12.1.3 Control Unit

The control unit includes the hardware and control software algorithm necessary to monitor and control the required electric motor drive system. The main aim of the control unit is to create at any instant the required gating signals of the power converter semiconductor switches through a software algorithm. The software algorithm decides the type of gating signals to be applied to the semiconductor switches created upon its input data about the status of the motor drive system. The control unit could be implemented using a microprocessor (μP) or a digital signal processor (DSP) unit. As can be

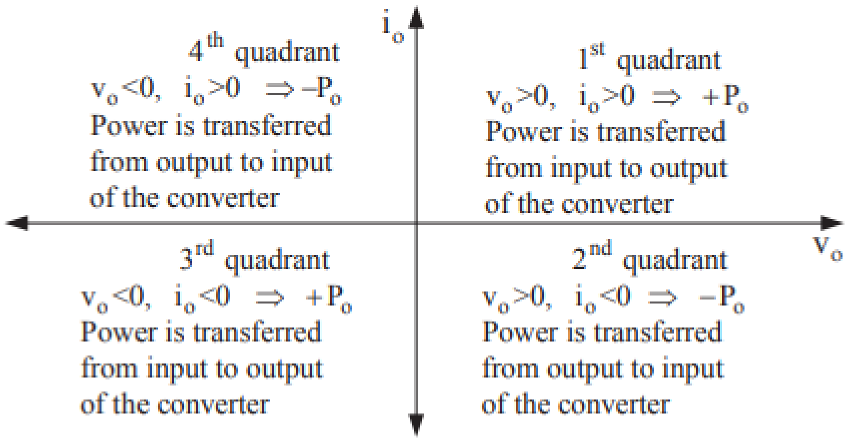


Figure 12.2 Possible four quadrants of operation of a power electronics converter.

seen from Fig. 12.1(a) the microprocessor receives all necessary information of the motor drive system through the measurements and observer unit. This information consists of voltages, currents, rotational speed, magnetic flux of the motor, or any other information which is needed by control technique power electronics, amplitude and frequency of the input voltage of the electric motor, so that the electric motor drive system has the desired response. Each of these units is required to operate with maximum performance and best possible cooperation with other units so that the overall system to function optimally.

12.2 DC Motor Drive Systems

The dc motor historically precedes the ac motor. Because of its ability to provide easy speed and torque control, was for decades the only choice for electric drive systems requiring variable speed operation. The ease of control is that generally in a dc motor rotation speed is proportional to the applied voltage to the armature and the developing torque is proportional to the armature current (this is clearly only for separately excited dc motors). Therefore, as is already evident, it is quite simple to control such a motor (e.g., through a dc−dc converter). The dc motors have been used for many years in electric drive systems and was considered indispensable for variable speed. Only the last 20 years been able to replace them by the asynchronous ac motors (induction motors) driven by advanced control techniques.

The operation of a conventional electric dc motor is based on the interaction of the stator and rotor fields. The stator field is generated by permanent magnets (PMs) (excitation) that are usually firmly fixed to the stator of the motor. The rotor field is generated in the rotor of the motor by rotating winding armature, and this constitutes an electromagnet. Key role in the operation of the motor plays the dc commutator. The role of the commutator is to keep the torque of a dc motor from reversing every time the coil moves through the plane perpendicular to the magnetic field. The commutator is a split-ring device that operates like a mechanical rectifier. This is necessary because the armature winding is rotating and without the commutator the motor will stop immediately the first time two opposite poles will lie across each other.

There is also the possibility that the dc motor does not have PMs in the stator, but electromagnet coil (as excitation), which is the most common practice. The motor is called a dc motor field winding. Therefore, by varying the current flowing through the electromagnet (called the field or alternatively winding excitation coil) may change the typical speed−torque motor. Depending on how the field winding of a dc motor is excited (powered) there are four types of dc motors as follows:

a) Separately excited dc motors (Fig. 12.5(a))
b) Shunt excited dc motors (Fig. 12.5(b))
c) Series excited dc motors (Fig. 12.5(c))
d) Compound excited dc motors (Fig. 12.5(d))

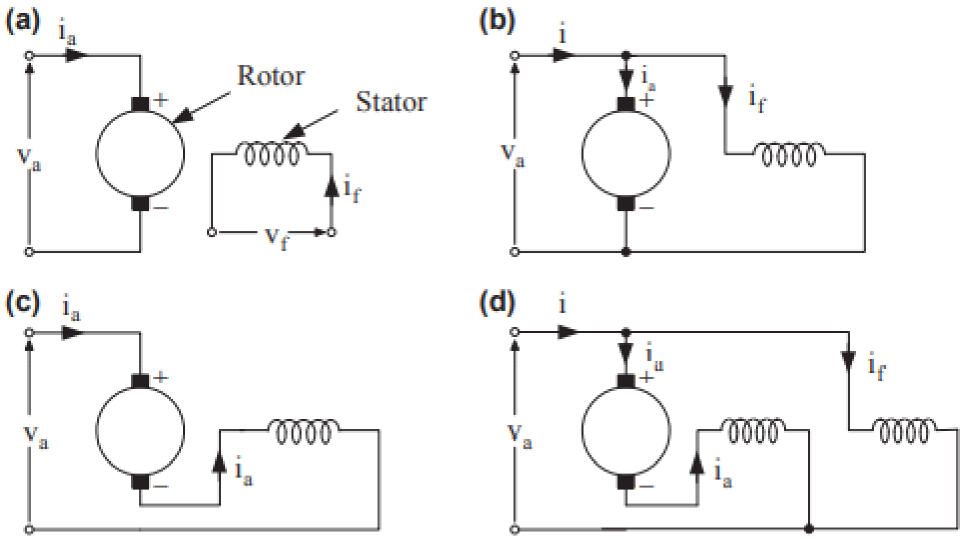


Figure 12.5 Types of dc motors. (a) Separately excited dc motors; (b) shunt excited dc motors; (c) series excited dc; (d) compound excited dc motors.

Fig. 12.6 shows the cross section of a dc motor with windings or PMs on the rotor. Also, Fig. 12.6(c) shows the interaction between the stator and rotor field that creates the rotor forces and, consequently, the rotation of the electric motor.

Fig. 12.7 shows the equivalent circuit of a separate excited dc motor.

The equations that describe a separate excited dc motor can be found from Fig. 12.7 and are the following:

$$v_f = \text{rotor field voltage} = R_f i_f + L_f \frac{di_f}{dt} \tag{12.1}$$

$$v_a = \text{armature voltage} = R_a i_a + L_a \frac{di_a}{dt} + e_g \tag{12.2}$$

$$e_g = \text{electromotive force or back emf} = K_v \omega_m i_f \tag{12.3}$$

$$T_e = \text{developed electromagnetic torque} = K_t i_f i_a \tag{12.4}$$

$$T_e = J\frac{d\omega_m}{dt} + B\omega_m + T_L \tag{12.5}$$

where

i_a = armature current, A
i_f = rotor current, A
R_a = armature internal resistance, Ω
R_f = rotor internal resistance, Ω

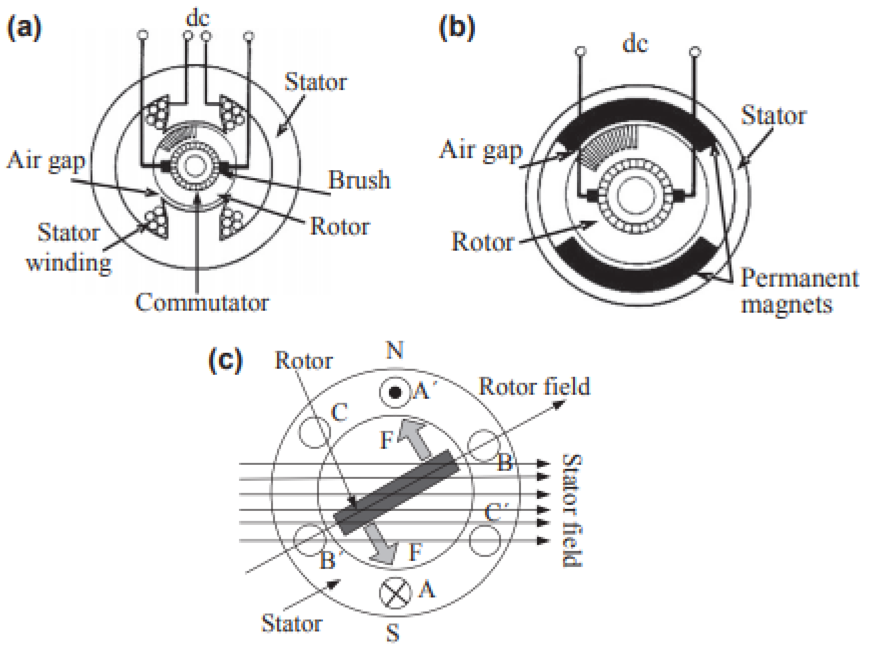


Figure 12.6 Cross section of a dc motor. (a) With windings on the rotor; (b) with permanent magnets on the rotor; (c) interaction between the stator field and the rotor field that creates the rotation of the electric motor.

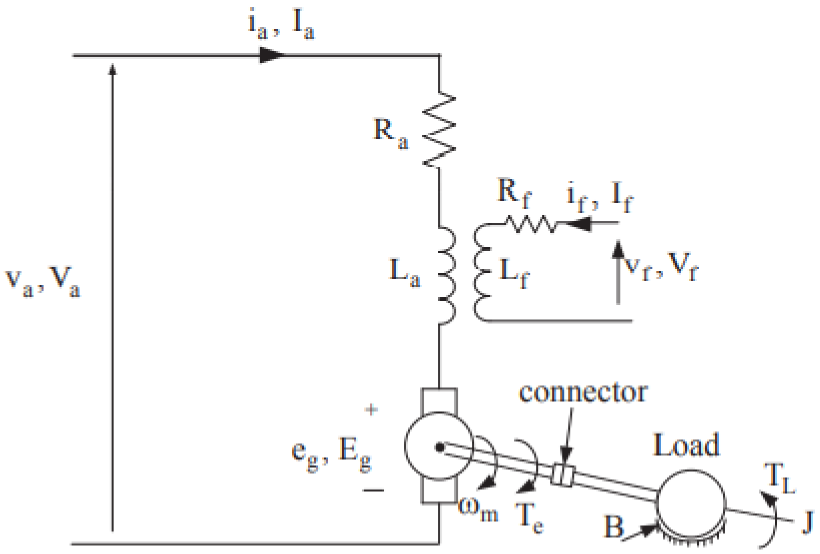


Figure 12.7 Equivalent circuit of a separate excited dc motor (capital character variables are used for the steady-state analysis).

T_L = load torque, Nm
K_v = back-EMF constant, volt·s/rad
K_f = field constant, volt·s/rad
K_t = torque constant, Nm/W
B = frictions constant, Nm/rad/s
J = rotor moment of inertia, kg m^2.

Assuming that the armature voltage v_a has a constant value V_a, the armature current i_a has a constant value I_a and the respective field values are also constant, then using Eq. 12.7 and the previous equations, the following equations are found for the steady-state operation of a dc motor:

$$T_e = \text{developed electromagnetic torque of the motor}$$
$$= K_a \Phi_f I_a = \frac{E_g I_a}{\omega_m} \quad \text{Nm} \tag{12.6}$$

where

$$Ka = \text{design constant of the armature}$$

$$\omega_m = \text{motor mechanical speed} = \frac{V_a - I_a R_a}{K_v \Phi_f} \text{ rad/s} \tag{12.7}$$

$$E_g I_a = T_e \omega_m \quad (\text{electrical energy conversion to mechanical}) \tag{12.8}$$

$$\Phi_f = \text{magnetic flux of the rotor} = K_f I_f \quad \text{Wb} \tag{12.9}$$

$$V_a = \text{armature voltage} = E_g + R_a I_a \quad \text{V} \tag{12.10}$$

$$E_g = \text{back emf} = K_v \Phi_f \omega_m \quad \text{V} \tag{12.11}$$

$$P_i = \text{Input active power of the motor} = I_a V_a + I_f V_f \quad \text{W} \tag{12.12}$$

$$P_{s\,cu} = \text{rotor copper losses} = I_f^2 R_f \quad \text{W} \tag{12.13}$$

$$P_{r\,cu} = \text{armature copper losses} = I_a^2 R_a \quad \text{W} \tag{12.14}$$

$$P_{rotational} = \text{mechanical losses}(\text{friction} + \text{ventilation} + \ldots) \quad \text{W} \tag{12.15}$$

$$P_{sl} = \text{losses due to ununiform current of the windings}$$
$$\approx 1\% \text{ of the output power W} \tag{12.16}$$

$$P_{magn} = \text{core losses} = 1\% \text{ to } 5\% \text{ of output power W} \tag{12.17}$$

$$P_e = \text{rotor electromagnetic power} \\ = I_a E_g = T_e \omega_m = P_i - P_{scu} - P_{rcu} - P_{sl} \ \ W \tag{12.18}$$

$$P_m = \text{mechanical power developed on the motor shaft} \\ = P_e - P_{rotational} = I_a E_g - P_{rot} = T_m \omega_m \tag{12.19}$$

$$T_m = \text{mechanical torque on the motor shaft} \\ = T_e - \frac{P_{rotational}}{\omega_r} \ \ Nm \tag{12.20}$$

$$\eta\% = \text{motor efficiency} = \frac{P_m}{P_i} 100 \tag{12.21}$$

Eqs. (12.6) and (12.7) indicate that the developed speed and torque of a dc motor can be controlled by the average armature voltage V_a. This is done by utilizing a power electronic converter between the power source and the motor armature. Fig. 12.8 shows the power flow and losses in a dc motor.

Fig. 12.9 shows the torque and power characteristics of a separate excited dc motor. As can be seen from this figure, the speed of the motor is controlled up to its rated value through the armature voltage and for values above the rated speed is controlled through its field current.

A dc machine is able to operate as a motor or as a generator. According to Eq. (12.18) when the torque has the same sign as that of the rotational speed, then the motor produces positive output mechanical power, which means that the motor absorbs electric power from the source through the converter and operate as a motor. However, when the torque and rotational speed have opposite signs, then the motor produces negative output mechanical power and operates as a generator transferring through the converter electric power to the input source. When the motor operates as a generator, load is that which gives the kinetic energy of the motor axis. This indicates that a dc machine can operate in any of the following four quadrant modes:

First quadrant mode ($P_e = (\omega_m)(T)_e$ = positive, motoring with clockwise rotation): In this operating mode the dc machine operates as a motor developing positive torque and

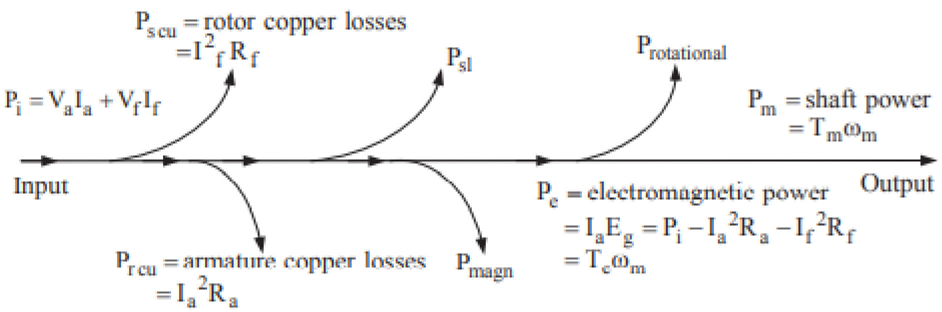


Figure 12.8 Power flow within a separate excited dc motor.

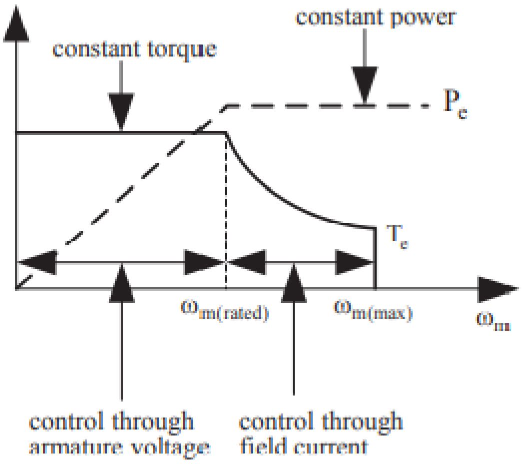


Figure 12.9 Torque and power characteristics of a separate excited dc motor.

rotates with positive speed (i.e., clockwise rotation). The quadrant of this operating mode is the first and is presented in Fig. 12.10. An example of this mode is when an electric vehicle runs uphill and electrical energy is taken from the batteries through a converter, which is applied to an electric motor, and the motor converting the electrical energy to mechanical moves the electric vehicle.

Second quadrant mode ($P_e = (+\omega_m)(-T_e) = -$ve, generating with clockwise rotation): In this operating mode, negative torque with positive speed is applied to the shaft of dc machine converting the motoring mode to generating mode. This means that the

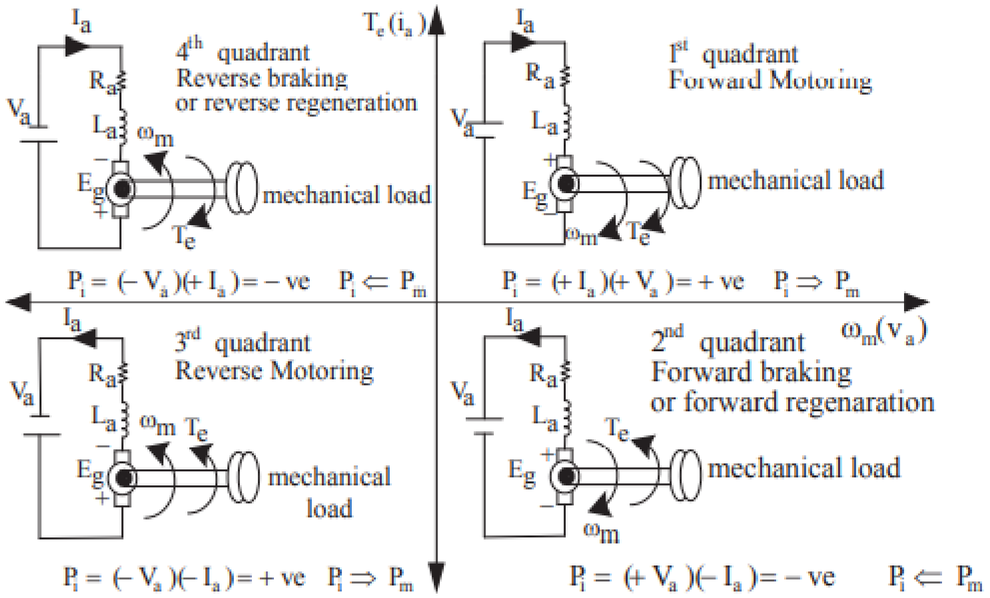


Figure 12.10 The four possible quadrant modes of operation of a separately excited dc machine.

mechanical load is providing kinetic energy to the shaft of the electric machine converting the motor to a generator that produces electrical energy which is fed to the input source through the armature and the power electronics converter. The quadrant of this operating mode is the second and is presented in Fig. 12.10. An example of this mode is when an electric vehicle travels downhill and the electric machine operates as a generator producing electrical power. During this mode the braking of the electric machine is achieved and since energy is saved during this mode this type of braking is called regenerative braking.

Third quadrant mode ($P_e = (-\omega_m)(-T_e)$ = positive, motoring with counterclockwise rotation): In this operating mode the dc machine operates as a motor developing negative torque and rotating with negative speed (i.e., counterclockwise rotation). The quadrant of this operating mode is the third and is presented in Fig. 12.10. An example of this mode is an electric vehicle when is running in backwards.

Fourth quadrant mode ($P_e = (-\omega_m)(+T_e) = -$ve, generating with counterclockwise rotation): In this operating mode the dc machine produces a positive torque and rotates with negative speed, which means that the mechanical load torque fed to the shaft of the electric motor causes the machine to become a generator and feeds electricity to the input source through the armature. The operating area of this mode is the fourth quadrant of Fig. 12.10. An example of this mode is when an electric vehicle travels downhill and the electric machine through the transmission systems gets kinetic energy, then the electric machine operates as a generator producing electrical energy that transfers to the input source through the power electronics converter. During this mode the braking of the machine is achieved, and since energy is saved during this mode this type of braking is called regenerative braking.

Depending on the motor used and the type of the input power supply the dc motor drive systems can employ one of the following power electronics topology:

Table 12.2 **DC motor drive systems employing regenerative pulse width modulation (PWM) (or so-called switching) rectifier topologies**

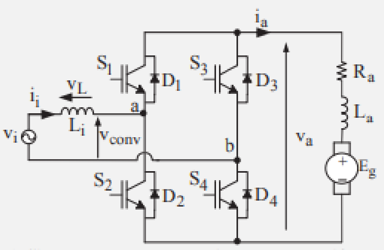
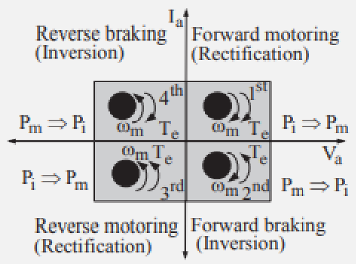
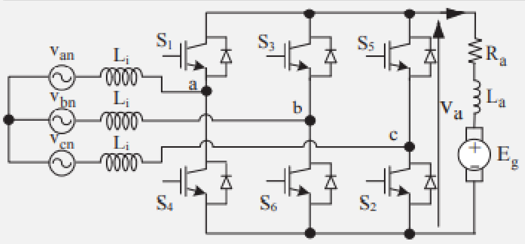
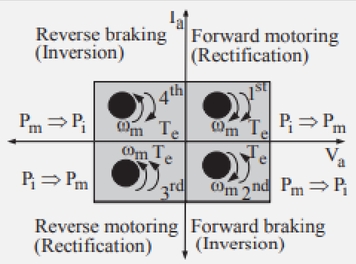
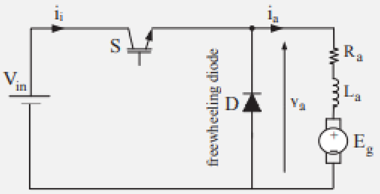
Switching rectifier topology	Quadrant(s) operation
 **a) Single-phase regenerative PWM rectifier**	
 **b) Three-phase regenerative PWM rectifier**	

Table 12.3 DC motor drive systems employing dc−dc converters—cont'd

<table>
<tr><th>dc−dc converter topology</th><th>Quadrant(s) of operation</th></tr>
<tr><td>


a) Step-down chopper or first quadrant chopper

When S is on and D is off: forward motoring.
When S is off and D conducts: motor current decreases through freewheeling diode and the motor still in forward rotation.

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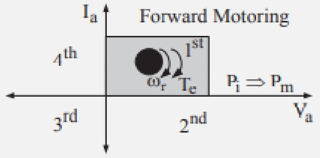

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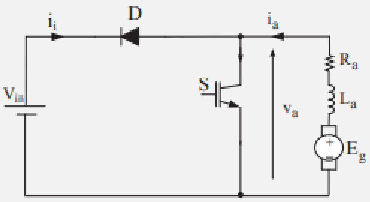
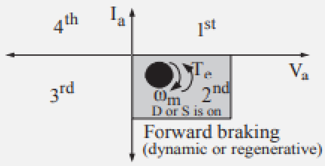
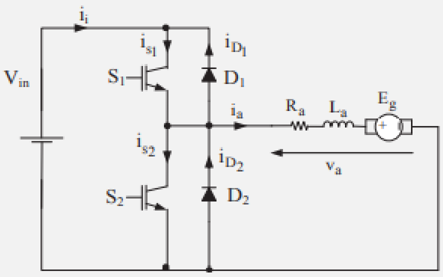


b) Step-up chopper or second quadrant chopper

When S is on and D is off: electrical energy is stored in L_a. During this mode forward dynamic braking is achieved.
When S is off and D contacts: electrical power is transferred to the dc source and the motor operates in the forward braking regeneration mode.

</td><td>


</td></tr>
<tr><th>dc−dc converter topology</th><th>Quadrant(s) of operation</th></tr>
<tr><td>


c) Two-quadrant chopper

When S_1 is on: forward motoring
When D_1 is on: forward regenerative braking
When S_2 is on: forward dynamic braking
When D_2 is on: motor current decreases through freewheeling diode and the motor still in forward rotation.

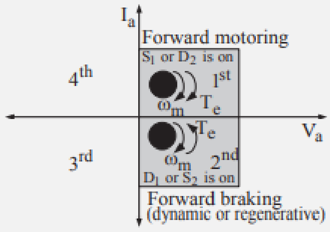
</td><td>


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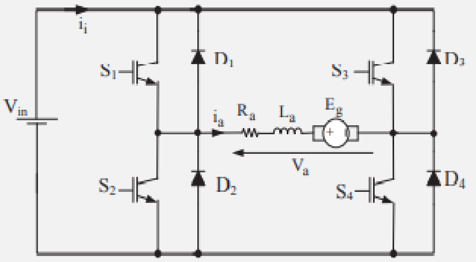
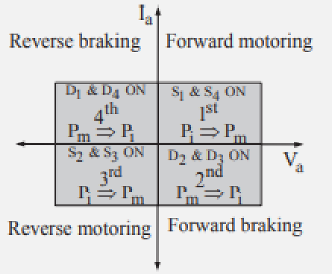
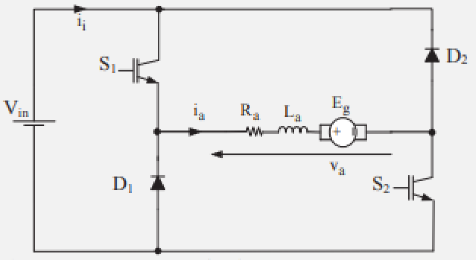
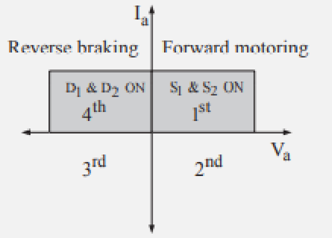
Table 12.3 **DC motor drive systems employing dc–dc converters**

dc–dc converter topology	Quadrant(s) of operation
 **d) Four-quadrant full-bridge chopper**	
 **e) Two-quadrant or half-bridge chopper**	

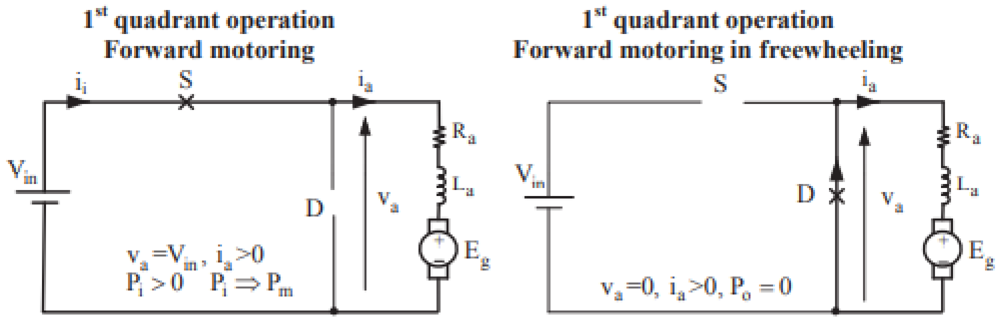


Figure 12.12 Operating modes of the step-down or first quadrant chopper presented in Table 12.3.

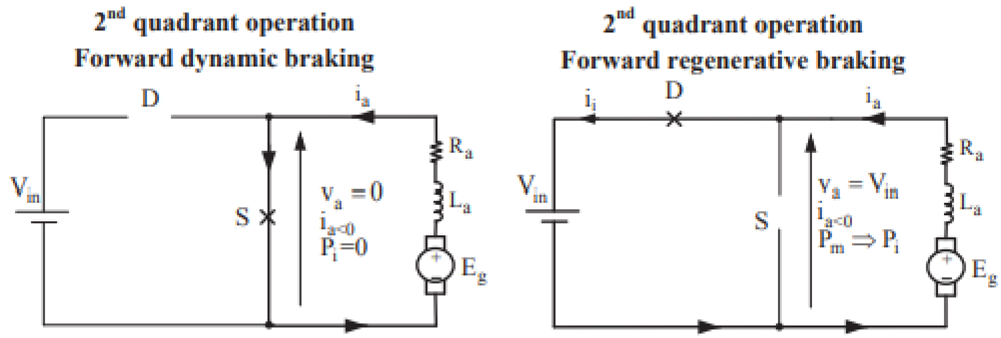


Figure 12.13 Operating modes of the step-up or second quadrant chopper presented in Table 12.3.

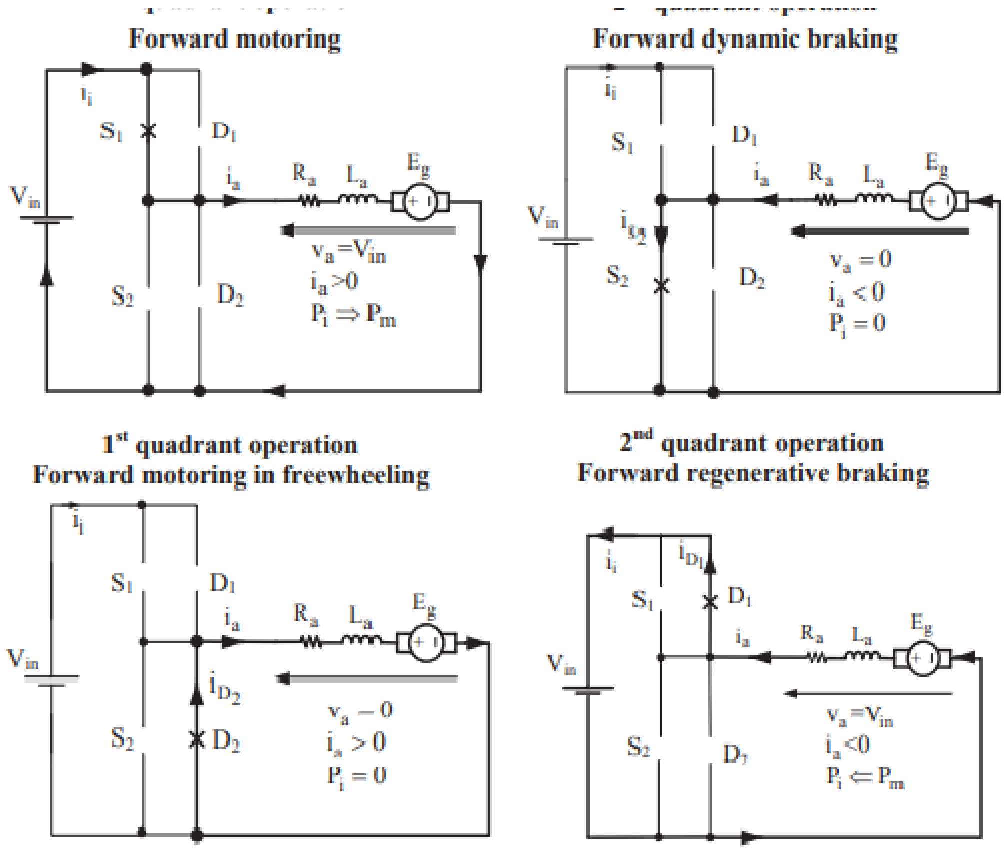


Figure 12.14 Operating modes of the two-quadrant converter presented in Table 12.3.

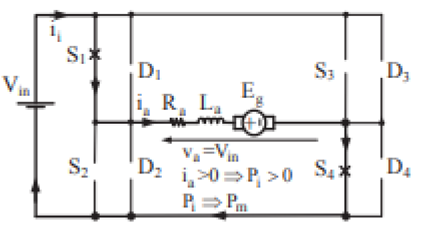
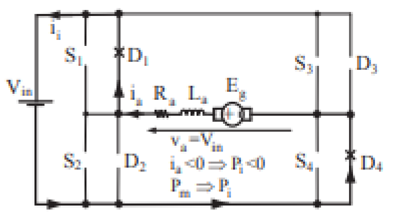
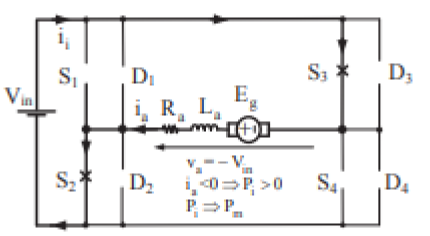
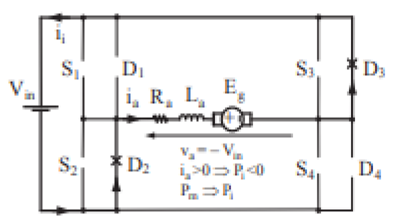
1st quadrant operation	2nd quadrant operation
 During this mode of operation only the switches S_1 and S_4 are conducting applying positive voltage and current to the armature. The machine is this mode operates as a motor with clockwise rotation. Keeping S_1 in conduction and applying PWM to S_4 speed control is achieved. Freewheeling current is achieved through S_1 and D_3.	 During this mode of operation only the diodes D_1 and D_4 are conducting applying positive voltage to the armature but creating a negative flow current. The machine in this mode operates as a generator with clockwise speed and, consequently, a regenerative braking mode is achieved.
3rd quadrant operation	**4th quadrant operation**
 During this mode of operation only the switches S_2 and S_3 are conducting applying negative voltage to the armature and creating a negative armature current. The machine is this mode operates as a motor with counter clockwise rotation. Keeping S_2 in conduction and applying PWM to S_3 speed control is achieved. Freewheeling current is achieved through S_2 and D_4.	 During this mode of operation only the diodes D_2 and D_3 are conducting applying negative voltage to the armature but creating a positive flow of armature current. The machine in this mode operates as a generator with counter clockwise speed and, consequently, a regenerative braking mode is achieved.

Figure 12.15 Operating modes of the full-bridge four-quadrant converter presented in Table 12.3.

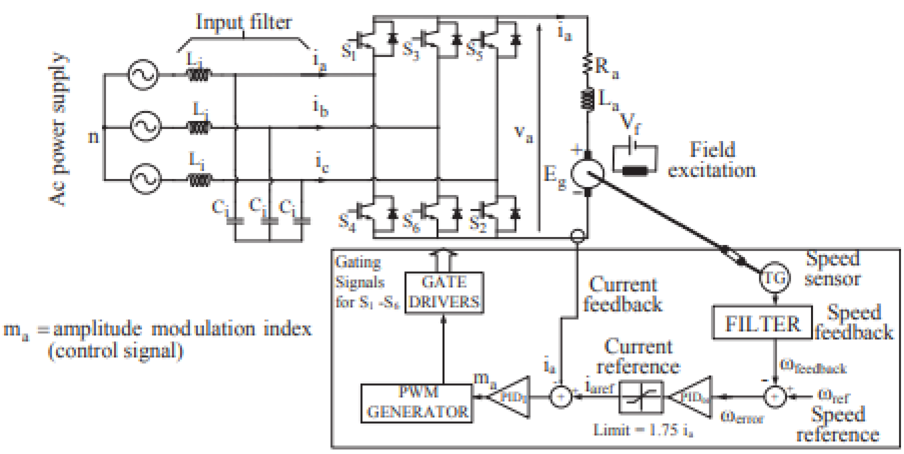


Figure 12.16 DC motor drive system implemented with a three-phase pulse width modulation rectifier.

Example 12.1

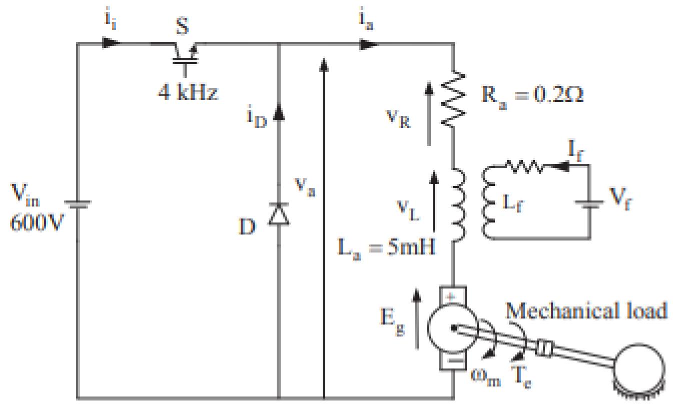
The dc separately excited motor drive system shown below has the following specifications:

Motor data		**Chopper data**	
Base speed:	1000 rpm	dc supply voltage:	600 V
Rated voltage:	500 V	Switching frequency:	4 kHz
Rated current:	200 A		
Armature resistance:	0.2 Ω		
Armature inductance:	5 mH		
Constant field voltage			

If the motor is operating at 50% of its base speed and 20% of its rated torque, then find the following:

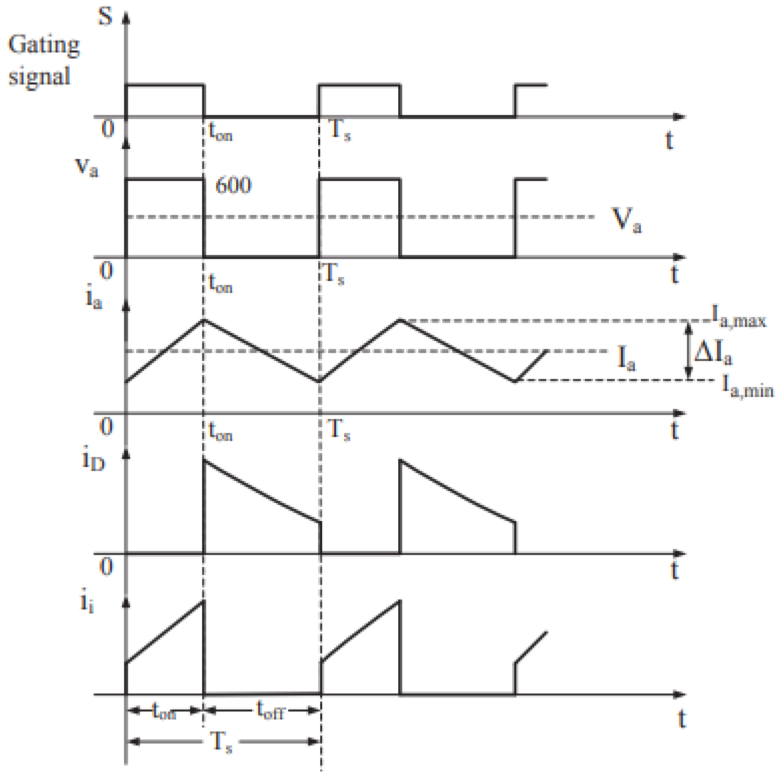
a) Calculate the duty cycle of the chopper for this operation condition.
b) Estimate the current ripple for the duty cycle calculated in part a.

Example 12.1—cont'd



Solution

a) The waveforms of the dc motor drive system are shown below.



Key waveforms of example 12.1

Using the output voltage waveform the average output voltage is:

$$
\begin{aligned}
V_a &= \text{armature average voltage} = \frac{1}{T_s}\int_0^{T_s} v_a(t)dt \\
&= \frac{1}{T_s}\left[\int_0^{t_{on}} V_{in}dt + \int_0^{T_s} 0\cdot dt\right] = V_{in}\frac{t_{on}}{T_s} = V_{in}t_{on}f_s = V_{in}D
\end{aligned} \tag{1}
$$

where
$D = \text{duty cycle} = \frac{t_{on}}{T_s}$.
From the above circuit, the following equation exists:

$$
v_a = \text{armature voltage} = R_a i_a + L_a\frac{di_a}{dt} + e_g \tag{2}
$$

Moreover, at steady state the armature voltage and current and back electromagnetic force (EMF) have constant values and, consequently, then Eq. (2) becomes:

$$
V_a = R_a I_a + E_g \quad \text{since} \quad \frac{dI_a}{dt} = 0 \tag{3}
$$

The speed of the dc motor is given by:

$$
\omega_m = \text{motor mechanical speed} = \frac{E_g}{K_v I_f} = \frac{V_a - R_a I_a}{K_v I_f} \tag{4}
$$

Eq. (4) indicates that the speed of the motor can be varied by varying the armature voltage V_a. Therefore, if the rated speed of the motor is to be reduced by 50%, then the armature voltage must be reduced to half (i.e., to 300 V) and, consequently, the duty cycle will be:

$$
D = \frac{V_a}{V_{in}} = \frac{300}{600} = 0.5 \tag{5}
$$

The developed torque is $T_d = K_t I_f I_a$, and the field current is assumed to be always constant. Therefore, when the rated torque is reduced to 20% of its rated value the armature current will be reduced to 20% of its rated value which is (200)(0.2) = 40 A.

The mechanical speed of the motor is $\omega_m = \frac{E_g}{K_v I_f}$, and the field current according to the specifications of the motor is always constant. Therefore, according to this equation when ω_m drops to half the back-EMF will drop to half of its rated value.

$$
E_{g(rated)} = V_{a(rated)} - R_a I_{a(rated)} = 600 - (0.2)(200) = 560V
$$

$$
E_{g(speed50\%)} = \frac{E_{g(rated)}}{2} = 280V
$$

When the switch is on then the voltage across the inductor is given by:

$$v_L = L\frac{di_a}{dt} = V_{in} - R_aI_a - E_g \quad (6)$$

Therefore, using the above equation the range of change of the motor armature current is

$$\frac{di_a}{dt} = \frac{V_{in} - R_aI_a - E_g}{L} \quad \text{or} \quad \frac{\Delta I_a}{t_{on}} = \frac{V_{in} - R_aI_a - E_g}{L} \quad (7)$$

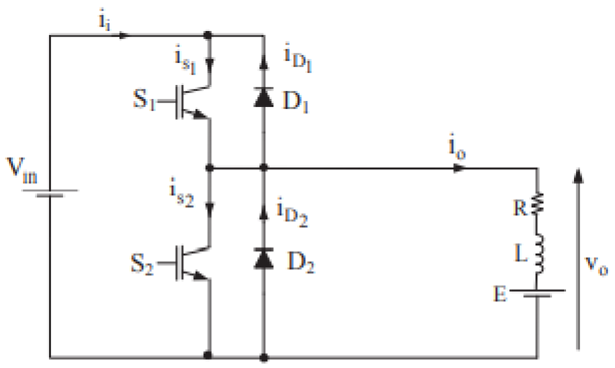
Therefore, the armature peak to peak current ripple is given by:

$$\Delta I_{a(on)} = \frac{V_{in} - R_aI_a - E_g}{L}t_{on} = \frac{V_{in} - R_aI_a - E_g}{L}DT_s$$

$$= \frac{V_{in} - R_aI_a - E_g}{Lf_s}D = \frac{600 - (40)(0.2) - 280}{(5 \times 10^{-3})(4000)}(0.5) = 7.8\text{ A}$$

Example 12.2

Perform the analysis of the following two-quadrant chopper under continuous output current operation.



Solution

As it was shown from the equivalent circuits of Fig. 12.14, the main characteristic of this converter is that although the output voltage is always positive, the output current can take positive and negative values, contrary to the first quadrant converters, that allow only positive current values at their output. For this reason, the two-quadrant chopper has the advantage to transfer power from the load to the input source and vice versa. As can be seen from the above converter, the semiconductor devices S_1 and D_2 comprise a buck converter that operates during forward motoring mode (i.e.,

in the first quadrant) and the semiconductor devices S_2 and D_1 comprise a boost co verter that operates during forward regenerative mode (i.e., in the second quadran The two figures below present the key waveforms of the converter when operati in the first and second quadrants, respectively.

For the given converter, the following equations hold:

$$i_o = \frac{V_{in} - E}{R}\left(1 - e^{-t/\tau}\right) + I_{min} e^{-t/T_s} \quad \text{for } 0 \le t \le t_{on}$$

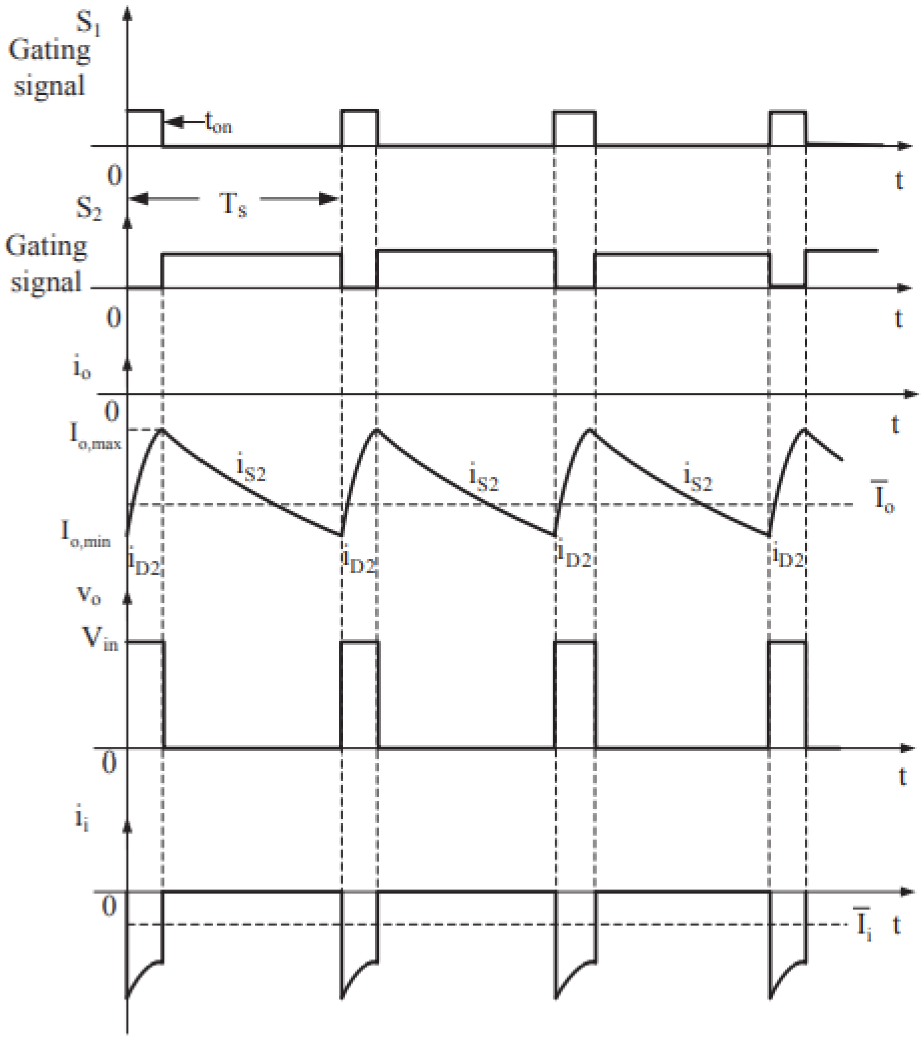
and

$$i_o = -\frac{E}{R}\left(1 - e^{-t'/\tau}\right) + I_{max} e^{-t'/\tau} \quad \text{for } t_{on} \le t \le T_s$$

where

$$I_{o,max} = \frac{V_{in}}{R}\frac{\left(1 - e^{-t_{on}/\tau}\right)}{\left(1 - e^{-T_s/\tau}\right)} - \frac{E}{R} \quad I_{o,min} = \frac{V_{in}}{R}\frac{\left(e^{t_{on}/\tau} - 1\right)}{\left(e^{T_s/\tau} - 1\right)} - \frac{E}{R}$$

$$\tau = \frac{L}{R}s \quad t' = t - t_{on}$$



(b): Second quadrant operation waveforms with continuous output current

If $I_{o,min} > 0$, the converter operates only in the first quadrant and, consequently, the semiconductor devices S_2 and D_2 are not conducting. If $I_{o,max} < 0$, the converter operates only in the second quadrant, and, consequently, the semiconductor devices S_1 and D_1 are not conducting. If $I_{o,max} > 0$ and $I_{o,min} < 0$ the converter operates partly in the first and partly in the second quadrant and, consequently, all semiconductor devices are participating in the operation of the converter.

The average output voltage for continuous output current when the converter operates in the first quadrant (i.e., as buck converter) is given by:

$$\overline{V}_o = V_{in}D$$

The average input voltage for continuous output current when the converter operates in the forward regenerative quadrant (i.e., as boost converter supplying energy from the load to the input source) is given by:

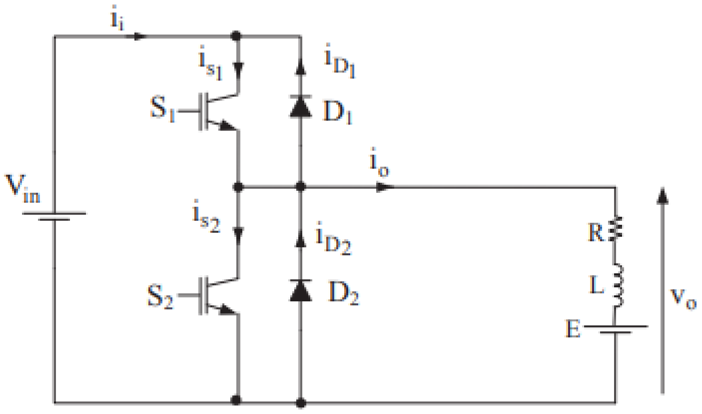
$$V_{in} = \frac{\overline{V}_o}{1 - D}$$

Example 12.3

The converter shown below has the following parameters: $V_{in} = 110$ V, $L = 0.2$ mH, $R = 0.25\ \Omega$, $E = 40$ V, $T_s = 2500\ \mu s$, and $t_{on} = 1250\ \mu s$.

Perform the following

a) Calculate the average output voltage and current.
b) Calculate the values of $I_{o,min}$ and $I_{o,max}$.
c) Draw the waveforms of variables v_o, i_o, i_{S1}, i_{S2}, i_{D1}, i_{D2}, and i_i.



Solution

a) Using the equation $\overline{V}_o = \dfrac{t_{on}}{T_s} V_{in}$

$$\overline{V}_o = \frac{1250}{2500} \times 110 = 55\text{ V} \quad \overline{I}_o = \frac{\overline{V}_o - E}{R} = \frac{55 - 40}{0.25} = 60\text{ A}$$

Since the output current is positive the converter operates in the first quadrant delivering power from the input source to the load.

b) The time constant of the circuit is:

$$\tau = \frac{L}{R} = \frac{0.2 \times 10^{-3}}{0.25} = 800\ \mu s$$

Moreover:

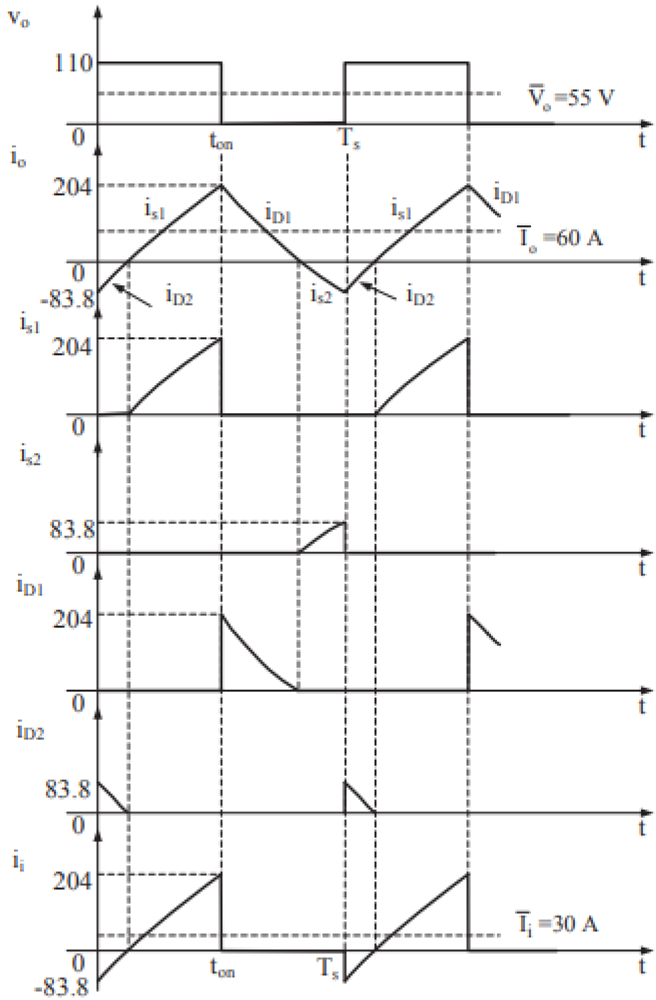
$$\frac{t_{on}}{\tau} = \frac{1250}{800} = 1.562 \quad \frac{T_s}{\tau} = \frac{2500}{800} = 3.125$$

Therefore, the following results are found:

$$I_{o,max} = \frac{110\left(1 - e^{-1.562}\right)}{0.25\left(1 - e^{-3.125}\right)} - \frac{40}{0.25} = 204 \text{ A}$$

$$I_{o,min} = \frac{110\left(e^{1.562} - 1\right)}{0.25\left(e^{3.125} - 1\right)} - \frac{40}{0.25} = -83.8 \text{ A}$$

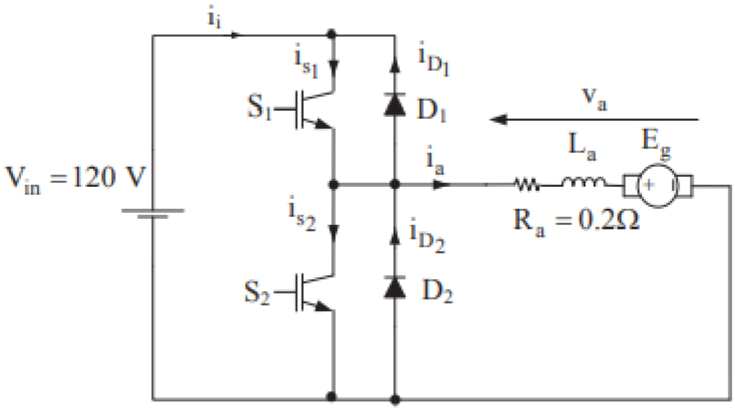
c) The waveforms of the corresponding variables are shown below.



Key waveforms of example 12.3

Example 12.4

A small electric vehicle is powered from a 120 V battery bank and uses a series wound dc motor. The dc motor during motoring and regenerative modes is controlled by the converter which is shown below. The armature resistance is 0.2 Ω and the armature voltage constant is 12 mV s/rad. During a downhill regeneration mode the motor speed is 900 rpm and the armature current is pure dc of 100 A. Calculate the required duty cycle and the available braking power.



Solution

As can be seen from the above converter, the semiconductor devices S_1 and D_2 comprise a step-down converter that operates during forward motoring mode and the semiconductor devices S_2 and D_1 comprise a step-up converter that operates during forward regenerative mode. The above converter is a two-quadrant converter that operates in the first and fourth quadrants. Due to the diode D_2 the motor armature voltage is always positive and only the armature current takes positive and negative values. According to the above figure, the following equations hold:

$$\mathrm{E_g} = \text{electromotive force} = \omega_m \mathrm{K_a I_a} = \left(900 \times \frac{2\pi}{60}\right) \times 12 \times 10^{-3} \times 100$$
$$= 113\ \mathrm{V}$$

$$\mathrm{V_a} = \mathrm{E_g} - \mathrm{I_a R_a} = 113 - 100 \times 0.2 = 63\ \mathrm{V}$$

Therefore, during forward regenerative mode, where the boost converter operates, the following transfer function holds:

$$\frac{\mathrm{V_a}}{\mathrm{V_{battery}}} = \frac{1}{1 - \mathrm{D}}$$

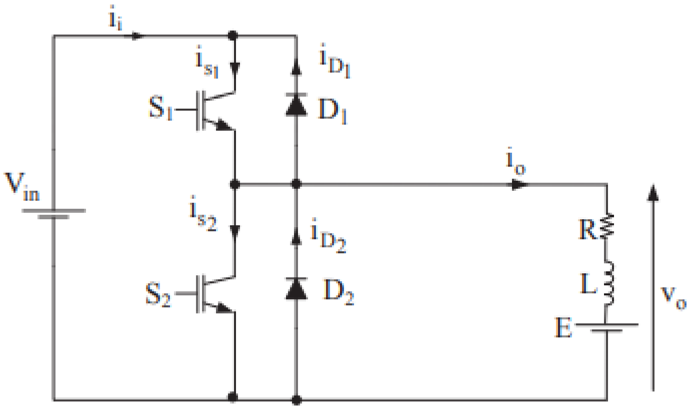
and consequently the duty cycle is given by : $\mathrm{D} = 1 - \frac{\mathrm{V_a}}{\mathrm{V_{battery}}} = 1 - \frac{63}{100} = 0.37.$

Also, the power delivered to the battery is given by $\mathrm{P_{delivered}} = \mathrm{V_a I_a} = 63 \times 100 = 6.3\ \ \mathrm{kW}.$

Example 12.5

The converter shown below has the following parameters: $V_{in} = 110$ V, $R = 0.15\ \Omega$, $E = 50$ V, $T_s = 4000\ \mu s$, and L has very large value so that the output current can be considered to be pure dc. Perform the following:

a) Find the time t_{on} if $i_o = \bar{I}_o = 100$ A. Also, draw the waveforms of variables v_o and i_i.
b) Show that the input power is equal to the output power when the converter is considered to be ideal.
c) Repeat (a) and (b) when $i_o = \bar{I}_o = -100$ A.



Solution

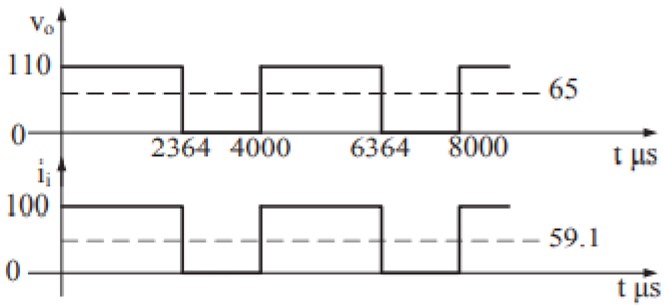
a) The average output voltage is:

$$\overline{V}_o = E + R\bar{I}_o = 50 + 0.15 \times 100 = 65\ \text{V}$$

Using the equation $\overline{V}_o = V_{in}D$ the value of the turn-on time is given by:

$$t_{on} = \frac{\overline{V}_o T_s}{V_{in}} = \frac{65 \times 4000 \times 10^{-6}}{110} = 2364\ \mu s$$

The waveforms of variables v_o and i_i are shown below.



b) The average input current is:

$$\bar{I}_i = \frac{1}{T}\int_o^T i_i dt = \frac{1}{T_s}\int_o^{t_{on}} i_i dt = \frac{1}{4000}\int_o^{2364} 100dt = \frac{2364}{4000} \times 100 = 59.1 \text{ A}$$

$$P_i = \text{input power} = V_{in}\bar{I}_i = 110 \times 59.1 = 6.50 \text{ kW}$$

$$P_o = \text{output power} = \bar{V}_o\bar{I}_o = 65 \times 100 = 6.50 \text{ kW}$$

Consequently, input power = output power.

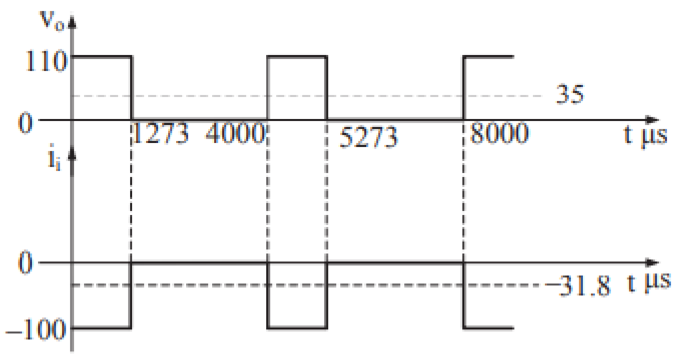
c) $\bar{V}_o = E + R\bar{I}_o = 50 - 0.15 \times 100 = 35 \text{ V}$

$$t_{on} = \frac{35 \times 4000 \times 10^{-6}}{110} = 1273 \text{ µs}$$

c) $\bar{V}_o = E + R\bar{I}_o = 50 - 0.15 \times 100 = 35 \text{ V}$

$$t_{on} = \frac{35 \times 4000 \times 10^{-6}}{110} = 1273 \text{ µs}$$

The waveforms of variables v_o and i_i are shown below.



$$\bar{I}_i = \frac{1}{T}\int_o^T i_i dt = \frac{1}{T_s}\int_o^{t_{on}} i_i dt = \frac{1}{4000}\int_o^{2364} (-100)dt = \frac{1273}{4000}(-100)$$
$$= -31.8 \text{ A}$$

$$P_i = \text{input power} = V_{in}\bar{I}_i = 110 \times (-31.8) = -3.5 \text{ kW}$$

$$P_o = \text{output power} = \bar{V}_o\bar{I}_o = 35 \times (-100) = -3.5 \text{ kW}$$

Example 21-10

The switch in Fig. 21.60a opens and closes at a frequency of 20 Hz and remains closed for 3 ms per cycle. A dc ammeter connected in series with the load E_0 indicates a current of 70 A.

a. If a dc ammeter is connected in series with the source, what current will it indicate?
b. What is the average current per pulse?

Solution

a. Using Eq. 21.20, we have

$$\text{period } T = \frac{1}{20} = 50 \text{ ms}$$

$$\text{duty cycle} = \frac{T_a}{T} = \frac{3}{50} = 0.06$$

$$\begin{aligned} I_S &= I_0 D \\ &= 70 \times 0.06 \\ &= 4.2 \text{ A} \end{aligned}$$

b. The average current during each *pulse* (duration T_a) is 70 A. Considering that the average current is only 4.2 A, the source has to be specially designed to supply such a high 70 A pulse. In most cases a large capacitor is connected across the terminals of the source. It can readily furnish the high current pulses as it discharges.

Example 21-11

We wish to charge a 120 V battery from a 600 V dc source using a dc chopper. The average battery current should be 20 A, with a peak-to-peak ripple of 2 A. If the chopper frequency is 200 Hz, calculate the following:

a. The dc current drawn from the source
b. The dc current in the diode
c. The duty cycle
d. The inductance of the inductor

Solution

The circuit diagram is shown in Fig. 21.61a and the desired battery current is given in Fig. 21.61b. It fluctuates between 19 A and 21 A, thus yielding an average of 20 A with a peak-to-peak ripple of 2 A.

a. The power supplied to the battery is

$$P = 120\text{ V} \times 20\text{ A} = 2400\text{ W}$$

The power supplied by the source is, therefore, 2400 W.
The dc current from the source is

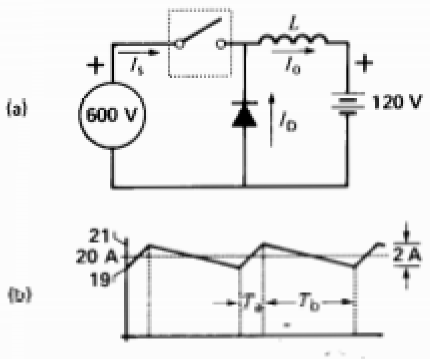


Figure 21.61
a. Circuit of Example 21-11.
b. Current in the load.

$$I_S = P/E_S = 2400/600 = 4\text{ A}$$

b. To calculate the average current in the diode, we refer to Fig. 21.61a. Current I_0 is 20 A and I_S was found to be 4 A. By applying Kirchhoff's current law to the diode/inductor junction, the average diode current I_D is

$$\begin{aligned} I_D &= I_0 - I_S \\ &= 20 - 4 \\ &= 16\text{ A} \end{aligned}$$

c. The duty cycle is

$$D = E_0/E_S = 120/600 = 0.2$$
$$T = 1/f = 1/200 = 5\text{ ms}$$

Consequently, the *on* time T_a is

$$T_a = DT = 0.2 \times 5\text{ ms} = 1\text{ ms}$$

The waveshapes of I_S and I_D are shown in Figs. 21.61c and 21.61d, respectively. Note the sharp pulses delivered by the source.

d. During interval T_a the average voltage across the inductor is $(600 - 120) = 480$ V. The volt-seconds accumulated by the inductor during this interval is $A_{(+)} = 480\text{ V} \times 1\text{ ms} = 480\text{ mV·s} = 0.48\text{ V·s}$. The change in current during the interval is 2 A; consequently,

$$\Delta I = A_{(+)}/L \qquad (2.28)$$
$$2 = 0.48/L$$
$$L = 0.24\text{ H}$$

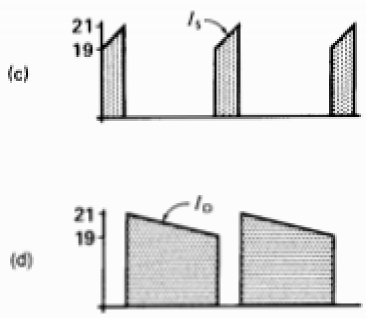


Figure 21.61
c. Current drawn from the source.
d. Current in the freewheeling diode.

Thus, the inductor should have an inductance of 0.24 H. If a larger inductance were used, the current ripple would be smaller, but the dc voltages and currents would remain the same.

Example 21-12

The chopper in Fig. 21.62 operates at a frequency of 4 kHz and the *on* time is 20 μs. Calculate the apparent resistance across the source, knowing that $R_0 = 12\ \Omega$.

Solution

The duty cycle is

$$D = T_a/T = T_a f = 20 \times 10^{-6} \times 4000 = 0.08$$

Applying Eq. 21.22, we have

$$\begin{aligned} R_S &= R_0/D^2 \\ &= 12/(0.08)^2 \\ &= 1875\ \Omega \end{aligned}$$

This example shows that the actual value of a resistor can be increased many times by using a chopper. Although a chopper can be compared to a transformer, there is an important difference between the two. The reason is that a transformer permits power flow in both directions—from the high-voltage side to the low-voltage side or vice versa. The step-down chopper we have just studied can transfer power only from the high-voltage side to the low-voltage side. Because power flow in both directions is often required, we now examine a dc-to-dc converter that achieves this result.

Example 21-13

The following data is given on a buck/boost converter (Fig. 21.65):

$$E_H = 100\ \text{V} \quad E_0 = 30\ \text{V} \quad R = 2\ \Omega \quad L = 10\ \text{mH}$$

switching frequency = 20 kHz with a duty cycle D of 0.2 for S1.

Determine the following:

a. The value and direction of the dc current I_L
b. The peak-to-peak ripple superposed on the dc current

Solution

Referring to Fig. 21.65, the value of $E_L = DE_H = 0.2 \times 100\ \text{V} = 20\ \text{V}$.

Because the battery voltage is greater than E_L, current I_L flows out of terminal 5 and into terminal 1. Its average value is

$$I_L = (30\ \text{V} - 20\ \text{V})/2\ \Omega = 5\ \text{A}$$

The duration of one cycle is

$$T = 1/f = 1/20\,000 = 50\ \mu\text{s}$$

Thus, S1 is closed for a time $T_a = 0.2 \times 50\ \mu\text{s} = 10\ \mu\text{s}$ and S2 is closed for 40 μs.

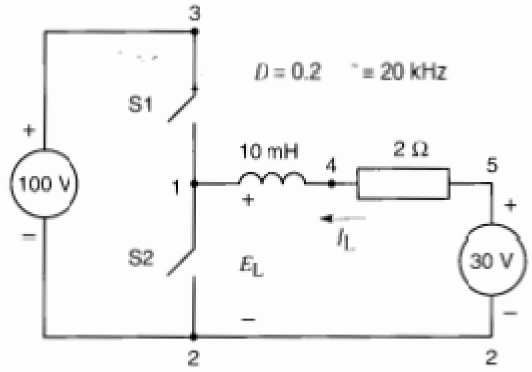


Figure 21.65
Circuit of Example 21-13.

To determine the peak-to-peak ripple, let us examine the situation when S2 is closed (Fig. 21.66). Assuming the current i is momentarily equal to its dc value of 5 A, the voltage E_{41} across the inductor is equal to the battery voltage minus the IR drop in the resistor: 30 V − 5 A × 2 Ω = 20 V. Knowing that I_L is flowing into terminal 4 and that terminal 4 is (+) with respect to terminal 1, it follows that I_L must be increasing. The inductor accumulates volt-seconds and during the 40 μs that S2 is closed, the magnetic "charge" totals 20 V × 40 μs = 800 V·μs. Therefore the current increases by an amount ΔI = 800 V·μs/10 mH = 0.08 A.

Let us now see what happens when S1 is closed and i is again momentarily 5 A (Fig. 21.67). The

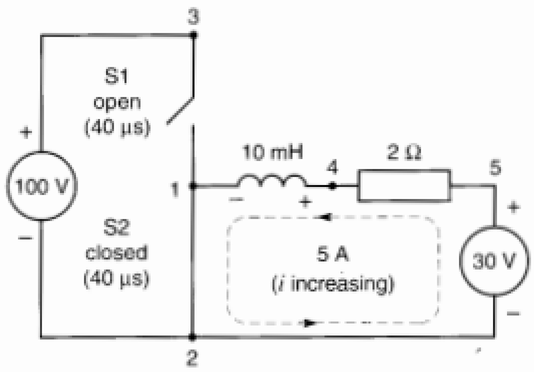


Figure 21.66
See Example 21-13.

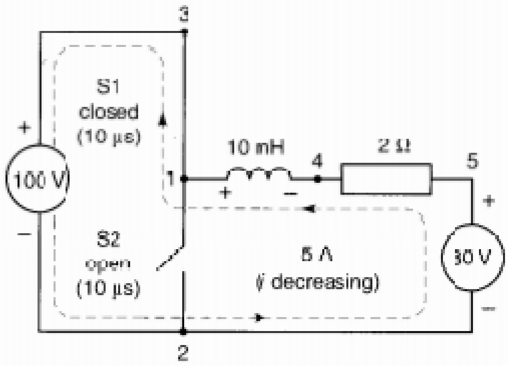


Figure 21.67
See Example 21-13.

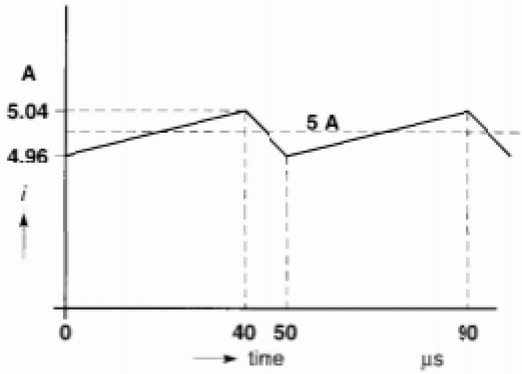


Figure 21.68

voltage across the inductor is now 100 V − (30 − 10) V = 80 V, but terminal 4 is *negative* with respect to terminal 1. The current i is therefore decreasing. The volt-seconds discharged during the 10 µs interval is 80 V × 10 µs = 800 V·µs. The change in current is ΔI = 800 V·µs/10 mH = 0.08 A.

We observe that the decrease in current when S1 is closed is the same as the previous increase when S1 was open. Consequently, the peak-to-peak ripple is 0.08 A. The dc current fluctuates between 5.04 A and 4.96 A (Fig. 21.68). Direct-current power flows from the lower voltage battery toward the higher voltage source. The converter is said to function in the boost mode.

Note that if the duty cycle were raised to 0.45, the value of E_L would increase to 100 × 0.45 = 45 V.

The current flowing into the battery would reverse and its value would become

$$(45\ \text{V} - 30\ \text{V})/2\ \Omega = 7.5\ \text{A}$$

Direct-current power now flows from the 100 V source toward the battery, causing the latter to charge up. Under these conditions, the converter is said to operate in the *buck* mode. Thus, the transition from boost to buck can be effected very smoothly by simply varying the duty cycle. Fig. 21.65 can be considered to be the mechanical equivalent of a buck/boost chopper.

21.41 Two-quadrant electronic converter

Figs. 21.66 and 21.67 show the direction of current flow in the case of a boost converter. If the converter operated in the buck mode, the currents would follow the same paths but in the opposite direction. With mechanical switches this creates no problem because they can carry current in either direction. But in the real world we have to deal with electronic switches, which inherently carry current in only one direction. Therefore, in order to get bidirectionality, diodes have to be placed in antiparallel with the respective semiconductor switches Q1 and Q2 (Fig. 21.69). The switch contacts are shown with an arrowhead to indicate the allowed direction of current flow. For example, when current flows into terminal 1, it can continue on to terminal 2 either by way of diode D1 and source E_H or by way of Q2, provided Q2 is closed.

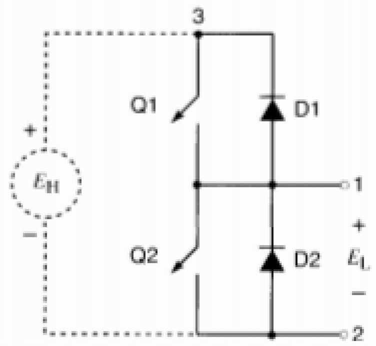


Figure 21.69
Two-quadrant electronic converter.

Similarly if current flows out of terminal 1, it can take the path through diode D2 or the path through Q1 and E_H, provided that Q1 is closed.

In this figure, Q1 and D1 together perform the same way as the mechanical switch S1. Similarly, Q2 and D2 together perform the same way as mechanical switch S2. Fig. 21.69 therefore represents the essence of a 2-quadrant electronic converter. If a dc voltage E_H is applied between terminals 3 and 2, the converter generates a dc voltage E_L between terminals 1 and 2 and the relationship is again $E_L = DE_H$, where D is the duty cycle of Q1.

It is important to note that Q1 and Q2 cannot be closed at the same time, otherwise a short-circuit will result across source E_H. Thus, for a very brief period, called *dead time*, both switches must be open. The current is carried by one of the two diodes during this instant.

Power can be made to flow from the higher voltage side to the lower voltage side or vice versa. The power transported in one direction or the other depends upon the respective voltages and the duty cycle. The 2-quadrant converter of Fig. 21.69 is the basic building block for most switch-mode converters.

21.42 Four-quadrant dc-to-dc converter

The 2-quadrant converter we have studied can only be used with a load whose voltage has a specific polarity. Thus, in Fig. 21.69, given the polarity of E_H, terminal 1 can only be (+) with respect to terminal 2. We can overcome this restriction by means of a *4-quadrant converter.* It consists of two identical 2-quadrant converters arranged as shown in Fig. 21.70. Switches Q1, Q2 in converter arm A open and close alternately, as do switches Q3, Q4 in converter arm B. The switching frequency (assumed to be 100 kHz) is the same for both. The switching sequence is such that Q1 and Q4 open and close simultaneously. Similarly, Q2 and Q3 open and close simultaneously. Consequently, if the duty cycle for Q1 is D, it will also be D for Q4. It follows that the duty cycle for Q2 and Q3 is $(1 - D)$.

The dc voltage E_A appearing between terminals A, 2 is given by

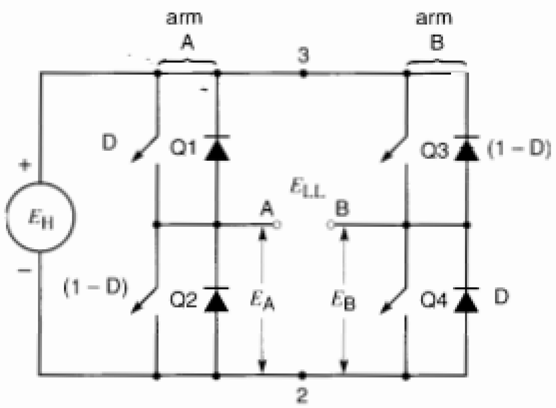


Figure 21.70
Four-quadrant dc-to-dc converter.

$$E_A = DE_H$$

The dc voltage E_B between terminals B, 2 is

$$E_B = (1 - D)E_H$$

The dc voltage E_{LL} between terminals A and B is the difference between E_A and E_B:

$$\begin{aligned} E_{LL} &= E_A - E_B \\ &= DE_H - (1 - D)E_H \end{aligned}$$

thus

$$E_{LL} = E_H(2D - 1) \qquad (21.24)$$

Equation 21.24 indicates that the dc voltage is zero when $D = 0.5$. Furthermore, the voltage changes linearly with D, becoming $+E_H$ when $D = 1$, and $-E_H$ when $D = 0$. The polarity of the output voltage can therefore be either positive or negative. Moreover, if a device is connected between terminals A, B, the direction of dc current flow can be either from A to B or from B to A. Consequently, the converter of Fig. 21.70 can function in all four quadrants.

The *instantaneous* voltages E_{A2} and E_{B2} oscillate constantly between zero and $+E_H$. Fig. 21.71 shows the respective waveshapes when $D = 0.5$. Similarly, Fig. 21.72 shows the waveshapes when $D = 0.8$. Note that the instantaneous voltage E_{AB} between the output terminals A, B oscillates between $+E_H$ and $-E_H$. In practice, the alternating

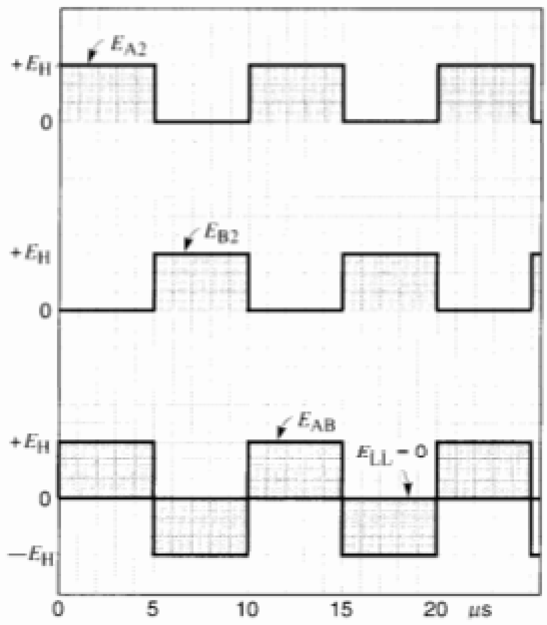


Figure 21.71
Voltage output when $D = 0.5$. The average voltage is zero.

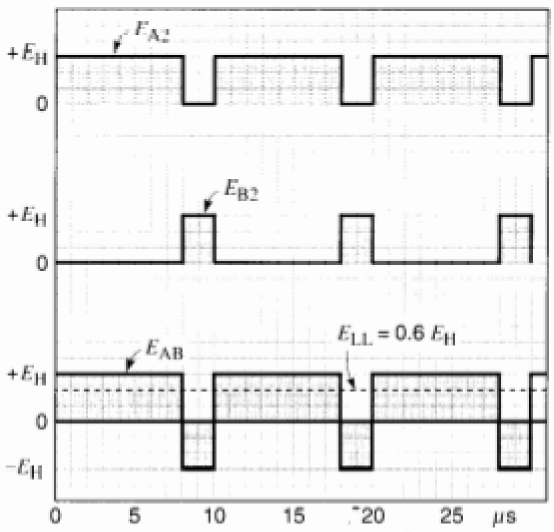


Figure 21.72
Voltage output when $D = 0.8$. The average voltage E_{LL} is 0.6 E_H.

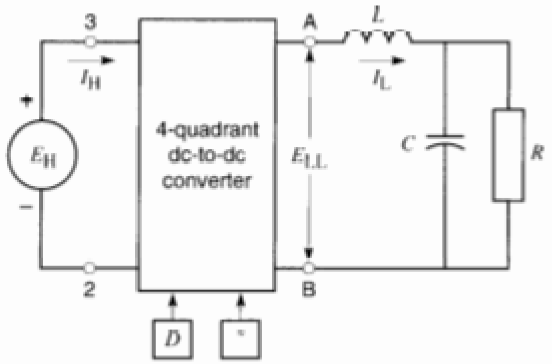


Figure 21.73
Four-quadrant dc-to-dc converter feeding a passive dc load *R*.

components that appear between terminals A, B are filtered out. Consequently, only the dc component E_{LL} remains as the active driving emf across the external device connected to terminals A, B.

Consider, for example, the block diagram of a converter feeding dc power to a passive load *R* (Fig. 21.73). The power is provided by source E_H. As we have seen, the magnitude and polarity of E_{LL} can be varied by changing the duty cycle *D*. The switching frequency *f* of several kilohertz is assumed to be constant. Inductor *L* and capacitor *C* act as filters so that the dc current flowing in the resistance has negligible ripple. Because the switching frequency is high, the inductance and capacitance can be small, thus making for inexpensive filter components.

The dc currents and voltages are related by the power-balance equation $E_H I_H = E_{LL} I_L$. We neglect the switching losses and the small control power associated with the *D* and *f* input signals.

Fig. 21.74 shows the converter connected to an active device E_0, which could be either a source or a load. If need be, the polarity of E_0 could be the reverse of that shown.

In all these applications we can force power to flow from E_H to E_0, or vice versa, by simply adjusting the duty cycle *D*. This 4-quadrant dc-to-dc converter is therefore an extremely versatile device.

The inductor *L* is a crucially important part of the converter. It alone is able to absorb energy at one

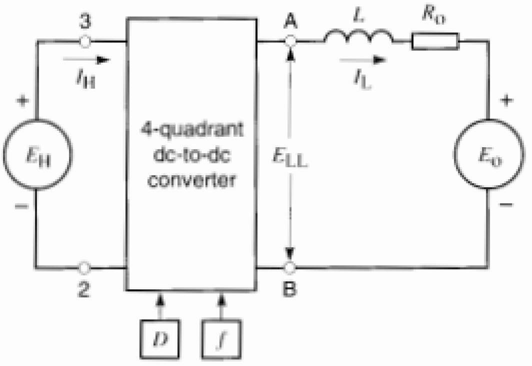


Figure 21.74
Four-quadrant dc-to-dc converter feeding an active dc source/sink E_o.

voltage level (high or low) and release it at another voltage level (low or high). And it performs this duty automatically, in response to the electronic switches and their duty cycle.

As indicated in Chap. 6 the use of series connected motors is restricted to only a few applications; for high dynamic performance drives, such as reversible servo drives, DC motors with separate excitation or permanent magnets are normally preferred; they require converters with four quadrant capability in the i_a/u_a-plane.

The bridge type converter shown in Fig. 9.13 a is connected to the armature circuit of a DC motor; it may be supplied with constant voltage u_D from a DC bus or a battery. The converter contains four electronic switches where two in each half-bridge are drawn in the form of a transfer switch (at the same time excluding accidental short circuits of the DC bus); the diodes which can be part of the electronic switches allow an inductive load current to continue during the short protective intervals, when all contacts are open (similar to the red-light-overlap on a signal crossing).

By assigning logic symbols S_1, S_2 to the otherwise ideally assumed switches, the voltage equation of the load circuit is

$$L_a \frac{i_a}{dt} + R_a i_a + e = u_a \,, \tag{9.8}$$

where, depending on the switching state

$$u_a = \frac{1}{2}(S_1 - S_2)\, u_D \text{ and } i_D = \frac{1}{2}(S_1 - S_2)\, i_a \tag{9.9}$$

holds.

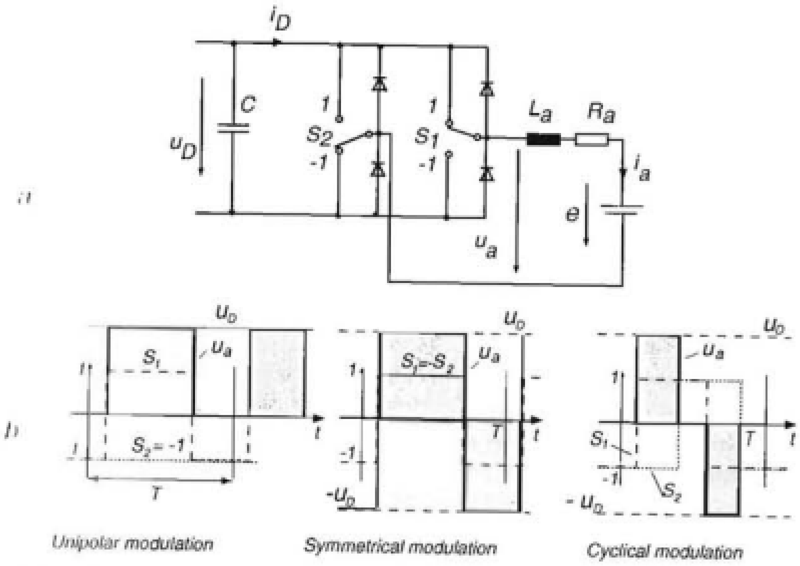


Fig. 9.13. Four-quadrant DC/DC converter with inductive load, (a) Circuit, (b)Different modulation patterns

The pulse-width-modulation (PWM) of the converter at the frequency $f = 1/T$ can follow different switching strategies, as illustrated in Fig. 9.13 b with the output voltage u_a during a switching period:

- Unipolar modulation
  One of the switches is assumed to be stationary, e. g. S_2 = -1 = const., whereas the other half-bridge is pulse-width-modulated, $S_2 = \pm 1$, so that the output voltage u_a assumes the values u_D or zero; the same applies with $S_1 = 1 =$ const. for negative output voltages.
- Symmetrical modulation
  With this modulation pattern the switches are operated in diagonal pairs, $S_1 = -S_2$, so that the short circuit interval is omitted and the output voltage alternates between the values u_D and -u_D. During the unavoidable (but in Fig. 9.13 neglected) protective intervals the diodes are carrying the load current.
- Cyclical modulation
  With this modulation scheme the two transfer switches are operated sequentially, so that the output is alternatively short circuited at the upper or lower supply bus. Hence the output voltage u_a assumes a ternary waveform, $u_a = u_D, 0, -u_D$. Whereas with symmetrical switching only the mean of the output voltage can be controlled in steady state, the cyclical modulation offers an additional degree of freedom that may for instance be used for eliminating harmonics of the output voltage u_a.

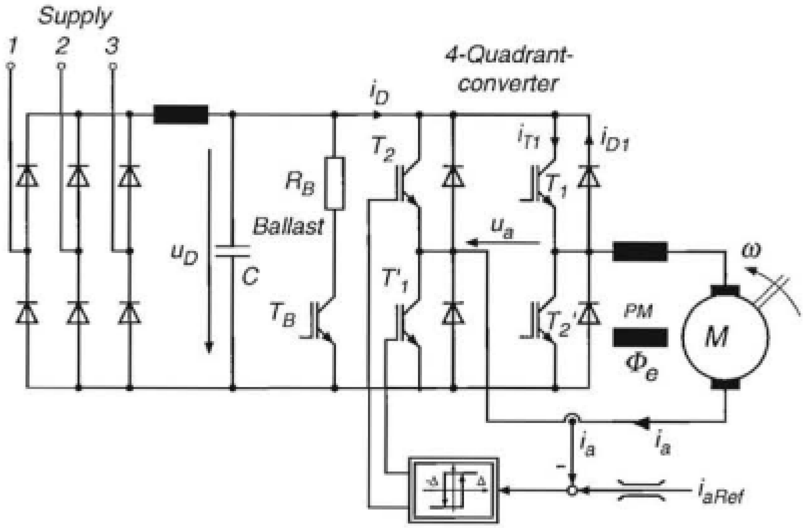


Fig. 9.14. DC Servo drive with voltage source DC/DC converter

A four-quadrant converter with IGBT-switches is depicted in Fig. 9.14 as frequently used for DC servo drives; protective circuitry is again omitted. For simplicity an On/Off device is drawn for current control but a linear current controller with constant frequency PWM would normally be preferred.

A diode rectifier followed by a smoothing filter, whose capacitor C absorbs also the modulation-induced ripple components of the link current i_D serves as the supply of the DC link with constant voltage u_D. For instance, when rapidly braking the drive, power released from the kinetic energy flows back into the DC-link causing negative current i_D and, because of the unidirectional line-side rectifiers, could result in an overcharge of the capacitor; this is prevented by dissipating the energy in a resistive ballast circuit that can also be pulse-width-modulated, depending on the link voltage. In view of the losses this is only practical with small drives or when it happens only occasionally; otherwise a reversible line-side supply (an active front-end converter) is preferable as will be shown in Fig. 9.18 and further discussed in Sect. 13.2.

Some of the steady state waveforms in a converter like the one in Fig. 9.14 are indicated in Fig. 9.15, showing the output voltage u_a alternating between u_D and -u_D and the alternating current components of i_D, which must be absorbed by the capacitor. The control can be arranged as before; an inner current loop controlling the converter via a pulse-width modulator is important for safe operation. The current controller in Fig. 9.14 is again drawn as an On/Off switch, but this is only an illustrative example.

The typical response time of the current loop, employing a switched transistor converter in combination with a DC disk motor, is 1 or 2 ms. For many

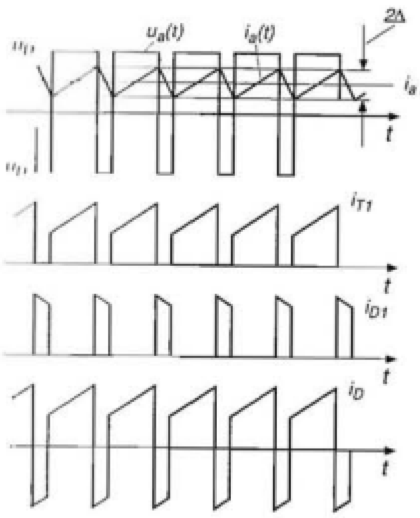


Fig. 9.15. Waveforms of DC/DC converter with symmetrical modulation

applications this justifies the assumption that the current control loop acts as controllable current source having instantaneous response. Current limit is achieved by limiting the current reference produced by the superimposed speed controller. The next higher level of control could be a position control loop as shown in Fig. 15.9, where the response may be further improved by feed forward signals from a reference generator.

Transistor converters have the important advantage that they can be switched at frequencies > 5 kHz, thus enlarging the control bandwidth as compared to line-commutated converters. With field effect transistors or IGBT's, the frequency can even be increased beyond the audible threshold > 16 kHz, so that the drive is no longer emitting objectionable acoustic noise.

The four quadrant converter shown in Fig. 9.14 is also suitable for producing AC in the form of pulse-with-modulated voltages and currents, provided its frequency is sufficiently below the switching frequency of the converter; this is seen in Fig. 9.16 with the example of approximately sinusoidal output current i_a in a passive ($e = 0$) inductive load circuit. With the assumed On/Off controller the modulation is symmetrical and the switching frequency varies but this could be remedied with a PWM modulator and fixed clock frequency. An inspection of the curves reveals that the converter passes cyclically though all quadrants of the i_a/u_a - plane, where energy is alternatively flowing from or into the DC link; naturally, in view of the losses, the reverse flow is reduced, as seen from the trace of i_D. We will meet this kind of converter again in connection with AC drives.

DC servo motors with permanent magnets are usually built in one of two forms:

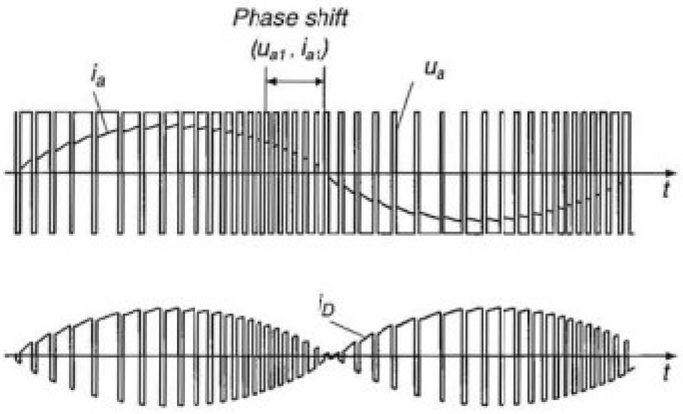


Fig. 9.16. Waveforms of symmetrically modulated four-quadrant converter with AC output

- Cylindrical motors with a slim rotor hence low inertia, but otherwise conventional design and
- Disk motors with axial magnetic field and radial armature winding, which is directly printed on a fiber reinforced disk; the brushes are sliding on the printed conductors of the commutator. These motors are charakterised by a very small armature time constant and low inertia; typical data are given in Fig. 14.1.

Because of the progress with controlled AC drives, servo motors are today mostly synchronous motors with permanent magnet excitation, Sect. 14.1.

When discussing phase controlled converters in Chap. 8, the problem of line-side interactions by harmonics and reactive current was mentioned. This applies also to uncontrolled rectifiers, such as in Fig. 9.14, particularly when a simple capacitive filter without smoothing choke is employed which causes line currents in the form of short spikes at the voltage peaks. With the growing use of power electronic equipment these effects are now receiving increased attention in technical standards and regulations.

With force controlled converters the reactive line currents can be greatly reduced while the harmonics are shifted to a higher frequency range, where it is easier to suppress them by filters; at the same time the problem of reverse power flow can be solved.

This is shown with the schematic diagram in Fig. 9.17 of a single phase converter which is of interest, apart from low power drives, on AC traction drives with intermediate DC link converter (an extension to three phases is discussed in Sect. 13.2). The circuit is essentially an inverted four-quadrant converter in Fig. 9.13, where the inductive impedance L_N is now represented by the leakage of the line-side transformer. The simplified equations are

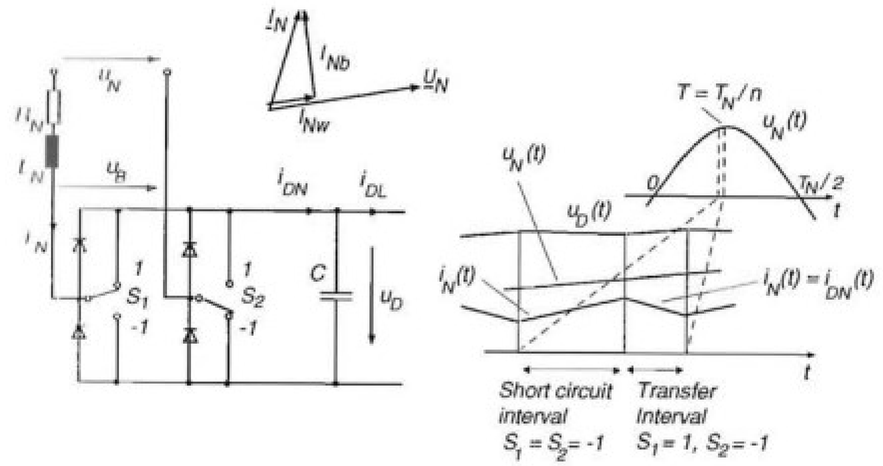


Fig. 9.17. Principle of a single phase line-fed four-quadrant converter for supplying a DC link

$$L_N \frac{i_N}{dt} + R_N\, i_N = u_N - u_B\ , \tag{9.10}$$

and

$$C \frac{u_D}{dt} = i_{DN} - i_{DL}\ , \tag{9.11}$$

where i_{DL} is the load-side link current; furthermore

$$u_B = \frac{1}{2}(S_1 - S_2)\, u_D \text{ und } i_{DN} = \frac{1}{2}(S_1 - S_2)\, i_N \tag{9.12}$$

holds. Approximately sinusoidal line current i_N can only be achieved if the DC link, usually operated at constant voltage u_D, can also be supplied with energy when the line voltage is much lower than u_D, i.e. near the zero intersections of u_N; hence it is necessary, as shown in Fig. 9.17 b, to create zero intervals of the voltage u_B for storing in the line-side inductance energy that is subsequently transferred in the form of a current pulse to the link capacitor. Thus it is necessary to employ a unipolar or cyclical modulation, where the converter bridge is temporarily short circuited at a DC bus.

The switching frequency of the converter is best chosen as an integer multiple n of the line frequency to avoid in steady state subharmonics of the line current; with a 50 Hz supply and a clock frequency for instance of 2 kHz, $n = 40$ switching cycles would fit into one line voltage period, resulting in a fairly smooth waveform of the current.

A possible control scheme is shown in Fig. 9.18, where a voltage control loop for u_D is superimposed on an inner current loop for i_N. The current reference $i_{N\,Ref}$ is derived from the line voltage for obtaining a sinusoidal line current reference, the magnitude of which is determined by the voltage controller. When shifting the current reference by t_0 with respect to the line voltage u_N, a reactive component of the line current results.

Caused by the single phase supply, periodic fluctuations of the link voltage u_D at twice line frequency cannot be avoided but are greatly diminished by inserting a resonant circuit tuned to double line frequency, as is indicated in Fig. 9.18; this is the usual practice on traction drives. The load-side DC link current i_{DL} acts as a disturbance to the voltage control, its effect may be reduced by feed-forward to the current reference.

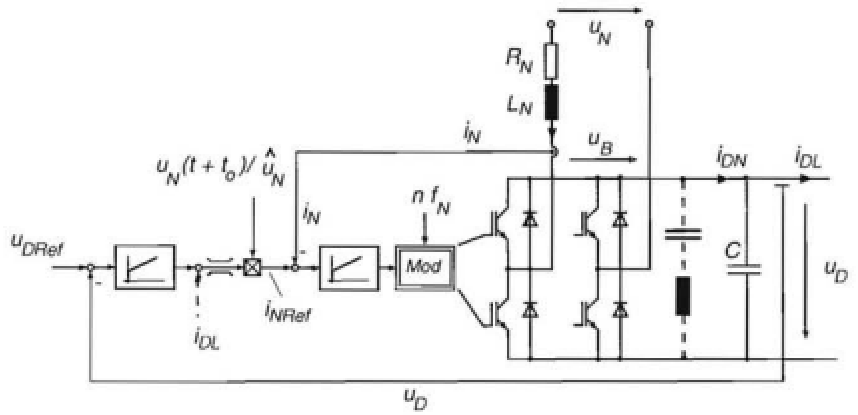


Fig. 9.18. Control of a single phase line-fed four-quadrant converter

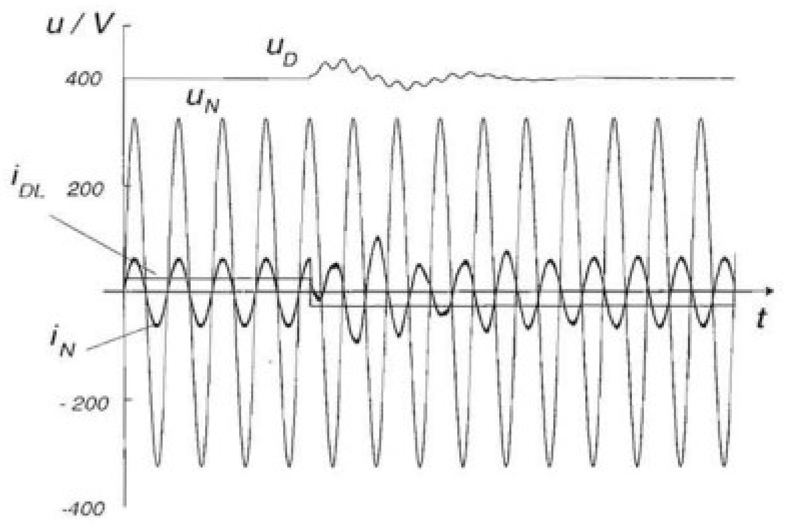


Fig. 9.19. Simulated load reversal of a line-fed four-quadrant converter

A simulated load reversal is plotted in Fig. 9.19, initiated by inverting the current i_{DL}, which results in a phase reversal of the line current in steady state, i. e. feeding power back to the line [I3]. The resonant circuit shown in Fig. 9.18 is included in the simulation model, thus reducing the harmonics content of the link voltage.

EXAMPLE 8.3 A load consisting of a direct-voltage source of 72 V in series with a resistance of 0.015 Ω and an inductance of 0.85 mH is to be supplied with an average current of 420 A from a 120-V direct-voltage source. A chopper capable of a frequency of 1 kHz is to be used. Determine the required ON time of the chopper.

Solution To assess what assumptions can reasonably be made, let us first check the time constant of the load.

$$\tau = \frac{L}{R} = \frac{0.85 \times 10^{-3}}{0.015} = 57 \qquad \text{ms}$$

This is much larger than the period of $T = 1$ ms of the chopper; thus, the current waveform segments can be assumed to be at constant slope ignoring the resistance.

The required average load voltage is, from Eq. (8.20),

$$\bar{v}_d = e_d + R\bar{i}_d = 72 + 0.015 \times 420 = 78.3 \qquad \text{V}$$

Then, from Eq. (8.19), the time t_{ON} is

$$t_{ON} = \frac{\bar{v}_d}{V} T = \frac{78.3}{120} \times 10^{-3} = 0.65 \qquad \text{ms}$$

Describe speed control of DC series motor using step down chopper.

Ans:

Speed control of DC series motor with step down chopper:

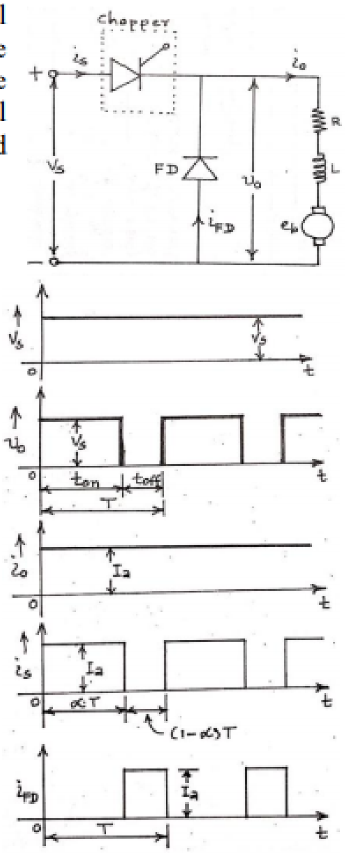


Figure shows the basic arrangement for speed control of DC series motor using step down chopper. Armature current is assumed continuous and ripple free. The waveforms for the source voltage V_s, Motor terminal voltage v_0, motor current i_0, dc source current i_s and freewheeling diode current i_{FD} are also shown.

Average motor voltage is given by,

$$V_0 = \frac{t_{on}}{T} V_s = \alpha V_s = f t_{on} V_s$$

where α = duty cycle = $\frac{t_{on}}{T}$

and f = Chopping frequency = $\frac{1}{T}$

Power delivered to motor is given by,

Power delivered to motor = Average motor voltage × Average motor current

$$= V_t I_a = \alpha\, V_s I_a$$

Motor voltage equation can be expressed as,

$$V_0 = \alpha\, V_s = E_b + I_a(R_a + R_{se})$$

The back emf is proportional to speed,

$$E_b \propto \omega_m \therefore E_b = K_m \omega_m$$

Thus voltage equation becomes,

$$V_0 = \alpha\, V_s = K_m \omega_m + I_a(R_a + R_{se})$$

The speed can be obtained as,

$$\omega_m = \frac{\alpha\, V_s - I_a(R_a + R_{se})}{K_m}$$

It is seen that by varying the duty cycle α of the chopper, armature terminal voltage can be controlled and thus speed of the dc series motor can be regulated.

Example 1: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle γ.

(b) The average value of the output voltage V_o.

(c) The rms value of the output voltage V_{orms}.

(d) The ripple factor RF.

(e) The output d.c. power.

Solution:

(a) $t_{on} = 1\text{ms}$, T=2.5 ms

$$\gamma = \frac{t_{on}}{T} = \frac{1\ ms}{2.5\ ms} = 0.4$$

(b) $V_{av} = V_o = \gamma\ V_d = 0.4 \times 100 = 40$ V.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.4} \times 100 = 63.2$ V.

(d) $RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$

(e)

$$I_a = \frac{V_o}{R} = \frac{40}{10} = 4A$$

$P_{av} = I_a V_o = 4 \times 40 = 160$ W

Example 2: An 80 V battery supplies RL load through a DC chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with 5Ω resistor. Load current, due to improper selection of frequency of chopping, varies widely between 9A and 10.2.

(a) Find the average load voltage, current and the duty cycle of the chopper.

(b) What is the operating frequency *f* ?

(c) Find the ripple current to maximum current ratio.

Solution:

(a) The average load voltage and current are:

$$V_{av} = V_o = \gamma V_d$$

$$I_{av} = \frac{1}{2}(I_2 + I_1) = \frac{9 + 10.2}{2} = 9.6A$$

$$I_{av} = \frac{V_{av}}{R} = \frac{\gamma V_d}{R} \qquad or \qquad \gamma = \frac{I_{av}\ R}{V_i} = \frac{9.6\ x\ 5}{80} = 0.6$$

$$V_{av} = 0.6 \text{ x } 80 = 48 \text{ V.}$$

(b) To find the operating (chopping) frequency:

During the ON period,

$$V_d = Ri + L\frac{di}{dt} \qquad \dots\dots\dots\dots\dots.(1)$$

Assuming $\frac{di}{dt} \cong constant$

$$\frac{di}{dt} \cong \frac{\Delta I}{t_{on}} = \frac{10.2 - 9}{\gamma T} = \frac{1.2}{\gamma T}$$

From eq.(1)

$$L\frac{di}{dt} \cong V_d - I_{av}\,R = 80 - 5 \times 9.6 = 32V$$

or

$$\frac{di}{dt} = \frac{32}{L} = \frac{32}{0.4} = 80\ A.s$$

but

$$\frac{di}{dt} = \frac{1.2}{\gamma T} = 80 = \frac{1.2}{0.6\,T}$$

$$\therefore T = \frac{1.2}{0.6x80} = 25\ ms$$

Hence

$$f = \frac{1}{T} = \frac{1}{25 \times 10^{-3}} = 40\ Hz$$

The maximum current I_m occurs at $\gamma = 1$,

$$\therefore\ I_m = \frac{\gamma V_d}{R} = \frac{1 \times 80}{5} = 16A$$

Ripple current $I_r = \Delta I = 10.2 - 9 = 1.2$ A

$$\therefore\ \frac{I_r}{I_m} = \frac{1.2}{16} = 0.075 \quad or\ \ 7.5\%\,.$$

Example 3: A DC Buck converter operates at frequency of 1 kHz from 100V DC source supplying a 10 Ω resistive load. The inductive component of the load is 50mH.For output average voltage of 50V volts, find:

(a) The duty cycle

(b) t_{on}

(c) The rms value of the output current

(d) The average value of the output current

(e) I_{max} and I_{min}

(f) The input power

(g) The peak-to-peak ripple current.

Solution:

(a) $V_{av} = V_o = \gamma V_d$

$$\gamma = \frac{V_{av}}{V_d} = \frac{50}{100} = 0.5$$

(b) $T = 1/f = 1 / 1000 = 1\text{ms}$

$$\gamma = \frac{t_{on}}{T}$$

$t_{on} = \gamma T = 05 \text{ x}1\text{ms} = 0.5 \text{ ms}$.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.5}\ x\ 100 = 70.07\ V$

(d)

$$I_{av} = \frac{V_{av}}{R} = \frac{50}{10} = 5\ A$$

(e)

$$I_{max} = \frac{V_{av}}{R} + \frac{t_{off}}{2L} V_{av} = \frac{50}{10} + \frac{(1 - 0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} x50$$

= 5 + 0.25 = 5,25 A

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av} = \frac{50}{10} - \frac{(1 - 0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} x50$$

= 5 - 0.25 = 4.75 A

(f)

$$I_{s(av)} = \frac{\gamma}{2} (I_{min} + I_{max}) = \gamma I_{av} = 0.5 \times 5 = 2.5A$$

$$P_{in} = I_{s(av)} V_d = 2.5 \times 100 = 250 \, W$$

(g) $$I_{p-p} = \Delta I = I_{max} - I_{min} = 5.25 - 4.75 = 0.5A$$

DC-DC CONVERTER

1. A class-A transistor chopper circuit shown in Fig.1 supplied with power from an ideal battery of terminal voltage 120 V .The load voltage waveforms consists of rectangular pulses of duration 1 ms in an overall cycle of 3 ms.

(a) Sketch the waveforms of v_L and i_L.
(b) Calculate the duty cycle γ,
(c) Calculate the average and r.m.s. values of the load voltage.
(d) Find the average value of the load current if R = 10 ohms.
(e) Calculate the input power and the ripple factor *RF*.

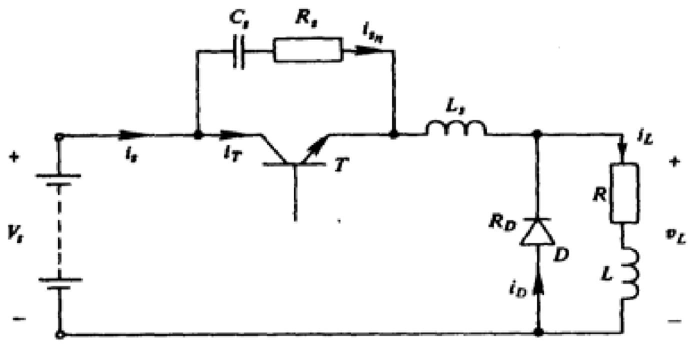


Fig.1

Question 1.

(a) Waveforms of Voltage v_L and current i_L are shown in Fig. Q2

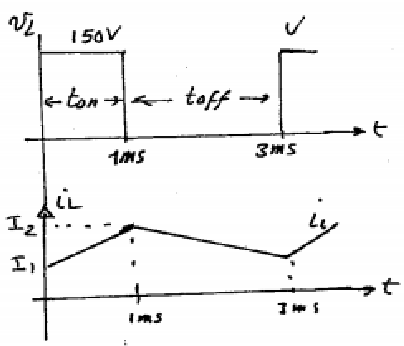


Fig. Q2.

(b) The duty cycle γ is

$$\gamma = \frac{t_{on}}{t_{on}+t_{off}} = \frac{1}{1+2} = \frac{1}{3}.$$

(c) $V_{av} = \gamma V_{in} = \frac{1}{3} \times 150 = 50V.$

$V_{Lr.m.s} = \sqrt{\gamma}\, V_{in} = \sqrt{\frac{1}{3}} \times 150 = 86.6V.$

(d) $I_{av} = \frac{V_{av}}{R} = \frac{50}{20} = 2.5\ A.$

(e) $I_{in} = I_{av}\gamma = 2.5 \times \frac{1}{3} = 0.833 A$

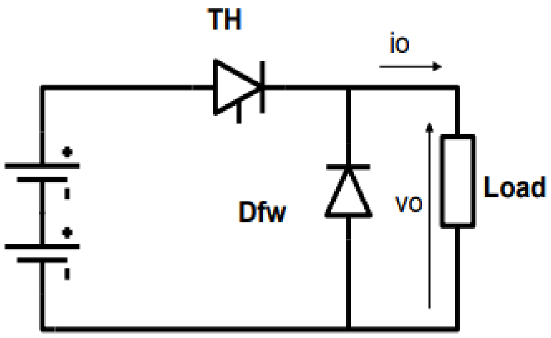
$P_{in} = V_s I_{in} = 0.833 \times 150 = 125 W.$

$$RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-\frac{1}{3}}{\frac{1}{3}}} = \sqrt{2} = 1.414$$

v

2. A class-A DC chopper shown in Fig.2 is operating at a frequency of 2kHz from 96V DC source to supply a load of resistance 8 ohms . The load time constant is 6 ms. If the mean load voltage is 57.6 V, find duty cycle (mark to space ratio), the mean load current, and the magnitude of the current ripple. Derive any formula used.

Fig.2



[Ans: γ =0.6 , Iav =7.2, Δ I = 0.24 A]

Solution:

$$T = \frac{1}{f} = \frac{1}{2000} = 0.5\ ms.$$

Load time constant $\tau = \frac{L}{R} = 6\ ms = 12T$. , hence the current variation is treated linear.

$$V_{av} = \gamma V_i$$

$57.6 = \gamma \times 96$ $\quad \therefore \gamma = \frac{57.6}{96} = 0.6$. $\equiv$ mark-space ratio.

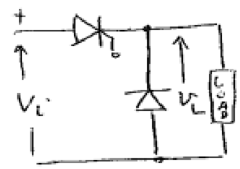
$$V_{0r.m.s} = V_i\sqrt{\gamma} = 96 \times \sqrt{0.6} = 74.36V.$$

$$I_{av} = \frac{V_{av}}{R} = \frac{57.6}{8} = 7.2A.$$

Current ripple $\Delta I = (V_i - V_{av})\frac{\Delta t}{L}$.

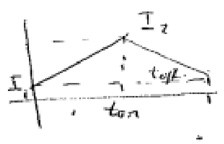
This comes from:

During conduction:



$$V_i - V_L = L\frac{di}{dt} \equiv L\frac{\Delta i}{\Delta t}.$$

$$\Delta i = (V_i - V_L)\frac{\Delta t}{L} = I_2 - I_1 \quad ---(1)$$



$$\Delta t = t_{on} - 0 = t_{on}.$$

$$\therefore I_2 - I_1 = (V_i - V_L)\frac{t_{on}}{L}.$$

During off period (from eq.(1)) $V_i = 0$

$$I_1 - I_2 = (0 - V_L)\frac{(T - t_{on})}{L}$$

or $I_2 - I_1 = V_L\frac{(T - t_{on})}{L} = V_L\frac{t_{off}}{L}$

Also $I_{av} = \frac{I_1 + I_2}{2}$

Hence

Hence

$$I_1 = I_{av} - V_L \frac{t_{off}}{2L} = I_{min}$$

$$I_2 = I_{av} + V_L \frac{t_{off}}{2L} = I_{max} \qquad V_L = V_{av}.$$

$\tau = \frac{L}{R} = 6\times10^{-3}$ $\quad \therefore L = 6\times10^{-3} \times R = 6\times10^{-3}\times 8 = 48\,mH.$

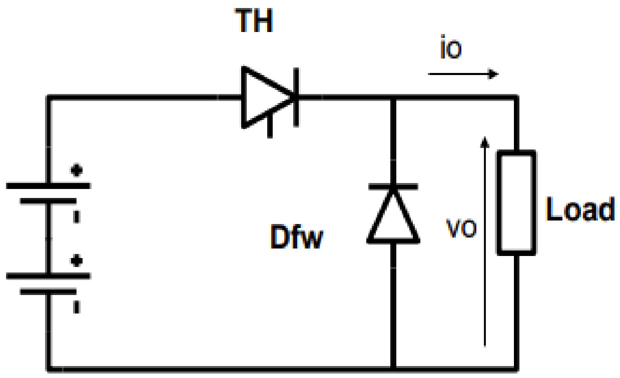
$$I_{max} = I_2 = 7.2 + \frac{57.6 \times 0.2\times10^{-3}}{2\times48\times10^{-3}} = 7.32\,A$$

$$I_{min} = I_1 = 7.2 - \frac{57.6\times0.2\times10^{-3}}{2\times48\times10^{-3}} = 7.08\,A$$

$$\Delta I = I_{max} - I_{min} = 7.32 - 7.08 = 0.24\,A.$$

3. A DC Buck converter (class-A chopper) supplies power to a load having 6 ohms resistance and 20 mH inductance. The source voltage is 100V d.c. and the output load voltage is 60V. If the ON time is 1.5 ms, find:

(a) Chopper switching frequency.
(b) Imax and Imin (I_2 and I_1).
(c) The average diode current.
(d) The average input current.
(e) Peak- to- peak ripple current.



[Ans: fc = 40Hz , Imax = 1.5A,Imin =8.5A, Iav(D) = Iav= 10A , ΔI= 3A]

Solution :

(a) $\gamma = \frac{V_{av}}{V_i} = \frac{60}{100} = 0.6$

$\gamma = \frac{t_{on}}{T} \Longrightarrow T = \frac{1.5}{0.6} = 2.5\ ms.$

$f = \frac{1}{T} = \frac{1}{2.5} \times 10^3 = 400\ Hz.$

(b) $I_{max} = \frac{V_{av}}{R} + V_{av}\left(\frac{t_{off}}{2L}\right) = \frac{60}{6} + 60\left(\frac{(2.5-1.5)\times 10^{-3}}{2\times 20\times 10^{-3}}\right)$

$= 10 + \frac{60}{40} = 10 + 1.5 = 11.5\ A.$

$I_{min} = \frac{V_{av}}{R} - V_{av}\left(\frac{t_{off}}{2L}\right) = 10 - 1.5 = 8.5\ A.$

(c) $I_{in(av)} = \gamma I_{av} = 0.6 \times \frac{V_{av}}{R} = 0.6 \times \frac{60}{60} = 6\ A.$

(d) $I_{Diode} = I_{av} = \frac{V_{av}}{R} = \frac{60}{6} = 10\ A.$

(e) p-p riple $\Delta I = I_{max} - I_{min} = 11.5 - 8.5 = 3.0\ A.$

4. In a class-A chopper circuit an ideal battery of terminal voltage 100V supplies a series load of resistance 0.5 Ohms and inductance of 1.0 mH . The thyristor is switched on for 1 ms in an overall period of 3 ms. Calculate the average values of the load voltage and current and the power taken from the battery. Assuming continuous current conduction .Also calculates the r.m.s value of the load current taking the firs two harmonics of the Fourier series.

[Ans: Vav = 33.3V , Iav = 66.7A, Pin= 2223W , $I_{Lr.m.s}$ = 69.1A]

Solition

$t_{on} = 1\ ms \quad , \quad T = 3\,ms$

$\therefore \gamma = \frac{t_{on}}{T} = \frac{1}{3}$

$\omega = 2\pi f = \frac{2\pi}{T} \qquad \therefore T = \frac{2\pi}{\omega} = \frac{3}{1000}$

Hence $\omega = \frac{2000\pi}{3} = 2094.4$ rad/s.

$\tau = \frac{L}{R} = \frac{1\times10^{-3}}{0.5} = 2\,mS.$

$V_{0av} = \gamma V_i = \frac{100}{3} = 33.33\,V.$

The Average load current $I_{av} = \frac{V_{o(av)}}{R} = \frac{33.33}{0.5} = 66.7A.$

The average supply (input Current) $I_{in(av)} = \gamma I_{av} = \frac{1}{3}\times 66.7$
$= 22.23A.$

Input power $P_{in} = V_i I_{in(av)}$
$= 100 \times 22.23 = \underline{2223}\ W.$

The impedance of the load to current of fundamental frequency: $Z_L = \sqrt{R^2+(\omega L)^2} = \sqrt{(0.5)^2+(2094.4\times10^{-3})^2} = 2.153\,\Omega$

The fundamental component of the load voltage:

$V_{o1(r.m.s)} = \frac{V_i}{\sqrt{2}\pi}\sqrt{\sin^2 2\pi\gamma + (1-\cos 2\pi\gamma)^2} = \frac{100}{\sqrt{2}\pi}\sqrt{0.75+2.25} = \frac{55.13}{\sqrt{2}}\,V.$

$\therefore I_{o1} = \frac{V_{o1(r.m.s)}}{Z_L} = \frac{55.13}{\sqrt{2}\times 2.153} = 18.1\,A$ - (r.m.s value of the fundamental Load current).

5. In a class-A chopper circuit an ideal battery of terminal voltage 100V supplies a series load of resistance 10 Ohms. The chopping frequency is f =1 kHz and the duty cycle is set to be 0.5 .Determine:

(a) The average output voltage.
(b) The rms output voltage.
(c) The chopper efficiency.
(d) The ripple factor.
(e) The fundamental component of output harmonic voltage.

[As: Va=110V, $V_{Lr.m.s}$ = 155.56V, ή=100%,RF=1.0, $V_{L1\ r.m.s}$ =99V]

Solution:

(a) $V_o = \gamma V_i = 0.5 \times 220 = 110V.$

(b) $V_{o_{r.m.s}} = \sqrt{\gamma}\, V_i = \sqrt{0.5} \times 220 = 155.56V.$

(c) $I_{av} = \frac{V_o}{R} = \frac{110}{10} = 11A.$

$P_o = I_{av}^2 R = 11^2 \times 10 = 1210 W.$

$I_{in(av)} = \gamma I_a = 0.5 \times 11 = 5.5A.$

$P_{in} = V_i \cdot I_{in(ave)} = 220 \times 5.5 = 1210 W$

$\therefore \eta = \frac{P_o}{P_i} = \frac{1210}{1210} = 100\%.$

(d) $RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-0.5}{0.5}} = 1.$

(e) $V_{o_{1(r.m.s)}} = \frac{V_i}{\sqrt{2}\pi}\sqrt{\sin^2 2\pi\gamma + (1-\cos 2\pi\gamma)^2}$

$= \frac{220}{\sqrt{2}\pi}\sqrt{\sin^2(2\pi \times \frac{1}{2}) + (1-\cos(2\pi \times \frac{1}{2}))^2}$

$= \frac{220}{\sqrt{2}\pi}\sqrt{0 + (2)^2} = \frac{220\sqrt{2}}{\pi} = 70.06\sqrt{2}$

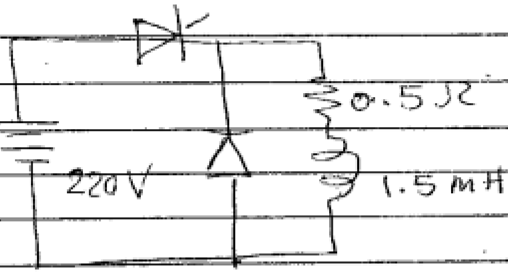
6. In the chopper circuit sown in Fig.1 (problem 1) Vi = 220V, L = 1.5 mH , R = 0.5 ohm and it operating with T = 3 ms and t_{on} = 1.5 ms.

(a) Determine the minimum, maximum and average values of load current.

(b) Express the load current variation in terms of ON and OFF periods.

[Ans: (a) Imin = 165A,Imax=275A, i_1= 165+36660 t , i_2 =165-36660t]

problem 6 -



$T = 3\ ms$

$t_{on} = 1.5\ ms$

I_{min} & I_{max}

sol. $t_{off} = T - t_{on} = 1.5\ ms$

$$\gamma = \frac{t_{on}}{T} = \frac{1.5}{3} = 0.5$$

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av}$$

$V_{av} = \gamma V_i = 0.5 \times 220 = 110\ V$

7. A separately excited d.c. motor with Ra = 1.2 ohms and La = 30 mH , is to be controlled using class-A transistor chopper .The d.c. supply is 120V .

(a) It is required to draw the speed torque characteristics for the motor when the duty cycle $\gamma = 1$. The motor design constant KeΦ has a value of 0.042 V/rpm.

(b) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle $\gamma = 0.5$.

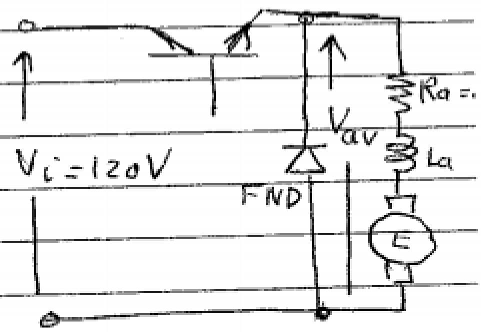
[Ans : n = 857 rpm]

Solution : In the steady state the armature inductance has no effect.

(a) At $\gamma = 1$

$V_{av} = \gamma V_{in} = 1 \times 120 = 120\ V.$

$$n = \frac{V_{av}}{K_e\phi} - \frac{\Sigma R_a\, T}{(K_e\phi)^2 \times 9.55}$$



when T=0

$$n_o = \frac{120}{0.042} = 2857\ rpm$$

when $n = 0$, $T = T_{st}$

$$\frac{\Sigma R_a}{9.55(K_e\phi)^2} \cdot T_{st} = \frac{V_{av}}{K_e\phi}$$

$$\therefore T_{st} = \frac{V_{av}}{\Sigma R_a} \frac{(K_e\phi)}{\frac{1}{9.55}} = \frac{120}{1.2} \times \frac{(0.042)}{\frac{1}{9.55}} \simeq 40\ N.m$$

Examlpe: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle D.

(b) The average value of the output voltage V_{dc}.

(c) The *rms* value of the output voltage V_{rms}.

(d) The ripple factor *RF*.

(e) The output dc power.

(a) $D = \frac{t_{ON}}{T} = \frac{1msec}{2.5msec} = 0.4$

(b) $V_{dc} = DV_s = 0.4 \times 100 = 40 \text{ V}$

(c) $V_{rms} = \sqrt{D}V_s = \sqrt{0.4} \times 100 = 63.2 \text{ V}$

(d) $RF = \sqrt{\frac{1-D}{D}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$

(e) $P_o = \frac{{V_{dc}}^2}{R} = \frac{40^2}{10} = 160 \text{ W}$

Examlpe: A dc chopper has a resistive load of 20Ω and input voltage V_S=220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.

$$V_s = 220\text{V}$$

$$D = \frac{t_{ON}}{T} = 0.8$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T}(V_s - V_{CH}) = 0.8(220 - 1.5) = 174.8 \text{ V}$$

$$T = \frac{1}{f} = \frac{1}{10 \times 10^{-3}} = 0.1\text{m } sec$$

$$t_{ON} = DT = 0.8 \times 0.1 \times 10^{-3} = 80\mu\, sec$$

Examlpe: buck dc-dc converter with Low Pass Filter has the following parameters:

$V_s = 50$ V $\quad L = 400\ \mu$H $\quad f = 20$ kHz
$D = 0.4 \quad C = 100\ \mu$F $\quad R = 20\ \Omega$

Assuming ideal components, calculate (a) the output voltage Vo, (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

(a) $V_o = V_s D = (50)(0.4) = 20$ V

(b)

$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75\text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$= 1 - \frac{1.5}{2} = 0.25\text{ A}$$

The average inductor current is 1 A, and Δi_L=1.5 A.

(c) $$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$

$$= 0.00469 = 0.469\%$$

Example: Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50. and the switching frequency is 25kHz.

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let L=120 μH.

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5\text{ A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2\text{ A}$$

$$I_{\max} = 1.5 + 1.2 = 2.7\text{ A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3\text{ A}$$

$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Designing a Buck Converter

Assume:

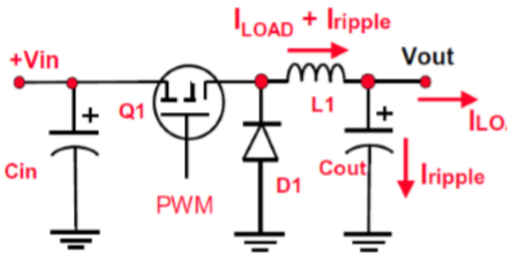
V_{in} = 12 V

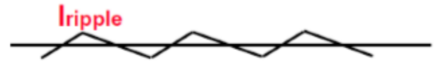
V_{OUT} = 5 volts

I_{LOAD} = 2 amps

F_{sw} = 400 KHz

D = V_{in} / $_{Vout}$ = 5V /12V = 0.416





Define Ripple current:

I_{ripple} = 0.3 • I_{LOAD} (typically 30%)

For an Inductor: V = L • ΔI / ΔT

Rearrange and substitute:

L = (Vin – Vout) • (D / Fsw) / Iripple

Calculate:

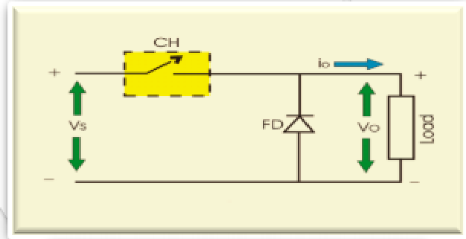
L = 7 V • (0.416 / 400 kHz) / 0.6A

L = 12.12 uh

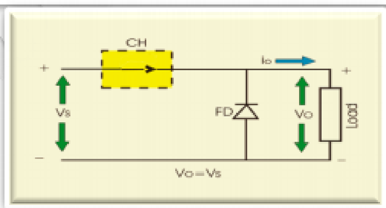
Select C, Diode (Schottky),
and the MOSFET
Calculate the Efficiency

The Buck (Step-Down) Converter

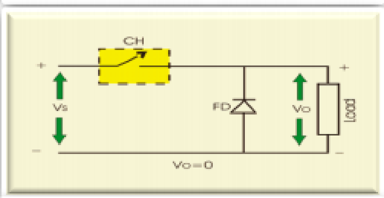
- Step down chopper as Buck converted is used to reduce the input voltage level at the output side. Circuit diagram of a step down chopper is shown in the figure.



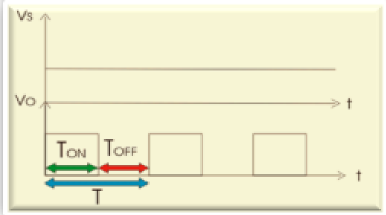
- When CH is turned ON, Vs directly appears across the load as shown in figure. So $V_O = V_S$.



- When CH is turned OFF, Vs is disconnected from the load. So output voltage $V_O = 0$.



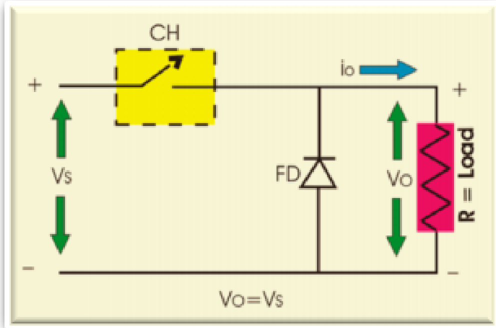
- The voltage waveform of step down chopper



- T_{ON} → It is the interval in which chopper is in ON state.
- T_{OFF} → It is the interval in which chopper is in OFF state.
- V_S → Source or input voltage.
- V_O → Output or load voltage.
- T → Chopping period = $T_{ON} + T_{OFF}$
- F=1/T is the frequency of chopper switching or chopping frequency

Operation of Step Down Chopper with Resistive Load

- When CH is ON, $V_O = V_S$ When CH is OFF, $V_O = 0$



The Average output voltage is

$$V_{dc} = V_o = \frac{1}{T}\int_0^{T_{ON}} V_s dt = \frac{V_s T_{ON}}{T} = DV_s$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{DV_s}{R}$$

$$D = \frac{T_{ON}}{T} \qquad\qquad T = T_{ON} + T_{OFF}$$

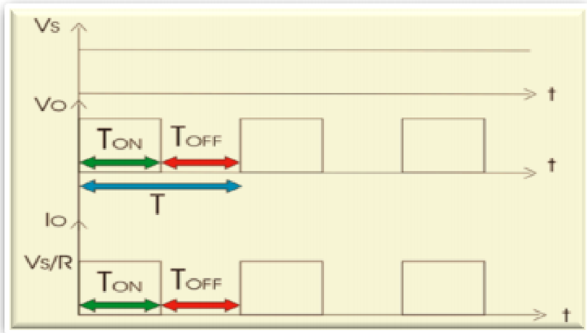
- ***Where,***
- ***D is <u>duty cycle</u> = T_{ON}/T. T_{ON} can be varied from 0 to T, so $0 \le D \le 1$.***
- ***The output voltage V_O can be varied from 0 to V_S.***

The *rms* output voltage is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T_{ON}} {V_s}^2 dt} = V_s\sqrt{\frac{T_{ON}}{T}} = \sqrt{D}V_s$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{D}V_s}{R}$$

$$P_o = V_{rms}I_{rms} = \frac{{V_{rms}}^2}{R} = D\frac{{V_s}^2}{R}$$



The output voltage is always less than the input voltage and hence the name step down chopper is justified.

Ripple factor (*RF*) can be found from

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{\frac{D{V_s}^2}{D^2{V_s}^2} - 1} = \sqrt{\frac{1}{D} - 1} = \sqrt{\frac{1-D}{D}}$$

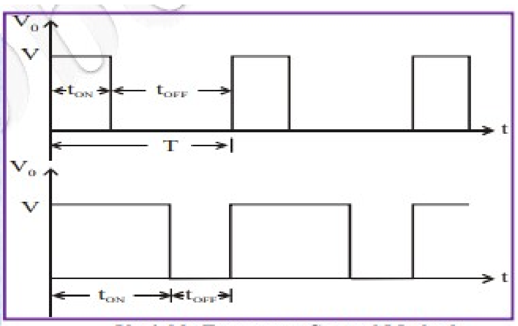
Methods of Control

1- Pulse Width Modulation

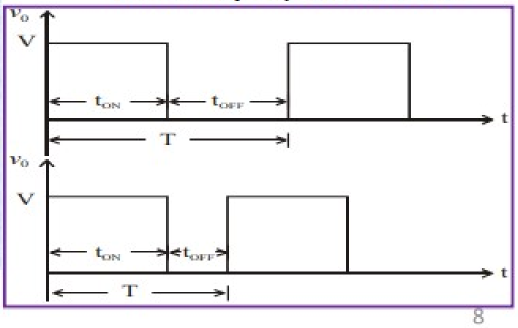
- t_{ON} is varied keeping chopping frequency '*f*' & chopping period 'T' constant.
- Output voltage is varied by varying the ON time t_{ON}

2- Variable Frequency Control

- Chopping frequency '*f*' is varied keeping either t_{ON} or t_{OFF} constant.
- To obtain full output voltage range, frequency has to be varied over a wide range.
- This method produces harmonics in the output and for large t_{OFF} load current may become discontinuous



Variable Frequency Control Method



8

Examlpe: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle D.

(b) The average value of the output voltage V_{dc}.

(c) The *rms* value of the output voltage V_{rms}.

(d) The ripple factor *RF*.

(e) The output dc power.

(a) $$D = \frac{t_{ON}}{T} = \frac{1msec}{2.5msec} = 0.4$$

(b) $$V_{dc} = DV_s = 0.4 \times 100 = 40 \text{ V}$$

(c) $$V_{rms} = \sqrt{D}V_s = \sqrt{0.4} \times 100 = 63.2 \text{ V}$$

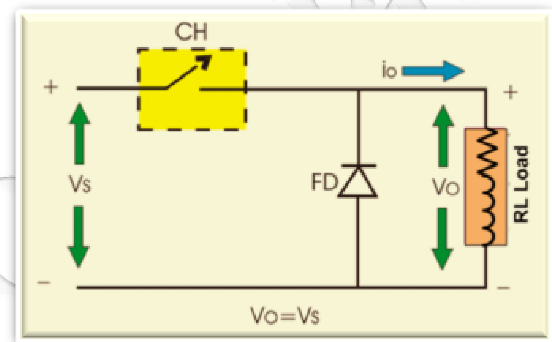
(d) $$RF = \sqrt{\frac{1-D}{D}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$$

(e) $$P_o = \frac{{V_{dc}}^2}{R} = \frac{40^2}{10} = 160 \text{ W}$$

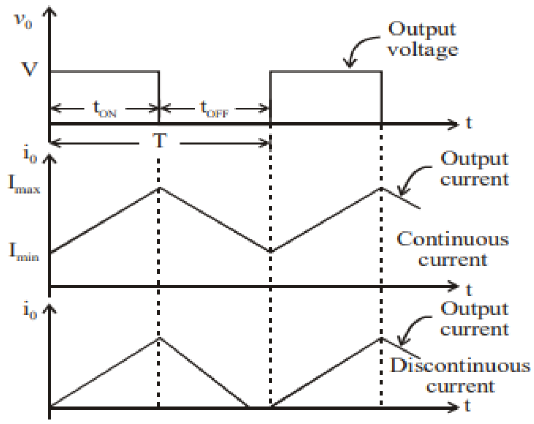
The Buck (Step-Down) Converter

Step Down Chopper with RL Load

- When chopper is ON, supply is connected across load. Current flows from supply to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in inductor ‘L’.

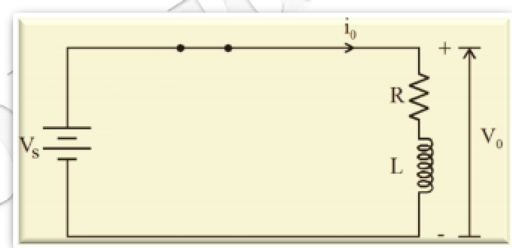


- Load current can be continuous or discontinuous depending on the values of ‘*L*’ and duty cycle ‘*D*’
- For a continuous current operation, load current varies between two limits I_{max} and I_{min}.
- When current becomes equal to I_{max} the chopper is turned-off and it is turned-on when current reduces to I_{min}.



Continuous Current Operation When Chopper Is ON ($0 \le t \le t_{ON}$)

- When the switch is closed in the buck converter, the circuit will be as shown in the figure, the diode is reverse-biased.

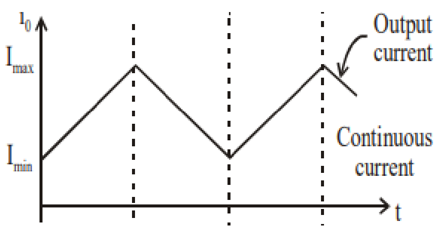


The voltage across the inductor is

$$V_s = V_R + V_L$$

$$V_s = V_R + L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{V_s - V_R}{L}$$

$$\Delta i = \int_0^{DT} \frac{V_s - V_R}{L} dt = \frac{V_s - V_R}{L} DT = \frac{V_s - V_R}{L} t_{ON} \qquad (1)$$

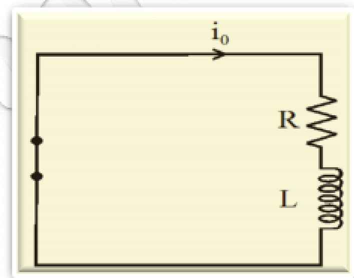


$$\frac{di}{dt} = \frac{\Delta i}{t_{ON}} = \frac{I_{max} - I_{min}}{t_{ON}} = \frac{V_s - V_R}{L}$$

From straight line equation

$$i_{o1} = I_{min} + \frac{I_{max} - I_{min}}{t_{ON}} t = I_{min} + \frac{I_{max} - I_{min}}{DT} t = I_{min} + \frac{V_s - V_R}{L} t \qquad (2)$$

Continuous Current Operation When Chopper Is <u>OFF</u> ($t_{ON} \leq t \leq T$)

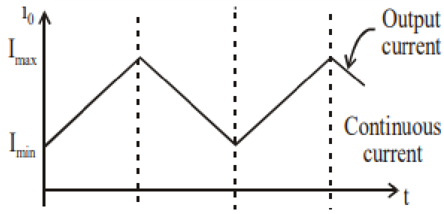


$$0 = V_R + V_L$$

$$0 = V_R + L\frac{di}{dt} \qquad \rightarrow \qquad \frac{di}{dt} = -\frac{V_R}{L}$$

$$\Delta i = \int_0^{t_{OFF}} -\frac{V_R}{L}dt = -\frac{V_R}{L}t_{OFF} \qquad (3)$$

$$\frac{di}{dt} = \frac{\Delta i}{t_{OFF}} = \frac{I_{min} - I_{max}}{t_{OFF}} = -\frac{I_{max} - I_{min}}{t_{OFF}} = -\frac{V_R}{L}$$



From straight line equation

$$i_{o2} = I_{max} + \frac{I_{min} - I_{max}}{t_{OFF}}(t - t_{ON}) = I_{max} - \frac{V_R}{L}(t - t_{ON}) \qquad (4)$$

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s - V_R}{L}t_{ON} - \frac{V_R}{L}t_{OFF} = 0 \longrightarrow \frac{V_s - V_R}{V_R} = \frac{t_{OFF}}{t_{ON}}$$

$$\frac{V_s}{V_R} - 1 = \frac{t_{OFF}}{t_{ON}} \longrightarrow \frac{V_s}{V_R} = \frac{t_{OFF}}{t_{ON}} + 1$$

$$\frac{V_s}{V_R} = \frac{t_{OFF} + t_{ON}}{t_{ON}} = \frac{T}{t_{ON}} \longrightarrow V_R = DV_s$$

From equation (1)

$$\boxed{\Delta i = \frac{V_s - DV_s}{L}DT = \frac{V_s(1-D)D}{Lf}}$$

since $\quad D = \dfrac{t_{ON}}{T}$

$$f = \frac{1}{T}$$

At steady state operation, the average inductor current must be the same as the average current in the load resistor.

$$I_L = I_R = \frac{V_R}{R}$$

The maximum and minimum values of the inductor current are computed as

$$I_{max} = I_L + \frac{\Delta i}{2}$$

$$\boxed{I_{max} = I_L + \frac{V_s(1-D)D}{2Lf} = I_L + \frac{V_R(1-D)}{2Lf}}$$

$$I_{min} = I_L - \frac{\Delta i}{2}$$

$$\boxed{I_{min} = I_L - \frac{V_s(1-D)D}{2Lf} = I_L - \frac{V_R(1-D)}{2Lf}}$$

The average dc output voltage and current can found as

$$V_{dc} = DV_s \qquad\qquad I_{dc} \cong \frac{I_{max} - I_{min}}{2}$$

Examlpe: A dc chopper has a resistive load of 20Ω and input voltage V_S=220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.

$$V_s = 220\text{V}$$

$$D = \frac{t_{ON}}{T} = 0.8$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T}(V_s - V_{CH}) = 0.8(220 - 1.5) = 174.8\text{ V}$$

$$T = \frac{1}{f} = \frac{1}{10 \times 10^{-3}} = 0.1\text{m}\,sec$$

$$t_{ON} = DT = 0.8 \times 0.1 \times 10^{-3} = 80\mu\,sec$$

Step Down Chopper with RL Load

Examlpe: A Chopper circuit is operating at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

$$V_s = 460\text{V}$$

Chopping period

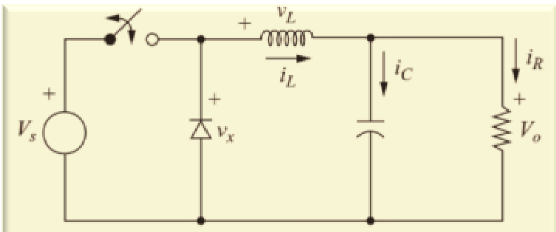
$$T = \frac{1}{f} = \frac{1}{2\times10^{-3}} = 0.5\text{m}\, sec$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T} V_s$$

$$t_{ON} = \frac{TV_{dc}}{V_s} = \frac{0.5\times10^{-3}\times350}{460} = 0.38\text{m}\, sec$$

Step Down Chopper with Low Pass Filter

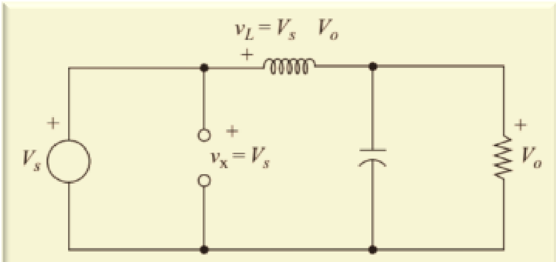
- **This converter is used if the objective is to produce an output that is purely DC.**
- **If the low-pass filter is ideal, the output voltage is the average of the input voltage to the filter.**



(a)

Analysis for the Switch Closed

When the switch is closed in the buck converter circuit of fig. a, the diode is reverse-biased and fig. b is an equivalent circuit. The voltage across the inductor is



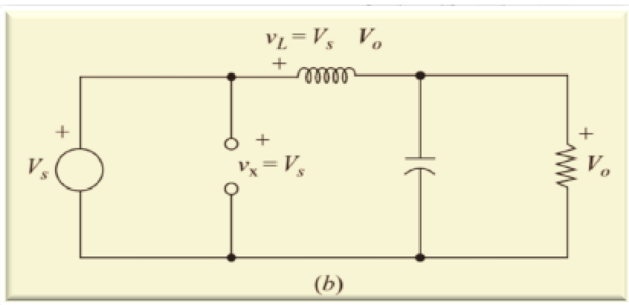
(b)

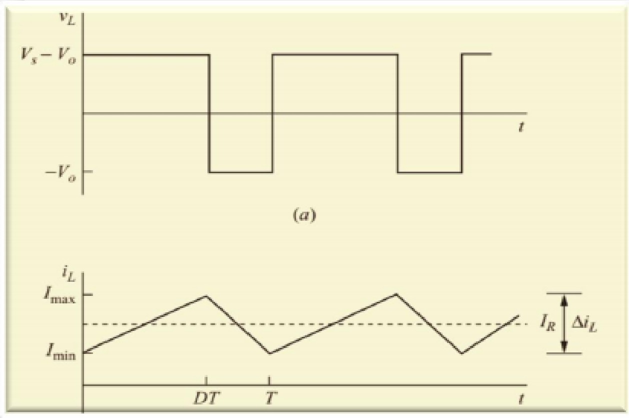
$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

Analysis for the Switch Closed

Since the derivative of the current is a positive constant, the current increases linearly. The change in current while the switch is closed is computed by modifying the preceding equation.





$$(\Delta i_L)_{closed} = \int_0^{DT} \frac{V_s - V_o}{L} dt = \frac{V_s - V_o}{L} DT$$

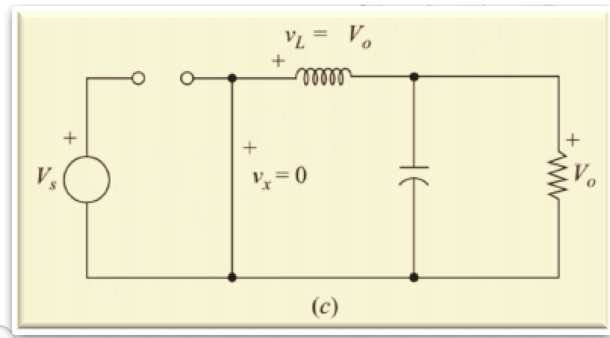
or

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L} \qquad (1)$$

$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L}\right) DT$$

Analysis for the Switch Opened

When the switch is open, the diode becomes forward-biased to carry the inductor current and the equivalent circuit of fig. c applies. The voltage across the inductor when the switch is open is



$$v_L = -V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V_o}{L}$$

The derivative of current in the inductor is a negative constant, and the current decreases linearly. The change in inductor current when the switch is open is

$$(\Delta i_L)_{opened} = \int_0^{(1-D)T} \frac{-V_o}{L} dt = \frac{-V_o}{L}(1-D)T$$

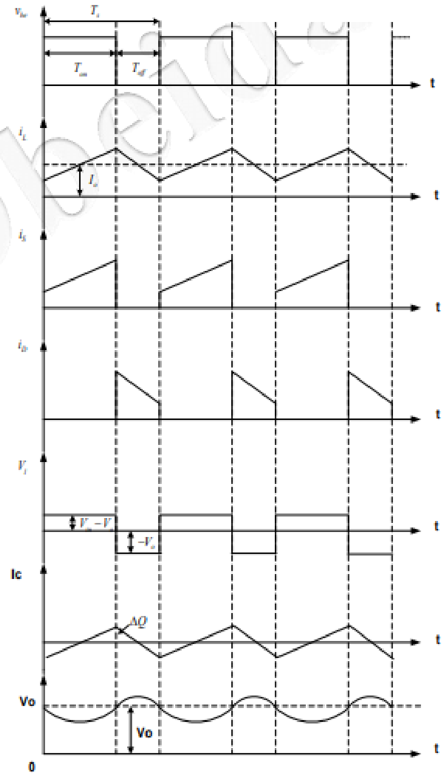
or

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T \qquad (2)$$

Dr. Firas

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires



$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

Using equations 1&2

$$\left(\frac{V_s - V_o}{L}\right)(DT) - \left(\frac{V_o}{L}\right)(1-D)T = 0$$

$$\boxed{V_o = V_s D}$$

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero for steady-state operation:

$$I_L = I_R = \frac{V_o}{R}$$

The maximum and minimum values of the inductor current are computed as

$$I_{\max} = I_L + \frac{\Delta i_L}{2}$$

$$= \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2}$$

$$= \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

Since I_{min}=0 is the boundary between continuous and discontinuous current,

$$I_{\min} = 0 = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$(Lf)_{\min} = \frac{(1-D)R}{2}$$

The minimum combination of inductance and switching frequency for continuous current in the buck converter is

$$L_{\min} = \frac{(1-D)R}{2f} \qquad \text{for continuous current}$$

where L_{min} is the minimum inductance required for continuous current. In practice, a value of inductance greater than L_{min} is desirable to ensure continuous current.

Since the converter components are assumed to be ideal, the power supplied by the source must be the same as the power absorbed by the load resistor.

$$P_s = P_o$$

$$V_s I_s = V_o I_o$$

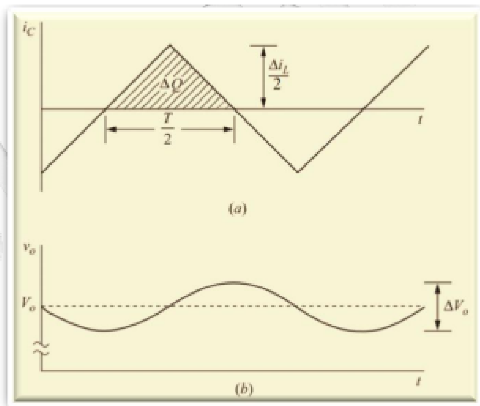
$$\frac{V_o}{V_s} = \frac{I_s}{I_o}$$

This relationship is similar to the voltage-current relationship for a transformer in AC applications. Therefore, the buck converter circuit is equivalent to a DC transformer.

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$

While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,



$$Q = CV_o$$

$$\Delta Q = C\Delta V_o$$

$$\Delta V_o = \frac{\Delta Q}{C}$$

The change in charge ΔQ is the area of the triangle above the time axis

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$$

$$\Delta V_o = \frac{T\Delta i_L}{8C}$$

Substitute $(\Delta i_L)_{open}$ in the above equation yields

$$\Delta V_o = \frac{T V_o}{8CL}(1 - D)T = \frac{V_o(1 - D)}{8LCf^2}$$

ΔV_o is the peak-to-peak ripple voltage at the output

The required capacitance in terms of specified voltage ripple:

$$C = \frac{1 - D}{8L(\Delta V_o/V_o)f^2}$$

Examlpe: buck dc-dc converter with Low Pass Filter has the following parameters:

$V_s = 50$ V $L = 400$ μH $f = 20$ kHz
$D = 0.4$ $C = 100$ μF $R = 20\ \Omega$

Assuming ideal components, calculate (a) the output voltage Vo, (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

(a) $V_o = V_s D = (50)(0.4) = 20\text{ V}$

(b)

$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75\text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$= 1 - \frac{1.5}{2} = 0.25\text{ A}$$

The average inductor current is 1 A, and Δi_L=1.5 A.

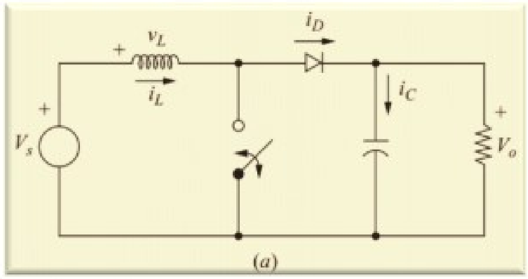
(c)

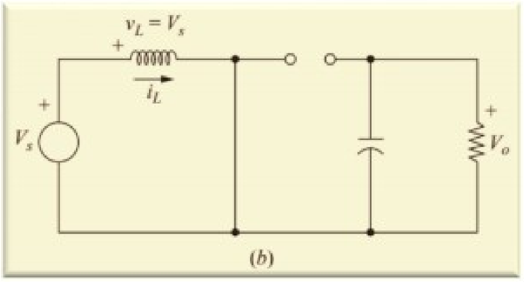
$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$

$$= 0.00469 = 0.469\%$$

The Boost (Step-Up) Converter

- It is called a boost converter because the output voltage is larger than the input.





Analysis for the Switch Closed

When the switch is closed, the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

$$v_L = V_s = L\frac{di_L}{dt} \qquad \text{or} \qquad \frac{di_L}{dt} = \frac{V_s}{L}$$

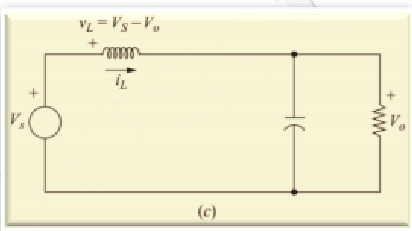
The rate of change of current is a constant, so the current increases linearly while the switch is closed. The change in inductor current is computed from

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L} \qquad \textbf{or}$$

$$(\Delta i_L)_{closed} = \int_0^{DT} \frac{V_s}{L}\,dt = \frac{V_s}{L}DT \qquad (1)$$

***

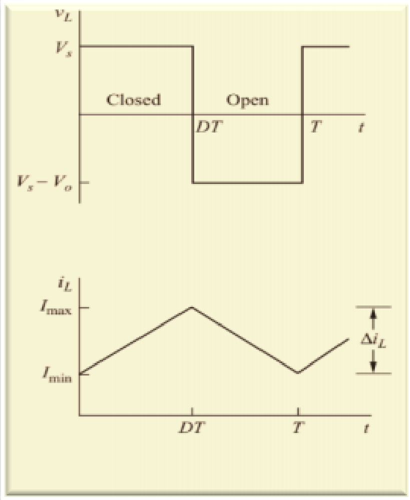
Analysis for the Switch opened



When the switch is opened, the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current. Assuming that the output voltage V_o is a constant, the voltage across the inductor is

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$



The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L} \qquad \text{or}$$

$$(\Delta i_L)_{opened} = \int_0^{(1-D)T} \frac{V_S - V_O}{L}\,dt = \frac{V_S - V_O}{L}(1-D)T \qquad (2)$$

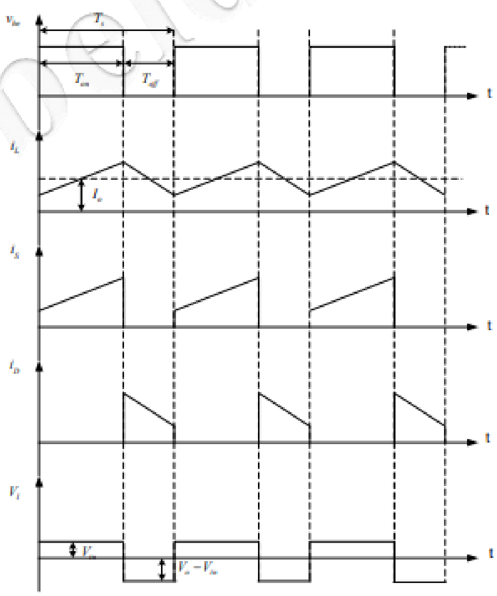
For steady-state operation, the net change in inductor current must be zero. Using equations 1&2

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s DT}{L} + \frac{(V_s - V_o)(1-D)T}{L} = 0$$

$$V_s(D + 1 - D) - V_o(1 - D) = 0$$

$$\boxed{V_o = \frac{V_s}{1-D}} \qquad (3)$$



If the switch is always open and D is zero, the output voltage is the same as the input. As the duty ratio is increased, the denominator of equation 3 becomes smaller, resulting in a larger output voltage. The boost converter produces an output voltage that is greater than or equal to the input voltage. However, the output voltage cannot be less than the input.

The average current in the inductor is determined by recognizing that the average power supplied by the source must be the same as the average power absorbed by the load resistor. Output power is

$$P_o = \frac{V_o^2}{R} = V_o I_o$$

Input power is $V_s I_s = V_s I_L$. Equating input and output powers and using eq. 3

$$V_s I_L = \frac{V_o^2}{R} = \frac{[V_s/(1-D)]^2}{R} = \frac{V_s^2}{(1-D)^2 R}$$

$$I_L = \frac{V_s}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s} \qquad (4)$$

Maximum and minimum inductor currents are determined by using the average value and the change in current from eq. 1.

$$I_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L} \qquad (5)$$

$$I_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \qquad (6)$$

Since I_{min}=0 is the boundary between continuous and discontinuous current,

$$I_{min} = 0 = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

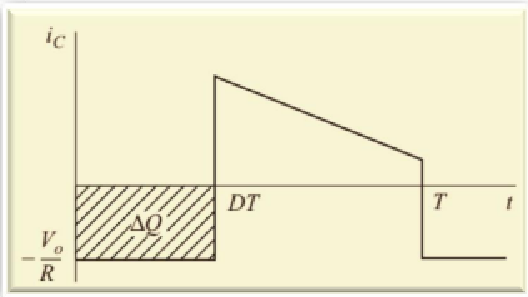
$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf}$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is

$$L_{min} = \frac{D(1-D)^2 R}{2f} \qquad (7)$$

The peak-to-peak output voltage ripple can be calculated from the capacitor current waveform. The change in capacitor charge can be calculated from

$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o$$



An expression for ripple voltage is then

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

expressing capacitance in terms of output voltage ripple yields

$$C = \frac{D}{R(\Delta V_o/V_o)f}$$

Example: Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50. and the switching frequency is 25kHz.

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let L=120 μH.

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5\ \text{A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2\ \text{A}$$

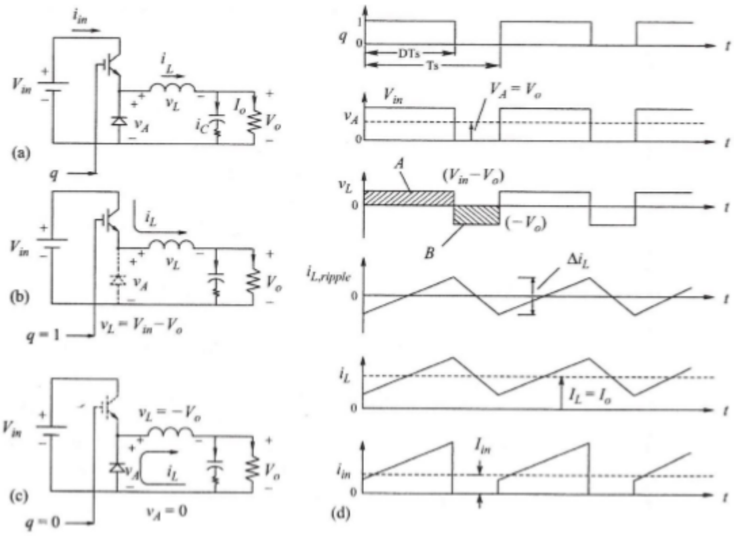
$$I_{\max} = 1.5 + 1.2 = 2.7\,\text{A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3\,\text{A}$$

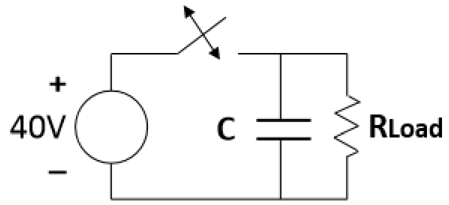
$$C \ge \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Buck Converter Analysis

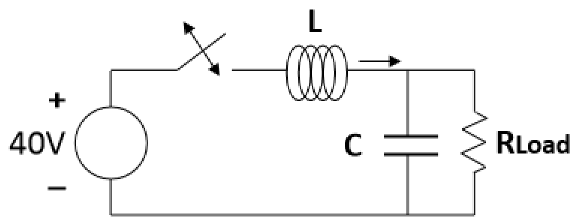
- $V_o = V_A = DV_{in}$; D = switch duty ratio
- $\Delta_{iL} = \frac{1}{L}(V_{in} - V_o)DT_s = \frac{1}{L}V_o(1-D)T_s$
- $I_L = Io = \frac{Vo}{R}$



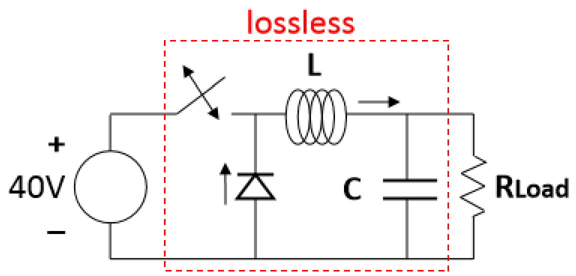
Examples of DC Conversion



Try adding a large C in parallel with the load to control ripple. But if the C has 13Vdc, then when the switch closes, the source current spikes to a huge value and burns out the switch.



Try adding an L to prevent the huge current spike. But now, if the L has current when the switch attempts to open, the inductor's current momentum and resulting *Ldi/dt* burns out the switch.



A DC-DC Buck Converter

By adding a "free wheeling" diode, the switch can open and the inductor current can continue to flow. With high-frequency switching, the load voltage ripple can be reduced to a small value.

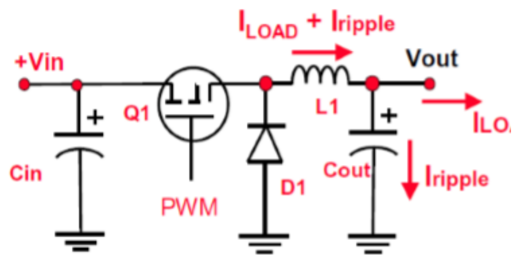
Designing a Buck Converter

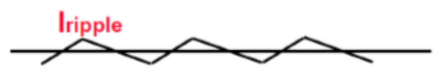
Assume:

- V_{in} = 12 V
- V_{OUT} = 5 volts
- I_{LOAD} = 2 amps
- F_{sw} = 400 KHz
- D = V_{in} / V_{out} = 5V /12V = 0.416

Define Ripple current:

$I_{ripple} = 0.3 \bullet I_{LOAD}$ (typically 30%)





For an Inductor: $V = L \bullet \Delta I / \Delta T$

Rearrange and substitute:

L = (Vin – Vout) • (D / Fsw) / Iripple

Calculate:

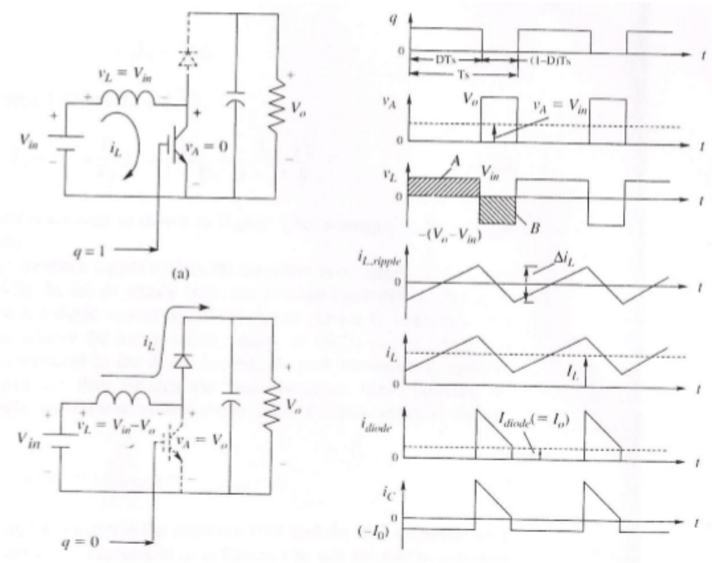
L = 7 V • (0.416 / 400 kHz) / 0.6A

L = 12.12 uh

Select C, Diode (Schottky),
and the MOSFET
Calculate the Efficiency

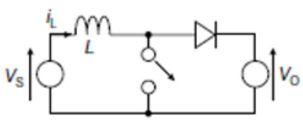
Boost Converter

- $\Delta_{iL} = \frac{1}{L}(V_{in})DT_s = \frac{1}{L}(V_o - V_{in})(1-D)T_s$
- $\frac{V_o}{V_{in}} = \frac{1}{1-D}$

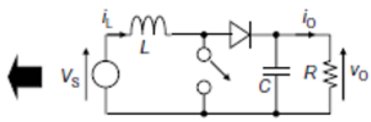


Boost (Step Up) Converter

- Step-up
- Same components
- Different topology!

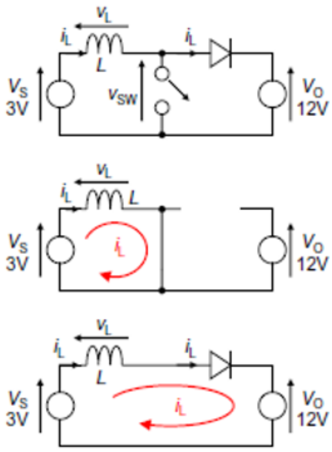


boost converter operating into a voltage source

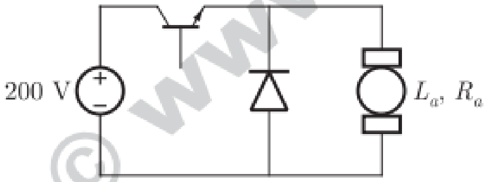


boost converter with resistive load

- See stages of operation



Q. 2 The separately excited dc motor in the figure below has a rated armature current of 20 A and a rated armature voltage of 150 V. An ideal chopper switching at 5 kHz is used to control the armature voltage. If $L_a = 0.1$ mH, $R_a = 1\,\Omega$, neglecting armature reaction, the duty ratio of the chopper to obtain 50% of the rated torque at the rated speed and the rated field current is



(A) 0.4

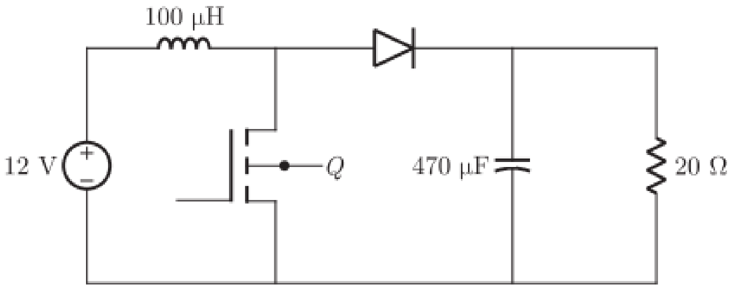
(B) 0.5

(C) 0.6

(D) 0.7

Common Data For Q. 3 and 4

In the figure shown below, the chopper feeds a resistive load from a battery source. MOSFET Q is switched at 250 kHz, with duty ratio of 0.4. All elements of the circuit are assumed to be ideal



Sol. 2 Option (D) is correct.

Given, the rated armature current

$$I_{a(rated)} = 20\text{ A}$$

as rated armature voltage

$$V_{a(rated)} = 150\text{ volt}$$

Also, for the armature, we have

$$L_a = 0.1\text{ mH}, \ R_a = 1\,\Omega$$

and $$T = 50\% \text{ of } T_{rated} \qquad (\text{T} \rightarrow \text{Torque})$$

So, we get

$$I = [I_{a(\text{rotated})}](0.5) = 10\text{ A}$$

$$N = N_{\text{rated}},$$

$$I_f = I_{f\,rated} \rightarrow \text{rated field current}$$

At the rated conditions,

$$E = V - I_{a(\text{rated})}R_a$$

$$= 150 - 20(1) = 130\text{ volt}$$

For given torque,

$$V = E + I_a R_a = 130 + (10)(1) = 140\text{ V}$$

Therefore, chopper output $= 140$ V

or, $$D(200) = 140$$

or, $$D = \frac{140}{200} = 0.7 \qquad (D \rightarrow \text{duty cycle})$$

Q. 3 The Peak to Peak source current ripple in amps is

(A) 0.96 (B) 0.144

(C) 0.192 (D) 0.228

Sol. 3 Option (C) is correct.

Here, as the current from source of 12 V is the same as that pass through inductor. So, the peak to peak current ripple will be equal to peak to peak inductor current.

Now, the peak to peak inductor current can be obtained as

$$I_L \text{ (Peak to Peak)} = \frac{V_s}{L} D T_s$$

where, $V_s \to$ source voltage $= 12$ volt,

$L \to$ inductance $= 100\mu\text{H} = 10^{-4}\text{H}$,

$D \to$ Duty ration $= 0.4$,

$T_S \to$ switching time period of MOSFET $= \frac{1}{f_S}$

and $f_s \to$ switching frequency $= 250$ kHz

Therefore, we get

$$I_{L(\text{Peak to Peak})} = \frac{12}{10^{-4}} \times 0.4 \times \frac{1}{250 \times 10^3} = 0.192 \text{ A}$$

This is the peak to peak source current ripple.

Sol. 4 Option (B) is correct.

Here, the average current through the capacitor will be zero. (since, it is a boost converter). We consider the two cases :

Case I : When MOSFET is ON

$$i_{c_1} = -i_0 \qquad (i_0 \text{ is output current})$$

(since, diode will be in cut off mode)

Case II : When MOSFET is OFF

Diode will be forward biased and so

$$i_{c_2} = I_s - i_0 \qquad (I_s \text{ is source current})$$

Therefore, average current through capacitor

$$I_{c,\,avg} = \frac{i_{c_1} + I_{c_2}}{2}$$

$$\Rightarrow \quad 0 = \frac{DT_s(-i_o) + (1-D)T_s(I_s - i_o)}{2} \quad (D \text{ is duty ratio})$$

Solving the equation, we get

$$I_s = \frac{i_0}{(1-D)} \qquad \text{....(1)}$$

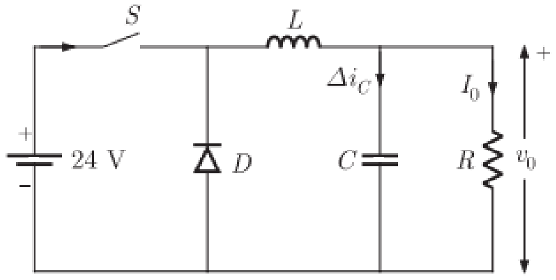
Since, the output load current can be given as

$$i_0 = \frac{V_0}{R} = \frac{V_s/_{1-D}}{R} = \frac{12/0.6}{20} = 1\text{A}$$

Hence, from Eq. (1)

$$I_s = \frac{i_0}{1-D} = \frac{1}{0.6} = \frac{5}{3}\text{A}$$

Q. 9 In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio of 50%. Given that Δi_c is 1.6 A peak-to-peak and I_0 is 5 A dc, the peak current in S, is

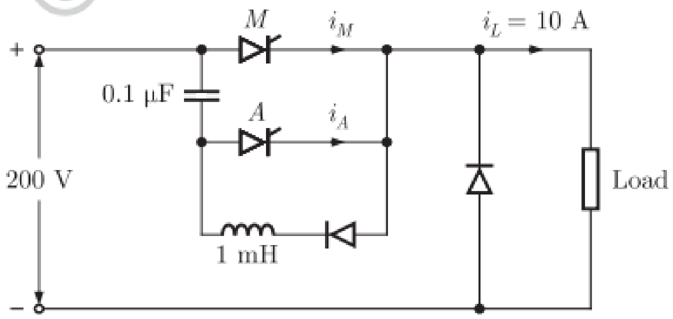


(A) 6.6 A
(B) 5.0 A
(C) 5.8 A
(D) 4.2 A

Sol. 9 Option (C) is correct.

$$I_S = I_0 + \frac{\Delta i_c}{2} = 5 + 0.8 = 5.8 \text{ A}$$

Q. 14 A voltage commutated chopper circuit, operated at 500 Hz, is shown below.



If the maximum value of load current is 10 A, then the maximum current through the main (M) and auxiliary (A) thyristors will be

(A) $i_{M\max} = 12$ A and $i_{A\max} = 10$ A

(B) $i_{M\max} = 12$ A and $i_{A\max} = 2$ A

(C) $i_{M\max} = 10$ A and $i_{A\max} = 12$ A

(D) $i_{M\max} = 10$ A and $i_{A\max} = 8$ A

Sol. 14 Option (A) is correct.

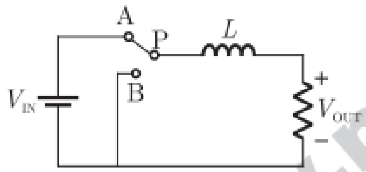
Maximum current through main thyristor

$$I_M(\max) = I_0 + V_s\sqrt{\frac{C}{L}} = 10 + 200\sqrt{\frac{0.1\times 10^{-6}}{1\times 10^{3}}} = 12 \text{ A}$$

Maximum current through auxiliary thyristor

$$I_A(\max) = I_0 = 10 \text{ A}$$

Q. 17 The power electronic converter shown in the figure has a single-pole double-throw switch. The pole P of the switch is connected alternately to throws A and B. The converter shown is a



(A) step down chopper (buck converter)

(B) half-wave rectifier

(C) step-up chopper (boost converter)

(D) full-wave rectifier

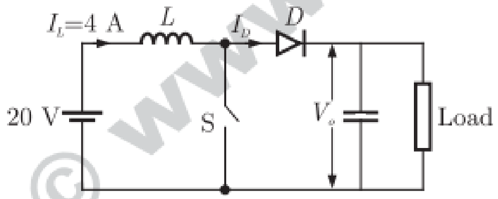
Sol. 17 Option (A) is correct.

The figure shows a step down chopper circuit.

$\because$ $$V_{out} = DV_{in}$$

where, $D =$ Duty cycle and $D < 1$

Q. 33 In the circuit shown in the figure, the switch is operated at a duty cycle of 0.5. A large capacitor is connected across the load. The inductor current is assumed to be continuous.



The average voltage across the load and the average current through the diode will respectively be

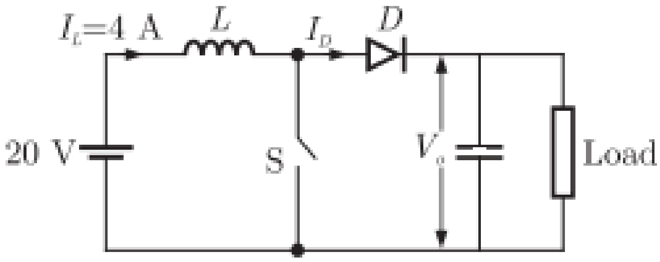
(A) 10 V, 2 A

(B) 10 V, 8 A

(C) 40 V 2 A

(D) 40 V, 8 A

Sol. 33 Option (C) is correct.



In the given diagram

when switch S is open $I_0 = I_L = 4$ A, $V_s = 20$ V

when switch S is closed $I_D = 0$, $V_0 = 0$ V

Duty cycle $= 0.5$ so average voltage is $\frac{V_s}{1-\delta}$

$$\text{Average current } = \frac{0+4}{2} = 2 \text{ amp}$$

$$\text{Average voltage } = \frac{20}{1-0.5} = 40 \text{ V}$$

Q. 44 The minimum approximate volt-second rating of pulse transformer suitable for triggering the SCR should be : (volt-second rating is the maximum of product of the voltage and the width of the pulse that may applied)

(A) 2000 μV-s (B) 200 μV-s

(C) 20 μV-s (D) 2 μV-s

Sol. 44 Option (A) is correct.

We know that the pulse width required is equal to the time taken by i_a to rise upto i_L

so, $$V_s = L\frac{di}{dt} + R_i(V_T \approx 0)$$

$$i_a = \frac{200}{1}[1 - e^{-t/0.15}]$$

Here also $t = T$, $i_a = i_L = 0.25$

$$0.25 = 200[1 - e^{-T/0.5}]$$

$$T = 1.876 \times 10^{-4} = 187.6 \text{ µs}$$

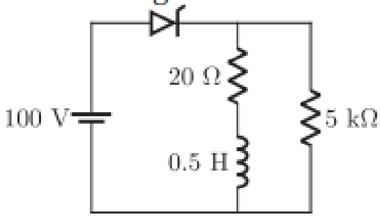
$$\text{Width of pulse} = 187.6 \text{ µs}$$

$$\text{Magnitude of voltage} = 10 \text{ V}$$

$$V_{sec} \text{ rating of P.T.} = 10 \times 187.6 \text{ µs}$$

$$= 1867 \text{ µV-s is approx to } 2000 \text{ µV-s}$$

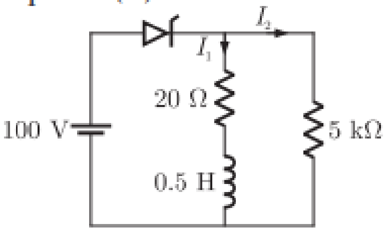
Q. 52 An SCR having a turn ON times of 5 μsec, latching current of 50 A and holding current of 40 mA is triggered by a short duration pulse and is used in the circuit shown in figure. The minimum pulse width required to turn the SCR ON will be



(A) 251 μsec (B) 150 μsec

(C) 100 μsec (D) 5 μsec

Sol. 52 Option (B) is correct.



In this given circuit minimum gate pulse width time= Time required by i_a rise up to i_L

$$i_2 = \frac{100}{5 \times 10^3} = 20 \text{ mA}$$

$$i_1 = \frac{100}{20}[1 - e^{-40t}]$$

$\therefore$ anode current $I = I_1 + I_2 = 0.02 + 5[1 - e^{-40t}]$

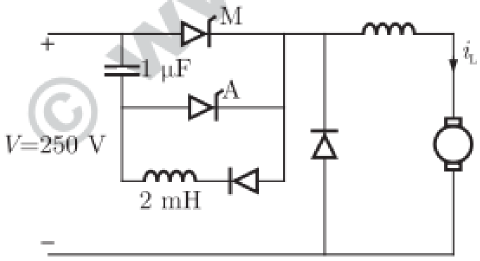
$$0.05 = 0.05 + 5[1 - e^{-40t}]$$

$$1 - e^{-40t} = \frac{0.03}{5}$$

$$T = 150 \text{ µs}$$

Common Data For Q. 53 and 54

A voltage commutated chopper operating at 1 kHz is used to control the speed of dc as shown in figure. The load current is assumed to be constant at 10 A



Q. 53 The minimum time in μsec for which the SCR M should be ON is.

(A) 280 (B) 140

(C) 70 (D) 0

Q. 54 The average output voltage of the chopper will be

(A) 70 V

(B) 47.5 V

(C) 35 V

(D) 0 V

Sol. 53 Option (B) is correct.

Given $I_L = 10$ A. So in the +ve half cycle, it will charge the capacitor, minimum time will be half the time for one cycle.

so min time required for charging

$$= \frac{\pi}{\omega_0} = \pi\sqrt{LC} = 3.14 \times \sqrt{2 \times 10^{-3} \times 10^{-6}} = 140\ \mu\text{sec}$$

Sol. 54 Option (C) is correct.

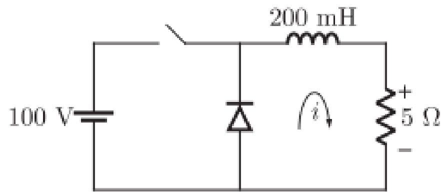
Given $T_{on} = 140\ \mu\text{sec}$

$$\text{Average output} = \frac{T_{on}}{T_{total}} \times V$$

$$T_{total} = 1/f = \frac{1}{103} = 1\ \text{msec}$$

$$\text{so average output} = \frac{140 \times 10^{-6}}{1 \times 10^{-3}} \times 250 = 35\ \text{V}$$

Q. 60 The given figure shows a step-down chopper switched at 1 kHz with a duty ratio $D = 0.5$. The peak-peak ripple in the load current is close to



(A) 10 A
(B) 0.5 A
(C) 0.125 A
(D) 0.25 A

Sol. 60 Option (C) is correct.

$$\text{Duty ratio } \alpha = 0.5$$

here
$$T = \frac{1}{1 \times 10^{-3}} = 10^{-3} \text{ sec}$$

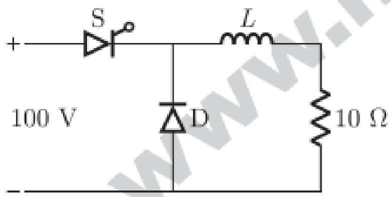
$$T_a = \frac{L}{R} = \frac{200 \text{ mH}}{5} = 40 \text{ msec}$$

$$\text{Ripple} = \frac{V_s}{R}\left[\frac{(1 - e^{-\alpha T/T_s})(1 - e^{-(1-\alpha)T/T_a})}{1 - e^{-T/T_s}}\right]$$

$$(\Delta I)_{\max} = \frac{V_s}{4fL} = \frac{100}{4 \times 10^3 \times 200 \times 10^{-3}}$$

$$= 0.125 \text{ A}$$

Q. 69 Figure shows a chopper operating from a 100 V dc input. The duty ratio of the main switch S is 0.8. The load is sufficiently inductive so that the load current is ripple free. The average current through the diode D under steady state is

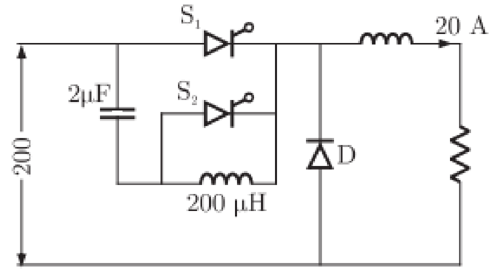


(A) 1.6 A
(B) 6.4 A
(B) 8.0 A
(D) 10.0 A

Sol. 69 Option (C) is correct.

$V_s = 100$ V, duty ratio $= 0.8$, $R = 10\ \Omega$

Q. 70 Figure shows a chopper. The device S_1 is the main switching device. S_2 is the auxiliary commutation device. S_1 is rated for 400 V, 60 A. S_2 is rated for 400 V, 30 A. The load current is 20 A. The main device operates with a duty ratio of 0.5. The peak current through S_1 is



(A) 10 A (B) 20 A

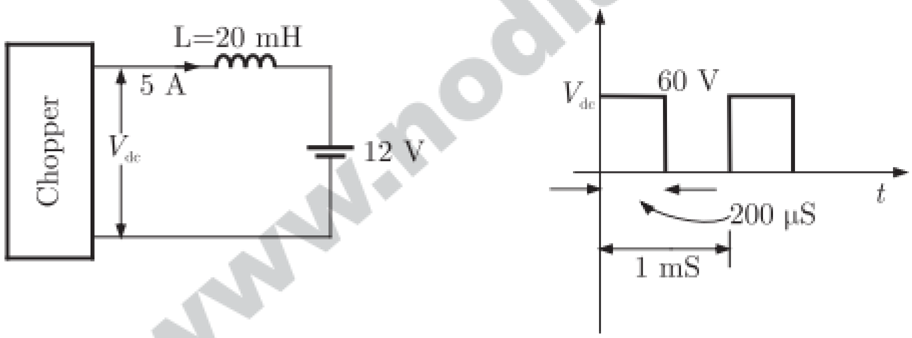
(C) 30 A (D) 40 A

Sol. 70 Option (D) is correct.

Peak current through S_1

$$I = I_0 + V_S\sqrt{C/L} = 20 + 200\sqrt{\frac{2\times 10^{-6}}{200\times 10^{-6}}} = 40\text{ A}$$

Q. 78 A chopper is employed to charge a battery as shown in figure. The charging is 5 A. The duty ratio is 0.2. The chopper output voltage is also shown figure. The peak to peak ripple current in the charging current is



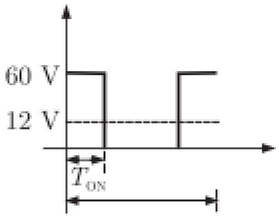
(A) 0.48 A (B) 1.2 A

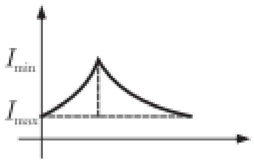
(C) 2.4 A (D) 1 A

Sol. 78 Option (A) is correct.

In the chopper during turn on of chopper V-t area across L is,

$$\int_0^{T_{on}} V_L\, dt = \int_0^{T_{on}} L\left(\frac{di}{dt}\right) dt = \int_{i_{min}}^{i_{max}} L di = L(i_{max} - i_{min}) = L(\Delta I)$$



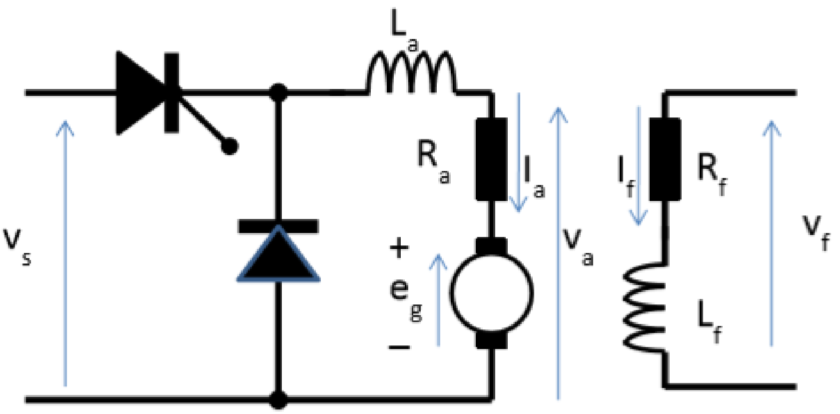


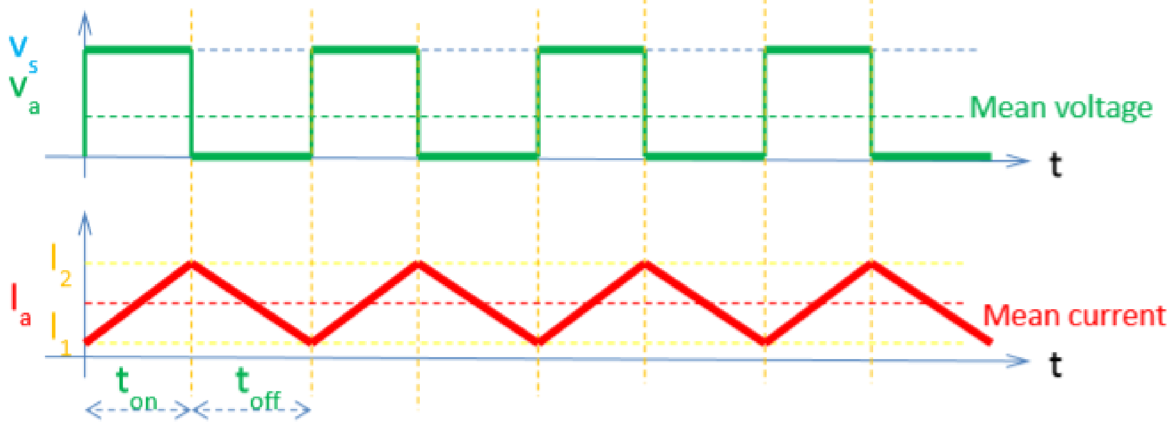
V-t are applied to 'L' is $= (60 - 12)\, T_{on} = 48\, T_{on}$

DC-DC MOTOR DRIVES (Choppers)

A chopper directly converts a fixed-voltage DC supply to a variable-voltage DC supply.

Step-Down Chopper (Motoring)





During t_{on} time		During t_{off} time	
$Vs = L\frac{di}{dt} + Va$	(1)	$0 = -L\frac{di}{dt} + Va$	(5)
$Vs - Va = L\frac{di}{dt}$	(2)	$Va = L\frac{di}{dt}$	(6)
$di = \frac{Vs-Va}{L}dt$	(3)	$di = \frac{Va}{L}dt$	(7)
$\Delta I = I_2 - I_1 = \frac{Vs-Va}{L}t_{on}$	(4)	$\Delta I = \frac{Va}{L}t_{off}$	(8)

$$t_{on} = DT,\ t_{off} = (1-D)T \quad where D\ is\ the\ Duty\ cycle$$

Equating ΔIs

$$\Delta I = \frac{Vs-Va}{L}t_{on} = \frac{Va}{L}t_{off} = \frac{Vs-Va}{L}DT = \frac{Va}{L}(1-D)T \Rightarrow \boxed{Va = DVs}$$

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \qquad (9) \qquad\qquad I_1 + I_2 = 2I_{mean} \quad (10)$$

adding (10) to (4) or (8)

$$2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{on} \qquad or \qquad 2I_2 = 2I_{mean} + \frac{Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{on}} \qquad or \qquad \boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{on}} \qquad or \qquad \boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{off}}$$

Example : A simple DC step-down chopper is operated at a frequency of 2KHz from a 120 V DC source to supply a motor load with Ra = 0.85 ohms, La = 0.32 mH. The required torque genrated by the motor is 20 Nm, at 1000 rpm, and field current is measured to be 1A. If Kv = 0.8345 V/A-rad/S, determine (a) the duty cycle for the switching pulse, (b) the mean load current, and (c) the max & min load currents.

(a) $V_a = DV_s = I_aR_a + E_g \qquad I_a = ? \qquad E_g = ?$

$$T = K_V I_a I_f \Rightarrow I_a = \frac{20}{0.8345 \ x \ 1} = 23.96 \ A$$

$$E_g = K_V \omega I_f = 0.8345 \ x \ 2\pi \ x \ \frac{1000}{60} = 87.38 \ V$$

$V_a = 23.96 \ x \ 0.85 + 87.38 = 107.75$ therefore $D = \frac{107.74}{120} = 0.89 = 89\%$

(b) $I_{mean} = I_a = 23.96 \ A$

(c) $I_{max} = I_{mean} + \frac{Vs-Va}{2L}t_{on} \qquad or \qquad I_{max} = I_{mean} + \frac{Va}{2L}t_{off}$

We know $t_{on} = DT, \ \ t_{off} = (1-D)T \quad and \quad T = 1/f$

$$I_{max} = 23.96A + \frac{120V - 107.75V}{2*0.32mH} * 0.89 * \frac{1}{2000} = 32.47A$$

$$I_{min} = I_{mean} - \frac{Vs-Va}{2L}t_{on} \qquad or \qquad I_1 = I_{mean} - \frac{Va}{2L}t_{off}$$

$$I_{min} = 23.96A - \frac{120V - 107.75V}{2*0.32mH} * 0.89 * \frac{1}{2000} = 15.45 \ A$$

Example : A separately excited DC motor is powerd by a DC chopper from a 600 V dc source. The armature resistance Ra = 0.05 ohms. The back e.m.f.constant of the motor is kv=1.527 V/A-rads/s. The armature voltage is continuous and ripple free. If the duty cycle of the copper is 60%, determine (a) the input power from the source, (b) the equivalent input resistance of the chopper drive, (c) the motor speed, and (d) the developed torque.

(a) P_{input}= ?, $P_{input} = DV_sI_a = 0.6 \ x \ 600 \ x \ 250 = 90$ kw

(b) Req= ?, $Req = \frac{Vs}{Is} = \frac{Vs}{DIs} = \frac{600V}{0.6 \; x \; 250A} = 4 \; \Omega$

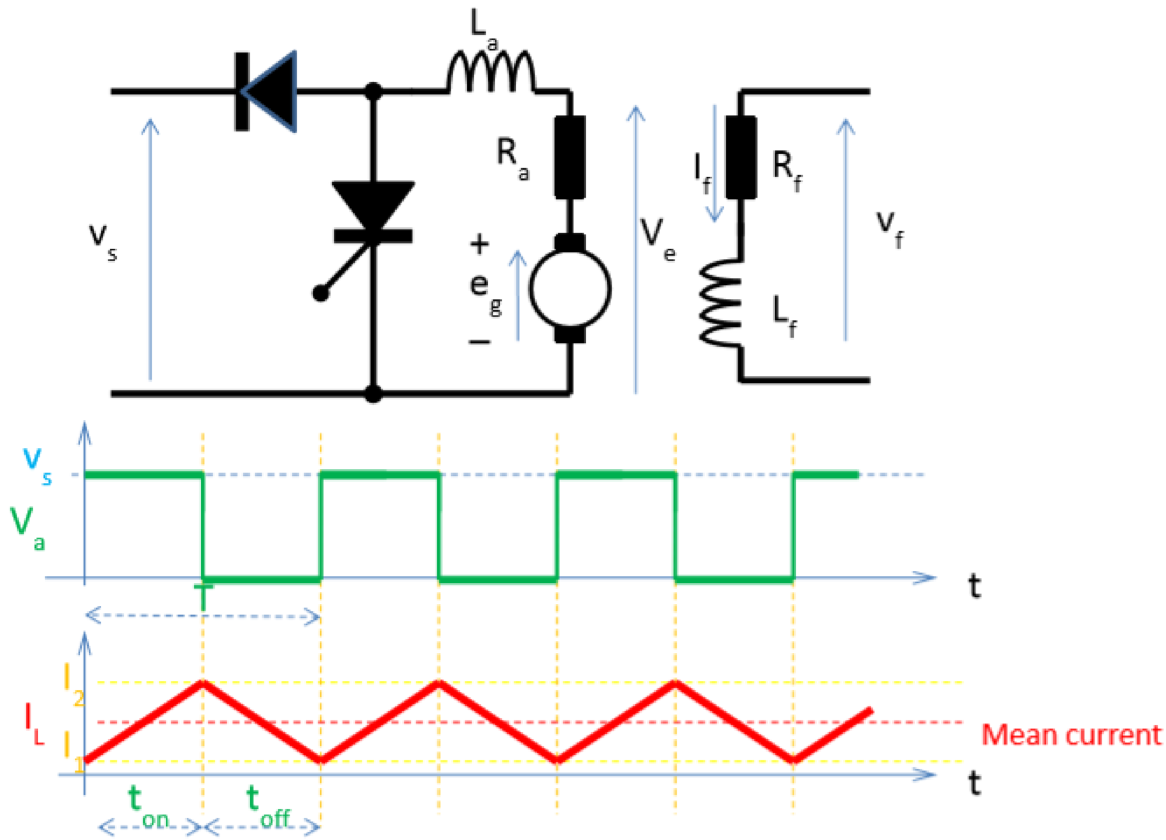
(c) ω = ?, $E_g = k_v \omega$ I, E_g= ?, $V_a = I_a R_a + E_g$, V_a = ?, $V_a = DV_s$ = 0.6 x 600 = 360 V

E_g = 360 – 250x0.05 = 347.5 V

$\omega = \frac{347.5V}{1.525 \; x \; 2.5A} = 91.03$ rad/s or $91.03 \; x \; \frac{60}{2\pi}$=869.3 rpm

(d) T_D=?, $T_D = k_v I_f I_a$ = 1.527 x 250 x 2.5 = 954.38 Nm

Step-Up Chopper – (Regenerative Braking)



During t_{on} time

$$Ve = L\frac{di}{dt} \qquad (1)$$

$$di = \frac{Ve}{L}dt \qquad (2)$$

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} \qquad (3)$$

During t_{off} time

$$Ve = -L\frac{di}{dt} + Vs \qquad (4)$$

$$di = \frac{Vs-Ve}{L}\,dt \qquad (5)$$

$$\Delta I = I_2 - I_1 = \frac{Vs-Ve}{L}t_{off} \qquad (6)$$

Equating ΔIs

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} = \frac{Vs-Ve}{L}t_{off} => \frac{Ve}{L}DT = \frac{Vs-Ve}{L}(1-D)T \qquad (7)$$

$$VeD = Vs - Ve - VsD + VeD => Vs = \frac{Ve}{1-D}$$

Since average voltage across L is zero, therefore Ve = Va and $\boxed{Vs = \frac{Va}{1-D}}$ (8)

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \qquad (9) \qquad\qquad I_1 + I_2 = 2I_{mean} \qquad (10)$$

adding (10) to (3)or(6) remembering Ve = Va

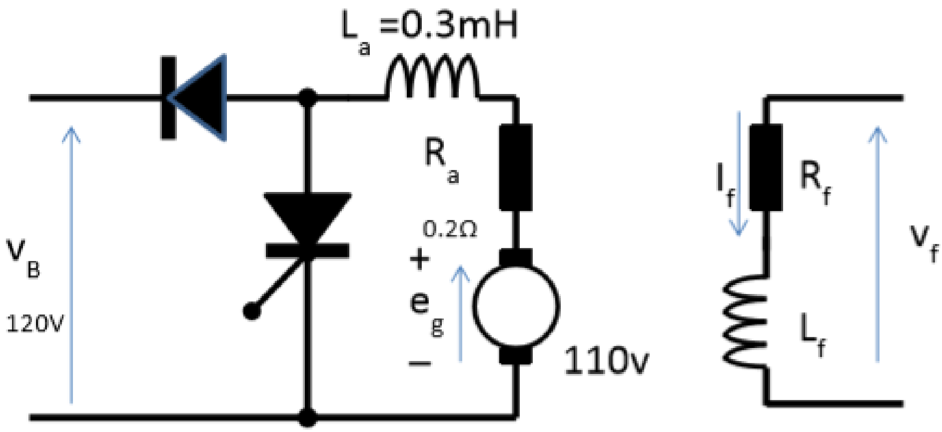
$$2I_2 = 2I_{mean} + \frac{Va}{L}t_{on} \qquad or \qquad 2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{on}} \qquad or \qquad \boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{on}} \qquad or \qquad \boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{off}}$$

Example : In a battery powered car, operating a frequency of 5 KHz, the battery voltage is 120 V. It is driven by a DC motor and employs chopper control. The resistance of the motor is 0.2 ohms and its inductance is 0.3 mH. During braking, the chopper configuration is changed to voltage step-up mode. While going down the hill at a certain speed, the back emf of the motor is 110 V and the breaking current is 10 A. Detrermine (a) the copper duty cycle, and (b) Max and Min values of the current.



(a) D = ?, $V_B = \frac{Va}{1-D} = \frac{IaRa+Eg}{1-D} > 1-D = \frac{IaRa+Eg}{V_B} = \frac{110V+0.2x10}{120} => D = 6.67\%$

(b) $I_{max} = I_{mean} + \frac{Va}{2L}t_{on} = 10A + \frac{112x0.0667}{2x.03x10^{-3}x2000} = 16.22\ A$

$$I_{min} = I_{mean} - \frac{Va}{2L}t_{on} = 10A - \frac{112x0.0667}{2x.03x10^{-3}x2000} = 3.78\ A$$

Problem-Repeat above for 50V back emf.

Motoring and Regenerative Braking `Two-Quadrant Chopper (buck-boost`

Example 13.1: ***DC chopper with load back emf (first quadrant)***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine, with and without (rotor standstill) the back emf:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the average load output current, average switch current, and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, (and electromagnetic efficiency for $E > 0$);
viii. sketch the output current and voltage waveforms.

5. The speed of a separately excited DC motor is controlled by a chopper. The DC supply voltage is 120 V, armature circuit resistance is 0.5 Ω, armature circuit inductance is 20 mH, and back emf constant is 0.05 V/RPM. The motor drives a constant torque load requiring an average current of 20A. Assuming the motor current to be continuous, determine the range of speed control and the range of duty cycle.

Given Data:

V_s=120 volts, R_a=0.5 ohms, L_a=20mH, K=0.05 V/RPM. I_a=20A

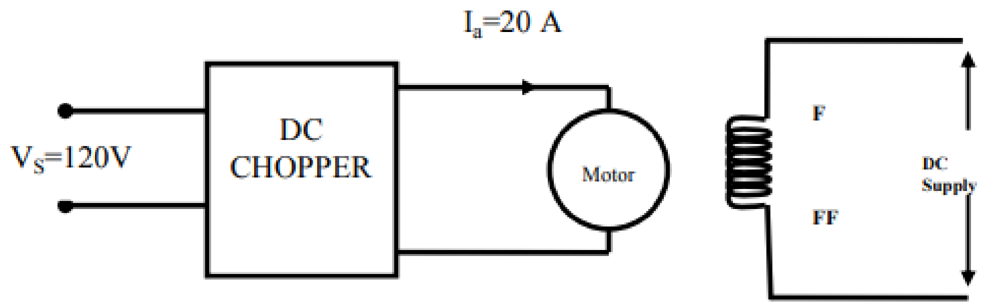
Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution



(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because V_a = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 20A and minimum possible speed is **ZERO**

$$\alpha x 120 = (0.05) x 0 + (20 x 0.05)$$

$$120\alpha = 10$$

$$\alpha = \frac{10}{120} = 0.08$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.08 \leq \alpha \leq 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x120 = 0.05xN + (20x0.5)$$
$$120 = 0.05N + 10$$
$$N = \frac{120 - 10}{0.05} = 2200rpm$$

The range of speed control is $0 \leq N \leq 2200RPM$

6. A 230 volts, 960 rpm, 200 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a dc source of 230 volts through a chopper. Assuming continuous conduction

a) Calculate the duty ratio of chopper for monitoring operation at rated torque and 350 rpm

b) If maximum duty ratio of chopper is limited to 0.95 and maximum permissible motor speed obtainable without field weakening

Given Data

V_s=230 volts, N=960 rpm, I_a=200 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =350rpm.

(ii) If $\alpha = 0.95$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (200x0.02) = 226 volts$$
$$E \text{ at 350 rpm (ie) } \mathrm{E}_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$220 = Kx\omega_1$$
$$\omega_1 = \frac{960x2\pi}{60} = 100.53 rad/\sec$$
$$\therefore K = \frac{226}{100.53} = 2.24 Volts.\sec/rad$$

E_2 at 350 rpm is given by

$$\omega_2 = \frac{350x2\pi}{60} = 36.651$$
$$\therefore E_2 = 36.65x2.24 = 82.1 Volts$$

Motor terminal voltage at 350 rpm is

$$V_{350rpm} = 82.1 + (200x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{350\text{ rpm}}}{\mathrm{V}_{960\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.95\text{x}230 = 218.5 \text{ Volts}$$

$$\therefore E = V_a + I_a R_a = 218.5 + (200x0.02) = 222.5 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{222.5}{2.24} = 99.330 rad/\sec$$
$$N = \frac{99.330x60}{2\pi} = 948.53 rpm$$

7. A DC series motor is fed from a 600 volts source through a chopper. The DC motor has the following parameters armature resistance is equal to 0.04 Ω, field resistance is equal to 0.06 Ω, constant $k = 4 \text{ x } 10^{-3} Nm/Amp^2$. The average armature current of 300 Amps is ripple free. For a chopper duty cycle of 60% determine

i. Input power drawn from the source.
ii. Motor speed and
iii. Motor torque.

Given Data

V_s=600 volts, I_a=300 amps, R_a=0.04 ohms, R_f=0.06 ohms, K=$4x10^{-3} Nm/amp^2$ $\delta = 0.6$

DC SERIES motor.

Solution

a. Power input to the motor = P = $V_a I_a$

$$V_a = \delta V_s = 0.6x600 = 360 Volts$$
$$\therefore P = 360x300 = 108KW$$

b. For a DC series motor

$$E_a = K_a \phi \omega_m$$
$$= \text{KI}_a \omega_m [\because \phi = I_a]$$
$$= 4x10^{-3} x300x\omega_m$$
$$\therefore V_a = E + I_a(R_a + R_s) = KI_a\omega_m + I_a(R_a + R_s)$$
$$\Rightarrow 0.6x600 = 4x10^{-3}x300x\omega_m + 300(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 27.5 rad/\sec (or) 2626 rpm$$
$$Motor \text{ Torque T} = \text{Ka}\phi \text{I}_a = KI_a^2$$
$$= 4\text{x}10^{-3} x300^2$$
$$= 360 \text{ N - M}$$

8. A 230 V, 1100 rpm, 220 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a chopper, which provides both motoring and braking operations. Calculate

i. The duty ratio of chopper for motoring operation at rated torque and 400 rpm

ii. The maximum permissible motor speed obtainable without field weakening, if the maximum duty ratio of the chopper is limited to 0.9 and the maximum permissible motor current is twice the rated current.

Given Data

V_s=230 volts, N=1100 rpm, I_a=220 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =400rpm.

(ii) If $\alpha = 0.9$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (220x0.02) = 225.6 volts$$
$$E \text{ at 400 rpm (ie) } E_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$\omega_1 = \frac{1110x2\pi}{60} = 115.192 rad/\sec$$
$$\therefore K = \frac{225.6}{115.192} = 1.95 Volts.\sec/rad$$

E_2 at 400 rpm is given by

$$\omega_2 = \frac{400x2\pi}{60} = 41.887 rad/\sec$$
$$\therefore E_2 = 41.887x1.95 = 81.68 Volts$$

Motor terminal voltage at 400 rpm is

$$V_{400rpm} = 81.68 + (220x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{400\text{ rpm}}}{V_{1100\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.9\text{x}230 = 207 \text{ Volts}$$

$$\therefore E = V_a + I_a R_a = 207 + (2x220x0.02) = 215.8 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$

$$\omega = \frac{215.8}{1.95} = 110.667 rad/\text{sec}$$

$$N = \frac{110.667x60}{2\pi} = 1056.78 rpm$$

9. A DC chopper is used to control the speed of a separately excited dc motor. The DC voltage is 220 V, R_a= 0.2 Ω and motor constant $K_e\phi$=0.08 V/rpm. The motor drives a constant load requiring an average armature current of 25 A. Determine

    iv. The range of speed control

    v. The range of duty cycle. Assume continuous conduction

Given Data:

V_s=220 volts, R_a=0.2 ohms, L_a=20mH, K=0.08 V/RPM. I_a=25A

Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution

(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because \mathrm{V_a} = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 25A and minimum possible speed is **ZERO**

$$\alpha x220 = (0.08)x0 + (25x0.2)$$

$$220\alpha = 10$$

$$\alpha = \frac{10}{220} = 0.04$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.04 \le \alpha \le 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x220 = 0.08xN + (25x0.2)$$

$$220 = 0.08N + 5$$

$$N = \frac{220-5}{0.08} = 2687.5rpm$$

The range of speed control is $0 \le N \le 2687.5RPM$

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

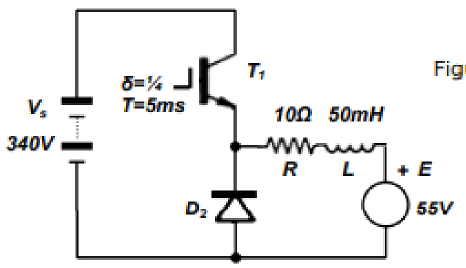


Figure Example 13.1. *Circuit diagram.*

i. From equations (13.2) and (13.3) the average and rms output voltages are both independent of the back emf, namely

$$\overline{V}_o = \frac{t_T}{T} V_s = \delta V_s$$
$$= ¼ \times 340\text{V} = 85\text{V}$$

$$V_r = \sqrt{\frac{t_T}{T}}\, V_s = \sqrt{\delta}\, V_s$$
$$= \sqrt{¼} \times 240\text{V} = 120\text{V rms}$$

ii. The rms ripple voltage hence ripple factor are given by equations (13.4) and (13.5), that is

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$
$$= 340\text{V}\sqrt{¼ \times (1 - ¼)} = 147.2\text{V ac}$$

and

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1}$$
$$= \sqrt{\frac{1}{¼} - 1} = \sqrt{3} = 1.732$$

No back emf, $E = 0$

iii. From equation (13.13), with E=0, the maximum and minimum currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} = 11.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} = 5.62\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{p-p} = \hat{I} - \check{I}$$
$$= 11.90\text{A} - 5.62\text{A} = 6.28\text{A}$$

Alternatively the ripple can be extracted from figure 13.4 using $T/\tau = 1$ and $\delta = ¼$.

iv. From equations (13.11) and (13.12), with $E = 0$, the time domain load current equations are

$$i_o = \frac{V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 34\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 5.62\times e^{\frac{-t}{5\text{ms}}}$$

$$= 34 - 28.38\times e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = \hat{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 11.90\times e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.17), with $E = 0$, is

$$\overline{I}_o = \overline{V}_o/R = 85\text{V}/10\Omega = 8.5\text{A}$$

The average switch current, which is the average supply current, is

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{¼\times(340\text{V - }0)}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(11.90\text{A - }5.62\text{A}) = 2.22\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$
$$= 8.50\text{A} - 2.22\text{A} = 6.28\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s\overline{I}_i = 340\text{V}\times 2.22\text{A} = 754.8\text{W}$$
$$P_{out} = P_{in} = 754.8\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o_{rms}} = \sqrt{\frac{P_{out}}{R}}$$
$$= \sqrt{\frac{754.8\text{W}}{10\Omega}} = 8.7\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$
$$= \frac{340\text{V}}{2.22\text{A}} = 153.2\,\Omega$$

Load back emf, E = 55V

i. and ii. The average output voltage, rms output voltage, ac ripple voltage, and ripple factor are independent of back emf, provided the load current is continuous. The earlier answers for $E = 0$ are applicable.

iii. From equation (13.13), the maximum and minimum load currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-\delta T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} - \frac{55\text{V}}{10\Omega} = 6.40\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{\delta T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} - \frac{55\text{V}}{10\Omega} = 0.12\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{pp} = \hat{I} - \check{I}$$
$$= 6.4\text{A} - 0.12\text{A} = 6.28\text{A}$$

The ripple value is the same as the $E = 0$ case, which is as expected since ripple current is independent of back emf with continuous output current.
Alternatively the ripple can be extracted from figure 13.4 using $T/\tau = 1$ and $\delta = ¼$.

iv. The time domain load current is defined by

$$i_o = \frac{V_s - E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}e^{\frac{-t}{\tau}}$$

$$i_o(t) = 28.5\times\left(1-e^{\frac{-t}{5ms}}\right) + 0.12e^{\frac{-t}{5ms}}$$

$$= 28.5 - 28.38e^{\frac{-t}{5ms}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = -\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}e^{\frac{-t}{\tau}}$$

$$i_o(t) = -5.5\times\left(1-e^{\frac{-t}{5ms}}\right) + 6.4e^{\frac{-t}{5ms}}$$

$$= -5.5 + 11.9e^{\frac{-t}{5ms}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.37) is

$$\overline{I}_o = {V_o - E}\big/{R}$$

$$= 85\text{V-}55\text{V}\big/10\Omega = 3\text{A}$$

The average switch current is the average supply current,

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{1/4 \times (340\text{V} - 55\text{V})}{10\Omega} - \frac{5\text{ms}}{5\text{ms}} \times (6.40\text{A} - 0.12\text{A}) = 0.845\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$

$$= 3\text{A} - 0.845\text{A} = 2.155\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s \overline{I}_i = 340\text{V} \times 0.845\text{A} = 287.3\text{W}$$

$$P_{out} = P_{in} = 287.3\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o_{rms}} = \sqrt{\frac{P_{out} - E\overline{I}_o}{R}}$$

$$= \sqrt{\frac{287.3\text{W} - 55\text{V} \times 3\text{A}}{10\Omega}} = 3.5\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$

$$= \frac{340\text{V}}{0.845\text{A}} = 402.4\ \Omega$$

The electromagnetic efficiency is given by equation (13.22), that is

$$\eta = \frac{E\overline{I}_o}{P_{in}}$$

$$= \frac{55\text{V} \times 3\text{A}}{287.3\text{W}} = 57.4\%$$

viii. The output voltage and current waveforms for the first-quadrant chopper, with and without back emf, are shown in the figure to follow.

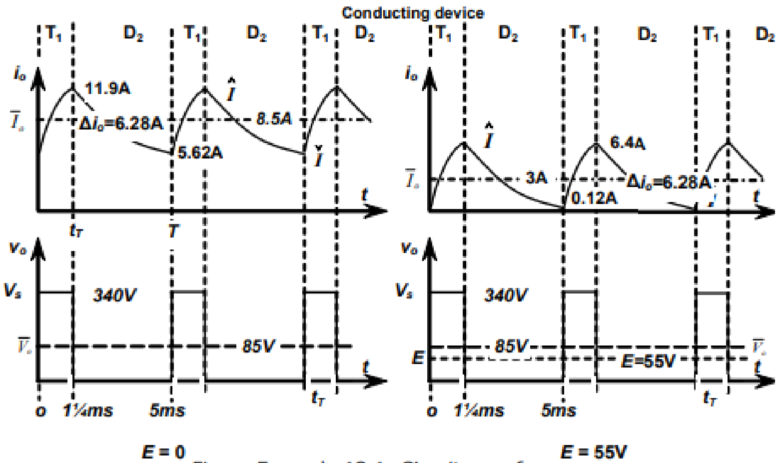


Figure Example 13.1. *Circuit waveforms.*

6. A step-down chopper supplies a separately excited dc motor with a supply voltage $E = 240$ V and back emf $E_b = 100$ V. Other data are total inductance L

$= 30$ mH, armature resistance $R_a = 2.5\ \Omega$, chopper frequency $= 200$, and duty cycle $= 50\%$. Assuming continuous current determine $I_{\max}$, $I_{\min}$, and the current ripple.

Solution

The circuit and waveforms are given in Fig. 3.3. From Eqn (3.17),

$$I_{\max} = \frac{E}{R_a}\left(\frac{1 - e^{-\tau_{ON}/T_a}}{1 - e^{-\tau/T_a}}\right) - \frac{E_b}{R_a}$$

$$\frac{\tau_{ON}}{\tau} = 0.5 \quad \text{and} \quad \tau = \frac{1}{200} = 0.005$$

Hence,

$$\tau_{ON} = \frac{0.5}{200} = 0.0025$$

$$T_a = \frac{L_a}{R_a} = \frac{30 \times 10^{-3}}{2.5} = 0.012$$

$$\frac{\tau_{ON}}{T_a} = \frac{0.0025}{0.012}$$

$$\frac{\tau}{T_a} = \frac{0.005}{0.012}$$

$$(1 - e^{-\tau_{ON}/T_a}) = 0.188, \quad (e^{-\tau_{ON}/T_a} - 1) = 0.2316$$

$$(1 - e^{-\tau/T_a}) = 0.34, \quad (e^{-\tau/T_a} - 1) = 0.5169$$

Hence,

$$I_{\max} = \frac{240}{2.5} \times \frac{0.188}{0.34} - \frac{100}{2.5} = 13.0 \text{ A}$$

From Eqn (3.18),

$$I_{\min} = \frac{E}{R_a}\left[\frac{e^{\tau_{ON}/T_a} - 1}{e^{\tau/T_a} - 1}\right] - \frac{E_b}{R_a}$$

$$= \frac{240}{2.5} \times \frac{0.2316}{0.5169} - \frac{100}{2.5} = 3.0 \text{ A}$$

The current ripple is

$$\Delta i_{ld} = \frac{I_{\max} - I_{\min}}{2} = \frac{13 - 3}{2} = 5 \text{ A}$$

7. A step-down chopper feeds a dc motor load. The data pertaining to this chopper-based drive is $E = 210$ V, $R_a = 7\ \Omega$, L (including armature inductance) = 12 mH. Chopper frequency = 1.5 kHz, duty cycle = 0.55, and $E_b = 55$ V. Assuming continuous conduction, determine the (a) average load current, (b) current ripple, (c) RMS value of current through chopper, (d) RMS value of current through D_{FW}, and (e) effective input resistance seen by the source, and (f) RMS value of load current.

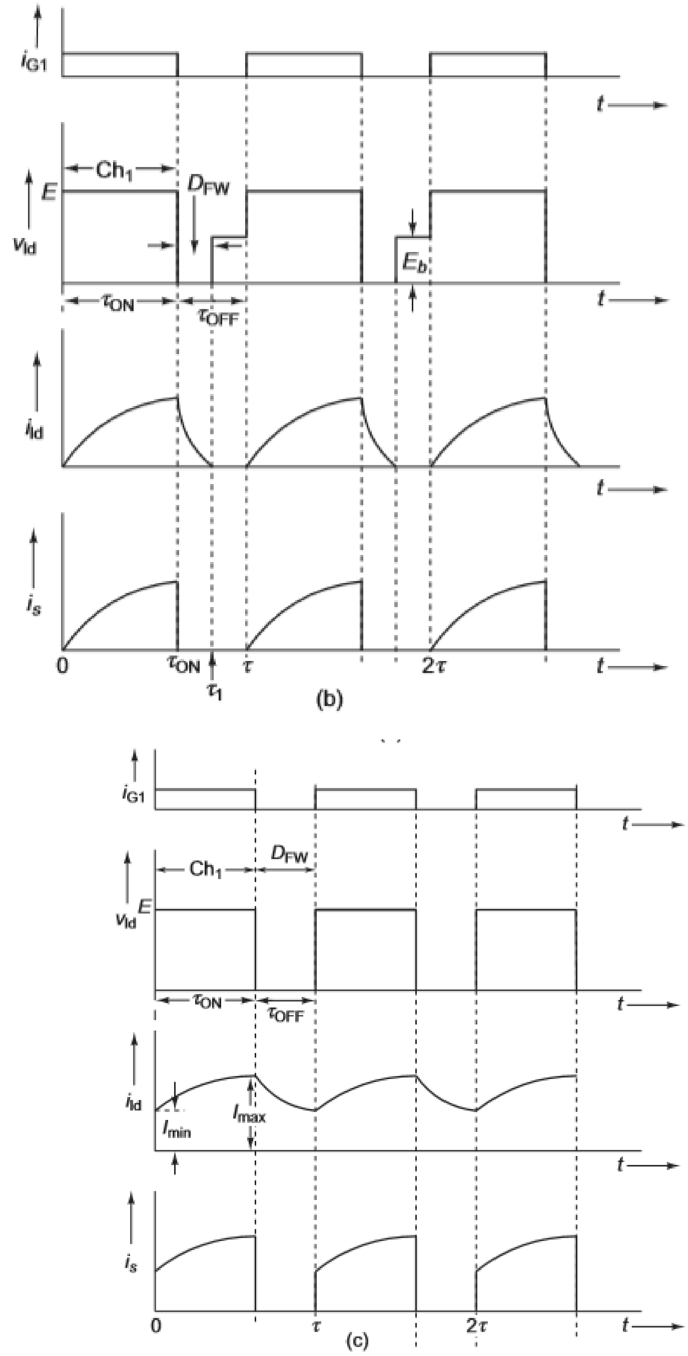
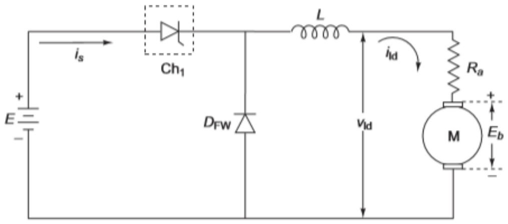


Fig. 3.3 Step-down chopper: (a) circuit, (b) waveforms for discontinuous conduction, (c) waveforms for continuous conduction



(a)

Fig. 3.3(a)

Solution

(a) The circuit and waveforms are given in Fig. 3.3. The average load current is given by Eqn (3.21) as

$$I_{ld} = \frac{E}{R_a}\frac{\tau_{ON}}{\tau} - \frac{E_b}{R_a}$$

Substitution of values gives

$$I_{ld} = \frac{210}{7} \times 0.55 - \frac{55}{7}$$
$$= 8.64 \text{ A}$$

$$I_{max} = \frac{E}{R_a}\frac{(1 - e^{-\tau_{ON}/T_a})}{(1 - e^{-\tau/T_a})} - \frac{E_b}{R_a} \tag{3.17}$$

and

$$I_{min} = \frac{E}{R_a}\frac{(e^{\tau_{ON}/T_a} - 1)}{(e^{\tau/T_a} - 1)} - \frac{E_b}{R_a} \tag{3.18}$$

The current ripple can now be obtained as

$$\Delta i_{ld} = \frac{I_{max} - I_{min}}{2} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - e^{\tau_{ON}/T_a} - e^{\tau_{OFF}/T_a}}{e^{\tau/T_a} - 1}\right] \tag{3.19}$$

(b) The current ripple is given by Eqn (3.19) as

$$\Delta i_{ld} = \frac{E}{2R_a}\left\{\frac{1 + e^{\tau/T_a} - e^{\tau_{ON}/T_a} - e^{\tau_{OFF}/T_a}}{e^{\tau/T_a} - 1}\right\}$$

Here,

$$T_a = \frac{L_a}{R} = \frac{12 \times 10^{-3}}{7}$$

$$\tau = \frac{1}{1.5 \times 10^3} = \frac{1 \times 10^{-3}}{1.5}$$

$$\frac{\tau}{T_a} = \frac{7}{1.5 \times 12} = 0.389$$

$$\frac{\tau_{ON}}{T_a} = \frac{0.55 \times 10^{-3}}{1.5} \times \frac{7}{12} \times 10^{-3} = 0.214$$

$$\frac{\tau_{OFF}}{T_a} = \frac{0.45 \times 10^{-3}}{1.5} \times \frac{7}{12} \times 10^{-3} = 0.175$$

Substitution of values gives

$$\Delta i_{ld} = \frac{210}{2 \times 7} \times \frac{1 + 1.475 - 1.239 - 1.191}{1.475 - 1} = 1.42 \text{ A}$$

(c) It is assumed that the load current increases linearly from $I_{\min}$ to $I_{\max}$ during $(0, \tau_{ON})$. Thus the instantaneous current i_{ld} can be expressed as

$$i_{ld} = I_{\min} + \frac{I_{\max} - I_{\min}}{\tau_{ON}} t, \quad 0 \le t \le \tau_{ON}$$

The RMS value of the current through the chopper can now be found as

$$I_{ch(RMS)} = \sqrt{\frac{1}{\tau}\int_0^{\tau_{ON}} (i_{ld})^2 dt}$$

Here,

$$I_{ch(RMS)} = \left\{\sqrt{\frac{\tau_{ON}}{\tau}\left[I_{\min}^2 + I_{\min}(I_{\max} - I_{\min}) + \frac{(I_{\max} - I_{\min})^2}{3}\right]}\right\}$$

where

$$I_{\min} = \frac{E}{R_a}\left[\frac{e^{\tau_{ON}/T_a} - 1}{e^{\tau/T_a} - 1}\right] - \frac{E_b}{R_a}$$

$$= \frac{210}{7}\left[\frac{1.239 - 1}{1.475 - 1}\right] - \frac{55}{7} = 15.095 - 7.857 = 7.24 \text{ A}$$

$$I_{\max} = \frac{E}{R_a}\left[\frac{1 - e^{-\tau_{ON}/T_a}}{1 - e^{-\tau/T_a}}\right] - E_b/R_a$$

$$= \frac{210}{7}\frac{(1 - 0.807)}{(1 - 0.678)} - \frac{55}{7} = 17.981 - 7.857 = 10.12 \text{ A}$$

Thus,

$$I_{ch(RMS)} = \sqrt{\frac{\tau_{ON}}{\tau}\left[7.24^2 + 7.24(10.12 - 7.24) + \frac{(10.12 - 7.24)^2}{3}\right]}$$

$$= \sqrt{0.55 \times 76.03}$$

$$= 6.46 \text{ A}$$

(d) The RMS value of current through D_{FW} can be found as

$$I_{D_{FW}(RMS)} = \sqrt{\frac{1}{\tau}\int_0^{\tau_{OFF}} (i_{ld})^2 dt'} \quad (t' = t - \tau_{ON})$$

$$= \sqrt{\frac{\tau_{OFF}}{\tau}\left[(10.12)^2 + \frac{(2.88)^2}{3} - 10.12 \times 2.88\right]}$$

(where $I_{\max} - I_{\min} = 10.12 - 7.24 = 2.88$ A)

$$= \sqrt{0.45 \times 76.02} = 5.85 \text{ A}$$

(e) Average source current $= (\tau_{ON}/\tau) \times$ average load current

$$= 0.55 \times \frac{(7.24 + 10.12)}{2} = 0.55 \times 8.68 = 4.77 \text{ A}$$

Hence, the effective input resistance seen by the source = 210/4.77 = 44 Ω.

(f) RMS value of load current $= \sqrt{\frac{1}{\tau}\left[\int_0^{\tau_{ON}} \{i_{ld}(t)\}^2 dt + \int_0^{\tau_{OFF}} \{i_{ld}(t')\}^2 dt'\right]} = 8.72$ A

with $t' = t - \tau_{ON}$. This is seen to be nearly equal to the average load current, namely, 8.68 A.

8. A step-down chopper has the following data: $R_{ld} = 0.40\ \Omega$, $E = 420$ V, $E_b = 25$ V. The average load current is 175 A and the chopper frequency is 280 Hz. Assuming the load current to be continuous, and linearly rising to the maximum and then linearly falling, calculate the inductance L which would limit the maximum ripple in the load current to 12% of the average load current.

Solution

The circuit is given in Fig. 3.3(a). The expression for the current ripple in Eqn (3.19) can be written with substitutions $\tau_{ON} = \delta\tau$ and $\tau_{OFF} = (1-\delta)\tau$, where δ is in the range $0 < \delta < 1$. Thus,

$$\Delta i_{ld} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - e^{\delta\tau/T_a} - e^{(1-\delta)\tau/T_a}}{e^{\tau/T_a} - 1}\right]$$

with $\delta = \tau_{ON}/\tau$. Differentiating the ripple current with respect to δ and equating this to zero gives the value of δ for maximum ripple:

$$\frac{d(\Delta i_{ld})}{d\delta} = \left(-\frac{\tau}{T_a}e^{\delta\tau/T_a}\right) + \frac{\tau}{T_a}e^{(1-\delta)\tau/T_a} = 0$$

This yields

$$e^{\delta\tau/T_a} = e^{(1-\delta)\tau/T_a}$$

or

$$\delta = 1 - \delta$$

This gives $\delta = 0.5$. Substituting this value of δ in the expression for Δi_{ld} gives

$$\Delta i_{ld} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - 2e^{0.5\tau/T_a}}{e^{\tau/T_a} - 1}\right]$$

$$= E/2R_a\left[\frac{(e^{0.5\tau/T_a} - 1)^2}{(e^{0.5\tau/T_a} - 1)(e^{0.5\tau/T_a} + 1)}\right]$$

$$= \frac{E}{2R_a}\left[\frac{e^{0.5\tau/T_a} - 1}{e^{0.5\tau/T_a} + 1}\right]$$

$$= \frac{E}{2R_a}\tanh\frac{R_a}{4fL}$$

where $f = 1/\tau$ is the chopper frequency. If $4fL >> R_a$, then $\tanh(R_a/4fL) \approx R_a/4fL$. Thus,

$$\Delta I_{ld(max)} = \frac{E}{2R_a}\frac{R_a}{4fL} = \frac{E}{8fL}$$

The condition $\Delta I_{ld(max)} = 12\% I_{ld}$ gives

$$\frac{E}{8fL} = 0.12 \times 175$$

Hence,

$$L = \frac{E}{8f \times 0.12 \times 175} = \frac{420}{8 \times 280 \times 0.12 \times 175} = 0.0089 \text{ H}$$

$$= 8.9 \text{ mH}$$

9. A 240-V, separately excited dc motor has an armature resistance of 2.2 Ω and an inductance of 4 mH. It is operated at constant load torque. The initial speed is 600 rpm and the armature current is 28 A. Its speed is now controlled by a step-down chopper with a frequency of 1 kHz, the input voltage remaining at 240 V. (a) If the speed is reduced to 300 rpm, determine the duty cycle of the chopper. (b) Compute the current ripple with this duty cycle.

Solution
(a) The equation for the motor is

$$V = E_b + I_a R_a$$

Substitution of values gives

$$240 = E_b + 28 \times 2.2$$

Thus,

$$E_b = 240 - 28 \times 2.2 = 178.4 \text{ V}$$

Other quantities on the right-hand side of the expression for E_b remaining constant, it can be expressed as

$$E_b = kN$$

or

$$178.4 = k600$$

$$k = \frac{178.4}{600} = 0.297$$

The new speed is 300 rpm. Hence,

$$E_{b(\text{new})} = 0.297 \times 300 = 89.1 \text{ V}$$

The new applied voltage with the same load torque, that is, the same armature current, is

$$V_{\text{new}} = E_b + I_a R_a = 89.1 + 28 \times 2.2 = 150.7 \text{ V}$$

The duty cycle of the chopper (τ_{ON}/τ) can be determined from the relation

$$V_{\text{new}} = \frac{\tau_{\text{ON}}}{\tau} \times \text{input voltage}$$

This gives

$$\frac{\tau_{\text{ON}}}{\tau} = \frac{V_{\text{new}}}{\text{input voltage}} = \frac{150.7}{240} = 0.628$$

(b) $T_a = L/R_a = 4 \times 10^{-3}/2.2$; $\tau = 1/1000 = 10^{-3}$. Therefore,

$$\frac{\tau}{T_a} = \frac{2.2}{4} = 0.55$$

$$\frac{\tau_{\text{ON}}}{T_a} = \frac{\tau_{\text{ON}}}{\tau}\frac{\tau}{T_a} = 0.628 \times 0.55 = 0.345$$

$$\frac{\tau_{\text{OFF}}}{T_a} = \frac{\tau}{T_a} - \frac{\tau_{\text{ON}}}{T_a} = 0.55 - 0.345 = 0.205$$

The current ripple is given as

$$\Delta i_{\text{ld}} = \frac{E}{2R_a}\frac{(1 + e^{\tau/T_a} - e^{\tau_{\text{ON}}/T_a} - e^{\tau_{\text{OFF}}/T_a})}{e^{\tau/T_a} - 1}$$
$$= \frac{150.7}{2 \times 2.2} \times \frac{1 + 1.733 - 1.412 - 1.227}{1.733 - 1}$$
$$= 4.39 \text{ A}$$

10. A dc chopper is used for regenerative braking of a separately excited dc motor as shown in Fig. 7.24(b). The data are $E = 400$ V, $R = 0.2\ \Omega$, and $L_a = 0.2$ mH. The back emf constant K_b^1 $(= K_b\phi_f)$, assuming ϕ_f to be constant, is equal to 1.96 V/rad s, and the average load current is 200 A. The frequency of the chopper is 1 kHz and $(\tau_{ON}/\tau)_a$ is 0.5. Compute the (a) average load voltage, (b) motor speed, (c) power regenerated and fed back to the battery, (d) equivalent resistance viewed from the motor side when it is working as a generator, and (e) minimum and maximum permissible speeds for regenerative braking.

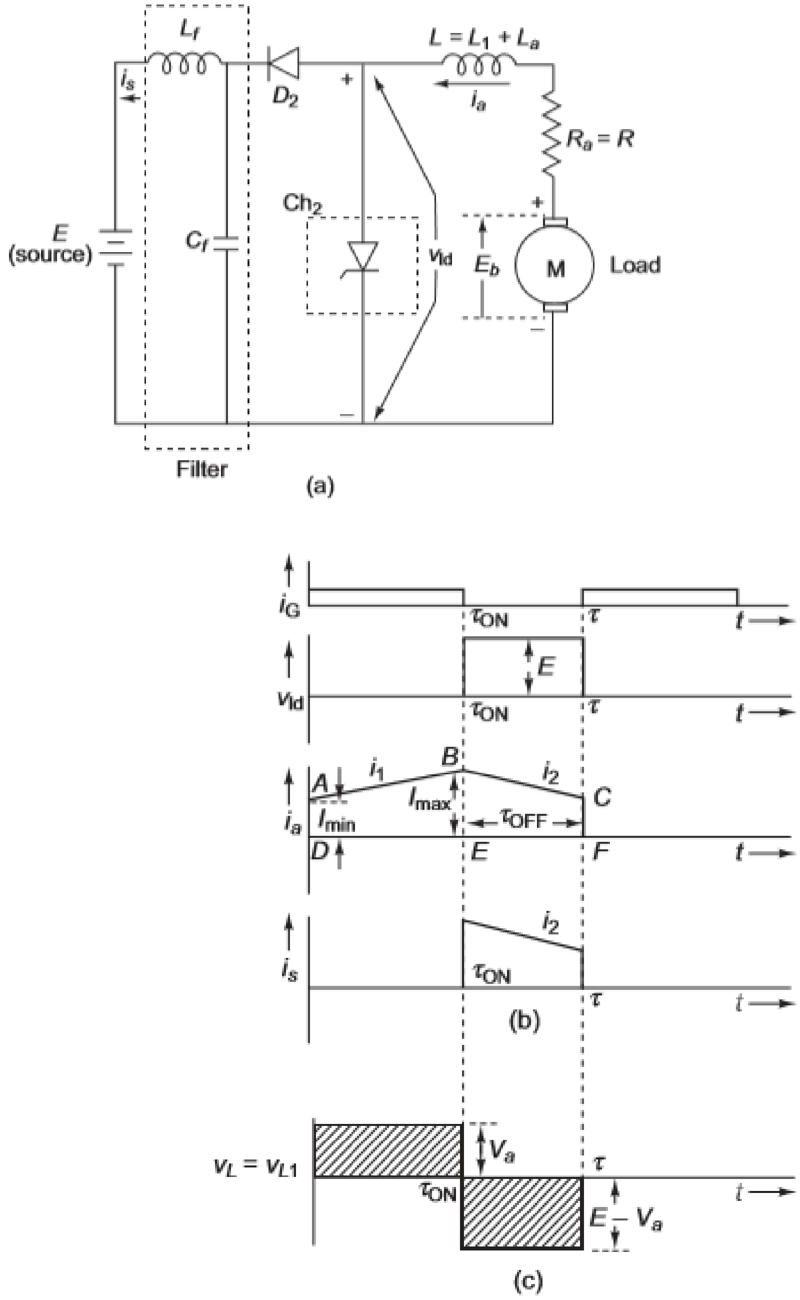


Fig. 7.26 Regenerative braking of a chopper-based dc drive: (a) circuit diagram, (b) waveforms, (c) voltage across inductor L for one period (τ)

Solution

(a) The circuit and waveforms are shown in Figs 7.26(a) and (b). The average load voltage is

$$V_a = E\left(1 - \frac{\tau_{ON}}{\tau}\right)_a = 400 \times 0.5 = 200 \text{ V}$$

(b)

$$I_a = \frac{I_{max} + I_{min}}{2} = 200 \text{ A}$$

or $I_{max} + I_{min} = 400$ A. From Eqns (7.64) and (7.65),

$$I_{max} + I_{min} = \frac{2E_b}{R} - \frac{E}{R}\left[\frac{1 - e^{-\tau_{OFF}/T_a}}{1 - e^{-\tau/T_a}} + \frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$

Here,

$$T_a = \frac{L_a}{R} = \frac{0.0002}{0.2} = 0.001$$

$$\tau = \frac{1}{1000} = 0.001$$

Hence,

$$\frac{\tau}{T_a} = 1 \text{ and } \frac{\tau_{ON}}{T_a} = \frac{0.0005}{0.001} = 0.5$$

Also

$$\tau_{OFF}/T_a = 0.5$$

Substituting the above values in the equation for $I_{max} + I_{min}$ gives

$$400 = \frac{2E_b}{0.2} - \frac{400}{0.2}\left[\frac{1 - 0.606}{1 - 0.368} + \frac{1.649 - 1}{2.718 - 1}\right]$$
$$= 10E_b - 2000(0.623 + 0.377)$$

This gives the value of the back emf as

$$E_b = 240$$

$$\text{Speed in rad/s} = \frac{240}{K_b^1} = \frac{240}{1.96} = 122.4$$

$$\text{Speed in rpm} = \frac{60}{2\pi} \times 122.4 = 1169 \text{ rpm}$$

(c) From Eqn (7.69), the power regenerated is

$$P_{reg} = \frac{E}{\tau}\left\{\frac{(E_b - E)}{R}[\tau_{OFF} + T_a(e^{-\tau_{OFF}/T_a} - 1)] - T_a I_{max}(e^{-\tau_{OFF}/T_a} - 1)\right\}$$

$$I_{max} = \frac{E_b}{R} - \frac{E}{R}\left[\frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$
$$= \frac{240}{0.2} - \frac{400}{0.2}\left[\frac{1.649 - 1}{2.718 - 1}\right]$$
$$= 1200 - 2000 \times \frac{0.649}{1.718} = 444 \text{ A}$$

Substitution of all values in the expression for P_{reg} gives

$$P_{\text{reg}} = 400\left\{\frac{240-400}{0.2}\left[\frac{\tau_{\text{OFF}}}{\tau} + \frac{T_a}{\tau}(e^{-0.5}-1) - \frac{T_a}{\tau} I_{\max}(e^{-0.5}-1)\right]\right\}$$

$$= 400\left\{\frac{-160}{0.2}[0.5 + 1(0.606-1)] - 1 \times 444(0.606-1)\right\}$$

$$= 400\left\{\frac{(-160 \times 0.106)}{0.2} + 444 \times 0.394\right\}$$

$$= 400 \times 90 = 36{,}000 \text{ W or } 36 \text{ kW}$$

(d) Average generated voltage

$$E_b = E(1-\delta) + I_a R_a = 400(1-0.5) + 200 \times 0.2 = 240 \text{ V}$$

Average current through the motor, $I_a = 200$ A. Hence the equivalent resistance is

$$\frac{E_b}{I_a} = \frac{240}{200} = 1.2\ \Omega$$

(e) Minimum and maximum permissible speeds for regeneration are obtained from the inequality

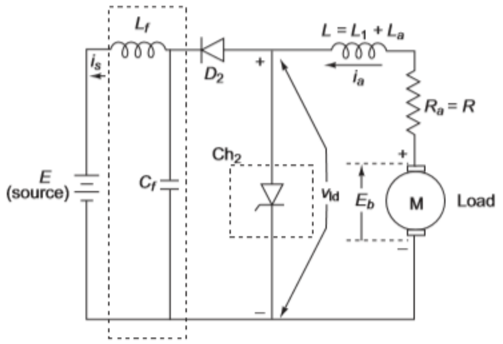
$$0 \le \omega_m \le \frac{E}{K_b^1}$$

or

$$0 \le \omega_m \le \frac{400}{1.96} = 204$$

Thus the maximum permissible speed is 204 rad/s or 1949 rpm. Also the minimum permissible speed is zero rpm.

13. A dc chopper is used for regenerative braking of a dc series motor as shown in Fig. 7.26(a). The data are $E = 400$ V, $R_a = 0.05\ \Omega$, $R_f = 0.04\ \Omega$, $L = L_a + L_f = 0.09$ mH. The product $K_b\phi_f$ may be assumed to be constant at 1.5 V/rad s and the average load current is 200 A. The frequency of the chopper is 1 kHz and $\tau_{ON}/\tau = 0.6$. Compute the (a) average load voltage, (b) motor speed, (c) power regenerated and fed back to the battery, (d) equivalent resistance viewed from the motor side when it is working as a generator, and (e) minimum and maximum permissible speeds.



(a)

Solution

(a) $$V_a = E\left(1 - \frac{\tau_{ON}}{\tau}\right) = 400(1 - 0.6) = 160 \text{ V}$$

(b) The correct value of E_b is arrived at as follows:

$$I_a = \frac{I_{max} + I_{min}}{2} = 200 \text{ A}$$

or

$$I_{max} + I_{min} = 400 \text{ A}$$

$$T_a = \frac{L_a + L_f}{R_a + R_f} = \frac{0.09 \times 10^{-3}}{0.09} = 0.001 \text{ s}$$

$$\tau = \frac{1}{f_{Ch}} = \frac{1}{1000} = 0.001 \text{ s}$$

From Eqns (7.64) and (7.65),

$$2I_a = 400 = I_{max} + I_{min} = \frac{2E_b}{R} - \frac{E}{R}\left[\frac{1 - e^{-\tau_{OFF}/T_a}}{1 - e^{-\tau/T_a}} + \frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$

Thus,

$$400 = \frac{2E_b}{0.09} - \frac{400}{0.09}\left[\frac{0.33}{0.632} + \frac{0.492}{1.718}\right]$$

Rearranging terms gives

$$\frac{2E_b}{0.09} = 400 + \frac{400}{0.09} \times 0.808$$

and we obtain

$$E_b = 179.6 \text{ V}$$

This gives

$$\text{Speed } N = 119.5 \times \frac{60}{2\pi} = 1141 \text{ rpm}$$

$$\begin{aligned}
I_{\max} &= \frac{E_b}{R} - \frac{E}{R}\left[\frac{e^{\tau_{\mathrm{OFF}}/T_a} - 1}{e^{\tau/T_a} - 1}\right] \\
&= \frac{179.6}{0.09} - \frac{400}{0.09} \times \frac{0.492}{1.728} \\
&= 724.5 \text{ A}
\end{aligned}$$

$$\begin{aligned}
I_{\min} &= \frac{E_b}{R} - \frac{E}{R}\frac{(1 - e^{-\tau_{\mathrm{OFF}}/T_a})}{(1 - e^{-\tau/T_a})} \\
&= \frac{179.6}{0.09} - \frac{400}{0.09} \times \frac{0.33}{0.632} \\
&= -325 \text{ A}
\end{aligned}$$

(c) $P_{\mathrm{reg}} = \dfrac{E}{\tau}\left\{\dfrac{(E_b - E)}{R}\left[\dfrac{\tau_{\mathrm{OFF}}}{\tau} + \dfrac{T_a}{\tau}(e^{-\tau_{\mathrm{OFF}}/T_a} - 1)\right] - \dfrac{T_a I_{\max}}{\tau}(e^{-\tau_{\mathrm{OFF}}/T_a} - 1)\right\}$

$$= 400\left\{\frac{(179.6 - 400)}{0.09}[0.4 + 1(0.67 - 1)] - 1 \times 724.5(0.67 - 1)\right\}$$

$$\begin{aligned}
&= 400\left\{\frac{-220.4}{0.09}[0.4 - 0.33] - 724.5(-0.33)\right\} \\
&= 400\left\{\frac{-220.4}{0.09} \times 0.07 + 239\right\} \\
&= 400\{-171.4 + 239\} \\
&= 400 \times 67.6 = 27{,}031 \text{ W} \approx 27 \text{ kW}
\end{aligned}$$

(d) Equivalent resistance $=$ back emf/average current $= 179.6/200 \approx 0.9\,\Omega$

(e) The minimum and maximum speeds are given as

$$0 \le N \le \frac{E}{K_b\phi_f}\frac{60}{2\pi}$$

or

$$0 \le N \le \frac{400}{1.5}\frac{60}{2\pi}$$

This gives

$$0 \le N \le 2546 \text{ rpm}$$

Hence the minimum and maximum speeds are 0 and 2546 rpm, respectively.

10. A 220-V, 80-A, separately excited dc motor operating at 800 rpm has an armature resistance of 0.18 Ω. The motor speed is controlled by a chopper operating at 1000 Hz. If the motor is regenerating, (a) determine the motor speed at full load current with a duty ratio of 0.7, this being the minimum permissible ratio (b) Repeat the calculation with a duty ratio of 0.1.

Solution

(a) When the machine is working as a motor, E_b is obtained from the equation

$$E_b = E - I_a R_a = 220 - 80 \times 0.18 = 220 - 14.4 = 205.6 \text{ V}$$

From the equation

$$E_b = kN$$

$$k = \frac{205.6}{800} = 0.257$$

When it is regenerating, the step-up configuration of Fig. 3.7(c) holds good. Thus,

$$E_b = E(1 - \delta) + I_a R_a = 220(1 - 0.7) + 80 \times 0.18 = 66 + 14.4 = 80.4 \text{ V}$$

$$N = \frac{E_b}{k} = \frac{80.4}{0.257} = 313 \text{ rpm}$$

(b) The speed for $\delta = 0.1$ is obtained as follows:

$$E_b = 220(1 - 0.10) + 80 \times 0.18 = 198 + 14.4 = 212.4 \text{ V}$$

Therefore the speed is

$$N = \frac{212.4}{0.257} = 827 \text{ rpm}$$

11. A 250-V, 105-A, separately excited dc motor operating at 600 rpm has an armature resistance of 0.18 Ω. Its speed is controlled by a two-quadrant chopper with a chopping frequency of 550 Hz. Compute (a) the speed for motor operation

with a duty ratio of 0.5 at 7/8 times the rated torque and (b) the motor speed if it regenerates at $\delta = 0.7$ with rated current.

Solution

(a) The initial back emf is to be determined from the equation

$$E_b = E - I_a R_a = 250 - 105 \times 0.18 = 231.1 \text{ V}$$

Hence the back emf constant $k = 231.1/600 = 0.385$. A fraction 7/8 of the rated current $I'_a = 7/8 \times 105 = 91.875$ A. The new E_b is obtained as

$$E'_b = E\delta - I'_a R_a$$

where $\delta = 0.5$ and $I'_a = 91.875$. Its numerical value is

$$E'_b = 250 \times 0.5 - 91.875 \times 0.18 = 108.46\text{V}$$

The new speed is

$$N = \frac{E'_b}{k} = \frac{108.46}{0.385} = 282 \text{ rpm}$$

(b) When it is regenerating, Eqn (7.54) is to be used. Thus,

$$I_a = \frac{E_b - E(1-\delta)}{R_a}$$

or

$$E_b = E(1-\delta) + I_a R_a$$

Substituting values gives

$$E_b = 250(1 - 0.7) + 105 \times 0.18 = 93.9 \text{ V}$$

From this, the speed $N = 93.9/0.385 = 244$ rpm

12. A 300-V, 100-A, separately excited dc motor operating at 600 rpm has an armature resistance and inductance of 0.25 Ω and 16 mH, respectively. It is controlled by a four-quadrant chopper with a chopper frequency of 1 kHz. (a) If the motor is to operate in the second quadrant at 4/5 times the rated current, at 450 rpm, calculate the duty ratio. (b) Compute the duty ratio if the motor is working in the third quadrant at 500 rpm and at 60% of the rated torque.

Solution

(a) $E_b - I_a R_a = 300 - 100 \times 0.25 = 275$ V. Back emf constant $k = E_b/N = 275/600 = 0.458$. Operation in the second quadrant implies that the motor works as a generator. Hence the motor terminal voltage V_a is written as

$$V_a = E(1 - \delta)$$

New current

$$I_a = \frac{4}{5} \times 100 = 80 \text{ A}$$

Hence,

$$E_b = V_a + I_a R_a$$
$$kN = E(1 - \delta) + I_a R_a$$

Substitution of values gives

$$0.458 \times 450 = 300(1 - \delta) + 80 \times 0.25$$

This yields $\delta = 0.38$.

(b) In the third quadrant, the machine works in the motoring mode but with reverse voltage and reverse current. The voltage equation relevant in this case is

$$E_b = V_a - I_a R_a$$

where

$$E_b = kN = 0.458 \times 500$$

$$V_a = E\delta = 300\delta$$

and

$$I_a = 0.6 \times 100 = 60 \text{ A}$$

By substituting numerical values, the equation becomes

$$0.458 \times 500 = 300 \times \delta - 60 \times 0.25$$

This gives $\delta = 0.813$.

8. A separately excited dc motor having a rating of 60 h.p. and running at 1200 rpm is supplied by a dc chopper whose source is a battery of 500 V. The field is also supplied by a chopper whose source is another battery of 300 V. The data pertaining to this chopper-based drive are as follows: $R_a = 0.18\ \Omega$, $K_b = 70$ V/(Wb rad/s), $\phi_f = 0.16 I_f$, $R_f = 120\ \Omega$, and $(\tau_{ON}/\tau)_f$ for the field chopper is 0.85. Assume that the load has sufficient inductance to make the load current continuous. If (τ_{ON}/τ) a for the armature is 0.65, compute the (a) mean armature current, (b) torque developed by the motor, (c) equivalent resistance for the armature circuit, and (d) total input power.

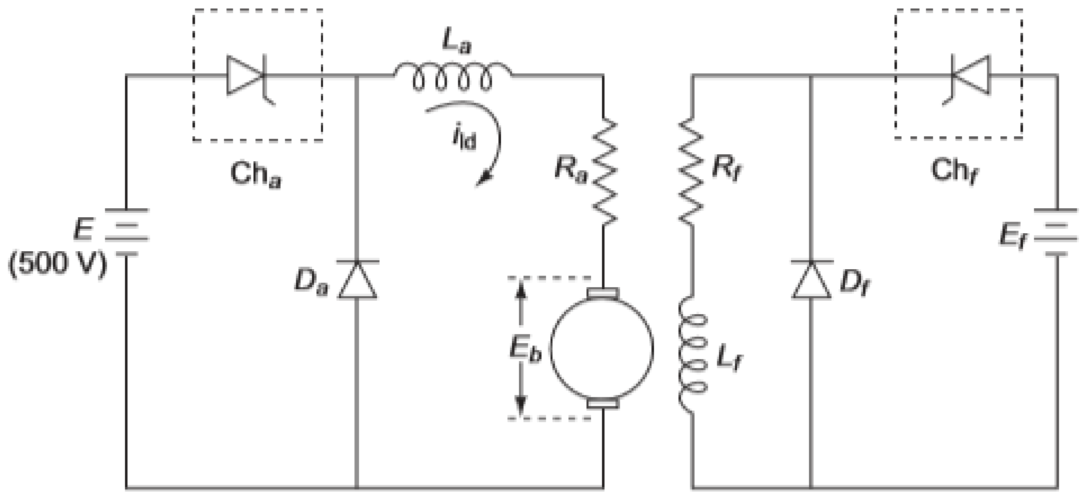


Fig. 7.33

Solution

(a) Let the source currents at the armature and field sides be denoted, respectively, as I_{sa} and I_{sf}. The circuit is shown in Fig. 7.33. Equation (3.23) gives the torque developed as

$$\mathcal{T}_d = K_t \phi_f I_{\text{ld}}(\omega) = K_b K_1 I_f I_a$$

Here,

$$K_b K_1 = 70 \times 0.016 = 1.12$$

$$I_f = E_f \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f \frac{1}{R_f} = \frac{300 \times 0.85}{120} = 2.125 \text{ A}$$

Hence the average torque is

$$\mathcal{T}_d = 1.12 \times 2.125 I_a = 2.38 I_a$$

$$\omega = \frac{2\pi \times 1200}{60} = 125.7$$

$$E_b = K_b K_1 I_f \omega = 2.38 \times 125.7 = 299 \text{ V}$$

$$V_a = E \left(\frac{\tau_{\text{ON}}}{\tau}\right)_a = 500 \times 0.65 = 325 \text{ V}$$

The mean armature current is

$$I_a = \frac{V_a - E_b}{R_a} = \frac{325 - 299}{0.18} = 144.4 \text{ A}$$

(b) Torque developed $\mathcal{T}_d = 2.38 \times 144.4 = 343.8$ N m.
(c) Input (or source) current is

$$I_{sa} = I_a \left(\frac{\tau_{\text{ON}}}{\tau}\right)_a = 144.4 \times 0.65 = 93.9 \text{ A}$$

Also,

$$I_{sf} = I_f \times \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f = 2.125 \times 0.85 = 1.81 \text{ A}$$

$$\begin{aligned}\text{Armature source equivalent resistance} &= \frac{\text{armature source voltage}}{\text{armature source current}} \\ &= \frac{500}{93.9} \\ &= 5.32\ \Omega\end{aligned}$$

(d) Total input power = power input to armature + power input to field $= E_a I_{sa} + E_f I_{sf}$. Hence, it is given as $P_i = 500 \times 93.9 + 300 \times 1.81 = 46{,}950 + 543 = 47{,}493$ W ≈ 47.5 kW.

9. A separately excited dc motor has a rating of 50 h.p. and when supplied by a battery of 480 V through a chopper, it has a mean armature current of 120 A. The field is also supplied by a chopper whose source is a battery of 250 V. Other data for this chopper-based drive are $R_a = 0.2\ \Omega$, $R_f = 125\ \Omega$, $K_b = 72$ V/(Wb rad/s), $\phi_f = 0.015 I_f$, $(\tau_{ON}/\tau)_a = 0.7$, and $(\tau_{ON}/\tau)_f = 0.9$. The armature circuit has sufficient inductance to make the current continuous. Compute the (a) speed of the motor, (b) torque developed by the motor, (c) equivalent resistance, and (d) total input power.

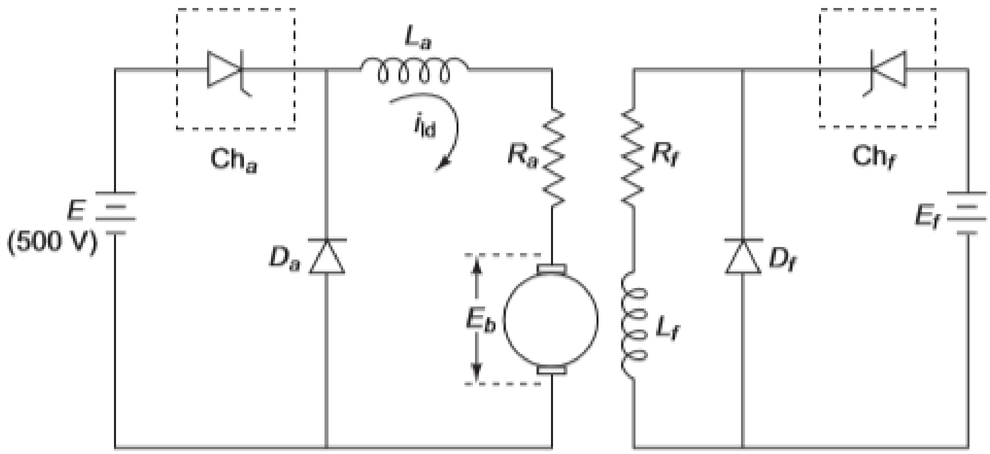


Fig. 7.33

Solution

(a) The circuit is the same as that given in Fig. 7.33.

$$V_a = 480\left(\frac{\tau_{ON}}{\tau}\right)_a = 480 \times 0.7 = 336\ \text{V}$$

$$E_b = V_a - I_a R_a = 336 - 120 \times 0.2 = 312\ \text{V}$$

$$\omega = \frac{E_b}{K_b \phi_f} = \frac{E_b}{K_b \times 0.015 I_f}$$

Here,

$$I_{sf} = 250\left(\frac{\tau_{ON}}{\tau}\right)_f \frac{1}{R_f} = \frac{250 \times 0.9}{125} = 1.8\ \text{A}$$

Hence,

$$\omega = \frac{E_b}{K_b \times 0.015 I_f} = \frac{312}{72 \times 0.015 \times 1.8} = 160 \text{ rad/s}$$

Also,

$$\text{speed} = \frac{60\omega}{2\pi} = \frac{60 \times 160}{2\pi} = 1528 \text{ rpm}$$

(b)
$$\begin{aligned}\text{Torque developed} &= K_b \times 0.015 I_f I_a \\ &= 72 \times 0.015 \times 1.8 \times 120 \\ &= 233.3 \text{ N m}\end{aligned}$$

The source current on the armature side is

$$I_{sa} = I_a \left(\frac{\tau_{\text{ON}}}{\tau}\right)_a = 120 \times 0.7 = 84 \text{ A}$$

(c)
$$\begin{aligned}\text{Armature source equivalent resistance} &= \frac{\text{armature source voltage}}{\text{armature source current}} \\ &= \frac{480}{84} = 5.7\ \Omega\end{aligned}$$

(d) Total input power

$$\begin{aligned}P_i &= \text{power input to armature} + \text{power input to field} \\ &= E_a I_{sa} + E_f I_{sf}\end{aligned}$$

where

$$I_{sf} = I_f \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f = 1.8 \times 0.9 = 1.62 \text{ A}$$

Hence,

$$P_i = 480 \times 84 + 250 \times 1.62 = 40{,}320 + 405 = 40{,}725 \text{ W} \approx 40.7 \text{ kW}$$

12. A dc series motor is supplied by a battery of 420 V with a dc chopper interposed between the battery and the motor. It has a mean armature current of 120 A. Other data for this chopper-based drive are $R_a = 0.05\ \Omega$, $R_f = 0.06\ \Omega$, $K_b = 0.72$ V/(Wb rad/s), and $\phi_f = 0.016\ I_a$. The duty ratio τ_{ON}/τ is 0.65. Compute the (a) speed of the motor, (b) torque developed by the motor, (c) equivalent input resistance, and (d) total input power.

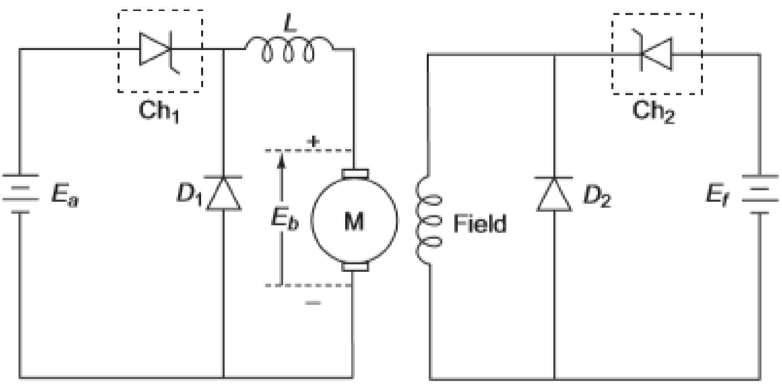


Fig. 7.25 Circuit diagram of a dc drive with the armature and field fed by separate choppers

Solution

The set-up is as shown in Fig. 7.25:

$$V_a = 420\frac{\tau_{ON}}{\tau} = 420 \times 0.65 = 273 \text{ V}$$

$$\begin{aligned} E_b &= V_a - I_a(R_a + R_f) \\ &= 273 - 120(0.05 + 0.06) \\ &= 259.8 \text{ V} \end{aligned}$$

$$\begin{aligned} \omega &= \frac{E_b}{K_b \times 0.016 I_a} \\ &= \frac{259.8}{0.72 \times 0.016 \times 120} \\ &= 188 \text{ rad/s} \end{aligned}$$

$$\begin{aligned} \text{Speed } N &= \frac{188 \times 60}{2\pi} \\ &= 1795 \text{ rpm} \end{aligned}$$

$$\begin{aligned} \text{Torque developed by the motor} &= K_b \times 0.016(I_a)^2 \\ &= 0.72 \times 0.016 \times (120)^2 \\ &= 165.9 \text{ N m} \end{aligned}$$

The source current is

$$I_s = I_a\frac{\tau_{ON}}{\tau} = 120 \times 0.65 = 78 \text{ A}$$

$$\text{Input power} = EI_s = 420 \times 78 = 32{,}760 \text{ W} = 32.76 \text{ kW}$$

Example 4

A separately excited d.c. motor with R_a = 1.2 ohms and L_a = 30 mH , is to be controlled using class-A thyristor chopper as shown in Fig.9.11 .The d.c. supply V_d = 120V . By ignoring the effect of the armature inductance L_a , it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle γ =1.

(b) Draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant $K e\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to γ = 0.5.

Solution

The average armature voltage is

$$V_{av} = \gamma\, V_d \;= 1 \times\, 120 = 120\, V$$

The motor's speed:

$$n \;= \frac{V_{av}}{K_e \emptyset} - \;\frac{R_a}{K_T K_e \emptyset^2} T_d$$

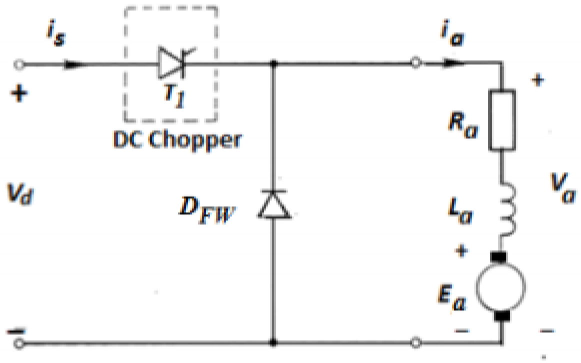


Fig. 9.11 Thyristor chopper drive.

At no load T_d = 0, hence

or $$n_o \;= \frac{\gamma V_d}{K_e \emptyset} = \frac{120}{0.042} = 2857\, rpm$$

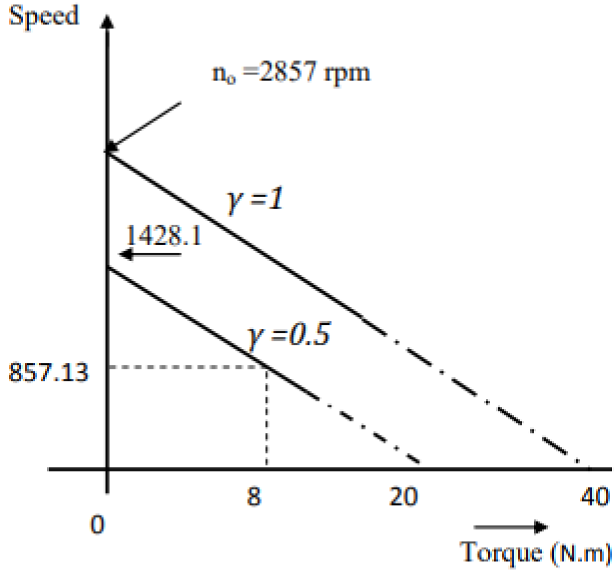
At starting, n = 0. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e \emptyset} - \;\frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$\therefore T_{st} \;= \frac{9.55\;\gamma V_d}{R_a} K_e \emptyset$$

$$T_{st} \;= \frac{9.55\; x120}{1.2} \times\; 0.042 = 40\; N.m$$

Fig.9.11 Speed-torque characteristics



(b) At $\gamma = 0.5$

$$V_a = \gamma V_d = 0.5\ x\ 120 = 60\ V$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5\ rpm$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20\ \mathrm{N.m}$$

At $\gamma = 0.5$, $T_L = 8$ N.m

$$n = \frac{60}{0.042} - \frac{1.2}{9.55(0.042)^2} \times 8 = 857.13\ \mathrm{rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 5

In the microcomputer -controlled class –A IGBT transistor DC chopper shown in Fig.12.6, the input voltage V_d = 260V, the load is a separately excited d.c. motor with Ra = 0.28 Ω and L_a = 30 mH . The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor *n* (rpm) when the chopper is switched fully ON such that the duty cycle γ = 1.0.

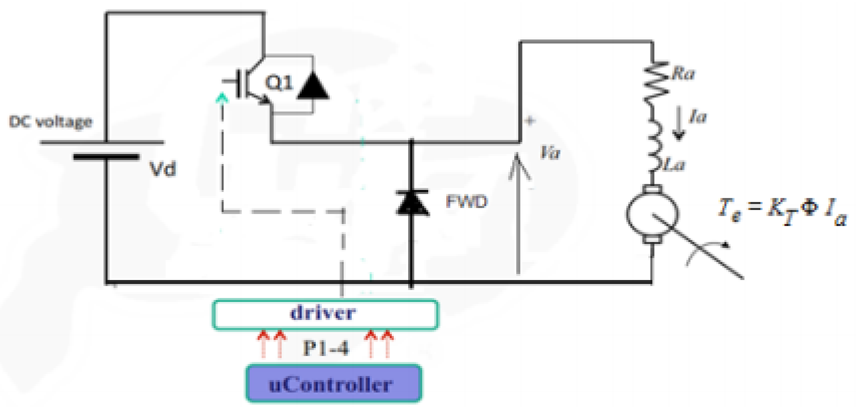


Fig.12.6 IGBT Chopper drive.

Solution

(a) With steady – state operation of the motor, the armature inductance L_a behaves like a short circuit and therefore has no effect at all.

At stand still n = 0 , and therefore $E_a = 0$, hence from Eq.(12.22)

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30\ A$$

$$\therefore\ V_{a0} = 0.28 \times 30 = 8.4\ V$$

At full speed n = 2500 rpm ,

$$E_{a2500} = K_e\, \phi\, n = 0.1 \times 2500 = 250\ V$$

For separately excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4\ V$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. γ =1, then $V_a = V_d = 260$ V .
At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e\, \phi\, n + I_a R_a \ = 260\ \ \text{V}$$

$$0.1\, n + 30\ x\ 0.28\ = 260 \qquad \rightarrow \qquad n = 2516\ \text{rpm}$$

Example 1: A separately-excited d.c. motor with $R_a = 0.3\Omega$, and $L_a = 15\,mH$ is to be speed controlled over a range 0-2000 r.p.m. The d.c. supply is 220V. The load torque is constant and requires an average armature current of 25A.

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi = 0.1002$ V/rpm.

Solution: In the steady-state, the armature inductance has no effect. The required motor terminal voltages are:

At $n = 0$, $E_b = 0$, so that

$$V_{dc} = E_b + I_a R_a = I_a R_a = 25 \times 0.3 = 7.5\,V.$$

At $n = 2000$ rpm,

$$E_b = K_e\Phi n = 0.1002 \times 2000 = 200.4\,V.$$

$$\therefore V_{dc} = E_b + I_a R_a = 200.4 + 25 \times 0.3 = 207.9\,V.$$

$$V_o = \gamma V_d$$

To give $V_o = 7.5\,V$ $\therefore$ $7.5 = \gamma_o \times 220$ or $\gamma_o = \frac{7.5}{220} = 0.034$

To give $V_o = 207.9\,V$ $\therefore$ $207.9 = \gamma_{2000} \times 220$

or $\gamma_{2000} = \frac{207.9}{220} = 0.943$.

Range of γ: $0.034 \leq \gamma \leq 0.943$.

(b) If the chopper was to be switched fully on, what is the speed of the motor when $\gamma = 1$.

Sol. when $\gamma = 1$, $V_o = 220$.

$$\therefore n = \frac{E_b}{K_e\Phi}, \quad E_b = V_o - I_a R_a = 220 - 25 \times 0.3 = 212.5\,V$$

$$n = \frac{212.5}{0.1002} = \underline{\underline{2121}} \text{ rpm}$$

Example :

An electrically-driven automobile is powered by d.c. series motor rated at 100V, 200A. The motor resistance and inductance are respectively 0.65 Ω and 6 mH. power is supplied from ideal battry of 120V via class-A d.c. chopper having a fixed frequency of 100 Hz. The machine constant $K_e\phi = 0.00025$ V/rpm and the motor speed is 2500 rpm. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor when running at 2500 rpm with duty cycle γ of 3/5.

Solution:

Chopping period $T = \frac{1}{f} = \frac{1}{100} = 10$ ms.

$$t_{on} = \gamma T = \frac{3}{5} \times 10 = 6\text{ ms}$$

$$\therefore \; t_{off} = 10 - 6 = 4\text{ ms.}$$

$$I_{max} = \frac{V_{av}}{R_a} + \frac{t_{off}}{2L_a} V_{av} = \frac{\gamma V_i}{R_a} + \frac{t_{off}}{2L_a} \gamma V_i$$

$$= \frac{\frac{3}{5} \times 120}{0.65} + \frac{4 \times 10^{-3}}{2 \times 6 \times 10^{-3}} \left(\frac{3}{5} \times 120\right)$$

$$= 110.7 + 24 = 134.7\text{ A}$$

$$I_{min} = \frac{V_{av}}{R_a} - \frac{t_{off}}{2L_a} V_{av} = 86.76\text{ A.}$$

$$= 110.7 - 24$$

For series motor:

Mean torque: $T_e = K_T \phi I_{av}^2 = 9.55\, K_e \Phi \left(\frac{I_{max} + I_{min}}{2}\right)^2$

$$= 9.55 \times 0.00025 \left(\frac{134.7 + 86.76}{2}\right)^2$$

$$= 30\text{ N.m.}$$

Mean power: $P_e = \omega T_e = \frac{2\pi}{60} \times 2500 \times 30 = 7.85$ KW

$$= 10.5\text{ hp.}$$

Example 4.5

A chopper is used to control the speed of a dc motor as shown in Fig. 4.20*a*. The motor is accelerated under current control (i.e., constant torque). Assume that the armature current remains constant at I_a amperes during start-up.

1 Show that the maximum rms ripple current in the chopper current i_{CH} occurs at a duty cycle of one-half, that is, at $\alpha = 0.5$.

2 For the duty cycle $\alpha = 0.5$, determine the values of the input filter components L and C for the following conditions: Supply voltage = 120 V. Chopper frequency $f_{CH} = 400$ Hz. Start-up motor current $I_a = 100$ A. Rms fundamental current to be allowed in the supply is 10% of the dc component of the source current. Electrolytic capacitor of rating 1000 μF and 300 V dc can take 5 A rms ripple current. For the design $f_{CH} > 2f_r$.

3 For the values of L and C obtained in part 2, determine the average and first three harmonic currents (in rms) in the supply.

Solution

1 The chopper current i_{CH} is in the form of square pulses of magnitude I_a and width α as shown in Fig. 4.21. Therefore, the dc component $I_{CH_{dc}}$, rms current I_{rms}, and ripple current I_{ripple} are as follows:

$$I_{CH_{dc}} = I_a\alpha$$

$$I_{rms} = \left(\int_0^{\alpha} I_a^2\, d\alpha\right)^{1/2} = I_a\sqrt{\alpha}$$

$$I_{ripple} = \left[\left(I_a\sqrt{\alpha}\right)^2 - (I_a\alpha)^2\right]^{1/2}$$

$$= I_a(\alpha - \alpha^2)^{1/2}$$

For maximum ripple current

$$\frac{dI_{ripple}}{d\alpha} = 0$$

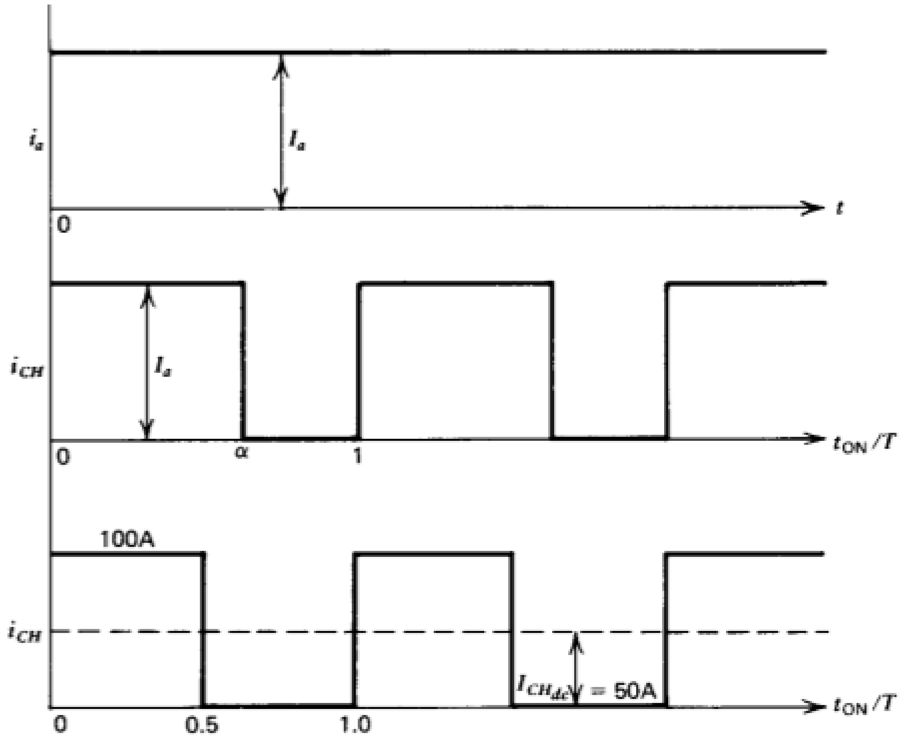


Fig. 4.21 Example 4.5

or

$$\frac{I_a(1-2\alpha)}{2(\alpha-\alpha^2)^{1/2}} = 0$$

from which

$$\alpha = 0.5$$

2 The *L-C* filter should be selected for the worst case, which corresponds to $\alpha = 0.5$. At this duty cycle (bottom wave form in Fig. 4.21) the Fourier series for the chopper current is

$$i_{CH} = I_{CH_{dc}} + \frac{4}{\pi}\frac{I_a}{2n}(\sin\omega t + \sin 3\omega t + \sin 5\omega t + \cdots)$$

Now,

$$I_{CH_{dc}} = \frac{100}{2} = 50\ \text{A}$$

The fundamental, third, and fifth harmonic currents in the chopper are

$$I_{CH_1} = \frac{4 \times 100}{\sqrt{2} \times \pi \times 2} = 45 \text{ A}$$

$$I_{CH_3} = 15 \text{ A}$$

$$I_{CH_5} = 9 \text{ A}$$

The dc component of the chopper current comes from the supply only. The capacitor cannot provide a dc current. Therefore, the dc component I_0 of the supply current is

$$I_0 = I_{CH_{dc}} = 50 \text{ A}$$

If the fundamental supply current I_1 is not to exceed 10% of the dc current I_0, then

$$I_1 = 5 \text{ A}$$

From equation 4.55

$$I_1 = \frac{X_C}{X_L - X_C} I_{CH_1}$$

$$5 = \frac{X_C}{X_L - X_C} \times 45$$

or

$$X_L = 10X_C$$

Fundamental capacitor current I_{C_1} is

$$I_{C_1} = \frac{X_L}{X_L - X_C} I_{CH_1}$$

$$= \frac{10X_C}{10X_C - X_C} \times 45$$

$$= 50 \text{ A}$$

Each electrolytic capacitor can take 5 A of current. Therefore 10 capacitors connected in parallel are required.

$$C = 10{,}000\ \mu\text{F}$$

$$X_C = \frac{1}{2\pi 400 \times 10^4 \times 10^{-6}}\ \Omega = 3.98 \times 10^{-2}\ \Omega$$

$$X_L = 10X_C = 3.98 \times 10^{-1}\ \Omega$$

$$L = \frac{0.398}{2\pi 400} = 158\ \mu\text{H}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = 127\ \text{Hz}$$

which makes

$$f_{CH} = \frac{400}{127} f_r = 3.15 f_r$$

3 From equation 4.58

$$I_1 = \frac{45}{(3.15)^2 - 1} = 5.0\ \text{A}$$

$$I_3 = \frac{15}{(3 \times 3.15)^2 - 1} = 0.17\ \text{A}$$

$$I_5 = \frac{9}{(5 \times 3.15)^2 - 1} = 0.036\ \text{A}$$

and

$$I_0 = 50\ \text{A}$$

Example 4.1

The speed of a separately excited dc motor is controlled by a chopper as shown in Fig. 4.8*a*. The dc supply voltage is 120 V, armature circuit resistance is $R_a = 0.5\ \Omega$, armature circuit inductance is $L_a = 20$ mH, and motor constant is $K_a\Phi = 0.05$ V/rpm. The motor drives a constant-torque load requiring an average armature current of 20 A. Assume that motor current is continuous.

Determine:

1 the range of speed control;
2 the range of the duty cycle α.

Solution

Minimum speed is zero at which $E_g = 0$. Therefore from equation 2.17

$$E_a = I_a R_a = 20 \times 0.5 = 10\,\text{V}$$

From equation 4.1

$$10 = 120\alpha$$

$$\alpha = \frac{1}{12}$$

Maximum speed corresponds to $\alpha = 1$ at which $E_a = E = 120$ V.

Therefore

$$\begin{aligned} E_g &= E_a - I_a R_a \\ &= 120 - (20 \times 0.5) \\ &= 110\,\text{V} \end{aligned}$$

From equation 2.13

$$N = \frac{E_g}{K_a\Phi} = \frac{110}{0.05} = 2200\,\text{rpm}$$

The range of speed is $0 < N < 2200$ rpm, and the range of the duty cycle is $1/12 < \alpha < 1$.

Example 4.1

A dc motor is driven from a chopper with a source voltage of 24V dc and at a frequency of 1 kHz. Determine the variation in duty cycle required to have a speed variation of 0 to 1 p.u. delivering a constant 2 p.u. load. The motor details are as follows:

1 hp, 10 V, 2500 rpm, 78.5 % efficiency, $R_a = 0.01\ \Omega$, $L_a = 0.002$ H, $K_b = 0.03819$ V/rad/sec

The chopper is one-quadrant, and the on-state drop voltage across the device is assumed to be 1 V regardless of the current variation.

Solution (i) Calculation of rated and normalized values

$$V_b = 10\text{ V}$$

$$V_n = \frac{V_s}{V_b} = \frac{24-1}{10} = 2.3\text{ p.u.}$$

$$\omega_{mr} = \frac{2500 \times 2\pi}{60} = 261.79\text{ rad/sec}$$

$$I_{ar} = \frac{\text{Output}}{\text{Voltage} \times \text{Efficiency}} = \frac{1 \times 746}{10 \times 0.785} = 95\text{ A} = I_b$$

$$R_{an} = \frac{I_b R_a}{V_b} = \frac{95 \times 0.001}{10} = 0.095\text{ p.u.}$$

$$T_{en} = 2\text{ p.u.}$$

(ii) Calculation of duty cycle

The minimum and maximum duty cycles occur at 0 and 1 p.u. speed, respectively, and at 2 p.u. load. From equation (4.9),

$$d = \frac{T_{en}R_{an} + \omega_{mn}}{V_n}$$

Example 4.2

The critical duty cycle can be changed by varying either the electrical time constant or the chopping frequency in the chopper. Draw a set of curves showing the effect of these variations on the critical duty cycle for various values of E/V_s.

Solution

$$d_c = \left(\frac{T_a}{T}\right)\log_e\left[1 + \frac{E}{V_s}\left(e^{\frac{T}{T_a}} - 1\right)\right]$$

In terms of chopping frequency,

$$d_c = f_c T_a \log_e\left[1 + \frac{E}{V_s}\left(e^{\frac{1}{T_a f_c}} - 1\right)\right]$$

Assigning various values of E/V_s and varying f_cT_a would yield a set of critical duty cycles. The graph between d_c and f_cT_a for varying values of E/V_s is shown in Figure 4.16. The maximum value of f_cT_a is chosen to be 10.

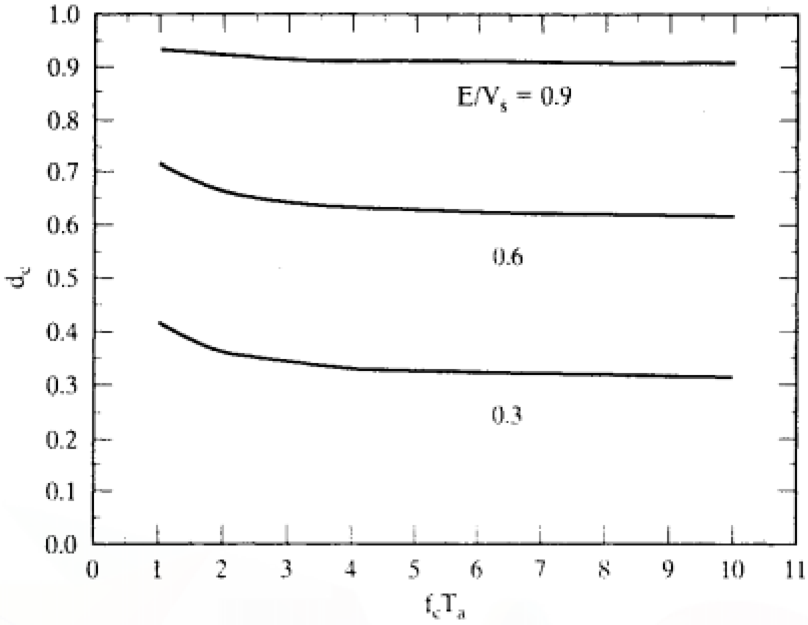


Figure 4.16 Critical duty cycles vs. the product of chopping frequency and electrical time constant of the dc motor, as a function of induced emf and source voltage

Example 4.3

A 200-hp, 230-V, 500-rpm separately-excited dc motor is controlled by a chopper. The chopper is connected to a bridge-diode rectifier supplied from a 230-V, 3-ϕ, 60-Hz ac main. The motor chopper details are as follows:

$$R_a = 0.04\ \Omega,\ L_a = 0.0015\ \text{H},\ K_b = 4.172\ \text{V/rad/sec},\ f_c = 2\ \text{kHz}.$$

The motor is running at 300 rpm with 55% duty cycle in the chopper. Determine the average current from steady-state current waveform and the electromagnetic torque produced in the motor. Compare these results with those obtained by averaging.

Solution The critical duty cycle is evaluated to determine the current continuity at the given duty cycle of 0.55.

$$d_c = \left(\frac{T_a}{T}\right)\log_e\left[1 + \frac{E}{V_s}\left(e^{\frac{1}{T_r}} - 1\right)\right]$$

$$T_a = \frac{0.0015}{0.04} = 0.0375\ \text{s}$$

$$T = \frac{1}{f_c} = \frac{1}{2 \times 10^3} = 0.5\ \text{ms}$$

$$\frac{T_a}{T} = 75$$

$$V_s = 1.35\ V\cos\alpha = 1.35 \times 230 \times \cos 0° = 310.5\ \text{V}$$

$$E = K_b\omega_m = 4.172 \times \frac{2\pi \times 300}{60} = 131.1\ \text{V}$$

$$d_c = 75\log_e\left[1 + \frac{131.1}{310.5}\left(e^{\frac{1}{75}} - 1\right)\right] = 0.423$$

The given value of d is greater than the critical duty cycle; hence, the armature current is continuous.

$$I_{a0} = \frac{V_s(e^{dT/T_a} - 1)}{R_a(e^{T/T_a} - 1)} - \frac{E}{R_a} = \frac{310.5(e^{0.55/75} - 1)}{0.04(e^{1/75} - 1)} - \frac{131.1}{0.04} = 979 \text{ A}$$

$$I_{a1} = \frac{V_s(1 - e^{-dT/T_a})}{R_a(1 - e^{-T/T_a})} - \frac{E}{R_a} = \frac{310.5(1 - e^{-0.55/75})}{0.04(1 - e^{-1/75})} - \frac{131.1}{0.04} = 1004.7 \text{ A}$$

The average current is

$$I_{av} = \frac{1}{T}\left[\int_0^{dT}\left(\frac{V_s - E}{R_a}(1 - e^{-t/T_a}) + I_{a0}e^{-t/T_a}\right)dt + \int_0^{(1-d)T}\left(-\frac{E}{R_a}(1 - e^{-t/T_a}) + I_{a1}e^{-t/T_a}\right)dt\right]$$

$$= \frac{1}{T}\left[\begin{array}{l}\frac{V_s - E}{R_a}\{dT + T_a(e^{-dT/T_a} - 1)\} + I_{a0}T_a(1 - e^{-dT/T_a}) \\ -\frac{E}{R_a}\{(1 - d)T - T_a + T_a e^{-(1-d)T/T_a}\} + I_{a1}T_a(1 - e^{-(1-d)T/T_a})\end{array}\right] = 991.8 \text{ A}$$

$$T_{av} = K_b I_{av} = 4.172 \times 991.8 = 4137.7 \text{ N·m}$$

Steady state by averaging

$$I_{av} = \frac{(dV_s - K_b\omega_m)}{R_a} = \frac{0.55 \times 310.5 - 4.172 \times 31.42}{0.04} = 991.88 \text{ A}$$

$$T_{av} = K_b I_{av} = 4138.1 \text{ N·m}$$

There is hardly any significant difference in the results by these two methods.

Example 4.4

A separately-excited dc motor is controlled by a chopper whose input dc voltage is 180 V. This motor is considered for low-speed applications requiring less than 2% pulsating torque at 300 rpm. (i) Evaluate its suitability for that application. (ii) If it is found unsuitable, what is the chopping frequency that will bring the pulsating torque to the specification? (iii) Alternatively, a series inductor in the armature can be introduced to meet the specification. Determine the value of that inductor. The motor and chopper data are as follows:

3 hp, 120 V, 1500 rpm, $R_a = 0.8\ \Omega$, $L_a = 0.003$ H, $K_b = 0.764$ V/rad/sec, $f_c = 500$ Hz

Solution Rated torque.

$$T_{er} = \frac{3 \times 745.6}{2\pi \times 1500/60} = 14.25 \text{ N}\cdot\text{m}$$

Maximum pulsating torque permitted $= 0.02 \times T_{er} = 0.02 \times 14.25 = 0.285$ N·m

To find the harmonic currents, it is necessary to know the duty cycle. That is approximately determined by the averaging-analysis technique, by assuming that the motor delivers rated torque at 300 rpm.

$$I_b = \frac{T_{er}}{K_b} = \frac{14.25}{0.764} = 18.65 \text{ A}$$

$$V_a = E + I_{av}R_a = K_b\omega_m + I_{av}R_a = 0.764 \times \frac{2\pi \times 300}{60} + 18.65 \times 0.8 = 38.91 \text{ V}$$

$$d = \frac{V_a}{V_s} = \frac{38.91}{180} = 0.216$$

(i) The fundamental pulsating torque is assumed to be predominant for this analysis.

$$i_{a1} = \frac{2V_s}{\pi\sqrt{R_a^2 + \omega_c^2 L_a^2}} \sin(\pi d) = \frac{2 \times 180}{\pi\sqrt{0.8^2 + (2\pi \times 500 \times 0.003)^2}} \sin(0.216\pi) = 7.6 \text{ A}.$$

$$T_{e1} = K_b i_{a1} = 0.764 \times 7.6 = 5.8 \text{ N}\cdot\text{m}$$

This pulsating torque exceeds the specification, and, hence, in the present condition, the drive is unsuitable for use.

(ii) The fundamental current to produce 2% pulsating torque is

$$i_{a1(spec)} = \frac{T_{e1(spec)}}{K_b} = \frac{0.285}{0.764} = 0.373 \text{ A}$$

$$i_{a1(spec)} = \frac{2V_s}{\pi\sqrt{R_a^2 + \omega_c^2 L_a^2}} \sin(\pi d)$$

from which the angular switching frequency to meet the fundamental current specification is obtained as

$$\omega_{c1} = \sqrt{\frac{4V_s^2 \sin^2(\pi d)}{\pi^2 L_a^2 i_{a1(spec)}^2} - \frac{R_a^2}{L_a^2}} = \sqrt{(4 \times 180^2 \sin^2(0.216\pi)/(\pi^2(0.003)^2(0.373)^2) - \left(\frac{0.8}{0.003}\right)^2}$$

$$= 64{,}278 \text{ rad/s}$$

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = 10.23 \text{ kHz}$$

Note that f_{c1} is the chopping frequency in Hz, which decreases the pulsating torque to the specification.

(iii) Let L_{ex} be the inductor introduced in the armature circuit. Then its value is

$$L_{ex} = \sqrt{\frac{4V_s^2 \sin^2(\pi d)}{\pi^2 \omega_c^2 i_{al(spec)}^2} - \frac{R_a^2}{\omega_c^2}} - L_a$$

$$= \sqrt{(4 \times 180^2 \sin^2(0.216\pi))/(\pi^2(2\pi \times 500)^2(0.373)^2) - \left(\frac{0.8}{2\pi \times 500}\right)^2} - 0.003 = 71.5 \text{ mH}$$

Example 4.5

Calculate (i) the maximum harmonic resistive loss and (ii) the derating of the motor drive given in Example 4.4. The motor is operated with a base current of 18.65 A, which is inclusive of the fundamental-harmonic current. Consider only the dominant-harmonic component, to simplify the calculation.

Solution

$$I_{rms}^2 = I_b^2 = I_{av}^2 + I_1^2$$

By dividing by the square of the base current, the equation is expressed in terms of malized currents as

$$I_{avn}^2 + I_{1n}^2 = 1 \text{ p.u.}$$

where

$$I_{1n} = \frac{\sqrt{2}}{\pi} \frac{V_s}{\sqrt{R_a^2 + \omega_c^2 L_a^2}} \sin(\pi d) \frac{1}{I_b} = \frac{\sqrt{2}}{\pi} \frac{V_{sn}}{Z_{an}} \sin(\pi d)$$

where

$$V_{sn} = \frac{V_s}{V_b}; Z_{an} = \frac{Z_a}{Z_b} = \frac{\sqrt{R_a^2 + \omega_c^2 L_a^2}}{Z_b}$$

$$Z_b = \frac{V_b}{I_b} = \frac{120}{18.65} = 6.43\ \Omega$$

$$V_{sn} = \frac{180}{120} = 1.5 \text{ p.u.}$$

$$Z_a = \sqrt{0.8^2 + (2\pi * 500 * 0.003)^2} = 9.42\ \Omega$$

$$Z_{an} = \frac{9.42}{6.43} = 1.464 \text{ p.u.}$$

For a duty cycle of 0.5, the dominant-harmonic current is maximum and is given as

$$I_{1n} = \frac{\sqrt{2}}{\pi} \frac{1.5}{1.464} = 0.46 \text{ p.u.}$$

(i) The dominant-harmonic armature resistive loss is

$$P_{1n} = I_{1n}^2 R_{an} = 0.46^2 \frac{0.8}{6.43} = 0.02628 \text{ p.u.}$$

(ii) For equality of losses in the machine with pure and chopped-current operation, the average current in the machine with chopped-current operation is derived as

$$I_{avn} = \sqrt{1 - I_{1n}^2} = \sqrt{1 - .46^2} = 0.887 \text{ p.u.}$$

which translates into an average electromagnetic torque of 0.887 p.u., resulting in 11.3% derating of the torque and hence of output power.

Example 4.1

A 250-V separately excited motor dc has an armature resistance of 2.5 Ω. When driving a load at 600 rpm with constant torque, the armature takes 20 A. This motor is controlled by a chopper circuit with a frequency of 400 Hz and an input voltage of 250 V.

1. What should be the value of the duty ratio if one desires to reduce the speed from 600 to 400 rpm, with the load torque maintained constant?
2. What should be the minimum value of the armature inductance, if the maximum armature current ripple expressed as a percentage of the rated current is not to exceed 10 percent?

Solution: With an input voltage of 250 V and at a constant torque, the motor will run at 600 rpm when $\delta = 1$.

1. At 600 rpm

$$E = V_a - I_a R_a = 250 - 20 \times 2.5 = 200 \text{ V}$$

At 400 rpm, the back emf

$$E_1 = 200 \times \frac{400}{600} = 133 \text{ V}$$

The average chopper output voltage

$$V_{a1} = E_1 + I_a R_a = 133 + 20 \times 2.5 = 183 \text{ V}.$$

Now $\delta V = V_{a1}$ or $\delta = V_{a1}/V = 183/250 = 0.73$.

2.

$$\Delta i_a = \frac{V}{2R_a}\left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1}\right] \tag{4.14}$$

Per-unit current ripple $= (\Delta i_a)_p = \dfrac{\Delta i_a}{I_{rated}}$

$$= \frac{V}{2R_aI_{rated}}\left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1}\right] \tag{E4.1}$$

For the maximum value of the per-unit ripple

$$\frac{d(\Delta i_a)p}{d\delta} = 0\,,$$

therefore from equation (E4.1)

$$-\frac{T}{\tau_a}e^{\delta T/\tau_a} + \frac{T}{\tau_a}e^{(1-\delta)T/\tau_a} = 0 \quad \text{or} \quad \delta = 1 - \delta \quad \text{or} \quad \delta = 0.5\,.$$

Substituting in equation (E4.1), the maximum value of the per-unit ripple $(\Delta i_a)pm$ is given by the following equation:

$$(\Delta i_a)pm = \frac{V}{2R_aI_{rated}}\left[\frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1}\right] \tag{E4.2}$$

For $(\Delta i_a)pm = 0.1$

$$\frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1} = \frac{0.2R_aI_{rated}}{V} = \frac{0.2 \times 2.5 \times 20}{250} = 0.04$$

$$\text{or} \quad e^{0.5T/\tau_a} = 1.08 \quad \text{or} \quad 0.5T/\tau_a = \ell n(1.08) = 0.08 \quad \text{or} \quad \tau_a = \frac{T}{0.16}$$

$$\text{or} \quad L_a = \frac{R_aT}{0.16} = \frac{2.5}{400 \times 0.16} = 39.1 \text{ mH}\,.$$

Example 4.2

A 220-V, 100A dc series motor has an armature resistance and an inductance of 0.06 Ω and 2 mH, respectively. The field winding resistance and inductance are 0.04 Ω and 18 mH, respectively. Running on no load as a generator, with the field winding connected to a separate source, it gives the following magnetization characteristic at 700 rpm:

Field current	25	50	75	100	125	150	175	A
Terminal voltage	66.5	124	158.5	181	198.5	211	221.5	V

The motor is controlled by a chopper operating at 400 Hz and 220 V. Calculate the motor speed for a duty ratio of 0.7 and a load torque equal to 1.5 times the rated torque.

Solution: The speed at which the magnetization characteristic was measured = $700 \times 2\pi/60 = 73.3$ rad/sec.

voltage induced $E = K_e \Phi \omega_m$

$$K_e \Phi = K = \frac{E}{\omega_m}$$

$$\text{Torque } T_a = K I_a = \frac{E I_a}{\omega_m} \qquad \text{(E4.3)}$$

From equation (E4.3) and the magnetization characteristic

I_a	25	50	75	100	125	150	175	A
T_a	22.7	84.6	162.2	246.9	338.5	431.8	528.8	N-m

The rated torque (torque at 100A) = 247 N-m

$1.5 \times$ Rated torque $= 1.5 \times 247 = 370.5$ N-m

From the above T_a/I_a table the current at $1.5 \times$ Rated torque = 133 A

Also K at 133 A = $370.5/133 = 2.79$

$$R_a = 0.06 + 0.04 = 0.1\ \Omega$$

$$\text{Now } \omega_m = \frac{\delta V - I_a R_a}{K} = \frac{0.7 \times 220 - 100 \times 0.1}{2.79} = 51.6 \text{ rad/sec.} = 492.7 \text{ rpm}$$

Example 4.3

A 230 V, 500 rpm, 90 A separately excited dc motor has the armature resistance and inductance of 0.115 Ω and 11 mH respectively. The motor is controlled by a chopper operating at 400 Hz. If the motor is regenerating,

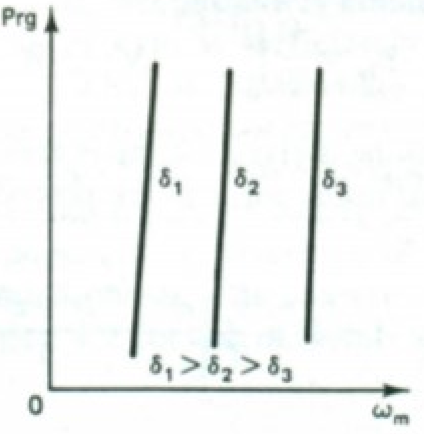


Figure 4.7 Regenerative braking performance curves of TRC chopper-fed dc separately excited motor.

1. Find the motor speed and the regenerated power at the rated current and a duty ratio of 0.5.
2. Calculate the maximum safe speed if the minimum value of the duty ratio is 0.1.

Solution: At rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. In regenerative braking

$$(1 - \delta)V = E - I_aR_a \quad \text{or} \quad E = (1 - \delta)V + I_aR_a \tag{E4.4}$$

At $\delta = 0.5$ and $I_a = 90$ A

$$E = 0.5 \times 230 + 90 \times 0.115 = 125 \text{ V}$$

Since $\dfrac{N}{N_r} = \dfrac{E}{E_r}$

where N_r = rated speed in rpm and N = speed to be calculated

Thus

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125}{219.7} = 284.5 \text{ rpm}$$

$$\tau_a = \frac{11 \times 10^{-3}}{0.115} = 95.65 \text{ mS}, \qquad T = \frac{1}{400} = 2.5 \text{ mS}$$

$$T/\tau_a = \frac{2.5 \times 10^{-3}}{95.65 \times 10^{-3}} = 0.026 \quad \text{and} \quad \tau_a/T = 38.3$$

Equation (4.30) is repeated here:

$$P_{rg} = \frac{V^2}{R_a}\left[\left(\frac{E}{V} - 1\right) \cdot (1-\delta) + \frac{\tau_a}{T}\left\{\frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}}\right\}\right] \tag{4.30}$$

Now

$$\frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}} = x = \frac{e^{0.5T/\tau_a} + e^{0.5T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}}$$

$$= \frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1} = \frac{0.013}{2.013} = 0.0065$$

From equation (4.30),

$$P_{rg} = \frac{V^2}{R_a}\left[\left(\frac{E}{V} - 1\right) \cdot (1-\delta) + \frac{\tau_a}{T}x\right]$$

$$= \frac{230^2}{0.115}\left[\left(\frac{125}{230} - 1\right)(1 - 0.5) + 38.3 \times 0.0065\right]$$

$$= 9.52 \text{ kW}$$

2. The maximum safe speed will be obtained at the minimum value of δ and the rated armature current. For higher speeds, the armature current will exceed the rated motor current and this operation will not be safe for the motor. At the maximum safe speed N_m, the back emf E_m is given by

$$E_m = (1 - \delta_{min})V + I_{ar}R_a = 0.9 \times 230 + 90 \times 0.115 = 217$$

$$N_m = \frac{N_r}{E_r} \times E_m = \frac{500}{219.7} \times 217 = 494 \text{ rpm}$$

Example 4.4

The motor of example 4.3 is controlled by a class C two-quadrant chopper operating with a source voltage of 230 V and a frequency of 400 Hz.

1. Calculate the motor speed for a motoring operation at $\delta = 0.5$ and half of rated torque.
2. What will be the motor speed when regenerating at $\delta = 0.5$ and rated torque?

Solution: At the rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. From equation (4.33),

$$\delta V = E + I_a R_a \tag{E4.5}$$

At half the rated torque, $I_a = 45$ A

At $\delta = 0.5$

$$E = \delta V - I_a R_a = 0.5 \times 230 - 45 \times 0.115 = 109.8 \text{ V}$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 109.8}{219.7} = 250 \text{ rpm}$$

2. In the regenerative braking at the rated torque, $I_a = -90$ A

From equation (E4.5),

$$E = \delta V - I_a R_a = 0.5 \times 230 + 90 \times 0.115 = 125.4$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125.4}{219.7} = 285 \text{ rpm}$$

Example 4.5

The motor of example 4.3 is fed by a four-quadrant chopper controlled by method III. The source voltage is 230 V and the frequency of operation is 400 Hz.

1. If the motor operation is required in the second quadrant at the rated torque and 300 rpm, calculate the duty ratio.
2. What should be the value of the duty ratio if the motor is working in the third quadrant at 400 rpm and half of the rated torque?

Solution: At the rated conditions of operation

$$E_r = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. Equation (4.39), which is applicable to method III is reproduced here:

$$I_a = \frac{2V(\delta - 0.5) - E}{R_a} \tag{4.39}$$

The motor is working in the second quadrant, therefore,

$$I_a = -90 \text{ A}$$

$$E = \frac{300}{500} \times E_r = \frac{300}{500} \times 219.7 = 131.8 \text{ V}$$

Substituting in equation (4.39), gives

$$-90 = \frac{2 \times 230(\delta - 0.5) - 131.8}{0.115}$$

or

$$\delta = 0.5 + \frac{121}{460} = .76 .$$

2. At half the rated torque and in the third quadrant

$$I_a = -45 \text{ A}$$

$$E = -\frac{400}{500} \times 219.7 = -175.7 \text{ V}$$

Substituting in equation (4.39), gives

$$-45 = \frac{2 \times 230(\delta - 0.5) + 175.7}{0.115}$$

or

$$\delta = 0.5 - \frac{181}{460} = 0.11 .$$

Example 4.23 The armature voltage of a separately excited dc motor is controlled by a one-quadrant chopper with chopping frequency of 200 pulses per second from a 300 V dc source. The motor runs at a speed of 800 rpm when the chopper's time ratio is 0.8. Assume that the armature circuit resistance and inductance are 0.08 Ω and 15 mH, respectively, and that the motor develops a torque of 2.72 N·m per ampere of armature current.

Find the mode of operation of the chopper, the output torque, and horsepower under the specified conditions.

Solution From the problem specifications at 800 rpm, using Eq. (4.201), we get,

$$E_c = (2.72)\frac{2\pi}{60}(800) = 227.9 \text{ V}$$

$$E_c = K_1\phi_f\omega \tag{4.201}$$

$$\omega = \frac{t_{on}}{t_x}\frac{V_i}{K_1\phi_f} - \frac{R_a}{(K_1\phi_f)^2}\frac{T}{t_x}T_0 \tag{4.202}$$

In the continuous mode of operation, with $t_{on} > t^*_{on}$, we have $t_x = T$. As a result Eq. (4.202) reduces to

$$\omega = \frac{(t_{on}/T)V_i - (R_a/K_1\phi_f)T_o}{K_1\phi_f)} \tag{4.203}$$

The armature circuit time constant is obtained as

$$\tau = \frac{L_a}{R_a} = \frac{15 \times 10^{-3}}{0.08} = 187.5 \times 10^{-3}\text{ s}$$

The chopping period is given by

$$T = \frac{1}{200} = 5 \times 10^{-3}\text{s}$$

We obtain the critical on-time using Eq. (4.197) as

$$\begin{aligned} t^*_{on} &= 187.5 \times 10^{-3}\, ln\left[1 + \frac{227.9}{300}(e^{5/187.5} - 1)\right] \\ &= 3.8 \times 10^{-3}\text{s} \end{aligned}$$

We know that $t_{on} = 0.8 \times 5 \times 10^{-3} = 4 \times 10^{-3}$. As a result, we conclude that the chopper output current is continuous.

To obtain the torque output, we use Eq. (4.203) rearranged as

$$T_o = \frac{K_1\phi_f}{R_a}\left(\frac{t_{on}}{T}V_i - K_1\phi_f\omega\right)$$

Thus we obtain

$$\begin{aligned} T_o &= \frac{2.72}{0.08}[0.8(300) - 227.9] \\ &= 411.4\,\mathrm{N \cdot m} \end{aligned}$$

The power output is obtained as

$$\begin{aligned} P_o &= (411.4)\frac{2\pi}{60}(800) = 34.5 \times 10^3\ \mathrm{W} \\ &= 46.2\ \mathrm{hp} \end{aligned}$$

To illustrate the principle of field control, we have the following example.

Example 4.24 Assume for the motor of Example 4.23 that field chopper control is employed to run the motor at a speed of 1500 rpm while delivering the same power output as obtained at 800 rpm and drawing the same armature current.

Solution Although we can use Eq. (4.205), we use basic formulas instead,

$$E_c = \frac{P_a}{I_a} = \frac{34.5 \times 10^3}{151.3} = 227.9\,\text{V}$$

This is the same back EMF. Recall that

$$E_c = K_1 \phi_f \omega$$

Thus the required field flux is obtained as

$$\phi_{f_n} = \phi_{f_0} \frac{\omega_0}{\omega_n} = \frac{8}{15} \phi_{f_0}$$

where the subscript n denotes the present case, and the subscript 0 denotes the field flux for Example 4.23. Assume that ϕ_{f_0} corresponds to full applied field flux; then

$$\frac{\phi_{f_0}}{\phi_{f_n}} = \frac{V_i}{V_o} = \frac{15}{8}$$

The required chopped output voltage is V_o. Now we have

$$\frac{V_o}{V_i} = \frac{t_{on}}{T}$$

Thus

$$\frac{t_{on}}{T} = \frac{8}{15}$$

Assuming that $T = 5 \times 10^{-3}$ s, we get

$$t_{on} = 2.67 \times 10^{-3}\,\text{s}$$

EXAMPLE 7.1 A separately excited dc motor runs at 1000 rpm from a 200 V dc supply. What will be the speed of the motor when power is supplied from a single-phase full converter working at $\alpha = 60°$. The supply voltage is 230 V (rms).

Solution

Neglecting the $I_a R_a$ drop and assuming constant excitation. The ratio of speeds will be same as the ratio of the applied voltages at the given conditions. Therefore,

$$\frac{V_{a1}}{V_{a2}} = \frac{N_1}{N_2}$$

$$N_2 = \frac{V_{a2} N_1}{V_{a1}}$$

$$= \frac{2 \times 230 \times \sqrt{2} \times \cos 60° \times 1000}{\pi \times 200}$$

$$= 518 \text{ rpm}$$

EXAMPLE 7.2 A dc drive works at 1100 rpm when fed from a 220 V dc source. The same drive is supplied from a chopper connected to 220 V dc mains. What will be the duty ratio to obtain 900 rpm. Neglect the $I_a R_a$ drop.

Solution

The ratio of speeds can be equated to the ratio of applied voltages.

$$\frac{V_{a1}}{V_{a2}} = \frac{N_1}{N_2}$$

$$V_{a2} = V_{a1} \times \frac{N_2}{N_1}$$

$$= 220 \times \frac{900}{1100} = 180 \text{ volts}$$

Since the field is constant,

Hence,

$$\text{Duty ratio } \delta = \frac{180}{220} = 0.82$$

EXAMPLE 7.1

A 230-V, 6-A, 1,500-rpm separately excited dc motor has an armature resistance of 5.1 Ω. A 1-quadrant chopper supplied from a 300-V dc bus is operating at a duty ratio of 60% and supplies power to the motor armature at rated current. Compute the motor speed.

SOLUTION:

At rated condition,

$$\begin{aligned} E_{b\ rated} &= V_a - I_{a\ rated} \times r_a \\ &= 230 - (5.1 \times 6) \\ &= 199.4\ V \end{aligned}$$

$$E_{b\ rated} \propto 1,500\ rpm\,(N_{rated})$$

At 60% duty ratio, $E_{b1} \propto N_1$.

$$\begin{aligned} \text{Armature voltage} &= 300 \times (60/100) \\ &= 180\ V \end{aligned}$$

$$\begin{aligned} \text{Back-emf}, E_{b1} &= 180 - (5.1 \times 6) \\ &= 149.4\ V \end{aligned}$$

Therefore,

$$\begin{aligned} N_1 &= \frac{E_{b1}}{E_{b\ rated}} N_{rated} \\ &= \frac{149.4}{199.4} \cdot 1500 \approx 1124\ rpm \end{aligned}$$

EXAMPLE 7.2

A first-quadrant dc/dc converter is fed from a 300-V dc bus. When the converter supplies power to a separately excited dc motor at 40% duty ratio, the average armature current is 5 A at 1,560 rpm. What is the duty ratio required to reduce the speed to 1,300 rpm for the same armature current? The armature resistance is 5.1 Ω.

SOLUTION:

At 40% duty ratio, armature voltage is

$$\begin{aligned} V_a &= 300 \times \frac{40}{100} \\ &= 120\ \text{V} \end{aligned}$$

$$\begin{aligned} \text{Back-emf at 40\% duty ratio, } E_{b1} &= 120 - (I_a r_a) \\ &= 120 - (5 \times 5.1) \\ &= 94.5\ \text{V} \end{aligned}$$

$$E_{b1} \propto 1{,}560\ \text{rpm}$$

Now back-emf, E_{b2}, at 1,300 rpm can be related as $E_{b2} \propto 1{,}300$ rpm.

$$E_{b2} = \frac{N_2}{N_1} E_{b1} = 78.75\ \text{V}$$

$$\text{Armature voltage, } V_a = E_{b2} + I_a r_a$$

$$V_a = 300 \times D$$

Therefore,

$$D = 34.75\%$$

EXAMPLE 7.3

A separately excited dc motor is fed from a 440-V dc source through a single-quadrant chopper, $r_a = 0.2\ \Omega$, and armature current is 175 A. The voltage and torque constants are equal at 1.2 V/rad/s. The field current is 1.5 A. The duty cycle of chopper is 0.5. Find (a) speed and (b) torque.

SOLUTION:

a.

$$\begin{aligned} E_b &= 220 - 175 \times 0.2 = 185\text{ V} \\ &= 1.2 \times \omega_r \times I_f \\ \omega_r &= 185/(1.2 \times 1.5) \\ &= 102.77\text{ rad/s} \\ &= 981.38\text{ rpm} \end{aligned}$$

b.

$$\text{Torque} = 1.2 \times 1.5 \times 175 = 315\text{ N-m}$$

EXAMPLE 7.4 A single quadrant dc chopper is used to control the speed of a separately excited armature controlled dc motor. The chopper has a supply voltage of 230 V dc. The on time and off time of the chopper are 10 ms and 25 ms respectively. Also, $R_a = 2\ \Omega$. Assuming continuous conduction of the motor current, determine the average armature current and torque developed by the motor when it is running at speed of 1400 rpm. The back emf constant of the motor is 0.5 V/rad/s.

Solution

$$\begin{aligned} \text{Back emf, } E_b &= 1400 \times 2\pi \times \frac{0.5}{60} \\ &= 73.30\text{ volts} \end{aligned}$$

$$V_a = V_s \times \frac{T_{on}}{T} = 230 \times \frac{10}{25} = 92\text{ volts}$$

$$I_a = \frac{(92 - 73.3)}{2} = 9.35\text{ A}$$

$$T = \text{Torque constant} \times I_a$$

Also,

$$\text{Torque constant} = 0.5\text{ Nm A}^{-1}$$

Therefore,

$$\begin{aligned} \text{Torque } T &= 9.35 \times 0.5 \\ &= 4.675\text{ Nm} \end{aligned}$$

EXAMPLE 7.4

A separately excited dc motor has the following name plate data: 220 V, 100 A, 2,200 rpm. The armature resistance is 0.1 Ω, and inductance is 5 mH. The motor is fed by a chopper that is operating from a dc supply of 250 V. Due to restrictions in the power circuit, the chopper can be operated over a duty cycle ranging from 30% to 70%. Determine the range of speeds over which the motor can be operated at rated torque.

SOLUTION:

Because e torque is constant, i_a is the same for all the values of D.

$$V_{o(av)} = DV_{dc}$$

At $D = 0.3$,

$$\begin{aligned} V_{o(av)} &= 0.3 \times 250 \\ &= 75 \text{ V} \\ E_{b(0.3)} &= V_{o(av)} - I_a r_a \\ &= 75 - (100 \times 0.1) \end{aligned}$$

At $D = 0.7$,

$$\begin{aligned} V_{o(av)} &= 0.7 \times 250 \\ &= 175 \text{ V} \\ E_{b(0.7)} &= 175 - 10 \\ &= 165 \text{ V} \end{aligned}$$

Under rated conditions, $V_a = 220$ V, $I_a = 100$ A, $r_a = 0.1\ \Omega$

$$\begin{aligned} E_{b(rated)} &= 220 - (100 \times 0.1) \\ &= 210 \text{ V} \end{aligned}$$

$$N_r = 2200 \text{ rpm}$$

$$\frac{N_{0.7}}{N_r} = \frac{E_{b(0.7)}}{E_{b(rated)}}$$

$$\begin{aligned} N_{(0.7)} &= \frac{165}{210} \times 2200 \\ &= 1{,}728.5714 \text{ rpm} \end{aligned}$$

$$\begin{aligned} N_{(0.3)} &= \frac{65}{210} \times 2200 \\ &= 680.95 \text{ rpm} \end{aligned}$$

Hence speed can be varied in the range $680.95 \le N \le 1728.5714$.

EXAMPLE 7.5

A separately excited dc motor has an armature resistance 2.3 Ω, and armature current is 100 A. (a) Find the voltage across the braking resistance for a duty ratio of 25%. (b) Find the power dissipated in braking resistance.

SOLUTION:

$$\text{Average current} = I_a(1-D) = 100(1-0.25) = 75\ \text{A}$$

$$\text{Average Voltage} = I_{b(avg)} \times R_b = 75 \times 2.3 = 172.5\ \text{V}$$

$$P_b = I_a^2 R_b (1-D)$$

$$P_b = 100^2 \times 2.3 \times (1-0.25) = 17250\ \text{W}$$

EXAMPLE 7.6

A separately excited dc motor has the following name plate data: 200 V, 75 A, and 1,500 rpm. The armature resistance is 0.2 Ω. If dynamic baking takes place at 600 rpm at rated torque, compute the duty ratio. The braking resistance is 5 Ω.

SOLUTION:

E_b under rated condition,

$$\begin{aligned} E_{b(rated)} &= V - I_a r_a \\ &= 200 - (75 \times 0.2) \\ &= 185\ \text{V} \end{aligned}$$

$$\frac{E_{b(600)}}{E_{b(rated)}} = \frac{N}{N_{rated}}$$

$$E_{b(600)} = \left(\frac{N}{N_{rated}}\right) E_{b\,rated}$$

$$E_{b(600)} = \left(\frac{600}{1500}\right) \times 185 = 74\ \text{V}$$

Now,

$$E_{b(600)} = I_a\left(r_a + R_b(1-D)\right)$$

Because braking takes place at rated torque, $I_a = I_{arated} = 75$ A,

$$\text{i.e., } E_{b(600)} = 75\left(0.2 + 5(1-D)\right)$$

$$74 = 75 \times 0.2 + 75 \times 5(1-D)$$

$$\therefore D = 0.84$$

EXAMPLE 7.7

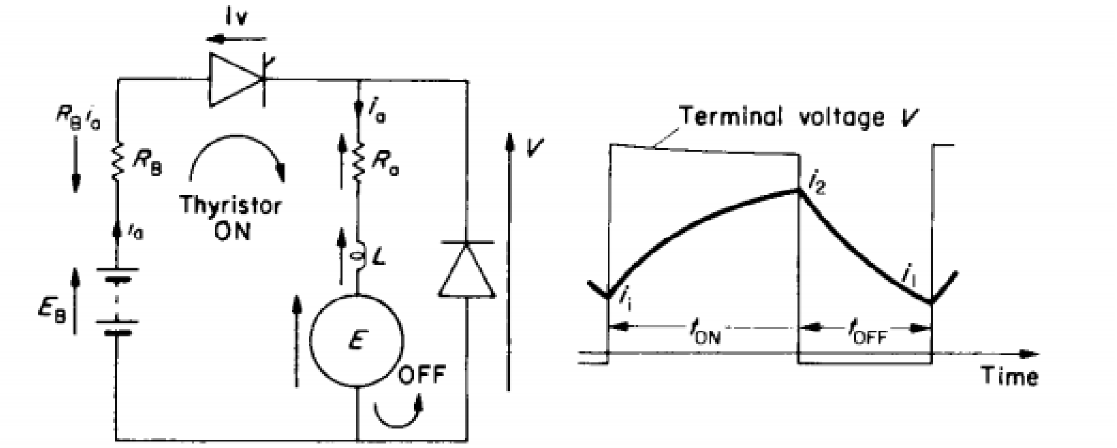
A dual-input dc/dc converter is supplied from two dc sources: 12-V and 24-V batteries. The duty ratio of the power switch connected to the first source is 40%, while that of the second source is 25%. Compute the average output voltage. The load consists of large inductance and resistance.

SOLUTION:

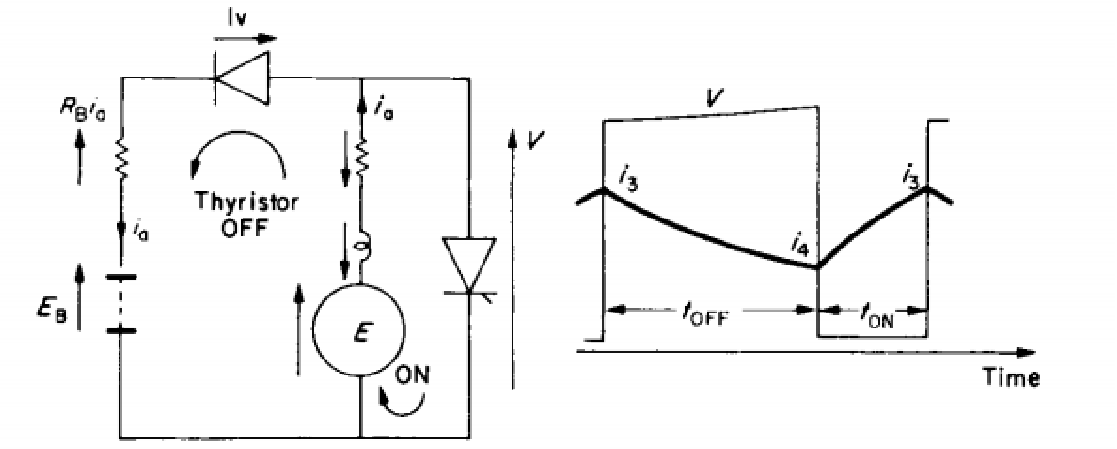
$$\begin{aligned} V_{o(av)} &= 12 \times \frac{40}{100} + 24 \times \frac{25}{100} \\ &= 4.8 + 6 \\ &= 10.8\ \text{V} \end{aligned}$$

Example 7.1

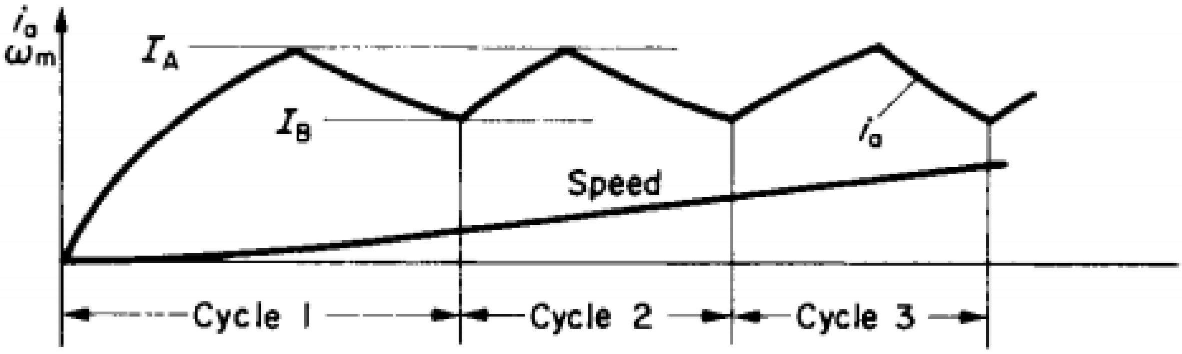
An electrically-driven automobile is powered by a d.c. series motor rated at 72 V, 200 A. The motor resistance and inductance are respectively 0.04 Ω and 6 milli-henrys. Power is supplied via an ON/OFF controller having a fixed frequency of 100 Hz. When the machine is running at 2500 rev/min the generated-e.m.f. per field-ampere, k_{fs}, is 0.32 V which may be taken as a mean "constant" value over the operating range of current. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor, when operating at this particular speed and with a duty-cycle ratio δ of 3/5. Mechanical, battery and semi-conductor losses may be neglected when considering the relevant diagrams of Fig. 7.1a.



(a) MOTORING (Motoring conventions)



(b) GENERATING (Generating conventions)



(c) Acceleration between limits

FIG. 7.1. Chopper-fed d.c. machine.

Chopping period = 1/100 = 10 msec and for $\delta = 3/5$; ON + OFF = 6 + 4 msec.

The equations are:

for ON period: $V = k_{fs}i + Ri + Lpi$—from eqn (7.2a).

Substituting: $72 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$.

For OFF period: $0 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$—from eqn (7.2b).

Rearranging:

ON $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 200 = I_{max}$,

OFF $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 0 = I_{min}$.

Current oscillates between a "low" of i_1 and a "high" of i_2, with $\tau = 0.0167$ second

ON $i_2 = i_1 + (200 - i_1)(1 - e^{-0.006/0.0167})$,

$i_2 = 200 - (200 - i_1)e^{-0.36} = \underline{60.46 + 0.698 i_1}$.

OFF $i_1 = i_2 + (0 - i_2)(1 - e^{-0.004/0.0167})$

$i_1 = i_2\, e^{-0.24} = \underline{0.787 i_2}$.

Hence, by substituting: $i_2 = 60.46 + 0.698 \times 0.787 i_2$,

from which $\underline{i_2 = 134.1\text{ A}}$ and $\underline{i_1 = 105.6\text{ A}}$.

$$\text{Torque} = k_\phi i = \frac{k_{fs} i}{\omega_m} \times i = \frac{k_{fs}}{\omega_m} i^2.$$

$$\text{Mean torque} = \frac{0.32}{2500 \times 2\pi/60}\left(\frac{134.1^2 + 105.6^2}{2}\right) = \underline{17.8\text{ Nm}}.$$

$$\text{Mean power} = \omega_m T_e = \frac{2\pi}{60} \times 2500 \times 17.8 = \underline{4.66\text{ kW}} = \underline{6.25\text{ hp}}.$$

Example 7.2

The chopper-controlled motor of the last question is to be separately excited at a flux corresponding to its full rating. During acceleration, the current pulsation is to be maintained as long as possible between 170 and 220 A. During deceleration the figures are to be 150 and 200 A. The total mechanical load referred to the motor shaft corresponds to an armature current of 100 A and rated flux. The total inertia referred to the motor shaft is 1.2 kg m^2. The battery resistance is 0.06 Ω and the semiconductor losses may be neglected. Determine the ON and OFF periods for both motoring and regenerating conditions and hence the chopping frequency when the speed is 1000 rev/min.

Calculate the accelerating and decelerating rates in rev/min per second and assuming these rates are maintained, determine the time to accelerate from zero to 1000 rev/min and to decelerate to zero from 1000 rev/min. Reference to all the diagrams of Fig. 7.1 will be helpful.

Rated flux at rated speed of 2500 rev/min corresponds to an e.m.f.:

$$E = V - RI_a = 72 - 0.04 \times 200 = 64\ \text{V}$$

At a speed of 1000 rev/min therefore, full flux corresponds to $64 \times 1000/2500 = 25.6$ V

Acceleration Total resistance $= R_a + R_B = 0.04 + 0.06 = 0.1\ \Omega$

For ON period $E_B = E + Ri_a + Lpi_a$,

$$72 = 25.6 + 0.1\, i_a + 0.006\, pi_a.$$

Rearranging: $0.06\ di_a/dt + i_a = 464 = I_{max}$.

Solution is: $i_2 = i_1 + (I_{max} - i_1)(1 - e^{-t_{ON}/\tau})$

and since i_1 and i_2 are known: $220 = 170 + (464 - 170)(1 - e^{-t_{ON}/0.06})$.

$$\frac{220 - 170}{464 - 170} = 1 - e^{-t_{ON}/0.06}$$

from which: $\underline{t_{ON} = 0.01118.}$

For OFF period $0 = 25.6 + 0.04 i_a + 0.006 pi_a$ (note resis. $= R_a$).

Rearranging: $0.15\, di_a/dt + i_a = -640 = I_{min}$.

Solution is: $i_1 = i_2 + (I_{min} - i_2)(1 - e^{-t_{OFF}/\tau})$.

Substituting i_1 and i_2: $170 = 220 + (-640 - 220)(1 - e^{-t_{OFF}/0.15})$,

$$\frac{170 - 220}{-640 - 220} = 1 - e^{-t_{OFF}/0.15},$$

from which: $\underline{t_{OFF} = 0.008985}$ $\underline{t_{ON} + t_{OFF} = 0.02017\ \text{second.}}$

Duty cycle $\delta = 0.01118/0.02017 = \underline{0.554.}$ Chopping frequency $= 1/0.02017 =$ 49.58 Hz.

Deceleration

Thyristor ON $\quad 0 = E - R_a i_a - Lpi_a.$

Substituting: $\quad = 25.6 - 0.04 i_a - 0.006 pi_a.$

Rearranging: $\quad 0.15\,di_a/dt + i_a = 640 = I_{max}.$

Solution is: $\quad i_3 = i_4 + (I_{max} - i_4)(1 - e^{-t_{ON}/\tau}).$

Substituting: $\quad 200 = 150 + (640 - 150)(1 - e^{-t_{ON}/0.15}),$

$$\frac{200-150}{640-150} = 1 - e^{-t_{ON}/0.15},$$

from which: $\quad \underline{t_{ON} = 0.01614.}$

Thyristor OFF $\quad E_B = E - Ri_a - Lpi_a,$

$$72 = 25.6 - 0.1 i_a - 0.006 Lpi_a.$$

Rearranging: $\quad 0.06\,di_a/dt + i_a = -464 = I_{min}.$

Solution is: $\quad i_4 = i_3 + (I_{min} - i_3)(1 - e^{-t_{OFF}/\tau}).$

Substituting: $\quad 150 = 200 + (-464 - 200)(1 - e^{-t_{OFF}/0.06}),$

$$\frac{150-200}{-464-200} = 1 - e^{-t_{OFF}/0.06}$$

from which: $\quad \underline{t_{OFF} = 0.004697} \quad \underline{t_{ON} + t_{OFF} = 0.02084 \text{ second.}}$

Duty cycle $\delta = 0.01614/0.02084 = \underline{0.774.}$ Chopping frequency $= 1/0.02084 =$ $\underline{47.98 \text{ Hz.}}$

Accelerating time

$$\text{Load torque} = k_\phi I_a = \frac{E}{\omega_m} I_a = \frac{64}{2500 \times 2\pi/60} \times 100 = 24.45\text{ Nm.}$$

During acceleration:

$$k_\phi I_{mean} = \frac{64}{2500 \times 2\pi/60} \times \frac{220+170}{2} = 0.2445 \times 195 = 47.67\text{ Nm.}$$

$$\text{Constant } d\omega_m/dt = \frac{T_e - T_m'}{J} = \frac{47.67 - 24.45}{1.2} = 19.35\text{ rad/s per second}$$

$$= 19.35 \times \frac{60}{2\pi} = \underline{184.8\text{ rev/min per sec.}}$$

$$\text{Accelerating time to 1000 rev/min} = \frac{1000}{184.8} = \underline{5.41\text{ seconds.}}$$

Decelerating time

During deceleration: $k_\phi I_{mean} = 0.2445(-200 - 150)/2 = -42.8\text{ Nm}.$

Note that this electromagnetic torque is now in the same sense as T_m, opposing rotation.

The mechanical equation is: $\quad T_e = T_m + J\,d\omega_m/dt,$

$$-42.8 = 24.45 + 1.2\,d\omega_m/dt.$$

from which:

$$\frac{d\omega_m}{dt} = \frac{-67.25}{1.2} = -56.04\text{ rad/s per second} = \underline{-535.1\text{ rev/min per second.}}$$

Time to stop from 1000 rev/min with this torque maintained $= 1000/535.1 =$ $\underline{1.87\text{ seconds.}}$

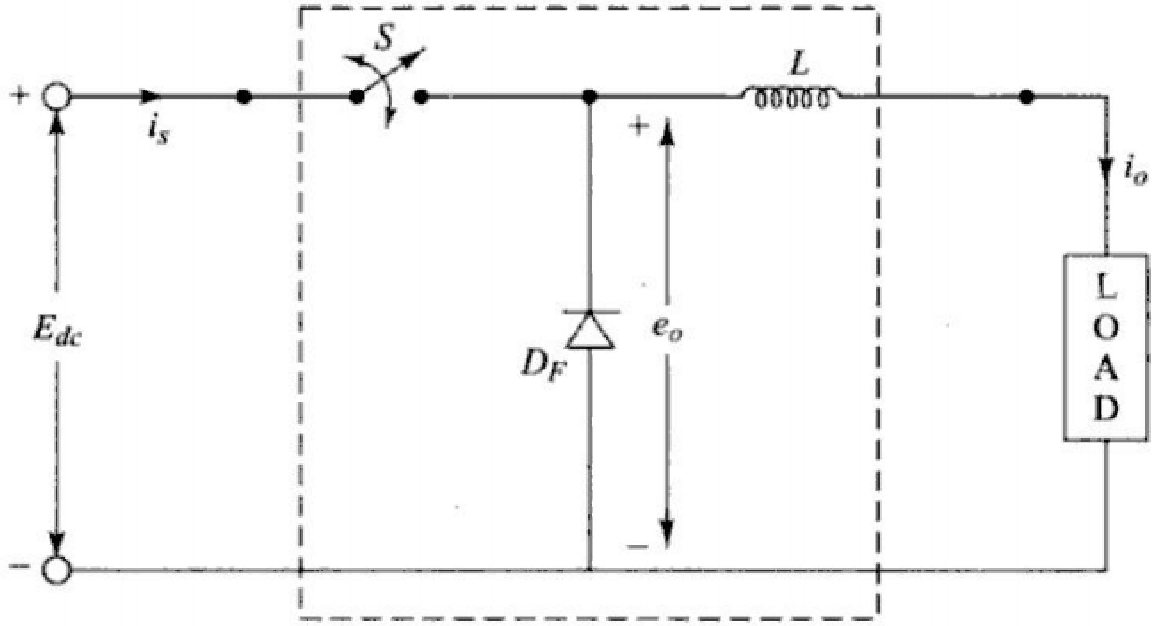


Fig. 8.4 *Basic chopper circuit*

Example 8.1 A d.c. chopper circuit connected to a 100 V d.c. source supplies an inductive load having 40 mH in series with a resistance of 5 Ω. A freewheeling diode is placed across the load. The load current varies between the limits of 10 A and 12 A. Determine the time ratio of the chopper.

Solution: The average value of the load current $= \dfrac{I_1 + I_2}{2} = \dfrac{10+12}{2} = 11$ A.

The maximum value of the load current $= \dfrac{100}{5} = 20$ A

Now, the average value of the voltage, $E_{0av} = 100 \times \dfrac{11}{20} = 55$ V

Also, $$E_{dc} \cdot \frac{T_{on}}{T_{on} + T_{off}} = E_{0av} \quad \text{or} \quad \frac{T_{on}}{T_{on} + T_{off}} = \frac{E_{0av}}{E_{dc}}$$

$$\frac{T_{on}}{T_{on} + T_{off}} = \frac{55}{100} = 0.55 \quad \therefore T_{on} = 0.55\,(T_{on} + T_{off})$$

$$\therefore \qquad \frac{T_{on}}{T_{off}} = \frac{0.55}{0.45} = 1.222.$$

Example 8.2 For the chopper circuit shown in Fig. Ex. 8.2, express the following variables as functions of E_{dc}, R, and duty cycle α.

(i) Average output voltage and current.

(ii) Output current at the instant of commutation.

(iii) Average and RMS freewheeling diode currents.

(iv) RMS value of the output voltage.

(v) RMS and average load currents.

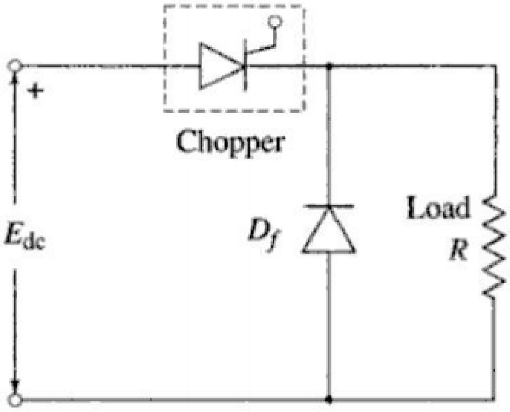


Fig. Ex. 8.2

Solution: With resistive load, load current waveforms are similar to load voltage waveforms.

$\therefore$ (i) Average output voltage $E_0 = E_{dc}\dfrac{T_{on}}{T} = E_{dc}\cdot\alpha.$

Average output current, $I_{0av} = \dfrac{E_0}{R} = \dfrac{E_{dc}}{R}\alpha\,.$

(ii) Output current at the instant of commutation $= \dfrac{E_{dc}}{R}$.

(iii) Freewheeling diode does not come into picture for a resistive load. Hence, average and RMS values of freewheeling diode currents are zero.

(iv) RMS value of output voltage

$$= \left[\frac{T_{on}}{T}E_{dc}^2\right]^{1/2} = \sqrt{\alpha}\cdot E_{dc}$$

(v) Now, average thyristor current

$$= \frac{E_{dc}}{R}\cdot\frac{T_{on}}{T} = \alpha\frac{E_{dc}}{R}$$

$$\text{RMS thyristor current} = \left(\alpha\cdot\left(\frac{E_{dc}}{R}\right)^2\right)^{1/2} = \sqrt{\alpha}\cdot\frac{E_{dc}}{R}$$

Example 8.3 A step-down dc chopper has a resistive load of $R = 15$ ohm and input voltage $E_{dc} = 200$ V. When the chopper remains ON, its voltage drop is 2.5 V. The chopper frequency is 1 kHz. If the duty cycle is 50%, determine:

(a) Average output voltage
(b) RMS output voltage
(c) Chopper efficiency
(d) Effective input resistance of chopper

Solution:

Given: Input voltage $E_{dc} = 200$ V, duty cycle $\alpha = 0.5$

$$R = 15\ \Omega,\ F = 1\ \text{kHz},\ \text{Chopper drop } E_d = 2.5\ \text{V}$$

(a) Average output voltage $E_0 = \alpha \cdot (E_{dc} - E_d)$

$$= 0.5\ (200 - 2.5) = 98.75\ \text{V}$$

(b) RMS output voltage

$$E_{0(rms)} = \sqrt{\alpha}\ (E_{dc} - E_d) = \sqrt{0.5}\ (200 - 2.5) = 139.653\ \text{V}$$

(c) Chopper efficiency

Output power, $P_0 = E_{0rms} \cdot I_{0rms}$

Now, $$I_{0rms} = \frac{E_{0rms}}{R} = \frac{\sqrt{\alpha} \cdot E_{dc}}{R}$$

$\therefore$ $$P_0 = \sqrt{\alpha} \cdot E_{dc} \cdot \frac{\sqrt{\alpha} \cdot E_{dc}}{R} = \frac{\alpha E_{dc}^2}{R}$$

If E_d is the chopper drop, then

$$P_0 = \frac{\alpha (E_{dc} - E_d)^2}{R} = \frac{0.5\,(200 - 2.5)^2}{15} = 1300.21\ \omega$$

Now, the input power to the chopper is given by

$$P_i = \frac{1}{T}\int_0^T E_{dc}\ i_s\ \mathrm{d}t = \frac{1}{T}\int_0^{T_{on}} E_{dc}\ \frac{(E_{dc} - E_d)}{R}\ \mathrm{d}t = \frac{1}{T}\int_0^{\alpha \cdot T} \frac{E_{dc}\ (E_{dc} - E_d)}{R}\ \mathrm{d}t$$

$$= \frac{E_{dc}\ (E_{dc} - E_d)}{T.R.}\ (t)_o^{\alpha_T} = \frac{\alpha\, E_{dc}\ (E_{dc} - E_d)}{R.} = \frac{0.5\,(200)\,(200 - 2.5)}{15} = 1316.67\ \text{W}$$

$\therefore$ Chopper efficiency, $$\eta = \frac{P_o}{P_i} = \frac{1300.21}{1316.67} = 0.9874 = 98.74\%$$

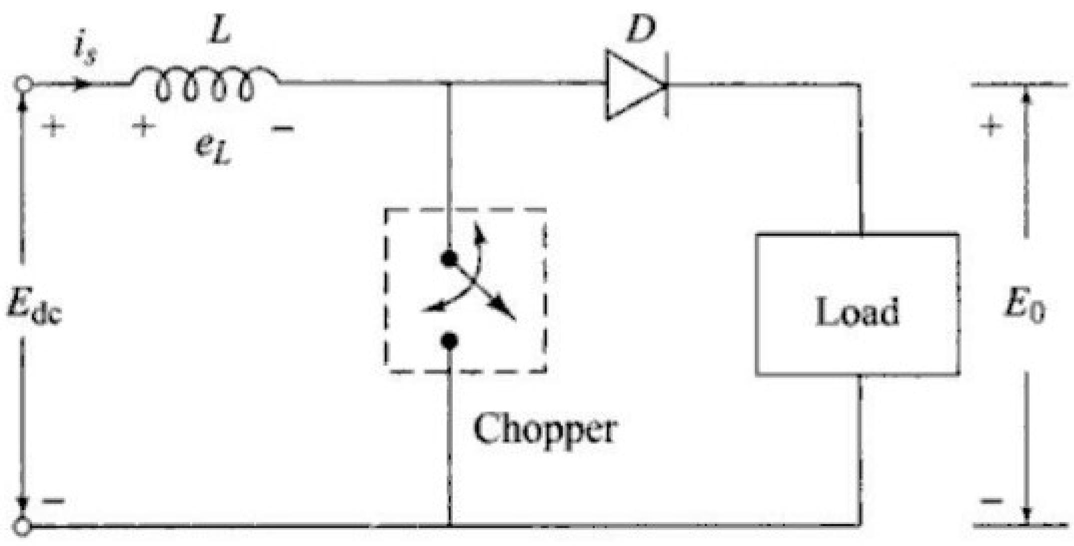


Fig. 8.6 *Step-up chopper or boost choppers*

Example 8.4 A step-up chopper is used to deliver load voltage of 500 V from a 220V d.c. source. If the blocking period of the thyristor is 80 μs, compute the required pulse width.

Solution: From Eq. (8.10) we have, $E_0 = E_{dc}\,\dfrac{T_{on} + T_{off}}{T_{off}}$

$$\therefore \quad 500 = 220\,\frac{T_{on} + 80 \times 10^{-6}}{80 \times 10^{-6}}, \qquad \therefore T_{on} = 101.6 \times 10^{-6} = 101.6\ \mu\text{s}.$$

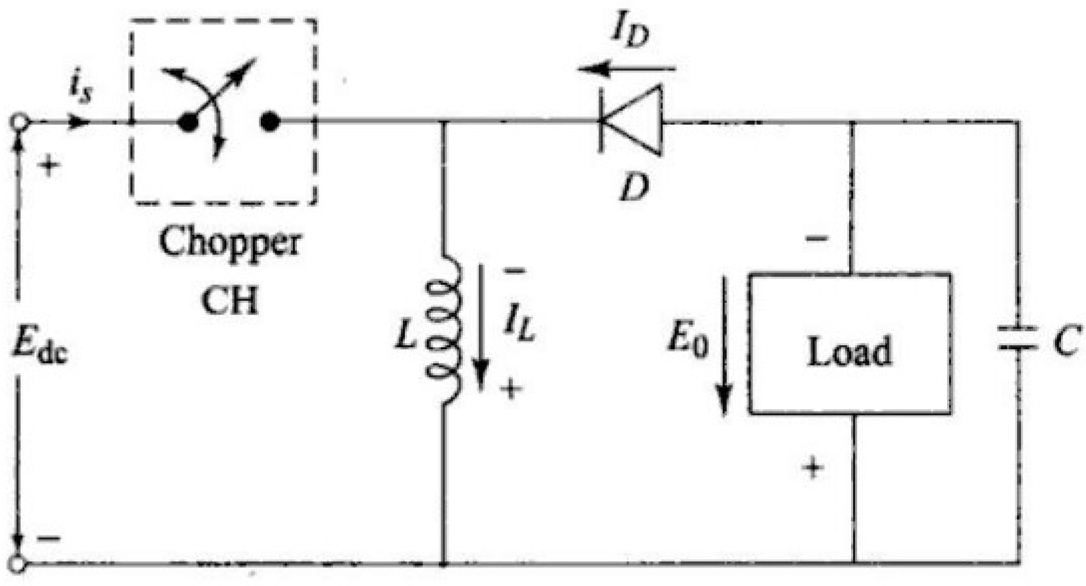


Fig. 8.7 *Step-up/down chopper*

8.4.1 Time-Ratio Control (TRC)

Example 8.5 A chopper circuit is operating on TRC principle at a frequency of 2 kHz on a 220 V d.c. supply. If the load voltage is 170 V, compute the conduction and blocking period of thyristor in each cycle.

Solution: From Eq. (8.2), $E_0 = E_{dc} \cdot T_{on} \cdot f$

Given : $f = 2$ kHz, $E_{dc} = 220$ V, $E_o = 170$.

Conduction period, $T_{on} = \dfrac{E_0}{E_{dc} \cdot f} = \dfrac{170}{220 \times 2 \times 10^3} = T_{on} = 0.386$ ms.

But, chopping period, $T = \dfrac{1}{f} = \dfrac{1}{2 \times 10^3} = 0.5$ ms

$\therefore$ Blocking period of SCR, $T_{off} = T - T_{on} = 0.5 - 0.386 = 0.114$ m sec.

8.4.2 Current Limit Control

Example 8.6 In a 110 V dc chopper drive using the CLC scheme, the maximum possible value of the accelerating current is 300 A, the lower-limit of the current pulsation is 140 A. The ON- and OFF periods are 15 ms and 12 ms, respectively. Calculate the limit of current pulsation, chopping frequency, duty cycle and the output voltage.

Solution: Given: $T_{on} = 15$ ms, $T_{off} = 12$ ms, $I_{0max} = 300\,A$, $I_{0min} = 140\,A$

Now, maximum limit of current pulsation = 300 – 140 = 160 A.

Chopping frequency $= \dfrac{1}{T} = \dfrac{1}{15+12} = 37$ Hz & ratio, $\alpha = \dfrac{T_{on}}{T} = \dfrac{15}{27} = 0.56$

Output voltage, $E_0 = \alpha E_{dc} = 0.56 \times 110 = 61.60$ V

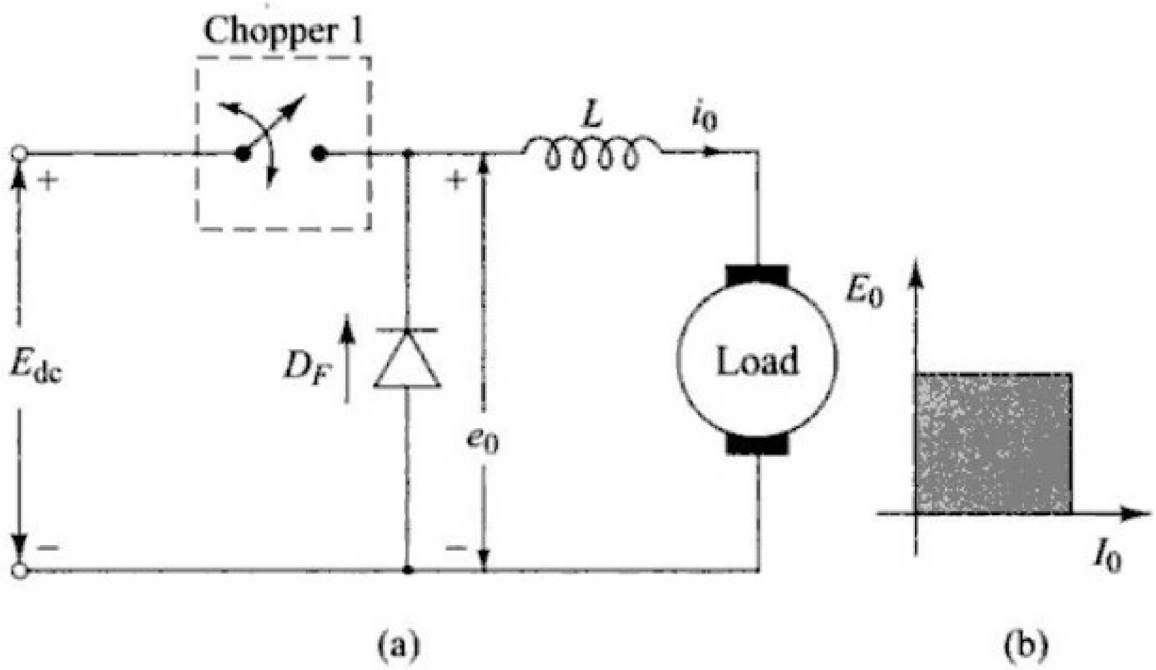


Fig. 8.12 *Type A chopper circuit and $E_0 - I_0$ characteristic*

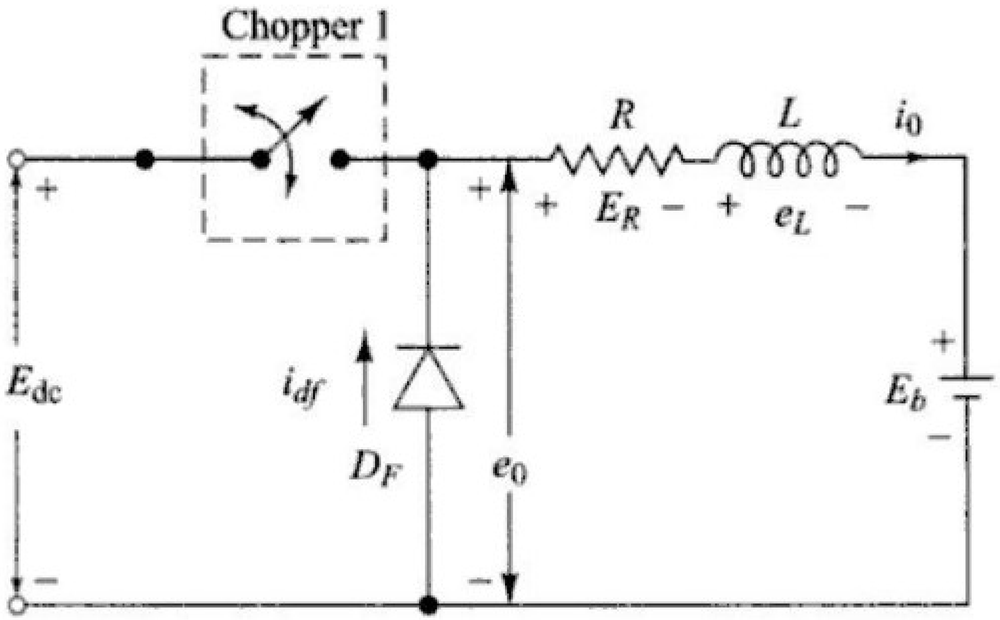


Fig. 8.13 *First quadrant chopper with $R - L$ load*

Example 8.7 For the ideal type A chopper circuit, following conditions are given: $E_{dc} = 220$ V, chopping frequency, $f = 500$ Hz; duty cycle $\alpha = 0.3$ and $R = 1\ \Omega$; $L = 3$ mH; and $E_b = 23$ V. Compute the following quantities.

(i) Check whether the load current is continuous or not.
(ii) Average output current.
(iii) Maximum and minimum values of steady-state output current.
(iv) RMS values of first, second and third harmonics of load current.
(v) Average value of source current.
(vi) The input power, power absorbed by the back emf E_b and power loss in the resistor.
(vii) RMS value of output current using the result of (ii) and (iv).
(viii) The RMS value of load current using the results of (iv). Compare the result with that obtained in part (vii) above.

Solution:

(i) We know from the chopper theory, that the load current is continuous only when actual value of duty cycle α is greater than α'. Therefore, first calculate α'. From Eq. (8.44), we write, $\alpha' = \left(\dfrac{\tau}{T}\right) \ln\left[1 + g\left(e^{T/\tau} - 1\right)\right]$

where $\quad \tau = L/R = \dfrac{3 \times 10^{-3}}{1} = 3 \times 10^{-3}$ s.

$$T = \frac{1}{f} = \frac{1}{500} = 2000\ \mu\text{s} \cdot g = \frac{E_b}{E_{dc}} = \frac{23}{220} = 0.105.$$

$$\therefore\ \alpha' = \left(\frac{3 \times 10^{-3}}{2000 \times 10^{-6}}\right) \ln\left[1 + 0.105\left(e^{0.67} - 1\right)\right] = 1.5 \ln[1.100]\ \alpha' = 0.143$$

Since $\alpha > \alpha'$, load current is continuous.

(ii) Average output current,

$$I_{0av} = \frac{\alpha \cdot E_{dc} - E_b}{R} = \frac{0.3 \times 220 - 23}{1} = 43 \text{ A}.$$

(iii) From Eq. (8.30), maximum value of output current is given by

$$I_{0max} = \frac{E_{dc}}{R}\left[\frac{1 - e^{-T_{on}/\tau}}{1 - e^{-T/\tau}}\right] - \frac{E_b}{R}$$

Now, $\quad \alpha = \dfrac{T_{on}}{T}\ 0.3 = \dfrac{T_{on}}{2000 \times 10^{-6}} \quad \therefore\ T_{on} = 600\ \mu$s.

Also, $\quad \dfrac{T_{on}}{\tau} = \dfrac{600 \times 10^{-6}}{3 \times 10^{-3}} = 200 \times 10^{-3}$.

$$\therefore \qquad I_{0\max} = \frac{220}{1}\left[\frac{1-e^{-200\times10^{-3}}}{1-e^{-0.67}}\right]-\frac{23}{1} \qquad \therefore I_{0\max} = 58.64 \text{ A.}$$

From Eq. (8.31), $I_{0\min} = \dfrac{E_{dc}}{R}\left[\dfrac{e^{T_{on}/\tau}-1}{e^{T/\tau}-1}\right]-\dfrac{E_b}{R}$

$$= \frac{220}{1}\left[\frac{e^{0.2}-1}{e^{0.67}-1}\right]-\frac{23}{1} = 28.05 \text{ A.}$$

(iv) The RMS value of first harmonic voltage is given by

$$E_1 = \frac{2E_{dc}}{\sqrt{2}\pi}\sin\pi = \frac{2\times220}{\sqrt{2}\pi}\sin(\pi\times0.3) = 80.121 \text{ V.}$$

Now, $\quad Z_1 = \sqrt{R^2+(WL)^2} = \sqrt{(1)^2+(2\pi\times500\times3\times10^{-3})^2} = 9.48\,\Omega.$

$$\therefore \qquad I_1 = \frac{E_1}{Z_1} = \frac{80.121}{9.48} = 8.452 \text{ A.}$$

Similarly, $\quad I_2 = \dfrac{2\times220}{2\sqrt{2}\pi}\sin 2\pi\alpha\dfrac{1}{\sqrt{1^2+\left(2\pi\times500\times2\times3\times10^{-3}\right)^2}} = 2.494 \text{ A}$

$$I_3 = \frac{2\times220}{3\sqrt{2}\pi}\sin(162^\circ)\frac{1}{\sqrt{1^2+\left(2\pi\times3\times500\times3\times10^{-3}\right)^2}} = 0.624 \text{ A.}$$

(v) From Eq. (8.45), the average value of source current is given by

$$I_{TAV} = \frac{(E_{dc}-E_b)\alpha}{R} - \frac{L}{RT}(I_{0\max}-I_{0\min})$$

$$= \frac{(220-23)0.3}{1} - \frac{3\times10^{-3}}{1\times2000\times10^{-6}}(58.64-28.05)$$

$$= 59.1 - 1.5\,(30.59) = 13.215$$

(vi) Input power = E_{dc} × average source current = 220 × 13.215 = 2907.3 W

Power absorbed by load emf = E_b × average load current = 23 × 43 = 989 W.

Power loss in resistor R = Input power – power absorbed by load emf

$$= 2907.3 - 989 = 1918.3 \text{ W.}$$

(vii) $I_{rms} = \sqrt{I_{0av}^2+I_1^2+I_2^2+I_3^2} = \sqrt{(43)^2+(8.452)^2+(2.494)^2+(0.624)^2}$

$$= 1927.05 \text{ A} = 43.89 \text{ A.}$$

(viii) Power loss in resistor I^2R = 1918.3 W

$$\therefore \qquad I_{rms} = \sqrt{\frac{1918.3}{1}} = 43.798 \text{ A}$$

The value of I_{rms} in both parts is nearly the same.

Example 8.8 An ideal chopper operating at a chopping period of 2 ms supplies a load of 4 Ω having an inductance of 8 mH from a 80 V battery. Assuming the load is shunted by a perfect commutating diode, and battery to be lossless,

(a) compute the load current waveform $f_{on} \frac{T_{on}}{T_{off}}$ values of

(ii) 1/1 (ii) 4/1 (iii) 1/4.

(b) Also, calculate the mean value of load voltage and current at each setting.

Solution:

(a) During the on-period, the battery is switched to a series R_L load, having an initial current I_{0min}.

During the off-period, the load current decays in the R–L load through the diode, having an initial value of I_{0max}. Fig. Ex. 8.8 shows the waveforms for each setting.

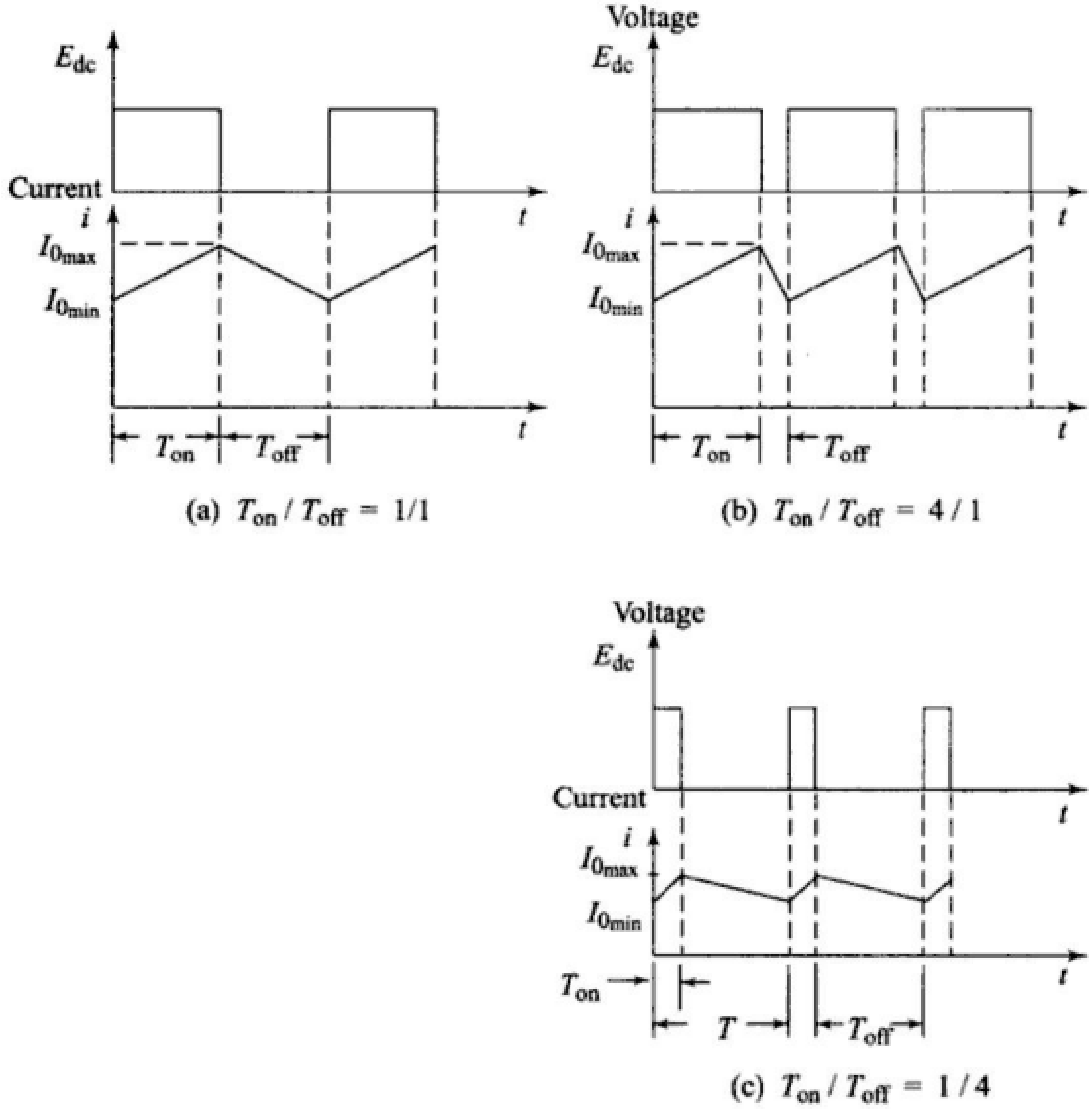


Fig. Ex. 8.8 *Three different waveforms*

When chopper is ON, $i = I_{0\min} \cdot \left(\frac{E_{dc}}{R} - I_{0\min}\right)\left(1 - e^{-T_{on}/\tau}\right) = I_{0\max}$ (a)

When chopper is OFF, $i = I_{0\max}\, e^{-T_{off}/\tau} = I_{0\min}$ (b)

From Eqs (a) and (b), we can write $I_{0\max} = \frac{E_{dc}}{R}\left(\frac{1-e^{-T_{on}/\tau}}{1-e^{-T/\tau}}\right)$ (c)

and $I_{0\,\min} = I_{0\,\max}\, e^{-T_{off}/\tau}$ (d)

Now, $\frac{E_{dc}}{R} = \frac{80}{4} = 20$ A. $\tau = L/R = \frac{0.008}{4} = 0.002$ s. $T = 2$ ms. $= 0.002$ s

(i) When $\frac{T_{on}}{T_{off}} = 1/1$,

$T_{on} = T_{off} = 1$ ms $= 0.001$ s. $\therefore I_{0\,\max} = 20\left(\frac{1-e^{-\frac{0.001}{0.002}}}{1-e^{-0.002/0.002}}\right) = 12.45$ A.

and $I_{0\,\min} = 12.45\, e^{-0.001/0.002} = 7.55$ A.

(ii) When $\frac{T_{on}}{T_{off}} = 4/1, T_{on} = 4\,T_{off}$

$\therefore \quad T_{on} = 0.0016$ s, and $T_{off} = 0.0004$ s

$$\therefore \quad I_{0\,\max} = 20\left(\frac{1-e^{-\frac{0.0016}{0.002}}}{1-e^{-0.002/0.002}}\right) = 17.42 \text{ A.}$$

$$I_{0\,\min} = 17.42\,(e^{-0.004/0.002}) = 2.36 \text{ A}$$

(iii) When $\frac{T_{on}}{T_{off}} = 1/4, T_{off} = 0.001\,6$ s, $T_{on} = 0.0004$ s.

$$\therefore \quad I_{0\,\max} = 20\left(\frac{1-e^{-\frac{0.004}{0.002}}}{1-e^{-0.002/0.002}}\right) = 5.73 \text{ A.}$$

$$I_{0\,\min} = 5.73\,(e^{-0016/0.002}) = 2.57 \text{ A.}$$

Example 8.9 An R–L E_b type load is operating in a chopper circuit from a 400 volts d.c. source. For the load, $L = 0.05$ H and $R = 0$. For a duty cycle of 0.3, find the chopping frequency to limit the amplitude of load current excursion to 8 A.

Solution: The related circuit diagram is shown in Fig. Ex. 8.9.

The average output voltage is given by, $E_0 = \alpha \cdot E_{dc}$

As the average value of voltage drop across inductance L is zero,

$$E_b = E_0 = \alpha \cdot E_{dc} = 0.3 \times 400 = 120 \text{ V}$$

During the on-period of the chopper T_{on}, the difference in source voltage E_{dc} and load back emf E_b, i.e. $(E_{dc} - E_b)$, appears across L, as shown in Fig. Ex. 8.9.

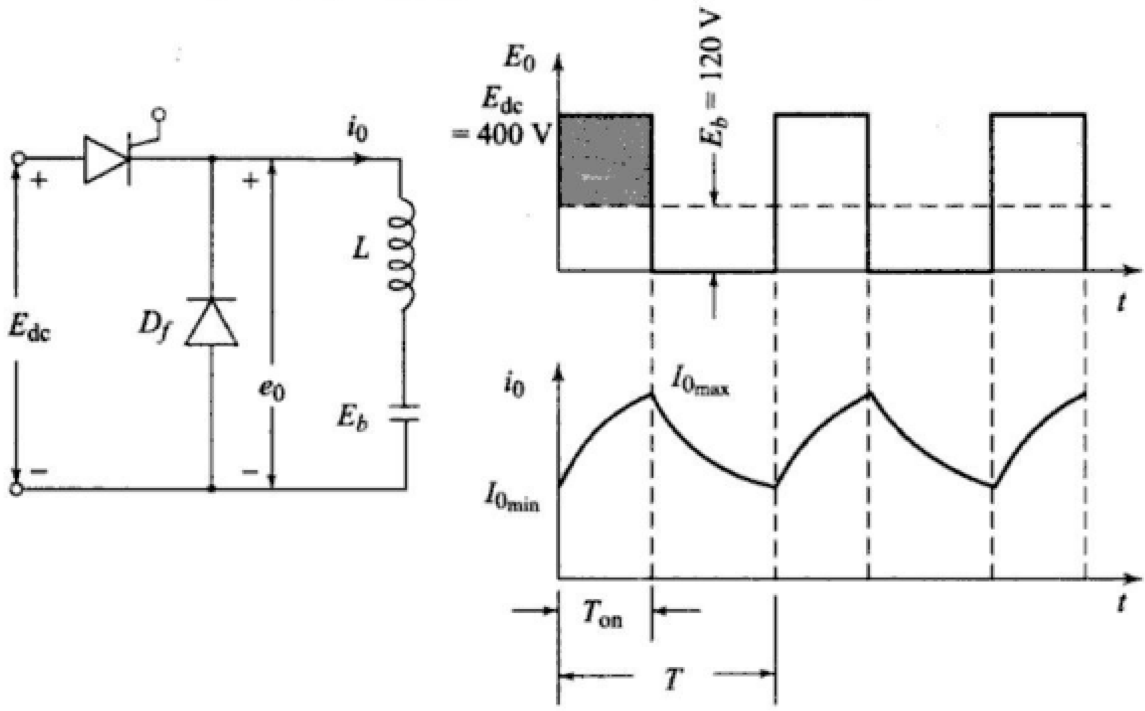


Fig. Ex. 8.9 *Chopper circuit and waveforms*

$\therefore$ During T_{on}, volt–time area applied to inductance = (400 – 120) T_{on} = 280 T_{on} $V-s$ (a)

As shown in Fig. Ex. 8.9, the current through inductance L rises from $I_{0\,min}$ to $I_{0\,max}$. From this, volt-time area across L during current change is given by

$$\int_0^{T_{on}} E_L dt = \int_0^{T_{on}} L \cdot \frac{di_0}{dt} dt = \int_{T_{0max}}^{T_{0max}} L \cdot di = L\,(I_{0\,max} - I_{0\,min}) = L\,\Delta\, I_0 \qquad \text{(b)}$$

During T_{on}, the volt–time areas given by Eqs (a) and (b) must be equal.

$$\therefore \qquad 280\, T_{on} = L\,\Delta\, I_0$$

$$\therefore \quad T_{on} = \frac{0.05 \times 8}{280} = 1.43 \text{ ms}$$

$\therefore$ Chopping frequency, $f = 1/T = \dfrac{\alpha}{T_{on}} = \dfrac{0 \cdot 3}{1.43 \times 10^{-3}} = 209.79$ Hz.

Example 8.10 A simple d.c. chopper is operating at a frequency of 2 kHz from a 96 V d.c. source to supply a load resistance of 8 Ω. The load time constant is 6 ms. If the average load voltage is 57.6 V, find the T_{on} period of the chopper, the average load current, the magnitude of the ripple current and its RMS value.

Solution: Chopping period, $T = 1/f = \dfrac{1}{2000} = 0.5$ ms.

Given load time constant = 6 ms

$\therefore$ Load time constant = 12 T, therefore treat as a linear current variation.

(i) Now, $E_0 = E_{dc} \cdot \frac{T_{on}}{T}$ $\quad \therefore \frac{57.6}{96} = \frac{T_{on}}{0.5 \times 10^{-3}}$ $\quad \therefore T_{on} = 0.3$ ms.

(ii) The RMS value of the load voltage is given by

$$F_{L_{RMS}} = E_{dc}\left(\frac{T_{on}}{T}\right)^{1/2} = 96 \times \left(\frac{0.3}{0.5}\right)^{1/2} = 74.36 \text{ V}$$

(iii) Therefore, the average load current $= \frac{E_0}{R} = \frac{57.6}{8} = 7.2$ A

(iv) Now, current ripple $= \Delta_i = \frac{(E_{dc} - E_0)\Delta t}{L}$

Load time constant $\tau = L/R$, $\quad \therefore L = 6 \times 10^{-3} \times 8 = 48$ mH.

$$\therefore \qquad \Delta_i = \frac{(96 - 57.6) \times 0.3 \times 10^{-3}}{48 \times 10^{-3}} = 0.24 \text{ A}$$

(v) From Example (8.8), we have

$$I_{0\,max} = \frac{E_{dc}}{R}\left(\frac{1 - e^{-T_{on}/\tau}}{1 - e^{-T/\tau}}\right) = \frac{96}{8}\left(\frac{1 - e^{-\frac{0.3\text{ ms}}{6\text{ ms}}}}{1 - e^{-0.5/6\text{ ms}}}\right) = 7.32 \text{ A.}$$

Similarly,

$$I_{0\,min} = I_{0\,max}\, e^{-T_{off}/\tau} = 7.32\, e^{-0.2\text{ ms}/6\text{ ms}} = 7.08 \text{ A}$$

The RMS value of the ripple current is given by

$$I_{r,\,RMS} = \frac{(I_{0\,max} - I_{0\,min})}{2\sqrt{3}} = \frac{(7.32 - 7.08)}{2\sqrt{3}} = 0.0693 \text{ A.}$$

Example 8.11 A d.c. motor with armature resistance $R_a = 0.4\ \Omega$ and armature inductance $t_a = 8$ mH, is having a back emf of 80V while carrying a current of 10 A. The motor is connected to a d.c. source of 180 V by the main SCR of the chopper. If the SCR turns off after 1 ms, compute the current in the motor

(i) at the instant the thyristor turns off, and
(ii) 8 ms after SCR turns off.

Solution: The differential equation with the given chopper conditions is given by

$E_{dc} = R_a \cdot i_a + L_a \dfrac{di_a}{dt} + E_{b,}$ Now, $\tau = L/R = 8$ mH/0.4 = 20 ms.

The solution of the above equation is given by

$$i(t) = \frac{E_{dc} - E_b}{R_a}\left(1 - e^{-t/\tau}\right) + I_0 \cdot e^{-t/\tau} = \frac{180 - 80}{0.4}\left(1 - e^{-t/20 \times 10^{-3}}\right) + 10(e^{-t/20 \times 10^{-3}})$$

$$= 250\left(1 - e^{-t/20 \times 10^{-3}}\right) + 10\, e^{-t/20 \times 10^{-3}}$$

(i) At $t = 1 \times 10^{-3}$ s, $I(t) = 250\left(1 - e^{-1 \times 10^{-3}/20 \times 10^{-3}}\right) + 10\, e^{-1/10^{-3}/20 \times 10^{-3}}$

$$I(t) = 12.193 + 9.512\, I(t) = 21.705 \text{ A.}$$

(ii) Current is freewheeled through the load for the period of 9 ms.

$$\therefore \qquad i_f = I(t) \cdot e^{-t/\tau} = i_f = 21.705 e^{-9 \times 10^{-3}/20 \times 10^{-3}} = 13.84 \text{ A}$$

Example 8.12 A DC chopper operates on 230 V dc and frequency of 400 Hz, feeds an R-L load. Determine the ON time of the chopper for output of 150 V.

Solution:

Given: $E_{dc} = 230$ V, $f = 400$ Hz, $E_0 = 150$ V

We have, $E_0 = \alpha \cdot E_{dc}$ $\therefore$ $150 = \alpha \cdot 230$, $\therefore$ $\alpha = 0.65$

Time period of output voltage wave is given by

$$T = 1/F = 1/400 = 2.5 \times 10^{-3} \text{ sec}$$

Now, $\alpha = \dfrac{T_{on}}{T}$, $\therefore t_{on} = \alpha \cdot T = 0.65\,(2.5 \times 10^{-3})$

Example 8.13 A single-quadrant type A chopper is operated with the following specifications:

(i) ideal battery of 220 V (ii) on-time t_{on} = 1 msec (iii) off-time t_{off} = 1.5 msec

Determine: (a) Average and RMS output voltages (b) Ripple and form factor

Solution:

Time period $T = t_{on} + t_{off} = (1 + 1.5) = 2.5$ msec, Duty cycle $\alpha = \frac{T_{on}}{T} = \frac{1}{2.5} = 0.4$

(a) Average output voltage, $E_0 = \alpha \cdot E_{dc} = (0.4)(220) = 88$ V.

$$\text{RMS output voltage } E_{0rms} = \sqrt{\alpha} \cdot E_{dc} = \sqrt{0.4}\ (220) = 139.14 \text{ V}$$

$$\text{(b) Form-factor (FF)} = \frac{\text{RMS Value}}{\text{Average Value}} = \frac{\sqrt{\alpha} \cdot E_{dc}}{\alpha \cdot E_{dc}} = \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.4}} = 1.58$$

$$\text{Ripple factor (RF)} = \sqrt{(FF)^2 - 1} = \sqrt{\frac{1}{\alpha} - 1} = \sqrt{\frac{1-\alpha}{\alpha}} = \sqrt{\frac{1-0.4}{0.4}} = 1.23$$

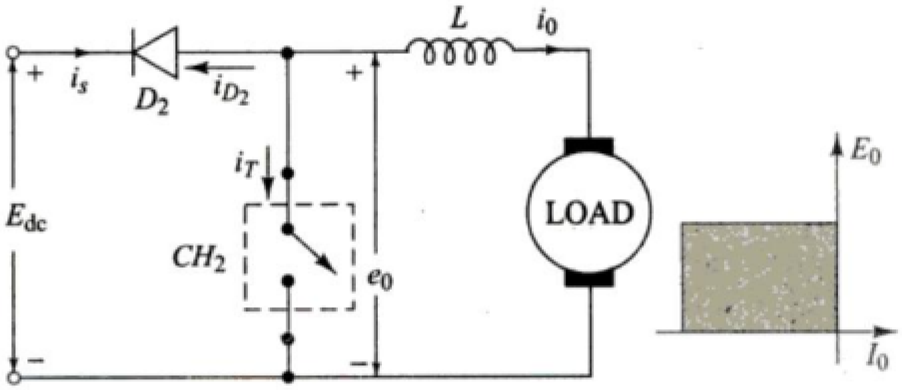


Fig. 8.18(*a*) *Type-B chopper circuit and E_0–I_0 characteristic*

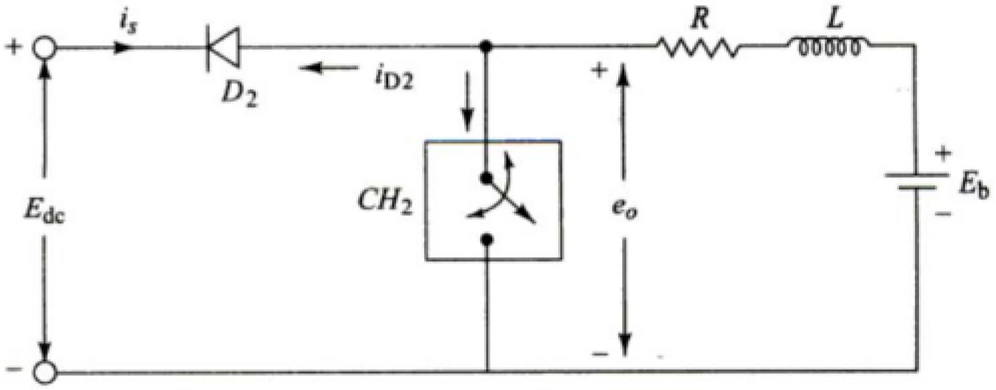


Fig. 8.18(*b*) *Class B with R–L load*

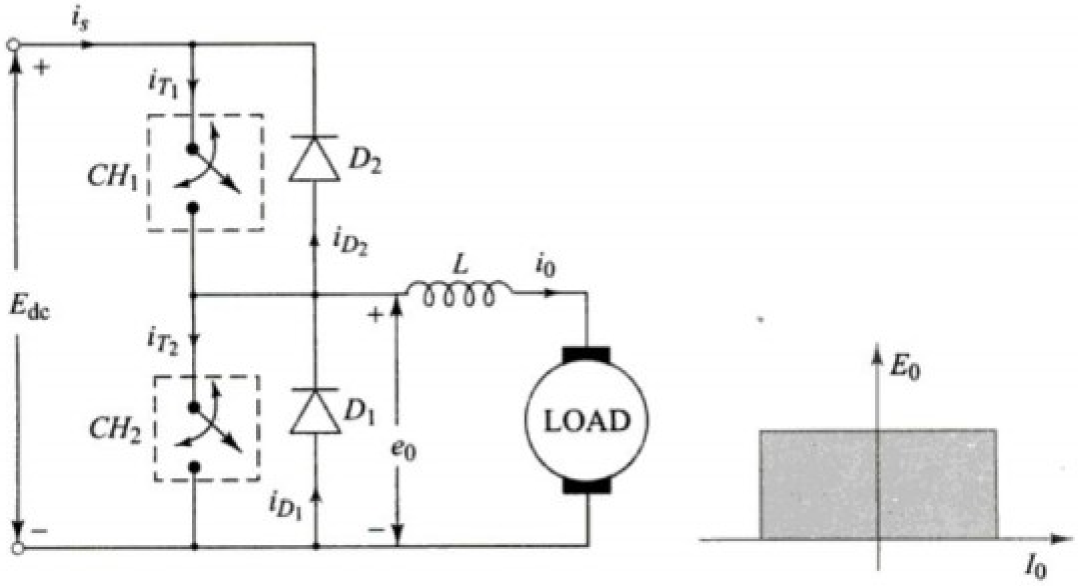


Fig. 8.20(a) *Type-C chopper circuit and $E_0 - I_0$ characteristics*

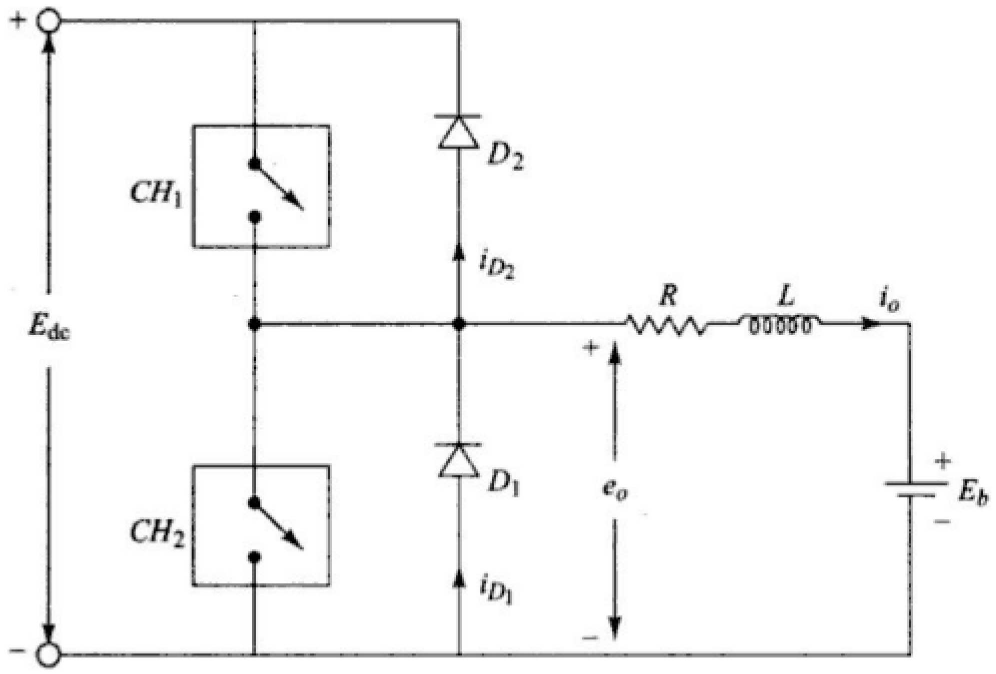


Fig. 8.20(b) *Class-C chopper with R-L load*

Example 8.14 A Class C chopper is operated from a 220 V battery. The load is a dc motor with $R = 0.1\ \Omega$, $L = 10$ mH and $E_b = 100$ V. Determine the following:

(i) Duty cycle for the motoring mode
(ii) Critical duty-cycle for the regenerative mode
(iii) Duty cycle to achieve regenerative braking at the rated current of 10 Amp
(iv) Power returned to the source during braking

Solution:

Given $E_{dc} = 220$ V, $R = 0.1\ \Omega$, $L = 10$ mH, $E_b = 100$

(i) The average load current is given by

$$i_0 = \frac{E_o - E_b}{R} = \frac{\alpha \cdot E_{dc} - E_b}{R}$$

$$\therefore \quad \alpha = \frac{i_o \cdot R + E_b}{E_{dc}} = \frac{(10 \times 0.1) + 100}{220}, \quad \therefore \quad \alpha = 0.459$$

(ii) Critical duty cycle for regenerative braking $= \dfrac{E_b}{E_{dc}} = 0.4545$

(iii) Duty cycle to achieve regenerative braking at the rated current of $i_0 = 10$ Amp.

$$\therefore \quad \text{Rated load current, } i_0 = \frac{\alpha \cdot E_{dc} - E_b}{R}$$

For regeneration this current should be negative.

$$\therefore \quad -10 = \frac{(\alpha \cdot 220) - 100}{0.1}, \quad \therefore \quad D = 0.45$$

(iv) Power returned to source during braking

$$= E_b\, i_0 - i_0^2 R = 100 \times 10 - (10^2 \times 0.1) = 990 \text{ Watts}$$

Example 8.15 A two-quadrant chopper operating in the first and fourth quadrant is operated from a 220 V battery. The load is dc motor with $R = 0.1\ \Omega$, L = 10 mH and $E_b = 100$ V, determine:

(i) Duty cycle α_m for motoring mode
(ii) Critical duty cycle for regenerative braking
(iii) Duty-cycle to achieve regenerative braking at the rated current of 10 Amp
(iv) Power returned to the source during braking
(v) The switching frequency of the devices if the output frequency is 5 kHz.

Solution:

Given: $E_{dc} = 220$ V, $R = 0.1\ \Omega$, $L = 10$ mH, $E_b = 100$ V, $i_0 = 10$ Amp

Class-D chopper operates in first and fourth quadrant

(i) Duty-cycle for motoring mode (α_m):

From equation (8.82), $E_0 = (2 \cdot \alpha_m - 1)\, E_{dc}$

Average-load current $I_0 = \dfrac{E_0 - E_b}{R}$

$$10 = \frac{(2\,\alpha_m - 1) \cdot (220) - (110)}{0.1} \qquad \therefore \alpha_m = 0.73$$

(ii) Critical duty-cycle for regenerative braking is given by

$$\alpha_c = (1 - \alpha_m) = (1 - 0.73) = 0.27$$

(iii) Duty-cycle for regeneration of rated armature current (α_R) :

Rated average armature current $= \dfrac{E_b - E_0}{R}$

But $\qquad E_0 = -(2\alpha_R - 1)$ during braking

$$\therefore \quad 10 = \frac{100 + (2\,\alpha_R - 1)\, 220}{0.1} \qquad \therefore \alpha_R = 0.275$$

(iv) Power-returned to the source during braking interval:

$P = E_b \cdot i_0 - i_0^2 R_a = (100 \times 10) - (100 \times 0.1) = 990$ W

(v) Switching frequency of device:

In Class-D chopper, the switching frequency of power switch is half the output frequency.

$\therefore \qquad f_s = 2.5$ kHz.

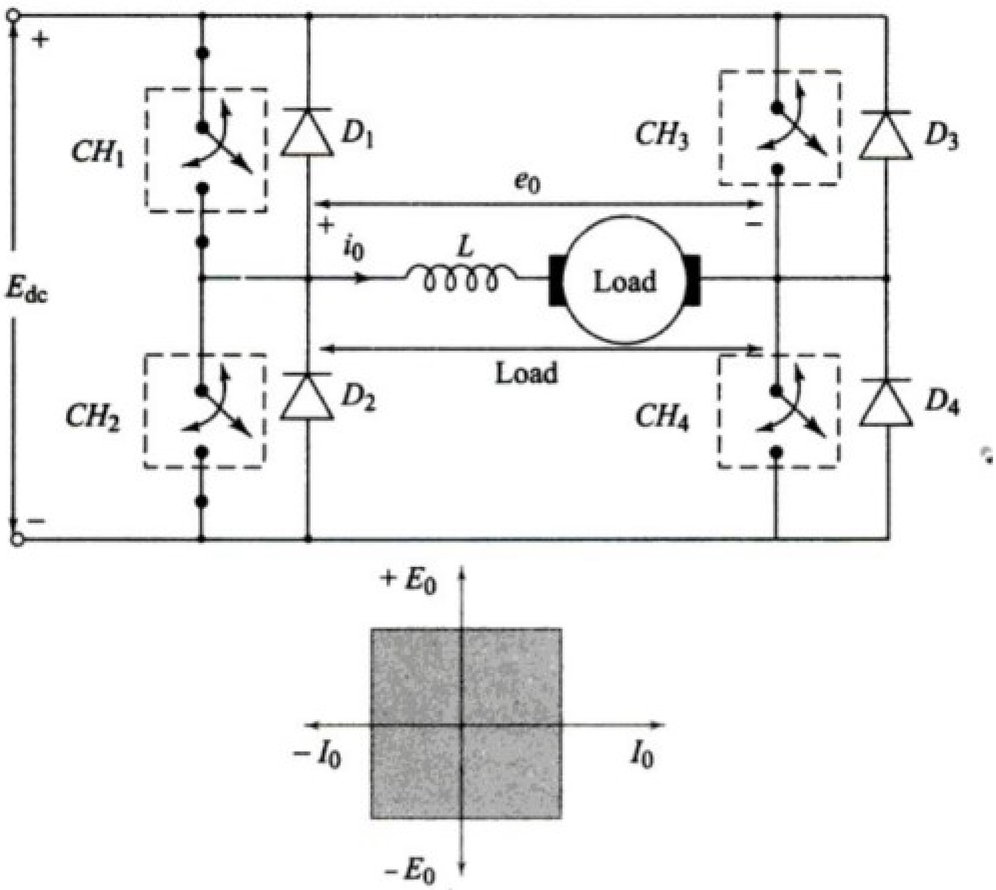


Fig. 8.25(a) *Type E chopper circuit and characteristic*

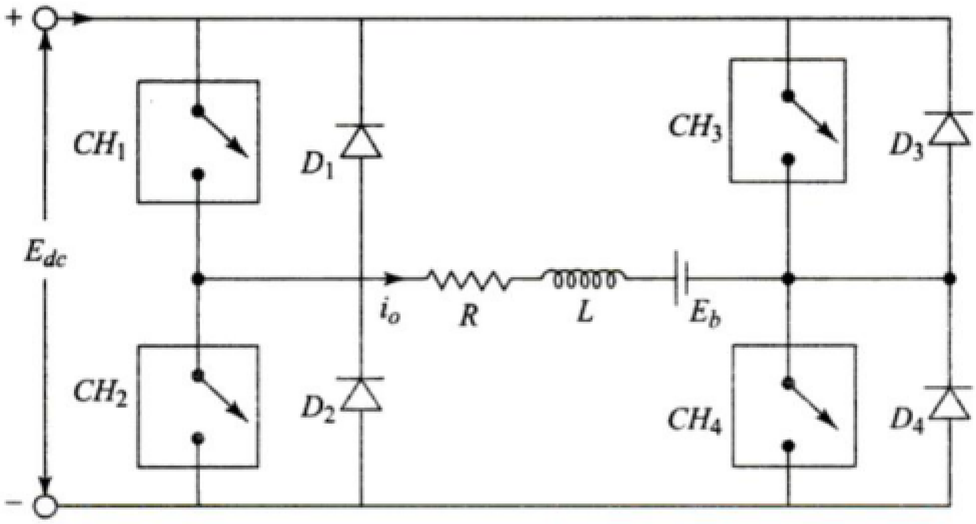


Fig. 8.25(b) *Class E chopper with R–L load*

SOLVED EXAMPLES

Example 8.16 A four-quadrant chopper is driving a separately excited dc motor load. The motor parameters are $R = 0.1$ ohm, $L = 10$ mH. The supply voltage is 200 V d.c. If the rated current of the motor is 10 A and if the motor is driving the rated torque. Determine:

(i) the duty cycle of the chopper if $E_b = 150$ V.
(ii) the duty cycle of the chopper if $E_b = -110$ V.

Solution:

For a four-quadrant chopper, the average voltage in all the four-modes is given by

$$E_0 = 2\,E_{dc} \cdot (\alpha - 0.5)$$

(i) The average current, $i_0 = \dfrac{E_0 - E_b}{R} = \dfrac{2\,E_{dc} \cdot (\alpha - 0.5) - E_b}{R}$

$$10 = \frac{2 \times 200\,(\alpha - 0.5) - 150}{0.1} \qquad \therefore\ \alpha = 0.876$$

Since, $\alpha > 0.5$, this mode is forward-motoring

(ii) Now, $10 = \dfrac{2 \times 200\,(\alpha - 0.5) - 110}{0.1}, \quad \therefore\ \alpha = 0.228$

PROBLEMS

10.1 Determine the ripple factor (RF), defined as $\mathrm{RF} = V_{rip}/V_{dc} = (V_{rms}^2 - V_{dc}^2)^{1/2}/V_{dc}$, for the following circuits.

(a) Fig. 10.17.

(b) Fig. 10.18, for $\alpha = 90°$.

What is the significance of the ripple factor?

CHAPTER: 10

10.1 (a) From equation 10.1

$$V_{dc} = \frac{\sqrt{2}\,V_p}{\pi}$$

From Fig 10.17

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(\sqrt{2}\,V_p \sin\theta)^2 d\theta} = \frac{V_p}{\sqrt{2}}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{\sqrt{2}} \times \frac{\pi}{\sqrt{2}\,V_p} = \frac{\pi}{2} = 1.5706$$

$$RF = \left\{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1\right\}^{\frac{1}{2}} = (1.5706^2 - 1)^{\frac{1}{2}} = 1.2114$$

(b) From equation 10.2

$$V_{dc} = \frac{V_p}{\sqrt{2}\,\pi}(1 + \cos 90^\circ) = \frac{V_p}{\sqrt{2}\,\pi}$$

$$V_{rms} = \left\{\frac{1}{2\pi}\int_{\frac{\pi}{2}}^{\pi}(\sqrt{2}\,V_p \sin\theta)^2 d\theta\right\}^{\frac{1}{2}} = \frac{V_p}{2}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{2} \times \frac{\sqrt{2}\,\pi}{V_p} = \frac{\pi}{\sqrt{2}} = 2.2218$$

$$RF = (2.2218^2 - 1)^{\frac{1}{2}} = 1.984$$

RF is a measure of ripple content

EXAMPLE 10.5

The two-quadrant chopper shown in Fig. 10.38*a* is used to control the speed of the dc motor and also for regenerative braking of the motor. The motor constant is $K\Phi = 0.1$ V/rpm ($E_a = K\Phi n$). The chopping frequency is $f_c = 250$ Hz and the motor armature resistance is $R_a = 0.2\ \Omega$. The inductance L_a is sufficiently large and the motor current i_0 can be assumed to be ripple-free. The supply voltage is 120 V.

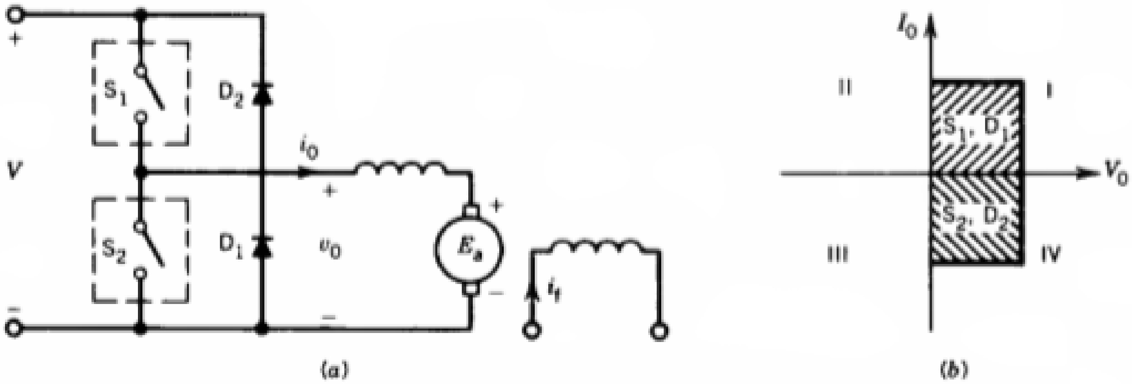


FIGURE 10.38 Two-quadrant chopper. (*a*) Circuit. (*b*) Quadrant operation.

(a) Chopper S_1 and diode D_1 are operated to control the speed of the motor. At $n = 400$ rpm and $i_0 = 100$ A (ripple-free),

- **(i)** Draw waveforms of v_0, i_0, and i_s.
- **(ii)** Determine the turn-on time (t_{on}) of the chopper.
- **(iii)** Determine the power developed by the motor, power absorbed by R_a, and power from the source.

(b) In the two-quadrant chopper S_2 and diode D_2 are operated for regenerative braking of the motor. At $n = 350$ rpm and $i_0 = -100$ A (ripple-free),

- **(i)** Draw waveforms of v_0, i_0, and i_s.
- **(ii)** Determine the turn-on time (t_{on}) of the chopper.
- **(iii)** Determine the power developed (and delivered) by the motor, power absorbed by R_a, and power to the source.

Solution

(a) **(i)** The waveforms are shown in Fig. E10.5*a*.

(ii) From Fig. 10.38*a*

$$V_0 = E_a + I_a R_a$$

$$= 0.1 \times 400 + 100 \times 0.2$$

$$= 60 \text{ V}$$

$$60 = \frac{t_{on}}{T} \text{V} = \frac{t_{on}}{T} 120$$

$$t_{on} = \frac{T}{2}$$

(iii)

$$P_{motor} = E_a I_0 = 0.1 \times 400 \times 100 = 4000 \text{ W}$$

$$P_R = (i_0)^2_{rms} R_a = 100^2 \times 0.2 = 2000 \text{ W}$$

$$P_s = V(i_s)_{avg} = 120 \times 100 \times \tfrac{2}{4} = 6000 \text{ W}$$

(b) **(i)** The waveforms are shown in Fig. E10.5*b*.

(ii)

$$V_0 = E_a + (-I_0 R_a)$$

$$= 0.1 \times 350 - 100 \times 0.2$$

$$= 15 \text{ V}$$

From Fig. E10.5*b*

$$V_0 = \frac{T - t_{on}}{T} V$$

$$15 = \left(1 - \frac{t_{on}}{T}\right) 120$$

$$\frac{t_{on}}{T} = \frac{7}{8}$$

$$t_{on} = \tfrac{7}{8} \times 4 = 3.5 \text{ msec}$$

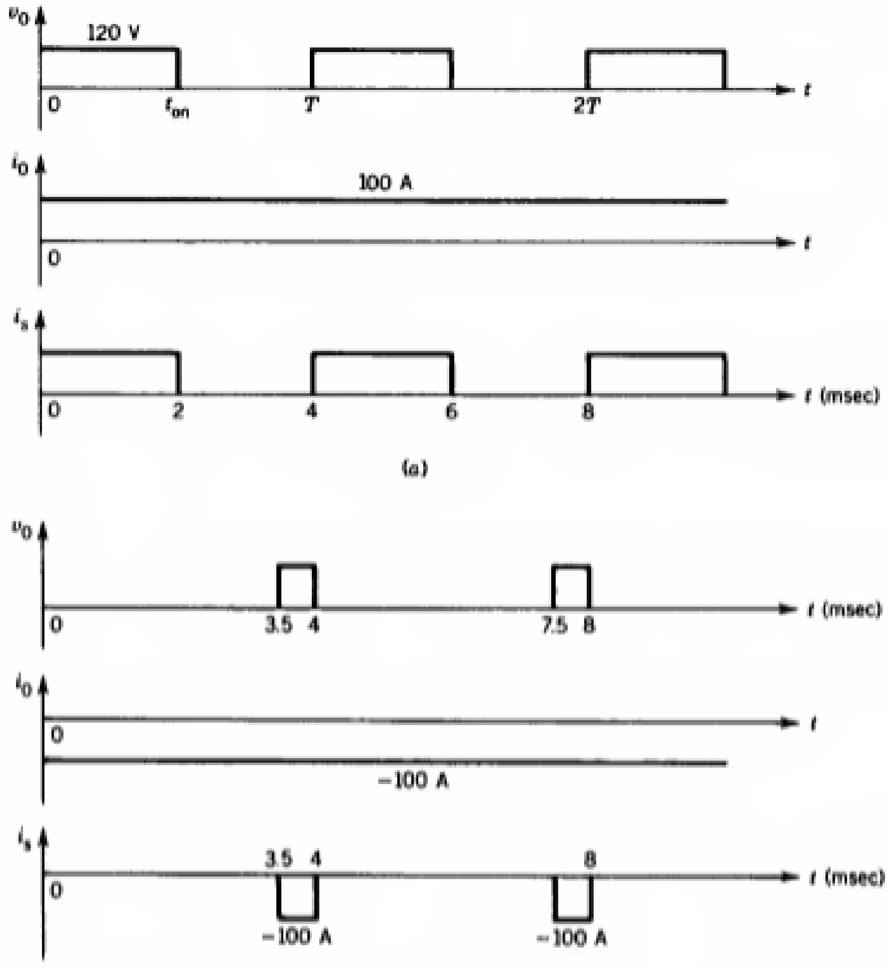


FIGURE E10.5

(iii)

$$P_{\text{motor}} = E_a I_0 = 0.1 \times 350(-100) = -3500 \text{ W}$$

$$P_R = 100^2 \times 0.2 = 2000 \text{ W}$$

$$P_s = V(i_s)_{\text{avg}} = 120(-100 \times \tfrac{1}{8}) = -1500 \text{ W}$$

10.21 A one-quadrant chopper, such as that shown in Fig. 10.34*a*, is used to control the speed of a dc motor.

Supply dc voltage = 120 V

$R_a = 0.15\ \Omega$

Motor back emf constant = 0.05 V/rpm

Chopper frequency = 250 Hz

At a speed of 1200 rpm, the motor current is 125 A. The motor current can be assumed to be ripple-free.

(a) Determine the duty ratio (α) of the chopper and the chopper on time t_{on}.

(b) Draw waveforms of v_0, i_0, and i_s.

(c) Determine the torque developed by the armature, power taken by the motor, and power drawn from the supply.

$\boxed{10.21}$ (a)

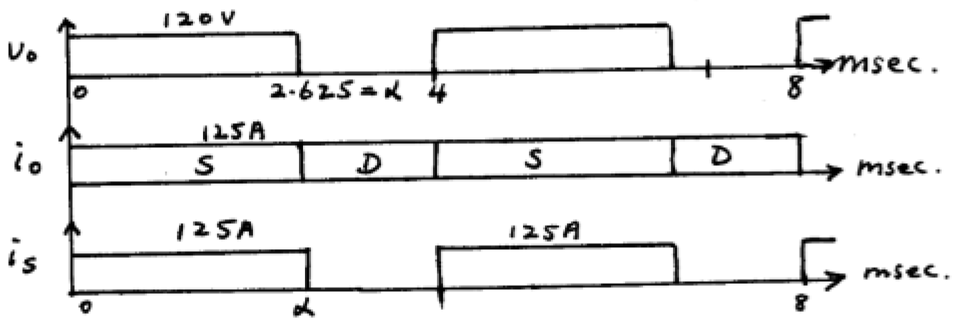
$$V_o = E_a + I_a R_a$$
$$= 0.05 \times 1200 + 125 \times 0.15$$
$$= 78.75\,V$$

$$T = \frac{10^3}{250}\,\text{msec} = 4\ \text{msec}$$

$$\alpha = \frac{78.75}{120} = 0.6563$$

$$t_{on} = \alpha T = 0.6563 \times 4 = 2.625\ \text{msec.}$$

(b)



(c)

$$E_a I_o = 60 \times 125 = 7500\,W$$

$$T = \frac{7500}{1200/60 \times 2\pi} = 59.683\ N\cdot m$$

$$P_o = V_o I_o = 78.75 \times 125 = 9844\,W$$

$$I_s = 125 \times 0.6563 = 82.03\ A$$

$$P_s = 120 \times 82.03 = 9844\ W$$

10.22 The power circuit configuration during regenerative braking of a subway car is shown in Fig. P10.22. The dc motor voltage constant is 0.3 V/rpm, and the dc bus voltage is 600 V. At a motor speed of 800 rpm and average motor current of 300 A,

(a) Draw the waveforms of v_0, i_a, and i_s for a particular value of the duty cycle α $(= t_{on}/T)$.

(b) Determine the duty ratio α of the chopper for the operating condition.

(c) Determine the power fed back to the bus.

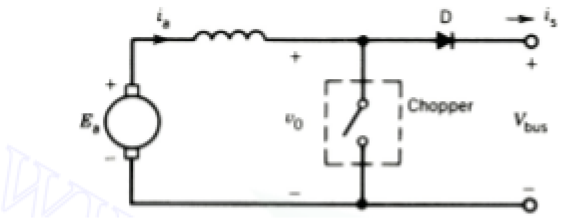
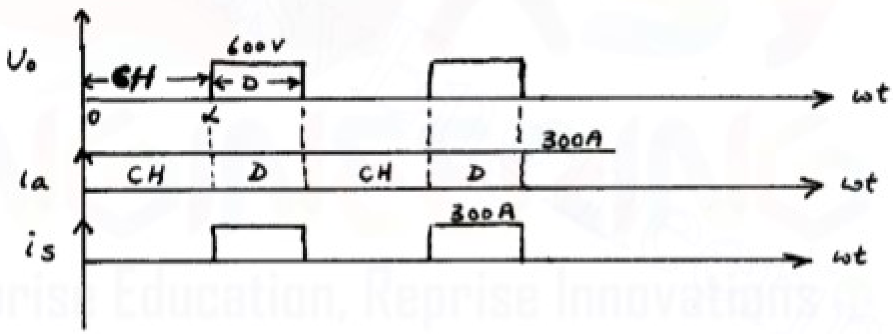


FIGURE P10.22

$\boxed{10.22}$ (a)



(b) $V_o = (1-\alpha)\,600 = E_a = 0.3 \times 800 = 240\text{ V}$

$\alpha = 1 - \dfrac{240}{600} = 0.6$

(c) $I_s = (1-\alpha)\,300 = (1-0.6)\,300 = 120\text{ A}$

$P_s = 600 \times 120 = 72\text{ kW}$

Or $P_s = P_a = E_a I_a = 240 \times 300 = 72\text{ kW}$

10.23 In the chopper circuit shown in Fig. P10.23, the two switches are simultaneously turned on for time t_{on} and turned off for time $t_{off} = T - t_{on}$, where T is the chopping period. Assume voltage v_0 to be ripple-free and current i_L to be continuous.

(a) Derive an expression for V_0 as a function of the duty cycle $\alpha = t_{on}/T$ and the supply voltage V. Determine V_0 for $\alpha = 0, 0.5, 1.0$.

(b) Draw waveforms of v_0, v_L, i_L, i_0, and i for $\alpha =$ [illegible].

(c) What are the advantages and disadvantages of this circuit?

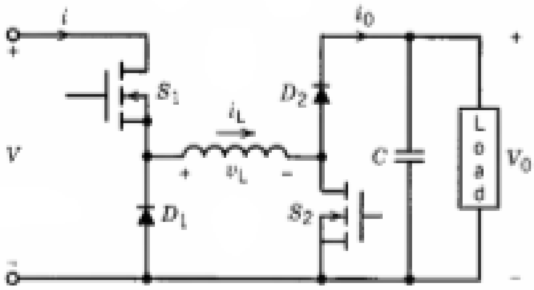


FIGURE P10.23

$\boxed{10.23}$ (a) during ton $\rightarrow$ $v_L = v = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}$

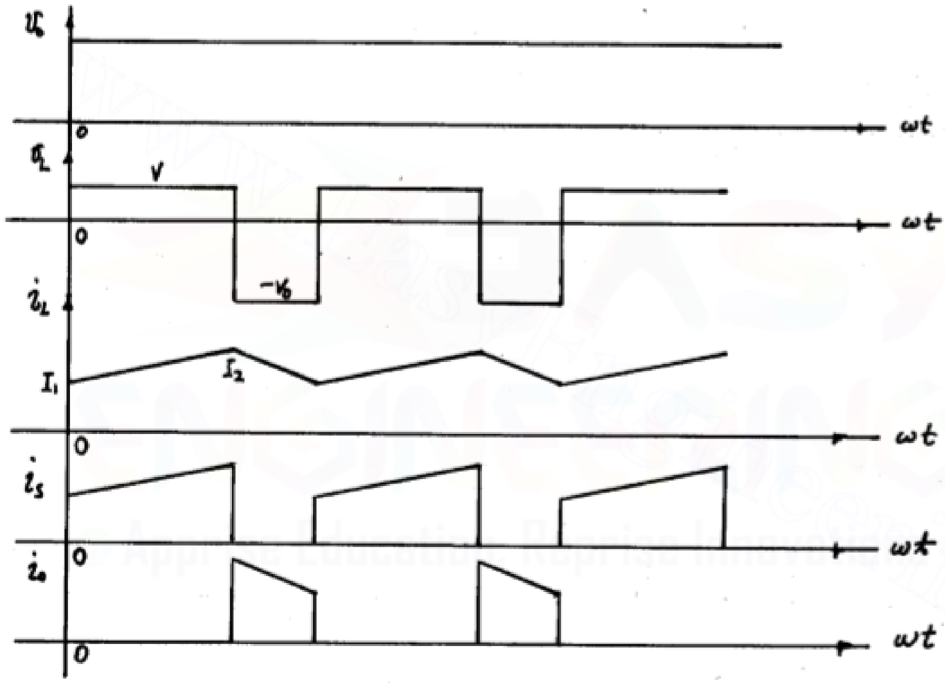
during toff $\rightarrow$ $v_L = -v_o = L\frac{I_1 - I_2}{t_{off}} = -L\frac{\Delta I}{t_{off}}$

$$\frac{V t_{on}}{L} = \frac{V_o t_{off}}{L}$$

$$V_o = \frac{t_{on}}{t_{off}} V = \frac{t_{on}}{T - t_{on}} V = \frac{\alpha}{1-\alpha} V$$

α	V_o
0	0
0.5	V
1.0	∞

(b)



(c)
- This is a step-down, step-up chopper
- Polarity of V_o is the same as that of V
- Higher conduction losses → two devices conduct at a time

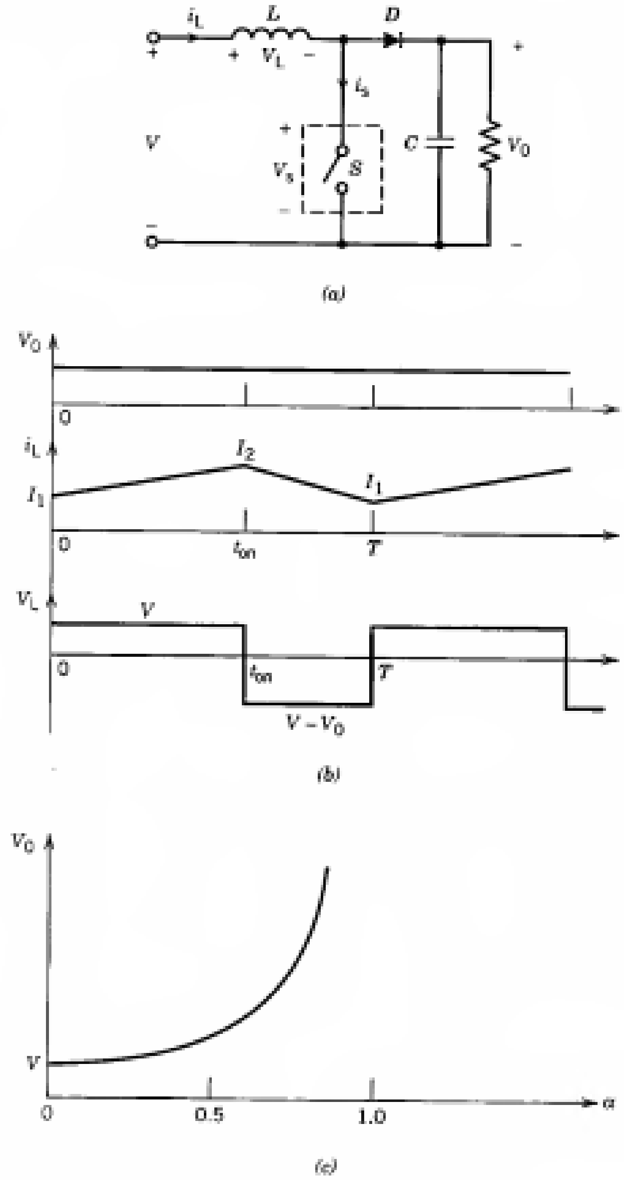


FIGURE 10.36 Boost converter. (a) Circuit. (b) Waveforms. (c) V_0 versus α.

10.24 The boost converter of Fig. 10.36 is used to charge a battery bank from a dc voltage source with $V = 160$ V. Assume ideal switch and no-loss operation, and neglect the ripple at the output voltage. The battery bank consists of 100 identical batteries. Each battery has an internal resistance $R_b = 0.1\ \Omega$. At the beginning of the charging process, each battery voltage is $V_{b1} = 2.5$ V. When each battery is charged up to $V_{b2} = 3.2$ V, the charging process is completed. The average charging current is kept constant at 0.5 A.

(a) Calculate the variation of duty ratio α for the charging process.

(b) Draw qualitatively the waveforms of v_L, i_L, v_S, i_S, v_D, i_D for $V_{b1} = 2.5$ V.

$\boxed{10.24}$ (a) $I = 0.5A \quad R_t = 100 \times R_b = 100 \times 0.1 = 10\Omega, \quad V_R = 0.5 \times 10 = 5V$

$V_{o1} = 100 \times V_{b1} + V_R = 100 \times 2.5 + 5 = 255V$

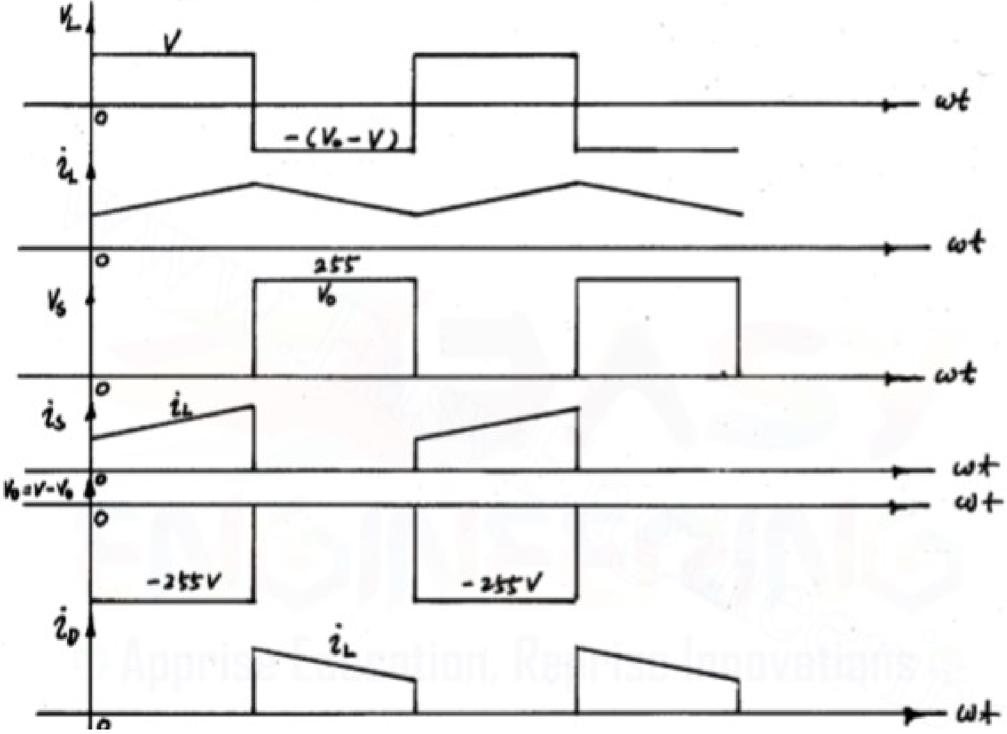
$V_{o2} = 100 \times V_{b2} + V_R = 100 \times 3.2 + 5 = 325V$

$V_o = \frac{1}{1-\alpha} V \rightarrow \alpha = \frac{V_o - V}{V_o}$

$\alpha_1 = \frac{255 - 150}{255} = 0.4118 \qquad \alpha_2 = \frac{325 - 150}{325} = 0.5385$

α changes from 0.4118 to 0.5385

(b)



10.25 For the battery charging system of Problem 10.24:

(a) If the supply voltage available is $V = 150$ V (dc), which dc to dc converter would be used? Draw the circuit.

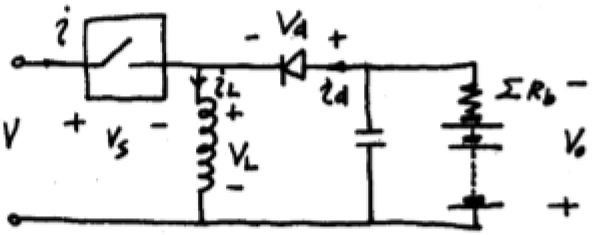
(b) Calculate the variation of the duty ratio α for the charging process.

(c) Draw qualitatively the waveforms of inductor voltage (v_L), inductor current (i_L), voltage across the chopper switch (v_s), current through the chopper switch (i_s), voltage across the diode (v_d), and current through the diode (i_d), for $v_{bl} = 1.2$ V.

$\boxed{10.25}$ (a) $V_{o1} = 100V_{b1} + IR = 100 \times 1.2 + 100 \times a1 \times 0.5 = 125V$

$V_{o2} = 100 \times 3.2 + 5 = 325\ V$

$V = 150V$ Need a Buck-Boost Converter

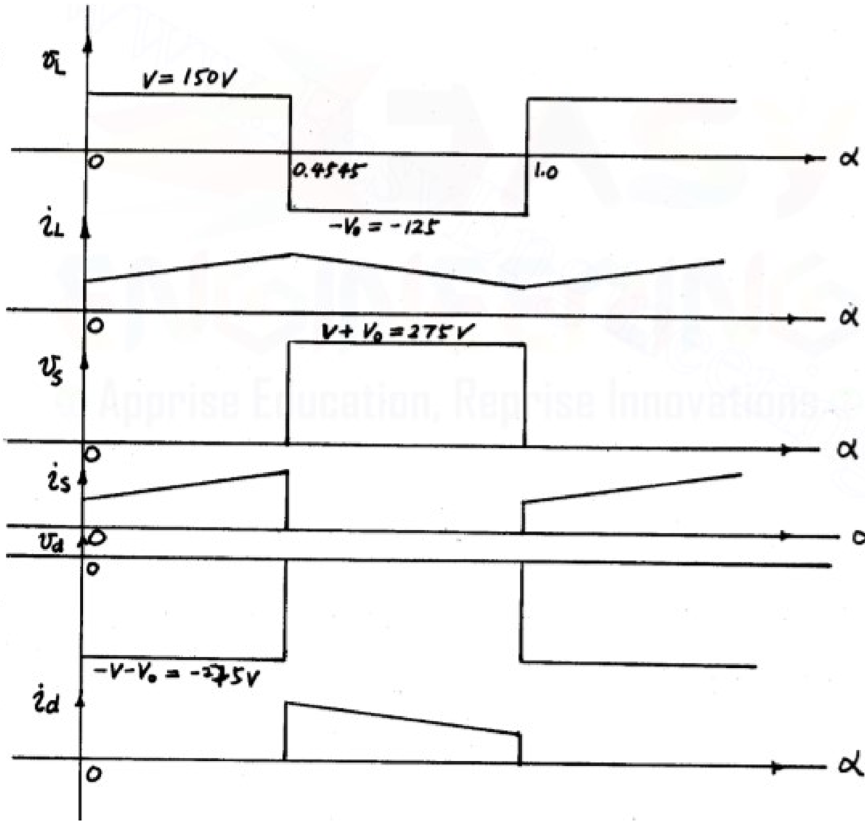


(b) $V_o = \dfrac{\alpha}{1-\alpha} V$

$\alpha = \dfrac{V_o}{V + V_o}$

$\alpha_1 = \dfrac{125}{150 + 125} = 0.4545$ $\quad \alpha_2 = \dfrac{325}{152 + 325} = 0.6842$

(c)



10.26 Consider the two-quadrant chopper systems shown in Fig. P10.26. The two choppers S_1 and S_2 are turned on for time t_{on} and turned off for time $T - t_{on}$, where T is the chopping period.

(a) Draw the waveform of the output voltage v_0. Assume continuous output current i_0.

(b) Derive an expression for the average output voltage V_o in terms of the supply voltage V and the duty ratio α ($= t_{on}/T$).

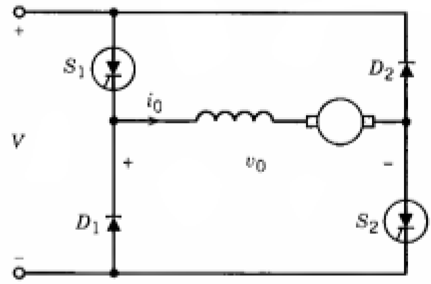
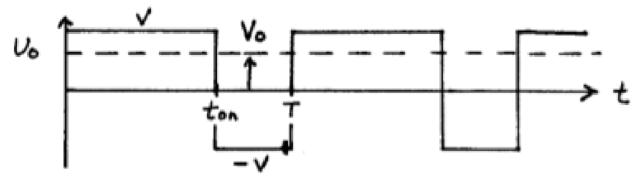


FIGURE P10.26

$\boxed{10.26}$ (a)



(b) $V_o = \frac{1}{T}\int_0^T v_o \, dt = \frac{1}{T}\left[\int_0^{t_{on}} V\,dt + \int_{t_{on}}^{T} -V\,dt\right]$

$= V(2\alpha - 1)$

Example 12.1

The speed of a separately-excited d.c. motor with $R_a = 1.2\ \Omega$ and $L_a = 30$ mH, is to be controlled using class-A thyristor chopper as shown in Fig.12.4. The d.c. supply $V_d = 120$ V. By ignoring the effect of the armature inductance L_a, it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle $\gamma = 1$.

(b) Draw the speed-torque characteristics for the motor when the duty cycle $\gamma = 1$. The motor design constant $Ke\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to $\gamma = 0.5$.

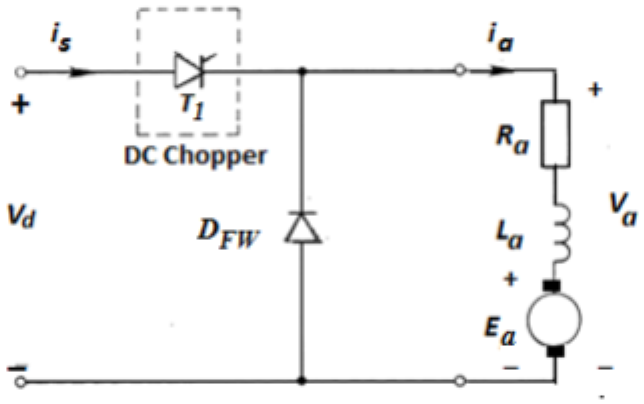


Fig. 12.4 Thyristor chopper drive.

Solution

The average armature voltage for $\gamma = 1$ is

$$V_{av} = \gamma V_d = 1 \times 120 = 120 \text{ V}$$

The motor's speed:

$$n = \frac{V_{av}}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

At no load $T_d = 0$, hence

$$\text{or} \qquad n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{120}{0.042} = 2857 \text{ rpm}$$

At starting, $n = 0$. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$T_{st} = \frac{9.55\, \gamma V_d}{R_a} K_e \emptyset$$

$$\therefore \qquad T_{st} = \frac{9.55 \times 120}{1.2} \times 0.042 = 40 \text{ Nm}$$

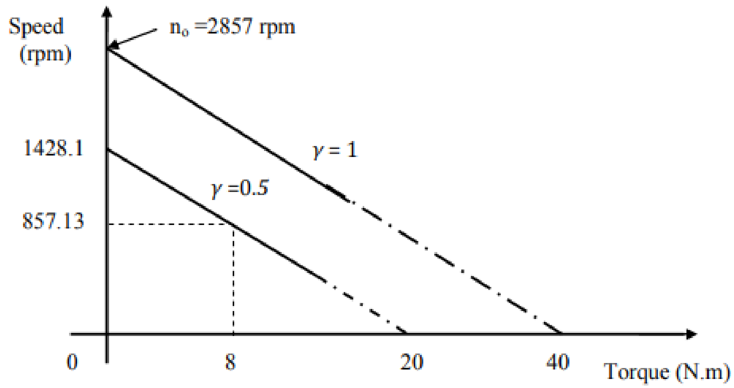


Fig. 12.5 Speed-torque characteristics.

(b) At $\gamma = 0.5$,

$$V_a = \gamma V_d = 0.5 \times 120 = 60 \text{ V}$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5 \text{ rpm}$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20 \text{ Nm}$$

At $\gamma = 0.5$, $T_L = 8$ Nm

$$n = \frac{60}{0.042} - \frac{1.2}{9.55 \times (0.042)^2} \times 8 = 857.13 \text{ rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 12.2

A d.c. motor is driven from a class-A d.c. chopper with source voltage of 220 V and at frequency of 1000 Hz. Determine the range of duty cycle to obtain a speed variation from 0 to 2000 rpm while the motor delivered a constant load of 70 Nm. The motor details as follows:
1kW, 200 V, 2000 rpm, 80% efficiency, R_a = 0.1 Ω, L_a = 0.02 H, and $K\emptyset$ = 0.54 V/rad /s.

Solution

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 2000}{60} = 209.3 \text{ rad/s}$$

$$I_{av} = \frac{P_{out}}{\text{Voltage } \times \eta} = \frac{1000}{200 \times 0.80} = 6.25 \text{ A}$$

$$I_{av} = \frac{\gamma V_d - E_a}{R_a} = \frac{\gamma V_d - K\emptyset\ \omega}{R_a}$$

$$T_{av} = K\emptyset\ I_{av}$$

$$T_{av} = K\emptyset\ \left(\frac{\gamma V_d - K\emptyset\ \omega}{R_a}\right) \text{Nm}$$

For $\omega_m = 0$

$$T_{av} = \frac{\gamma\ K\emptyset\ V_d}{R_a}$$

$$\gamma = \frac{T_{av}\ R_a}{K\emptyset\ V_d} = \frac{70 \times 0.1}{0.54 \times 220} = 0.058$$

$$\therefore\ \gamma_{min} = 0.058$$

and $$\gamma_{max} = \frac{T_{av} R_a}{K\emptyset V_d} + \frac{K\emptyset \omega_m}{V_d} = \frac{70 \times 0.1}{0.54 \times 220} + \frac{0.54 \times 209.3}{220} = 0.571$$

Hence the range of γ is $0.058 - 0.571$.

Example 12.3

In the microcomputer-controlled class-A IGBT transistor d.c. chopper shown in Fig.12.6, the input voltage $V_d = 260$ V, the load is a separately-excited d.c. motor with $R_a = 0.28\ \Omega$ and $L_a = 30$ mH. The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30 A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor n (rpm) when the chopper is switched fully ON such that the duty cycle $\gamma = 1.0$.

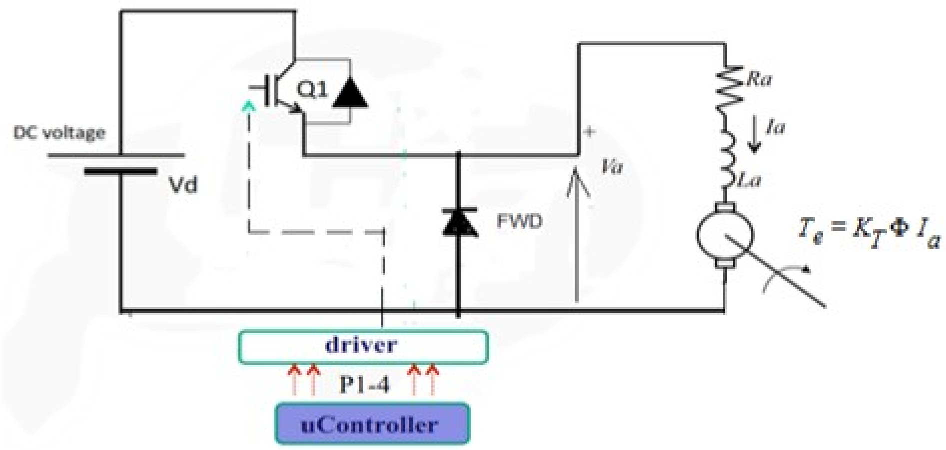


Fig.12.6 IGBT d.c. chopper drive.

Solution

(a) With steady-state operation of the motor, the armature induct behaves like a short circuit and therefore has no effect at all.

At stand still $n = 0$, and therefore , $E_a = 0$, hence from Eq.(12.22),

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30 \text{ A}$$

$$\therefore\ V_{a0} = 0.28 \times 30 = 8.4 \text{ V}$$

At full speed $n = 2500$ rpm ,

$$E_{a2500} = K_e\, \phi\, n = 0.1\ \times\ 2500 = 250 \text{ V}$$

For separately-excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4 \text{ V}$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. $\gamma = 1$, then

$$V_a = V_d = 260 \text{ V}.$$

At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e\, \phi\, n + I_a R_a\ = 260\text{V}$$

$$0.1\, n + 30\ \times\ 0.28\ = 260 \qquad \rightarrow \qquad n = 2516 \text{ rpm}$$

Example 12.4

A separately-excited d.c. motor has the following parameters:

$R_a = 0.5\ \Omega$, $L_a = 5.0$ mH , $K_e\,\Phi = 0.078$ V/rpm.

The motor speed is controlled by a class-A d.c. chopper fed from an ideal 200 V d.c. source. The motor is driven at a speed of 2180 rpm by switching on the thyristor for a period of 4 ms in each overall period of 6 ms .

(a) State whether the motor will operate in continuous or discontinuous current mode,
(b) Calculate the extinction angle of the current if it exist,
(c) Sketch the armature voltage and current waveforms,
(d) Calculate the maximum and minimum values of the armature current,
(e) Calculate the average armature voltage and current.

Solution

(a) To find whether the motor operates in continuous or discontinuous current modes ,we have to find the values of γ and γ' :

$$\gamma = \frac{t_{on}}{t_{on} - t_{off}} = \frac{t_{on}}{T} = \frac{4ms}{6ms} = 0.667$$

$$T = 6\,x\,10^{-3}\text{ s} \quad \rightarrow \quad \omega = 2\pi\,\frac{1}{T} = 1046.6\ \text{rad/s}$$

The armature circuit tim constant is $\tau = \frac{L_a}{R_a} = \frac{5}{0.5} = 10$ ms

Therefore, $\frac{2\pi}{\omega\tau} = \frac{2 \times 3.14}{1046.7 \times 10\ \times\ 10^{-3}} = 0.6$

At speed of 2180 rpm , $E_a = K_e\,\phi\,n = 0.078\ \times 2180 = 170$ V

The critical value of γ' will be , (using Eq.(12. 31))

$$\frac{E_a}{V_d} = \frac{e^{2\pi\gamma'/\omega\tau} - 1}{e^{2\pi/\omega\tau} - 1} = \frac{170}{200} = \frac{e^{0.6\,\gamma'} - 1}{e^{0.6} - 1}$$

From which $\gamma' = 0.0.8829$, therefore , $\gamma' > \gamma$, hence the motor is operating in discontinuous current mode.

(b) The extinction angle x of the current is calculated from Eq.(12.29) as,

$$x = \omega\tau \ln\left[e^{(2\pi\gamma)/\omega\tau}\left\{1 + (\frac{V_d}{E_a} - 1)\left(1 - e^{-2\pi\gamma/\omega\tau}\right)\right\}\right]$$

$$\omega\tau = 1046.7x10\ x\ 10^{-3} = 10.467 \text{ rad}$$

$$x = 10.467 \ln\left[e^{(2\pi x0.6)/10.467}\left\{1 + (\frac{200}{170} - 1)\left(1 - e^{-2\pi x0.6/10.467}\right)\right\}\right]$$

From which $x = 4.8$ rad $\rightarrow$ $x = 275.16°$

(c) The armature voltage and current waveforms are shown in Fig.12.7.

(d) The maximum and minimum values of the armature currents are:

$I_{min} = 0$, since it is discontinuous.
I_{maxD} is calculated from Eq.(12.26) as,

$$I_{max_D} = \frac{V_d - E_a}{R_a}\left(1 - e^{-2\pi\gamma/\omega\tau}\right)$$

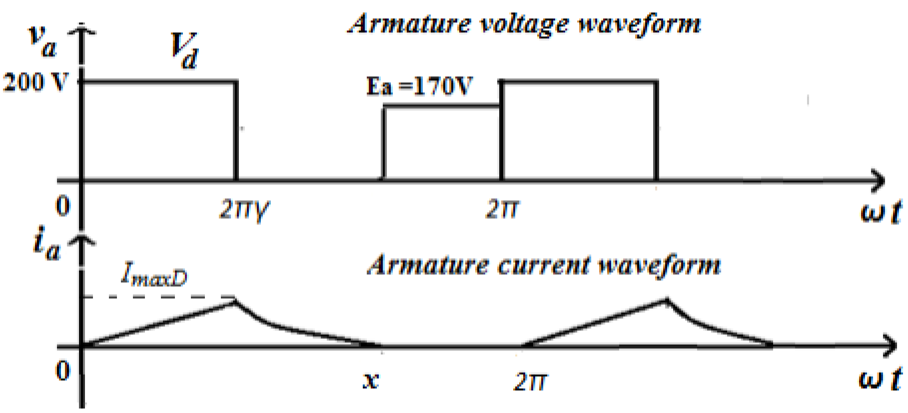


Fig.12.7 Armature voltage and current waveforms.

Example 12.5

A separately-excited d.c. motor with R_a = 0.1 Ω and L_a = 20 mH, is to be controlled using class-A thyristor chopper. The d.c. supply is a battery with V_d = 400 V. The motor voltage constant is 5 V.s/rad. In the steady-state

operation the average armature current I_a = 100 A and it is assumed to be continuous and ripple-free.

(a) For a duty cycle of 0.5, it is required to calculate (i) the input power to the motor, (ii) the speed of the motor, (iii) the developed torque. Mechanical, battery and semiconductor losses may be neglected.

(b) If the duty cycle of the chopper is varied between 20% and 80%, find the difference in speed resulting from this variation.

Solution

(a) Input power to the motor, speed of the motor and the developed torque are calculated as follows:

(i) For continuous current operation the input power is

$$P_{in} = V_a I_a = \gamma V_d I_a = 0.5 \times 400 \times 100 = 20 \text{ kW}$$

(ii) Speed of the motor can be calculated as,

The voltage across the armature circuit

$$V_a = \gamma V_d = 0.5 \times 400 = 200 \text{ V}$$

The induce voltage $E_a = K\emptyset\, \omega$

$$K\emptyset = 5 \text{ V.s/rad.}$$

$$E_a = V_a - I_a R_a = 200 - 100 \times 0.1 = 190 \text{ V}$$

$$\omega = \frac{E_a}{K\emptyset} = \frac{190}{5} = 38 \text{ rad/s}$$

To find the speed n in rpm

$$n = \frac{60}{2\pi}\omega = \frac{60}{2\pi} \times 38 = 363 \text{ rpm}$$

(iii) The torque produced by the motor,

$$T_m = \frac{E_a I_a}{\omega} = \frac{190 \times 100}{38} = 500 \text{ Nm}$$

(b) For duty cycle of 20%,

$$E_{a20\%} = \gamma V_d - I_a R_a = 0.2 \times 400 - 100 \times 0.1 = 70\ V$$

$$\omega_{20\%} = \frac{E_{a20\%}}{K\emptyset} = \frac{70}{5} = 14 \ \frac{\text{rad}}{\text{s}} \quad \rightarrow \ n_{20\%} = 14 \times \frac{60}{2\pi} = 133.7 \text{ rpm}$$

For duty cycle of 80%,

$$E_{a80\%} = \gamma V_d - I_a R_a = 0.8 \times 400 - 100 \times 0.1 = 310 \text{ V}$$

$$\omega_{80\%} = \frac{E_{a80\%}}{K\emptyset} = \frac{310}{5} = 62 \text{ rad/s} \ \rightarrow \ n_{80\%} = 62 \times \frac{60}{2\pi} = 592.3 \text{ rpm}$$

Hence the difference in speed is

$$n_{80\%} - n_{20\%} = 592.3 - 133.7 = 458.6 \text{ rpm}$$

Example 12.6

A class-A d.c. chopper operating at a frequency of 500 Hz and feeding a separately-excited d.c. motor from 200 V d.c. source. The load torque is 35 Nm and speed is 1000 rpm. Motor resistance and inductance are 0.15 Ω and 1.0 mH respectively. The *emf* and torque constant of motor are 1.6 V/rad/s and 1.4 Nm /A respectively. Find (a) Maximum and minimum values of motor armature current, and (b) Variation of armature current. Neglect chopper losses.

Solution

(a) Let duty cycle = γ

$$V_d = 200 \text{ V}$$

$$V_{av} = \gamma V_d = \gamma \times 200$$

Average armature current $I_a = T / K\varphi = 35/1.4 = 25$ A

Back *emf* $E_a = K\varphi\omega = 1.6 \times (950 \times 2\pi/60) = 159.16$ V

$$V_{av} = E_a + I_a R_a$$

$$200\gamma = 159.16 + 25 \times 0.15 = 162.29 \text{ V}$$

$$\gamma = 0.8145$$

$$T = 1/500 = 2 \text{ ms}$$

$$t_{on} = \gamma T = 2 \times 0.8145 = 1.629 \text{ ms}$$

$$t_{off} = 2 - 1.629 = 0.371 \text{ ms}$$

From Eq.s (12.19) and (12.20) ,The maximum and minimum currents are calculated as

Let:

$$T = 2\pi \;, \quad t_{on} = 2\pi\gamma = \gamma T \;, \quad \tau = \frac{R_a}{L_a} \;,\text{and} \quad -t_{on}/\tau = \frac{-\gamma T R_a}{L_a}$$

Hence Eq.(12.19) and (12.20) can be re-written as

$$I_{max} = \frac{V_d}{R_a}\left(\frac{1 - e^{-t_{on}/\tau}}{1 - e^{-T/\tau}}\right) - \frac{E_a}{R_a}$$

and

$$I_{min} = \frac{V_d}{R_a}\left(\frac{e^{t_{on}/\tau} - 1}{e^{T/\tau} - 1}\right) - \frac{E_a}{R_a}$$

$$\frac{TR_a}{L_a} = \frac{2 \times 10^{-3} \times 0.15}{1 \times 10^{-3}} = 0.30$$

$$e^{-\frac{\gamma TR_a}{L_a}} = e^{-0.8145 \times 0.3} = e^{-0.24435} = 0.7832$$

$$e^{-\frac{TR_a}{L_a}} = e^{-0.3} = 0.7408$$

$$I_{max} = \frac{200}{0.15}\left(\frac{1 - 0.7832}{1 - 0.7408}\right) - \frac{159.15}{0.15}$$

$$= 1333.34 \times \left(\frac{0.2168}{0.2592}\right) - 1061 = 54{,}2A$$

$$I_{min} = \frac{200}{0.15}\left(\frac{1.2767 - 1}{1.3498 - 1}\right) - \frac{159.15}{0.15}$$

$$= 1333.34 \times \left(\frac{0.2767}{0.3498}\right) - 1061 = 0$$

(b) Variation of armature current = $I_{max} - I_{min} = 54{,}2 - 0 = 54{,}2\text{A}$

Example 12.16. *A dc series motor is fed from 600 V dc source through a chopper. The dc motor has the following parameters :*

$$r_a = 0.04\ \Omega, \quad r_s = 0.06\ \Omega, \quad k = 4 \times 10^{-3}\ Nm/amp^2$$

The average armature current of 300 A is ripple free. For a chopper duty cycle of 60%, determine :

(a) input power from the source

(b) motor speed and (c) motor torque.

Solution. (*a*) Power input to motor

$$= V_t \cdot I_a = \alpha V_s \cdot I_a$$
$$= 0.6 \times 600 \times 300 = 108 \text{ kW}.$$

(*b*) For a dc series motor,

$$\alpha V_s = E_a + I_a R = k\, I_a\, \omega_m + I_a R$$
$$0.6 \times 600 = 4 \times 10^{-3} \times 300 \times \omega_m + 300\,(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 275 \text{ rad/sec or } 2626.1 \text{ rpm}$$

(*c*) Motor torque, $\quad T_e = k\, I_a^2 = 4 \times 10^{-3} \times 300^2 = 360 \text{ Nm}.$

Example 12.17. *The chopper used for on-off control of a dc separately-excited motor has supply voltage of 230V dc, an on- time of 10 m sec and off-time of 15 m sec. Neglecting armature inductance and assuming continuous conduction of motor current, calculate the average load current when the motor speed is 1500 rpm and has a voltage constant of K_v = 0.5 V/rad per sec. The armature resistance is 3 Ω.* [*I.A.S., 1985*]

Solution. Chopper duty cycle

$$\alpha = \frac{T_{on}}{T_{on} + T_{off}} = \frac{10}{10 + 15} = 0.4$$

For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a\, r_a = K_m \cdot \omega_m + I_a r_a$$
$$0.4 \times 230 = 0.5 \times \frac{2\pi \times 1500}{60} + I_a \times 3$$

$\therefore$ Motor load current, $I_a = \dfrac{92 - 25 \times \pi}{3} = 4.487 \text{ A}$

Example 12.18. *A dc chopper is used to control the speed of a separately-excited dc motor. The dc supply voltage is 220 V, armature resistance r_a = 0.2 Ω and motor constant $K_a\,\phi$ = 0.08 V/rpm.*

This motor drives a constant torque load requiring an average armature current of 25 A. Determine (a) the range of speed control (b) the range of duty cycle α. Assumed the motor current to be continuous. [I.A.S., 1990]

Solution. For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a$$

As motor drives a constant torque load, motor torque T_e is constant and therefore armature current remains constant at 25 A.

Minimum possible motor speed is $N = 0$. Therefore,

$$\alpha \times 220 = 0.08 \times 0 + 25 \times 2.0 = 5$$

$$\alpha = \frac{5}{220} = \frac{1}{44}$$

Maximum possible motor speed corresponds to $\alpha = 1$, *i.e.* when 220 V dc is directly applied and no chopping is done.

$$\therefore \quad 1 \times 220 = 0.08 \times N + 25 \times 0.2$$

or $$N = \frac{220 - 5}{0.08} = 2687.5 \text{ rpm}$$

∴ Range of speed control : $0 < N < 2687.5$ rpm and corresponding range of duty cycle : $\frac{1}{44} < \alpha < 1$.

Example 12.21. *A dc chopper is used for regenerative braking of a separately-excited dc motor. The dc supply voltage is 400V. The motor has $r_a = 0.2\ \Omega$, $K_m = 1.2$ V-s/rad. The average armature current during regenerative braking is kept constant at 300 A with negligible ripple.*

For a duty cycle of 60% for a chopper, determine

(a) power returned to the dc supply

(b) minimum and maximum permissible braking speeds and

(c) speed during regenerative braking.

Solution. (*a*) Average armature terminal voltage,

$$V_t = (1 - \alpha)\, V_s = (1 - 0.6) \times 400 = 160 \text{ V}.$$

Power returned to the dc supply

$$= V_t I_a = 160 \times 300 \text{ W} = 48 \text{ kW}$$

(*b*) From Eq. (12.41), minimum braking speed is

$$\omega_{mn} = \frac{I_a \cdot r_a}{K_m} = \frac{300 \times 0.2}{1.2} = 50 \text{ rad/s or } 477.46 \text{ rpm}$$

From Eq. (12.42), maximum braking speed is

$$\omega_{mx} = \frac{V_s + I_a \cdot r_a}{K_m} = \frac{400 + 300 \times 0.2}{1.2}$$

$$= 383.33 \text{ rad/s or } 3660.6 \text{ rpm}$$

(*c*) When working as a generator during regenerative braking, the generated emf is $E_a'' = K_m\, \omega_m = V_t + I_a r_a = 160 + 300 \times 0.2 = 220$ V

$\therefore$ Motor speed, $\omega_m = \dfrac{220}{1.2}$ rad/s or 1750.7 rpm

Example 12.19. *A separately-excited dc motor is fed from 220 V dc source through a chopper operating at 400 Hz. The load torque is 30 Nm at a speed of 1000 rpm. The motor has $r_a = 0$, $L_a = 2$ mH and $K_m = 1.5$ V-sec/rad. Neglecting all motor and chopper losses, calculate*

(*a*) *the minimum and maximum values of armature current and the armature current excursion,*

(*b*) *the armature current expressions during on and off periods.*

Solution. As the armature resistance is neglected, armature current varies linearly between its minimum and maximum values.

(*a*) Average armature current, $I_a = \frac{T_e}{K_m} = \frac{30}{1.5} = 20$ A

Motor emf, $E_a = K_m \cdot \omega_m = 1.5 \times \frac{2\pi \times 1000}{60} = 157.08$ V

Motor input voltage, $\alpha V_s = V_t = E_a + I_a r_s = 157.08 + 0$

$\therefore \quad \alpha = \frac{157.08}{220} = 0.714$

Periodic time, $T = \frac{1}{f} = \frac{1}{400} = 2.5$ ms

On-period, $T_{on} = \alpha T = 0.714 \times 2.5 = 1.785$ ms

Off-period, $T_{off} = (1 - \alpha) T = 0.715$ ms

During on-period T_{on}, armature current will rise which is governed by the equation,

$$0 + L \frac{di_a}{dt} + E_a = V_s$$

or $\quad \frac{di_a}{dt} = \frac{V_s - E_a}{L} = \frac{220 - 157.08}{0.02} = 3146$ A/s

During off period, $\quad \frac{di_a}{dt} = -\frac{E_a}{L} = \frac{-157.08}{0.02} = -7854$ A/s

With current rising linearly, it is seen from Fig. 12.21 that

$$I_{mx} = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times T_{on}$$

$$= I_{mn} + 3146 \times 1.785 \times 10^{-3}$$

or $$I_{mx} = I_{mn} + 5.616 \qquad ...(i)$$

For linear variation between I_{mn} and I_{mx}, average value of armature current

$$I_a = \frac{I_{mx} + I_{mn}}{2} = 20 \text{ A}$$

or $$I_{mx} = 40 - I_{mn} \qquad ...(ii)$$

Solving Eqs. (*i*) and (*ii*), we get $I_{mx} = 22.808$ A

and $$I_{mn} = 17.912 \text{ A.}$$

$\therefore$ Armature current excursion $= I_{mx} - I_{mn} = 22.808 - 17.912 = 5.616$ A

(*b*) Armature current expression during turn-on,

$$i_a(t) = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times t$$

$$= 17.192 + 3146\, t \quad \text{for } 0 \le t \le T_{on}$$

Armature current expression during turn-off,

$$i_a(t) = I_{mx} + \left(\frac{di_a}{dt} \text{ during } T_{off}\right) \times t$$

$$= 22.808 - 7854\, t \quad \text{for } 0 \le t \le T_{off}$$

Example 12.20. *Repeat Example 12.19, in case motor has a resistance of 0.2 Ω for its armature circuit.*

Solution. (*a*) From Example 12.19, armature current, $I_a = 20$ A and motor emf, $E_a = 157.08$ V; source voltage, $V_s = 220$ V.

For armature circuit, $\alpha V_s = V_0 = V_t = E_a + I_a r_a = 157.08 + 20 \times 0.2 = 161.08$ V

$\therefore$ $$\alpha = \frac{161.08}{220} = 0.7322$$

$$T_{on} = \alpha T = 0.7322 \times 2.5 = 1.831 \text{ ms}$$

$$T_{off} = T - T_{on} = 0.669 \text{ ms}, \frac{R}{L} = \frac{0.2}{0.02} = 10$$

During T_{on}, from Eq. (12.34), armature current is

$$i_a(t) = \frac{220 - 157.08}{0.2}(1 - e^{-10t}) + I_{mn} \cdot e^{-10t}$$

At $t = T_{on} = 1.831$ ms, current become I_{mx}. This gives

$$i_a(t) = I_{mx} = 5.7079 + 0.98187\, I_{mn} \qquad ...(i)$$

During T_{off}, from Eq. (12.35), armature current is

$$i_a(t) = \frac{-157.08}{0.2}(1 - e^{-10t}) + I_{mx} \cdot e^{-10t}$$

At $t = 0.669$ ms, $i_a(t) = I_{mn}$. This gives

$$i_a(t) = I_{mn} = -5.237 + 0.9933\, I_{mx} \qquad ...(ii)$$

Solving Eqs. (i) and (ii), we get

$$I_{mx} = 5.7079 + 0.98187(-5.237 + 0.9933\, I_{mx})$$

$$= 0.5658 + 0.9753\, I_{mx}$$

or

$$I_{mx} = \frac{0.5658}{0.0247} = 22.907 \text{ A}$$

$$I_{mn} = -5.237 + 0.9933 \times 22.907 = 17.516 \text{ A}$$

$\therefore$ Armature current excursion

$$= I_{mx} - I_{mn} = 22.907 - 17.516 = 5.39 \text{ A}$$

(b) Armature current expression during turn-on period is

$$i_a(t) = 314.6(1 - e^{-10t}) + 17.516\, e^{-10t}$$

Armature current expression during turn-off period is

$$i_a(t) = -785.4(1 - e^{-10t}) + 22.907\, e^{-10t}$$

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(The Industrial Electronics Handbook) Bogdan M. Wilamowski, J. David Irwin-Power Electronics and Motor Drives-CRC Press (2011)

Example 13.1

A buck converter operating at 50-Hz switching frequency has T_{on} = 5 ms. Determine the average source current if the load current is 40 A.

Solution:

In the buck converter, we expect the source side at higher voltage to draw lower current than the load side at lower voltage.

Switching period $T = 1/50 = 0.02$ s = 20 ms

Duty ratio $D = 5/20 = 0.25$

Average source current = $40 \times 0.25 = 10$ A.

Example 13.2

A buck converter is operating at 1-kHz switching frequency from a 120-V dc source. The inductance is 50 mH. If the output voltage is 60 V and the load resistance is 12 Ω, we determine the following:

Duty ratio $D = 60$ V ÷ 120 V = 0.50   period $T = 1/1000 = 0.001$ s = 1 ms.

$T_{on} = 0.50 \times 1 = 0.5$ ms $T_{off} = 1 - 0.5 = 0.5$ ms.

Average load current = 60 V ÷ 12 Ω = 5 A   Output power = $60 \times 5 = 300$ W.

Average source current = $5 \times 0.50 = 2.5$ A   Input power = $120 \times 2.5 = 300$ W.

Equation 13.4 gives peak-to-peak ripple current:

$$\Delta I_L = V_{out} \times T_{off} \div L = 60 \times 0.0005 \div 0.050 = 0.6 \text{ A.}$$

If the switching frequency were 10 kHz, $T = 1/10{,}000 = 0.1$ ms, and $T_{off} = 0.05$ ms, the inductance required for the same ripple current = 60 V × 0.05 ms/0.6 A = 5 mH, which is 1/10 the size of that required at 1 kHz. This illustrates the benefit of high switching frequency.

Example 13.3

A buck converter has 120-V input voltage, $R_L = 12$ Ω, switching frequency of 1 kHz, and on-time of 0.5 ms. If the average source current is 2 A, we determine the following:

For 1-kHz switching frequency, $T = 1/1000$ s = 1 ms, duty ratio = 0.5/1 = 0.5, T_{on} = 0.5 ms, and $T_{off} = 1 - 0.5 = 0.5$ ms

average output voltage = $0.5 \times 120 = 60$ V

average output current = $2/0.5 = 4$ A

average output power = $V_{out} \times I_{out} = 60 \times 4 = 240$ W.

For continuous conduction, using Equation 13.9, we have L_{min} = 0.5 ms × 12 ÷ 2 = 3 mH.

Example 13.4

For a buck converter with V_{out} = 5 V, f_s = 20 kHz, L = 1 mH, C = 470 μF, V_{in} = 12.6 V, and I_{out} = 0.2 A, determine the peak-to-peak ripple ΔV_{out} in the output voltage.

Solution:

Switching period T = 1/20,000 = 0.00005 s, and $D = V_{out}/V_{in}$ = 5/12.5 = 0.4. Using Equation 13.10, we obtain

$$\Delta V_{out} = \frac{0.00005^2}{8 \times 470 \times 10^{-6}} \times \frac{5}{0.001} \times (1 - 0.40) = 0.002\,\text{V}.$$

This is quite a low ripple voltage, giving a smooth dc voltage output in this converter.

Example 13.5

A boost converter powers a 4-Ω resistor and 1-mH inductor load. The input voltage is 60 V, and the output load voltage is 80 V. If the on-time is 2 ms, we determine the following.

Using Equation 13.12, 80 ÷ 60 = 1 ÷ (1 − D), which gives the duty ratio D = 0.25.

Therefore, T_{on} = 0.25 T, that is, 2 ms = 0.25 T, which gives T = 8 ms, where $T = T_{on} + T_{off}$.

Switching frequency = 1/T = 1/0.008 = 125 Hz.

Output current = 80/4 = 20 A Input current = 20 (1 − 0.25) = 15 A.

Example 13.6

For a boost converter with switching frequency of 500 Hz, input of 50 V dc, output of 75 V dc, inductor of 2 mH, and load resistance of 2.5 Ω, we determine the following:

$$\text{period } T = 1/500 = 0.002 \text{ s} = 2 \text{ ms}$$

$$V_{out}/V_{in} = 75/50 = 1/(1 - D), \text{ which gives } D = 0.333$$

$$T_{on} = 0.333 \times 2 \text{ ms} = 0.666 \text{ ms} \quad T_{off} = 2 - 0.666 = 1.334 \text{ ms}$$

$$I_{out} = 75/2.5 = 30 \text{ A} \quad I_{in} = 30 \div (1 - 0.333) = 45 \text{ A}$$

$$\Delta I_{Ripple.pk\text{-}pk} = V_{in} \times T_{on} \div L = 50 \text{ V} \times 0.666 \text{ ms} \div 2 \text{ mH} = 16.65 \text{ A (a large ripple).}$$

Example 13.1: ***DC chopper with load back emf (first quadrant)***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine, with and without (rotor standstill) the back emf:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the average load output current, average switch current, and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, (and electromagnetic efficiency for $E > 0$);
viii. sketch the output current and voltage waveforms.

Solution
The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

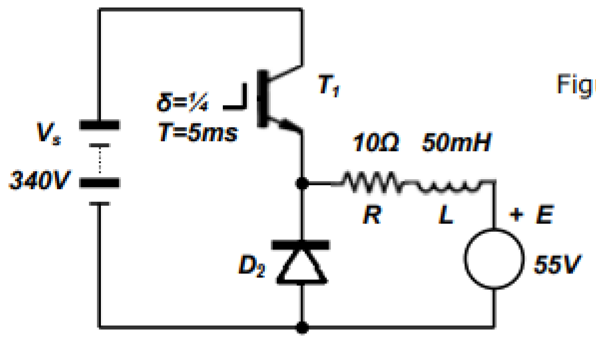

Figure Example 13.1.
Circuit diagram.

i. From equations (13.2) and (13.3) the average and rms output voltages are both independent of the back emf, namely

$$\overline{V}_o = \frac{t_T}{T} V_s = \delta V_s$$
$$= ¼ \times 340\text{V} = 85\text{V}$$
$$V_r = \sqrt{\frac{t_T}{T}}\, V_s = \sqrt{\delta}\, V_s$$
$$= \sqrt{¼} \times 240\text{V} = 120\text{V rms}$$

ii. The rms ripple voltage hence ripple factor are given by equations (13.4) and (13.5), that is

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$
$$= 340\text{V}\sqrt{¼ \times (1 - ¼)} \;=\; 147.2\text{V ac}$$

and

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1}$$
$$= \sqrt{\frac{1}{¼} - 1} \;=\; \sqrt{3} = 1.732$$

No back emf, $E = 0$

iii. From equation (13.13), with E=0, the maximum and minimum currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} = 11.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} = 5.62\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{p-p} = \hat{I} - \check{I}$$
$$= 11.90\text{A} - 5.62\text{A} = 6.28\text{A}$$

Alternatively the ripple can be extracted from figure 13.4 using $T/\tau = 1$ and $\delta = ¼$.

iv. From equations (13.11) and (13.12), with $E = 0$, the time domain load current equations are

$$i_o = \frac{V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 34\times\left(1-e^{\frac{-t}{5ms}}\right) + 5.62\times e^{\frac{-t}{5\text{ms}}}$$

$$= 34 - 28.38\times e^{\frac{-t}{5\text{ms}}} \quad (\text{A}) \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = \hat{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 11.90\times e^{\frac{-t}{5\text{ms}}} \quad (\text{A}) \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.17), with $E = 0$, is

$$\overline{I}_o = \overline{V}_o / R = 85\text{V} / 10\Omega = 8.5\text{A}$$

The average switch current, which is the average supply current, is

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{¼\times(340\text{V - }0)}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(11.90\text{A - }5.62\text{A}) = 2.22\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$
$$= 8.50\text{A} - 2.22\text{A} = 6.28\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s \overline{I}_i = 340\text{V} \times 2.22\text{A} = 754.8\text{W}$$
$$P_{out} = P_{in} = 754.8\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o_{rms}} = \sqrt{\frac{P_{out}}{R}}$$
$$= \sqrt{\frac{754.8\text{W}}{10\Omega}} = 8.7\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$
$$= \frac{340\text{V}}{2.22\text{A}} = 153.2\,\Omega$$

Load back emf, $E = 55$V

i. and ii. The average output voltage, rms output voltage, ac ripple voltage, and ripple factor are independent of back emf, provided the load current is continuous. The earlier answers for $E = 0$ are applicable.

iii. From equation (13.13), the maximum and minimum load currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega} \times \frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} - \frac{55\text{V}}{10\Omega} = 6.40\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega} \times \frac{e^{1/4}-1}{e^{1}-1} - \frac{55\text{V}}{10\Omega} = 0.12\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{pp} = \hat{I} - \check{I}$$
$$= 6.4\text{A} - 0.12\text{A} = 6.28\text{A}$$

The ripple value is the same as the $E = 0$ case, which is as expected since ripple current is independent of back emf with continuous output current.
Alternatively the ripple can be extracted from figure 13.4 using $T/\tau = 1$ and $\delta = 1/4$.

iv. The time domain load current is defined by

$$i_o = \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I}\, e^{\frac{-t}{\tau}}$$

$$i_o(t) = 28.5 \times \left(1 - e^{\frac{-t}{5ms}}\right) + 0.12 e^{\frac{-t}{5ms}}$$

$$= 28.5 - 28.38 e^{\frac{-t}{5ms}} \qquad \text{(A)} \qquad\qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\, e^{\frac{-t}{\tau}}$$

$$i_o(t) = -5.5 \times \left(1 - e^{\frac{-t}{5ms}}\right) + 6.4 e^{\frac{-t}{5ms}}$$

$$= -5.5 + 11.9 e^{\frac{-t}{5ms}} \qquad \text{(A)} \qquad\qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.37) is

$$\bar{I}_o = {V_o - E}\big/{R}$$

$$= 85\text{V-}55\text{V}\big/10\Omega = 3\text{A}$$

The average switch current is the average supply current,

$$\bar{I}_i = \bar{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{¼ \times (340\text{V - }55\text{V})}{10\Omega} - \frac{5\text{ms}}{5\text{ms}} \times (6.40\text{A - }0.12\text{A}) = 0.845\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\bar{I}_{diode} = \bar{I}_o - \bar{I}_i$$

$$= 3\text{A - }0.845\text{A} = 2.155\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s \bar{I}_i = 340\text{V} \times 0.845\text{A} = 287.3\text{W}$$

$$P_{out} = P_{in} = 287.3\text{W}$$

From equation (13.18) the rms load current is given by

$$\bar{I}_{o_{rms}} = \sqrt{\frac{P_{out} - E\bar{I}_o}{R}}$$

$$= \sqrt{\frac{287.3\text{W - }55\text{V} \times 3\text{A}}{10\Omega}} = 3.5\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$

$$= \frac{340\text{V}}{0.845\text{A}} = 402.4\,\Omega$$

The electromagnetic efficiency is given by equation (13.22), that is

$$\eta = \frac{E\overline{I}_o}{P_{in}}$$

$$= \frac{55\text{V}\times 3\text{A}}{287.3\text{W}} = 57.4\%$$

viii. The output voltage and current waveforms for the first-quadrant chopper, with and without back emf, are shown in the figure to follow.

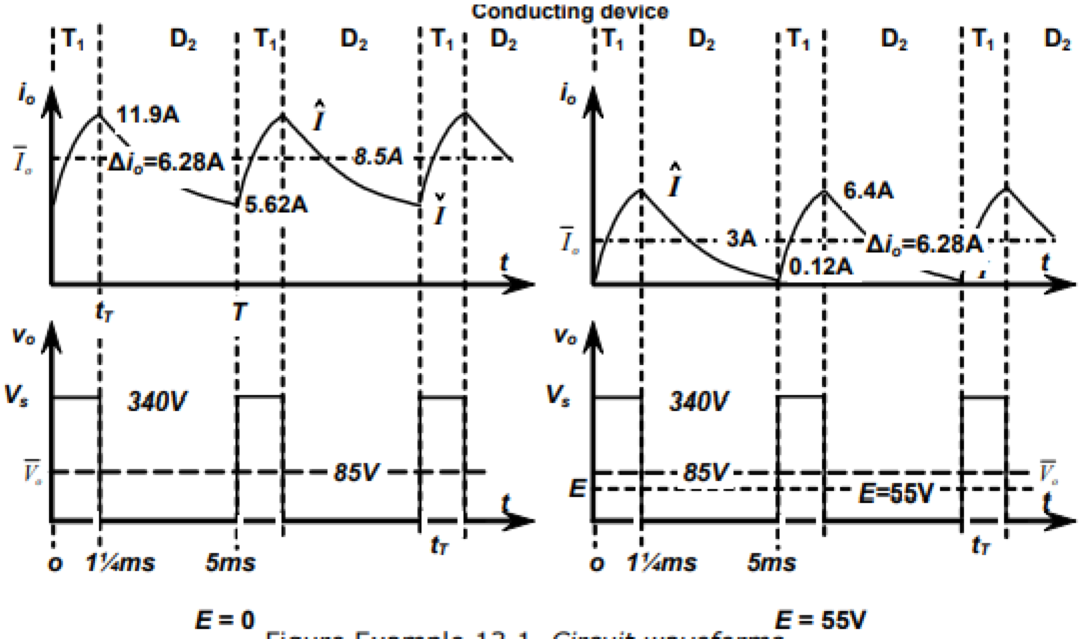

Figure Example 13.1. *Circuit waveforms.*

Example 13.2: ***DC chopper with load back emf - verge of discontinuous conduction***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

i. the maximum back emf before discontinuous load current conduction commences with δ=¼;
ii. with 55V back emf, what is the minimum duty cycle before discontinuous load current conduction; and
iii. minimum switching frequency at E=55V and t_T = 1.25ms before dis-continuous conduction.

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

First it is necessary to establish whether the given conditions represent continuous or discontinuous load current. The current extinction time t_x for discontinuous conduction is given by equation (13.24), and yields

$$t_x = t_T + \tau\,\ell n\left(1+\frac{V_s-E}{E}\left(1-e^{\frac{-t_T}{\tau}}\right)\right)$$

$$= 1.25\text{ms} + 5\text{ms}\times\ell n\left(1+\frac{340\text{V} - 55\text{V}}{55\text{V}}\times\left(1-\text{e}^{\frac{-1.25\text{ms}}{5\text{ms}}}\right)\right) = 5.07\text{ms}$$

Since the cycle period is 5ms, which is less than the necessary time for the current to fall to zero (5.07ms), the load current is continuous. From example 13.1 part iv, with E=55V the load current falls from 6.4A to near zero (0.12A) at the end of the off-time, thus the chopper is operating near the verge of discontinuous conduction. A small increase in E, decrease in the duty cycle δ, or increase in switching period T, would be expected to result in discontinuous load current.

i. $\hat{E}$

The necessary back emf can be determined graphically or analytically.

Graphically:

The bounds of continuous and discontinuous load current for a given duty cycle, switching period, and load time constant can be determined from figure 13.5.

Using δ=¼, T/τ = 1 with τ=5ms, and T = 5ms, figure 13.5 gives E/V_s=0.165. That is, $E = 0.165\times V_s = 0.165\times 340\text{V} = 56.2\text{V}$

Analytically:

The chopper is operating too close to the boundary between continuous and discontinuous load current conduction for accurate readings to be obtained from the graphical approach, using figure 13.5. Examination of the expression for minimum current, equation (13.13), gives

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = 0$$

Rearranging to give the back emf, E, produces

$$E = V_s\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1}$$

$$= 340\text{V}\times\frac{\text{e}^{\frac{1.25\text{ms}}{5\text{ms}}}-1}{\text{e}^{\frac{5\text{ms}}{5\text{ms}}}-1} = 56.2\text{V}$$

That is, if the back emf increases from 55V to 56.2V then at that voltage, discontinuous load current commences.

ii. $\check{\delta}$

If equation (13.13) is solved for $\check{I} = 0$ then

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}} - 1}{e^{\frac{T}{\tau}} - 1} - \frac{E}{R} = 0$$

Rearranging to isolate t_T gives

$$\begin{aligned} t_T &= \tau \; \ell n\left(1 + \frac{E}{V_s}\left(e^{\frac{T}{\tau}} - 1\right)\right) \\ &= 5\text{ms} \times \ell n\left(1 + \frac{55\text{V}}{340\text{V}}\left(\text{e}^{\frac{5\text{ms}}{5\text{ms}}} - 1\right)\right) \\ &= 1.226\text{ms} \end{aligned}$$

If the switch on-state period is reduced by 0.024ms, from 1.250ms to 1.226ms (24.52%), operation is then on the verge of discontinuous conduction.

iii. $\hat{T}$

If the switching frequency is decreased such that $T = t_x$, then the minimum period for discontinuous load current is given by equation (13.24). That is,

$$t_x = T = t_T + \tau \; \ell n\left(1 + \frac{V_s - E}{E}\left(1 - e^{\frac{-t_T}{\tau}}\right)\right)$$

$$T = 1.25\text{ms} + 5\text{ms} \times \ell n\left(1 + \frac{340\text{V} - 55\text{V}}{55\text{V}} \times \left(1 - \text{e}^{\frac{-1.25\text{ms}}{5\text{ms}}}\right)\right) = 5.07\text{ms}$$

Discontinuous conduction operation occurs if the period is increased by 0.07ms.

In conclusion, for the given load, for continuous conduction to cease, the following operating conditions can be changed

- increase the back emf E from 55V to 56.2V
- decrease the duty cycle δ from 25% to 24.52% (t_T decreased from 1.25ms to 1.226ms)
- increase the switching period T by 0.07ms, from 5ms to 5.07ms (from 200Hz to 197.2Hz), with the switch on-time, t_T, unchanged from 1.25ms.

Appropriate simultaneous smaller changes in more than one parameter would suffice.

♣

Example 13.3: ***DC chopper with load back emf – discontinuous conduction***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 100V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

1. the load average and rms voltages;
2. the rms ripple voltage, hence ripple factor;
3. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;

iv. the current in the time domain;
v. the load average current, average switch current and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, and electromagnetic efficiency; and
viii. Sketch the circuit, load, and output voltage and current waveforms.

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

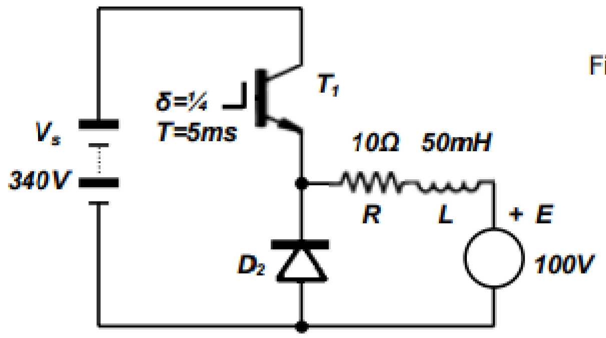

Figure Example 13.3.
Circuit diagram.

Confirmation of discontinuous load current can be obtained by evaluating the minimum current given by equation (13.13), that is

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}} - 1}{e^{\frac{T}{\tau}} - 1} - \frac{E}{R}$$

$$\check{I} = \frac{340\text{V}}{10\Omega} \times \frac{e^{\frac{1.25\text{ms}}{5\text{ms}}} - 1}{e^{\frac{5\text{ms}}{5\text{ms}}} - 1} - \frac{100\text{V}}{10\Omega} = 5.62\text{A} - 10\text{A} = -4.38\text{A}$$

The minimum practical current is zero, so clearly discontinuous current periods exist in the load current. The equations applicable to discontinuous load current need to be employed.

The current extinction time is given by equation (13.24), that is

$$t_x = t_T + \tau\, \ell n\left(1 + \frac{V_s - E}{E}\left(1 - e^{\frac{-t_T}{\tau}}\right)\right)$$

$$= 1.25\text{ms} + 5\text{ms} \times \ell n\left(1 + \frac{340\text{V} - 100\text{V}}{100\text{V}} \times \left(1 - e^{\frac{-1.25\text{ms}}{5\text{ms}}}\right)\right)$$

$$= 1.25\text{ms} + 2.13\text{ms} = 3.38\text{ms}$$

i. From equations (13.26) and (13.27) the load average and rms voltages are

$$\overline{V}_o = \delta V_s + \frac{T-t_x}{T}E$$

$$= \tfrac{1}{4}\times 340\,\text{V} + \frac{5\text{ms - }3.38\text{ms}}{5\text{ms}}\times 100\text{V} = 117.4\text{V}$$

$$V_{rms} = \sqrt{\delta V_s^2 + \frac{T-t_x}{T}E^2}$$

$$= \sqrt{\tfrac{1}{4}\times 340^2 + \frac{5\text{ms - }3.38\text{ms}}{5\text{ms}}\times 100^2} = 179.3\text{V rms}$$

ii. From equations (13.28) and (13.29) the rms ripple voltage, hence ripple factor, are

$$V_r = \sqrt{V_{rms}^2 - V_o^2}$$

$$= \sqrt{179.3^2 - 117.4^2} = 135.5\text{V ac}$$

$$RF = \frac{V_r}{\overline{V}_o} = \frac{135.5\text{V}}{117.4\text{V}} = 1.15$$

iii. From equation (13.36), the maximum and minimum output current, hence the peak-to-peak output ripple in the current, are

$$\hat{I} = \frac{V_s - E}{R}\left(1 - e^{\frac{-t_T}{\tau}}\right)$$

$$= \frac{340\text{V-}100\text{V}}{10\Omega}\times\left(1 - e^{\frac{-1.25\text{ms}}{5\text{ms}}}\right) = 5.31\text{A}$$

The minimum current is zero so the peak-to-peak ripple current is $\Delta i_o = 5.31\text{A}$.

iv. From equations (13.32) and (13.33), the current in the time domain is

$$i_o(t) = \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$= \frac{340\text{V - }100\text{V}}{10\Omega}\times\left(1 - e^{\frac{-t}{5\text{ms}}}\right)$$

$$= 24\times\left(1 - e^{\frac{-t}{5\text{ms}}}\right) \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o(t) = -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\,e^{\frac{-t}{\tau}}$$

$$= -\frac{100\text{V}}{10\Omega}\times\left(1 - e^{\frac{-t}{5\text{ms}}}\right) + 5.31e^{\frac{-t}{5\text{ms}}}$$

$$= 15.31\times e^{\frac{-t}{5\text{ms}}} - 10 \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 2.13\text{ms}$$

$$i_o(t) = 0 \qquad \text{for} \quad 3.38\text{ms} \le t \le 5\text{ms}$$

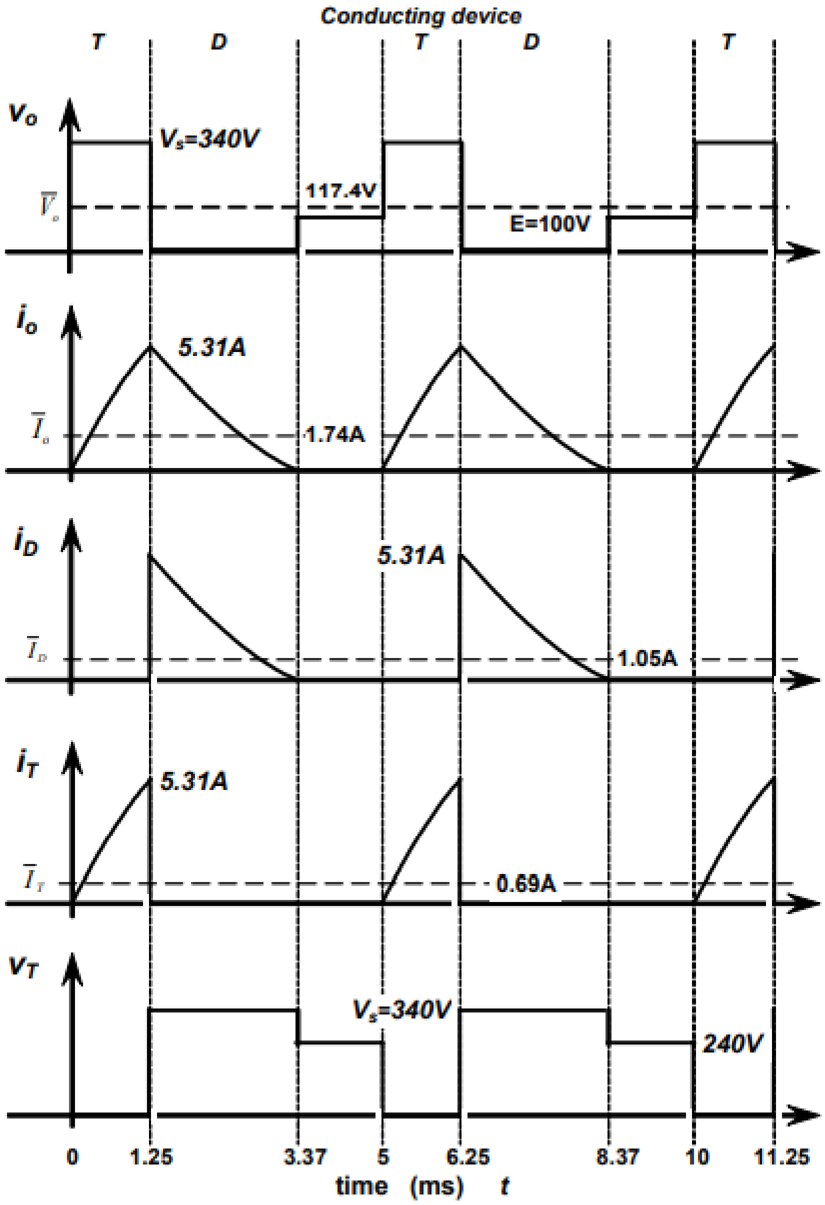

Figure Example 13.3. *Circuit waveforms.*

v. From equations (13.37) to (13.40), the load average current, average switch current, and average diode current are

$$\overline{I}_o = \overline{V}_o - E \big/ R$$

$$= 117.4\text{V - }100\text{V} \big/ 10\Omega = 1.74\text{A}$$

$$\overline{I}_{diode} = \frac{\tau}{T}\hat{I} - \frac{E\left(\frac{t_x}{T} - \delta\right)}{R}$$

$$= \frac{5\text{ms}}{5\text{ms}} \times 5.31\text{A - } \frac{100\text{V} \times \left(\frac{3.38\text{ms}}{5\text{ms}} \text{ - } 0.25\right)}{10\Omega} = 1.05\text{A}$$

$$\bar{I}_i = \bar{I}_o - \bar{I}_{diode}$$
$$=1.74\text{A} - 1.05\text{A} = 0.69\text{A}$$

vi. From equation (13.38), the input power, hence output power and rms output current are

$$P_{in} = V_s \bar{I}_i = 340\text{V}\times 0.69\text{A} = 234.6\text{W}$$
$$P_{in} = P_{out} = I_{o\,rms}^2\, R + E\bar{I}_o$$

Rearranging gives

$$I_{o_{rms}} = \sqrt{P_{in} - E\bar{I}_o} \;/\; R$$
$$=\sqrt{234.6\text{W} - 100\text{V}\times 0.69\text{A}} \;/\; 10\Omega = 1.29\text{A}$$

vii. From equations (13.42) and (13.43), the effective input impedance, and electromagnetic efficiency for $E > 0$

$$Z_{in} = \frac{V_s}{\bar{I}_i} = \frac{340\text{V}}{0.69\text{A}} = 493\Omega$$

$$\eta = \frac{E\bar{I}_o}{P_{in}} = \frac{E\bar{I}_o}{V_s\bar{I}_i} = \frac{100\text{V}\times 1.74\text{A}}{340\text{V}\times 0.69\text{A}} = 74.2\%$$

viii. The circuit, load, and output voltage and current waveforms are plotted in figure example 13.3.

13.3 Second-Quadrant dc chopper

The second-quadrant dc-to-dc chopper shown in figure 13.2b transfers energy from the load, back to the dc energy source, called *regeneration*. Its operating principles are the same as those for the boost switch mode power supply analysed in chapter 15.4. The two energy transfer modes are shown in figure 13.6. Energy is transferred from the back emf E to the supply V_s, by varying the switch T_2 on-state duty cycle. Two modes of transfer can occur, as with the first-quadrant chopper already considered. The current in the load inductor can be either continuous or discontinuous, depending on the specific circuit parameters and operating conditions.

In this analysis it is assumed that:

- No source impedance;
- Constant switch duty cycle;
- Steady-state conditions have been reached;
- Ideal semiconductors; and
- No load impedance temperature effects.

Load waveforms for continuous load current conduction are shown in figure 13.7a. The output voltage v_o, load voltage, or switch voltage, is defined by

$$v_o(t) = \begin{cases} 0 & \text{for } 0 \le t \le t_T \\ V_s & \text{for } t_T \le t \le T \end{cases} \tag{13.44}$$

The mean load voltage is

$$\begin{aligned} \overline{V}_o &= \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{T}\int_{t_T}^T V_s\,dt \\ &= \frac{T-t_T}{T}V_s = (1-\delta)V_s \end{aligned} \tag{13.45}$$

where the switch on-state duty cycle $\delta = t_T/T$ is defined in figure 13.7a.

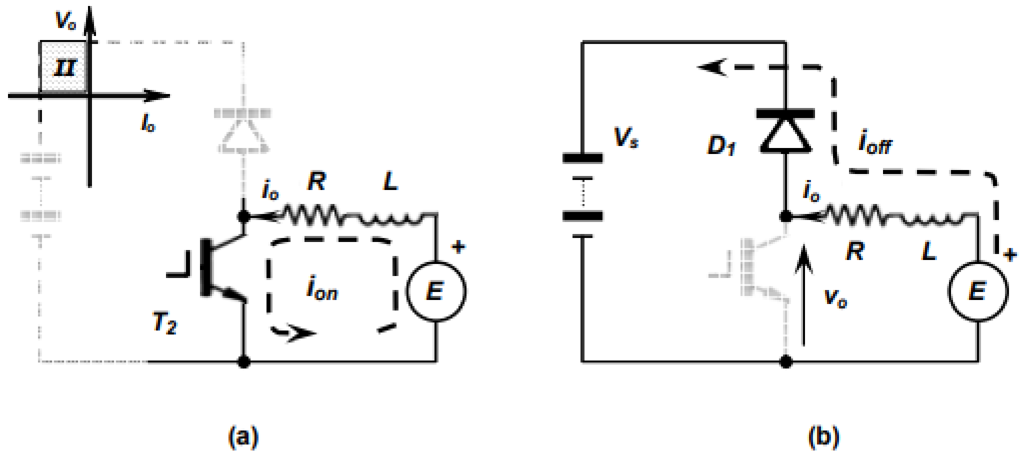

Figure 13.6. *Stages of operation for the second-quadrant chopper: (a) switch-on, boosting current and (b) switch-off, energy into V_s.*

Alternatively the voltage across the dc source V_s is

$$V_s = \frac{1}{1-\delta}\overline{V}_o \tag{13.46}$$

Since $0 \le \delta \le 1$, the step-up voltage ratio, to regenerate into V_s, is continuously adjustable from unity to infinity.

The average output current is

$$\overline{I}_o = \frac{E-\overline{V}_o}{R} = \frac{E-V_s(1-\delta)}{R} \tag{13.47}$$

The average output current can also be found by integration of the time domain output current i_o. By solving the appropriate time domain differential equations, the continuous load current i_o shown in figure 13.7a is defined by

During the **switch on-period**, when v_o=0

$$L\frac{di_o}{dt} + R\,i_o = E$$

which yields

$$i_o(t) = \frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\, e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le t_T \tag{13.48}$$

During the **switch off-period**, when $v_o = V_s$

$$L\frac{di_o}{dt} + Ri_o + V_s = E$$

which, after shifting the zero time reference to t_T, gives

$$i_o(t) = \frac{E-V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}\, e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le T - t_T \tag{13.49}$$

$$\text{where} \quad \hat{I} = \frac{E}{R} - \frac{V_s}{R}\,\frac{e^{\frac{-t_T}{\tau}} - e^{\frac{-T}{\tau}}}{1-e^{\frac{-T}{\tau}}} \qquad \text{(A)}$$

$$\text{and} \quad \check{I} = \frac{E}{R} - \frac{V_s}{R}\,\frac{1-e^{\frac{-T+t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} \qquad \text{(A)} \tag{13.50}$$

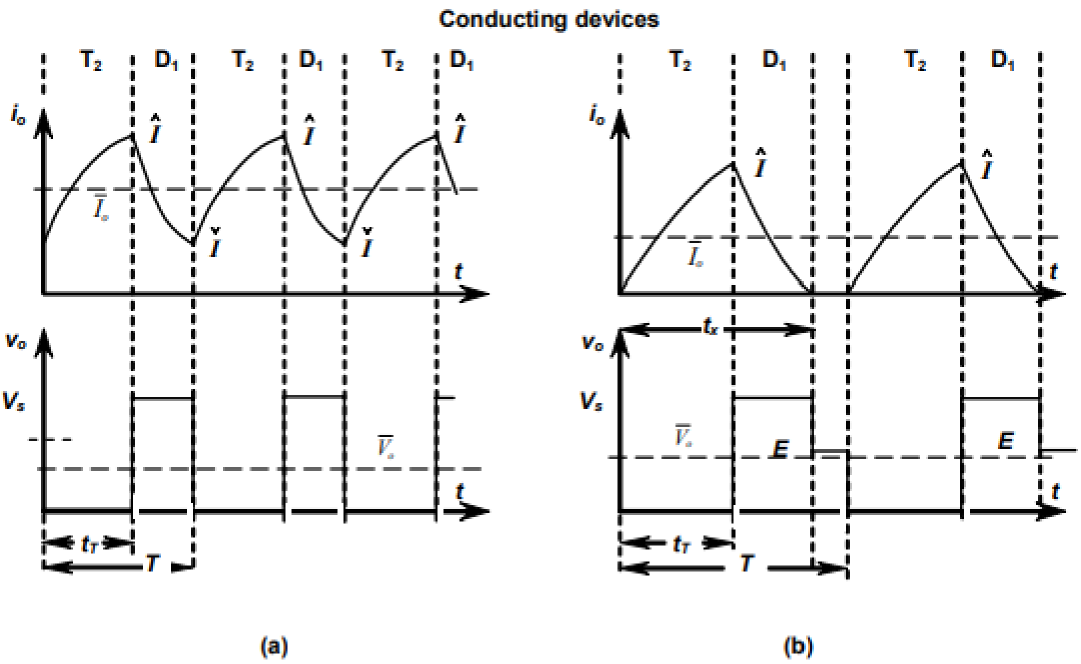

Figure 13.7. *Second-quadrant chopper output modes of current operation: (a) continuous inductor current and (b) discontinuous inductor current.*

The output ripple current, for continuous conduction, is independent of the back emf E and is given by

$$I_{p-p} = \hat{I} - \check{I} = \frac{V_s}{R}\,\frac{\left(1+e^{\frac{-T}{\tau}}\right) - \left(e^{\frac{-t_T}{\tau}} + e^{\frac{-T+t_T}{\tau}}\right)}{1-e^{\frac{-T}{\tau}}} \tag{13.51}$$

which in terms of the on-state duty cycle, $\delta = t_T/T$, becomes

$$I_{p-p} = \frac{V_s}{R} \frac{(1-e^{\frac{-\delta T}{\tau}})(1+e^{\frac{-T}{\tau}})}{1-e^{\frac{-T}{\tau}}} \tag{13.52}$$

This is the same expression derived in 13.2.1 for the first-quadrant chopper. The normalised ripple current design curves in figure 13.3 are valid for the second-quadrant chopper.

The average switch current, $\overline{I}_{switch}$, can be derived by integrating the switch current given by equation (13.48), that is

$$\begin{aligned} \overline{I}_{switch} &= \frac{1}{T}\int_0^{t_T} i_o(t)\,dt \\ &= \frac{1}{T}\int_0^{t_T} \left(\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}}\right)dt \\ &= \frac{\delta E}{R} - \frac{\tau}{T}\left(\hat{I}-\check{I}\right) \end{aligned} \tag{13.53}$$

The term $\hat{I}-\check{I} = I_{p-p}$ is the peak-to-peak ripple current, which is given by equation (13.51). By Kirchhoff's current law, the average diode current $\overline{I}_{diode}$ is the difference between the average output current $\overline{I}_o$ and the average switch current,

difference between the average output current $\overline{I}_o$ and the average switch current,

$\overline{I}_{switch}$, that is

$$\begin{aligned} \overline{I}_{diode} &= \overline{I}_o - \overline{I}_{switch} \\ &= \frac{E-V_s(1-\delta)}{R} - \frac{\delta E}{R} + \frac{\tau}{T}\left(\hat{I}-\check{I}\right) \\ &= \frac{\tau}{T}\left(\hat{I}-\check{I}\right) - \frac{(V_s-E)(1-\delta)}{R} \end{aligned} \tag{13.54}$$

The average diode current can also be found by integrating the diode current given in equation (13.49), as follows

$$\begin{aligned} \overline{I}_{diode} &= \frac{1}{T}\int_0^{T-t_T} \left(\frac{E-V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}\,e^{\frac{-t}{\tau}}\right)dt \\ &= \frac{\tau}{T}\left(\hat{I}-\check{I}\right) - \frac{(V_s-E)(1-\delta)}{R} \end{aligned} \tag{13.55}$$

The power produced by the back emf source E is

$$P_E = E\overline{I}_o = E\left(\frac{E-V_s(1-\delta)}{R}\right) \tag{13.56}$$

The power delivered to the dc source V_s is

$$P_{V_s} = V_s\overline{I}_{diode} = V_s\left(\frac{\tau}{T}\left(\hat{I}-\check{I}\right) - \frac{(V_s-E)(1-\delta)}{R}\right) \tag{13.57}$$

The difference between the two powers is the power lost in the load resistor, R, that is

$$\begin{aligned} P_E &= P_{V_s} + I_{o_{rms}}^2 R \\ I_{o_{rms}} &= \sqrt{\frac{E\overline{I}_o - V_s\overline{I}_{diode}}{R}} \end{aligned} \tag{13.58}$$

The efficiency of energy transfer between the back emf E and the dc source V_s is

$$\eta = \frac{P_{V_s}}{P_E} = \frac{V_s \overline{I}_{diode}}{E \overline{I}_o} \tag{13.59}$$

13.3.2 Discontinuous inductor current

With low duty cycles, δ, low inductance, L, or a relatively high dc source voltage, V_s, the minimum output current may reach zero at t_x, before the period T is complete, as shown in figure 13.7b. Equation (13.50) gives a boundary identity that must be satisfied for zero current,

$$\check{I} = \frac{E}{R} - \frac{V_s}{R}\frac{1-e^{\frac{T-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} = 0 \tag{13.60}$$

That is

$$\frac{E}{V_s} = \frac{1-e^{\frac{-T+t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} \tag{13.61}$$

Alternatively, the time domain equations (13.48) and (13.49) can be used, such that $\check{I}$ =0. An expression for the extinction time t_x can be found by substituting $t=t_T$ into equation (13.48). The resulting expression for $\hat{I}$ is then substituted into equation (13.49) which is set to zero. Isolating the time variable, which becomes t_x, yields

$$\hat{I} = \frac{E}{R}\left(1-e^{\frac{-t_T}{\tau}}\right)$$

$$0 = \frac{E-V_s}{R}\left(1-e^{\frac{-t_x}{\tau}}\right) + \frac{E}{R}\left(1-e^{\frac{-t_T}{\tau}}\right)e^{\frac{-t_x}{\tau}}$$

which yields

$$t_x = t_T + \tau\,\ell n\left(1 + \frac{E}{V_s - E}\left(1-e^{\frac{-t_T}{\tau}}\right)\right) \tag{13.62}$$

This equation shows that $t_x \geq t_T$. Load waveforms for discontinuous load current conduction are shown in figure 13.7b.

The output voltage v_o, load voltage, or switch voltage, is defined by

$$v_o(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq t_T \\ V_s & \text{for } t_T \leq t \leq t_x \\ E & \text{for } t_x \leq t \leq T \end{cases} \tag{13.63}$$

The mean load voltage is

$$\overline{V}_o = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{T}\left(\int_{t_T}^{t_x} V_s\,dt + \int_{t_x}^{T} E\,dt\right)$$

$$= \frac{t_x - t_T}{T} V_s + \frac{T - t_x}{T} E = \left(\frac{t_x}{T} - \delta \right) V_s + \left(1 - \frac{t_x}{T} \right) E$$

$$\overline{V}_o = E - \delta V_s + \frac{t_x}{T} (V_s - E) \tag{13.64}$$

where the switch on-state duty cycle $\delta = t_T/T$ is defined in figure 13.7b.
The average output current is

$$\overline{I}_o = \frac{E - \overline{V}_o}{R} = \frac{\delta V_s - \frac{t_x}{T}(V_s - E)}{R} \tag{13.65}$$

The average output current can also be found by integration of the time domain output current i_o. By solving the appropriate time domain differential equations, the continuous load current i_o shown in figure 13.7a is defined by
During the **switch on-period**, when $v_o = 0$

$$L \frac{di_o}{dt} + R i_o = E$$

which yields

$$i_o(t) = \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) \qquad \text{for} \quad 0 \le t \le t_T \tag{13.66}$$

During the **switch off-period**, when $v_o = V_s$

$$L \frac{di_o}{dt} + R i_o + V_s = E$$

which, after shifting the zero time reference to t_T, gives

$$i_o(t) = \frac{E - V_s}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I} e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le t_x - t_T \tag{13.67}$$

$$\text{where} \quad \hat{I} = \frac{E}{R}\left(1 - e^{\frac{-t_T}{\tau}}\right) \qquad \text{(A)}$$

$$\text{and} \quad \check{I} = 0 \qquad \text{(A)} \tag{13.68}$$

After t_x, $v_o(t) = E$ and the load current is zero, that is

$$i_o(t) = 0 \qquad \text{for} \quad t_x \le t \le T \tag{13.69}$$

The output ripple current, for discontinuous conduction, is dependent of the back emf E and is given by equation (13.68),

$$I_{p-p} = \hat{I} = \frac{E}{R}\left(1 - e^{\frac{-t_T}{\tau}}\right) \tag{13.70}$$

The average switch current, $\overline{I}_{switch}$, can be derived by integrating the switch current given by equation (13.66), that is

$$\begin{aligned} \overline{I}_{switch} &= \frac{1}{T}\int_0^{t_T} i_o(t)\,dt \\ &= \frac{1}{T}\int_0^{t_T} \left(\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)\right) dt \\ &= \frac{\delta E}{R} - \frac{\tau}{T}\hat{I} \end{aligned} \tag{13.71}$$

The term $\hat{I} = I_{p-p}$ is the peak-to-peak ripple current, which is given by equation (13.70). By Kirchhoff's current law, the average diode current $\overline{I}_{diode}$ is the difference between the average output current $\overline{I}_o$ and the average switch current, $\overline{I}_{switch}$, that is

$$\begin{aligned}\overline{I}_{diode} &= \overline{I}_o - \overline{I}_{switch} \\ &= \frac{\delta V_s - \frac{t_x}{T}(V_s - E)}{R} - \frac{\delta E}{R} + \frac{\tau}{T}\hat{I} \\ &= \frac{\tau}{T}\hat{I} - \frac{\left(\frac{t_x}{T} - \delta\right)(V_s - E)}{R}\end{aligned} \tag{13.72}$$

The average diode current can also be found by integrating the diode current given in equation (13.67), as follows

$$\begin{aligned}\overline{I}_{diode} &= \frac{1}{T}\int_0^{t_x - \delta T}\left(\frac{E - V_s}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\, e^{\frac{-t}{\tau}}\right)dt \\ &= \frac{\tau}{T}\hat{I} - \frac{\left(\frac{t_x}{T} - \delta\right)(V_s - E)}{R}\end{aligned} \tag{13.73}$$

The power produced by the back emf source E is

$$P_E = E\overline{I}_o \tag{13.74}$$

The power delivered to the dc source V_s is

$$P_{V_s} = V_s\overline{I}_{diode} \tag{13.75}$$

Alternatively, the difference between the two powers is the power lost in the load resistor, R, that is

$$\begin{aligned}P_E &= P_{V_s} + I_{o_{rms}}^2 R \\ I_{o_{rms}} &= \sqrt{\frac{E\overline{I}_o - V_s\overline{I}_{diode}}{R}}\end{aligned} \tag{13.76}$$

The efficiency of energy transfer between the back emf and the dc source is

$$\eta = \frac{P_{V_s}}{P_E} = \frac{V_s\overline{I}_{diode}}{E\overline{I}_o} \tag{13.77}$$

Example 13.4: ***Second-quadrant DC chopper – continuous conduction***

A dc-to-dc chopper capable of second-quadrant operation is used in a 200V dc battery electric vehicle. The machine armature has 1 ohm resistance and 1mH inductance.

i. The machine is used for regenerative braking. At a constant speed downhill, the back emf is 150V, which results in a 10A braking current. What is the switch on-state duty cycle if the machine is delivering continuous output current? What is the minimum chopping frequency for these conditions?

ii. At this speed, (that is, E=150V), determine the minimum duty cycle for continuous inductor current, if the switching frequency is 1kHz. What is the average braking current at the critical duty cycle? What is the regenerating efficiency and the rms machine output current?

iii. If the chopping frequency is increased to 5kHz, at the same speed, (that is, E=150V), what is the critical duty cycle and the corresponding average machine current?

Solution

The main circuit operating parameters are

- V_s=200V
- E=150V
- load time constant $\tau = L/R = 1\text{H}/1\Omega = 1\text{ms}$

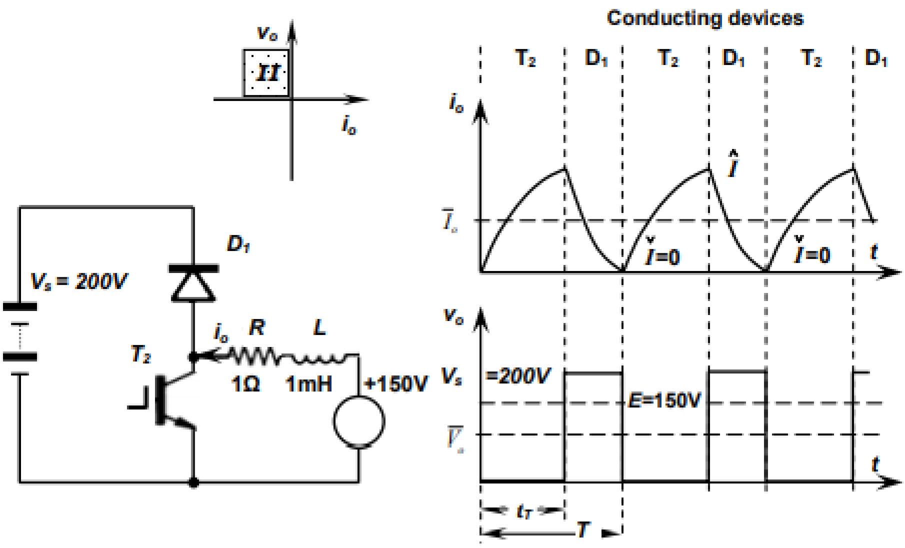

Figure Example 13.4. *Circuit diagram and waveforms.*

i. The relationship between the dc supply V_s and the machine back emf E is given by equation (13.47), that is

$$\overline{I}_o = \frac{E - \overline{V}_o}{R} = \frac{E - V_s(1-\delta)}{R}$$

$$10\text{A} = \frac{150\text{V} - 200\text{V} \times (1 - \delta)}{1\Omega}$$

that is

$$\delta = 0.3 \equiv 30\% \quad \text{and} \quad \overline{V}_o = 140\text{V}$$

The expression for the average machine output current is based on continuous armature inductance current. Therefore the switching period must be shorter than the time t_x predicted by equation (13.62) for the current to reach zero, before the next switch on-period. That is, for t_x=T and δ=0.3

$$t_x = t_T + \tau\, \ell n\left(1 + \frac{E}{V_s - E}\left(1 - e^{\frac{-t_T}{\tau}}\right)\right)$$

This simplifies to

$$1 = 0.3 + \frac{1\text{ms}}{T}\ell n\left(1 + \frac{150\text{V}}{200\text{V - 150V}}\left(1 - e^{\frac{-0.3T}{1\text{ms}}}\right)\right)$$

$$e^{0.7T} = 4 \quad 3e^{-0.3T}$$

Iteratively solving this transcendental equation gives T= 0.4945ms. That is the switching frequency must be greater than $f_c = 1/T$ = 2.022kHz, else machine output current discontinuities occur, and equation (13.47) is invalid. The switching frequency can be reduced if the on-state duty cycle is increased as in the next part of this example.

ii. The operational boundary condition giving by equation (13.61), using $T=1/f_s$ =1/1kHz = 1ms, yields

$$\frac{E}{V_s} = \frac{1 - e^{\frac{-T + t_T}{\tau}}}{1 - e^{\frac{-T}{\tau}}}$$

$$\frac{150\text{V}}{200\text{V}} = \frac{1 - \text{e}^{\frac{(\delta-1)\times 1\text{ms}}{1\text{ms}}}}{1 - \text{e}^{\frac{-1\text{ms}}{1\text{ms}}}}$$

Solving gives δ=0.357. That is, the on-state duty cycle must be at least 35.7% for continuous machine output current at a switching frequency of 1kHz.
For continuous inductor current, the average output current is given by equation (13.47), that is

$$\overline{I}_o = \frac{E - \overline{V}_o}{R} = \frac{E - V_s(1-\delta)}{R}$$

$$= \frac{150\text{V - }\overline{V}_o}{1\Omega} = \frac{150\text{V - 200V}\times(1\text{ - }0.357)}{1\Omega} = 21.4\text{A}$$

$$\overline{V}_o = 150\text{V - }21.4\text{A}\times 1\Omega = 128.6\text{V}$$

The average machine output current of 21.4A is split between the switch and the diode (which is in series with V_s).
The diode current is given by equation (13.54)

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_{switch}$$

$$= \frac{\tau}{T}\left(\hat{I} - \check{I}\right) - \frac{(V_s - E)(1-\delta)}{R}$$

The minimum output current is zero while the maximum is given by equation (13.68).

$$\hat{I} = \frac{E}{R}\left(1 - e^{\frac{-t_T}{\tau}}\right) = \frac{150\text{V}}{1\Omega}\times\left(1\text{ - e}^{\frac{-0.357\times 1\text{ms}}{1\text{ms}}}\right) = 45.0\text{A}$$

Substituting into the equation for the average diode current gives

$$\overline{I}_{diode} = \frac{1\text{ms}}{1\text{ms}}\times(45.0\text{A - 0A})\text{ - }\frac{(200\text{V - 150V})\times(1\text{ - }0.357)}{1\Omega} = 12.85\text{A}$$

The power delivered by the back emf E is

$$P_E = E\overline{I}_o = 150\text{V}\times 21.4\text{A} = 3210\text{W}$$

While the power delivered to the 200V battery source V_s is

$$P_{V_s} = V_s\overline{I}_{diode} = 200\text{V}\times 12.85\text{A} = 2570\text{W}$$

The regeneration transfer efficiency is

$$\eta = \frac{P_{V_s}}{P_E} = \frac{2570\text{W}}{3210\text{W}} = 80.1\%$$

The energy generated deficit, 640W (3210W - 2570W)), is lost in the armature resistance, as I^2R heat dissipated. The output rms current is

$$I_{o_{rms}} = \sqrt{\frac{P}{R}} = \sqrt{\frac{640\text{W}}{1\Omega}} = 25.3\text{A rms}$$

iii. At an increased switching frequency of 5kHz, the duty cycle would be expected to be much lower than the 35.7% as at 1kHz. The operational boundary between continuous and discontinuous armature inductor current is given by equation (13.61), that is

$$\frac{E}{V_s} = \frac{1-e^{\frac{-T+t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}}$$

$$\frac{150\text{V}}{200\text{V}} = \frac{1 - \text{e}^{\frac{(-1+\delta)\times 0.2\text{ms}}{1\text{ms}}}}{1 - \text{e}^{\frac{-0.2\text{ms}}{1\text{ms}}}}$$

which yields $\delta = 26.9\%$.

The machine average output current is given by equation (13.47)

$$\overline{I}_o = \frac{E-\overline{V}_o}{R} = \frac{E-V_s(1-\delta)}{R}$$

$$= \frac{150\text{V}-\overline{V}_o}{1\Omega} = \frac{150\text{V} - 200\text{V}\times(1 - 0.269)}{1\Omega} = 3.8\text{A}$$

such that the average output voltage $\overline{V}_o$ is 146.2V.

Example 13.5: ***Two-quadrant DC chopper with load back emf***

The two-quadrant dc-to-dc chopper in figure 13.8a feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 100V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the current crossover times, if applicable;
vi. the load average current, average switch current and average diode current for all devices;
vii. the input power, hence output power and rms output current;
viii. effective input impedance and electromagnetic efficiency; and
ix. Sketch the circuit, load, and output voltage and current waveforms.

Subsequently determine the necessary change in

x. Duty cycle δ to result in zero average output current and
xi. Back emf E to result in zero average load current

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

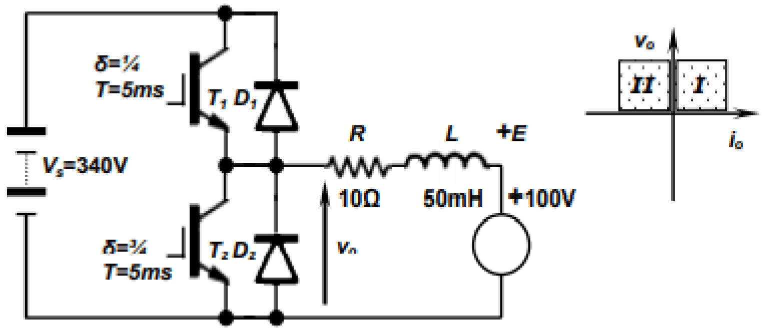

Figure Example 13.5. *Circuit diagram.*

i. From equations (13.79) and (13.80) the load average and rms voltages are

$$v_o = \frac{t_T}{T} V_s = \frac{1.25\text{ms}}{5\text{ms}} \times 340\text{V} = ¼ \times 340\text{V} = 85\text{V}$$

$$V_{rms} = \sqrt{\delta}\, V_s = \sqrt{¼} \times 340\text{V} = 170\text{V} \quad \text{rms}$$

ii. The rms ripple voltage, hence ripple factor, from equations (13.81) and (13.82) are

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$

$$= \sqrt{170^2 \text{ - } 85^2} = 340\text{V}\sqrt{¼ \times (1 \text{ - } ¼)} = 147.2\text{V}$$

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1} = \sqrt{\frac{1}{¼} \text{ - } 1} \;=\; 1.732$$

iii. From equations (13.85) and (13.86), the maximum and minimum output current, hence the peak-to-peak output ripple in the current are given by

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega} \times \frac{1 \text{ - } e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1 \text{ - } e^{\frac{-5\text{ms}}{5\text{ms}}}} \text{ - } \frac{100\text{V}}{10\Omega} = 1.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega} \times \frac{e^{\frac{1.25\text{ms}}{5\text{ms}}} \text{ - } 1}{e^{\frac{5\text{ms}}{5\text{ms}}} \text{ - } 1} \text{ - } \frac{100\text{V}}{10\Omega} = \text{ - } 4.38\text{A}$$

The peak-to-peak ripple current is therefore $\Delta i_o = 1.90\text{A - - }4.38\text{A} = 6.28\text{A}$ p-p.

$$i_o(t) = 24 \text{ - } 28.38 \times e^{\frac{-t}{5\text{ms}}} = 0 \qquad \text{where} \quad 0 \le t = t_{xT} \le 1.25\text{ms}$$

$$t_{xT} = 5\text{ms} \times \ell n\frac{28.38}{24} = 0.838\text{ms}$$

During the switch off-time

$$i_o(t) = -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}} = 0 \qquad \text{where} \quad 0 \le t = t_{xD} \le 3.75\text{ms}$$

$$t_{xD} = 5\text{ms} \times \ell n\frac{11.90}{10} = 0.870\text{ms}$$

(1.250ms + 0.870ms = 2.12ms with respect to switch turn-on)

iv. The current in the time domain is given by equations (13.83) and (13.84)

$$i_o(t) = \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I}\, e^{\frac{-t}{\tau}}$$

$$= \frac{340\text{V-}100\text{V}}{10\Omega} \times \left(1 - e^{\frac{-t}{5\text{ms}}}\right) - 4.38 \times e^{\frac{-t}{5\text{ms}}}$$

$$= 24 \times \left(1 - e^{\frac{-t}{5\text{ms}}}\right) - 4.38 \times e^{\frac{-t}{5\text{ms}}}$$

$$= 24 - 28.38 \times e^{\frac{-t}{5\text{ms}}} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o(t) = -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\, e^{\frac{-t}{\tau}}$$

$$= -\frac{100V}{10\Omega} \times \left(1 - e^{\frac{-t}{5\text{ms}}}\right) + 1.90 \times e^{\frac{-t}{5\text{ms}}}$$

$$= -10 \times \left(1 - e^{\frac{-t}{5\text{ms}}}\right) + 1.90 \times e^{\frac{-t}{5\text{ms}}}$$

$$= -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. Since the maximum current is greater than zero (1.9A) and the minimum is less that zero (- 4.38A), the current crosses zero during the switch on-time and off-time. The time domain equations for the load current are solved for zero to give the cross over times t_{xT} and t_{xD}, as given by equation (13.89), or solved from the time domain output current equations as follows.

During the switch on-time

$$i_o(t) = 24 - 28.38 \times e^{\frac{-t}{5\text{ms}}} = 0 \qquad \text{where} \quad 0 \le t = t_{xT} \le 1.25\text{ms}$$

$$t_{xT} = 5\text{ms} \times \ell n \frac{28.38}{24} = 0.838\text{ms}$$

During the switch off-time

$$i_o(t) = -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}} = 0 \qquad \text{where} \quad 0 \le t = t_{xD} \le 3.75\text{ms}$$

$$t_{xD} = 5\text{ms} \times \ell n \frac{11.90}{10} = 0.870\text{ms}$$

(1.250ms + 0.870ms = 2.12ms with respect to switch turn-on)

vi. The load average current, average switch current, and average diode current for all devices;

$$\overline{I}_o = \left(\overline{V}_o - E\right)/R = \left(\delta V_s - E\right)/R$$

$$\left(85\text{V} - 100\text{V}\right)/10\Omega = \text{-1.5A}$$

When the output current crosses zero current, the conducting device changes. Table 13.1 gives the necessary current equations and integration bounds for the condition $\hat{I} > 0, \check{I} < 0$. Table 13.1 shows that all four semiconductors are involved in the output current cycle.

$$\overline{I}_{T1} = \frac{1}{T}\int_{t_{xT}}^{t_T} \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I}\, e^{\frac{-t}{\tau}}\, dt$$

$$= \frac{1}{5\text{ms}}\int_{0.838\text{ms}}^{1.25\text{ms}} 24 \text{ - } 28.38 \times e^{\frac{-t}{5\text{ms}}}\, dt = 0.081\text{A}$$

$$\overline{I}_{D1} = \frac{1}{T}\int_{0}^{t_{xT}} \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I}\, e^{\frac{-t}{\tau}}\, dt$$

$$= \frac{1}{5\text{ms}}\int_{0}^{0.84\text{ms}} 24 \text{ - } 28.38 \times e^{\frac{-t}{5\text{ms}}}\, dt = -0.357\text{A}$$

$$\overline{I}_{T2} = \frac{1}{T}\int_{t_{xD}}^{T-t_T} -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\, e^{\frac{-t}{\tau}}\, dt$$

$$= \frac{1}{5\text{ms}}\int_{0870\text{ms}}^{3.75\text{ms}} -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}}\, dt = -1.382\text{A}$$

$$\overline{I}_{D2} = \frac{1}{T}\int_{0}^{t_{xD}} -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\, e^{\frac{-t}{\tau}}\, dt$$

$$= \frac{1}{5\text{ms}}\int_{0}^{0.870\text{ms}} -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}}\, dt = 0.160\text{A}$$

Check $\overline{I}_o + \overline{I}_{T1} + \overline{I}_{D1} + \overline{I}_{T2} + \overline{I}_{D2} = \text{- 1.5A} + 0.080\text{A - }0.357\text{A - }1.382\text{A} + 0.160\text{A} = 0$

vii. The input power, hence output power and rms output current;

$$P_{in} = P_{V_s} = V_s \overline{I}_i = V_s\left(\overline{I}_{T1} + \overline{I}_{D1}\right)$$

$$= 340\text{V} \times \left(0.080\text{A - }0.357\text{A}\right) = \text{-95.2W, (charging } V_s\text{)}$$

$$P_{out} = P_E = E\overline{I}_o = 100\text{V} \times \left(\text{-1.5A}\right) = \text{-150W, that is generating 150W}$$

From

$$V_s I_s = I_{o_{rms}}^2 R + E\overline{I}_o$$

$$I_{o_{rms}} = \sqrt{\frac{P_{out} - P_{in}}{R}} = \sqrt{\frac{150\text{W - }92.5\text{W}}{10\Omega}} = 2.34\text{A rms}$$

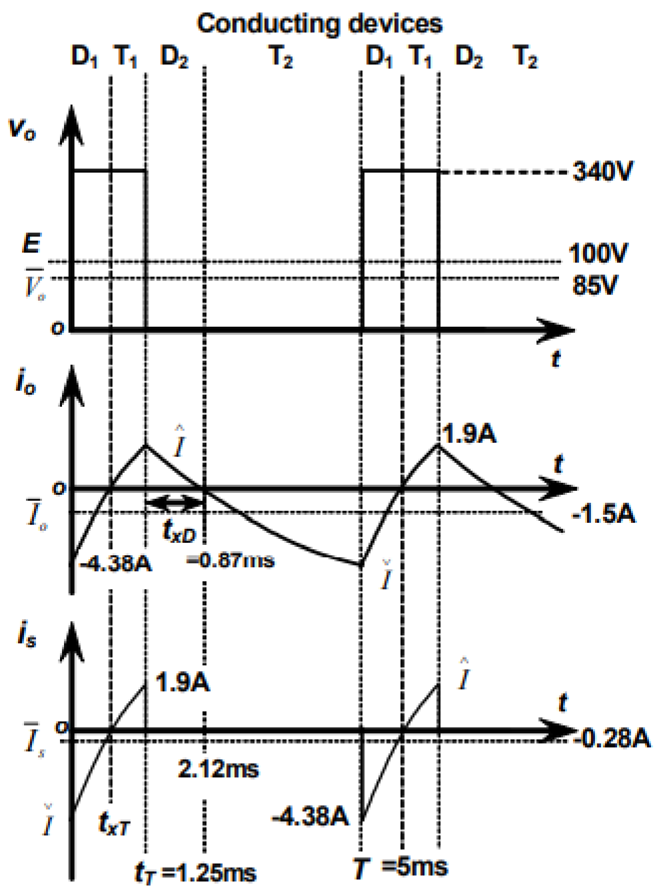

Figure Example 13.5. *Circuit waveforms*

viii. Since the average output current is negative, energy is being transferred from the back emf E to the dc source V_s, the electromagnetic efficiency of conversion is given by

$$\eta = \frac{V_s \overline{I}_i}{E\overline{I}_o} \quad \text{for } \overline{I}_o < 0$$
$$= \frac{95.2\text{W}}{150\text{W}} = 63.5\%$$

The effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i} = \frac{V_s}{\overline{I}_{T1} + \overline{I}_{D1}} = \frac{340\text{V}}{0.080\text{A} - 0.357\text{A}} = -1214\Omega$$

ix. The circuit, load, and output voltage and current waveforms are sketched in the figure for example 13.5.

x. Duty cycle δ to result in zero average output current can be determined from the expression for the average output current, equation (13.87), that is

$$\overline{I}_o = \frac{\delta V_s - E}{R} = 0$$

that is

$$\delta = \frac{E}{V_s} = \frac{100\text{V}}{340\text{V}} = 29.4\%$$

xi. As in part x, the average load current equation can be rearranged, this time to give the back emf E that results in zero average load current

$$\overline{I}_o = \frac{\delta V_s - E}{R} = 0$$

that is

$$E = \delta V_s = \tfrac{1}{4} \times 340\text{V} = 85\text{V}$$

Example 13.6: ***Asymmetrical, half H-bridge, dc chopper***

The asymmetrical half H-bridge, dc-to-dc chopper in figure 13.9 feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. The chopper output current is controlled in a hysteresis mode within a current band between limits 5A and 10A. Determine the period of the current shape shown in the figure example 13.6,

i. when only $\pm V_s$ loops are used and
ii. when a zero volt loop is used to maintain tracking within the 5A band.

In each case calculate the switching frequency if the current were to be maintained within the hysteresis band for a prolonged period.
How do the on-state losses compare between the two control approaches?

Solution

The main circuit and operating parameters are

- $E = 55\text{V}$ and $V_s = 340\text{V}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$
- $I^+ = 10\text{A}$ and $I^- = 5\text{A}$

Examination of the figure shows that only one period of the cycle differs, namely the second period, t_2, where the current is required to fall to the lower hysteresis band level, -5A. The period of the other three regions (t_1, t_3, and t_4) are common and independent of the period of the second region, t_2.

$\boldsymbol{t_1}$**:** The first period, the initial rise time, $t^+ = t_1$ is given by equation (13.100), where I^+=10A and $\check{I}$ = 0A.

$$t^+ = \tau \ell n\left(\frac{V_s - E - \check{I}R}{V_s - E - I^+R}\right)$$

$$\text{that is}\quad t_1 = 5\text{ms} \times \ell n\left(\frac{340\text{V} - 55\text{V} - 0\text{A}\times 10\Omega}{340\text{V} - 55\text{V} - 10\text{A}\times 10\Omega}\right) = 2.16\text{ms}$$

t_3**:** In the third period, the current rises from the lower hysteresis band limit of 5A to the upper band limit 10A. The duration of the current increase is given by equation (13.100) again, but with $\check{I} = I^{-} = 5\text{A}$.

$$t^{+} = \tau \ell n\left(\frac{V_s - E - \check{I}\,R}{V_s \quad E \quad I^{+}R}\right)$$

$$\text{that is}\quad t_3 = 5\text{ms} \times \ell n\left(\frac{340\text{V} - 55\text{V} - 5\text{A}\times 10\Omega}{340\text{V} - 55\text{V} - 10\text{A}\times 10\Omega}\right) = 1.20\text{ms}$$

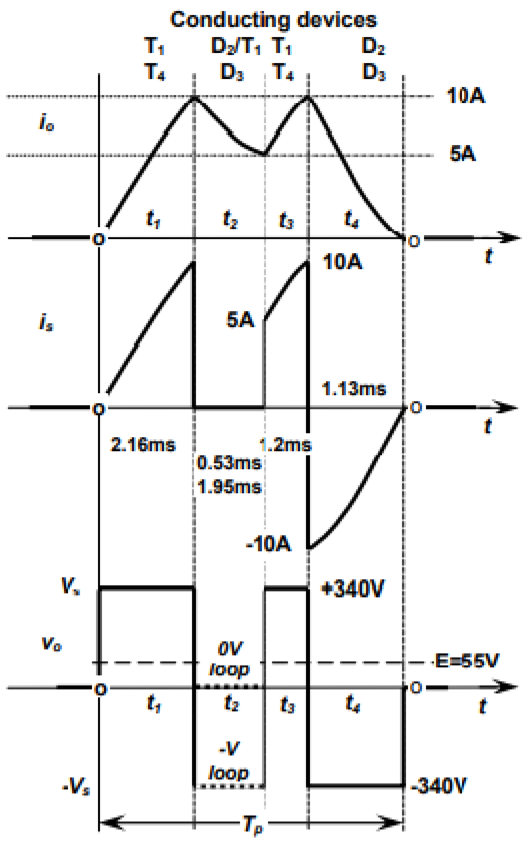

Figure Example 13.6. *Circuit waveforms.*

t_4**:** The fourth and final period is a negative voltage loop where the current falls from the upper band limit of 10A to I^{-} which equals zero. From equation (13.104) with $\hat{I} = I^{+} = 10\text{A}$ and $I^{-} = 0\text{A}$

$$t^{-} = \tau \ell n\left(\frac{V_s + E + \hat{I}\,R}{V_s + E + I^{-}R}\right)$$

$$\text{that is}\quad t_4 = 5\text{ms} \times \ell n\left(\frac{340\text{V} + 55\text{V} + 10\text{A}\times 10\Omega}{340\text{V} + 55\text{V} + 0\text{A}\times 10\Omega}\right) = 1.13\text{ms}$$

The current pulse period is given by

$$\begin{aligned} T_p &= t_1 + t_2 + t_3 + t_4 \\ &= 2.16\text{ms} + t_2 + 1.20\text{ms} + 1.13\text{ms} \\ &= 4.49\text{ms} + t_2 \end{aligned}$$

i. $\boldsymbol{t_2}$**:** When only -V_s paths are used to decrease the current, the time t_2 is given by equation (13.104), with I^{-}=5A and $\hat{I}$ =10A,

$$t^{-} = \tau \ell n\left(\frac{V_s + E + \hat{I}\,R}{V_s + E + I^{-}R}\right)$$

$$\text{that is}\quad t_2 = 5\text{ms} \times \ell n\left(\frac{340\text{V} + 55\text{V} + 10\text{A}\times 10\Omega}{340\text{V} + 55\text{V} + 5\text{A}\times 10\Omega}\right) = 0.53\text{ms}$$

The total period, T_p, of the chopped current pulse when a 0V loop is not used, is

$$\begin{aligned} T_p &= t_1 + t_2 + t_3 + t_4 \\ &= 2.16\text{ms} + 0.53\text{ms} + 1.20\text{ms} + 1.13\text{ms} = 5.02\text{ms} \end{aligned}$$

ii. $\boldsymbol{t_2}$**:** When a zero voltage loop is used to maintain the current within the hysteresis band, the period time t_2 is given by equation (13.102), with I^{-} = 5A and $\hat{I}$ =10A,

$$t^{o} = \tau \ell n\left(\frac{E + \hat{I}\,R}{E + I^{-}R}\right)$$

$$\text{that is}\quad t_2 = 5\text{ms} \times \ell n\left(\frac{55\text{V} + 10\text{A}\times 10\Omega}{55\text{V} + 5\text{A}\times 10\Omega}\right) = 1.95\text{ms}$$

The total period, T_p, of the chopped current pulse when a 0V loop is used, is

$$\begin{aligned} T_p &= t_1 + t_2 + t_3 + t_4 \\ &= 2.16\text{ms} + 1.95\text{ms} + 1.20\text{ms} + 1.13\text{ms} = 6.44\text{ms} \end{aligned}$$

The current falls significantly faster within the hysteresis band if negative voltage loops are employed rather that zero voltage loops, 0.53ms versus 1.95ms.
The switching frequency within the current bounds has a period $t_2 + t_3$, and each case is summarized in the following table.
Using zero voltage current loops reduces the switching frequency of the H-bridge switches by a factor of almost four, for a given peak-to-peak ripple current.
If the on-state voltage drop of the switches and the diodes are similar for the same current level, then the on-state losses are similar, and evenly distributed for both control methods. The on-state losses are similar because each of the three states always involves the same current variation flowing through two semiconductors.
The principal difference is in the significant increase in switching losses when only $\pm V$ loops are used (1:3.42).

Table Example 13.6. Switching losses.

Voltage loops	$t_2 + t_3$	Current ripple frequency	Switch frequency	Switch loss ratio
±V	0.53ms+1.20ms = 1.73ms	578Hz	578Hz	$\frac{578}{169} = 3.42$
+V and zero	1.95ms+1.20ms = 3.15ms	317Hz	169Hz	1

Example 13.7: ***Four-quadrant dc chopper***

The H-bridge, dc-to-dc chopper in figure 13.13 feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated with a 200Hz multilevel carrier as in figure 13.14 a and b, with a modulation depth of $\delta = \frac{1}{4}$, determine:

i. the average output voltage and switch T_1 on-time
ii. the rms output voltage and ac ripple voltage
iii. the average output current, hence quadrant of operation
iv. the electromagnetic power being extracted from the back emf *E*.

If the mean load current is to be halved, what is

v. the modulation depth, δ, requirement
vi. the average output voltage and the corresponding switch T_1 on-time
vii. the electromagnetic power being extracted from the back emf *E*?

Solution

The main circuit and operating parameters are

- modulation depth $\delta = \frac{1}{4}$
- period $T_{carrier} = 1/f_{carrier} = 1/200\text{Hz} = 5\text{ms}$
- E=55V and V_s=340V dc
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

i. The average output voltage is given by equation (13.114), and for $\delta < \frac{1}{2}$,

$$\overline{V}_o = \left(\frac{t_T}{T} - 1\right)V_s = (2\delta - 1)V_s$$

$$= 340\text{V} \times (2 \times \tfrac{1}{4} - 1) = -170\text{V}$$

where

$$t_T = 2\delta T = 2 \times \tfrac{1}{4} \times (\tfrac{1}{2} \times 5\text{ms}) = 1.25\text{ms}$$

Figure 13.14 reveals that the carrier frequency is half the switching frequency, thus the 5ms in the above equation has been halved. The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 3.75ms.

ii. The rms load voltage, from equation (13.118), is

$$V_{rms} = \sqrt{1-2\delta}\ V_s$$
$$= 340\text{V}\times\sqrt{1 - 2\times¼} = 240\text{V rms}$$

From equation (13.119), the output ac ripple voltage is

$$V_r = \sqrt{2}\,V_s\sqrt{\delta\left(1-2\delta\right)}$$
$$=\sqrt{2}\times 340\text{V}\times\sqrt{¼\times\left(1 - 2\times¼\right)} = 170\text{V ac}$$

iii. The average output current is given by equation (13.117)

$$\overline{I}_o = \frac{\overline{V}_o - E}{R} = \frac{\left(2\delta-1\right)V_s - E}{R}$$
$$= \frac{340\text{V}\times\left(2\times¼ - 1\right) - 55\text{V}}{10\Omega} = -22.5\text{A}$$

Since both the average output current and voltage are negative (-170V and -22.5A) the chopper with a modulation depth of δ = ¼, is operating in the third quadrant.

iv. The electromagnetic power developed by the back emf E is given by

$$P_E = E\overline{I}_o = 55\text{V}\times\left(-22.5\text{A}\right) = -1237.5\text{W}$$

v. The average output current is given by

$$\overline{I}_o = {\left(\overline{V}_o - E\right)}\Big/_R = {\left(\left(2\delta-1\right)V_s - E\right)}\Big/_R$$

when the mean current is -11.25A, δ = 0.415, as derived in part vi.

vi. Then, if the average current is halved to -11.25A

$$\overline{V}_o = E + \overline{I}_o R$$
$$=55\text{V} - 11.25\text{A}\times 10\Omega = -57.5\text{V}$$

The average output voltage rearranged in terms of the modulation depth δ gives

$$\delta = ½\left(1+\frac{\overline{V}_o}{V_s}\right)$$
$$= ½\times\left(1 + \frac{-57.5\text{V}}{340\text{V}}\right) = 0.415$$

The switch on-time when δ < ½ is given by

$$t_T = 2\delta T = 2\times 0.415\times\left(½\times 5\text{ms}\right) = 2.07\text{ms}$$

From figure 13.14b both T_1 and T_4 are turned on for 2.07ms, although, from table 13.3B, for negative load current, $\overline{I}_o = -11.25\text{A}$, the parallel connected freewheel diodes D_1 and D_4 conduct alternately, rather than the switches (assuming $\hat{I}_o < 0$). The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 2.93ms.

vii. The electromagnetic power developed by the back emf E is halved and is given by

$$P_E = E\overline{I}_o = 55\text{V}\times\left(-11.25\text{A}\right) = -618.75\text{W}$$