# The Laplace Transform

The *Laplace transform* technique is useful for solving both 0-D and 1-D transient conduction problems

The technique is broadly useful across all engineering disciplines

The Laplace transform maps a problem from the time domain to the s domain through a mathematical operation

Why do this?  The Laplace transform removes time derivatives from the problem.

The problem in the *s* domain is much simpler than it was in the *t* domain:

0-D problem:  ODE for $T(t)$ becomes an algebraic expression for $\hat{T}(s)$

1-D problem:  PDE for $T(x,t)$ becomes an ODE for $\hat{T}(x,s)$

# The Laplace Transform

$$\boxed{\hat{T}(s)} = \boxed{\langle T(t) \rangle} = \underbrace{\int_0^\infty \exp(-s\,t)\,T(t)\,dt}_{\text{the Laplace transform operation}}$$

the Laplace transform of *T*

take the Laplace transform of

For example, we can determine the Laplace transform of a constant, *C*:

$$T(t) = C \qquad \hat{T}(s) = \langle C \rangle = \int_0^\infty \exp(-s\,t)\,C\,dt$$

$$\hat{T}(s) = -\frac{C}{s}\left[\exp(-s\,t)\right]_0^\infty = -\frac{C}{s}\left[\underbrace{\exp(-s\,\infty)}_{0} - \underbrace{\exp(-s\,0)}_{1}\right]$$

$$\hat{T}(s) = \frac{C}{s}$$

The Laplace transform of many functions are tabulated for you.

# Laplace Transforms with Table 3-3

| Function in $t$ | Function in $s$ | Function in $t$ | Function in $s$ |
|---|---|---|---|
| $C$ | $\frac{C}{s}$ | $\operatorname{erfc}\left(\frac{C}{2\sqrt{t}}\right)$ | $\frac{\exp\left(-C\sqrt{s}\right)}{s}$ |
| $Ct$ | $\frac{C}{s^2}$ | $\frac{2\sqrt{t}}{\sqrt{\pi}}\exp\left(-\frac{C^2}{4t}\right)-C\operatorname{erfc}\left(\frac{C}{2\sqrt{t}}\right)$ | $\frac{\exp\left(-C\sqrt{s}\right)}{s\sqrt{s}}$ |
| $\exp(Ct)$ | $\frac{1}{s-C}$ | $\left(t+\frac{C^2}{2}\right)\operatorname{erfc}\left(\frac{C}{2\sqrt{t}}\right)-$ $\frac{C\sqrt{t}}{\sqrt{\pi}}\exp\left(-\frac{C^2}{4t}\right)$ | $\frac{\exp\left(-C\sqrt{s}\right)}{s^2}$ |
| $\sin(Ct)$ | $\frac{C}{s^2+C^2}$ | $\frac{C_1}{2\sqrt{\pi t^3}}\exp\left(-\frac{C_1^2}{4t}-C_2 t\right)$ | $\exp\left(-C_1\sqrt{s+C_2}\right)$ |
| $\cos(Ct)$ | $\frac{s}{s^2+C^2}$ | $\frac{\exp(-C_1 t)-\exp(-C_2 t)}{C_2-C_1}$ | $\frac{1}{(s+C_1)(s+C_2)}$ |
| $\sinh(Ct)$ | $\frac{C}{s^2-C^2}$ | $t\exp(-C_1 t)$ | $\frac{1}{(s+C_1)^2}$ |
| $\cosh(Ct)$ | $\frac{s}{s^2-C^2}$ | $\frac{\left[\begin{array}{l}(C_3-C_2)\exp(-C_1 t)+\\(C_1-C_3)\exp(-C_2 t)+\\(C_2-C_1)\exp(-C_3 t)\end{array}\right]}{(C_1-C_2)(C_2-C_3)(C_3-C_1)}$ | $\frac{1}{(s+C_1)(s+C_2)(s+C_3)}$ |
| $\frac{C}{2\sqrt{\pi t^3}}\exp\left(-\frac{C^2}{4t}\right)$ | $\exp\left(-C\sqrt{s}\right)$ | $\frac{\exp(-C_2 t)-\exp(-C_1 t)\left[1-(C_2-C_1)t\right]}{(C_2-C_1)^2}$ | $\frac{1}{(s+C_1)^2(s+C_2)}$ |
| $\frac{1}{\sqrt{\pi t^3}}\exp\left(-\frac{C^2}{4t}\right)$ | $\frac{\exp\left(-C\sqrt{s}\right)}{\sqrt{s}}$ | $\frac{t^2\exp(-Ct)}{2}$ | $\frac{1}{(s+C)^3}$ |

# Properties of the Laplace Transform

These properties are proved in Section 3.4.4 and are summarized in Table 3-4

| |
|---|
| $\langle T_1(t)+T_2(t)\rangle=\tilde{T}_1(s)+\tilde{T}_2(s)$ |
| $\langle C\,T(t)\rangle=C\,\tilde{T}(s)$ |
| $\left\langle \dfrac{dT(t)}{dt}\right\rangle=s\,\tilde{T}(s)-T_{t=0}$ |
| $\left\langle \dfrac{\partial T(x,t)}{\partial t}\right\rangle=s\,\tilde{T}(x,s)-T_{x,t=0}$ |
| $\left\langle \dfrac{\partial^n T(x,t)}{\partial x^n}\right\rangle=\dfrac{\partial^n \tilde{T}(x,s)}{\partial x^n}$ |

The Laplace tranform is linear

The useful property of the Laplace transform

- makes ODE in *t* algebraic in *s*
- makes PDE in *x* & *t* an ODE in *x* and *s*

*x*-derivatives are unchanged

# Inverse Laplace Transform

Once you solve the problem in the *s* domain you have to transform it back to the *t* domain using the <u>*inverse Laplace transform*</u>

- this is typically the hardest part of the problem
- practically, this is done by recognizing the Laplace transform in the table

The typical form of the solution in the *s* domain is a polynomial fraction

For example: $\widehat{T}(s)=\dfrac{s^2-3s+4}{(s+1)(s-1)(s+2)}$

The Laplace transform of this function cannot be found in the table

- we need to review the method of partial fractions in order to break this large fraction into its components
- the simpler components can be found in the table

# Method of Partial Fractions

The method of partial fractions proceeds based on the form of the denominator

*Distinct factors in the denominator*

$$\hat{T}(s) = \frac{s^2 - 3s + 4}{\boxed{(s+1)(s-1)(s+2)}}$$

distinct factors

- Express the function as the sum of individual, lower order fractions corresponding to the distinct factors

$$\frac{s^2 - 3s + 4}{(s+1)(s-1)(s+2)} = \boxed{\frac{C_1}{(s+1)} + \frac{C_2}{(s-1)} + \frac{C_3}{(s+2)}}$$

lower order fractions
with undetermined constants

These can be found in Table 3-3

- Clear the denominator

$$\text{multiply both sides by: } (s+1)(s-1)(s+2)$$

# Distinct Factors (continued)

$$\left[\frac{s^2-3s+4}{(s+1)(s-1)(s+2)}=\frac{C_1}{(s+1)}+\frac{C_2}{(s-1)}+\frac{C_3}{(s+2)}\right](s+1)(s-1)(s+2)$$

$$s^2-3s+4=C_1(s-1)(s+2)+C_2(s+1)(s+2)+C_3(s+1)(s-1)$$

$$s^2-3s+4=C_1\left(s^2+s-2\right)+C_2\left(s^2+3s+2\right)+C_3\left(s^2-1\right)$$

In order for this to be true for all values of *s*, the coefficient for each power of *s* must be satisfied:

$$\left.\begin{aligned} s^2 &: 1=C_1+C_2+C_3 \\ s^1 &: -3=C_1+3C_2 \\ s^0 &: 4=-2C_1+2C_2-C_3 \end{aligned}\right\}\text{3 equations in 3 unknowns}$$

# Distinct Factors (continued)

A shortcut that works for this situation (but not others):

$$s^2 - 3s + 4 = C_1(s-1)(s+2) + C_2(s+1)(s+2) + C_3(s+1)(s-1)$$

Set $s$ = -1:

$$(-1)^2 - 3(-1) + 4 = C_1(-1-1)(-1+2) + C_2\underbrace{(-1+1)}_{=0}(-1+2) + C_3\underbrace{(-1+1)}_{=0}(-1-1)$$

$$8 = C_1(-2) \quad \boxed{C_1 = -4}$$

Set $s$ = 1:

$$(1)^2 - 3(1) + 4 = C_1\underbrace{(1-1)}_{=0}(1+2) + C_2(1+1)(1+2) + C_3(1+1)\underbrace{(1-1)}_{=0}$$

$$2 = C_2 6 \quad \boxed{C_2 = 1/3}$$

Set $s$ = -2:

$$(-2)^2 - 3(-2) + 4 = C_1(-2-1)\underbrace{(-2+2)}_{=0} + C_2(-2+1)\underbrace{(-2+2)}_{=0} + C_3(-2+1)(-2-1)$$

$$14 = C_3 3 \quad \boxed{C_3 = 14/3}$$

$$\hat{T}(s) = \frac{-4}{(s+1)} + \frac{1}{3(s-1)} + \frac{14}{3(s+2)}$$

# Repeated Factors

*Repeated factors in the denominator*

$$\hat{T}(s) = \frac{s^2 - 3s + 4}{\boxed{(s+1)^2}(s-1)}$$

repeated factors

- include each power of the repeated term

both powers of the repeated factor

$$\frac{s^2 - 3s + 4}{(s+1)^2(s-1)} = \boxed{\frac{C_1}{(s+1)} + \frac{C_2}{(s+1)^2}} + \frac{C_3}{(s-1)}$$

- Clear the denominator multiply both sides by: $(s+1)^2(s-1)$

$$s^2 - 3s + 4 = C_1(s+1)(s-1) + C_2(s-1) + C_3(s+1)^2$$

$$s^2 - 3s + 4 = C_1(s^2 - 1) + C_2(s-1) + C_3(s^2 + 2s + 1)$$

$$\left.\begin{aligned} s^2 &: 1 = C_1 + C_3 \\ s^1 &: -3 = C_2 + 2C_3 \\ s^0 &: 4 = -C_1 - C_2 + C_3 \end{aligned}\right\}$$

3 equations in 3 unknowns

# Polynomial Factors

*Polynomial factors in the denominator*

$$\widehat{T}(s)=\frac{s^2-3s+4}{\boxed{\left(s^2+2\right)}(s-1)}$$

polynomial factor

- include a lower order polynomial in the numerator

lower order polynomial

$$\frac{s^2-3s+4}{\left(s^2+2\right)(s-1)}=\frac{\boxed{C_1 s+C_2}}{\left(s^2+2\right)}+\frac{C_3}{(s-1)}$$

- Clear the denominator  multiply both sides by: $\left(s^2+2\right)(s-1)$

$$s^2-3s+4=\left(C_1 s+C_2\right)(s-1)+C_3\left(s^2+2\right)$$

$$s^2-3s+4=C_1 s^2-C_1 s+C_2 s-C_2+C_3 s^2+2C_3$$

$$\left.\begin{aligned}1&=C_1+C_3\\-3&=-C_1+C_2\\4&=-C_2+2C_3\end{aligned}\right\}$$

3 equations in 3 unknowns

# 0-D Transient Problem

A small particle is subjected to a volumetric generation that exponentially decays with time:

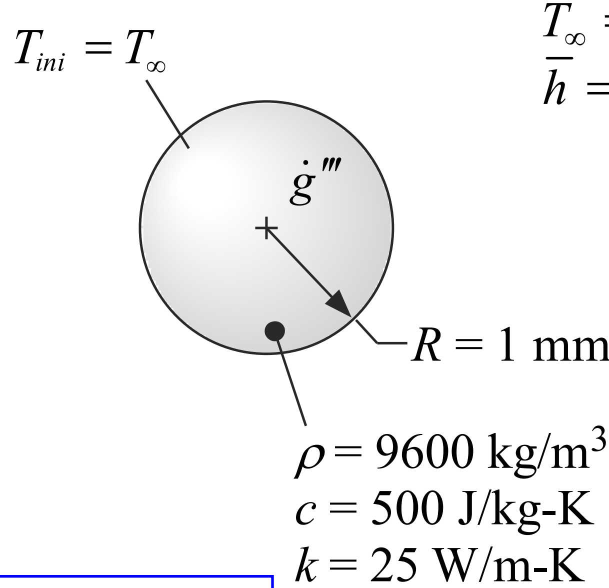
$$\dot{g}''' = \dot{g}'''_{max} \exp\left(\frac{-t}{a}\right)$$

$$\dot{g}'''_{max} = 1\text{x}10^9 \text{ W/m}^3$$

$$a = 2 \text{ s}$$

$T_\infty = 25°\text{C}$

$\overline{h} = 1000 \text{ W/m}^2\text{-K}$




```
$UnitSystem SI MASS RAD PA  K J
$TABSTOPS   0.2 0.4 0.6 0.8 3.5 in

"Inputs"
R=1 [mm]*convert(mm,m)                    "radius of sphere"
rho=9000 [kg/m^3]                         "density"
c=500 [J/kg-K]                            "specific heat capacity"
k=25 [W/m-K]                              "thermal conductivity"
T_infinity=converttemp(C,K,25[C])         "ambient temperature"
h_bar=1000 [W/m^2-K]                      "heat transfer coefficient"
gv_max=1e9 [W/m^3]                        "maximum volumetric generation"
a=2 [s]                                   "time constant of generation"
```

# Biot Number

Is the lumped capacitance approximation appropriate for this problem:

$$R_{cond,r} = \frac{L_{cond}}{k\,A_s} \quad \text{where } L_{cond} = \frac{V}{A_s},\; V = \frac{4}{3}\pi R^3,\; \text{and } A_s = 4\pi R^2$$

$$R_{conv} = \frac{1}{\overline{h}\,A_s}$$

$$Bi = \frac{R_{cond,r}}{R_{conv}}$$

```
V=4*pi*R^3/3                    "volume"
A_s=4*pi*R^2                    "surface area"
L_cond=V/A_s                    "conduction length"
R_cond_r=L_cond/(k*A_s)         "resistance to radial conduction, approximate"
R_conv=1/(A_s*h_bar)            "resistance to convection"
Bi=R_cond_r/R_conv              "Biot number"
```

$$Bi = 0.013 << 1$$

# Time Constant

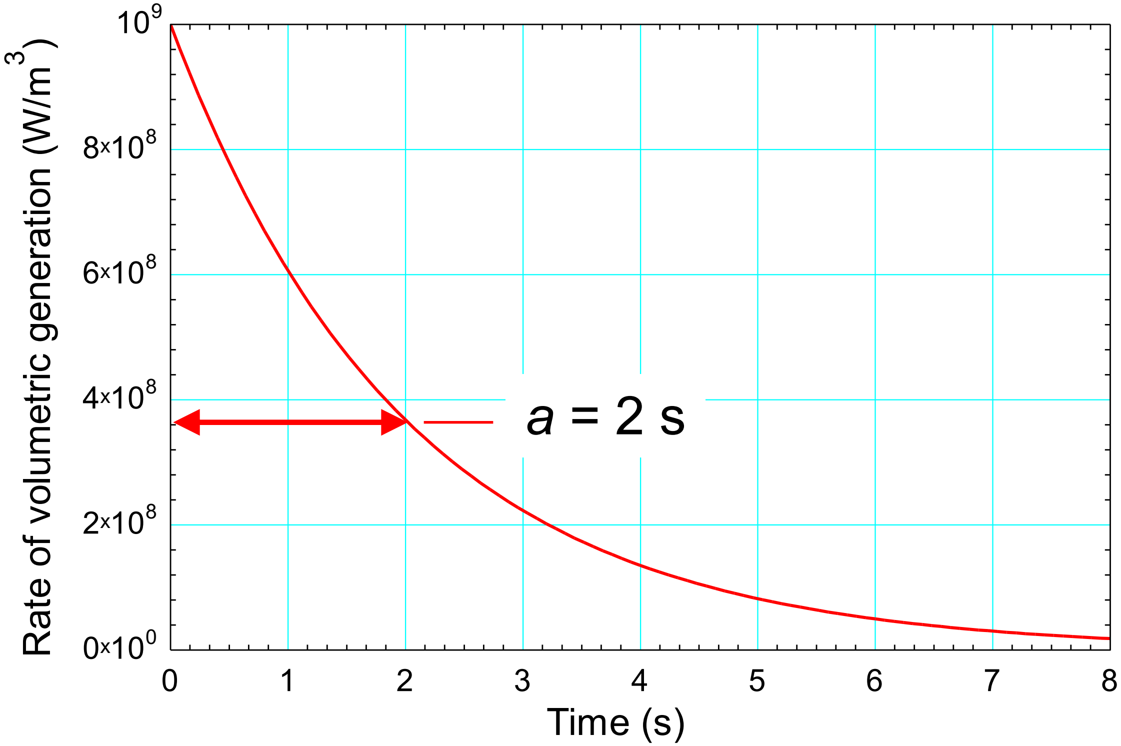
What is the time constant associated with the particle?

$$\tau = C\,R_{conv} \quad \text{where } C = V\,\rho\,c$$

```
Cap=V*rho*c                    "total heat capacity"
tau=R_conv*Cap                 "time constant"
```

$$\tau = 1.5\text{ s}$$

Based on the time constant, what is the anticipated form of the solution?



# Anticipated Solution

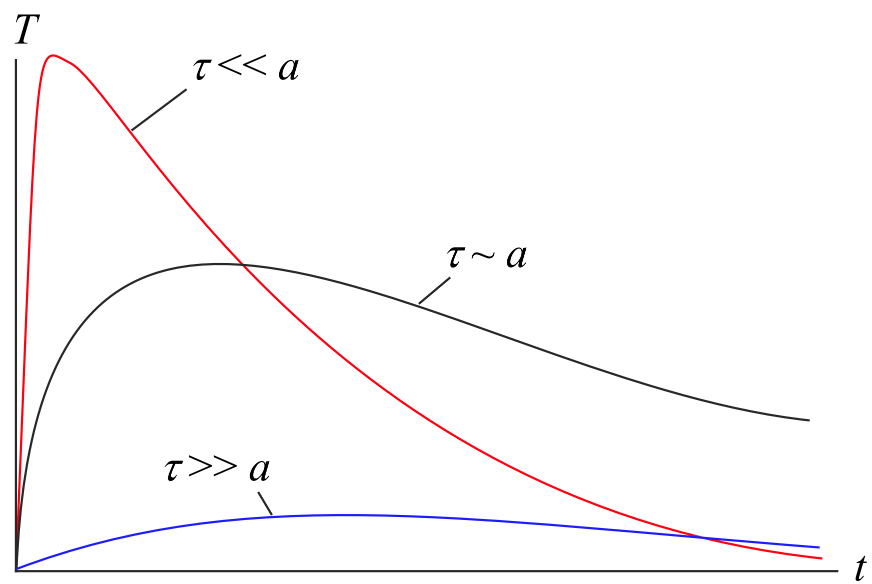
If $\tau$ << *a* then what happens?

- particle quickly comes to a quasi-steady state condition where generation balances convection: $\dot{g}'''V \approx \bar{h}\, A_s \left(T - T_\infty\right)$
- temperature ~ exponential decay following the generation

If *t* >> *a* then what happens?

- particle is slow to react to the generation
- temperature rises slightly and then cools off slowly

We are somewhere in the middle of these extremes since $\tau$ ~ *a*



# Derive Governing Differential Equation

Energy balance on entire particle:

$$\underbrace{\dot{g}'''V}_{\text{generation}} = \underbrace{\bar{h}\,A_s\left(T - T_\infty\right)}_{\text{convection}} + \underbrace{C\frac{dT}{dt}}_{\text{storage}}$$

$$\frac{dT}{dt} + \frac{\bar{h}\,A_s}{C}T = \frac{\bar{h}\,A_s}{C}T_\infty + \frac{\dot{g}'''_{max}\,V\exp\left(\frac{-t}{a}\right)}{C}$$

$$\boxed{\frac{dT}{dt} + \frac{1}{\tau}T = \frac{1}{\tau}T_\infty + \frac{\dot{g}'''_{max}\exp\left(\frac{-t}{a}\right)}{\rho\,c} \quad \text{with } T_{t=0} = T_{ini}}$$

# Laplace Transform Solution

Three step process for 0-D problem:

1. transform problem to *s* domain
2. solve algebraic equation in *s* domain
3. transform back to *t* domain

**Table 3-4: Useful properties of the Laplace transforms**

| |
|---|
| $\left\langle T_1(t) + T_2(t) \right\rangle = \hat{T}_1(s) + \hat{T}_2(s)$ |
| $\left\langle C\,T(t) \right\rangle = C\,\hat{T}(s)$ |
| $\left\langle \frac{dT(t)}{dt} \right\rangle = s\,\hat{T}(s) - T_{t=0}$ |
| $\left\langle \frac{\partial T(x,t)}{\partial t} \right\rangle = s\,\hat{T}(x,s) - T_{x,t=0}$ |
| $\left\langle \frac{\partial^n T(x,t)}{\partial x^n} \right\rangle = \frac{\partial^n \hat{T}(x,s)}{\partial x^n}$ |

Transform problem to *s* domain

$$\left\langle \frac{dT}{dt} + \frac{1}{\tau}T = \frac{1}{\tau}T_\infty + \frac{\dot{g}'''_{max}\exp\left(\frac{-t}{a}\right)}{\rho c} \right\rangle$$

take Laplace transform of entire ODE

because the Laplace transform is linear, the transform can be accomplished one term at a time (see Table 3-3)

# Transform to the *s* Domain

$$\left\langle \boxed{\frac{dT}{dt}} + \frac{1}{\tau} T = \frac{1}{\tau} T_\infty + \frac{\dot{g}'''_{max} \exp\left(\frac{-t}{a}\right)}{\rho c} \right\rangle$$

$$\left\langle \frac{dT}{dt} \right\rangle = s\hat{T}(s) - \boxed{T_{t=0}} = s\hat{T}(s) - T_\infty$$

**Table 3-4: Useful properties of the Laplace transforms**

| |
|---|
| $\langle T_1(t) + T_2(t) \rangle = \hat{T}_1(s) + \hat{T}_2(s)$ |
| $\langle C\,T(t) \rangle = C\hat{T}(s)$ |
| $\boxed{\left\langle \frac{dT(t)}{dt} \right\rangle = s\hat{T}(s) - T_{t=0}}$ |
| $\left\langle \frac{\partial T(x,t)}{\partial t} \right\rangle = s\hat{T}(x,s) - T_{x,t=0}$ |
| $\left\langle \frac{\partial^n T(x,t)}{\partial x^n} \right\rangle = \frac{\partial^n \hat{T}(x,s)}{\partial x^n}$ |

Note that the initial condition is carried into the *s* domain with the derivative and does not have to be transformed separately

# Transform to the $s$ Domain

$$\left\langle \frac{dT}{dt} + \frac{1}{\tau}T = \frac{1}{\tau}T_\infty + \frac{\dot{g}'''_{max}\exp\left(\frac{-t}{a}\right)}{\rho c} \right\rangle$$

**Table 3-4: Useful properties of the Laplace transforms**

| |
|---|
| $\langle T_1(t)+T_2(t)\rangle = \hat{T}_1(s)+\hat{T}_2(s)$ |
| $\boxed{\langle C\,T(t)\rangle = C\hat{T}(s)}$ |
| $\left\langle \frac{dT(t)}{dt}\right\rangle = s\hat{T}(s) - T_{t=0}$ |
| $\left\langle \frac{\partial T(x,t)}{\partial t}\right\rangle = s\hat{T}(x,s) - T_{x,t=0}$ |
| $\left\langle \frac{\partial^n T(x,t)}{\partial x^n}\right\rangle = \frac{\partial^n \hat{T}(x,s)}{\partial x^n}$ |

$$\left\langle \frac{1}{\tau}T \right\rangle = \frac{1}{\tau}\hat{T}(s)$$

$$\left\langle \frac{1}{\tau}T_\infty \right\rangle = \frac{1}{s}\frac{T_\infty}{\tau}$$

| Function in $t$ | Function in $s$ |
|---|---|
| $C$ | $\frac{C}{s}$ |

$$\left\langle \frac{\dot{g}'''_{max}\exp\left(\frac{-t}{a}\right)}{\rho c} \right\rangle = \frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s+\frac{1}{a}\right)}$$

| | |
|---|---|
| $\exp(Ct)$ | $\frac{1}{s-C}$ |

# Problem in the *s* Domain

$$\left\langle \frac{dT}{dt} + \frac{1}{\tau} T = \frac{1}{\tau} T_\infty + \frac{\dot{g}'''_{max} \exp\left(\frac{-t}{a}\right)}{\rho c} \right\rangle$$

$$\boxed{s\hat{T}(s) - T_\infty} + \boxed{\frac{1}{\tau}\hat{T}(s)} = \boxed{\frac{1}{s}\frac{T_\infty}{\tau}} + \boxed{\frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s + \frac{1}{a}\right)}}$$

The problem is an algebraic equation in the *s* domain

# Solve in the *s* Domain

$$s\hat{T}(s) - T_\infty + \frac{1}{\tau}\hat{T}(s) = \frac{1}{s}\frac{T_\infty}{\tau} + \frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s+\frac{1}{a}\right)}$$

$$\hat{T}\left(s+\frac{1}{\tau}\right) = \frac{1}{s}\frac{T_\infty}{\tau} + \frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s+\frac{1}{a}\right)} + T_\infty$$

These two do not appear in the table

this one does - it is easy to transform back to *t* domain

$$\hat{T} = \boxed{\frac{T_\infty}{\tau s\left(s+\frac{1}{\tau}\right)}} + \boxed{\frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s+\frac{1}{a}\right)\left(s+\frac{1}{\tau}\right)}} + \boxed{\frac{T_\infty}{\left(s+\frac{1}{\tau}\right)}}$$

Solution in the *s* domain is easy to get

Transformation back to the *t* domain is harder - we need to transform the two, complex polynomials to their simpler components using the method of partial fractions

# Apply Method of Partial Fractions

$$\frac{T_\infty}{\underbrace{\tau\ s\left(s+\frac{1}{\tau}\right)}_{\text{2 distinct factors}}} = \frac{C_1}{s} + \frac{C_2}{\left(s+\frac{1}{\tau}\right)} \rightarrow$$

clear the denominator

$$\left[\frac{T_\infty}{\tau\, s\left(s+\frac{1}{\tau}\right)} = \frac{C_1}{s} + \frac{C_2}{\left(s+\frac{1}{\tau}\right)}\right]\tau\, s\left(s+\frac{1}{\tau}\right)$$

$$T_\infty = C_1\,\tau\left(s+\frac{1}{\tau}\right) + C_2\,\tau\, s$$

$$s^1 : 0 = C_1\,\tau + C_2\,\tau$$

$$s^0 : T_\infty = C_1$$

$$\frac{T_\infty}{\tau\, s\left(s+\frac{1}{\tau}\right)} = \boxed{\frac{T_\infty}{s} - \frac{T_\infty}{\left(s+\frac{1}{\tau}\right)}}$$

these transforms can be found in the table

# Apply Method of Partial Fractions

$$\frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\underbrace{\left(s+\frac{1}{a}\right)\left(s+\frac{1}{\tau}\right)}_{\text{2 distinct factors}}}=\frac{C_3}{\left(s+\frac{1}{a}\right)}+\frac{C_4}{\left(s+\frac{1}{\tau}\right)}$$

clear the denominator

$$\left[\frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s+\frac{1}{a}\right)\left(s+\frac{1}{\tau}\right)}=\frac{C_3}{\left(s+\frac{1}{a}\right)}+\frac{C_4}{\left(s+\frac{1}{\tau}\right)}\right]\rho c\left(s+\frac{1}{a}\right)\left(s+\frac{1}{\tau}\right)$$

$$\dot{g}'''_{max}=C_3\,\rho c\left(s+\frac{1}{\tau}\right)+C_4\,\rho c\left(s+\frac{1}{a}\right)$$

$$\boxed{\begin{aligned} s^1 &: 0=C_3\,\rho c+C_4\,\rho c \\ s^0 &: \dot{g}'''_{max}=\frac{C_3\,\rho c}{\tau}+\frac{C_4\,\rho c}{a}\end{aligned}}$$

2 equations in 2 unknowns - let EES solve them

```
"partial fraction expansion of volumetric generation term"
gv_max=C_3*rho*c/tau+C_4*rho*c/a
0=C_3+C_4
```

# Inverse Transform

| Function in $t$ | Function in $s$ |
|---|---|
| $C$ | $\frac{C}{s}$ |
| $Ct$ | $\frac{C}{s^2}$ |
| $\exp(Ct)$ | $\frac{1}{s-C}$ |
| $\sin(Ct)$ | $\frac{C}{s^2+C^2}$ |
| $\cos(Ct)$ | $\frac{s}{s^2+C^2}$ |
| $\sinh(Ct)$ | $\frac{C}{s^2-C^2}$ |
| $\cosh(Ct)$ | $\frac{s}{s^2-C^2}$ |

$$\hat{T} = \frac{T_\infty}{\tau s\left(s+\frac{1}{\tau}\right)} + \frac{\dot{g}'''_{max}}{\rho c}\frac{1}{\left(s+\frac{1}{a}\right)\left(s+\frac{1}{\tau}\right)} + \frac{T_\infty}{\left(s+\frac{1}{\tau}\right)}$$

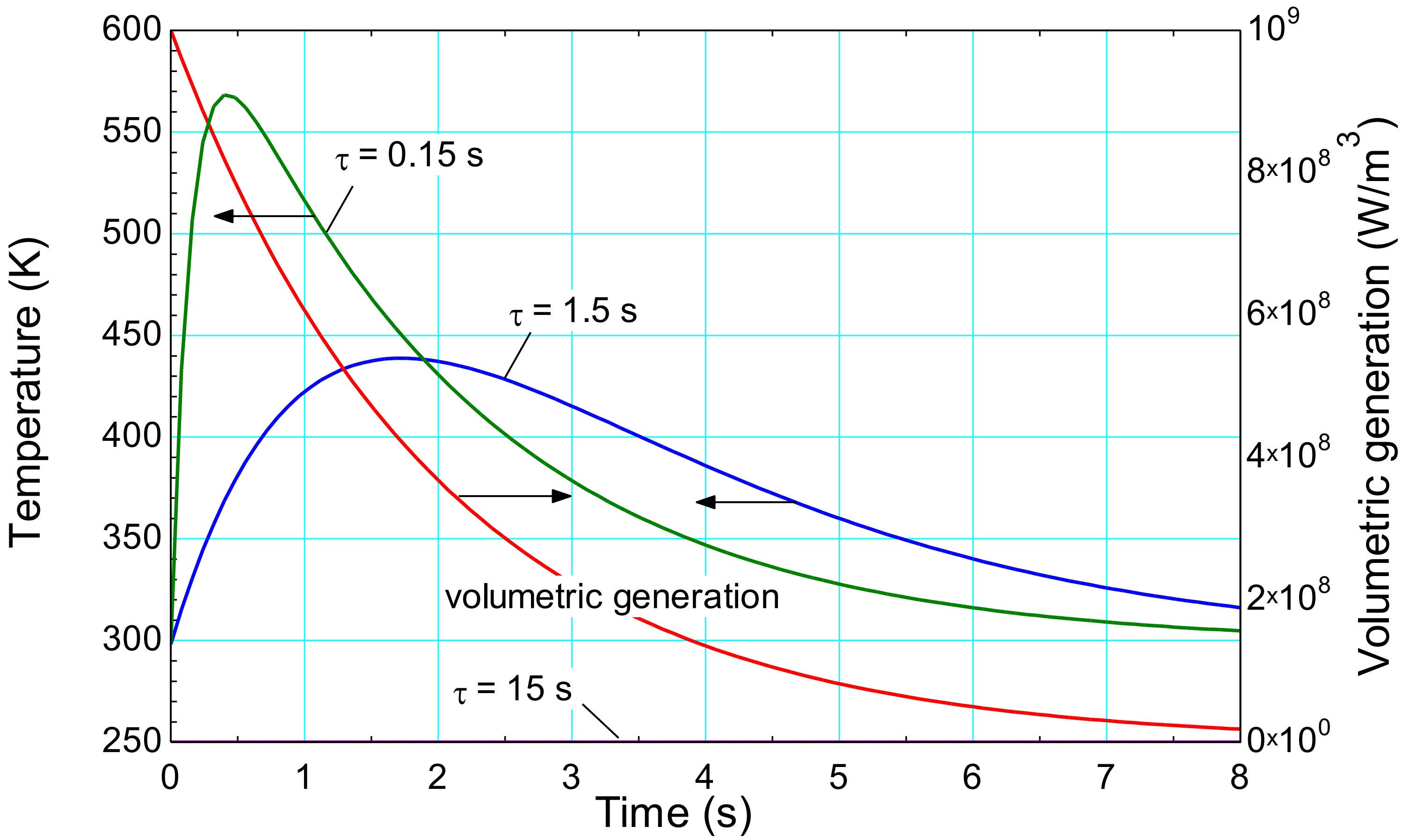
$$\hat{T} = \boxed{\frac{T_\infty}{s} - \cancel{\frac{T_\infty}{\left(s+\frac{1}{\tau}\right)}}} + \boxed{\frac{C_3}{\left(s+\frac{1}{a}\right)} + \frac{C_4}{\left(s+\frac{1}{\tau}\right)}} + \boxed{\cancel{\frac{T_\infty}{\left(s+\frac{1}{\tau}\right)}}}$$

$$\hat{T} = \boxed{\frac{T_\infty}{s}} + \boxed{\frac{C_3}{\left(s+\frac{1}{a}\right)}} + \boxed{\frac{C_4}{\left(s+\frac{1}{\tau}\right)}}$$

$$\hat{T} = T_\infty + C_3\exp\left(-\frac{t}{a}\right) + C_4\exp\left(-\frac{t}{\tau}\right)$$

# Solution

```
time=0 [s]
gv=gv_max*exp(-time/a)                              "volumetric generation"
T=T_infinity+C_3*exp(-time/a)+C_4*exp(-time/tau)"solution for temperature"
```




Note that the solution behaves as we initially anticipated