitude n=mg

Some Applications of Newton's Laws

Ex/ A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tensions in the cables.



T_3 has only y-component

$$\Sigma F_y = 0 \qquad T_3 - F_g = 0$$

$$T_3 = F_g = 122 \text{ N}$$

T_1 and T_2 have x- and y-components

$$\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$$

$$\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0$$

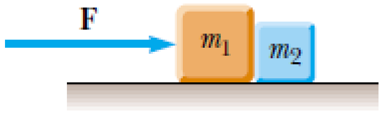
$$T_2 = T_1\left(\frac{\cos 37.0^\circ}{\cos 53.0^\circ}\right) = 1.33 T_1$$

$$T_1 \sin 37.0° + (1.33 T_1)(\sin 53.0°) - 122 \text{ N} = 0$$

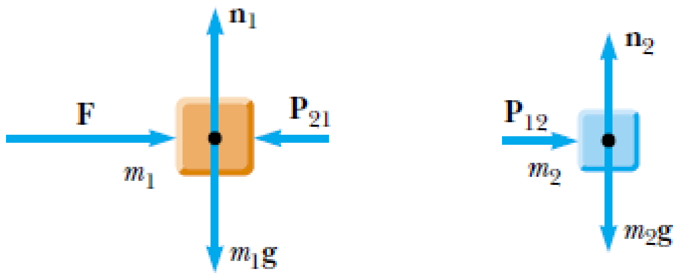
$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33 T_1 = 97.4 \text{ N}$$

Ex/



$$\sum F_x(\text{system}) = F = (m_1 + m_2)a_x$$



For m_2

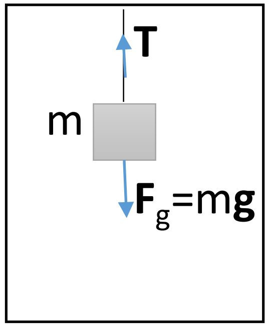
$$\sum F_x = P_{12} = m_2 a_x \qquad P_{12} = m_2 a_x = \left(\frac{m_2}{m_1 + m_2}\right)F$$

For m_1

$$\sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

Weighing in an Elevator



Acceleration for the elevator;

1) a=0

F_y=ma ; T-mg=0 ; T=mg

2) Acceleration is upward ; a $\uparrow$

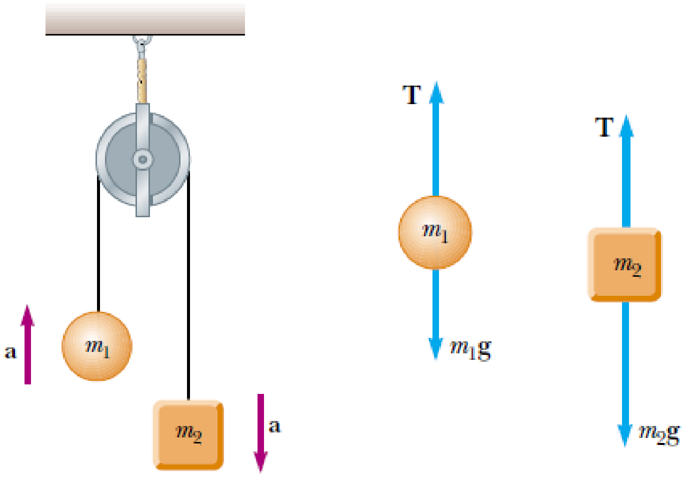
F_y=ma ; T-mg=ma ; T=m(g+a)

2) Acceleration is downward ; a $\downarrow$

F_y=ma ; mg-T=ma ; T=m(g-a)

The Atwood Machine

Ex/



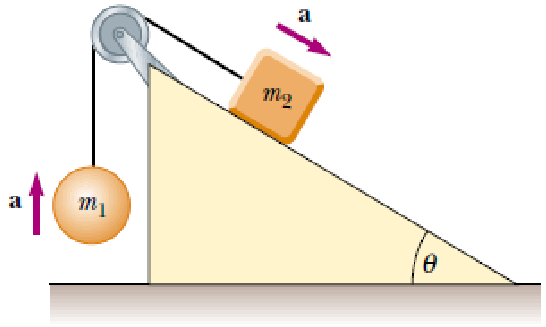
$$\sum F_y = T - m_1 g = m_1 a_y$$

$$\sum F_y = m_2 g - T = m_2 a_y$$

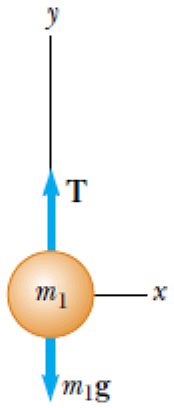
$$-m_1 g + m_2 g = m_1 a_y + m_2 a_y$$

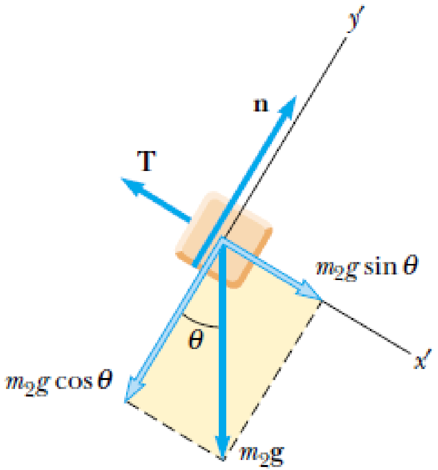
$$a_y = \boxed{\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g}$$

Ex/ A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.



Free body diagrams





For m_1

$$\sum F_x = 0$$

$$\sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

for m_2

$$\sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

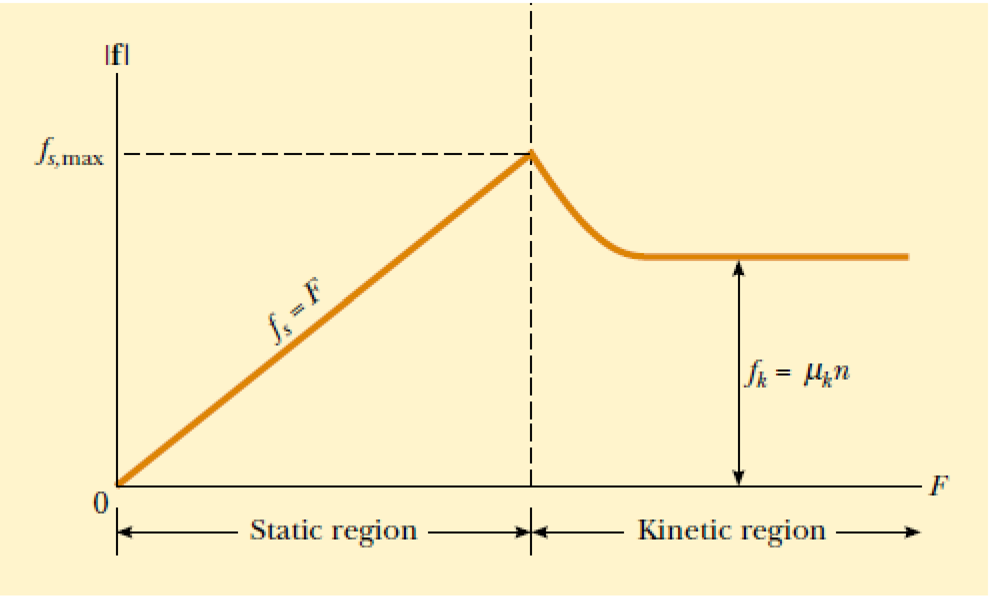
$$\sum F_{y'} = n - m_2 g\cos\theta = 0$$

$$a = \boxed{\frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}}$$

$$T = \boxed{\frac{m_1 m_2 g(\sin\theta + 1)}{m_1 + m_2}}$$

Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a force of friction.



- force of ***static friction*** $\mathbf{f}_s$ acts on a standing object.

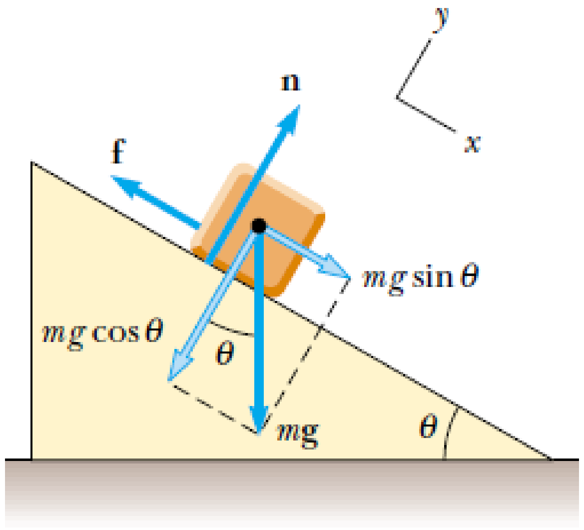
$f_s = F$

- force of ***kinetic friction*** $\mathbf{f}_k$ acts on a moving object.

$$f_s \leq \mu_s n \qquad f_k = \mu_k n$$

- μ_s : the coefficient of static friction
- μ_k : the coefficient of kinetic friction.
- **n** : the normal force

Ex/ The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure. The incline angle is increased until the block starts to move. By measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s.



$$\sum F_x = mg\sin\theta - f_s = ma_x = 0$$

$$\sum F_y = n - mg\cos\theta = ma_y = 0$$

$$f_s = mg\sin\theta = \left(\frac{n}{\cos\theta}\right)\sin\theta = n\tan\theta$$

$$\mu_s n = n\tan\theta_c$$

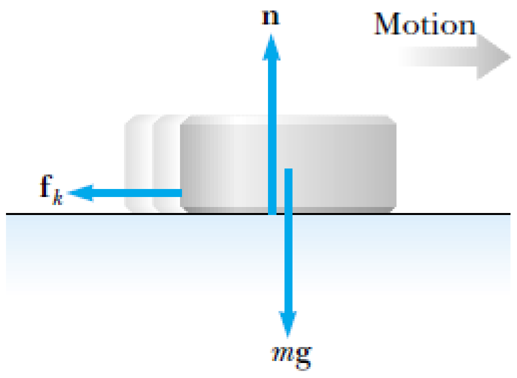
$$\mu_s = \tan\theta_c$$

$$\mu_k = \tan\theta_c'$$

in which

$$\theta_c' < \theta_c.$$

Ex/ A hockey puck on a frozen pond is given an initial speed of 20 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.



$$\sum F_x = -f_k = ma_x$$

$$\sum F_y = n - mg = 0 \qquad (a_y = 0)$$

$$f_k = \mu_k n, \qquad n = mg.$$

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

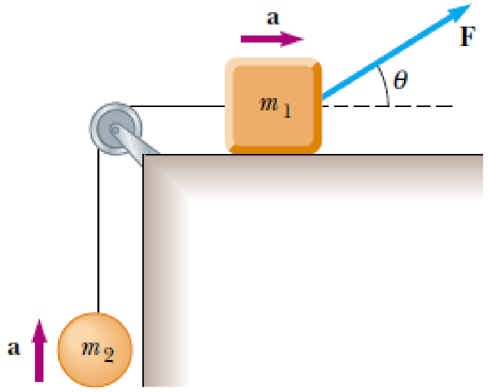
$$x_i = 0 \text{ and } v_f = 0$$

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(115 \text{ m})} = 0.117$$

Ex/ A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a lightweight cord over a lightweight, frictionless pulley, as shown in Figure. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.



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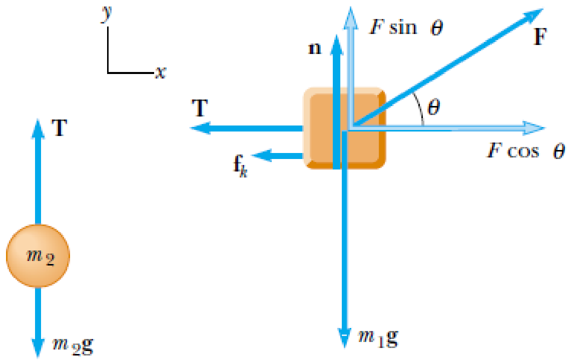

Motion of block: (1) $\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

(2) $\sum F_y = n + F\sin\theta - m_1 g = m_1 a_y = 0$

Motion of ball: $\sum F_x = m_2 a_x = 0$

(3) $\sum F_y = T - m_2 g = m_2 a_y = m_2 a$

$f_k = \mu_k n,$ $\quad n = m_1 g - F\sin\theta$ $\quad f_k = \mu_k(m_1 g - F\sin\theta)$

$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a + g) = m_1 a$$

$$a = \frac{F(\cos\theta + \mu_k\sin\theta) - g(m_2 + \mu_k m_1)}{m_1 + m_2}$$