Heat and Mass Transfer, 3rd Edition
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Chapter 7
EXTERNAL FORCED CONVECTION

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Objectives

- Evaluate the heat transfer associated with flow over a flat plate for both laminar and turbulent flow, and flow over cylinders and spheres
- Distinguish between internal and external flow,
- Develop an intuitive understanding of friction drag and pressure drag, and evaluate the average drag and convection coefficients in external flow,
- Evaluate the drag and heat transfer associated with flow over a flat plate for both laminar and turbulent flow,
- Calculate the drag force exerted on cylinders during cross flow, and the average heat transfer coefficient, and
- Determine the pressure drop and the average heat transfer coefficient associated with flow across a tube bank for both in-line and staggered configurations.

Drag and Heat Transfer in External flow

- Fluid flow over solid bodies cause physical phenomena such as
  - ✓ *drag force*
    - automobiles
    - power lines
  - ✓ *lift force*
    - airplane wings
  - ✓ *cooling* of metal or plastic sheets.
- **Free-stream velocity** — the velocity of the fluid relative to an immersed solid body sufficiently far from the body.
- The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface.

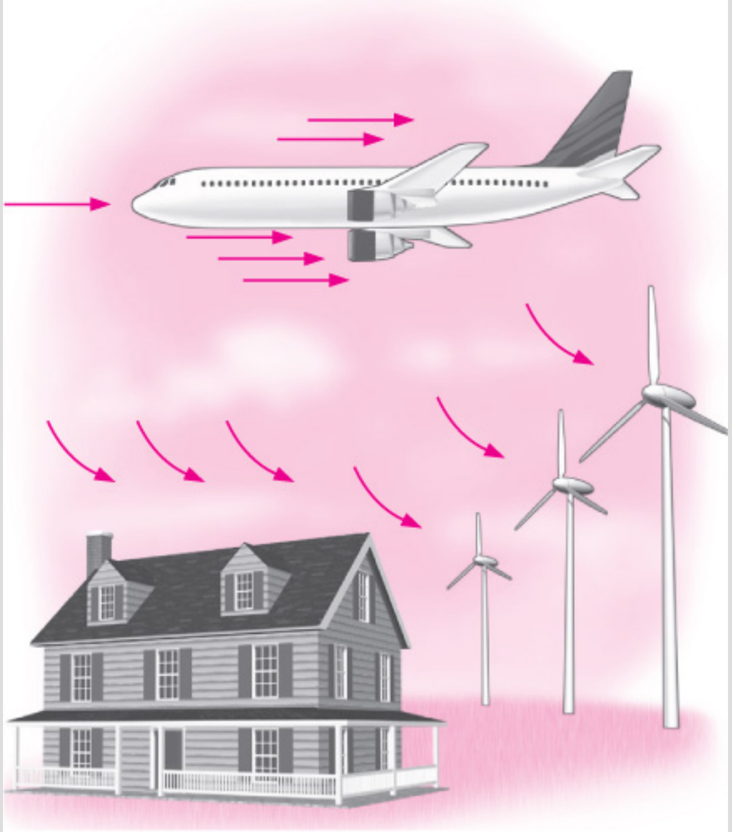


FIGURE 7–1
Flow over bodies is commonly encountered in practice.

Friction and Pressure Drag

- The force a flowing fluid exerts on a body in the flow direction is called **drag**.
- Drag is compose of:
  - ✓ pressure drag,
  - ✓ friction drag (skin friction drag).
- The drag force F_D depends on the
  - ✓ density ρ of the fluid,
  - ✓ the upstream velocity V, and
  - ✓ the size, shape, and orientation of the body.
- The dimensionle… **efficient** C_D *is* defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

$$C_D = C_{D,\,\text{friction}} + C_{D,\,\text{pressure}}$$

For flat plate: $C_D = C_{D,\,\text{friction}} = C_f$

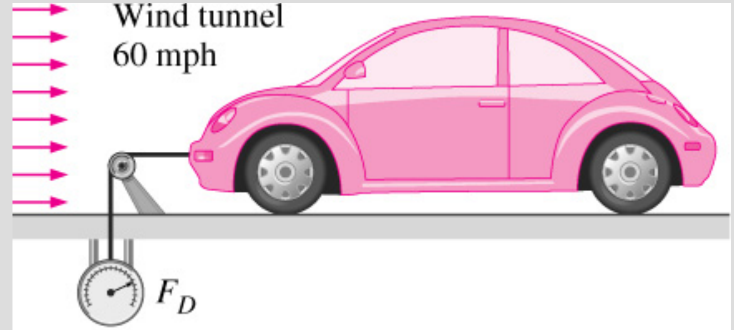


FIGURE 7–2
Schematic for measuring the drag force acting on a car in a wind tunnel.

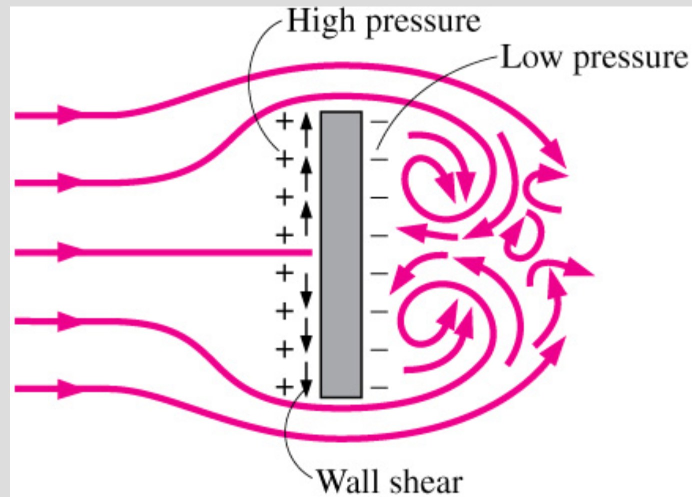


FIGURE 7–3
Drag force acting on a flat plate normal to the flow depends on the pressure only and is independent of the wall shear, which acts normal to the free-stream flow.

- At low Reynolds numbers, most drag is due to friction drag.
- The friction drag is also proportional to the surface area.
- The pressure drag is proportional to the frontal area and to the *difference* between the pressures acting on the front and back of the immersed body.
- The pressure drag is usually dominant for blunt bodies and negligible for streamlined bodies.
- When a fluid separates from a body, it forms a separated region between the body and the fluid stream.
- The larger the separated region, the larger the pressure drag.

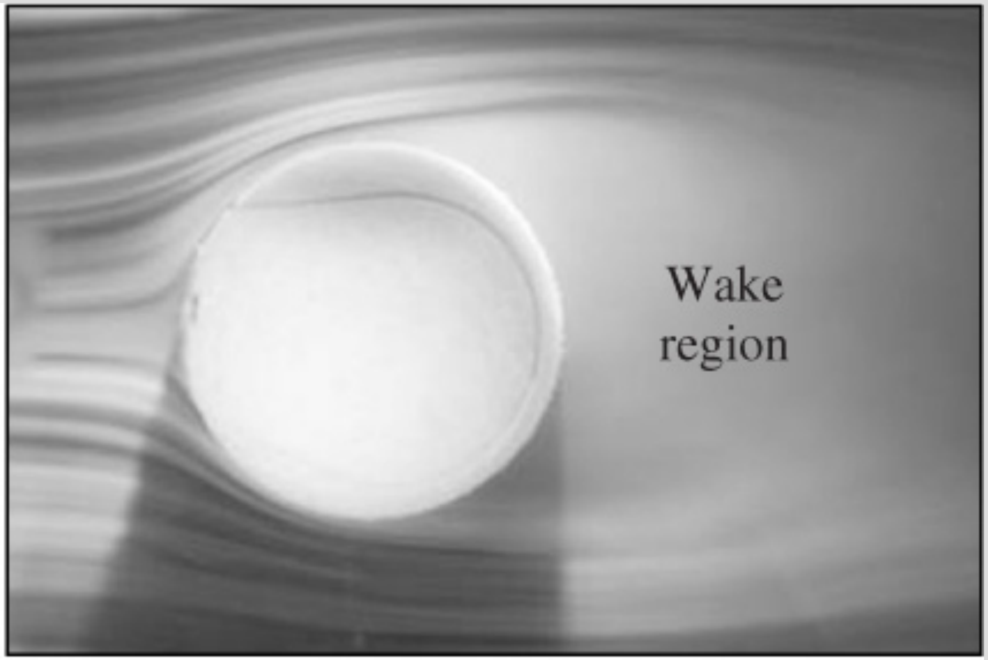


FIGURE 7–5
Separation during flow over a tennis ball and the wake region.
Courtesy of NASA and Cislunar Aerospace, Inc.

Heat Transfer

Local and average Nusselt numbers:

$$\mathrm{Nu}_x = f_1(x^*, \mathrm{Re}_x, \mathrm{Pr}) \quad \text{and} \quad \mathrm{Nu} = f_2(\mathrm{Re}_L, \mathrm{Pr})$$

Average Nusselt number:

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m \mathrm{Pr}^n$$

Film temperature:

$$T_f = \frac{T_s + T_\infty}{2}$$

Average friction coefficient:

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx$$

Average heat transfer coefficient:

$$h = \frac{1}{L}\int_0^L h_x\,dx$$

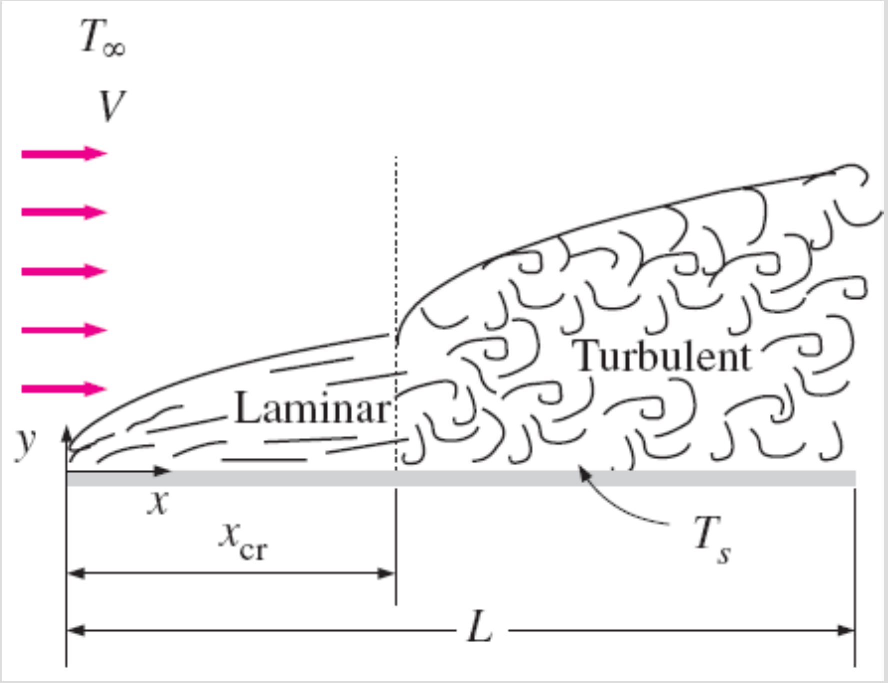
The heat transfer rate:

$$\dot{Q} = hA_s(T_s - T_\infty)$$

PARALLEL FLOW OVER FLAT PLATES

The transition from laminar to turbulent flow depends on the *surface geometry*, *surface roughness*, *upstream velocity*, *surface temperature*, and the *type of fluid*, among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance *x* from the leading edge of a flat plate is expressed as

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$



Laminar and turbulent regions of the boundary layer during flow over a flat plate.

A generally accepted value for the Critical Reynold number

$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from 10^5 to 3×10^6, depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

Friction Coefficient

Laminar: $\delta_{v,x} = \dfrac{4.91x}{\mathrm{Re}_x^{1/2}}$ and $C_{f,x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}}$, $\mathrm{Re}_x < 5 \times 10^5$

Turbulent: $\delta_{v,x} = \dfrac{0.38x}{\mathrm{Re}_x^{1/5}}$ and $C_{f,x} = \dfrac{0.059}{\mathrm{Re}_x^{1/5}}$, $5 \times 10^5 \le \mathrm{Re}_x \le 10^7$

Laminar: $C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}}$ $\mathrm{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}}$ $5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x \text{ laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,x,\text{ turbulent}}\,dx\right)$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log \dfrac{\varepsilon}{L}\right)^{-2.5}$

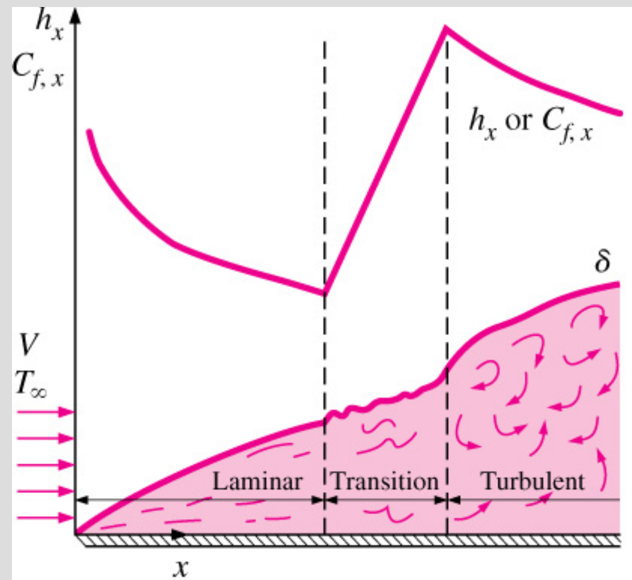
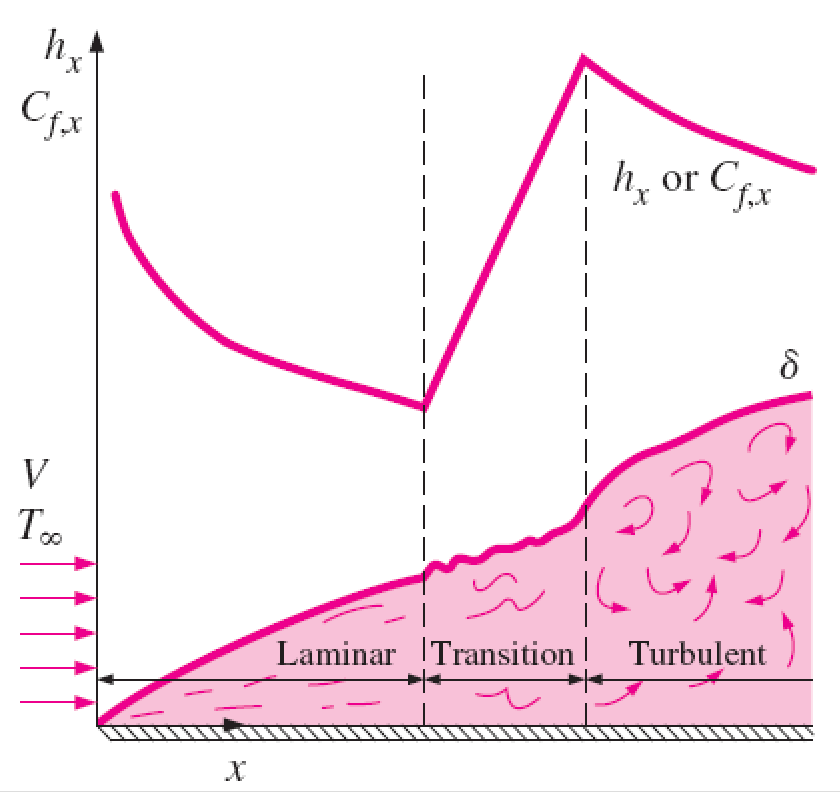


FIGURE 7–9
The variation of the local friction and heat transfer coefficients for flow over a flat plate.

The local Nusselt number at a location *x* for laminar flow over a flat plate may be obtained by solving the differential energy equation to be

Laminar: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

Turbulent: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$



These relations are for *isothermal* and *smooth* surfaces

The local friction and heat transfer coefficients are higher in turbulent flow than they are in laminar flow.

Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction.

The variation of the local friction and heat transfer coefficients for flow over a flat plate.

Nusselt numbers for average heat transfer coefficients

Laminar: $\mathrm{Nu} = \dfrac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Re}_L < 5 \times 10^5$

Turbulent: $\mathrm{Nu} = \dfrac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $\; 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60, \quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Laminar + turbulent

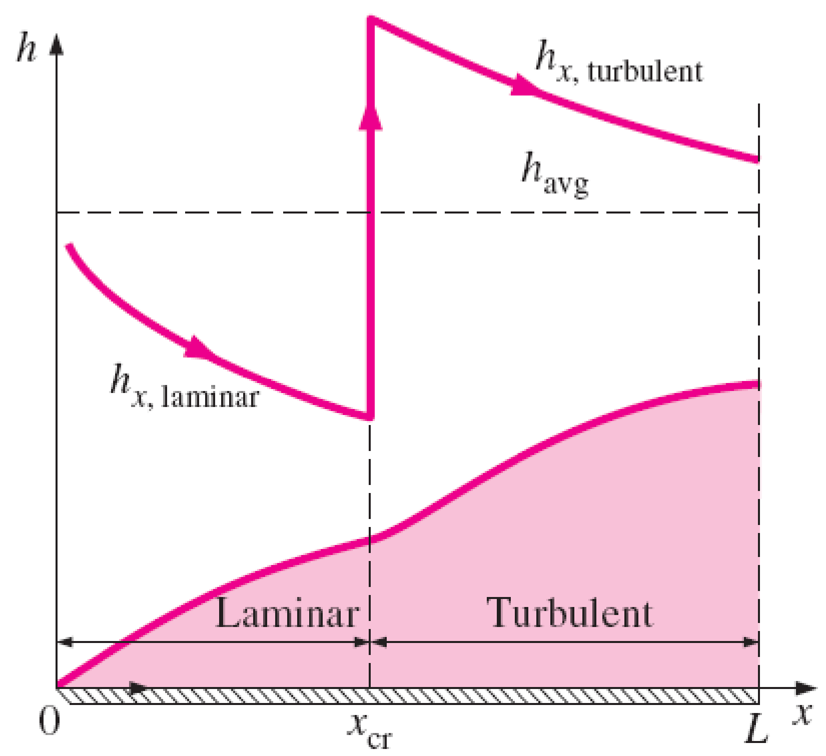
$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\,\mathrm{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x,\,\mathrm{turbulent}}\,dx\right)$$

For liquid metals

$$\mathrm{Nu}_x = 0.565(\mathrm{Re}_x\,\mathrm{Pr})^{1/2} \qquad \mathrm{Pr} < 0.05$$

For all liquids, all Prandtl numbers

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$



Graphical representation of the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

Flat Plate with Unheated Starting Length

Local Nusselt numbers

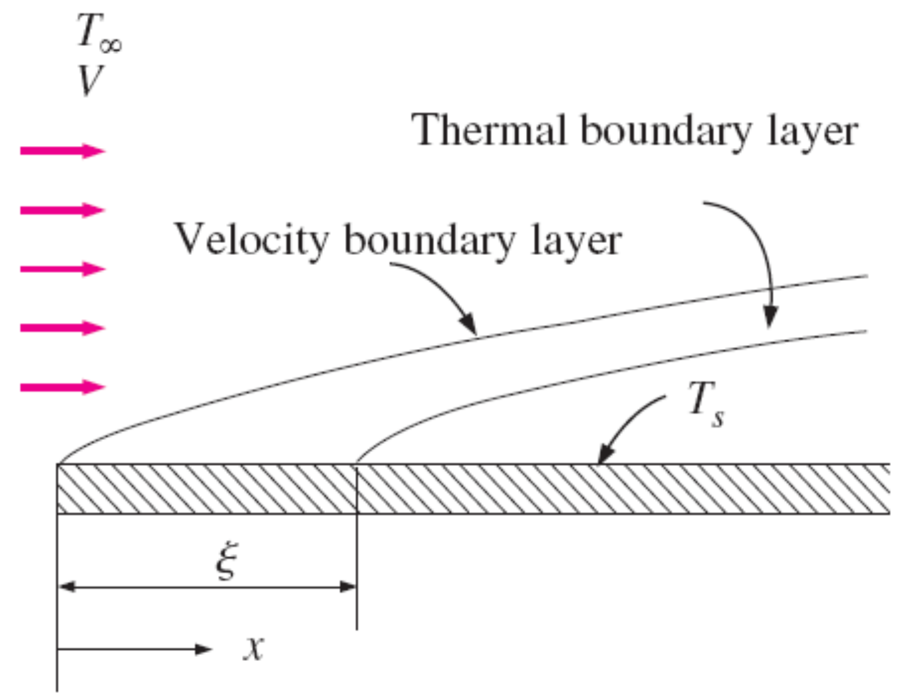
Laminar: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

Turbulent: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

Average heat transfer coefficients

Laminar: $$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent: $$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L}$$



Flow over a flat plate with an unheated starting length.

Uniform Heat Flux

For a flat plate subjected to *uniform heat flux*

Laminar: $$\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

Turbulent: $$\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

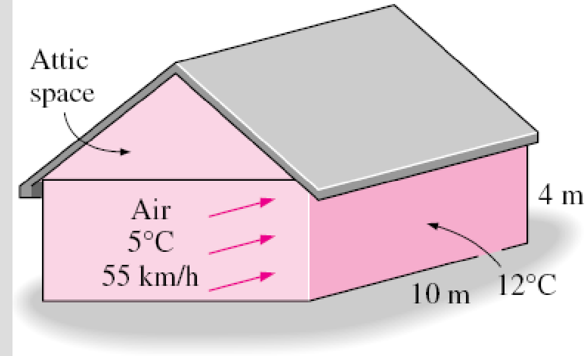
These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case.

When heat flux is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance *x* are determined from

$$\dot{Q} = \dot{q}_s A_s$$

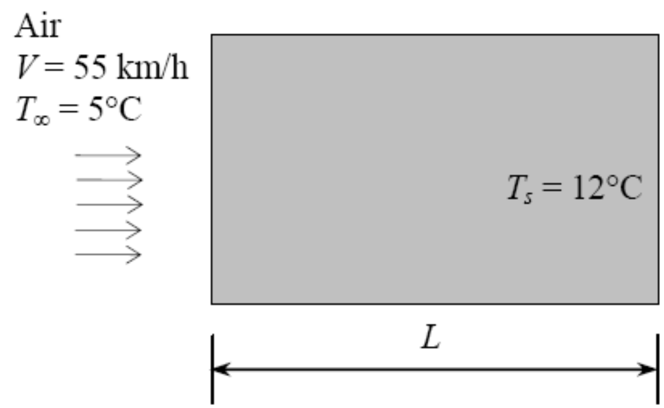
$$\dot{q}_s = h_x[T_s(x) - T_\infty] \qquad \rightarrow \qquad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$

7–16 During a cold winter day, wind at 55 km/h is blowing parallel to a 4-m-high and 10-m-long wall of a house. If the air outside is at 5°C and the surface temperature of the wall is 12°C, determine the rate of heat loss from that wall by convection. What would your answer be if the wind velocity was doubled? [redacted]



Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (12+5)/2 =$ 8.5°C are (Table A-15)

$$k = 0.02428 \text{ W/m}\cdot°\text{C}$$
$$\nu = 1.413\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7340$$



Analysis *Air flows parallel to the 10 m side:*

The Reynolds number in this case is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(55\times1000/3600)\text{m/s}](10\text{ m})}{1.413\times10^{-5}\text{ m}^2/\text{s}} = 1.081\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient and then heat transfer rate are determined to be

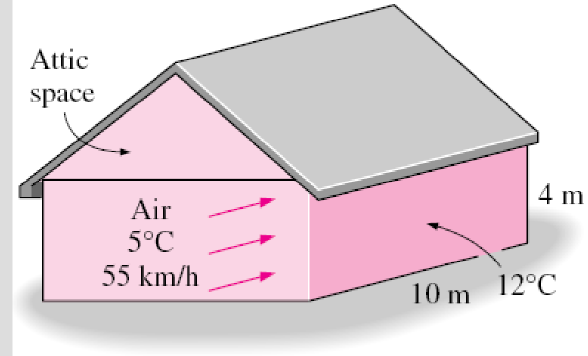
$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{\,0.8} - 871)\,\text{Pr}^{1/3} = [0.037(1.081\times10^7)^{0.8} - 871](0.7340)^{1/3} = 1.336\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.02428\text{ W/m.°C}}{10\text{ m}}(1.336\times10^4) = 32.43\text{ W/m}^2\text{.°C}$$

$$A_s = wL = (4\text{ m})(10\text{ m}) = 40\text{ m}^2$$

$$\dot{Q} = hA_s(T_\infty - T_s) = (32.43\text{ W/m}^2\text{.°C})(40\text{ m}^2)(12-5)°\text{C} = 9080\text{ W} = \mathbf{9.08\ kW}$$

7–16 During a cold winter day, wind at 55 km/h is blowing parallel to a 4-m-high and 10-m-long wall of a house. If the air outside is at 5°C and the surface temperature of the wall is 12°C, determine the rate of heat loss from that wall by convection. What would your answer be if the wind velocity was doubled? [redacted]



If the wind velocity is doubled:

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{[(110 \times 1000/3600)\,\mathrm{m/s}](10\,\mathrm{m})}{1.413 \times 10^{-5}\ \mathrm{m^2/s}} = 2.162 \times 10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\mathrm{Re}_L{}^{0.8} - 871)\,\mathrm{Pr}^{1/3} = [0.037(2.162 \times 10^7)^{0.8} - 871](0.7340)^{1/3} = 2.384 \times 10^4$$

$$h = \frac{k}{L} Nu = \frac{0.02428\ \mathrm{W/m.°C}}{10\,\mathrm{m}}(2.384 \times 10^4) = 57.88\ \mathrm{W/m^2.°C}$$

$$\dot{Q} = hA_s(T_\infty - T_s) = (57.88\ \mathrm{W/m^2.°C})(40\,\mathrm{m^2})(12-5)°\mathrm{C} = 16{,}210\ \mathrm{W} = \mathbf{16.21\,kW}$$

FLOW OVER CYLINDERS AND SPHERES

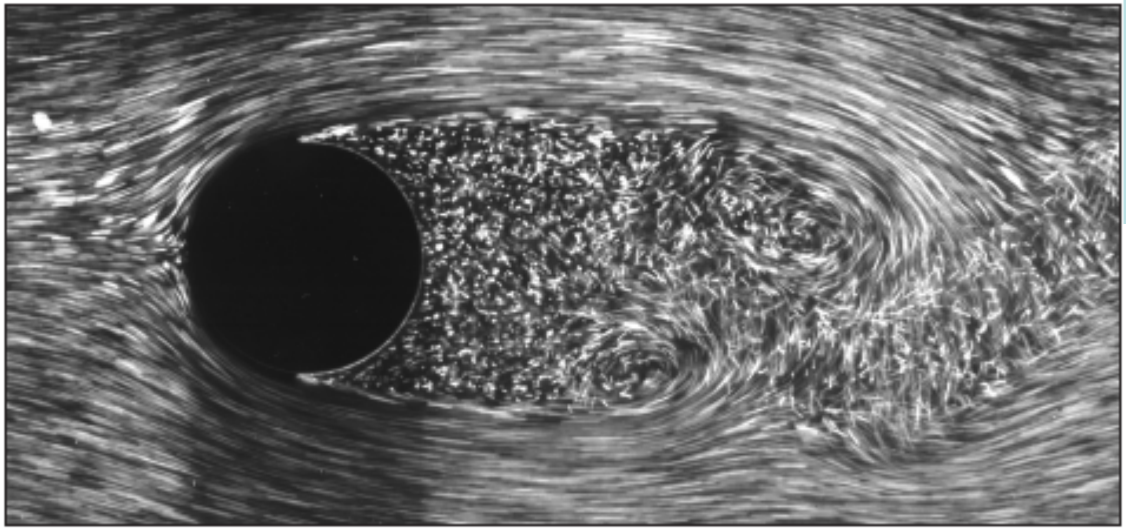
Flow over cylinders and spheres is frequently encountered in practice.

The tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes.

Many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $\mathrm{Re} = VD/\nu$ where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\mathrm{Re}_{cr} \cong 2 \times 10^5$. That is, the boundary layer remains laminar for $\mathrm{Re} \lesssim 2 \times 10^5$, is "transitional" for $2 \times 10^5 \lesssim \mathrm{Re} \lesssim 2 \times 10^6$, and becomes fully turbulent for $\mathrm{Re} \gtrsim 2 \times 10^6$.

At very low velocities, the fluid completely wraps around the cylinder. Flow in the wake region is characterized by periodic vortex formation and low pressures.



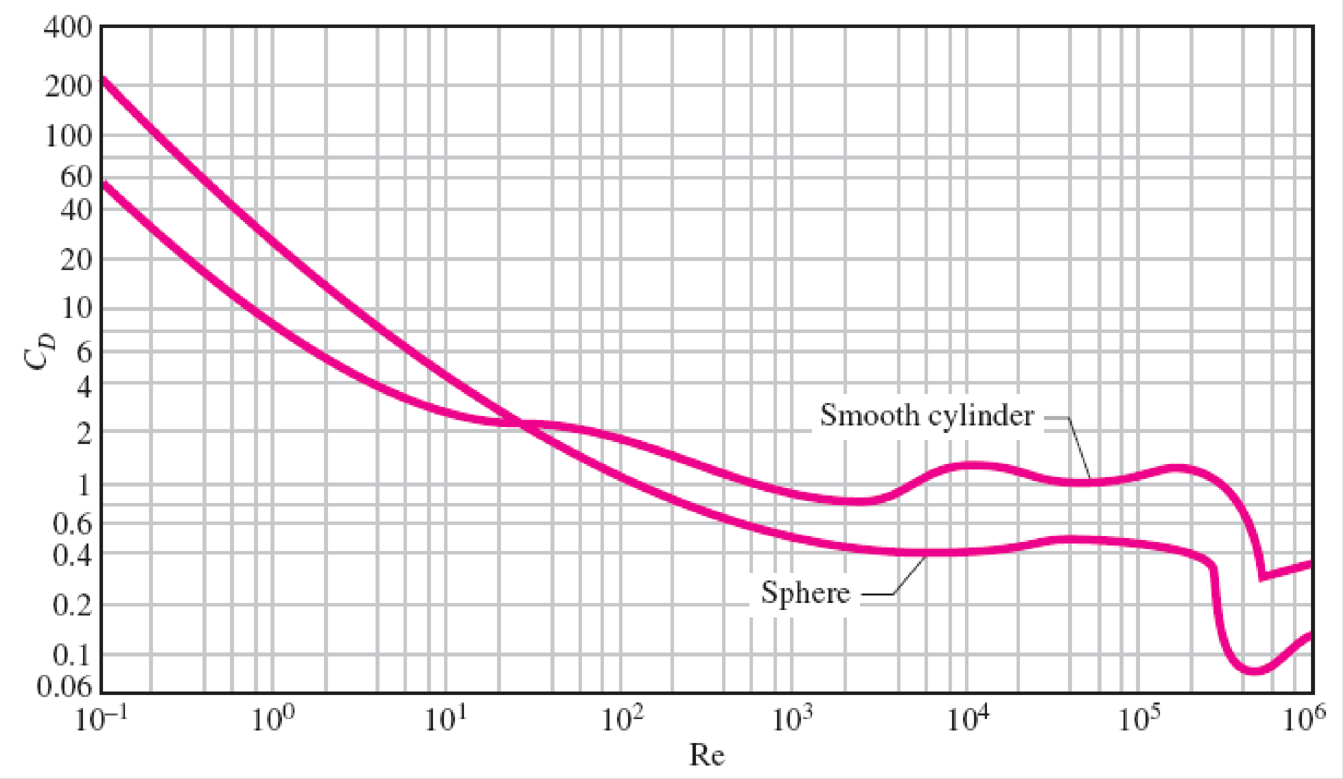
Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder at Re=2000.

For flow over cylinder or sphere, both the *friction drag* and the *pressure drag* can be significant.

The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow.

The drag force is primarily due to friction drag at low Reynolds numbers (Re<10) and to pressure drag at high Reynolds numbers (Re>5000).

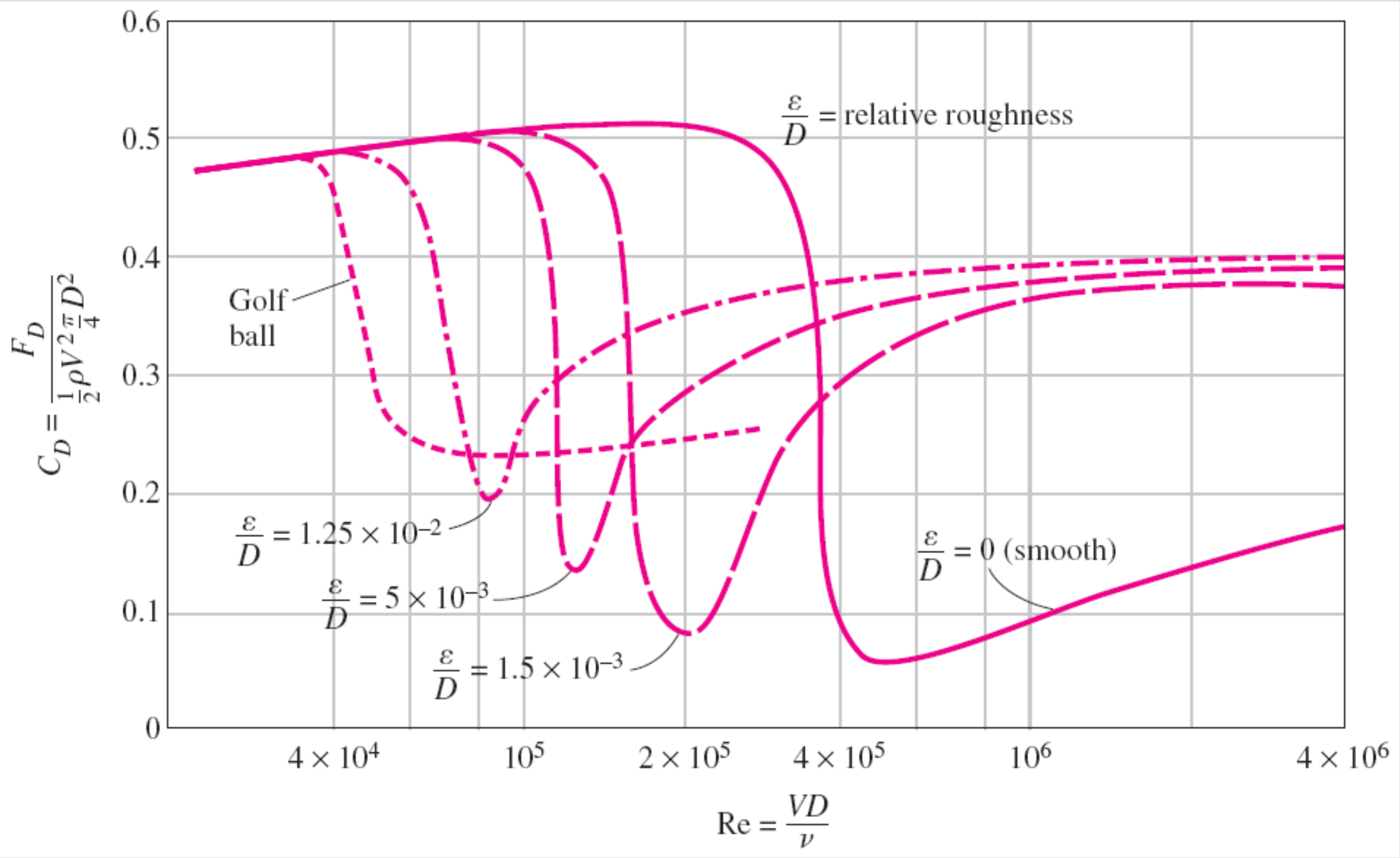
Both effects are significant at intermediate Reynolds numbers.



Average drag coefficient for cross-flow over a smooth circular cylinder and a smooth sphere.

Effect of Surface Roughness

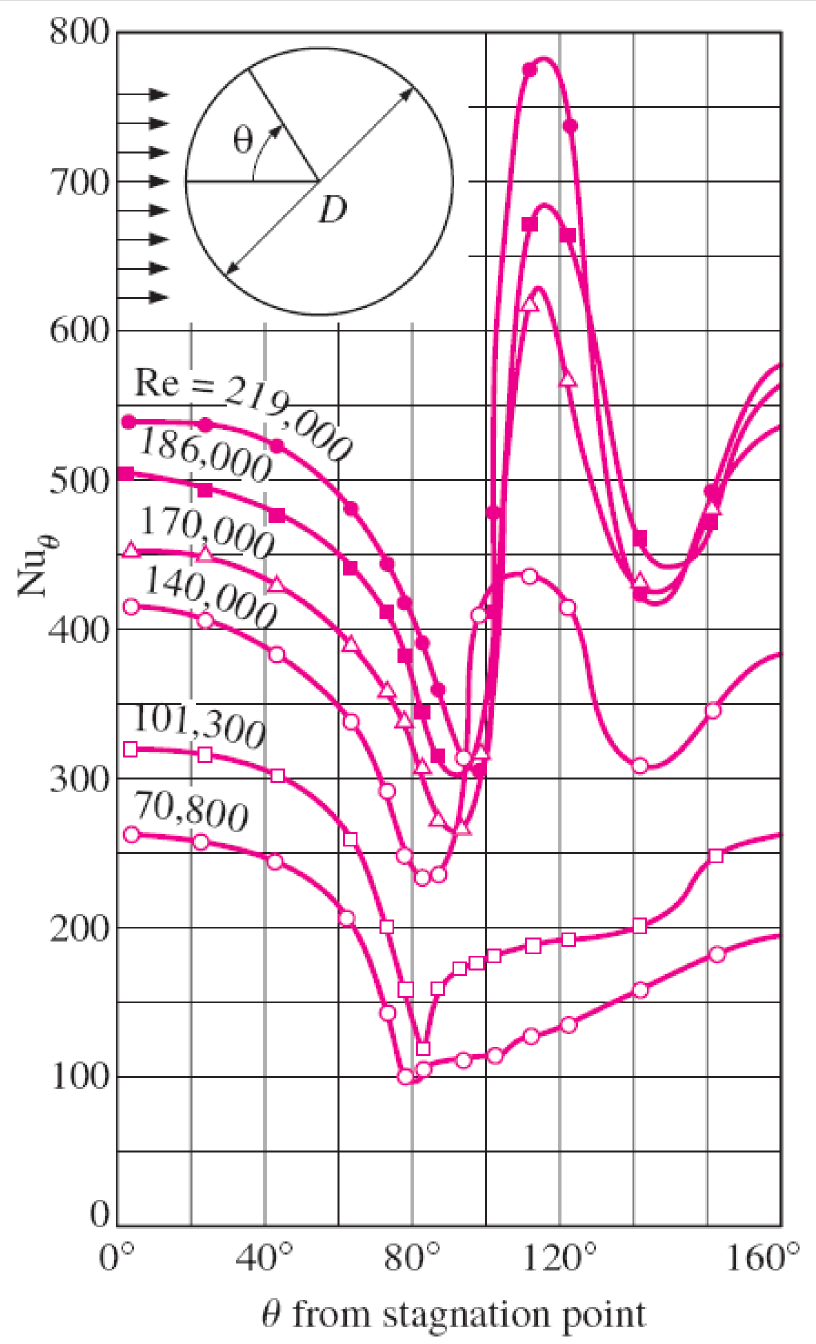
Surface roughness, in general, increases the drag coefficient in turbulent flow.
This is especially the case for streamlined bodies.
For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *increase* or *decrease* the drag coefficient depending on Reynolds number.



The effect of surface roughness on the drag coefficient of a sphere.

FLOW ACROSS CYLINDERS AND SPHERES

- Flows across cylinders and spheres, in general, involve *flow separation*, which is difficult to handle analytically.
- Flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.



Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross flow of air

For flow over a *cylinder*

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$\mathrm{RePr} > 0.2$

The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$

For flow over a *sphere*

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$

The fluid properties are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s.

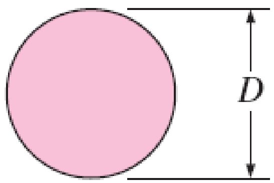
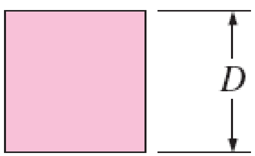
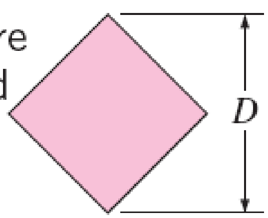
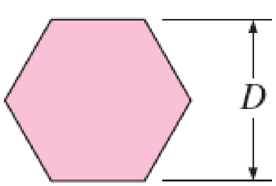
$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$n = \frac{1}{3}$

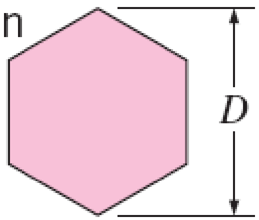
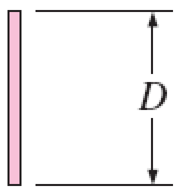
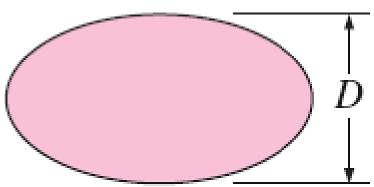
Constants C and m are given in the table.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. They are applicable to *smooth* surfaces.

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$

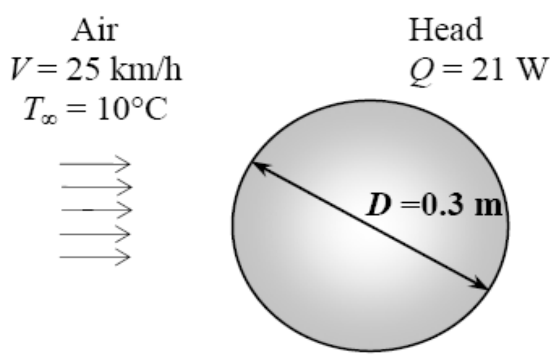
Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160 Re^{0.638} Pr^{1/3}$ $Nu = 0.0385 Re^{0.782} Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228 Re^{0.731} Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248 Re^{0.612} Pr^{1/3}$

7–49 An average person generates heat at a rate of 84 W while resting. Assuming one-quarter of this heat is lost from the head and disregarding radiation, determine the average surface temperature of the head when it is not covered and is subjected to winds at 10°C and 25 km/h. The head can be approximated as a 30-cm-diameter sphere. [redacted]

Properties We assume the surface temperature to be 15°C for viscosity. The properties of air at 1 atm pressure and the free stream temperature of 10°C are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2\text{/s}$$
$$\mu_\infty = 1.778\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@15°C} = 1.802\times10^{-5} \text{ kg/m.s}$$
$$\Pr = 0.7336$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(25\times1000/3600)\text{ m/s}](0.3\text{ m})}{1.426\times10^{-5}\text{ m}^2\text{/s}} = 1.461\times10^5$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1.461\times10^5)^{0.5} + 0.06(1.461\times10^5)^{2/3}\right](0.7336)^{0.4}\left(\frac{1.778\times10^{-5}}{1.802\times10^{-5}}\right)^{1/4} = 283.2$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02439\text{ W/m.°C}}{0.3\text{ m}}(283.2) = 23.02\text{ W/m}^2\text{.°C}$$

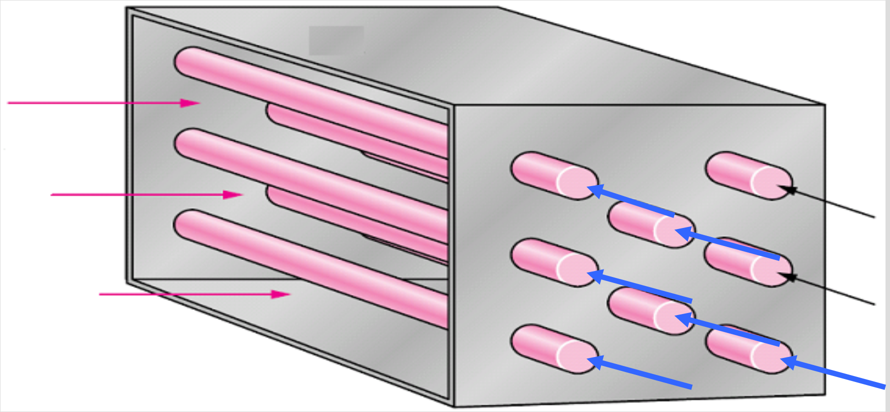
Then the surface temperature of the head is determined to be

$$A_s = \pi D^2 = \pi(0.3\text{ m})^2 = 0.2827\text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 10\text{ °C} + \frac{(84/4)\text{ W}}{(23.02\text{ W/m}^2\text{.°C})(0.2827\text{ m}^2)} = \mathbf{13.2\text{ °C}}$$

FLOW ACROSS TUBE BANKS

- Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment, e.g., heat exchangers.
- In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.
- Flow *through* the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes.
- For flow *over* the tubes the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them are altered.



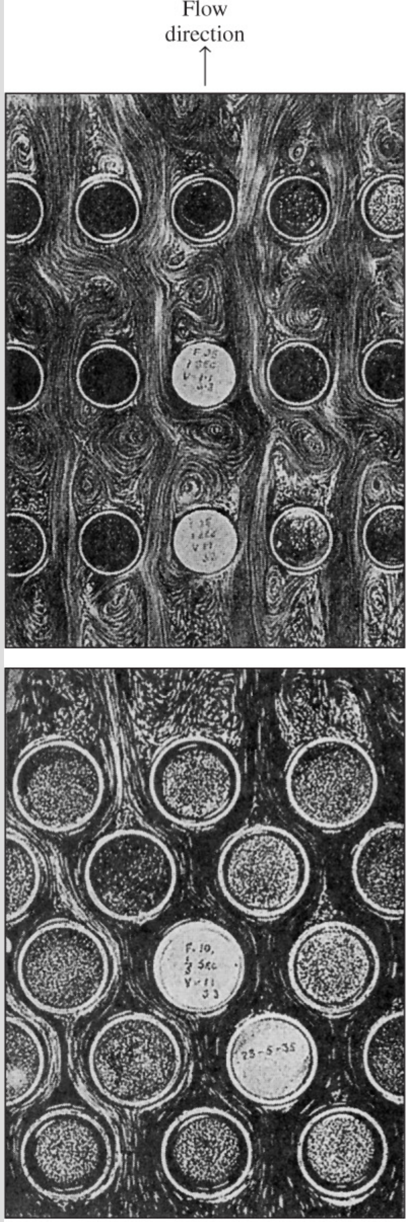
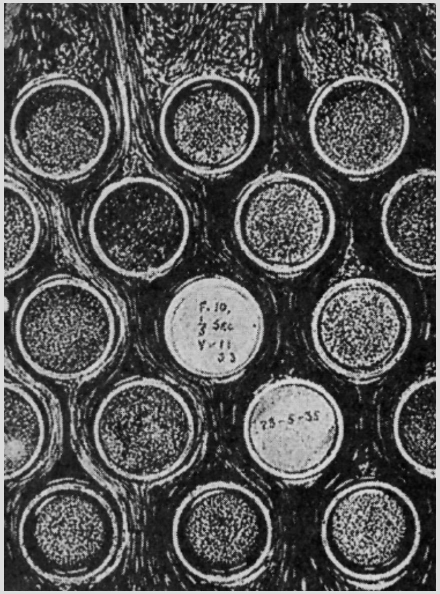


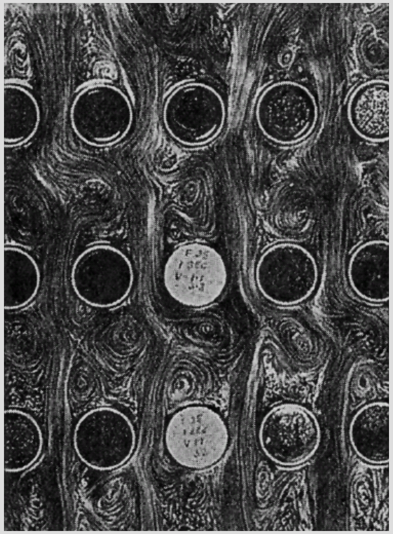
FIGURE 7–25

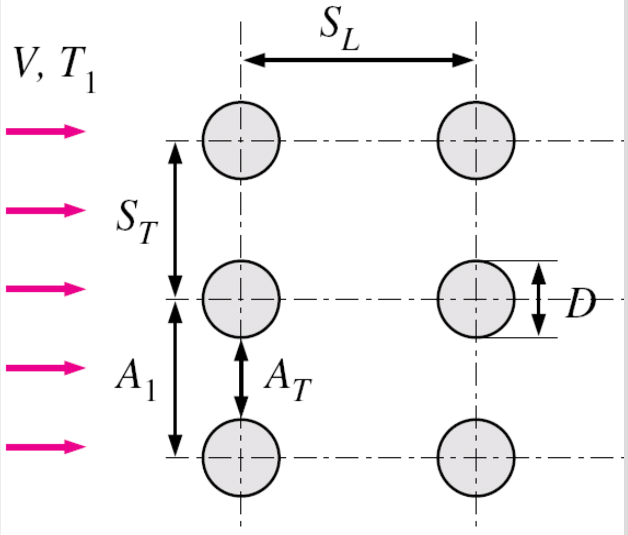
Flow patterns for staggered and in-line tube banks (photos by R. D. Willis).

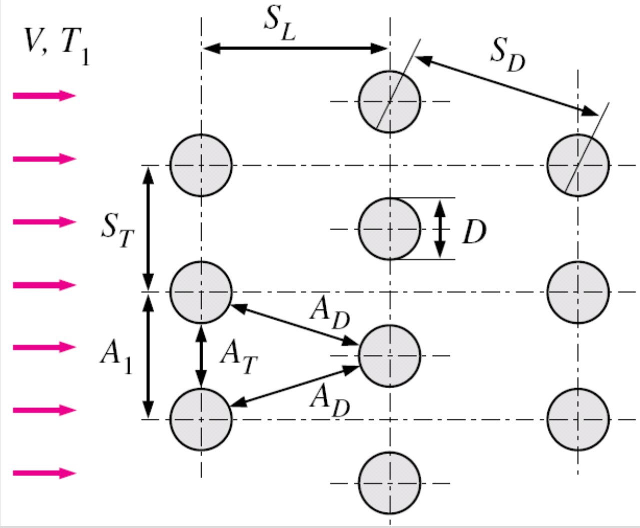
- Typical arrangement
  - ✓ in-line
  - ✓ staggered
- The outer tube diameter *D* is the characteristic length.
- The arrangement of the tubes are characterized by the
  - ✓ *transverse pitch S_T,*
  - ✓ *longitudinal pitch S_L*, and the
  - ✓ *diagonal pitch S_D* between tube centers

In-line **Staggered**









- The diagonal pitch: $S_D = \sqrt{S_L^2 + (S_T/2)^2}$
- Re number based on max. velocity: $\mathrm{Re}_D = \dfrac{\rho V_{\max} D}{\mu} = \dfrac{V_{\max} D}{\nu}$
- Max. velocity (in-line): $V_{\max} = \dfrac{S_T}{S_T - D} V$
- Max. velocity (staggered): *Staggered and* $S_D < (S_T + D)/2$: $\quad V_{\max} = \dfrac{S_T}{2(S_D - D)} V$
- Nusselt number (Table 7-2): $\mathrm{Nu}_D = \dfrac{hD}{k} = C\,\mathrm{Re}_D^m \mathrm{Pr}^n (\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
- Average temperature of inlet and exit (for property evaluation): $T_m = \dfrac{T_i + T_e}{2}$
- Nusselt number (< 16 rows): $\mathrm{Nu}_{D,N_L} = F\,\mathrm{Nu}_D$
- Log mean temp. dif. $\Delta T_{\ln} = \dfrac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \dfrac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$
- Exit temperature: $T_e = T_s - (T_s - T_i)\exp\left(\pm \dfrac{A_s h}{\dot{m} c_p}\right)$
- Heat transfer rate: $\dot{Q} = h A_s \Delta T_{\ln} = \dot{m} c_p (T_e - T_i)$

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, 1987)*

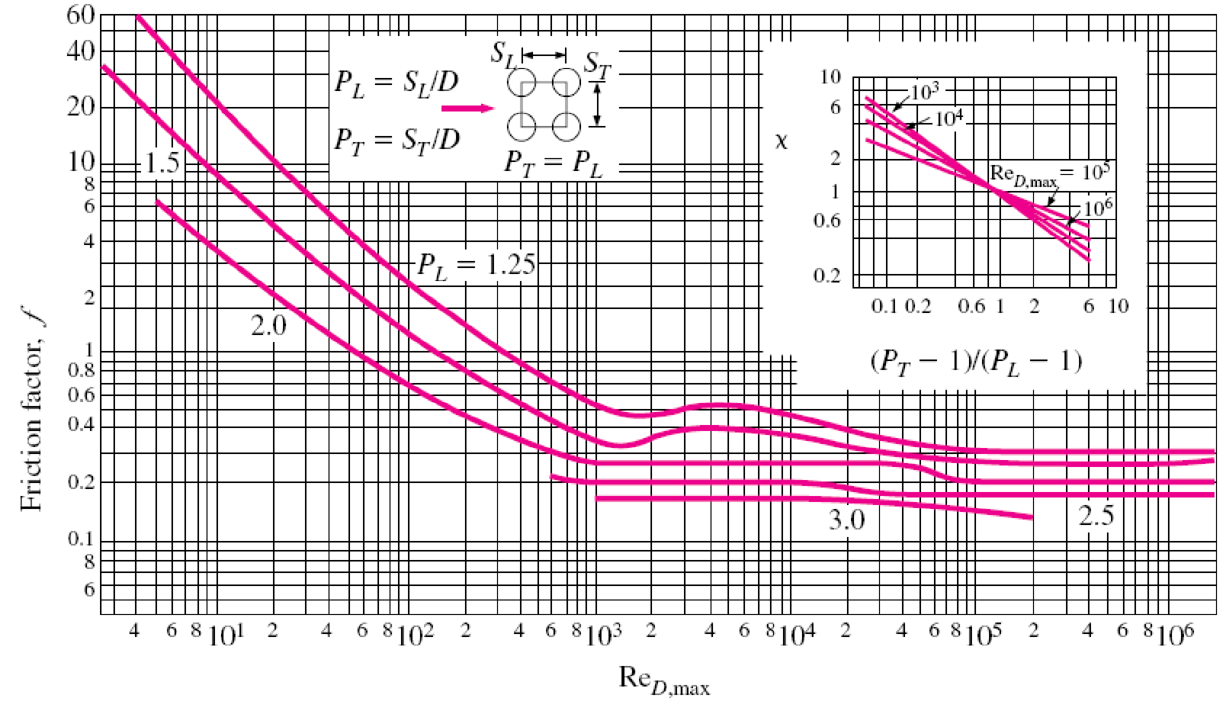
Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\, Re_D^{0.63} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\, Re_D^{0.8} Pr^{0.4} (Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\, Re_D^{0.4} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\, Re_D^{0.5} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35 (S_T/S_L)^{0.2} Re_D^{0.6} Pr^{0.36} (Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031 (S_T/S_L)^{0.2} Re_D^{0.8} Pr^{0.36} (Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

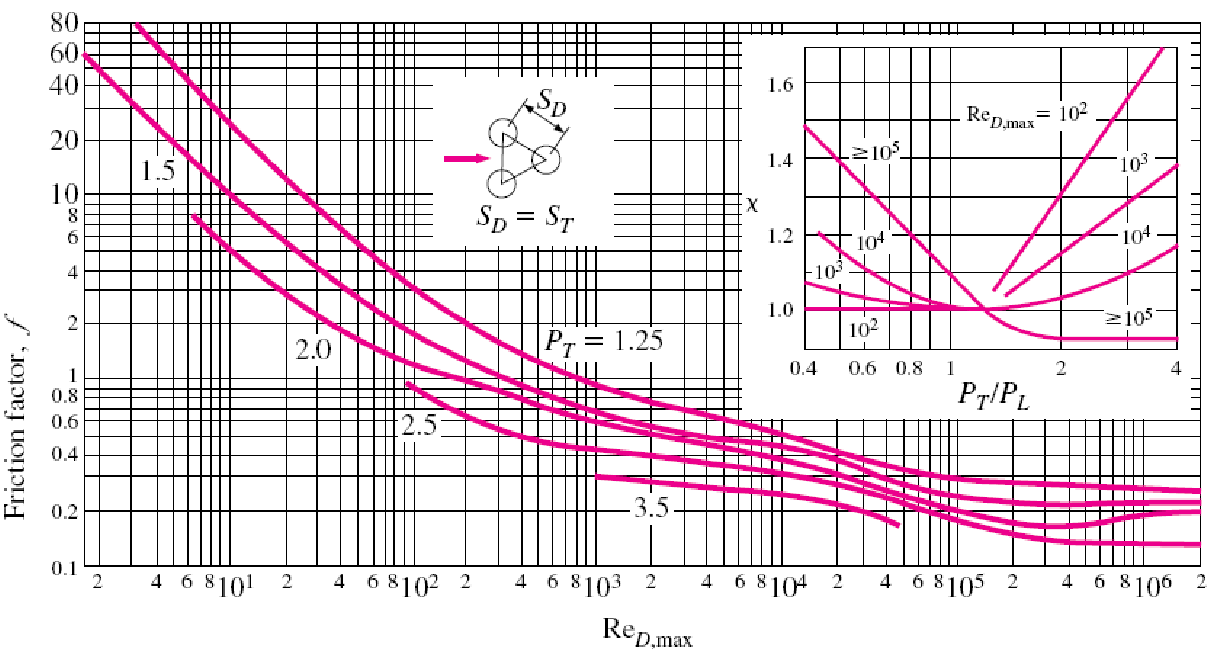
TABLE 7–3

Correction factor F to be used in $Nu_{D, N_L} = F Nu_D$ for $N_L < 16$ and $Re_D > 1000$ (from Zukauskas, 1987)

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99



(a) In-line arrangement



(b) Staggered arrangement

Pressure drop:

$$\Delta P = N_L f \chi \frac{\rho V_{max}^2}{2}$$

$$\dot{W}_{pump} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho}$$

$$\dot{V} = V(N_T S_T L)$$

$$\dot{m} = \rho \dot{V} = \rho V(N_T S_T L)$$

- f is the friction factor and c is the correction factor.
- The correction factor χ given is used to account for the effects of deviation from square arrangement (in-line) and from equilateral arrangement (staggered).

Summary

- Parallel Flow Over Flat Plates
  - ✓ Flat Plate with Unheated Starting Length, Uniform Heat Flux
- Flow Across Cylinders and Spheres
- Flow across Tube Banks