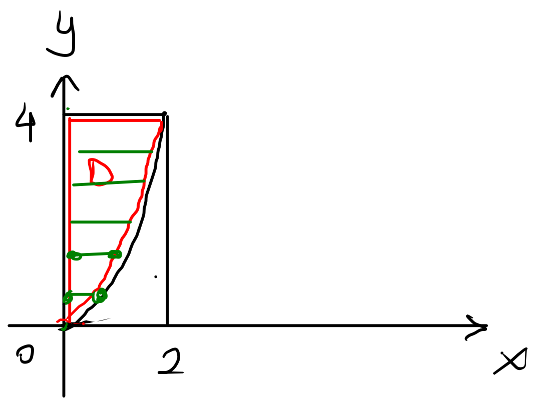
Örnekler

1) $I=\int_0^2\int_{x^2}^4 x.\sin(y^2)\,dy\,dx$ integralini hesaplayınız.

$\int_0^2 x\left[\underbrace{\int_{x^2}^4 \sin(y^2)\,dy}_{\text{Eliptik.}}\right]dx$ integrasyon sırası değiştirilmelidir.

$0\le x\le 2$

$x^2\le y\le 4$



$I=\int_0^4\int_0^{\sqrt{y}} x.\sin(y^2)\,dx\,dy = \int_0^4 \sin(y^2)\left(\int_0^{\sqrt{y}} x\,dx\right)dy$

$=\int_0^4 \sin(y^2).\left(\frac{x^2}{2}\Big|_0^{\sqrt{y}}\right)dy$

$=\frac{1}{2}\int_0^4 y.\sin(y^2)\,dy = \frac{1}{2}\int_0^{16}\sin t.\frac{dt}{2}$

$=\frac{1}{4}\left(-\cos t\Big|_0^{16}\right) = \frac{1}{4}\left[-\cos 16+1\right]$

$y^2=t$

$2y\,dy=dt \to y\,dy=\frac{dt}{2}$

$y=0 \Rightarrow t=0$

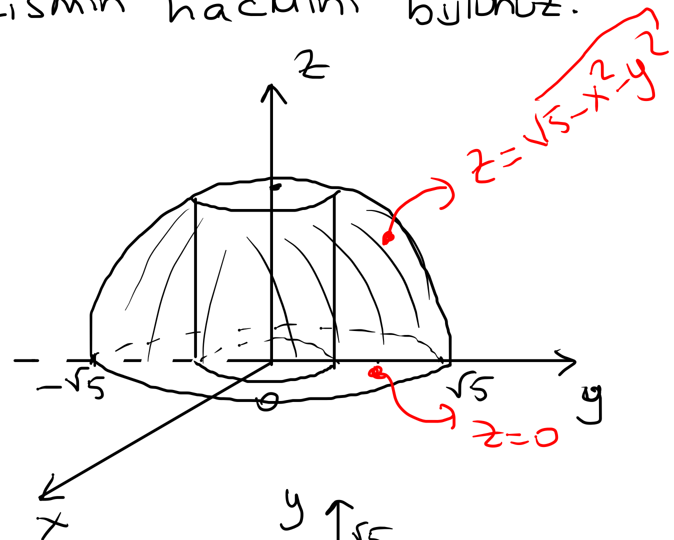
$y=4 \Rightarrow t=16$

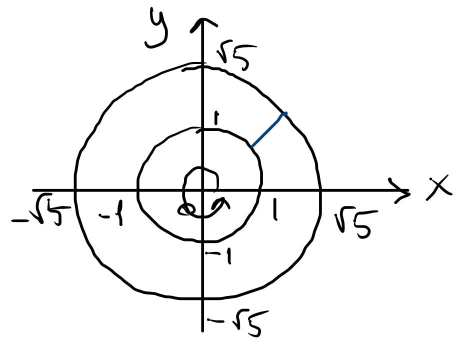
2) $z=\sqrt{5-x^2-y^2}$ yarı-küresi ile üstten ve $x^2+y^2=1$ silindiri ve de alttan $z=0$ düzlemi ile sınırlı cismin hacmini bulunuz.

$x=\rho\cos\theta$

$y=\rho\sin\theta$

$J=\rho$





$$V=\iint_D \left[\sqrt{5-x^2-y^2}-0\right] dA$$

$$V=\int_0^{2\pi}\int_1^{\sqrt{5}} \sqrt{5-\rho^2}\,\rho\, d\rho\, d\theta$$

$$=\int_0^{2\pi}\int_4^{0} \sqrt{t}\cdot\left(-\frac{dt}{2}\right) d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\left[\int_0^4 \sqrt{t}\, dt\right] d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\left(\frac{2}{3}t^{3/2}\Big|_0^4\right) d\theta = \frac{1}{3}\cdot 8\int_0^{2\pi} d\theta$$

$$=\frac{8}{3}\cdot 2\pi = \frac{16\pi}{3}\, br^3$$

$5-\rho^2=t$

$-2\rho\, d\rho = dt$

$\rho\, d\rho = -\frac{dt}{2}$

$\rho=1 \Rightarrow t=4$

$\rho=\sqrt{5} \Rightarrow t=0$

3) $z = 5 - x - y$ yüzeyi altında $D: 0 \le x \le 1$, $0 \le y \le 3$ $\left([0,1] \times [0,3] \right)$ bölgesi üstünde bulunan cismin hacmini bulunuz.

$$V = \iint_D (5 - x - y)\, dA = \int_0^1 \left[\int_0^3 (5 - x - y)\, dy \right] dx = \int_0^1 \left[5y - xy - \frac{y^2}{2} \Big|_0^3 \right] dx$$

$$= \int_0^1 \left[15 - 3x - \frac{9}{2} \right] dx$$

$$= \frac{21x}{2} - \frac{3x^2}{2} \Big|_0^1 = \frac{21 - 3}{2} = 9 \, br^3$$