# Electronic Devices and Circuit Theory

Boylestad

## BJT AC Analysis

## Chapter 5

# BJT Transistor Modeling

A model is an equivalent circuit that represents the AC characteristics of the transistor.

A model uses circuit elements that approximate the behavior of the transistor.

There are two models commonly used in small signal AC analysis of a transistor:

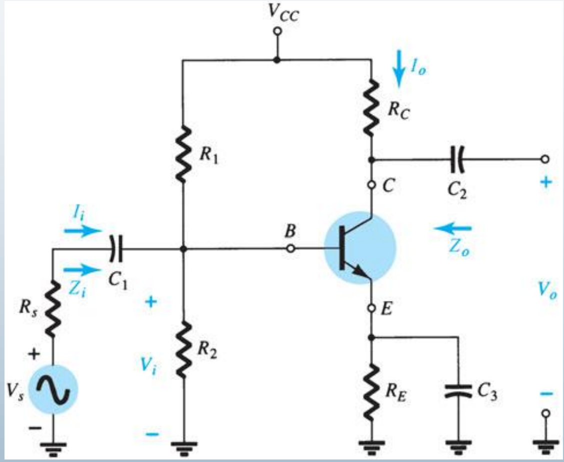
**$r_e$ model**

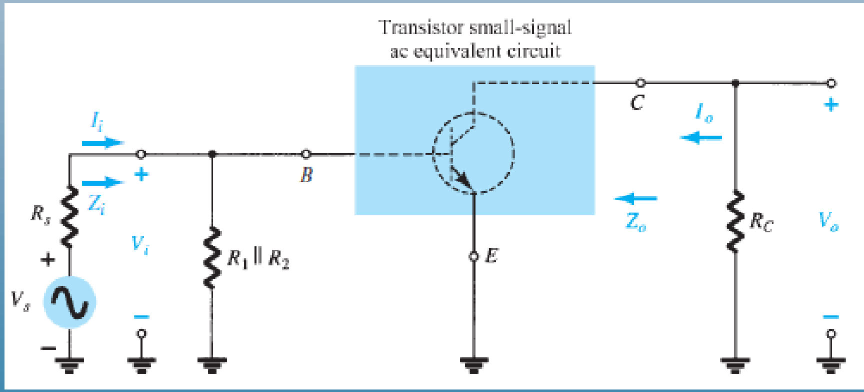
**Hybrid equivalent model**

# The $r_e$ Transistor Model

BJTs are basically current-controlled devices; therefore the $r_e$ model uses a diode and a current source to duplicate the behavior of the transistor.

One disadvantage to this model is its sensitivity to the DC level. This model is designed for specific circuit conditions.



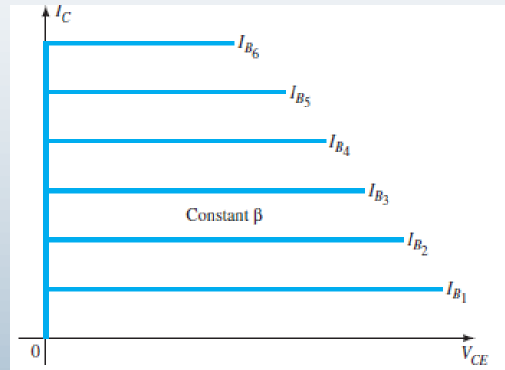



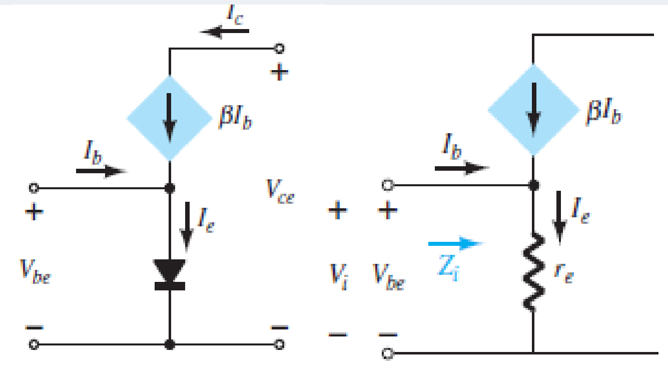
# Common-Emitter Configuration

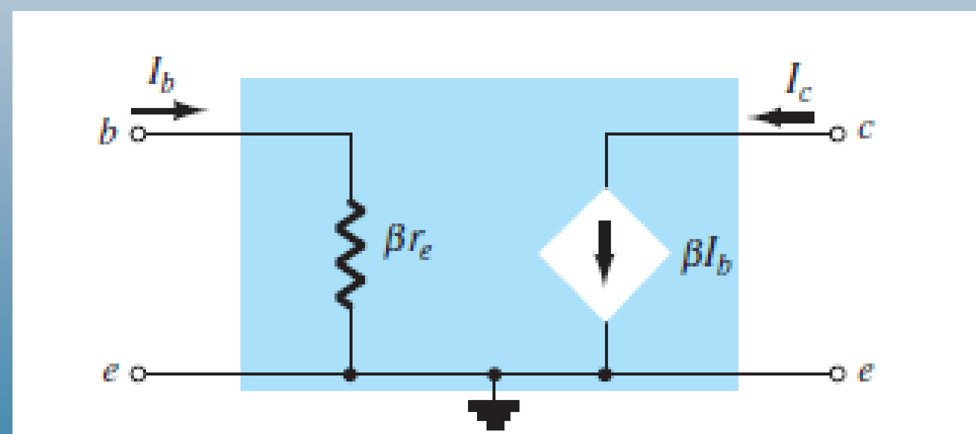
The diode $r_e$ model can be replaced by the resistor $r_e$.

$$I_e = (\beta + 1) I_b \cong \beta I_b$$

$$r_e = \frac{26\,mV}{I_e}$$



# Common-Emitter Configuration

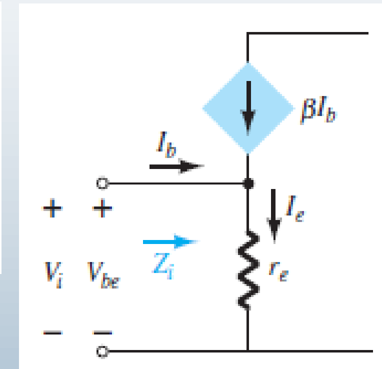
**Input impedance:** $Z_i = \beta r_e$

$$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$$

$$V_{be} = I_e r_e = (I_c + I_b) r_e = (\beta I_b + I_b) r_e$$

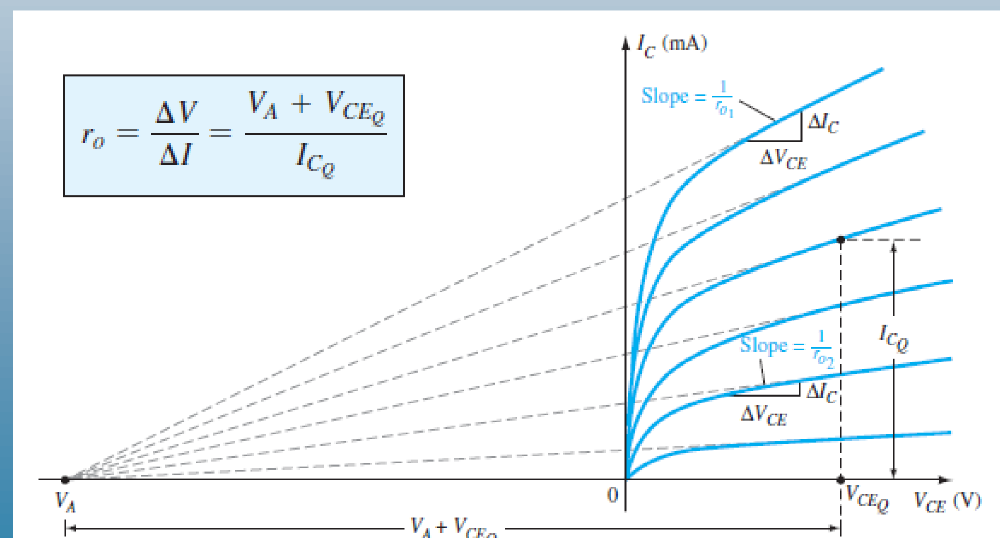
$$= (\beta + 1) I_b r_e$$

$$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta + 1) I_b r_e}{I_b}$$




**Output impedance:**

$$Z_o = r_o \cong \infty \Omega$$



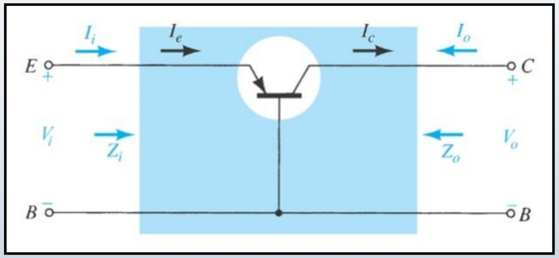

**Voltage gain:**

$$A_V = -\frac{R_L}{r_e}$$

**Current gain:**

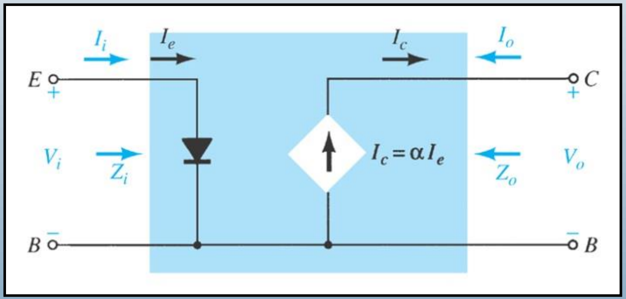
$$A_i = \beta \big|_{r_o = \infty}$$

# Common-Base Configuration




## Input impedance:

$$r_e = \frac{26\,mV}{I_e} \qquad Z_i = r_e$$

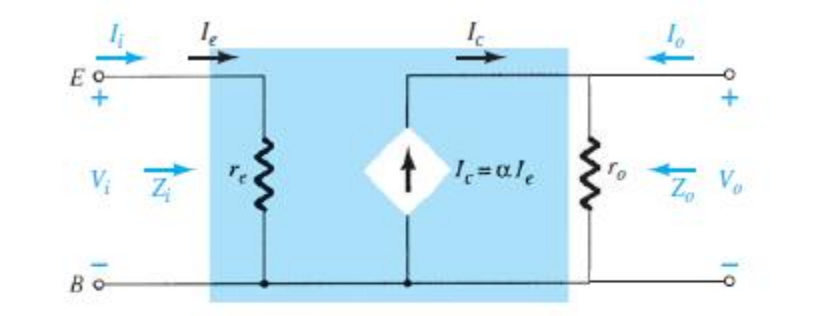



## Output impedance:

$$Z_o \cong \infty\Omega$$

## Voltage gain:

$$A_V = \frac{\alpha R_L}{r_e} \cong \frac{R_L}{r_e}$$



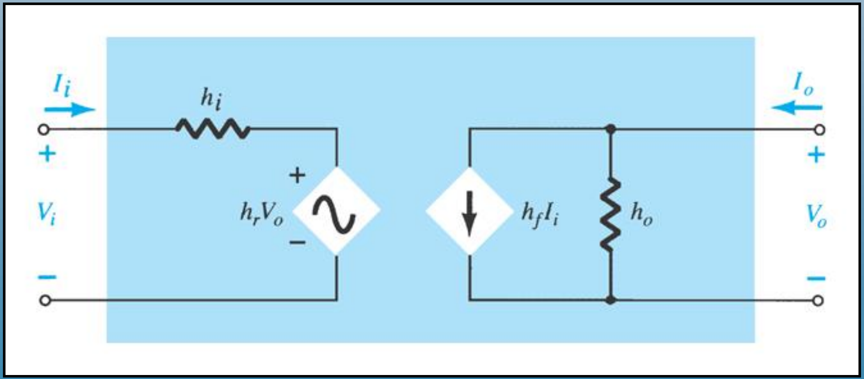

## Current gain:

$$A_i = -\alpha \cong -1$$

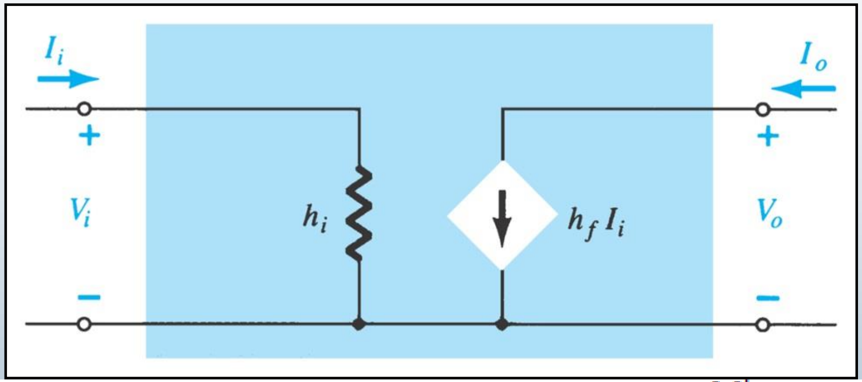
# The Hybrid Equivalent Model

Hybrid parameters are developed and used for modeling the transistor. These parameters can be found on a transistor's specification sheet:

$h_i$ = input resistance
$h_r$ = reverse transfer voltage ratio $(V_i/V_o) \cong 0$
$h_f$ = forward transfer current ratio $(I_o/I_i)$
$h_o$ = output conductance



# Simplified General h-Parameter Model




| | | Min. | Max. | |
|---|---|---|---|---|
| Input impedance ($I_C$ = 1 mA dc, $V_{CE}$ = 10 V dc, $f$ = 1 kHz) 2N4400 | $h_{ie}$ | 0.5 | 7.5 | kΩ |
| Voltage feedback ratio ($I_C$ = 1 mA dc, $V_{CE}$ = 10 V dc, $f$ = 1 kHz) | $h_{re}$ | 0.1 | 8.0 | $\times 10^{-4}$ |
| Small-signal current gain ($I_C$ = 1 mA dc, $V_{CE}$ = 10 V dc, $f$ = 1 kHz) 2N4400 | $h_{fe}$ | 20 | 250 | — |
| Output admittance ($I_C$ = 1 mA dc, $V_{CE}$ = 10 V dc, $f$ = 1 kHz) | $h_{oe}$ | 1.0 | 30 | 1μS |

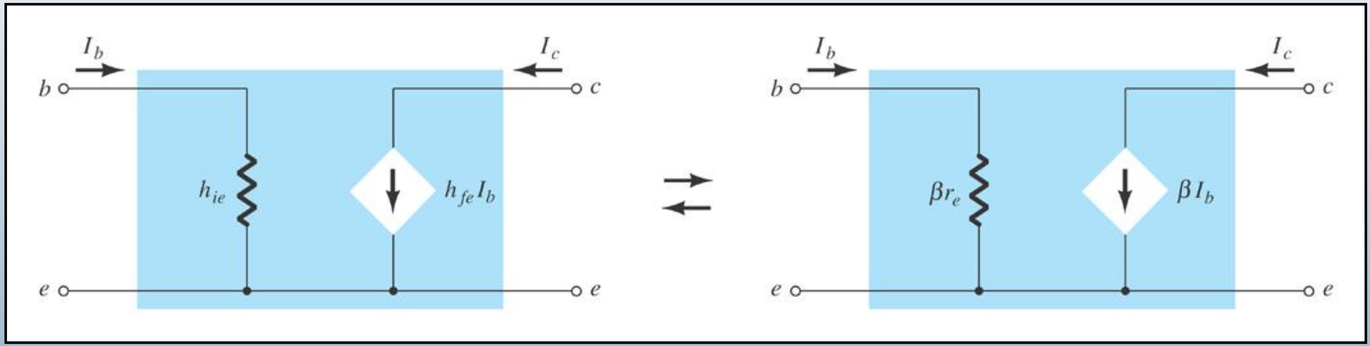
$h_i$ = input resistance

$h_f$ = forward transfer current ratio ($I_o/I_i$)

# h-Parameter vs. $r_e$ Model

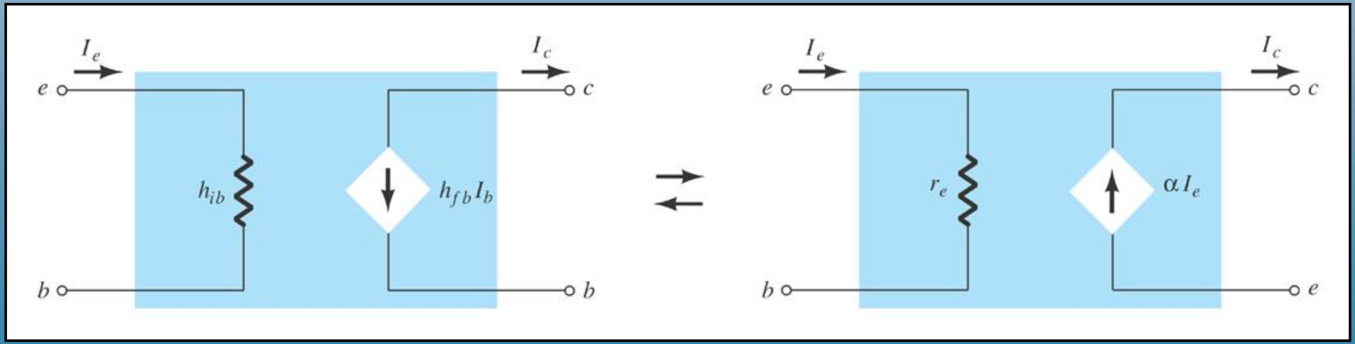
## Common-Emitter

$$h_{ie} = \beta r_e$$
$$h_{fe} = \beta_{ac}$$




## Common-Base

$$h_{ib} = r_e$$
$$h_{fb} = -\alpha \cong -1$$



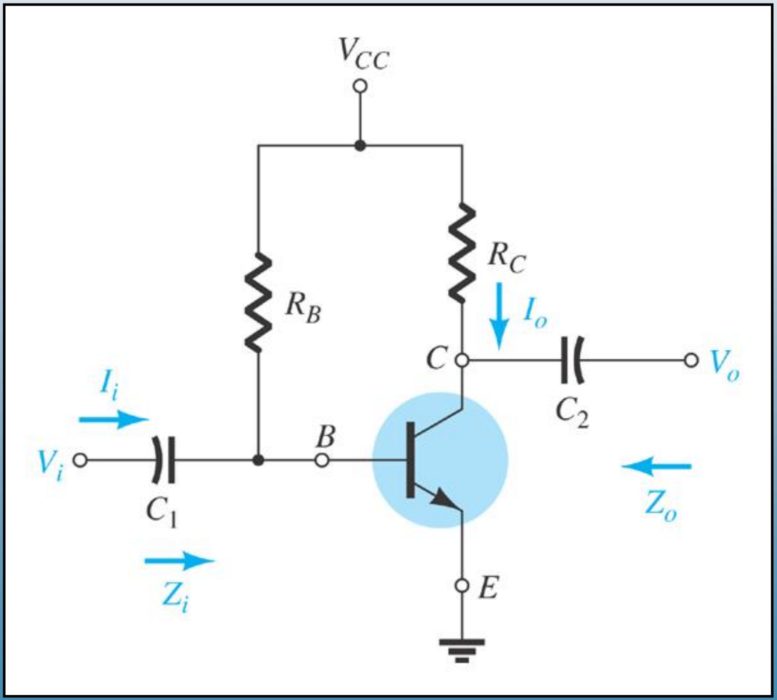
# Common-Emitter Fixed-Bias Configuration

The input is applied to the base
The output is taken from the collector

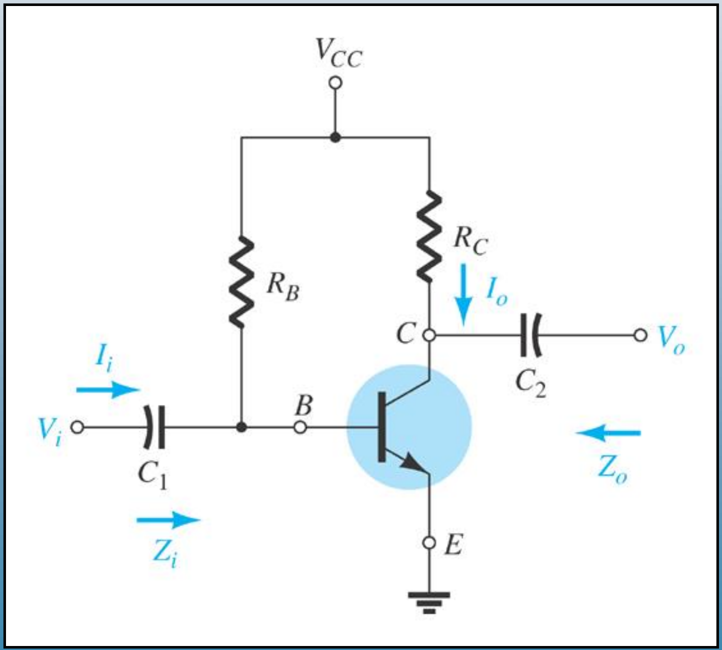
High input impedance
Low output impedance

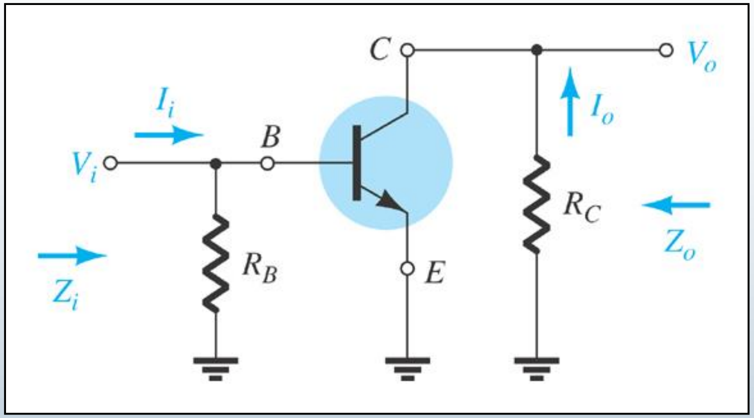
High voltage and current gain

Phase shift between input and output is 180°

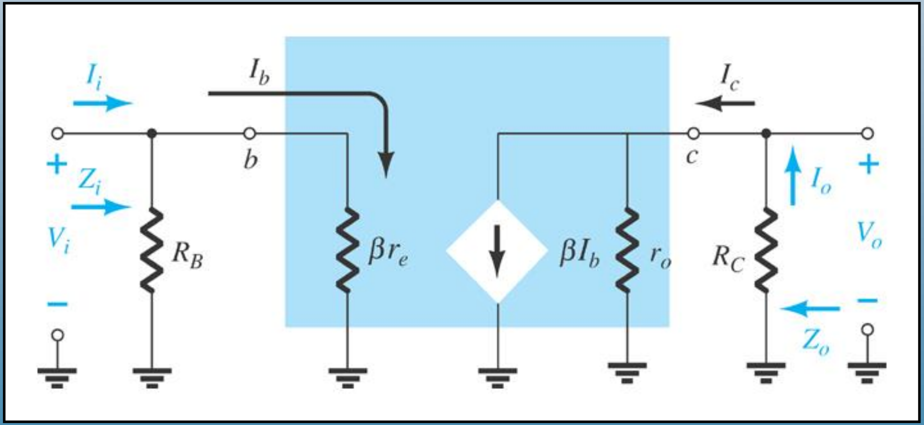


# Common-Emitter Fixed-Bias Configuration



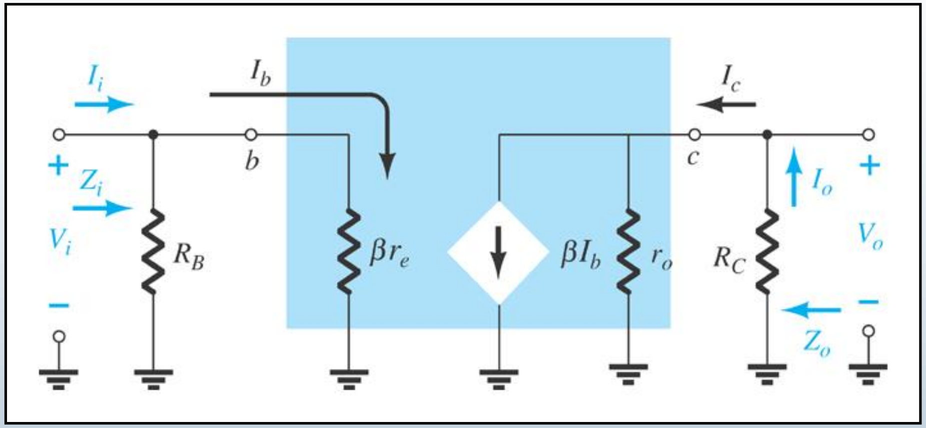




AC equivalent




$r_e$ model

# Common-Emitter Fixed-Bias Calculations




Input impedance:

$$Z_i = R_B \,||\, \beta r_e$$
$$Z_i \cong \beta r_e \Big|_{R_B \geq 10\beta r_e}$$

Output impedance:

$$Z_o = R_C || r_o$$
$$Z_o \cong R_C \Big|_{r_o \geq 10 R_C}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{(R_C || r_o)}{r_e}$$
$$A_v = -\frac{R_C}{r_e} \Big|_{r_o \geq 10 R_C}$$

Current gain:

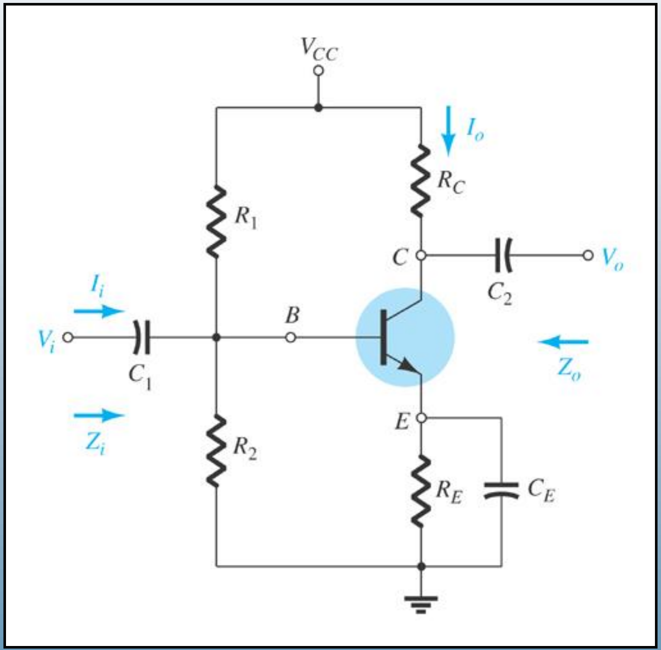
$$A_i \cong \beta \Big|_{r_o \geq 10 R_C,\, R_B \geq 10\beta r_e}$$

$$A_{i_L} = \frac{I_o}{I_i} = \frac{-\dfrac{V_o}{R_L}}{\dfrac{V_i}{Z_i}} = -\frac{V_o}{V_i} \cdot \frac{Z_i}{R_L}$$

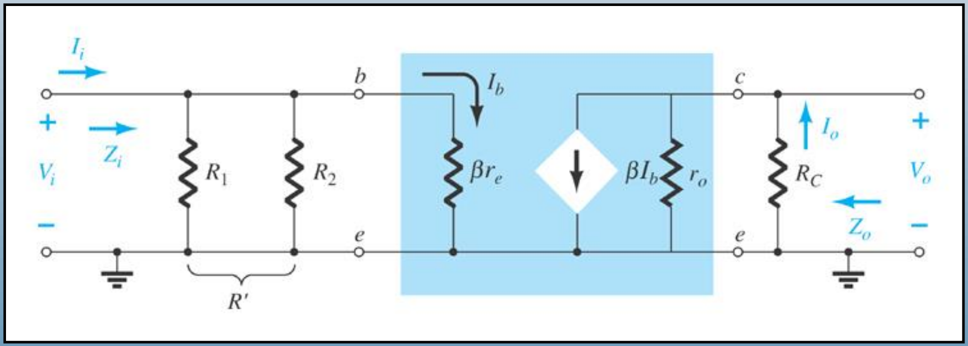
Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$

# Common-Emitter Voltage-Divider Bias




$r_e$ model requires you to determine β, $r_e$, and $r_o$.



# Common-Emitter Voltage-Divider Bias Calculations

$$A_{i_L} = \frac{I_o}{I_i} = \frac{-\dfrac{V_o}{R_L}}{\dfrac{V_i}{Z_i}} = -\frac{V_o}{V_i} \cdot \frac{Z_i}{R_L}$$

Input impedance

$$R' = R_1 \,||\, R_2$$
$$Z_i = R' \,||\, \beta r_e$$

Output impedance

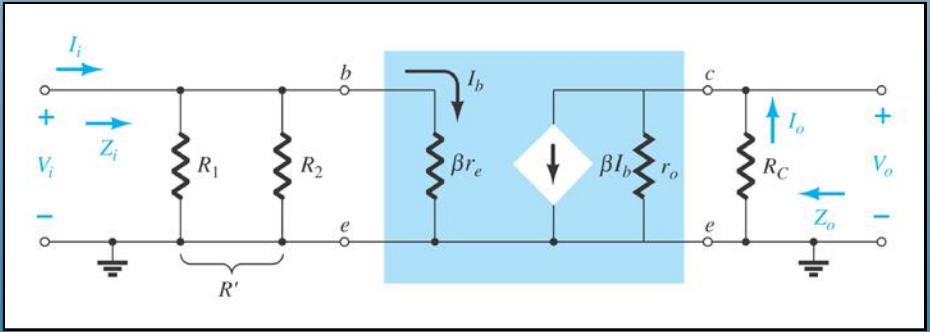
$$Z_o = R_C \,||\, r_o$$
$$Z_o \cong R_C \Big|_{r_o \geq 10R_C}$$

Current gain from $A_v$

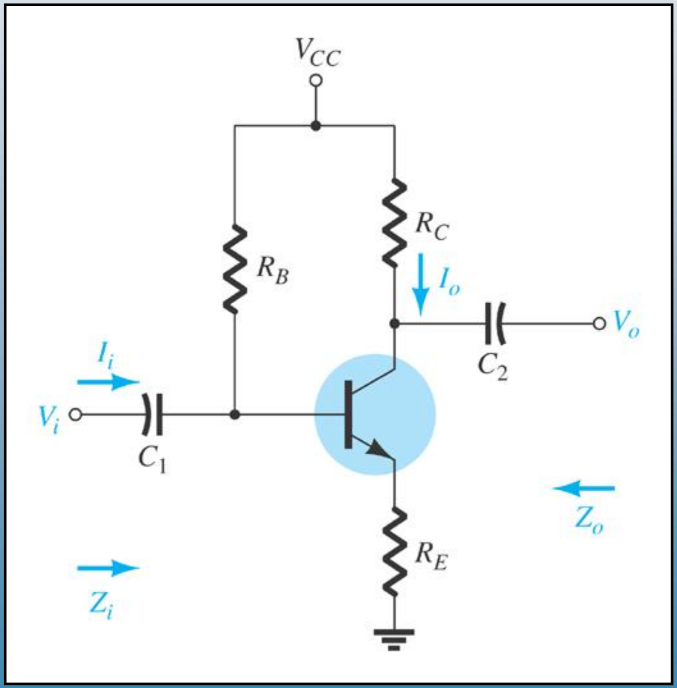
$$A_i = -A_v \frac{Z_i}{R_C}$$

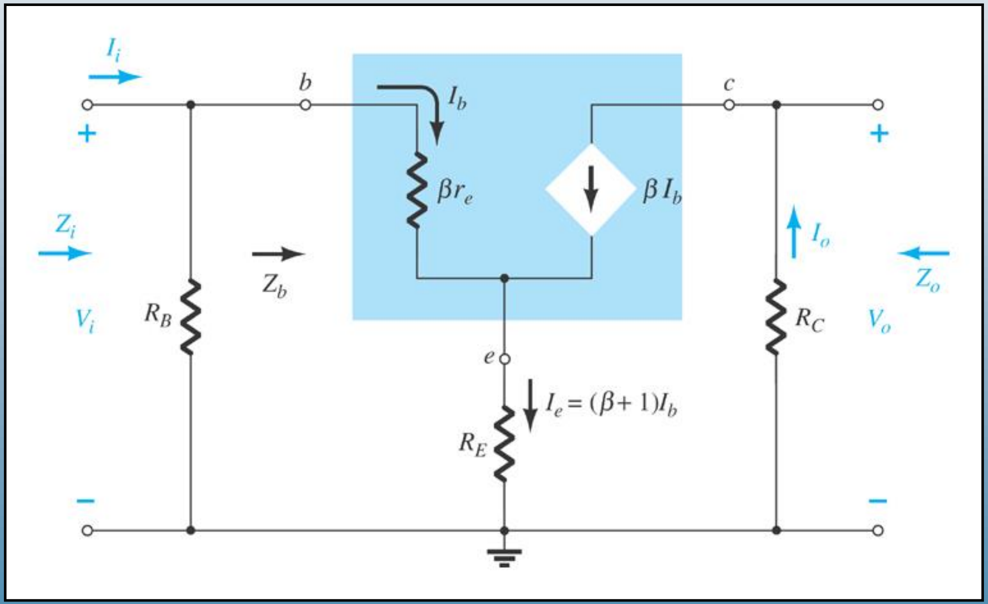
Voltage gain

$$A_v = \frac{V_o}{V_i} = \frac{-R_C \,||\, r_o}{r_e}$$
$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e}\Big|_{r_o \geq 10R_C}$$



# Common-Emitter Emitter-Bias Configuration

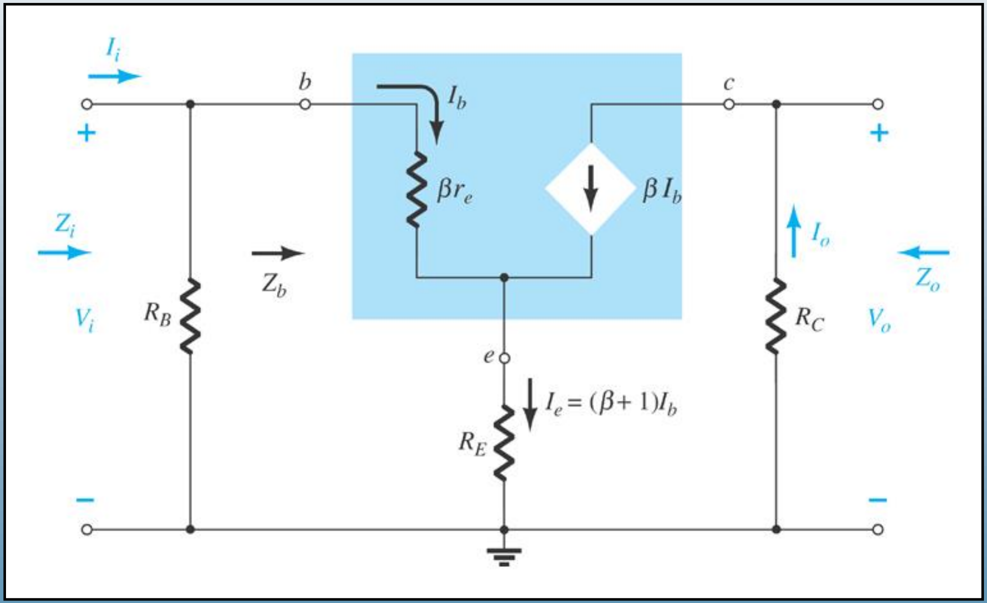


# Impedance Calculations

## Input impedance:

$$Z_i = R_B \,||\, Z_b$$
$$Z_b = \beta r_e + (\beta + 1)R_E$$
$$Z_b \cong \beta(r_e + R_E)$$
$$Z_b \cong \beta R_E$$




## Output impedance:

$$Z_o = R_C$$

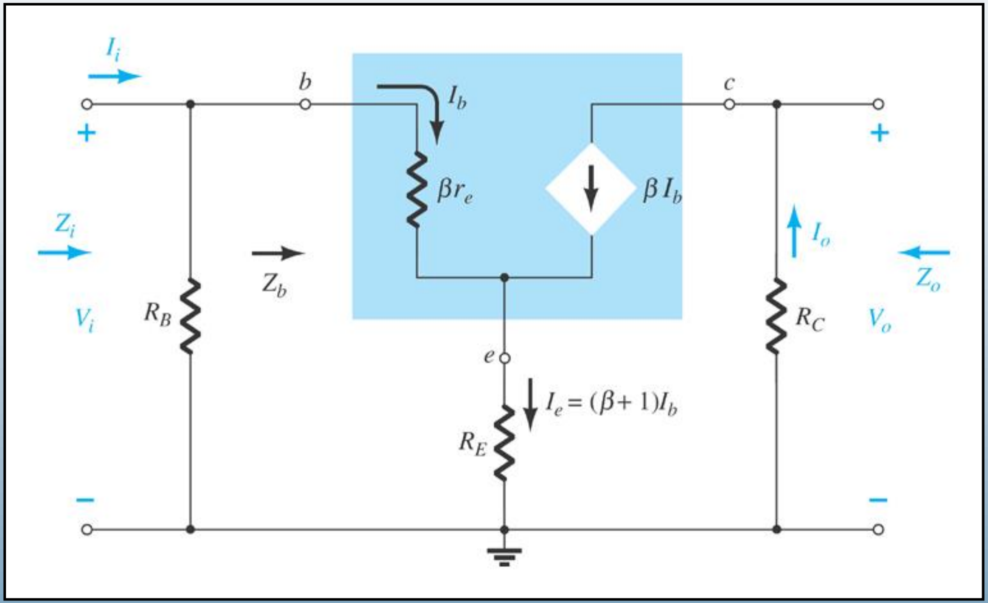
# Gain Calculations

## Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}$$

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e + R_E}\bigg|_{Z_b = \beta(r_e + R_E)}$$

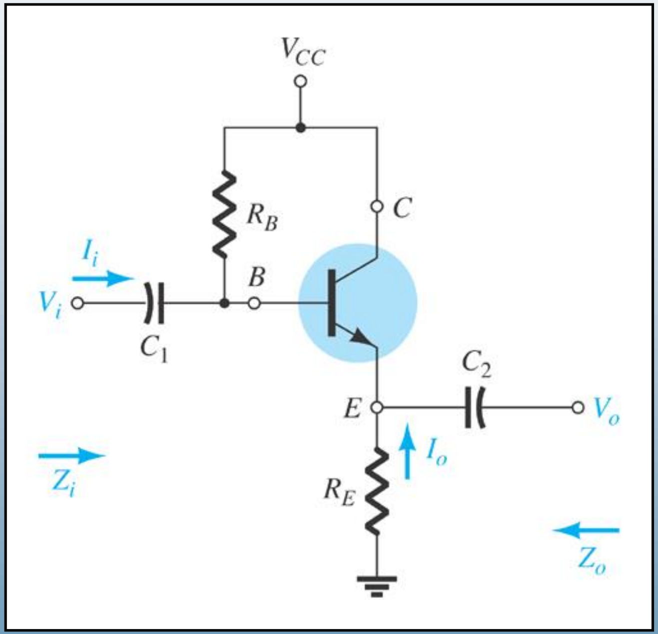
$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{R_E}\bigg|_{Z_b \cong \beta R_E}$$

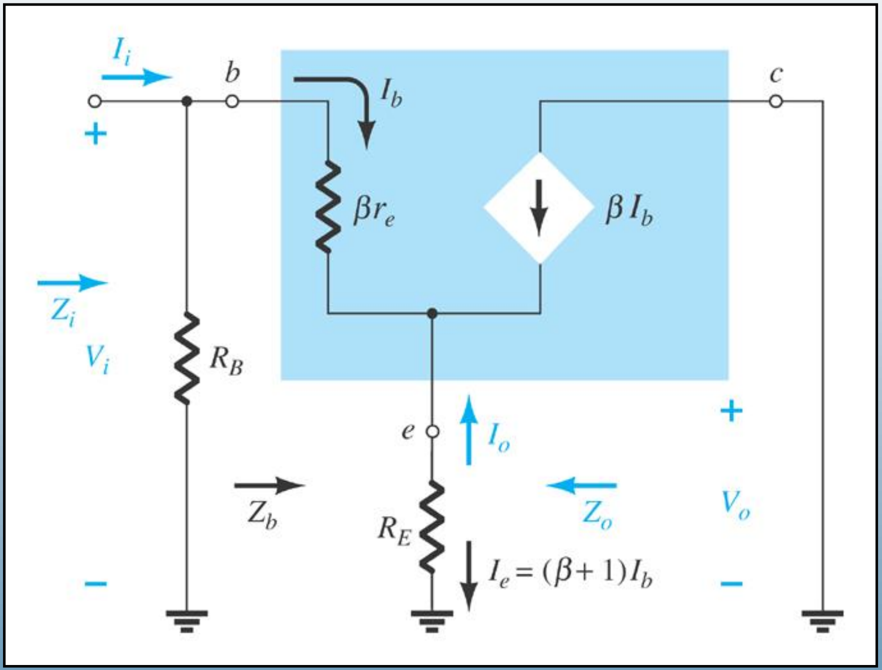



## Current gain from $A_v$:

$$A_i = -A_v \frac{Z_i}{R_C}$$

# Emitter-Follower Configuration



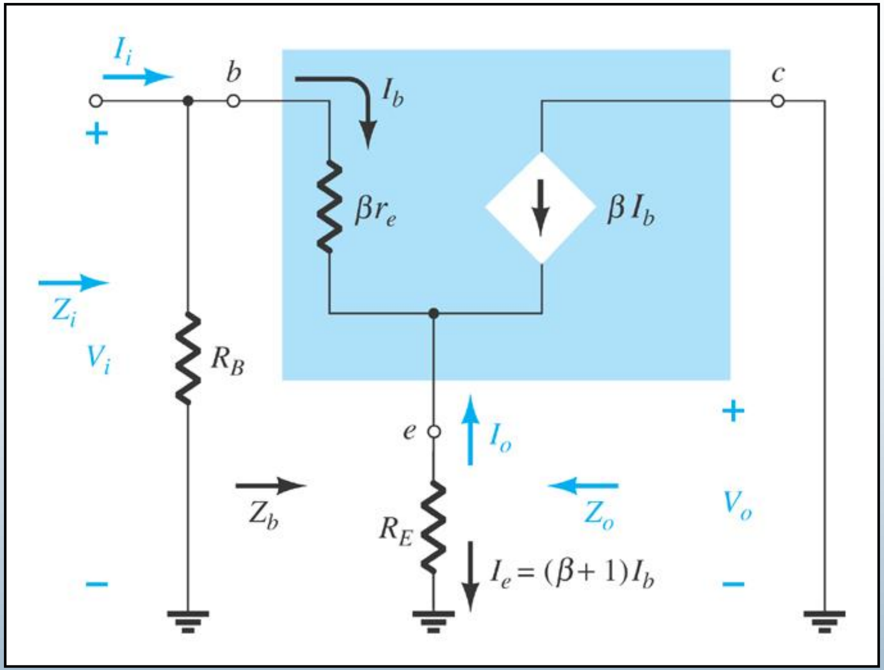




This is also known as the *common-collector* configuration.

The input is applied to the base and the output is taken from the emitter.

There is no phase shift between input and output.

# Impedance Calculations

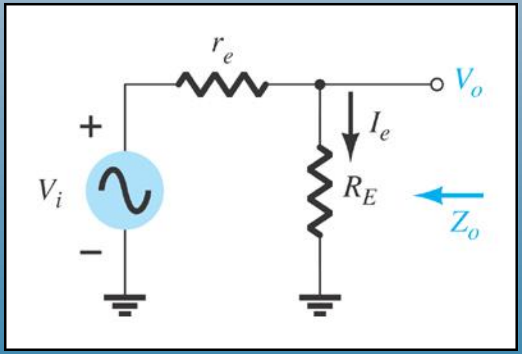



## Input impedance:

$$Z_i = R_B \| Z_b$$
$$Z_b = \beta r_e + (\beta + 1) R_E$$
$$Z_b \cong \beta (r_e + R_E)$$
$$Z_b \cong \beta R_E$$

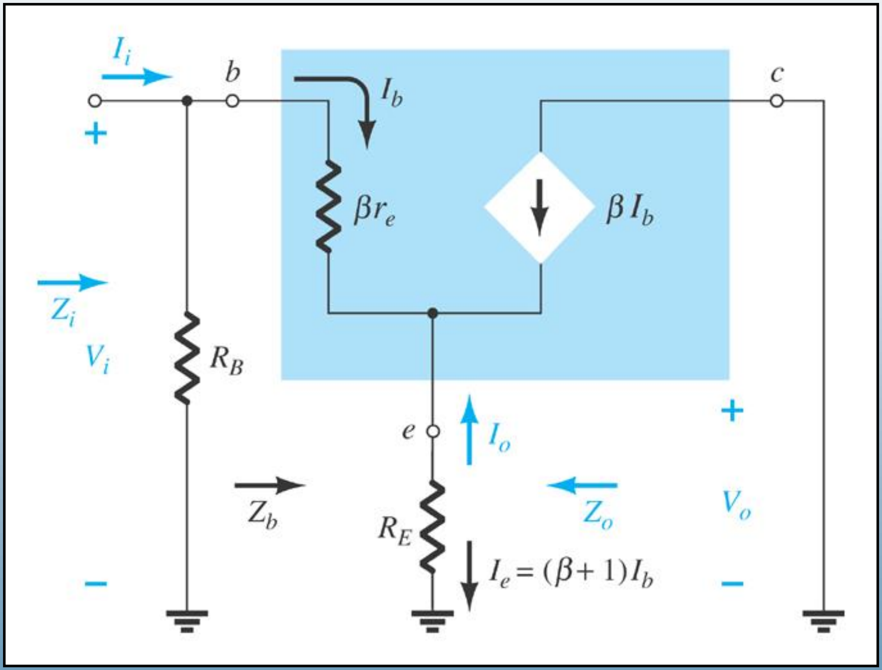
## Output impedance:

$$Z_o = R_E \| r_e$$
$$Z_o \cong r_e \big|_{R_E >> r_e}$$



# Gain Calculations

## Voltage gain:



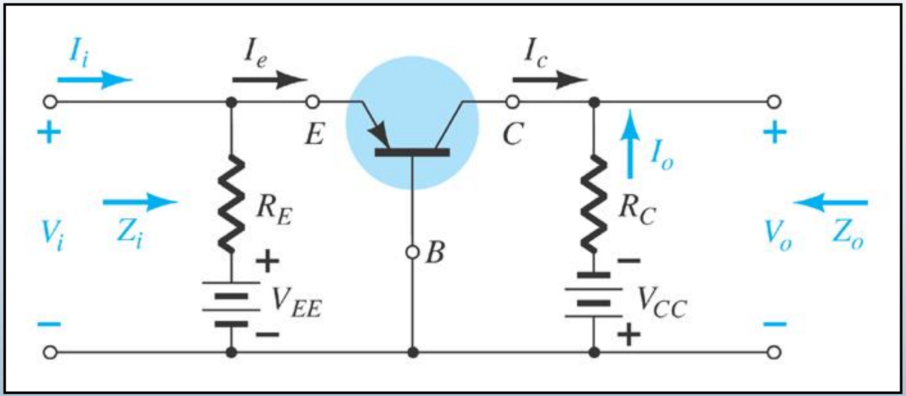

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

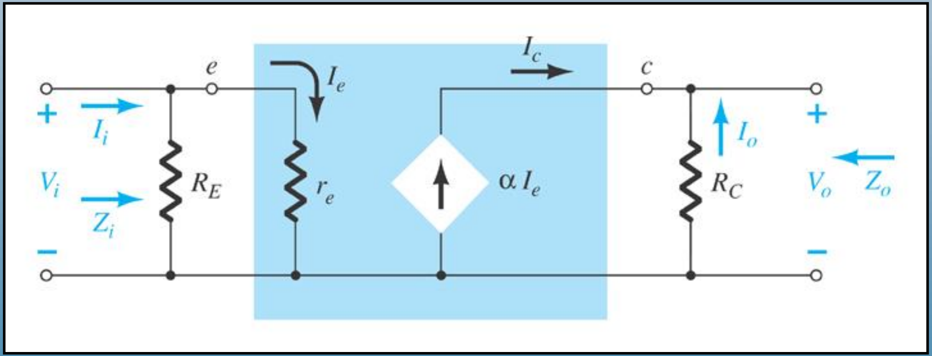
$$A_v = \frac{V_o}{V_i} \cong 1 \Big|_{R_E >> r_e,\ R_E + r_e \cong R_E}$$

## Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_E}$$

# Common-Base Configuration




The input is applied to the emitter

The output is taken from the collector

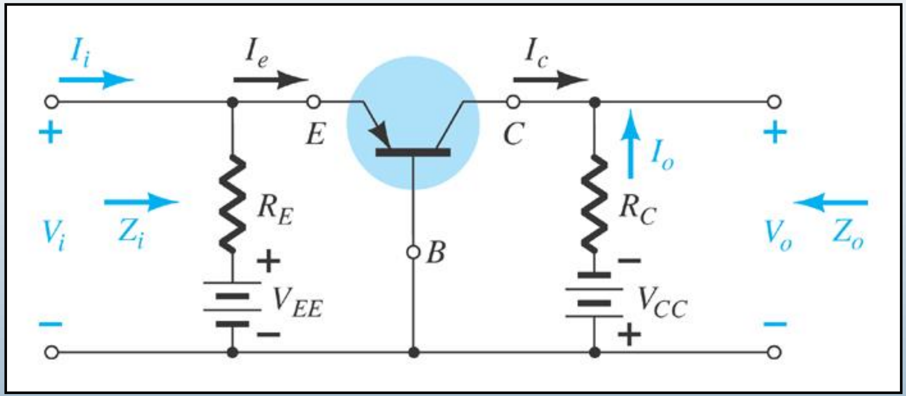
Low input impedance.
High output impedance

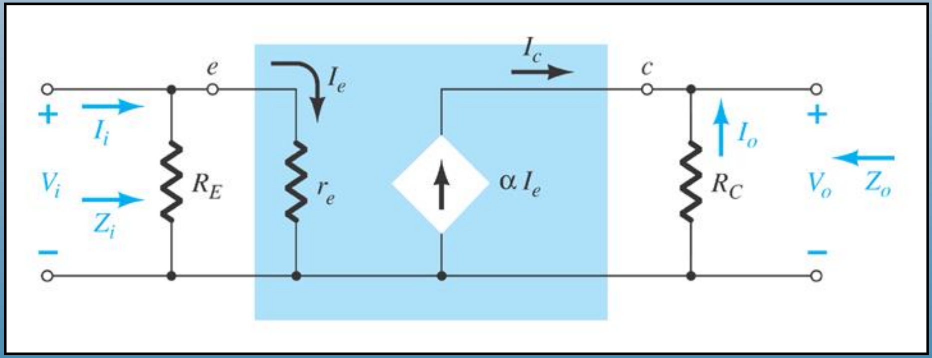
Current gain less than unity

Very high voltage gain

No phase shift between input and output

# Calculations




**Input impedance:**

$$Z_i = R_E \,||\, r_e$$

**Output impedance:**

$$Z_o = R_C$$

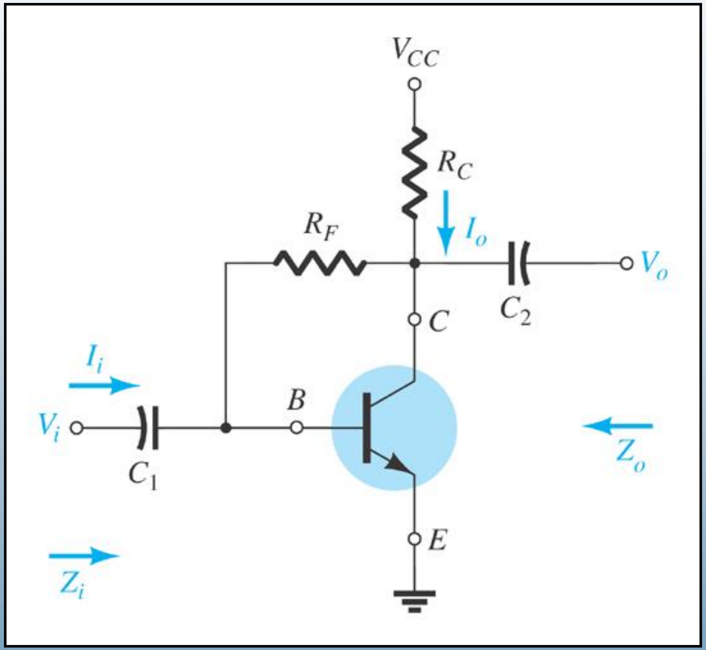
**Voltage gain:**

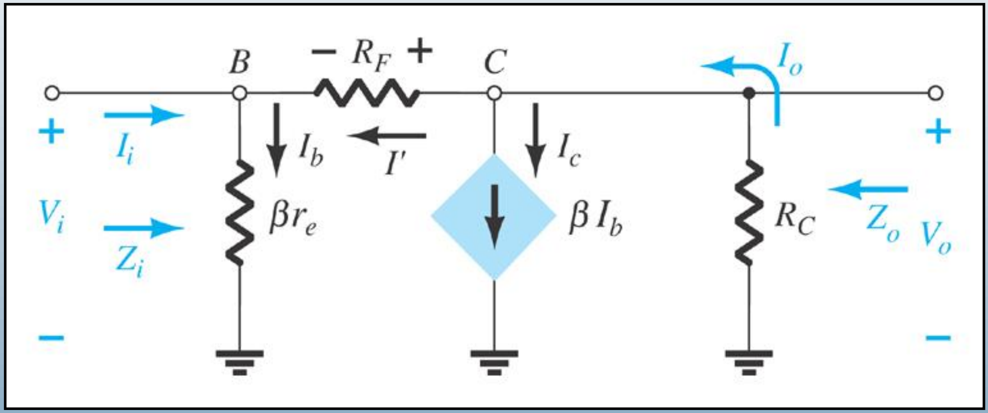
$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \cong \frac{R_C}{r_e}$$

**Current gain:**

$$A_i = \frac{I_o}{I_i} = -\alpha \cong -1$$

# Common-Emitter Collector Feedback Configuration




- A variation of the common-emitter fixed-bias configuration
- Input is applied to the base
- Output is taken from the collector
- There is a 180° phase shift between the input and output

# Calculations

**Input impedance:**

$$Z_i = \frac{r_e}{\dfrac{1}{\beta} + \dfrac{R_C}{R_F}}$$

**Output impedance:**

$$Z_o \cong R_C \,||\, R_F$$

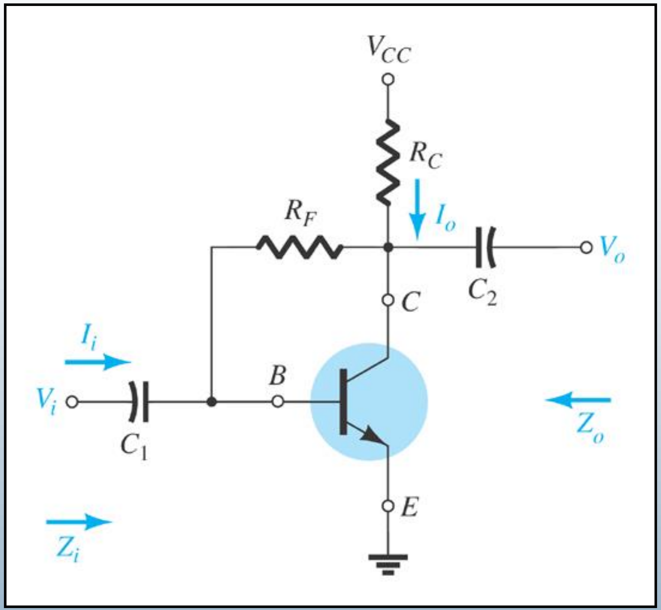
**Voltage gain:**

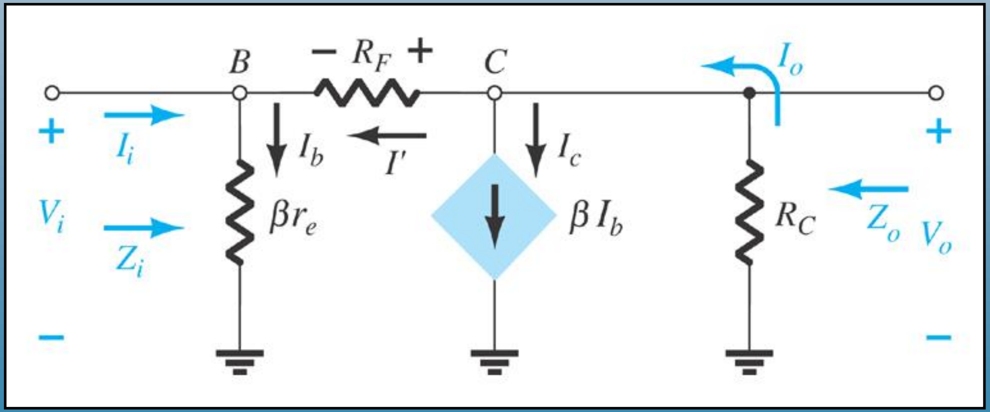
$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e}$$

**Current gain:**

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{R_F + \beta R_C}$$

$$A_i = \frac{I_o}{I_i} \cong \frac{R_F}{R_C}$$



# Two-Port Systems Approach

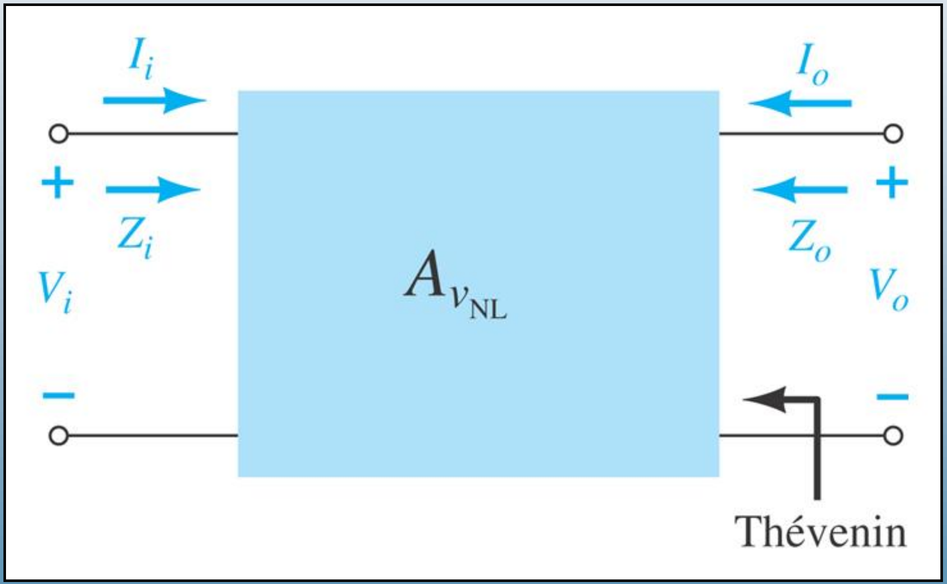
**With $V_i$ set to 0 V:**

$$Z_{Th} = Z_o = R_o$$

**The voltage across the open terminals is:**

$$E_{Th} = A_{vNL} V_i$$

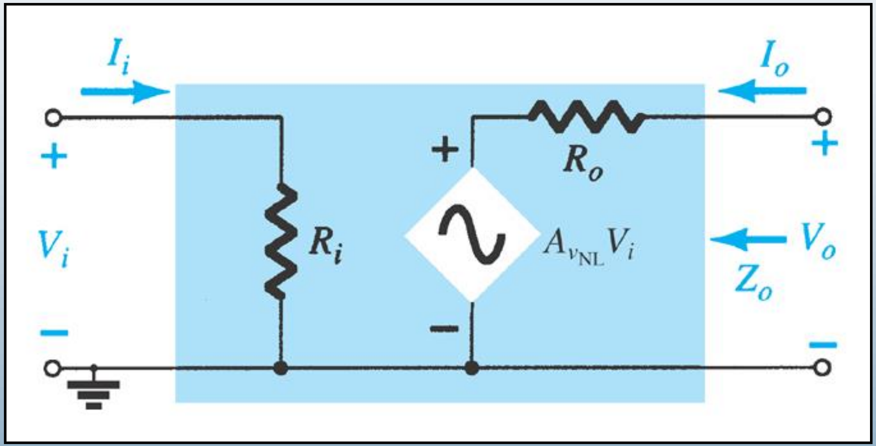
where $A_{vNL}$ is the no-load voltage gain



# Effect of Load Impedance on Gain

This model can be applied to any current- or voltage-controlled amplifier.

Adding a load reduces the gain of the amplifier:



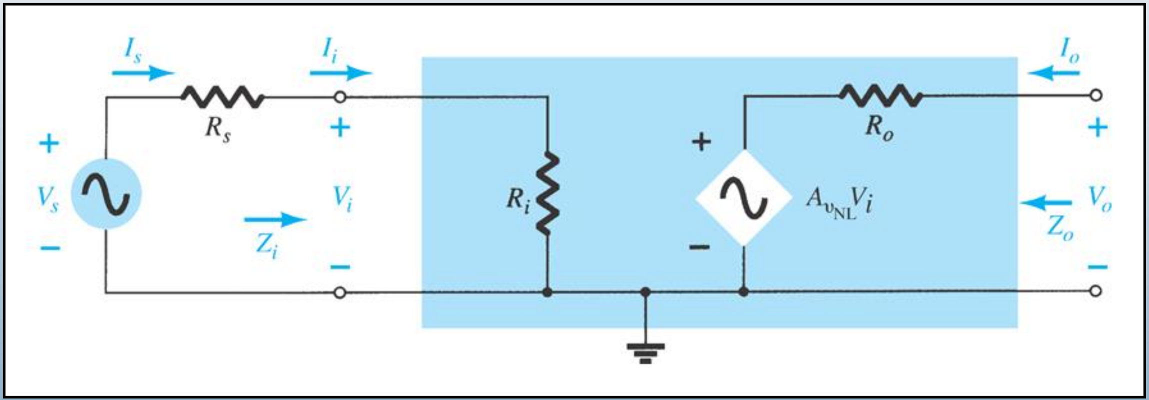

$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_i = -A_v \frac{Z_i}{R_L}$$

# Effect of Source Impedance on Gain

The amplitude of the applied signal that reaches the input of the amplifier is:

$$V_i = \frac{R_i V_s}{R_i + R_s}$$




The internal resistance of the signal source reduces the overall gain:

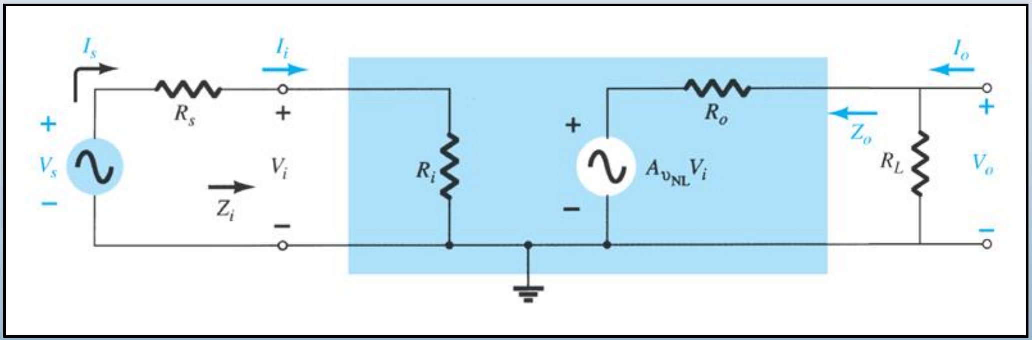
$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} A_{vNL}$$

# Combined Effects of $R_S$ and $R_L$ on Voltage Gain

## Effects of $R_L$:

$$A_v = \frac{V_o}{V_i} = \frac{R_L A_{vNL}}{R_L + R_o}$$

$$A_i = -A_v \frac{R_i}{R_L}$$




## Effects of $R_L$ and $R_S$:

$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s}\frac{R_L}{R_L + R_o}A_{vNL}$$

$$A_{is} = -A_{vs}\frac{R_s + R_i}{R_L}$$

# Cascaded Systems

- The output of one amplifier is the input to the next amplifier
- The overall voltage gain is determined by the product of gains of the individual stages
- The DC bias circuits are isolated from each other by the coupling capacitors
- The DC calculations are independent of the cascading
- The AC calculations for gain and impedance are interdependent

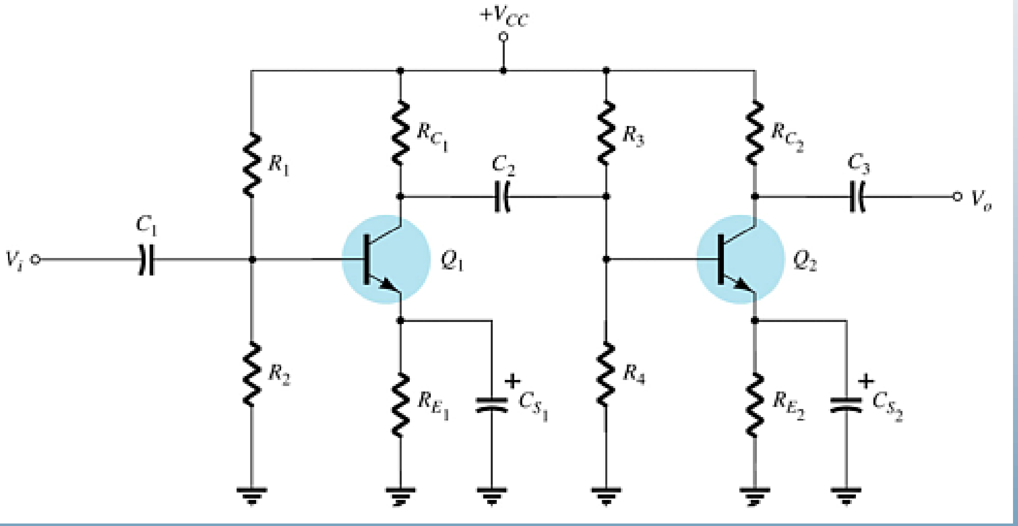
# R-C Coupled BJT Amplifiers

**Voltage gain:**

$$A_{v1} = \frac{R_C \,||\, R_1 \,||\, R_2 \,||\, \beta R_e}{r_e}$$

$$A_{v2} = \frac{R_C}{r_e}$$

$$A_v = A_{v1} A_{v2}$$




**Input impedance, first stage:**

$$Z_i = R_1 \,||\, R_2 \,||\, \beta R_e$$

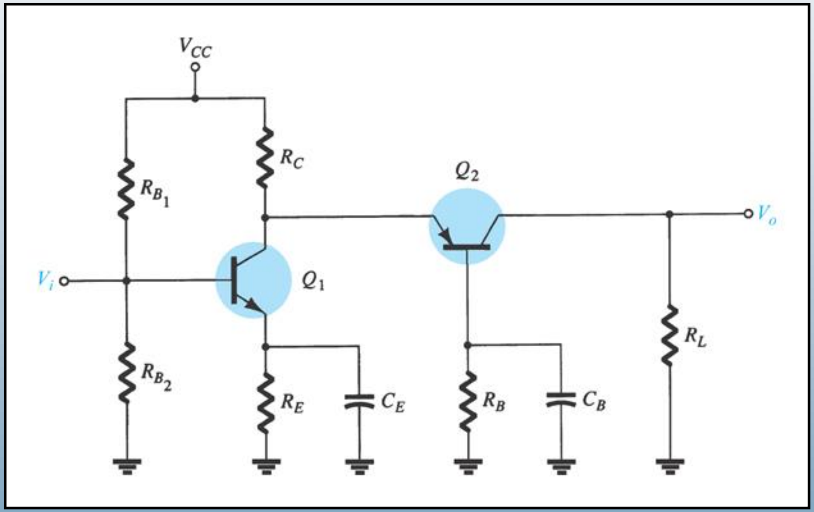
**Output impedance, second stage:**

$$Z_o = R_C$$

# Cascade Connection

This example is a CE–CB combination. This arrangement provides high input impedance but a low voltage gain.

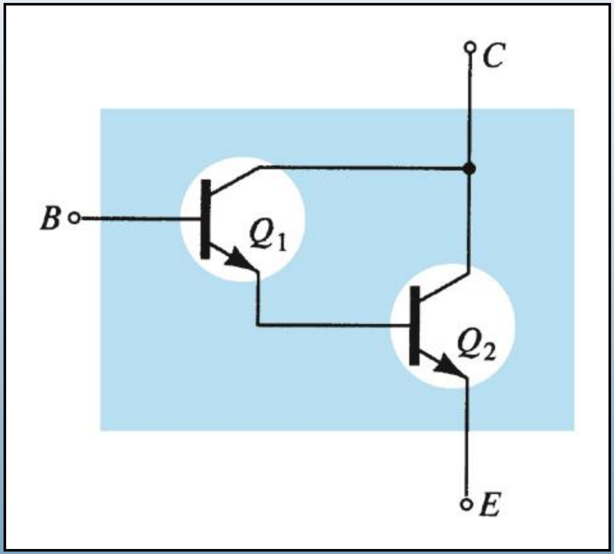
The low voltage gain of the input stage reduces the Miller input capacitance, making this combination suitable for high-frequency applications.



# Darlington Connection

The Darlington circuit provides very high current gain, equal to the product of the individual current gains:

$$\beta_D = \beta_1 \beta_2$$




The practical significance is that the circuit provides a very high input impedance.

# DC Bias of Darlington Circuits

**Base current:**

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E}$$

**Emitter current:**

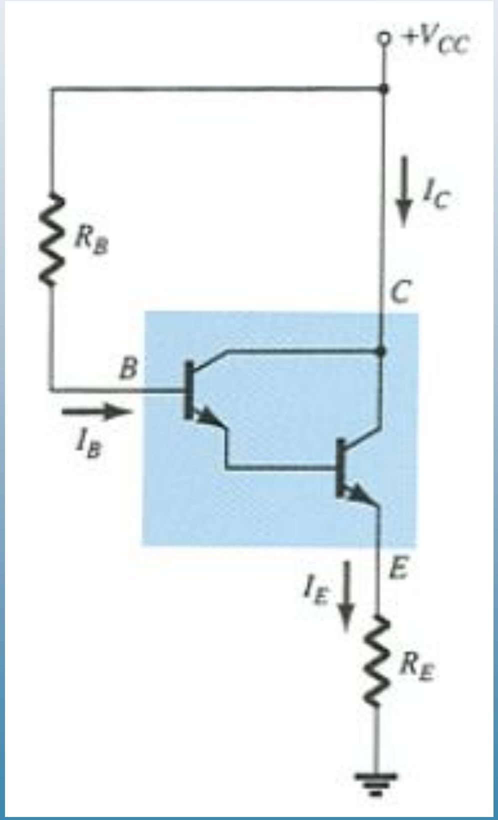
$$I_E = (\beta_D + 1) I_B \cong \beta_D I_B$$

**Emitter voltage:**

$$V_E = I_E R_E$$

**Base voltage:**

$$V_B = V_E + V_{BE}$$

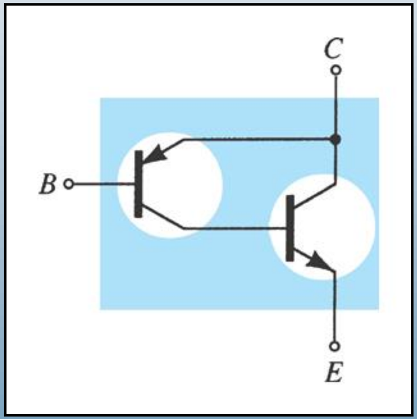


# Feedback Pair

This is a two-transistor circuit that operates like a Darlington pair, *but it is not a Darlington pair*.

It has similar characteristics:

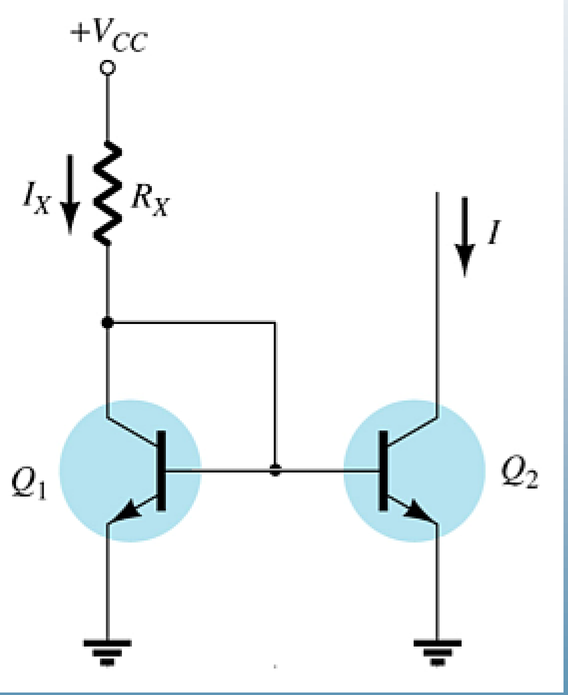
- High current gain
- Voltage gain near unity
- Low output impedance
- High input impedance




The difference is that a Darlington uses a pair of like transistors, whereas the feedback-pair configuration uses complementary transistors.

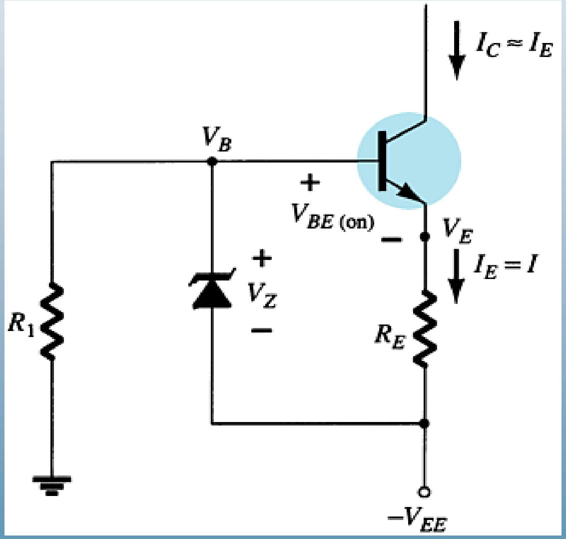
# Current Mirror Circuits

Current mirror circuits provide constant current in integrated circuits.

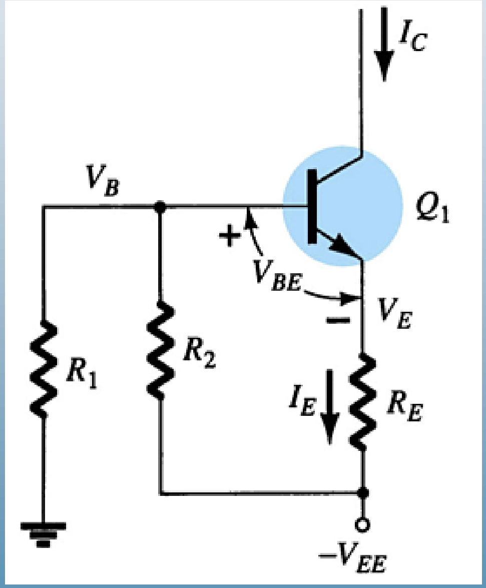


# Current Source Circuits

*Constant-current sources can be built using FETs, BJTs, and combinations of these devices.*



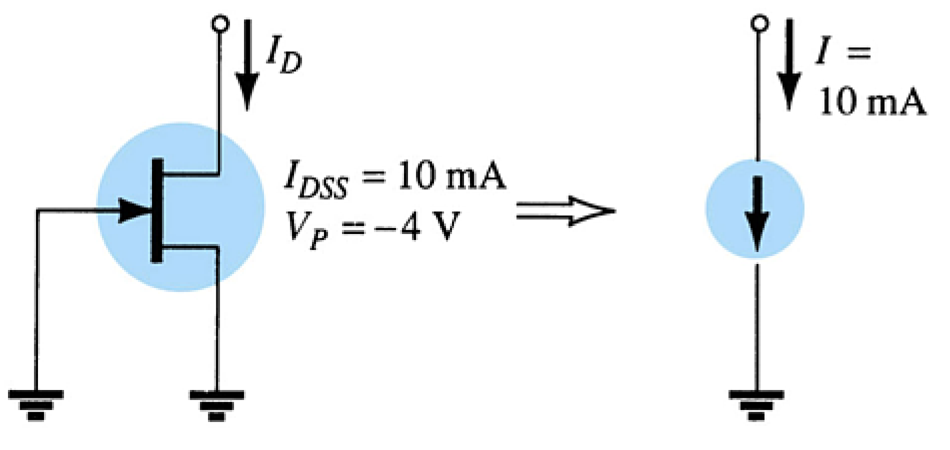

$$I \cong I_E = \frac{V_Z - V_{BE}}{R_E}$$



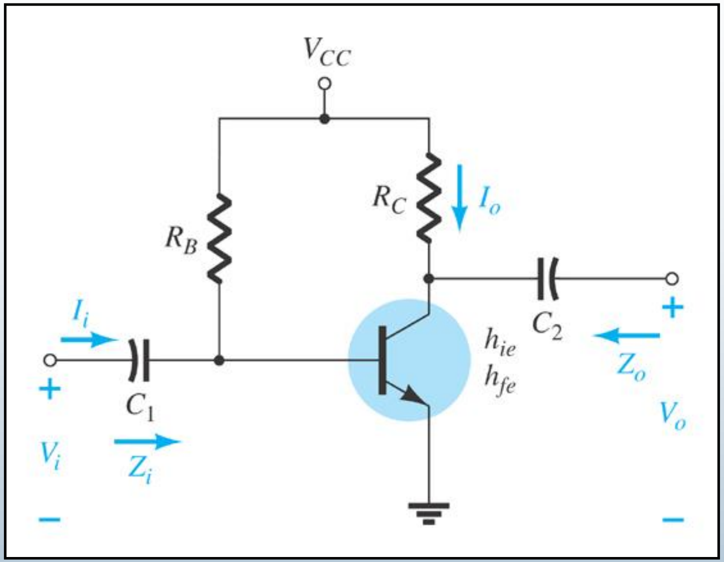

$$I_E \cong I_C$$

# Current Source Circuits

$V_{GS} = 0V$
$I_D = I_{DSS} = 10$ mA

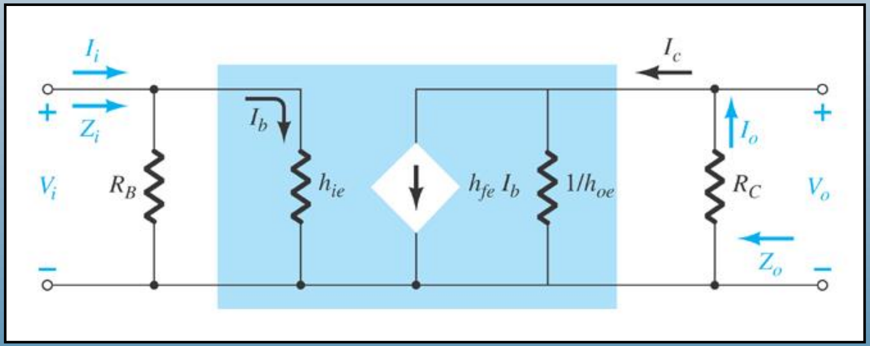


# Fixed-Bias




## Input impedance:

$$Z_i = R_B \,||\, h_{ie}$$




## Output impedance:

$$Z_o = R_C \,||\, 1/h_{oe}$$

## Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fe}(R_C \,||\, 1/h_oe)}{h_{ie}}$$

## Current gain:

$$A_i = \frac{I_o}{I_i} \cong h_{fe}$$

# Voltage-Divider Configuration

**Input impedance:**

$$Z_i = R' \,||\, h_{ie}$$

**Output impedance:**

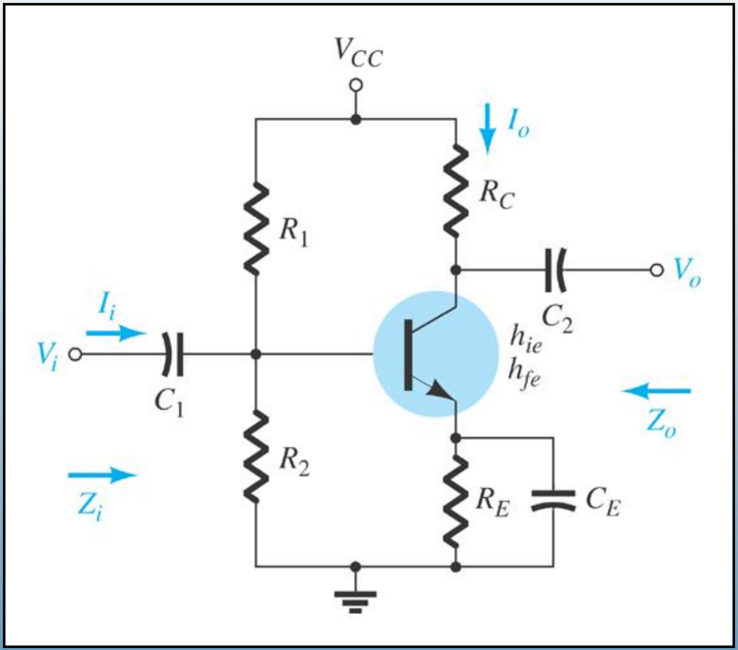
$$Z_o \cong R_C$$

**Voltage gain:**

$$A_v = -\frac{h_{fe}(R_C \,||\, 1/h_{oe})}{h_{ie}}$$

**Current gain:**

$$A_i = -\frac{h_{fe}R'}{R' + h_{ie}}$$



# Emitter-Follower Configuration

## Input impedance:

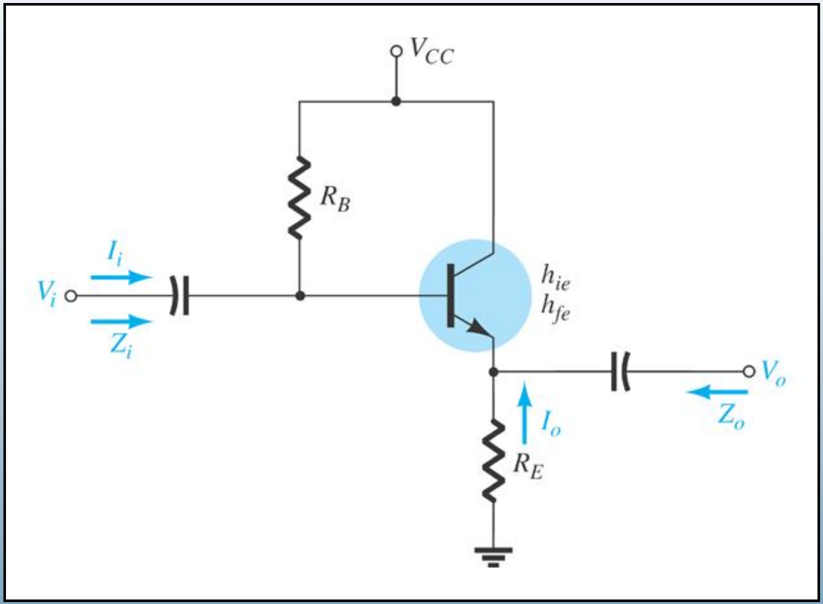
$$Z_b = h_{fe} R_E$$
$$Z_i = R_o \parallel Z_b$$

## Output impedance:

$$Z_o \cong R_E \parallel \frac{h_{ie}}{h_{fe}}$$

## Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + h_{ie} / h_{fe}}$$




## Current gain:

$$A_i = \frac{h_{fe} R_B}{R_B + Z_b}$$
$$A_i = -A_v \frac{Z_i}{R_E}$$

# Common-Base Configuration

**Input impedance:**

$$Z_i = R_E \,||\, h_{ib}$$

**Output impedance:**

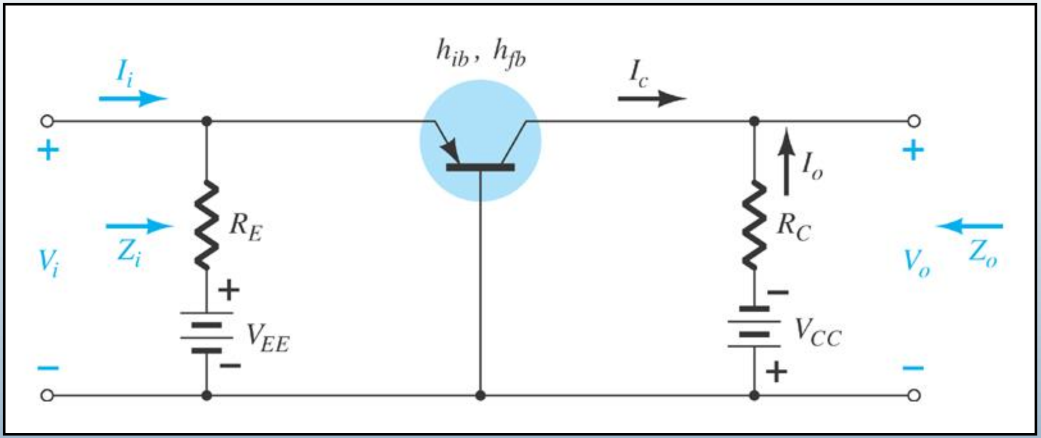
$$Z_o = R_C$$

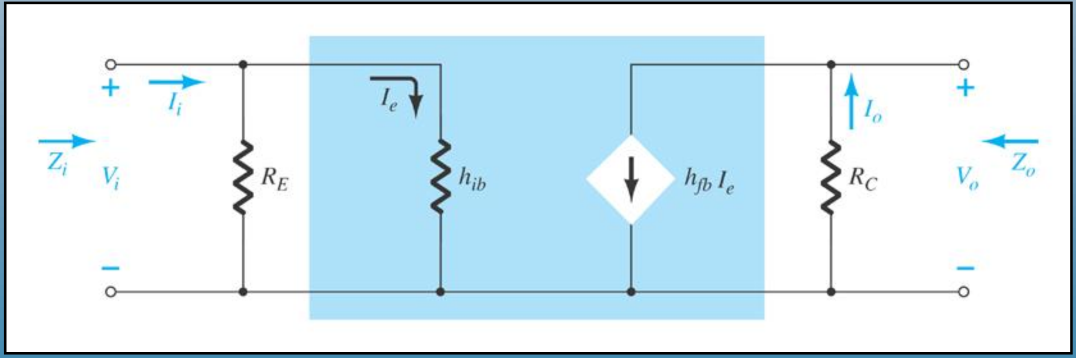
**Voltage gain:**

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fb}R_C}{h_{ib}}$$

**Current gain:**

$$A_i = \frac{I_o}{I_i} = h_{fb} \cong -1$$



# Troubleshooting

Check the DC bias voltages

- ✓ If not correct, check power supply, resistors, transistor. Also check the coupling capacitor between amplifier stages.

Check the AC voltages

- ✓ If not correct check transistor, capacitors and the loading effect of the next stage.