$\boxed{1.)}$

The differential equation $y' - \frac{y}{x} = \sqrt{\frac{x}{y}}$ is transformed into a seperable differential equation using a suitable transformation. Which of the following differential equation is this new seperable differential equation ?

$$\frac{y}{x} = u \Rightarrow y = ux \Rightarrow y' = u'x + u$$

$$u'x + \cancel{u} - \cancel{u} = \sqrt{\frac{1}{u}}$$

$$\frac{du}{dx} \cdot x = \frac{1}{\sqrt{u}} \Rightarrow u' = \frac{1}{x\sqrt{u}}$$

A:

$u' = \frac{1}{x\sqrt{u}}$

B :

$u' = \frac{1}{\sqrt{xu}}$

C :

$u' = x\sqrt{u}$

D :

$u' = \frac{1}{xu}$

E :

$u' = \sqrt{xu}$

Doğru Cevap : A

2.)

Which of the following statement is true about the roots of the characteristic equation of the differential equation $y''' + ay'' + by' = 0$ with $a \in \mathbb{R},\ b < 0$?

A:

It has at least one pair of complex root.

B :

It has 2 coincident roots.

C :

It has 3 real and distinct roots.

D :

It has 1 real, 1 pair of complex roots.

E :

It has 3 coincident roots.

Doğru Cevap : C

3)

Which of the following is the differential equation that accepts the solution as the family of curves $y = e^{c_1 arctanx}$, where c_1 is a constant?

$$\ln y = \ln e^{c_1 \arctan x}$$

$$\ln y = c_1 \arctan x \Rightarrow c_1 = \frac{\ln y}{\arctan x}$$

$$\frac{y'}{y} = c_1 \cdot \frac{1}{1+x^2}$$

$$y' = y \cdot \frac{\ln y}{\arctan x} \cdot \frac{1}{1+x^2}$$

$$\arctan x \, (1+x^2) \cdot y' = y \ln y$$

A:

$yIny = (1 + x^2)y'arctanx$

B :

$\frac{1}{y}Iny = (1 + x^2)y'arctanx$

C :

$arctanx.Iny = (1 + x^2)y'y$

D :

$Iny = (1 + x^2)y'arctanx$

E :

$yIny = (1 + x^2)arctanx$

Doğru Cevap : A

4)

Let $f(x^2)$ be a differentiable function. Which of the following function is the integrating factor $\lambda = \lambda(x)$ of the differential equation $dx + 2f(x^2)dy = 0$?

$P(x,y) = 1$

$Q(x,y) = 2f(x^2)$

$\frac{\partial P}{\partial y} = 0$

$\frac{\partial Q}{\partial x} = 2.2x.f'(x^2) = 4x.f'(x^2)$

A:

$f(x^2)$

B :

$\frac{1}{f(x^2)}$

$$\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = 0 - 4x f'(x^2)$$

C :

$f\left(\frac{1}{x^2}\right)$

$$\ln\lambda = \int \frac{\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}}{Q} dx = \int \frac{-4x f'(x^2)}{2f(x^2)} dx$$

D :

$\frac{1}{f\left(\frac{1}{x^2}\right)}$

$$= -\int \frac{2x.f'(x^2)}{f(x^2)} dx$$

E :

None of them

$$= -\ln f(x^2)$$

$$= \ln \frac{1}{f(x^2)}$$

Doğru Cevap : B

$$\lambda = \frac{1}{f(x^2)}$$

5)

For $y' = p$, which of the following statement is true ?

A:

The general solution of the differential equation $y = xy' + (y')^3$ is $y = cx + c^2$

B :

The differential equation $y(y')^2 + x(y')^3 = 0$ is a Clairaut differential equation.

C :

The differential equation $y = (2 + y')x + (y')^2$ transformed into a linear differential equation is $\frac{dx}{dp} + \frac{x}{2} = -p$.

D :

The differential equation $yy' = -x(y')^2 + 4$ is a Clairaut differential equation.

E :

The general solution of the differential equation $y - xy' - 3\sqrt{1 + (y')^3} = 0$ is $y = cx + \sqrt{1 + c^3}$ dir.

Doğru Cevap : C

6)

Which of the following function is the integrating factor which transforms the differential equation $xInx\frac{dy}{dx}+y=2Inx$ into an exact differential equation?

$$x\ln x\frac{dy}{dx}+y=2\ln x$$

$$x\ln x\frac{dy}{dx}+y-2\ln x=0$$

$$\underbrace{x\ln x}_{Q}\,dy+\underbrace{(y-2\ln x)}_{P}\,dx=0$$

A:
$In(Inx)$

B:
x

C:
x^2

D:
$\frac{1}{x}$

$$\frac{\partial P}{\partial y}=1\neq\frac{\partial Q}{\partial x}=\ln x+1$$

$$\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}=1-\ln x-1=-\ln x$$

E:
$xInx$

$$\ln\lambda=\int\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}dx=\int\frac{-\cancel{\ln x}}{x\cancel{\ln x}}dx$$

$$=\int-\frac{dx}{x}=-\ln x$$

Doğru Cevap : D

$$\lambda=\frac{1}{x}$$

7)

For which value of b, the differential equation $(e^x siny+bx^2y^2)dx+(e^x cosy+x^3y)dy=0$ is an exact differential equation?

$$\underbrace{(e^x\sin y+bx^2y^2)}_{P}dx+\underbrace{(e^x\cos y+x^3y)}_{Q}dy=0$$

A:
2

B:
0

$$\frac{\partial P}{\partial y}=e^x\cos y+2bx^2y$$

C:
$\frac{1}{2}$

$$\frac{\partial Q}{\partial x}=e^x\cos y+3x^2y$$

D:
1

$$\cancel{e^x\cos y}+2bx^2y=\cancel{e^x\cos y}+3x^2y$$

$$2b=3\Rightarrow b=\frac{3}{2}$$

E:
$\frac{3}{2}$

Doğru Cevap : E

8)

Let $y_1(t)$ and $y_2(t)$ be two solutions of a second order homogeneous linear differential equation. The Wronskian determinant of the two solutions is $W\big(y_1(t), y_2(t)\big) = e^{-t}$. Then, which of the following statement is false?

A:

$y_1(t)$ and $y_2(t)$ are linearly dependent functions.

B :

The function $2y_1(t) - 3y_2(t)$ is also a solution of this differential equation.

C :

$y_1(t)$ and $y_2(t)$ construct a fundamental set of solutions.

D :

All the solutions of this differential equation can be represented as $c_1 y_1(t) + c_2 y_2(t)$, where c_1 and c_2 are constants .

E :

$W\big(2y_1(t), 3y_2(t)\big) = 6e^{-t}$

Doğru Cevap : A

9)

Which of the following is true?

A:

$\sin(t)\, y'' + (1 - t^2)y' + \cos(y)\, y = 0$ is a second order linear differential equation.

B :

$y' = \frac{t}{y^2}$ is a first order linear differential equation.

C :

$y'' + (y')^3 + y = 0$ is a second order nonlinear differential equation.

D :

$y'' + y' + y = t$ is a second order homogeneous differential equation.

E :

$\frac{dy}{dt} + ty = 0$ is a nonlinear differential equation.

Doğru Cevap : C

10)

Which of the following is the differential equation that accepts the solution as the family of curves $y = C_1 x + C_2 x.\ell nx$, where C_1 and C_2 are constants?

$y = c_1 x + c_2 x \ln x$

$y' = c_1 + c_2 \ln x + c_2 \cancel{x} \cdot \frac{1}{\cancel{x}}$

$= c_1 + c_2 + c_2 \ln x$

A:

$y'' - xy' - y = 0$

B:

$x^2y'' - xy' + y = 0$

$y'' = \frac{c_2}{x} \Rightarrow c_2 = xy''$

C:

$x^2y'' + x^2y' + xy = 0$

$y' = c_1 + xy'' \ln x + xy''$

D:

$xy'' - y' + x^2y = 0$

$c_1 = y' - xy'' \ln x - xy''$

E:

$y'' - xy' + x^2y = 0$

$\Rightarrow y = xy' - \cancel{x^2y'' \ln x} - x^2y'' + \cancel{x^2y'' \ln x}$

$\Rightarrow y = -x^2y'' + xy' \Rightarrow x^2y'' - xy' - y = 0$

Doğru Cevap : B

11)

If x^2, $e^{2x}\cos x$ and e^{-3x} are some solutions of a sixth-order homogeneous differential equation with constant coefficient, then which of the following is the general solution of this differential equation ?

A:

$$y = c_0 + c_1 x + c_2 x^2 + c_3 e^x\, cosx + c_4 e^{-3x}$$

B :

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 e^x \sin x + c_5 e^{-3x}$$

C :

$$y = c_0 + c_1 x + c_2 x^2 + e^{2x}(c_3 \sin x + c_4 \cos x) + c_5 e^{-3x}$$

D :

$$y = c_0 + c_1 x + c_2 x^2 + c_3 e^{2x} cosx + c_4 e^{-3x} + c_5 x e^{-3x}$$

E :

$$y = c_0 + c_1 x + c_2 x^2 + c_3 e^{2x} \sin x + c_4 e^{-3x} + c_5 x e^{-3x}$$

Doğru Cevap : C

12)

Which of the following transformation is the transformation that will convert the differential equation $y' = \frac{2x-y+5}{x+y+1}$ into a homogeneous differential equation?

$$\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$$

A:
$x = x_1 - 2$
$y = y_1 + 1$

B:
$x = x_1 - 2$
$y = y_1 - 1$

C:
$x = x_1 + 2$
$y = y_1 - 1$

D:
$x = x_1 + 2$
$y = y_1 + 1$

E:
$x = x_1 + 1$
$y = y_1 + 2$

Doğru Cevap : A

$$\frac{dy}{dx} = \frac{2x-y+5}{x+y+1}$$

$$\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2+1 = 3 \neq 0$$ Homojen Hale Getirilebilen

$x = X + h \Rightarrow dx = dX$
$y = Y + k \Rightarrow dy = dY$

$$\frac{dY}{dX} = \frac{2[X+h]-[Y+k]+5}{[X+h]+[Y+k]+1} = \frac{2X-Y+(2h-k+5)}{X+Y+[h+k+1]}$$

$2h-k+5=0$
$h+k+1=0$
$3h=-6$
$h=-2 \Rightarrow k=1$

$x = X-2$
$y = Y+1$

13)

For $x \in \left[0, \frac{\pi}{2}\right]$, What is the particular solution of the differential equation $(y + y\sin x)y' = \sqrt{\cos^2 x - y^4 \cos^2 x}$ with the initial condition $y(0) = 0$?

A:

$\frac{1}{2} Arccos y^2 = \ln(1 + \sin x)$

B:

$\frac{1}{2} Arcsin y^2 = \ln(1 + \cos x)$

C:

$\frac{1}{2} Arcsin y = \ln(1 + \cos x)$

D:

$\frac{1}{2} Arcsin y^2 = \ln(1 + \sin x)$

E:

$\frac{1}{2} Arcsin y^2 = \ln(1 - \sin x)$

Doğru Cevap : D

14)

Which of the following functions construct a linearly independent function set?

A:

$y_1 = e^{3x}, y_2 = 4e^{3x}, y_3 = e^{-x}$

B:

$y_1 = 3x^2 + 2, y_2 = x^2, y_3 = 1$

C:

$y_1 = e^{2x}, y_2 = e^{3x}, y_3 = e^{4x}$

D:

$y_1 = e^{2x}, y_2 = 2e^{x}, y_3 = e^{x+2}$

E:

$y_1 = e^{-x}, y_2 = 2e^{x}, y_3 = e^{x+2}$

Doğru Cevap : C

15)

If $y_1 = \frac{A}{x}$ is a particular solution of the differential equation $y' + y^2 = \frac{2}{x^2}$, then which of the following is the Bernoulli differential equation to be obtained for the largest value of A?

A:

$u' + \frac{4}{x}u = -u^2$

B :

$u' - x^2 u = -xu^2$

C :

$u' - \frac{4}{x}u = u^2$

D :

$u' + \frac{4}{x}u = u^2$

E :

$u' - 4xu = -u^2$

Doğru Cevap : A

16)

A curve whose slope is $\frac{c}{y.\sqrt{1-x^2}}$ at any point passes from the points $A(0,0)$ and B(1, $\sqrt{\frac{\pi}{2}}$) . According to this information, What is the value of C?

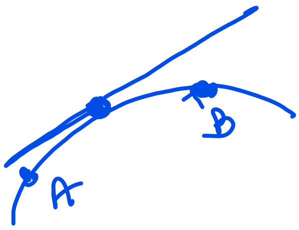
$y' = \frac{c}{y\sqrt{1-x^2}}$

$y\,dy = \frac{c.dx}{\sqrt{1-x^2}}$

$\frac{y^2}{2} = c.\arcsin x + c_1$

$y^2 = 2c\arcsin x + 2c_1$

$0 = 2c_1 \Rightarrow c_1 = 0$



A:

π

B :

1

C :

$\frac{1}{2}$

$y^2 = 2c\arcsin x$

$\frac{\pi}{2} = 2c\arcsin 1 \Rightarrow \frac{\pi}{2} = \frac{2c\pi}{2}$

$2c = 1$

$c = 1/2$

D :

0

E :

$\frac{\pi}{2}$

Doğru Cevap : C

17)

If $y = c_1e^{5x} + c_2xe^{5x}$ is the general solution of the differential equation $y'' + ky' + my = 0$, then what is the sum of $k + m$?

A:
10

B :
15

C :
20

D :
0

E :
5

Doğru Cevap : B

1

18)

An algorithm that can learn by itself is to model a meteorite approaching the Earth with its sensors as $Q''(x) - 3Q'(x) + 2Q(x) = 0,\ Q(0) = 1,\ Q'(0) = 2$. So, what does the algoritm find the function Q(x)?

A:
$2e^{2x}$

B :
$4e^{2x}$

C :
0

D :
e^{x}

E :
e^{2x}

Doğru Cevap : E

19)

Which of the following is the linear differential equation that is transformed from the differential equation $y = -x(y')^3 + \ln(1 + (y')) + 5$ and the integrating factor which obtained from this lineer differential equation?

A:

$\frac{dx}{dp} + \frac{3p}{(1+p^2)}x = \frac{1}{p(1+p)(1+p^2)}$; $\mu = (1 + p^2)^{\frac{5}{2}}$

B :

$\frac{dx}{dp} + \frac{5p}{(1+p^2)}x = \frac{1}{p(1+p)(1+p^2)}$; $\mu = (1 + p^2)^{\frac{5}{2}}$

C :

$\frac{dx}{dp} + \frac{5p}{(1+p^2)}x = \frac{1}{p(1+p)(1+p^2)}$; $\mu = (1 + p^2)^{\frac{5}{4}}$

D :

$\frac{dx}{dp} + \frac{3p}{(1+p^2)}x = \frac{1}{p(1+p)(1+p^2)}$; $\mu = (1 + p^2)^{\frac{3}{2}}$

E :

$\frac{dx}{dp} + \frac{2p}{3(1+p^2)}x = \frac{1}{(1+p)(1+p^2)}$; $\mu = (1 + p^2)^{\frac{1}{3}}$

Doğru Cevap : D

20)

Which of the following differential equation is the new form of the $y' = 6y + 3x(\sqrt[3]{y})$ differential eqution transformed into a linear differential equation using a suitable transformation?

A:

$\frac{dz}{dx} - z = x^3$

B :

$\frac{dz}{dx} - 4z = 2x$

C :

$\frac{dz}{dx} - 9z = \frac{9x}{2}$

D :

$\frac{dz}{dx} - \frac{3z}{2} = 6x$

E :

$\frac{dz}{dx} - 6z = \sqrt[3]{x}$

Doğru Cevap : B