/ D : (0,0) (1,0) (1,1) üçgeninde $\iint_T xy\, dA = ?$ $\frac{1}{8}$

/ $y=0,\ z=0,\ z=a-x+y$ $a>0$ $y=a-\frac{x^2}{a}$ hacim. $\frac{28}{15}a^3$

İki katlı integralde değişken dönüşümü:

1) Kartezyen Koordinatlarda Değişken Dönüşümü:

$\iint_D f(x,y)\,dx\,dy$ integralinde $x=x(u,v)$ $y=y(u,v)$ değişken dönüşümü yapıldığında D bölgesi bir D' bölgesine dönüşür ve integralin yeni şekli $\iint_{D'} f[x(u,v),y(u,v)]\cdot|J|\,du\,dv$ olur.

$$J=\frac{D(x,y)}{D(u,v)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix} \qquad J=\frac{D(x,y,z)}{D(u,v,w)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w}\\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w}\end{vmatrix}$$

$$\frac{1}{J}=\frac{D(u,v)}{D(x,y)}$$

$$J\cdot\frac{1}{J}=1$$

ÖR ⇒ $\int_1^2\int_0^{\sqrt{3-y}}\frac{x}{y}\,dx\,dy$ integralinde $x^2=u-v$ $y=v$ değişken dönüşümünü yaparak integrali hesaplayınız.

$x^2=u-v$

$y=v$

$D: 1\le y\le 2$

$0\le x\le\sqrt{3-y}$

$x=0 \Rightarrow u-v=0 \Rightarrow u=v$

$y=1 \Rightarrow v=1$

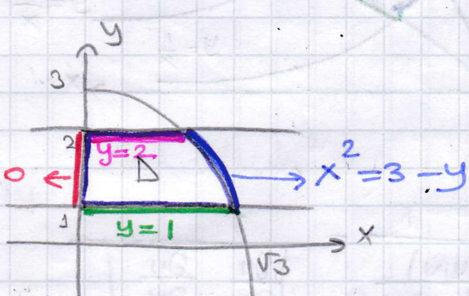
$y=2 \Rightarrow v=2$

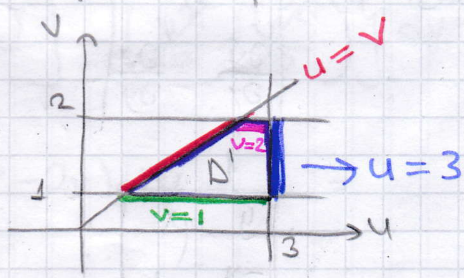
$x^2=3-y \Rightarrow u=3$

$x^2=3-y$

$x^2+y=3$

$x^2+y=u$





$$\left.\begin{matrix}x^2=u-v\\ y=v\end{matrix}\right\} J=\frac{D(x,y)}{D(u,v)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}$$

$$\left.\begin{matrix}x^2+y=u\\ y=v\end{matrix}\right\} \Rightarrow J=\frac{D(u,v)}{D(x,y)}$$

$$=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}=\begin{vmatrix}2x & 1\\ 0 & 1\end{vmatrix}=\underline{\underline{2x}}$$

$$I=\iint_{D'} \frac{x}{y}\cdot|J|\cdot du\,dv$$

$$=\iint_{D'} \frac{x}{y}\cdot\frac{1}{2x}\,du\,dv$$

$$=\iint_{D'} \frac{1}{v}\,du\,dv$$

$$=\int_1^2\int_v^3 \frac{1}{v}\,du\,dv$$

NOT $x=x(u,v)$

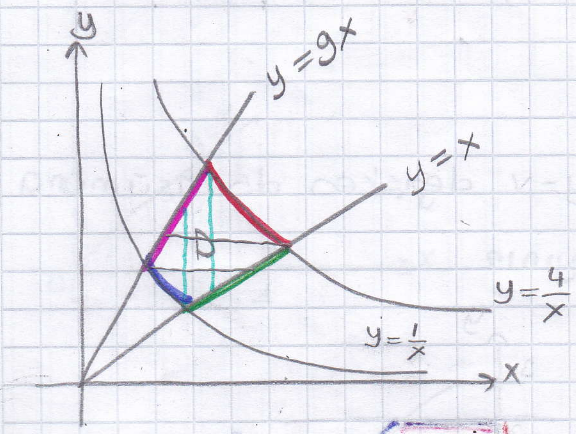
$y=y(u,v)$

$$\boxed{J=\frac{D(x,y)}{D(u,v)}}$$

$$\boxed{J=\frac{1}{\dfrac{D(u,v)}{D(x,y)}}}$$

ÖR ⇒ D: $x\cdot y=1$ $x\cdot y=4$ $\frac{y}{x}=1$ $\frac{y}{x}=9$ eğrileri ile sınırlı bölgenin alanını hesaplayınız.

birinci dörte bir bölge



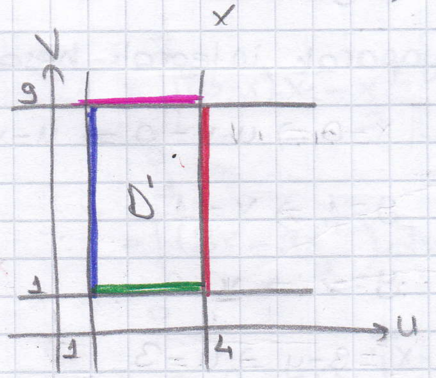
$$\left.\begin{aligned} x\cdot y&=u\\ \frac{y}{x}&=v\end{aligned}\right\}$$

$xy=1 \Rightarrow u=1$

$xy=4 \Rightarrow u=4$

$\frac{y}{x}=1 \Rightarrow v=1$

$\frac{y}{x}=9 \Rightarrow v=9$



$$J=\frac{1}{\dfrac{D(u,v)}{D(x,y)}}=\frac{1}{\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}}$$

$$=\frac{1}{\begin{vmatrix} y & x\\ -\frac{y}{x^2} & \frac{1}{x}\end{vmatrix}}$$

$$=\frac{1}{\frac{y}{x}+\frac{y}{x}}$$

$$=\frac{1}{2\frac{y}{x}}=\underline{\frac{1}{2v}}$$

$$A=\int_1^9\int_1^4 \frac{1}{2v}\,du\,dv=\int_1^9\left[\frac{u}{2v}\Big|_1^4\right]dv$$

$$=\frac{3}{2}\int_1^9\frac{dv}{v}=\frac{3}{2}\left[\ln v\Big|_1^9\right]$$

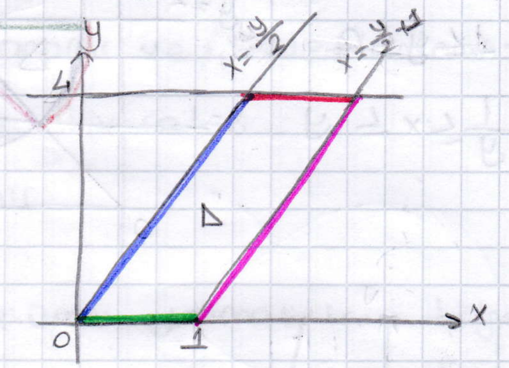
$$=\frac{3}{2}\ln 8$$

$$=\underline{\frac{9}{2}\ln 2\ br^2}$$

ÖR ⇒ $I=\int_0^4\int_{\frac{y}{2}}^{\frac{y}{2}+1}\left(\frac{2x-y}{2}\right)dx\,dy$ integralinde $y=2v$ $x=\frac{u+v}{2}$ değişken dönüşümü yaparak integrali hesaplayınız.

$0\le y\le 4$

$\frac{y}{2}\le x\le \frac{y}{2}+1$

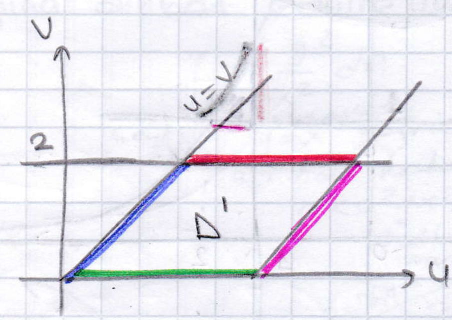


$x=\frac{u+v}{2}\Big\}\ x=\frac{y}{2}\Rightarrow v=\frac{u+v}{2}\Rightarrow u=v$

$x=\frac{y}{2}+1\Rightarrow v+1=\frac{u+v}{2}\Rightarrow u=v+2$

$y=2v\Big\}\ y=0\Rightarrow v=0$

$y=4\Rightarrow v=2$



$$J=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=\begin{vmatrix}\frac{1}{2} & \frac{1}{2}\\ 0 & 2\end{vmatrix}$$

$=\underline{1}$

$$I=\iint_{\Delta'}\left[\frac{u+v}{2}-v\right]1\,du\,dv$$

$$I=\int_0^2\int_v^{v+2}\left(u-\frac{v}{2}\right)du\,dv=\int_0^2\left[\frac{u^2}{2}-\frac{uv}{2}\Big|_v^{v+2}\right]dv$$

$$=\int_0^2\left\{\left[\left(\frac{v+2}{2}\right)^2-\frac{v(v+2)}{2}\right]-\left(\cancel{\frac{v^2}{2}}-\cancel{\frac{v^2}{2}}\right)\right\}dv$$

$$=\frac{1}{2}\int_0^2(v^2+4v+4-v^2-2v)\,dv$$

$$=\frac{1}{2}\int_0^2(2v+4)\,dv$$

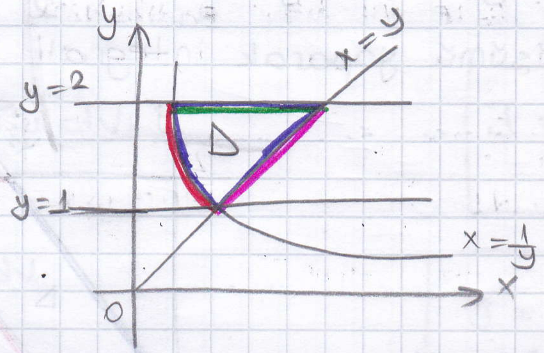
$$=\frac{1}{2}\left(v^2+4v\Big|_0^2\right)=\frac{1}{2}.12$$

$=6//$

ÖR ⇒ $\int_1^2\int_{\frac{1}{y}}^{y} \sqrt{\frac{y}{x}}\cdot e^{\sqrt{xy}}\,dx\,dy$ integralini hesaplayınız

$1 \le y \le 2$

$\frac{1}{y} \le x \le y$



$\sqrt{xy} = u \qquad xy = u^2$

$\sqrt{\frac{y}{x}} = v, \qquad \frac{y}{x} = v^2$

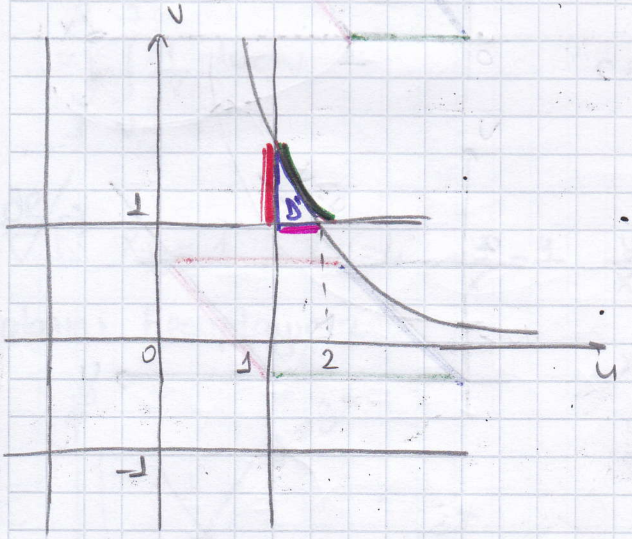
$y^2 = u^2 v^2$

$y = 2 \Rightarrow u^2v^2 = 4 \Rightarrow u^2 = \frac{4}{v^2}$

$u = \mp \frac{2}{v}$

$x = y \Rightarrow v^2 = 1 \Rightarrow v = \mp 1$

$x = \frac{1}{y} \Rightarrow u^2 = 1 \Rightarrow u = \mp 1$



$$J = \frac{1}{\frac{D(u,v)}{D(x,y)}} = \frac{1}{\begin{vmatrix} \frac{y}{2\sqrt{xy}} & \frac{x}{2\sqrt{xy}} \\ \frac{-y/x^2}{2\sqrt{\frac{y}{x}}} & \frac{1/x}{2\sqrt{\frac{y}{x}}} \end{vmatrix}} = \underline{\underline{2x}}$$

$$I = \iint_{D'} \sqrt{\frac{y}{x}}\cdot e^{\sqrt{xy}}\, 2x\, du\,dv$$

$$= 2\iint_{D'} u\cdot e^u\, du\,dv$$

$$= 2\int_1^2\int_1^{2/u} u\,e^u\, dv\,du = 2\int_1^2 \left[u e^u v \Big|_1^{\frac{2}{u}} \right] du$$

$$= 2\int_1^2 (e^u - u e^u)\, du$$

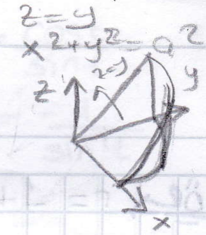
$$\underbrace{\begin{matrix} u = t & e^u du = ds \\ du = dt & e^u = s \end{matrix}}$$

$$I = 2e^u\Big|_1^2 - 2\left[u e^u \Big|_1^2 - \int_1^2 e^u du \right]$$

$$= 2(e^2 - e) - 2\left[(2e^2 - e) - (e^u\Big|_1^2)\right]$$

$$= 2e^2 - 2e - 4e^2 + 2e + 2e^2 - 2e$$

$$= \underline{\underline{-2e}}$$



Bölge çembersel veya eliptik ise kutupsal koordinatlar kullanılır

2) Kutupsal Koordinatlarda Değişken Dönüşümü :

$\iint_D f(x,y)\,dA$ integralinde $x=r\cos\theta$ $y=r\sin\theta$ dönüşümü yapıldığında D bölgesi bu kutupsal koordinatlara göre taranır ve J determinantı hesaplanır. $dx\,dy$ daima $r\,dr\,d\theta$ şeklinde alınarak

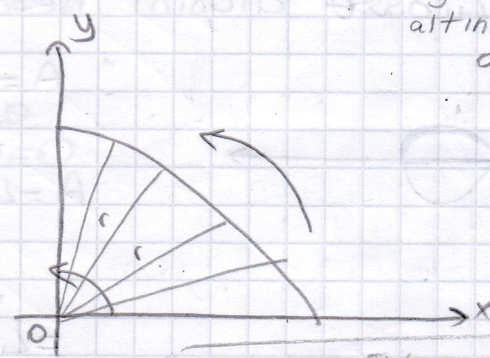
$\iint_D f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$ şeklinde hesaplanır.

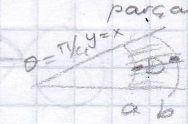
NOT: Kutupsal koordinatlarda değişken dönüşümü yapmak için bölgenin çembersel ya da eliptik bölge olup olmadığına dikkat edilmelidir.

$$\iint_D f(x,y)\,dA = \iint_D f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

$\iint_D \frac{y^2}{x^2}\,dA$ · Geyrek düzlemde $y=x$ doğrusu altında kalan $0<a^2\le x^2+y^2\le b^2$ halkasının parçasında

Çembersel:	Eliptik
$x=r\cos\theta$	$x=ar\cos\theta$
$y=r\sin\theta$	$y=br\sin\theta$
$J=r$	$J=abr$
$(x^2+y^2=a^2)$	$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\right)$





$0\le\theta\le\pi/2$ $0\le r\le a$

$z=f(r,\theta)=r\sin\theta$

$$\boxed{A=\iint_D r\,dr\,d\theta} \rightarrow \text{Alan hesabı}$$

ÖR. ⇒ $$I=\int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx \Rightarrow I=\int_0^{\pi/2}\int_0^1 e^{r^2} r\,dr\,d\theta$$

$r^2=t$

$2r\,dr=dt$

$r\,dr=\frac{dt}{2}$

D: $0\le x\le 1$

$0\le y\le\sqrt{1-x^2}$

$$=\int_0^{\pi/2}\int_0^1 e^t\frac{dt}{2}\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}(e-1)\,d\theta$$

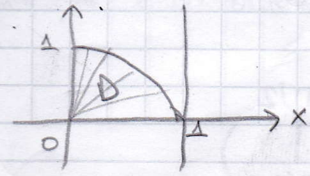
$x=r\cos\theta$ $x^2+y^2=1$

$y=r\sin\theta$ $r^2=1$

$r=1$

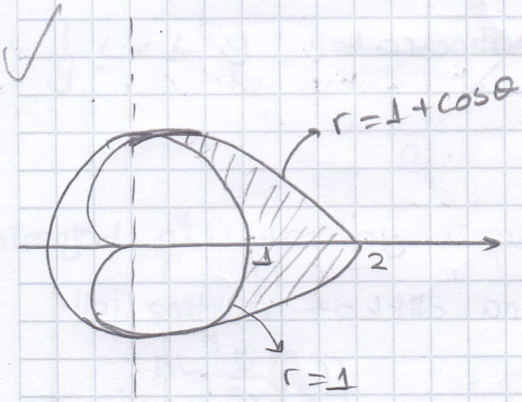
$J=r$

$$=\frac{e-1}{2}\theta\Big|_0^{\pi/2}=\frac{(e-1)\pi}{4}$$

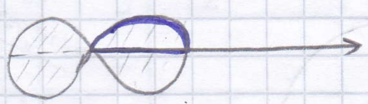


ÖR ⇒ $r = 1+\cos\theta$ kardioyidinin içinde $r=1$ çemberinin dışında kalan bölge üzerinde $f(r,\theta)$'nın integralini almak için integralin sınırlarını bulunuz.

$$I = \int_{-\pi/2}^{\pi/2}\int_{1}^{1+\cos\theta} f(r,\theta)\, r\,dr\,d\theta$$



ÖR ⇒ $r^2 = 4\cos 2\theta$ alanını hesaplayınız $\quad r = 2\sqrt{\cos 2\theta}$



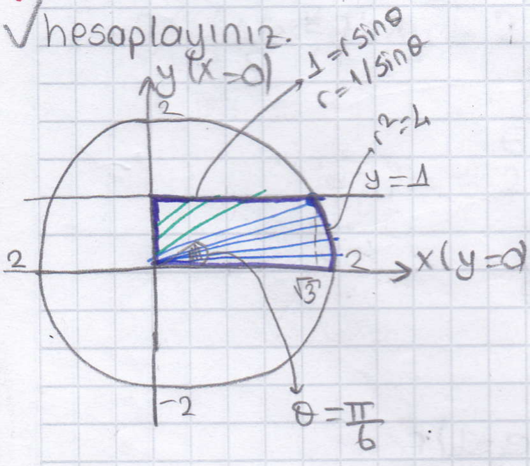
$$A = \iint r\,dr\,d\theta \qquad \theta = 0 \Rightarrow r = 2$$

$$A = 4\int_0^{\pi/4}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta \qquad 2\theta = \frac{\pi}{2}, \quad \theta = \frac{\pi}{4}$$

$$= 4\int_0^{\pi/4}\left[\frac{r^2}{2}\Big|_0^{2\sqrt{\cos 2\theta}}\right] d\theta$$

$$= 4\int_0^{\pi/4} 2\cos 2\theta\, d\theta = \frac{8}{2}\sin 2\theta\Big|_0^{\pi/4} = \underline{4\ br^2}$$

ÖR ⇒ $x^2+y^2=4$, $y=1$, $x=0$, $y=0$ ile sınırlı bölgenin alanını hesaplayınız.



$x = r\cos\theta$
$y = r\sin\theta$
$J = r$

$$A = \int_0^{\pi/6}\int_0^{2} r\,dr\,d\theta + \int_{\pi/6}^{\pi/2}\int_0^{1/\sin\theta} r\,dr\,d\theta$$

$$= \int_0^{\pi/6}\left[\frac{r^2}{2}\Big|_0^{2}\right]d\theta + \int_{\pi/6}^{\pi/2}\left[\frac{r^2}{2}\Big|_0^{1/\sin\theta}\right]d\theta$$

$$= 2\int_0^{\pi/6} d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2}\frac{1}{\sin^2\theta}\,d\theta$$

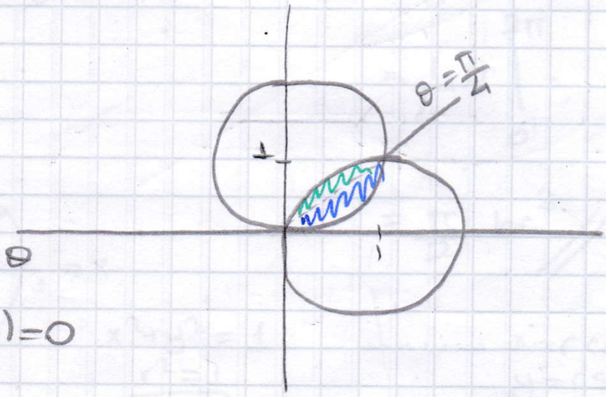
$$= 2\theta\Big|_0^{\pi/6} + \frac{1}{2}\left(-\cot\theta\Big|_{\pi/6}^{\pi/2}\right)$$

$$\left\{ \begin{array}{l} \text{cotan}\,t = \dfrac{\cos t}{\sin t} \\ (\text{cotan}\,t)' = -\dfrac{1}{\sin^2 t} \end{array} \right.$$

$$2\left(\frac{\pi}{6}\right) - \frac{1}{2}\left[\text{cotan}\frac{\pi}{2} - \text{cotan}\frac{\pi}{6}\right] = \frac{\pi}{3} - \frac{1}{2}\left[0 - \sqrt{3}\right]$$

$$= \frac{\pi}{3} + \frac{\sqrt{3}}{2} \text{ br}^2$$

Örn ⇒ $x^2+y^2=2x$ $x^2+y^2=2y$ çemberlerinin sınırlamış olduğu bölgenin alanını bulunuz



$x^2+y^2=2x \Rightarrow (x-1)^2+y^2=1$

$x^2+y^2=2y \Rightarrow x^2+(y-1)^2=1$

$x = r\cos\theta$
$y = r\sin\theta$
$J = r$

$x^2+y^2=2y \Rightarrow r^2 = 2r\sin\theta$

$r(r-2\sin\theta)=0$

$r = 2\sin\theta$

$x^2+y^2=2x \Rightarrow r^2 = 2r\cos\theta$

$r(r-2\cos\theta)=0$

$r = 2\cos\theta$

$$A = \int_0^{\pi/4}\int_0^{2\sin\theta} r\,dr\,d\theta + \int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} r\,dr\,d\theta$$

$$= \frac{\pi}{2} - 1 \text{ br}^2$$

İki Katlı İntegraller için Ortalama Değer Teoremi:

Kapalı bir aralık üzerinde tek değişkenli integrallenebilir bir fonksiyonun ortalama değeri verilen aralık üzerinde fonksiyonun integralinin aralığın uzunluğuna bölünmesidir. Düzlemde kapalı bir bölge üzerinde tanımlanmış iki değişkenli integrallenebilir bir fonksiyon için ortalama değer bölge üzerindeki integralin bölgenin alanına bölümüdür.

D üzerinde f'in ortalama değeri: $\hat{f}(x,y) = \dfrac{\iint_D f(x,y)\,dA}{\iint_D dA}$

ör $\Rightarrow$ $f(x,y) = x\cos(xy)$'nin $0 \le x \le \pi$ $0 \le y \le 1$ dikdörtgeni üzerinde ortalama değerini bulunuz.

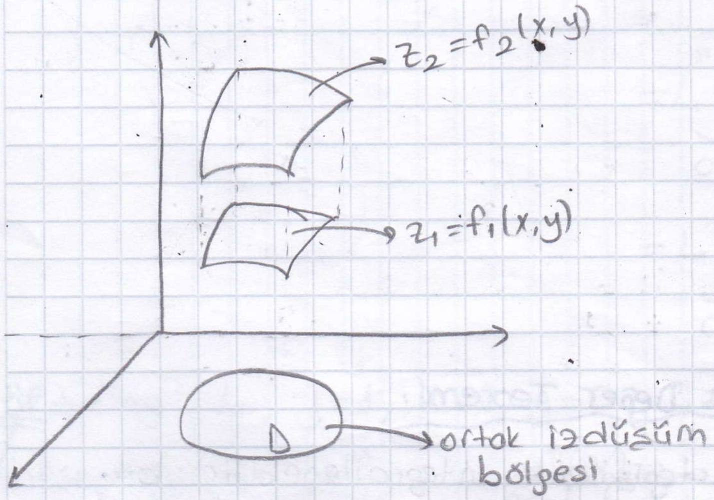
$$\hat{f}(x,y) = \frac{\int_0^{\pi}\int_0^{1} x\cos(xy)\,dy\,dx}{\int_0^{\pi}\int_0^{1} dy\,dx} = \frac{\int_0^{\pi}\left[\frac{x}{x}\sin(xy)\Big|_0^1\right]dx}{\int_0^{\pi}\left[y\Big|_0^1\right]dx} = \frac{\int_0^{\pi}\sin x\,dx}{\int_0^{\pi}dx}$$

$$= \frac{-\cos x\Big|_0^{\pi}}{x\Big|_0^{\pi}}$$

$$= \frac{-(\cos\pi - \cos 0)}{\pi}$$

$$= \frac{2}{\pi}$$ //

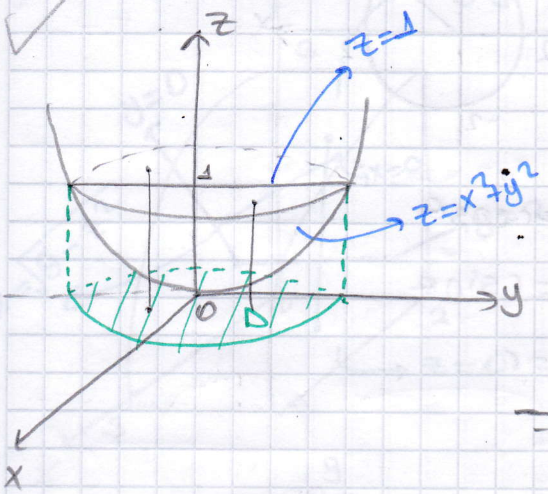
İki Yüzey Arasında Kalan Cismin Hacmi



$$\boxed{V = \iint_D (z_2 - z_1)\,dA}$$

Ör: $z=x^2+y^2$ paraboloidinin $z=1$ ile kesilmiş kısmının hacmini hesaplayınız

$D: x^2+y^2=1$

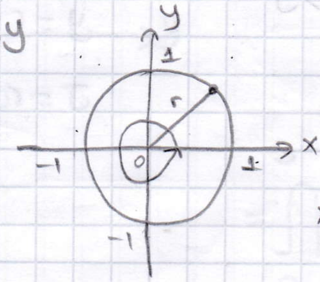


$$V=\iint_D \left[1-(x^2+y^2)\right]dA$$

$$=\int_0^{2\pi}\int_0^1 (1-r^2)\,r\,dr\,d\theta=\int_0^{2\pi}\left[\frac{r^2}{2}-\frac{r^4}{4}\Big|_0^1\right]d\theta$$

$$=\frac{1}{4}\theta\Big|_0^{2\pi}$$

$$=\frac{\pi}{2}\ br^3$$



$x^2+y^2=1$

$r^2=1$

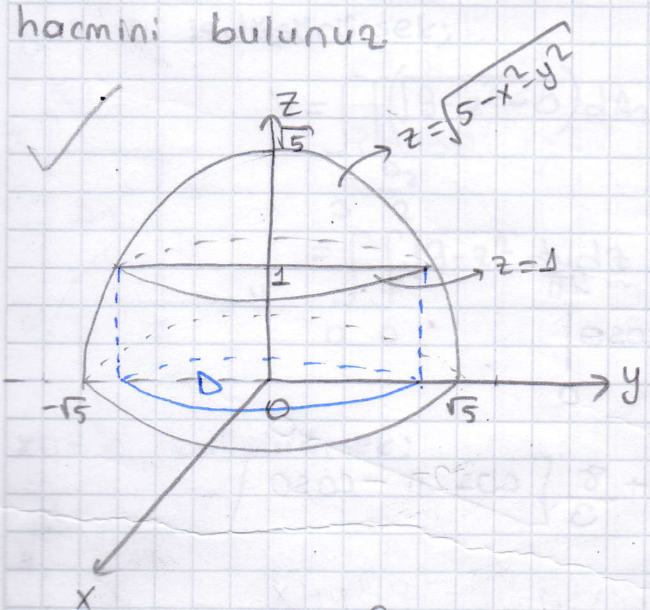
$\boxed{r=1}$

$x=r\cos\theta$

$y=r\sin\theta$

$J=r$

Ör: $x^2+y^2+z^2=5$ küresinin $z=1$ düzlemi üstünde kalan kısmının hacmini bulunuz



$$V=\iint_D\left(\sqrt{5-x^2-y^2}-1\right)dA=\int_0^{2\pi}\int_0^2\left[\sqrt{5-r^2}-1\right]r\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^2\sqrt{5-r^2}\,r\,dr\,d\theta-\int_0^{2\pi}\int_0^2 r\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_5^1\sqrt{t}\left(-\frac{dt}{2}\right)d\theta-\int_0^{2\pi}\left[\frac{r^2}{2}\Big|_0^2\right]d\theta$$

$$=\frac{1}{2}\int_0^{2\pi}\int_1^5\sqrt{t}\,dt\,d\theta-2\left(\theta\Big|_0^{2\pi}\right)$$

$$=\frac{1}{2}\int_0^{2\pi}\left[\frac{2}{3}t^{3/2}\Big|_1^5\right]d\theta-4\pi$$

$$=\frac{1}{3}(5\sqrt{5}-1)\theta\Big|_0^{2\pi}-4\pi$$

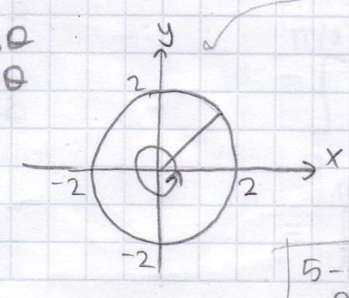
$$=\frac{10\sqrt{5}\pi-14\pi}{3}\ br^3$$

$\left.\begin{matrix}x^2+y^2+z^2=5\\ z=1\end{matrix}\right\}D: x^2+y^2=4$

$x=r\cos\theta$

$y=r\sin\theta$

$J=r$



$5-r^2=t$

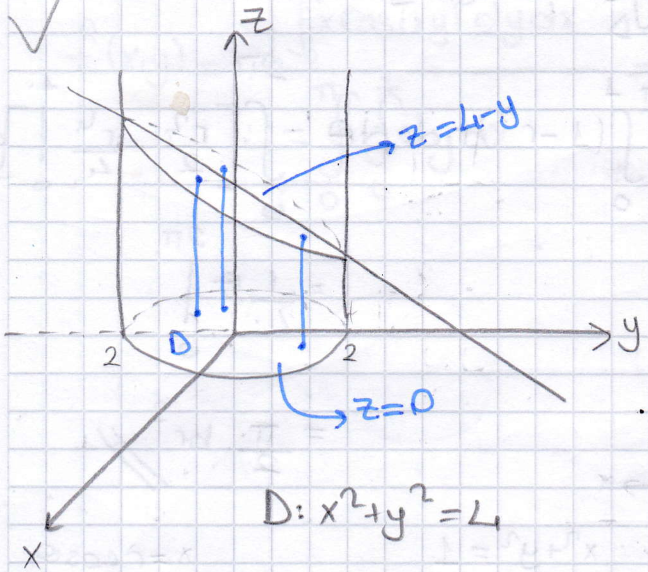
$-2r\,dr=dt$

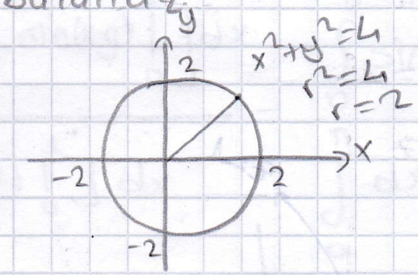
$r\,dr=-\frac{dt}{2}$

$r=0\Rightarrow t=5$

$r=2\Rightarrow t=1$

ÖR $x^2+y^2=4$ silindir yüzeyi ile $y+z=4$, $z=0$ düzlemleri arasında kalan cismin hacmini bulunuz.





$x=r\cos\theta$

$y=r\sin\theta$

$J=r$

$D: x^2+y^2=4$

$$V=\iint_D (4-y-0)\,dA$$

$$=\int_0^{2\pi}\int_0^{2}(4-r\sin\theta)\,r\,dr\,d\theta$$

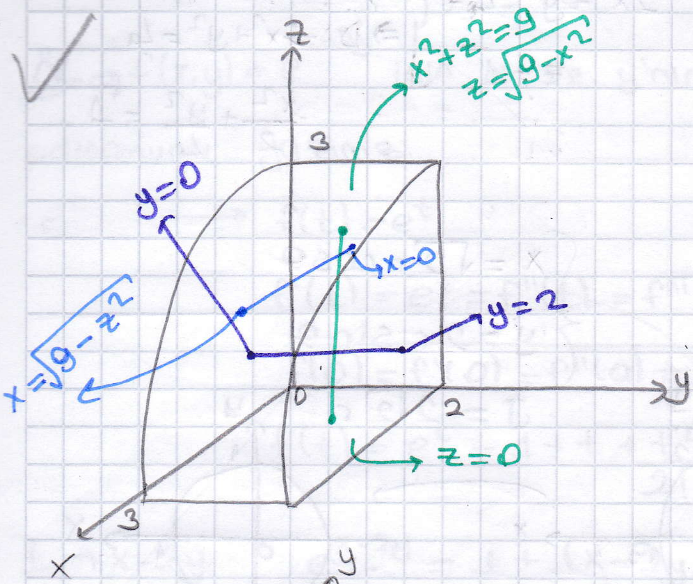
$$=\int_0^{2\pi}\left[\frac{4r^2}{2}-\frac{r^3}{3}\sin\theta\Big|_0^2\right]d\theta$$

$$=\int_0^{2\pi}\left[8-\frac{8}{3}\sin\theta\right]d\theta = 8\theta+\frac{8}{3}\cos\theta\Big|_0^{2\pi}$$

$$=8(2\pi-0)+\frac{8}{3}\left[\cancel{\cos 2\pi-\cos 0}\right]^{0}$$

$$=\underline{\underline{16\pi\ br^3}}$$

ÖR → $x^2+z^2=9$ silindiri ile $y=2$ $y=0$ $x=0$ $z=0$ düzlemleri tarafından sınırlandırılan cismin hacmini bulunuz.



xoy'ye izdüşürelim;

$$V=\iint_{D_1}\left[\sqrt{9-x^2}-0\right]dA_1$$

$$=\int_0^3\int_0^2\sqrt{9-x^2}\,dy\,dx$$

$x=3\sin t$
$dx=3\cos t\,dt$

$x=0 \Rightarrow t=0$
$x=3 \Rightarrow t=\pi/2$

$$=\int_0^3\left[\sqrt{9-x^2}\,y\Big|_0^2\right]dx$$

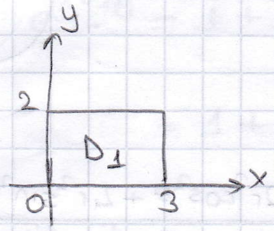
$$=2\int_0^3\sqrt{9-x^2}\,dx$$

$$=2\int_0^{\pi/2}\sqrt{9-9\sin^2 t}\cdot 3\cos t\,dt$$

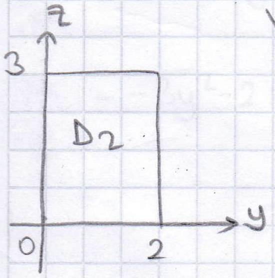
$$=18\int_0^{\pi/2}\cos^2 t\,dt$$

$$=18\int_0^{\pi/2}\left[\frac{1+\cos 2t}{2}\right]dt$$

$$=9\left[t+\frac{1}{2}\sin 2t\Big|_0^{\pi/2}\right]=\frac{9\pi}{2}\,br^3$$



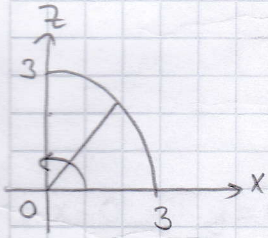
yoz'e izdüşürürsek;



$$V=\iint_{D_2}\left(\sqrt{9-z^2}-0\right)dA_2$$

$$=\int_0^3\int_0^2\sqrt{9-z^2}\,dy\,dz$$

xoz'e izdüşürürsek;



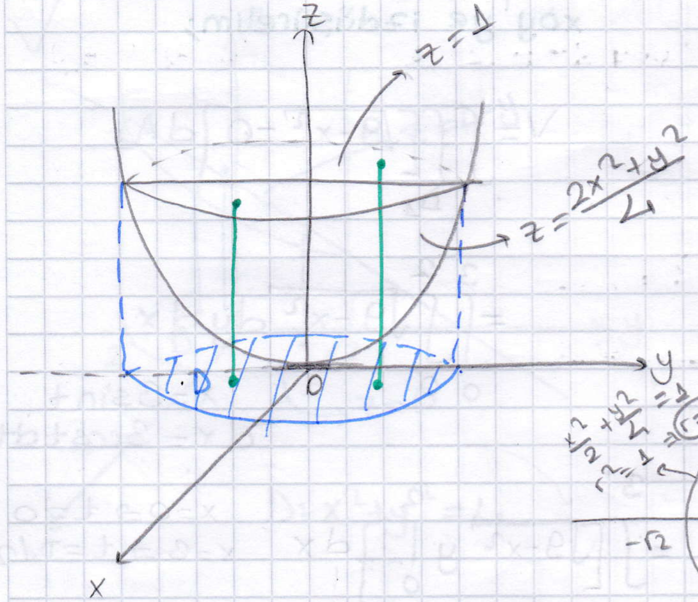
$x=r\cos\theta$
$y=r\sin\theta$
$J=r$

$$V=\iint_{D_3}(2-0)\,dA_3$$

$$=\int_0^{\pi/2}\int_0^3 2r\,dr\,d\theta$$

$$=\int_0^{\pi/2}\left[r^2\Big|_0^3\right]d\theta=9\int_0^{\pi/2}d\theta=\frac{9\pi}{2}\,br^3$$

ÖR ⇒ $2x^2+y^2=4z$ paraboloidi ile $z=1$ düzlemi arasında kalan cismin hacmini bulunuz

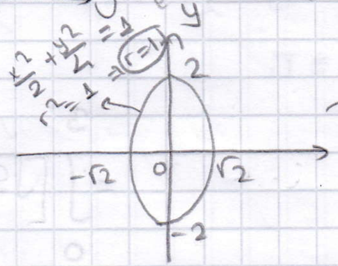


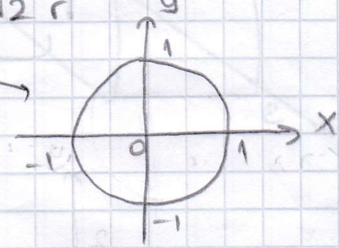
$$\left.\begin{array}{l} 2x^2+y^2=4z \\ z=1 \end{array}\right\} \Rightarrow D: 2x^2+y^2=4$$

$$\frac{x^2}{2}+\frac{y^2}{4}=1$$

$$\left\{\begin{array}{l} x=\sqrt{2}\, r\cos\theta \\ y=2r\sin\theta \end{array}\right.$$

$$J=2\sqrt{2}\, r$$





$$V=\iint_D\left[1-\frac{2x^2+y^2}{4}\right]dA=\int_0^{2\pi}\int_0^{1}\left[1-\frac{2.2r^2\cos^2\theta+4r^2\sin^2\theta}{4}\right]2\sqrt{2}\, r\,dr\,d\theta$$

$$=2\sqrt{2}\int_0^{2\pi}\int_0^{1}(1-r^2)\,r\,dr\,d\theta$$

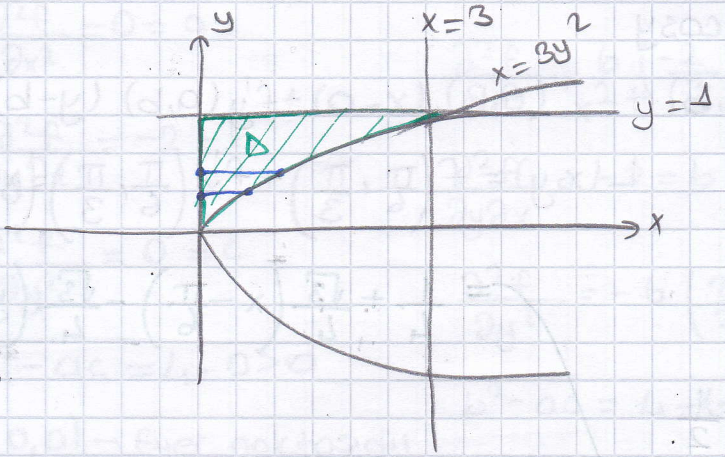
$$=2\sqrt{2}\int_0^{2\pi}\left[\frac{r^2}{2}-\frac{r^4}{4}\Big|_0^1\right]d\theta$$

$$=\frac{\sqrt{2}}{2}\,\theta\Big|_0^{2\pi}$$

$$=\underline{\sqrt{2}\pi}\ br^3$$

Eliptik integral olduğundan integrali verildiği şekli ile çözemeyiz. Bölgeyi çizip, integrasyon sırasını değiştirerek çözmeliyiz

ÖR ⇒ $I=\int_0^3\int_{\sqrt{\frac{x}{3}}}^1 e^{y^3}\,dy\,dx$ integralini hesaplayınız.



$0\le x\le 3$

$\sqrt{\frac{x}{3}}\le y\le 1$

$x=3y^2$

$y=\mp\sqrt{\frac{x}{3}}$

$$I=\int_0^1\int_0^{3y^2} e^{y^3}\,dx\,dy$$

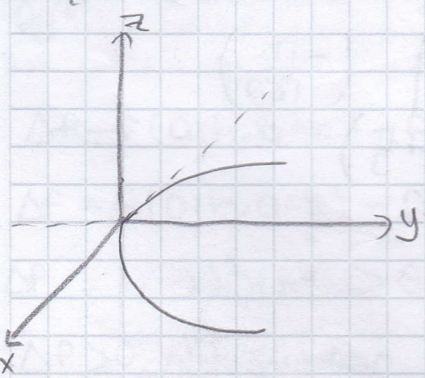
$$=\int_0^1\left[x\,e^{y^3}\Big|_0^{3y^2}\right]dy$$

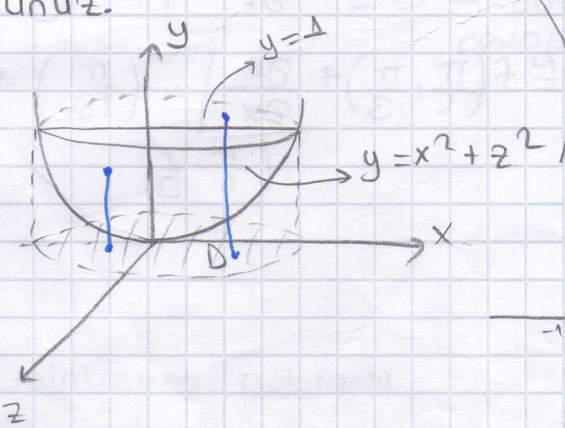
$$=\int_0^1 3y^2 e^{y^3}\,dy \longrightarrow \begin{array}{l} y^3=t \\ 3y^2dy=dt \\ y=0\Rightarrow t=0 \\ y=1\Rightarrow t=1\end{array}$$

$$=\int_0^1 e^t\,dt$$

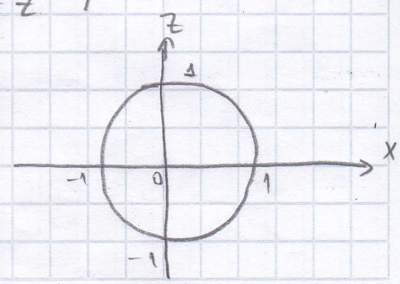
$$=e^t\Big|_0^1=\underline{\underline{e-1}}$$

ÖR ⇒ $y=x^2+z^2$ paraboloidinin $y=1$ düzlemi ile kesilmesi sonucu oluşan cismin hacmini bulunuz.





$x^2+z^2=1 : D$



$x=r\cos\theta$

$y=r\sin\theta$

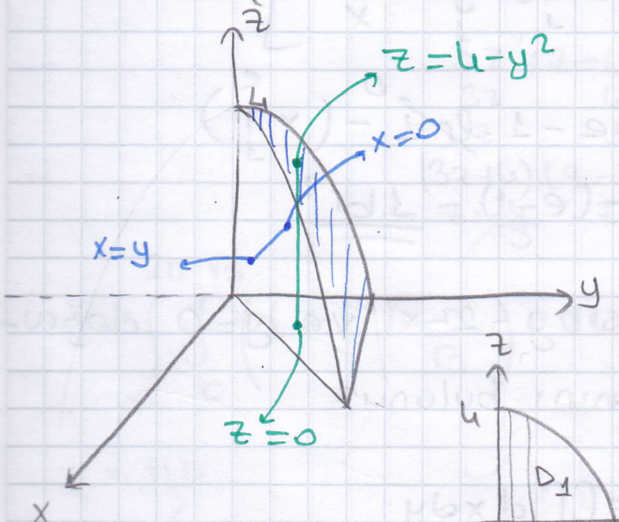
$J=r$

$$V=\iint_D \left[1-(x^2+z^2)\right] dA$$

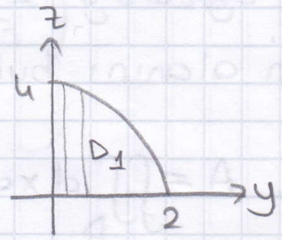
$$=\int_0^{2\pi}\int_0^1 (1-r^2)\,r\,dr\,d\theta$$

$$=\int_0^{2\pi}\left[\left(\frac{r^2}{2}-\frac{r^4}{4}\right)\Big|_0^1\right]d\theta=\frac{1}{4}\theta\Big|_0^{2\pi}=\frac{\pi}{2}\,br^3 \;//$$

ÖR ⇒ $x\geq 0$, $y=x$, $z=0$ düzlemleri ile $z=4-y^2$ parabolik silindiri ile sınırlanmış cismin hacmini bulunuz (Birinci bölgede)



yoz izdüşüm bölgesi



$$V=\iint_{D_1}(y-0)\,dA_1$$

$$=\int_0^2\int_0^{4-y^2} y\,dz\,dy$$

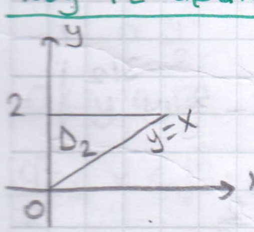
$$=\int_0^2 y\left(z\Big|_0^{4-y^2}\right)dy$$

$$=\int_0^2 y(4-y^2)\,dy=\frac{4y^2}{2}-\frac{y^4}{4}\Big|_0^2$$

$$=8-4$$

$$=4\,br^3$$

xoy izdüşüm bölgesi



$$V=\iint_{D_2}(4-y^2-0)\,dA_2$$

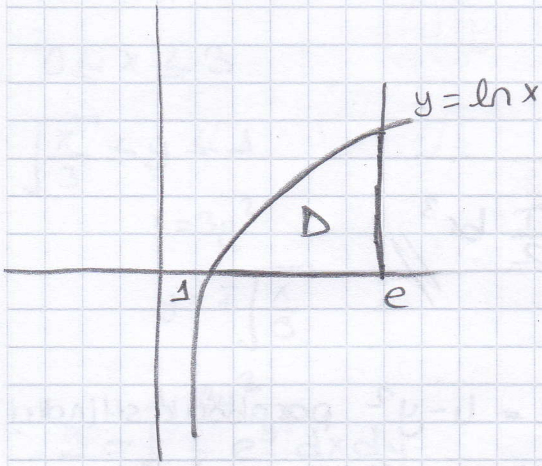
$$=\int_0^2\int_x^2 (4-y^2)\,dy\,dx$$

$$=\int_0^2\left[4y-\frac{y^3}{3}\Big|_x^2\right]dx=\int_0^2\left[\left(8-\frac{8}{3}\right)-4x+\frac{x^3}{3}\right]dx$$

$$=\int_0^2\left(\frac{16}{3}-4x+\frac{x^3}{3}\right)dx$$

$$=4\,br^3 \;//$$

ÖR ⇒ x-ekseni $x=e$ doğrusu ve $y=\ln x$ eğrisi arasında kalan alanını iki katlı integral ile bulunuz



$$A=\iint_D dA$$

$$=\int_1^e\int_0^{\ln x} dy\,dx$$

$$=\int_1^e \ln x\,dx$$

$\ln x = u \qquad dx = dv$

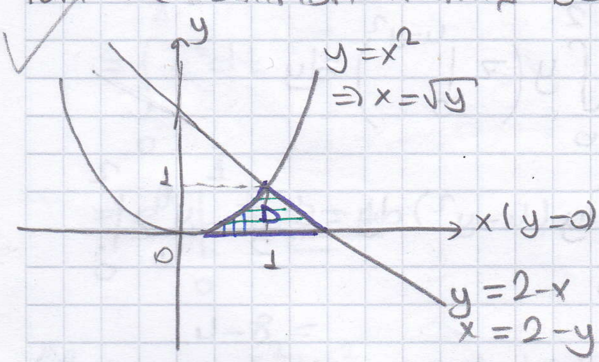
$\frac{dx}{x} = du \qquad x = v$

$$=\left[x\ln x\Big|_1^e - \int_1^e x\frac{dx}{x}\right]$$

$$= e\ln e - 1\cdot\cancel{\ln 1}^{0} - \left(x\Big|_1^e\right)$$

$$= e-(e-1) = \underline{\underline{1\,br^2}}$$

ÖR ⇒ İki katlı integral ile $y=x^2$ eğrisi $y=2-x$ ve $y=0$ doğruları ile sınırlandırılmış bölgenin alanını bulunuz



$$A=\iint_D dx\,dy$$

$$=\int_0^1\int_{\sqrt{y}}^{2-y} dx\,dy = \int_0^1 (2-y-\sqrt{y})\,dy$$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$

$x=1$

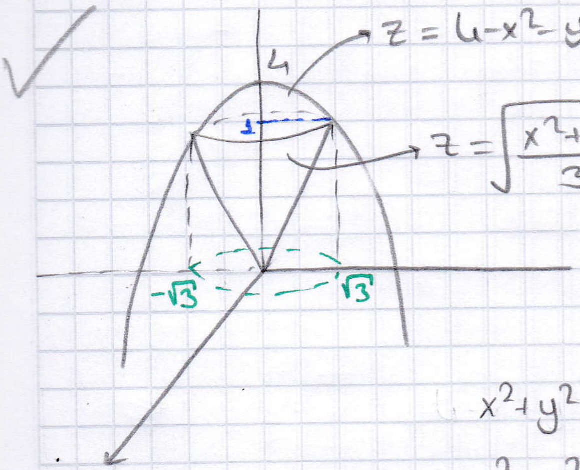
$$= 2y-\frac{y^2}{2}-\frac{2}{3}y^{3/2}\Big|_0^1$$

$$= 2-\frac{1}{2}-\frac{2}{3}$$

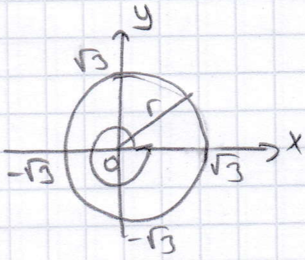
$$=\frac{5}{6}\,br^2$$

$z=4-x^2-y^2$ $\quad z\geq 0$ $\quad$ koni

ÖR ⇒ $x^2+y^2=4+z$ paraboloidi ile $3z^2=x^2+y^2$ yüzeylerinin sınırladığı cismin hacmini bulunuz.



$$V=\iint_D \left[(4-x^2-y^2)-\sqrt{\frac{x^2+y^2}{3}}\right]dA$$



$x=r\cos\theta$

$y=r\sin\theta$

$J=r$

$x^2+y^2=4+z$

$x^2+y^2=3z^2$

$3z^2+z-4=0$

$3z \qquad 4$

$z \qquad -1$

$(3z+4)(z-1)=0$

$\cancel{z=-\frac{4}{3}} \qquad z=1 \Rightarrow x^2+y^2=3$

$$V=\int_0^{2\pi}\int_0^{\sqrt{3}}\left((4-r^2)-\frac{r}{\sqrt{3}}\right)r\,dr\,d\theta$$

$$=\frac{11\pi}{2}$$

ÖR ⇒ $\displaystyle\int_0^{\sqrt{2}}\int_{\frac{y^2}{2}}^{\sqrt{3-y^2}} f(x,y)\,dx\,dy$ integralinde integrasyon sırasını değiştiriniz

$$\int_0^1\int_0^{\sqrt{2x}} f(x,y)\,dy\,dx+\int_1^{\sqrt{3}}\int_0^{\sqrt{3-x^2}} f(x,y)\,dy\,dx$$

$0\leq y\leq \sqrt{2}$

$\frac{y^2}{2}\leq x\leq\sqrt{3-y^2} \Rightarrow x=\frac{y^2}{2}$

$x^2+y^2=3$

$x^2+2x-3=0$

$3 \quad -1$

$(x+3)(x-1)=0$

$\cancel{x=-3} \quad x=1$