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ENGR3930U-HEAT TRANSFER

Steady Heat Conduction

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Chapter 3
STEADY HEAT CONDUCTION

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OUTLINE

- Steady Heat Conduction in Plane Walls
- Thermal Contact Resistance
- Generalized Thermal Resistance Networks
- Heat Conduction in Cylinders and Spheres
- Critical Radius of Insulation
- Heat Transfer from Finned Surfaces
- Heat Transfer in Common Configurations
- Conclusions

Steady Heat Conduction In Plane Walls

Heat transfer through the wall is in the *normal direction* to the wall surface, and no significant heat transfer takes place in the wall in other directions.

Heat transfer in a certain direction is driven by the *temperature gradient* in that direction.

There will be no heat transfer in a direction in which there is no change in temperature.

If the air temperatures in and outside the house remain constant, then heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*.

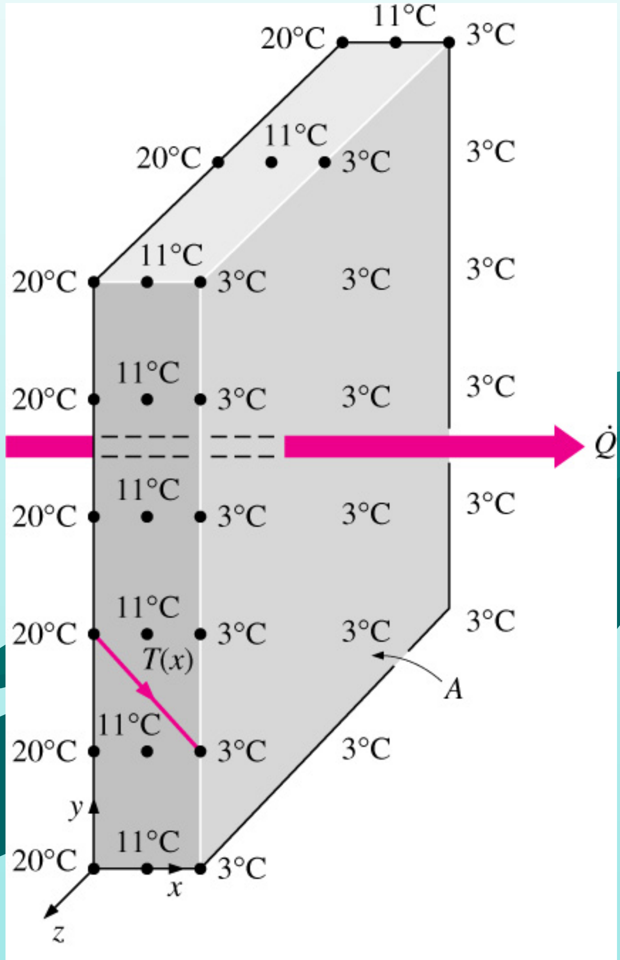


FIGURE 3–1
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

Integrating and rearranging

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

- Energy balance:

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt}$$

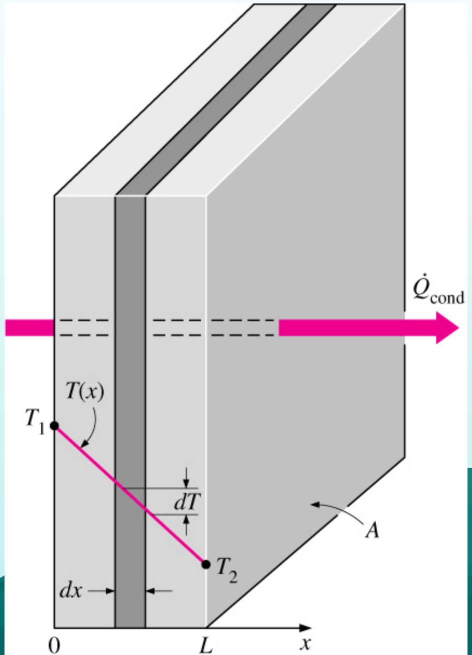


FIGURE 3–2
Under steady conditions, the temperature distribution in a plane wall is a straight line.

$dE_{\text{wall}}/dt = 0$ for *steady* operation (no change in the temperature of the wall with time at any point) and $\dot{Q}_{\text{cond, wall}} = \text{constant}$

- The Fourier's law of heat conduction for the wall:

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

where *dT/dx*= constant and *T varies linearly with x.*

The Thermal Resistance Concept

Heat conduction through a plane wall is

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

where

$$R_{\text{wall}} = \frac{L}{kA} \qquad (^\circ\text{C/W})$$

is the *thermal resistance* of the wall against heat conduction (**conduction resistance**). The thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

Taking into account analogous to the relation for *electric current flow I:*

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$

where $R_e = L/\sigma_e A$ is the *electric resistance* and $\mathbf{V}_1 - \mathbf{V}_2$ is the *voltage difference* across the resistance (σ_e is the electrical conductivity).

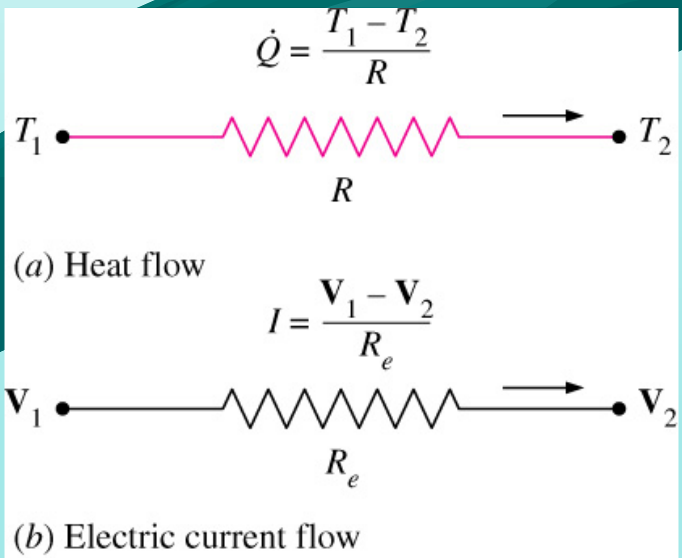


FIGURE 3–3
Analogy between thermal and electrical resistance concepts.

Newton's law of cooling for convection heat transfer rate:

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

can be rearranged as

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \quad \text{(W)}$$

with

$$R_{conv} = \frac{1}{hA_s} \quad (^\circ\text{C/W})$$

which is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface.

When the convection heat transfer coefficient is very large $(h \to \infty)$, the convection resistance becomes *zero* and $T_s \approx T_\infty$. That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.

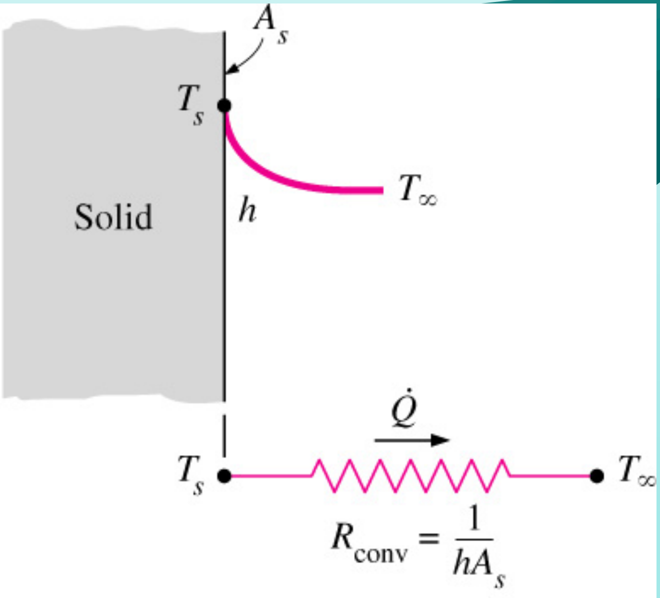


FIGURE 3–4
Schematic for convection resistance at a surface.

The rate of radiation heat transfer between a surface of emissivity ε and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma\, A_s(T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}}\, A_s(T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \qquad \text{(W)}$$

with $$R_{\text{rad}} = \frac{1}{h_{\text{rad}}\, A_s} \qquad \text{(K/W)}$$ which is the *radiation resistance.*

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \qquad (\text{W/m}^2 \cdot \text{K})$$

is the radiation heat transfer coefficient.

Both T_s and T_{surr} *must* be in K in the evaluation of h_{rad}.

When $T_{\text{surr}} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} \qquad (\text{W/m}^2 \cdot \text{K})$$

where $h_{combined}$ is the combined heat transfer coefficient.

Thermal Resistance Network

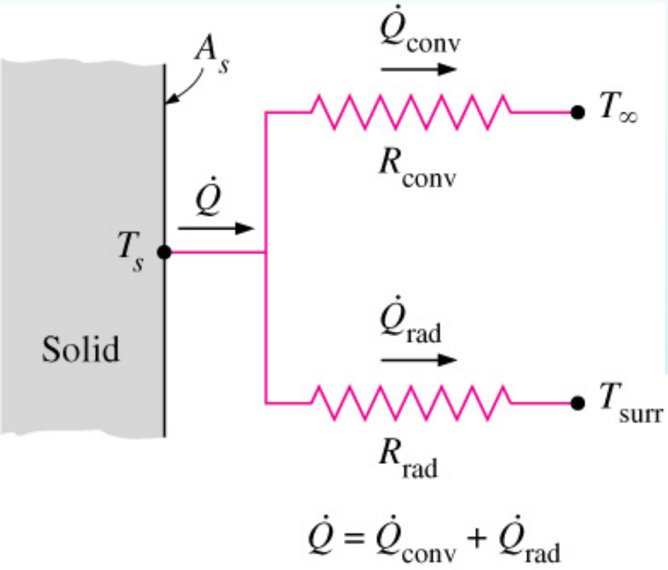


FIGURE 3–5
Schematic for convection and radiation resistances at a surface.

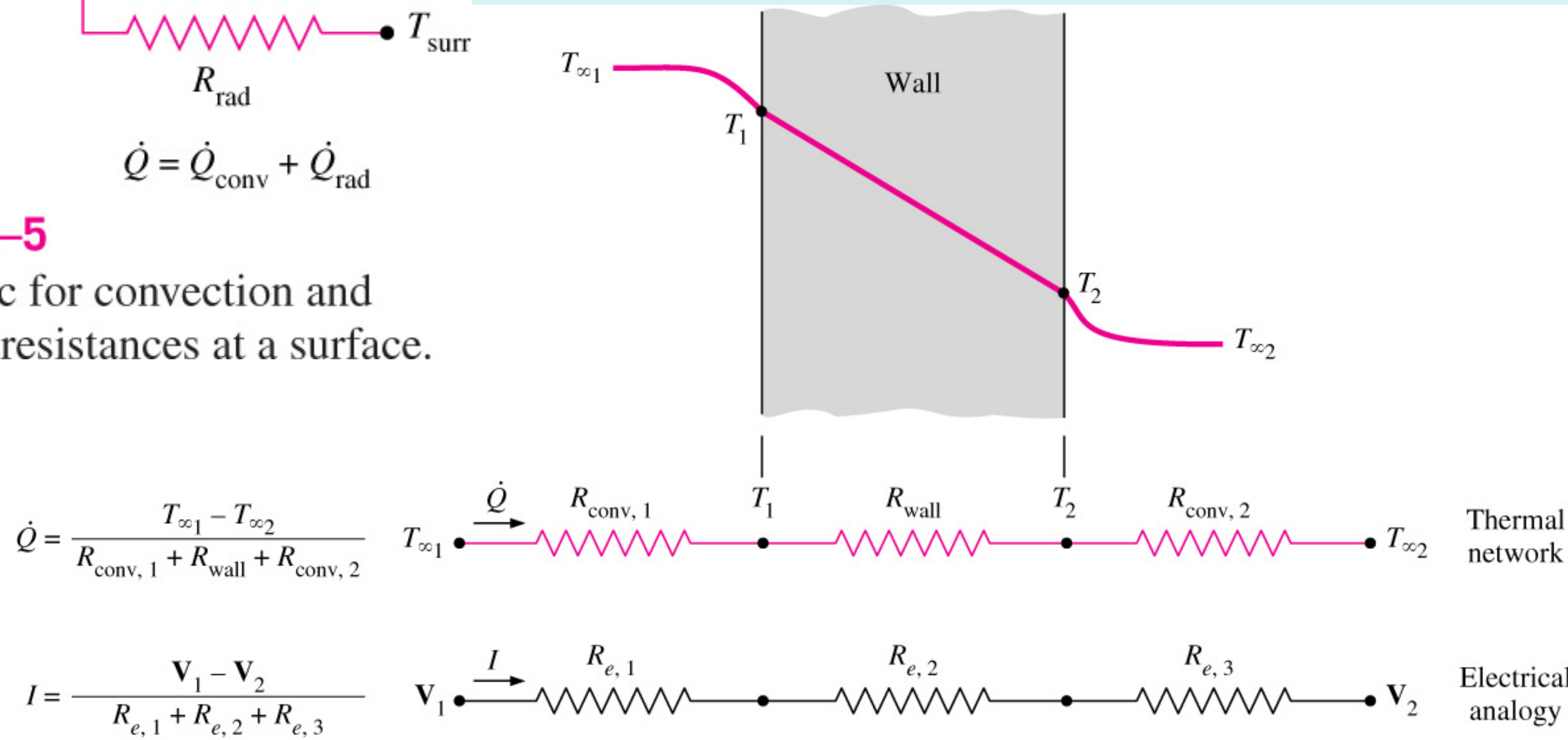


FIGURE 3–6
The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Under steady conditions

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

Adding the numerators and denominators yields

$$\dot{Q} = \frac{T_{\infty} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)}$$

where

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (^\circ\text{C/W})$$

The thermal resistances are in *series,* and the equivalent thermal resistance is determined by simply *adding* the individual resistances, just like the electrical resistances connected in series.

The equation $\dot{Q} = \Delta T / R$ can be rearranged as $\Delta T = \dot{Q} R \quad (^\circ\text{C})$

Here, the *temperature drop* across any layer is equal to the *rate of heat transfer* times the *thermal resistance* across that layer.

If

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n} = c$$

then

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = c$$

For example,

$$\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = 0.25$$

and

$$\frac{1+2+5}{4+8+20} = 0.25$$

FIGURE 3–7
A useful mathematical identity.

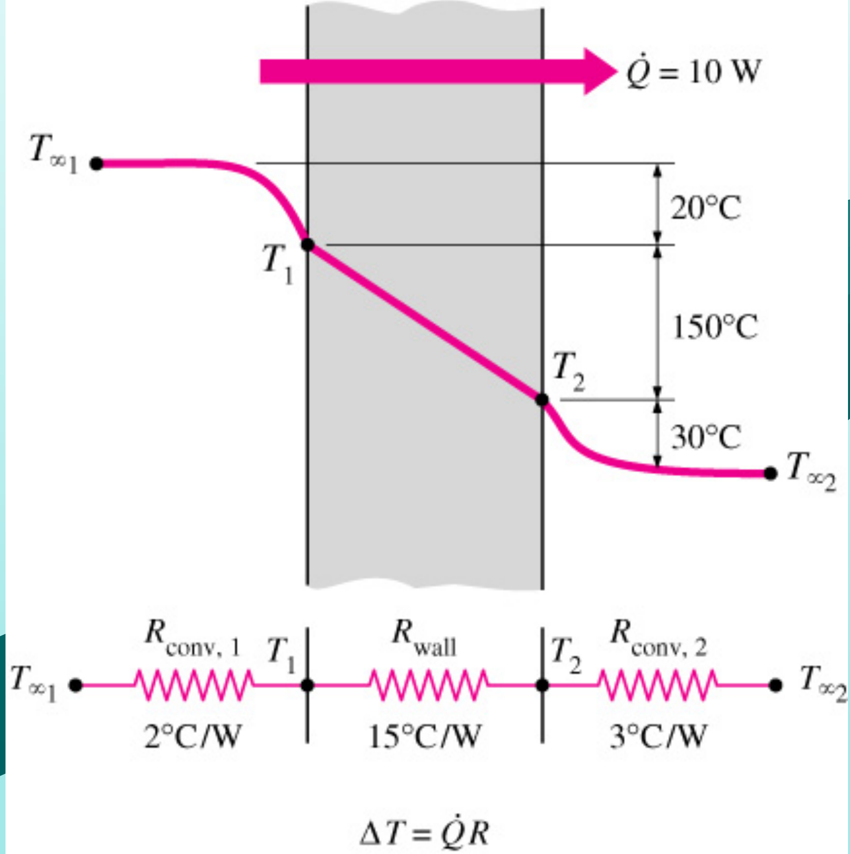


FIGURE 3–8
The temperature drop across a layer is proportional to its thermal resistance.

Analogous to Newton's law of cooling as $\dot{Q} = UA\,\Delta T \qquad (W)$

U: the overall heat transfer coefficient

$$UA = \frac{1}{R_{total}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,\,1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$

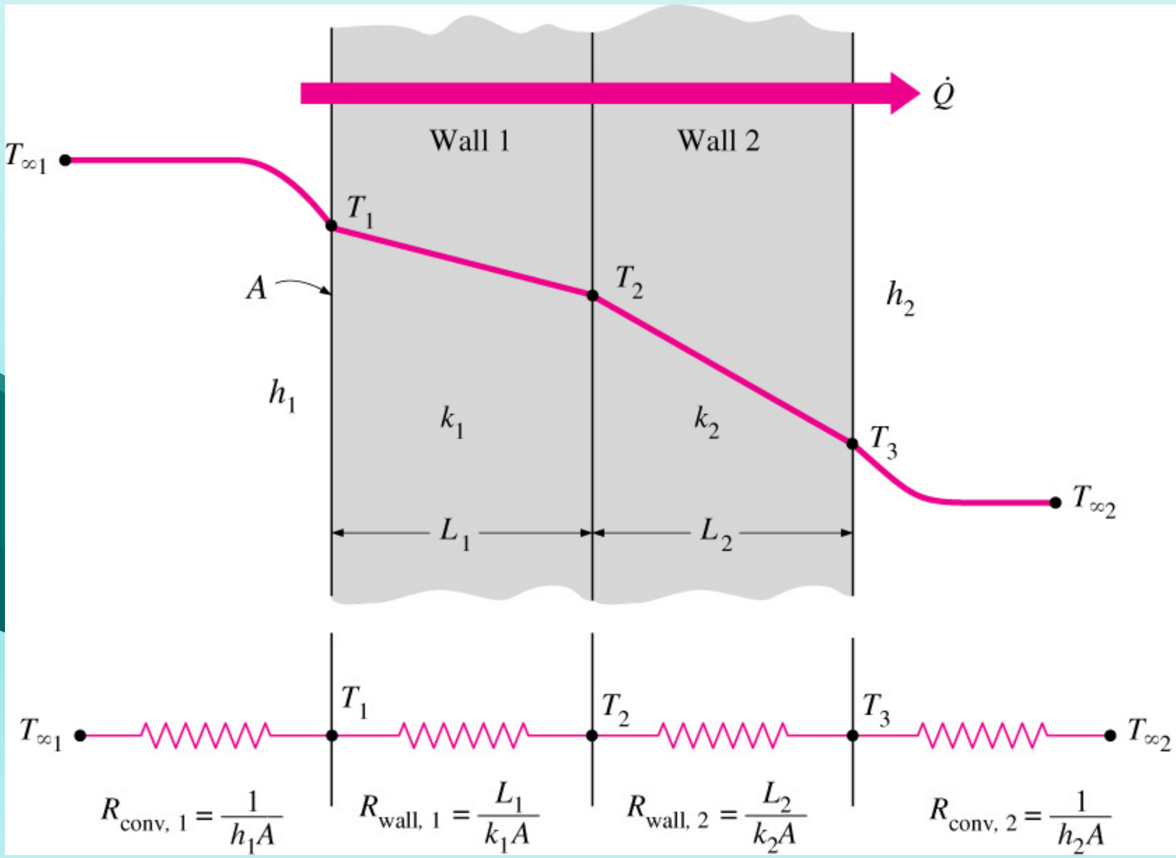


FIGURE 3–9
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

Multilayer Plane Walls

The rate of steady heat transfer through a plane wall consisting of two layers

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

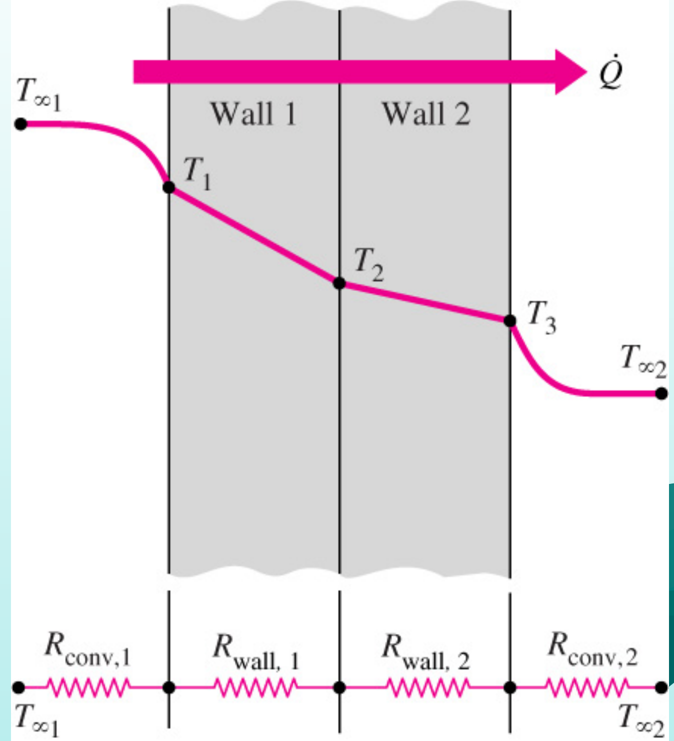
R_{total}: the *total thermal resistance*

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall},1} + R_{\text{wall},2} + R_{\text{conv},2} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

for the resistances *in series*.

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$

☞ It is limited to systems involving *steady heat transfer* with *no heat generation*.



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

FIGURE 3–10
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

EXAMPLE 3–1 **Heat Loss through a Wall**

Consider a 17-m-high, 5-m-wide, and 0.17-m-thick wall whose thermal conductivity is $k = 0.9$ W/m · °C (Fig. 17–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9$ W/m · °C.

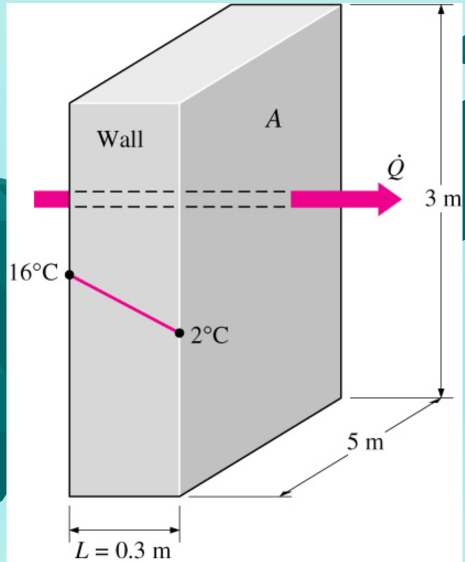


FIGURE 3–11
Schematic for Example 3–1.

Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 17–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9 \text{ W/m} \cdot {}^\circ\text{C})(15 \text{ m}^2)\frac{(16-2)^\circ\text{C}}{0.3 \text{ m}} = 630 \text{ W}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m} \cdot {}^\circ\text{C})(15 \text{ m}^2)} = 0.02222^\circ\text{C/ W}$$

Substituting, we get

$$\dot{Q} = \frac{(16-2)^\circ\text{C}}{0.02222^\circ\text{C/ W}} = 630 \text{ W}$$

Discussion This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples.

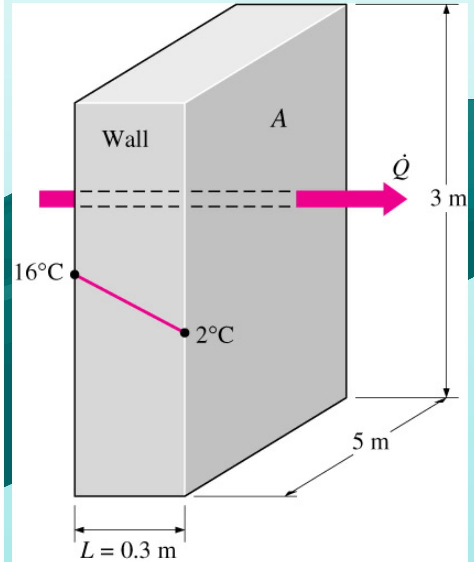


FIGURE 3–11
Schematic for Example 3–1.

EXAMPLE 3–2 **Heat Loss through a Single-Pane Window**

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 40$ W/m^2 · °C, which includes the effects of radiation.

SOLUTION Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m · °C.

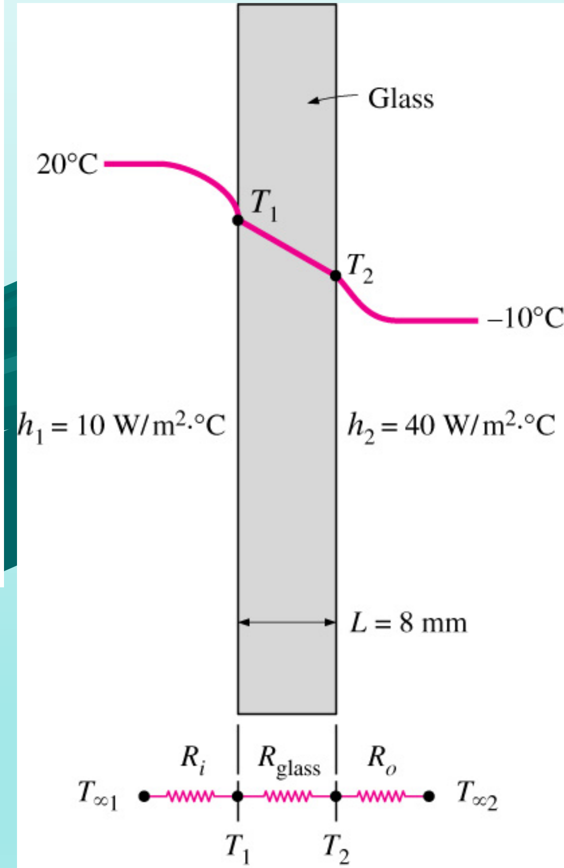


FIGURE 3–12
Schematic for Example 3–2.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 17–12. Noting that the area of the window is $A = 0.8\ \text{m} \times 1.5\ \text{m} = 1.2\ \text{m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008\ \text{m}}{(0.78\ \text{W/m} \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2\ \text{m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = 266\ \text{W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} \longrightarrow \begin{aligned} T_1 &= T_{\infty 1} - \dot{Q} R_{\text{conv, 1}} \\ &= 20^\circ\text{C} - (266\ \text{W})(0.08333^\circ\text{C/W}) \\ &= -2.2^\circ\text{C} \end{aligned}$$

Discussion Note that the inner surface temperature of the window glass will be -2.2°C even though the temperature of the air in the room is maintained at 20°C. Such low surface temperatures are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

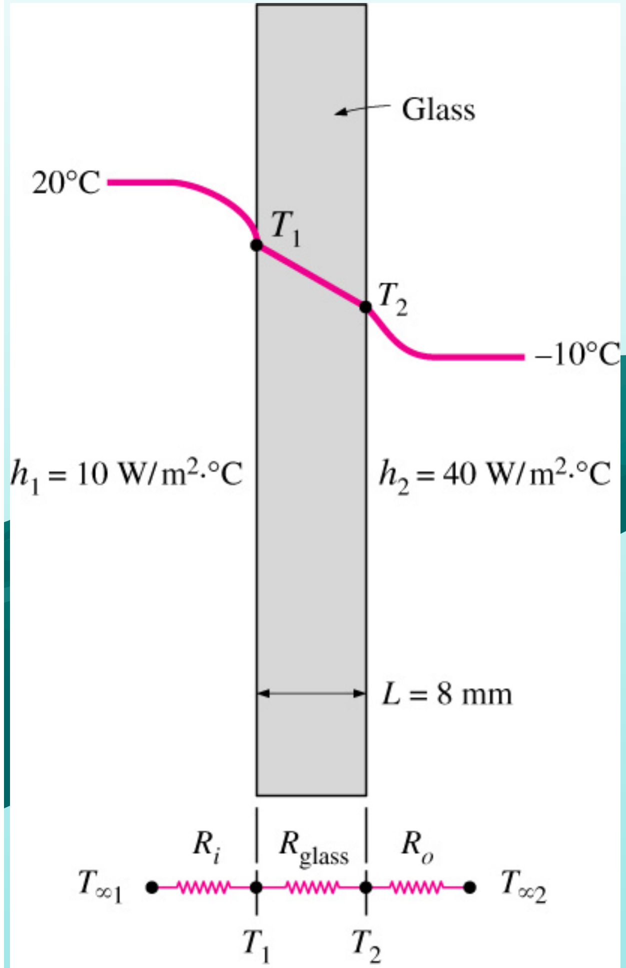


FIGURE 3–12
Schematic for Example 3–2.

EXAMPLE 3–3 Heat Loss through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass (k = 0.78 W/m · °C) separated by a 10-mm-wide stagnant air space (k = 0.026 W/m · °C). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be h_1 = 10 W/m^2 · °C and h_2 = 40 W/m^2 · °C, which includes the effects of radiation.

SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

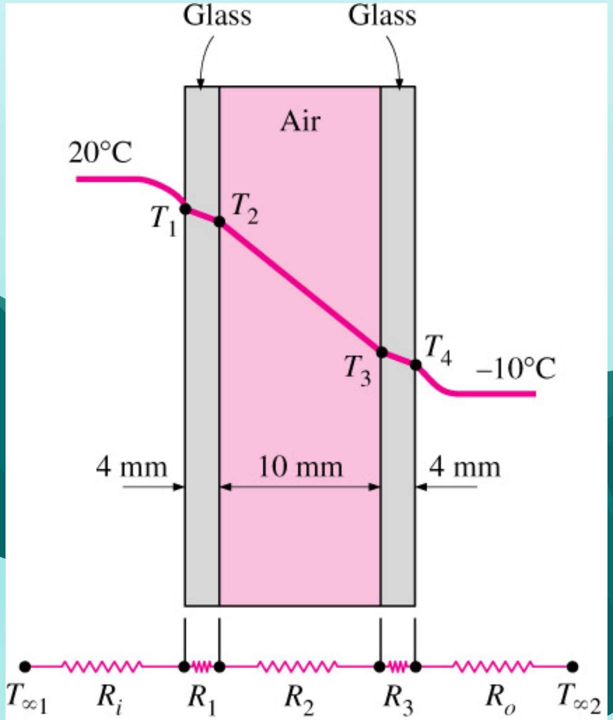


FIGURE 3–13
Schematic for Example 3–3.

Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem will involve two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 17–13. Noting that the area of the window is again $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m} \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.00427^\circ\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m} \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.3205^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass, 1}} + R_{\text{air}} + R_{\text{glass, 2}} + R_{\text{conv, 2}} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = 69.2 \text{ W}$$

which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv, 1}} = 20^\circ\text{C} - (69.2 \text{ W})(0.08333^\circ\text{C/W}) = 14.2^\circ\text{C}$$

which is considerably higher than the -2.2°C obtained in the previous example. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce the heat gain in summer, and thus reduce the air-conditioning costs.

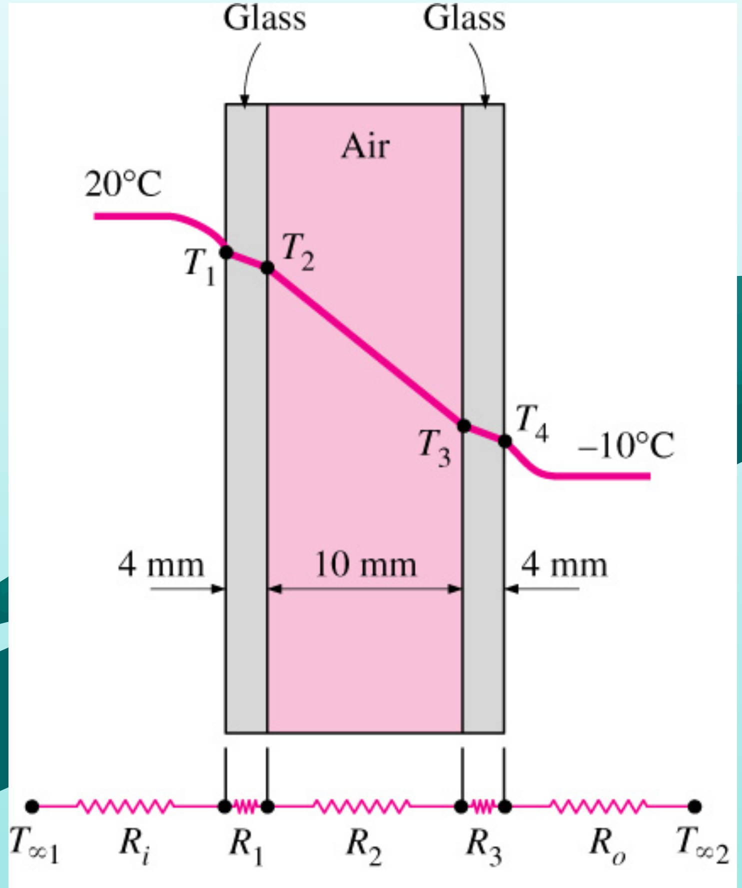


FIGURE 3–13

Schematic for Example 3–3.

THERMAL CONTACT RESISTANCE

In the analysis of heat conduction through multilayer solids, we assumed "perfect contact" at the interface of two layers, and thus no temperature drop at the interface.

Thermal Contact Resistance (*Rc*.): the resistance per unit interface area

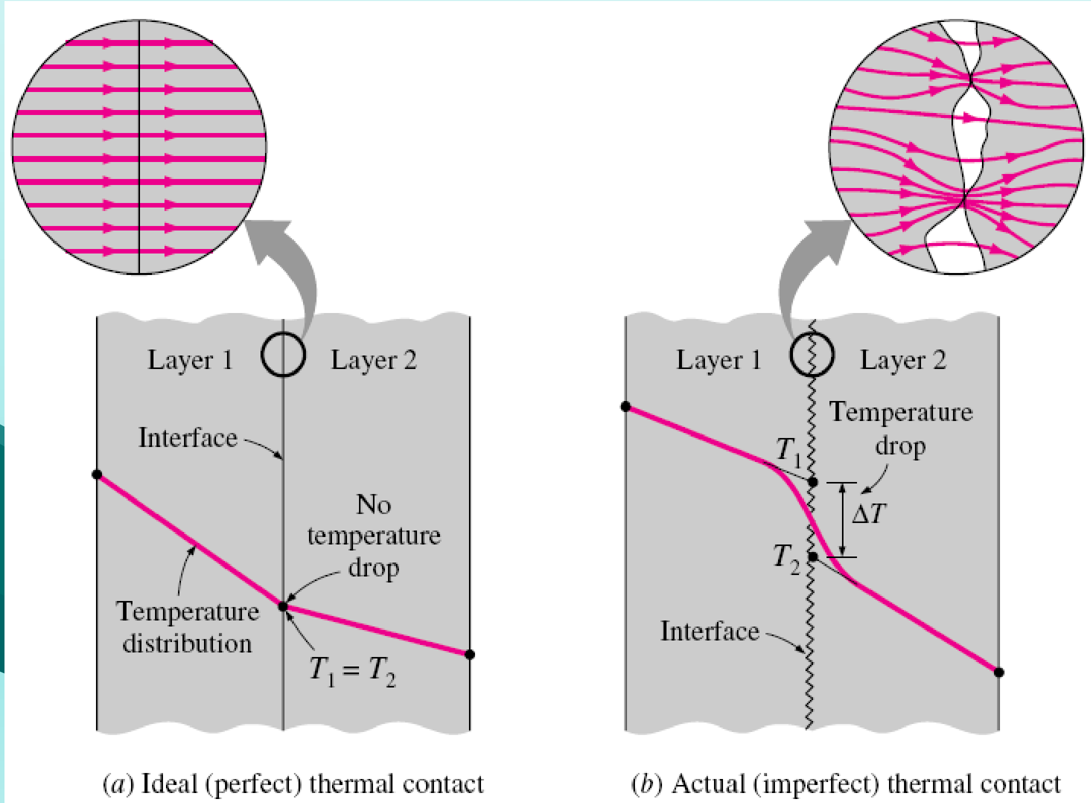


FIGURE 3–14
Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

Heat transfer through the interface of two metal rods of cross-sectional area A is the sum of the heat transfers through the *solid contact spots* and the *gaps* in the noncontact areas and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}}$$

An analogous manner to Newton's law of cooling:

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}}$$

A: the apparent interface area (which is the same as the cross-sectional area of the rods

$T_{\text{interface}}$: the effective temperature difference at the interface

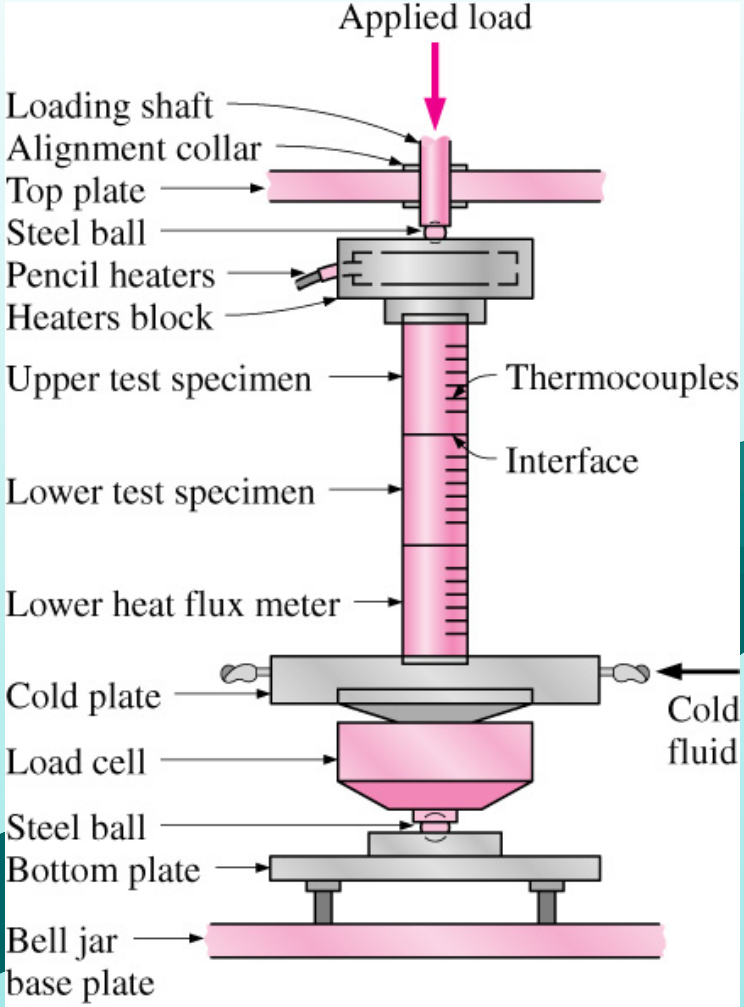


FIGURE 3–15

A typical experimental setup for the determination of thermal contact resistance (from Song et al.).

The thermal contact conductance is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C})$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {}^\circ\text{C/W})$$

The thermal resistance of a 1-cm-thick layer of an insulating material per unit surface area is

$$R_{c,\,\text{insulation}} = \frac{L}{k} = \frac{0.01\ \text{m}}{0.04\ \text{W/m} \cdot {}^\circ\text{C}} = 0.25\ \text{m}^2 \cdot {}^\circ\text{C/W}$$

whereas for a 1-cm-thick layer of copper, it is

$$R_{c,\,\text{copper}} = \frac{L}{k} = \frac{0.01\ \text{m}}{386\ \text{W/m} \cdot {}^\circ\text{C}} = 0.000026\ \text{m}^2 \cdot {}^\circ\text{C/W}$$

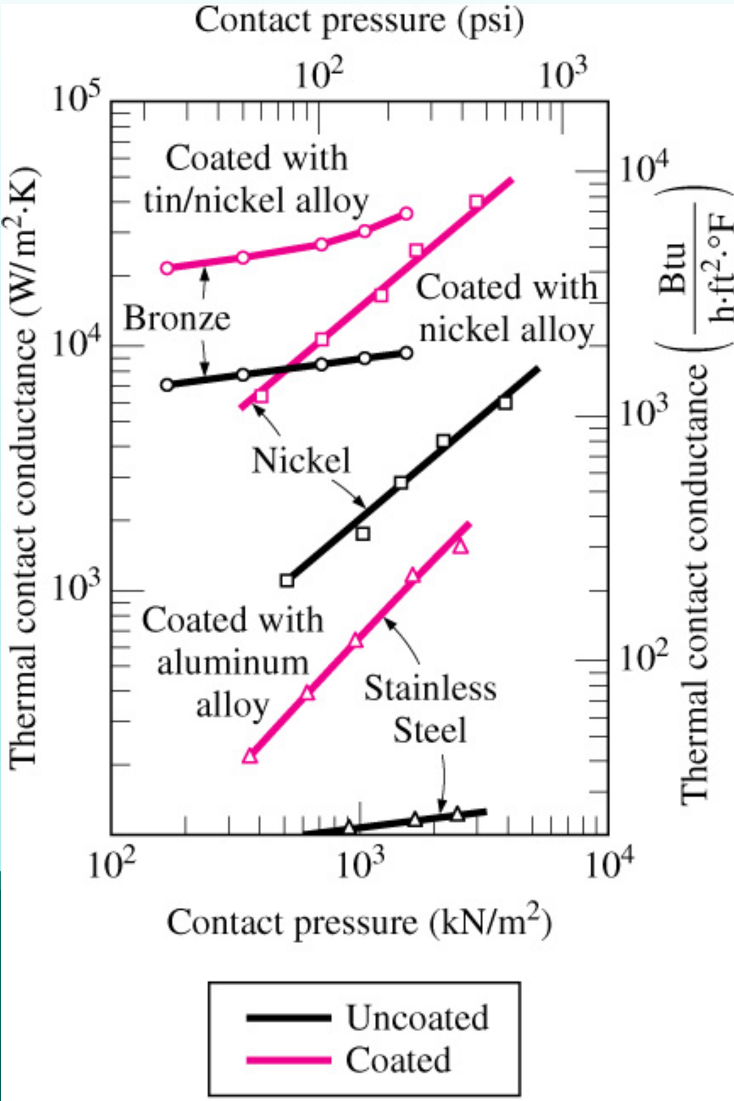


FIGURE 3–16
Effect of metallic coatings on thermal contact conductance (from Peterson).

TABLE 3–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried).

Fluid at the interface	Contact conductance, h_c, $W/m^2 \cdot K$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

TABLE 3–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m^2 · °C
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar Metal Pairs					
Stainless steel–				10	2900
Aluminum		20–30	20	20	3600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4800
Aluminum	Milled	4.5–7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.17–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	22,000

*Divide the given values by 5.678 to convert to Btu/h · ft^2 · °F.

The *thermal contact conductance* is *highest* (with the lowest contact resistance) for *soft metals* with *smooth surfaces* at *high pressure.*

EXAMPLE 3–4 Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be 11,000 W/m² · °C. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (Fig. 17–17).

SOLUTION The thickness of the aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of aluminum at room temperature is $k = 237$ W/m · °C (Table A–25).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11{,}000 \text{ W/m}^2 \cdot {}^\circ\text{C}} = 0.909 \times 10^{-4} \text{ m}^2 \cdot {}^\circ\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (237 \text{ W/m} \cdot {}^\circ\text{C})(0.909 \times 10^{-4} \text{ m}^2 \cdot {}^\circ\text{C/W}) = 0.0215 \text{ m} = \mathbf{2.15 \text{ cm}}$$

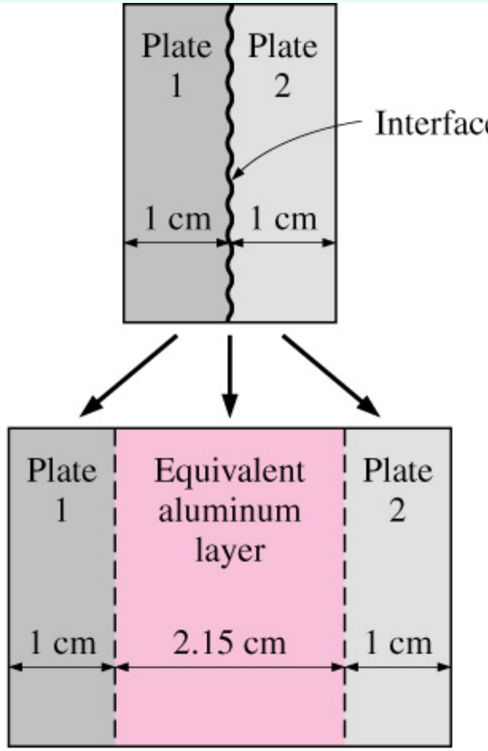


FIGURE 3–17
Schematic for Example 3–4.

EXAMPLE 3–5 Contact Resistance of Transistors

Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm × 20-cm square copper plate (k = 386 W/m · °C) by screws that exert an average pressure of 6 MPa (Fig. 17–18). The base area of each transistor is 8 cm^2, and each transistor is placed at the center of a 10-cm × 10-cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 W/m^2 · °C. If the case temperature of the transistor is not to exceed 70°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.

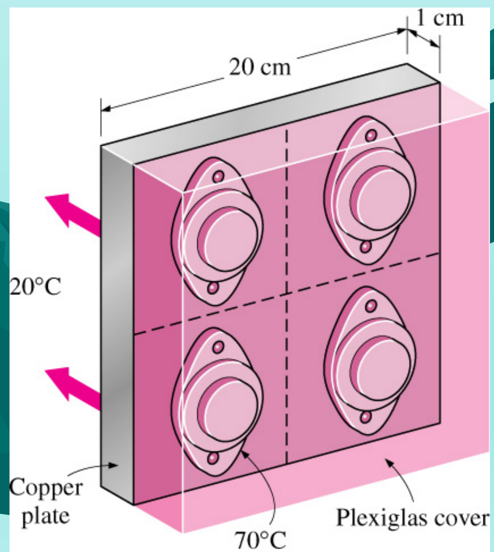


FIGURE 3–18
Schematic for Example 3–5.

SOLUTION Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. **3** All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. **4** Thermal conductivities are constant.

Properties The thermal conductivity of copper is given to be k = 386 W/m · °C. The contact conductance is obtained from Table 17–2 to be h_c = 42,000 W/m² · °C, which corresponds to copper-aluminum interface for the case of 1.17–1.4 μm roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis The contact area between the case and the plate is given to be 8 cm², and the plate area for each transistor is 100 cm². The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{\text{interface}} = \frac{1}{h_c A_c} = \frac{1}{(42{,}000\ \text{W/m}^2\cdot{}^\circ\text{C})(8\times 10^{-4}\ \text{m}^2)} = 0.030^\circ\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(386\ \text{W/m}\cdot{}^\circ\text{C})(0.01\ \text{m}^2)} = 0.0026^\circ\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{h_o A} = \frac{1}{(25\ \text{W/m}^2\cdot{}^\circ\text{C})(0.01\ \text{m}^2)} = 4.0^\circ\text{C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{plate}} + R_{\text{ambient}} = 0.030 + 0.0026 + 4.0 = 4.0326°\text{C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(70 - 20)°\text{C}}{4.0326°\text{C/W}} = \mathbf{12.4\ W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C.

The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q}R_{\text{interface}} = (12.4\text{ W})(0.030°\text{C/W}) = \mathbf{0.37°C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we will lower the operating temperature of the transistor in this case by less than 0.4°C.

GENERALIZED THERMAL RESISTANCE NETWORKS

For the composite wall consisting of two parallel layers, the total heat transfer is the sum of the heat transfers through each layer.

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

With the electrical analogy

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

with

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$

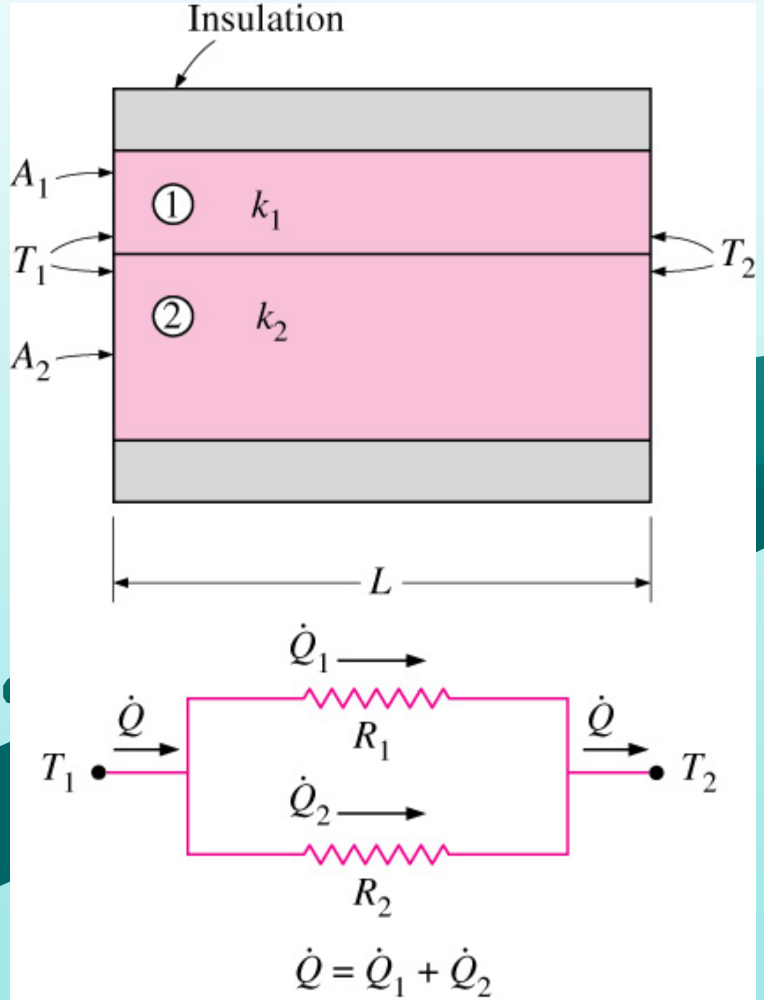


FIGURE 3–19
Thermal resistance network for two parallel layers.

For the combined series-parallel arrangement, the total rate of heat transfer through this composite system is

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

with

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

and

$$R_1 = \frac{L_1}{k_1 A_1}, \qquad R_2 = \frac{L_2}{k_2 A_2}, \qquad R_3 = \frac{L_3}{k_3 A_3}, \qquad R_{\text{conv}} = \frac{1}{hA_3}$$

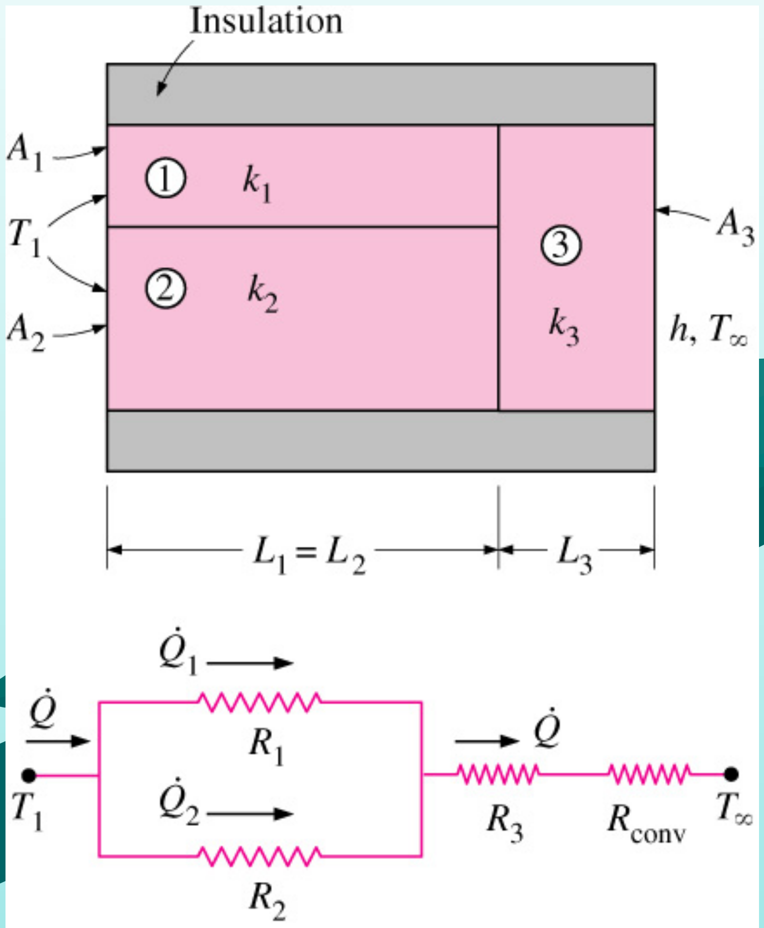


FIGURE 3–20
Thermal resistance network for combined series-parallel arrangement.

Two assumptions:
(i) any plane wall normal to the *x*-axis is *isothermal* and
(ii) any plane parallel to the *x*-axis is *adiabatic.*

These assumptions result in different resistance networks, while the actual result lies between two assumptions.

HEAT CONDUCTION IN CYLINDERS AND SPHERES

The Fourier's law of heat conduction for heat transfer through the cylindrical layer is

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

Here, $A = 2\pi rL$ is the heat transfer area at location r

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

We obtain

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)}$$

since $\dot{Q}_{\text{cond, cyl}}$ = constant.

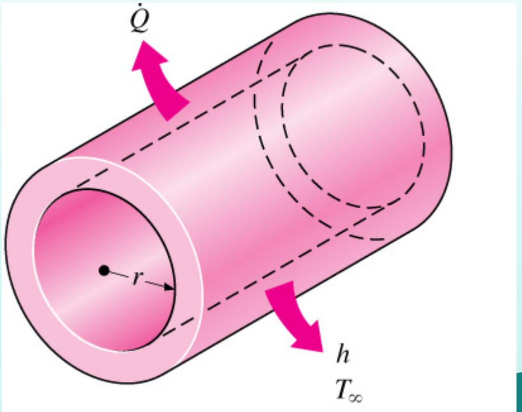


FIGURE 3–23
Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

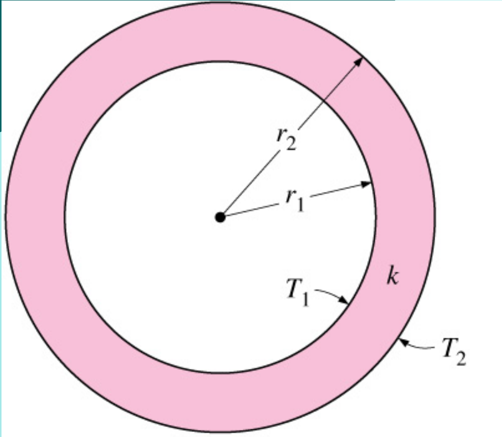


FIGURE 3–24
A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

The *thermal resistance* of the cylindrical layer against heat conduction, or simply the **conduction resistance** of the cylinder layer.

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})}$$

Repeating the analysis for a *spherical layer* by taking $A = 4\pi r^2$

$$\dot{Q}_{cond,\ sph} = \frac{T_1 - T_2}{R_{sph}}$$

with

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

which is the *thermal resistance* of the spherical layer against heat conduction, or simply the **conduction resistance** of the spherical layer.

The rate of heat transfer through a cylindrical or spherical layer under steady conditions:

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

where

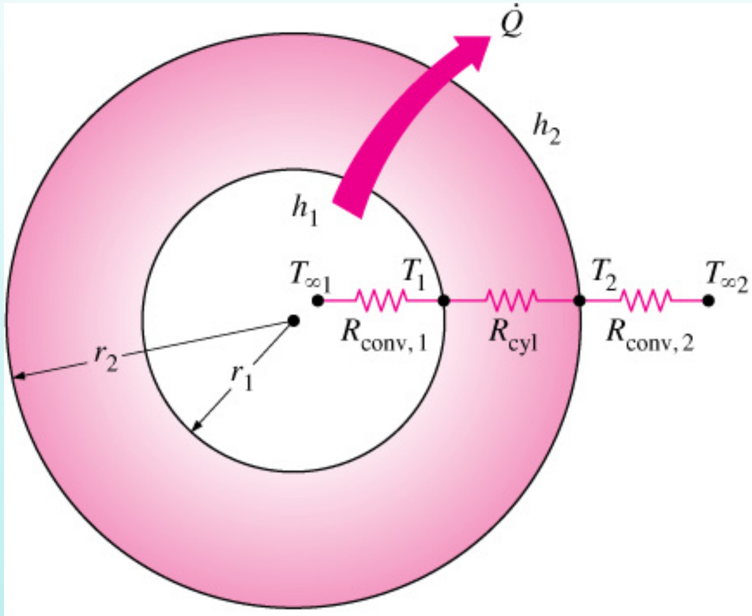
$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$

for a *cylindrical* layer, and

$$R_{total} = R_{conv,1} + R_{sph} + R_{conv,2} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

for a *spherical* layer.

A in the convection resistance relation $R_{conv} = 1/hA$ is the *surface area at which convection occurs.* It is equal to $A = 2\pi rL$ for a cylindrical surface and $A = 4\pi r^2$ for a spherical surface of radius *r*.



$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$$

FIGURE 3–25
The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

Multilayered Cylinders and Spheres

Steady heat transfer through multilayered cylindrical or spherical shells is treated like multilayered plane walls.

$$\boxed{\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}}$$

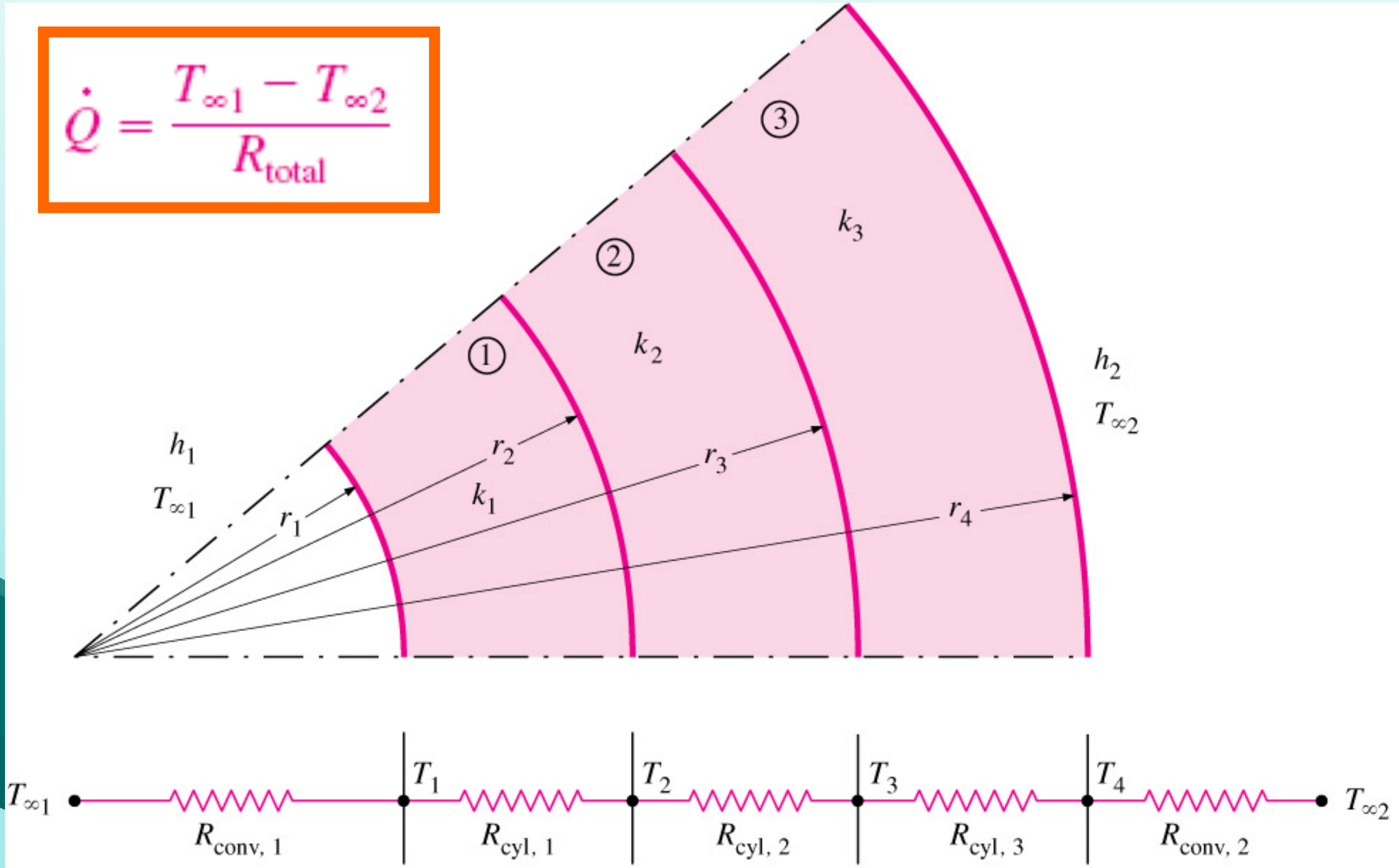


FIGURE 3–26

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

R_{total} is the *total thermal resistance*, expressed as

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

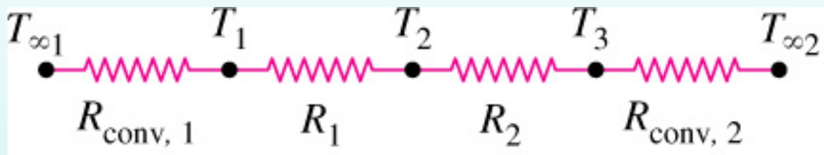
Here, $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$

The total thermal resistance is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{cyl,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

We can also calculate T_2 from

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{conv,2}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$



$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$$
$$= \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_1}$$
$$= \frac{T_1 - T_3}{R_1 + R_2}$$
$$= \frac{T_2 - T_3}{R_2}$$
$$= \frac{T_2 - T_{\infty 2}}{R_2 + R_{conv,2}}$$
$$= \cdots$$

FIGURE 3–27

The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

EXAMPLE 3–7 Heat Transfer to a Spherical Container

A 17-m internal diameter spherical tank made of 2-cm-thick stainless steel ($k = 15$ W/m · °C) is used to store iced water at $T_{\infty 1} = 0$°C. The tank is located in a room whose temperature is $T_{\infty 2} = 22$°C. The walls of the room are also at 22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are $h_1 = 80$ W/m² · °C and $h_2 = 10$ W/m² · °C, respectively. Determine (*a*) the rate of heat transfer to the iced water in the tank and (*b*) the amount of ice at 0°C that melts during a 24-h period.

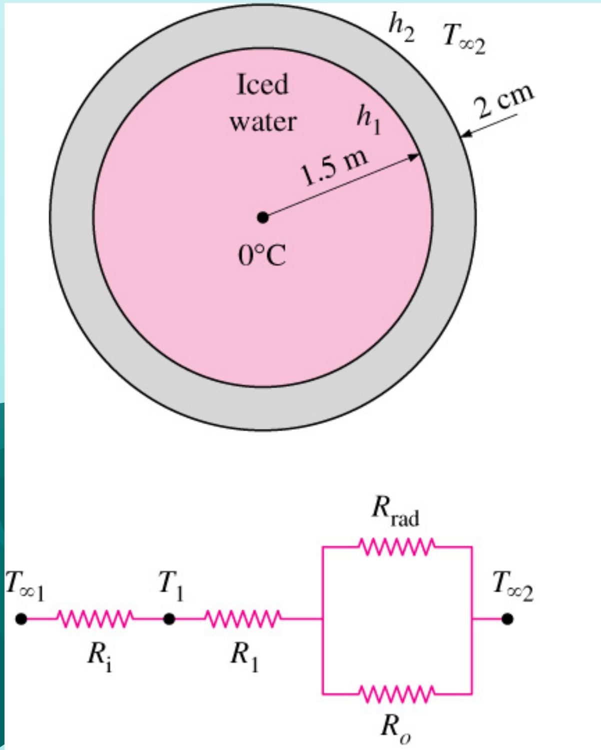


FIGURE 3–28
Schematic for Example 3–7.

SOLUTION A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be $k = 15$ W/m · °C. The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. The outer surface of the tank is black and thus its emissivity is $\varepsilon = 1$.

Analysis (*a*) The thermal resistance network for this problem is given in Fig. 17–28. Noting that the inner diameter of the tank is $D_1 = 3$ m and the outer diameter is $D_2 = 3.04$ m, the inner and the outer surface areas of the tank are

$$A_1 = \pi D_1^2 = \pi(3 \text{ m})^2 = 28.3 \text{ m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04 \text{ m})^2 = 29.0 \text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{\text{rad}} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking T_2 = 5°C = 278 K, the radiation heat transfer coefficient is determined to be

$$h_{\text{rad}} = (1)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(295\ \text{K})^2 + (278\ \text{K})^2][(295 + 278)\ \text{K}]$$
$$= 5.34\ \text{W/m}^2 \cdot \text{K} = 5.34\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A_1} = \frac{1}{(80\ \text{W/m}^2 \cdot {}^\circ\text{C})(28.3\ \text{m}^2)} = 0.000442^\circ\text{C/W}$$

$$R_1 = R_{\text{sphere}} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50)\ \text{m}}{4\pi\ (15\ \text{W/m} \cdot {}^\circ\text{C})(1.52\ \text{m})(1.50\ \text{m})}$$
$$= 0.000047^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_2} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(29.0\ \text{m}^2)} = 0.00345^\circ\text{C/W}$$

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A_2} = \frac{1}{(5.34\ \text{W/m}^2 \cdot {}^\circ\text{C})(29.0\ \text{m}^2)} = 0.00646^\circ\text{C/W}$$

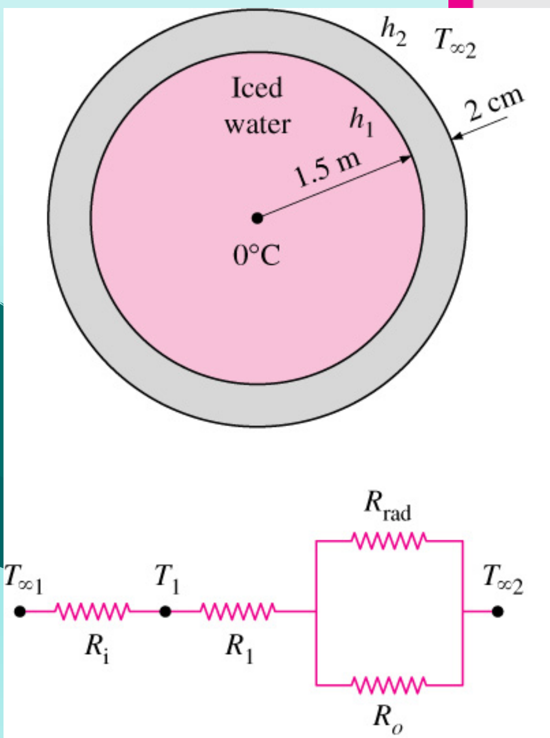


FIGURE 3–28
Schematic for Example 3–7.

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{equiv}} = \frac{1}{R_o} + \frac{1}{R_{rad}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7 \text{ W/°C}$$

which gives

$$R_{equiv} = 0.00225\text{°C/ W}$$

Now all the resistances are in series, and the total resistance is determined to be

$$R_{total} = R_i + R_1 + R_{equiv} = 0.000442 + 0.000047 + 0.00225 = 0.00274\text{°C/ W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(22 - 0)\text{°C}}{0.00274\text{°C/ W}} = \mathbf{8029\ W} \qquad (\text{or } \dot{Q} = 8.027 \text{ kJ/s})$$

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equiv}} \longrightarrow \quad T_2 = T_{\infty 2} - \dot{Q}R_{equiv}$$
$$= 22\text{°C} - (8029 \text{ W})(0.00225\text{°C/ W}) = 4\text{°C}$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.

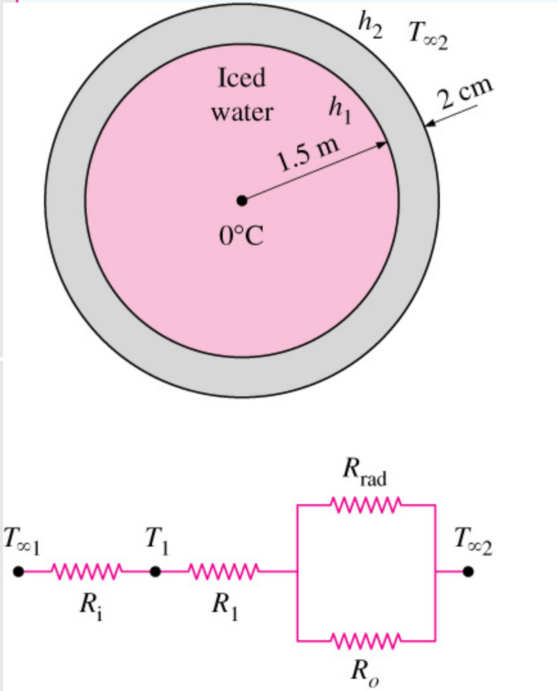


FIGURE 3–28
Schematic for Example 3–7.

(*b*) The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\,\Delta t = (8.029 \text{ kJ/s})(24 \times 3600 \text{ s}) = 673{,}700 \text{ kJ}$$

Noting that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the amount of ice that will melt during a 24-h period is

$$m_{\text{ice}} = \frac{Q}{h_{if}} = \frac{673{,}700 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{2079 \text{ kg}}$$

Therefore, about 2 metric tons of ice will melt in the tank every day.

Discussion An easier way to deal with combined convection and radiation at a surface when the surrounding medium and surfaces are at the same temperature is to add the radiation and convection heat transfer coefficients and to treat the result as the convection heat transfer coefficient. That is, to take $h = 10 + 5.34 = 15.34 \text{ W/m}^2 \cdot °\text{C}$ in this case. This way, we can ignore radiation since its contribution is accounted for in the convection heat transfer coefficient. The convection resistance of the outer surface in this case would be

$$R_{\text{combined}} = \frac{1}{h_{\text{combined}} A_2} = \frac{1}{(15.34 \text{ W/m}^2 \cdot °\text{C})(29.0 \text{ m}^2)} = 0.00225°\text{C/ W}$$

which is identical to the value obtained for the equivalent resistance for the parallel convection and the radiation resistances.

CRITICAL RADIUS OF INSULATION

The rate of heat transfer from the insulated pipe to the surrounding air is

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}}$$

Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body to be

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)}$$

The critical radius of insulation for a spherical shell is

$$r_{\text{cr, sphere}} = \frac{2k}{h}$$

k : the thermal conductivity of the insulation

h : the convection heat transfer coefficient on the outer surface

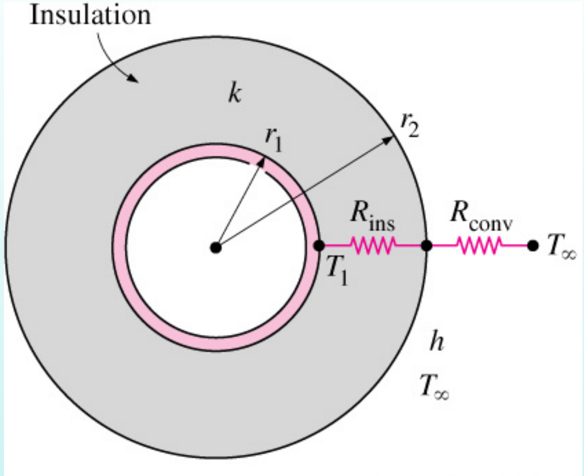


FIGURE 3–30
An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

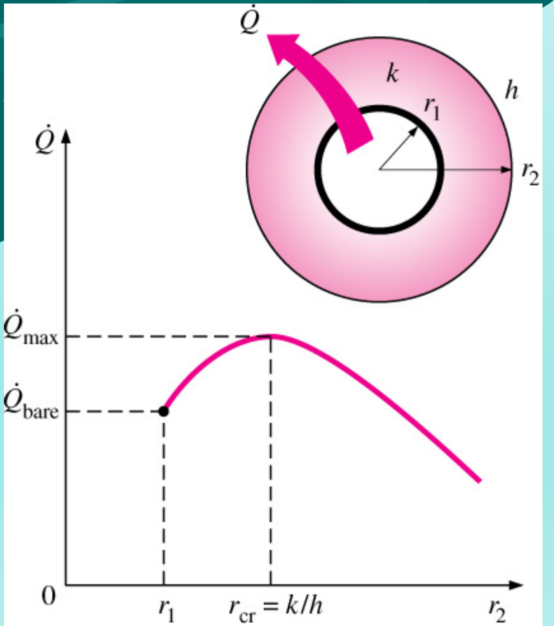


FIGURE 3–31

EXAMPLE 3–9 Heat Loss from an Insulated Electric Wire

A 17-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30$°C with a heat transfer coefficient of $h = 12$ W/m^2 · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

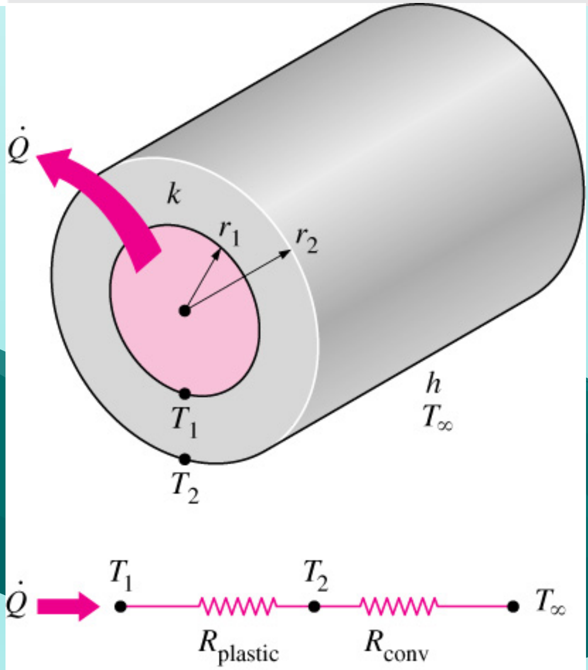


FIGURE 3–32
Schematic for Example 3–9.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ W/m · °C.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 17–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot{}^\circ\text{C})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot{}^\circ\text{C})(5\text{ m})} = 0.18^\circ\text{C/W}$$

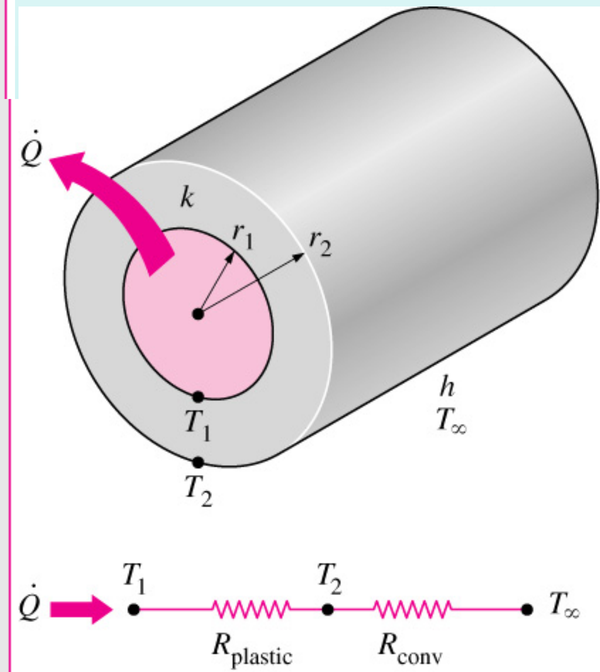


FIGURE 3–32
Schematic for Example 3–9.

and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94°\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}}$$

$$= 30°\text{C} + (80\text{ W})(0.94°\text{C/W}) = \mathbf{105°C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 17–50 to be

$$r_{\text{cr}} = \frac{k}{h} = \frac{0.15\text{ W/m}\cdot°\text{C}}{12\text{ W/m}^2\cdot°\text{C}} = 0.0125\text{ m} = 12.5\text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations above for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.

HEAT TRANSFER FROM FINNED SURFACES

The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T is given by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

As : the heat transfer surface area
h : the convection heat transfer coefficient

There are *two ways* to increase the rate of heat transfer:
1) to increase the *convection heat transfer coefficient h*
2) to increase the *surface area A_s*

Increasing *h* may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate.

The alternative is to increase the surface area by attaching to the surface *extended surfaces* called *fins* made of highly conductive materials such as aluminum.

Consider *steady* operation with *no heat generation* in the fin with the following assumptions:

- The thermal conductivity k of the material remains constant.
- The convection heat transfer coefficient h *is constant* and *uniform* over the entire surface of the fin for convenience in the analysis.

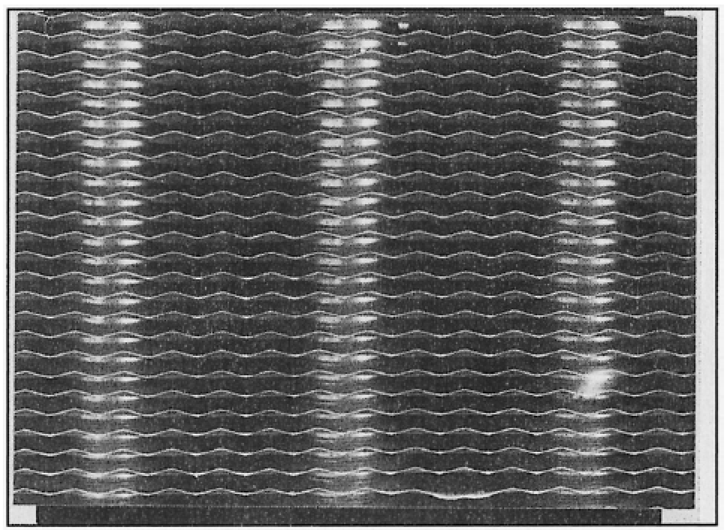


FIGURE 3–33
The thin plate fins of a car radiator greatly increase the rate of heat transfer to the air. (© Yunus Çengel, photo by James Kleiser.)

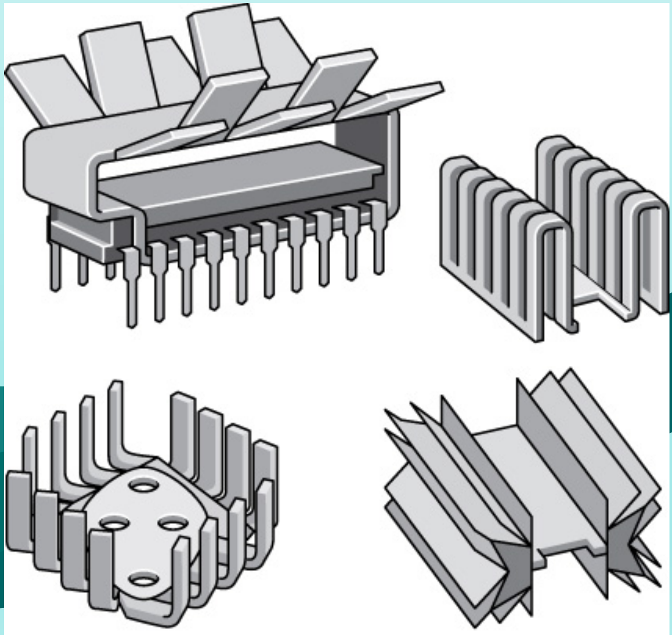


FIGURE 3–34
Some innovative fin designs.

Fin Equation

Under steady conditions, the energy balance on this volume element can be expressed as

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$

or

$$\dot{Q}_{\text{cond},\,x} = \dot{Q}_{\text{cond},\,x+\Delta x} + \dot{Q}_{\text{conv}}$$

with

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{\text{cond},\,x+\Delta x} - \dot{Q}_{\text{cond},\,x}}{\Delta x} + hp(T - T_\infty) = 0$$

Taking the limit as $\Delta x \rightarrow 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0$$

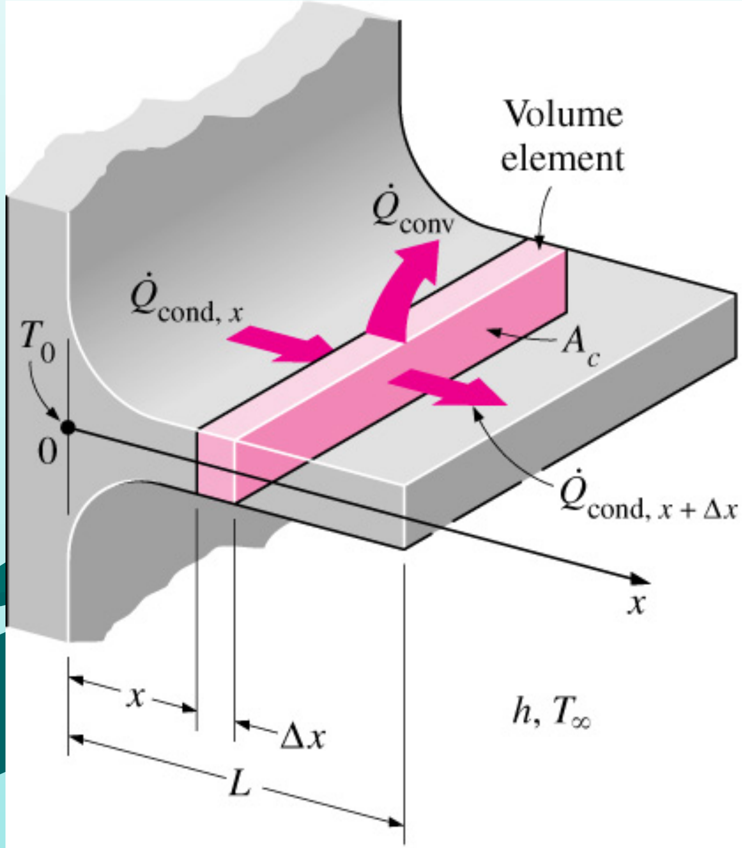


FIGURE 3–35
Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

From Fourier's law of heat conduction we have $\dot{Q}_{\text{cond}} = -kA_c \frac{dT}{dx}$

where A_c : the cross-sectional area of the fin at location x

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

In the special case (with *constant cross section* and *thermal conductivity*):

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0 \quad \text{with} \quad a^2 = \frac{hp}{kA_c}$$

and $\theta = T - T_\infty$ is the *temperature excess*.

At the fin base we have $\theta_b = T_b - T_\infty$.

The function u and its second derivative must be *constant multiples* of each other.

$$\theta(x) = C_1 e^{ax} + C_2 e^{-ax}$$

where C_1 and C_2 are arbitrary constants.

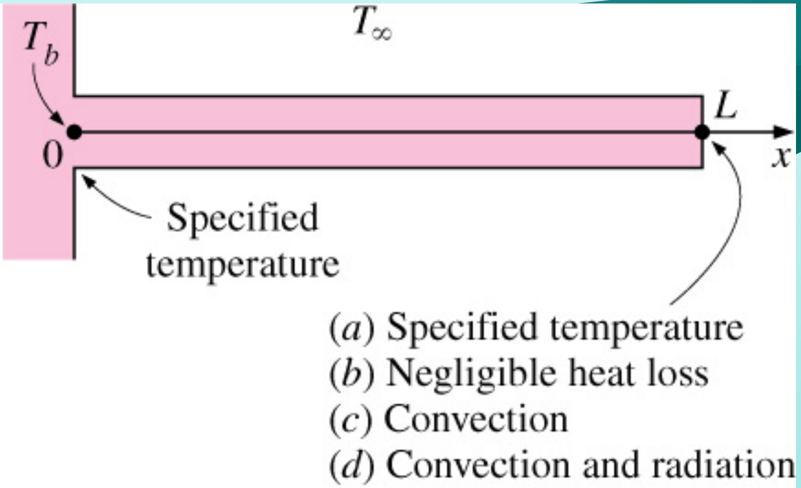


FIGURE 3–36
Boundary conditions at the fin base and the fin tip.

Boundary condition at fin base: $\theta(0) = \theta_b = T_b - T_\infty$

Infinitely Long Fin ($T_{fin\ tip} = T_\infty$)

For a sufficiently long fin of *uniform* cross section (*Ac* constant):

Boundary condition at fin tip: $\theta(L) = T(L) - T_\infty = 0$ as $L \to \infty$

Very long fin: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}}$$

Very long fin: $$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

p : the perimeter

A_c : the cross-sectional area of the fin

x : the distance from the fin base

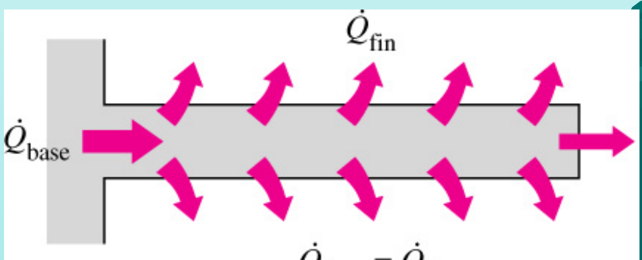
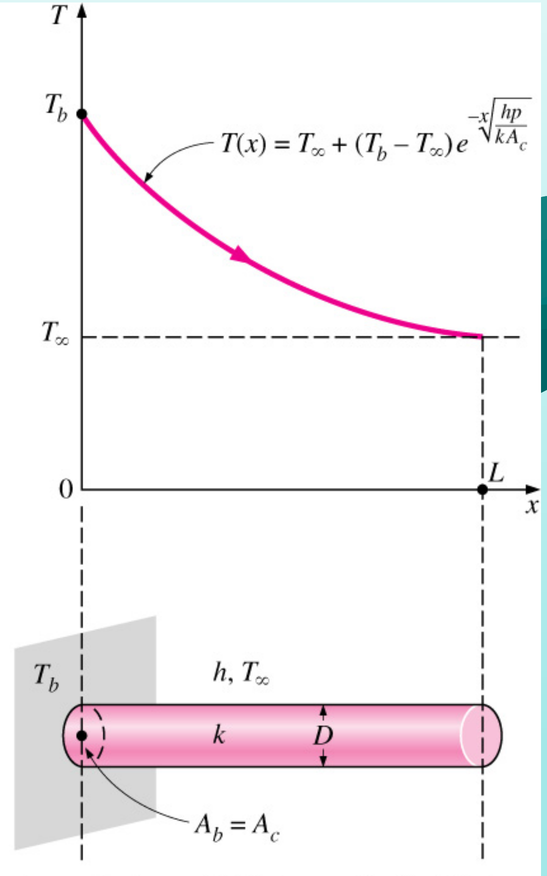


FIGURE 3–38
Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.



($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

FIGURE 3–37
A long circular fin of uniform cross section and the variation of temperature along it.

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty]\, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x)\, dA_{\text{fin}}$$

Negligible Heat Loss from the Fin Tip (Insulated fin tip, $\dot{Q}_{\text{fin tip}} = 0$)

The fin tip can be assumed to be insulated, and the condition at the fin tip can be expressed as

Boundary condition at fin tip: $$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

Adiabatic fin tip: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL}$$

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction:

Adiabatic fin tip: $$\dot{Q}_{\text{insulated tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL$$

The heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor tanh *aL*, which approaches 1 as *L* becomes very large.

Convection (or Combined Convection and Radiation) from Fin Tip

A practical way of accounting for the heat loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case by a **corrected length** defined as

Corrected fin length: $$L_c = L + \frac{A_c}{p}$$

t: the thickness of the rectangular fins
D: the diameter of the cylindrical fins.

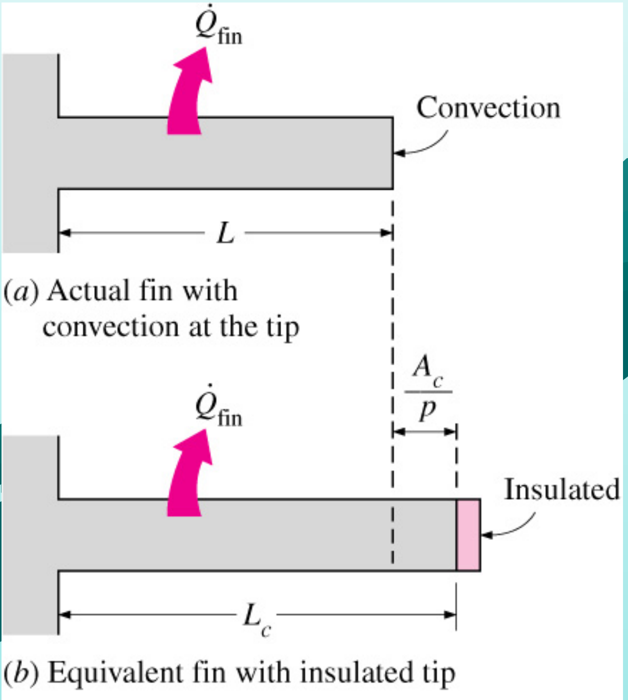


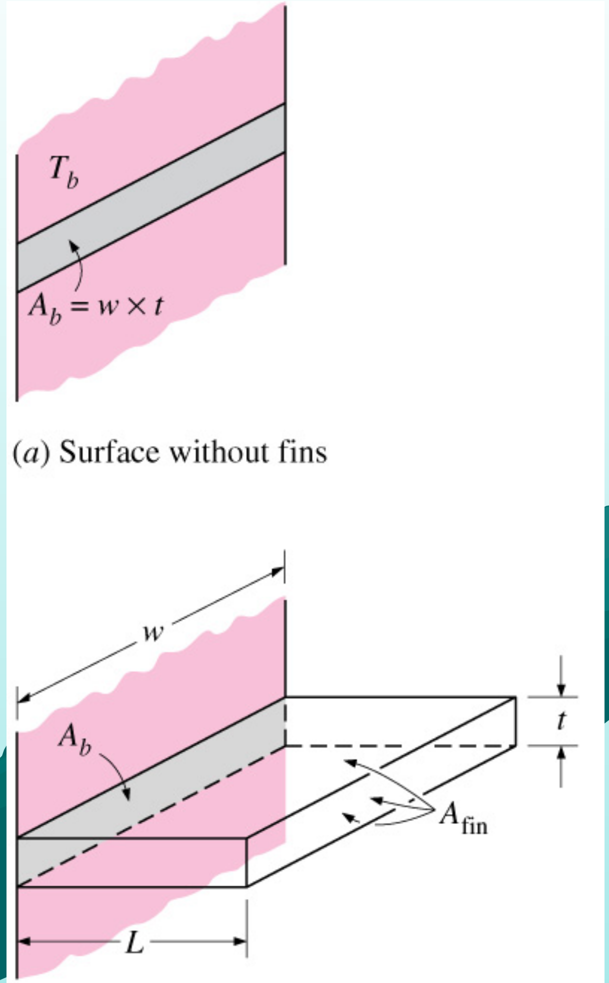
FIGURE 3–39
Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.

Fin Efficiency

In the limiting case of *zero thermal resistance* or *infinite thermal conductivity*, $(k \to \infty)$ the temperature of the fin will be uniform at the base value of T_b.

The heat transfer from the fin will be *maximum* in this case and can be expressed as

$$\dot{Q}_{\text{fin, max}} = hA_{\text{fin}}\,(T_b - T_\infty)$$



(*a*) Surface without fins

(*b*) Surface with a fin

$$A_{\text{fin}} = 2 \times w \times L + w \times t$$
$$\cong 2 \times w \times L$$

FIGURE 3–40
Fins enhance heat transfer from a surface by enhancing surface area.

Fin efficiency can be defined as:

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

or

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\, \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}\, hA_{\text{fin}}\, (T_b - T_\infty)$$

For the cases of constant cross section of *very long fins* and *fins with insulated tips*, the fin efficiency can be expressed as

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{aL}$$

and

$$\eta_{\text{insulated tip}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{\tanh aL}{aL}$$

since $A_{\text{fin}} = pL$ for fins with constant cross section.

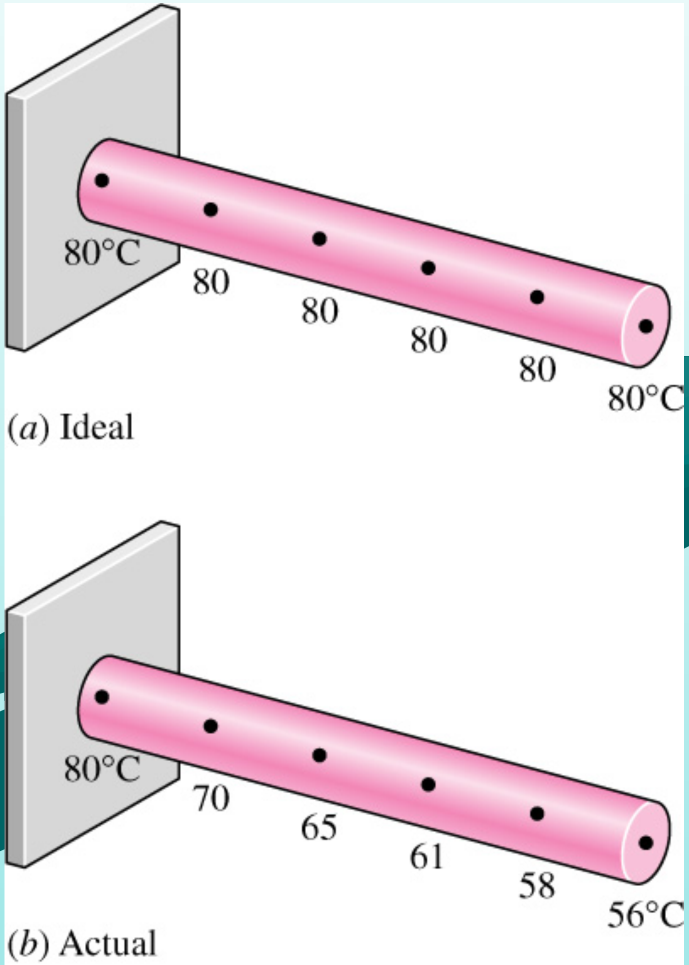


FIGURE 3–41
Ideal and actual temperature distribution along a fin.

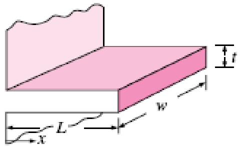
TABLE 3–3

Efficiency and surface areas of common fin configurations

Straight rectangular fins

$m = \sqrt{2h/kt}$
$L_c = L + t/2$
$A_{fin} = 2wL_c$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$

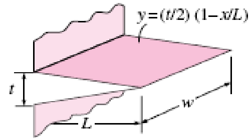


Straight triangular fins

$m = \sqrt{2h/kt}$

$A_{fin} = 2w\sqrt{L^2 + (t/2)^2}$

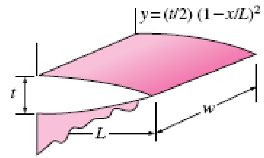
$$\eta_{fin} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



Straight parabolic fins

$m = \sqrt{2h/kt}$
$A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)]$
$C_1 = \sqrt{1 + (t/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$

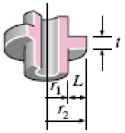


Circular fins of rectangular profile

$m = \sqrt{2h/kt}$
$r_{2c} = r_2 + t/2$
$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$

$$\eta_{fin} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

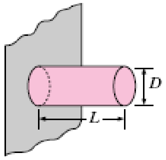
$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



Pin fins of rectangular profile

$m = \sqrt{4h/kD}$
$L_c = L + D/4$
$A_{fin} = \pi DL_c$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$

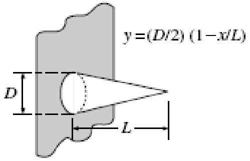


Pin fins of triangular profile

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$

$$\eta_{fin} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$



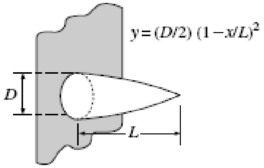
Pin fins of parabolic profile

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi L^3}{8D}[C_3C_4 - \frac{L}{2D}ln(2DC_4/L + C_3)]$

$C_3 = 1 + 2(D/L)^2$
$C_4 = \sqrt{1 + (D/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$

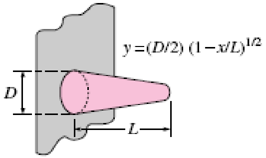


Pin fins of parabolic profile (blunt tip)

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi D^4}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$

$$\eta_{fin} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$



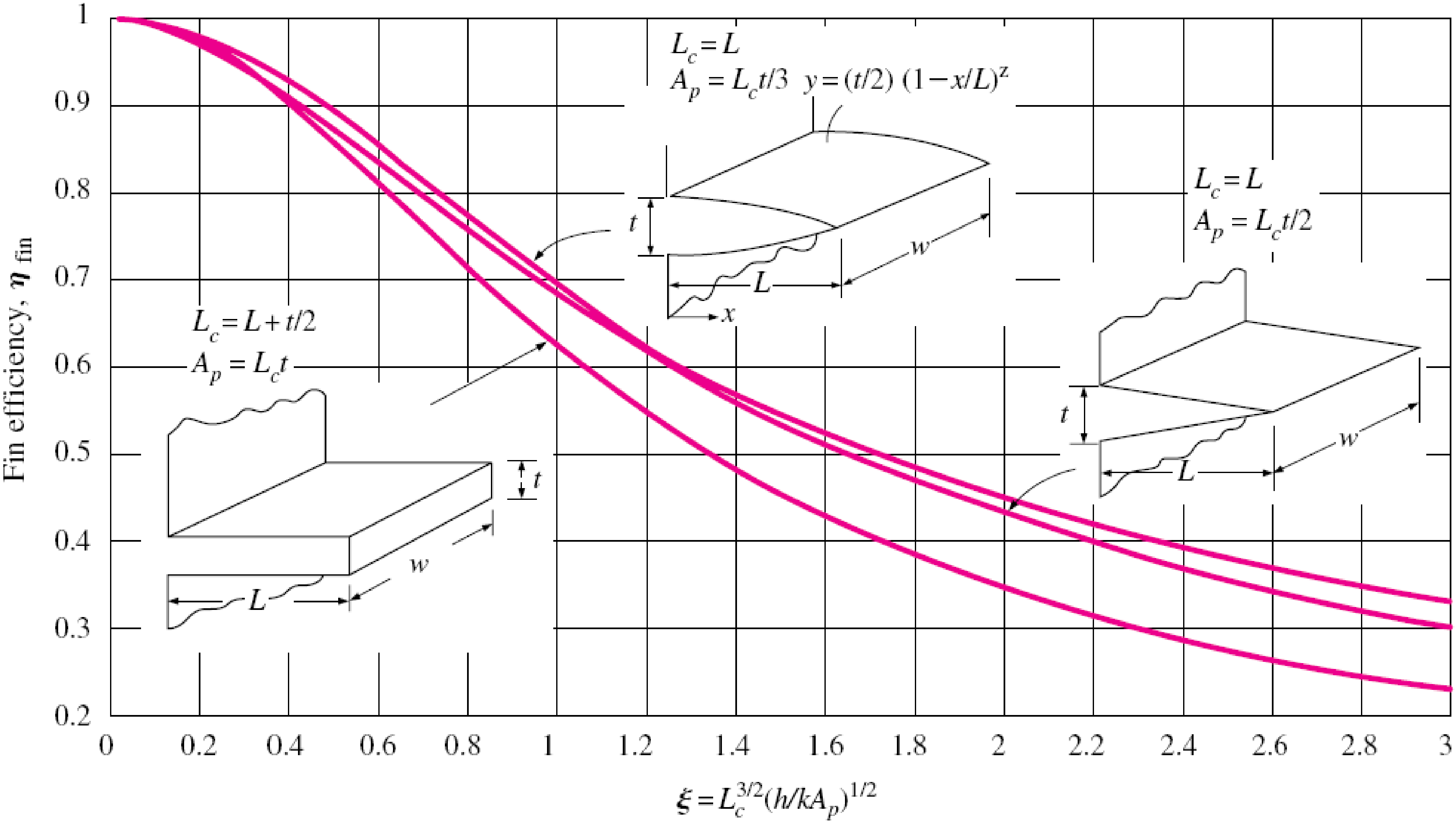


FIGURE 3–42

Efficiency of straight fins of rectangular, triangular, and parabolic profiles.

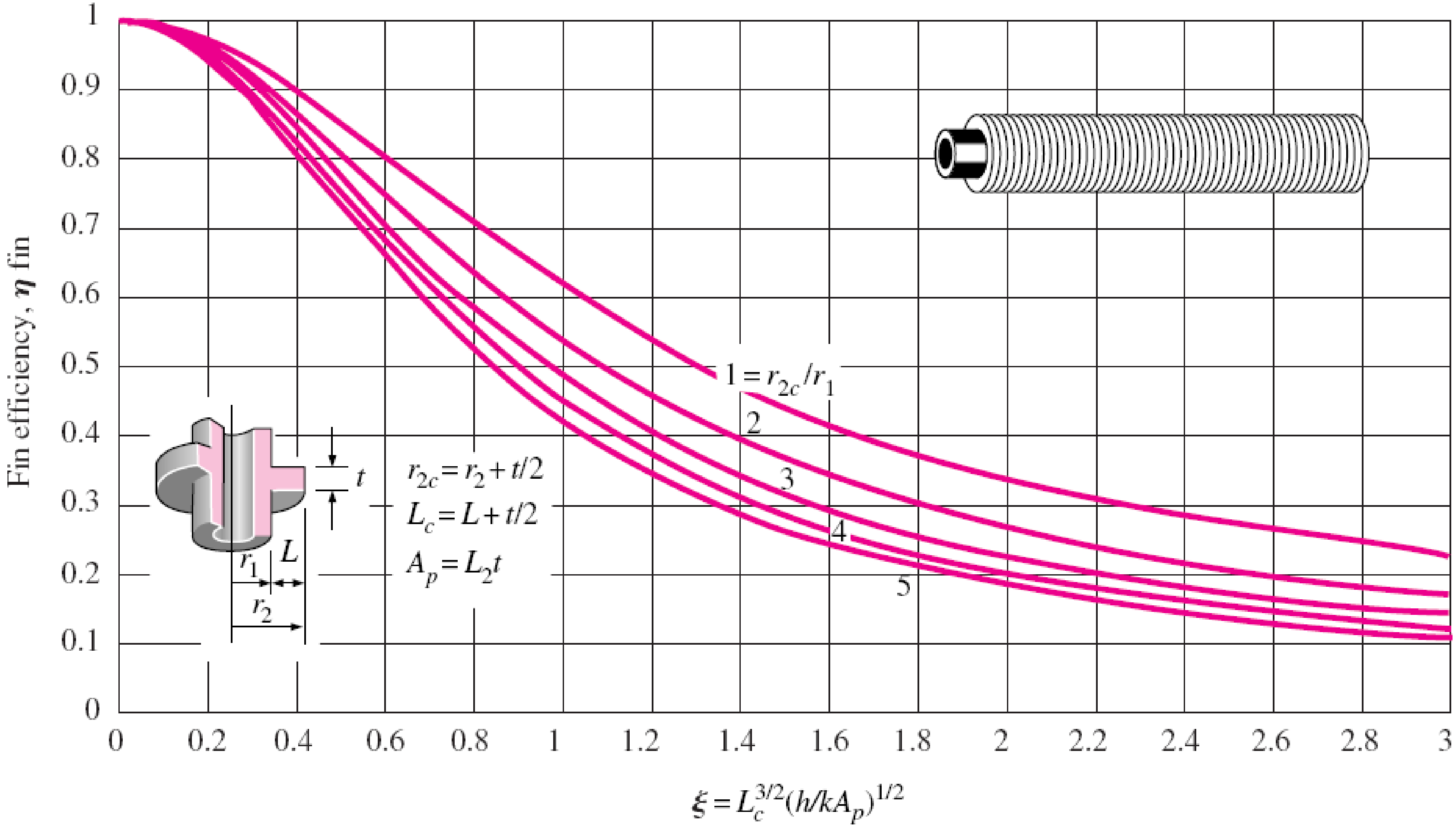


FIGURE 3–43

Efficiency of annular fins of constant thickness t.

- Fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

- An important consideration in the design of finned surfaces is the selection of the proper *fin length L*. Normally the *longer* the fin, the *larger* the heat transfer area and thus the *higher* the rate of heat transfer from the fin.

- The larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction. Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.

- Fin lengths that cause the fin efficiency to drop below 60% percent usually cannot be justified economically and should be avoided. The efficiency of most fins used in practice is above 90%.

Fin Effectiveness

The performance of fins expressed in terms of the *fin effectiveness* ε_{fin} is defined

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

A_b : the cross-sectional area of the fin at the base

$\dot{Q}_{\text{no fin}}$: the rate of heat transfer from this area if no fins are attached to the surface.

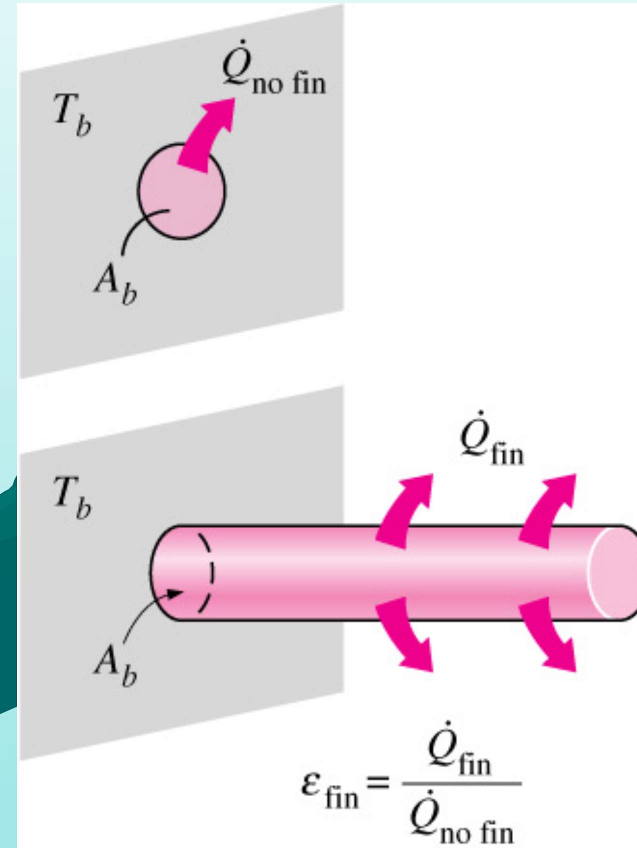


FIGURE 3–44
The effectiveness of a fin.

An effectiveness of $\varepsilon_{fin} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all.

An effectiveness of $\varepsilon_{fin} < 1$ indicates that the fin actually acts as *insulation,* slowing down the heat transfer from the surface.

An effectiveness of $\varepsilon_{fin} > 1$ indicates that fins are *enhancing* heat transfer from the surface, as they should.

Finned surfaces are designed on the basis of *maximizing* effectiveness for a specified cost or *minimizing* cost for a desired effectiveness.

The fin efficiency and fin effectiveness are related to each other by

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\,\eta_{\text{fin}}$$

The effectiveness of a sufficiently *long* fin of *uniform* cross section under steady conditions is determined to be

$$\varepsilon_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

since $A_c = A_b$.

In the design and selection of the fins, the following should be taken into account:

- The *thermal conductivity k* of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminum, and iron being the most common ones. Perhaps the most widely used fins are made of aluminum because of its low cost and weight and its resistance to corrosion.

- The ratio of the *perimeter* to the *cross-sectional area* of the fin p/A_c should be as high as possible. This criterion is satisfied by *thin* plate fins and *slender* pin fins.

- The use of fins is *most effective* in applications involving a *low convection heat transfer coefficient.*

The rate of heat transfer for a surface containing n fins can be expressed as

$$\begin{aligned}\dot{Q}_{\text{total, fin}} &= \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}} \\ &= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)\end{aligned}$$

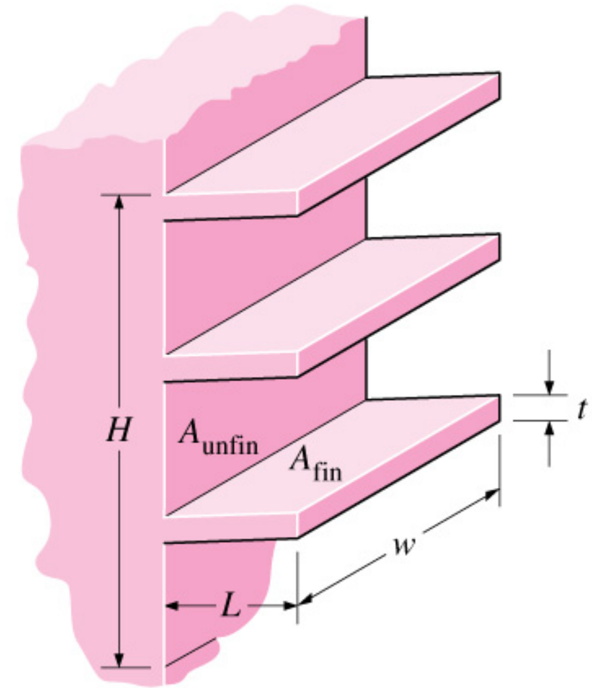
The **overall effectiveness** for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins.

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$

$A_{\text{no fin}}$: the area of the surface when there are no fins

A_{fin} : the total surface area of all the fins on the surface

A_{unfin} : the area of the unfinned portion of the surface



$$\begin{aligned}A_{\text{no fin}} &= w \times H \\ A_{\text{unfin}} &= w \times H - 3 \times (t \times w) \\ A_{\text{fin}} &= 2 \times L \times w + t \times w \\ &\cong 2 \times L \times w \text{ (one fin)}\end{aligned}$$

FIGURE 3–45

Various surface areas associated with a rectangular surface with three fins.

Proper Length of a Fin

To get a sense of the proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fin under the same conditions. The ratio of these two heat transfers is

Heat transfer ratio:

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL$$

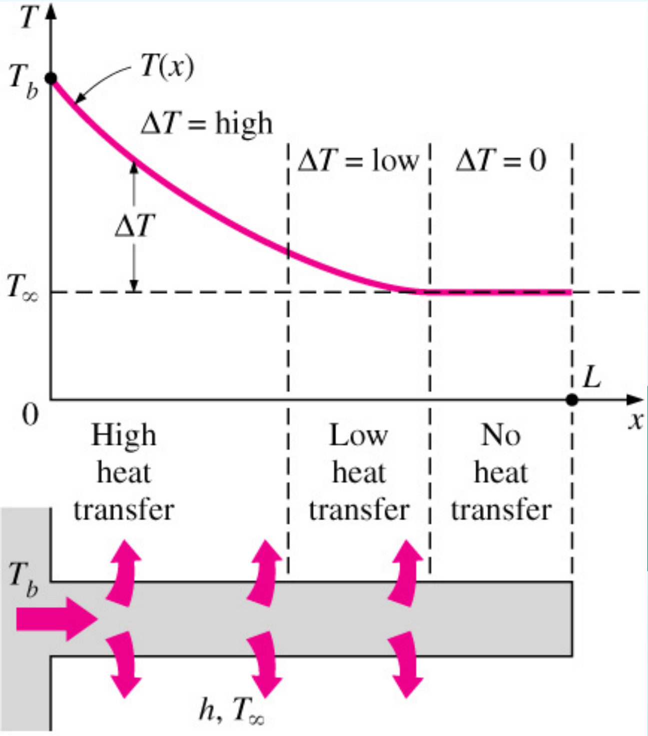


FIGURE 3–46
Because of the gradual temperature drop along the fin, the region near the fin tip makes little or no contribution to heat transfer.

Studies have shown that the error involved in one-dimensional fin analysis is negligible (less than about 1%) when

$$\frac{h\delta}{k} < 0.2$$

The heat transfer performance of heat sinks is usually expressed in terms of their *thermal resistances R* in °C/W, which is defined as

$$\dot{Q}_{\text{fin}} = \frac{T_b - T_\infty}{R} = hA_{\text{fin}}\,\eta_{\text{fin}}\,(T_b - T_\infty)$$

EXAMPLE 3–11 Selecting a Heat Sink for a Transistor

A 60-W power transistor is to be cooled by attaching it to one of the commercially available heat sinks shown in Table 17–4. Select a heat sink that will allow the case temperature of the transistor not to exceed 90°C in the ambient air at 30°C.

SOLUTION A commercially available heat sink from Table 17–4 is to be selected to keep the case temperature of a transistor below 90°C.

Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

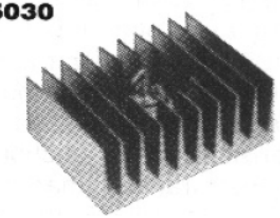
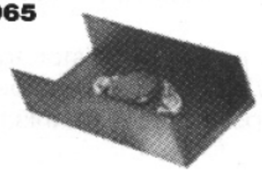
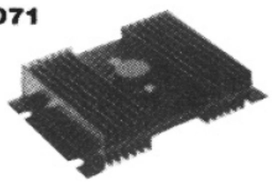
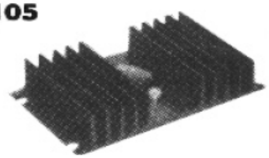
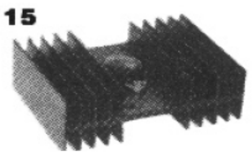
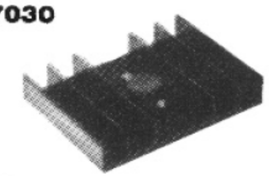
Analysis The rate of heat transfer from a 60-W transistor at full power is $\dot{Q} = 60$ W. The thermal resistance between the transistor attached to the heat sink and the ambient air for the specified temperature difference is determined to be

$$\dot{Q} = \frac{\Delta T}{R} \longrightarrow R = \frac{\Delta T}{\dot{Q}} = \frac{(90 - 30)°\text{C}}{60 \text{ W}} = 1.0°\text{C/W}$$

Therefore, the thermal resistance of the heat sink should be below 1.0°C/W. An examination of Table 17–4 reveals that the HS 5030, whose thermal resistance is 0.9°C/W in the vertical position, is the only heat sink that will meet this requirement.

TABLE 3–6

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long.

HS 5030		R = 0.9°C/W (vertical) R = 1.2°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm^2
HS 6065		R = 5°C/W Dimensions: 76 mm × 38 mm × 24 mm Surface area: 387 cm^2
HS 6071		R = 1.4°C/W (vertical) R = 1.8°C/W (horizontal) Dimensions: 76 mm × 92 mm × 26 mm Surface area: 968 cm^2
HS 6105		R = 1.8°C/W (vertical) R = 2.1°C/W (horizontal) Dimensions: 76 mm × 127 mm × 91 mm Surface area: 677 cm^2
HS 6115		R = 1.1°C/W (vertical) R = 1.3°C/W (horizontal) Dimensions: 76 mm × 102 mm × 25 mm Surface area: 929 cm^2
HS 7030		R = 2.9°C/W (vertical) R = 3.1°C/W (horizontal) Dimensions: 76 mm × 97 mm × 19 mm Surface area: 290 cm^2

EXAMPLE 3–12 **Effect of Fins on Heat Transfer from Steam Pipes**

Steam in a heating system flows through tubes whose outer diameter is $D_1 = 3$ cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins ($k = 180$ W/m · °C) of outer diameter $D_2 = 6$ cm and constant thickness $t = 2$ mm are attached to the tube, as shown in Fig. 17–48. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at $T_\infty = 25$°C, with a combined heat transfer coefficient of $h = 60$ W/m^2 · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

SOLUTION Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be $k = 180$ W/m · °C.

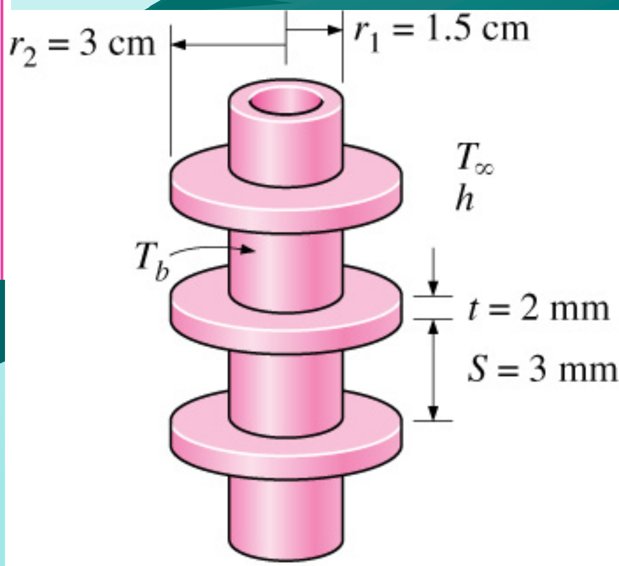


FIGURE 3–48
Schematic for Example 3–12.

Analysis In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$$
\begin{aligned}
A_{\text{no fin}} &= \pi D_1 L = \pi(0.03\ \text{m})(1\ \text{m}) = 0.0942\ \text{m}^2 \\
\dot{Q}_{\text{no fin}} &= hA_{\text{no fin}}(T_b - T_\infty) \\
&= (60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.0942\ \text{m}^2)(120 - 25)^\circ\text{C} \\
&= 537\ \text{W}
\end{aligned}
$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 17–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$
\left.
\begin{aligned}
\frac{r_2 + \frac{1}{2}t}{r_1} &= \frac{(0.03 + \frac{1}{2}\times 0.002)\ \text{m}}{0.015\ \text{m}} = 2.07 \\
(L + \tfrac{1}{2}t)\sqrt{\frac{h}{kt}} &= (0.015 + \tfrac{1}{2}\times 0.002)\ \text{m} \times \sqrt{\frac{60\ \text{W/m}^2\cdot{}^\circ\text{C}}{(180\ \text{W/m}\cdot{}^\circ\text{C})(0.002\ \text{m})}} = 0.207
\end{aligned}
\right\}\ \eta_{\text{fin}} = 0.95
$$

$$
\begin{aligned}
A_{\text{fin}} &= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t \\
&= 2\pi[(0.03\ \text{m})^2 - (0.015\ \text{m})^2] + 2\pi(0.03\ \text{m})(0.002\ \text{m}) \\
&= 0.00462\ \text{m}^2
\end{aligned}
$$

$$
\begin{aligned}
\dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} h A_{\text{fin}}(T_b - T_\infty) \\
&= 0.95(60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.00462\ \text{m}^2)(120 - 25)^\circ\text{C} \\
&= 25.0\ \text{W}
\end{aligned}
$$

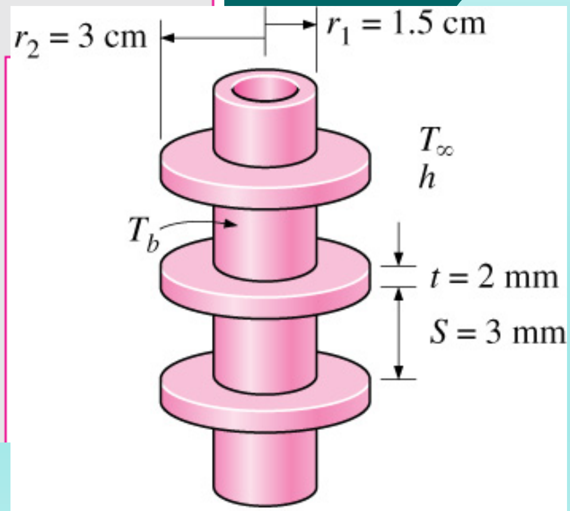


FIGURE 3–48
Schematic for Example 3–12.

Heat transfer from the unfinned portion of the tube is

$$\begin{aligned}
A_{\text{unfin}} &= \pi D_1 S = \pi(0.03 \text{ m})(0.003 \text{ m}) = 0.000283 \text{ m}^2 \\
\dot{Q}_{\text{unfin}} &= hA_{\text{unfin}}(T_b - T_\infty) \\
&= (60 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.000283 \text{ m}^2)(120 - 25)^\circ\text{C} \\
&= 1.60 \text{ W}
\end{aligned}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6) \text{ W} = 5320 \text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \mathbf{4783 \text{ W}} \qquad \text{(per m tube length)}$$

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320 \text{ W}}{537 \text{ W}} = 9.9$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

HEAT TRANSFER IN COMMON CONFIGURATIONS

• We have dealt with 1-D simple geometries.

☞ The question: What happens if we have 2- or 3-D complicated geometries?

• The steady rate of heat transfer between two surfaces at *constant* temperatures T_1 and T_2 is expressed as

$$Q = Sk(T_1 - T_2)$$

S : the conduction shape factor **(which** has the dimension of *length)*

k : the thermal conductivity of the medium between the surfaces

☞ The conduction shape factor depends on the *geometry* of the system only.

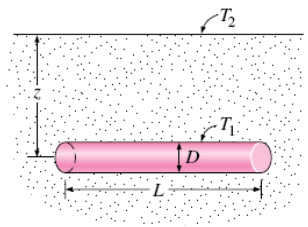
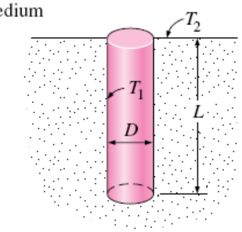
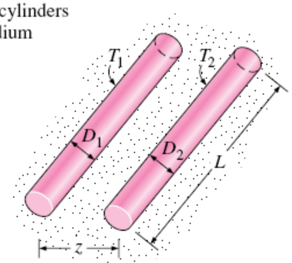
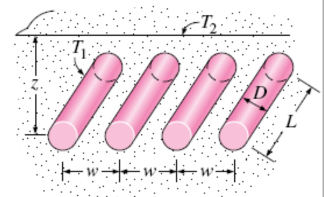
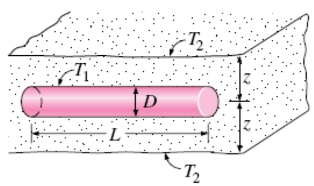
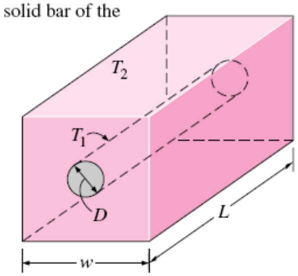
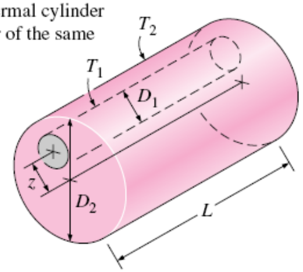
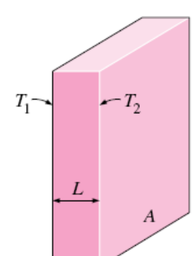
A comparison of the following equations reveals that the conduction shape factor S is related to the thermal resistance R by $R = 1/kS$ or $S = 1/kR$.

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

$$Q = Sk(T_1 - T_2)$$

TABLE 3–7

Conduction shape factors S for several configurations for use in $\dot{Q} = kS(T_1 - T_2)$ to determine the steady rate of heat transfer through a medium of thermal conductivity k between the surfaces at temperatures T_1 and T_2

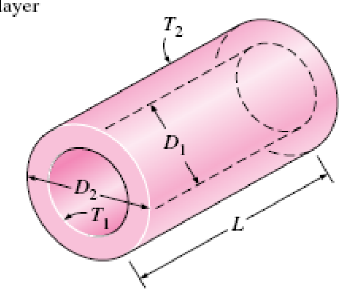
(1) Isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$ and $z > 1.5D$) $S = \dfrac{2\pi L}{\ln(4z/D)}$  	(2) Vertical isothermal cylinder of length L buried in a semi-infinite medium ($L >> D$) $S = \dfrac{2\pi L}{\ln(4L/D)}$ 
(3) Two parallel isothermal cylinders placed in an infinite medium ($L >> D_1, D_2, z$) $S = \dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$  	(4) A row of equally spaced parallel isothermal cylinders buried in a semi-infinite medium ($L >> D, z,$ and $w > 1.5D$) $S = \dfrac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$ (per cylinder) 
(5) Circular isothermal cylinder of length L in the midplane of an infinite wall ($z > 0.5D$) $S = \dfrac{2\pi L}{\ln(8z/\pi D)}$  	(6) Circular isothermal cylinder of length L at the center of a square solid bar of the same length $S = \dfrac{2\pi L}{\ln(1.08\,w/D)}$ 
(7) Eccentric circular isothermal cylinder of length L in a cylinder of the same length ($L > D_2$) $S = \dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)}$  	(8) Large plane wall $S = \dfrac{A}{L}$ 

(continued)

TABLE 3–7 (*Continued*)

(9) A long cylindrical layer

$$S = \frac{2\pi L}{\ln (D_2/D_1)}$$



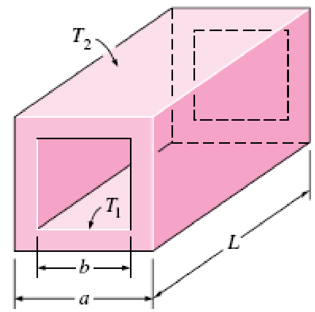
(10) A square flow passage

(*a*) For $a/b > 1.4$,

$$S = \frac{2\pi L}{0.93 \ln (0.948 a/b)}$$

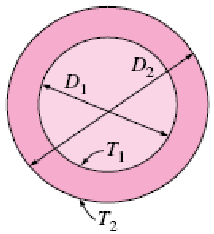
(*b*) For $a/b < 1.41$,

$$S = \frac{2\pi L}{0.785 \ln (a/b)}$$



(11) A spherical layer

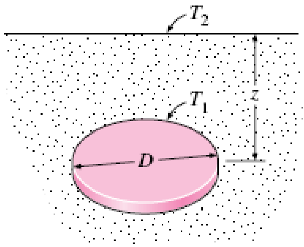
$$S = \frac{2\pi D_1 D_2}{D_2 - D_1}$$



(12) Disk buried parallel to the surface in a semi-infinite medium ($z >> D$)

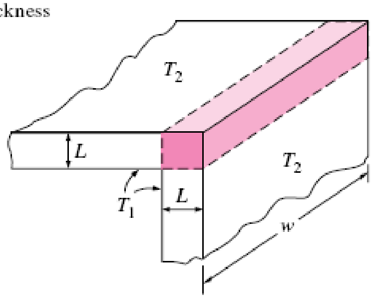
$$S = 4D$$

($S = 2D$ when $z = 0$)



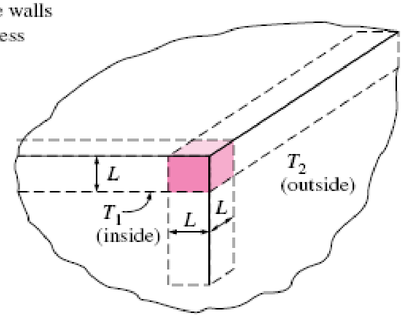
(13) The edge of two adjoining walls of equal thickness

$$S = 0.54\, w$$



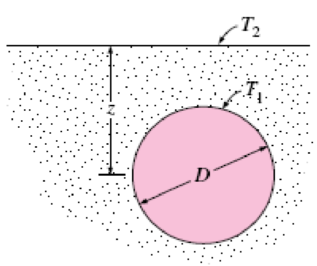
(14) Corner of three walls of equal thickness

$$S = 0.15 L$$



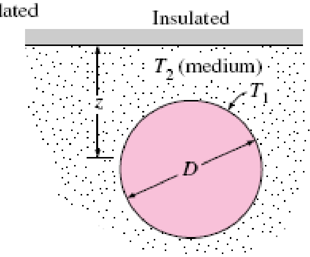
(15) Isothermal sphere buried in a semi-infinite medium

$$S = \frac{2\pi D}{1 - 0.25D/z}$$



(16) Isothermal sphere buried in a semi-infinite medium at T_2 whose surface is insulated

$$S = \frac{2\pi D}{1 + 0.25D/z}$$



EXAMPLE 3–13 Heat Loss from Buried Steam Pipes

A 30-m-long, 10-cm-diameter hot-water pipe of a district heating system is buried in the soil 50 cm below the ground surface, as shown in Fig. 17–49. The outer surface temperature of the pipe is 80°C. Taking the surface temperature of the earth to be 10°C and the thermal conductivity of the soil at that location to be 0.9 W/m · °C, determine the rate of heat loss from the pipe.

SOLUTION The hot-water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be $k = 0.9$ W/m · °C.

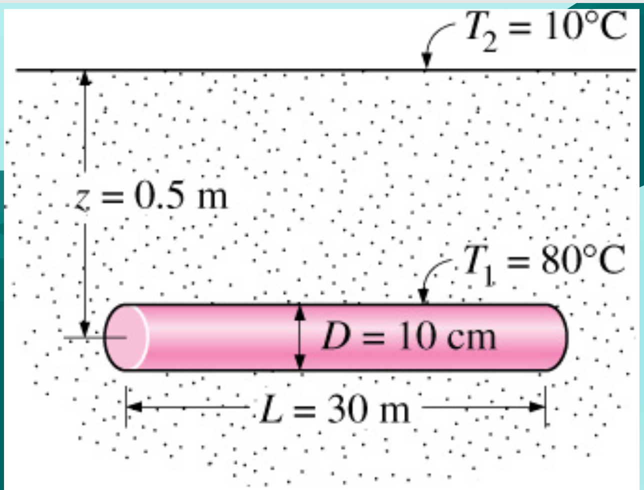


FIGURE 3–49
Schematic for Example 3–13.

Analysis The shape factor for this configuration is given in Table 17–5 to be

$$S = \frac{2\pi L}{\ln(4z/D)}$$

since $z > 1.5D$, where z is the distance of the pipe from the ground surface, and D is the diameter of the pipe. Substituting,

$$S = \frac{2\pi \times (30\text{ m})}{\ln(4 \times 0.5/0.1)} = 62.9\text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (62.9\text{ m})(0.9\text{ W/m}\cdot{}^\circ\text{C})(80 - 10)^\circ\text{C} = \mathbf{3963\text{ W}}$$

Discussion Note that this heat is conducted from the pipe surface to the surface of the earth through the soil and then transferred to the atmosphere by convection and radiation.

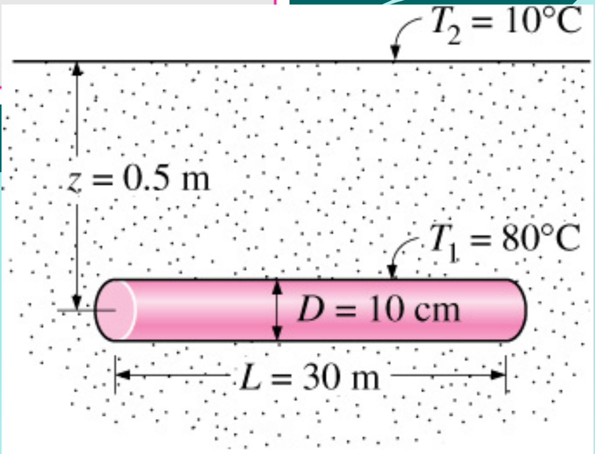


FIGURE 3–49
Schematic for Example 3–13.

Concluding Points:

- Steady and One-Dimensional Modeling of Heat Transfer through a Wall
- Conduction and Convection Resistances
- Analogy between Thermal and Electrical Resistances
- Radiation and Combined Heat Transfer Coefficients
- Overall Heat Transfer Coefficient
- Heat Transfer through a Plane and Multilayer Plane Walls
- Thermal Contact Resistance
- Generalized Thermal Resistance Networks
- Heat Conduction in Multilayered Cylinders and Spheres
- Critical Radius of Insulation for Cylindrical and Spherical Bodies
- Heat Transfer from Finned Surfaces
- Fin Efficiency, Fin Effectiveness and Overall Effectiveness
- Important Considerations in the Design and Selection of Fins
- Heat Transfer in Common Configurations and Conduction Shape Factors