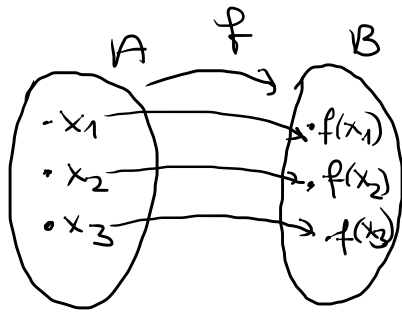
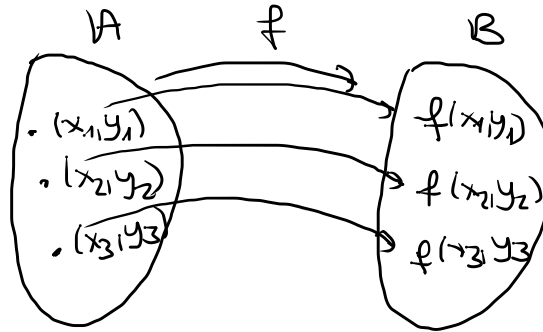


Çok Değişkenli Fonksiyonlar

D n -boyutlu bir uzay olsun. A, D 'nin bir alt kümesi olmak üzere A 'nın her noktasına bir (ya da bir) sayı karşılık getiren kurala fonksiyon denir.



$$\begin{aligned} x_i &\in A & y &= f(x) \\ f(x_i) &\in B & & \text{(tek değişkenli)} \end{aligned}$$



$$\begin{aligned} (x_i, y_i) &\in A & z &= f(x, y) \\ f(x_i, y_i) &\in B & & \text{(iki değişkenli)} \end{aligned}$$

$$A \subset D \quad \left\{ \begin{array}{l} P \in A \rightarrow f(P) \\ P(x_1, x_2, \dots, x_n) \end{array} \right.$$

$$(x_1, x_2, \dots, x_n) \rightarrow f(x_1, x_2, \dots, x_n) \equiv f(P) \rightarrow \text{Çok değişkenli fonksiyon}$$

$$\left. \begin{array}{l} y = f(x) \\ z = f(x, y) \\ \vdots \\ u = f(x_1, x_2, \dots, x_n) \end{array} \right\} \text{açık form.}$$

$$\left. \begin{array}{l} F(x, y) = 0 \\ F(x, y, z) = 0 \\ F(x_1, x_2, \dots, x_n, u) = 0 \end{array} \right\} \text{kapalı form}$$

Ö5/

$$y = \sqrt{25 - x^2} \xrightarrow{\text{açık form}} x^2 + y^2 = 25 \rightarrow \underbrace{x^2 + y^2 - 25 = 0}_{f(x,y) = 0}$$

$$\underbrace{y - \sqrt{25 - x^2} = 0}_{f(x,y) = 0}$$

kapalı form.

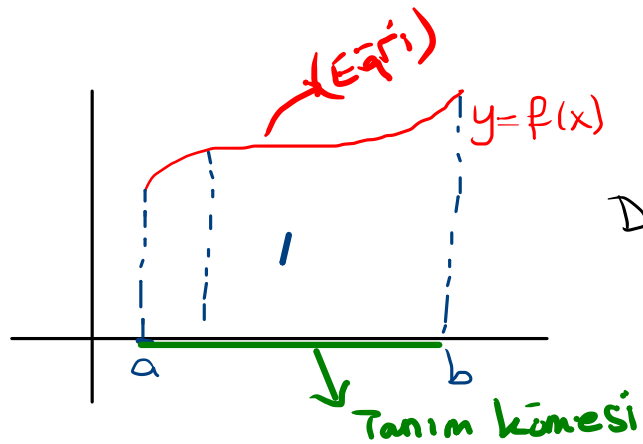
$$x^2 + y^2 = 25$$

$$2x + 2yy' = 0$$

Ö5/

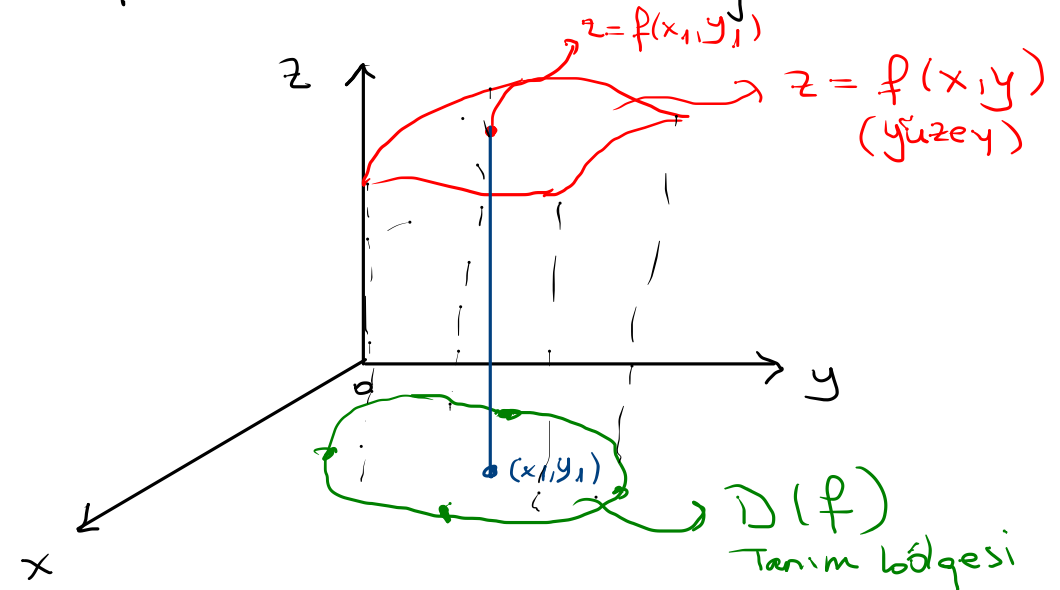
$$x^2 + y^2 + z^2 = 4 \Rightarrow F(x,y,z) = 0$$

$$x^2 + y^2 + z^2 - 4 = 0 \quad \text{kapalı form}$$



$$z = f(x, y)$$

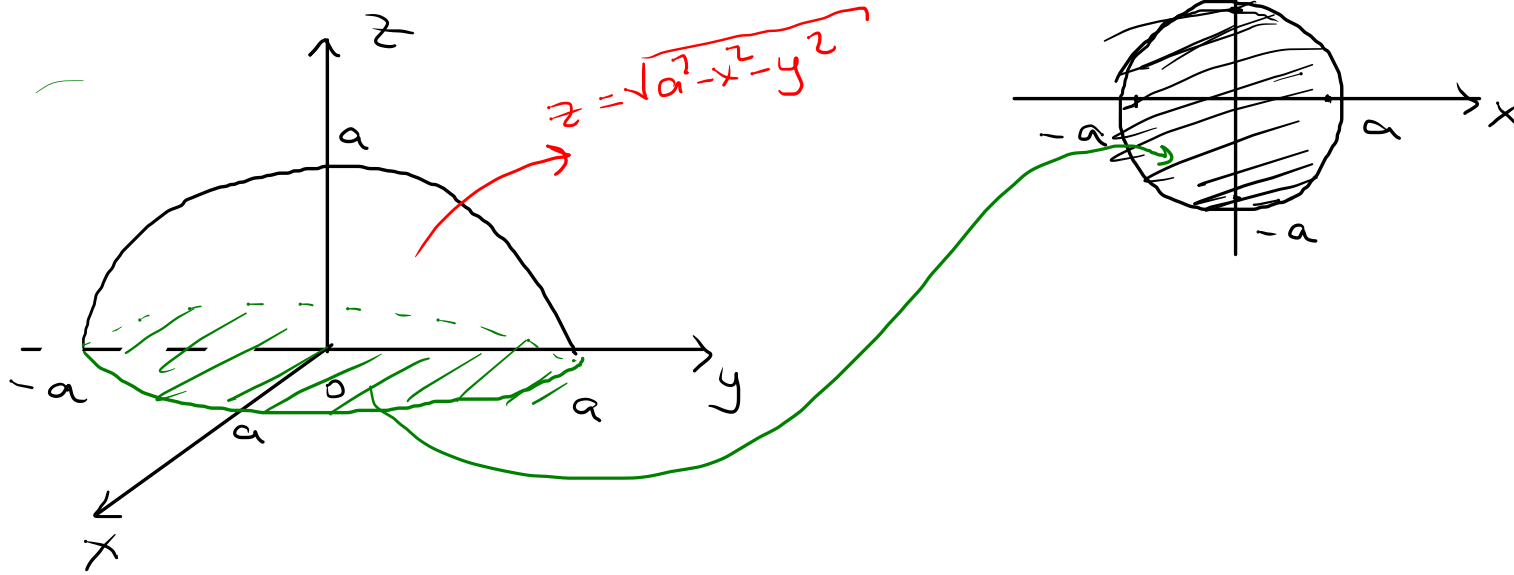
$$z = \sqrt{4 - x^2 - y^2}$$



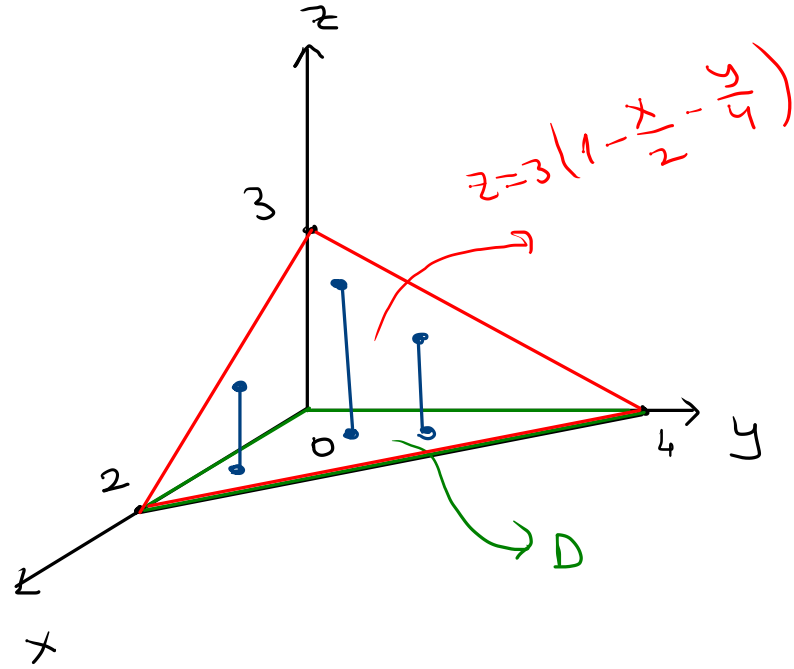
Elemanları (x,y) sayı çiftinden oluşan D kümesini gözönüne alalım. Bu kümenin her elemanı, üzerinde birbirine dik iki koordinat eksenini seçilmiş bir düzlemde, P gibi bir nokta ile gösterilir. Böylece D kümesi, düzlemde bir nokta kümesi ile bire bir eşleşmiş olur. O halde $z = f(x,y)$ fonksiyonunun tanım bölgesi olan A kümesi düzlemde bir nokta kümesidir. Bu nokta kümesi düzlemin tamamı olabileceği gibi bir veya birden fazla eği ile sınırlanmış bir nokta kümesidir.

Ör $f(x,y) = \sqrt{a^2 - x^2 - y^2}$ fonksiyonunun tanım kümesini bulunuz.

$$a^2 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq a^2$$



Ör $z = f(x, y) = 3 \left(1 - \frac{x}{2} - \frac{y}{4} \right)$ ($0 \leq x \leq 2$, $0 \leq y \leq 4 - 2x$) yüzeyi ve tanım bölgesini gösteriniz.



$$z = 3 \left(1 - \frac{x}{2} - \frac{y}{4} \right)$$

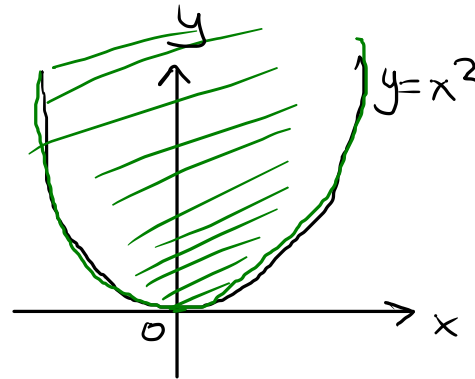
$$\frac{z}{3} = 1 - \frac{x}{2} - \frac{y}{4} \Rightarrow \boxed{\frac{x}{2} + \frac{y}{4} + \frac{z}{3} = 1} \rightarrow \text{Düzlem}$$

$$y = z = 0 \Rightarrow x = 2 \quad (2, 0, 0)$$

$$x = z = 0 \Rightarrow y = 4 \quad (0, 4, 0)$$

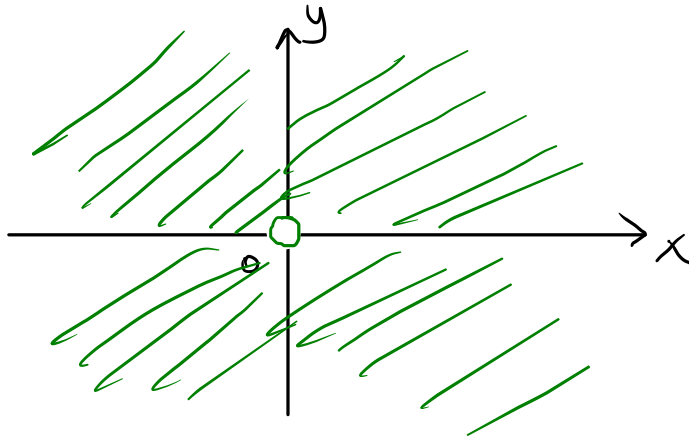
$$x = y = 0 \Rightarrow z = 3 \quad (0, 0, 3)$$

Ör $z = \sqrt{y - x^2} \rightarrow$ tanım kümesi nedir?
 $y - x^2 \geq 0$
 $x^2 \leq y$



Ör/ $z = \frac{1}{x \cdot y}$ tanım kümesi nedir?

$$xy \neq 0$$



Ör/ $z = \ln [x \cdot \ln(y-x)]$ tanım kümesini bulunuz.

$$x \cdot \ln(y-x) > 0$$

$$x > 0$$

$$\ln(y-x) > 0$$

$$y-x > 1$$

$$y > x+1$$

$$\bullet y-x > 0$$

$$y > x$$

$$x < 0$$

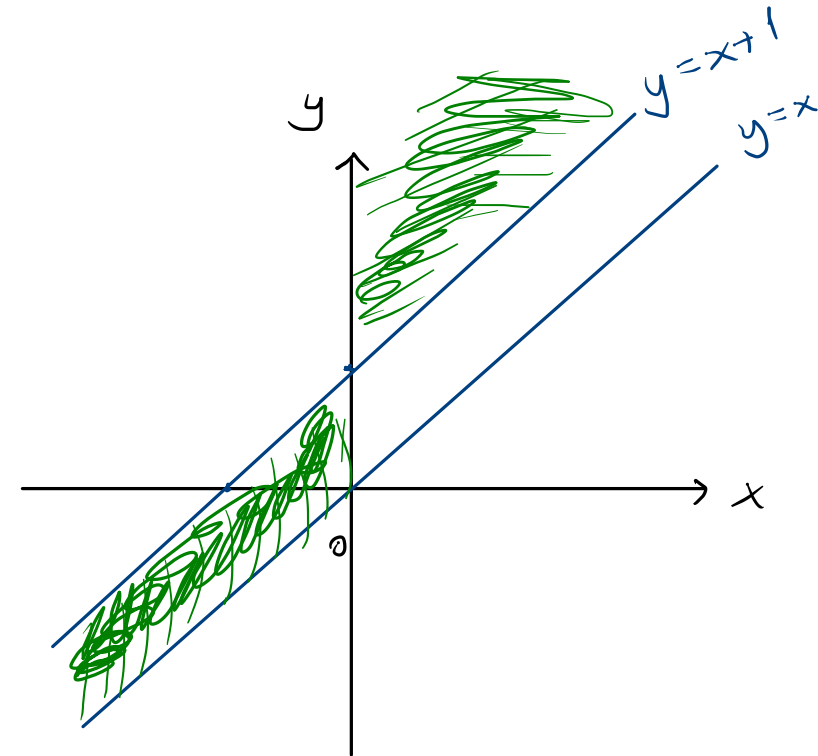
$$\ln(y-x) < 0$$

$$y-x < 1$$

$$y < x+1$$

$$\bullet y-x > 0$$

$$y > x$$



or

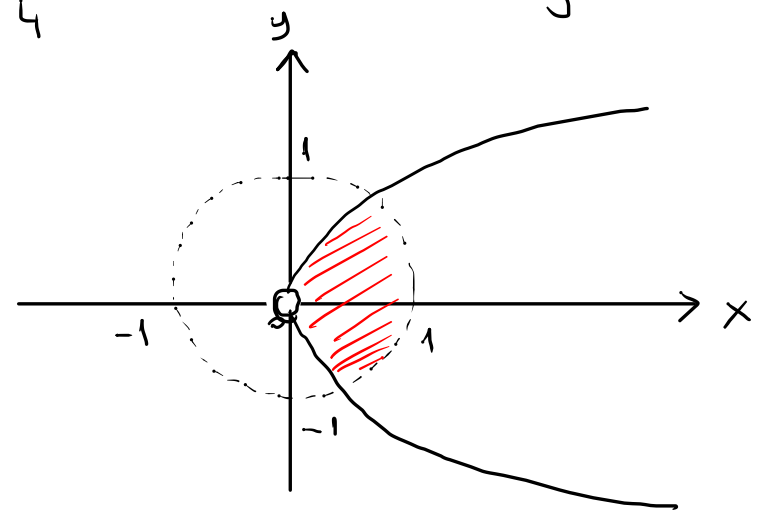
$$z = \frac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$$

$$4x-y^2 \geq 0$$
$$4x \geq y^2$$
$$x \geq \frac{y^2}{4}$$

tanım kümesini bulunuz.

$$\ln(1-x^2-y^2) \neq 0$$
$$1-x^2-y^2 \neq 1$$
$$x^2+y^2 \neq 0$$

$$1-x^2-y^2 > 0$$
$$x^2+y^2 < 1$$



Lineer ve Kuadratik Yüzeyler

Düzlem

- $ax+by+cz+d=0 \rightarrow$ Düzlemin genel denklemini

$$ax+by+cz=-d$$

$$-\frac{a}{d}x - \frac{b}{d}y - \frac{c}{d}z = 1$$

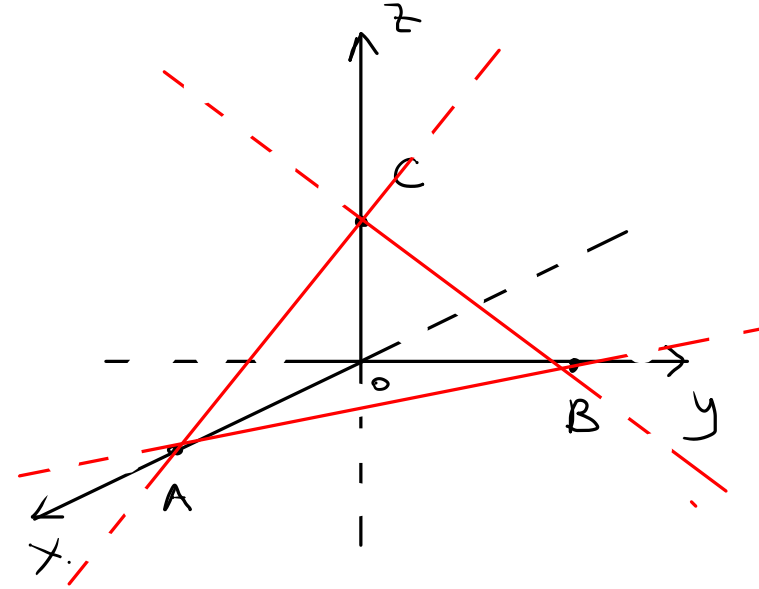
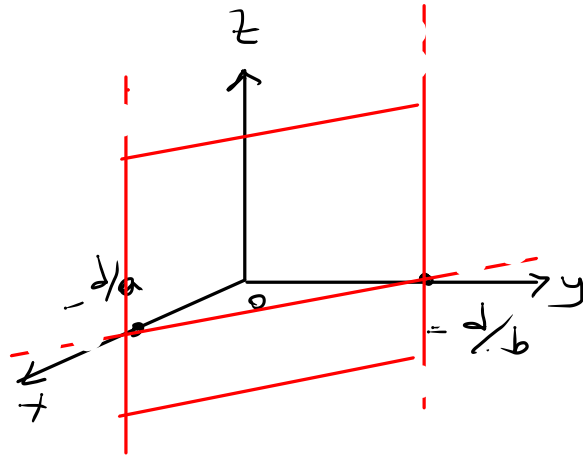
$$\left. \begin{array}{l} -\frac{a}{d} = A \\ -\frac{b}{d} = B \\ -\frac{c}{d} = C \end{array} \right\} \Rightarrow \frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1 \Rightarrow \text{Düzlemin kesen formu}$$

- $ax+by+d=0$

$$z=0$$

$$y=0 \Rightarrow x = -\frac{d}{a}$$

$$x=0 \Rightarrow y = -\frac{d}{b}$$



x-eksenini A'da
y-eksenini B'de
z-eksenini C'de
kesen düzlem

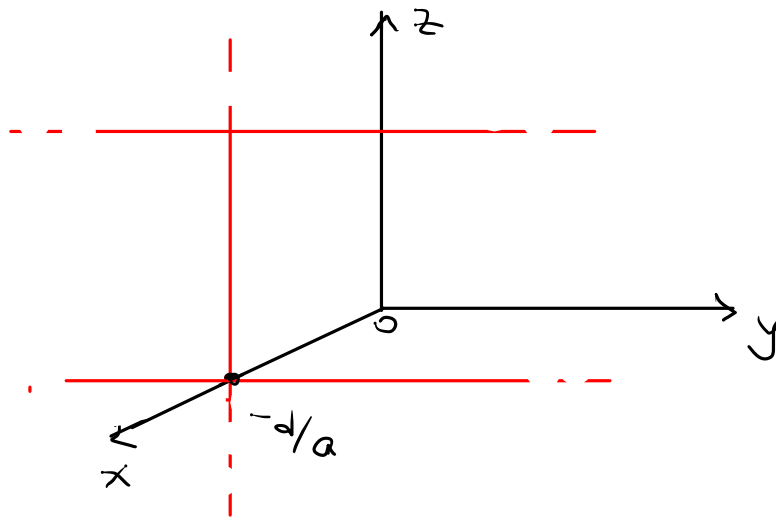
x-eksenini $-\frac{d}{a}$ 'da, y-eksenini $-\frac{d}{b}$ 'de
kesen ve z-eksenine paralel olan
bir düzlem

- $ax + d = 0$

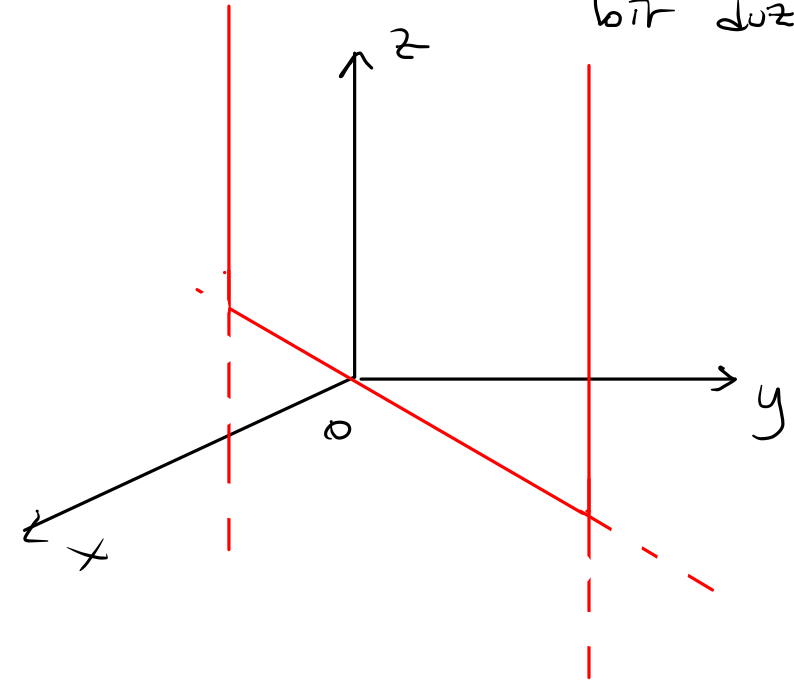
$$x = -\frac{d}{a}$$

x-eksenini $-\frac{d}{a}$ 'da

kesen yoz-düzlemine
paralel bir düzlem
gösterir.



- $ax + by + cz = 0$ (orijinden geçen
bir düzlem)



Küre

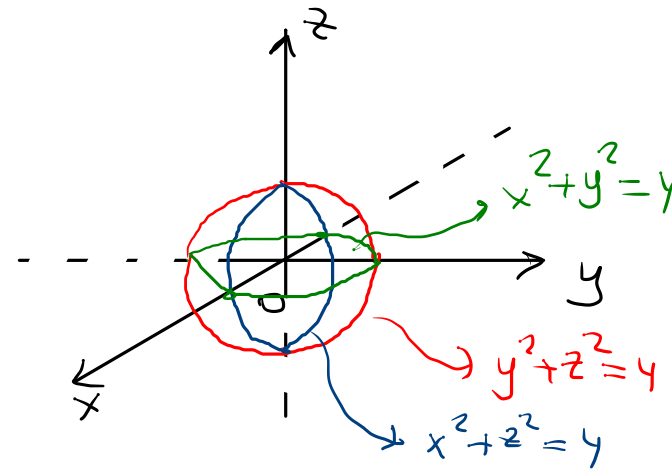
$$(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2 \quad M(a,b,c), \quad r=R \rightarrow \text{kürenin genel denklemi}$$

$$x^2 + y^2 + z^2 = 4 \quad M(0,0,0) \quad r=2$$

$$x=0 \Rightarrow y^2 + z^2 = 4$$

$$y=0 \Rightarrow x^2 + z^2 = 4$$

$$z=0 \Rightarrow x^2 + y^2 = 4$$



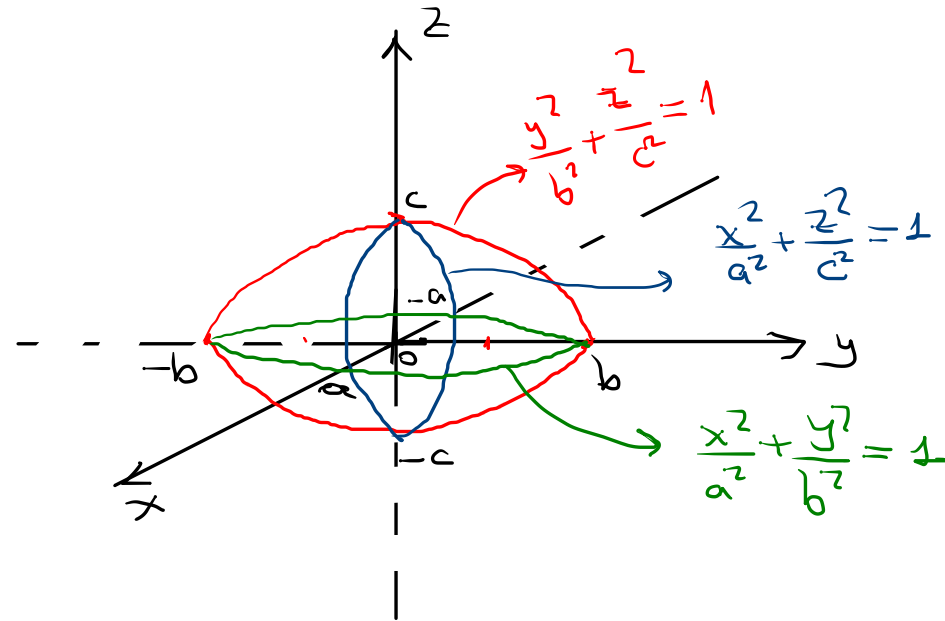
Elipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$x=0 \Rightarrow \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$y=0 \Rightarrow \frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$

$$z=0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



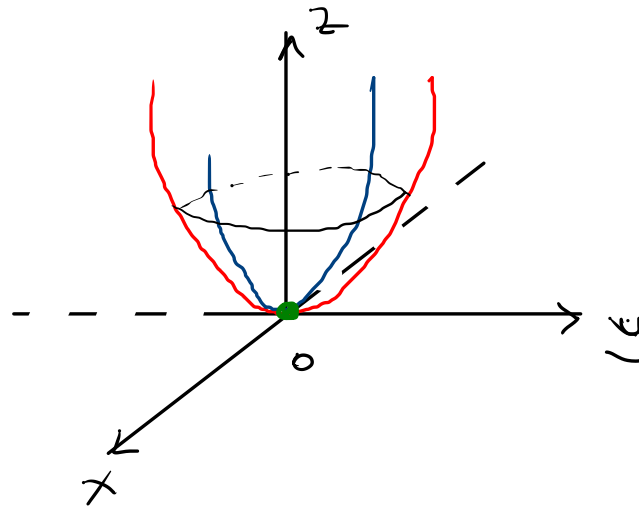
Paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2cz$$

$$x=0 \Rightarrow \frac{y^2}{b^2} = 2cz \Rightarrow z = \frac{y^2}{2cb^2}$$

$$y=0 \Rightarrow \frac{x^2}{a^2} = 2cz \Rightarrow z = \frac{x^2}{2ca^2}$$

$$z=0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \Rightarrow \text{origin.}$$



Koni

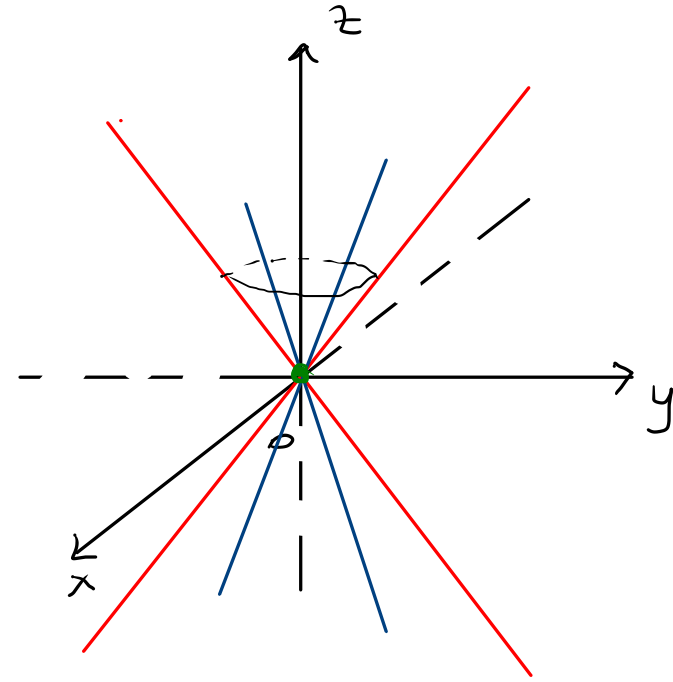
$a^2x^2 + b^2y^2 = c^2z^2 \rightarrow$ koninin genel denklemi

$$x^2 + y^2 = z^2$$

$$x=0 \Rightarrow y^2 = z^2 \Rightarrow y = \pm z$$

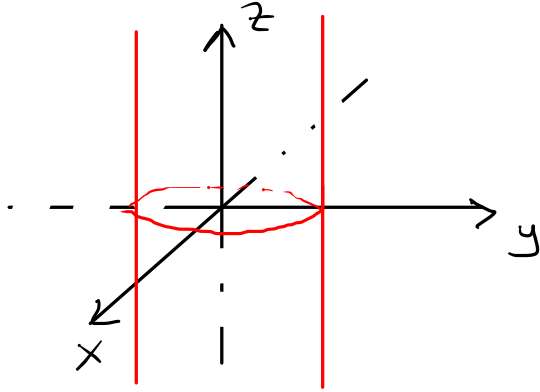
$$y=0 \Rightarrow x^2 = z^2 \Rightarrow x = \pm z$$

$$z=0 \Rightarrow x^2 + y^2 = 0 \Rightarrow \text{orjin}$$



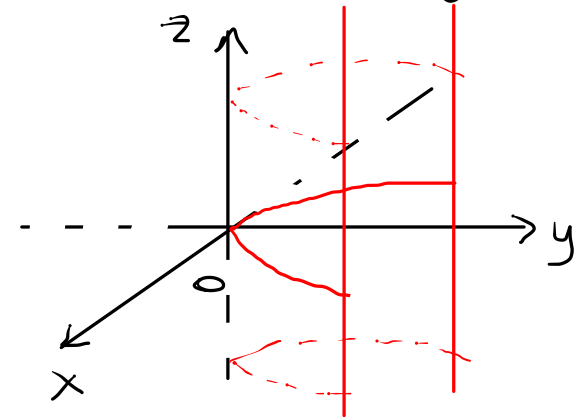
Silindir

$x^2 + y^2 = 4$ üç boyutlu uzayda silindir gösterir.



Parabolik silindir

Üç boyutlu uzayda parabol denklemi parabolik silindir gösterir. Ör/ $y = x^2$



Hiperbolik paraboloid

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c} \rightarrow \text{Genel denklem}$$

$$x=0 \Rightarrow \frac{y^2}{b^2} = \frac{z}{c} \Rightarrow z = \frac{c}{b^2} y^2$$

$$y=0 \Rightarrow -\frac{x^2}{a^2} = \frac{z}{c} \Rightarrow z = -\frac{c}{a^2} x^2$$

$$z=0 \Rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = 0 \Rightarrow \text{orj\u00f6n}$$

