Four Quadrant Chopper or Type E Chopper or Class E Chopper

In this topic, you study Four Quadrant Chopper or Type-E Chopper or Class E Chopper v-i plane, working principle, quadrant operation, and Circuit diagrams.

Type E chopper is a four-quadrant chopper.

$v_O - i_O$ plane

The Type E chopper operates in the four quadrants of $v_O - i_O$ plane as shown in Figure 2. Here v_O is the output voltage, V_O is the average output voltage, i_O is the output current and I_O is the average output current of Type E chopper circuit.

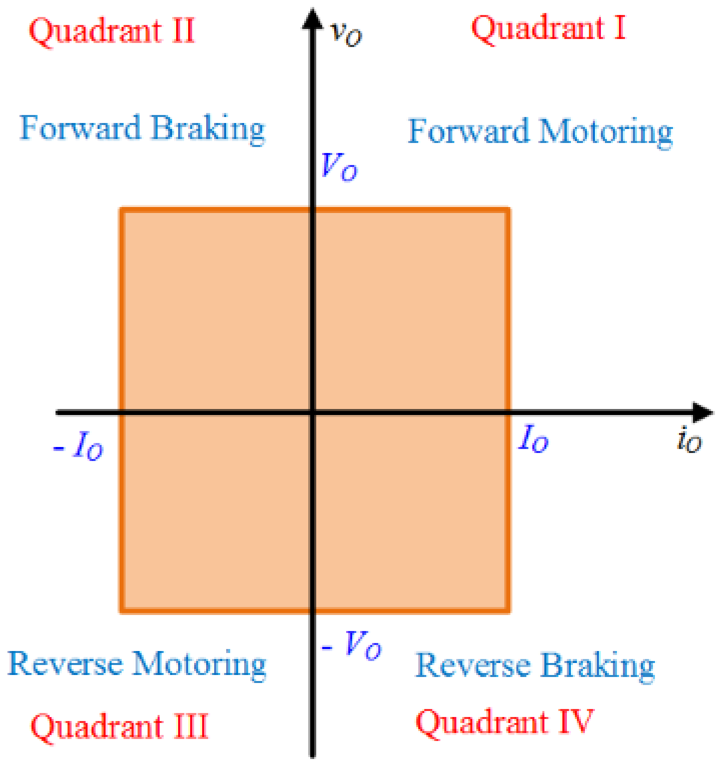


Figure 1 Type E chopper v_O - i_O plane

Circuit Diagram

The Type E chopper circuit diagram as shown in Figure 1. Here the motor load is assumed, R_a and L_a armature resistance and inductance of the motor respectively. E_b is the back emf of the motor.

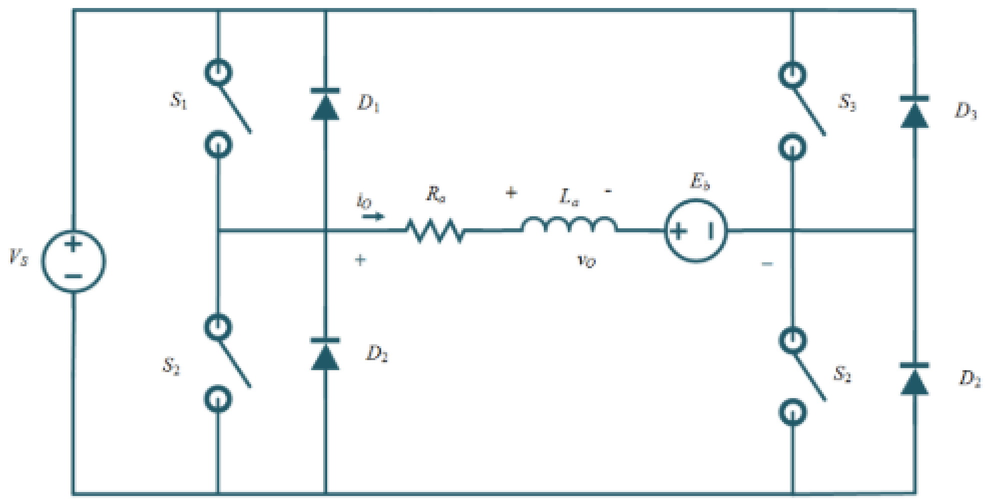


Figure 2 Circuit diagram of Type E chopper

Quadrant I operation when Switch S_1 turned on

The Type E chopper equivalent circuit diagram for Quadrant I is shown in Figure 3. Here switch S_1 operated, Switches S_1 and S_2 conduct, output voltage v_O and the output current i_O both are positives, power flows from source to load and inductor stores energy, the motor rotates in the forward direction hence called forward motoring.

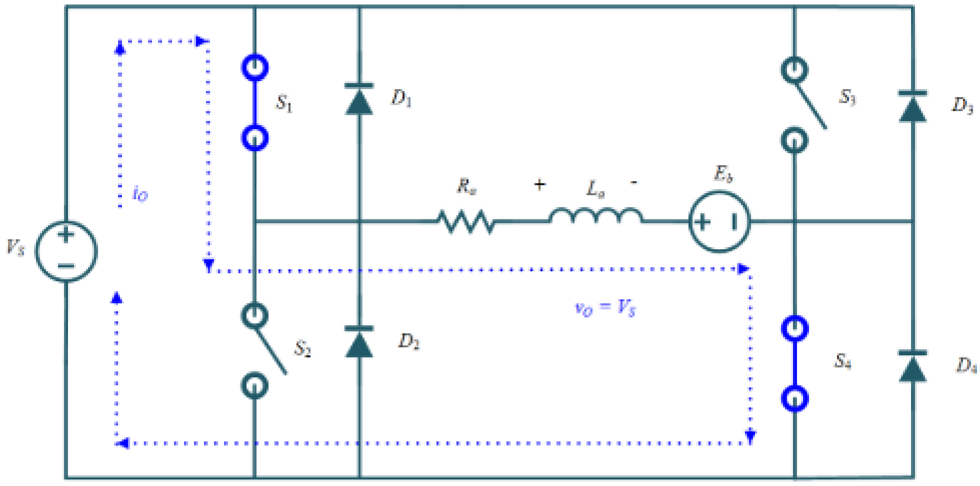


Figure 3 Equivalent circuit diagram I of Type E chopper

Quadrant I operation when Switch S_1 turned off

The Type E chopper equivalent circuit diagram for Quadrant I is shown in Figure 4. Switch S_1 turned off but switch S_4 and diode D_2 conducts, output current i_O is positive and the output voltage v_O becomes zero, inductor release energy and freewheeling action using diode D_2 takes place, the motor rotates in the forward direction hence called Forward motoring.

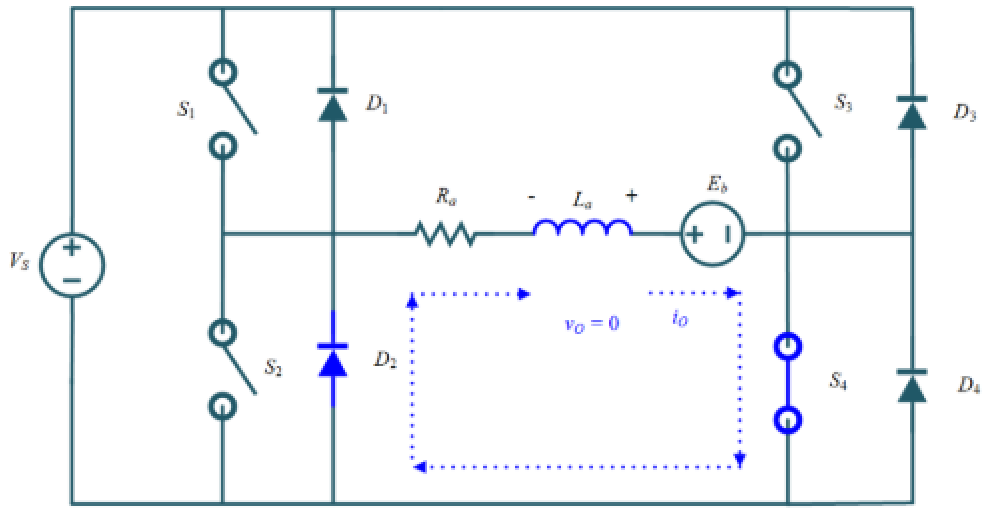


Figure 4 Forward Motoring Equivalent circuit diagram II of Type E chopper

Quadrant II operation when Switch S_2 turned on

The Type E chopper equivalent circuit diagram for Quadrant II is shown in Figure 5. Let us assume that the motor is running in the forward direction. Here switch S_2 operated, Switch S_2 and diode D_4 conducts, output voltage v_O is zero and E_b is responsible for the negative output current i_O, machine behave as generator and inductor stores energy.

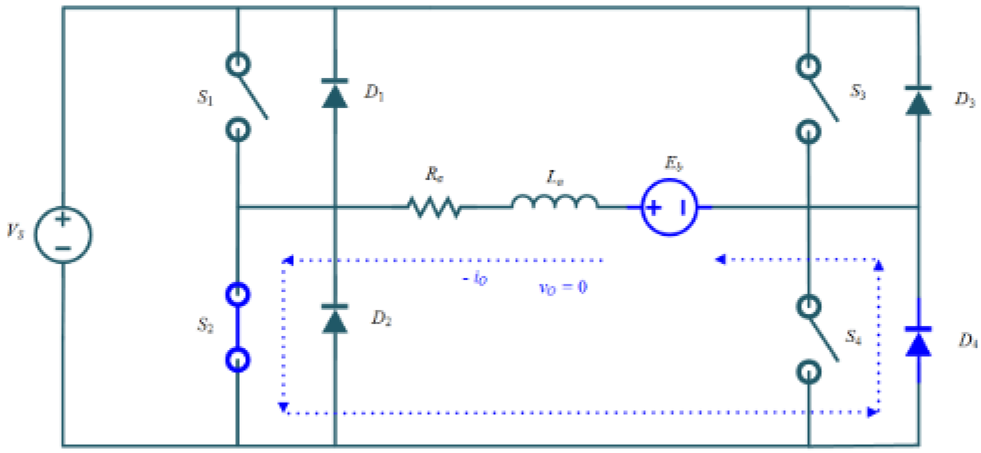


Figure 5 Forward Braking Equivalent circuit diagram III of Type E chopper

The Type E chopper equivalent circuit diagram for Quadrant II is shown in Figure 6. Switch S_2 turned off, diode D_1 and diode D_4 conducts, output voltage v_O becomes positive and the output current i_O is negative, inductor release energy using diodes D_1 and D_4, power flows from load to source and hence called as reverse braking.

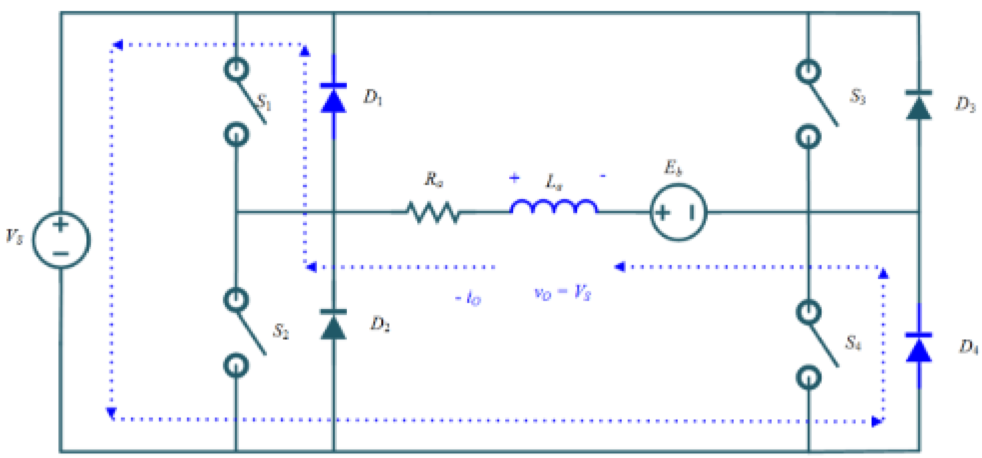


Figure 6 Forward Braking Equivalent circuit diagram IV of Type E chopper

Quadrant III operation when Switch S_3 turned on

The Type E chopper equivalent circuit diagram for Quadrant III is shown in Figure 7. The polarity of back emf E_b must be reversed. Here switch S_3 operated, Switches S_3 and S_2 conducts, output voltage v_O and the output current i_O both are negatives, power flows from source to load and inductor stores energy, the motor rotates in the reverse direction hence called as reverse motoring.

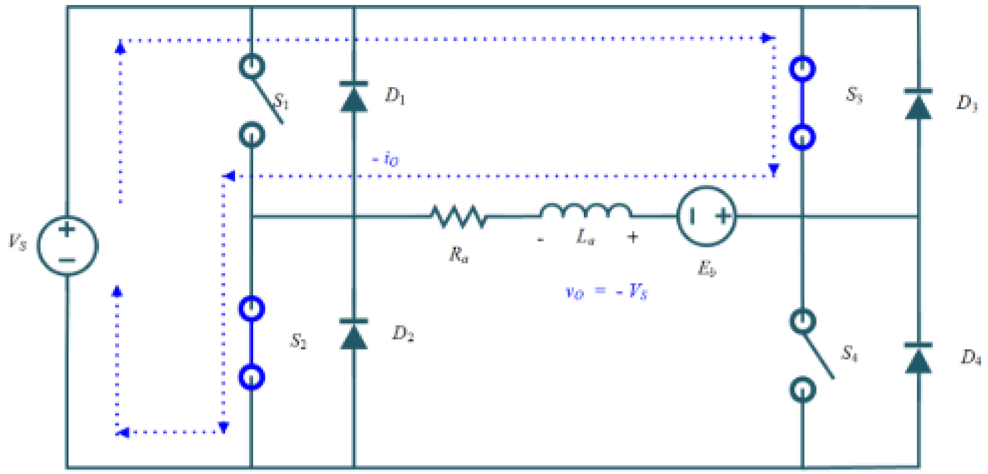


Figure 7 Equivalent circuit diagram V of Type E chopper

Quadrant III operation when Switch S_3 turned off

The Type E chopper equivalent circuit diagram for Quadrant III is shown in Figure 8. The polarity of back emf E_b must be reversed. Switch S_3 turned off but switch S_2 and diode D_4 conducts, output current i_O is negative and the output voltage v_O becomes zero, inductor release energy and freewheeling action using diode D_4 takes place, the motor rotates in the reverse direction hence called as Reverse motoring.

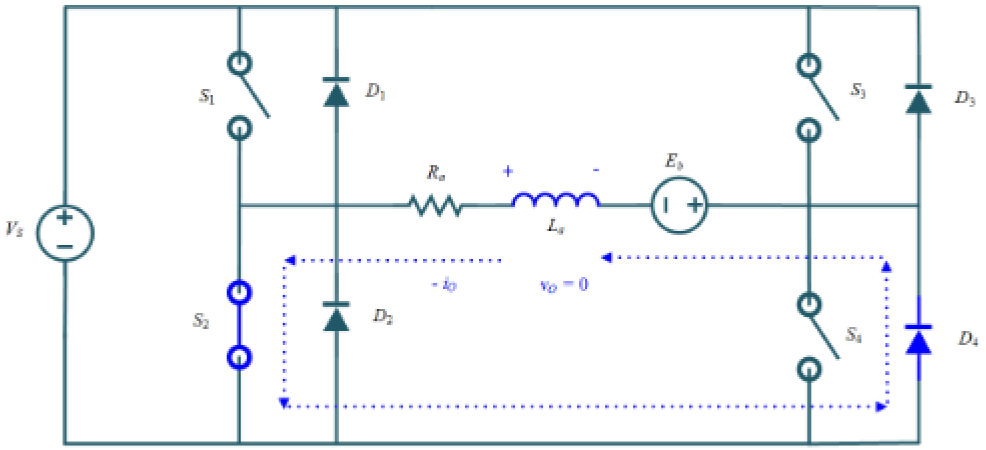


Figure 8 Reverse Motoring Equivalent circuit diagram VI of Type E chopper

Quadrant IV operation when Switch S_4 turned on

The Type E chopper equivalent circuit diagram for Quadrant IV is shown in Figure 9. The polarity of back emf E_b must be reversed. Let us assume that the motor is running in the reverse direction. Here switch S_4 operated, Switches S_4 and diode D_2 conducts, output voltage v_O is zero and E_b is responsible for the positive output current i_O, machine behave as generator and inductor stores energy.

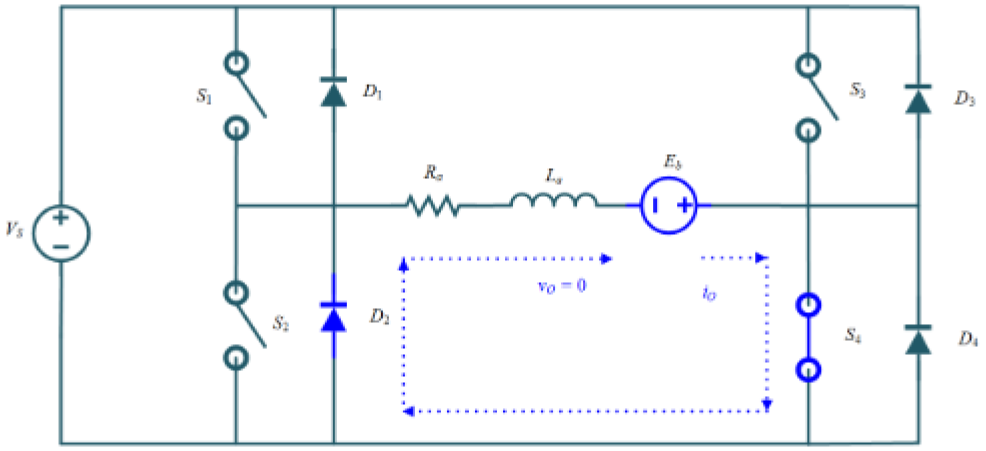


Figure 9 Reverse Braking Equivalent circuit diagram VII of Type E chopper

Quadrant IV operation when Switch S_4 turned off

The Type E chopper equivalent circuit diagram for Quadrant IV is shown in Figure 10. The polarity of back emf E_b must be reversed. Switch S_4 turned off, diode D_2 and diode D_3 conducts, output voltage v_O becomes negative and output current i_O is positive, inductor release energy using diodes D_2 and D_3, power flows from load to source and hence called as reverse braking.

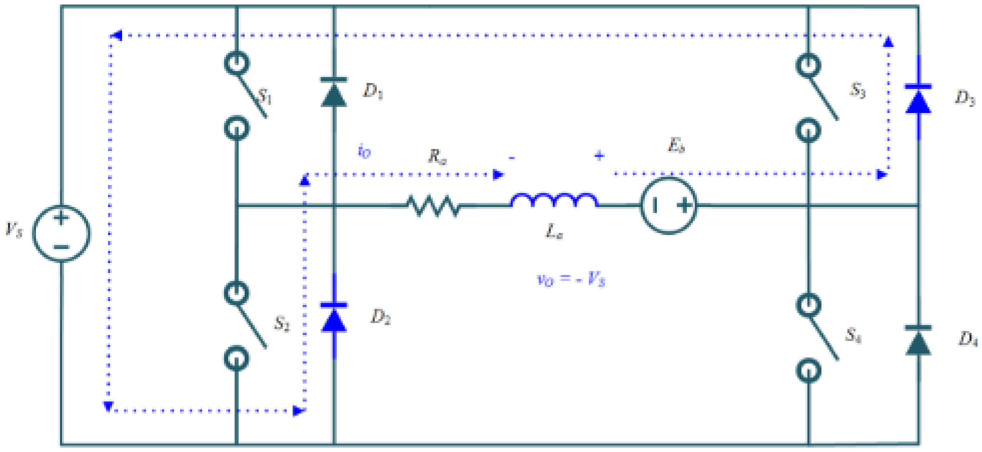
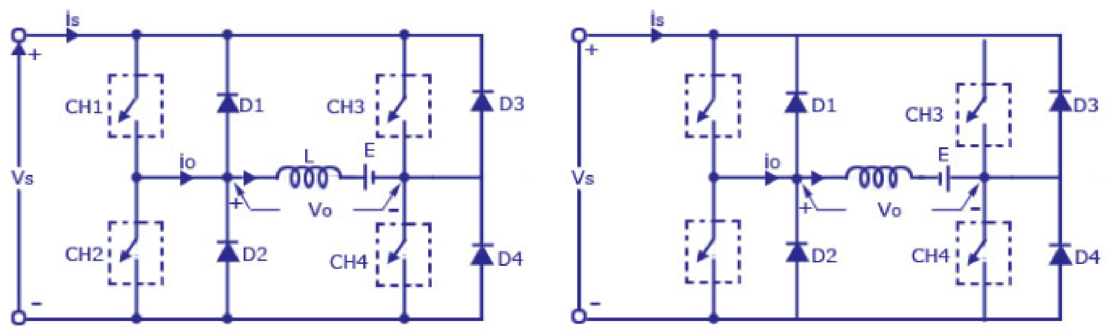


Figure 10 Reverse Braking Equivalent circuit diagram VIII of Type E chopper

Type –E chopper or the Fourth-Quadrant Chopper

Type E or the fourth quadrant chopper consists of four semiconductor switches and four diodes arranged in antiparallel. The 4 choppers are numbered according to which quadrant they belong. Their operation will be in each quadrant and the corresponding chopper only be active in its quadrant.

E-type Chopper Circuit Diagram With Load emf E and E Reversed



E-type Chopper Circuit diagram with load emf E and E Reversed

- **First Quadrant**

During the first quadrant operation the chopper CH4 will be on . Chopper CH3 will be off and CH1 will be operated. AS the CH1 and CH4 is on the load voltage v_0 will be equal to the source voltage V_s and the load current i_0 will begin to flow . v_0 and i_0 will be positive as the first quadrant operation is taking place. As soon as the chopper CH1 is turned off, the positive current freewheels through CH4 and the diode D2 . The type E chopper acts as a step- down chopper in the first quadrant.

- **Second Quadrant**

In this case the chopper CH2 will be operational and the other three are kept off. As CH2 is on negative current will starts flowing through the inductor L . CH2 ,E and D4. Energy is stored in the inductor L as the chopper CH2 is on. When CH2 is off the current will be fed back to the source through the diodes D1 and D4. Here (E+L.di/dt) will be more than the source voltage V_s . In second quadrant the chopper will act as a step-up chopper as the power is fed back from load to source

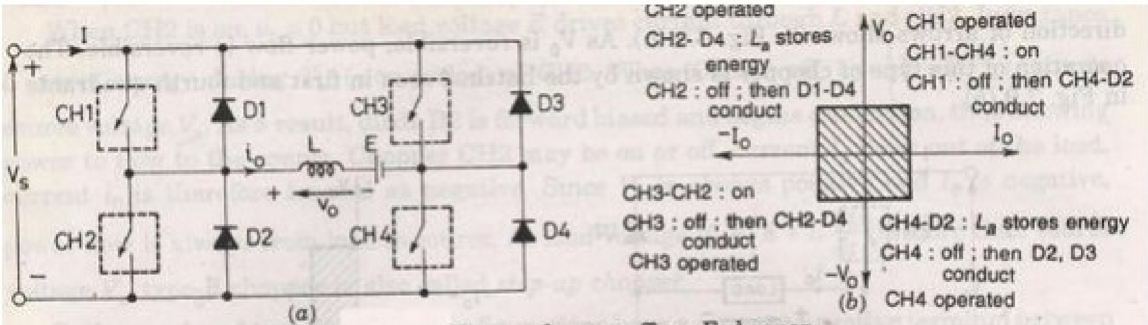
- **Third Quadrant**

In third quadrant operation CH1 will be kept off , CH2 will be on and CH3 is operated. For this quadrant working the polarity of the load should be reversed. As the chopper CH3 is on, the load gets connected to the source V_s and v_0 and i_0 will be negative and the third quadrant operation will takes place. This chopper acts as a step-down chopper

- **Fourth Quadrant**

CH4 will be operated and CH1, CH2 and CH3 will be off. When the chopper CH4 is turned on positive current starts to flow through CH4, D2 ,E and the inductor L will store energy. As the CH4 is turned off the current is feedback to the source through the diodes D2 and D3 , the operation will be in fourth quadrant as the load voltage is negative but the load current is positive. The chopper acts as a step up chopper as the power is fed back from load to source.

FOUR QUADRANT CHOPPER, OR TYPE E CHOPPER



Four-qudrant, or Type-E chopper
(a) circuit diagram and (b) operation of conducting devices.

FIRST QUADRANT:

CH4 is kept ON

CH3 is off

CH1 is operarted

V_0=V_s

i_0 = positive

when CH1 is off positive current free wheels through CH4,D2

so V_0 and I_2 is in first quadrant.

SECOND QUADRANT:

CH1,CH3,CH4 are off.

CH2 is operated.

Reverse current flows and I is negative through L CH2 D4 and E.

When CH2 off D1 and D4 is ON and current id fed back to source. So

$E + L\frac{di}{dt}$ is more than source voltage V_s.

As i_0 is negative and V_0 is positive, so second quadrant operation.

THIRD QUADRANT:

CH1 OFF, CH2 ON

CH3 operated. So both V_0 and i_0 is negative.

When CH3 turned off negative current freewheels through CH2 and D4.

FOURTH QUADRANT:

CH4 is operated other are off.

Positive current flows through CH4 E L D2.

Inductance L stores energy when current fed to source through D3 and D2.V_0 is negative.

1.2.8 Four-Quadrant Chopper

The four-quadrant chopper is shown in Figure 1.6c. The input voltage is positive, and the output voltage can be either positive or negative. The switches and diode status for the operation are shown in Table 1.1. The output voltage can be calculated by the formula

$$V_2 = \begin{cases} kV_1 & QI\_operation \\ (1-k)V_1 & QII\_operation \\ -kV_1 & QIII\_operation \\ -(1-k)V_1 & QIV\_operation \end{cases} \quad (1.7)$$

TABLE 1.1
Switches and Diodes' Status for Four-Quadrant Operation

Switch or Diode	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
S_1	Works	Idle	Idle	Works
D_1	Idle	Works	Works	Idle
S_2	Idle	Works	Works	Idle
D_2	Works	Idle	Idle	Works
S_3	Idle	Idle	On	Idle
D_3	Idle	Idle	Idle	On
S_4	On	Idle	Idle	Idle
D_4	Idle	On	Idle	Idle
Output	V_{2+}, I_{2+}	V_{2+}, I_{2-}	V_{2-}, I_{2-}	V_{2-}, I_{2+}

2.12.4 Four quadrant Chopper or Type E Chopper

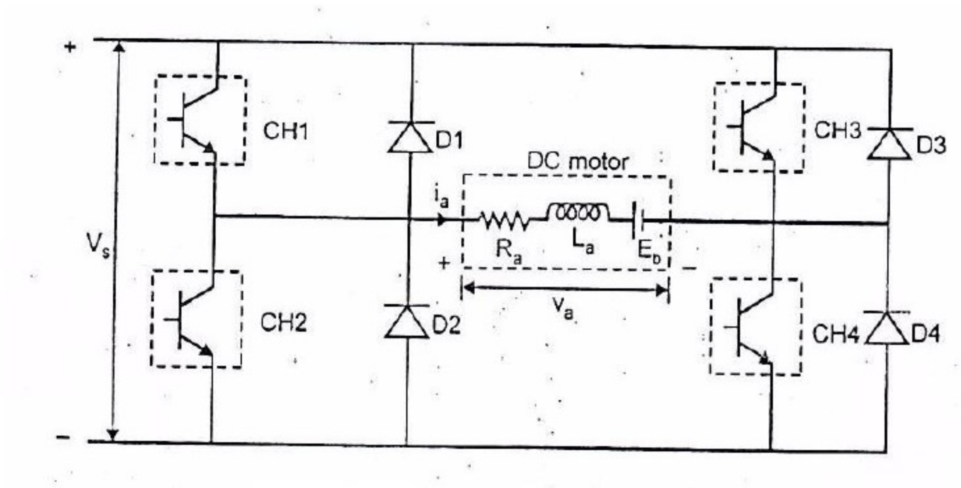


Fig (2.12.4) Four quadrant Chopper or Type E Chopper

Forward Motoring Mode

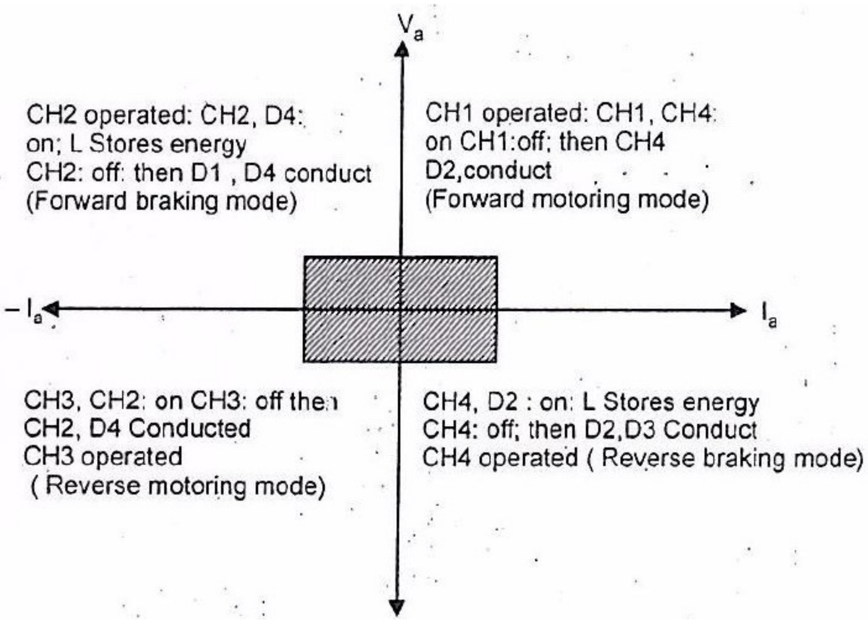
For first quadrant operation of figure CH4 is kept on, Cl-l3 is kept off and CH1 is operated. when CHI and Cl-I4 are on, load voltage is equal to supply voltage i,e, Va = Vs and load current ia begins to flow. Here both output voltage va and load current ia are positive giving first quadrant operation. When CH4 is turned of£ positive current freewheels through CH-4,D2 in this way, both output voltage va, load current ia can be controlled in the first quadrant. First quadrant operation gives the forward motoring mode.

Forward Braking Mode

Here CH2 is operated and CH1, CH3 and CH4 are kept off. With CH2 on, reverse (or negative) current flows through L, CH2, D4 and E. During the on time of CH2 the inductor L stores energy. When CH2 is turned off current is fedback to source through diodes D1, D4 note that there [E+L di/dt] is greater than the source voltage Vs. As the load voltage Va is positive and load current ia is negative, it indicates the second quadrant operation of chopper. Also power flows from load to source, second quadrant operation gives forward braking mode.

Reverse Motoring Mode

For third quadrant operation of figure, CHI is kept off, CH2 is kept on and CH3 is operated. Polarity of load emf E-must be reversed for this quadrant operation. With CH3 on, load gets connected to source Vs so that both output voltage Va and load current ia are negative. it gives third quadrant operation. lt is also known as reverse motoring mode. When CH3 is turned off, negative current freewheels through CH2, D4. ln this way, output voltage Va and load current ia can be controlled in the third quadrant.



Reverse Braking Mode

Here CH4 is operated and other devices are kept off.Load emf E must have its polarity reversed, it'is shown in figure . With CH4 on, positive current flows through CH4, D2, L and E. During the on time of CH4, the inductor L stores energy.

When CH4 is turned off; current is feedback to source through diodes D2, D3. Here load voltage is negative, but load current is positive leading to the chopper operation in the fourth quadrant.

Also power is flows from load to source. The fourth quadrant operation gives reverse braking mode.

2.13 Braking

In braking, the motor works as a generator developing a negative torque which oppose the motion. It is of three types

1. Regenerative braking
2. Plugging or Reverse voltage braking
3. Dynamic braking or Rheostatic braking

2.13.1Regenerative braking

In regenerative braking, generated energy is supplied to the source,for this to happen following condition should be satisfied

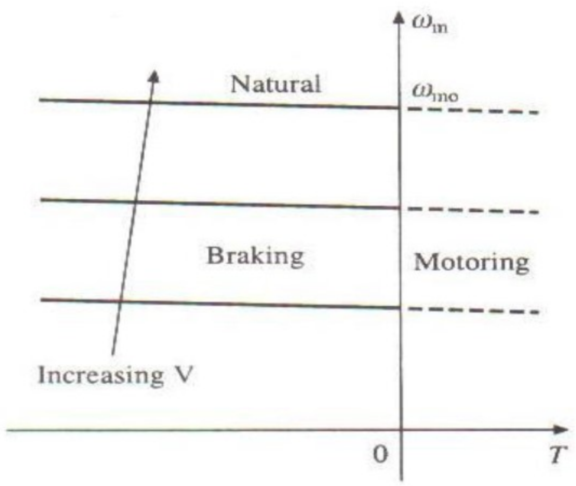
E > V and negative Ia

Field flux cannot be increased substantially beyond rated because of saturation, therefore according to equation ,for a source of fixed voltage of rated value regenerative braking is possible only for speeds higher than rated and with a variable voltage source it is also possible below rated speeds .

The speed –torque characteristics shown in fig. for a separately excited motor.

In series motor as speed increases, armature current, and therefore flux decreases

Condition of equation cannot be achieved .Thus regenerative braking is not possible



2.13.2 Plugging

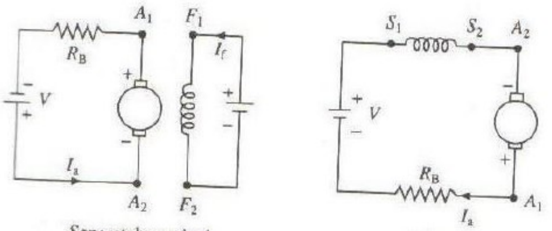
The supply voltage of a separately excited motor is reversed so that it assists the emf in forcing armature current in reverse direction .A resistance R_B is also connected in series with armature to limit the current.For plugging of a series motor armature is reversed.

A particular case of plugging for motor rotation in reverse direction arises .when a motor connected for forward motoring,is driven by an active load in the reverse direction.Here again back emf and applied voltage act in the same direction.However the direction of torque remains positive.

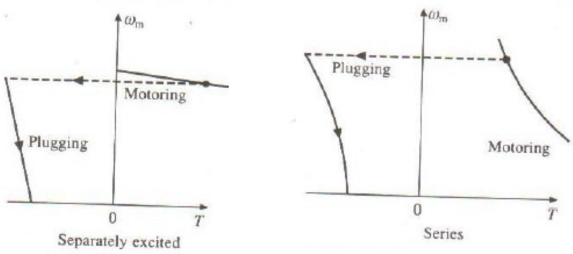
This type of situation arises in crane and the braking is then called counter – torque braking.

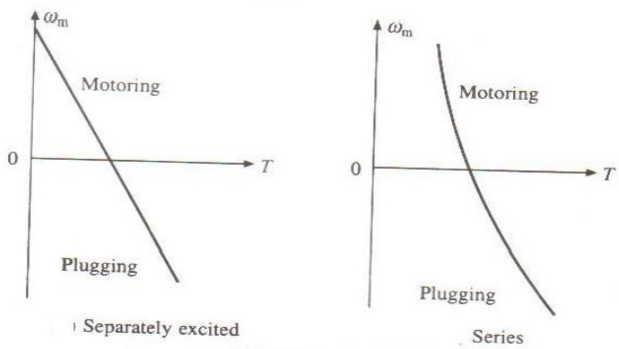
Plugging gives fast braking due to high average torque,even with one section of braking resistance RB.Since torque ia not zero speed,when used for stopping a load,the supply must be disconnected when close to zero speed.

Centifugal switches are employed to disconnect the supply.Plugging is highly inefficient because in addition to the generated power,the power supplied by the source is also wasted in resistances.



Plugging operation of dc motors





Counter-torque braking

2.13.3 Dynamic braking

In dynamic braking ,the motor is made to act as a generator,the armature is

disconnected from the supply ,but it continues to rotate and generate a voltage.The polarity of the generated voltage remains unchanged if the direction if field excitation is unaltered.

But if a resistance is connected across the coasting motor,the direction of the armature current is reversed ,because the armature represents a source of power rather than a load.

Thus a braking torque is developed ,exactly as in the generator,tending to oppose the motion.

The braking torque can be controlled by the field excitation and armature current.

3.3.3 Four-quadrant Chopper

The circuit of a four-quadrant chopper is shown in Fig. 3.15 in which the inductor L is assumed to be composed of the armature inductance and an external inductor.

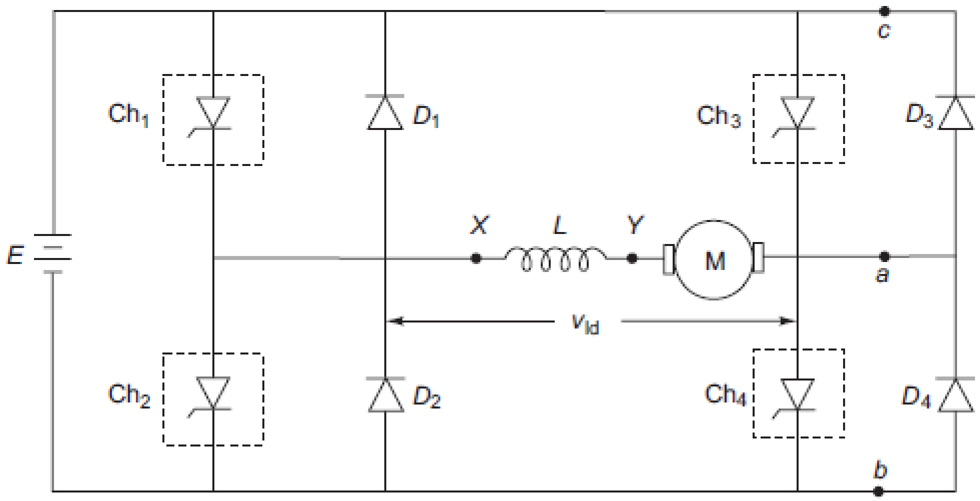


Fig. 3.15 Circuit of a four-quadrant chopper

Three methods of control are possible for the operation of this chopper as elaborated below.

Method 1 The circuit is operated as a two-quadrant chopper to obtain (a) first- and second-quadrant operation as well as (b) third- and fourth-quadrant operation.

Sequence 1 To obtain mode (a), Ch_4 is permanently kept on; terminals a and b are always kept shorted by ensuring conduction by either Ch_4 or D_4 and terminals a and c are always kept open. The choppers Ch_1 and Ch_2 are controlled as per the following four steps.

(a) If Ch_1 and Ch_4 are turned on at $t = 0$, the battery voltage E will be applied to the load circuit and current will flow from X to Y as shown in Fig. 3.16(a); this direction is the positive one. Thus the load voltage during this interval is kept at $+E$.

(b) When Ch_1 is turned off at τ_{ON}, the current due to the stored $(1/2)Li^2$ energy of the inductor L drives the current through D_2 and Ch_4 as shown in Fig. 3.16(b). Ch_2, which is turned on at τ_{ON}, does not conduct because it is shorted by D_2.

(c) Ch_2, which is on, conducts the current when it reverses, as shown in Fig. 3.16(c).

(d) Finally, when Ch_2 is turned off at τ, current flows through the path consisting of the negative of the battery, D_4, the motor, L, D_1, and the positive of the battery as shown in Fig. 3.16(d). If the machine were to be operated as a generator, this circuit facilitates regenerative braking. The zero crossing instants of the current waveform depend upon the values of E, E_b, L, and the armature resistance R_a of the motor. It is seen that Ch_1 does not conduct till i_{ld} becomes positive and Ch_2 does not conduct till i_{ld} flows in the negative direction. Also, D_4 conducts the reverse current and applies a reverse bias against Ch_4. The devices that conduct during each of the intervals are shown in Fig. 3.17(a), which gives the waveforms of this mode.

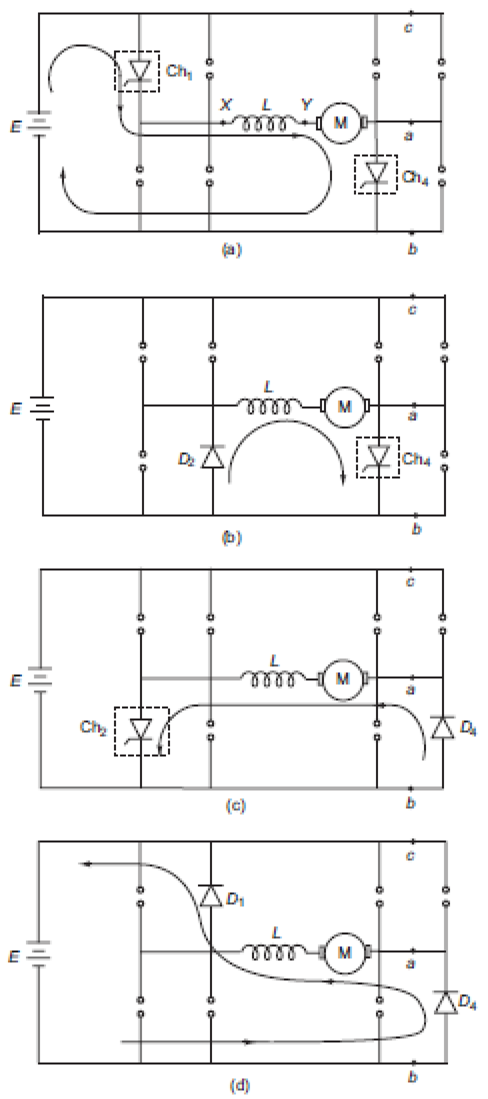


Fig. 3.16 Circuit conditions of a four-quadrant chopper with method 1: (a) Ch_1 and Ch_4 turned on; (b) Ch_1 turned off, Ch_4 remaining on; (c) Ch_2 turned on, Ch_4 shorted by D_4; (d) Ch_2 turned off, Ch_4, shorted by D_4

Sequence 2 For the circuit to provide third- and fourth-quadrant operation, Ch_3 is permanently kept on. Terminals *a* and *c* remain shorted due to conduction by either Ch_3 or D_3; terminals *a* and *b* remain open throughout. The relevant waveforms are shown in Fig. 3.17(b) (in the figure, I_B denotes the current through the battery).

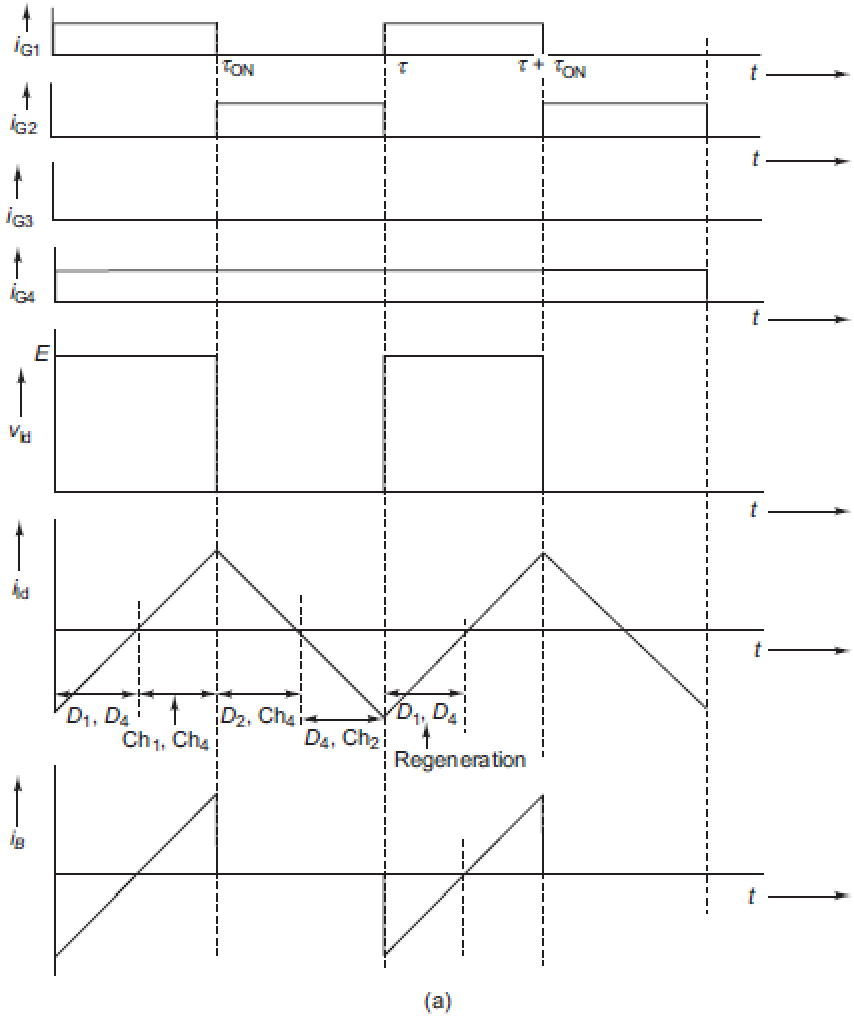


Fig. 3.17(a)

Fig. 3.17(a)

(a) Ch_2 is triggered on at $t = 0$ but starts conduction only when a reverse current flows through the path consisting of the positive of the battery, Ch_3, the motor, L, Ch_2, and the negative of the battery. A voltage equal to $-E$ is applied at the load terminals.

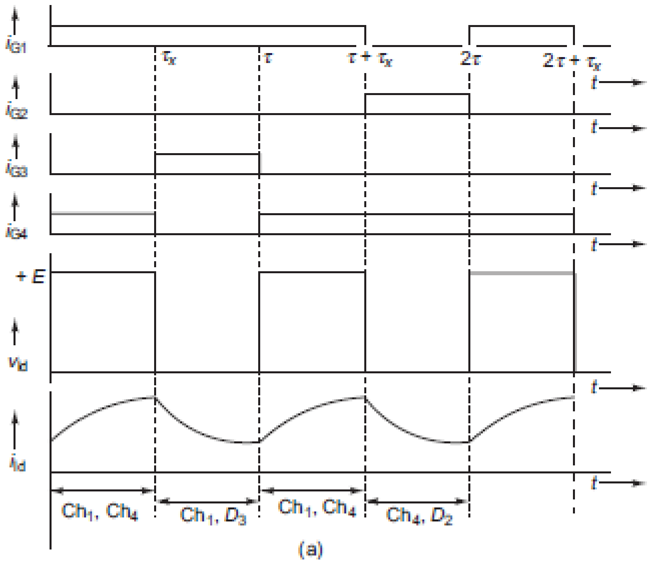
(b) When Ch_2 is turned off at τ_{ON}, the inductor continues to drive the current in the reverse direction through the path consisting of Ch_3, the motor, L, and D_1. The load voltage then becomes zero.

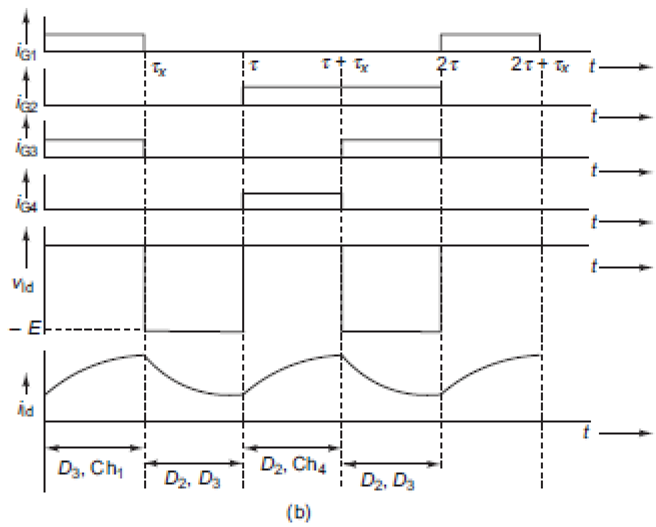
(c) Ch_1 is triggered at τ_{ON} but starts conduction only when the current flows in the positive direction, flowing through the closed circuit consisting of Ch_1, L, the motor, and D_3.

(d) When Ch_1 is turned off at τ, a negative battery voltage is applied to the load but positive current flows through the negative terminal of the battery, D_2, L, the motor, D_3, and back to the positive terminal of the battery.

It is seen that either Ch_1 or Ch_2 conducts current when i_{ld} becomes positive or negative, respectively. This is because, even though their control signals are present prior to the zero crossing of the load current, the conducting diodes D_1 and D_2 apply a reverse bias, respectively, across Ch_1 and Ch_2. The devices that conduct during each interval are given in Fig. 3.17(b).

This circuit suffers from the disadvantage that either Ch_3 or Ch_4 are kept on for a long time, which may lead to commutation problems. An important precaution to be taken is that the choppers Ch_1 and Ch_2 should not conduct simultaneously, as otherwise the source gets shorted through them. To ensure this, a small interval of time has to be provided between the turn-off of Ch_2 and the turn-on of Ch_1 and vice versa; this feature, however, limits the maximum chopper frequency.





Figs 3.18(a) and (b)

Method 2 In this method, the four-quadrant chopper provides first- and fourth-quadrant operation similar to method 2 of the two-quadrant type-B chopper. Thus the chopper pair Ch_1, Ch_4 and the diode pair D_2, D_3 conduct alternately; the other chopper pair is permanently kept off. Accordingly the waveforms will be identical to those of Figs 3.13(a) and (b), respectively. Likewise, for obtaining second- and third-quadrant operation, the chopper pair Ch_2, Ch_3 and the diode pair D_1, D_4 of Fig. 3.15 conduct in alternate intervals with the chopper pair Ch_1, Ch_4 always kept off. The waveforms in this case will be similar to those of Figs 3.13(a) and (b) except for the fact that the instantaneous current in both cases is always negative. Thus the operating point will be in either the second or the third quadrant.

Method 3 This method consists of operating the same combinations of chopper pairs, as in method 2, to provide four-quadrant operation. However, the chopper pairs are controlled in such a way that if one of them conducts during some interval, the other pair is off. The waveforms for the first, fourth, second, and third quadrants are given, respectively, in Figs 3.18(a), (b), (c), and (d).

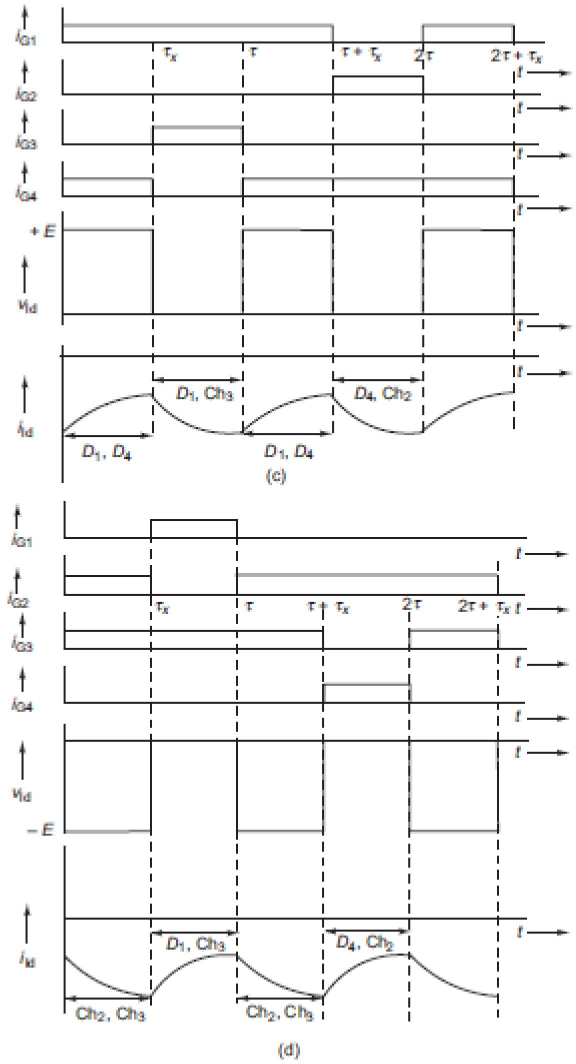


Fig. 3.18 Waveforms for four-quadrant chopper operation (method 3): (a) first-quadrant operation, (b) fourth-quadrant operation, (c) second-quadrant operation, (d) third-quadrant operation

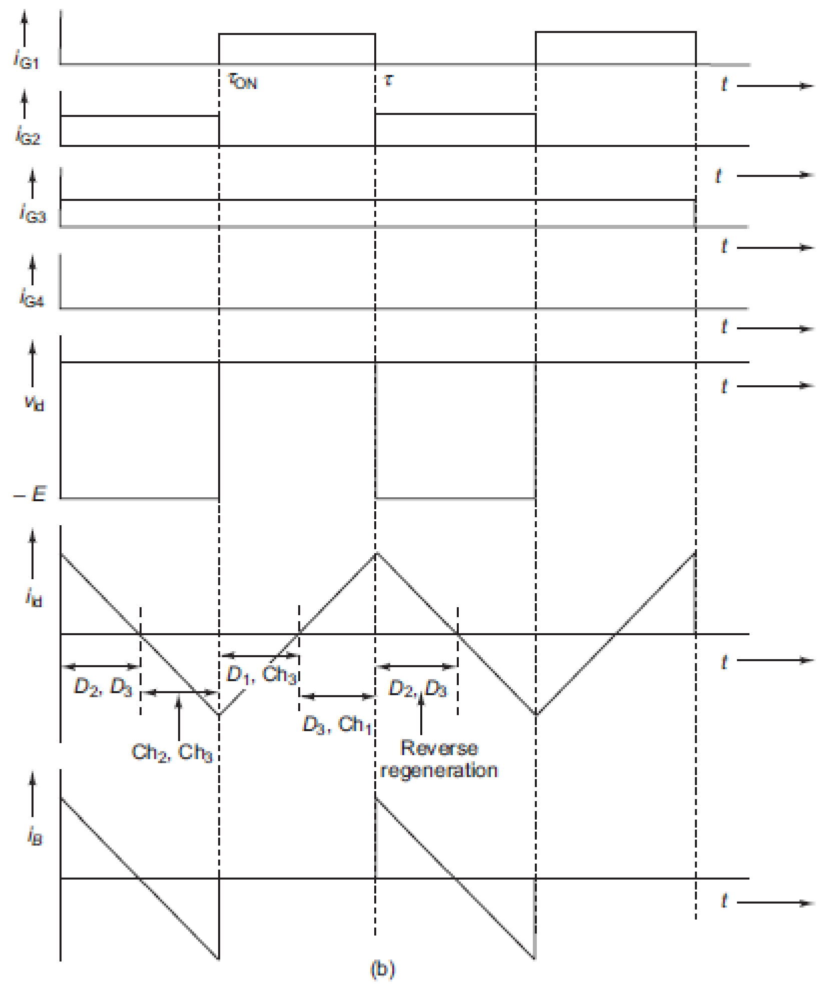


Fig. 3.17 Four-quadrant chopper: (a) waveforms for sequence 1—first- and second-quadrant operation, (b) waveforms for sequence 2—third- and fourth-quadrant operation

Example 4.23 The armature voltage of a separately excited dc motor is controlled by a one-quadrant chopper with chopping frequency of 200 pulses per second from a 300 V dc source. The motor runs at a speed of 800 rpm when the chopper's time ratio is 0.8. Assume that the armature circuit resistance and inductance are 0.08 Ω and 15 mH, respectively, and that the motor develops a torque of 2.72 N·m per ampere of armature current.

Find the mode of operation of the chopper, the output torque, and horsepower under the specified conditions.

Solution From the problem specifications at 800 rpm, using Eq. (4.201), we get,

$$E_c = (2.72)\frac{2\pi}{60}(800) = 227.9 \text{ V}$$

$$E_c = K_1 \phi_f \omega \tag{4.201}$$

$$\omega = \frac{t_{on}}{t_x}\frac{V_i}{K_1\phi_f} - \frac{R_a}{(K_1\phi_f)^2}\frac{T}{t_x}T_0 \qquad (4.202)$$

In the continuous mode of operation, with $t_{on} > t^*_{on}$, we have $t_x = T$. As a result Eq. (4.202) reduces to

$$\omega = \frac{(t_{on}/T)V_i - (R_a/K_1\phi_f)T_o}{K_1\phi_f)} \qquad (4.203)$$

The armature circuit time constant is obtained as

$$\tau = \frac{L_a}{R_a} = \frac{15 \times 10^{-3}}{0.08} = 187.5 \times 10^{-3}\ \text{s}$$

The chopping period is given by

$$T = \frac{1}{200} = 5 \times 10^{-3}\text{s}$$

We obtain the critical on-time using Eq. (4.197) as

$$\begin{aligned} t^*_{on} &= 187.5 \times 10^{-3}\, ln\left[1 + \frac{227.9}{300}(e^{5/187.5} - 1)\right] \\ &= 3.8 \times 10^{-3}\text{s} \end{aligned}$$

We know that $t_{on} = 0.8 \times 5 \times 10^{-3} = 4 \times 10^{-3}$. As a result, we conclude that the chopper output current is continuous.

To obtain the torque output, we use Eq. (4.203) rearranged as

$$T_o = \frac{K_1\phi_f}{R_a}\left(\frac{t_{on}}{T}V_i - K_1\phi_f\omega\right)$$

Thus we obtain

$$\begin{aligned} T_o &= \frac{2.72}{0.08}[0.8(300) - 227.9] \\ &= 411.4\,\mathrm{N \cdot m} \end{aligned}$$

The power output is obtained as

$$\begin{aligned} P_o &= (411.4)\frac{2\pi}{60}(800) = 34.5 \times 10^3\,\mathrm{W} \\ &= 46.2\,\mathrm{hp} \end{aligned}$$

To illustrate the principle of field control, we have the following example.

Example 4.24 Assume for the motor of Example 4.23 that field chopper control is employed to run the motor at a speed of 1500 rpm while delivering the same power output as obtained at 800 rpm and drawing the same armature current.

Solution Although we can use Eq. (4.205), we use basic formulas instead,

$$E_c = \frac{P_a}{I_a} = \frac{34.5 \times 10^3}{151.3} = 227.9\,\text{V}$$

This is the same back EMF. Recall that

$$E_c = K_1 \phi_f \omega$$

Thus the required field flux is obtained as

$$\phi_{f_n} = \phi_{f_0} \frac{\omega_0}{\omega_n} = \frac{8}{15} \phi_{f_0}$$

where the subscript n denotes the present case, and the subscript 0 denotes the field flux for Example 4.23. Assume that ϕ_{f_0} corresponds to full applied field flux; then

$$\frac{\phi_{f_0}}{\phi_{f_n}} = \frac{V_i}{V_o} = \frac{15}{8}$$

The required chopped output voltage is V_o. Now we have

$$\frac{V_o}{V_i} = \frac{t_{\text{on}}}{T}$$

Thus

$$\frac{t_{\text{on}}}{T} = \frac{8}{15}$$

Assuming that $T = 5 \times 10^{-3}$ s, we get

$$t_{\text{on}} = 2.67 \times 10^{-3}\,\text{s}$$

Example 4.1

The speed of a separately excited dc motor is controlled by a chopper as shown in Fig. 4.8*a*. The dc supply voltage is 120 V, armature circuit resistance is $R_a = 0.5\ \Omega$, armature circuit inductance is $L_a = 20$ mH, and motor constant is $K_a\Phi = 0.05$ V/rpm. The motor drives a constant-torque load requiring an average armature current of 20 A. Assume that motor current is continuous.

Determine:

1 the range of speed control;
2 the range of the duty cycle α.

Solution

Minimum speed is zero at which $E_g = 0$. Therefore from equation 2.17

$$E_a = I_a R_a = 20 \times 0.5 = 10\,\text{V}$$

From equation 4.1

$$10 = 120\alpha$$

$$\alpha = \frac{1}{12}$$

Maximum speed corresponds to $\alpha = 1$ at which $E_a = E = 120$ V.

Therefore

$$\begin{aligned} E_g &= E_a - I_a R_a \\ &= 120 - (20 \times 0.5) \\ &= 110\,\text{V} \end{aligned}$$

From equation 2.13

$$N = \frac{E_g}{K_a\Phi} = \frac{110}{0.05} = 2200\,\text{rpm}$$

The range of speed is $0 < N < 2200$ rpm, and the range of the duty cycle is $1/12 < \alpha < 1$.

12. A 300-V, 100-A, separately excited dc motor operating at 600 rpm has an armature resistance and inductance of 0.25 Ω and 16 mH, respectively. It is controlled by a four-quadrant chopper with a chopper frequency of 1 kHz. (a) If the motor is to operate in the second quadrant at 4/5 times the rated current, at 450 rpm, calculate the duty ratio. (b) Compute the duty ratio if the motor is working in the third quadrant at 500 rpm and at 60% of the rated torque.

Solution

(a) $E_b - I_a R_a = 300 - 100 \times 0.25 = 275$ V. Back emf constant $k = E_b/N = 275/600 = 0.458$. Operation in the second quadrant implies that the motor works as a generator. Hence the motor terminal voltage V_a is written as

$$V_a = E(1 - \delta)$$

New current

$$I_a = \frac{4}{5} \times 100 = 80 \text{ A}$$

Hence,

$$E_b = V_a + I_a R_a$$
$$kN = E(1 - \delta) + I_a R_a$$

Substitution of values gives

$$0.458 \times 450 = 300(1 - \delta) + 80 \times 0.25$$

This yields $\delta = 0.38$.

(b) In the third quadrant, the machine works in the motoring mode but with reverse voltage and reverse current. The voltage equation relevant in this case is

$$E_b = V_a - I_a R_a$$

where

$$E_b = kN = 0.458 \times 500$$

$$V_a = E\delta = 300\delta$$

and

$$I_a = 0.6 \times 100 = 60 \text{ A}$$

By substituting numerical values, the equation becomes

$$0.458 \times 500 = 300 \times \delta - 60 \times 0.25$$

This gives $\delta = 0.813$.

6.5. THE FOUR-QUADRANT CHOPPER

A d.c. brush motor with separate excitation is fed through a four-quadrant chopper (Table 6.1e). Show the waveforms of voltage and current in the third and fourth quadrants.

Solution:

The basic circuit of a four-quadrant chopper is shown in Figure 6.9.

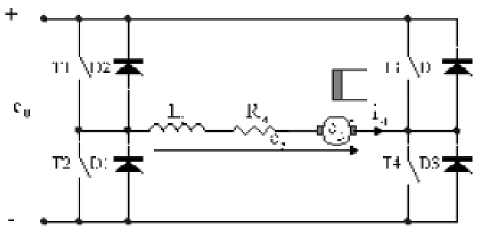


Figure 6.9. D.c. brush motor fed through a four-quadrant chopper

If T_4 is on all the time, T_1-D_1 and T2-D2 provide first- and (respectively) second-quadrant operations as shown in previous paragraphs. With T_2 on all the time and T_3-D_3 and, respectively, T_4-D_4 the third- and fourth-quadrant operations is obtained (Figure 6.10). So, in fact, we have 2 two-quadrant choppers acting in turns.

However, only 2 out of 4 main switches are turned on and off with the frequency f_{ch} while the third main switch is kept on all the time and the fourth one is off all the time.

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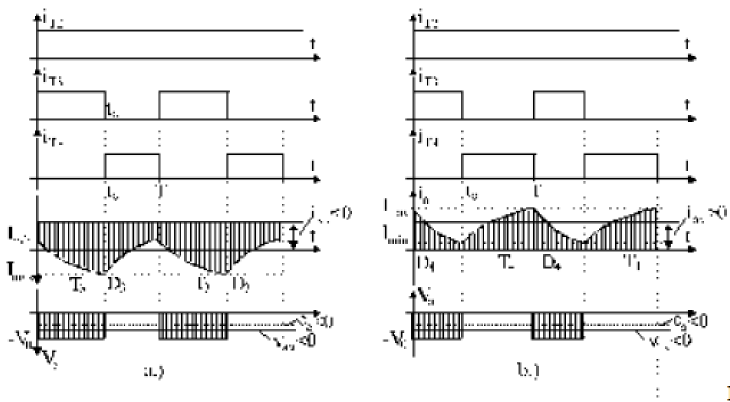


Figure 6.10. Four-quadrant chopper supplying a d.c. brush motor

a.) Third quadrant: iav<0, Vav<0; b.) Fourth quadrant: iav>0, Vav<0.

Four-quadrant operation is required for fast response reversible variable speed drives.

As expected, discontinuous current mode is also possible but it should be avoided by increasing the switching frequency f_{ch} or adding an inductance in series with the motor.

Let us assume that:

A d.c. brush motor, fed through a four-quadrant chopper, works as a motor in the third quadrant (reverse motion). The main data are $V_0 = 120V$, $R_a = 0.5\Omega$, $L_a = 2.5mH$, rated current $I_{an} = 20A$; rated speed $n_n = 3000$ rpm; separate excitation.

a. Calculate the rated e.m.f., e_g, and rated electromagnetic torque, T_e.

b. For $n = -1200$ rpm and rated average current ($i_{av} = -I_{an}$) determine the average voltage V_{av}, $t_c / T = \alpha_{on}$, and maximum and minimum values of motor current I_{max} and I_{min} for 1kHz switching frequency.

Solution:

a. The motor voltage equation for steady state is:

$$V_{av} = R_a i_a + e_g \tag{6.54}$$

for rated values $V_{av} = V_0 = 120V$, $i_a = i_{an} = 20A$, thus

$$e_{gn} = K_a \lambda_f n_n = V_{av} - R_a i_a = 120 - 20 \cdot 0.5 = 110 \text{ V} \tag{6.55}$$

$$K_a \lambda_f = \frac{e_{gn}}{n_n} = \frac{110}{50} = 2.2 \text{ Wb} \tag{6.56}$$

b. The motor equation in the third quadrant is

$$V_{av} = R_a i_a + e_g = 0.5 \cdot (-20) + 2.2 \cdot (-20) = -54 \text{ V}, \tag{6.57}$$

the conducting time t_c for T_3 (Figure 6.10a) is

$$\frac{t_c}{T} = \frac{V_{av}}{-V_0} = \frac{-54}{-120} = 0.45 \tag{6.58}$$

$$t_c = T \cdot 0.45 = \frac{1}{f_{ch}} \cdot 0.45 = \frac{1}{10^3} \cdot 0.45 = 0.45 \cdot 10^{-3} \text{ s} \tag{6.59}$$

Form ((6.40)-(6.41)) the motor current variation (Figure 6.10a) is described by

$$i_a = \frac{V_0' - e_g}{R_a} + A \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t \le t_c \tag{6.60}$$

$$i_a' = -\frac{e_g}{R_a} + A' \cdot e^{-(t - t_c)\frac{R_a}{L_a}}; \quad t_c < t \le T \tag{6.61}$$

The current continuity condition ($i_a(t_c) = i_a'(t_c)$) provides

$$t_c = -\frac{L_a}{R_a}\cdot \ln\left[\left(A' - \frac{V_0'}{R_a}\right)/A\right] \tag{6.62}$$

The second condition is obtained from the average current expression

$$i_{av} = \frac{1}{T}\left[\int_0^{t_c} i_a dt + \int_{t_c}^{T} i_a' dt\right] =$$

$$\frac{1}{T}\left\{\frac{V_0' - e_g}{R_a}t_c - \frac{e_g}{R_a}(T - t_c) + \frac{L_a}{R_a}\left[\left(1 - e^{-t_c\frac{R_a}{L_a}}\right)A + A'\left(1 - e^{-(T-t_c)\frac{R_a}{L_a}}\right)\right]\right\} \tag{6.63}$$

From (6.62) and (6.63) we obtain:

$$\left(A' - \frac{V_0'}{R_a}\right)/A = e^{-t_c\frac{R_a}{L_a}};$$

$$V_0' = -V_0;\quad e_g = K_e\lambda_p n = 2.2\cdot(-20) = -44\ \mathrm{V}$$

$$\left(A' + \frac{(-120)}{0.5}\right)/A = e^{-0.45\cdot 10^{-3}\frac{0.5}{2.5\cdot 10^{-3}}} = 0.914 \tag{6.64}$$

$$-20 = 10^3\left\{\frac{-120-(-44)}{0.5}0.45\cdot 10^{-3} - \frac{(-44)}{0.5}0.55\cdot 10^{-3}\right.$$
$$\left.+\frac{2.5\cdot 10^{-3}}{0.5}\left[\left(1 - e^{-0.45\cdot 10^{-3}\frac{0.5}{2.5\cdot 10^{-3}}}\right)A + A'\left(1 - e^{-0.55\cdot 10^{-3}\frac{0.5}{2.5\cdot 10^{-3}}}\right)\right]\right\} \tag{6.65}$$

$$-20 = -20 + 0.43A + 0.5205A' \tag{6.66}$$

$$0.43A + 0.5205A' = 0 \tag{6.67}$$

$$A' + 240 = 0.914A \tag{6.68}$$

$$A = 137.62;\ A' = -113.92 \tag{6.69}$$

Now we may calculate $I_{min} = i_a(0)$

$$I_{min} = A + \frac{V_0' - e_g}{R_a} = 137.92 + \frac{-120-(-44)}{0.5} = -15.08\ \mathrm{A} \tag{6.70}$$

Also $I_{max} = i_a'(t_c)$

$$I_{max} = A' - \frac{e_g}{R_a} = -113.92 + \frac{-(-44)}{0.5} = -25.92\ \mathrm{A} \tag{6.71}$$

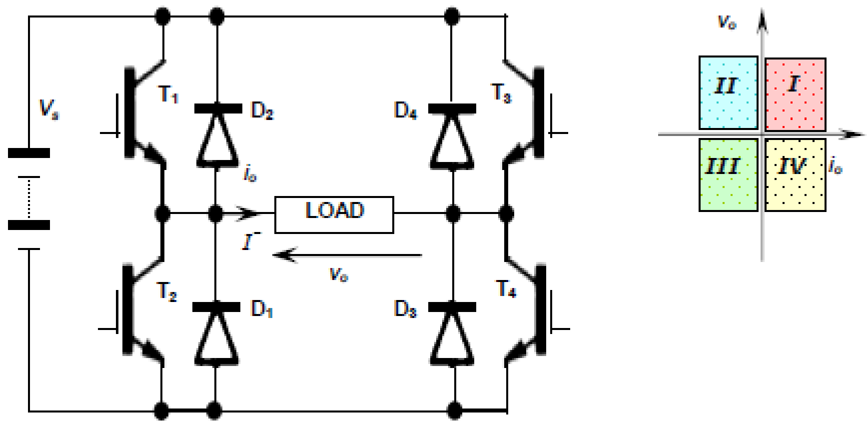
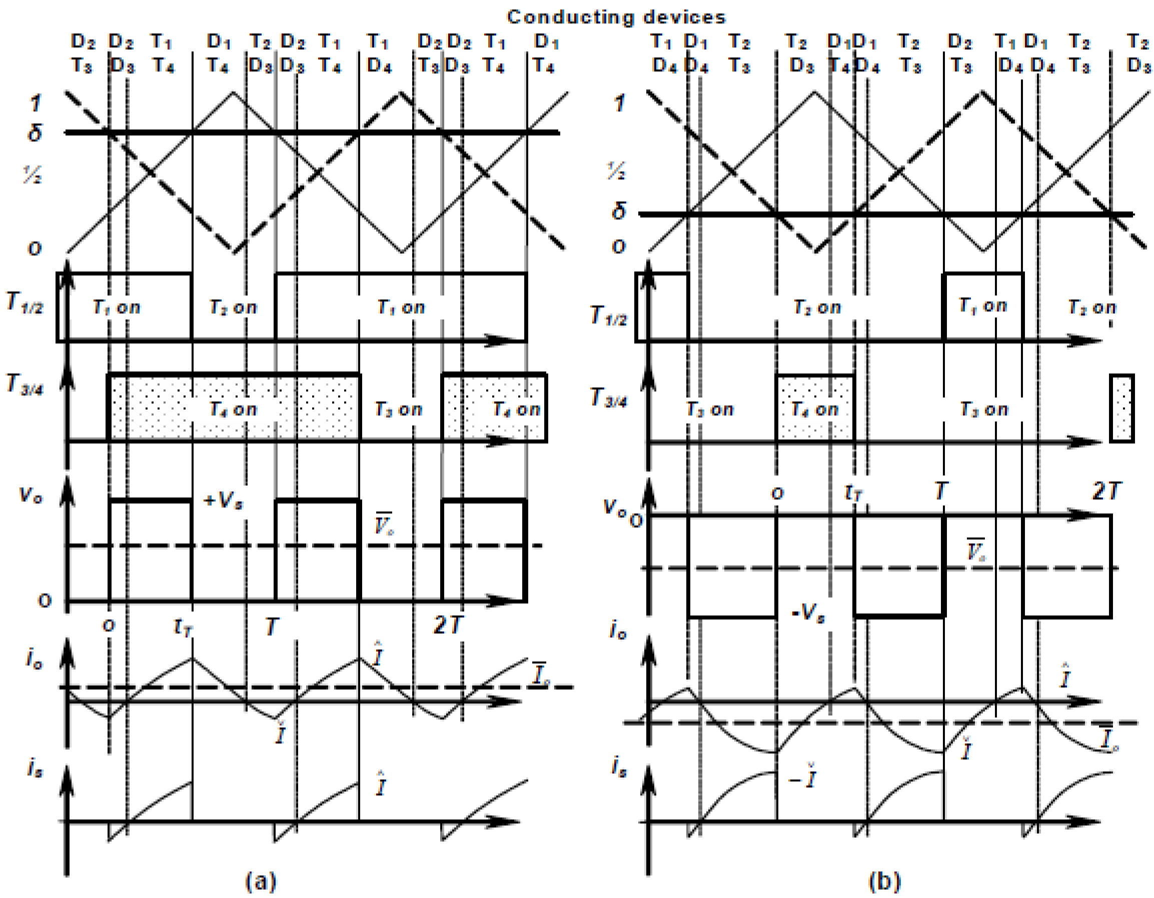


Figure 14.21. *Four-quadrant dc chopper circuit, showing first quadrant i_o and v_o references.*



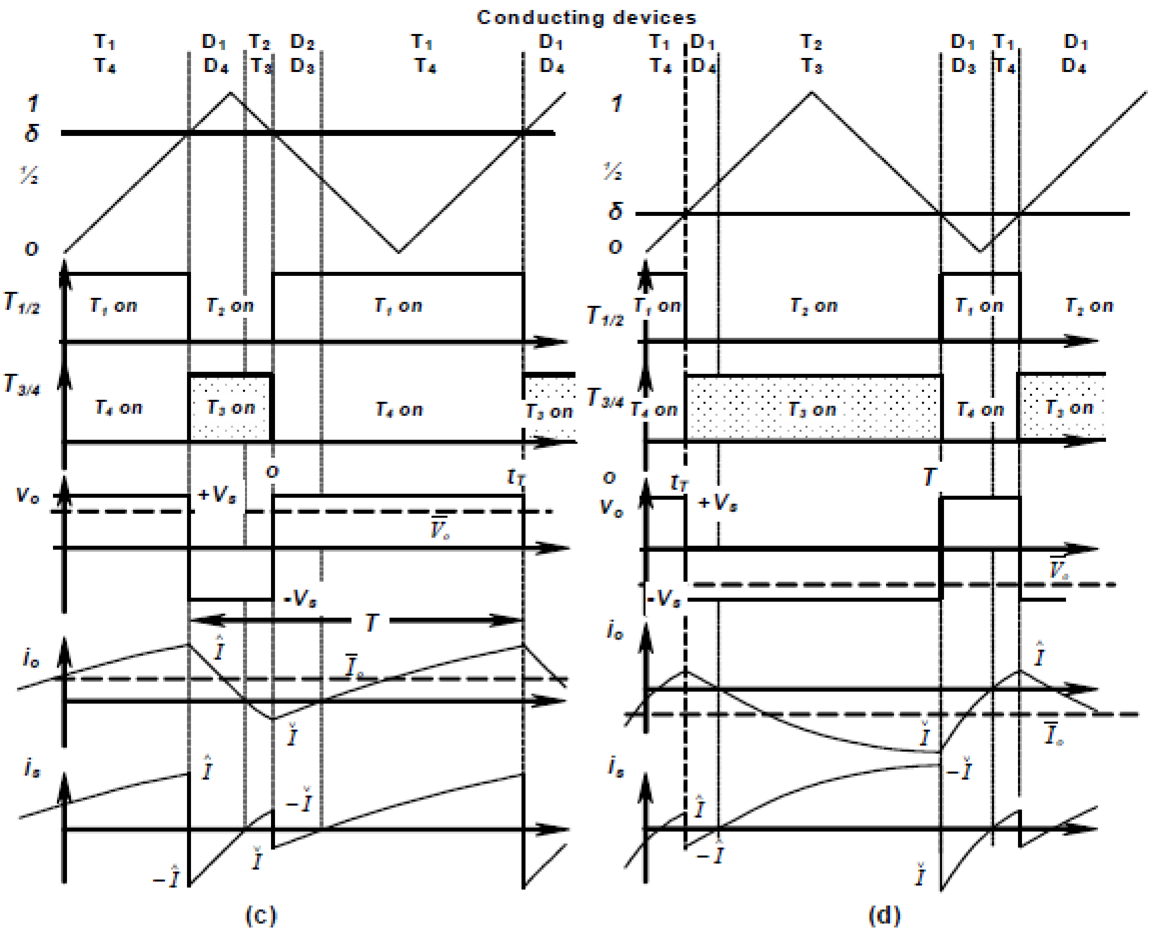


Figure 14.22. *Four-quadrant dc chopper circuit waveforms: multilevel (three-level) output voltage (a) with $\overline{V}_o > 0$ and $\overline{I}_o > 0$; (b) with $\overline{V}_o < 0$ and $\overline{I}_o < 0$; bipolar (two-level) output voltage (c) with $\overline{V}_o > 0$ and $\overline{I}_o > 0$; (d) with $\overline{V}_o < 0$ and $\overline{I}_o < 0$.*

Example 14.6: ***Asymmetrical, half H-bridge, dc chopper***

The asymmetrical half H-bridge, dc-to-dc chopper in figure 14.18 feeds an inductive load of 10 Ω resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc voltage source. The chopper output current is controlled in a hysteresis mode within a current band between limits 5A and 10A. Determine the period of the current shape shown in the figure 14.20:

i. when only $\pm V_s$ loops are used and
ii. when a zero volt loop is used to maintain tracking within the 5A band.

In each case calculate the switching frequency if the current were to be maintained within the hysteresis band for a prolonged period.

How do the on-state losses compare between the two control approaches?

Solution

The main circuit and operating parameters are

- E = 55V and V_s = 340V
- load time constant $\tau = L/R$ = 0.05mH/10Ω = 5ms
- I^+ = 10A and I^- = 5A

Examination of the figure 14.20 shows that only one period of the cycle differs, namely the second period, t_2, where the current is required to fall to the lower hysteresis band level, -5A. The period of the other three regions (t_1, t_3, and t_4) are common and independent of the period of the second region, t_2.

t_1: The first period, the initial rise time, $t^+ = t_1$ is given by equation (14.102), where I^+=10A and $\check{I}$ = 0A.

$$t^+ = \tau\ \ell n\left(\frac{V_s - E - \check{I}R}{V_s - E - I^+R}\right)$$

that is $$t_1 = 5\text{ms} \times \ell n\left(\frac{340\text{V - 55V - 0A}\times 10\Omega}{340\text{V - 55V - 10A}\times 10\Omega}\right) = 2.16\text{ms}$$

t_3: In the third period, the current rises from the lower hysteresis band limit of 5A to the upper band limit 10A. The duration of the current increase is given by equation (14.102) again, but with $\check{I} = I^-$ = 5A.

$$t^+ = \tau\ \ell n\left(\frac{V_s - E - \check{I}R}{V_s - E - I^+R}\right)$$

that is $$t_3 = 5\text{ms} \times \ell n\left(\frac{340\text{V - 55V - 5A}\times 10\Omega}{340\text{V - 55V - 10A}\times 10\Omega}\right) = 1.20\text{ms}$$

t_4: The fourth and final period is a negative voltage loop where the current falls from the upper band limit of 10A to I^- which equals zero. From equation (14.106) with $\hat{I} = I^+$=10A and I^- = 0A

$$t^- = \tau \,\ell n\left(\frac{V_s + E + \hat{I}R}{V_s + E + I^- R}\right)$$

that is $$t_4 = 5\text{ms} \times \ell n\left(\frac{340\text{V} + 55\text{V} + 10\text{A}\times 10\Omega}{340\text{V} + 55\text{V} + 0\text{A}\times 10\Omega}\right) = 1.13\text{ms}$$

The current pulse period is given by

$$\begin{aligned} T_p &= t_1 + t_2 + t_3 + t_4 \\ &= 2.16\text{ms} + t_2 + 1.20\text{ms} + 1.13\text{ms} \\ &= 4.49\text{ms} + t_2 \end{aligned}$$

i. **t_2:** When only $-V_s$ paths are used to decrease the current, the time t_2 is given by equation (14.106), with I^- =5A and $\hat{I}$ =10A,

$$t^- = \tau \,\ell n\left(\frac{V_s + E + \hat{I}R}{V_s + E + I^- R}\right)$$

that is $$t_2 = 5\text{ms} \times \ell n\left(\frac{340\text{V} + 55\text{V} + 10\text{A}\times 10\Omega}{340\text{V} + 55\text{V} + 5\text{A}\times 10\Omega}\right) = 0.53\text{ms}$$

The total period, T_p, of the chopped current pulse when a 0V loop is not used, is

$$\begin{aligned} T_p &= t_1 + t_2 + t_3 + t_4 \\ &= 2.16\text{ms} + 0.53\text{ms} + 1.20\text{ms} + 1.13\text{ms} = 5.02\text{ms} \end{aligned}$$

ii. **t_2:** When a zero voltage loop is used to maintain the current within the hysteresis band, the current decays slowly, and the period time t_2 is given by equation (14.104), with I^- = 5A and $\hat{I}$ =10A,

$$t^o = \tau \,\ell n\left(\frac{E + \hat{I}R}{E + I^- R}\right)$$

that is $$t_2 = 5\text{ms} \times \ell n\left(\frac{55\text{V} + 10\text{A}\times 10\Omega}{55\text{V} + 5\text{A}\times 10\Omega}\right) = 1.95\text{ms}$$

The total period, T_p, of the chopped current pulse when a 0V loop is used, is

$$\begin{aligned} T_p &= t_1 + t_2 + t_3 + t_4 \\ &= 2.16\text{ms} + 1.95\text{ms} + 1.20\text{ms} + 1.13\text{ms} = 6.44\text{ms} \end{aligned}$$

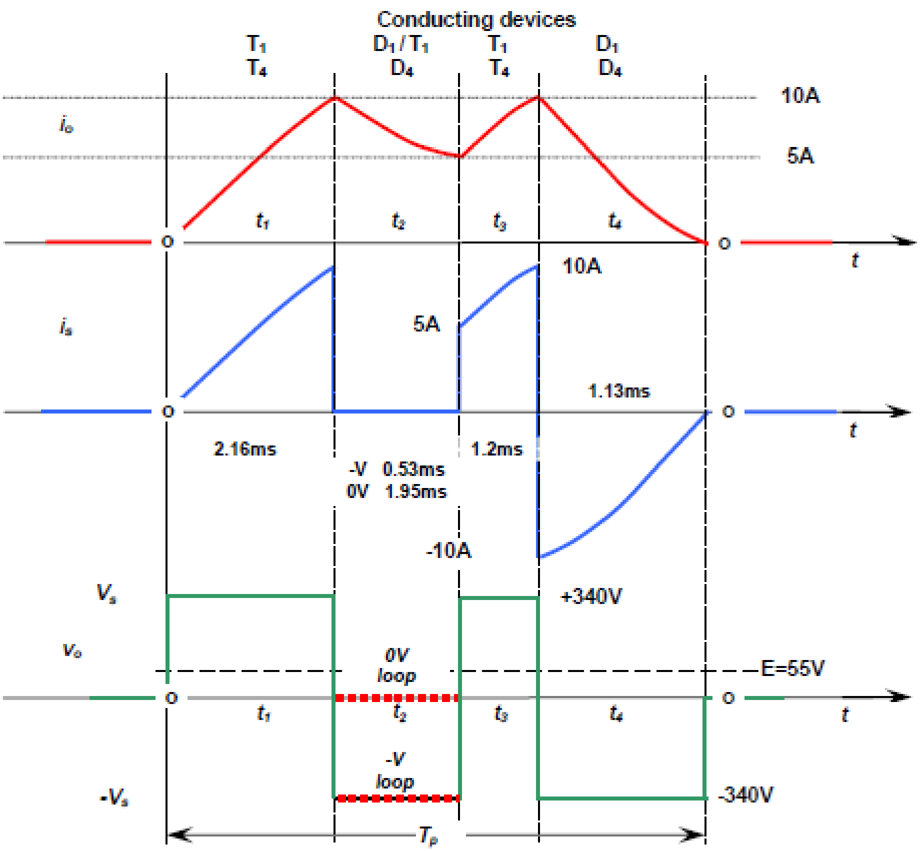


Figure 14.20. Example 14.6. *Circuit waveforms.*

The current falls significantly faster within the hysteresis band if negative voltage loops are employed rather that zero voltage loops, 0.53ms versus 1.95ms.
The switching frequency within the current bounds has a period $t_2 + t_3$, and each case is summarized in the following table. For longer current chopping, t_2 and t_3 dominate the switching frequency.
Using zero voltage current loops (alternated) reduces the switching frequency of the H-bridge switches by a factor of over three, for a given peak-to-peak ripple current.
If the on-state voltage drop of the switches and the diodes are similar for the same current level, then the on-state losses are similar, and evenly distributed for both control methods. The on-state losses are similar because each of the three states always involves the same current variation flowing through two semiconductors. The principal difference is in the significant increase in switching losses when only ±*V* loops are used (1:3.42).

Table Example 14.6. Switching losses.

Voltage loops	$t_2 + t_3$	Current ripple frequency	Switch frequency	Switch loss ratio
±V	0.53ms+1.20ms =1.73ms	578Hz	578Hz	$\frac{578}{169} = 3.42$
+V and zero	1.95ms+1.20ms = 3.15ms	317Hz	169Hz	1

♣

Example 14.7: ***Four-quadrant dc chopper***

The H-bridge, dc-to-dc chopper in figure 14.21 feeds an inductive load of 10 Ω resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated with a 200Hz multilevel carrier as in figure 14.22 a and b, with a modulation depth of δ = ¼, determine:

i. the average output voltage and switch T_1 on-time
ii. the rms output voltage and ac ripple voltage, hence voltage ripple and form factors
iii. the average output current, hence quadrant of operation
iv. the electromagnetic power being extracted from the back emf *E*.

If the mean load current is to be halved, what is
v. the modulation depth, δ, requirement
vi. the average output voltage and the corresponding switch T_1 on-time
vii. the electromagnetic power being extracted from the back emf *E*?

Solution

The main circuit and operating parameters are

- modulation depth δ = ¼
- period $T_{carrier} = 1/f_{carrier}$ = 1/200Hz = 5ms
- E = 55V and V_s = 340V dc
- load time constant $\tau = L/R$ = 0.05mH/10Ω = 5ms

i. The average output voltage is given by equation (14.116), and for δ < ½,

$$\overline{V}_o = \left(\frac{t_T}{T} - 1\right)V_s = (2\delta - 1)V_s$$
$$= 340\text{V}\times(2\times¼ - 1) = -170\text{V}$$

where

$$t_T = 2\delta T = 2\times¼\times(½\times5\text{ms}) = 1.25\text{ms}$$

Figure 14.22 reveals that the carrier frequency is half the switching frequency, thus the 5ms in the above equation has been halved. The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 3.75ms.

ii. The rms load voltage, from equation (14.120), is

$$V_{rms} = \sqrt{1-2\delta}\ V_s$$
$$= 340\text{V}\times\sqrt{1 - 2\times¼} = 240\text{V rms}$$

From equation (14.121), the output ac ripple voltage, hence voltage ripple factor, are

$$V_r = \sqrt{2}\,V_s\sqrt{\delta(1-2\delta)}$$
$$= \sqrt{2}\times340\text{V}\times\sqrt{¼\times(1 - 2\times¼)} = 170\text{V ac}$$

$$RF = \frac{V_r}{\overline{V}_o} = \frac{170\text{V}}{|-170\text{V}|} = 1 \qquad FF = \sqrt{RF^2+1} = \sqrt{2} = 1.41$$

iii. The average output current is given by equation (14.119)

$$\overline{I}_o = \frac{\overline{V}_o - E}{R} = \frac{(2\delta-1)V_s - E}{R}$$
$$= \frac{340\text{V}\times(2\times¼ - 1) - 55\text{V}}{10\Omega} = -22.5\text{A}$$

Since both the average output current and voltage are negative (-170V and -22.5A) the chopper with a modulation depth of δ = ¼, is operating in the third quadrant.

iv. The electromagnetic power developed by the back emf *E* is given by

$$P_E = E\overline{I}_o = 55\text{V}\times(\text{-}22.5\text{A}) = \text{-}1237.5\text{W}$$

v. The average output current is given by

$$\overline{I}_o = {(\overline{V}_o - E)}\big/{R} = {((2\delta-1)V_s - E)}\big/{R}$$

when the mean current is -11.25A, δ = 0.415, as derived in part vi.

vi. Then, if the average current is halved to -11.25A

$$\overline{V}_o = E + \overline{I}_o R$$
$$= 55\text{V} - 11.25\text{A}\times 10\Omega = \text{-}57.5\text{V}$$

The average output voltage rearranged in terms of the modulation depth δ gives

$$\delta = \tfrac{1}{2}\left(1 + \frac{\overline{V}_o}{V_s}\right)$$
$$= \tfrac{1}{2}\times\left(1 + \frac{\text{-}57.5\text{V}}{340\text{V}}\right) = 0.415$$

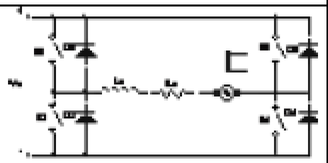
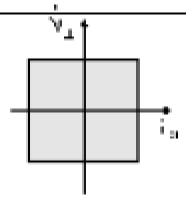
The switch on-time when $\delta < \tfrac{1}{2}$ is given by

$$t_T = 2\delta T = 2\times 0.415\times(\tfrac{1}{2}\times 5\text{ms}) = 2.07\text{ms}$$

From figure 14.22b both T_1 and T_4 are turned on for 2.07ms, although, from table 14.3B, for negative load current, $\overline{I}_o = $-11.25A, the parallel connected freewheel diodes D_2 and D_3 conduct alternately, rather than the switches (assuming $\hat{I}_o < 0$). The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 2.93ms.

vii. The electromagnetic power developed by the back emf *E* is halved and is given by

$$P_E = E\overline{I}_o = 55\text{V}\times(\text{-}11.25\text{A}) = \text{-}618.75\text{W}$$

Four quadrant chopper			S4 on & S3 off S1 & S2 operated Va>0 ia - reversible S2 on & S1 off S3 & S4 operated Va<0 ia - reversible

Example 4.5

The motor of example 4.3 is fed by a four-quadrant chopper controlled by method III. The source voltage is 230 V and the frequency of operation is 400 Hz.

1. If the motor operation is required in the second quadrant at the rated torque and 300 rpm, calculate the duty ratio.
2. What should be the value of the duty ratio if the motor is working in the third quadrant at 400 rpm and half of the rated torque?

Solution: At the rated conditions of operation

$$E_r = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. Equation (4.39), which is applicable to method III is reproduced here:

$$I_a = \frac{2V(\delta - 0.5) - E}{R_a} \tag{4.39}$$

The motor is working in the second quadrant, therefore,

$$I_a = -90 \text{ A}$$

$$E = \frac{300}{500} \times E_r = \frac{300}{500} \times 219.7 = 131.8 \text{ V}$$

Substituting in equation (4.39), gives

$$-90 = \frac{2 \times 230(\delta - 0.5) - 131.8}{0.115}$$

or

$$\delta = 0.5 + \frac{121}{460} = .76 .$$

2. At half the rated torque and in the third quadrant

$$I_a = -45 \text{ A}$$

$$E = -\frac{400}{500} \times 219.7 = -175.7 \text{ V}$$

Substituting in equation (4.39), gives

$$-45 = \frac{2 \times 230(\delta - 0.5) + 175.7}{0.115}$$

or

$$\delta = 0.5 - \frac{181}{460} = 0.11 .$$

4.3 FOUR-QUADRANT CHOPPER CIRCUIT

A four-quadrant chopper with transistor switches is shown in Figure 4.2. Each transistor has a freewheeling diode across it and a snubber circuit to limit the rate of rise of the voltage. The snubber circuit is not shown in the figure.

The load consists of a resistance, an inductance, and an induced emf. The source is dc, and a capacitor is connected across it to maintain a constant voltage. The base drive circuits of the transistors are isolated, and they reproduce and amplify the control signals at the output. For the sake of simplicity, it is assumed that the switches are ideal and hence, the base drive signals can be used to draw the load voltage.

First-quadrant operation corresponds to a positive output voltage and current. This is obtained by triggering T_1 and T_2 together, as is shown in Figure 4.3; then the load voltage is equal to the source voltage. To obtain zero load voltage, either T_1 or T_2 can be turned off. Assume that T_1 is turned off; then the current will decrease in the power switch and inductance. As the current tries to decrease in the inductance, it will have a voltage induced across it in proportion to the rate of fall of current with a polarity opposite to the load-induced emf, thus forward-biasing diode D_4. D_4 provides the path for armature current continuity during this time. Because of this, the circuit configuration changes as shown in Figure 4.4. The load is short-circuited, reducing its voltage to zero. The current and voltage waveforms for continuous and

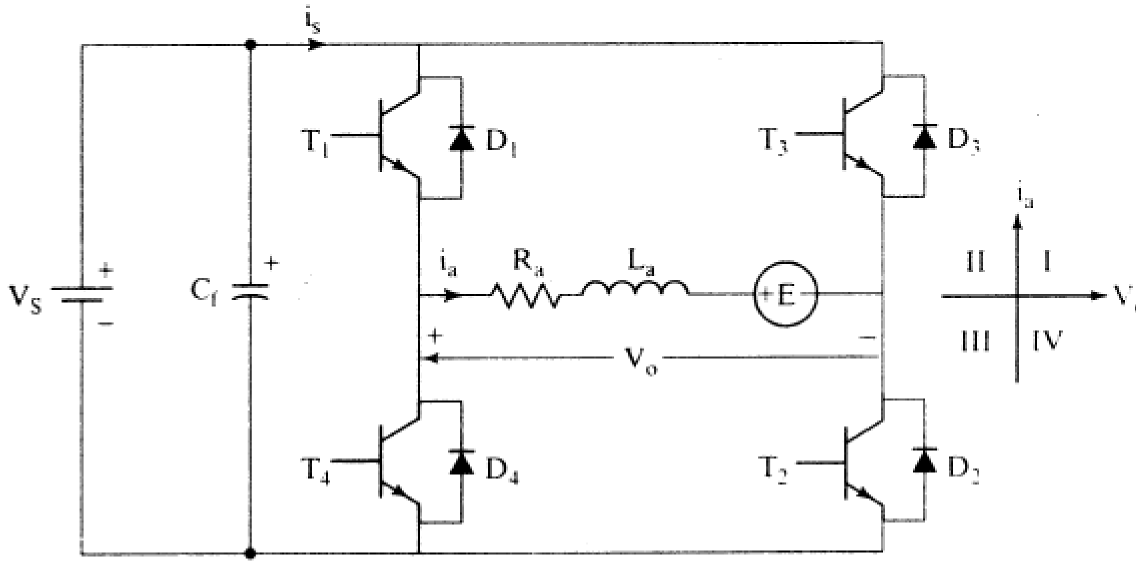


Figure 4.2 A four-quadrant chopper circuit

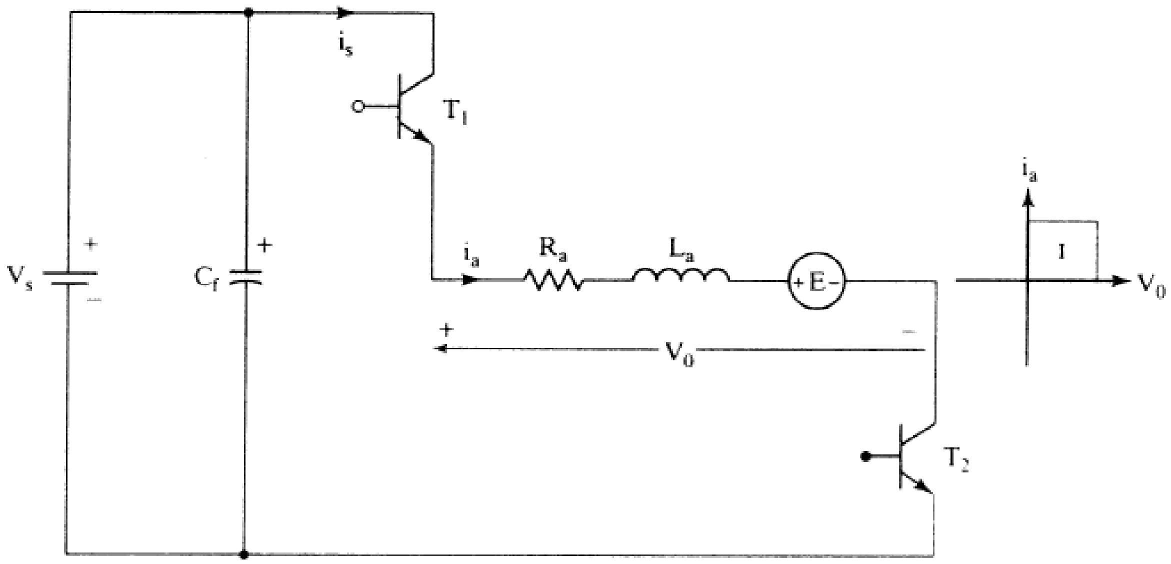


Figure 4.3 First-quadrant operation with positive voltage and current in the load

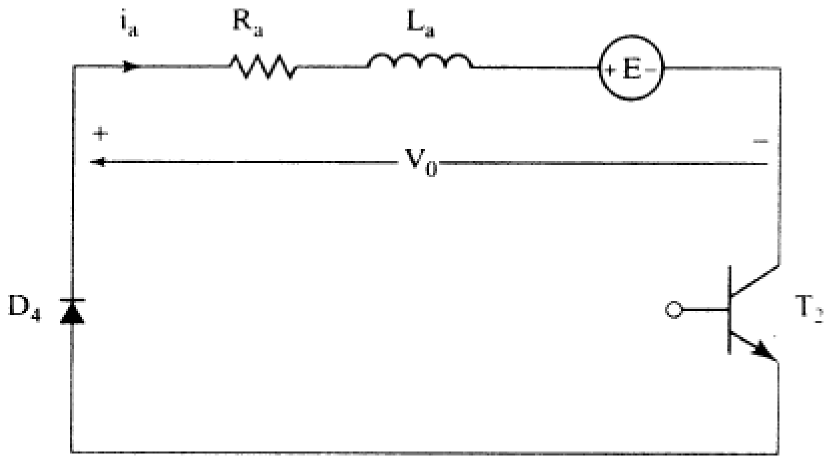


Figure 4.4 First-quadrant operation with zero voltage across the load

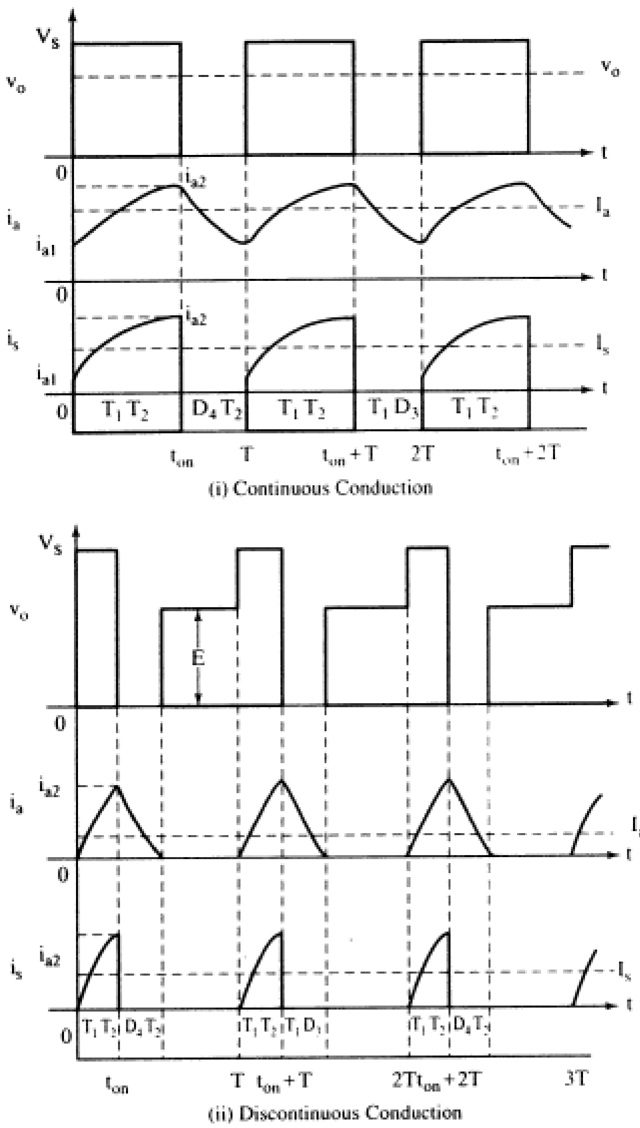


Figure 4.5 Voltage and current waveforms in first-quadrant operation

discontinuous current conduction are shown in Figure 4.5. Note that, in the discontinuous current-conduction mode; the induced emf of the load appears across the load when the current is zero. The load voltage, therefore, is a stepped waveform. The operation discussed here corresponds to motoring in the clockwise direction, or *forward motoring*. It can be observed that the average output voltage will vary from 0 to V_s; the duty cycle can be varied only from 0 to 1.

The output voltage can also be varied by another switching strategy. Armature current is assumed continuous. Instead of providing zero voltage during turn-off time to the load, consider that T1 and T2 are simultaneously turned off, to enable conduction by diodes D3 and D4. The voltage applied across the load then is equal to the negative source voltage, resulting in a reduction of the average output voltage. The disadvantages of this switching strategy are as follows:

(i) Switching losses double, because two power devices are turned off instead of one only.

(ii) The rate of change of voltage across the load is twice that of the other strategy. If the load is a dc machine, then it has the deleterious effect of causing higher dielectric losses in the insulation and therefore reduced life. Note that the dielectric is a capacitor with a resistor in series.

(iii) The rate of change of load current is high, contributing to vibration of the armature in the case of the dc machine.

(iv) Since a part of the energy is being circulated between the load and source in every switching cycle, the switching harmonic current is high, resulting in additional losses in the load and in the cables connecting the source and converter.

Therefore, this switching strategy is not considered any further in this chapter.

4.3.2 Second-Quadrant Operation

Second-quadrant operation corresponds to a positive current with a negative voltage across the load terminals. Assume that the load's emf is negative. Consider that T_1 or T_2 is conducting at a given time. The conducting transistor is turned off. The current in the inductive load has to continue to flow until the energy in it is depleted to zero. Hence, the diodes D_3 and D_4 will take over, maintaining the load current in the same direction, but the load voltage is negative in the new circuit configuration, as is shown in Figure 4.6. The voltage and current waveforms are shown in Figure 4.7. When diodes D_3 and D_4 are conducting, the source receives power from the load. If the source cannot absorb this power, provision has to be made to consume the power. In that case, the overcharge on the filter capacitor is periodically dumped into a resistor connected across the source by controlling the on-time of a transistor in series with a resistor. This form of recovering energy from the load is known as regenerative braking and is common in low-HP motor drives, where the saving in energy might not be considerable or cost-effective. When the current in the load is decreasing, T_2 is turned on. This allows the short-circuiting of the load through T_2 and D_4, resulting in an increase in the load current. Turning off T_2 results in a pulse of current flowing into the source via D_3 and D_4. This operation allows the priming up of the current and a building up of the energy in the inductor from the load's emf. thus enabling the transfer of energy from the load to the source. Note that it is possible to transfer energy from load to source even when E is lower in magnitude than V_s. This particular operational feature is sometimes referred to as *boost operation* in dc-to-dc power supplies. Priming up the load current can also be achieved alternatively, by using T_1 instead of T_2.

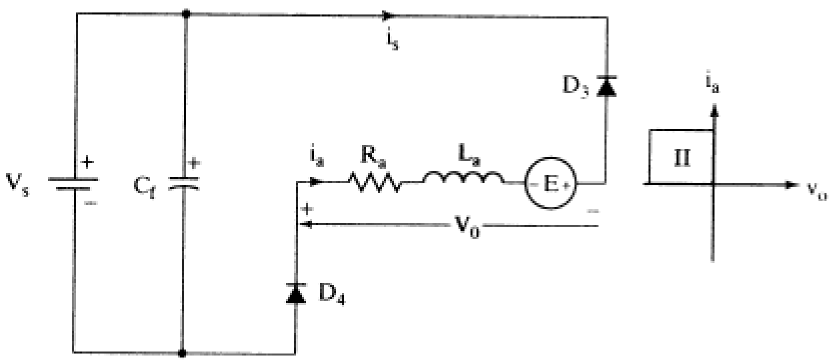


Figure 4.6 Second-quadrant operation, with negative load voltage and positive current

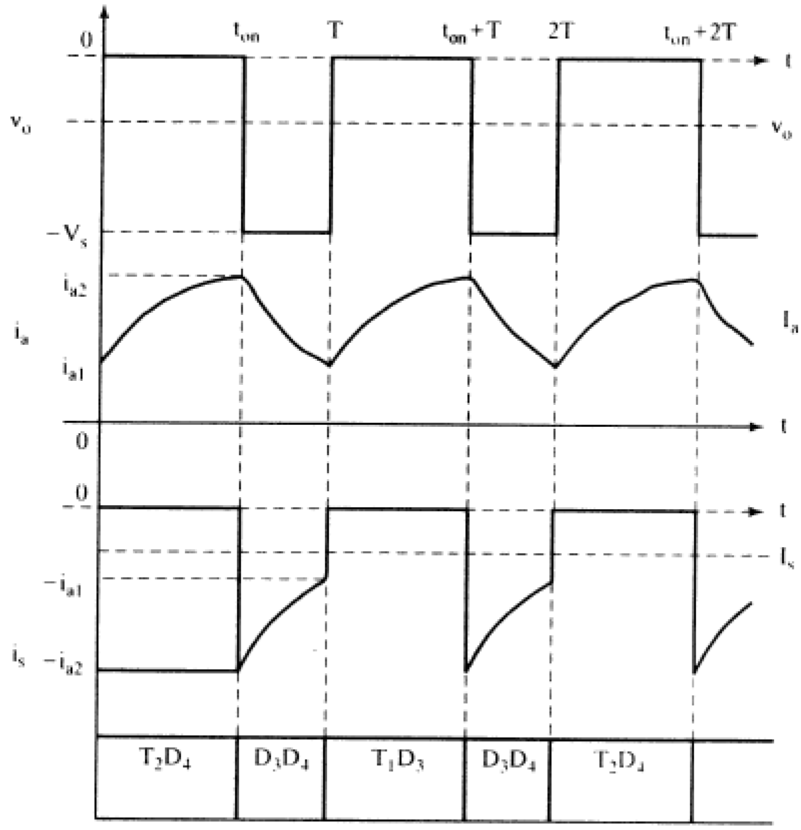
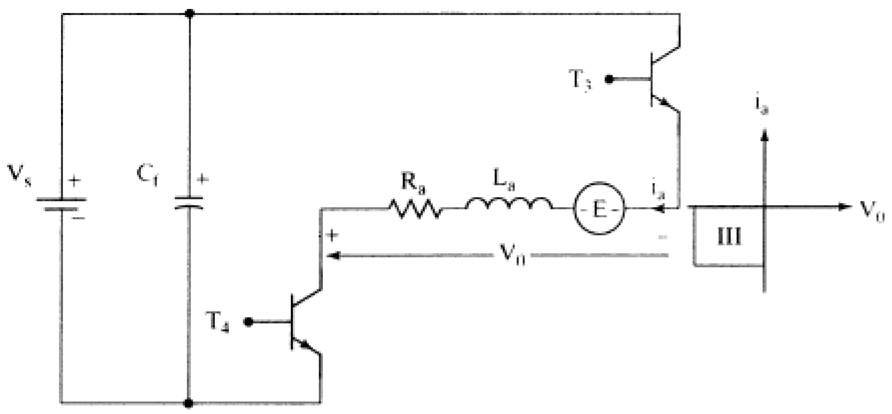


Figure 4.7 Second-quadrant operation of the chopper

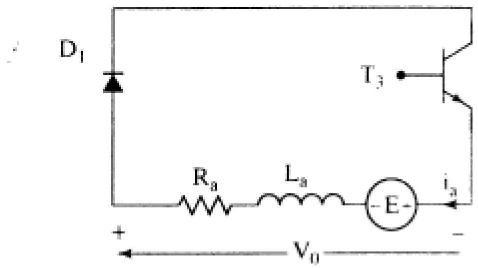
4.3.3 Third-Quadrant Operation

Third-quadrant operation provides the load with negative current and voltage. A negative emf source, $-E$, is assumed in the load. Switching on T_3 and T_4 increases the current in the load, and turning off one of the transistors short-circuits the load, decreasing the load current. That way, the load current can be controlled within the externally set limits. The circuit configurations for the switching instants are shown in Figure 4.8. The voltage and current waveforms under continuous and discontinuous

current-conduction modes are shown in Figure 4.9. Note the similarity between first- and third-quadrant operation.

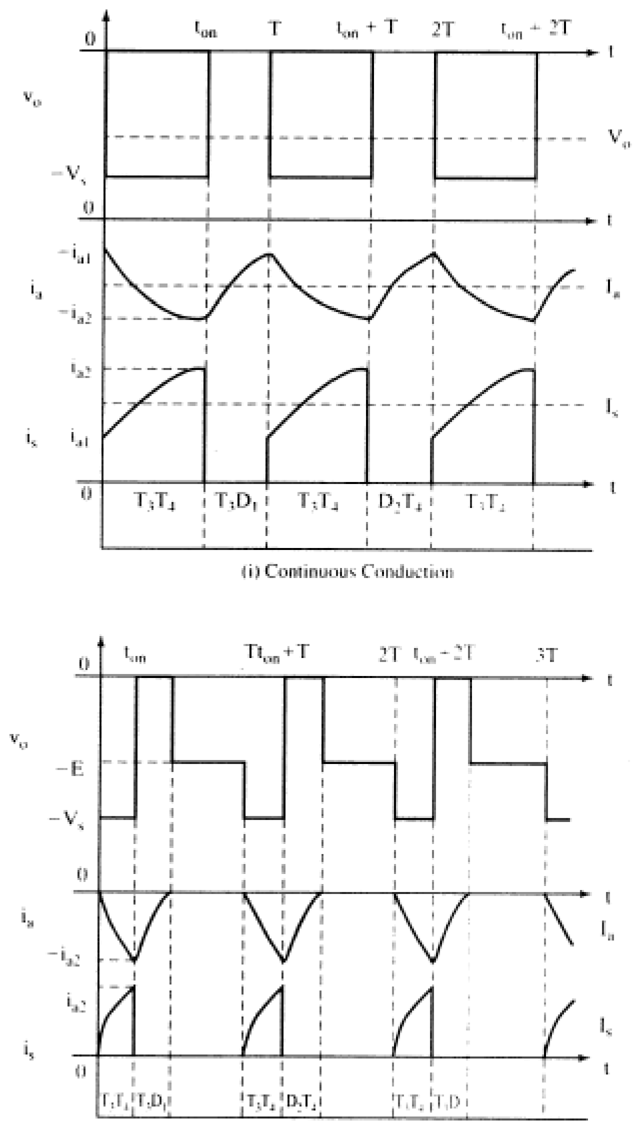


(i) Increasing load current



(ii) Decreasing load current

Figure 4.8 Modes of operation in the third quadrant



(i) Continuous Conduction

(ii) Discontinuous Conduction

Figure 4.9 Third-quadrant operation

4.3.4 Fourth-Quadrant Operation

Fourth-quadrant operation corresponds to a positive voltage and a negative current in the load. A positive load-emf source E is assumed. To send energy to the dc source from the load, note that the armature current has to be established to flow

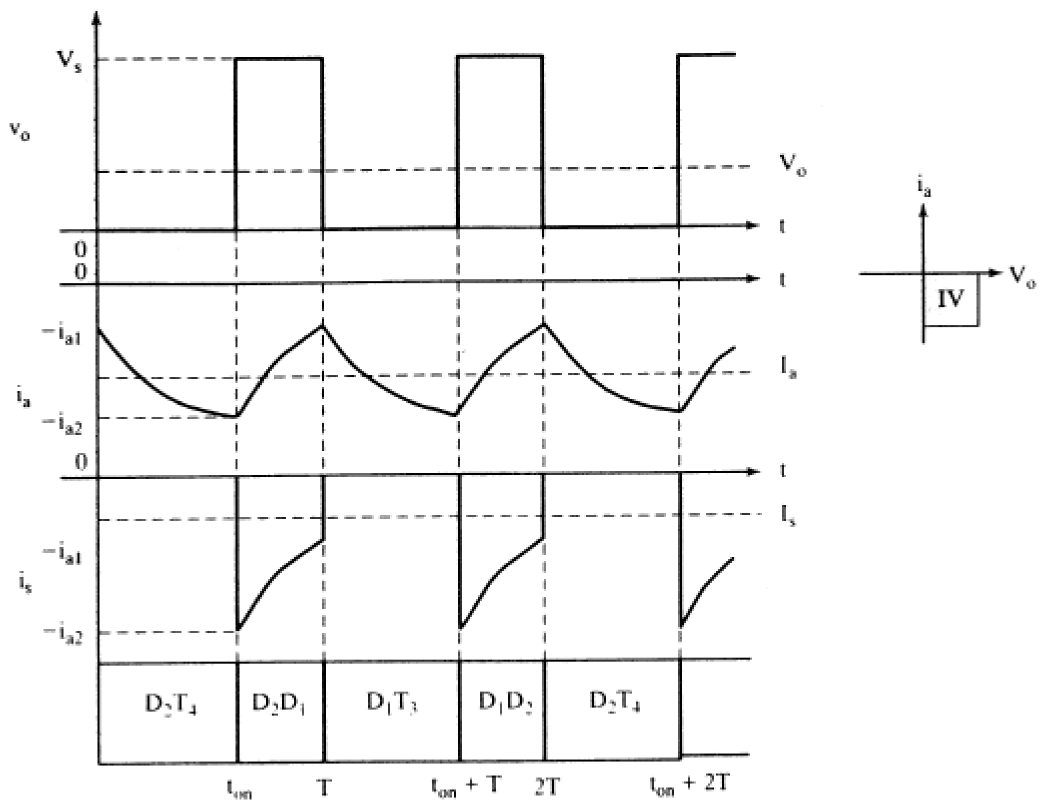


Figure 4.10 Fourth-quadrant operation of the chopper

from the right side to the left side as seen in Figure 4.2. By the convention adopted in this book, that direction of current is negative. Assume that the machine has been operating in quadrant I with a positive current in the armature. When a brake command is received, the torque and armature current command goes negative. The armature current can be driven negative from its positive value through zero. Opening T_1 and T_2 will enable D_3 and D_4 to allow current via the source, reducing the current magnitude rapidly to zero. To establish a negative current, T_4 is turned on. That will short-circuit the load, making the emf source build a current through T_4 and D_2. When the current has reached a desired peak, T_4 is turned off. That forces D_1 to become forward-biased and to carry the load current to the dc input source via D_2 and the load. When the current falls below a lower limit, T_4 is again turned on, to build up the current for subsequent transfer to the source. The voltage and current waveforms are shown in Figure 4.10. The average voltage across the load is positive, and the average load current is negative, indicating that power is transferred from the load to the source. The source power is the product of average source current and average source voltage, and it is negative, as is shown in Figure 4.10.

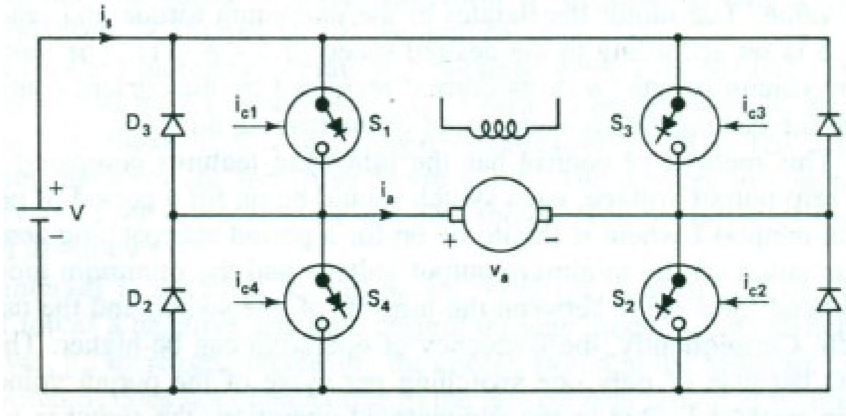


Figure 4.15 Class E four-quadrant chopper.

4.7.3 Four-Quadrant Control

The four-quadrant operation can be obtained by using the class E chopper shown in 4.15. The chopper can be controlled using the following methods.

Method I. If S_2 is kept closed continuously and S_1 and S_4 are controlled, one gets a two-quadrant chopper as shown in figure 4.12a. This provides a variable positive terminal voltage and the armature current in either direction, giving the motor control in quadrants I and II.

Now if S_3 is kept closed continuously and S_1 and S_4 are controlled, a two-quadrant chopper is obtained, which can supply a variable negative terminal voltage and the armature current in either direction, giving motor control in quadrants III and IV.

For the changeover from forward motoring to reverse motoring, the following sequence of steps is followed.

In the first quadrant S_2 is on continuously, and S_1 and S_4 are being controlled. For the changeover, δ is reduced to its minimum value. The motor current reverses [equation (4.33)] and reaches the maximum permissible value. The current control loop restricts it from exceeding the maximum permissible value. The motor decelerates at the maximum torque and reaches zero speed. Now S_2 is opened, S_3 is continuously closed and δ for the pair S_1, S_4 is adjusted corresponding to the desired speed. The motor now accelerates at the maximum torque in the reverse direction and its current is regulated by the current-control loop. Finally it settles at the desired speed.

This method of control has the following features: The utilization factor of the switches is low due to the asymmetry in the circuit operation. Switches S_3 and S_2 should remain on for a long period. This can create commutation problems when the switches are realized using thyristors. The minimum output voltage depends directly on the minimum time for which the switch can be closed. Since there is always a restriction on the minimum time for which the switch can be closed, particularly in thyristor choppers, the minimum available output voltage, and, therefore, the minimum available motor speed, is restricted.

To ensure that the switches S_1 and S_4, and S_3 and S_4 are not on at the same time, some fixed time interval must elapse between the turn-off of one switch and the turn-on of another switch. This restricts the maximum permissible frequency of operation. It also requires two switching operations during a cycle of the output voltage.

Method II. Switches S_1 and S_2 with diodes D_1 and D_2 provide a circuit identical to the chopper of figure 4.13. This chopper can provide a positive current and a variable voltage in either direction, thus allowing motor control in quadrants I and IV. Switches S_3 and S_4 with diodes D_3 and D_4 form another chopper, which can provide a negative current and a variable voltage in either direction, thus allowing the motor control in quadrants II and III.

The switch-over from quadrant I to quadrant III can be carried out using the following sequence of steps. In quadrant I, the switches S_1 and S_2 are controlled with $0.5 < \delta < 1.0$. The armature current has the direction shown in figure 4.15. For the changeover, S_1 and S_2 are turned off. The armature current now flows through diode D_1, source V, and diode D_2, and quickly falls to zero. The motor back emf has the polarity with the left terminal positive. Now the switches S_3 and S_4 are controlled with δ in the range $0 < \delta < 0.5$, but approaching 0.5. The motor current flows in the reverse direction and reaches the maximum value [equation (4.39)]. The current-control loop regulates δ to keep the current from exceeding the maximum permissible value. The motor decelerates at the maximum torque and reaches zero speed. Now δ is set according to the desired speed ($0.5 < \delta < 1$). The motor accelerates at the maximum torque, with its current regulated by the current-control loop and settles at the desired steady-state speed in the reverse direction.

This method of control has the following features compared to method I: At near-zero output voltage, each switch should be on for a period of nearly T sec., unlike in method I where it should be on for a period approaching zero. Thus, there is no limitation on the minimum output voltage and the minimum motor speed. There is no need for a delay between the turn-off of one switch and the turn-on of another switch. Consequently, the frequency of operation can be higher. The switching loss is less because of only one switching per cycle of the output voltage compared to two in method I. Due to the symmetrical operation, the switches have a better utilization factor.

Method III. This method is a modification of method II. In method II, switches S_1 and S_2 with diodes D_1 and D_2 form one chopper, which allows motor control in quadrants I and IV. The second chopper, providing operation in quadrants II and III is formed by switches S_3 and S_4, and diodes D_3 and D_4. In method II, these choppers are controlled separately. In the present method, these choppers are controlled simultaneously as follows.[9]

The control signals for the switches S_1–S_4 are denoted by i_{c1}, i_{c2}, i_{c3}, and i_{c4}, respectively. As with the convention adopted, a switch conducts if its control signal is present and it is forward biased; otherwise it remains open. The control signal i_{c1} to i_{c4}, and the waveform of v_a, i_a, and i_s for forward motoring and forward regeneration are shown in figure 4.16a and b, respectively. Switches S_1 and S_2 are given control signals with a phase difference of T secs. Switch S_1 receives a control signal from $t = 0$ to $t = 2\delta T$, where $\delta = t_{on}/2T$. The control signal for switch S_2 is present from $t = T$ to $t = T + 2\delta T$. Switches S_1 and S_4, and S_2 and S_3 form complementary pairs in the sense that the switches of the same pair receive control signals alternately. Usually some interval must elapse between the turn-off of one switch and the turn-on of another switch of the same pair to ensure that they are not on at the same time. This interval has been neglected in drawing the waveforms of figure 4.16.

In a duration of 2T seconds, which is also the time period of each switch, the chopper operates in four intervals, which are marked as I, II, III, and IV in figures 4.16a and b. The devices under conduction during these intervals are also shown. The operation of the machine in quadrant I can be explained as follows.

In interval I, switches S_1 and S_2 are conducting. The motor is subjected to a positive voltage equal to the source voltage and the armature current increases. At the end of interval I, S_2 is turned off. In interval II, switches S_1 and S_3 receive control signals. Since the motor is carrying a positive current, it flows through a path consisting of D_1 and S_1. Now v_a is zero and i_a is decreasing. Switch S_3 remains off as it is reverse biased by the voltage drop of the conducting diode D_1. At the beginning of interval III, S_2 is turned on again. Now $v_a = V$ and i_a is increasing. At the end of interval III, switch S_1 is turned off. In interval IV, switches S_2 and S_4 receive control signals. The positive motor current flows through S_2 and D_2, and S_4 does not conduct due to the reverse bias applied by the drop of diode D_2.

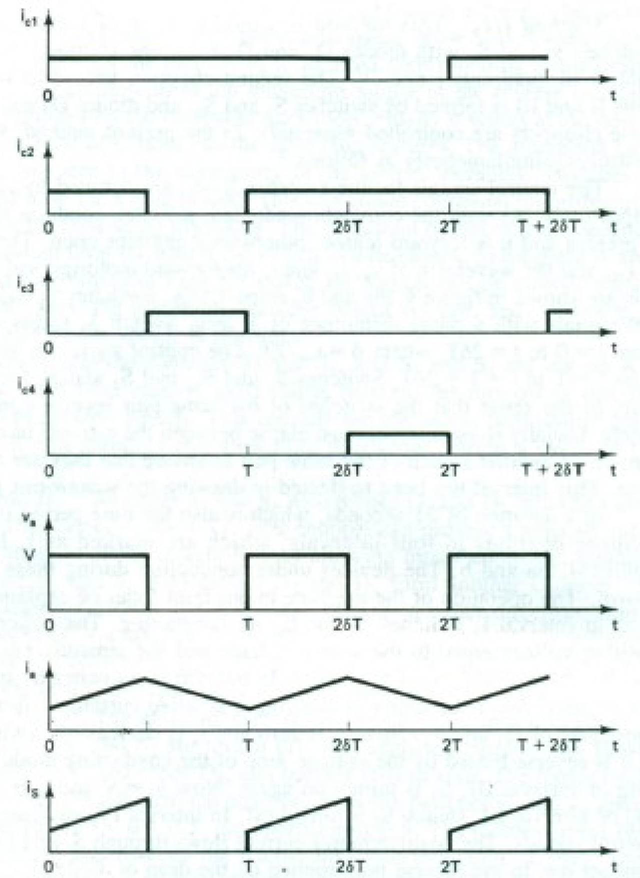
Note that the output voltage waveform is identical to that of figure 4.14a. Hence, equations (4.38) and (4.39) are applicable.

The forward motoring operation is obtained when I_a is positive. The operation can be transferred from forward motoring to forward regeneration by decreasing δ or increasing E to make $V_a < E$ or I_a negative [equation (4.39)]. The waveforms for forward regeneration are shown in figure 4.16b. The devices in conduction in the four intervals of the chopper cycle are also shown. The operation of the chopper is explained as follows.

In interval I, switches S_1 and S_3 are receiving control signals. The positive back emf forces a negative armature current through diode D_3 and switch S_3. During this interval, $|i_a|$ increases, increasing the energy stored in the armature circuit inductance. Switch S_1 does not conduct due to the reverse bias provided by the drop of the conducting diode D_3. Switch S_3 is opened at the end of interval I. The armature current is forced through diode D_3, source V, and diode D_4, and the energy is fed to the source. Although switches S_1 and S_2 are receiving the control signals, they remain open due to the reverse bias provided by the voltage drops of diodes D_3 and D_4. The motor terminal voltage is now V and $|i_a|$ is decreasing. S_4 is turned on in interval III. The armature current now flows through switch S_4 and diode D_4. Switch S_2 also receives a control signal; however, it does not conduct due to the reverse bias applied by diode D_4. The armature current magnitude again builds up. S_4 is turned off at the end of interval III. The armature current is forced again through diode D_3, the source, and diode D_4, and the energy is fed to the source.

The motoring and regenerative braking operations in the reverse direction are obtained when $0 < \delta < 0.5$, for which V_a is negative. Reverse motoring is obtained by setting δ such that $|V_a| > |E|$ and reverse regeneration is realized when $|E| > |V_a|$.

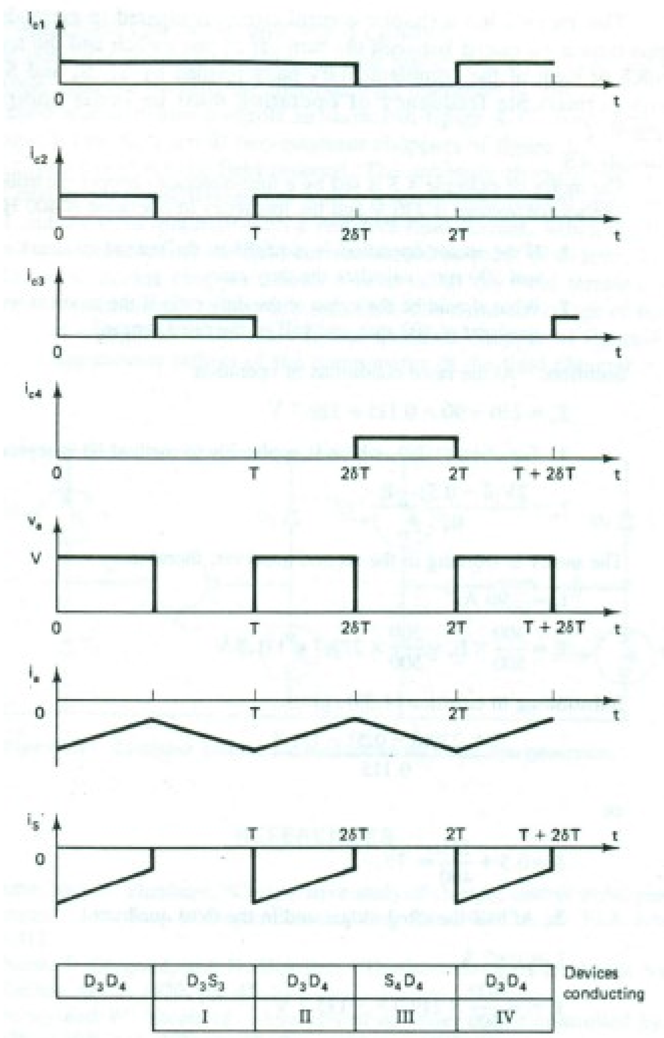
This method has a simpler control circuit compared to methods I and II. Since some time must elapse between the turn-off of one switch and the turn-on of another switch of each of the complementary pairs formed by S_1, S_4 and S_2, S_3, the maximum permissible frequency of operation must be lower compared to that of method II.



S_1S_2	S_1D_1	S_1S_2	S_2D_2	S_1S_2	Devices conducting
I	II	III	IV		

(a) Forward motoring, $0.5 \leq \delta \leq 1.0$ and $V_a > E$

Figure 4.16 Waveforms of the four quadrant chopper of Fig. 4.15 using *method III* *(continued on next page)*.



(b) Forward regeneration, $0.5 \leq \delta \leq 1.0$ and $V_a < E$

Figure 4.16 (continued).

Example 4.5

The motor of example 4.3 is fed by a four-quadrant chopper controlled by method III. The source voltage is 230 V and the frequency of operation is 400 Hz.

1. If the motor operation is required in the second quadrant at the rated torque and 300 rpm, calculate the duty ratio.
2. What should be the value of the duty ratio if the motor is working in the third quadrant at 400 rpm and half of the rated torque?

Solution: At the rated conditions of operation

$$E_r = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. Equation (4.39), which is applicable to method III is reproduced here:

$$I_a = \frac{2V(\delta - 0.5) - E}{R_a} \tag{4.39}$$

The motor is working in the second quadrant, therefore,

$$I_a = -90 \text{ A}$$

$$E = \frac{300}{500} \times E_r = \frac{300}{500} \times 219.7 = 131.8 \text{ V}$$

Substituting in equation (4.39), gives

$$-90 = \frac{2 \times 230(\delta - 0.5) - 131.8}{0.115}$$

or

$$\delta = 0.5 + \frac{121}{460} = .76\,.$$

2. At half the rated torque and in the third quadrant

$$I_a = -45 \text{ A}$$

$$E = -\frac{400}{500} \times 219.7 = -175.7 \text{ V}$$

Substituting in equation (4.39), gives

$$-45 = \frac{2 \times 230(\delta - 0.5) + 175.7}{0.115}$$

or

$$\delta = 0.5 - \frac{181}{460} = 0.11\,.$$

Four-quadrant Operation with Field Control

When field control is required for getting speeds higher than base speed and the transient response need not be fast, the four-quadrant operation is obtained by a combination of field and armature controls as shown in figure 4.17. Both armature and field are supplied by the class D two-quadrant choppers of figure 4.13. The reversal switch RS is employed for the field reversal. The armature chopper provides operation in the first and the fourth quadrant with a positive field current and operation in the second and the third quadrant with a negative field current. When the field connection is to be reversed, first the field current should be reduced to zero. The use of the class D two-quadrant chopper allows a reversal of the field terminal voltage, which forces the field current to become zero fast. The main advantage of this circuit is the lower cost compared to the class E four-quadrant chopper of figure 4.13, because of the lower current ratings of the components of the field chopper.

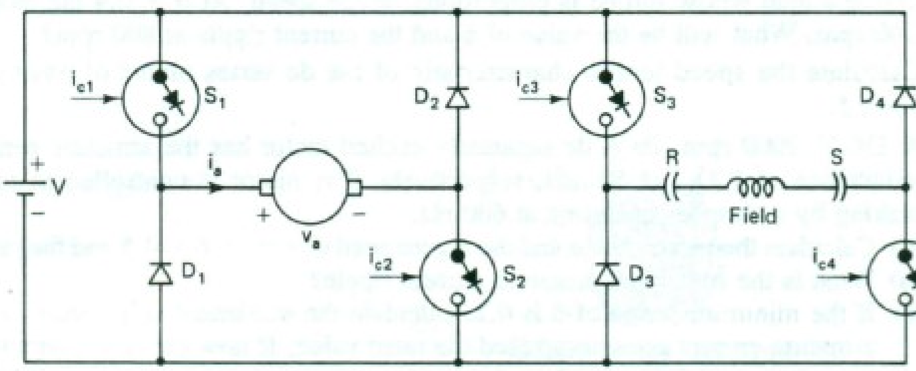


Figure 4.17 Combined armature and field control for four-quadrant operation.

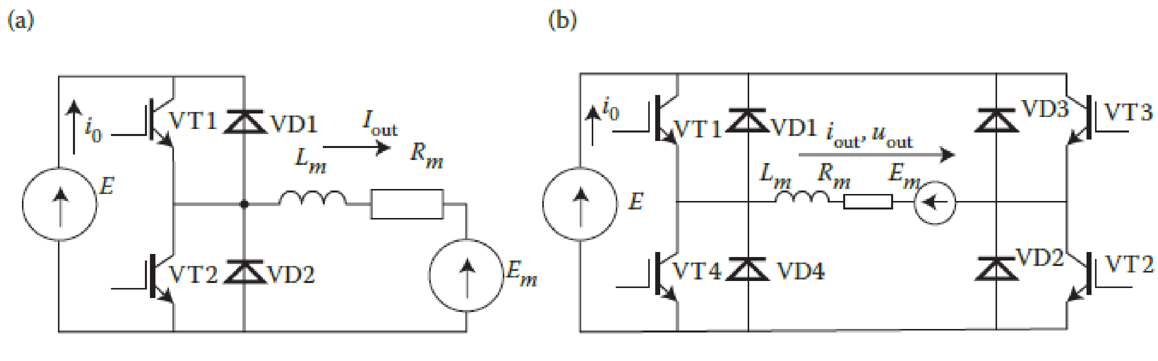


Figure 5.12 (a) Two-quadrant and (b) four-quadrant dc voltage converters.

5.4.2 *Four-quadrant converter*

The rotation of a dc electrical machine may be reversed by means of a four-quadrant dc converter (Figure 5.12b).

The converter may operate in four modes.

Quadrant I. The electrical machine operates in the motor mode, with forward rotation. An output-voltage pulse is formed at the motor input when transistors VT1 and VT2 are switched on simultaneously, and $u_{out} = E$. To create an inactive interval, it is sufficient to switch off one of the transistors—say, VT2. Then the motor current flows through VT1 and diode VD3; $U_{out} = 0$ and $i_0 = 0$. In this quadrant, the converter resembles a step-down dc/dc converter: $U_{out} = \gamma E$.

Quadrant II. The motor turns in the same direction, but with recuperative braking. Consequently, the machine operates in the generative mode, and the current i_{out} is reversed. Two modes alternate in the converter.

- An interval of length γT_{sw}, in which all the transistors are on; current passes through diodes VD1 and VD2; and the motor current flows through source E, to which energy is returned.
- An interval of length $(1 - \gamma)T_{sw}$, in which transistor VT3 is on; the load current passes through the circuit VT3–VD1, bypassing the source; and $i_0 = 0$. The same results may be obtained by switching on VT4, which forms a circuit with diode VD2.

In the second quadrant, the converter resembles a step-up dc/dc converter, in which the energy source is the emf E_m.

Quadrant III. The direction of rotation is reversed; the directions of the voltages and currents in the electrical machine are the opposite to those shown in Figure 5.12b. When transistors VT3 and VT4 are switched on simultaneously, $u_{out} = -E$; energy is sent from the source to the motor. When one of those transistors is switched off, current flows through the circuit consisting of a transistor and a diode, bypassing the source: $u_{out} = 0$; $i_0 = 0$; and $u_{out} = -\gamma E$.

Quadrant IV. Recuperative braking occurs. When all the transistors are switched off, the current in the electrical machine, whose direction is as in Figure 5.12b, passes through the circuit VD3–VD4, returning energy to source E. When transistor VT1 is turned on, current i_{out} flows through diode VD3, bypassing the source. The same result may be obtained by switching on transistor VT2, which forms a circuit with diode VD4.

We may note the similarities between multiquadrant voltage converters and voltage source inverters (Section 6.1). The circuit in Figure 5.12a corresponds to a half-bridge voltage inverter with asymmetric connection of the load and the circuit in Figure 5.12b to a single-phase bridge inverter.

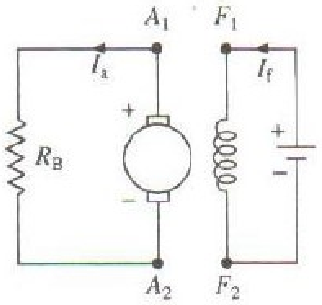
5.4.1 *Two-quadrant converter*

In Figure 5.12a, we show the circuit diagram of a two-quadrant dc converter. The load considered is a dc motor, which is replaced by an equivalent circuit consisting of the motor's counteremf E_m, its resistance R_m, and its inductance L_m.

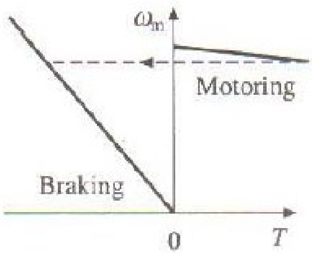
When an electrical machine operates in the motor mode, only transistor VT1 operates in the converter, and transistor VT2 is always off. When transistor VT1 is turned on, it connects source E to the motor; motor current $i_{out} = i_0$ passes through VT1. When transistor VT1 is turned off, the motor current passes through diode VD2, and the voltage applied to the motor is zero. It is readily evident that the conducting section of the circuit corresponds to a step-down dc/dc converter (Section 5.2.1). In that case, neglecting the losses, we write the output voltage in the form

$$U_{out} = E\frac{T_p}{T_{sw}} = \gamma E. \tag{5.32}$$

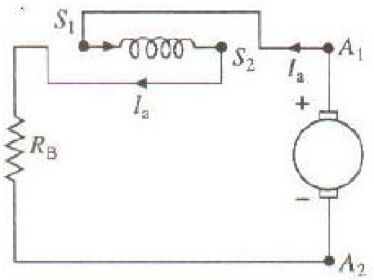
In recuperative braking, the motor continues to turn, and the polarity E_m is unchanged. However, on switching to the generator mode, the polarity of current i_{out} will be the opposite of that in Figure 5.12a. In this mode, no control pulses are sent to transistor VT1. When transistor VT2 is off, current flows through diode VD1; the polarity of current i_0 is reversed, and the motor energy is recuperated to source E. When transistor VT2 is turned on, it transmits the current i_{out}. Operation in the recuperation mode corresponds to a step-up dc/dc converter (Section 5.2.2), if we assume that the energy source is the motor's counteremf E_m.



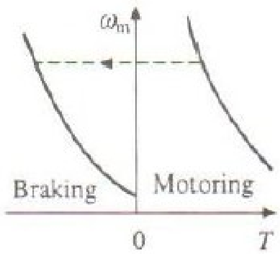
Separately excited motor



Separately excited motor



(b) Series motor



Series motor

6.5. THE FOUR-QUADRANT CHOPPER

A d.c. brush motor with separate excitation is fed through a four-quadrant chopper (Table 6.1e). Show the waveforms of voltage and current in the third and fourth quadrants.

Solution:

The basic circuit of a four-quadrant chopper is shown in Figure 6.9.

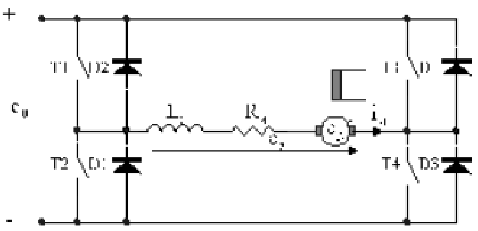


Figure 6.9. D.c. brush motor fed through a four-quadrant chopper

If T_4 is on all the time, T_1-D_1 and T2-D2 provide first- and (respectively) second-quadrant operations as shown in previous paragraphs. With T_2 on all the time and T_3-D_3 and, respectively, T_4-D_4 the third- and fourth-quadrant operations is obtained (Figure 6.10). So, in fact, we have 2 two-quadrant choppers acting in turns.

However, only 2 out of 4 main switches are turned on and off with the frequency f_{ch} while the third main switch is kept on all the time and the fourth one is off all the time.

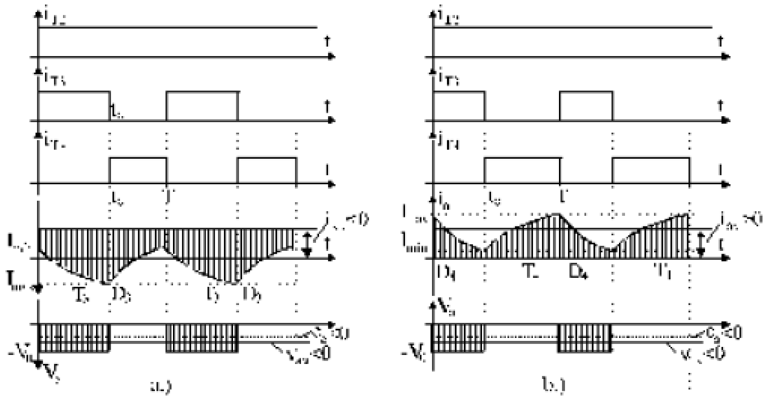


Figure 6.10. Four-quadrant chopper supplying a d.c. brush motor

a.) Third quadrant: iav<0, Vav<0; b.) Fourth quadrant: iav>0, Vav<0.

Four-quadrant operation is required for fast response reversible variable speed drives.

As expected, discontinuous current mode is also possible but it should be avoided by increasing the switching frequency f_{ch} or adding an inductance in series with the motor.

Let us assume that:

A d.c. brush motor, fed through a four-quadrant chopper, works as a motor in the third quadrant (reverse motion). The main data are $V_0 = 120V$, $R_a = 0.5\Omega$, $L_a = 2.5mH$, rated current $I_{an} = 20A$; rated speed $n_n = 3000$ rpm; separate excitation.

a. Calculate the rated e.m.f., e_g, and rated electromagnetic torque, T_e.

b. For $n = -1200$ rpm and rated average current ($i_{av} = -I_{an}$) determine the average voltage V_{av}, $t_c / T = \alpha_{on}$, and maximum and minimum values of motor current I_{max} and I_{min} for 1kHz switching frequency.

Solution:

a. The motor voltage equation for steady state is:

$$V_{av} = R_a i_a + e_g \tag{6.54}$$

for rated values $V_{av} = V_0 = 120V$, $i_a = i_{an} = 20A$, thus

$$e_{gn} = K_e \lambda_p n_n = V_{av} - R_a i_a = 120 - 20 \cdot 0.5 = 110 \text{ V} \tag{6.55}$$

$$K_e \lambda_p = \frac{e_{gn}}{n_n} = \frac{110}{50} = 2.2 \text{ Wb} \tag{6.56}$$

b. The motor equation in the third quadrant is

$$V_{av} = R_a i_a + e_g = 0.5 \cdot (-20) + 2.2 \cdot (-20) = -54 \text{ V}, \tag{6.57}$$

the conducting time t_c for T_3 (Figure 6.10a) is

$$\frac{t_c}{T} = \frac{V_{av}}{-V_0} = \frac{-54}{-120} = 0.45 \tag{6.58}$$

$$t_c = T \cdot 0.45 = \frac{1}{f_{ch}} \cdot 0.45 = \frac{1}{10^3} \cdot 0.45 = 0.45 \cdot 10^{-3} \text{ s} \tag{6.59}$$

Form ((6.40)-(6.41)) the motor current variation (Figure 6.10a) is described by

$$i_a = \frac{V_0' - e_g}{R_a} + A \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t \le t_c \tag{6.60}$$

$$i_a' = -\frac{e_g}{R_a} + A' \cdot e^{-(t-t_c)\frac{R_a}{L_a}}; \quad t_c < t \le T \tag{6.61}$$

The current continuity condition ($i_a(t_c) = i_a'(t_c)$) provides

$$t_c = -\frac{L_a}{R_a} \cdot \ln\left[\left(A' - \frac{V_0'}{R_a}\right) / A\right] \tag{6.62}$$

The second condition is obtained from the average current expression

$$i_{av} = \frac{1}{T}\left[\int_0^{t_c} i_a dt + \int_{t_c}^{T} i_a' dt\right] =$$

$$\frac{1}{T}\left\{\frac{V_0' - e_g}{R_a} t_c - \frac{e_g}{R_a}(T - t_c) + \frac{L_a}{R_a}\left[\left(1 - e^{-t_c\frac{R_a}{L_a}}\right)A + A'\left(1 - e^{-(T-t_c)\frac{R_a}{L_a}}\right)\right]\right\} \tag{6.63}$$

From (6.62) and (6.63) we obtain:

$$\left(A' - \frac{V_0'}{R_a}\right) / A = e^{-t_c\frac{R_a}{L_a}};$$

$$V_0' = -V_0; \quad e_g = K_e \lambda_p n = 2.2 \cdot (-20) = -44 \text{ V}$$

$$\left(A' + \frac{(-120)}{0.5}\right) / A = e^{-0.45 \cdot 10^{-3} \frac{0.5}{5 \cdot 10^{-3}}} = 0.914 \tag{6.64}$$

$$-20 = 10^{3}\left\{\frac{-120-(-44)}{0.5}0.45\cdot 10^{-3} - \frac{(-44)}{0.5}0.55\cdot 10^{-3}\right.$$
$$\left. + \frac{2.5\cdot 10^{-3}}{0.5}\left[\left(1-e^{-0.45\cdot 10^{-3}\frac{0.5}{2.5\cdot 10^{-3}}}\right)A + A'\left(1-e^{-0.55\cdot 10^{-3}\frac{0.5}{2.5\cdot 10^{-3}}}\right)\right]\right\} \tag{6.65}$$

$$-20 = -20 + 0.43A + 0.5205A' \tag{6.66}$$

$$0.43A + 0.5205A' = 0 \tag{6.67}$$

$$A' + 240 = 0.914A \tag{6.68}$$

$$A = 137.62;\ A' = -113.92 \tag{6.69}$$

Now we may calculate $I_{min} = i_a(0)$

$$I_{min} = A + \frac{V_0' - e_g}{R_a} = 137.92 + \frac{-120-(-44)}{0.5} = -15.08\ \text{A} \tag{6.70}$$

Also $I_{max} = i_a'(t_c)$

$$I_{max} = A' - \frac{e_g}{R_a} = -113.92 + \frac{-(-44)}{0.5} = -25.92\ \text{A} \tag{6.71}$$

7.4 4-Quadrant Chopper

The circuit diagram to facilitate 4-quadrant operation is given in Fig. 7.5(a). This circuit contains four controlled switches and four diodes. Here, v_a and i_a are indicated with reference to motor terminals A and AA. The controlled switches are realized using IGBTs in this chapter. This circuit is capable of providing motoring, regenerative braking, operation of the motor in the reverse speed and regenerative braking in that direction corresponding to the four quadrants of V-I diagram, which is shown in Fig. 7.5(b). The following section explains the converter fed drive characteristics in four different quadrants.

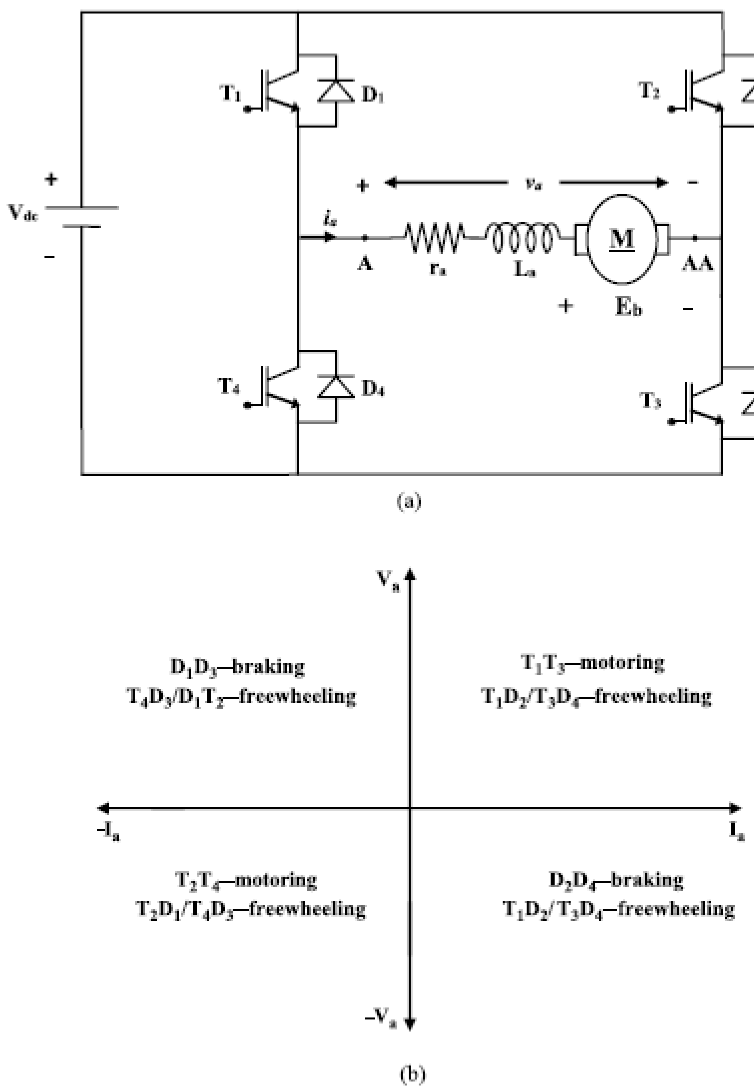


FIGURE 7.5
(a) 4-quadrant chopper circuit. (b) V-I diagram.

7.4.1 Motoring in the Forward Direction

Fig. 7.6(a) shows the devices conducting during the motoring of the drive in the forward direction. During the period T_{on} of the converter, the IGBTs T_1 and T_3 are simultaneously gated so that $v_a = V_{dc}$ and the armature current i_a is positive. During the OFF period of the dc/dc converter, T_1 alone is switched OFF, such that the armature current i_a now freewheels through T_3 and D_4, making the motor terminal voltage zero. The motor terminal voltage and current waveforms for continuous mode are shown in Fig. 7.6(b). As seen in this figure, the average values of voltage and current are positive, thus the average output power is always positive, leading to first-quadrant operation as indicated in Fig. 7.6(c). It may be noted that freewheeling of the armature current is also possible through D_2 and T_1, which is indicated by a dotted line.

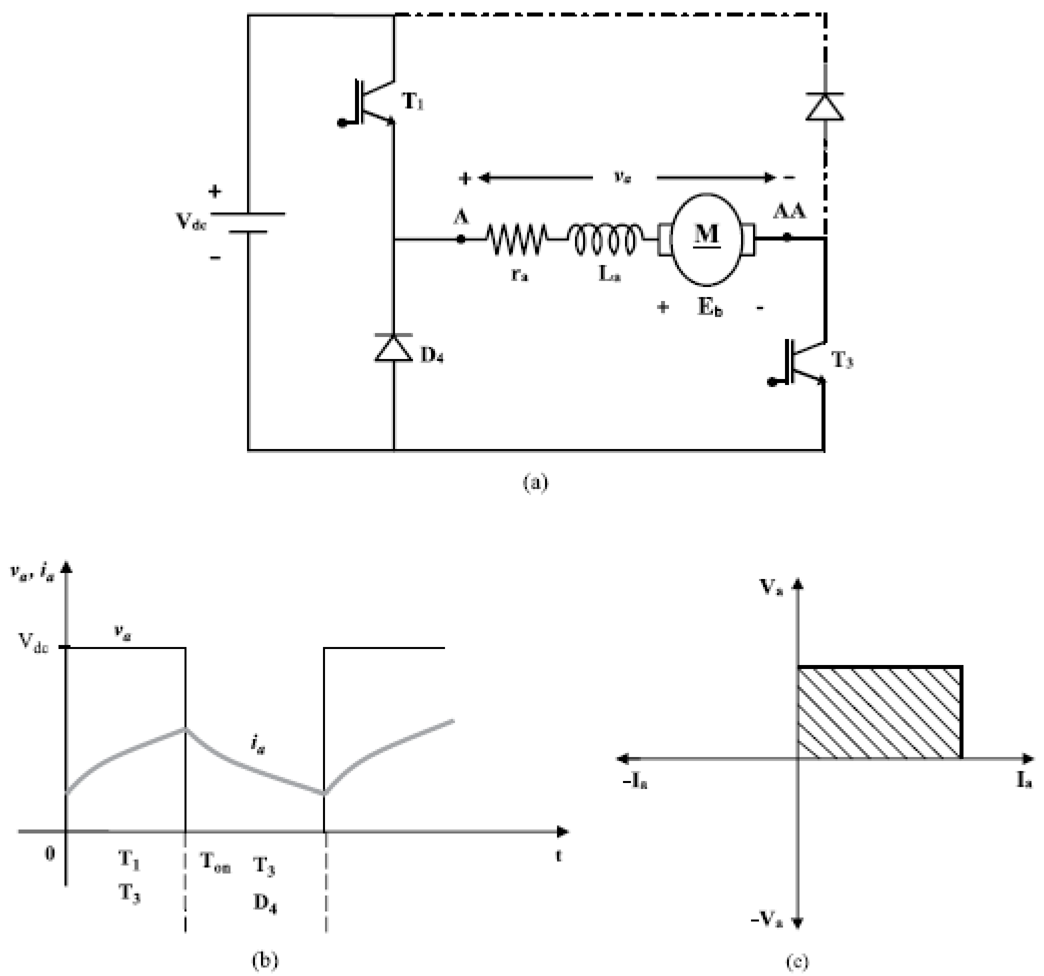


FIGURE 7.6
First-quadrant circuit. (a) Equivalent circuit. (b) Motor voltage and current waveforms. (c) V-I diagram.

7.4.2 Regenerative Braking after Forward Rotation

When regenerative braking is required, the IGBT T_1 is switched OFF. For the regenerative braking to take place, the electromagnetic torque (T_e) must be made negative. Because $T_e \propto (\Phi_m I_a)$, reversal of polarity of T_e is possible by changing the polarity of either Φ_m or I_a. We consider the case of reversal of I_a alone because reversal of Φ_m requires more time due to the increased field time constant.

Consider Fig. 7.7(a). Assume that freewheeling was taking place through T_3 and D_4 and that the regenerative braking command has come during the freewheeling action. To initiate the armature current reversal process during regenerative braking, T_4 is triggered and the gating signal to T_3 is continued till armature current goes to zero. Although T_4 is gated, the forward voltage drop across D_4 prevents T_4 from conducting and, as such, T_3-D_4 continues to cause freewheeling armature current. When this current drops to zero, the back-emf E_b causes reversal of armature current through T_4 and D_3. Armature current, i_a now flows from A to AA and is in the negative direction as shown in Fig. 7.7(b). This current rises exponentially, and when T_4 is turned OFF, the armature current maintains its direction

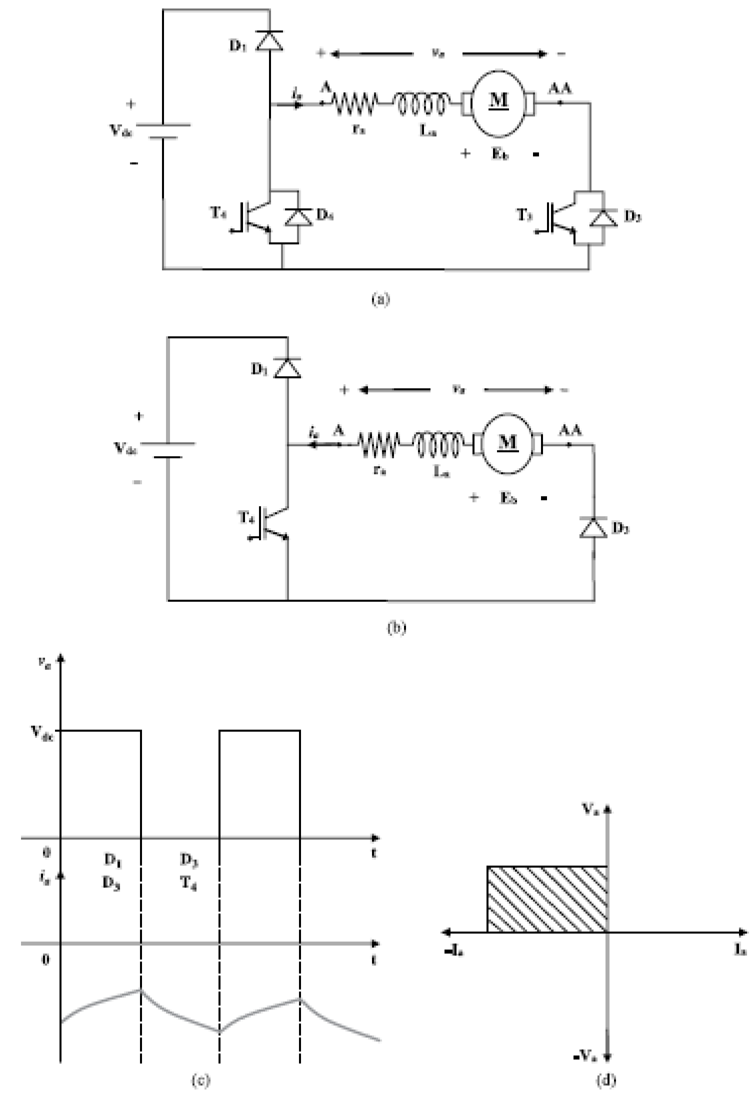


FIGURE 7.7
Second-quadrant operation. (a) Circuit during transition. (b) Circuit during regenerative braking. (c) Motor voltage and current waveforms. (d) V-I diagram.

through D_3-D_1 flowing to the source leading to regenerative braking. Pulse-width modulation of T_4 results in uniform braking. Armature voltage and current during regeneration are sketched in Fig. 7.7(c). This is a second-quadrant operation in the V-I diagram and is given in Fig. 7.7(d). It may be noted that free-wheeling of motor current can also take place through D_1-T_2.

7.4.3 Motoring in the Reverse Direction/Third-Quadrant Operation

Now the motor should be accelerated in the reverse direction. To achieve this, IGBTs T_2 and T_4 are switched ON, and both v_a and i_a get reversed. The product of v_a and i_a is positive, indicating that the process is in the motoring operation. Because i_a is reversed, the direction of electromagnetic torque is also reversed, so the motor is accelerated in the opposite direction. Freewheeling of the armature current can take place either through T_4-D_3 or D_1-T_2. The power circuit, steady-state voltage, current waveforms, and V-I diagram are given in Fig. 7.8.

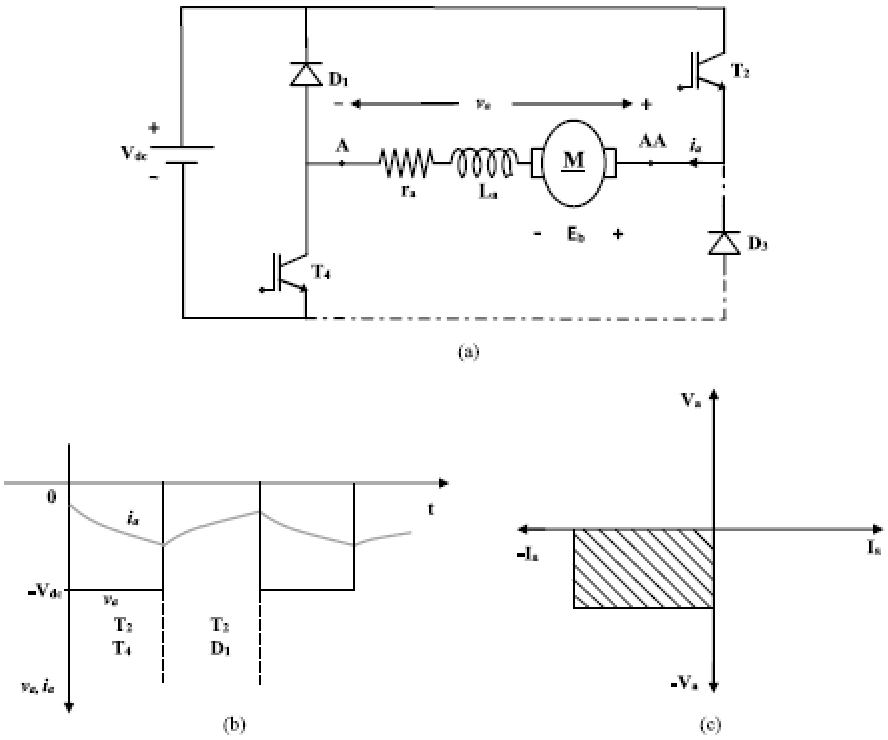


FIGURE 7.8
Motoring in reverse-braking. (a) Equivalent circuit. (b) Motor voltage and current waveforms. (c) V-I diagram.

7.4.4 Regenerative Braking after Speed Reversal

A process similar to that for second-quadrant operation takes place for the regenerative braking mode. Referring to Fig. 7.9(a), D_1-T_2 is the path of freewheeling, and when T_1 is triggered for regenerative braking, T_1 is prevented from conducting because of the forward voltage drop across D_1. The devices D_1-T_2 stop conducting when the armature current becomes zero, and the back-emf now drives the armature current through D_2 and T_1. Now armature current i_a flows from A to AA and hence is positive. When T_1 is turned OFF,

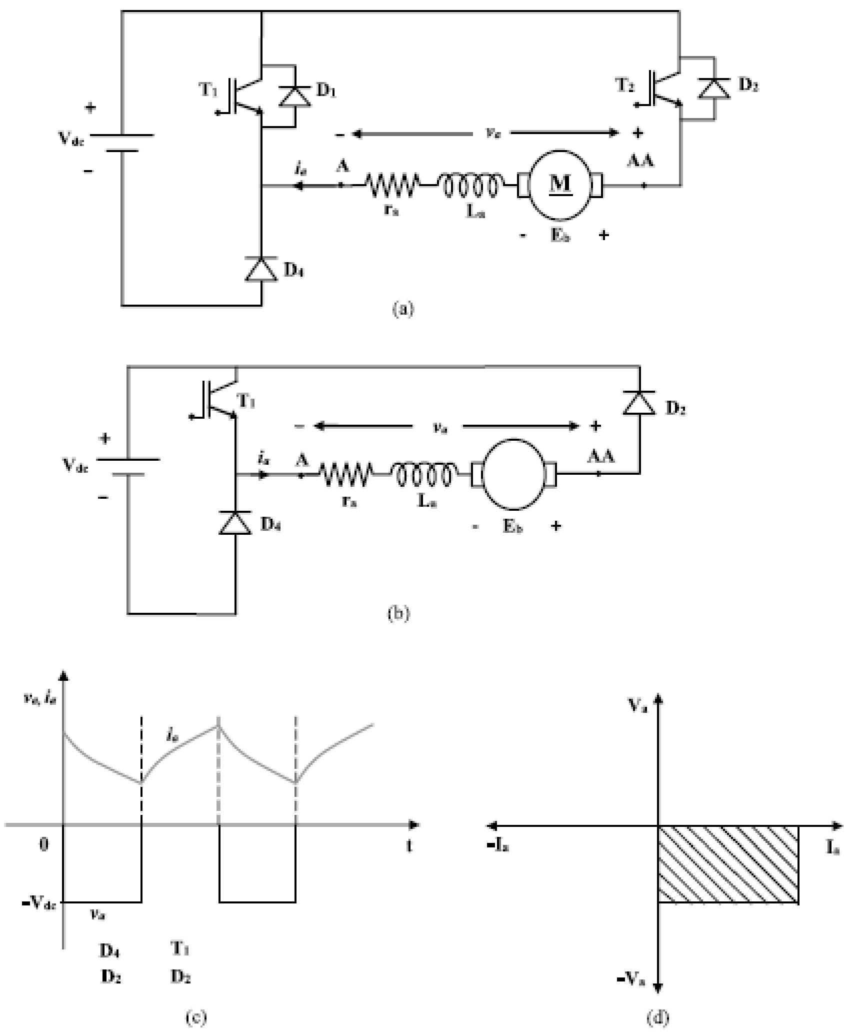


FIGURE 7.9
Fourth-quadrant operation. (a) Circuit during transition. (b) Circuit during regenerative braking. (c) Motor voltage and current waveforms. (d) V-I diagram.

regeneration takes place through D_4-D_2 and leads to fourth-quadrant operation. The power circuit, typical steady-state waveforms, and V-I diagram are given in Fig. 7.9.

7.4.5. Four-quadrant Chopper, or Type-E Chopper

The power circuit diagram for a four-quadrant chopper is shown in Fig. 7.10 (*a*). It consists of four semiconductor switches CH1 to CH4 and four diodes D1 to D4 in antiparallel. Workin of this chopper in the four quadrants is explained as under :

First quadrant : For first-quadrant operation of Fig. 7.10 (*a*), CH4 is kept on, CH3 is kept off and *CH1 is operated.* With CH1, CH4 on, load voltage $v_0 = V_s$ (source voltage) and load current i_0 begins to flow. Here both v_0 and i_0 are positive giving first quadrant operation. When CH1 is turned off, positive current freewheels through CH4, D2. In this manner, both V_0, I_0 can be controlled in the first quadrant.

Second quadrant : *Here CH2 is operated* and CH1, CH3 and CH4 are kept off. With CH2 on, reverse (or negative) current flows through L, CH2, D4 and E. Inductance L stores energy during the time CH2 is on. When CH2 is turned off, current is fed back to source through diodes D1, D4. Note that here $\left(E + L\frac{di}{dt}\right)$ is more than the source voltage V_s. As load voltage V_0 is positive and I_0 is negative, it is second quadrant operation of chopper. Also, power is fed back from load to source.

Third quadrant : For third-quadrant operation of Fig. 7.10 (*a*), CH1 is kept off, CH2 is kept on and *CH3 is operated.* Polarity of load emf E must be reversed for this quadrant working. With CH3 on, load gets connected to source V_s so that both v_0, i_0 are negative leading to third quadrant operation. When CH3 is turned off, negative current freewheels through CH2, D4. In this manner, v_0 and i_0 can be controlled in the third quadrant.

Fourth quadrant : *Here CH4 is operated* and other devices are kept off. Load emf E must have its polarity reversed to that shown in Fig. 7.10 (*a*) for operation in the fourth quadrant. With CH4 on, positive current flows through CH4, D2, L and E. Inductance L stores energy during the time CH4 is on. When CH4 is turned off, current is fed back to source through diodes D2, D3. Here load voltage is negative, but load current is positive leading to the chopper operation in the fourth quadrant. Also power is fed back from load to source.

The devices conducting in the four quadrants are indicated in Fig. 7.10 (*b*).

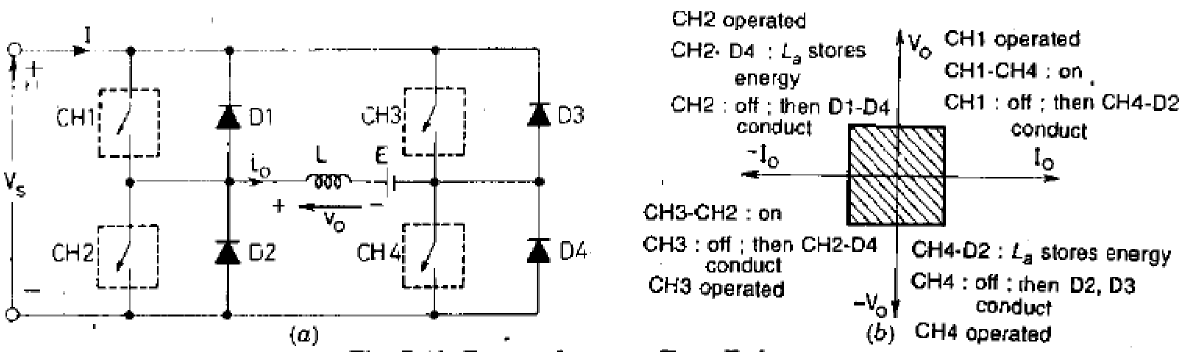


Fig. 7.10. Four-qudrant, or Type-E chopper
(*a*) circuit diagram and (*b*) operation of conducting devices.

SOLVED EXAMPLES

Example 8.16 A four-quadrant chopper is driving a separately excited dc motor load. The motor parameters are $R = 0.1$ ohm, $L = 10$ mH. The supply voltage is 200 V d.c. If the rated current of the motor is 10 A and if the motor is driving the rated torque. Determine:

(i) the duty cycle of the chopper if $E_b = 150$ V.
(ii) the duty cycle of the chopper if $E_b = -110$ V.

Solution:

For a four-quadrant chopper, the average voltage in all the four-modes is given by

$$E_0 = 2\,E_{dc} \cdot (\alpha - 0.5)$$

(i) The average current, $i_0 = \dfrac{E_0 - E_b}{R} = \dfrac{2\,E_{dc} \cdot (\alpha - 0.5) - E_b}{R}$

$$10 = \frac{2 \times 200\,(\alpha - 0.5) - 150}{0.1} \qquad \therefore\ \alpha = 0.876$$

Since, $\alpha > 0.5$, this mode is forward-motoring

(ii) Now, $10 = \dfrac{2 \times 200\,(\alpha - 0.5) - 110}{0.1}, \quad \therefore\ \alpha = 0.228$

As $\alpha < 0.5$, this mode is reverse motoring mode.

8.5.5 Four-Quadrant Chopper (or Class E Chopper)

Figure 8.25(a) shows the basic power circuit of Type E chopper. From Fig. 8.25, it is observed that the four-quadrant chopper system can be considered as the parallel combination of two Type C choppers. In this chopper configuration, with motor load, the sense of rotation can be reversed without reversing the polarity of excitation. In Fig. 8.25, CH_1, CH_4 D_2 and D_3 constitute one Type *C* chopper and CH_2, CH_3, D_1 and D_4 form another Type *C* chopper circuit. Figure 8.25(b) shows Class-E with *R–L* load.

If chopper CH_4 is turned on continuously, the antiparallel connected pair of devices CH_4 and D_4 constitute a short-circuit. Chopper CH_3 may not be turned on at the same time as CH_4 because that would short circuit source E_{dc}.

With CH_4 continuously on, and CH_3 always off, operation of choppers CH_1 and CH_2 will make E_0 positive and I_0 reversible, and operation in the first and second quadrants is possible. On the other hand, with CH_2 continuously on and CH_1 always off, operation of CH_3 and CH_4 will make E_0 negative and I_0 reversible, and operation in the third and fourth quadrants is possible.

The operation of the four-quadrant chopper circuit is explained in detail as follows:

When choppers CH_1 and CH_4 are turned-on, current flows through the path, $E_{dc+} - CH_1$ – load – $CH_4 - E_{dc-}$. Since both E_0 and I_0 are positive, we get the first quadrant operation. When both the choppers CH_1 and CH_4 are turned-off, load dissipates its energy through the path load–$D_3 - E_{dc+} - E_{dc-} - D_2$ – load. In this case, E_0 is negative while I_0 is positive, and fourth-quadrant operation is possible.

When choppers CH_2 and CH_3 are turned-on, current flows through the path, $E_{dc+} - CH_3$ – load – $CH_2 - E_{dc-}$. Since both E_0 and I_0 are negative, we get the third-quadrant operation. When both choppers CH_2 and CH_3 are turned-off, load dissipates its energy through the path load– $D_1 - E_{dc+} - E_{dc-} - D_2$ – load. In this case, E_0 is positive and I_0 is negative, and second-quadrant operation is possible.

This four-quadrant chopper circuit consists of two bridges, forward bridge and reverse bridge. Chopper bridge CH_1 to CH_4 is the forward bridge which permits energy flow from source to load. Diode bridge D_1 to D_4 is the reverse bridge which permits the energy flow from load-to-source. This four-quadrant chopper configuration can be used for a reversible regenerative d.c. drive.

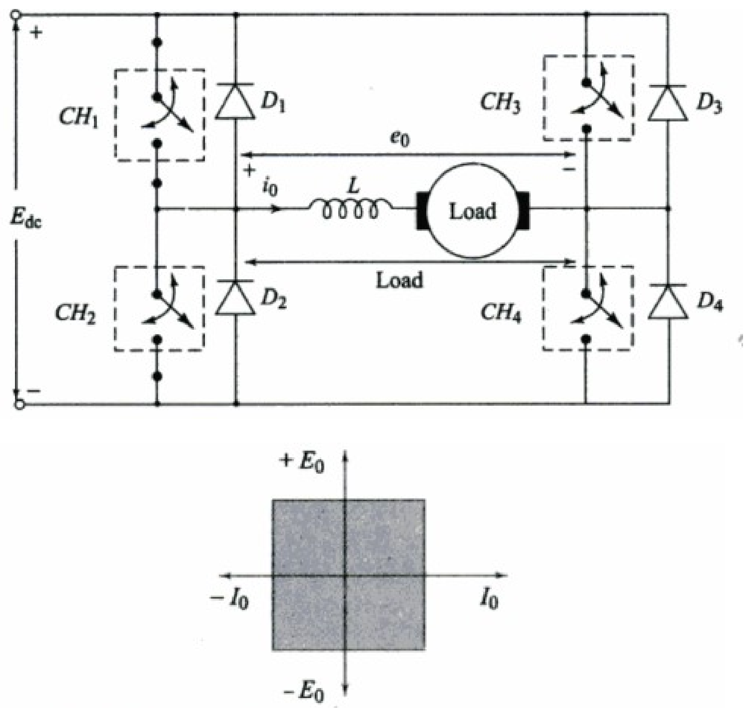


Fig. 8.25(a) *Type E chopper circuit and characteristic*

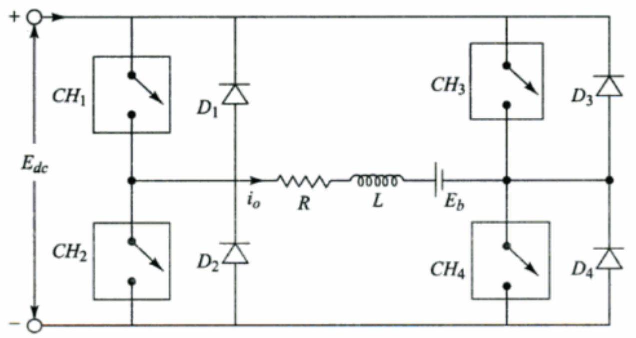


Fig. 8.25(b) *Class E chopper with R–L load*

The bridge type converter shown in Fig. 9.13 a is connected to the armature circuit of a DC motor; it may be supplied with constant voltage u_D from a DC bus or a battery. The converter contains four electronic switches where two in each half-bridge are drawn in the form of a transfer switch (at the same time excluding accidental short circuits of the DC bus); the diodes which can be part of the electronic switches allow an inductive load current to continue during the short protective intervals, when all contacts are open (similar to the red-light-overlap on a signal crossing).

By assigning logic symbols S_1, S_2 to the otherwise ideally assumed switches, the voltage equation of the load circuit is

$$L_a \frac{i_a}{dt} + R_a i_a + e = u_a \,, \tag{9.8}$$

where, depending on the switching state

$$u_a = \frac{1}{2}(S_1 - S_2)\, u_D \text{ and } i_D = \frac{1}{2}(S_1 - S_2)\, i_a \tag{9.9}$$

holds.

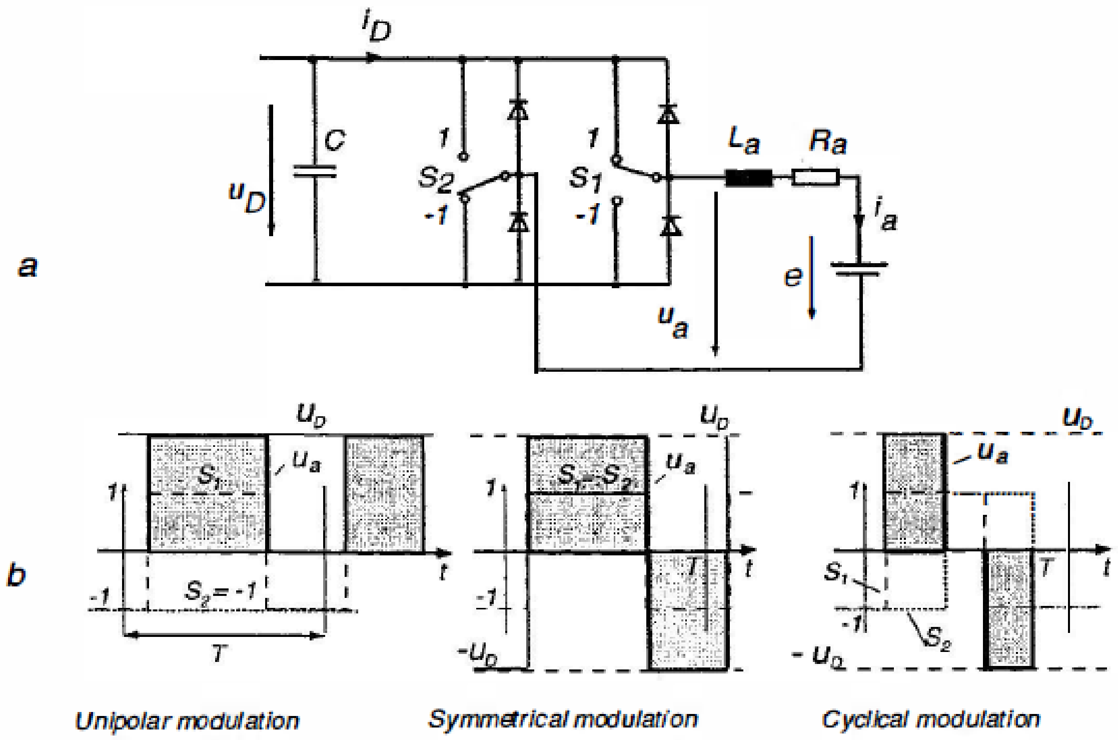


Fig. 9.13. Four-quadrant DC/DC converter with inductive load, **(a)** Circuit, **(b)** Different modulation patterns

The pulse-width-modulation (PWM) of the converter at the frequency $f = 1/T$ can follow different switching strategies, as illustrated in Fig. 9.13 b with the output voltage u_a during a switching period:

- Unipolar modulation
  One of the switches is assumed to be stationary, e.g. S_2 = -1 = const., whereas the other half-bridge is pulse-width-modulated, $S_2 = \pm 1$, so that the output voltage u_a assumes the values u_D or zero; the same applies with $S_1 = -1$ = const. for negative output voltages.
- Symmetrical modulation
  With this modulation pattern the switches are operated in diagonal pairs, S_1 = - S_2, so that the short circuit interval is omitted and the output voltage alternates between the values u_D and -u_D. During the unavoidable (but in Fig. 9.13 neglected) protective intervals the diodes are carrying the load current.
- Cyclical modulation
  With this modulation scheme the two transfer switches are operated sequentially, so that the output is alternatively short circuited at the upper or lower supply bus. Hence the output voltage u_a assumes a ternary waveform, $u_a = u_D$, 0, -u_D. Whereas with symmetrical switching only the mean of the output voltage can be controlled in steady state, the cyclical modulation offers an additional degree of freedom that may for instance be used for eliminating harmonics of the output voltage u_a.

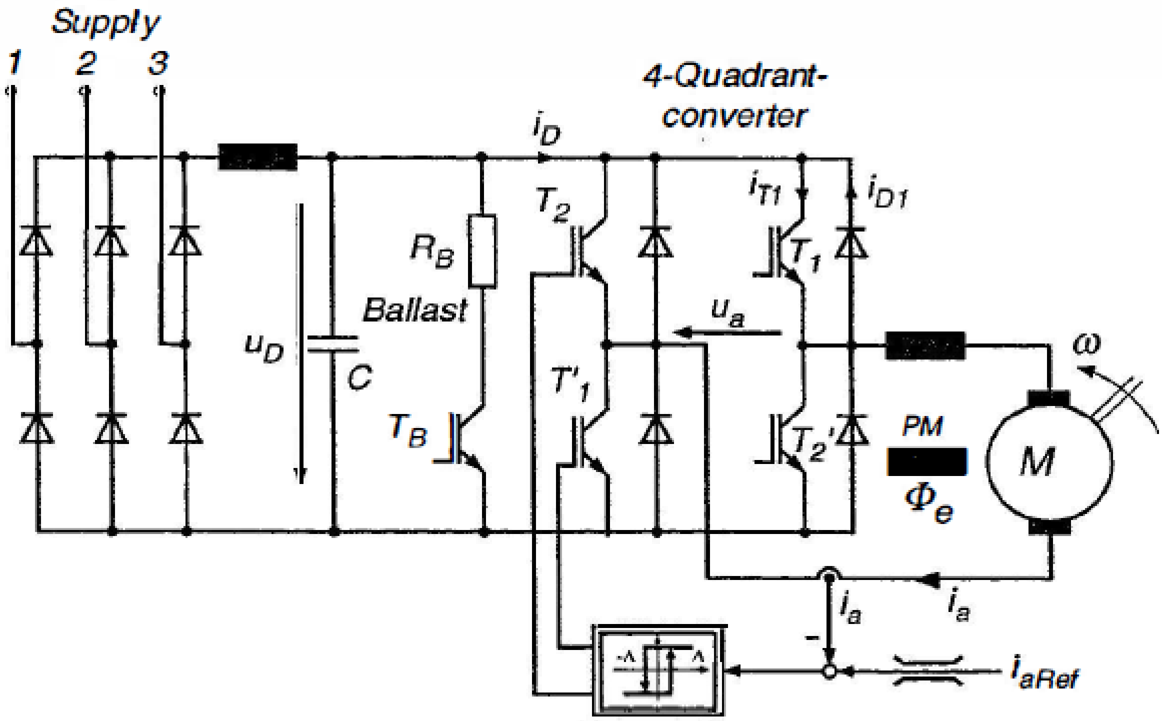


Fig. 9.14. DC Servo drive with voltage source DC/DC converter

A four-quadrant converter with IGBT-switches is depicted in Fig. 9.14 as frequently used for DC servo drives; protective circuitry is again omitted. For simplicity an On/Off device is drawn for current control but a linear current controller with constant frequency PWM would normally be preferred.

A diode rectifier followed by a smoothing filter, whose capacitor C absorbs also the modulation-induced ripple components of the link current i_D serves as the supply of the DC link with constant voltage u_D. For instance, when rapidly braking the drive, power released from the kinetic energy flows back into the DC-link causing negative current i_D and, because of the unidirectional line-side rectifiers, could result in an overcharge of the capacitor; this is prevented by dissipating the energy in a resistive ballast circuit that can also be pulse-width-modulated, depending on the link voltage. In view of the losses this is only practical with small drives or when it happens only occasionally; otherwise a reversible line-side supply (an active front-end converter) is preferable as will be shown in Fig. 9.18 and further discussed in Sect. 13.2.

Some of the steady state waveforms in a converter like the one in Fig. 9.14 are indicated in Fig. 9.15, showing the output voltage u_a alternating between u_D and $-u_D$ and the alternating current components of i_D, which must be absorbed by the capacitor. The control can be arranged as before; an inner current loop controlling the converter via a pulse-width modulator is important for safe operation. The current controller in Fig. 9.14 is again drawn as an On/Off switch, but this is only an illustrative example.

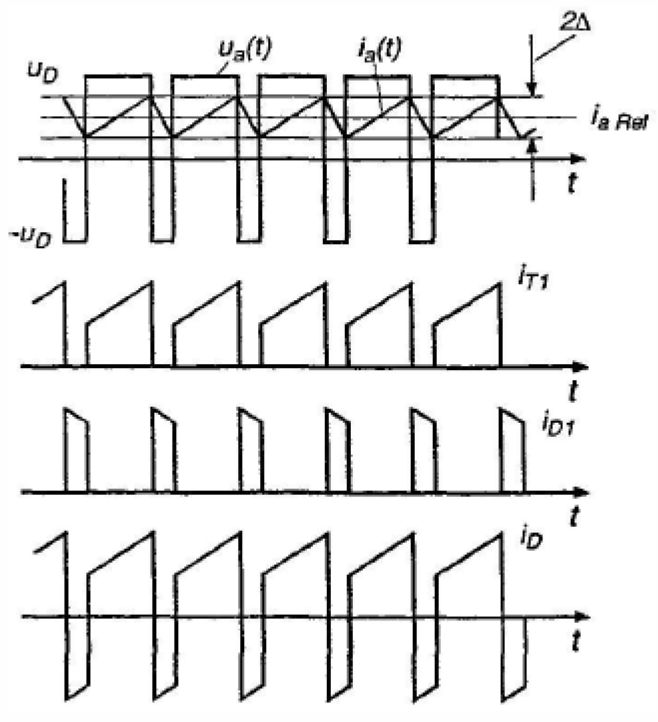
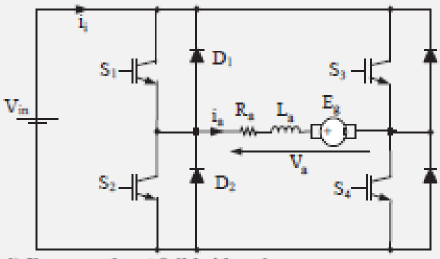
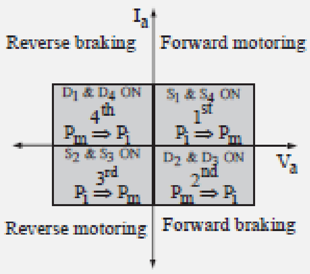
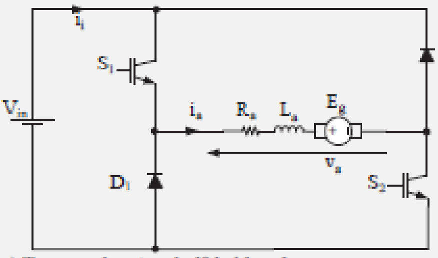
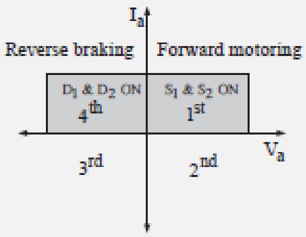


Fig. 9.15. Waveforms of DC/DC converter with symmetrical modulation

The typical response time of the current loop, employing a switched transistor converter in combination with a DC disk motor, is 1 or 2 ms. For many applications this justifies the assumption that the current control loop acts as controllable current source having instantaneous response. Current limit is achieved by limiting the current reference produced by the superimposed speed controller. The next higher level of control could be a position control loop as shown in Fig. 15.9, where the response may be further improved by feed-forward signals from a reference generator.

Transistor converters have the important advantage that they can be switched at frequencies > 5 kHz, thus enlarging the control bandwidth as compared to line-commutated converters. With field effect transistors or IGBT's, the frequency can even be increased beyond the audible threshold > 16 kHz, so that the drive is no longer emitting objectionable acoustic noise.

Table 12.3 DC motor drive systems employing dc–dc converters

dc–dc converter topology	Quadrant(s) of operation
 d) Four-quadrant full-bridge chopper	
 e) Two-quadrant or half-bridge chopper	

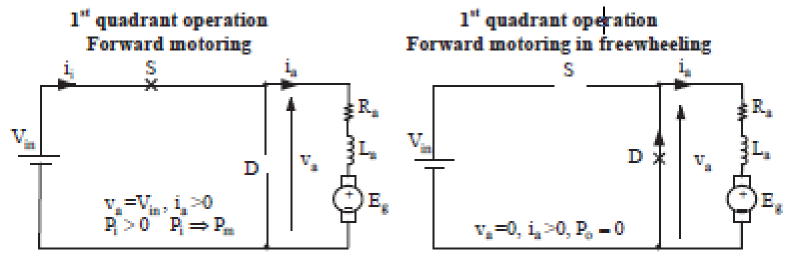


Figure 12.12 Operating modes of the step-down or first quadrant chopper presented in Table 12.3.

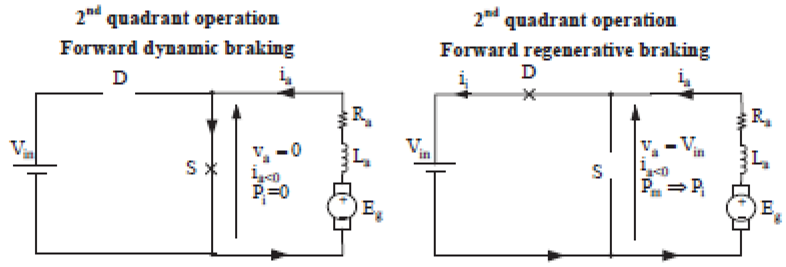


Figure 12.13 Operating modes of the step-up or second quadrant chopper presented in Table 12.3.

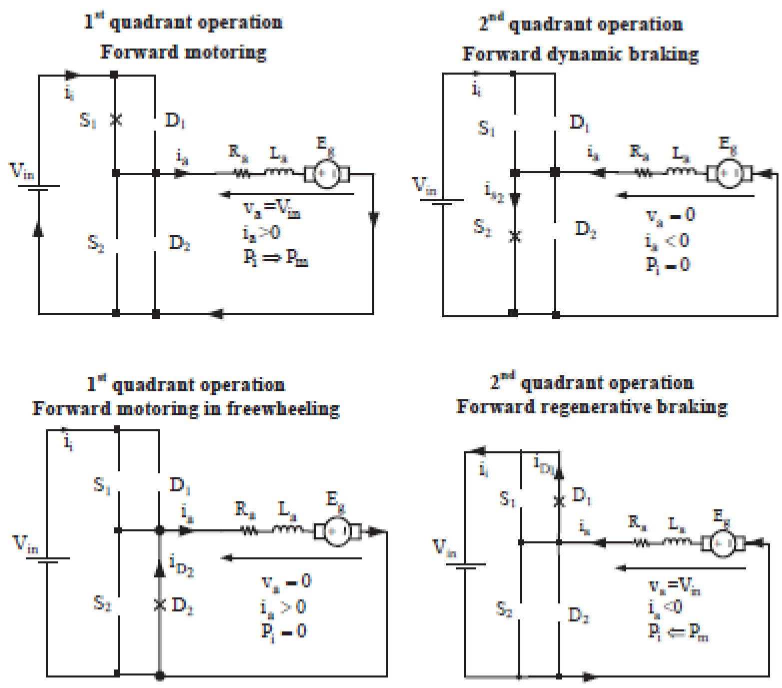


Figure 12.14 Operating modes of the two-quadrant converter presented in Table 12.3.

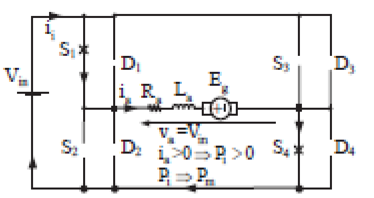
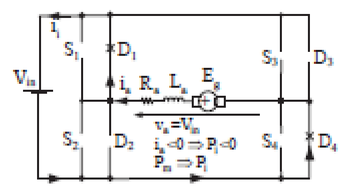
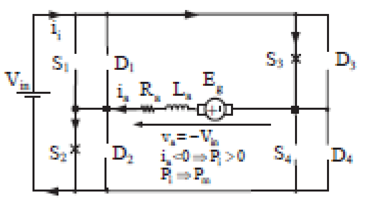
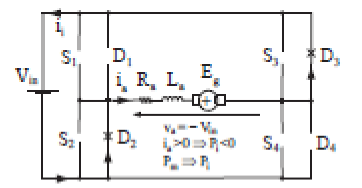
1st quadrant operation	2nd quadrant operation
 During this mode of operation only the switches S_1 and S_4 are conducting applying positive voltage and current to the armature. The machine is this mode operates as a motor with clockwise rotation. Keeping S_1 in conduction and applying PWM to S_4 speed control is achieved. Freewheeling current is achieved through S_1 and D_3.	 During this mode of operation only the diodes D_1 and D_4 are conducting applying positive voltage to the armature but creating a negative flow current. The machine in this mode operates as a generator with clockwise speed and, consequently, a regenerative braking mode is achieved.
3rd quadrant operation	**4th quadrant operation**
 During this mode of operation only the switches S_2 and S_3 are conducting applying negative voltage to the armature and creating a negative armature current. The machine is this mode operates as a motor with counter clockwise rotation. Keeping S_2 in conduction and applying PWM to S_3 speed control is achieved. Freewheeling current is achieved through S_2 and D_4.	 During this mode of operation only the diodes D_2 and D_3 are conducting applying negative voltage to the armature but creating a positive flow of armature current. The machine in this mode operates as a generator with counter clockwise speed and, consequently, a regenerative braking mode is achieved.

Figure 12.15 Operating modes of the full-bridge four-quadrant converter presented in Table 12.3.

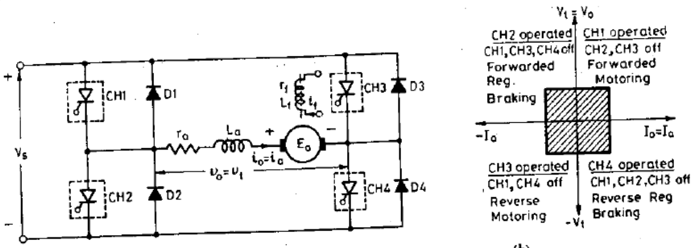
12.5.4. Four-quadrant Chopper Drives

In four-quadrant dc chopper drives, a motor can be made to work in forward-motoring mode (first quadrant), forward regenerative braking mode (second quadrant), reverse motoring mode (third quadrant) and reverse regenerative-braking mode (fourth quadrant). The circuit shown in Fig. 12.24 (*a*) offers four-quadrant operation of a separately-excited dc motor. This circuit consists of four choppers, four diodes and a separately-excited dc motor. Its operation in the four quadrants can be explained as under :

Forward motoring mode. During this mode or first-quadrant operation, choppers CH2, CH3 are kept off, CH4 is kept on whereas *CH1 is operated*. When CH1, CH4 are on, motor voltage is positive and positive armature current rises. When CH1 is turned off, positive armature current free-wheels and decreases as it flows through CH4, D2. In this manner, controlled motor operation in first quadrant is obtained.

Forward regenerative-braking mode. A dc motor can work in the regenerative-braking mode only if motor generated emf is made to exceed the dc source voltage. For obtaining this mode, CH1, CH3 and CH4 are kept off whereas *CH2 is operated.* When CH2 is turned on, negative armature current rises through CH2, D4, E_a, L_a, r_a. When CH2 is turned off, diodes D1, D2 are turned on and the motor acting as a generator returns energy to the dc source. This results in forward regenerative-braking mode in the second-quadrant.

Reverse motoring mode. This operating mode is opposite to forward motoring mode. Choppers CH1, CH4 are kept off, CH2 is kept on whereas *CH3 is operated.* When CH3 and CH2 are on, armature gets connected to source voltage V_s so that both armature voltage V_t and armature current i_a are negative. As armature current is reversed, motor torque is reversed and consequently motoring mode in third quadrant is obtained. When CH3 is turned off, negative armature current freewheels through CH2, D4, E_a, L_a, r_a; armature current decreases and thus speed control is obtained in third quadrant. Note that during this mode, polarity of E_a is opposite to that shown in Fig. 12.24 (a).

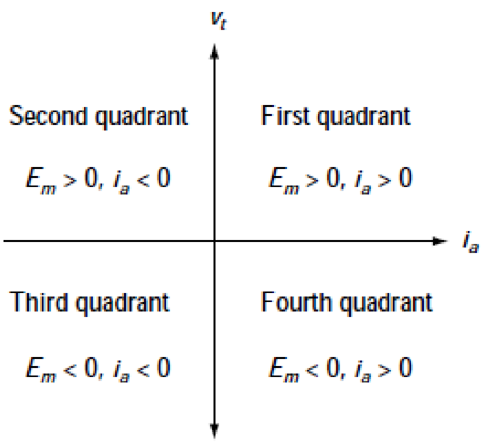


(a) (b)

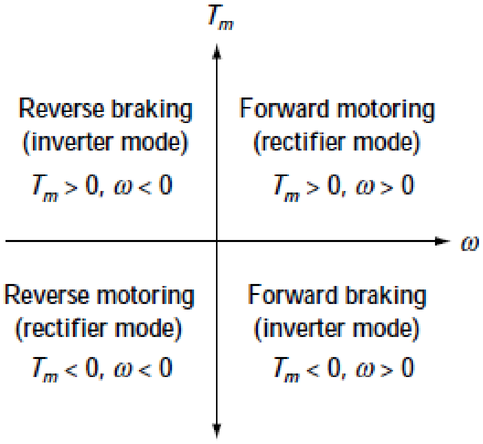
Fig. 12.24. Four-quadrant dc chopper drive (a) circuit diagram and (b) four-quadrant diagram.

Reverse Regenerative-braking mode. As in forward braking mode, reverse regenerative-braking mode is feasible only if motor generated emf is made to exceed the dc source voltage. For this operating mode, CH1, CH2 and CH3 are kept off whereas *CH4 is operated.* When CH4 is turned on, positive armature current i_a rises through CH4, D2, r_a, L_a, E_a. When CH4 is turned off, diodes D2, D3 begin to conduct and motor acting as a generator returns energy to the dc source. This leads to reverse regenerative-braking operation of the dc separately-excited motor in fourth quadrant.

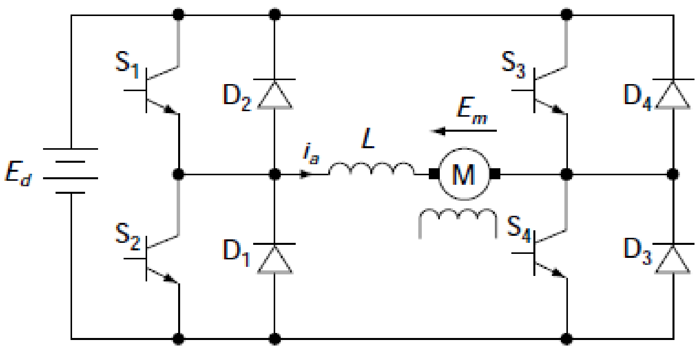
Note that in Fig. 12,24 (a), the numbering of choppers is done to agree with the quadrants in which these are operated. For example, CH1 is operated for first quadrant, ..., CH4 for fourth quadrant etc.



(a) Operation modes by voltage and current polarities



(b) Operation modes by torque and rotating direction



(c) Circuit

12.6 FOUR-QUADRANT dc/dc CONVERTER DRIVE

A simplified schematic diagram of a four-quadrant chopper drive system is shown in Fig. 12.6-1. Typical steady-state waveforms that depict the operation of the converter are shown in Fig. 12.6-2. As the name implies, four-quadrant operation (current versus voltage) is possible. That is, the instantaneous armature current i_a and the instantaneous armature voltage may be positive or negative. In fact, four-quadrant operation is depicted in each switching period in Fig. 12.6-2. In particular, I_1 is negative and I_2 is positive and v_a is v_S during interval A and $-v_S$ during interval B; however, the average v_a and the average i_a are positive. Therefore, from an average-value

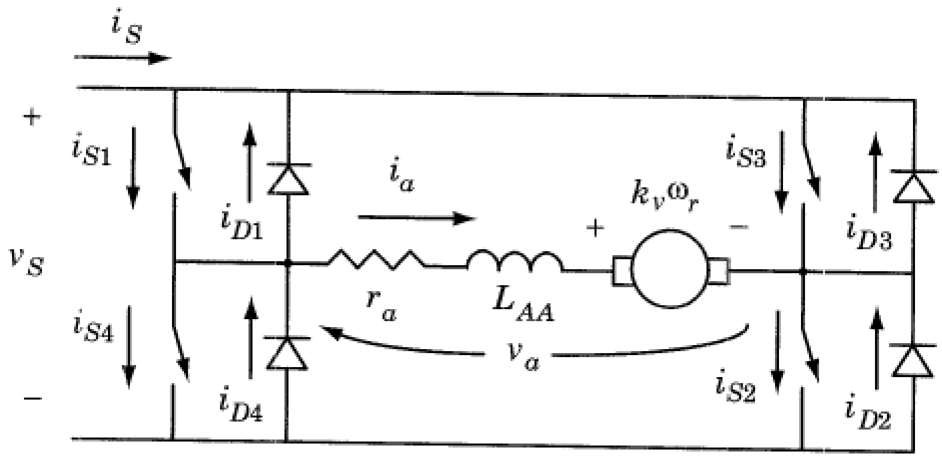


Figure 12.6-1 Four-quadrant chopper drive system.

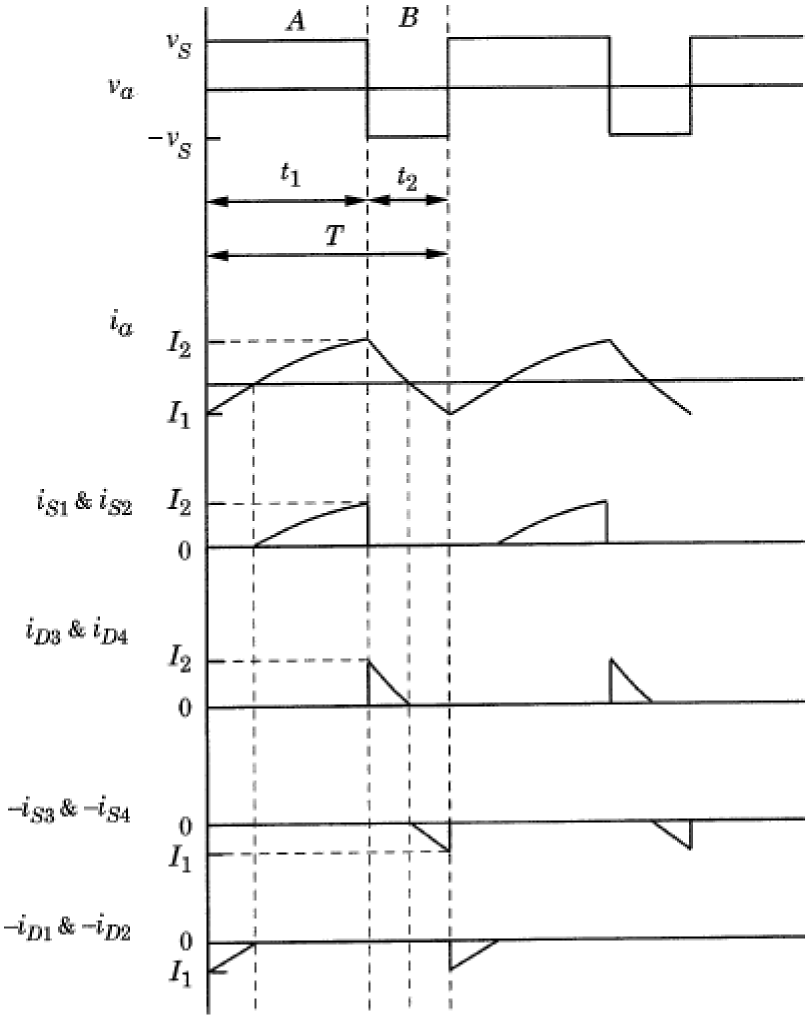


Figure 12.6-2 Typical waveforms for steady-state operation of a four-quadrant chopper drive.

point of view, the dc drive system depicted in Fig. 12.6-2 is operating as a motor if the rotor speed ω_r is positive (ccw). This is 1st quadrant operation of an average-current versus average-voltage plot even though four-quadrant operation of i_a versus v_a occurs each switching period. One must distinguish between four-quadrant operation during a period and four-quadrant average-value operation.

If v_a is positive and i_a is negative (average or instantaneous), then operation is in the 4th quadrant of an i_a versus v_a plot; and if ω_r is positive (ccw), the machine is operating as a generator. In the 2nd quadrant, v_a is negative and i_a is positive; and if ω_r is negative (cw), we have generator operation. In the 3rd quadrant, v_a is negative and i_a is negative; and if ω_r is negative (cw), we have motor operation. It is important to emphasize that one-, two-, and four-quadrant chopper operation has been defined from the plot of i_a versus v_a. However, the T_e versus ω_r plot is also used to define drive operation where the 1st and 3rd quadrants depict motor operation and the 2nd and 4th quadrants depict generator operation. At first glance, one might assume that the quadrants of the i_a versus v_a plot can be assigned the same modes of operation. Actually this is only the case if ω_r is positive (ccw) when v_a is positive and negative (ccw) when v_a is negative as stipulated above. In order to illustrate this, let us assume that v_a and i_a are both positive and ω_r is positive (ccw); the machine is operating as a motor, and operation is in the 1st quadrant of i_a versus v_a and in the 1st quadrant of T_e versus ω_r. If v_a and i_a are positive and ω_r is zero, the power $(v_a i_a)$ is being dissipated in r_a and the machine is neither a motor nor a generator. If, however, ω_r is made slightly negative by supplying an input torque, v_a and i_a can still both be positive (1st quadrant) and yet generator action is occurring because T_e is positive and ω_r is negative (2nd quadrant of T_e versus ω_r). Therefore, we must know the direction (sign) of ω_r when assigning motor or generator action to the four quadrants of the i_a versus v_a plot.

There are numerous switching strategies that might be used with a four-quadrant chopper. The switching depicted in Fig. 12.6-2 is perhaps one of the least involved. In this case, there are only two states. In the first state, which occurs over interval A, $S1$ and $S2$ are closed and $S3$ and $S4$ are open. The second state occurs over interval B, wherein the $S3$ and $S4$ are closed and $S1$ and $S2$ are open. As in the case of the previous dc/dc converters, we will consider the switches and diodes as being ideal.

During interval A, $S1$ and $S2$ are closed and $S3$ and $S4$ are open. At the beginning of the interval, i_a is negative (I_1) in Fig. 12.6-2. Because $S1$ and $S2$ cannot carry negative armature current, I_1 must flow through diodes $D1$ and $D2$. Note in Fig. 12.6-2 that $-i_{D1}$, $-i_{D2}$, $-i_{S3}$, and $-i_{S4}$ are plotted for the purpose of a direct comparison with i_a. During interval A, the armature voltage v_a is v_S; and because v_S is larger than the counter emf, the armature current increases from the negative value of I_1 toward zero. During this part of the interval, the source current is $-i_{D1}$, which is also $-i_{D2}$. When i_a reaches zero, $D1$ and $D2$ block positive armature current flow; however, $S1$ and $S2$ are closed ready to carry a positive i_a. Hence, the current increases from zero to I_2 through $S1$ and $S2$. During this part of the interval, the source current i_S is i_{S1}, which is also i_{S2}.

During interval B, v_a is $-v_S$ and are $S1$ and $S2$ are open with $S3$ and $S4$ closed. At the beginning of interval B, i_a is positive (I_2); however, $S3$ and $S4$ cannot conduct a positive armature current. Hence at the beginning of interval B the positive I_2 flows through diodes $D3$ and $D4$. This continues until i_a is driven to zero by $-v_S$. During this part of interval B, the source current i_S is $-i_{D3}$ or $-i_{D4}$. When i_a reaches zero, diodes $D3$ and $D4$ block negative i_a; thus, $S3$ and $S4$ carry the negative armature current to the end of interval B where $i_a = I_1$, which is negative. During this part of interval B, the source current i_S is i_{S3} or i_{S4}. We have completed a switching cycle.

Expressions for I_1 and I_2 can be derived by a procedure similar to that used in the case of the previous choppers. It can be shown that

$$I_1 = \frac{v_s}{r_a}\left[\frac{2e^{-(1-k)T/\tau_a} - e^{-T/\tau_a} - 1}{1 - e^{-T/\tau_a}}\right] - \frac{k_v\omega_r}{r_a} \tag{12.6-1}$$

$$I_2 = \frac{v_s}{r_a}\left[\frac{1 - 2e^{-kT/\tau_a} + e^{-T/\tau_a}}{1 - e^{-T/\tau_a}}\right] - \frac{k_v\omega_r}{r_a} \tag{12.6-2}$$

If k and v_S do not change significantly from one switching period to the next, the average armature voltage may be expressed as

$$\begin{aligned}\bar{v}_a &= \frac{1}{T}\left[\int_0^{kT} v_S\, d\xi + \int_{kT}^{T} -v_S\, d\xi\right]\\ &= \frac{1}{T}[kTv_S - (1-k)Tv_S]\\ &= (2k-1)v_S \qquad (12.6\text{-}3)\end{aligned}$$

Note that when $k = 0$, $v_a = -v_S$ and when $k = 1$, $v_a = v_S$. It is clear that the time-domain block diagram for the four-quadrant chopper drive is the same as that shown in Fig. 12.5-3 for the two-quadrant chopper drive with $v_a = kv_S$ replaced with $v_a = (2k-1)v_S$ which is (12.6-3).

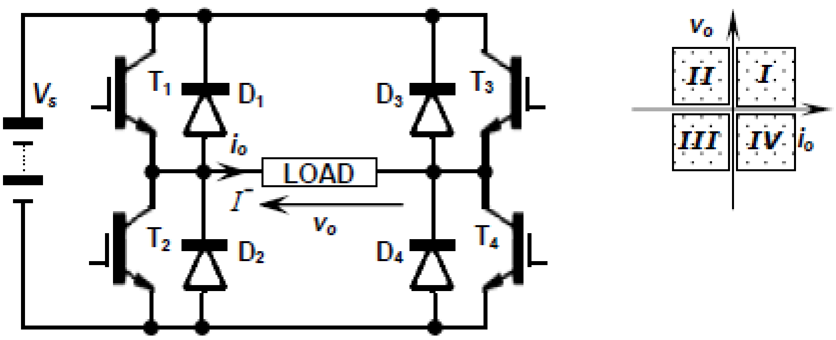


Figure 13.13. *Four-quadrant dc chopper circuit, showing first quadrant i_o and v_o references.*

13.6 Four-quadrant dc chopper

The four-quadrant H-bridge dc chopper is shown in figure 13.13 where the load current and voltage are referenced with respect to T_1, so that the quadrant of operation with respect to the switch number is persevered.
The H-bridge is a flexible basic configuration where its use to produce single-phase ac is considered in chapter 14.1.1, while its use in smps applications is considered in chapter 15.8.2. It can also be used as a dc chopper for the four-quadrant control of a dc machine.
With the flexibility of four switches, a number of different control methods can be used to produce four-quadrant output voltage and current (bidirectional voltage and current). All practical methods should employ complementary device switching in each leg (either T_1 or T_4 on but not both and either T_2 or T_3 on, but not both) so as to minimise distortion by ensuring current continuity around zero current output.
One control method involves controlling the H-bridge as two virtually independent two-quadrant choppers, with the over-riding restriction that no two switches in the same leg conduct simultaneously. One chopper is formed with T_1 and T_4 grouped with D_2 and D_3, which gives positive current i_o but bidirectional voltage $\pm v_o$ (QI and QIV operation). The second chopper is formed by grouping T_2 and T_3 with D_1 and D_4, which gives negative output current $-i_o$, but bi-direction voltage $\pm v_o$ (QII and QIII operation).

The second control method is to unify the operation of all four switches within a generalised control algorithm.

With both control methods, the chopper output voltage can be either multilevel or bipolar, depending on whether zero output voltage loops are employed or not. Bipolar output states increase the ripple current magnitude, but do facilitate faster current reversal, without crossover distortion. Operation is independent of the direction of the output current i_o.
Since the output voltage is reversible for each control method, a triangular based modulation control method, as used with the asymmetrical H-bridge dc chopper in figure 13.9, is applicable in each case. Two generalised unified H-bridge control approaches are considered.

13.6.1 Unified four-quadrant dc chopper - bipolar voltage output switching

The simpler output to generate is bipolar output voltages, which use one reference carrier triangle as shown in figure 13.14 parts (c) and (d). The output voltage switches between + V_s and – V_s and the relative duration of each state depends on the magnitude of the modulation index δ.

If δ = 0 then T_1 and T_4 never turn-on since T_2 and T_3 conduct continuously which impresses – V_s across the load.

At the other extreme, **if δ = 1** then T_1 and T_4 are on continuously and V_s is impressed across the load.

If δ = ½ then T_1 and T_4 are turned on for half of the period T, while T_2 and T_3 are on for the remaining half of the period. The output voltage is – V_s for half of the time and + V_s for the remaining half of any period. The average output voltage is therefore zero, but disadvantageously, the output current needlessly ripples about zero (with an average value of zero).

The chopper output voltage is defined in terms of the triangle voltage reference level v_Δ by

- $v_\Delta > \delta,\ v_o = -V_s$
- $v_\Delta < \delta,\ v_o = +V_s$

From figure 13.14c and d, the average output voltage varies linearly with δ such that

$$\begin{aligned}\overline{V}_o &= \frac{1}{T}\left(\int_0^{t_T} +V_s\,dt + \int_{t_T}^{T} -V_s\,dt\right)\\ &= \frac{1}{T}(2t_T - T)V_s = \left(2\frac{t_T}{T} - 1\right)V_s\end{aligned} \qquad (13.105)$$

Examination of figures 13.14c and d reveals that the relationship between t_T and δ must produce

$$\begin{aligned}&\text{when } \delta = 0,\ t_T = 0 \text{ and } v_o = -V_s\\ &\text{when } \delta = ½,\ t_T = ½T \text{ and } v_o = 0\\ &\text{when } \delta = 1,\ t_T = T \text{ and } v_o = +V_s\end{aligned}$$

that is

$$\delta = \frac{t_T}{T}$$

which on substituting for t_T / T in equation (13.105) gives

$$\overline{V}_o = \left(2\frac{t_T}{T} - 1\right) V_s \qquad (13.106)$$
$$= (2\delta - 1)\, V_s \quad \text{for} \quad 0 \le \delta \le 1$$

The average output voltage can be positive or negative, depending solely on δ. No current discontinuity occurs since the output voltage is never zero. Even when the average is zero, ripple current flows though the load.

The rms output voltage is independent of the duty cycle and is V_s.

The output ac ripple voltage is

$$V_r = \sqrt{V_{rms}^2 - V_o^2}$$
$$= \sqrt{V_s^2 - (2\delta - 1)V_s^2} = 2\,V_s\sqrt{\delta(1-\delta)} \qquad (13.107)$$

The ac ripple voltage is zero at $\delta = 0$ and $\delta = 1$, when the output voltage is pure dc, namely - V_s or V_s, respectively. The maximum ripple voltage occurs at $\delta = ½$, when $V_r = V_s$.

The output ripple factor is

$$RF = \frac{V_r}{V_o} = \frac{2\,V_s\sqrt{\delta(1-\delta)}}{(2\delta-1)V_s}$$
$$= \frac{2\sqrt{\delta(1-\delta)}}{(2\delta-1)} \qquad (13.108)$$

Circuit operation is characterized by two time domain equations

During the **on-period for T1 and T4**, when $v_o(t)=V_s$

$$L\frac{di_o}{dt} + R\,i_o + E = V_s$$

which yields

$$i_o(t) = \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le t_T \qquad (13.109)$$

During the **on-period for T2 and T3**, when $v_o(t)= - V_s$

$$L\frac{di_o}{dt} + R\,i_o + E = -V_s$$

which, after shifting the zero time reference to t_T, gives

$$i_o(t) = -\frac{V_s + E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\,e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le T - t_T \qquad (13.110)$$

The initial conditions $\hat{I}$ and $\check{I}$ are determined by using the usual steady-state boundary condition method.

$$\text{where} \quad \hat{I} = \frac{V_s}{R}\,\frac{1 - 2e^{\frac{-t_T}{\tau}} + e^{\frac{-T}{\tau}}}{1 - e^{\frac{-T}{\tau}}} - \frac{E}{R} \quad \text{(A)}$$

$$\text{and} \quad \check{I} = \frac{V_s}{R}\,\frac{2e^{\frac{t_T}{\tau}} - 1 + e^{\frac{-T}{\tau}}}{1 - e^{\frac{-T}{\tau}}} - \frac{E}{R} \quad \text{(A)} \qquad (13.111)$$

Example 13.7: *Four-quadrant dc chopper*

The H-bridge, dc-to-dc chopper in figure 13.13 feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated with a 200Hz multilevel carrier as in figure 13.14 a and b, with a modulation depth of $\delta = ¼$, determine:

i. the average output voltage and switch T_1 on-time
ii. the rms output voltage and ac ripple voltage
iii. the average output current, hence quadrant of operation
iv. the electromagnetic power being extracted from the back emf E.

If the mean load current is to be halved, what is

v. the modulation depth, δ, requirement
vi. the average output voltage and the corresponding switch T_1 on-time
vii. the electromagnetic power being extracted from the back emf E?

Solution

The main circuit and operating parameters are

- modulation depth $\delta = ¼$
- period $T_{carrier} = 1/f_{carrier} = 1/200\text{Hz} = 5\text{ms}$
- E=55V and V_s=340V dc
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

i. The average output voltage is given by equation (13.114), and for $\delta < ½$,

$$\overline{V}_o = \left(\frac{t_T}{T} - 1\right)V_s = (2\delta - 1)V_s$$
$$= 340\text{V}\times(2\times¼ - 1) = -170\text{V}$$

where

$$t_T = 2\delta T = 2\times¼\times(½\times5\text{ms}) = 1.25\text{ms}$$

Figure 13.14 reveals that the carrier frequency is half the switching frequency, thus the 5ms in the above equation has been halved. The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 3.75ms.

ii. The rms load voltage, from equation (13.118), is

$$V_{rms} = \sqrt{1-2\delta}\;V_s$$
$$= 340\text{V}\times\sqrt{1 - 2\times¼} = 240\text{V rms}$$

From equation (13.119), the output ac ripple voltage is

$$V_r = \sqrt{2}\,V_s\sqrt{\delta(1-2\delta)}$$
$$=\sqrt{2}\times 340\text{V}\times\sqrt{¼\times(1 - 2\times¼)} = 170\text{V ac}$$

iii. The average output current is given by equation (13.117)

$$\overline{I}_o = \frac{\overline{V}_o - E}{R} = \frac{(2\delta-1)V_s - E}{R}$$
$$= \frac{340\text{V}\times(2\times¼ - 1) - 55\text{V}}{10\Omega} = -22.5\text{A}$$

Since both the average output current and voltage are negative (-170V and -22.5A) the chopper with a modulation depth of $\delta = ¼$, is operating in the third quadrant.

iv. The electromagnetic power developed by the back emf E is given by

$$P_E = E\overline{I}_o = 55\text{V}\times(-22.5\text{A}) = -1237.5\text{W}$$

v. The average output current is given by

$$\overline{I}_o = {(\overline{V}_o - E)}/{R} = {((2\delta-1)V_s - E)}/{R}$$

when the mean current is -11.25A, $\delta = 0.415$, as derived in part vi.

vi. Then, if the average current is halved to -11.25A

$$\overline{V}_o = E + \overline{I}_o R$$
$$=55\text{V} - 11.25\text{A}\times 10\Omega = -57.5\text{V}$$

The average output voltage rearranged in terms of the modulation depth δ gives

$$\delta = ½\left(1+\frac{\overline{V}_o}{V_s}\right)$$
$$= ½\times\left(1 + \frac{-57.5\text{V}}{340\text{V}}\right) = 0.415$$

The switch on-time when $\delta < ½$ is given by

$$t_T = 2\delta T = 2\times 0.415\times(½\times 5\text{ms}) = 2.07\text{ms}$$

From figure 13.14b both T_1 and T_4 are turned on for 2.07ms, although, from table 13.3B, for negative load current, $\overline{I}_o = -11.25$A, the parallel connected freewheel diodes D_1 and D_4 conduct alternately, rather than the switches (assuming $\hat{I}_o < 0$). The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 2.93ms.

vii. The electromagnetic power developed by the back emf E is halved and is given by

$$P_E = E\overline{I}_o = 55\text{V}\times(-11.25\text{A}) = -618.75\text{W}$$

If the output current never goes positive, that is $\hat{I}$ is negative, then T_1, T_4, D_2, and D_3 do not conduct, thus do not appear in the output device sequence. The conducting sequence is as shown in table 13.3B for $\hat{I} < 0$.

Unlike the bipolar control method, the output sequence is affected by the average output voltage level, as well as the polarity of the output current swing. The transition between the six possible sequences due to load voltage and current polarity changes, is seamless. The only restriction is that devices in any leg do not conduct simultaneously. This is ensured by inserting a brief dead-time between a switch turning off and its leg complement being turned on.

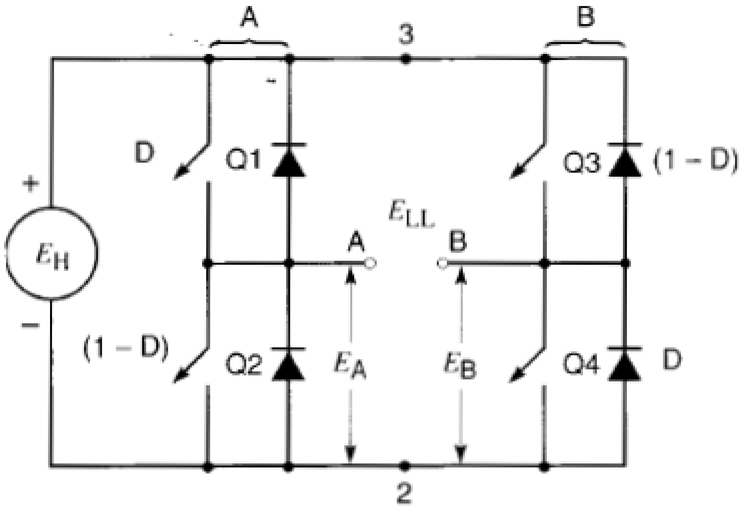


Figure 21.70
Four-quadrant dc-to-dc converter.

21.42 Four-quadrant dc-to-dc converter

The 2-quadrant converter we have studied can only be used with a load whose voltage has a specific polarity. Thus, in Fig. 21.69, given the polarity of E_H, terminal 1 can only be (+) with respect to terminal 2. We can overcome this restriction by means of a *4-quadrant converter.* It consists of two identical 2-quadrant converters arranged as shown in Fig. 21.70. Switches Q1, Q2 in converter arm A open and close alternately, as do switches Q3, Q4 in converter arm B. The switching frequency (assumed to be 100 kHz) is the same for both. The switching sequence is such that Q1 and Q4 open and close simultaneously. Similarly, Q2 and Q3 open and close simultaneously. Consequently, if the duty cycle for Q1 is D, it will also be D for Q4. It follows that the duty cycle for Q2 and Q3 is $(1 - D)$.

The dc voltage E_A appearing between terminals A, 2 is given by

$$E_A = DE_H$$

The dc voltage E_B between terminals B, 2 is

$$E_B = (1 - D)E_H$$

The dc voltage E_{LL} between terminals A and B is the difference between E_A and E_B:

$$\begin{aligned} E_{LL} &= E_A - E_B \\ &= DE_H - (1 - D)E_H \end{aligned}$$

thus

$$E_{LL} = E_H (2D - 1) \qquad (21.24)$$

Equation 21.24 indicates that the dc voltage is zero when $D = 0.5$. Furthermore, the voltage changes linearly with D, becoming $+E_H$ when $D = 1$, and $-E_H$ when $D = 0$. The polarity of the output voltage can therefore be either positive or negative. Moreover, if a device is connected between terminals A, B, the direction of dc current flow can be either from A to B or from B to A. Consequently, the converter of Fig. 21.70 can function in all four quadrants.

The *instantaneous* voltages E_{A2} and E_{B2} oscillate constantly between zero and $+E_H$. Fig. 21.71 shows the respective waveshapes when $D = 0.5$. Similarly, Fig. 21.72 shows the waveshapes when $D = 0.8$. Note that the instantaneous voltage E_{AB} between the output terminals A, B oscillates between $+E_H$ and $-E_H$. In practice, the alternating components that appear between terminals A, B are filtered out. Consequently, only the dc component E_{LL} remains as the active driving emf across the external device connected to terminals A, B.

Consider, for example, the block diagram of a converter feeding dc power to a passive load R (Fig. 21.73). The power is provided by source E_H. As we have seen, the magnitude and polarity of E_{LL} can be varied by changing the duty cycle D. The switching frequency f of several kilohertz is assumed to be constant. Inductor L and capacitor C act as filters so that the dc current flowing in the resistance has negligible ripple. Because the switching frequency is high, the inductance and capacitance can be small, thus making for inexpensive filter components.

The dc currents and voltages are related by the power-balance equation $E_H I_H = E_{LL} I_L$. We neglect the switching losses and the small control power associated with the D and f input signals.

Fig. 21.74 shows the converter connected to an active device E_0, which could be either a source or a load. If need be, the polarity of E_0 could be the reverse of that shown.

In all these applications we can force power to flow from E_H to E_0, or vice versa, by simply adjusting the duty cycle D. This 4-quadrant dc-to-dc converter is therefore an extremely versatile device.

The inductor L is a crucially important part of the converter. It alone is able to absorb energy at one voltage level (high or low) and release it at another voltage level (low or high). And it performs this duty automatically, in response to the electronic switches and their duty cycle.

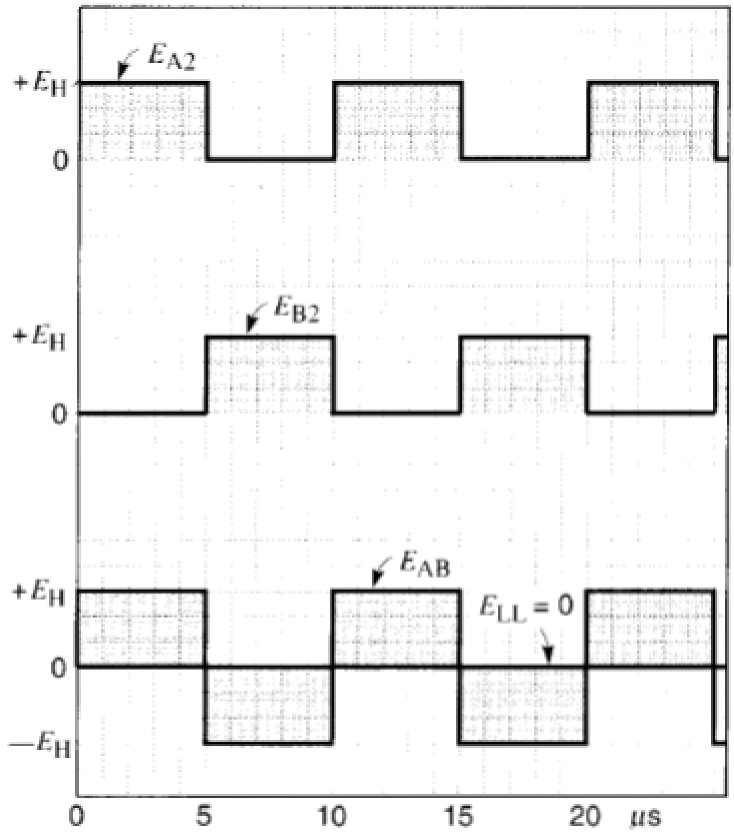


Figure 21.71
Voltage output when $D = 0.5$. The average voltage is zero.

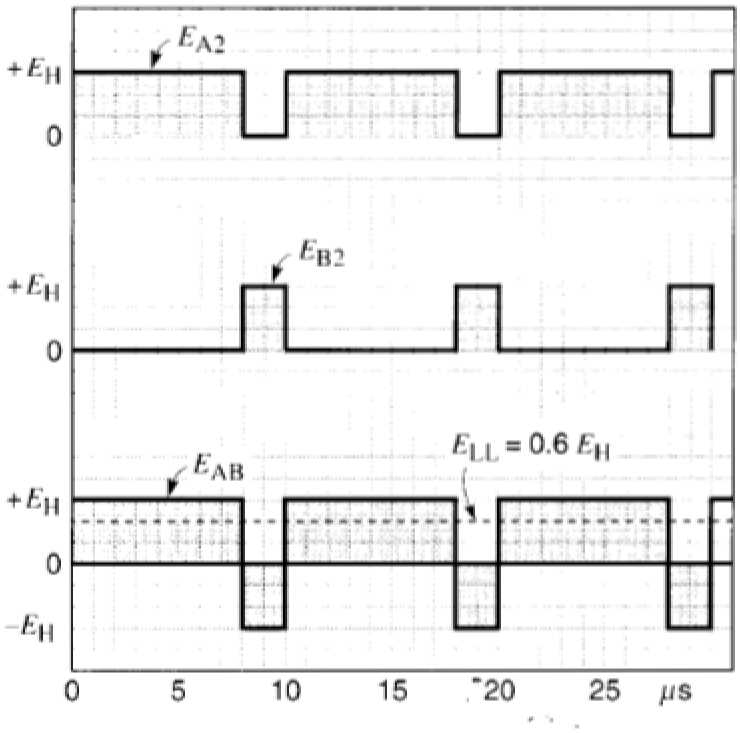


Figure 21.72
Voltage output when $D = 0.8$. The average voltage E_{LL} is 0.6 E_H.

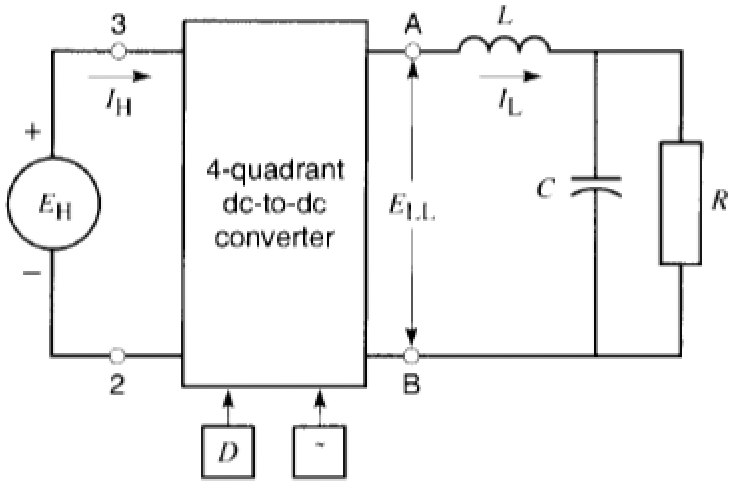


Figure 21.73
Four-quadrant dc-to-dc converter feeding a passive dc load R.

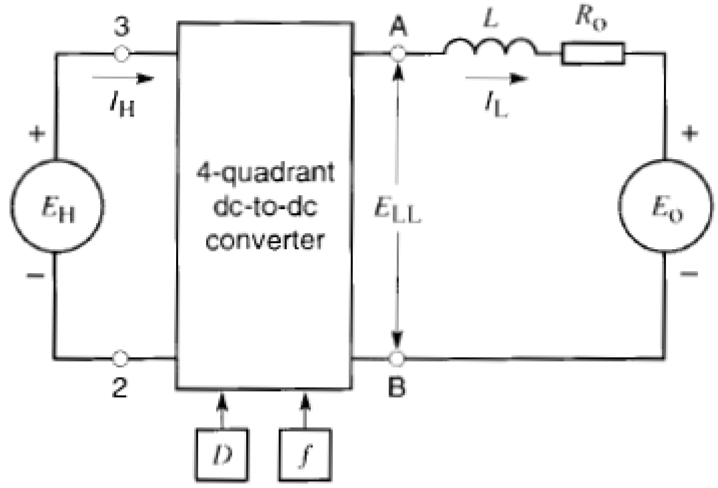


Figure 21.74
Four-quadrant dc-to-dc converter feeding an active dc source/sink E_o.

Example 22-7

A 25 hp, 250 V, 900 r/min dc motor is connected to a dc-to-dc converter that operates at a switching frequency of 2 kHz. The converter is fed by a 6-pulse rectifier connected to a 240 V, 3-phase, 60 Hz line (Fig. 22.21a). A 500 μF capacitor C and an inductor L_d act as filters. The armature resistance and inductance are respectively 150 mΩ and 4 mH. The rated dc armature current is 80 A. We wish to determine the following:

a. The required duty cycle when the motor develops its rated torque at rated speed
b. The waveshape of currents I_1, I_2, and I_a
c. The waveshape of voltages E_{12} and E_{AB}.

V

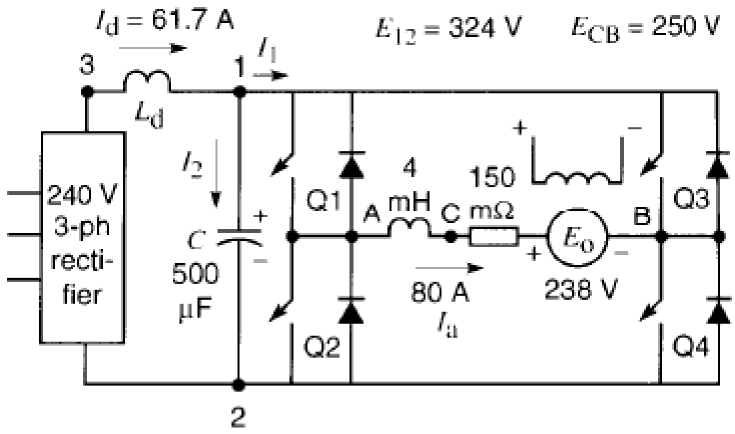


Figure 22.21a
See Example 22-7.

thus,

$$250 = 324\,(2\,D - 1)$$

and so,

$$D = 0.886$$

The 250 V appears between terminals A, B (Fig. 22.21a).

Because the motor develops rated torque, the armature draws its rated current, namely 80 A. The voltage drop in the armature resistance is

$$80\text{ A} \times 0.15\ \Omega = 12\text{ V}$$

The induced armature voltage, or counter emf, at 900 r/min is, therefore,

$$E_o = 250 - 12 = 238 \text{ V}$$

The dc power input to the motor is

$$P = 250 \text{ V} \times 80 \text{ A} = 20\,000 \text{ W}$$

Neglecting the losses in the converter, and recalling that the dc output of the rectifier is 324 V, it follows that current I_d is given by

$$324\, I_d = 20\,000$$
$$I_d = 61.7 \text{ A}$$

The frequency of the converter is 2 kHz and so the period of one cycle is

$$T = 1/f = 1/2000 = 500\ \mu\text{s}$$

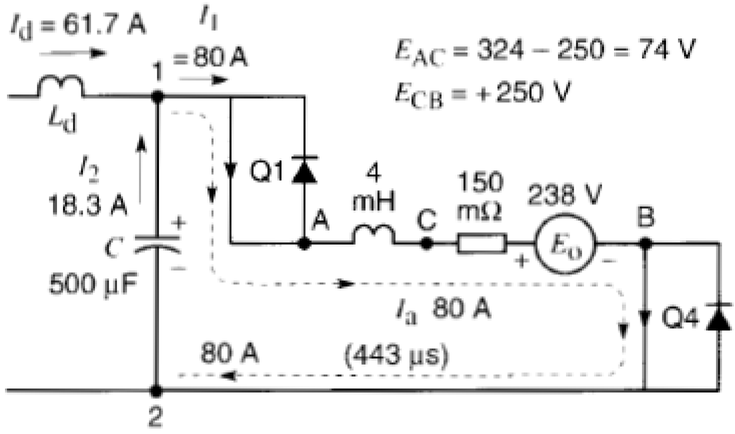


Figure 22.21b
Circuit when Q1 and Q4 are "on." Current I_a is increasing. $E_{CA} = -74$ V.

The *on* and *off* times of Q1 (and Q4) are, respectively,

$$T_a = DT = 0.886 \times 500 = 443\ \mu\text{s}$$
$$T_b = 500 - 443 = 57\ \mu\text{s}$$

It follows that the corresponding *on* and *off* times of Q2 (and Q3) are 57 μs and 443 μs.

We recall that Q1 and Q4 operate simultaneously, followed by Q2 and Q3, which also open and close simultaneously.

When Q1 and Q4 are conducting, the armature current follows the path shown in Fig. 22.21b. This lasts for 443 μs and during this time I_1 (= 80 A) flows in the positive direction. Note however, that the rectifier only furnishes 61.7 A, whereas the armature current is 80 A. It follows that the difference (80 − 61.7) = 18.3 A must come from the capacitor. The capacitor discharges, causing the voltage across it to drop by an amount ΔE given by

$$\Delta E = Q/C = 18.3 \text{ A} \times 443 \text{ μs}/500 \text{ μF} = 16 \text{ V}$$

Q1 and Q4 then open for 57 μs. During this interval Q2 and Q3 are closed (Fig. 22.21c), but they cannot carry the armature current because it is flowing opposite to the direction permitted by these IGBTs. However, the current *must* continue to flow because of the armature inductance. Fortunately, a path is offered by the diodes D2 and D3 associated with Q2 and Q3, as shown in the figure. Note that I_1

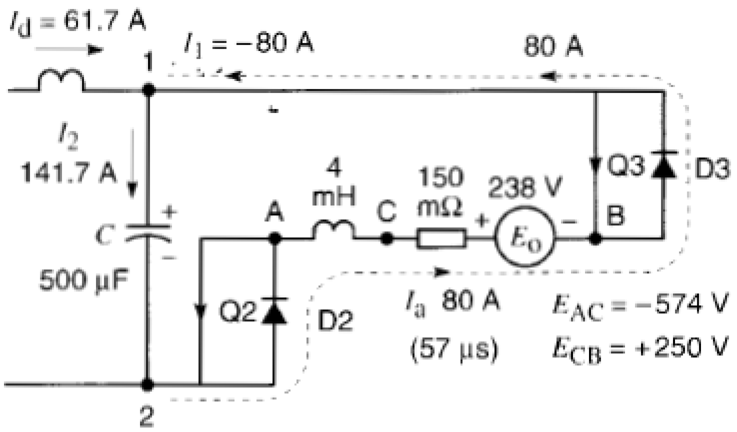


Figure 22.21c
Circuit when D2 and D3 are conducting. Current I_a is decreasing.

(= 80 A) now flows toward terminal 1, which is opposite to the direction it had in Fig. 22.21b.

Meanwhile, current I_d furnished by the rectifier continues to flow unchanged because of the presence of inductor L_d. As a result, by Kirchhoff's current law, the current I_2 must flow into the capacitor and its value is (80 + 61.7) = 141.7 A. This highlights the absolute necessity of having a capacitor in the circuit. Without it, the flow of armature current would be inhibited during this 57 μs interval. The capacitor charges up and the increase in voltage ΔE is given by

$$\Delta E = Q/C = 141.7 \text{ A} \times 57 \text{ μs}/500 \text{ μF} = 16 \text{ V}$$

Note that the increase in voltage across the capacitor during the 57 μs interval is exactly equal to the decrease during the 443 μs interval. The peak-to-peak ripple across the capacitor is, therefore, 16 V. Thus, the voltage between points **1** and **2** fluctuates between (324 + 8) = 332 V and (324 − 8) = 316 V. This 2.5 percent fluctuation does not affect the operation of the motor.

Let us now look more closely at the armature current, particularly as regards the ripple. In Fig. 22.21b the voltage across the armature inductance can be found by applying KVL:

$$E_{AC} + E_{CB} + E_{B2} + E_{21} + E_{1A} = 0$$
$$E_{AC} + 250 + 0 - 324 + 0 = 0$$

Hence

$$E_{AC} = 74 \text{ V}$$

Therefore, the volt seconds accumulated during this 443 μs interval is 74 × 443 = 32 782 μs·V. The resulting increase in armature current ΔI_a is

$$\Delta I_a = A/L_a = 32\,782 \times 10^{-6}/0.004 = 8 \text{ A} \quad (2.28)$$

Next, consider Fig. 22.21c. The voltage across the armature inductance can again be found by applying KVL:

$$E_{AC} + E_{CB} + E_{B1} + E_{12} + E_{2A} = 0$$
$$E_{AC} + 250 + 0 + 324 + 0 = 0$$

Hence, $E_{AC} = -574$ V. This negative voltage causes a very rapid decrease in the armature current. The decrease during the 57 μs interval is given by

$$\Delta I_a = 574 \times 57 \times 10^{-6}/0.004 = 8 \text{ A} \quad (2.28)$$

The 8 A decrease during the 57 μs interval is precisely equal to the increase during the previous 443 μs interval. The peak-to-peak ripple is, therefore, 8 A, which means that the armature current fluctuates between (80 + 4) = 84 A and (80 − 4) = 76 A. Figure 22.21d shows the waveshapes of the various voltages and currents.

Example 22-8

We now consider the question of dynamic braking. The same motor is used as in Example 22-7, and we assume it is running at 900 r/min at the moment that braking is applied. We further assume that the inertia of the motor and its load is very large. As a result, the speed cannot change quickly. The connection between the converter and the 6-pulse rectifier is removed and a braking resistance of 20 Ω is connected between terminals 1 and 2, along with the 500 μF capacitor (Fig. 22.22). We assume that a braking torque equal to 75 percent of nominal torque is sufficient. Consequently, the required armature current is $0.75 \times 80 \text{ A} = 60 \text{ A}$. The switching frequency remains unchanged at 2 kHz. We wish to determine the following:

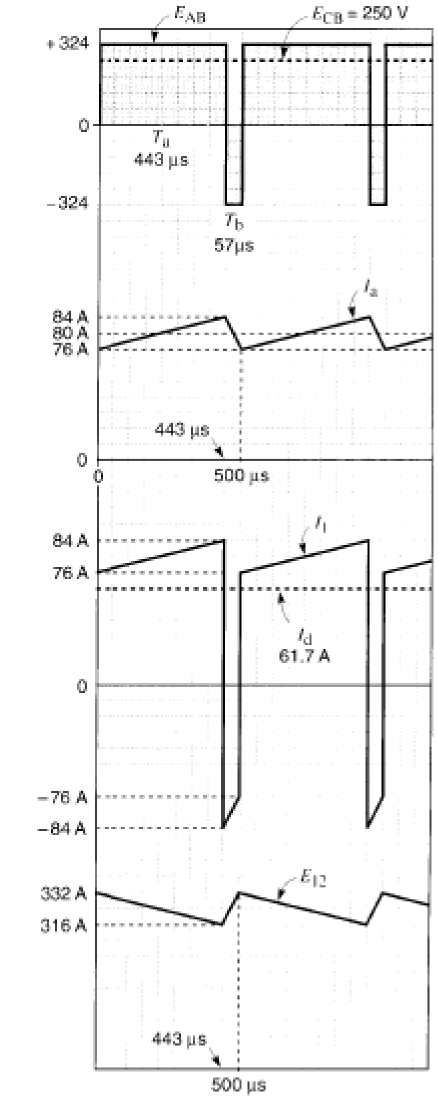


Figure 22.21d
Waveshapes of currents and voltages in Example 22-7.

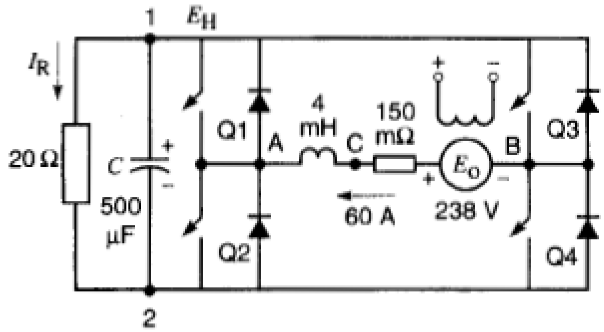


Figure 22.22
Dynamic braking. See Example 22-8.

a. The voltage across the resistor
b. The duty cycle required
c. The braking behavior of the system

Solution

a. Because the motor is turning at 900 r/min at the moment that braking is applied, the induced voltage E_0 remains at 238 V. However, the motor must now operate as a generator and so the 60 A braking current flows out of the (+) terminal, as shown in Fig. 22.22.

The voltage drop across the armature resistance is $0.15\ \Omega \times 60\ \text{A} = 9\ \text{V}$.

The dc voltage between terminals A, B is $(238 - 9) = 229$ V, which is the required *average* output voltage E_{LL} of the converter.

To calculate the dc input voltage E_H between terminals 1, 2 of the converter, we reason as follows:

Due to the large inertia, the speed will remain essentially constant at 900 r/min for, say, 10 cycles of the converter switching frequency.

The power output of the generator during this 10-cycle period is equal to the power absorbed by the 20 Ω braking resistor. Thus,

$$229\ \text{V} \times 60\ \text{A} = (E_H)^2/20\ \Omega$$

hence $$E_H = E_{12} = 524\ \text{V}$$

This voltage is much higher than the previous operating voltage of 324 V. It is actually an advantage because the higher voltage automatically prevents the input rectifier from continuing to feed power to the drive system. On the other hand, the voltage should not be too high, otherwise it could exceed the withstand capability of the switching IGBT devices.

The *average* current in the resistor is $524\ \text{V}/20\ \Omega = 26\ \text{A}$.

b. Knowing the input and output voltages of the converter, we can determine the value of the duty cycle:

$$E_{LL} = E_H(2\,D - 1) \qquad (21.24)$$
$$229 = 524(2\,D - 1)$$

Therefore

$$D = 0.72$$

The *on* and *off* times of Q1 (and Q4) are, therefore,

$$T_a = DT = 0.72 \times 500 = 360\ \mu s$$
$$T_b = 500 - 360 = 140\ \mu s$$

It follows that the corresponding *on* and *off* times of Q2 (and Q3) are 140 μs and 360 μs. Q1 and Q4 still operate simultaneously, as do Q2 and Q3.

When Q2 and Q3 are closed, the armature current follows the path shown in Fig. 22.23. This lasts for 140 μs and during this time I_1 (= 60 A) flows out of terminal 1. The current in the resistor is still 26 A. It follows that a current (60 + 26) = 86 A must come from the capacitor. The capacitor discharges, causing the voltage across it to drop by an amount ΔE given by

$$\Delta E = Q/C = 86\ \text{A} \times 140\ \mu\text{s}/500\ \mu\text{F} = 24\ \text{V}$$

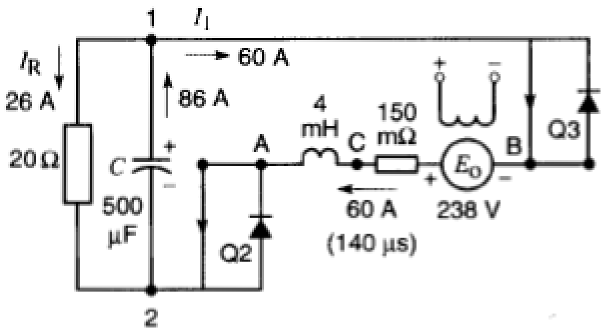


Figure 22.23
Current flows through IGBTs Q2 and Q3.

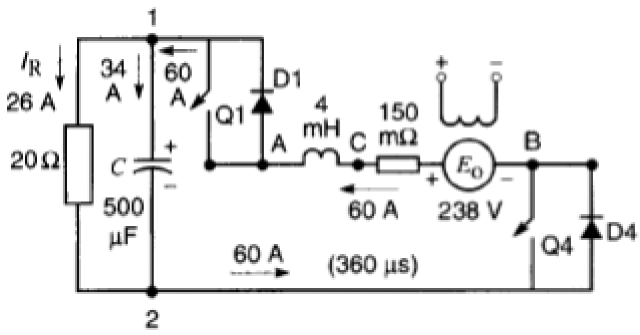


Figure 22.24
Current flows through diodes D1 and D4.

Next, when Q2, Q3 open and Q1, Q4 close, the current has to circulate via diodes D1 and D4 (Fig. 22.24). Applying KCL, a current of $(60 - 26) = 34$ A must flow into the capacitor during 360 μs. The resulting increase in voltage is

$$\Delta E = Q/C = 34 \text{ A} \times 360 \text{ μs}/500 \text{ μF} = 24 \text{ V}$$

Thus, the increase in voltage during 360 μs is exactly equal to the decrease during the remaining 140 μs of the switching cycle. The voltage across the resistor fluctuates between $524 + 12 = 536$ V and $524 - 12 = 512$ V.

This example shows that the converter can transfer power to the passive braking resistor. In so doing, the motor will slow down and the voltage between terminals A, B will decrease progressively. By continually adjusting the duty cycle during the deceleration period, it is possible to maintain the 60 A braking current until the speed is only a fraction of its rated value. This adjustment is of course done automatically by means of an electronic control circuit.

10.22 The power circuit configuration during regenerative braking of a subway car is shown in Fig. P10.22. The dc motor voltage constant is 0.3 V/rpm, and the dc bus voltage is 600 V. At a motor speed of 800 rpm and average motor current of 300 A,

(a) Draw the waveforms of v_0, i_a, and i_s for a particular value of the duty cycle α ($= t_{on}/T$).

(b) Determine the duty ratio α of the chopper for the operating condition.

(c) Determine the power fed back to the bus.

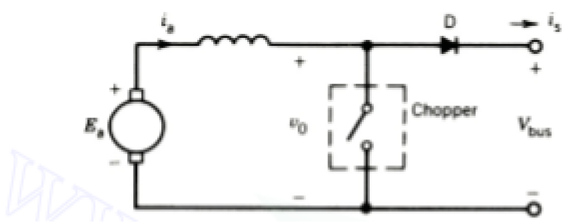
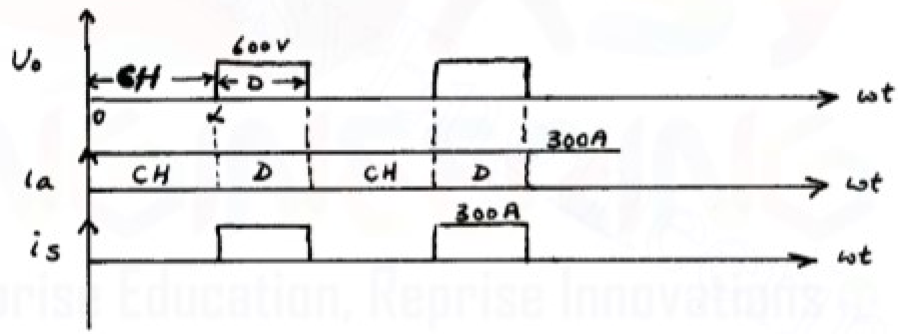


FIGURE P10.22

$\boxed{10.22}$ (a)



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(b) $V_o = (1-\alpha)\,600 = E_a = 0.3 \times 800 = 240\ \text{V}$

$\alpha = 1 - \frac{240}{600} = 0.6$

(c) $I_s = (1-\alpha)\,300 = (1-0.6)\,300 = 120\ A$

$P_s = 600 \times 120 = 72\ \text{KW}$

or $P_s = P_a = E_a I_a = 240 \times 300 = 72\ \text{kW}$

10. A 220-V, 80-A, separately excited dc motor operating at 800 rpm has an armature resistance of 0.18 Ω. The motor speed is controlled by a chopper operating at 1000 Hz. If the motor is regenerating, (a) determine the motor speed at full load current with a duty ratio of 0.7, this being the minimum permissible ratio (b) Repeat the calculation with a duty ratio of 0.1.

Solution

(a) When the machine is working as a motor, E_b is obtained from the equation

$$E_b = E - I_a R_a = 220 - 80 \times 0.18 = 220 - 14.4 = 205.6 \text{ V}$$

From the equation

$$E_b = kN$$

$$k = \frac{205.6}{800} = 0.257$$

When it is regenerating, the step-up configuration of Fig. 3.7(c) holds good. Thus,

$$E_b = E(1 - \delta) + I_a R_a = 220(1 - 0.7) + 80 \times 0.18 = 66 + 14.4 = 80.4 \text{ V}$$

$$N = \frac{E_b}{k} = \frac{80.4}{0.257} = 313 \text{ rpm}$$

(b) The speed for $\delta = 0.1$ is obtained as follows:

$$E_b = 220(1 - 0.10) + 80 \times 0.18 = 198 + 14.4 = 212.4 \text{ V}$$

Therefore the speed is

$$N = \frac{212.4}{0.257} = 827 \text{ rpm}$$

11. A 250-V, 105-A, separately excited dc motor operating at 600 rpm has an armature resistance of 0.18 Ω. Its speed is controlled by a two-quadrant chopper with a chopping frequency of 550 Hz. Compute (a) the speed for motor operation with a duty ratio of 0.5 at 7/8 times the rated torque and (b) the motor speed if it regenerates at $\delta = 0.7$ with rated current.

Solution

(a) The initial back emf is to be determined from the equation

$$E_b = E - I_a R_a = 250 - 105 \times 0.18 = 231.1 \text{ V}$$

Hence the back emf constant $k = 231.1/600 = 0.385$. A fraction 7/8 of the rated current $I'_a = 7/8 \times 105 = 91.875$ A. The new E_b is obtained as

$$E'_b = E\delta - I'_a R_a$$

where $\delta = 0.5$ and $I'_a = 91.875$. Its numerical value is

$$E'_b = 250 \times 0.5 - 91.875 \times 0.18 = 108.46\text{V}$$

The new speed is

$$N = \frac{E'_b}{k} = \frac{108.46}{0.385} = 282 \text{ rpm}$$

(b) When it is regenerating, Eqn (7.54) is to be used. Thus,

$$I_a = \frac{E_b - E(1-\delta)}{R_a}$$

or

$$E_b = E(1-\delta) + I_a R_a$$

Substituting values gives

$$E_b = 250(1 - 0.7) + 105 \times 0.18 = 93.9 \text{ V}$$

From this, the speed $N = 93.9/0.385 = 244$ rpm

Example 12.21. *A dc chopper is used for regenerative braking of a separately-excited dc motor. The dc supply voltage is 400V. The motor has $r_a = 0.2\ \Omega$, $K_m = 1.2$ V-s/rad. The average armature current during regenerative braking is kept constant at 300 A with negligible ripple.*

For a duty cycle of 60% for a chopper, determine

(a) power returned to the dc supply

(b) minimum and maximum permissible braking speeds and

(c) speed during regenerative braking.

Solution. (a) Average armature terminal voltage,

$$V_t = (1 - \alpha)\, V_s = (1 - 0.6) \times 400 = 160 \text{ V}.$$

Power returned to the dc supply

$$= V_t I_a = 160 \times 300 \text{ W} = 48 \text{ kW}$$

(b) From Eq. (12.41), minimum braking speed is

$$\omega_{mn} = \frac{I_a \cdot r_a}{K_m} = \frac{300 \times 0.2}{1.2} = 50 \text{ rad/s or } 477.46 \text{ rpm}$$

From Eq. (12.42), maximum braking speed is

$$\omega_{mx} = \frac{V_s + I_a \cdot r_a}{K_m} = \frac{400 + 300 \times 0.2}{1.2}$$

$$= 383.33 \text{ rad/s or } 3660.6 \text{ rpm}$$

(c) When working as a generator during regenerative braking, the generated emf is $E_a'' = K_m\, \omega_m = V_t + I_a r_a = 160 + 300 \times 0.2 = 220$ V

$\therefore$ Motor speed, $\omega_m = \dfrac{220}{1.2}$ rad/s or 1750.7 rpm

Principle of Step Down Chopper (Buck Converter)

In this topic, you study the Principle of Step Down Chopper and its associated circuit diagram, Waveforms, Modes of operation, & theory.

The buck converter produces a lower average output voltage than the dc source input voltage.

Circuit diagram

The working of a buck regulator is explained using the circuit diagram as shown in Figure 1. The switch S_1 shown in the circuit diagram can be a conventional thyristor i.e., SCR, a GTO thyristor, a power transistor, or a MOSFET.

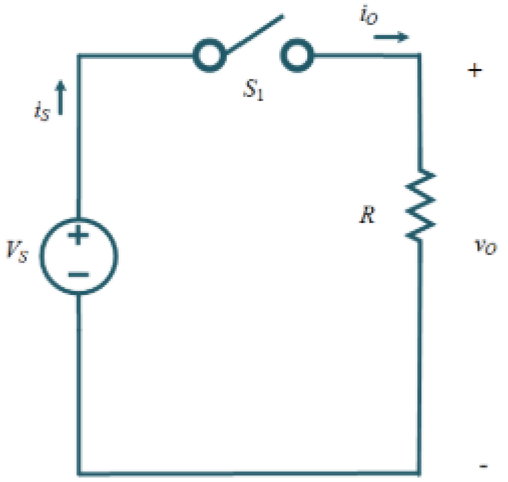


Figure 1 Circuit diagram of step-down chopper (buck converter) with resistive load.

Waveforms

The typical waveforms in the converter are shown in Figure 2.

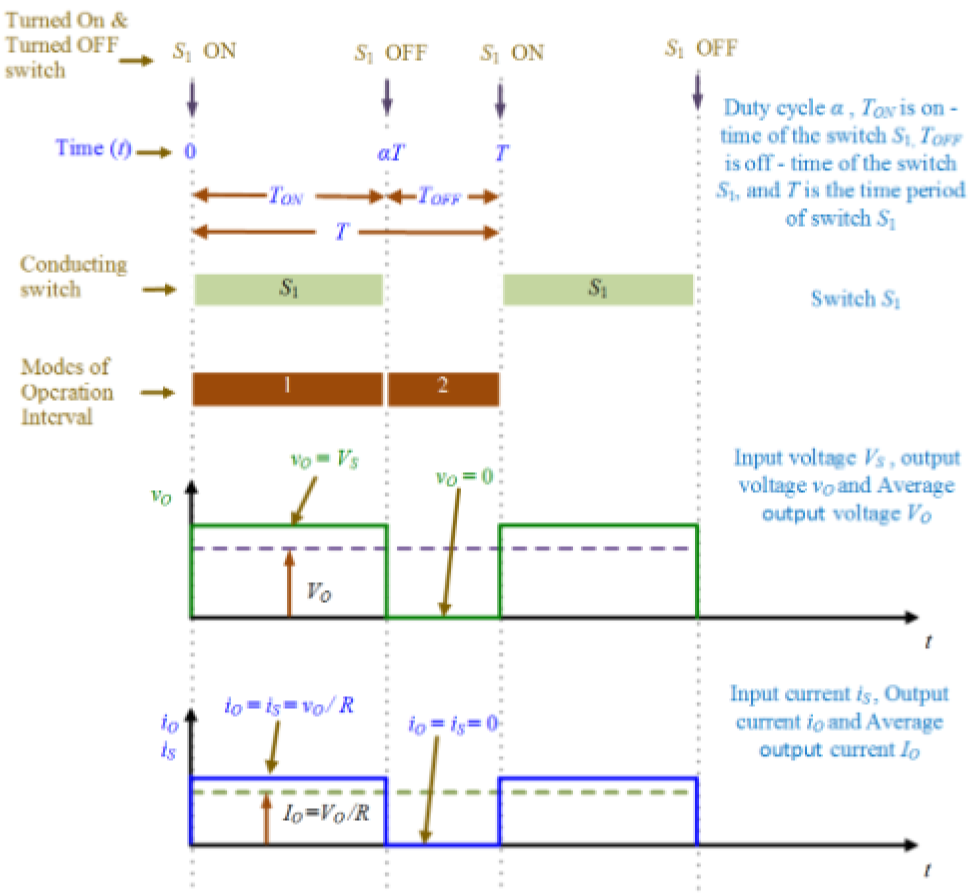


Figure 2 Waveforms of step-down chopper (buck converter) with resistive load.

Modes of Operation Interval

The two modes in steady state operations are

Mode of Operation Interval 1: –

The time interval is $0 \le t \le T_{ON}$. The circuit diagram for Mode of Operation Interval 1 is shown in Figure 3 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned on and the resistive R load directly connects to input dc source voltage V_S and hence $v_O = V_S$, the source (or input) current flows through the Resistive load so $i_S = i_O = v_O/R$.

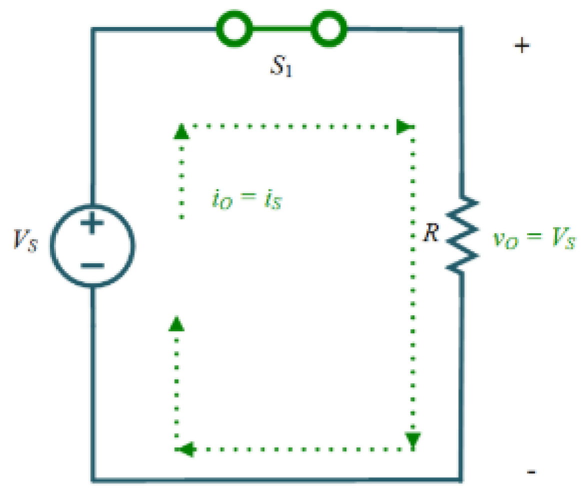


Figure 3 Circuit diagram of step-down chopper (buck converter) with resistive load when switch S_1 ON.

Mode of Operation Interval 2: –

The time interval is $T_{ON} \leq t \leq T_{OFF}$. The circuit diagram for Mode of Operation Interval 2 is shown in Figure 4 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned off and the resistive R load disconnects from input dc source voltage V_S and hence v_O = 0, also the source (or input) current flows through the Resistive load will be i_S = i_O = 0.

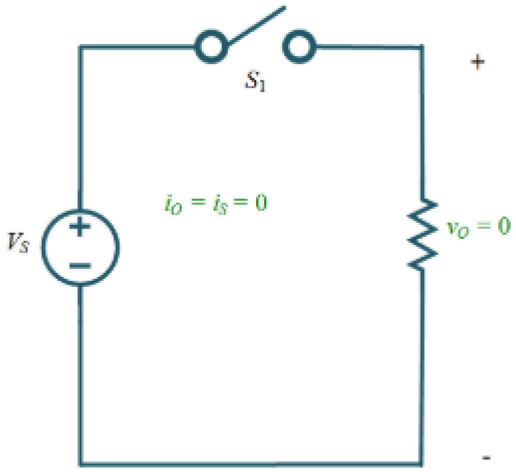


Figure 4 Circuit diagram of step-down chopper (buck converter) with resistive load when switch S_1 OFF.

Average output voltage V_O

Using the output voltage waveform as shown in Figure 2, the average value of the output voltage write as

$$V_O = \frac{T_{ON}}{T_{ON} + T_{OFF}} V_S \ldots (1)$$

Also,

$$T = T_{ON} + T_{OFF} \ldots (2)$$

Using Equation 1 and Equation 2 gives

$$V_O = \frac{T_{ON}}{T} V_S = \alpha V_S$$

So,

$$V_O = \alpha V_S \ldots (3)$$

where,

$\alpha = T_{ON}/T,\ \alpha$ is the duty cycle of the chopper and the value of α lies between $0 \leq \alpha \leq 1$. T_{ON} is the on – time of the switch S_1 or chopper , T_{OFF} is the off – time of the switch S_1 or chopper, T is the chopping period, and the chopping frequency $f = 1/T$.

RMS output voltage V_{orms}

Using the output voltage waveform as shown in Figure 2, the RMS value of the output voltage write as

$$V_{orms} = \left[\frac{T_{ON}}{T_{ON} + T_{OFF}} V_S^2 \right]^{1/2}$$

or

$$V_{orms} = \sqrt{\alpha}\, V_S$$

Example 12.16. *A dc series motor is fed from 600 V dc source through a chopper. The dc motor has the following parameters :*

$$r_a = 0.04\ \Omega, \quad r_s = 0.06\ \Omega, \quad k = 4 \times 10^{-3}\ Nm/amp^2$$

The average armature current of 300 A is ripple free. For a chopper duty cycle of 60%, determine :

(*a*) *input power from the source*

(*b*) *motor speed and* (*c*) *motor torque.*

Solution. (*a*) Power input to motor

$$= V_t \cdot I_a = \alpha V_s \cdot I_a$$
$$= 0.6 \times 600 \times 300 = 108 \text{ kW}.$$

(*b*) For a dc series motor,

$$\alpha V_s = E_a + I_a R = k I_a \omega_m + I_a R$$
$$0.6 \times 600 = 4 \times 10^{-3} \times 300 \times \omega_m + 300\,(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 275 \text{ rad/sec or } 2626.1 \text{ rpm}$$

(*c*) Motor torque, $T_e = k I_a^2 = 4 \times 10^{-3} \times 300^2 = 360$ Nm.

Example 12.17. *The chopper used for on-off control of a dc separately-excited motor has supply voltage of 230V dc, an on- time of 10 m sec and off-time of 15 m sec. Neglecting armature inductance and assuming continuous conduction of motor current, calculate the average load current when the motor speed is 1500 rpm and has a voltage constant of $K_v = 0.5$ V/rad per sec. The armature resistance is 3 Ω.* [*I.A.S., 1985*]

Solution. Chopper duty cycle

$$\alpha = \frac{T_{on}}{T_{on} + T_{off}} = \frac{10}{10 + 15} = 0.4$$

For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a = K_m \cdot \omega_m + I_a r_a$$

$$0.4 \times 230 = 0.5 \times \frac{2\pi \times 1500}{60} + I_a \times 3$$

$\therefore$ Motor load current, $I_a = \dfrac{92 - 25 \times \pi}{3} = 4.487$ A

Example 12.18. *A dc chopper is used to control the speed of a separately-excited dc motor. The dc supply voltage is 220 V, armature resistance $r_a = 0.2\ \Omega$ and motor constant $K_a\ \phi = 0.08$ V/rpm.*

This motor drives a constant torque load requiring an average armature current of 25 A. Determine (a) the range of speed control (b) the range of duty cycle α. Assumed the motor current to be continuous. [*I.A.S., 1990*]

Solution. For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a$$

As motor drives a constant torque load, motor torque T_e is constant and therefore armature current remains constant at 25 A.

Minimum possible motor speed is $N = 0$. Therefore,

$$\alpha \times 220 = 0.08 \times 0 + 25 \times 2.0 = 5$$

$$\alpha = \frac{5}{220} = \frac{1}{44}$$

Maximum possible motor speed corresponds to $\alpha = 1$, *i.e.* when 220 V dc is directly applied and no chopping is done.

$$\therefore \quad 1 \times 220 = 0.08 \times N + 25 \times 0.2$$

or $$N = \frac{220 - 5}{0.08} = 2687.5 \text{ rpm}$$

$\therefore$ Range of speed control : $0 < N < 2687.5$ rpm and corresponding range of duty cycle : $\frac{1}{44} < \alpha < 1$.

Example 12.19. *A separately-excited dc motor is fed from 220 V dc source through a chopper operating at 400 Hz. The load torque is 30 Nm at a speed of 1000 rpm. The motor has $r_a = 0$, $L_a = 2$ mH and $K_m = 1.5$ V-sec/rad. Neglecting all motor and chopper losses, calculate*

(a) the minimum and maximum values of armature current and the armature current excursion,

(b) the armature current expressions during on and off periods.

Solution. As the armature resistance is neglected, armature current varies linearly between its minimum and maximum values.

(*a*) Average armature current, $I_a = \frac{T_e}{K_m} = \frac{30}{1.5} = 20$ A

Motor emf, $E_a = K_m \cdot \omega_m = 1.5 \times \frac{2\pi \times 1000}{60} = 157.08$ V

Motor input voltage, $\alpha V_s = V_t = E_a + I_a r_s = 157.08 + 0$

$\therefore$ $\alpha = \frac{157.08}{220} = 0.714$

Periodic time, $T = \frac{1}{f} = \frac{1}{400} = 2.5$ ms

On-period, $T_{on} = \alpha T = 0.714 \times 2.5 = 1.785$ ms

Off-period, $T_{off} = (1 - \alpha) T = 0.715$ ms

During on-period T_{on}, armature current will rise which is governed by the equation,

$$0 + L \frac{di_a}{dt} + E_a = V_s$$

or

$$\frac{di_a}{dt} = \frac{V_s - E_a}{L} = \frac{220 - 157.08}{0.02} = 3146 \text{ A/s}$$

During off period, $\frac{di_a}{dt} = -\frac{E_a}{L} = \frac{-157.08}{0.02} = -7854$ A/s

With current rising linearly, it is seen from Fig. 12.21 that

$$I_{mx} = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times T_{on}$$

$$= I_{mn} + 3146 \times 1.785 \times 10^{-3}$$

or $$I_{mx} = I_{mn} + 5.616 \quad ...(i)$$

For linear variation between I_{mn} and I_{mx}, average value of armature current

$$I_a = \frac{I_{mx} + I_{mn}}{2} = 20 \text{ A}$$

or $$I_{mx} = 40 - I_{mn} \quad ...(ii)$$

Solving Eqs. (*i*) and (*ii*), we get $I_{mx} = 22.808$ A

and $I_{mn} = 17.912$ A.

$\therefore$ Armature current excursion $= I_{mx} - I_{mn} = 22.808 - 17.912 = 5.616$ A

(b) Armature current expression during turn-on,

$$i_a(t) = I_{mn} + \left(\frac{di_a}{dt}\text{ during } T_{on}\right)\times t$$

$$= 17.192 + 3146\,t \quad \text{for } 0 \le t \le T_{on}$$

Armature current expression during turn-off,

$$i_a(t) = I_{mx} + \left(\frac{di_a}{dt}\text{ during } T_{off}\right)\times t$$

$$= 22.808 - 7854\,t \quad \text{for } 0 \le t \le T_{off}$$

Example 12.20. *Repeat Example 12.19, in case motor has a resistance of 0.2 Ω for its armature circuit.*

Solution. (a) From Example 12.19, armature current, $I_a = 20$ A and motor emf, $E_a = 157.08$ V; source voltage, $V_s = 220$ V.

For armature circuit, $\alpha V_s = V_0 = V_t = E_a + I_a r_a = 157.08 + 20 \times 0.2 = 161.08$ V

$$\therefore \qquad \alpha = \frac{161.08}{220} = 0.7322$$

$$T_{on} = \alpha T = 0.7322 \times 2.5 = 1.831 \text{ ms}$$

$$T_{off} = T - T_{on} = 0.669 \text{ ms}, \frac{R}{L} = \frac{0.2}{0.02} = 10$$

During T_{on}, from Eq. (12.34), armature current is

$$i_a(t) = \frac{220 - 157.08}{0.2}(1 - e^{-10t}) + I_{mn}\cdot e^{-10t}$$

At $t = T_{on} = 1.831$ ms, current become I_{mx}. This gives

$$i_a(t) = I_{mx} = 5.7079 + 0.98187\,I_{mn} \qquad \ldots(i)$$

During T_{off}, from Eq. (12.35), armature current is

$$i_a(t) = \frac{-157.08}{0.2}(1 - e^{-10t}) + I_{mx}\cdot e^{-10t}$$

At $t = 0.669$ ms, $i_a(t) = I_{mn}$. This gives

$$i_a(t) = I_{mn} = -5.237 + 0.9933\,I_{mx} \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$I_{mx} = 5.7079 + 0.98187\,(-5.237 + 0.9933\,I_{mx})$$
$$= 0.5658 + 0.9753\,I_{mx}$$

or

$$I_{mx} = \frac{0.5658}{0.0247} = 22.907 \text{ A}$$
$$I_{mn} = -5.237 + 0.9933 \times 22.907 = 17.516 \text{ A}$$

$\therefore$ Armature current excursion

$$= I_{mx} - I_{mn} = 22.907 - 17.516 = 5.39 \text{ A}$$

(b) Armature current expression during turn-on period is

$$i_a(t) = 314.6\,(1 - e^{-10t}) + 17.516\,e^{-10t}$$

Armature current expression during turn-off period is

$$i_a(t) = -785.4\,(1 - e^{-10t}) + 22.907\,e^{-10t}$$

Example 4.3

A 230 V, 500 rpm, 90 A separately excited dc motor has the armature resistance and inductance of 0.115 Ω and 11 mH respectively. The motor is controlled by a chopper operating at 400 Hz. If the motor is regenerating,

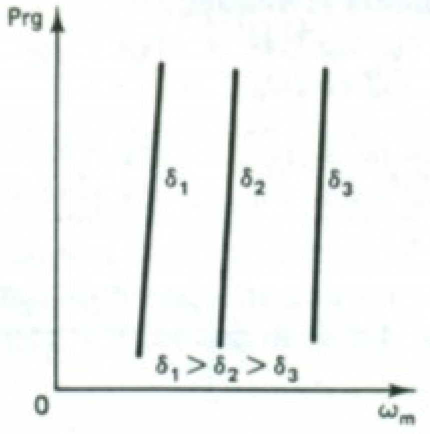


Figure 4.7 Regenerative braking performance curves of TRC chopper-fed dc separately excited motor.

1. Find the motor speed and the regenerated power at the rated current and a duty ratio of 0.5.
2. Calculate the maximum safe speed if the minimum value of the duty ratio is 0.1.

Solution: At rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. In regenerative braking

$$(1 - \delta)V = E - I_aR_a \quad \text{or} \quad E = (1 - \delta)V + I_aR_a \tag{E4.4}$$

At $\delta = 0.5$ and $I_a = 90$ A

$$E = 0.5 \times 230 + 90 \times 0.115 = 125 \text{ V}$$

Since $\dfrac{N}{N_r} = \dfrac{E}{E_r}$

where N_r = rated speed in rpm and N = speed to be calculated

Thus

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125}{219.7} = 284.5 \text{ rpm}$$

$$\tau_a = \frac{11 \times 10^{-3}}{0.115} = 95.65 \text{ mS}, \qquad T = \frac{1}{400} = 2.5 \text{ mS}$$

$$T/\tau_a = \frac{2.5 \times 10^{-3}}{95.65 \times 10^{-3}} = 0.026 \quad \text{and} \quad \tau_a/T = 38.3$$

Equation (4.30) is repeated here:

$$P_{rg} = \frac{V^2}{R_a}\left[\left(\frac{E}{V} - 1\right) \cdot (1 - \delta) + \frac{\tau_a}{T}\left\{\frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}}\right\}\right] \tag{4.30}$$

Now

$$\frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}} = x = \frac{e^{0.5T/\tau_a} + e^{0.5T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}}$$

$$= \frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1} = \frac{0.013}{2.013} = 0.0065$$

From equation (4.30),

$$P_{rg} = \frac{V^2}{R_a}\left[\left(\frac{E}{V} - 1\right) \cdot (1 - \delta) + \frac{\tau_a}{T} x\right]$$

$$= \frac{230^2}{0.115}\left[\left(\frac{125}{230} - 1\right)(1 - 0.5) + 38.3 \times 0.0065\right]$$

$$= 9.52 \text{ kW}$$

2. The maximum safe speed will be obtained at the minimum value of δ and the rated armature current. For higher speeds, the armature current will exceed the rated motor current and this operation will not be safe for the motor. At the maximum safe speed N_m, the back emf E_m is given by

$$E_m = (1 - \delta_{min})V + I_{ar}R_a = 0.9 \times 230 + 90 \times 0.115 = 217$$

$$N_m = \frac{N_r}{E_r} \times E_m = \frac{500}{219.7} \times 217 = 494 \text{ rpm}$$

Example 4.4

The motor of example 4.3 is controlled by a class C two-quadrant chopper operating with a source voltage of 230 V and a frequency of 400 Hz.

1. Calculate the motor speed for a motoring operation at $\delta = 0.5$ and half of rated torque.
2. What will be the motor speed when regenerating at $\delta = 0.5$ and rated torque?

Solution: At the rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. From equation (4.33),

$$\delta V = E + I_a R_a \tag{E4.5}$$

At half the rated torque, $I_a = 45$ A
At $\delta = 0.5$

$$E = \delta V - I_a R_a = 0.5 \times 230 - 45 \times 0.115 = 109.8 \text{ V}$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 109.8}{219.7} = 250 \text{ rpm}$$

2. In the regenerative braking at the rated torque, $I_a = -90$ A
From equation (E4.5),

$$E = \delta V - I_a R_a = 0.5 \times 230 + 90 \times 0.115 = 125.4$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125.4}{219.7} = 285 \text{ rpm}$$

Example 4.5

The motor of example 4.3 is fed by a four-quadrant chopper controlled by method III. The source voltage is 230 V and the frequency of operation is 400 Hz.

1. If the motor operation is required in the second quadrant at the rated torque and 300 rpm, calculate the duty ratio.
2. What should be the value of the duty ratio if the motor is working in the third quadrant at 400 rpm and half of the rated torque?

Solution: At the rated conditions of operation

$$E_r = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. Equation (4.39), which is applicable to method III is reproduced here:

$$I_a = \frac{2V(\delta - 0.5) - E}{R_a} \tag{4.39}$$

The motor is working in the second quadrant, therefore,

$$I_a = -90 \text{ A}$$

$$E = \frac{300}{500} \times E_r = \frac{300}{500} \times 219.7 = 131.8 \text{ V}$$

Substituting in equation (4.39), gives

$$-90 = \frac{2 \times 230(\delta - 0.5) - 131.8}{0.115}$$

or

$$\delta = 0.5 + \frac{121}{460} = .76 .$$

2. At half the rated torque and in the third quadrant

$$I_a = -45 \text{ A}$$

$$E = -\frac{400}{500} \times 219.7 = -175.7 \text{ V}$$

Substituting in equation (4.39), gives

$$-45 = \frac{2 \times 230(\delta - 0.5) + 175.7}{0.115}$$

or

$$\delta = 0.5 - \frac{181}{460} = 0.11 .$$

Example 4.1

A dc motor is driven from a chopper with a source voltage of 24V dc and at a frequency of 1 kHz. Determine the variation in duty cycle required to have a speed variation of 0 to 1 p.u. delivering a constant 2 p.u. load. The motor details are as follows:

1 hp, 10 V, 2500 rpm, 78.5 % efficiency, $R_a = 0.01\ \Omega$, $L_a = 0.002$ H, $K_b = 0.03819$ V/rad/sec

The chopper is one-quadrant, and the on-state drop voltage across the device is assumed to be 1 V regardless of the current variation.

Solution (i) Calculation of rated and normalized values

$$V_b = 10\ \text{V}$$

$$V_n = \frac{V_s}{V_b} = \frac{24 - 1}{10} = 2.3\ \text{p.u.}$$

$$\omega_{mr} = \frac{2500 \times 2\pi}{60} = 261.79\ \text{rad / sec}$$

$$I_{ar} = \frac{\text{Output}}{\text{Voltage} \times \text{Efficiency}} = \frac{1 \times 746}{10 \times 0.785} = 95\ \text{A} = I_b$$

$$R_{an} = \frac{I_b R_a}{V_b} = \frac{95 \times 0.001}{10} = 0.095\ \text{p.u.}$$

$$T_{en} = 2\ \text{p.u.}$$

(ii) Calculation of duty cycle

The minimum and maximum duty cycles occur at 0 and 1 p.u. speed, respectively, and at 2 p.u. load. From equation (4.9),

$$d = \frac{T_{en} R_{an} + \omega_{mn}}{V_n}$$

$$d_{min} = \frac{2 \times 0.095 + 0}{2.3} = 0.0826$$

$$d_{max} = \frac{2 \times 0.095 + 1}{2.3} = 0.517$$

The range of duty cycle variation required, then, is

$$0.0826 \leq d \leq 0.517$$

Example 4.2

The critical duty cycle can be changed by varying either the electrical time constant or the chopping frequency in the chopper. Draw a set of curves showing the effect of these variations on the critical duty cycle for various values of E/V_s.

Solution

$$d_c = \left(\frac{T_a}{T}\right)\log_e\left[1 + \frac{E}{V_s}\left(e^{\frac{T}{T_a}} - 1\right)\right]$$

In terms of chopping frequency,

$$d_c = f_c T_a \log_e\left[1 + \frac{E}{V_s}\left(e^{\frac{1}{T_a f_c}} - 1\right)\right]$$

Assigning various values of E/V_s and varying $f_c T_a$ would yield a set of critical duty cycles. The graph between d_c and $f_c T_a$ for varying values of E/V_s is shown in Figure 4.16. The maximum value of $f_c T_a$ is chosen to be 10.

Example 4.3

A 200-hp, 230-V, 500-rpm separately-excited dc motor is controlled by a chopper. The chopper is connected to a bridge-diode rectifier supplied from a 230-V, 3-ϕ, 60-Hz ac main. The motor chopper details are as follows:

$$R_a = 0.04\ \Omega,\ L_a = 0.0015\ \text{H},\ K_b = 4.172\ \text{V/rad/sec},\ f_c = 2\ \text{kHz}.$$

The motor is running at 300 rpm with 55% duty cycle in the chopper. Determine the average current from steady-state current waveform and the electromagnetic torque produced in the motor. Compare these results with those obtained by averaging.

Solution The critical duty cycle is evaluated to determine the current continuity at the given duty cycle of 0.55.

$$d_c = \left(\frac{T_a}{T}\right)\log_e\left[1 + \frac{E}{V_s}(e^{\frac{T}{T_a}} - 1)\right]$$

$$T_a = \frac{0.0015}{0.04} = 0.0375\ \text{s}$$

$$T = \frac{1}{f_c} = \frac{1}{2 \times 10^3} = 0.5\ \text{ms}$$

$$\frac{T_a}{T} = 75$$

$$V_s = 1.35\ V\cos\alpha = 1.35 \times 230 \times \cos 0^\circ = 310.5\ \text{V}$$

$$E = K_b\omega_m = 4.172 \times \frac{2\pi \times 300}{60} = 131.1\ \text{V}$$

$$d_c = 75\log_e\left[1 + \frac{131.1}{310.5}(e^{\frac{1}{75}} - 1)\right] = 0.423$$

The given value of d is greater than the critical duty cycle; hence, the armature current is continuous.

$$I_{a0} = \frac{V_s(e^{dT/T_a} - 1)}{R_a(e^{T/T_a} - 1)} - \frac{E}{R_a} = \frac{310.5(e^{0.55/75} - 1)}{0.04(e^{1/75} - 1)} - \frac{131.1}{0.04} = 979 \text{ A}$$

$$I_{a1} = \frac{V_s(1 - e^{-dT/T_a})}{R_a(1 - e^{-T/T_a})} - \frac{E}{R_a} = \frac{310.5(1 - e^{-0.55/75})}{0.04(1 - e^{-1/75})} - \frac{131.1}{0.04} = 1004.7 \text{ A}$$

The average current is

$$I_{av} = \frac{1}{T}\left[\int_0^{dT}\left(\frac{V_s - E}{R_a}(1 - e^{-t/T_a}) + I_{a0}e^{-t/T_a}\right)dt + \int_0^{(1-d)T}\left(-\frac{E}{R_a}(1 - e^{-t/T_a}) + I_{a1}e^{-t/T_a}\right)dt\right]$$

$$= \frac{1}{T}\left[\begin{array}{l} \frac{V_s - E}{R_a}\{dT + T_a(e^{-dT/T_a} - 1)\} + I_{a0}T_a(1 - e^{-dT/T_a}) \\ -\frac{E}{R_a}\{(1 - d)T - T_a + T_a e^{-(1-d)T/T_a}\} + I_{a1}T_a(1 - e^{-(1-d)T/T_a}) \end{array}\right] = 991.8 \text{ A}$$

$$T_{av} = K_b I_{av} = 4.172 \times 991.8 = 4137.7 \text{ N·m}$$

Steady state by averaging

$$I_{av} = \frac{(dV_s - K_b\omega_m)}{R_a} = \frac{0.55 \times 310.5 - 4.172 \times 31.42}{0.04} = 991.88 \text{ A}$$

$$T_{av} = K_b I_{av} = 4138.1 \text{ N·m}$$

There is hardly any significant difference in the results by these two methods.

Example 4.4

A separately-excited dc motor is controlled by a chopper whose input dc voltage is 180 V. This motor is considered for low-speed applications requiring less than 2% pulsating torque at 300 rpm. (i) Evaluate its suitability for that application. (ii) If it is found unsuitable, what is the chopping frequency that will bring the pulsating torque to the specification? (iii) Alternatively, a series inductor in the armature can be introduced to meet the specification. Determine the value of that inductor. The motor and chopper data are as follows:

3 hp, 120 V, 1500 rpm, $R_a = 0.8\ \Omega$, $L_a = 0.003$ H, $K_b = 0.764$ V/rad/sec, $f_c = 500$ Hz

Solution Rated torque.

$$T_{er} = \frac{3 \times 745.6}{2\pi \times 1500/60} = 14.25\ \text{N}\cdot\text{m}$$

$$\text{Maximum pulsating torque permitted} = 0.02 \times T_{er} = 0.02 \times 14.25 = 0.285\ \text{N}\cdot\text{m}$$

To find the harmonic currents, it is necessary to know the duty cycle. That is approximately determined by the averaging-analysis technique, by assuming that the motor delivers rated torque at 300 rpm.

$$I_b = \frac{T_{er}}{K_b} = \frac{14.25}{0.764} = 18.65\ \text{A}$$

$$V_a = E + I_{av}R_a = K_b\omega_m + I_{av}R_a = 0.764 \times \frac{2\pi \times 300}{60} + 18.65 \times 0.8 = 38.91\ \text{V}$$

$$d = \frac{V_a}{V_s} = \frac{38.91}{180} = 0.216$$

(i) The fundamental pulsating torque is assumed to be predominant for this analysis.

$$i_{a1} = \frac{2V_s}{\pi\sqrt{R_a^2 + \omega_c^2L_a^2}}\sin(\pi d) = \frac{2 \times 180}{\pi\sqrt{0.8^2 + (2\pi \times 500 \times 0.003)^2}}\sin(0.216\pi) = 7.6\ \text{A}.$$

$$T_{e1} = K_b i_{a1} = 0.764 \times 7.6 = 5.8\ \text{N}\cdot\text{m}$$

This pulsating torque exceeds the specification, and, hence, in the present condition, the drive is unsuitable for use.

(ii) The fundamental current to produce 2% pulsating torque is

$$i_{a1(spec)} = \frac{T_{e1(spec)}}{K_b} = \frac{0.285}{0.764} = 0.373\ \text{A}$$

$$i_{a1(spec)} = \frac{2V_s}{\pi\sqrt{R_a^2 + \omega_c^2L_a^2}}\sin(\pi d)$$

from which the angular switching frequency to meet the fundamental current specification is obtained as

$$\omega_{c1} = \sqrt{\frac{4V_s^2\sin^2(\pi d)}{\pi^2L_a^2i_{a1(spec)}^2} - \frac{R_a^2}{L_a^2}} = \sqrt{(4 \times 180^2\sin^2(0.216\pi)/(\pi^2(0.003)^2(0.373)^2) - \left(\frac{0.8}{0.003}\right)^2}$$

$$= 64{,}278\ \text{rad/s}$$

$$f_{c1} = \frac{\omega_{c1}}{2\pi} = 10.23\ \text{kHz}$$

Note that f_{c1} is the chopping frequency in Hz, which decreases the pulsating torque to the specification.

(iii) Let L_{ex} be the inductor introduced in the armature circuit. Then its value is

$$L_{ex} = \sqrt{\frac{4V_s^2 \sin^2(\pi d)}{\pi^2 \omega_c^2 i_{a1(spec)}^2} - \frac{R_a^2}{\omega_c^2}} - L_a$$

$$= \sqrt{(4 \times 180^2 \sin^2(0.216\pi))/(\pi^2(2\pi \times 500)^2(0.373)^2) - \left(\frac{0.8}{2\pi \times 500}\right)^2} - 0.003 = 71.5 \text{ mH}$$

Example 4.5

Calculate (i) the maximum harmonic resistive loss and (ii) the derating of the motor drive given in Example 4.4. The motor is operated with a base current of 18.65 A, which is inclusive of the fundamental-harmonic current. Consider only the dominant-harmonic component, to simplify the calculation.

Solution

$$I_{rms}^2 = I_b^2 = I_{av}^2 + I_1^2$$

By dividing by the square of the base current, the equation is expressed in terms of the normalized currents as

$$I_{avn}^2 + I_{1n}^2 = 1 \text{ p.u.}$$

where

$$I_{1n} = \frac{\sqrt{2}}{\pi} \frac{V_s}{\sqrt{R_a^2 + \omega_c^2 L_a^2}} \sin(\pi d) \frac{1}{I_b} = \frac{\sqrt{2}}{\pi} \frac{V_{sn}}{Z_{an}} \sin(\pi d)$$

where

$$V_{sn} = \frac{V_s}{V_b};\ Z_{an} = \frac{Z_a}{Z_b} = \frac{\sqrt{R_a^2 + \omega_c^2 L_a^2}}{Z_b}$$

$$Z_b = \frac{V_b}{I_b} = \frac{120}{18.65} = 6.43\ \Omega$$

$$V_{sn} = \frac{180}{120} = 1.5 \text{ p.u.}$$

$$Z_a = \sqrt{0.8^2 + (2\pi * 500 * 0.003)^2} = 9.42\ \Omega$$

$$Z_{an} = \frac{9.42}{6.43} = 1.464 \text{ p.u.}$$

For a duty cycle of 0.5, the dominant-harmonic current is maximum and is given as

$$I_{1n} = \frac{\sqrt{2}}{\pi} \frac{1.5}{1.464} = 0.46 \text{ p.u.}$$

(i) The dominant-harmonic armature resistive loss is

$$P_{1n} = I_{1n}^2 R_{an} = 0.46^2 \frac{0.8}{6.43} = 0.02628 \text{ p.u.}$$

(ii) For equality of losses in the machine with pure and chopped-current operation, the average current in the machine with chopped-current operation is derived as

$$I_{avn} = \sqrt{1 - I_{1n}^2} = \sqrt{1 - .46^2} = 0.887 \text{ p.u.}$$

which translates into an average electromagnetic torque of 0.887 p.u., resulting in 11.3% derating of the torque and hence of output power.

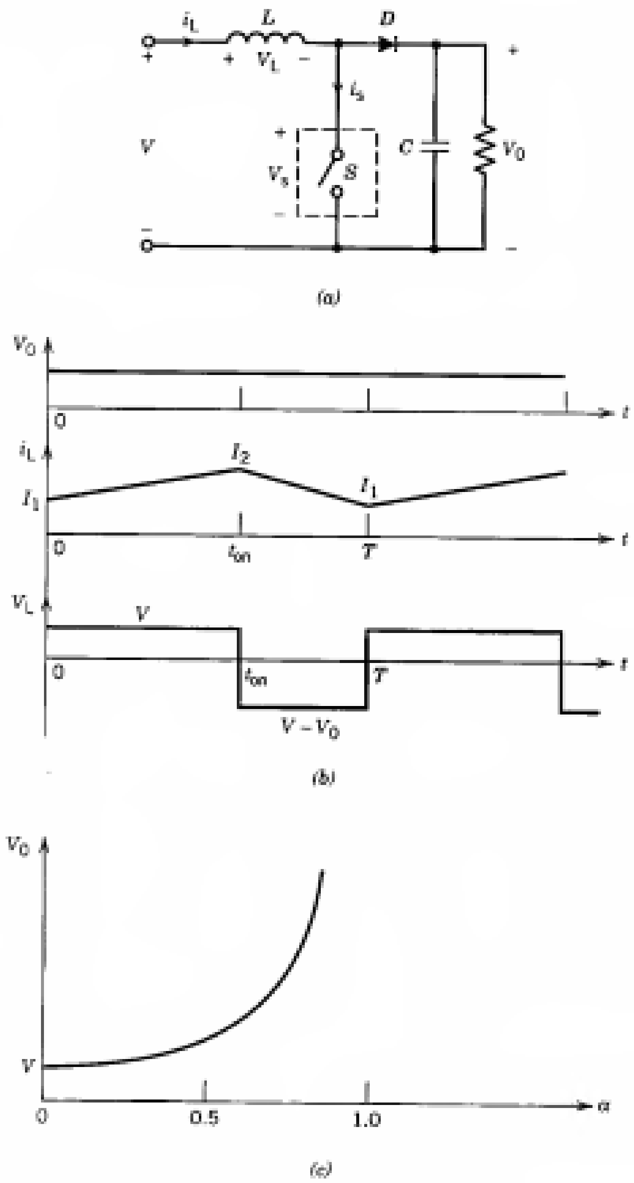


FIGURE 10.36 Boost converter. (*a*) Circuit. (*b*) Waveforms. (*c*) V_0 versus α.

10.24 The boost converter of Fig. 10.36 is used to charge a battery bank from a dc voltage source with $V = 160$ V. Assume ideal switch and no-loss operation, and neglect the ripple at the output voltage. The battery bank consists of 100 identical batteries. Each battery has an internal resistance $R_b = 0.1\,\Omega$. At the beginning of the charging process, each battery voltage is $V_{b1} = 2.5$ V. When each battery is charged up to $V_{b2} = 3.2$ V, the charging process is completed. The average charging current is kept constant at 0.5 A.

(a) Calculate the variation of duty ratio α for the charging process.

(b) Draw qualitatively the waveforms of v_L, i_L, v_S, i_S, v_D, i_D for $V_{b1} = 2.5$ V.

10.24 (a) $I = 0.5A \quad R_t = 100 \times R_b = 100 \times 0.1 = 10\Omega, \quad V_R = 0.5 \times 10 = 5V$

$$V_{o1} = 100 \times V_{b1} + V_R = 100 \times 2.5 + 5 = 255V$$

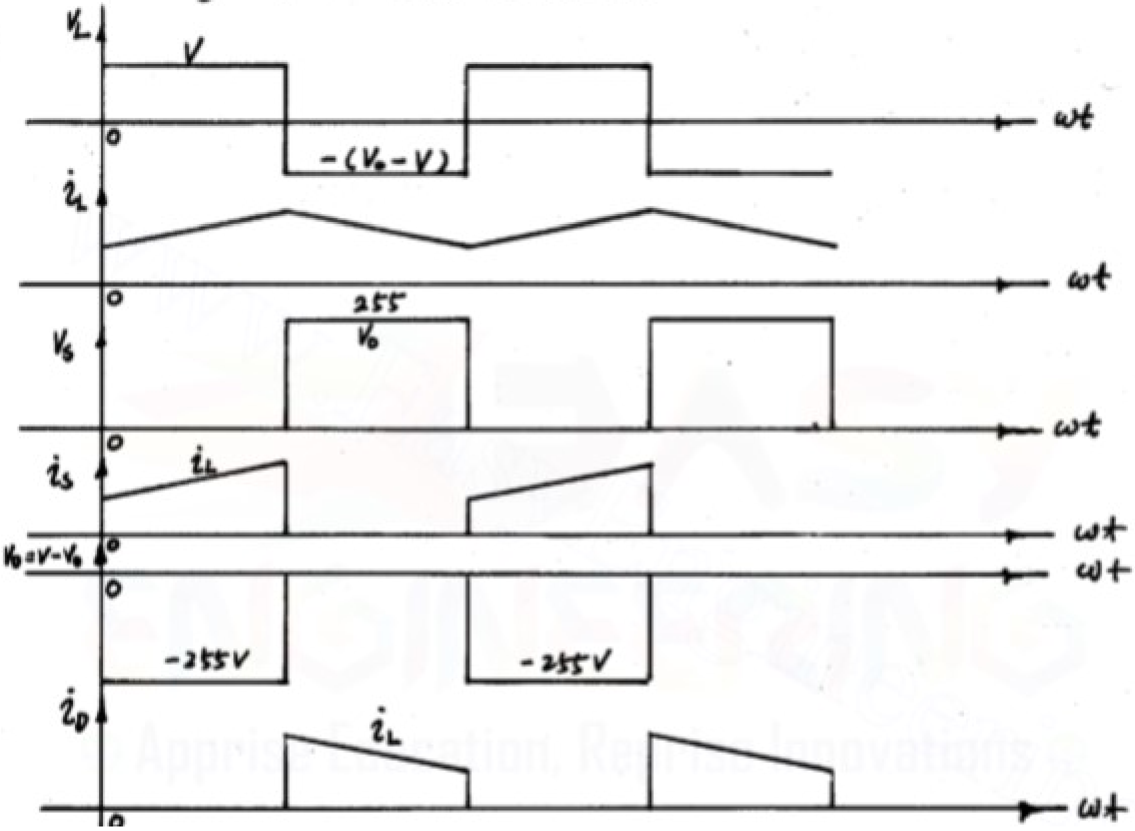
$$V_{o2} = 100 \times V_{b2} + V_R = 100 \times 3.2 + 5 = 325V$$

$$V_o = \frac{1}{1-\alpha} V \longrightarrow \alpha = \frac{V_o - V}{V_o}$$

$$\alpha_1 = \frac{255 - 150}{255} = 0.4118 \qquad \alpha_2 = \frac{325 - 150}{325} = 0.5385$$

α changes from 0.4118 to 0.5385

(b)



10.25 For the battery charging system of Problem 10.24:

(a) If the supply voltage available is $V = 150$ V (dc), which dc to dc converter would be used? Draw the circuit.

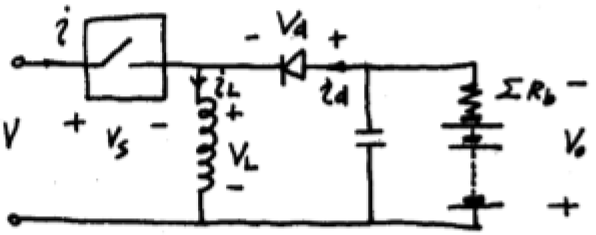
(b) Calculate the variation of the duty ratio α for the charging process.

(c) Draw qualitatively the waveforms of inductor voltage (v_L), inductor current (i_L), voltage across the chopper switch (v_s), current through the chopper switch (i_s), voltage across the diode (v_d), and current through the diode (i_d), for $v_{bl} = 1.2$ V.

$\boxed{10.25}$ (a) $V_{o1} = 100 V_{b1} + IR = 100\times 1.2 + 100\times 0.1\times 0.5 = 125V$

$V_{o2} = 100\times 3.2 + 5 = 325\ V$

$V = 150V$ Need a Buck-Boost Converter

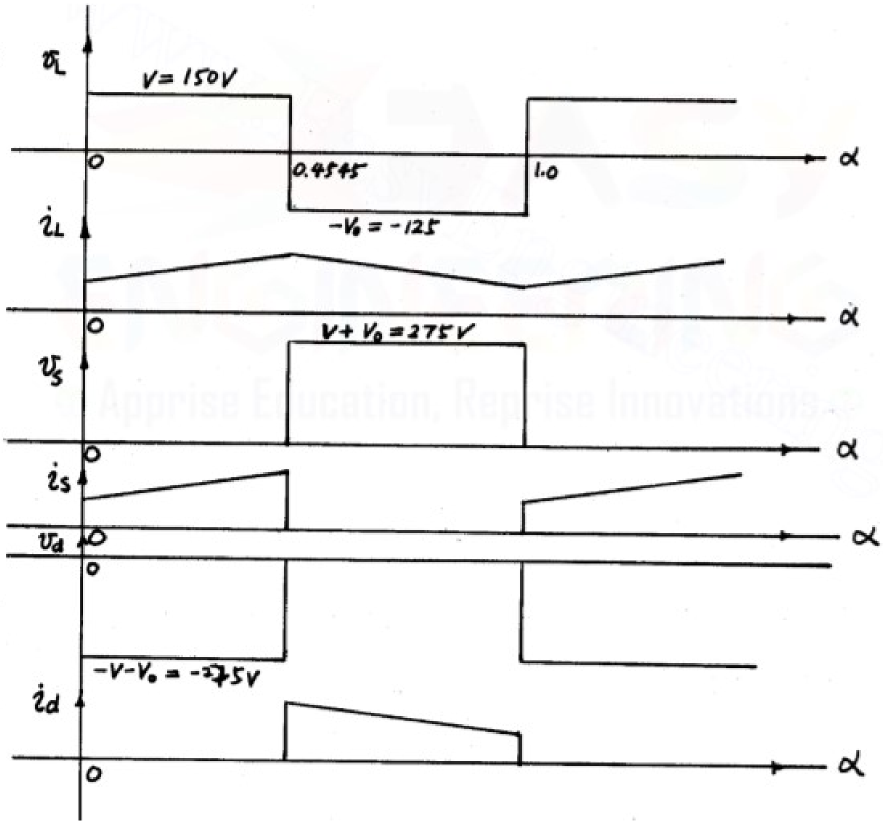


(b) $V_o = \dfrac{\alpha}{1-\alpha} V$

$\alpha = \dfrac{V_o}{V + V_o}$

$\alpha_1 = \dfrac{125}{150 + 125} = 0.4545 \qquad \alpha_2 = \dfrac{325}{150 + 325} = 0.6842$

(c)



Example 14.1: *DC chopper (first quadrant) with load back emf*

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 Ω resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine, with and without (rotor standstill, E = 0) the back emf:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the average load output current, average switch current, and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, (and electromagnetic efficiency for E > 0); and
viii. sketch the output current and voltage waveforms.

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s$ = 1/200Hz = 5ms
- on-period of the switch t_T = 1.25ms
- load time constant $\tau = L/R$ = 0.05mH/10Ω = 5ms

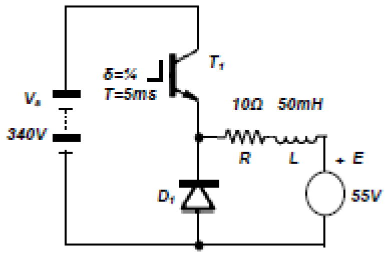


Figure 14.6. Example 14.1. *Circuit diagram.*

i. From equations (14.2) and (14.3), assuming continuous load current, the average and rms output voltages are both independent of the back emf, namely

$$\overline{V}_o = \frac{t_T}{T} V_s = \delta V_s$$
$$= ¼ \times 340\text{V} = 85\text{V}$$
$$V_r = \sqrt{\frac{t_T}{T}}\, V_s = \sqrt{\delta}\, V_s$$
$$= \sqrt{¼} \times 240\text{V} = 120\text{V rms}$$

ii. The rms ripple voltage hence ripple factor are given by equations (14.4) and (14.5), that is

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$
$$= 340\text{V}\sqrt{¼\times(1 - ¼)} = 147.2\text{V ac}$$

and

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1} = \sqrt{FF^2 - 1}$$
$$= \sqrt{\frac{1}{¼} - 1} = \sqrt{3} = 1.732 \qquad FF = 2$$

No back emf, E = 0

iii. From equation (14.13), with $E = 0$, the maximum and minimum currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-\delta T}{\tau}}}{1-e^{\frac{-T}{\tau}}} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} = 11.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{\delta T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} = 5.62\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{p-p} = \hat{I}-\check{I}$$
$$= 11.90\text{A} - 5.62\text{A} = 6.28\text{A}$$

Alternatively the ripple can be extracted from figure 14.4 using $T/\tau = 1$ and $\delta = ¼$.

iv. From equations (14.11) and (14.12), with $E = 0$, the time domain load current equations are

$$i_o = \frac{V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right)+\check{I}e^{\frac{-t}{\tau}}$$

$$i_o(t) = 34\times\left(1-e^{\frac{-t}{5\text{ms}}}\right)+5.62\times e^{\frac{-t}{5\text{ms}}}$$

$$= 34-28.38\times e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0\le t\le 1.25\text{ms}$$

$$i_o = \hat{I}e^{\frac{-t}{\tau}}$$

$$i_o(t) = 11.90\times e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0\le t\le 3.75\text{ms}$$

v. The average load current from equation (14.17), with $E = 0$, is

$$\bar{I}_o = \bar{V}_o/R = 85\text{V}/10\Omega = 8.5\text{A}$$

The average switch current, which is the average supply current, is

$$\bar{I}_i = \bar{I}_{switch} = \frac{\delta(V_s-E)}{R}-\frac{\tau}{T}\left(\hat{I}-\check{I}\right)$$

$$= \frac{¼\times(340\text{V} - 0)}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(11.90\text{A} - 5.62\text{A}) = 2.22\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\bar{I}_{diode} = \bar{I}_o - \bar{I}_i$$
$$= 8.50\text{A} - 2.22\text{A} = 6.28\text{A}$$

vi. The input power is the dc supply voltage multiplied by the average input current, that is

$$P_{in} = V_s\bar{I}_i = 340\text{V}\times 2.22\text{A} = 754.8\text{W}$$
$$P_{out} = P_{in} = 754.8\text{W}$$

From equation (14.18) the rms load current is given by

$$I_{o_{rms}} = \sqrt{\frac{P_{out}}{R}}$$
$$= \sqrt{\frac{754.8\text{W}}{10\Omega}} = 8.7\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{I_s} = \frac{340\text{V}}{2.22\text{A}} = 153.2\,\Omega$$

Load back emf, *E* = 55V

i. and *ii.* The average output voltage (85V), rms output voltage (120V rms), ac ripple voltage (147.2V ac), and ripple factor (1.732) are independent of back emf, provided the load current is continuous. The earlier answers for E = 0 are applicable.

iii. From equation (14.13), the maximum and minimum load currents are

$$\hat{I} = \frac{V_s}{R}\,\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} - \frac{55\text{V}}{10\Omega} = 6.40\text{A}$$

$$\check{I} = \frac{V_s}{R}\,\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} - \frac{55\text{V}}{10\Omega} = 0.12\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{p-p} = \hat{I} - \check{I} = 6.4\text{A} - 0.12\text{A} = 6.28\text{A}$$

The ripple value is the same as the E = 0 case, which is as expected since ripple current is independent of back emf with continuous output current.
Alternatively the ripple can be extracted from figure 14.4 using $T/\tau = 1$ and $\delta = 1/4$.

iv. The time domain load current is defined by

$$i_o = \frac{V_s - E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 28.5\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 0.12e^{\frac{-t}{5\text{ms}}}$$

$$= 28.5 - 28.38e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = -\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = -5.5\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 6.4e^{\frac{-t}{5\text{ms}}}$$

$$= -5.5 + 11.9e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (14.39) is

$$\bar{I}_o = {V_o - E}/{R}$$

$$= 85\text{V-}55\text{V}/{10\Omega} = 3\text{A}$$

The average switch current is the average supply current,

$$\bar{I}_i = \bar{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{¼\times(340\text{V} - 55\text{V})}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(6.40\text{A} - 0.12\text{A}) = 0.845\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\bar{I}_{diode} = \bar{I}_o - \bar{I}_i$$

$$= 3\text{A} - 0.845\text{A} = 2.155\text{A}$$

vi. The input power is the dc supply voltage multiplied by the average input current, that is

$$P_{in} = V_s\bar{I}_i = 340\text{V}\times0.845\text{A} = 287.3\text{W}$$

$$P_{out} = P_{in} = 287.3\text{W}$$

From equation (14.18) the rms load current is given by

$$\bar{I}_{o_{rms}} = \sqrt{\frac{P_{out} - E\bar{I}_o}{R}}$$

$$= \sqrt{\frac{287.3\text{W} - 55\text{V}\times3\text{A}}{10\Omega}} = 3.5\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\bar{I}_i}$$

$$= \frac{340\text{V}}{0.845\text{A}} = 402.4\,\Omega$$

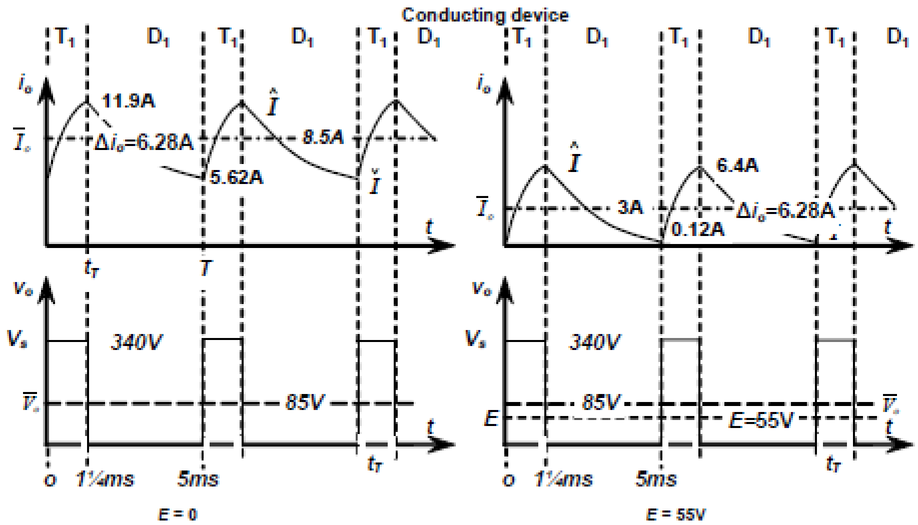


Figure 14.7. Example 14.1. *Circuit waveforms.*

Example 14.2: *DC chopper with load back emf - verge of discontinuous conduction*

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 Ω resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc voltage source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

i. the maximum back emf before discontinuous load current conduction commences with δ=¼;

ii. with 55V back emf, what is the minimum duty cycle before discontinuous load current conduction; and

iii. minimum switching frequency at E = 55V and t_T = 1.25ms before discontinuous conduction.

Solution

The main circuit and operating parameters are

- on-state duty cycle δ = ¼
- period $T = 1/f_s$ = 1/200Hz = 5ms
- on-period of the switch t_T = 1.25ms
- load time constant $\tau = L/R$ = 0.05mH/10Ω = 5ms

First it is necessary to establish whether the given conditions represent continuous or discontinuous load current. The current extinction time t_x for discontinuous conduction is given by equation (14.24), and yields

$$t_x = t_T + \tau\ \ell n\left(1+\frac{V_s - E}{E}\left(1-e^{\frac{-t_T}{\tau}}\right)\right)$$

$$= 1.25\text{ms} + 5\text{ms}\times \ell n\left(1+\frac{340\text{V} - 55\text{V}}{55\text{V}}\times\left(1 - e^{\frac{-1.25\text{ms}}{5\text{ms}}}\right)\right) = 5.07\text{ms}$$

Since the cycle period is 5ms, which is less than the necessary time for the current to fall to zero (5.07ms), the load current is continuous. From example 14.1 part iv, with E = 55V the load current falls from 6.4A to near zero (0.12A) at the end of the off-time, thus the chopper is operating near the verge of discontinuous conduction. A small increase in E, decrease in the duty cycle δ, or increase in switching period T, would be expected to result in discontinuous load current.

i. $\hat{E}$

The necessary back emf can be determined graphically or analytically.

Graphically:

The bounds of continuous and discontinuous load current for a given duty cycle, switching period, and load time constant can be determined from figure 14.5.

Using δ = ¼, T/τ = 1 with τ = 5ms, and T = 5ms, figure 14.5 gives E / V_s = 0.165. That is, $E = 0.165\times V_s$ = 0.165×340V = 56.2V

Analytically:

The chopper is operating too close to the boundary between continuous and discontinuous load current conduction for accurate readings to be obtained from the graphical approach, using figure 14.5. Examination of the expression for minimum current, equation (14.13), gives

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = 0$$

Rearranging to give the back emf, E, produces

$$E = V_s\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1}$$

$$= 340\text{V}\times\frac{e^{\frac{1.25\text{ms}}{5\text{ms}}}-1}{e^{\frac{5\text{ms}}{5\text{ms}}}-1} = 56.2\text{V}$$

That is, if the back emf increases from 55V to 56.2V then at and above that voltage, discontinuous load current commences.

ii. $\check{\delta}$

Again, if equation (14.13) is solved for $\check{I} = 0$ then

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = 0$$

Rearranging to isolate t_T gives

$$t_T = \tau\ \ell n\left(1+\frac{E}{V_s}\left(e^{\frac{T}{\tau}}-1\right)\right)$$

$$= 5\text{ms}\times \ell n\left(1+\frac{55\text{V}}{340\text{V}}\left(e^{\frac{5\text{ms}}{5\text{ms}}}-1\right)\right)$$

$$= 1.226\text{ms}$$

If the switch on-state period is reduced by 0.024ms, from 1.250ms to 1.226ms (δ = 24.52%), operation is then on the verge of discontinuous conduction.

iii. $\hat{T}$

If the switching frequency is decreased such that $T = t_x$, then the minimum period for discontinuous load current is given by equation (14.24). That is,

$$t_x = T = t_T + \tau \, \ell n \left(1 + \frac{V_s - E}{E} \left(1 - e^{\frac{-t_T}{\tau}} \right) \right)$$

$$T = 1.25\text{ms} + 5\text{ms} \times \ell n \left(1 + \frac{340\text{V} - 55\text{V}}{55\text{V}} \times \left(1 - e^{\frac{-1.25\text{ms}}{5\text{ms}}} \right) \right) = 5.07\text{ms}$$

Discontinuous conduction operation occurs if the period is increased by more than 0.07ms.

In conclusion, for the given load, for continuous conduction to cease, the following operating conditions can be changed

- increase the back emf E from 55V to 56.2V
- decrease the duty cycle δ from 25% to 24.52% (t_T decreased from 1.25ms to 1.226ms)
- increase the switching period T by 0.07ms, from 5ms to 5.07ms (from 200Hz to 197.2Hz), with the switch on-time, t_T, unchanged from 1.25ms.

Appropriate simultaneous smaller changes in more than one parameter would suffice.

♣

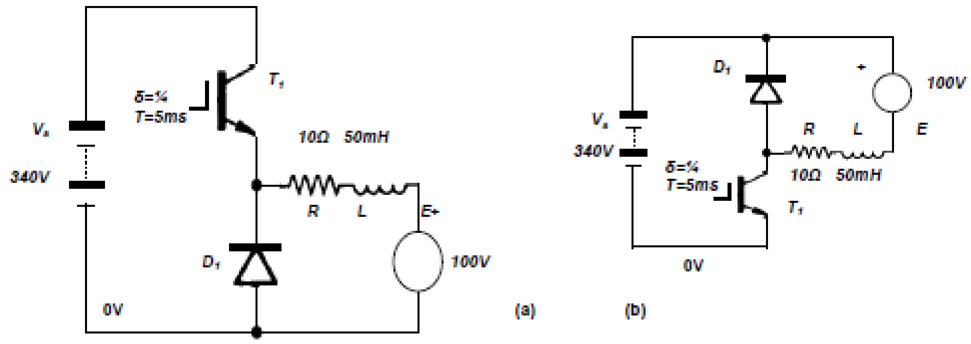


Figure 14.8. Example 14.3. *Circuit diagram:*
(a) with load connected to ground and (b) load connected so that machine flash-over to ground (0V), *by-passes the switch* T_1.

Example 14.3: *DC chopper with load back emf – discontinuous conduction*

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 Ω resistance, 50mH inductance, and an opposing back emf of 100V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple and form factors;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the load average current, average switch current and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, and electromagnetic efficiency; and
viii. sketch the circuit, load, and output voltage and current waveforms.

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{mH}/10\Omega = 5\text{ms}$

Confirmation of discontinuous load current can be obtained by evaluating the minimum current given by equation (14.13), that is

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R}$$

$$\check{I} = \frac{340\text{V}}{10\Omega}\times\frac{e^{\frac{1.25\text{ms}}{5\text{ms}}}-1}{e^{\frac{5\text{ms}}{5\text{ms}}}-1} - \frac{100\text{V}}{10\Omega} = 5.62\text{A} - 10\text{A} = -4.38\text{A}$$

The minimum practical current is zero, so clearly discontinuous current periods exist in the load current. The equations applicable to discontinuous load current need to be employed.
The current extinction time is given by equation (14.24), that is

$$t_x = t_T + \tau\,\ell n\left(1+\frac{V_s - E}{E}\left(1-e^{\frac{-t_T}{\tau}}\right)\right)$$

$$= 1.25\text{ms} + 5\text{ms}\times\ell n\left(1+\frac{340\text{V} - 100\text{V}}{100\text{V}}\times\left(1 - e^{\frac{-1.25\text{ms}}{5\text{ms}}}\right)\right)$$

$$= 1.25\text{ms} + 2.13\text{ms} = 3.38\text{ms}$$

i. From equations (14.28) and (14.29) the load average and rms voltages are

$$\overline{V}_o = \delta V_s + \frac{T - t_x}{T}E$$

$$= ¼\times 340\text{V} + \frac{5\text{ms} - 3.38\text{ms}}{5\text{ms}}\times 100\text{V} = 117.4\text{V}$$

$$V_{rms} = \sqrt{\delta V_s^2 + \frac{T - t_x}{T}E^2}$$

$$= \sqrt{¼\times 340^2 + \frac{5\text{ms} - 3.38\text{ms}}{5\text{ms}}\times 100^2} = 179.3\text{V rms}$$

ii. From equations (14.30) and (14.31) the rms ripple voltage, hence voltage ripple factor, are

$$V_r = \sqrt{V_{rms}^2 - V_o^2}$$

$$= \sqrt{179.3^2 - 117.4^2} = 135.5\text{V ac}$$

$$RF = \frac{V_r}{\overline{V}_o} = \frac{135.5\text{V}}{117.4\text{V}} = 1.15 \qquad FF = \sqrt{RF^2+1} = \sqrt{1.15^2+1} = 1.52$$

iii. From equation (14.38), the maximum and minimum output current, hence the peak-to-peak output ripple in the current, are

$$\hat{I} = \frac{V_s - E}{R}\left(1-e^{\frac{-t_T}{\tau}}\right)$$

$$= \frac{340\text{V}-100\text{V}}{10\Omega}\times\left(1 - e^{\frac{-1.25\text{ms}}{5\text{ms}}}\right) = 5.31\text{A}$$

The minimum current is zero so the peak-to-peak ripple current is $\Delta i_o = 5.31\text{A}$.

iv. From equations (14.34) and (14.35), the current in the time domain is

$$i_o(t) = \frac{V_s - E}{R}\left(1-e^{\frac{-t}{\tau}}\right)$$

$$= \frac{340\text{V} - 100\text{V}}{10\Omega}\times\left(1-e^{\frac{-t}{5\text{ms}}}\right)$$

$$= 24\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o(t) = -\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}e^{\frac{-t}{\tau}}$$

$$= -\frac{100\text{V}}{10\Omega}\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 5.31e^{\frac{-t}{5\text{ms}}}$$

$$= 15.31\times e^{\frac{-t}{5\text{ms}}} - 10 \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 2.13\text{ms}$$

$$i_o(t) = 0 \qquad \text{for} \quad 3.38\text{ms} \le t \le 5\text{ms}$$

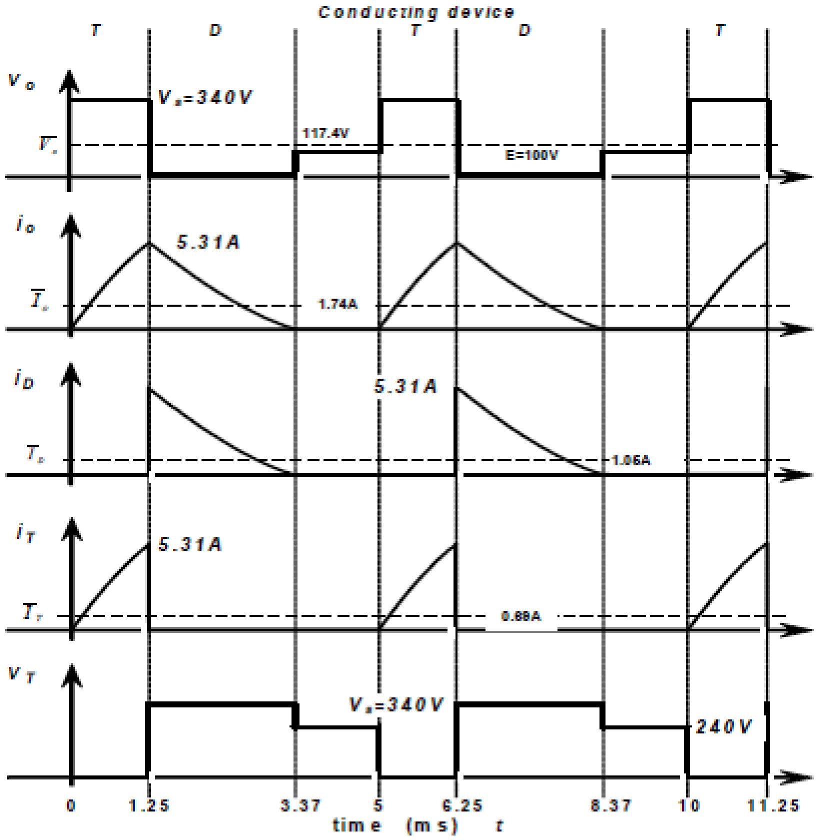


Figure 14.9. Example 14.3. *Chopper circuit waveforms.*

v. From equations (14.39) to (14.42), the average load current, average switch current, and average diode current are

$$\overline{I}_o = \overline{V}_o - E \big/ R$$
$$= 117.4\text{V} - 100\text{V} \big/ 10\Omega = 1.74\text{A}$$

$$\overline{I}_{diode} = \frac{\tau}{T}\hat{I} - \frac{E\left(\dfrac{t_x}{T} - \delta\right)}{R}$$

$$= \frac{5\text{ms}}{5\text{ms}} \times 5.31\text{A} - \frac{100\text{V} \times \left(\dfrac{3.38\text{ms}}{5\text{ms}} - 0.25\right)}{10\Omega} = 1.05\text{A}$$

$$\overline{I}_i = \overline{I}_o - \overline{I}_{diode} = 1.74\text{A} - 1.05\text{A} = 0.69\text{A}$$

vi. From equation (14.40), the input power, hence output power and rms output current are

$$P_{in} = V_s \overline{I}_i = 340\text{V} \times 0.69\text{A} = 234.6\text{W}$$
$$P_{in} = P_{out} = I^2_{o\,rms} R + E\overline{I}_o$$

Rearranging gives

$$I_{o_{rms}} = \sqrt{P_{in} - E\overline{I}_o} \,/\, R$$
$$= \sqrt{234.6\text{W} - 100\text{V} \times 0.69\text{A}} \,/\, 10\Omega = 1.29\text{A}$$

vii. From equations (14.44) and (14.45), the effective input impedance and electromagnetic efficiency, for $E > 0$ are

$$Z_{in} = \frac{V_s}{\overline{I}_i} = \frac{340\text{V}}{0.69\text{A}} = 493\Omega$$

$$\eta = \frac{E\overline{I}_o}{P_{in}} = \frac{E\overline{I}_o}{V_s \overline{I}_i} = \frac{100\text{V} \times 1.74\text{A}}{340\text{V} \times 0.69\text{A}} = 74.2\%$$

viii. The circuit, load, and output voltage and current waveforms are plotted in figure 14.9.

♣

EXAMPLE 6.1

A dc motor with a rated terminal voltage of 110 V is to be fed from a 220 V dc mains with the help of a dc/dc converter. Find the duty ratio required for the converter.

SOLUTION:

$$V_{in} = 220\ \text{V},\ V_o = 110\ \text{V}$$
$$V_{o(av)} = D \cdot V_{in}$$
$$D = \frac{110}{220} \times 100 = 50\%$$

EXAMPLE 6.2

A separately excited dc motor fed from a 250-V mains through a dc/dc converter is running at 1,500 rpm and produces a back-emf of 195 V. Given an armature resistance 1 Ω, and the duty ratio of the chopper is 80%. Calculate the armature current.

SOLUTION:

$$V_{in} = 250\ \text{V} \quad E_b = 195\ \text{V} \quad D = 80\%$$
$$V_{o(av)} = D \cdot V_{in} = 0.8 \times 250 = 200\ \text{V}$$
$$I_a = \frac{V_0 - E_b}{R_a} = \frac{200 - 195}{1} = 5\ \text{A}$$

EXAMPLE 6.3

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 110$ V, $L_a = 1$ mH, $r_a = 0.25\ \Omega$, $E_b = 11$ V, $T = 4{,}500\ \mu$s, $T_{on} = 1{,}000\ \mu$s. Find I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$I_{min} = \frac{110}{0.25}\left[\frac{e^{\frac{0.25}{1\times10^{-3}}1000\times10^{-6}} - 1}{e^{\frac{0.25}{1\times10^{-3}}4500\times10^{-6}} - 1}\right] - \frac{11}{0.25}$$

$$I_{min} = 16.0759\ \text{A}$$

If I_{min} is positive, then the motor current is continuous.

$$I_{max} = \frac{V_{dc}}{r_a}\left[\frac{1 - e^{\frac{-r_a}{L_a}T_{on}}}{1 - e^{\frac{-r_a}{L_a}T}}\right] - \frac{E_b}{r_a}$$

$$I_{min} = \frac{110}{0.25}\left[\frac{e^{-\frac{0.25}{1\times10^{-3}}1000\times10^{-6}} - 1}{e^{-\frac{0.25}{1\times10^{-3}}4500\times10^{-6}} - 1}\right] - \frac{11}{0.25}$$

$$I_{max} = 100.11\ \text{A}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T}$$

$$= 110 \times \frac{1000\times10^{-6}}{4500\times10^{-6}}$$

$$V_{o(av)} = 24.44\ \text{V}$$

EXAMPLE 6.4

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 200$ V, $L_a = 2$ mH, $r_a = 1.0\ \Omega$, $E_b = 40$ V, $T = 10$ ms, $T_{on} = 7$ ms. Calculate I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$= \frac{200}{1}\left[\frac{e^{\frac{1}{2\times10^{-3}}\times7\times10^{-3}} - 1}{e^{\frac{1}{2\times10^{-3}}\times10\times10^{-3}} - 1}\right] - \frac{40}{1}$$

$$I_{min} = 3.572 \text{ A}$$

Positive sign of I_{min} indicates the motor current is continuous.

$$I_{max} = \frac{V_{dc}}{r_a}\left[\frac{1 - e^{\frac{-r_a}{L_a}T_{on}}}{1 - e^{\frac{-r_a}{L_a}T}}\right] - \frac{E_b}{r_a}$$

$$= \frac{200}{1}\left[\frac{1 - e^{-\frac{1}{2\times10^{-3}}\times7\times10^{-3}}}{1 - e^{-\frac{1}{2\times10^{-3}}\times10\times10^{-3}}}\right] - \frac{40}{1}$$

$$I_{max} = 155.276 \text{ A}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T}$$

$$= 200 \times \frac{7\times10^{-3}}{10\times10^{-3}}$$

$$V_{o(av)} = 140 \text{ V}$$

EXAMPLE 6.5

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 200$ V, $L_a = 0.8$ mH, $r_a = 0.65\ \Omega$, $E_b = 120$ V, $T = 3$ ms, $T_{on} = 0.25$ ms. Find I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$= \frac{200}{0.65}\left[\frac{e^{\frac{0.65}{0.8\times10^{-3}}0.25\times10^{-3}} - 1}{e^{\frac{0.65}{0.8\times10^{-3}}3\times10^{-3}} - 1}\right] - \frac{120}{0.65}$$

$$I_{min} = -177.9857 \text{ A}$$

A negative sign indicates that the motor current is discontinuous.

Hence, $I_{min} = 0$. From Equation (6.6),

$$I_{max} = \frac{V_{dc} - E_b}{r_a}\left(1 - e^{-\frac{r_a}{L_a}T_{on}}\right)$$

$$= \frac{200 - 120}{0.65}\left(1 - e^{-\frac{0.65}{0.8\times10^{-3}}0.25\times10^{-3}}\right)$$

$$I_{max} = 22.6245 \text{ A}$$

$$t^x = -\frac{L_a}{r_a}\ln\left[\frac{E_b}{V_{dc}\left(1 - e^{-\frac{r_a}{L_a}T_{on}}\right) + E_b\left(e^{-\frac{r_a}{L_a}T_{on}}\right)}\right]$$

$$= -\frac{0.8\times10^{-3}}{0.65}\ln\left[\frac{120}{200\left(1 - e^{\frac{-0.65}{0.8\times10^{-3}}0.25\times10^{-3}}\right) + 120\left(e^{\frac{-0.65}{0.8\times10^{-3}}0.25\times10^{-3}}\right)}\right]$$

$$t^x = 0.1423 \text{ ms}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T} + E_b\left(1 - \frac{T_{on}}{T} - \frac{t^x}{T}\right)$$

$$= 200\frac{0.25\times10^{-3}}{3\times10^{-3}} + 120\left(1 - \frac{0.25\times10^{-3}}{3\times10^{-3}} - \frac{0.1423\times10^{-3}}{3\times10^{-3}}\right)$$

$$V_{o(av)} = 120.9812 \text{ V}$$

EXAMPLE 6.6

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 110$ V, $L_a = 0.2$ mH, $r_a = 0.25\ \Omega$, $E_b = 40$ V, $T = 2{,}500\ \mu$s, $T_{on} = 1{,}250\ \mu$s. Find I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$I_{min} = \frac{110}{0.25}\left[\frac{e^{\frac{0.25}{0.2\times10^{-3}}1250\times10^{-6}} - 1}{e^{\frac{0.25}{0.2\times10^{-3}}2500\times10^{-6}} - 1}\right] - \frac{40}{0.25}$$

$$I_{min} = -83.754 \text{ A}$$

A negative sign indicates that the motor current is discontinuous.

Hence $I_{min} = 0$. Now, using Equation (6.6),

$$I_{max} = \left(\frac{V_{dc} - E_b}{R}\right)\left(1 - e^{-\frac{R}{L}T_{on}}\right)$$

$$= \left(\frac{110 - 40}{025}\right)\left(1 - e^{-\frac{-0.25}{0.2\times10^{-3}}1250\times10^{-6}}\right)$$

$$I_{max} = 221.312 \text{ A}$$

$$t^x = -\frac{L_a}{r_a}\ln\left[\frac{E_b}{V_{dc}\left(1 - e^{-\frac{r_a}{L_a}T_{on}}\right) + E_b\left(e^{-\frac{r_a}{L_a}T_{on}}\right)}\right]$$

$$t^x = -\frac{0.2\times10^{-3}}{0.25}\ln\left[\frac{40}{110\left(1 - e^{\frac{-0.25}{0.2\times10^{-3}}1250\times10^{-6}}\right) + 40\left(e^{\frac{-0.25}{0.2\times10^{-3}}1250\times10^{-6}}\right)}\right]$$

$$t^x = 0.6948 \text{ ms}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T} + E_b\left(1 - \frac{T_{on}}{T} - \frac{t_x}{T}\right)$$

$$= 110\frac{1250\times10^{-6}}{2500\times10^{-6}} + 40\left(1 - \frac{1250\times10^{-6}}{2500\times10^{-6}} - \frac{0.6948\times10^{-3}}{2500\times10^{-6}}\right)$$

$$V_{o(av)} = 63.882 \text{ V}$$

EXAMPLE 6.7

A first-quadrant chopper is feeding a separately excited motor rated 200 V, 10 A, 1,500 rpm. The chopper is supplied from a constant bus bar voltage of 300 V. If the SCR turn-OFF time is 38 μs, compute the value of commutation components.

$$L = \frac{t_1}{F(x)} \frac{V_{dc}}{xI_a}$$

where t_1 = circuit turn-OFF time (and must be greater than SCR turn-OFF time) and V_{dc} = source voltage:

$$\omega_{LC} t_1 = \pi - 2\sin^{-1}\left(\frac{1}{x}\right) = F(x)$$

I_a: Maximum load current
I_m: Peak capacitor current
$V_{dc} = 300$ V
$t_1 = 38 + 38 = 76$ μs
$x = 1.5$
$I_a = 10$ A

$$L = \frac{76 \times 10^{-6} \times 300}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 1.5 \times 10}$$

$$L = 903.61\ \mu H$$

$$C = \frac{t_1}{F(x)} \frac{xI_a}{V_{dc}}$$

$$C = \frac{76 \times 10^{-6} \times 1.5 \times 10}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 300}$$

$$C = 2.259\ \mu F$$

EXAMPLE 6.8

First-quadrant chopper using current-commutated SCRs is used to drive a 230-V, 25-A, 1,450-rpm separately excited dc motor. If the motor is supplied from a 110-V busbar and the turn-OFF time of the SCR is 25 μs, design the commutation circuit.

SOLUTION:

$$L = \frac{t_1}{F(x)}\frac{V_{dc}}{xI_a}$$

$$\omega_{LC}t_1 = \pi - 2\sin^{-1}\left(\frac{1}{x}\right) = F(x)$$

$V_{dc} = 110$ V
$t_1 = 25 + 25 = 50$ μs
$x = 1.5$
$I_a = 25$ A

$$L = \frac{50 \times 10^{-6} \times 110}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 1.5 \times 25}$$

$$L = 87.19\ \mu H$$

$$C = \frac{t_1}{F(x)}\frac{xI_a}{V_{dc}}$$

$$C = \frac{50 \times 10^{-6} \times 1.5 \times 25}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 110}$$

$$C = 10.13\ \mu F$$

EXAMPLE 6.9

The speed of a 230-V, 25-A, 150-rpm separately excited dc motor is controlled by connecting a current-commutated first-quadrant chopper in the armature. The input voltage to the chopper is 300 V. The circuit turn-OFF time is fixed at 25 μs and the peak capacitor current is twice the load current. Compute the values of commutating components.

SOLUTION:

$V_{dc} = 300$ V
$t_1 = 25$ ms
$x = 2$
$I_a = 25$ A

$$L = \frac{t_1}{F(x)}\frac{V_{dc}}{xI_a}$$

$$L = \frac{25 \times 10^{-6} \times 300}{\left(\pi - 2\sin^{-1}\left(\frac{1}{2}\right)\right) \times 2 \times 25}$$

$$\mathbf{L = 71.62\ \mu H}$$

$$C = \frac{t_1}{F(x)}\frac{xI_a}{V_{dc}}$$

$$C = \frac{25 \times 10^{-6} \times 2 \times 25}{\left(\pi - 2\sin^{-1}\left(\frac{1}{2}\right)\right) \times 300}$$

$$\mathbf{C = 1.989\ \mu F}$$

EXAMPLE 7.1

A 230-V, 6-A, 1,500-rpm separately excited dc motor has an armature resistance of 5.1 Ω. A 1-quadrant chopper supplied from a 300-V dc bus is operating at a duty ratio of 60% and supplies power to the motor armature at rated current. Compute the motor speed.

SOLUTION:

At rated condition,

$$\begin{aligned} E_{b\,\text{rated}} &= V_a - I_{a\,\text{rated}} \times r_a \\ &= 230 - (5.1 \times 6) \\ &= 199.4\ \text{V} \end{aligned}$$

$$E_{b\,\text{rated}} \propto 1,500\ \text{rpm}(N_{\text{rated}})$$

At 60% duty ratio, $E_{b1} \propto N_1$.

$$\begin{aligned} \text{Armature voltage} &= 300 \times (60/100) \\ &= 180\ \text{V} \end{aligned}$$

$$\begin{aligned} \text{Back-emf}, E_{b1} &= 180 - (5.1 \times 6) \\ &= 149.4\ \text{V} \end{aligned}$$

Therefore,

$$\begin{aligned} N_1 &= \frac{E_{b1}}{E_{b\,\text{rated}}} N_{\text{rated}} \\ &= \frac{149.4}{199.4} \cdot 1500 \approx 1124\ \text{rpm} \end{aligned}$$

EXAMPLE 7.2

A first-quadrant dc/dc converter is fed from a 300-V dc bus. When the converter supplies power to a separately excited dc motor at 40% duty ratio, the average armature current is 5 A at 1,560 rpm. What is the duty ratio required to reduce the speed to 1,300 rpm for the same armature current? The armature resistance is 5.1 Ω.

SOLUTION:

At 40% duty ratio, armature voltage is

$$\begin{aligned} V_a &= 300 \times \frac{40}{100} \\ &= 120\ \text{V} \end{aligned}$$

$$\text{Back-emf at 40\% duty ratio, } E_{b1} = 120 - (I_a r_a)$$
$$= 120 - (5 \times 5.1)$$
$$= 94.5\text{ V}$$
$$E_{b1} \propto 1,560\text{ rpm}$$

Now back-emf, E_{b2}, at 1,300 rpm can be related as $E_{b2} \propto 1{,}300$ rpm.

$$E_{b2} = \frac{N_2}{N_1} E_{b1} = 78.75\text{ V}$$

$$\text{Armature voltage, } V_a = E_{b2} + I_a r_a$$

$$V_a = 300 \times D$$

Therefore,

$$D = 34.75\%$$

EXAMPLE 7.3

A separately excited dc motor is fed from a 440-V dc source through a single-quadrant chopper, $r_a = 0.2\ \Omega$, and armature current is 175 A. The voltage and torque constants are equal at 1.2 V/rad/s. The field current is 1.5 A. The duty cycle of chopper is 0.5. Find (a) speed and (b) torque.

SOLUTION:

a.

$$E_b = 220 - 175 \times 0.2 = 185\text{ V}$$
$$= 1.2 \times \omega_r \times I_f$$
$$\omega_r = 185/(1.2 \times 1.5)$$
$$= 102.77\text{ rad/s}$$
$$= 981.38\text{ rpm}$$

b.

$$\text{Torque} = 1.2 \times 1.5 \times 175 = 315\text{ N-m}$$

EXAMPLE 7.4

A separately excited dc motor has the following name plate data: 220 V, 100 A, 2,200 rpm. The armature resistance is 0.1 Ω, and inductance is 5 mH. The motor is fed by a chopper that is operating from a dc supply of 250 V. Due to restrictions in the power circuit, the chopper can be operated over a duty cycle ranging from 30% to 70%. Determine the range of speeds over which the motor can be operated at rated torque.

SOLUTION:

Because e torque is constant, i_a is the same for all the values of D.

$$V_{o(av)} = DV_{dc}$$

At D = 0.3,

$$V_{o(av)} = 0.3 \times 250$$
$$= 75 \text{ V}$$
$$E_{b(0.3)} = V_{o(av)} - I_a r_a$$
$$= 75 - (100 \times 0.1)$$
$$= 65 \text{ V}$$

At D = 0.7,

$$V_{o(av)} = 0.7 \times 250$$
$$= 175 \text{ V}$$
$$E_{b(0.7)} = 175 - 10$$
$$= 165 \text{ V}$$

Under rated conditions, $V_a = 220$ V, $I_a = 100$ A, $r_a = 0.1\ \Omega$

$$E_{b(rated)} = 220 - (100 \times 0.1)$$
$$= 210 \text{ V}$$
$$N_r = 2200 \text{ rpm}$$
$$\frac{N_{0.7}}{N_r} = \frac{E_{b(0.7)}}{E_{b(rated)}}$$
$$N_{(0.7)} = \frac{165}{210} \times 2200$$
$$= 1{,}728.5714 \text{ rpm}$$
$$N_{(0.3)} = \frac{65}{210} \times 2200$$
$$= 680.95 \text{ rpm}$$

Hence speed can be varied in the range $680.95 \le N \le 1728.5714$.

EXAMPLE 7.5

A separately excited dc motor has an armature resistance 2.3 Ω, and armature current is 100 A. (a) Find the voltage across the braking resistance for a duty ratio of 25%. (b) Find the power dissipated in braking resistance.

SOLUTION:

$$\text{Average current} = I_a(1 - D) = 100(1 - 0.25) = 75 \text{ A}$$

$$\text{Average Voltage} = I_{b(avg)} \times R_b = 75 \times 2.3 = 172.5 \text{ V}$$

$$P_b = I_a^2 R_b (1 - D)$$

$$P_b = 100^2 \times 2.3 \times (1 - 0.25) = 17250 \text{ W}$$

EXAMPLE 7.6

A separately excited dc motor has the following name plate data: 200 V, 75 A, and 1,500 rpm. The armature resistance is 0.2 Ω. If dynamic baking takes place at 600 rpm at rated torque, compute the duty ratio. The braking resistance is 5 Ω.

SOLUTION:

E_b under rated condition,

$$E_{b(\text{rated})} = V - I_a r_a$$
$$= 200 - (75 \times 0.2)$$
$$= 185 \text{ V}$$

$$\frac{E_{b(600)}}{E_{b(\text{rated})}} = \frac{N}{N_{\text{rated}}}$$

$$E_{b(600)} = \left(\frac{N}{N_{\text{rated}}}\right) E_{b\,\text{rated}}$$

$$E_{b(600)} = \left(\frac{600}{1500}\right) \times 185 = 74 \text{ V}$$

Now,

$$E_{b(600)} = I_a\left(r_a + R_b(1-D)\right)$$

Because braking takes place at rated torque, $I_a = I_{a\text{rated}} = 75$ A,

$$\text{i.e., } E_{b(600)} = 75\left(0.2 + 5(1-D)\right)$$
$$74 = 75 \times 0.2 + 75 \times 5(1-D)$$
$$\therefore D = 0.84$$

EXAMPLE 7.7

A dual-input dc/dc converter is supplied from two dc sources: 12-V and 24-V batteries. The duty ratio of the power switch connected to the first source is 40%, while that of the second source is 25%. Compute the average output voltage. The load consists of large inductance and resistance.

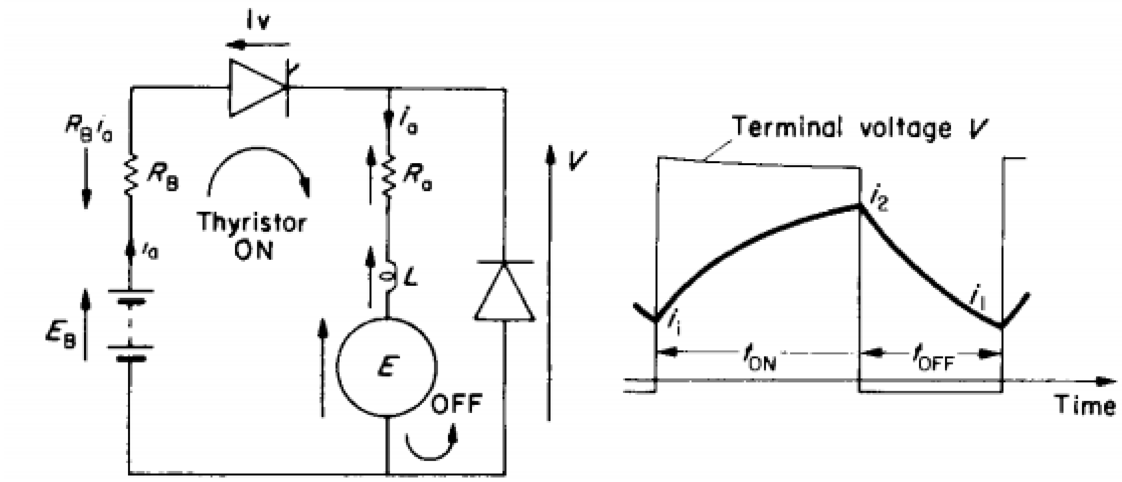
SOLUTION:

$$V_{o(av)} = 12 \times \frac{40}{100} + 24 \times \frac{25}{100}$$
$$= 4.8 + 6$$
$$= 10.8 \text{ V}$$

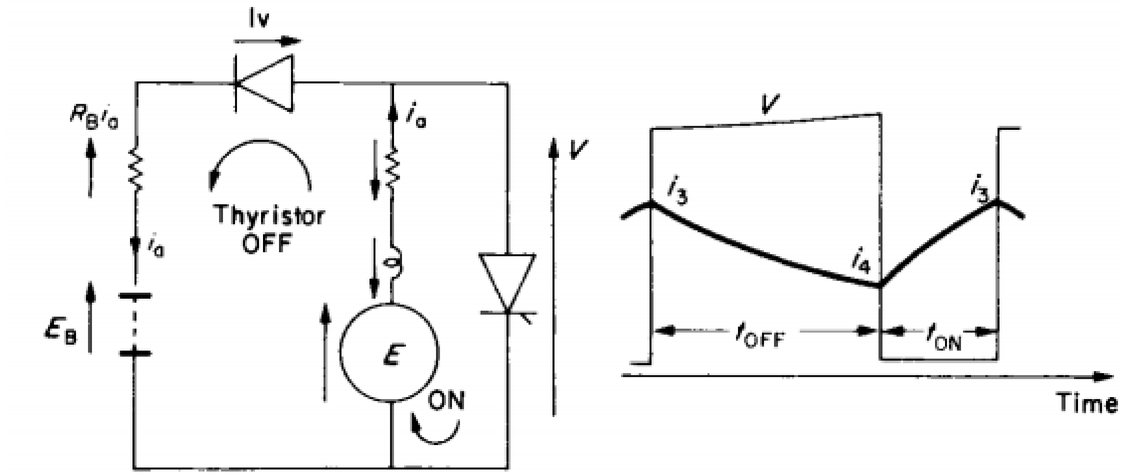
Example 7.1

An electrically-driven automobile is powered by a d.c. series motor rated at 72 V, 200 A. The motor resistance and inductance are respectively 0.04 Ω and 6 milli-henrys. Power is

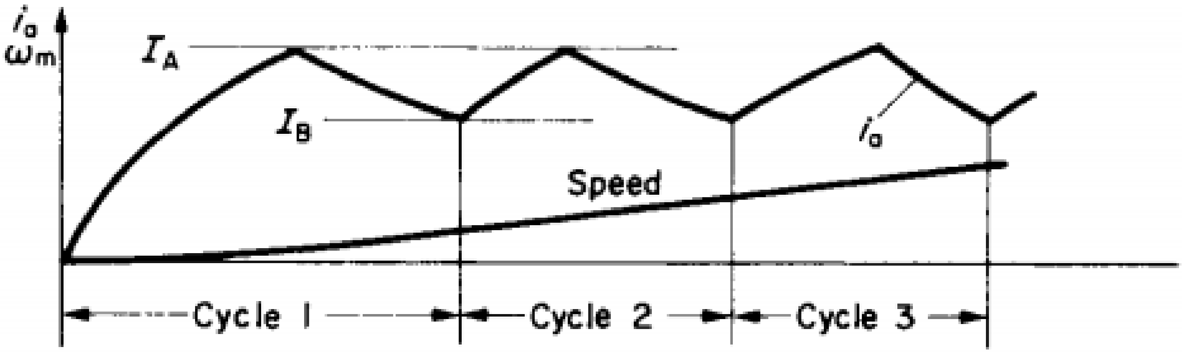
supplied via an ON/OFF controller having a fixed frequency of 100 Hz. When the machine is running at 2500 rev/min the generated-e.m.f. per field-ampere, k_{f_s}, is 0.32 V which may be taken as a mean "constant" value over the operating range of current. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor, when operating at this particular speed and with a duty-cycle ratio δ of 3/5. Mechanical, battery and semi-conductor losses may be neglected when considering the relevant diagrams of Fig. 7.1a.



(a) MOTORING (Motoring conventions)



(b) GENERATING (Generating conventions)



(c) Acceleration between limits

FIG. 7.1. Chopper-fed d.c. machine.

Chopping period = 1/100 = 10 msec and for $\delta = 3/5$; ON + OFF = 6 + 4 msec.

The equations are:

for ON period: $V = k_{fs}i + Ri + Lpi$—from eqn (7.2a).

Substituting: $72 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$.

For OFF period: $0 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$—from eqn (7.2b).

Rearranging:

ON $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 200 = I_{max}$,

OFF $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 0 = I_{min}$.

Current oscillates between a "low" of i_1 and a "high" of i_2, with $\tau = 0.0167$ second

ON $i_2 = i_1 + (200 - i_1)(1 - e^{-0.006/0.0167})$,

$i_2 = 200 - (200 - i_1)e^{-0.36} = \underline{60.46 + 0.698 i_1}$.

OFF $i_1 = i_2 + (0 - i_2)(1 - e^{-0.004/0.0167})$

$i_1 = i_2\,e^{-0.24} = \underline{0.787 i_2}$.

Hence, by substituting: $i_2 = 60.46 + 0.698 \times 0.787 i_2$,

from which $\underline{i_2 = 134.1\text{ A}}$ and $\underline{i_1 = 105.6\text{ A}}$.

$$\text{Torque} = k_\phi i = \frac{k_{fs} i}{\omega_m} \times i = \frac{k_{fs}}{\omega_m} i^2.$$

$$\text{Mean torque} = \frac{0.32}{2500 \times 2\pi/60}\left(\frac{134.1^2 + 105.6^2}{2}\right) = \underline{17.8\text{ Nm}}.$$

$$\text{Mean power} = \omega_m T_e = \frac{2\pi}{60} \times 2500 \times 17.8 = \underline{4.66\text{ kW}} = \underline{6.25\text{ hp}}.$$

Example 7.2

The chopper-controlled motor of the last question is to be separately excited at a flux corresponding to its full rating. During acceleration, the current pulsation is to be maintained as long as possible between 170 and 220 A. During deceleration the figures are to be 150 and 200 A. The total mechanical load referred to the motor shaft corresponds to an armature current of 100 A and rated flux. The total inertia referred to the motor shaft is

1.2 kg m^2. The battery resistance is 0.06 Ω and the semiconductor losses may be neglected. Determine the ON and OFF periods for both motoring and regenerating conditions and hence the chopping frequency when the speed is 1000 rev/min.

Calculate the accelerating and decelerating rates in rev/min per second and assuming these rates are maintained, determine the time to accelerate from zero to 1000 rev/min and to decelerate to zero from 1000 rev/min. Reference to all the diagrams of Fig. 7.1 will be helpful.

Rated flux at rated speed of 2500 rev/min corresponds to an e.m.f.:

$$E = V - RI_a = 72 - 0.04 \times 200 = 64\,\text{V}$$

At a speed of 1000 rev/min therefore, full flux corresponds to $64 \times 1000/2500 = 25.6$ V

Acceleration Total resistance $= R_a + R_B = 0.04 + 0.06 = 0.1\,\Omega$

For ON period $E_B = E + Ri_a + Lpi_a$,

$$72 = 25.6 + 0.1 i_a + 0.006 pi_a.$$

Rearranging: $0.06\, di_a/dt + i_a = 464 = I_{max}$.

Solution is: $i_2 = i_1 + (I_{max} - i_1)(1 - e^{-t_{ON}/\tau})$

and since i_1 and i_2 are known: $220 = 170 + (464 - 170)(1 - e^{-t_{ON}/0.06})$.

$$\frac{220 - 170}{464 - 170} = 1 - e^{-t_{ON}/0.06}$$

from which: $\underline{t_{ON} = 0.01118.}$

For OFF period $0 = 25.6 + 0.04 i_a + 0.006 pi_a$ (note resis. $= R_a$).

Rearranging: $0.15\, di_a/dt + i_a = -640 = I_{min}$.

Solution is: $i_1 = i_2 + (I_{min} - i_2)(1 - e^{-t_{OFF}/\tau})$.

Substituting i_1 and i_2: $170 = 220 + (-640 - 220)(1 - e^{-t_{OFF}/0.15})$,

$$\frac{170 - 220}{-640 - 220} = 1 - e^{-t_{OFF}/0.15},$$

from which: $\underline{t_{OFF} = 0.008985}$ $\underline{t_{ON} + t_{OFF} = 0.02017 \text{ second.}}$

Duty cycle $\delta = 0.01118/0.02017 = \underline{0.554.}$ Chopping frequency $= 1/0.02017 =$ 49.58 Hz.

Deceleration

Thyristor ON $\qquad 0 = E - R_a i_a - Lpi_a.$

Substituting: $\qquad = 25.6 - 0.04 i_a - 0.006 pi_a.$

Rearranging: $\qquad 0.15\,di_a/dt + i_a = 640 = I_{max}.$

Solution is: $\qquad i_3 = i_4 + (I_{max} - i_4)(1 - e^{-t_{ON}/\tau}).$

Substituting: $\qquad 200 = 150 + (640 - 150)(1 - e^{-t_{ON}/0.15}),$

$$\frac{200-150}{640-150} = 1 - e^{-t_{ON}/0.15},$$

from which: $\qquad \underline{t_{ON} = 0.01614.}$

Thyristor OFF $\qquad E_B = E - Ri_a - Lpi_a,$

$$72 = 25.6 - 0.1 i_a - 0.006 Lpi_a.$$

Rearranging: $\qquad 0.06\,di_a/dt + i_a = -464 = I_{min}.$

Solution is: $\qquad i_4 = i_3 + (I_{min} - i_3)(1 - e^{-t_{OFF}/\tau}).$

Substituting: $\qquad 150 = 200 + (-464 - 200)(1 - e^{-t_{OFF}/0.06}),$

$$\frac{150-200}{-464-200} = 1 - e^{-t_{OFF}/0.06}$$

from which: $\qquad \underline{t_{OFF} = 0.004697} \qquad \underline{t_{ON} + t_{OFF} = 0.02084\ \text{second.}}$

Duty cycle $\delta = 0.01614/0.02084 = \underline{0.774.}$ Chopping frequency $= 1/0.02084 =$ $\underline{47.98\ \text{Hz.}}$

Accelerating time

$$\text{Load torque} = k_\phi I_a = \frac{E}{\omega_m} I_a \quad = \frac{64}{2500 \times 2\pi/60} \times 100 = 24.45\ \text{Nm}.$$

During acceleration:

$$k_\phi I_{mean} = \frac{64}{2500 \times 2\pi/60} \times \frac{220+170}{2} = 0.2445 \times 195 = 47.67\ \text{Nm}.$$

$$\text{Constant } d\omega_m/dt \quad = \frac{T_e - T_m}{J} = \frac{47.67 - 24.45}{1.2} = 19.35\ \text{rad/s per second}$$

$$= 19.35 \times \frac{60}{2\pi} = \underline{184.8\ \text{rev/min per sec.}}$$

$$\text{Accelerating time to 1000 rev/min} = \frac{1000}{184.8} = \underline{5.41\ \text{seconds.}}$$

Decelerating time

During deceleration: $k_\phi I_{mean} = 0.2445(-200 - 150)/2 = -42.8\ \text{Nm}.$

Note that this electromagnetic torque is now in the same sense as T_m, opposing rotation.

The mechanical equation is: $\qquad T_e = T_m + J\,d\omega_m/dt,$

$$-42.8 = 24.45 + 1.2\,d\omega_m/dt.$$

from which:

$$\frac{d\omega_m}{dt} = \frac{-67.25}{1.2} = -56.04\ \text{rad/s per second} = \underline{-535.1\ \text{rev/min per second.}}$$

Time to stop from 1000 rev/min with this torque maintained $= 1000/535.1 =$ $\underline{1.87\ \text{seconds.}}$

Example 4

A separately excited d.c. motor with R_a = 1.2 ohms and L_a = 30 mH , is to be controlled using class-A thyristor chopper as shown in Fig.9.11 .The d.c. supply V_d = 120V . By ignoring the effect of the armature inductance L_a , it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle γ =1.

(b) Draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant $Ke\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to γ = 0.5.

Solution

The average armature voltage is

$$V_{av} = \gamma V_d = 1 \times 120 = 120\,V$$

The motor's speed:

$$n = \frac{V_{av}}{K_e\emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

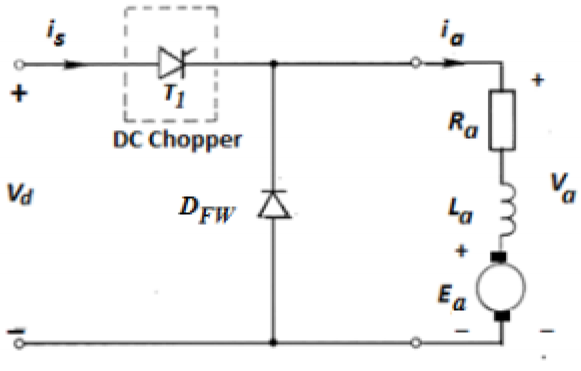


Fig. 9.11 Thyristor chopper drive.

At no load $T_d = 0$, hence

or $$n_o = \frac{\gamma V_d}{K_e\emptyset} = \frac{120}{0.042} = 2857\,rpm$$

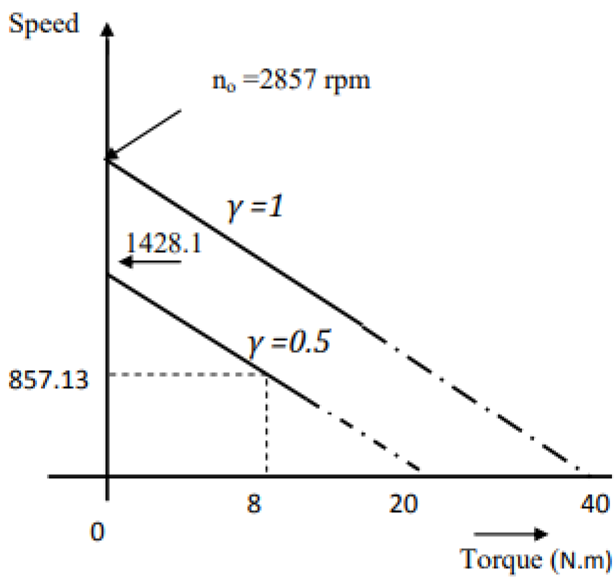
At starting, $n = 0$. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e\emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$\therefore T_{st} = \frac{9.55\,\gamma V_d}{R_a} K_e\emptyset$$

$$T_{st} = \frac{9.55\,x120}{1.2} \times 0.042 = 40\,N.m$$

Fig.9.11 Speed-torque characteristics



(b) At γ =*0.5*

$$V_a = \gamma V_d = 0.5 x 120 = 60 V$$

$$n_o = \frac{\gamma V_d}{K_e Ø} = \frac{60}{0.042} = 1428.5 rpm$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20 \text{ N.m}$$

At $\gamma = 0.5$, $T_L = 8$ N.m

$$n = \frac{60}{0.042} - \frac{1.2}{9.55(0.042)^2} \times 8 = 857.13 \text{ rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 5

In the microcomputer -controlled class –A IGBT transistor DC chopper shown in Fig.12.6, the input voltage V_d = 260V, the load is a separately excited d.c. motor with Ra = 0.28 Ω and L_a = 30 mH . The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor *n* (rpm) when the chopper is switched fully ON such that the duty cycle γ = 1.0.

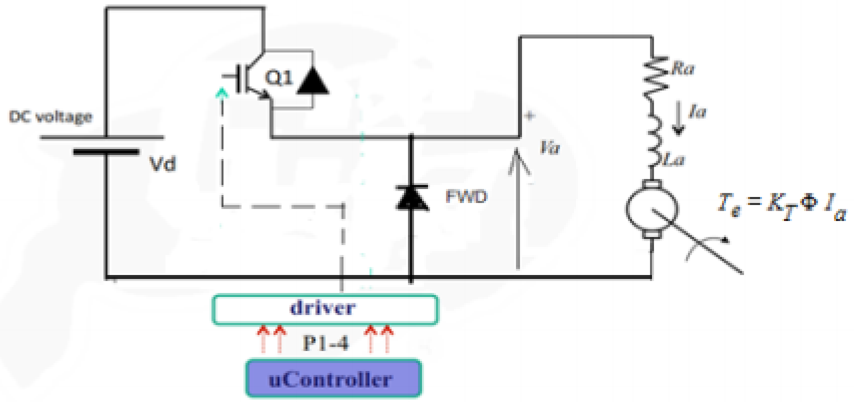


Fig.12.6 IGBT Chopper drive.

Solution

(a) With steady – state operation of the motor, the armature inductance L_a behaves like a short circuit and therefore has no effect at all.

At stand still n = 0 , and therefore $E_a = 0$, hence from Eq.(12.22)

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30\ A$$

$$\therefore\ V_{a0} = 0.28 \times 30 = 8.4\ V$$

At full speed n = 2500 rpm ,

$$E_{a2500} = K_e\,\phi\,n = 0.1 \times 2500 = 250\ V$$

For separately excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4\ V$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. γ =1, then $V_a = V_d = 260$ V .

At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e\ \phi\ n + I_a R_a\ = 260\ \text{ V}$$

$$0.1\ n + 30\ x\ 0.28\ = 260 \qquad \rightarrow \qquad n = 2516 \text{ rpm}$$

Example 1: A separately-excited d.c. motor with $R_a = 0.3\Omega$, and $L_a = 15$ mH is to be speed controlled over a range 0-2000 r.p.m. The d.c. supply is 220V. The load torque is constant and requires an average armature current of 25A.

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi = 0.1002$ V/rpm.

Solution: In the steady-state, the armature inductance has no effect. The required motor terminal voltages are:

At $n = 0$, $E_b = 0$, so that

$$V_{dc} = E_b + I_a R_a = I_a R_a = 25 \times 0.3 = 7.5\,V.$$

At $n = 2000$ rpm,

$$E_b = K_e\Phi n = 0.1002 \times 2000 = 200.4\,V.$$

$$\therefore V_{dc} = E_b + I_a R_a = 200.4 + 25 \times 0.3 = 207.9\,V.$$

$$V_o = \gamma V_d$$

To give $V_o = 7.5\,V$ $\therefore$ $7.5 = \gamma_o \times 220$ or $\gamma_o = \frac{7.5}{220} = 0.034$

To give $V_o = 207.9\,V$ $\therefore$ $207.9 = \gamma_{2000} \times 220$

or $\gamma_{2000} = \frac{207.9}{220} = 0.943$.

Range of γ: $0.034 \leq \gamma \leq 0.943$.

(b) If the chopper was to be switched fully on, what is the speed of the motor when $\gamma = 1$.

Sol. when $\gamma = 1$, $V_o = 220$.

$$\therefore n = \frac{E_b}{K_e\Phi}, \quad E_b = V_o - I_a R_a = 220 - 25 \times 0.3 = 212.5\,V$$

$$n = \frac{212.5}{0.1002} = \underline{\underline{2121}} \text{ rpm}$$

Example:

An electrically-driven automobile is powered by d.c. series motor rated at 100V, 200A. The motor resistance and inductance are respectively 0.65 Ω and 6 mH. power is supplied from ideal battry of 120V via class-A d.c. chopper having a fixed frequency of 100 Hz. The machine constant $K e\phi = 0.00025 \, V/rpm$ and the motor speed is 2500 rpm. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor when running at 2500 rpm with duty cycle γ of 3/5.

Solution:

Chopping period $T = \frac{1}{f} = \frac{1}{100} = 10 \text{ ms}.$

$$t_{on} = \gamma T = \frac{3}{5} \times 10 = 6 \text{ ms}$$

$$\therefore \; t_{off} = 10 - 6 = 4 \text{ ms}.$$

$$I_{max} = \frac{V_{av}}{R_a} + \frac{t_{off}}{2 L_a} V_{av} = \frac{\gamma V_i}{R_a} + \frac{t_{off}}{2 L_a} \gamma V_i$$

$$= \frac{\frac{3}{5} \times 120}{0.65} + \frac{4 \times 10^{-3}}{2 \times 6 \times 10^{-3}} \left(\frac{3}{5} \times 120\right)$$

$$= 110.7 + 24 = 134.7 \text{ A}$$

$$I_{min} = \frac{V_{av}}{R_a} - \frac{t_{off}}{2 L_a} V_{av} = 86.76 \text{ A}.$$

$$= 110.7 - 24$$

For series motor:

Mean torque: $T_e = K_T \phi I_{av}^2 = 9.55 \, K e \Phi \left(\frac{I_{max} + I_{min}}{2}\right)^2$

$$= 9.55 \times 0.00025 \left(\frac{134.7 + 86.76}{2}\right)^2$$

$$= 30 \text{ N.m}.$$

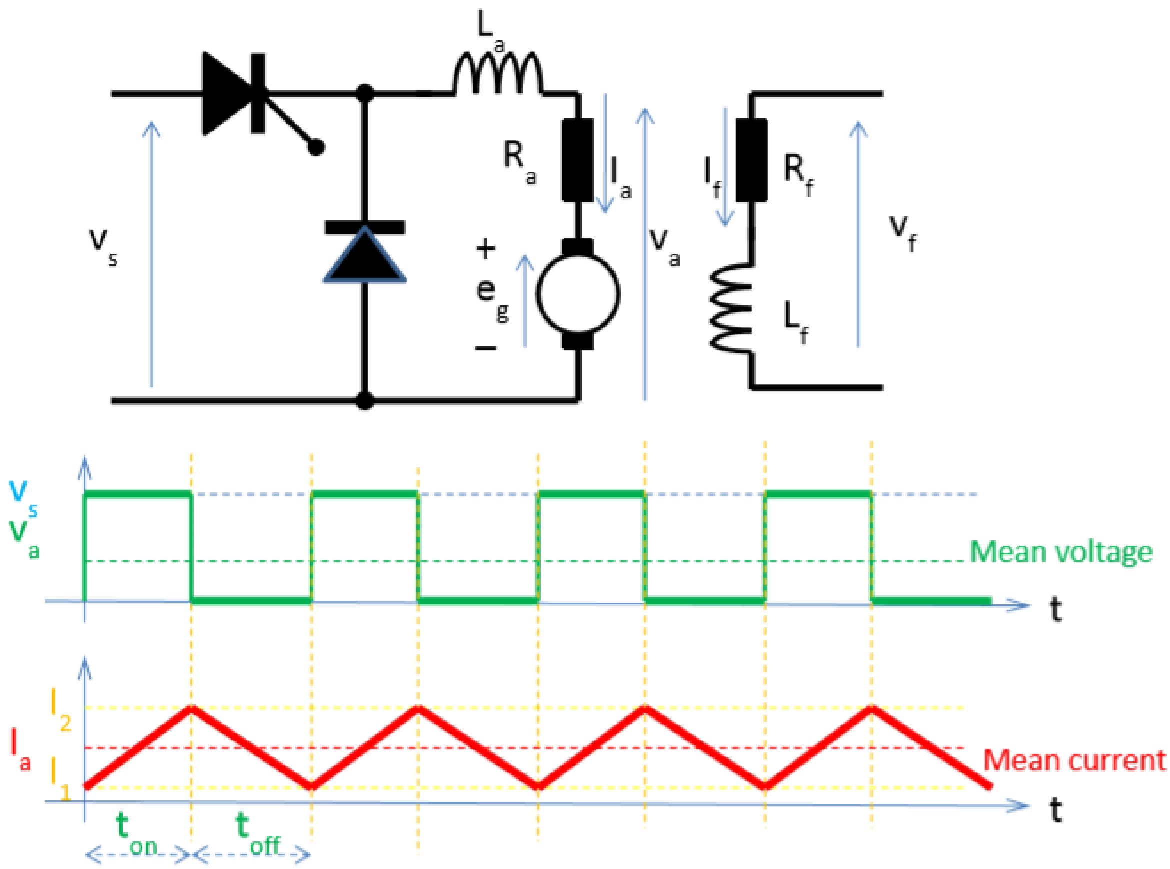
Mean power: $P_e = \omega T_e = \frac{2\pi}{60} \times 2500 \times 30 = 7.85 \text{ kW}$

$$= 10.5 \text{ hp}.$$

DC-DC MOTOR DRIVES (Choppers)

A chopper directly converts a fixed-voltage DC supply to a variable-voltage DC supply.

Step-Down Chopper (Motoring)



$$\textit{During } t_{on} \textit{ time} \qquad\qquad \textit{During } t_{off} \textit{ time}$$

$$Vs = L\frac{di}{dt} + Va \quad (1) \qquad\qquad 0 = -L\frac{di}{dt} + Va \quad (5)$$

$$Vs - Va = L\frac{di}{dt} \quad (2) \qquad\qquad Va = L\frac{di}{dt} \quad (6)$$

$$di = \frac{Vs-Va}{L}dt \quad (3) \qquad\qquad di = \frac{Va}{L}dt \quad (7)$$

$$\Delta I = I_2 - I_1 = \frac{Vs-Va}{L}t_{on} \quad (4) \qquad\qquad \Delta I = \frac{Va}{L}t_{off} \quad (8)$$

$$t_{on} = DT,\ t_{off} = (1-D)T \quad where D\ is\ the\ Duty\ cycle$$

Equating ΔIs

$$\Delta I = \frac{Vs-Va}{L}t_{on} = \frac{Va}{L}t_{off} = \frac{Vs-Va}{L}DT = \frac{Va}{L}(1-D)T => \boxed{Va = DVs}$$

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \quad (9) \qquad\qquad I_1 + I_2 = 2I_{mean} \quad (10)$$

adding (10) to (4) or (8)

$$2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{on} \quad or \quad 2I_2 = 2I_{mean} + \frac{Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{on}} \quad or \quad \boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{on}} \quad or \quad \boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{off}}$$

Example : A simple DC step-down chopper is operated at a frequency of 2KHz from a 120 V DC source to supply a motor load with Ra = 0.85 ohms, La = 0.32 mH. The required torque genrated by the motor is 20 Nm, at 1000 rpm, and field current is measured to be 1A. If Kv = 0.8345 V/A-rad/S, determine (a) the duty cycle for the switching pulse, (b) the mean load current, and (c) the max & min load currents.

(a) $V_a = DV_s = I_a R_a + E_g \qquad I_a = ? \qquad E_g = ?$

$$T = K_V I_a I_f \Rightarrow I_a = \frac{20}{0.8345 \times 1} = 23.96\ A$$

$$E_g = K_V \omega I_f = 0.8345 \times 2\pi \times \frac{1000}{60} = 87.38\ V$$

$V_a = 23.96 \times 0.85 + 87.38 = 107.75$ therefore $D = \frac{107.74}{120} = 0.89 = 89\%$

(b) $I_{mean} = I_a = 23.96\ A$

(c) $\boldsymbol{I_{max} = I_{mean} + \frac{Vs-Va}{2L} t_{on}} \quad$ ***or*** $\quad \boldsymbol{I_{max} = I_{mean} + \frac{Va}{2L} t_{off}}$

We know $t_{on} = DT,\ t_{off} = (1-D)T \quad and \quad T = 1/f$

$$I_{max} = 23.96A + \frac{120V - 107.75V}{2 * 0.32mH} * 0.89 * \frac{1}{2000} = 32.47A$$

$\boldsymbol{I_{min} = I_{mean} - \frac{Vs-Va}{2L} t_{on}} \quad$ ***or*** $\quad \boldsymbol{I_1 = I_{mean} - \frac{Va}{2L} t_{off}}$

$$I_{min} = 23.96A - \frac{120V - 107.75V}{2 * 0.32mH} * 0.89 * \frac{1}{2000} = 15.45\ A$$

Example : A separately excited DC motor is powerd by a DC chopper from a 600 V dc source. The armature resistance Ra = 0.05 ohms. The back e.m.f.constant of the motor is kv=1.527 V/A-rads/s. The armature voltage is continuous and ripple free. If the duty cycle of the copper is 60%, determine (a) the input power from the source, (b) the equivalent input resistance of the chopper drive, (c) the motor speed, and (d) the developed torque.

(a) P_{input}= ?, $P_{input} = DV_sI_a = 0.6 \times 600 \times 250 = 90$ kw

(b) Req= ?, $Req = \frac{Vs}{Is} = \frac{Vs}{DIs} = \frac{600V}{0.6 \times 250A} = 4\ \Omega$

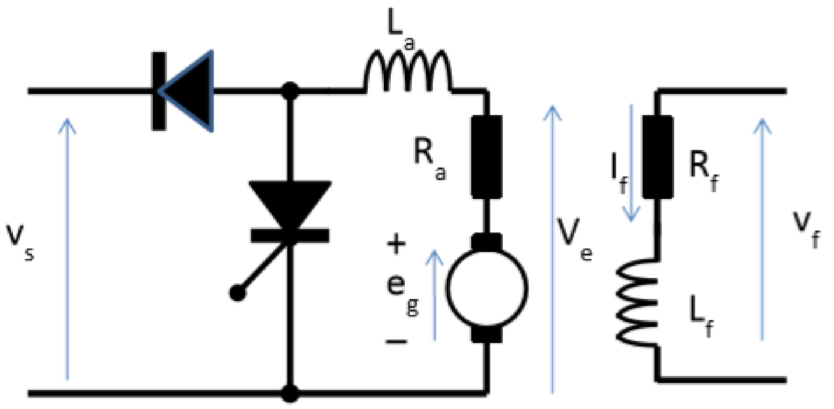
(c) ω = ?, $E_g = k_v \omega I$, E_g = ?, $V_a = I_a R_a + E_g$, V_a = ?, $V_a = DV_s = 0.6 \times 600 = 360$ V

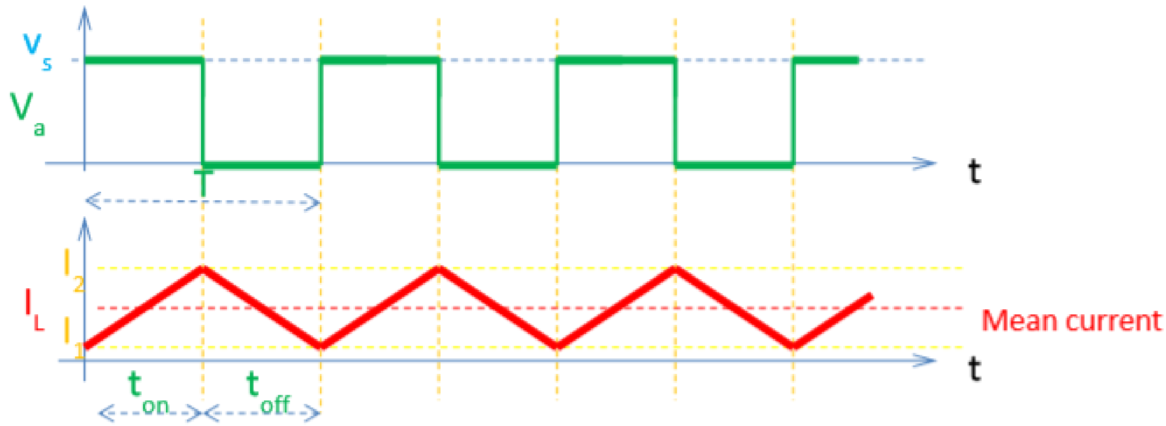
$E_g = 360 - 250 \times 0.05 = 347.5$ V

$\omega = \frac{347.5V}{1.525 \times 2.5A} = 91.03$ rad/s or $91.03 \times \frac{60}{2\pi} = 869.3$ rpm

(d) T_D=?, $T_D = k_v I_f I_a = 1.527 \times 250 \times 2.5 = 954.38$ Nm

Step-Up Chopper – (Regenerative Braking)





During t_{on} time

$$Ve = L\frac{di}{dt} \qquad (1)$$

$$di = \frac{Ve}{L}dt \qquad (2)$$

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} \qquad (3)$$

During t_{off} time

$$Ve = -L\frac{di}{dt} + Vs \qquad (4)$$

$$di = \frac{Vs-Ve}{L}dt \qquad (5)$$

$$\Delta I = I_2 - I_1 = \frac{Vs-Ve}{L}t_{off} \qquad (6)$$

Equating ΔIs

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} = \frac{Vs-Ve}{L}t_{off} \Rightarrow \frac{Ve}{L}DT = \frac{Vs-Ve}{L}(1-D)T \qquad (7)$$

$$VeD = Vs - Ve - VsD + VeD \Rightarrow Vs = \frac{Ve}{1-D}$$

Since average voltage across L is zero, therefore Ve = Va and $\boxed{Vs = \frac{Va}{1-D}}$ (8)

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \qquad (9)$$

$$I_1 + I_2 = 2I_{mean} \qquad (10)$$

adding (10) to (3)or(6) remembering Ve = Va

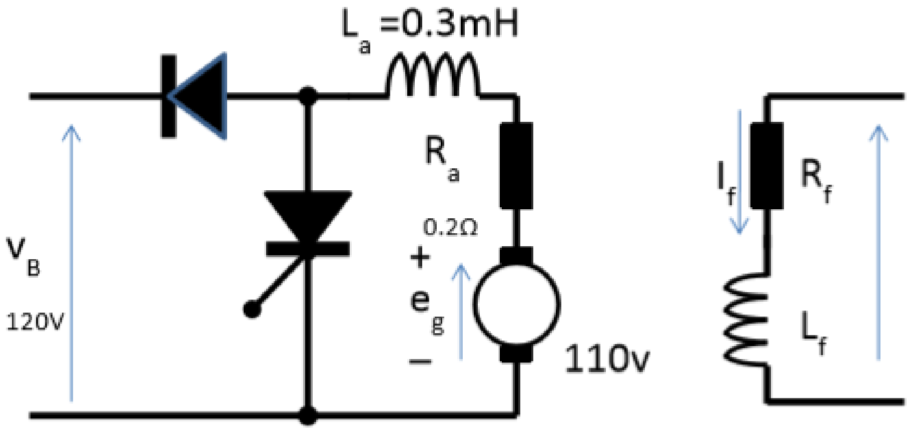
$$2I_2 = 2I_{mean} + \frac{Va}{L}t_{on} \qquad or \qquad 2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{on}} \qquad or \qquad \boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{on}} \qquad or \qquad \boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{off}}$$

Example : In a battery powered car, operating a frequency of 5 KHz, the battery voltage is 120 V. It is driven by a DC motor and employs chopper control. The resistance of the motor is 0.2 ohms and its inductance is 0.3 mH. During braking, the chopper configuration is changed to voltage step-up mode. While going down the hill at a certain speed, the back emf of the motor is 110 V and the breaking current is 10 A. Detrermine (a) the copper duty cycle, and (b) Max and Min values of the current.



(a) D = ?, $V_B = \frac{Va}{1-D} = \frac{IaRa+Eg}{1-D} > 1 - D = \frac{IaRa+Eg}{V_B} = \frac{110V+0.2x10}{120} => D = 6.67\%$

(b) $I_{max} = I_{mean} + \frac{Va}{2L}t_{on} = 10A + \frac{112x0.0667}{2x.03x10^{-3}x2000} = 16.22\ A$

$$I_{min} = I_{mean} - \frac{Va}{2L}t_{on} = 10A - \frac{112x0.0667}{2x.03x10^{-3}x2000} = 3.78\ A$$

Problem-Repeat above for 50V back emf.

Motoring and Regenerative Braking `Two-Quadrant Chopper (buck-boost`

Example 13.1: ***DC chopper with load back emf (first quadrant)***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine, with and without (rotor standstill) the back emf:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the average load output current, average switch current, and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, (and electromagnetic efficiency for $E > 0$);
viii. sketch the output current and voltage waveforms.

First Quadrant Chopper or Type A Chopper or Class A Chopper

In this topic, you study First Quadrant Chopper or Type A Chopper or Class A Chopper v-i plane, working principle, quadrant operation, Applications, and Circuit diagrams.

Type A chopper is basically a Step-Down Chopper.

Circuit Diagram

The Type A chopper circuit diagram as shown in Figure 1. Here the motor load is assumed, R_a and L_a armature resistance and inductance of the motor respectively. E_b is the back emf of the motor.

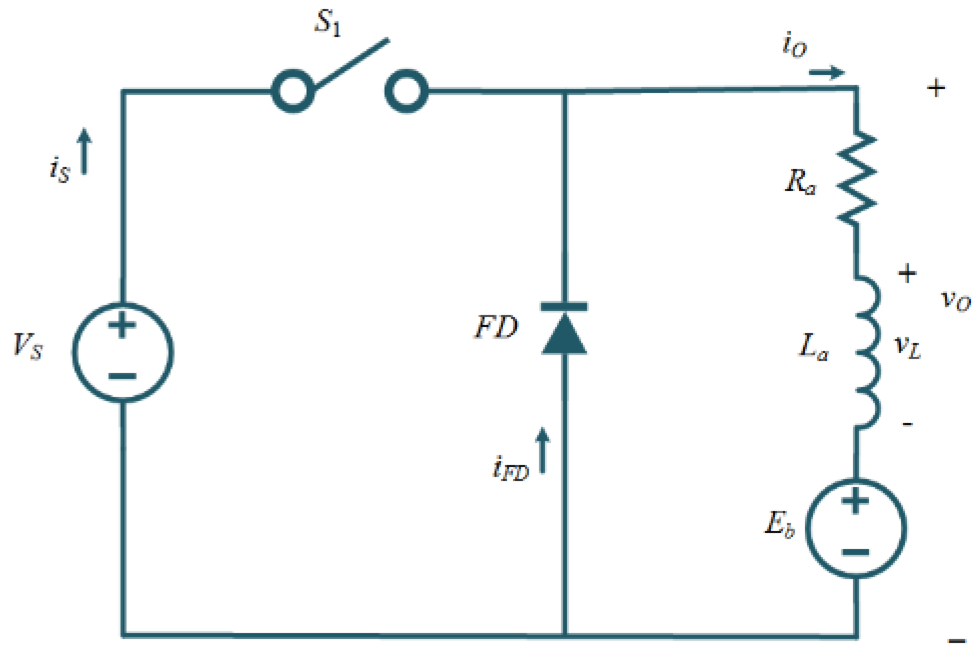


Figure 1 Circuit diagram of Type A chopper

$v_O - i_O$ plane

The Type A chopper operates in the first quadrant of $v_O - i_O$ plane as shown in Figure 2. Here v_O is the output voltage, V_O is the average output voltage, i_O is the output current and I_O is the average output current of Type A chopper circuit.

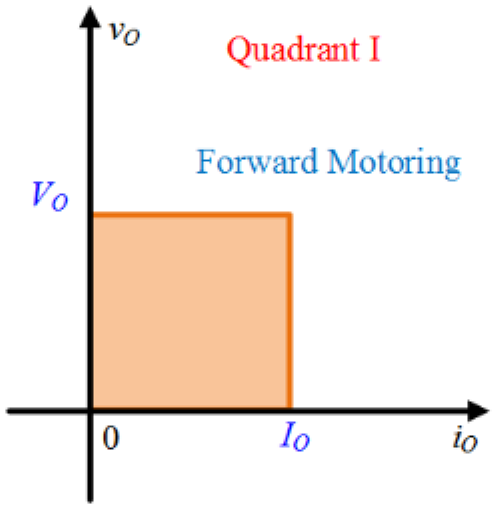


Figure 2 Type A chopper v_O - i_O plane

Quadrant I operation when Switch S_1 turned on

The Type A chopper equivalent circuit diagram for Quadrant I is shown in Figure 3. Here switch S_1 operated, Switch S_1 conducts, output voltage v_O and the output current i_O both are positives, power flows from source to load and inductor stores energy, the motor rotates in the forward direction hence called forward motoring.

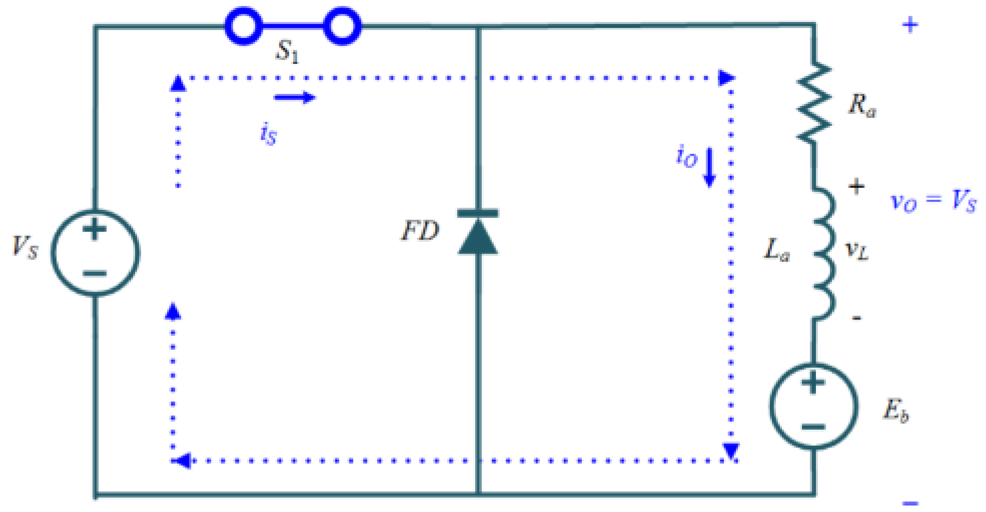


Figure 3 Equivalent Circuit diagram I of Type A chopper

Quadrant I operation when Switch S_1 turned off

The Type A chopper equivalent circuit diagram for Quadrant I is shown in Figure 4. Switch S_1 turned off diode *FD* conducts, output current i_O is positive and the output voltage v_O becomes zero, inductor release energy and freewheeling action using diode *FD* takes place, the motor rotates in the forward direction hence called Forward motoring.

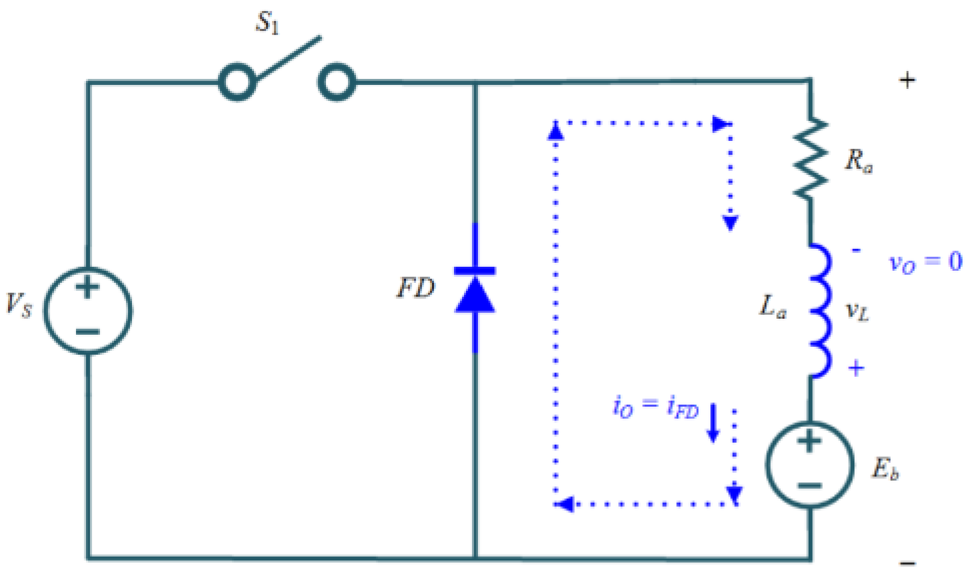


Figure 4 Equivalent Circuit diagram II of Type A chopper

Application

This chopper is suitable for motoring application only.

6.2. THE FIRST-QUADRANT (STEP-DOWN) CHOPPER

A d.c. brush motor with permanent magnet excitation with the data $R_a = 1\Omega$, $K_e\lambda_p = 0.055$ Wb / rpm, is fed through a first-quadrant chopper (Table 6.1a) from a 120 Vd.c. supply at a constant (ideal) armature current of 10A.

Determine:

a. the range of duty cycle α from zero to maximum speed;

b. the range of speed.

Solution:

a. The average output voltage V_a is

$$V_a = V_0 \cdot \alpha = 120\alpha \qquad (6.16)$$

At standstill n = 0 and thus

$$V_a = R_a i_a = 1 \cdot 10 = 10\text{V}$$

$$\text{Thus} \quad \alpha_{min} = \frac{(V_a)_{n=0}}{V_0} = \frac{10}{120} = \frac{1}{12} \qquad (6.17)$$

For maximum speed, the voltage is 120V ($\alpha = 1$). Consequently, α varies from 1/12 to 1.

b. The voltage equation for maximum speed is

$$V_{a_{max}} = R_a I_a + K_e \lambda_p n_{max} \qquad (6.18)$$

$$n_{max} = \frac{120 - 1 \cdot 10}{0.055} = 2000 \text{ rpm} \qquad (6.19)$$

The speed range is thus from zero to 2000 rpm.

c. For the d.c. brush motor and chopper as above and $\alpha = 0.3$, calculate the actual armature current waveform, its average value, and voltage average value at n = 1600 rpm, for the chopping frequency f_{ch} = 50Hz. Determine the chopping frequency for which the limit between discontinuous and continuous current is reached at same t_{on} as above.

Solution:

We now apply the armature current expressions (6.6)-(6.7) first for f_{ch} = 50Hz.

$$\text{The turn-on time interval} \quad t_{on} = \frac{1}{f_{ch}} \cdot \alpha = \frac{0.3}{50} = 6 \cdot 10^{-3} s = 6 \text{ ms} \qquad (6.20)$$

$$\text{with} \quad T = \frac{1}{f_{ch}} = \frac{1}{50} = 0.02 s = 20 \text{ ms}$$

$$i_a = A \cdot e^{-t\frac{1}{3\cdot 10^{-3}}} + \frac{120 - 0.055 \cdot 1600}{1} = A \cdot e^{-300t} + 32$$

$$i_a' = A' \cdot e^{-300(t - 6\cdot 10^{-3})} - \frac{88}{1} \qquad (6.21)$$

Assuming discontinuous current mode

$$(i_a)_{t=0} = 0; \quad A = -32 \qquad (6.22)$$

$$(i_a')_{t_{on}} = (i_a)_{t_{on}}; \quad A' - 88 = 32\left(1 - e^{-300 \cdot 6 \cdot 10^{-3}}\right) = 22.36$$

$$\text{Also} \quad A' = 11036 \qquad (6.23)$$

The current i_a' becomes zero at $t = t_1$

$$A' e^{-300(t_1 - 6\cdot 10^{-3})} - 88 = 0; \quad 110.36 \cdot e^{-300(t_1 - 6\cdot 10^{-3})} - 88 = 0$$

$$t_1 = 7.132 \cdot 10^{-3} s < T = 20 \text{ ms} \qquad (6.24)$$

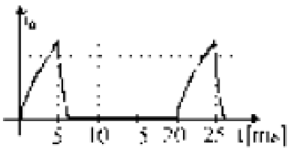


Figure 6.5. Discontinuous current

Thus indeed the current is discontinuous.
The average current i_{av} is:

$$i_{av} = \frac{1}{T}\left[\int_0^{t_{on}} i_a \, dt + \int_{t_{on}}^{t_1} i_a' \, dt\right] =$$

$$\frac{1}{2\cdot 10^{-2}}\left[32\int_0^{1\cdot 10^{-3}}\left(1 - e^{-200t}\right)dt + \int_{1\cdot 10^{-3}}^{7.132\cdot 10^{-3}}\left(110.36\cdot e^{-200\left(t-1\cdot 10^{-3}\right)} - 88\right)dt\right]$$

$$i_{av} = 7.8452 \text{ A} \tag{6.25}$$

The chopping frequency for the limit between the discontinuous and continuous current is obtained for

$$t_1 = T_s = 7.132\cdot 10^{-3}; \quad f_{ch}' = \frac{1}{T_s} = \frac{1}{7.132\cdot 10^{-3}} = 140.2 \text{ Hz} \tag{6.26}$$

In this case α becomes

$$\alpha_s = \frac{t_{on}}{T_s} = \frac{6\cdot 10^{-3}}{7.132\cdot 10^{-3}} = 0.8412 \tag{6.27}$$

As the current is discontinuous the average voltage is from (6.5)

$$V_{av} = V_0\frac{t_{on}}{T} + e_g\frac{\left(T - t_1\right)}{T} = 120\cdot 0.3 + 88\cdot\frac{20 - 7.132}{20}$$

$$= 40 + 56.619 = 96.619 \text{ V} \tag{6.28}$$

10.21 A one-quadrant chopper, such as that shown in Fig. 10.34*a*, is used to control the speed of a dc motor.

Supply dc voltage = 120 V

$R_a = 0.15\ \Omega$

Motor back emf constant = 0.05 V/rpm

Chopper frequency = 250 Hz

At a speed of 1200 rpm, the motor current is 125 A. The motor current can be assumed to be ripple-free.

(a) Determine the duty ratio (α) of the chopper and the chopper on time t_{on}.

(b) Draw waveforms of v_0, i_0, and i_s.

(c) Determine the torque developed by the armature, power taken by the motor, and power drawn from the supply.

$\boxed{10.21}$ (a)

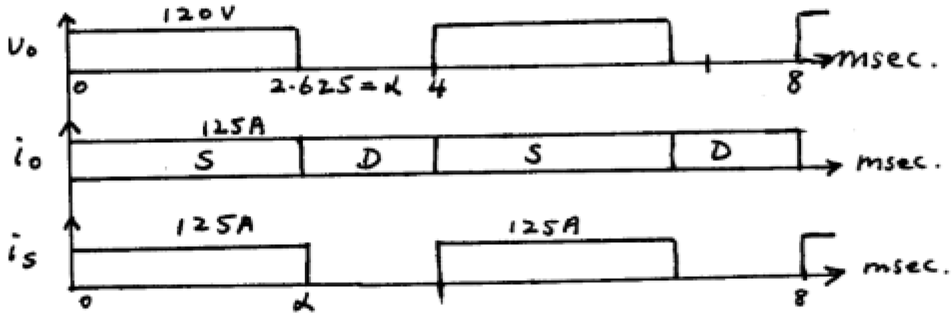
$$V_o = E_a + I_a R_a$$
$$= 0.05 \times 1200 + 125 \times 0.15$$
$$= 78.75\,V$$

$$T = \frac{10^3}{250}\,msec = 4\ msec$$

$$\alpha = \frac{78.75}{120} = 0.6563$$

$$t_{on} = \alpha T = 0.6563 \times 4 = 2.625\ msec.$$

(b)



(c)

$$E_a I_o = 60 \times 125 = 7500\,W$$

$$T = \frac{7500}{1200/60 \times 2\pi} = 59.683\ N\cdot m$$

$$P_o = V_o I_o = 78.75 \times 125 = 9844\,W$$

$$I_s = 125 \times 0.6563 = 82.03\ A$$

$$P_s = 120 \times 82.03 = 9844\ W$$

Example 4.23 The armature voltage of a separately excited dc motor is controlled by a one-quadrant chopper with chopping frequency of 200 pulses per second from a 300 V dc source. The motor runs at a speed of 800 rpm when the chopper's time ratio is 0.8. Assume that the armature circuit resistance and inductance are 0.08 Ω and 15 mH, respectively, and that the motor develops a torque of 2.72 N·m per ampere of armature current.

Find the mode of operation of the chopper, the output torque, and horsepower under the specified conditions.

Solution From the problem specifications at 800 rpm, using Eq. (4.201), we get,

$$E_c = (2.72)\frac{2\pi}{60}(800) = 227.9 \text{ V}$$

$$E_c = K_1\phi_f\omega \tag{4.201}$$

$$\omega = \frac{t_{on}}{t_x}\frac{V_i}{K_1\phi_f} - \frac{R_a}{(K_1\phi_f)^2}\frac{T}{t_x}T_0 \tag{4.202}$$

In the continuous mode of operation, with $t_{on} > t^*_{on}$, we have $t_x = T$. As a result Eq. (4.202) reduces to

$$\omega = \frac{(t_{on}/T)V_i - (R_a/K_1\phi_f)T_o}{K_1\phi_f)} \tag{4.203}$$

The armature circuit time constant is obtained as

$$\tau = \frac{L_a}{R_a} = \frac{15 \times 10^{-3}}{0.08} = 187.5 \times 10^{-3} \text{ s}$$

The chopping period is given by

$$T = \frac{1}{200} = 5 \times 10^{-3}\text{s}$$

We obtain the critical on-time using Eq. (4.197) as

$$\begin{aligned} t^*_{on} &= 187.5 \times 10^{-3}\, ln\left[1 + \frac{227.9}{300}(e^{5/187.5} - 1)\right] \\ &= 3.8 \times 10^{-3}\text{s} \end{aligned}$$

We know that $t_{on} = 0.8 \times 5 \times 10^{-3} = 4 \times 10^{-3}$. As a result, we conclude that the chopper output current is continuous.

To obtain the torque output, we use Eq. (4.203) rearranged as

$$T_o = \frac{K_1\phi_f}{R_a}\left(\frac{t_{on}}{T}V_i - K_1\phi_f\omega\right)$$

Thus we obtain

$$\begin{aligned} T_o &= \frac{2.72}{0.08}[0.8(300) - 227.9] \\ &= 411.4\,\mathrm{N\cdot m} \end{aligned}$$

The power output is obtained as

$$\begin{aligned} P_o &= (411.4)\frac{2\pi}{60}(800) = 34.5 \times 10^3\,\mathrm{W} \\ &= 46.2\,\mathrm{hp} \end{aligned}$$

To illustrate the principle of field control, we have the following example.

Example 4.24 Assume for the motor of Example 4.23 that field chopper control is employed to run the motor at a speed of 1500 rpm while delivering the same power output as obtained at 800 rpm and drawing the same armature current.

Solution Although we can use Eq. (4.205), we use basic formulas instead,

$$E_c = \frac{P_a}{I_a} = \frac{34.5 \times 10^3}{151.3} = 227.9\,\text{V}$$

This is the same back EMF. Recall that

$$E_c = K_1 \phi_f \omega$$

Thus the required field flux is obtained as

$$\phi_{f_n} = \phi_{f_0} \frac{\omega_0}{\omega_n} = \frac{8}{15} \phi_{f_0}$$

where the subscript n denotes the present case, and the subscript 0 denotes the field flux for Example 4.23. Assume that ϕ_{f_0} corresponds to full applied field flux; then

$$\frac{\phi_{f_0}}{\phi_{f_n}} = \frac{V_i}{V_o} = \frac{15}{8}$$

The required chopped output voltage is V_o. Now we have

$$\frac{V_o}{V_i} = \frac{t_{\text{on}}}{T}$$

Thus

$$\frac{t_{\text{on}}}{T} = \frac{8}{15}$$

Assuming that $T = 5 \times 10^{-3}$ s, we get

$$t_{\text{on}} = 2.67 \times 10^{-3}\,\text{s}$$

EXAMPLE 10.5

The two-quadrant chopper shown in Fig. 10.38*a* is used to control the speed of the dc motor and also for regenerative braking of the motor. The motor constant is $K\Phi = 0.1$ V/rpm ($E_a = K\Phi n$). The chopping frequency is $f_c = 250$ Hz and the motor armature resistance is $R_a = 0.2\ \Omega$. The inductance L_a is sufficiently large and the motor current i_0 can be assumed to be ripple-free. The supply voltage is 120 V.

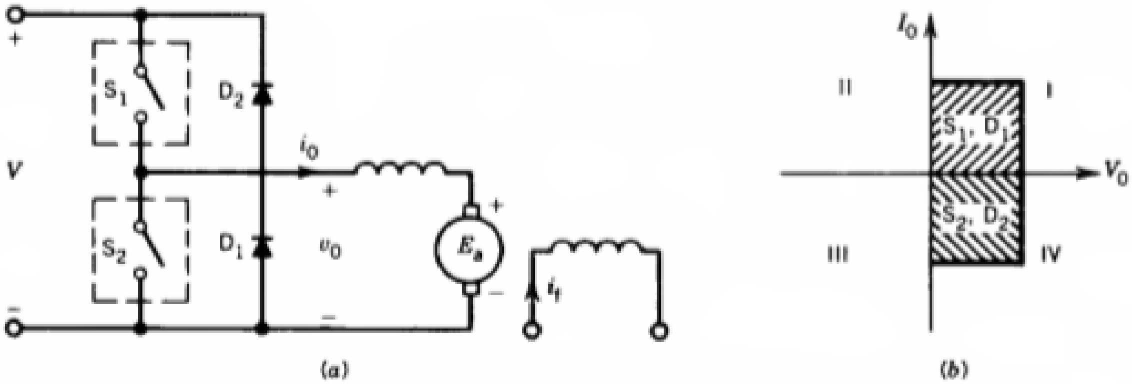


FIGURE 10.38 Two-quadrant chopper. (*a*) Circuit. (*b*) Quadrant operation.

(a) Chopper S_1 and diode D_1 are operated to control the speed of the motor. At $n = 400$ rpm and $i_0 = 100$ A (ripple-free),

- **(i)** Draw waveforms of v_0, i_0, and i_s.
- **(ii)** Determine the turn-on time (t_{on}) of the chopper.
- **(iii)** Determine the power developed by the motor, power absorbed by R_a, and power from the source.

(b) In the two-quadrant chopper S_2 and diode D_2 are operated for regenerative braking of the motor. At $n = 350$ rpm and $i_0 = -100$ A (ripple-free),

- **(i)** Draw waveforms of v_0, i_0, and i_s.
- **(ii)** Determine the turn-on time (t_{on}) of the chopper.
- **(iii)** Determine the power developed (and delivered) by the motor, power absorbed by R_a, and power to the source.

Solution

(a) **(i)** The waveforms are shown in Fig. E10.5*a*.

(ii) From Fig. 10.38*a*

$$V_0 = E_a + I_a R_a$$

$$= 0.1 \times 400 + 100 \times 0.2$$

$$= 60 \text{ V}$$

$$60 = \frac{t_{on}}{T} \text{V} = \frac{t_{on}}{T} 120$$

$$t_{on} = \frac{T}{2}$$

(iii)

$$P_{motor} = E_a I_0 = 0.1 \times 400 \times 100 = 4000 \text{ W}$$

$$P_R = (i_0)^2_{rms} R_a = 100^2 \times 0.2 = 2000 \text{ W}$$

$$P_s = V(i_s)_{avg} = 120 \times 100 \times \tfrac{2}{4} = 6000 \text{ W}$$

(b) **(i)** The waveforms are shown in Fig. E10.5*b*.

(ii)

$$V_0 = E_a + (-I_0 R_a)$$

$$= 0.1 \times 350 - 100 \times 0.2$$

$$= 15 \text{ V}$$

From Fig. E10.5*b*

$$V_0 = \frac{T - t_{on}}{T} V$$

$$15 = \left(1 - \frac{t_{on}}{T}\right) 120$$

$$\frac{t_{on}}{T} = \frac{7}{8}$$

$$t_{on} = \tfrac{7}{8} \times 4 = 3.5 \text{ msec}$$

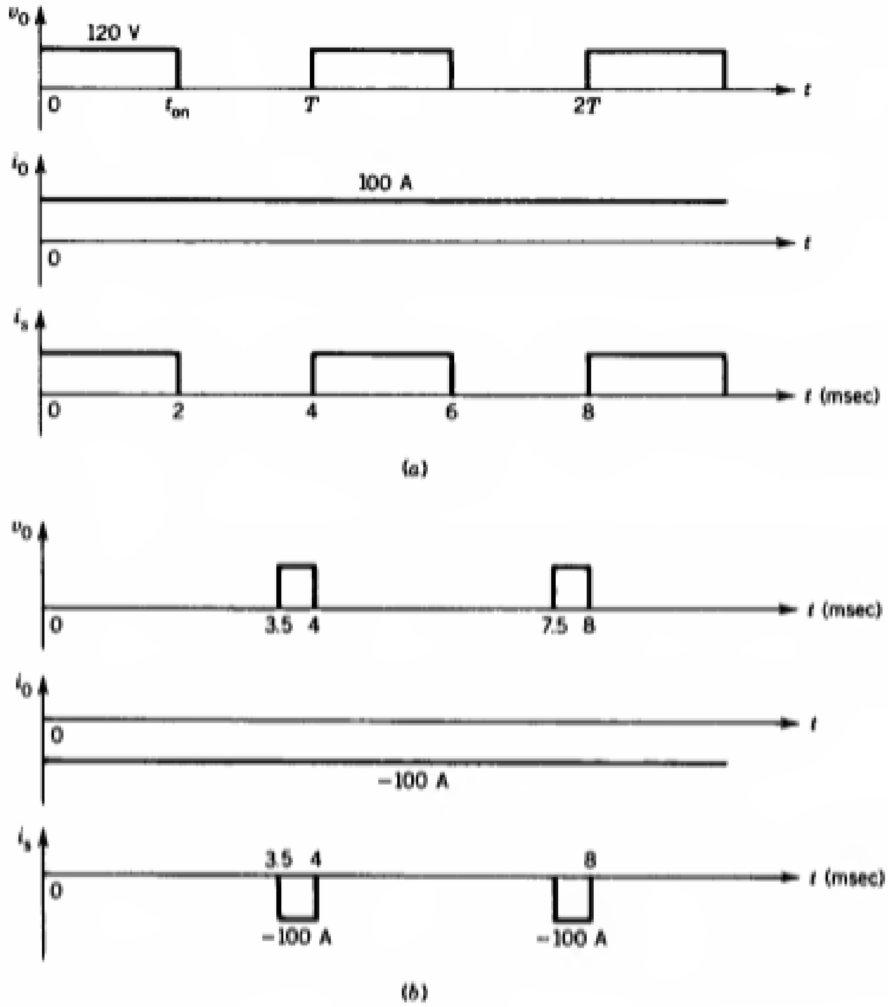


FIGURE E10.5

(iii)

$$P_{motor} = E_a I_0 = 0.1 \times 350(-100) = -3500 \text{ W}$$

$$P_R = 100^2 \times 0.2 = 2000 \text{ W}$$

$$P_s = V(i_s)_{avg} = 120(-100 \times \tfrac{1}{8}) = -1500 \text{ W}$$

Example 4.1

The speed of a separately excited dc motor is controlled by a chopper as shown in Fig. 4.8*a*. The dc supply voltage is 120 V, armature circuit resistance is $R_a = 0.5\ \Omega$, armature circuit inductance is $L_a = 20$ mH, and motor constant is $K_a\Phi = 0.05$ V/rpm. The motor drives a constant-torque load requiring an average armature current of 20 A. Assume that motor current is continuous.

Determine:

1 the range of speed control;

2 the range of the duty cycle α.

Solution

Minimum speed is zero at which $E_g = 0$. Therefore from equation 2.17

$$E_a = I_a R_a = 20 \times 0.5 = 10\,\mathrm{V}$$

From equation 4.1

$$10 = 120\alpha$$

$$\alpha = \frac{1}{12}$$

Maximum speed corresponds to $\alpha = 1$ at which $E_a = E = 120$ V.
Therefore

$$\begin{aligned} E_g &= E_a - I_a R_a \\ &= 120 - (20 \times 0.5) \\ &= 110\ \mathrm{V} \end{aligned}$$

From equation 2.13

$$N = \frac{E_g}{K_a \Phi} = \frac{110}{0.05} = 2200\,\mathrm{rpm}$$

The range of speed is $0 < N < 2200$ rpm, and the range of the duty cycle is $1/12 < \alpha < 1$.

Example 4.1

A 250-V separately excited motor dc has an armature resistance of 2.5 Ω. When driving a load at 600 rpm with constant torque, the armature takes 20 A. This motor is controlled by a chopper circuit with a frequency of 400 Hz and an input voltage of 250 V.

1. What should be the value of the duty ratio if one desires to reduce the speed from 600 to 400 rpm, with the load torque maintained constant?
2. What should be the minimum value of the armature inductance, if the maximum armature current ripple expressed as a percentage of the rated current is not to exceed 10 percent?

Solution: With an input voltage of 250 V and at a constant torque, the motor will run at 600 rpm when $\delta = 1$.

1. At 600 rpm

$$E = V_a - I_a R_a = 250 - 20 \times 2.5 = 200 \text{ V}$$

At 400 rpm, the back emf

$$E_1 = 200 \times \frac{400}{600} = 133 \text{ V}$$

The average chopper output voltage

$$V_{a1} = E_1 + I_a R_a = 133 + 20 \times 2.5 = 183 \text{ V}.$$

$$\text{Now } \delta V = V_{a1} \quad \text{or} \quad \delta = V_{a1}/V = 183/250 = 0.73.$$

2.

$$\Delta i_a = \frac{V}{2R_a}\left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1}\right] \tag{4.14}$$

$$\text{Per-unit current ripple} = (\Delta i_a)_p = \frac{\Delta i_a}{I_{rated}}$$

$$= \frac{V}{2R_a I_{rated}}\left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1}\right] \tag{E4.1}$$

For the maximum value of the per-unit ripple

$$\frac{d(\Delta i_a)p}{d\delta} = 0\,,$$

therefore from equation (E4.1)

$$-\frac{T}{\tau_a}e^{\delta T/\tau_a} + \frac{T}{\tau_a}e^{(1-\delta)T/\tau_a} = 0 \quad \text{or} \quad \delta = 1 - \delta \quad \text{or} \quad \delta = 0.5\,.$$

Substituting in equation (E4.1), the maximum value of the per-unit ripple $(\Delta i_a)pm$ is given by the following equation:

$$(\Delta i_a)pm = \frac{V}{2R_a I_{rated}}\left[\frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1}\right] \tag{E4.2}$$

For $(\Delta i_a)pm = 0.1$

$$\frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1} = \frac{0.2R_a I_{rated}}{V} = \frac{0.2 \times 2.5 \times 20}{250} = 0.04$$

$$\text{or} \quad e^{0.5T/\tau_a} = 1.08 \quad \text{or} \quad 0.5T/\tau_a = \ell n(1.08) = 0.08 \quad \text{or} \quad \tau_a = \frac{T}{0.16}$$

$$\text{or} \quad L_a = \frac{R_a T}{0.16} = \frac{2.5}{400 \times 0.16} = 39.1 \text{ mH}\,.$$

Example 4.2

A 220-V, 100A dc series motor has an armature resistance and an inductance of 0.06 Ω and 2 mH, respectively. The field winding resistance and inductance are 0.04 Ω and 18 mH, respectively. Running on no load as a generator, with the field winding connected to a separate source, it gives the following magnetization characteristic at 700 rpm:

Field current	25	50	75	100	125	150	175	A
Terminal voltage	66.5	124	158.5	181	198.5	211	221.5	V

The motor is controlled by a chopper operating at 400 Hz and 220 V. Calculate the motor speed for a duty ratio of 0.7 and a load torque equal to 1.5 times the rated torque.

Solution: The speed at which the magnetization characteristic was measured = $700 \times 2\pi/60 = 73.3$ rad/sec.

voltage induced $E = K_e\Phi\omega_m$

$$K_e\Phi = K = \frac{E}{\omega_m}$$

$$\text{Torque } T_a = KI_a = \frac{EI_a}{\omega_m} \tag{E4.3}$$

From equation (E4.3) and the magnetization characteristic

I_a	25	50	75	100	125	150	175	A
T_a	22.7	84.6	162.2	246.9	338.5	431.8	528.8	N-m

The rated torque (torque at 100A) = 247 N-m

$1.5 \times$ Rated torque $= 1.5 \times 247 = 370.5$ N-m

From the above T_a/I_a table the current at $1.5 \times$ Rated torque = 133 A

Also K at 133 A = 370.5/133 = 2.79

$$R_a = 0.06 + 0.04 = 0.1\ \Omega$$

$$\text{Now } \omega_m = \frac{\delta V - I_a R_a}{K} = \frac{0.7 \times 220 - 100 \times 0.1}{2.79} = 51.6 \text{ rad/sec.} = 492.7 \text{ rpm}$$

Example 12.1

The speed of a separately-excited d.c. motor with R_a = 1.2 Ω and L_a = 30 mH, is to be controlled using class-A thyristor chopper as shown in Fig.12.4. The d.c. supply V_d = 120 V. By ignoring the effect of the armature inductance L_a, it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle $\gamma = 1$.

(b) Draw the speed-torque characteristics for the motor when the duty cycle γ = 1. The motor design constant $Ke\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to $\gamma = 0.5$.

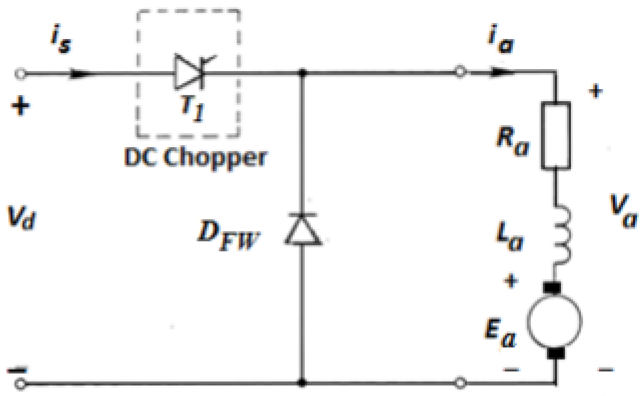


Fig. 12.4 Thyristor chopper drive.

Solution

The average armature voltage for $\gamma = 1$ is

$$V_{av} = \gamma V_d = 1 \times 120 = 120 \text{ V}$$

The motor's speed:

$$n = \frac{V_{av}}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

At no load $T_d = 0$, hence

$$\text{or} \quad n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{120}{0.042} = 2857 \text{ rpm}$$

At starting, $n = 0$. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$T_{st} = \frac{9.55\, \gamma V_d}{R_a} K_e \emptyset$$

$$\therefore \quad T_{st} = \frac{9.55 \times 120}{1.2} \times 0.042 = 40 \text{ Nm}$$

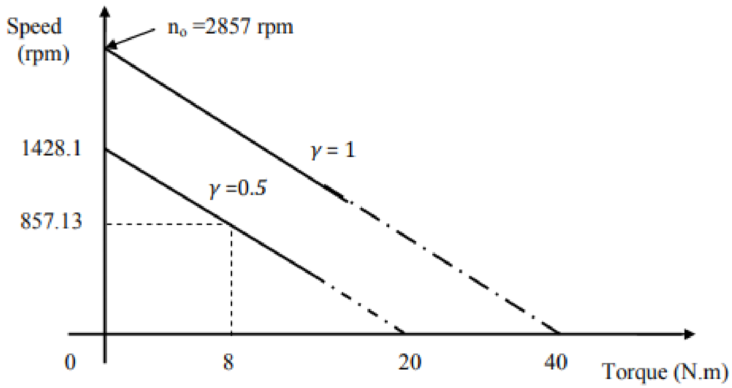


Fig. 12.5 Speed-torque characteristics.

(b) At $\gamma = 0.5$,

$$V_a = \gamma V_d = 0.5 \times 120 = 60 \text{ V}$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5 \text{ rpm}$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20 \text{ Nm}$$

At $\gamma = 0.5$, $T_L = 8$ Nm

$$n = \frac{60}{0.042} - \frac{1.2}{9.55 \times (0.042)^2} \times 8 = 857.13 \text{ rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 12.2

A d.c. motor is driven from a class-A d.c. chopper with source voltage of 220 V and at frequency of 1000 Hz. Determine the range of duty cycle to obtain a speed variation from 0 to 2000 rpm while the motor delivered a constant load of 70 Nm. The motor details as follows:
1kW, 200 V, 2000 rpm, 80% efficiency, R_a = 0.1 Ω, L_a = 0.02 H, and $K\emptyset$ = 0.54 V/rad /s.

Solution

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 2000}{60} = 209.3 \text{ rad/s}$$

$$I_{av} = \frac{P_{out}}{\text{Voltage} \times \eta} = \frac{1000}{200 \times 0.80} = 6.25 \text{ A}$$

$$I_{av} = \frac{\gamma V_d - E_a}{R_a} = \frac{\gamma V_d - K\emptyset\, \omega}{R_a}$$

$$T_{av} = K\emptyset\, I_{av}$$

$$T_{av} = K\emptyset \left(\frac{\gamma V_d - K\emptyset\, \omega}{R_a}\right) \text{Nm}$$

For $\omega_m = 0$

$$T_{av} = \frac{\gamma\, K\emptyset\, V_d}{R_a}$$

$$\gamma = \frac{T_{av}\, R_a}{K\emptyset\, V_d} = \frac{70 \times 0.1}{0.54 \times 220} = 0.058$$

$$\therefore\ \gamma_{min} = 0.058$$

and $$\gamma_{max} = \frac{T_{av} R_a}{K\emptyset V_d} + \frac{K\emptyset \omega_m}{V_d} = \frac{70 \times 0.1}{0.54 \times 220} + \frac{0.54 \times 209.3}{220} = 0.571$$

Hence the range of γ is $0.058 - 0.571$.

Example 12.3

In the microcomputer-controlled class-A IGBT transistor d.c. chopper shown in Fig.12.6, the input voltage $V_d = 260$ V, the load is a separately-excited d.c. motor with $R_a = 0.28\ \Omega$ and $L_a = 30$ mH. The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30 A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor n (rpm) when the chopper is switched fully ON such that the duty cycle $\gamma = 1.0$.

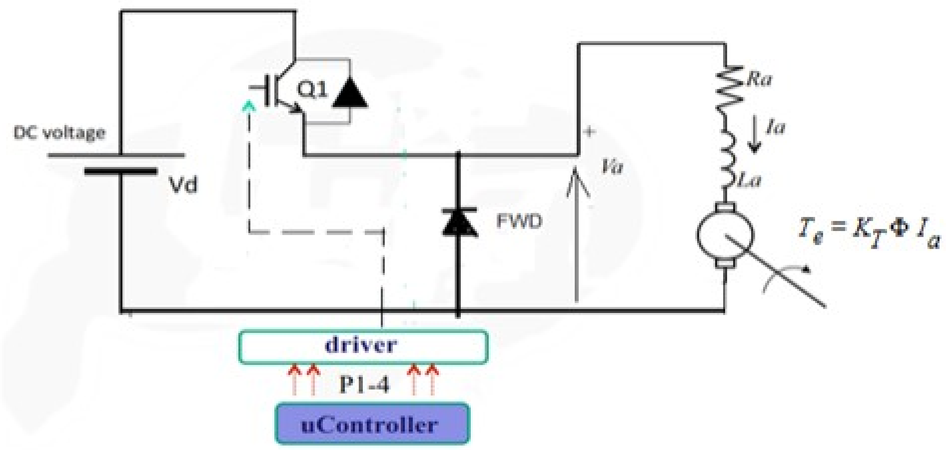


Fig.12.6 IGBT d.c. chopper drive.

Solution

(a) With steady-state operation of the motor, the armature induct behaves like a short circuit and therefore has no effect at all.

At stand still $n = 0$, and therefore , $E_a = 0$, hence from Eq.(12.22),

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30 \text{ A}$$

$$\therefore V_{a0} = 0.28 \times 30 = 8.4 \text{ V}$$

At full speed $n = 2500$ rpm ,

$$E_{a2500} = K_e \, \phi \, n = 0.1 \times 2500 = 250 \text{ V}$$

For separately-excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4 \text{ V}$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. $\gamma = 1$, then

$$V_a = V_d = 260 \text{ V} .$$

At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e \, \phi \, n + I_a R_a = 260\text{V}$$

$$0.1 \, n + 30 \times 0.28 = 260 \qquad \rightarrow \qquad n = 2516 \text{ rpm}$$

Example 12.4

A separately-excited d.c. motor has the following parameters:

$R_a = 0.5\ \Omega$, $L_a = 5.0$ mH , $K_e\,\Phi = 0.078$ V/rpm.

The motor speed is controlled by a class-A d.c. chopper fed from an ideal 200 V d.c. source. The motor is driven at a speed of 2180 rpm by switching on the thyristor for a period of 4 ms in each overall period of 6 ms .

(a) State whether the motor will operate in continuous or discontinuous current mode,
(b) Calculate the extinction angle of the current if it exist,
(c) Sketch the armature voltage and current waveforms,
(d) Calculate the maximum and minimum values of the armature current,
(e) Calculate the average armature voltage and current.

Solution

(a) To find whether the motor operates in continuous or discontinuous current modes ,we have to find the values of γ and γ' :

$$\gamma = \frac{t_{on}}{t_{on} - t_{off}} = \frac{t_{on}}{T} = \frac{4ms}{6ms} = 0.667$$

$$T = 6\,x\,10^{-3}\ \text{s} \qquad \rightarrow \qquad \omega = 2\pi\,\frac{1}{T} = 1046.6\ \ \text{rad/s}$$

The armature circuit tim constant is $\tau = \dfrac{L_a}{R_a} = \dfrac{5}{0.5} = 10$ ms

Therefore, $\dfrac{2\pi}{\omega\tau} = \dfrac{2 \times 3.14}{1046.7 \times 10\ \times\ 10^{-3}} = 0.6$

At speed of 2180 rpm , $E_a = K_e\,\phi\,n = 0.078\ \times 2180 = 170$ V

The critical value of γ' will be , (using Eq.(12. 31))

$$\frac{E_a}{V_d} = \frac{e^{2\pi\gamma'/\omega\tau} - 1}{e^{2\pi/\omega\tau} - 1} = \frac{170}{200} = \frac{e^{0.6\,\gamma'} - 1}{e^{0.6} - 1}$$

From which $\gamma' = 0.0.8829$, therefore , $\gamma' > \gamma$, hence the motor is operating in discontinuous current mode.

(b) The extinction angle x of the current is calculated from Eq.(12.29) as,

$$x = \omega\tau \ln\left[e^{(2\pi\gamma)/\omega\tau}\left\{1 + (\frac{V_d}{E_a} - 1)\left(1 - e^{-2\pi\gamma/\omega\tau}\right)\right\}\right]$$

$$\omega\tau = 1046.7x10\ x\ 10^{-3} = 10.467 \text{ rad}$$

$$x = 10.467 \ln\left[e^{(2\pi x0.6)/10.467}\left\{1 + (\frac{200}{170} - 1)\left(1 - e^{-2\pi x0.6/10.467}\right)\right\}\right]$$

From which $x = 4.8 \text{ rad} \rightarrow \quad x = 275.16°$

(c) The armature voltage and current waveforms are shown in Fig.12.7.

(d) The maximum and minimum values of the armature currents are:

$I_{min} = 0$, since it is discontinuous.
I_{maxD} is calculated from Eq.(12.26) as,

$$I_{max_D} = \frac{V_d - E_a}{R_a}\left(1 - e^{-2\pi\gamma/\omega\tau}\right)$$

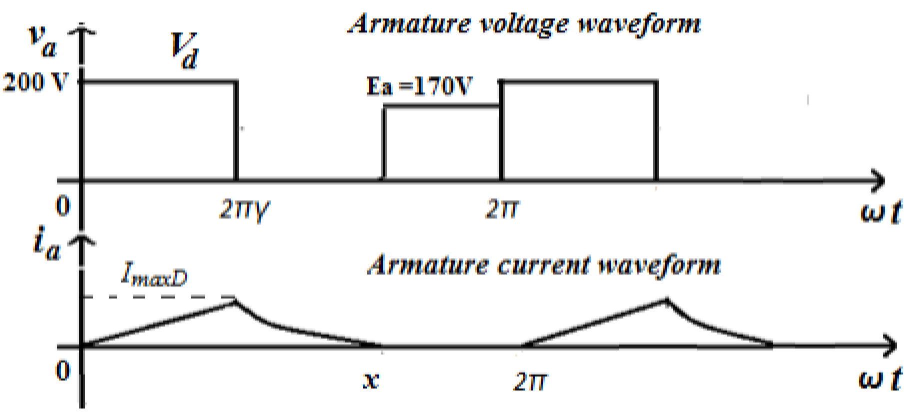


Fig.12.7 Armature voltage and current waveforms.

Example 12.5

A separately-excited d.c. motor with $R_a = 0.1\ \Omega$ and $L_a = 20$ mH, is to be controlled using class-A thyristor chopper. The d.c. supply is a battery with $V_d = 400$ V. The motor voltage constant is 5 V.s/rad. In the steady-state operation the average armature current $I_a = 100$ A and it is assumed to be continuous and ripple-free.

(a) For a duty cycle of 0.5, it is required to calculate (i) the input power to the motor, (ii) the speed of the motor, (iii) the developed torque. Mechanical, battery and semiconductor losses may be neglected.

(b) If the duty cycle of the chopper is varied between 20% and 80%, find the difference in speed resulting from this variation.

Solution

(a) Input power to the motor, speed of the motor and the developed torque are calculated as follows:

(i) For continuous current operation the input power is

$$P_{in} = V_a I_a = \gamma V_d I_a = 0.5 \times 400 \times 100 = 20 \text{ kW}$$

(ii) Speed of the motor can be calculated as,

The voltage across the armature circuit

$$V_a = \gamma V_d = 0.5 \times 400 = 200 \text{ V}$$

The induce voltage $E_a = K\emptyset\, \omega$

$$K\emptyset = 5 \text{ V.s/rad.}$$

$$E_a = V_a - I_a R_a = 200 - 100 \times 0.1 = 190 \text{ V}$$

$$\omega = \frac{E_a}{K\emptyset} = \frac{190}{5} = 38 \text{ rad/s}$$

To find the speed n in rpm

$$n = \frac{60}{2\pi}\omega = \frac{60}{2\pi} \times 38 = 363 \text{ rpm}$$

(iii) The torque produced by the motor,

$$T_m = \frac{E_a I_a}{\omega} = \frac{190 \times 100}{38} = 500 \text{ Nm}$$

(b) For duty cycle of 20%,

$$E_{a20\%} = \gamma V_d - I_a R_a = 0.2 \times 400 - 100 \times 0.1 = 70\ V$$

$$\omega_{20\%} = \frac{E_{a20\%}}{K\emptyset} = \frac{70}{5} = 14 \ \frac{\text{rad}}{\text{s}} \quad \rightarrow \quad n_{20\%} = 14 \times \frac{60}{2\pi} = 133.7 \text{ rpm}$$

For duty cycle of 80%,

$$E_{a80\%} = \gamma V_d - I_a R_a = 0.8 \times 400 - 100 \times 0.1 = 310\text{ V}$$

$$\omega_{80\%} = \frac{E_{a80\%}}{K\emptyset} = \frac{310}{5} = 62\text{ rad/s} \rightarrow n_{80\%} = 62 \times \frac{60}{2\pi} = 592.3\text{ rpm}$$

Hence the difference in speed is

$$n_{80\%} - n_{20\%} = 592.3 - 133.7 = 458.6\text{ rpm}$$

Example 12.6

A class-A d.c. chopper operating at a frequency of 500 Hz and feeding a separately-excited d.c. motor from 200 V d.c. source. The load torque is 35 Nm and speed is 1000 rpm. Motor resistance and inductance are 0.15 Ω and 1.0 mH respectively. The *emf* and torque constant of motor are 1.6 V/rad/s and 1.4 Nm /A respectively. Find (a) Maximum and minimum values of motor armature current, and (b) Variation of armature current. Neglect chopper losses.

Solution

(a) Let duty cycle = γ

$$V_d = 200 \text{ V}$$

$$V_{av} = \gamma V_d = \gamma \times 200$$

Average armature current $I_a = T / K\varphi = 35/1.4 = 25$ A

Back *emf* $E_a = K\varphi\omega = 1.6 \times (950 \times 2\pi/60) = 159.16$ V

$$V_{av} = E_a + I_a R_a$$

$$200\,\gamma = 159.16 + 25 \times 0.15 = 162.29 \text{ V}$$

$$\gamma = 0.8145$$

$$T = 1/500 = 2 \text{ ms}$$

$$t_{on} = \gamma T = 2 \times 0.8145 = 1.629 \text{ ms}$$

$$t_{off} = 2 - 1.629 = 0.371 \text{ ms}$$

From Eq.s (12.19) and (12.20) ,The maximum and minimum currents are calculated as

Let:

$$T = 2\pi \ , \quad t_{on} = 2\pi\gamma = \gamma T \ , \quad \tau = \frac{R_a}{L_a} \ \text{, and} \quad -t_{on}/\tau = \frac{-\gamma T R_a}{L_a}$$

Hence Eq.(12.19) and (12.20) can be re-written as

$$I_{max} = \frac{V_d}{R_a}\left(\frac{1 - e^{-t_{on}/\tau}}{1 - e^{-T/\tau}}\right) - \frac{E_a}{R_a}$$

and

$$I_{min} = \frac{V_d}{R_a}\left(\frac{e^{t_{on}/\tau} - 1}{e^{T/\tau} - 1}\right) - \frac{E_a}{R_a}$$

$$\frac{TR_a}{L_a} = \frac{2 \times 10^{-3} \times 0.15}{1 \times 10^{-3}} = 0.30$$

$$e^{-\frac{\gamma TR_a}{L_a}} = e^{-0.8145 \times 0.3} = e^{-0.24435} = 0.7832$$

$$e^{-\frac{TR_a}{L_a}} = e^{-0.3} = 0.7408$$

$$I_{max} = \frac{200}{0.15}\left(\frac{1 - 0.7832}{1 - 0.7408}\right) - \frac{159.15}{0.15}$$

$$= 1333.34 \times \left(\frac{0.2168}{0.2592}\right) - 1061 = 54{,}2A$$

$$I_{min} = \frac{200}{0.15}\left(\frac{1.2767 - 1}{1.3498 - 1}\right) - \frac{159.15}{0.15}$$

$$= 1333.34 \times \left(\frac{0.2767}{0.3498}\right) - 1061 = 0$$

(b) Variation of armature current = $I_{max} - I_{min} = 54{,}2 - 0 = 54{,}2\text{A}$

Example 12.16. *A dc series motor is fed from 600 V dc source through a chopper. The dc motor has the following parameters :*

$$r_a = 0.04\ \Omega, \quad r_s = 0.06\ \Omega, \quad k = 4 \times 10^{-3}\ Nm/amp^2$$

The average armature current of 300 A is ripple free. For a chopper duty cycle of 60%, determine :

(a) input power from the source

(b) motor speed and (c) motor torque.

Solution. (*a*) Power input to motor

$$= V_t \cdot I_a = \alpha\, V_s \cdot I_a$$

$$= 0.6 \times 600 \times 300 = 108 \text{ kW}.$$

(*b*) For a dc series motor,

$$\alpha V_s = E_a + I_a R = k\, I_a\, \omega_m + I_a R$$

$$0.6 \times 600 = 4 \times 10^{-3} \times 300 \times \omega_m + 300\,(0.04 + 0.06)$$

$$\omega_m = \frac{360 - 30}{1.2} = 275 \text{ rad/sec or } 2626.1 \text{ rpm}$$

(*c*) Motor torque, $\quad T_e = k\, I_a^{\,2} = 4 \times 10^{-3} \times 300^2 = 360$ Nm.

Example 12.17. *The chopper used for on-off control of a dc separately-excited motor has supply voltage of 230V dc, an on- time of 10 m sec and off-time of 15 m sec. Neglecting armature inductance and assuming continuous conduction of motor current, calculate the average load current when the motor speed is 1500 rpm and has a voltage constant of $K_v = 0.5$ V/rad per sec. The armature resistance is 3 Ω.* [*I.A.S., 1985*]

Solution. Chopper duty cycle

$$\alpha = \frac{T_{on}}{T_{on} + T_{off}} = \frac{10}{10 + 15} = 0.4$$

For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a = K_m \cdot \omega_m + I_a r_a$$

$$0.4 \times 230 = 0.5 \times \frac{2\pi \times 1500}{60} + I_a \times 3$$

$\therefore$ Motor load current, $I_a = \dfrac{92 - 25 \times \pi}{3} = 4.487$ A

Example 12.18. *A dc chopper is used to control the speed of a separately-excited dc motor. The dc supply voltage is 220 V, armature resistance r_a = 0.2 Ω and motor constant $K_a \phi$ = 0.08 V/rpm.*

This motor drives a constant torque load requiring an average armature current of 25 A. Determine (a) the range of speed control (b) the range of duty cycle α. Assumed the motor current to be continuous. [*I.A.S., 1990*]

Solution. For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a$$

As motor drives a constant torque load, motor torque T_e is constant and therefore armature current remains constant at 25 A.

Minimum possible motor speed is $N = 0$. Therefore,

$$\alpha \times 220 = 0.08 \times 0 + 25 \times 2.0 = 5$$

$$\alpha = \frac{5}{220} = \frac{1}{44}$$

Maximum possible motor speed corresponds to $\alpha = 1$, *i.e.* when 220 V dc is directly applied and no chopping is done.

$\therefore$ $$1 \times 220 = 0.08 \times N + 25 \times 0.2$$

or $$N = \frac{220 - 5}{0.08} = 2687.5 \text{ rpm}$$

$\therefore$ Range of speed control : $0 < N < 2687.5$ rpm and corresponding range of duty cycle : $\frac{1}{44} < \alpha < 1$.

Example 12.19. *A separately-excited dc motor is fed from 220 V dc source through a chopper operating at 400 Hz. The load torque is 30 Nm at a speed of 1000 rpm. The motor has r_a = 0, L_a = 2 mH and K_m = 1.5 V-sec/rad. Neglecting all motor and chopper losses, calculate*

(a) the minimum and maximum values of armature current and the armature current excursion,

(b) the armature current expressions during on and off periods.

Solution. As the armature resistance is neglected, armature current varies linearly between its minimum and maximum values.

(*a*) Average armature current, $I_a = \frac{T_e}{K_m} = \frac{30}{1.5} = 20$ A

Motor emf, $E_a = K_m \cdot \omega_m = 1.5 \times \frac{2\pi \times 1000}{60} = 157.08$ V

Motor input voltage, $\alpha V_s = V_t = E_a + I_a r_s = 157.08 + 0$

$\therefore$ $\alpha = \frac{157.08}{220} = 0.714$

Periodic time, $T = \frac{1}{f} = \frac{1}{400} = 2.5$ ms

On-period, $T_{on} = \alpha\, T = 0.714 \times 2.5 = 1.785$ ms

Off-period, $T_{off} = (1 - \alpha)\, T = 0.715$ ms

During on-period $T_{on,}$ armature current will rise which is governed by the equation,

$$0 + L\frac{di_a}{dt} + E_a = V_s$$

or $$\frac{di_a}{dt} = \frac{V_s - E_a}{L} = \frac{220 - 157.08}{0.02} = 3146 \text{ A/s}$$

During off period, $$\frac{di_a}{dt} = -\frac{E_a}{L} = \frac{-157.08}{0.02} = -7854 \text{ A/s}$$

With current rising linearly, it is seen from Fig. 12.21 that

$$I_{mx} = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times T_{on}$$

$$= I_{mn} + 3146 \times 1.785 \times 10^{-3}$$

or $$I_{mx} = I_{mn} + 5.616 \qquad ...(i)$$

For linear variation between I_{mn} and I_{mx}, average value of armature current

$$I_a = \frac{I_{mx} + I_{mn}}{2} = 20 \text{ A}$$

or $$I_{mx} = 40 - I_{mn} \qquad ...(ii)$$

Solving Eqs. (*i*) and (*ii*), we get $I_{mx} = 22.808$ A

and $I_{mn} = 17.912$ A.

$\therefore$ Armature current excursion $= I_{mx} - I_{mn} = 22.808 - 17.912 = 5.616$ A (*b*) Armature current expression during turn-on,

$$i_a(t) = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times t$$

$$= 17.192 + 3146\, t \quad \text{for } 0 \le t \le T_{on}$$

Armature current expression during turn-off,

$$i_a(t) = I_{mx} + \left(\frac{di_a}{dt} \text{ during } T_{off}\right) \times t$$

$$= 22.808 - 7854\, t \quad \text{for } 0 \le t \le T_{off}$$

Example 12.20. *Repeat Example 12.19, in case motor has a resistance of 0.2 Ω for its armature circuit.*

Solution. (*a*) From Example 12.19, armature current, $I_a = 20$ A and motor emf, $E_a = 157.08$ V; source voltage, $V_s = 220$ V.

For armature circuit, $\alpha V_s = V_0 = V_t = E_a + I_a\, r_a = 157.08 + 20 \times 0.2 = 161.08$ V

$$\therefore \qquad \alpha = \frac{161.08}{220} = 0.7322$$

$$T_{on} = \alpha T = 0.7322 \times 2.5 = 1.831 \text{ ms}$$

$$T_{off} = T - T_{on} = 0.669 \text{ ms}, \frac{R}{L} = \frac{0.2}{0.02} = 10$$

During $T_{on,}$ from Eq. (12.34), armature current is

$$i_a\,(t) = \frac{220 - 157.08}{0.2}\,(1 - e^{-10t}) + I_{mn} \cdot e^{-10t}$$

At $t = T_{on} = 1.831$ ms, current become I_{mx}. This gives

$$i_a\,(t) = I_{mx} = 5.7079 + 0.98187\, I_{mn} \qquad \ldots(i)$$

During T_{off}, from Eq. (12.35), armature current is

$$i_a\,(t) = \frac{-157.08}{0.2}\,(1 - e^{-10t}) + I_{mx} \cdot e^{-10\,t}$$

At $t = 0.669$ ms, $i_a(t) = I_{mn.}$ This gives

$$i_a(t) = I_{mn} = -5.237 + 0.9933\, I_{mx} \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$I_{mx} = 5.7079 + 0.98187\,(-5.237 + 0.9933\,I_{mx})$$
$$= 0.5658 + 0.9753\,I_{mx}$$

or

$$I_{mx} = \frac{0.5658}{0.0247} = 22.907 \text{ A}$$
$$I_{mn} = -5.237 + 0.9933 \times 22.907 = 17.516 \text{ A}$$

$\therefore$ Armature current excursion

$$= I_{mx} - I_{mn} = 22.907 - 17.516 = 5.39 \text{ A}$$

(b) Armature current expression during turn-on period is

$$i_a(t) = 314.6\,(1 - e^{-10t}) + 17.516\,e^{-10t}$$

Armature current expression during turn-off period is

$$i_a(t) = -785.4\,(1 - e^{-10t}) + 22.907\,e^{-10t}$$

Example 12.21. *A dc chopper is used for regenerative braking of a separately-excited dc motor. The dc supply voltage is 400V. The motor has $r_a = 0.2\ \Omega$, $K_m = 1.2$ V-s/rad. The average armature current during regenerative braking is kept constant at 300 A with negligible ripple.*

For a duty cycle of 60% for a chopper, determine

(a) power returned to the dc supply

(b) minimum and maximum permissible braking speeds and

(c) speed during regenerative braking.

Solution. (*a*) Average armature terminal voltage,

$$V_t = (1 - \alpha)\ V_s = (1 - 0.6) \times 400 = 160\ \text{V}.$$

Power returned to the dc supply

$$= V_t I_a = 160 \times 300\ \text{W} = 48\ \text{kW}$$

(*b*) From Eq. (12.41), minimum braking speed is

$$\omega_{mn} = \frac{I_a \cdot r_a}{K_m} = \frac{300 \times 0.2}{1.2} = 50\ \text{rad/s or } 477.46\ \text{rpm}$$

From Eq. (12.42), maximum braking speed is

$$\omega_{mx} = \frac{V_s + I_a \cdot r_a}{K_m} = \frac{400 + 300 \times 0.2}{1.2}$$

$$= 383.33\ \text{rad/s or } 3660.6\ \text{rpm}$$

(*c*) When working as a generator during regenerative braking, the generated emf is $E_a'' = K_m\ \omega_m = V_t + I_a r_a = 160 + 300 \times 0.2 = 220$ V

$\therefore$ Motor speed, $\omega_m = \dfrac{220}{1.2}$ rad/s or 1750.7 rpm

Example 21-10

The switch in Fig. 21.60a opens and closes at a frequency of 20 Hz and remains closed for 3 ms per cycle. A dc ammeter connected in series with the load E_0 indicates a current of 70 A.

a. If a dc ammeter is connected in series with the source, what current will it indicate?
b. What is the average current per pulse?

Solution

a. Using Eq. 21.20, we have

$$\text{period } T = \frac{1}{20} = 50 \text{ ms}$$

$$\text{duty cycle} = \frac{T_a}{T} = \frac{3}{50} = 0.06$$

$$\begin{aligned} I_S &= I_0 D \\ &= 70 \times 0.06 \\ &= 4.2 \text{ A} \end{aligned}$$

b. The average current during each *pulse* (duration T_a) is 70 A. Considering that the average current is only 4.2 A, the source has to be specially designed to supply such a high 70 A pulse. In most cases a large capacitor is connected across the terminals of the source. It can readily furnish the high current pulses as it discharges.

Example 21-11

We wish to charge a 120 V battery from a 600 V dc source using a dc chopper. The average battery current should be 20 A, with a peak-to-peak ripple of 2 A. If the chopper frequency is 200 Hz, calculate the following:

a. The dc current drawn from the source
b. The dc current in the diode
c. The duty cycle
d. The inductance of the inductor

Solution

The circuit diagram is shown in Fig. 21.61a and the desired battery current is given in Fig. 21.61b. It fluctuates between 19 A and 21 A, thus yielding an average of 20 A with a peak-to-peak ripple of 2 A.

a. The power supplied to the battery is

$$P = 120\text{ V} \times 20\text{ A} = 2400\text{ W}$$

The power supplied by the source is, therefore, 2400 W.

The dc current from the source is

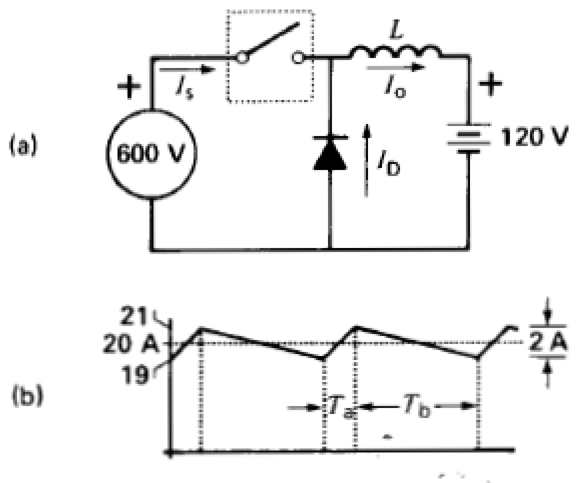


Figure 21.61

a. Circuit of Example 21-11.

b. Current in the load.

$$I_S = P/E_S = 2400/600 = 4 \text{ A}$$

b. To calculate the average current in the diode, we refer to Fig. 21.61a. Current I_0 is 20 A and I_S was found to be 4 A. By applying Kirchhoff's current law to the diode/inductor junction, the average diode current I_D is

$$\begin{aligned} I_D &= I_0 - I_S \\ &= 20 - 4 \\ &= 16 \text{ A} \end{aligned}$$

c. The duty cycle is

$$D = E_0/E_S = 120/600 = 0.2$$
$$T = 1/f = 1/200 = 5 \text{ ms}$$

Consequently, the *on* time T_a is

$$T_a = DT = 0.2 \times 5 \text{ ms} = 1 \text{ ms}$$

The waveshapes of I_S and I_D are shown in Figs. 21.61c and 21.61d, respectively. Note the sharp pulses delivered by the source.

d. During interval T_a the average voltage across the inductor is (600 − 120) = 480 V.
The volt-seconds accumulated by the inductor during this interval is $A_{(+)}$ = 480 V × 1 ms = 480 mV·s = 0.48 V·s. The change in current during the interval is 2 A; consequently,

$$\Delta I = A_{(+)}/L \qquad (2.28)$$
$$2 = 0.48/L$$
$$L = 0.24\ \text{H}$$

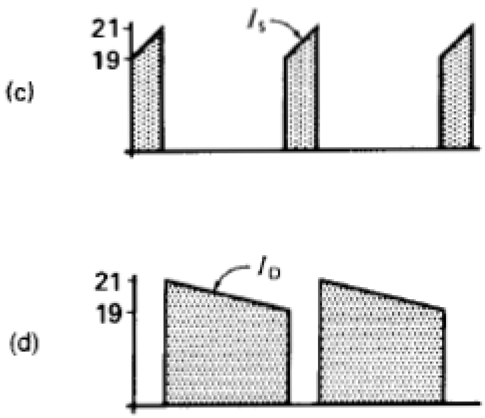


Figure 21.61
c. Current drawn from the source.
d. Current in the freewheeling diode.

Thus, the inductor should have an inductance of 0.24 H. If a larger inductance were used, the current ripple would be smaller, but the dc voltages and currents would remain the same.

Example 21-12

The chopper in Fig. 21.62 operates at a frequency of 4 kHz and the *on* time is 20 μs. Calculate the apparent resistance across the source, knowing that $R_0 = 12\ \Omega$.

Solution

The duty cycle is

$$D = T_a/T = T_a f = 20 \times 10^{-6} \times 4000 = 0.08$$

Applying Eq. 21.22, we have

$$\begin{aligned} R_S &= R_0/D^2 \\ &= 12/(0.08)^2 \\ &= 1875\ \Omega \end{aligned}$$

This example shows that the actual value of a resistor can be increased many times by using a chopper. Although a chopper can be compared to a transformer, there is an important difference between the two. The reason is that a transformer permits power flow in both directions—from the high-voltage side to the low-voltage side or vice versa. The step-down chopper we have just studied can transfer power only from the high-voltage side to the low-voltage side. Because power flow in both directions is often required, we now examine a dc-to-dc converter that achieves this result.

6.3. THE SECOND-QUADRANT (STEP-UP) CHOPPER FOR GENERATOR BRAKING

A d.c. brush motor with PM excitation is fed through a second-quadrant chopper for regenerative braking (Table 6.1b). The motor data are $R_a = 1\Omega$; $L_a = 20mH$; $e_g = 80V$ (given speed). The supply voltage V_0 is 120Vd.c. and $t_{on} = 5.10^{-3}$ s. Determine:

a. The waveform of motor current for zero initial current
b. The waveform of source current
c. The maximum average power generated

Solution:

a. The current waveforms are as shown in Figure 6.2 and 6.3 with $i_{a0} = 0$.

$$i_a = -\frac{e_g}{R_a} + B \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t < t_{on} \tag{6.29}$$

b.

$$i_a' = \frac{V_s - e_g}{R_a} + B' \cdot e^{-(t-t_{on})\frac{R_a}{L_a}}; \quad t_{on} < t < T \tag{6.30}$$

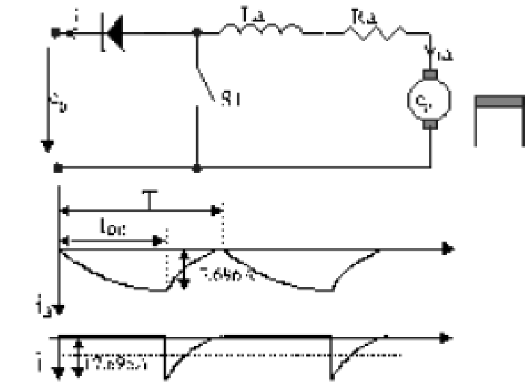


Figure 6.6. The step-up second-quadrant chopper.

The boundary conditions are

$$(i_a)_{t=0} = 0; \quad (i_a)_{t=t_{on}} = (i_a')_{t=t_{on}}; \quad (i_a')_{t=T} = 0 \tag{6.31}$$

The unknowns are B, B′ and T. Consequently,

$$B = \frac{e_g}{R_a} = \frac{80}{1} = 80 \tag{6.32}$$

$$80 \cdot e^{-5\cdot 10^{-3}\cdot 10} - 80 = \frac{(120 - 80)}{1} + B'; \quad B' = -57.696 \tag{6.33}$$

$$40 - 57.696 \cdot e^{-(T-t_{on})\cdot 10} = 0; \quad T - t_{on} = 7.326 \cdot 10^{-3}\,s$$
$$\text{thus } T = (7.326 + 5)\cdot 10^{-3}\,s = 12.326 \cdot 10^{-3}\,s \tag{6.34}$$

Note that the source current occurs during the S_1 turn-off and is negative, proving the regenerative operation. The average source current i_{av} is

$$i_{av}' = \frac{1}{T}\int_{t_{on}}^{T} i_a'\,dt = \frac{1}{T}\left[\frac{e_0 - e_g}{R_a}(T - t_{on}) - \frac{L_a}{R_a}B'\left(e^{-(T-t_{on})\frac{R_a}{L_a}} - 1\right)\right] =$$
$$= \frac{1}{12.326\cdot 10^{-3}}\left[\frac{40}{1}\cdot 7.326\cdot 10^{-3} + 20\cdot 10^{-3}\cdot 57.696\cdot\left(e^{-7.326\cdot 10^{-3}\cdot 10} - 1\right)\right];$$
$$i_{av}' = -4.938\text{ A} \tag{6.35}$$

The average power regenerated P_{av} is

$$P_{av} = -i_{av}' \cdot V_s = 4.938 \cdot 120 = 592.56\text{ W} \tag{6.36}$$

Example 14.4: *Second-quadrant DC chopper – continuous inductor current*

A dc-to-dc chopper capable of second-quadrant operation is used in a 200V dc battery electric vehicle. The machine armature has 1 Ω resistance in series with 1mH inductance.

i. The machine is used for regenerative braking. At a constant speed downhill, the back emf is 150V, which results in a 10A braking current. What is the switch on-state duty cycle if the machine is delivering continuous output current? What is the minimum chopping frequency for these conditions?

ii. At this speed, (that is, E = 150V), determine the minimum duty cycle for continuous inductor current, if the switching frequency is 1kHz. What is the average braking current at the critical duty cycle? What is the regenerating efficiency and the rms machine output current?

iii. If the chopping frequency is increased to 5kHz, at the same speed, (that is, E = 150V), what is the critical duty cycle and the corresponding average dc machine current?

Solution

The main circuit operating parameters are

- V_s = 200V
- E = 150V
- load time constant $\tau = L/R$ = 1mH/1Ω = 1ms

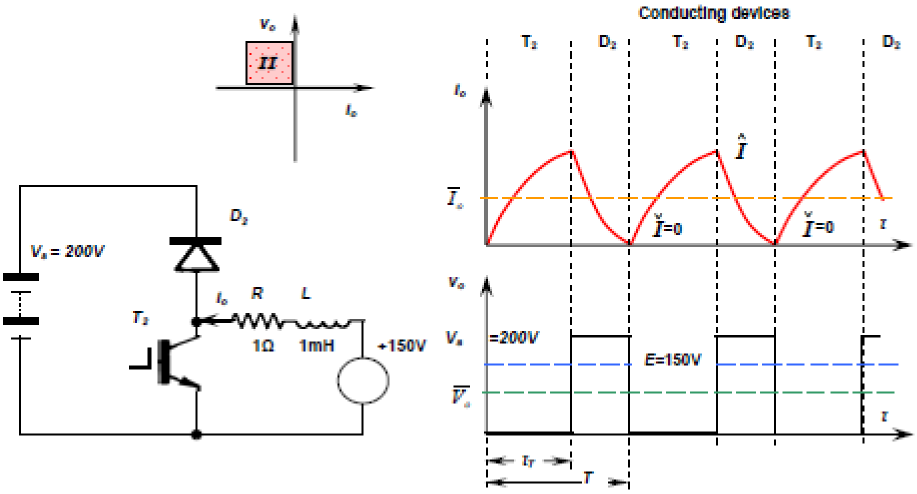


Figure 14.12. Example 14.4. *Circuit diagram and waveforms.*

i. The relationship between the dc supply V_s and the dc machine back emf E is given by equation (14.49), that is

$$\bar{I}_o = \frac{E - \bar{V}_o}{R} = \frac{E - V_s(1-\delta)}{R}$$

$$10\text{A} = \frac{150\text{V} - 200\text{V} \times (1-\delta)}{1\Omega}$$

that is

$$\delta = 0.3 \equiv 30\% \quad \text{and} \quad \bar{V}_o = 140\text{V}$$

The expression for the average dc machine output current is based on continuous armature inductance current. Therefore the switching period must be shorter than the time t_x predicted by equation (14.64) for the current to reach zero, before the next switch on-period. That is, for $t_x = T$ and $\delta = 0.3$

$$t_x = t_T + \tau\,\ell n\left(1 + \frac{E}{V_s - E}\left(1 - e^{\frac{-t_T}{\tau}}\right)\right)$$

This simplifies to

$$1 = 0.3 + \frac{1\text{ms}}{T}\,\ell n\left(1 + \frac{150\text{V}}{200\text{V} - 150\text{V}}\left(1 - e^{\frac{-0.3T}{1\text{ms}}}\right)\right)$$

$$e^{0.3T} = 4 - 3e^{-0.3T}$$

Iteratively solving this transcendental equation gives T = 0.4945ms. That is the switching frequency must be greater than $f_s = 1/T$ = 2.022kHz, else machine output current discontinuities occur, and equation (14.49) is invalid. The switching frequency can be reduced if the on-state duty cycle is increased as in the next part of this example.

ii. The operational boundary condition giving by equation (14.63), using $T=1/f_s=1/1\text{kHz} = 1\text{ms}$, yields

$$\frac{E}{V_s}=\frac{1-e^{\frac{-T+t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}}$$

$$\frac{150\text{V}}{200\text{V}} = \frac{1 - e^{\frac{(\delta-1)\times 1\text{ms}}{1\text{ms}}}}{1 - e^{\frac{-1\text{ms}}{1\text{ms}}}}$$

Solving gives $\delta = 0.357$. That is, the on-state duty cycle must be at least 35.7% for continuous machine output current at a switching frequency of 1kHz.
For continuous inductor current, the average output current is given by equation (14.49), that is

$$\bar{I}_o = \frac{E-\bar{V}_o}{R} = \frac{E-V_s(1-\delta)}{R}$$

$$= \frac{150\text{V} - \bar{V}_o}{1\Omega} = \frac{150\text{V} - 200\text{V}\times(1 - 0.357)}{1\Omega} = 21.4\text{A}$$

$$\bar{V}_o = 150\text{V} - 21.4\text{A}\times 1\Omega = 128.6\text{V}$$

The average machine output current of 21.4A is split between the switch and the diode (which is in series with V_s).
The diode current is given by equation (14.56)

$$\bar{I}_{diode} = \bar{I}_o - \bar{I}_{switch}$$

$$= \frac{\tau}{T}\left(\hat{I}-\check{I}\right) - \frac{(V_s-E)(1-\delta)}{R}$$

The minimum output current is zero while the maximum is given by equation (14.70).

$$\hat{I} = \frac{E}{R}\left(1-e^{\frac{-t_T}{\tau}}\right) = \frac{150\text{V}}{1\Omega}\times\left(1 - e^{\frac{-0.357\times 1\text{ms}}{1\text{ms}}}\right) = 45.0\text{A}$$

Substituting into the equation for the average diode current gives

$$\bar{I}_{diode} = \frac{1\text{ms}}{1\text{ms}}\times(45.0\text{A} - 0\text{A}) - \frac{(200\text{V} - 150\text{V})\times(1 - 0.357)}{1\Omega} = 12.85\text{A}$$

The power delivered by the dc machine back emf E is

$$P_E = E\bar{I}_o = 150\text{V}\times 21.4\text{A} = 3210\text{W}$$

while the power delivered to the 200V battery source V_s is

$$P_{V_s} = V_s\bar{I}_{diode} = 200\text{V}\times 12.85\text{A} = 2570\text{W}$$

The regeneration transfer efficiency is

$$\eta = \frac{P_{V_s}}{P_E} = \frac{2570\text{W}}{3210\text{W}} = 80.1\%$$

The energy generated deficit, 640W (3210W - 2570W)), is lost in the armature resistance, as I^2R heat dissipation. The output rms current is

$$I_{o_{rms}} = \sqrt{\frac{P}{R}} = \sqrt{\frac{640\text{W}}{1\Omega}} = 25.3\text{A rms}$$

iii. At an increased switching frequency of 5kHz, the duty cycle would be expected to be much lower than the 35.7% as at 1kHz. The operational boundary between continuous and discontinuous armature inductor current is given by equation (14.63), that is

$$\frac{E}{V_s}=\frac{1-e^{\frac{-T+t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}}$$

$$\frac{150\text{V}}{200\text{V}} = \frac{1 - e^{\frac{(-1+\delta)\times 0.2\text{ms}}{1\text{ms}}}}{1 - e^{\frac{-0.2\text{ms}}{1\text{ms}}}}$$

which yields $\delta = 26.9\%$.
The machine average output current is given by equation (14.49)

$$\bar{I}_o = \frac{E-\bar{V}_o}{R} = \frac{E-V_s(1-\delta)}{R}$$

$$= \frac{150\text{V}-\bar{V}_o}{1\Omega} = \frac{150\text{V} - 200\text{V}\times(1 - 0.269)}{1\Omega} = 3.8\text{A}$$

such that the average output voltage $\bar{V}_o$ is 146.2V.

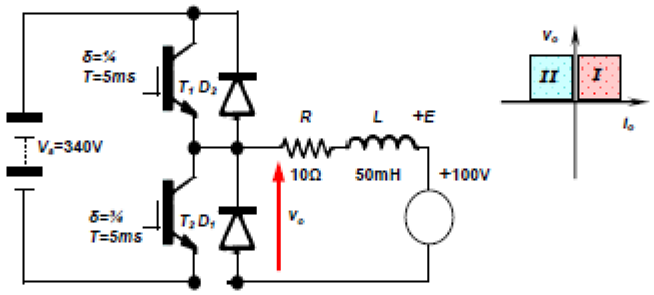


Figure 14.14. Example 14.5. *Circuit diagram.*

Second Quadrant Chopper or Type B Chopper or Class B Chopper

In this topic, you study Second Quadrant Chopper or Type B Chopper or Class B Chopper v-i plane, working principle, quadrant operation, Applications, waveforms, and Circuit diagrams.

Type B chopper is basically equivalent to Step-Up Chopper.

Circuit Diagram

The Type B chopper circuit diagram as shown in Figure 1. Here the motor load is assumed, R_a and L_a armature resistance and inductance of the motor respectively. E_b is the back emf of the motor.

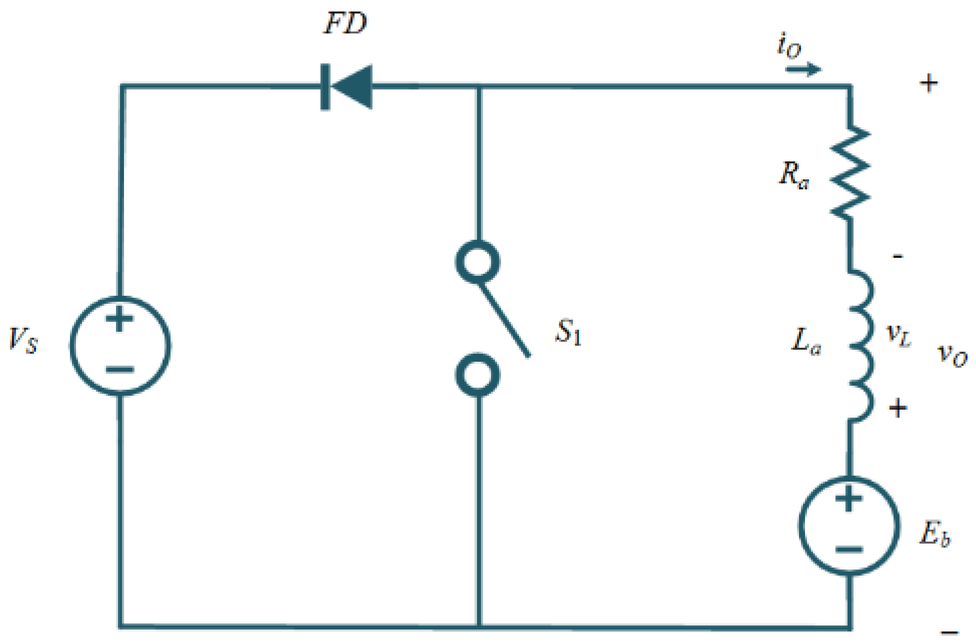


Figure 1 Circuit diagram of Type B chopper

$v_O - i_O$ plane

The Type B chopper operates in the Second quadrant of $v_O - i_O$ plane as shown in Figure 2. Here v_O is the output voltage, V_O is the average output voltage, i_O is the output current and I_O is the average output current of Type B chopper circuit.

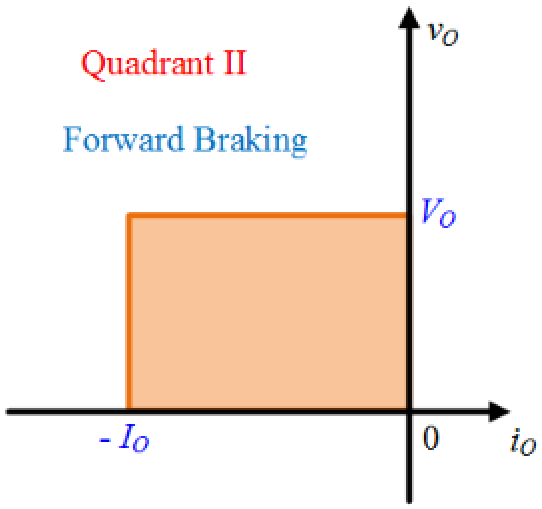


Figure 2 Type B chopper v_O - i_O plane

Quadrant II operation when Switch S_1 turned on

The Type B chopper circuit diagram for Quadrant II is shown in Figure 3 and the associated waveforms are shown below in Figure 5. Let us assume that the motor is running in the forward direction. When switch S_1 operated, Switch S_1 turned on and conducts, output voltage v_O is zero and E_b is responsible for the negative output current i_O, the machine behaves as generator and inductor stores energy.

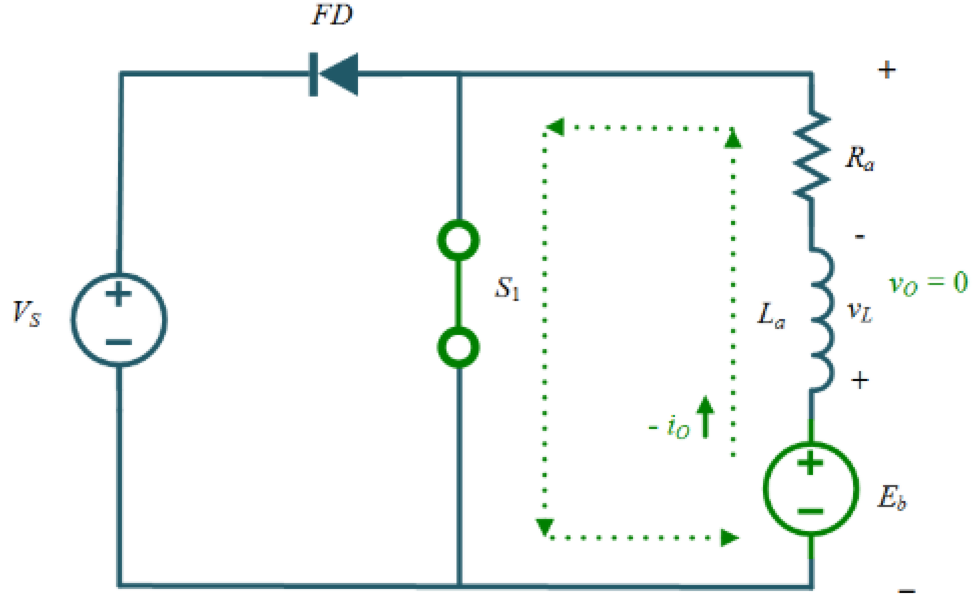


Figure 3 Equivalent circuit diagram I of Type B chopper

Quadrant II operation when Switch S_1 turned off

The Type E chopper equivalent circuit diagram for Quadrant II is shown in Figure 4 and the associated waveforms are below shown in Figure 5. Switch S_1 turned off, diode *FD* conducts, output voltage v_O becomes positive and the output current i_O is negative, inductor release energy using diode *FD*, power flows from load to source and hence called as reverse braking.

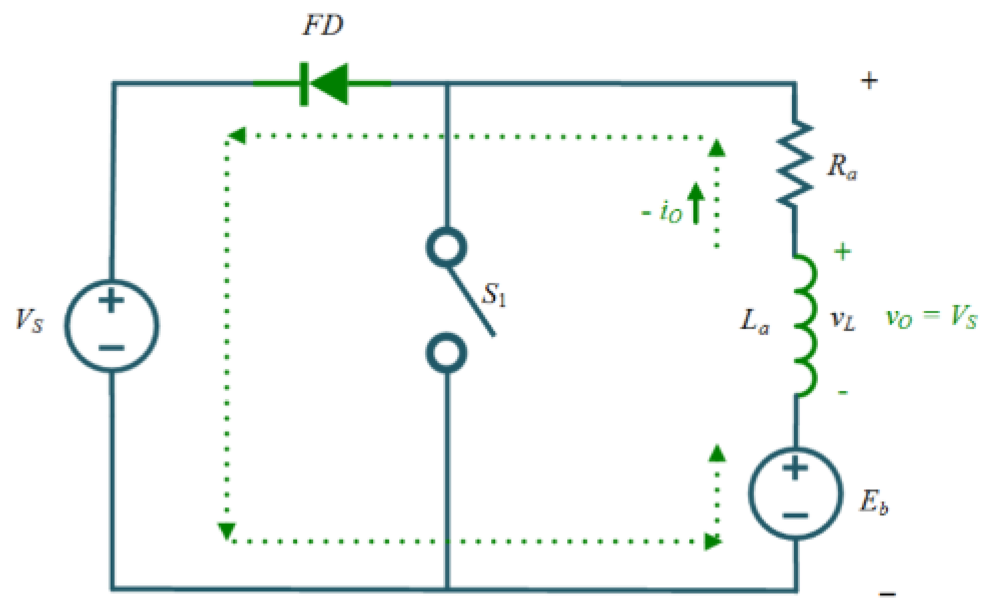


Figure 4 Equivalent circuit diagram II of Type B chopper

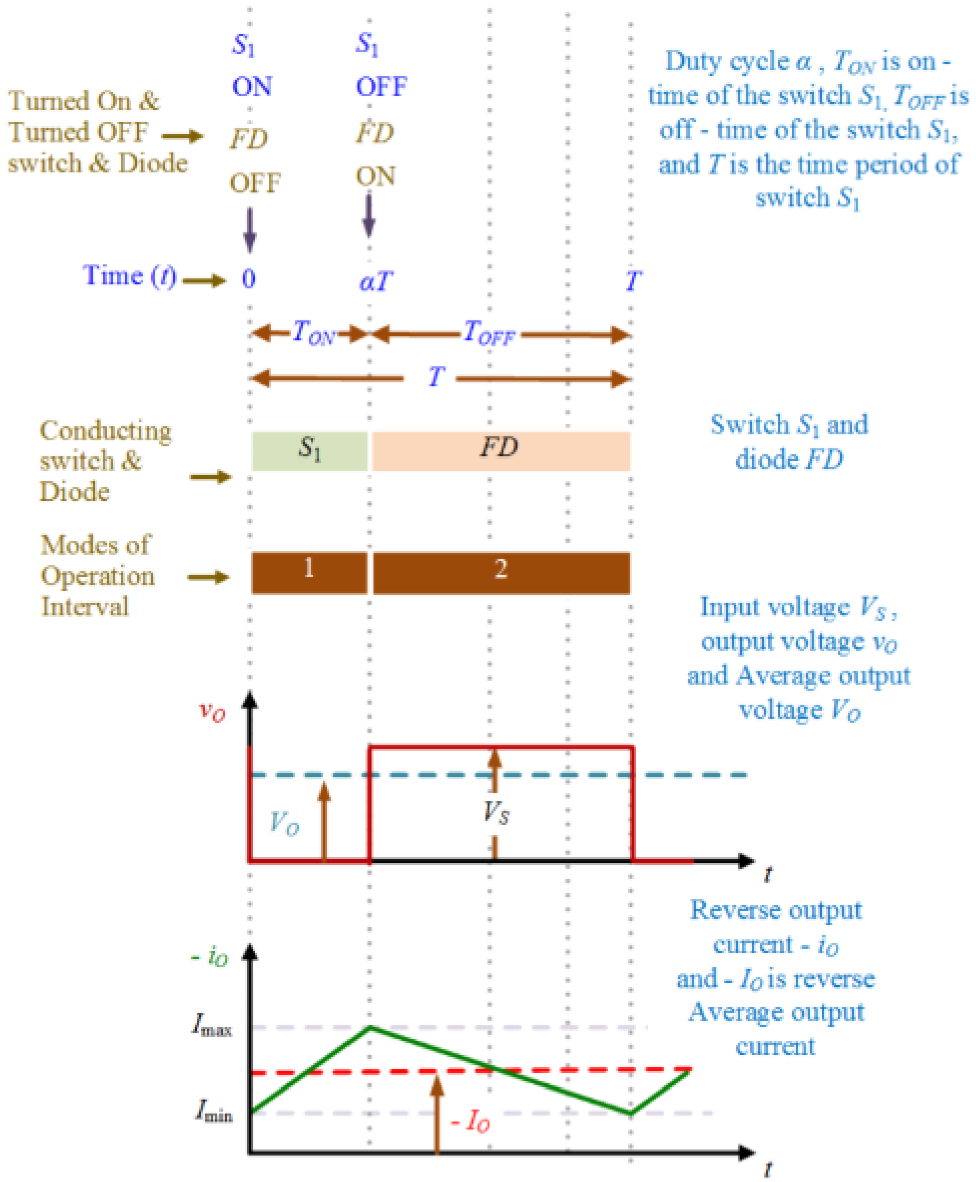


Figure 5 Waveform of Type B chopper

Waveforms

Type B chopper associated waveforms are shown in Figure 5.

Application

This chopper is suitable for regenerative breaking application only.

Mathematical Analysis

Using the waveform as shown in Figure 1, The average output voltage write as

$$V_O = V_S\left(\frac{T_{OFF}}{T}\right)\ldots.(1)$$

Also

$$\frac{T_{OFF}}{T} = 1-\alpha\ldots.(2)$$

Put Equation 2 in Equation 1 gives

$$V_O = V_S(1-\alpha)\ldots.(3)$$

Equation 3 describe the realtion between input dc source voltage and average output voltage for Type B chopper.

EXAMPLE 10.5

The two-quadrant chopper shown in Fig. 10.38*a* is used to control the speed of the dc motor and also for regenerative braking of the motor. The motor constant is $K\Phi = 0.1$ V/rpm ($E_a = K\Phi n$). The chopping frequency is $f_c = 250$ Hz and the motor armature resistance is $R_a = 0.2\ \Omega$. The inductance L_a is sufficiently large and the motor current i_0 can be assumed to be ripple-free. The supply voltage is 120 V.

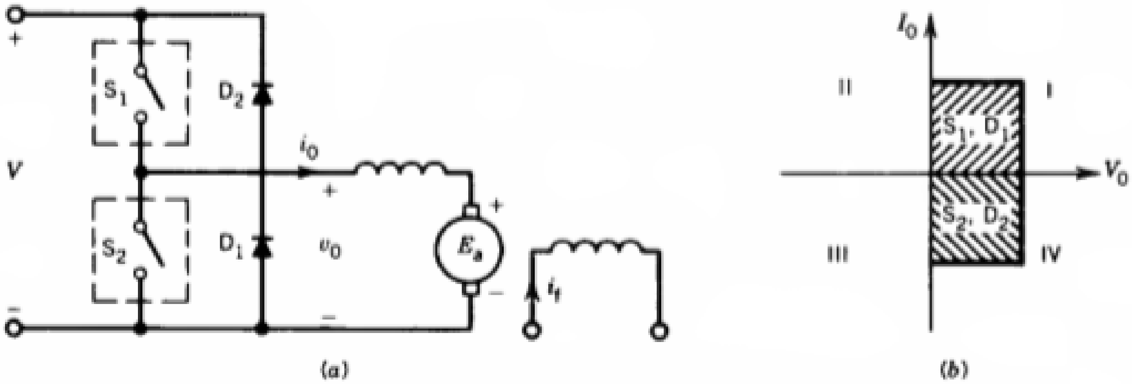


FIGURE 10.38 Two-quadrant chopper. (*a*) Circuit. (*b*) Quadrant operation.

(a) Chopper S_1 and diode D_1 are operated to control the speed of the motor. At $n = 400$ rpm and $i_0 = 100$ A (ripple-free),

- **(i)** Draw waveforms of v_0, i_0, and i_s.
- **(ii)** Determine the turn-on time (t_{on}) of the chopper.
- **(iii)** Determine the power developed by the motor, power absorbed by R_a, and power from the source.

(b) In the two-quadrant chopper S_2 and diode D_2 are operated for regenerative braking of the motor. At $n = 350$ rpm and $i_0 = -100$ A (ripple-free),

- **(i)** Draw waveforms of v_0, i_0, and i_s.
- **(ii)** Determine the turn-on time (t_{on}) of the chopper.
- **(iii)** Determine the power developed (and delivered) by the motor, power absorbed by R_a, and power to the source.

Solution

(a) **(i)** The waveforms are shown in Fig. E10.5*a*.

(ii) From Fig. 10.38*a*

$$V_0 = E_a + I_a R_a$$
$$= 0.1 \times 400 + 100 \times 0.2$$
$$= 60\text{ V}$$

$$60 = \frac{t_{on}}{T}\text{V} = \frac{t_{on}}{T}120$$

$$t_{on} = \frac{T}{2}$$

(iii)
$$P_{motor} = E_a I_0 = 0.1 \times 400 \times 100 = 4000\text{ W}$$
$$P_R = (i_0)^2_{rms} R_a = 100^2 \times 0.2 = 2000\text{ W}$$
$$P_s = V(i_s)_{avg} = 120 \times 100 \times \tfrac{2}{4} = 6000\text{ W}$$

(b) **(i)** The waveforms are shown in Fig. E10.5*b*.

(ii)
$$V_0 = E_a + (-I_0 R_a)$$
$$= 0.1 \times 350 - 100 \times 0.2$$
$$= 15\text{ V}$$

From Fig. E10.5*b*

$$V_0 = \frac{T - t_{on}}{T} V$$

$$15 = \left(1 - \frac{t_{on}}{T}\right)120$$

$$\frac{t_{on}}{T} = \frac{7}{8}$$

$$t_{on} = \tfrac{7}{8} \times 4 = 3.5\text{ msec}$$

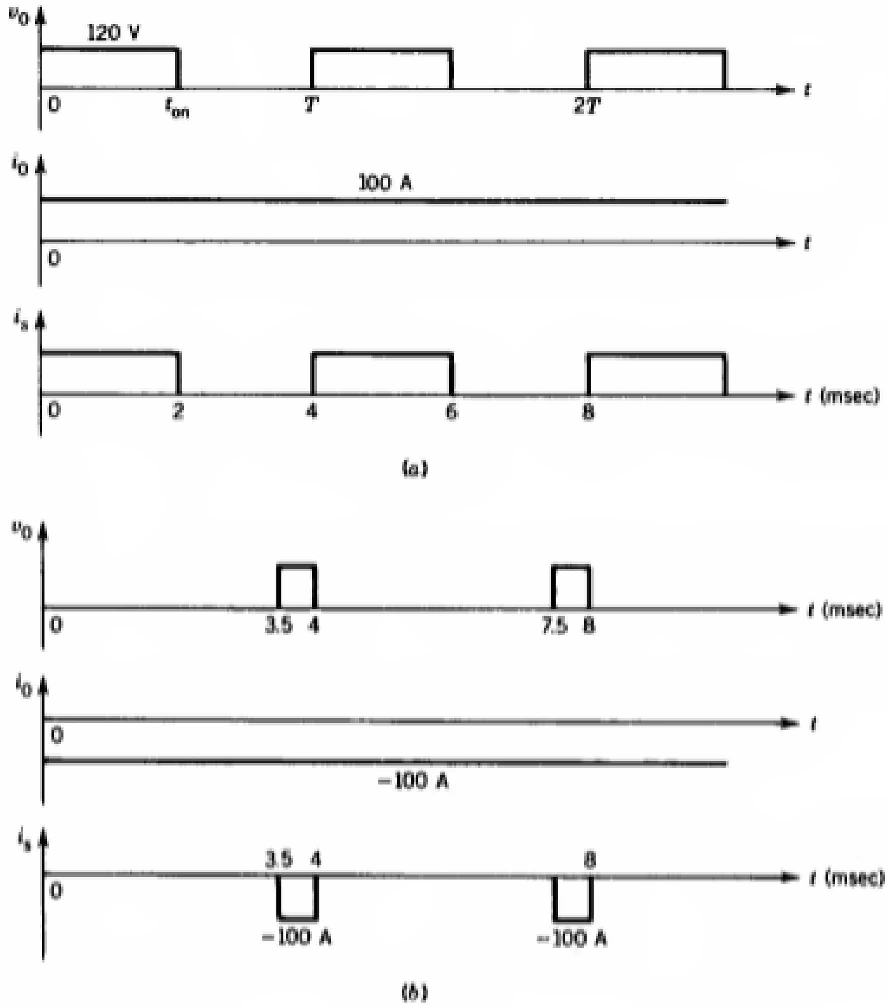


FIGURE E10.5

(iii)

$$P_{\text{motor}} = E_a I_0 = 0.1 \times 350(-100) = -3500\ \text{W}$$

$$P_R = 100^2 \times 0.2 = 2000\ \text{W}$$

$$P_s = V(i_s)_{\text{avg}} = 120(-100 \times \tfrac{1}{8}) = -1500\ \text{W}$$

10.23 In the chopper circuit shown in Fig. P10.23, the two switches are simultaneously turned on for time t_{on} and turned off for time $t_{off} = T - t_{on}$, where T is the chopping period. Assume voltage v_0 to be ripple-free and current i_L to be continuous.

(a) Derive an expression for V_0 as a function of the duty cycle $\alpha = t_{on}/T$ and the supply voltage V. Determine V_0 for $\alpha = 0, 0.5, 1.0$.

(b) Draw waveforms of v_0, v_L, i_L, i_0, and i for $\alpha = \frac{1}{3}$.

(c) What are the advantages and disadvantages of this circuit?

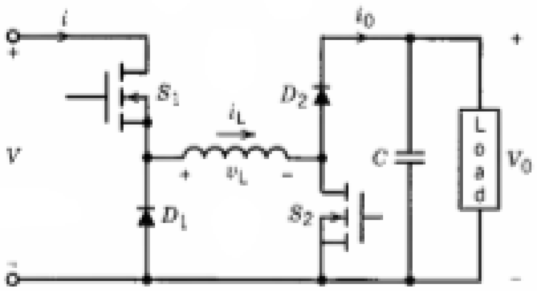


FIGURE P10.23

$\boxed{10.23}$ (a) during $t_{on} \rightarrow v_L = v = L\dfrac{I_2 - I_1}{t_{on}} = L\dfrac{\Delta I}{t_{on}}$

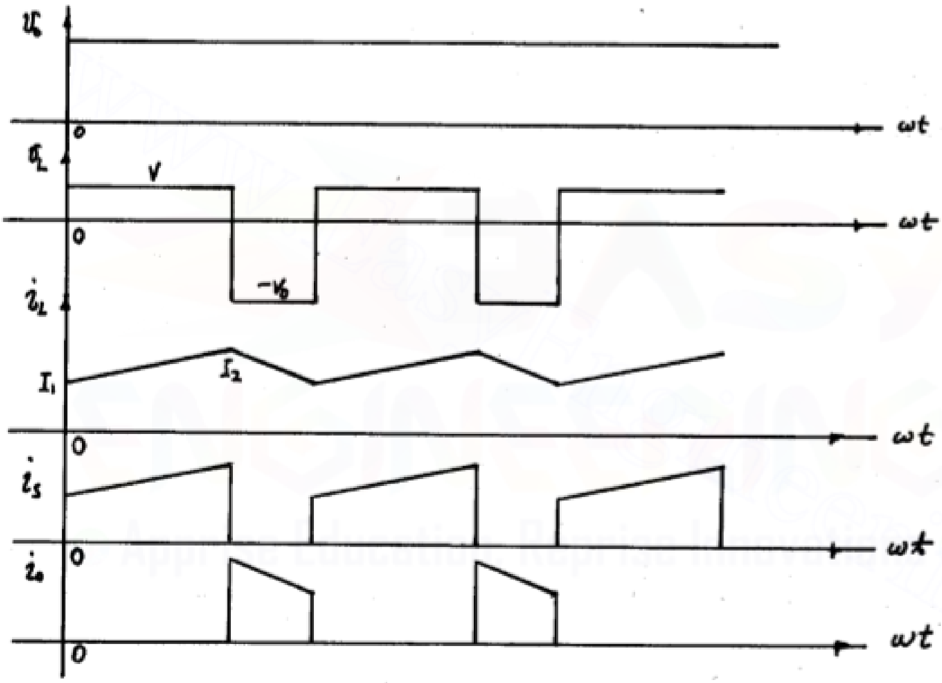
during $t_{off} \rightarrow v_L = -V_o = L\dfrac{I_1 - I_2}{t_{off}} = -L\dfrac{\Delta I}{t_{off}}$

$$\frac{V t_{on}}{L} = \frac{V_o t_{off}}{L}$$

$$V_o = \frac{t_{on}}{t_{off}} V = \frac{t_{on}}{T - t_{on}} V = \frac{\alpha}{1-\alpha} V$$

α	V_o
0	0
0.5	V
1.0	∞

(b)



(c) • This is a step-down , step-up chopper

• Polarity of V_o is the same as that of V

• Higher conduction losses → two devices conduct at a time

10.26 Consider the two-quadrant chopper systems shown in Fig. P10.26. The two choppers S_1 and S_2 are turned on for time t_{on} and turned off for time $T - t_{on}$, where T is the chopping period.

(a) Draw the waveform of the output voltage v_0. Assume continuous output current i_0.

(b) Derive an expression for the average output voltage V_0 in terms of the supply voltage V and the duty ratio α ($= t_{on}/T$).

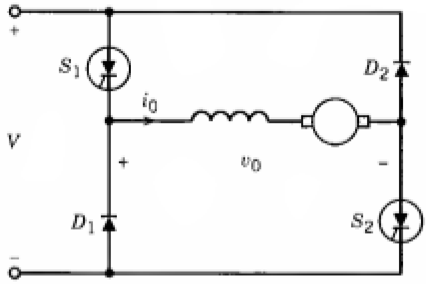
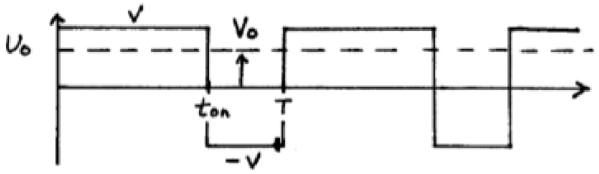


FIGURE P10.26

$\boxed{10.26}$ (a)



(b) $$V_o = \frac{1}{T}\int_0^T v_o\,dt = \frac{1}{T}\left[\int_0^{t_{on}} V\,dt + \int_{t_{on}}^{T} -V\,dt\right]$$

$$= V(2\alpha - 1)$$

6.4. THE TWO-QUADRANT CHOPPER

Consider a two-quadrant chopper (Figure 6.7) supplying a d.c. brush motor with separate excitation. The load current varies between $I_{max}>0$ and $I_{min} < 0$.

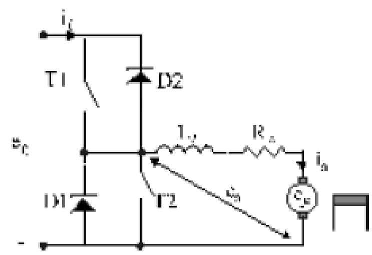


Figure 6.7. Two-quadrant chopper supplying a separately excited d.c. brush motor

Determine:

a. The voltage and current waveforms for the load current varying between an $I_{max}>0$ and $I_{min}<0$ with $I_{max} > |I_{min}|$.

b. Derive the expression of the conducting times t_{d2} and t_{d1} of the diodes D_1 and D_2 for case a.

Solution:

a. Let us first draw the load current which varies from a positive maximum to a negative minimum (Figure 6.8). The conduction intervals for each of the four switches T_1, D_2, D_1, T_2 are

$$\begin{aligned} &T_1 \text{ for } t_{d2} < t < t_c; \\ &T_2 \text{ for } t_{d1} < t < T; \\ &D_1 \text{ for } t_c < t < t_{d1}; \\ &D_2 \text{ for } 0 < t < t_{d2} \end{aligned} \tag{6.37}$$

b. The equations for the current are:

$$V_0 - e_g = R_a i_a + L_a \frac{di_a}{dt}; \quad \text{for } 0 < t < t_c \tag{6.38}$$

$$-e_g = R_a i_a + L_a \frac{di_a}{dt}; \quad \text{for } t_c < t < T \tag{6.39}$$

with the solutions:

$$i_a = \frac{V_0 - e_g}{R_a} + A \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t \le t_c \tag{6.40}$$

$$i_a' = -\frac{e_g}{R_a} + A' \cdot e^{-(t-t_c)\frac{R_a}{L_a}}; \quad t_c < t \le T \tag{6.41}$$

with the boundary conditions $i_a(0) = i_{min}$, $i_a'(T) = i_{min}$ and $i_a(t_c) = i_a'(t_c)$.

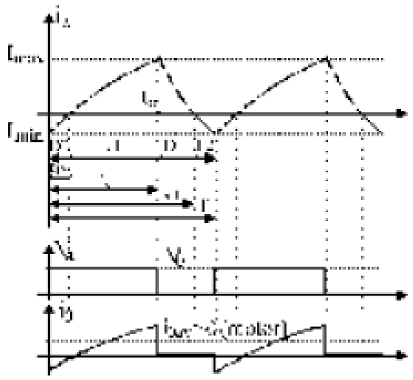


Figure 6.8. Voltage and current waveforms of two-quadrant chopper-fed d.c. motor

The unknowns A, A′ and t_c are obtained from

$$A = I_{min} - \frac{e_0 - e_g}{R_a} \tag{6.42}$$

$$A' = I_{max} + \frac{e_g}{R_a} \tag{6.43}$$

$$T - t_c = -\frac{L_a}{R_a} \cdot \ln\left[\left(A' - \frac{e_0}{R_a}\right) / A\right] \tag{6.44}$$

Consider a d.c. brush motor whose data are $e_0 = 120V$, $R_a = 1\Omega$, $L_a = 5mH$, $e_g = 80V$, $I_{max} = 5A$, $I_{min} = -2A$ and chopping frequency $f_{ch} = 0.5kHz$. For the two-quadrant chopper as above calculate:

a. The $t_c / T = \alpha_{on}$ ratio.

b. The conducting intervals of the 4 switches.

Solution:

a. The constants A, A′, t_c expressions developed above ((6.42)-(6.44)) yield

$$A = I_{min} - \frac{e_0 - e_g}{R_a} = -2 - \frac{120-80}{1} = -42 \text{ A} \tag{6.45}$$

$$A' = I_{max} + \frac{e_g}{R_a} = 5 + \frac{80}{1} = 85 \text{ A} \tag{6.46}$$

$$T = \frac{1}{f_{ch}} = \frac{1}{500} s = 0.002 s = 2 \text{ ms} \tag{6.47}$$

$$T - t_c = -\frac{L_a}{R_a} \cdot \ln\left[\left(A' - \frac{e_0}{R_a}\right)/A\right] =$$

$$-\frac{5 \cdot 10^{-3}}{1} \cdot \ln\left[\left(85 - \frac{120}{1}\right)/(-42)\right] = 0.9116 \cdot 10^{-3} s$$

$$t_c = 2 \cdot 10^{-3} - 0.5116 \cdot 10^{-3} = 1.0884 \cdot 10^{-3} s \tag{6.48}$$

b. The conducting interval of D_2, t_{d2}, corresponds to $i_a = 0$

$$\frac{V_0 - e_g}{R_a} + A \cdot e^{-t_{d2}\frac{R_a}{L_a}} = 0$$

$$t_{d2} = --\frac{L_a}{R_a} \cdot \ln\left[\frac{V_0 - e_g}{-A \cdot R_a}\right] = \frac{-5 \cdot 10^{-3}}{1} \ln\left(\frac{120-80}{-(-42) \cdot 1}\right) = 0.2439 \cdot 10^{-3} s \tag{6.49}$$

Thus the main switch T_1 conducts for a time interval

$$t_c - t_{d2} = (1.0884 - 0.2439) \cdot 10^{-3} = 0.84445 \cdot 10^{-3} s \tag{6.50}$$

To calculate the conducting time of the diode D_1 we apply the condition $i_a'(t_{d1}) = 0$

$$\frac{-e_g}{R_a} + A' \cdot e^{-(t_{d1} - t_c)\frac{R_a}{L_a}} = 0 \tag{6.51}$$

$$t_{d1} - t_c = -\frac{L_a}{R_a} \cdot \ln\left[\frac{e_g}{A' \cdot R_a}\right] = \frac{-5 \cdot 10^{-3}}{1} \ln\left(\frac{80}{85 \cdot 1}\right) = 0.303 \cdot 10^{-3} s \tag{6.52}$$

Consequently, the diode D_1 conducts for 0.303 ms. Finally, the static switch T_2 conducts for the time interval

$$T - t_{d1} = (2 - 1.0884 - 0.303) \cdot 10^{-3} = 0.6084 \cdot 10^{-3} s \tag{6.53}$$

Note: As seen above, the two-quadrant operation of the chopper resides in the variation of t_c / T as the main switches command signals last t_c and, respectively, T−t_c intervals though they conduct less time than that, allowing the diodes D_2 and D_1 to conduct. The two-quadrant chopper has the advantage of natural (continuous) transition from motor to generator action.

Example 14.5: *Two-quadrant DC chopper with load back emf*

The two-quadrant dc-to-dc chopper in figure 14.13a feeds an inductive load of 10 Ω resistance, 50mH inductance, and back emf of 100V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple and form factors;
iii. the maximum and minimum output current, hence peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the current crossover times, if applicable;
vi. the load average current, average switch current and average diode current for all devices;
vii. the input power, hence output power and rms output current;
viii. effective input impedance and electromagnetic efficiency; and
ix. sketch the circuit, load, and output voltage and current waveforms.

Subsequently determine the necessary change in

x. duty cycle δ to result in zero average output current and
xi. back emf E to result in zero average load current.

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{mH}/10\Omega = 5\text{ms}$

i. From equations (14.81) and (14.82) the load average and rms voltages are

$$v_o = \frac{t_T}{T}V_s = \frac{1.25\text{ms}}{5\text{ms}}\times 340\text{V} = ¼\times 340\text{V} = 85\text{V}$$

$$V_{rms} = \sqrt{\delta}\,V_s = \sqrt{¼}\times 340\text{V} = 170\text{V}\quad\text{rms}$$

ii. The rms ripple voltage, hence voltage ripple factor, from equations (14.83) and (14.84) are

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$

$$= \sqrt{170^2 - 85^2} = 340\text{V}\sqrt{¼\times(1-¼)} = 147.2\text{V}$$

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta}-1} = \sqrt{\frac{1}{¼}-1} = 1.732 \qquad FF = 1/\sqrt{\delta} = 1/\sqrt{¼} = 2$$

iii. From equations (14.87) and (14.88), the maximum and minimum output current, hence the peak-to-peak output ripple in the load current are given by

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} - \frac{100\text{V}}{10\Omega} = 1.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{e^{\frac{1.25\text{ms}}{5\text{ms}}}-1}{e^{\frac{5\text{ms}}{5\text{ms}}}-1} - \frac{100\text{V}}{10\Omega} = -4.38\text{A}$$

The peak-to-peak ripple current is therefore $\Delta i_o = 1.90\text{A} - -4.38\text{A} = 6.28\text{A}$ p-p.

iv. The current in the time domain is given by equations (14.85) and (14.86)

$$i_o(t) = \frac{V_s - E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}e^{\frac{-t}{\tau}}$$

$$= \frac{340\text{V}-100\text{V}}{10\Omega}\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) - 4.38\times e^{\frac{-t}{5\text{ms}}}$$

$$= 24\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) - 4.38\times e^{\frac{-t}{5\text{ms}}}$$

$$= 24 - 28.38\times e^{\frac{-t}{5\text{ms}}} \qquad \text{for}\quad 0\le t\le 1.25\text{ms}$$

$$i_o(t) = -\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}e^{\frac{-t}{\tau}}$$

$$= -\frac{100V}{10\Omega}\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 1.90\times e^{\frac{-t}{5\text{ms}}}$$

$$= -10\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 1.90\times e^{\frac{-t}{5\text{ms}}}$$

$$= -10 + 11.90\times e^{\frac{-t}{5\text{ms}}} \qquad \text{for}\quad 0\le t\le 3.75\text{ms}$$

During the switch on-time

$$i_o(t) = 24 - 28.38 \times e^{\frac{-t}{5\text{ms}}} = 0 \qquad \text{where} \quad 0 \le t = t_{xT} \le 1.25\text{ms}$$

$$t_{xT} = 5\text{ms} \times \ell n \frac{28.38}{24} = 0.838\text{ms}$$

During the switch off-time

$$i_o(t) = -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}} = 0 \qquad \text{where} \quad 0 \le t = t_{xD} \le 3.75\text{ms}$$

$$t_{xD} = 5\text{ms} \times \ell n \frac{11.90}{10} = 0.870\text{ms}$$

$$(1.250\text{ms} + 0.870\text{ms} = 2.12\text{ms with respect to switch } T_1 \text{ turn-on})$$

vi. The load average current, average switch current, and average diode current for all devices;

$$\bar{I}_o = \left(\bar{V}_o - E\right) / R = \left(\delta V_s - E\right) / R$$

$$\left(85\text{V} - 100\text{V}\right) / 10\Omega = -1.5\text{A}$$

When the output current crosses zero current, the conducting device changes. Table 14.1 gives the necessary current equations and integration bounds for the condition $\hat{I} > 0, \check{I} < 0$. Table 14.1 shows that all four semiconductors are involved in the output current cycle.

$$\bar{I}_{T1} = \frac{1}{T}\int_{t_{xT}}^{t_T} \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I} e^{\frac{-t}{\tau}} dt$$

$$= \frac{1}{5\text{ms}}\int_{0.838\text{ms}}^{1.25\text{ms}} 24 - 28.38 \times e^{\frac{-t}{5\text{ms}}} dt = 0.081\text{A}$$

$$\bar{I}_{D1} = \frac{1}{T}\int_{0}^{t_{xT}} \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I} e^{\frac{-t}{\tau}} dt$$

$$= \frac{1}{5\text{ms}}\int_{0}^{0.84\text{ms}} 24 - 28.38 \times e^{\frac{-t}{5\text{ms}}} dt = -0.357\text{A}$$

$$\bar{I}_{T2} = \frac{1}{T}\int_{t_{xD}}^{T-t_T} -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I} e^{\frac{-t}{\tau}} dt$$

$$= \frac{1}{5\text{ms}}\int_{0.870\text{ms}}^{3.75\text{ms}} -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}} dt = -1.382\text{A}$$

$$\bar{I}_{D2} = \frac{1}{T}\int_{0}^{t_{xD}} -\frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I} e^{\frac{-t}{\tau}} dt$$

$$= \frac{1}{5\text{ms}}\int_{0}^{0.870\text{ms}} -10 + 11.90 \times e^{\frac{-t}{5\text{ms}}} dt = 0.160\text{A}$$

Check $\bar{I}_o + \bar{I}_{T1} + \bar{I}_{D1} + \bar{I}_{T2} + \bar{I}_{D2} = -1.5\text{A} + 0.080\text{A} - 0.357\text{A} - 1.382\text{A} + 0.160\text{A} = 0$

vii. The input power, hence output power and rms output current;

$$P_{in} = P_{V_s} = V_s \bar{I}_i = V_s\left(\bar{I}_{T1} + \bar{I}_{D1}\right)$$

$$= 340\text{V} \times \left(0.080\text{A} - 0.357\text{A}\right) = -95.2\text{W}, \text{(charging } V_s\text{)}$$

$$P_{out} = P_E = E\bar{I}_o = 100\text{V} \times \left(-1.5\text{A}\right) = -150\text{W}, \text{ that is generating 150W}$$

From

$$V_s I_s = I_{o_{rms}}^2 R + E\bar{I}_o$$

$$I_{o_{rms}} = \sqrt{\frac{P_{out} - P_{in}}{R}} = \sqrt{\frac{150\text{W} - 92.5\text{W}}{10\Omega}} = 2.34\text{A rms}$$

viii. Since the average output current is negative, energy is being transferred from the back emf E to the dc voltage source V_s, the electromagnetic efficiency of conversion is given by

$$\eta = \frac{V_s \bar{I}_i}{E\bar{I}_o} \quad \text{for } \bar{I}_o < 0$$

$$= \frac{95.2\text{W}}{150\text{W}} = 63.5\%$$

The effective input impedance is

$$Z_{in} = \frac{V_s}{\bar{I}_i} = \frac{V_s}{\bar{I}_{T1} + \bar{I}_{D1}} = \frac{340\text{V}}{0.080\text{A} - 0.357\text{A}} = -1214\Omega$$

ix. The circuit, load, and output voltage and current waveforms are sketched in the figure 14.15.

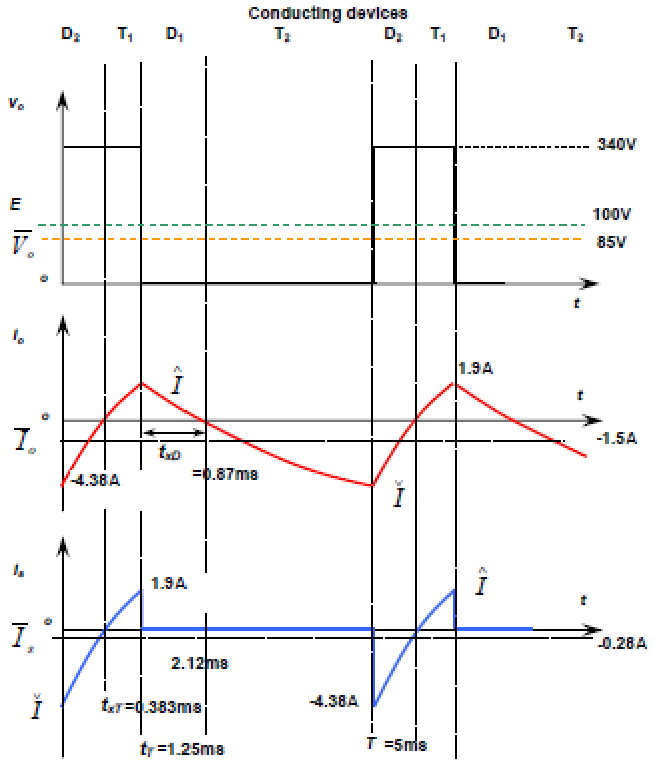


Figure 14.15. Example 14.5. *Circuit waveforms.*

x. Duty cycle δ to result in zero average output current can be determined from the expression for the average output current, equation (14.89), that is

$$\bar{I}_o = \frac{\delta V_s - E}{R} = 0$$

that is

$$\delta = \frac{E}{V_s} = \frac{100\text{V}}{340\text{V}} = 29.4\%$$

xi. As in part x, the average load current equation can be rearranged to give the back emf E that results in zero average load current

$$\bar{I}_o = \frac{\delta V_s - E}{R} = 0$$

that is

$$E = \delta V_s = \tfrac{1}{4} \times 340\text{V} = 85\text{V}$$

♣

Example 4.4

The motor of example 4.3 is controlled by a class C two-quadrant chopper operating with a source voltage of 230 V and a frequency of 400 Hz.

1. Calculate the motor speed for a motoring operation at $\delta = 0.5$ and half of rated torque.
2. What will be the motor speed when regenerating at $\delta = 0.5$ and rated torque?

Solution: At the rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. From equation (4.33),

$$\delta V = E + I_a R_a \tag{E4.5}$$

At half the rated torque, $I_a = 45$ A
At $\delta = 0.5$

$$E = \delta V - I_a R_a = 0.5 \times 230 - 45 \times 0.115 = 109.8 \text{ V}$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 109.8}{219.7} = 250 \text{ rpm}$$

2. In the regenerative braking at the rated torque, $I_a = -90$ A
From equation (E4.5),

$$E = \delta V - I_a R_a = 0.5 \times 230 + 90 \times 0.115 = 125.4$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125.4}{219.7} = 285 \text{ rpm}$$

10.21 A one-quadrant chopper, such as that shown in Fig. 10.34*a*, is used to control the speed of a dc motor.

Supply dc voltage = 120 V

$R_a = 0.15\,\Omega$

Motor back emf constant = 0.05 V/rpm

Chopper frequency = 250 Hz

At a speed of 1200 rpm, the motor current is 125 A. The motor current can be assumed to be ripple-free.

(a) Determine the duty ratio (α) of the chopper and the chopper on time t_{on}.

(b) Draw waveforms of v_0, i_0, and i_s.

(c) Determine the torque developed by the armature, power taken by the motor, and power drawn from the supply.

10.21 (a)

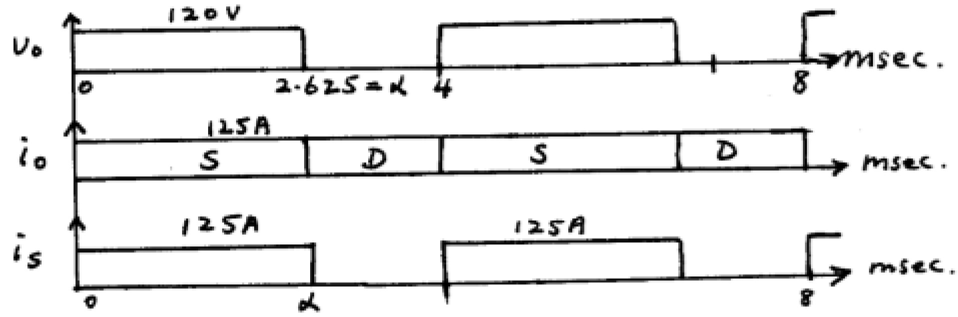
$$V_o = E_a + I_a R_a$$
$$= 0.05 \times 1200 + 125 \times 0.15$$
$$= 78.75\,V$$

$$T = \frac{10^3}{250}\,\text{msec} = 4\,\text{msec}$$

$$\alpha = \frac{78.75}{120} = 0.6563$$

$$t_{on} = \alpha T = 0.6563 \times 4 = 2.625\,\text{msec.}$$

(b)



(c)

$$E_a I_o = 60 \times 125 = 7500\,W$$

$$T = \frac{7500}{1200/60 \times 2\pi} = 59.683\ N\cdot m$$

$$P_o = V_o I_o = 78.75 \times 125 = 9844\,W$$

$$I_s = 125 \times 0.6563 = 82.03\ A$$

$$P_s = 120 \times 82.03 = 9844\ W$$

10.22 The power circuit configuration during regenerative braking of a subway car is shown in Fig. P10.22. The dc motor voltage constant is 0.3 V/rpm, and the dc bus voltage is 600 V. At a motor speed of 800 rpm and average motor current of 300 A,

(a) Draw the waveforms of v_0, i_a, and i_s for a particular value of the duty cycle α ($= t_{on}/T$).

(b) Determine the duty ratio α of the chopper for the operating condition.

(c) Determine the power fed back to the bus.

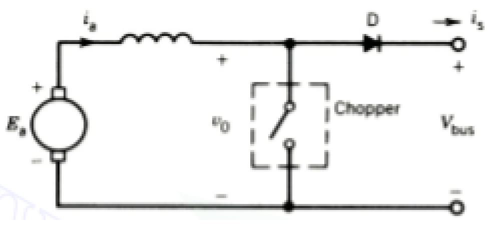
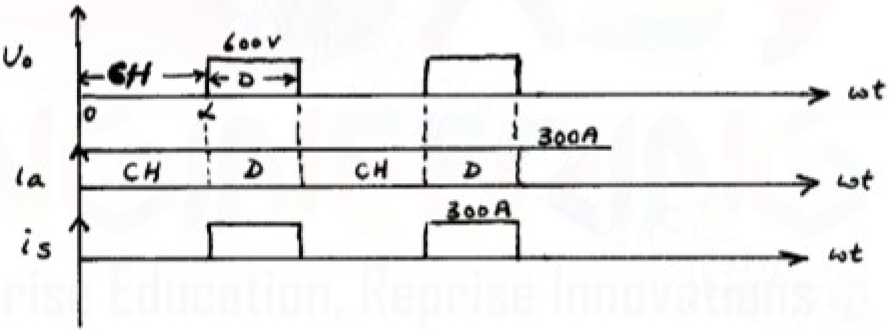


FIGURE P10.22

$\boxed{10.22}$ (a)



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(b) $V_o = (1-\alpha)\,600 = E_a = 0.3 \times 800 = 240\ \text{V}$

$\alpha = 1 - \frac{240}{600} = 0.6$

(c) $I_s = (1-\alpha)\,300 = (1-0.6)\,300 = 120\ \text{A}$

$P_s = 600 \times 120 = 72\ \text{KW}$

or $P_s = P_a = E_a I_a = 240 \times 300 = 72\ \text{kW}$

10.23 In the chopper circuit shown in Fig. P10.23, the two switches are simultaneously turned on for time t_{on} and turned off for time $t_{off} = T - t_{on}$, where T is the chopping period. Assume voltage v_0 to be ripple-free and current i_L to be continuous.

(a) Derive an expression for V_0 as a function of the duty cycle $\alpha = t_{on}/T$ and the supply voltage V. Determine V_0 for $\alpha = 0, 0.5, 1.0$.

(b) Draw waveforms of v_0, v_L, i_L, i_0, and i for $\alpha =$ [illegible].

(c) What are the advantages and disadvantages of this circuit?

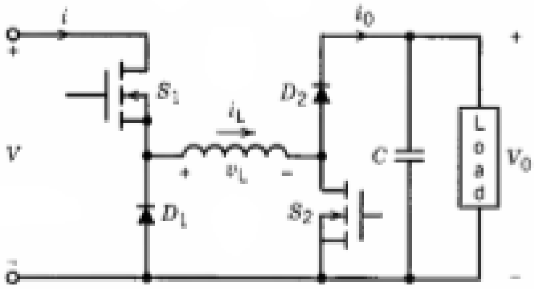


FIGURE P10.23

$\boxed{10.23}$ (a) during ton $\rightarrow$ $v_L = v = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}$

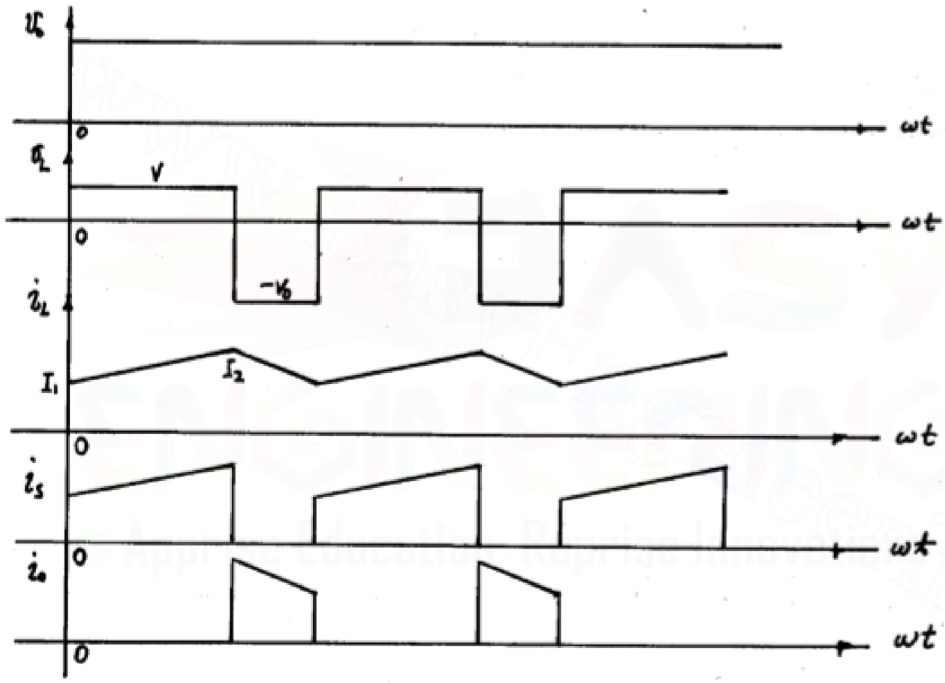
during toff $\rightarrow$ $v_L = -v_o = L\frac{I_1 - I_2}{t_{off}} = -L\frac{\Delta I}{t_{off}}$

$$\frac{V t_{on}}{L} = \frac{V_o t_{off}}{L}$$

$$V_o = \frac{t_{on}}{t_{off}} V = \frac{t_{on}}{T - t_{on}} V = \frac{\alpha}{1-\alpha} V$$

α	V_o
0	0
0.5	V
1.0	∞

(b)



(c) • This is a step-down, step-up chopper
• Polarity of V_o is the same as that of V
• Higher conduction losses $\rightarrow$ two devices conduct at a time

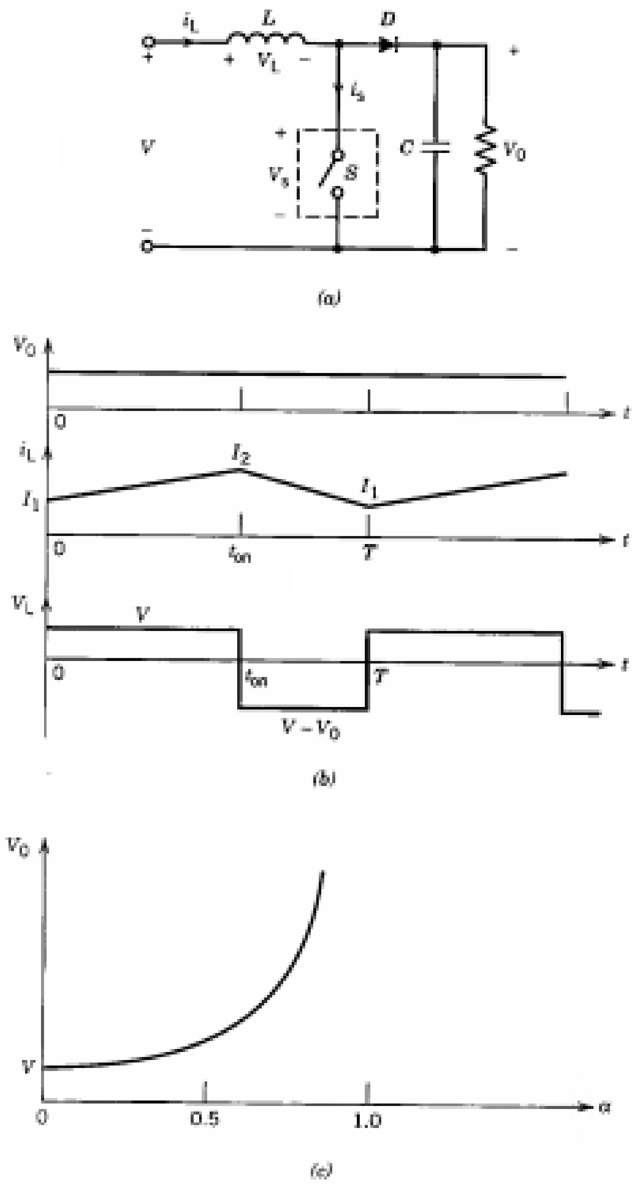


FIGURE 10.36 Boost converter. (*a*) Circuit. (*b*) Waveforms. (*c*) V_0 versus α.

10.24 The boost converter of Fig. 10.36 is used to charge a battery bank from a dc voltage source with $V = 160$ V. Assume ideal switch and no-loss operation, and neglect the ripple at the output voltage. The battery bank consists of 100 identical batteries. Each battery has an internal resistance $R_b = 0.1\,\Omega$. At the beginning of the charging process, each battery voltage is $V_{b1} = 2.5$ V. When each battery is charged up to $V_{b2} = 3.2$ V, the charging process is completed. The average charging current is kept constant at 0.5 A.

(a) Calculate the variation of duty ratio α for the charging process.

(b) Draw qualitatively the waveforms of v_L, i_L, v_s, i_s, v_D, i_D for $V_{b1} = 2.5$ V.

$\boxed{10.24}$ (a) $I = 0.5A \quad R_1 = 100 \times R_D = 100 \times 0.1 = 10\Omega \,, \quad V_R = 0.5 \times 10 = 5V$

$V_{o1} = 100 \times V_{D1} + V_R = 100 \times 2.5 + 5 = 255V$

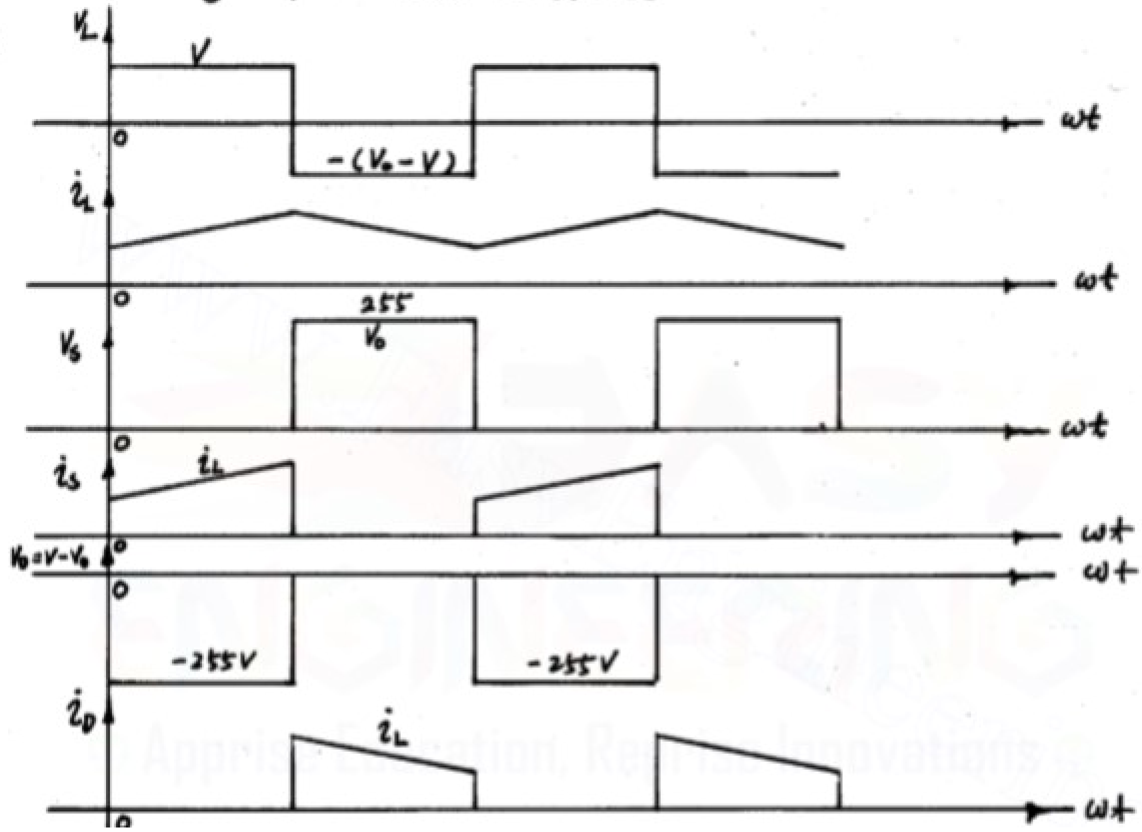
$V_{o2} = 100 \times V_{D2} + V_R = 100 \times 3.2 + 5 = 325V$

$V_o = \frac{1}{1-\alpha} V \longrightarrow \alpha = \frac{V_o - V}{V_o}$

$\alpha_1 = \frac{255 - 150}{255} = 0.4118 \qquad \alpha_2 = \frac{325 - 150}{325} = 0.5385$

α changes from 0.4118 to 0.5385

(b)



10.25 For the battery charging system of Problem 10.24:

(a) If the supply voltage available is $V = 150$ V (dc), which dc to dc converter would be used? Draw the circuit.

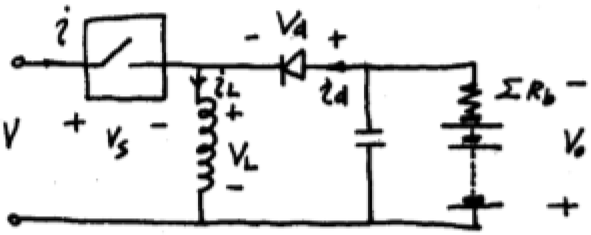
(b) Calculate the variation of the duty ratio α for the charging process.

(c) Draw qualitatively the waveforms of inductor voltage (v_L), inductor current (i_L), voltage across the chopper switch (v_s), current through the chopper switch (i_s), voltage across the diode (v_d), and current through the diode (i_d), for $v_{bl} = 1.2$ V.

$\boxed{10.25}$ (a) $V_{o1} = 100V_{b1} + IR = 100 \times 1.2 + 100 \times a \times 0.5 = 125V$

$V_{o2} = 100 \times 3.2 + 5 = 325\ V$

$V = 150V$ Need a Buck-Boost Converter

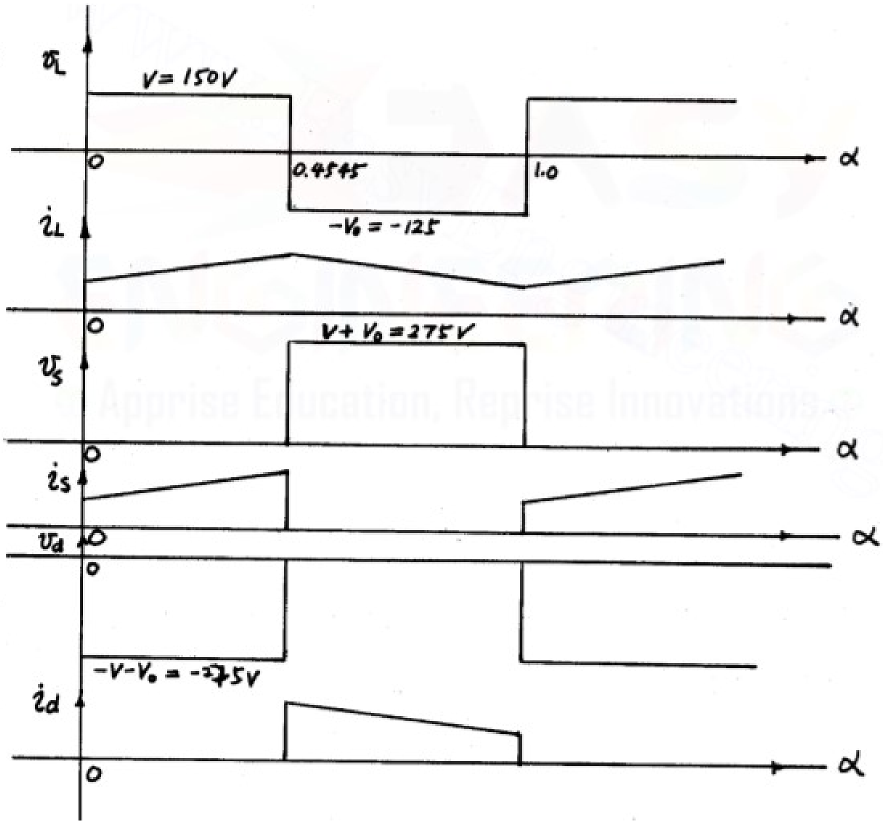


(b) $V_o = \dfrac{\alpha}{1-\alpha} V$

$\alpha = \dfrac{V_o}{V + V_o}$

$\alpha_1 = \dfrac{125}{150 + 125} = 0.4545$ $\quad \alpha_2 = \dfrac{325}{152 + 325} = 0.6842$

(c)



10.26 Consider the two-quadrant chopper systems shown in Fig. P10.26. The two choppers S_1 and S_2 are turned on for time t_{on} and turned off for time $T - t_{on}$, where T is the chopping period.

(a) Draw the waveform of the output voltage v_0. Assume continuous output current i_0.

(b) Derive an expression for the average output voltage V_o in terms of the supply voltage V and the duty ratio α ($= t_{on}/T$).

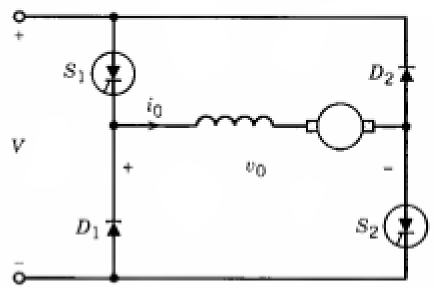
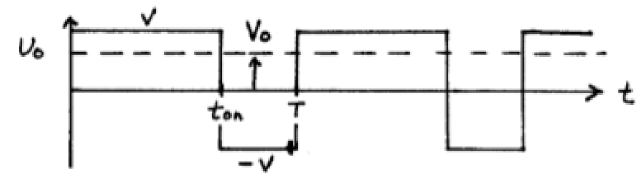


FIGURE P10.26

$\boxed{10.26}$ (a)



(b) $$V_o = \frac{1}{T}\int_0^T v_o\,dt = \frac{1}{T}\left[\int_0^{t_{on}} V\,dt + \int_{t_{on}}^{T} -V\,dt\right]$$

$$= V(2\alpha - 1)$$

8. A separately excited dc motor having a rating of 60 h.p. and running at 1200 rpm is supplied by a dc chopper whose source is a battery of 500 V. The field is also supplied by a chopper whose source is another battery of 300 V. The data pertaining to this chopper-based drive are as follows: $R_a = 0.18\ \Omega$, $K_b = 70$ V/(Wb rad/s), $\phi_f = 0.16 I_f$, $R_f = 120\ \Omega$, and $(\tau_{ON}/\tau)_f$ for the field chopper is 0.85. Assume that the load has sufficient inductance to make the load current continuous. If (τ_{ON}/τ) a for the armature is 0.65, compute the (a) mean armature current, (b) torque developed by the motor, (c) equivalent resistance for the armature circuit, and (d) total input power.

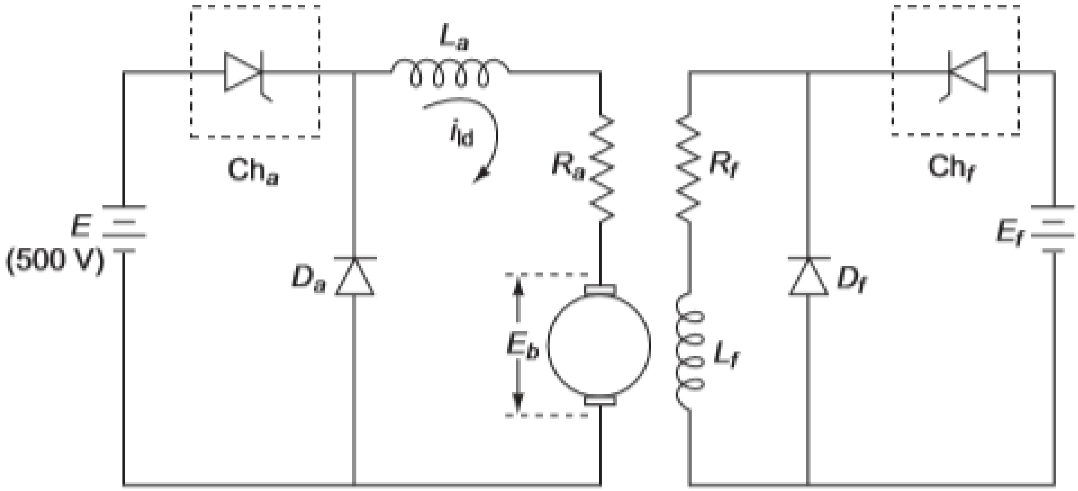


Fig. 7.33

Solution

(a) Let the source currents at the armature and field sides be denoted, respectively, as I_{sa} and I_{sf}. The circuit is shown in Fig. 7.33. Equation (3.23) gives the torque developed as

$$\mathcal{T}_d = K_t \phi_f I_{\text{ld}}(\omega) = K_b K_1 I_f I_a$$

Here,

$$K_b K_1 = 70 \times 0.016 = 1.12$$

$$I_f = E_f \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f \frac{1}{R_f} = \frac{300 \times 0.85}{120} = 2.125 \text{ A}$$

Hence the average torque is

$$\mathcal{T}_d = 1.12 \times 2.125 I_a = 2.38 I_a$$

$$\omega = \frac{2\pi \times 1200}{60} = 125.7$$

$$E_b = K_b K_1 I_f \omega = 2.38 \times 125.7 = 299 \text{ V}$$

$$V_a = E\left(\frac{\tau_{ON}}{\tau}\right)_a = 500 \times 0.65 = 325 \text{ V}$$

The mean armature current is

$$I_a = \frac{V_a - E_b}{R_a} = \frac{325 - 299}{0.18} = 144.4 \text{ A}$$

(b) Torque developed $\mathcal{T}_d = 2.38 \times 144.4 = 343.8$ N m.

(c) Input (or source) current is

$$I_{sa} = I_a\left(\frac{\tau_{ON}}{\tau}\right)_a = 144.4 \times 0.65 = 93.9 \text{ A}$$

Also,

$$I_{sf} = I_f \times \left(\frac{\tau_{ON}}{\tau}\right)_f = 2.125 \times 0.85 = 1.81 \text{ A}$$

$$\begin{aligned}\text{Armature source equivalent resistance} &= \frac{\text{armature source voltage}}{\text{armature source current}}\\ &= \frac{500}{93.9}\\ &= 5.32\ \Omega\end{aligned}$$

(d) Total input power = power input to armature + power input to field $= E_a I_{sa} + E_f I_{sf}$. Hence, it is given as $P_i = 500 \times 93.9 + 300 \times 1.81 = 46{,}950 + 543 = 47{,}493 \text{ W} \approx 47.5$ kW.

9. A separately excited dc motor has a rating of 50 h.p. and when supplied by a battery of 480 V through a chopper, it has a mean armature current of 120 A. The field is also supplied by a chopper whose source is a battery of 250 V. Other data for this chopper-based drive are $R_a = 0.2\ \Omega$, $R_f = 125\ \Omega$, $K_b = 72$ V/(Wb rad/s), $\phi_f = 0.015 I_f$, $(\tau_{ON}/\tau)_a = 0.7$, and $(\tau_{ON}/\tau)_f = 0.9$. The armature circuit has sufficient inductance to make the current continuous. Compute the (a) speed of the motor, (b) torque developed by the motor, (c) equivalent resistance, and (d) total input power.

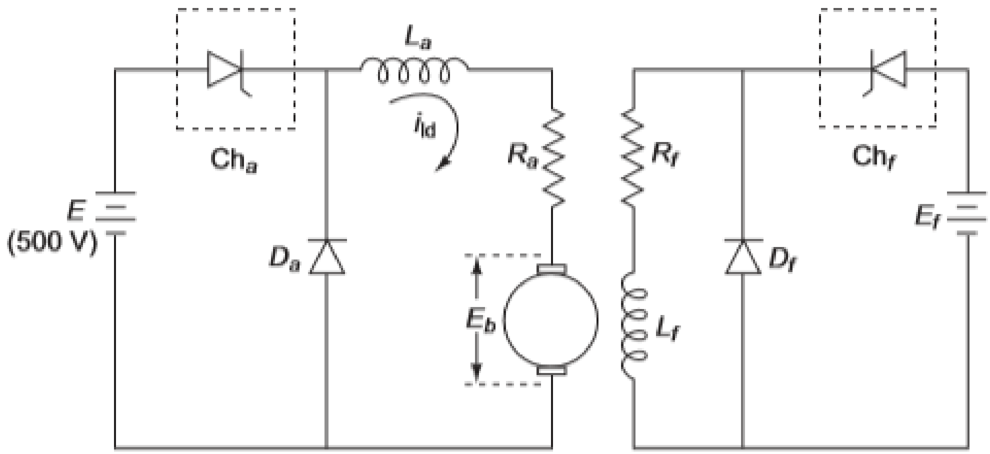


Fig. 7.33

Solution

(a) The circuit is the same as that given in Fig. 7.33.

$$V_a = 480\left(\frac{\tau_{ON}}{\tau}\right)_a = 480 \times 0.7 = 336 \text{ V}$$

$$E_b = V_a - I_a R_a = 336 - 120 \times 0.2 = 312 \text{ V}$$

$$\omega = \frac{E_b}{K_b \phi_f} = \frac{E_b}{K_b \times 0.015 I_f}$$

Here,

$$I_{sf} = 250\left(\frac{\tau_{ON}}{\tau}\right)_f \frac{1}{R_f} = \frac{250 \times 0.9}{125} = 1.8 \text{ A}$$

Hence,

$$\omega = \frac{E_b}{K_b \times 0.015 I_f} = \frac{312}{72 \times 0.015 \times 1.8} = 160 \text{ rad/s}$$

Also,

$$\text{speed} = \frac{60\omega}{2\pi} = \frac{60 \times 160}{2\pi} = 1528 \text{ rpm}$$

(b) $$\text{Torque developed} = K_b \times 0.015 I_f I_a$$
$$= 72 \times 0.015 \times 1.8 \times 120$$
$$= 233.3 \text{ N m}$$

The source current on the armature side is

$$I_{sa} = I_a \left(\frac{\tau_{\text{ON}}}{\tau}\right)_a = 120 \times 0.7 = 84 \text{ A}$$

(c) $$\text{Armature source equivalent resistance} = \frac{\text{armature source voltage}}{\text{armature source current}}$$
$$= \frac{480}{84} = 5.7\ \Omega$$

(d) Total input power

$$P_i = \text{power input to armature} + \text{power input to field}$$
$$= E_a I_{sa} + E_f I_{sf}$$

where

$$I_{sf} = I_f \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f = 1.8 \times 0.9 = 1.62 \text{ A}$$

Hence,

$$P_i = 480 \times 84 + 250 \times 1.62 = 40{,}320 + 405 = 40{,}725 \text{ W} \approx 40.7 \text{ kW}$$

10. A 220-V, 80-A, separately excited dc motor operating at 800 rpm has an armature resistance of 0.18 Ω. The motor speed is controlled by a chopper operating at 1000 Hz. If the motor is regenerating, (a) determine the motor speed at full load current with a duty ratio of 0.7, this being the minimum permissible ratio (b) Repeat the calculation with a duty ratio of 0.1.

Solution

(a) When the machine is working as a motor, E_b is obtained from the equation

$$E_b = E - I_a R_a = 220 - 80 \times 0.18 = 220 - 14.4 = 205.6 \text{ V}$$

From the equation

$$E_b = kN$$

$$k = \frac{205.6}{800} = 0.257$$

When it is regenerating, the step-up configuration of Fig. 3.7(c) holds good. Thus,

$$E_b = E(1 - \delta) + I_a R_a = 220(1 - 0.7) + 80 \times 0.18 = 66 + 14.4 = 80.4 \text{ V}$$

$$N = \frac{E_b}{k} = \frac{80.4}{0.257} = 313 \text{ rpm}$$

(b) The speed for $\delta = 0.1$ is obtained as follows:

$$E_b = 220(1 - 0.10) + 80 \times 0.18 = 198 + 14.4 = 212.4 \text{ V}$$

Therefore the speed is

$$N = \frac{212.4}{0.257} = 827 \text{ rpm}$$

11. A 250-V, 105-A, separately excited dc motor operating at 600 rpm has an armature resistance of 0.18 Ω. Its speed is controlled by a two-quadrant chopper with a chopping frequency of 550 Hz. Compute (a) the speed for motor operation with a duty ratio of 0.5 at 7/8 times the rated torque and (b) the motor speed if it regenerates at $\delta = 0.7$ with rated current.

Solution

(a) The initial back emf is to be determined from the equation

$$E_b = E - I_a R_a = 250 - 105 \times 0.18 = 231.1 \text{ V}$$

Hence the back emf constant $k = 231.1/600 = 0.385$. A fraction 7/8 of the rated current $I'_a = 7/8 \times 105 = 91.875$ A. The new E_b is obtained as

$$E'_b = E\delta - I'_a R_a$$

where $\delta = 0.5$ and $I'_a = 91.875$. Its numerical value is

$$E'_b = 250 \times 0.5 - 91.875 \times 0.18 = 108.46\text{V}$$

The new speed is

$$N = \frac{E'_b}{k} = \frac{108.46}{0.385} = 282 \text{ rpm}$$

(b) When it is regenerating, Eqn (7.54) is to be used. Thus,

$$I_a = \frac{E_b - E(1-\delta)}{R_a}$$

or

$$E_b = E(1-\delta) + I_a R_a$$

Substituting values gives

$$E_b = 250(1 - 0.7) + 105 \times 0.18 = 93.9 \text{ V}$$

From this, the speed $N = 93.9/0.385 = 244$ rpm

12. A 300-V, 100-A, separately excited dc motor operating at 600 rpm has an armature resistance and inductance of 0.25 Ω and 16 mH, respectively. It is controlled by a four-quadrant chopper with a chopper frequency of 1 kHz. (a) If the motor is to operate in the second quadrant at 4/5 times the rated current, at 450 rpm, calculate the duty ratio. (b) Compute the duty ratio if the motor is working in the third quadrant at 500 rpm and at 60% of the rated torque.

Solution

(a) $E_b - I_a R_a = 300 - 100 \times 0.25 = 275$ V. Back emf constant $k = E_b/N = 275/600 = 0.458$. Operation in the second quadrant implies that the motor works as a generator. Hence the motor terminal voltage V_a is written as

$$V_a = E(1-\delta)$$

New current

$$I_a = \frac{4}{5} \times 100 = 80 \text{ A}$$

Hence,

$$E_b = V_a + I_a R_a$$
$$kN = E(1-\delta) + I_a R_a$$

Substitution of values gives

$$0.458 \times 450 = 300(1-\delta) + 80 \times 0.25$$

This yields $\delta = 0.38$.

(b) In the third quadrant, the machine works in the motoring mode but with reverse voltage and reverse current. The voltage equation relevant in this case is

$$E_b = V_a - I_a R_a$$

where

$$E_b = kN = 0.458 \times 500$$

$$V_a = E\delta = 300\delta$$

and

$$I_a = 0.6 \times 100 = 60 \text{ A}$$

By substituting numerical values, the equation becomes

$$0.458 \times 500 = 300 \times \delta - 60 \times 0.25$$

This gives $\delta = 0.813$.

12. A dc series motor is supplied by a battery of 420 V with a dc chopper interposed between the battery and the motor. It has a mean armature current of 120 A. Other data for this chopper-based drive are $R_a = 0.05\ \Omega$, $R_f = 0.06\ \Omega$, $K_b = 0.72$ V/(Wb rad/s), and $\phi_f = 0.016\ I_a$. The duty ratio τ_{ON}/τ is 0.65. Compute the (a) speed of the motor, (b) torque developed by the motor, (c) equivalent input resistance, and (d) total input power.

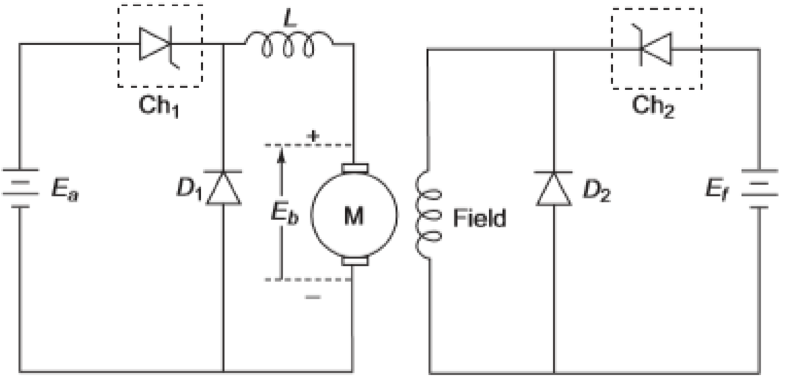


Fig. 7.25 Circuit diagram of a dc drive with the armature and field fed by separate choppers

Solution

The set-up is as shown in Fig. 7.25:

$$V_a = 420\frac{\tau_{ON}}{\tau} = 420 \times 0.65 = 273\text{ V}$$

$$\begin{aligned} E_b &= V_a - I_a(R_a + R_f) \\ &= 273 - 120(0.05 + 0.06) \\ &= 259.8\text{ V} \end{aligned}$$

$$\begin{aligned} \omega &= \frac{E_b}{K_b \times 0.016 I_a} \\ &= \frac{259.8}{0.72 \times 0.016 \times 120} \\ &= 188\text{ rad/s} \end{aligned}$$

$$\begin{aligned} \text{Speed } N &= \frac{188 \times 60}{2\pi} \\ &= 1795\text{ rpm} \end{aligned}$$

$$\begin{aligned} \text{Torque developed by the motor} &= K_b \times 0.016(I_a)^2 \\ &= 0.72 \times 0.016 \times (120)^2 \\ &= 165.9\text{ N m} \end{aligned}$$

The source current is

$$I_s = I_a\frac{\tau_{ON}}{\tau} = 120 \times 0.65 = 78\text{ A}$$

$$\text{Input power} = EI_s = 420 \times 78 = 32{,}760\text{ W} = 32.76\text{ kW}$$

First Quadrant Chopper or Type A Chopper or Class A Chopper

In this topic, you study First Quadrant Chopper or Type A Chopper or Class A Chopper v-i plane, working principle, quadrant operation, Applications, and Circuit diagrams.

Type A chopper is basically a Step-Down Chopper.

Circuit Diagram

The Type A chopper circuit diagram as shown in Figure 1. Here the motor load is assumed, R_a and L_a armature resistance and inductance of the motor respectively. E_b is the back emf of the motor.

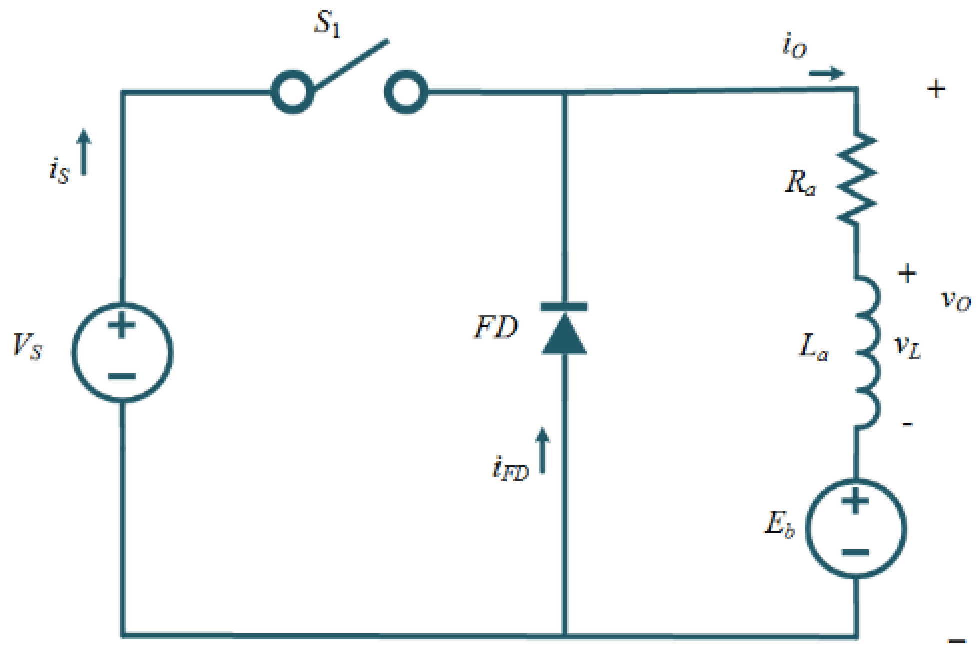


Figure 1 Circuit diagram of Type A chopper

$v_O - i_O$ plane

The Type A chopper operates in the first quadrant of $v_O - i_O$ plane as shown in Figure 2. Here v_O is the output voltage, V_O is the average output voltage, i_O is the output current and I_O is the average output current of Type A chopper circuit.

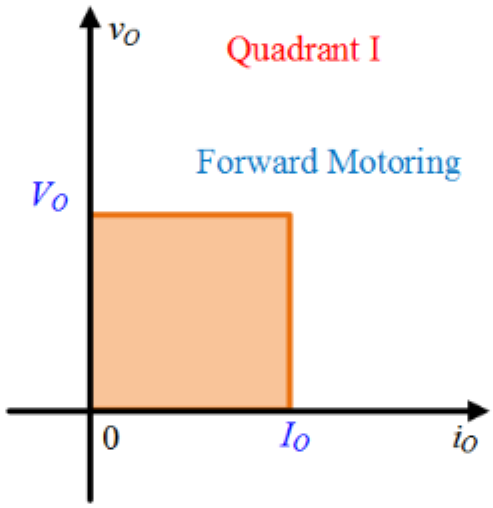


Figure 2 Type A chopper v_O - i_O plane

Quadrant I operation when Switch S_1 turned on

The Type A chopper equivalent circuit diagram for Quadrant I is shown in Figure 3. Here switch S_1 operated, Switch S_1 conducts, output voltage v_O and the output current i_O both are positives, power flows from source to load and inductor stores energy, the motor rotates in the forward direction hence called forward motoring.

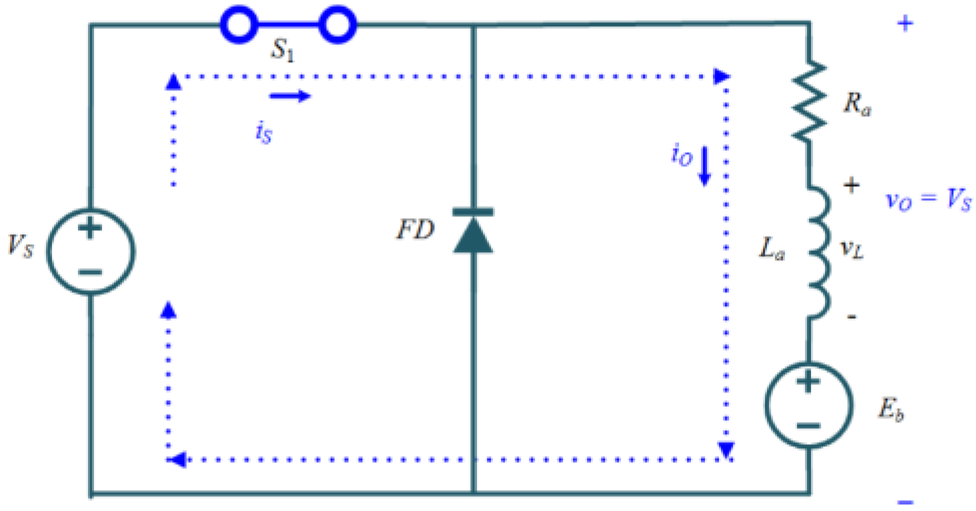


Figure 3 Equivalent Circuit diagram I of Type A chopper

Quadrant I operation when Switch S_1 turned off

The Type A chopper equivalent circuit diagram for Quadrant I is shown in Figure 4. Switch S_1 turned off diode *FD* conducts, output current i_O is positive and the output voltage v_O becomes zero, inductor release energy and freewheeling action using diode *FD* takes place, the motor rotates in the forward direction hence called Forward motoring.

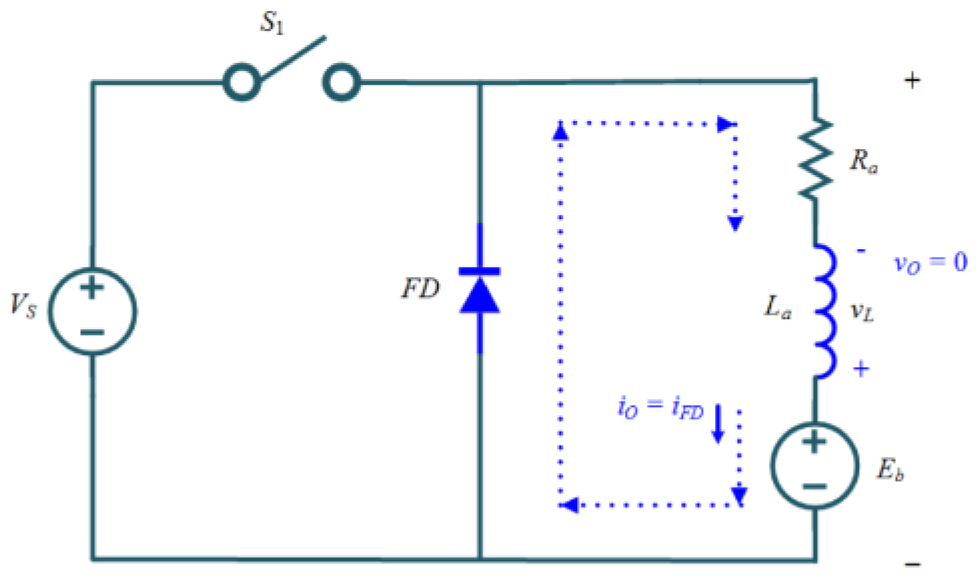


Figure 4 Equivalent Circuit diagram II of Type A chopper

Application

This chopper is suitable for motoring application only.

Principle of Step Down Chopper (Buck Converter)

In this topic, you study the Principle of Step Down Chopper and its associated circuit diagram, Waveforms, Modes of operation, & theory.

The buck converter produces a lower average output voltage than the dc source input voltage.

Circuit diagram

The working of a buck regulator is explained using the circuit diagram as shown in Figure 1. The switch S_1 shown in the circuit diagram can be a conventional thyristor i.e., SCR, a GTO thyristor, a power transistor, or a MOSFET.

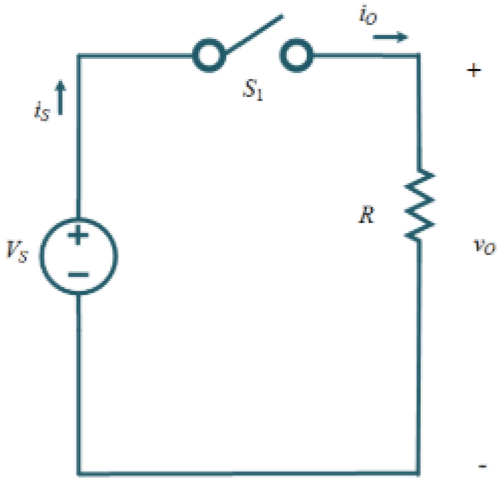


Figure 1 Circuit diagram of step-down chopper (buck converter) with resistive load.

Waveforms

The typical waveforms in the converter are shown in Figure 2.

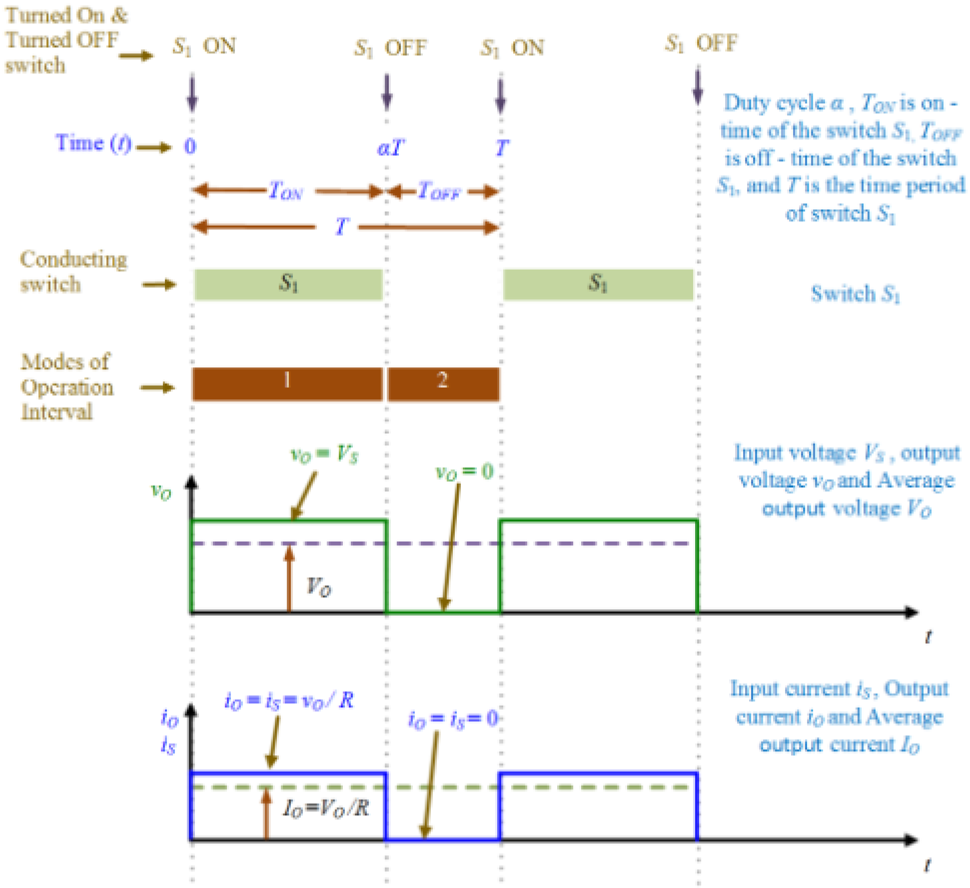


Figure 2 Waveforms of step-down chopper (buck converter) with resistive load.

Modes of Operation Interval

The two modes in steady state operations are

Mode of Operation Interval 1: –

The time interval is $0 \leq t \leq T_{ON}$. The circuit diagram for Mode of Operation Interval 1 is shown in Figure 3 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned on and the resistive R load directly connects to input dc source voltage V_S and hence $v_O = V_S$, the source (or input) current flows through the Resistive load so $i_S = i_O = v_O/R$.

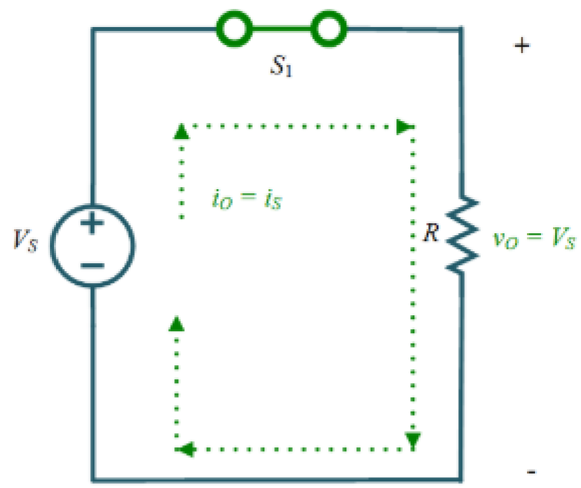


Figure 3 Circuit diagram of step-down chopper (buck converter) with resistive load when switch S_1 ON.

Mode of Operation Interval 2: –

The time interval is $T_{ON} \leq t \leq T_{OFF}$. The circuit diagram for Mode of Operation Interval 2 is shown in Figure 4 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned off and the resistive R load disconnects from input dc source voltage V_S and hence v_O = 0, also the source (or input) current flows through the Resistive load will be i_S = i_O = 0.

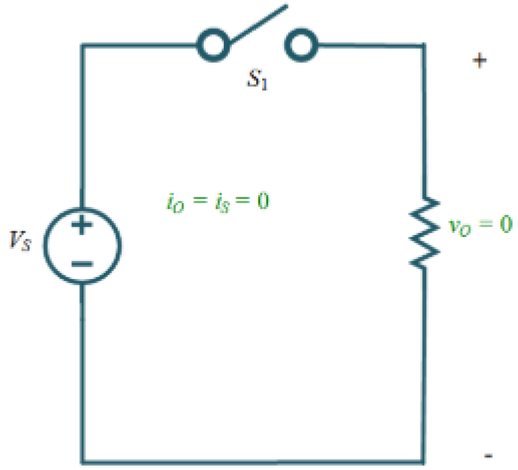


Figure 4 Circuit diagram of step-down chopper (buck converter) with resistive load when switch S_1 OFF.

Average output voltage V_O

Using the output voltage waveform as shown in Figure 2, the average value of the output voltage write as

$$V_O = \frac{T_{ON}}{T_{ON} + T_{OFF}} V_S \ldots (1)$$

Also,

$$T = T_{ON} + T_{OFF} \ldots (2)$$

Using Equation 1 and Equation 2 gives

$$V_O = \frac{T_{ON}}{T} V_S = \alpha V_S$$

So,

$$V_O = \alpha V_S \ldots (3)$$

where,

$\alpha = T_{ON}/T,\ \alpha$ is the duty cycle of the chopper and the value of α lies between $0 \leq \alpha \leq 1$. T_{ON} is the on – time of the switch S_1 or chopper , T_{OFF} is the off – time of the switch S_1 or chopper, T is the chopping period, and the chopping frequency $f = 1/T$.

RMS output voltage V_{orms}

Using the output voltage waveform as shown in Figure 2, the RMS value of the output voltage write as

$$V_{orms} = \left[\frac{T_{ON}}{T_{ON} + T_{OFF}} V_S^2 \right]^{1/2}$$

or

$$V_{orms} = \sqrt{\alpha}\, V_S$$

Buck Boost Regulator Peak to Peak Ripple Current of Inductor Expression Derivation

In this topic, you study How to derive an expression for Peak to Peak ripple current for Buck-Boost Regulator.

The buck-boost regulator can produce an average output voltage less than or greater than the dc source input voltage. Let us assume large filter capacitance C connected across the load so that output voltage remains almost constant. The Resistive load is considered.

Circuit diagram

The working of a buck-boost regulator is explained using the circuit diagram as shown in Figure 1. The switch S_1 shown in the circuit diagram can be a conventional thyristor i.e., SCR, a GTO thyristor, a power transistor, or a MOSFET.

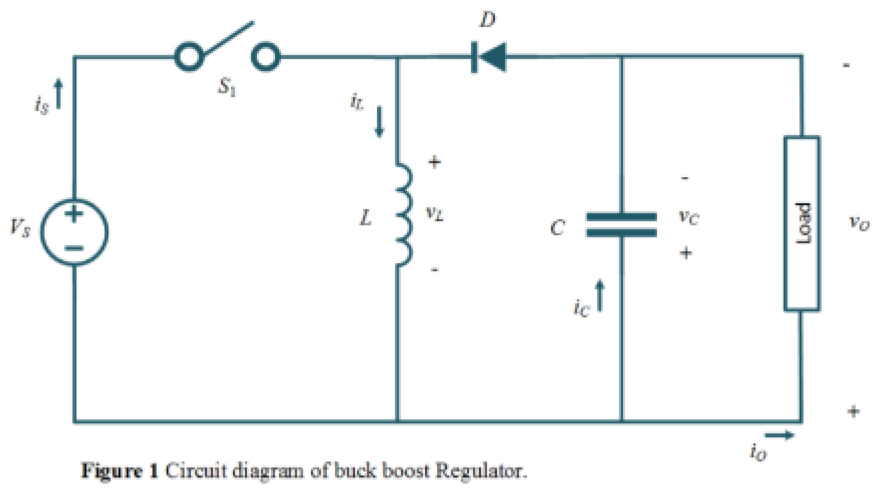


Figure 1 Circuit diagram of buck boost Regulator.

Waveforms

The typical waveforms in the converter are shown in Figure 2.

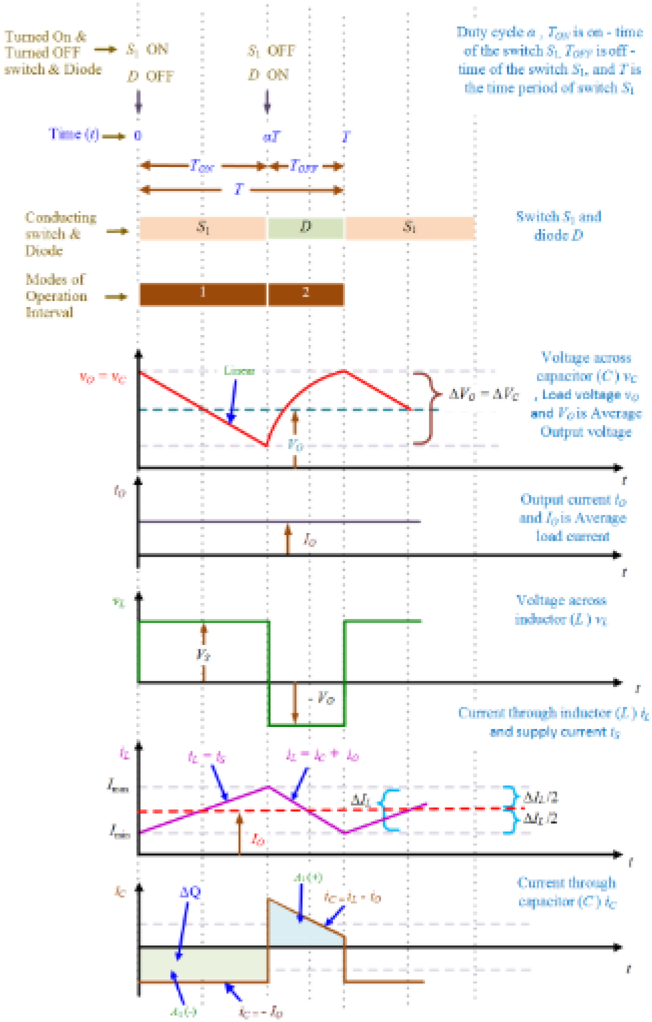


Figure 2 Waveforms of buck boost Regulator.

Mode of Operation Interval 1: –

The time interval is $0 \leq t \leq T_{ON}$. The switch S_1 is turned on. The circuit diagram for Mode of Operation Interval 1 is shown in Figure 3 and the corresponding waveforms are shown in Figure 2.

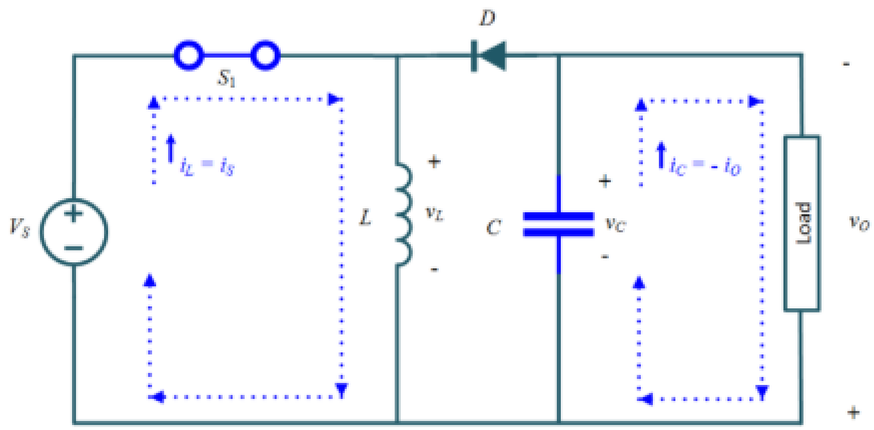


Figure 3 Circuit diagram of buck boost Regulator when switch S_1 ON.

From the waveform of voltage across the inductor, as shown in Figure 2, the equation for the inductor voltage write as

$$v_L = V_S \qquad \ldots.(1)$$

The general equation relates voltage across the inductor and current passes through it as

$$v_L = L\frac{di_L}{dt} \qquad \ldots.(2)$$

Put Equation 2 in Equation 1 gives

$$L\frac{di_L}{dt} = V_S \ldots(3)$$

The waveform for current passes through inductor L as shown in Figure

2, Integrate Equation 3 using the maximum and minimum value of inductor current gives

$$\int_{I_{\min}}^{I_{\max}} di_L = \frac{V_s}{L}\int_0^{T_{ON}} dt$$

or

$$I_{\max} - I_{\min} = \frac{V_S}{L}.T_{ON} \qquad \ldots.(4)$$

Here $\Delta I_L = I_{\max} - I_{\min}$ is the peak to peak ripple current of inductor L and hence Equation 4 can be write as

$$\Delta I_L = \frac{V_S}{L}.T_{ON} \qquad \ldots.(5)$$

Also

$$T_{ON} = \alpha T = \frac{\alpha}{f} \qquad \ldots.(6)$$

Using Equation 5 and Equation 6 gives

$$\Delta I_L = \frac{V_S}{L}.\frac{\alpha}{f} \qquad \ldots.(7)$$

Equation 7 describes the peak to peak ripple current of inductor L in buck-boost converter.

Buck Boost Regulator Circuit diagram, Waveform, Modes of Operation & Theory

In this topic, you study the Buck-Boost Regulator Circuit diagram, Waveforms, Modes of operation & theory.

The buck-boost regulator can produce an average output voltage less than or greater than the dc source input voltage. Let us assume large filter capacitance C connected across the load so that output voltage remains almost constant. The Resistive load is considered.

Circuit diagram

The working of a buck-boost regulator is explained using the circuit diagram as shown in Figure 1. The regulation is normally achieved by PWM (Pulse Width Modulation) at a fixed frequency and using the switch S_1 shown in the circuit diagram can be a conventional thyristor i.e., SCR, a GTO thyristor, a power transistor, or a MOSFET.

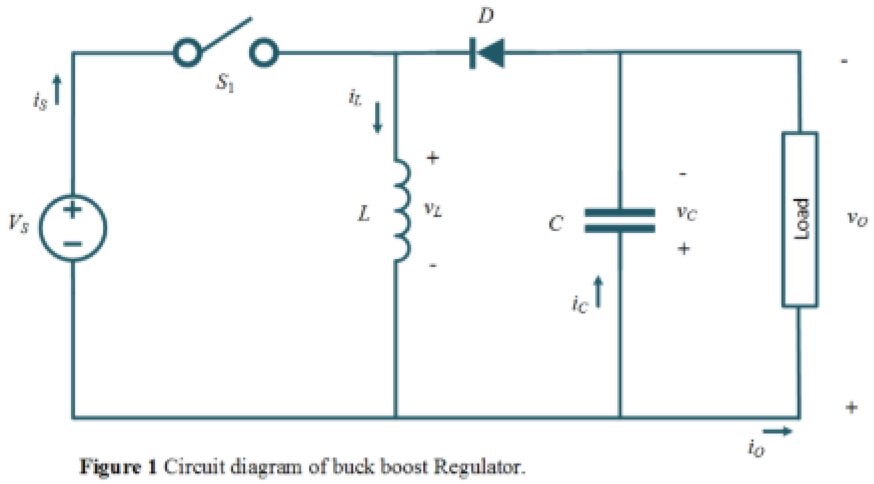


Figure 1 Circuit diagram of buck boost Regulator.

Waveforms

The typical waveforms in the converter are shown in Figure 2.

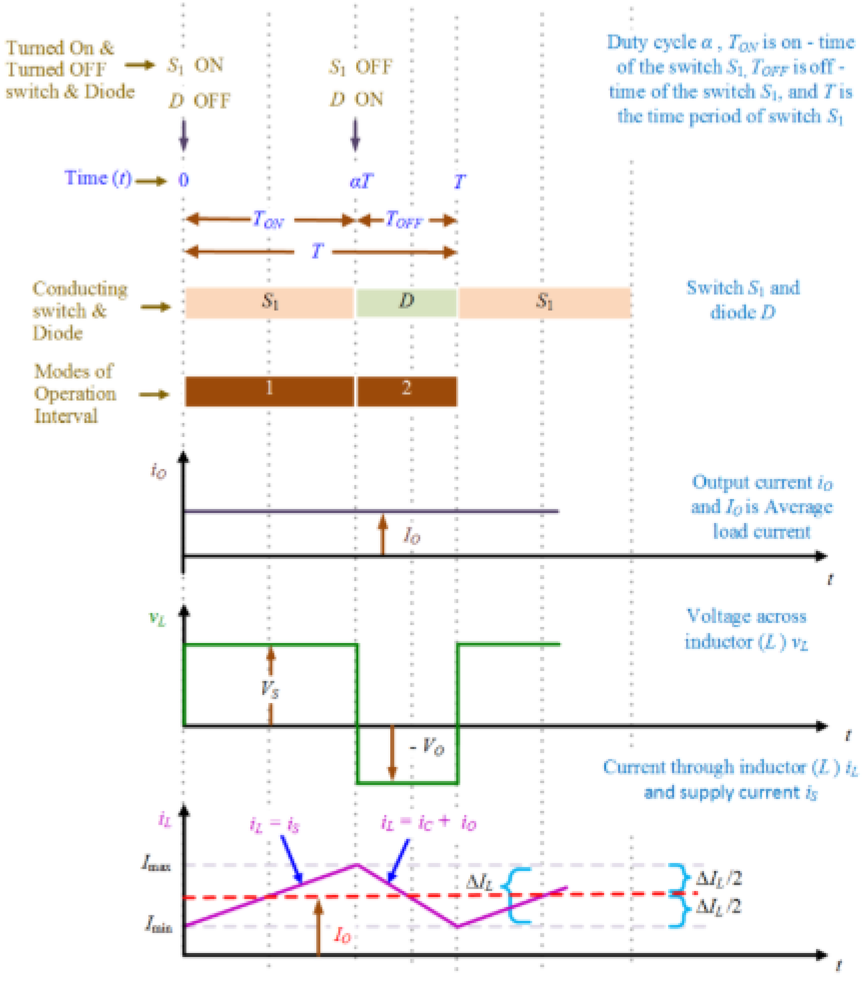


Figure 2 Waveforms of buck boost Regulator.

Modes of Operation Interval

The two modes in steady state operations are

Mode of Operation Interval 1: –

The time interval is $0 \le t \le T_{ON}$. The circuit diagram for Mode of Operation Interval 1 is shown in Figure 3 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned on, and the dc source directly connects to inductor L, the source (or input) current increases linearly and flows through the inductor L. The capacitor maintains the voltage v_O across the load and hence supplies current i_O to the load. The diode D gets reverse biased and behave as an open circuit. The inductor stores energy during this interval.

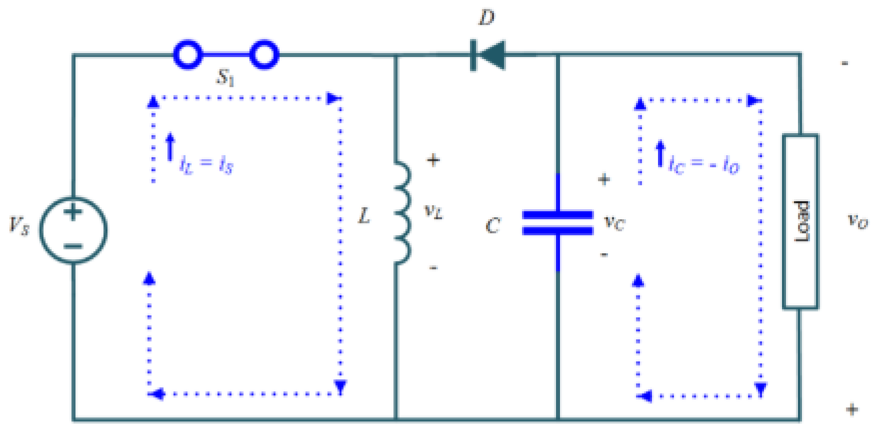


Figure 3 Circuit diagram of buck boost Regulator when switch S_1 ON.

Mode of Operation Interval 2: –

The time interval is $T_{ON} \leq t \leq T_{OFF}$. The circuit diagram for Mode of Operation Interval 2 is shown in Figure 4 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned OFF, the inductor current decreases linearly and flows through the Forward biased diode *D*, filter capacitor *C*, Resistive load. The inductor release energy during this interval.

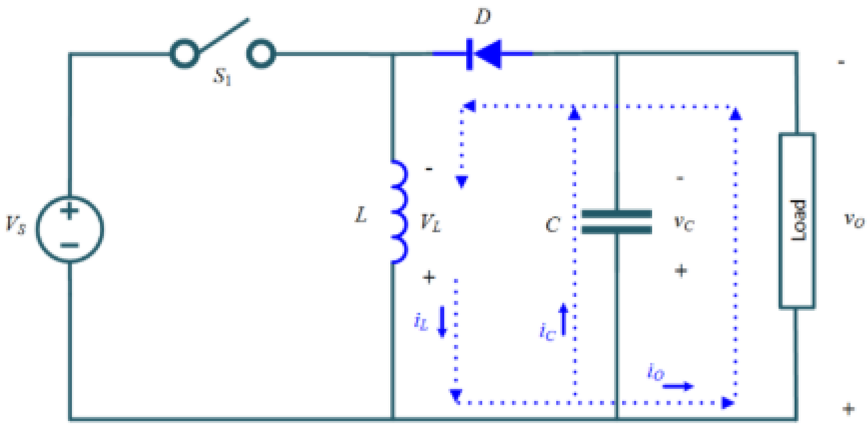


Figure 4 Circuit diagram of buck boost Regulator when switch S_1 OFF.

Buck Boost Regulator Peak to Peak Ripple Current of Inductor Expression Derivation

In this topic, you study How to derive an expression for Peak to Peak ripple current for Buck-Boost Regulator.

The buck-boost regulator can produce an average output voltage less than or greater than the dc source input voltage. Let us assume large filter capacitance *C* connected across the load so that output voltage remains almost constant. The Resistive load is considered.

Circuit diagram

The working of a buck-boost regulator is explained using the circuit diagram as shown in Figure 1. The switch S_1 shown in the circuit diagram can be a conventional thyristor i.e., SCR, a GTO thyristor, a power transistor, or a MOSFET.

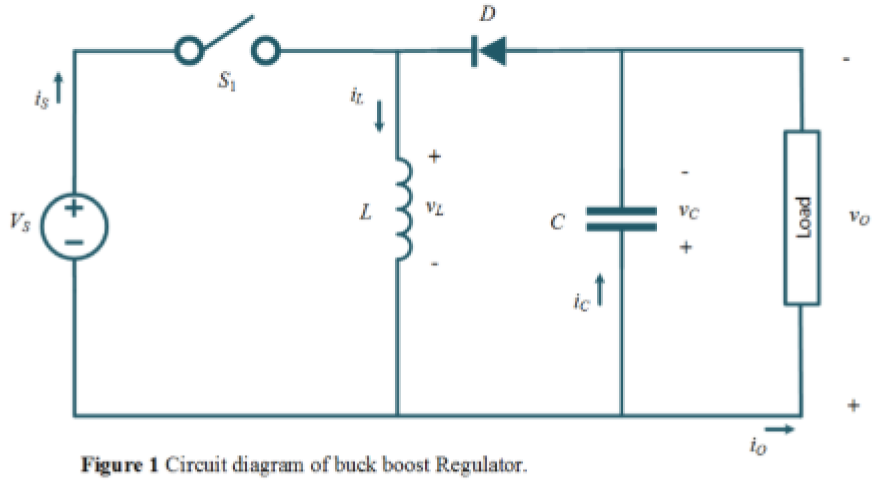


Figure 1 Circuit diagram of buck boost Regulator.

Waveforms

The typical waveforms in the converter are shown in Figure 2.

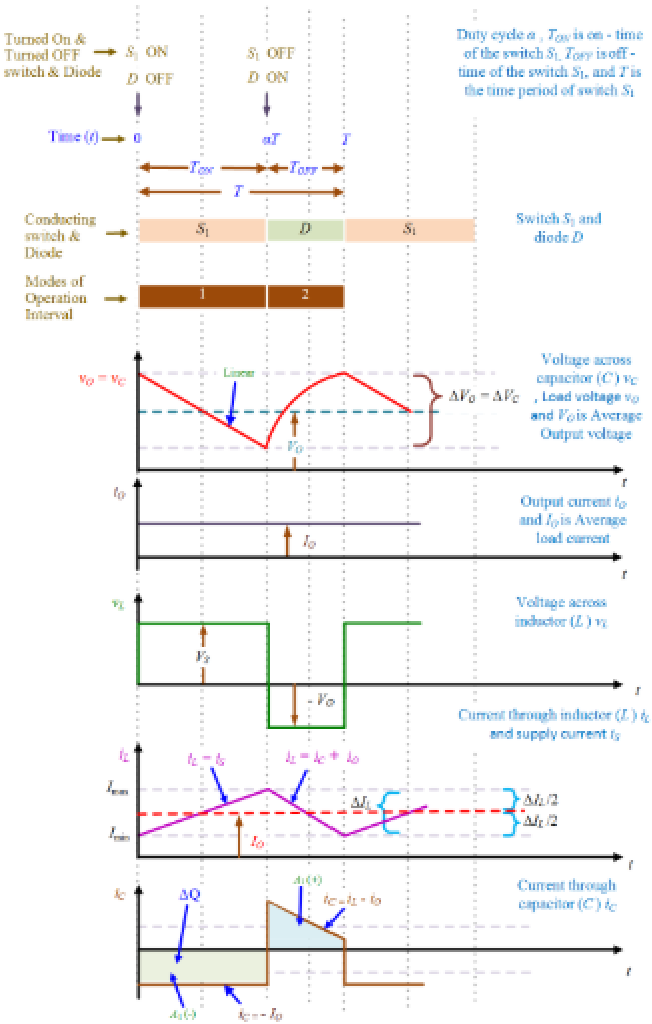


Figure 2 Waveforms of buck boost Regulator.

Mode of Operation Interval 1: –

The time interval is $0 \leq t \leq T_{ON}$. The switch S_1 is turned on. The circuit diagram for Mode of Operation Interval 1 is shown in Figure 3 and the corresponding waveforms are shown in Figure 2.

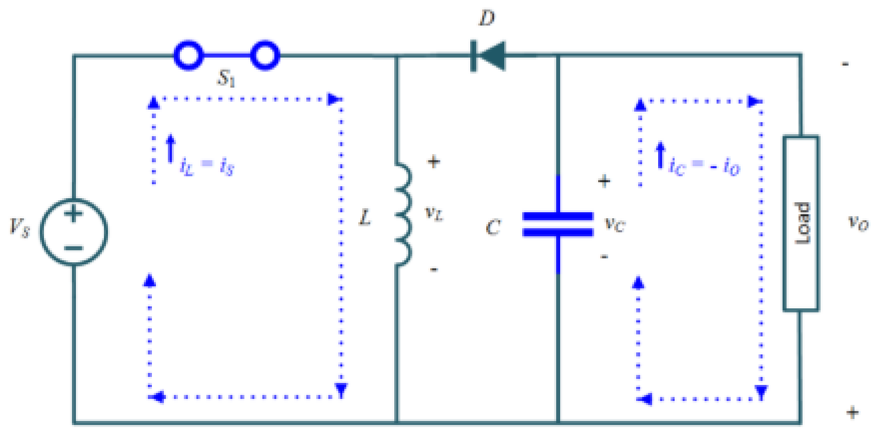


Figure 3 Circuit diagram of buck boost Regulator when switch S_1 ON.

From the waveform of voltage across the inductor, as shown in Figure 2, the equation for the inductor voltage write as

$$v_L = V_S \qquad \ldots . (1)$$

The general equation relates voltage across the inductor and current passes through it as

$$v_L = L\frac{di_L}{dt} \qquad \ldots . (2)$$

Put Equation 2 in Equation 1 gives

$$L\frac{di_L}{dt} = V_S \ldots (3)$$

The waveform for current passes through inductor L as shown in Figure

2, Integrate Equation 3 using the maximum and minimum value of inductor current gives

$$\int_{I_{\min}}^{I_{\max}} di_L = \frac{V_s}{L}\int_0^{T_{ON}} dt$$

or

$$I_{\max} - I_{\min} = \frac{V_S}{L}.T_{ON} \qquad \ldots.(4)$$

Here $\Delta I_L = I_{\max} - I_{\min}$ is the peak to peak ripple current of inductor L and hence Equation 4 can be write as

$$\Delta I_L = \frac{V_S}{L}.T_{ON} \qquad \ldots.(5)$$

Also

$$T_{ON} = \alpha T = \frac{\alpha}{f} \qquad \ldots.(6)$$

Using Equation 5 and Equation 6 gives

$$\Delta I_L = \frac{V_S}{L}.\frac{\alpha}{f} \qquad \ldots.(7)$$

Equation 7 describes the peak to peak ripple current of inductor L in buck-boost converter.

Buck Boost Regulator Circuit diagram, Waveform, Modes of Operation & Theory

In this topic, you study the Buck-Boost Regulator Circuit diagram, Waveforms, Modes of operation & theory.

The buck-boost regulator can produce an average output voltage less than or greater than the dc source input voltage. Let us assume large filter capacitance *C* connected across the load so that output voltage remains almost constant. The Resistive load is considered.

Circuit diagram

The working of a buck-boost regulator is explained using the circuit diagram as shown in Figure 1. The regulation is normally achieved by PWM (Pulse Width Modulation) at a fixed frequency and using the switch S_1 shown in the circuit diagram can be a conventional thyristor i.e., SCR, a GTO thyristor, a power transistor, or a MOSFET.

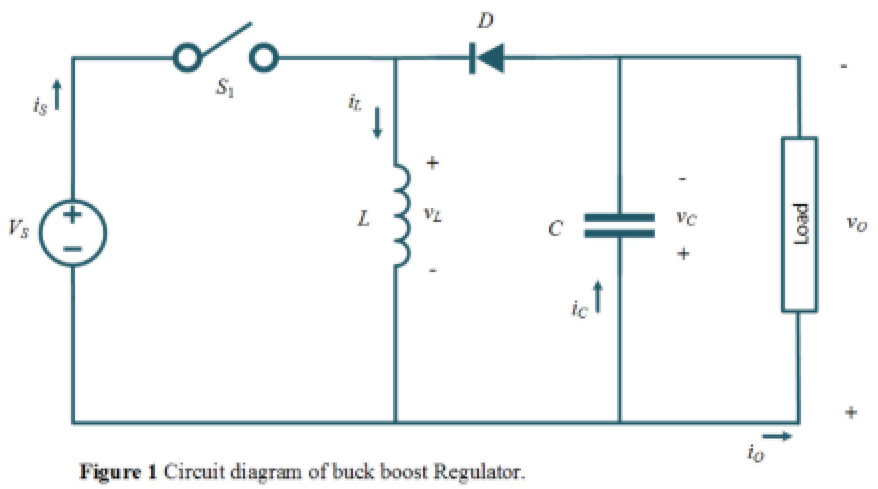


Figure 1 Circuit diagram of buck boost Regulator.

Waveforms

The typical waveforms in the converter are shown in Figure 2.

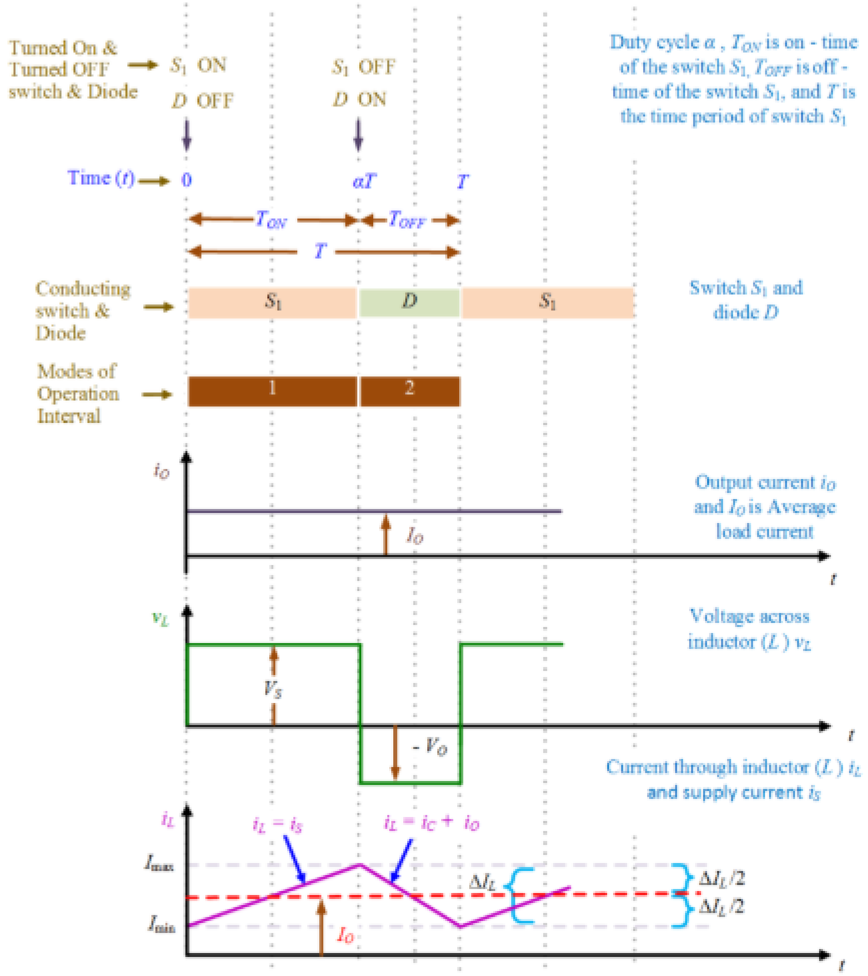


Figure 2 Waveforms of buck boost Regulator.

Modes of Operation Interval

The two modes in steady state operations are

Mode of Operation Interval 1: –

The time interval is $0 \le t \le T_{ON}$. The circuit diagram for Mode of Operation Interval 1 is shown in Figure 3 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned on, and the dc source directly connects to inductor L, the source (or input) current increases linearly and flows through the inductor L. The capacitor maintains the voltage v_O across the load and hence supplies current i_O to the load. The diode D gets reverse biased and behave as an open circuit. The inductor stores energy during this interval.

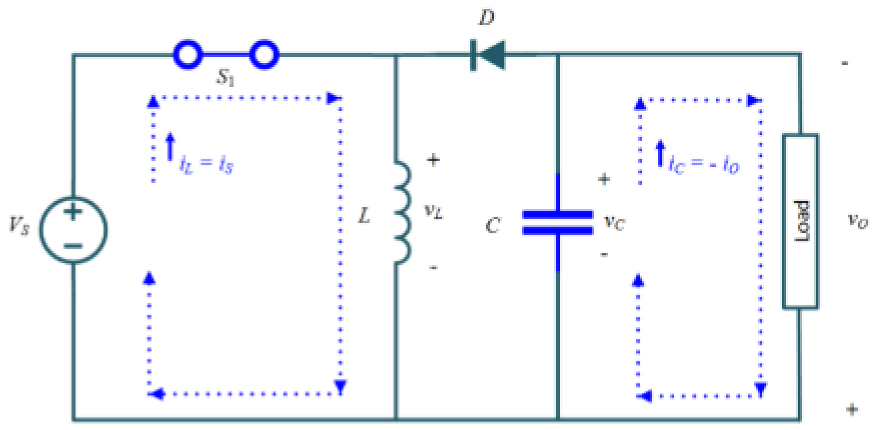


Figure 3 Circuit diagram of buck boost Regulator when switch S_1 ON.

Mode of Operation Interval 2: –

The time interval is $T_{ON} \le t \le T_{OFF}$. The circuit diagram for Mode of Operation Interval 2 is shown in Figure 4 and the corresponding waveforms are shown in Figure 2. The switch S_1 is turned OFF, the inductor current decreases linearly and flows through the Forward biased diode *D*, filter capacitor *C*, Resistive load. The inductor release energy during this interval.

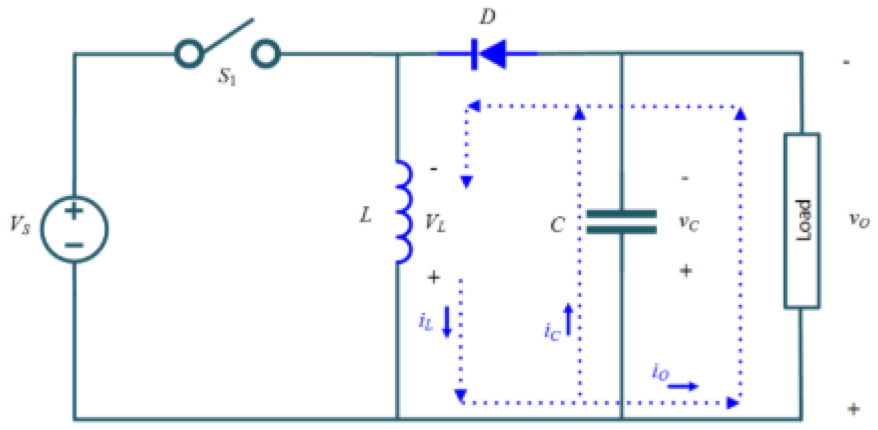


Figure 4 Circuit diagram of buck boost Regulator when switch S_1 OFF.

Second Quadrant Chopper or Type B Chopper or Class B Chopper

In this topic, you study Second Quadrant Chopper or Type B Chopper or Class B Chopper v-i plane, working principle, quadrant operation, Applications, waveforms, and Circuit diagrams.

Type B chopper is basically equivalent to Step-Up Chopper.

Circuit Diagram

The Type B chopper circuit diagram as shown in Figure 1. Here the motor load is assumed, R_a and L_a armature resistance and inductance of the motor respectively. E_b is the back emf of the motor.

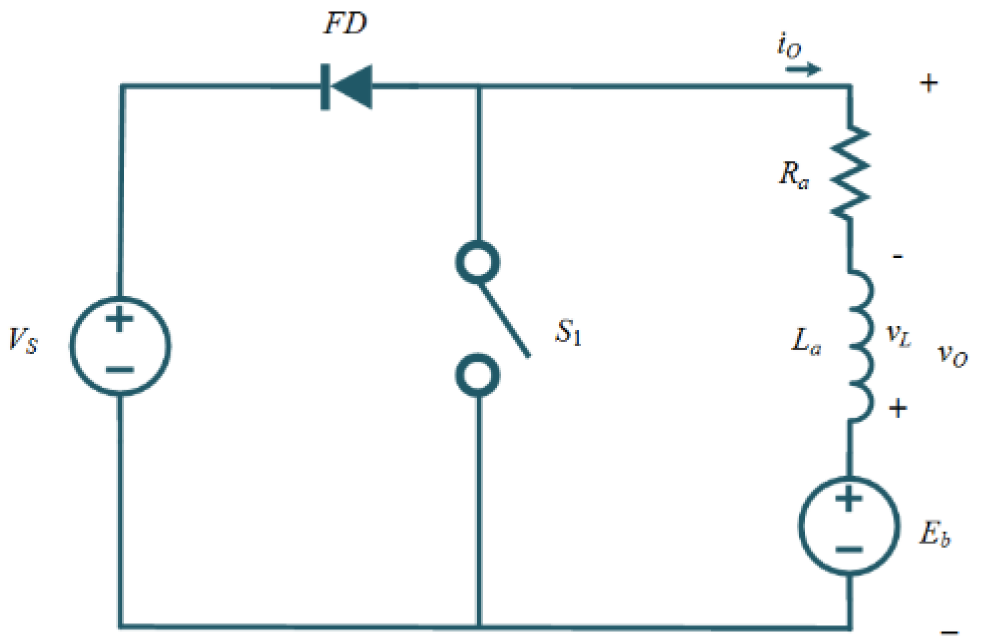


Figure 1 Circuit diagram of Type B chopper

$v_O - i_O$ plane

The Type B chopper operates in the Second quadrant of $v_O - i_O$ plane as shown in Figure 2. Here v_O is the output voltage, V_O is the average output voltage, i_O is the output current and I_O is the average output current of Type B chopper circuit.

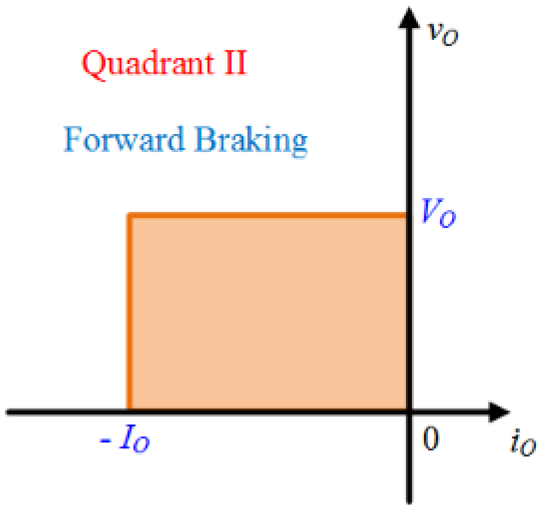


Figure 2 Type B chopper v_O - i_O plane

Quadrant II operation when Switch S_1 turned on

The Type B chopper circuit diagram for Quadrant II is shown in Figure 3 and the associated waveforms are shown below in Figure 5. Let us assume that the motor is running in the forward direction. When switch S_1 operated, Switch S_1 turned on and conducts, output voltage v_O is zero and E_b is responsible for the negative output current i_O, the machine behaves as generator and inductor stores energy.

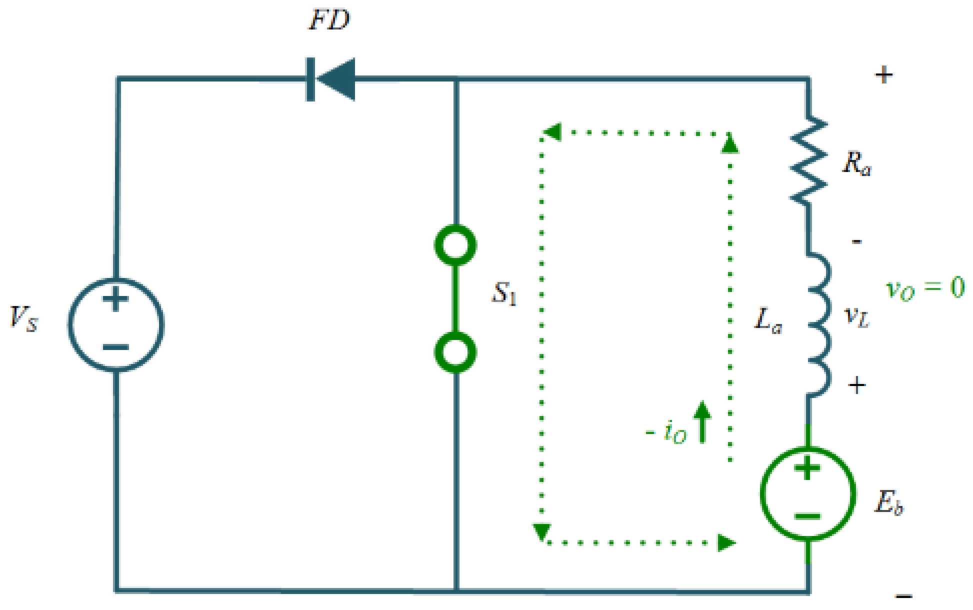


Figure 3 Equivalent circuit diagram I of Type B chopper

Quadrant II operation when Switch S_1 turned off

The Type E chopper equivalent circuit diagram for Quadrant II is shown in Figure 4 and the associated waveforms are below shown in Figure 5. Switch S_1 turned off, diode *FD* conducts, output voltage v_O becomes positive and the output current i_O is negative, inductor release energy using diode *FD*, power flows from load to source and hence called as reverse braking.

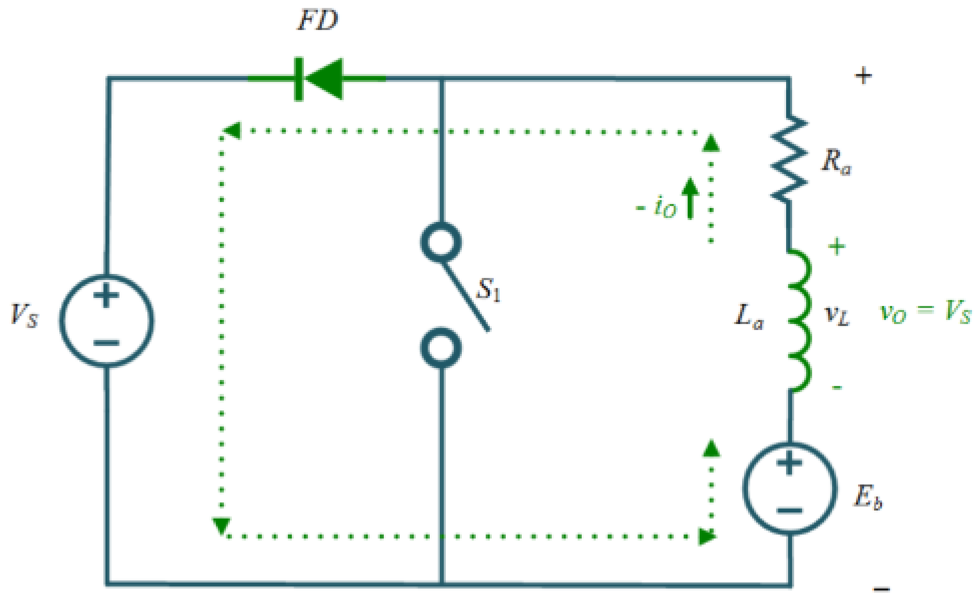


Figure 4 Equivalent circuit diagram II of Type B chopper

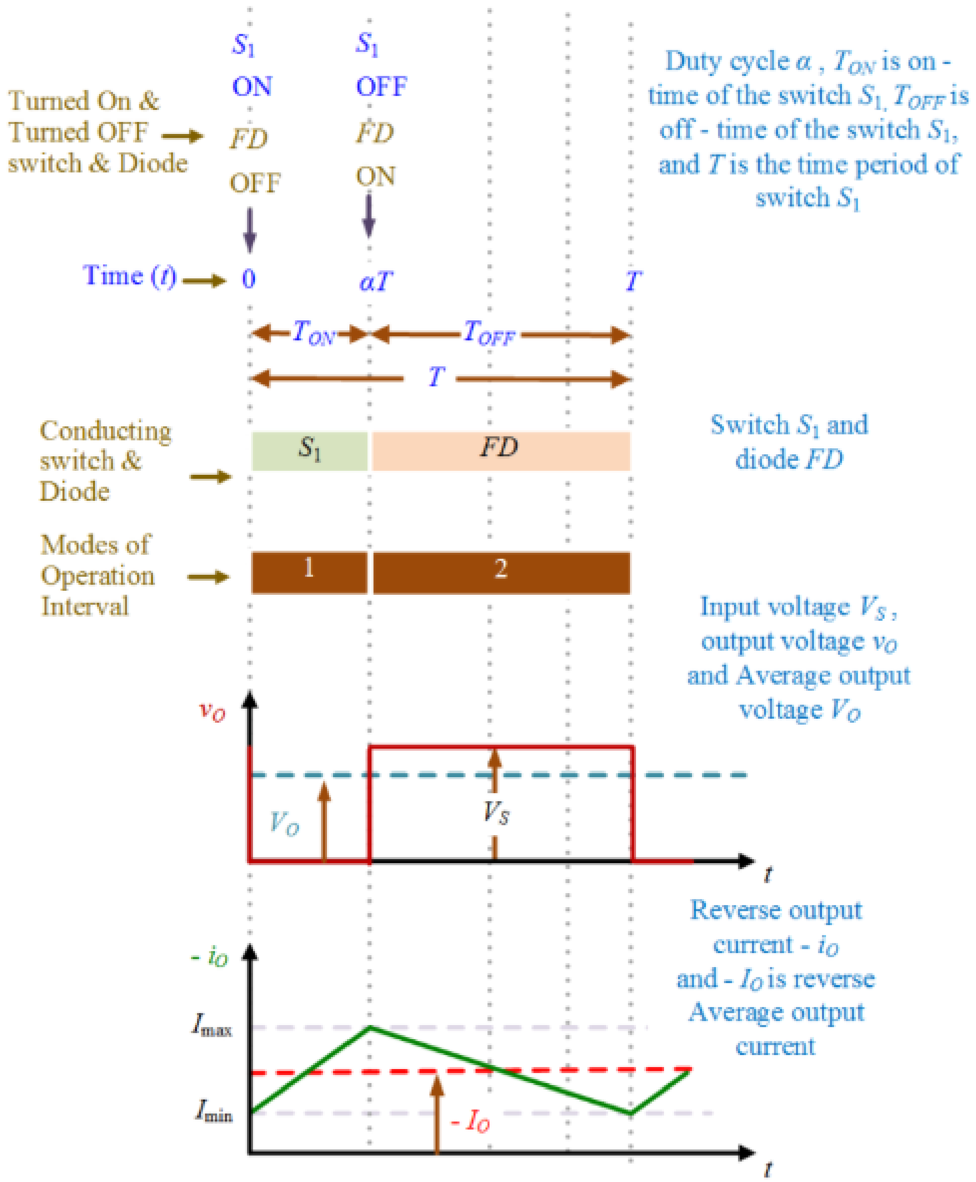


Figure 5 Waveform of Type B chopper

Waveforms

Type B chopper associated waveforms are shown in Figure 5.

Application

This chopper is suitable for regenerative breaking application only.

Mathematical Analysis

Using the waveform as shown in Figure 1, The average output voltage write as

$$V_O = V_S \left(\frac{T_{OFF}}{T} \right) \ldots . (1)$$

Also

$$\frac{T_{OFF}}{T} = 1 - \alpha \ldots . (2)$$

Put Equation 2 in Equation 1 gives

$$V_O = V_S (1 - \alpha) \ldots . (3)$$

Equation 3 describe the realtion between input dc source voltage and average output voltage for Type B chopper.

Four Quadrant Chopper or Type E Chopper or Class E Chopper

In this topic, you study Four Quadrant Chopper or Type-E Chopper or Class E Chopper v-i plane, working principle, quadrant operation, and Circuit diagrams.

Type E chopper is a four-quadrant chopper.

$v_O - i_O$ plane

The Type E chopper operates in the four quadrants of $v_O - i_O$ plane as shown in Figure 2. Here v_O is the output voltage, V_O is the average output voltage, i_O is the output current and I_O is the average output current of Type E chopper circuit.

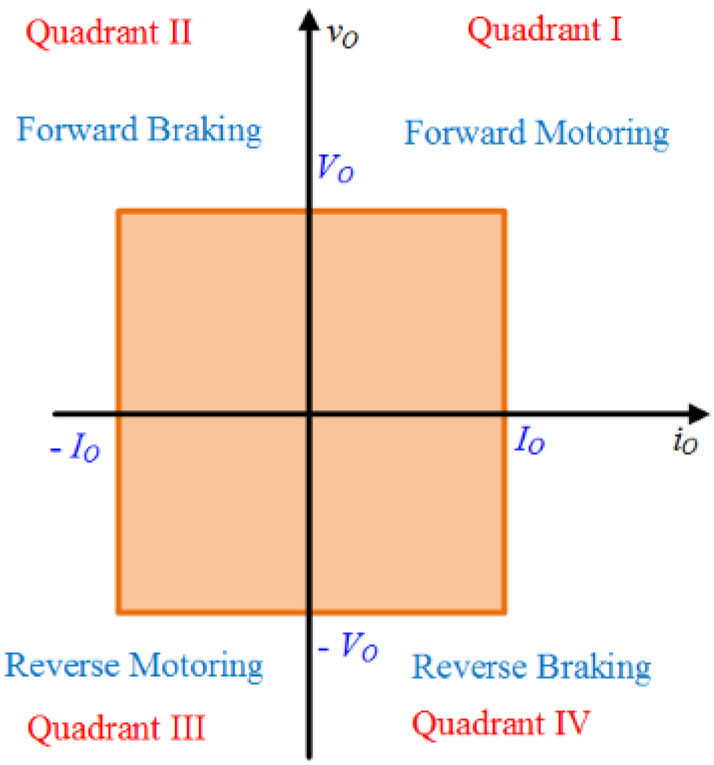


Figure 1 Type E chopper v_O - i_O plane

Circuit Diagram

The Type E chopper circuit diagram as shown in Figure 1. Here the motor load is assumed, R_a and L_a armature resistance and inductance of the motor respectively. E_b is the back emf of the motor.

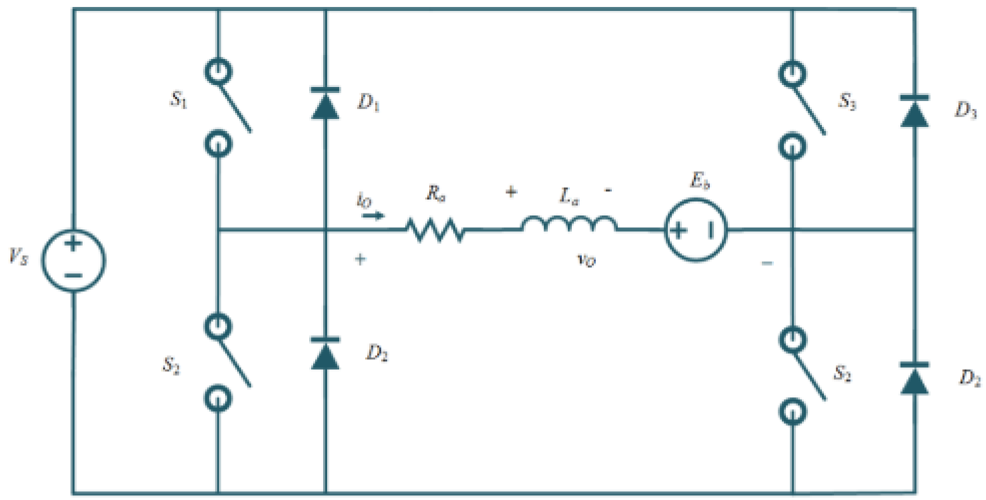


Figure 2 Circuit diagram of Type E chopper

Quadrant I operation when Switch S_1 turned on

The Type E chopper equivalent circuit diagram for Quadrant I is shown in Figure 3. Here switch S_1 operated, Switches S_1 and S_2 conduct, output voltage v_O and the output current i_O both are positives, power flows from source to load and inductor stores energy, the motor rotates in the forward direction hence called forward motoring.

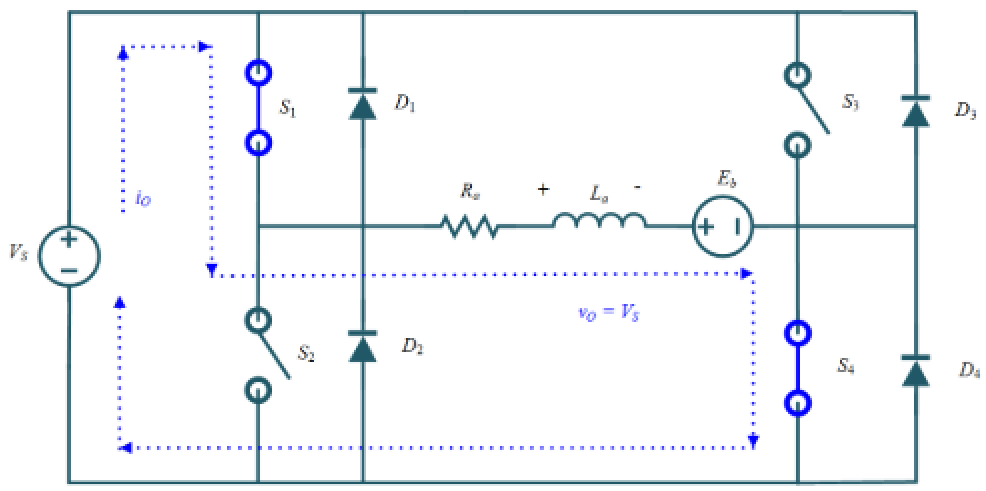


Figure 3 Equivalent circuit diagram I of Type E chopper

Quadrant I operation when Switch S_1 turned off

The Type E chopper equivalent circuit diagram for Quadrant I is shown in Figure 4. Switch S_1 turned off but switch S_4 and diode D_2 conducts, output current i_O is positive and the output voltage v_O becomes zero, inductor release energy and freewheeling action using diode D_2 takes place, the motor rotates in the forward direction hence called Forward motoring.

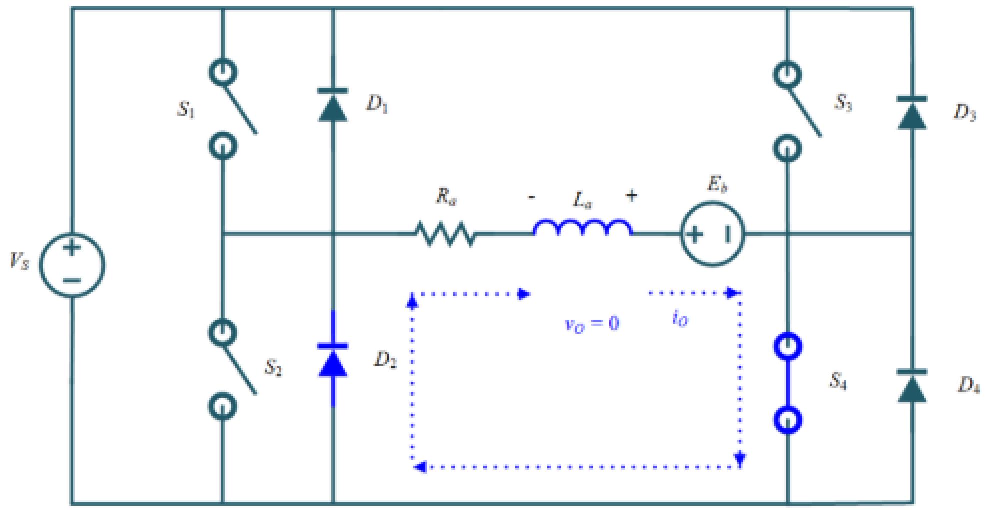


Figure 4 Forward Motoring Equivalent circuit diagram II of Type E chopper

Quadrant II operation when Switch S_2 turned on

The Type E chopper equivalent circuit diagram for Quadrant II is shown in Figure 5. Let us assume that the motor is running in the forward direction. Here switch S_2 operated, Switch S_2 and diode D_4 conducts, output voltage v_O is zero and E_b is responsible for the negative output current i_O, machine behave as generator and inductor stores energy.

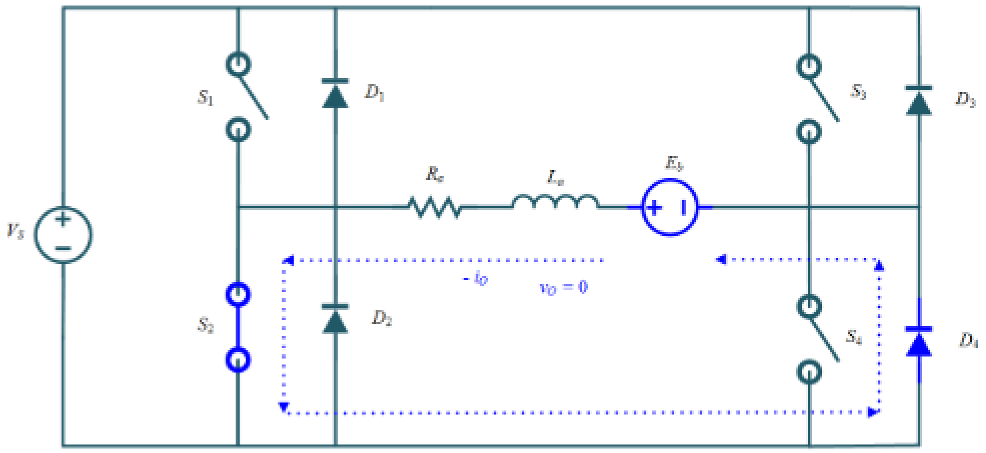


Figure 5 Forward Braking Equivalent circuit diagram III of Type E chopper

The Type E chopper equivalent circuit diagram for Quadrant II is shown in Figure 6. Switch S_2 turned off, diode D_1 and diode D_4 conducts, output voltage v_O becomes positive and the output current i_O is negative, inductor release energy using diodes D_1 and D_4, power flows from load to source and hence called as reverse braking.

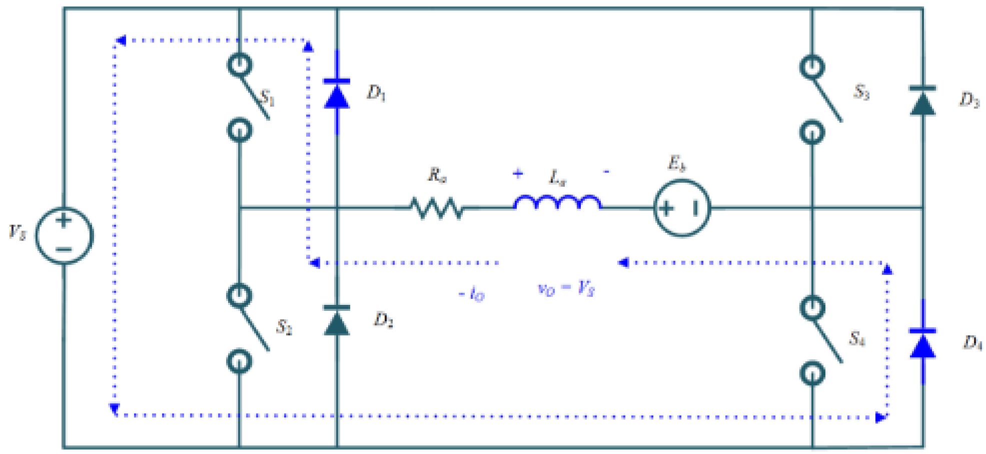


Figure 6 Forward Braking Equivalent circuit diagram IV of Type E chopper

Quadrant III operation when Switch S_3 turned on

The Type E chopper equivalent circuit diagram for Quadrant III is shown in Figure 7. The polarity of back emf E_b must be reversed. Here switch S_3 operated, Switches S_3 and S_2 conducts, output voltage v_O and the output current i_O both are negatives, power flows from source to load and inductor stores energy, the motor rotates in the reverse direction hence called as reverse motoring.

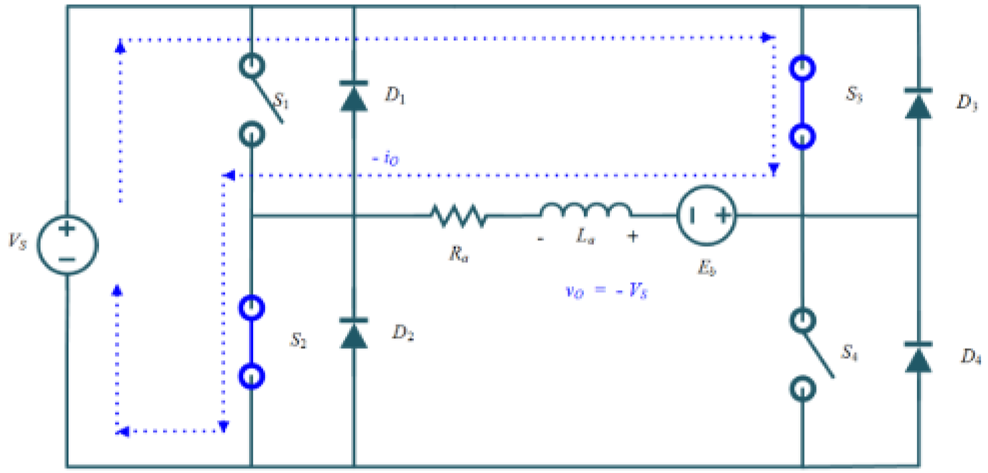


Figure 7 Equivalent circuit diagram V of Type E chopper

Quadrant III operation when Switch S_3 turned off

The Type E chopper equivalent circuit diagram for Quadrant III is shown in Figure 8. The polarity of back emf E_b must be reversed. Switch S_3 turned off but switch S_2 and diode D_4 conducts, output current i_O is negative and the output voltage v_O becomes zero, inductor release energy and freewheeling action using diode D_4 takes place, the motor rotates in the reverse direction hence called as Reverse motoring.

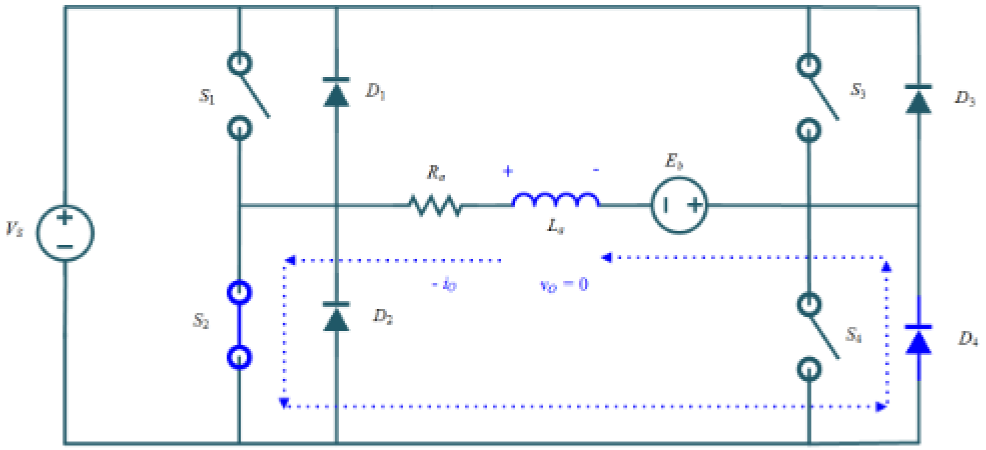


Figure 8 Reverse Motoring Equivalent circuit diagram VI of Type E chopper

Quadrant IV operation when Switch S_4 turned on

The Type E chopper equivalent circuit diagram for Quadrant IV is shown in Figure 9. The polarity of back emf E_b must be reversed. Let us assume that the motor is running in the reverse direction. Here switch S_4 operated, Switches S_4 and diode D_2 conducts, output voltage v_O is zero and E_b is responsible for the positive output current i_O, machine behave as generator and inductor stores energy.

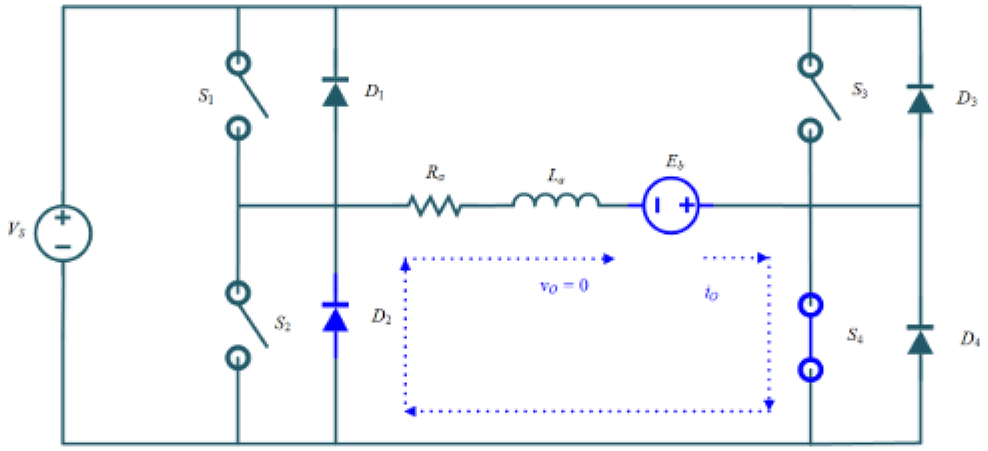


Figure 9 Reverse Braking Equivalent circuit diagram VII of Type E chopper

Quadrant IV operation when Switch S_4 turned off

The Type E chopper equivalent circuit diagram for Quadrant IV is shown in Figure 10. The polarity of back emf E_b must be reversed. Switch S_4 turned off, diode D_2 and diode D_3 conducts, output voltage v_O becomes negative and output current i_O is positive, inductor release energy using diodes D_2 and D_3, power flows from load to source and hence called as reverse braking.

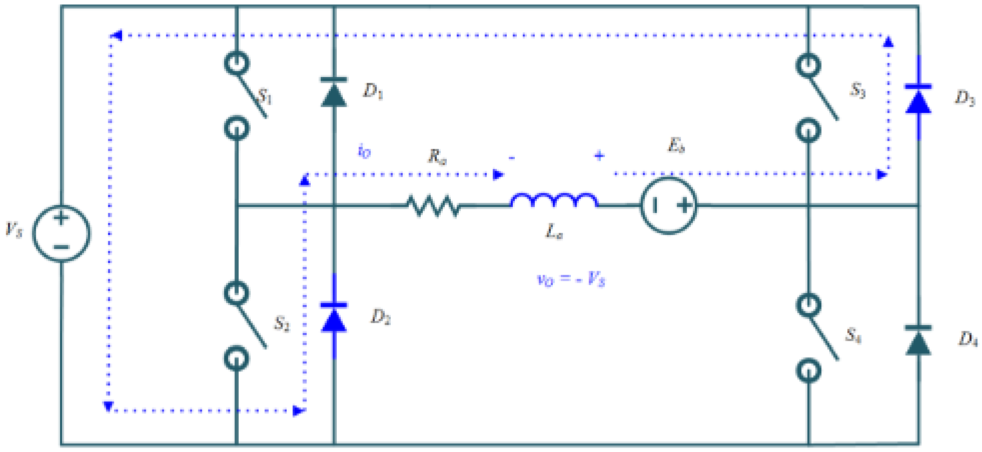
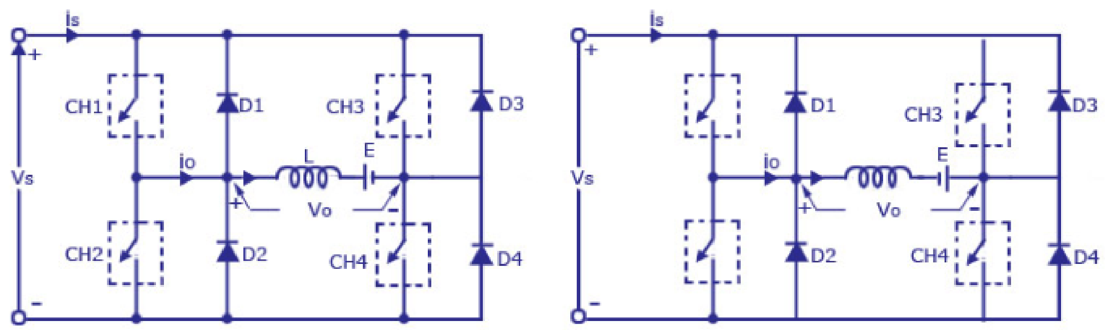


Figure 10 Reverse Braking Equivalent circuit diagram VIII of Type E chopper

Type –E chopper or the Fourth-Quadrant Chopper

Type E or the fourth quadrant chopper consists of four semiconductor switches and four diodes arranged in antiparallel. The 4 choppers are numbered according to which quadrant they belong. Their operation will be in each quadrant and the corresponding chopper only be active in its quadrant.

E-type Chopper Circuit Diagram With Load emf E and E Reversed



E-type Chopper Circuit diagram with load emf E and E Reversed

- **First Quadrant**

During the first quadrant operation the chopper CH4 will be on . Chopper CH3 will be off and CH1 will be operated. AS the CH1 and CH4 is on the load voltage v_0 will be equal to the source voltage V_s and the load current i_0 will begin to flow . v_0 and i_0 will be positive as the first quadrant operation is taking place. As soon as the chopper CH1 is turned off, the positive current freewheels through CH4 and the diode D2 . The type E chopper acts as a step- down chopper in the first quadrant.

- **Second Quadrant**

In this case the chopper CH2 will be operational and the other three are kept off. As CH2 is on negative current will starts flowing through the inductor L . CH2 ,E and D4. Energy is stored in the inductor L as the chopper CH2 is on. When CH2 is off the current will be fed back to the source through the diodes D1 and D4. Here (E+L.di/dt) will be more than the source voltage V_s . In second quadrant the chopper will act as a step-up chopper as the power is fed back from load to source

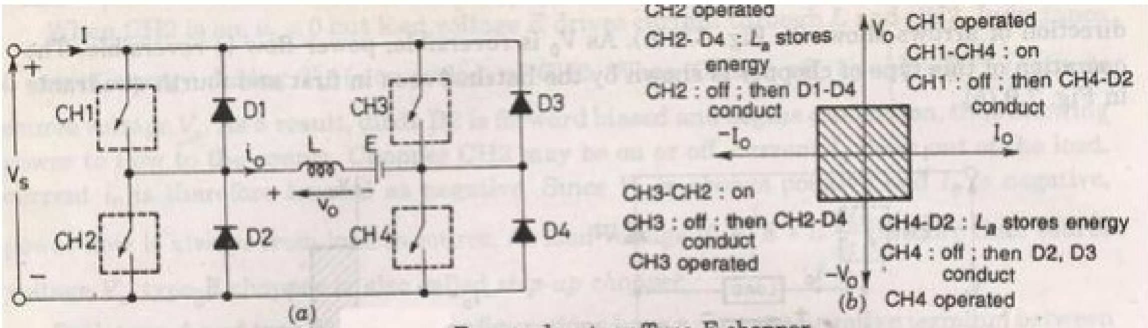
- **Third Quadrant**

In third quadrant operation CH1 will be kept off , CH2 will be on and CH3 is operated. For this quadrant working the polarity of the load should be reversed. As the chopper CH3 is on, the load gets connected to the source V_s and v_0 and i_0 will be negative and the third quadrant operation will takes place. This chopper acts as a step-down chopper

- **Fourth Quadrant**

CH4 will be operated and CH1, CH2 and CH3 will be off. When the chopper CH4 is turned on positive current starts to flow through CH4, D2 ,E and the inductor L will store energy. As the CH4 is turned off the current is feedback to the source through the diodes D2 and D3 , the operation will be in fourth quadrant as the load voltage is negative but the load current is positive. The chopper acts as a step up chopper as the power is fed back from load to source.

FOUR QUADRANT CHOPPER, OR TYPE E CHOPPER



Four-qudrant, or Type-E chopper
(a) circuit diagram and (b) operation of conducting devices.

FIRST QUADRANT:

CH4 is kept ON

CH3 is off

CH1 is operarted

$V_0=V_s$

i_0 = positive

when CH1 is off positive current free wheels through CH4,D2

so V_0 and I_2 is in first quadrant.

SECOND QUADRANT:

CH1,CH3,CH4 are off.

CH2 is operated.

Reverse current flows and I is negative through L CH2 D4 and E.

When CH2 off D1 and D4 is ON and current id fed back to source. So

$E + L\frac{di}{dt}$ is more than source voltage V_s.

As i_0 is negative and V_0 is positive, so second quadrant operation.

THIRD QUADRANT:

CH1 OFF, CH2 ON

CH3 operated. So both V_0 and i_0 is negative.

When CH3 turned off negative current freewheels through CH2 and D4.

FOURTH QUADRANT:

CH4 is operated other are off.

Positive current flows through CH4 E L D2.

Inductance L stores energy when current fed to source through D3 and D2.V_0 is negative.

1.2.8 Four-Quadrant Chopper

The four-quadrant chopper is shown in Figure 1.6c. The input voltage is positive, and the output voltage can be either positive or negative. The switches and diode status for the operation are shown in Table 1.1. The output voltage can be calculated by the formula

$$V_2 = \begin{cases} kV_1 & QI\_operation \\ (1-k)V_1 & QII\_operation \\ -kV_1 & QIII\_operation \\ -(1-k)V_1 & QIV\_operation \end{cases} \tag{1.7}$$

TABLE 1.1
Switches and Diodes' Status for Four-Quadrant Operation

Switch or Diode	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
S_1	Works	Idle	Idle	Works
D_1	Idle	Works	Works	Idle
S_2	Idle	Works	Works	Idle
D_2	Works	Idle	Idle	Works
S_3	Idle	Idle	On	Idle
D_3	Idle	Idle	Idle	On
S_4	On	Idle	Idle	Idle
D_4	Idle	On	Idle	Idle
Output	V_{2+}, I_{2+}	V_{2+}, I_{2-}	V_{2-}, I_{2-}	V_{2-}, I_{2+}

2.12.4 Four quadrant Chopper or Type E Chopper

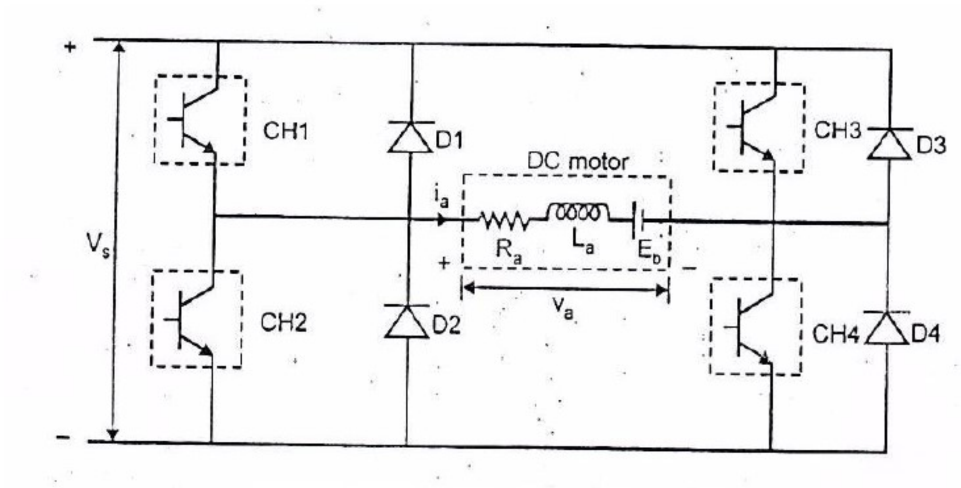


Fig (2.12.4) Four quadrant Chopper or Type E Chopper

Forward Motoring Mode

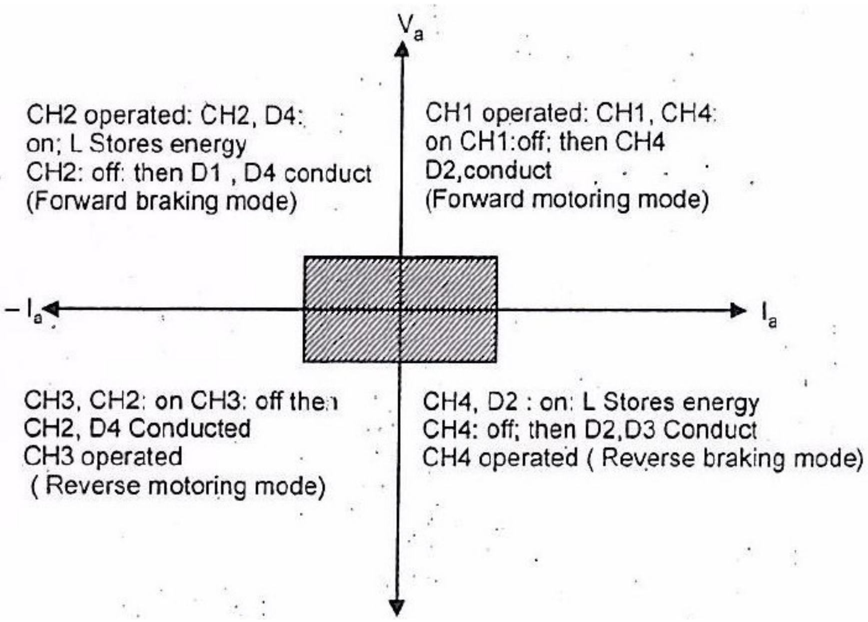
For first quadrant operation of figure CH4 is kept on, Cl-l3 is kept off and CH1 is operated. when CHI and Cl-I4 are on, load voltage is equal to supply voltage i,e, Va = Vs and load current ia begins to flow. Here both output voltage va and load current ia are positive giving first quadrant operation. When CH4 is turned of£ positive current freewheels through CH-4,D2 in this way, both output voltage va, load current ia can be controlled in the first quadrant. First quadrant operation gives the forward motoring mode.

Forward Braking Mode

Here CH2 is operated and CH1, CH3 and CH4 are kept off. With CH2 on, reverse (or negative) current flows through L, CH2, D4 and E. During the on time of CH2 the inductor L stores energy. When CH2 is turned off current is fedback to source through diodes D1, D4 note that there [E+L di/dt] is greater than the source voltage Vs. As the load voltage Va is positive and load current ia is negative, it indicates the second quadrant operation of chopper. Also power flows from load to source, second quadrant operation gives forward braking mode.

Reverse Motoring Mode

For third quadrant operation of figure, CHI is kept off, CH2 is kept on and CH3 is operated. Polarity of load emf E-must be reversed for this quadrant operation. With CH3 on, load gets connected to source Vs so that both output voltage Va and load current ia are negative. it gives third quadrant operation. lt is also known as reverse motoring mode. When CH3 is turned off, negative current freewheels through CH2, D4. ln this way, output voltage Va and load current ia can be controlled in the third quadrant.



Reverse Braking Mode

Here CH4 is operated and other devices are kept of£Load emf E must have its polarity reversed, it'is shown in figure . With CH4 on, positive current flows through CH4, D2, L and E. During the on time of CH4, the inductor L stores energy.

When CH4 is turned off; current is feedback to source through diodes D2, D3. Here load voltage is negative, but load current is positive leading to the chopper operation in the fourth quadrant.

Also power is flows from load to source. The fourth quadrant operation gives reverse braking mode.

2.13 Braking

In braking, the motor works as a generator developing a negative torque which oppose the motion. It is of three types

1. Regenerative braking
2. Plugging or Reverse voltage braking
3. Dynamic braking or Rheostatic braking

2.13.1Regenerative braking

In regenerative braking, generated energy is supplied to the source,for this to happen following condition should be satisfied

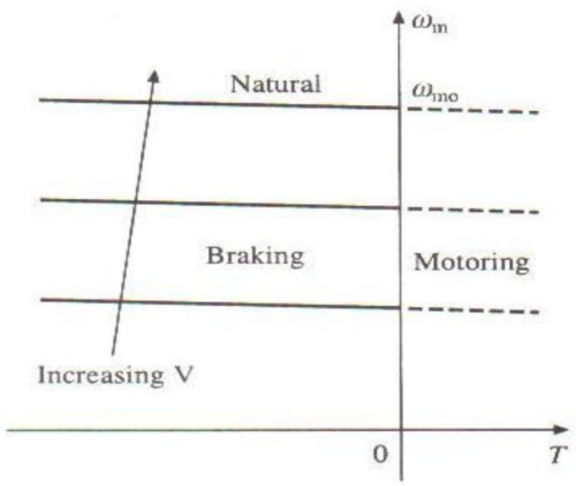
E > V and negative Ia

Field flux cannot be increased substantially beyond rated because of saturation, therefore according to equation ,for a source of fixed voltage of rated value regenerative braking is possible only for speeds higher than rated and with a variable voltage source it is also possible below rated speeds .

The speed –torque characteristics shown in fig. for a separately excited motor.

In series motor as speed increases, armature current, and therefore flux decreases

Condition of equation cannot be achieved .Thus regenerative braking is not possible



2.13.2 Plugging

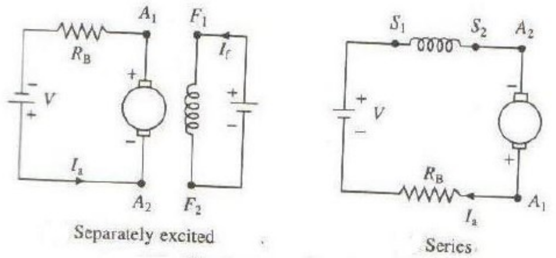
The supply voltage of a separately excited motor is reversed so that it assists the emf in forcing armature current in reverse direction .A resistance R_B is also connected in series with armature to limit the current.For plugging of a series motor armature is reversed.

A particular case of plugging for motor rotation in reverse direction arises .when a motor connected for forward motoring,is driven by an active load in the reverse direction.Here again back emf and applied voltage act in the same direction.However the direction of torque remains positive.

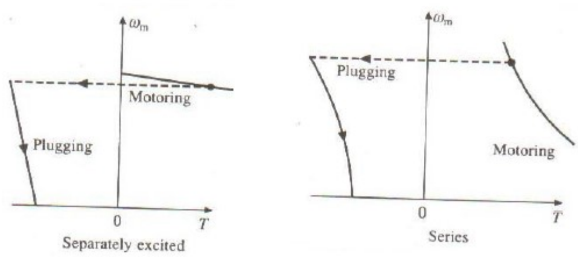
This type of situation arises in crane and the braking is then called counter – torque braking.

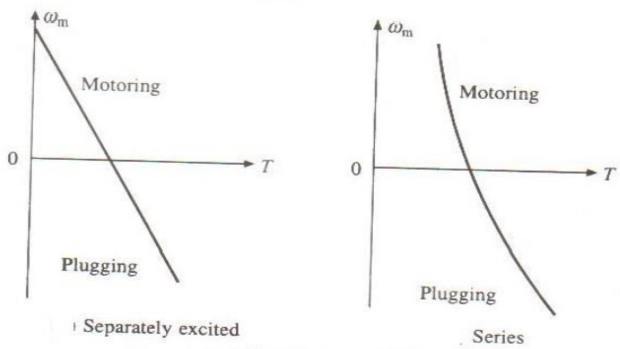
Plugging gives fast braking due to high average torque,even with one section of braking resistance RB.Since torque ia not zero speed,when used for stopping a load,the supply must be disconnected when close to zero speed.

Centifugal switches are employed to disconnect the supply.Plugging is highly inefficient because in addition to the generated power,the power supplied by the source is also wasted in resistances.



Plugging operation of dc motors





Counter-torque braking

2.13.3 Dynamic braking

In dynamic braking ,the motor is made to act as a generator,the armature is

disconnected from the supply ,but it continues to rotate and generate a voltage.The polarity of the generated voltage remains unchanged if the direction if field excitation is unaltered.

But if a resistance is connected across the coasting motor,the direction of the armature current is reversed ,because the armature represents a source of power rather than a load.

Thus a braking torque is developed ,exactly as in the generator,tending to oppose the motion.

The braking torque can be controlled by the field excitation and armature current.

3.3.3 Four-quadrant Chopper

The circuit of a four-quadrant chopper is shown in Fig. 3.15 in which the inductor L is assumed to be composed of the armature inductance and an external inductor.

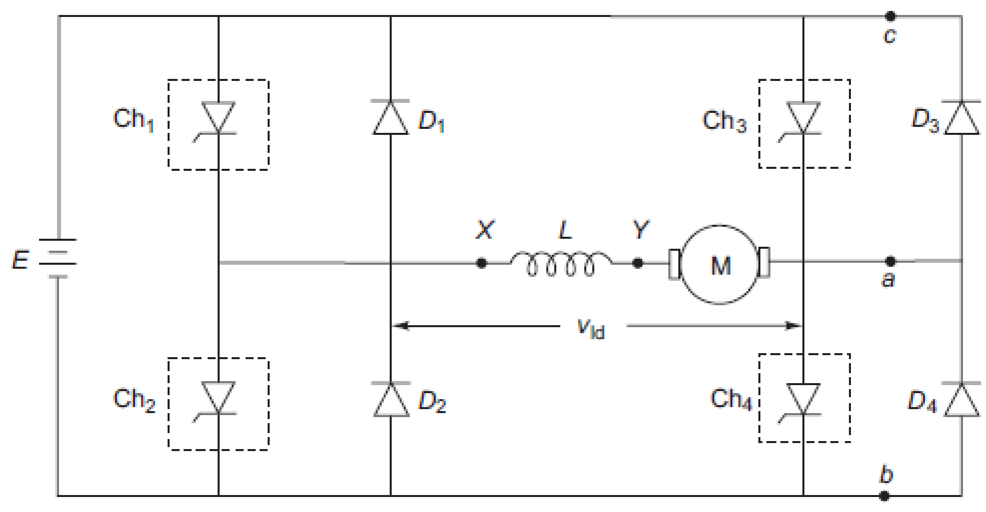


Fig. 3.15 Circuit of a four-quadrant chopper

Three methods of control are possible for the operation of this chopper as elaborated below.

Method 1 The circuit is operated as a two-quadrant chopper to obtain (a) first- and second-quadrant operation as well as (b) third- and fourth-quadrant operation.

Sequence 1 To obtain mode (a), Ch_4 is permanently kept on; terminals a and b are always kept shorted by ensuring conduction by either Ch_4 or D_4 and terminals a and c are always kept open. The choppers Ch_1 and Ch_2 are controlled as per the following four steps.

(a) If Ch_1 and Ch_4 are turned on at $t = 0$, the battery voltage E will be applied to the load circuit and current will flow from X to Y as shown in Fig. 3.16(a); this direction is the positive one. Thus the load voltage during this interval is kept at $+E$.

(b) When Ch_1 is turned off at τ_{ON}, the current due to the stored $(1/2)Li^2$ energy of the inductor L drives the current through D_2 and Ch_4 as shown in Fig. 3.16(b). Ch_2, which is turned on at τ_{ON}, does not conduct because it is shorted by D_2.

(c) Ch_2, which is on, conducts the current when it reverses, as shown in Fig. 3.16(c).

(d) Finally, when Ch_2 is turned off at τ, current flows through the path consisting of the negative of the battery, D_4, the motor, L, D_1, and the positive of the battery as shown in Fig. 3.16(d). If the machine were to be operated as a generator, this circuit facilitates regenerative braking. The zero crossing instants of the current waveform depend upon the values of E, E_b, L, and the armature resistance R_a of the motor. It is seen that Ch_1 does not conduct till i_{ld} becomes positive and Ch_2 does not conduct till i_{ld} flows in the negative direction. Also, D_4 conducts the reverse current and applies a reverse bias against Ch_4. The devices that conduct during each of the intervals are shown in Fig. 3.17(a), which gives the waveforms of this mode.

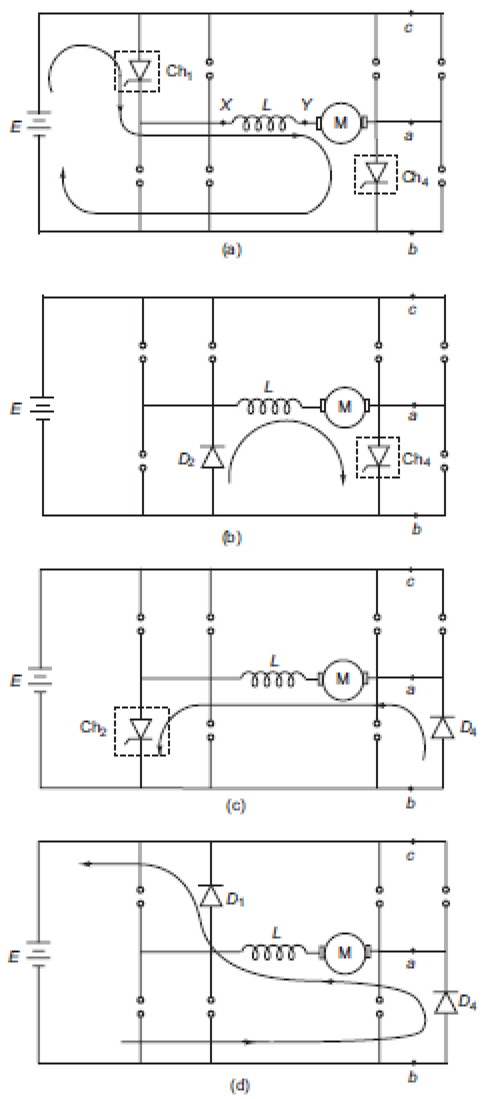


Fig. 3.16 Circuit conditions of a four-quadrant chopper with method 1: (a) Ch_1 and Ch_4 turned on; (b) Ch_1 turned off, Ch_4 remaining on; (c) Ch_2 turned on, Ch_4 shorted by D_4; (d) Ch_2 turned off, Ch_4, shorted by D_4

Sequence 2 For the circuit to provide third- and fourth-quadrant operation, Ch_3 is permanently kept on. Terminals a and c remain shorted due to conduction by either Ch_3 or D_3; terminals a and b remain open throughout. The relevant waveforms are shown in Fig. 3.17(b) (in the figure, I_B denotes the current through the battery).

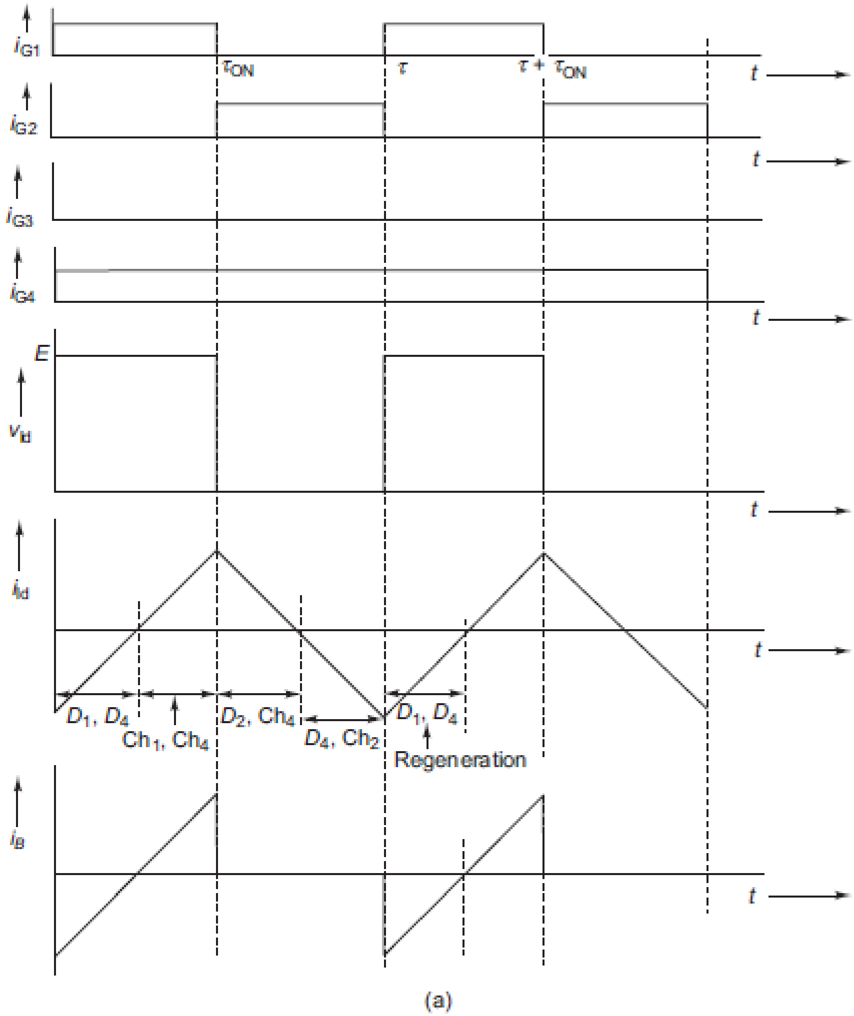


Fig. 3.17(a)

Fig. 3.17(a)

(a) Ch_2 is triggered on at $t = 0$ but starts conduction only when a reverse current flows through the path consisting of the positive of the battery, Ch_3, the motor, L, Ch_2, and the negative of the battery. A voltage equal to $-E$ is applied at the load terminals.

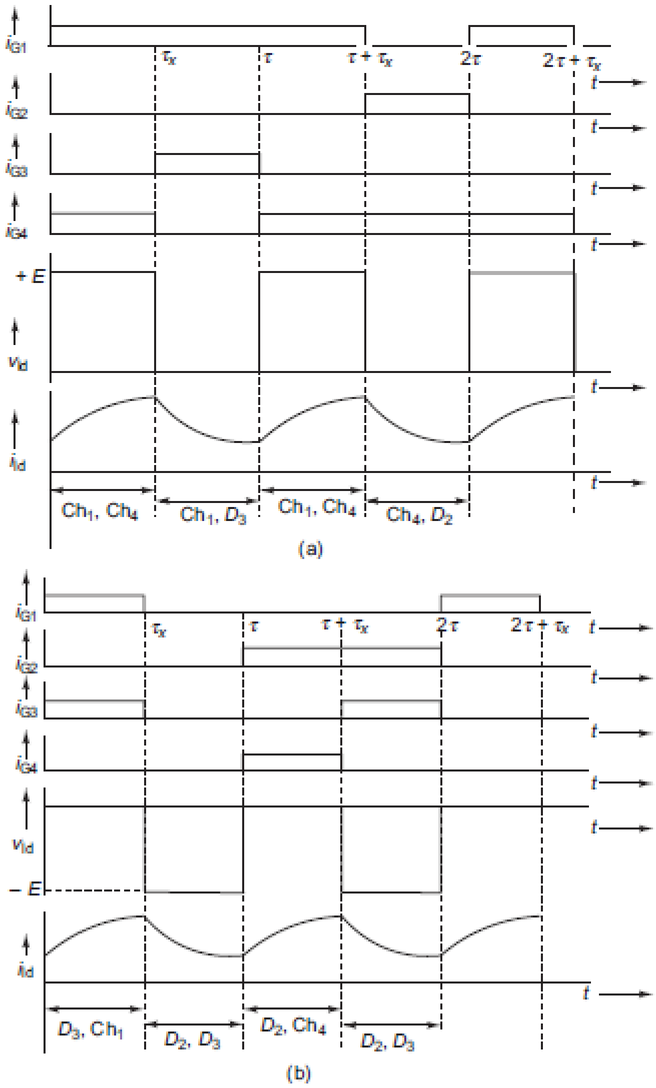
(b) When Ch_2 is turned off at τ_{ON}, the inductor continues to drive the current in the reverse direction through the path consisting of Ch_3, the motor, L, and D_1. The load voltage then becomes zero.

(c) Ch_1 is triggered at τ_{ON} but starts conduction only when the current flows in the positive direction, flowing through the closed circuit consisting of Ch_1, L, the motor, and D_3.

(d) When Ch_1 is turned off at τ, a negative battery voltage is applied to the load but positive current flows through the negative terminal of the battery, D_2, L, the motor, D_3, and back to the positive terminal of the battery.

It is seen that either Ch_1 or Ch_2 conducts current when i_{ld} becomes positive or negative, respectively. This is because, even though their control signals are present prior to the zero crossing of the load current, the conducting diodes D_1 and D_2 apply a reverse bias, respectively, across Ch_1 and Ch_2. The devices that conduct during each interval are given in Fig. 3.17(b).

This circuit suffers from the disadvantage that either Ch_3 or Ch_4 are kept on for a long time, which may lead to commutation problems. An important precaution to be taken is that the choppers Ch_1 and Ch_2 should not conduct simultaneously, as otherwise the source gets shorted through them. To ensure this, a small interval of time has to be provided between the turn-off of Ch_2 and the turn-on of Ch_1 and vice versa; this feature, however, limits the maximum chopper frequency.



Figs 3.18(a) and (b)

Method 2 In this method, the four-quadrant chopper provides first- and fourth-quadrant operation similar to method 2 of the two-quadrant type-B chopper. Thus the chopper pair Ch_1, Ch_4 and the diode pair D_2, D_3 conduct alternately; the other chopper pair is permanently kept off. Accordingly the waveforms will be identical to those of Figs 3.13(a) and (b), respectively. Likewise, for obtaining second- and third-quadrant operation, the chopper pair Ch_2, Ch_3 and the diode pair D_1, D_4 of Fig. 3.15 conduct in alternate intervals with the chopper pair Ch_1, Ch_4 always kept off. The waveforms in this case will be similar to those of Figs 3.13(a) and (b) except for the fact that the instantaneous current in both cases is always negative. Thus the operating point will be in either the second or the third quadrant.

Method 3 This method consists of operating the same combinations of chopper pairs, as in method 2, to provide four-quadrant operation. However, the chopper pairs are controlled in such a way that if one of them conducts during some interval, the other pair is off. The waveforms for the first, fourth, second, and third quadrants are given, respectively, in Figs 3.18(a), (b), (c), and (d).

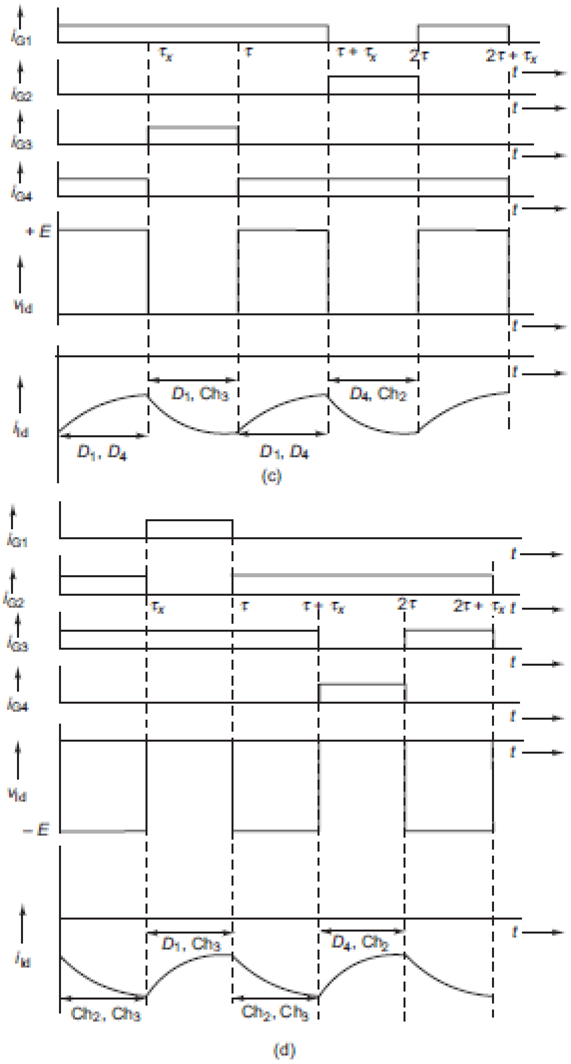


Fig. 3.18 Waveforms for four-quadrant chopper operation (method 3): (a) first-quadrant operation, (b) fourth-quadrant operation, (c) second-quadrant operation, (d) third-quadrant operation

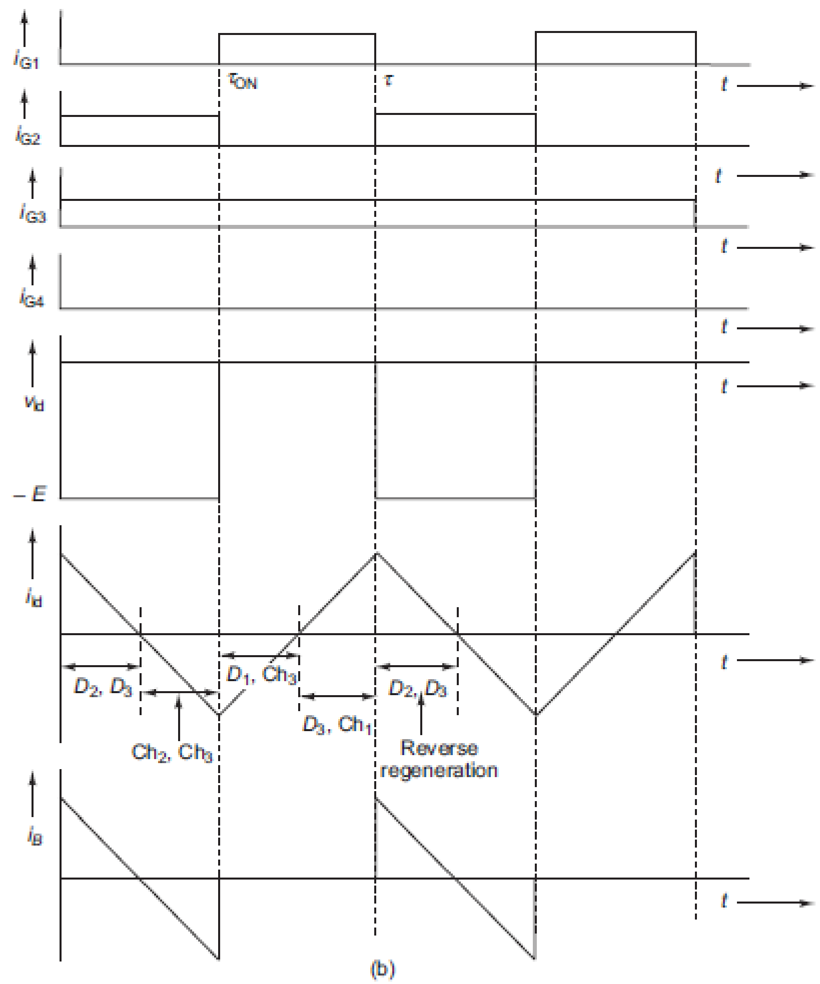


Fig. 3.17 Four-quadrant chopper: (a) waveforms for sequence 1—first- and second-quadrant operation, (b) waveforms for sequence 2—third- and fourth-quadrant operation

4.3 FOUR-QUADRANT CHOPPER CIRCUIT

A four-quadrant chopper with transistor switches is shown in Figure 4.2. Each transistor has a freewheeling diode across it and a snubber circuit to limit the rate of rise of the voltage. The snubber circuit is not shown in the figure.

The load consists of a resistance, an inductance, and an induced emf. The source is dc, and a capacitor is connected across it to maintain a constant voltage. The base drive circuits of the transistors are isolated, and they reproduce and amplify the control signals at the output. For the sake of simplicity, it is assumed that the switches are ideal and hence, the base drive signals can be used to draw the load voltage.

First-quadrant operation corresponds to a positive output voltage and current. This is obtained by triggering T_1 and T_2 together, as is shown in Figure 4.3; then the load voltage is equal to the source voltage. To obtain zero load voltage, either T_1 or T_2 can be turned off. Assume that T_1 is turned off; then the current will decrease in the power switch and inductance. As the current tries to decrease in the inductance, it will have a voltage induced across it in proportion to the rate of fall of current with a polarity opposite to the load-induced emf, thus forward-biasing diode D_4. D_4 provides the path for armature current continuity during this time. Because of this, the circuit configuration changes as shown in Figure 4.4. The load is short-circuited, reducing its voltage to zero. The current and voltage waveforms for continuous and

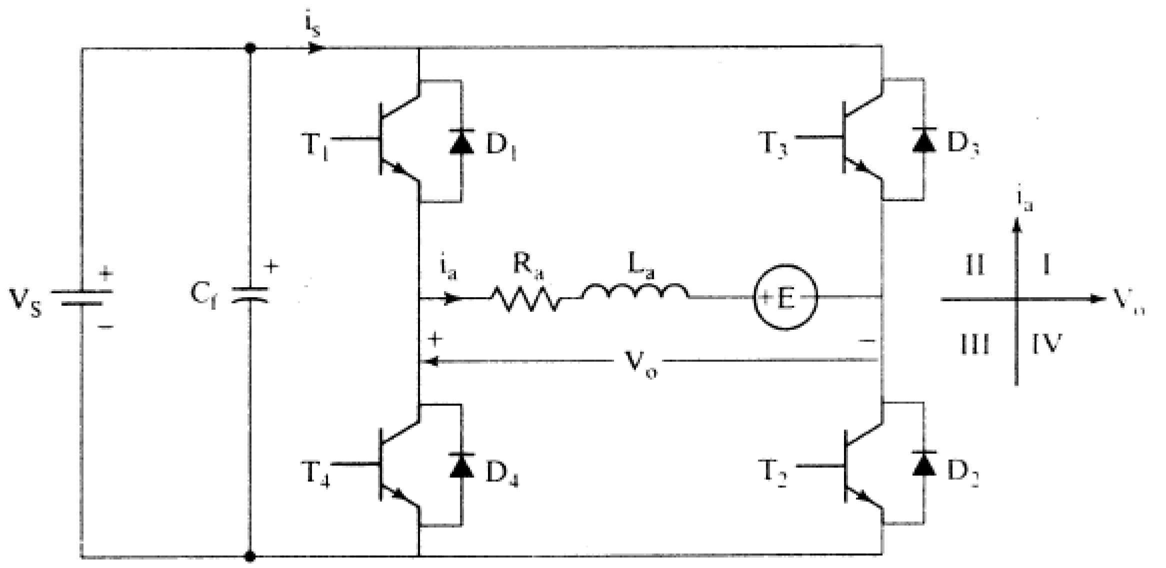


Figure 4.2 A four-quadrant chopper circuit

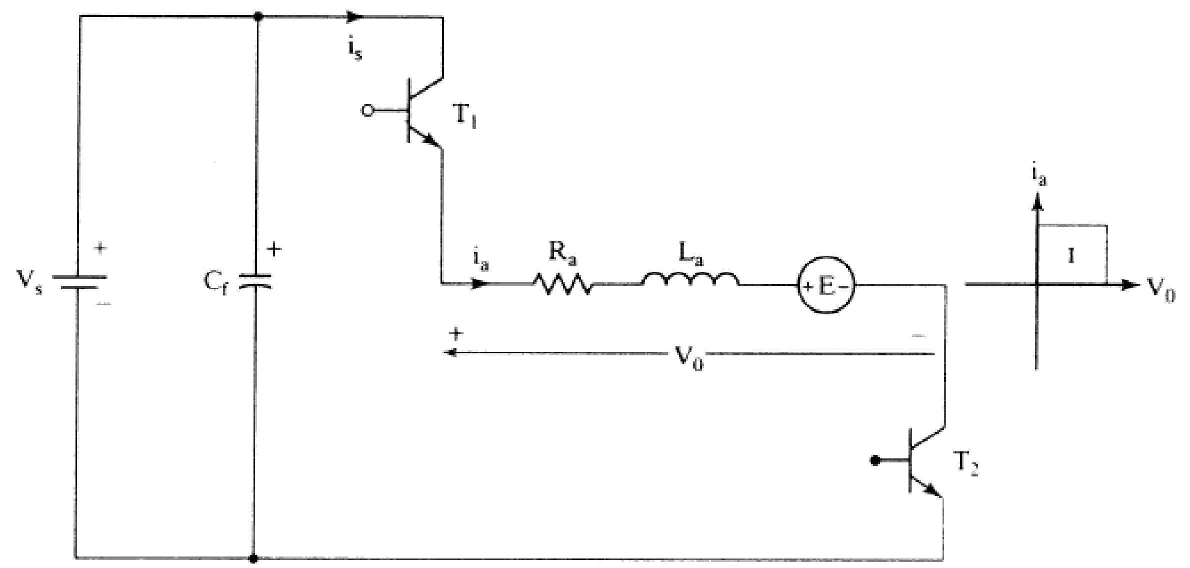


Figure 4.3 First-quadrant operation with positive voltage and current in the load

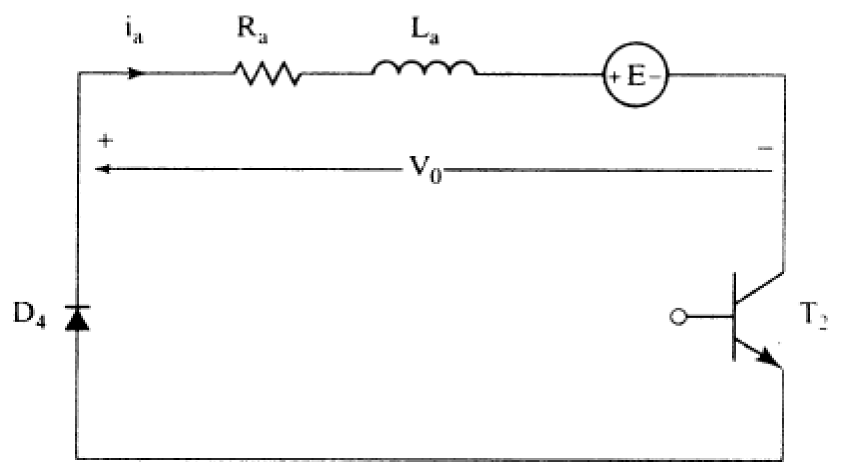


Figure 4.4 First-quadrant operation with zero voltage across the load

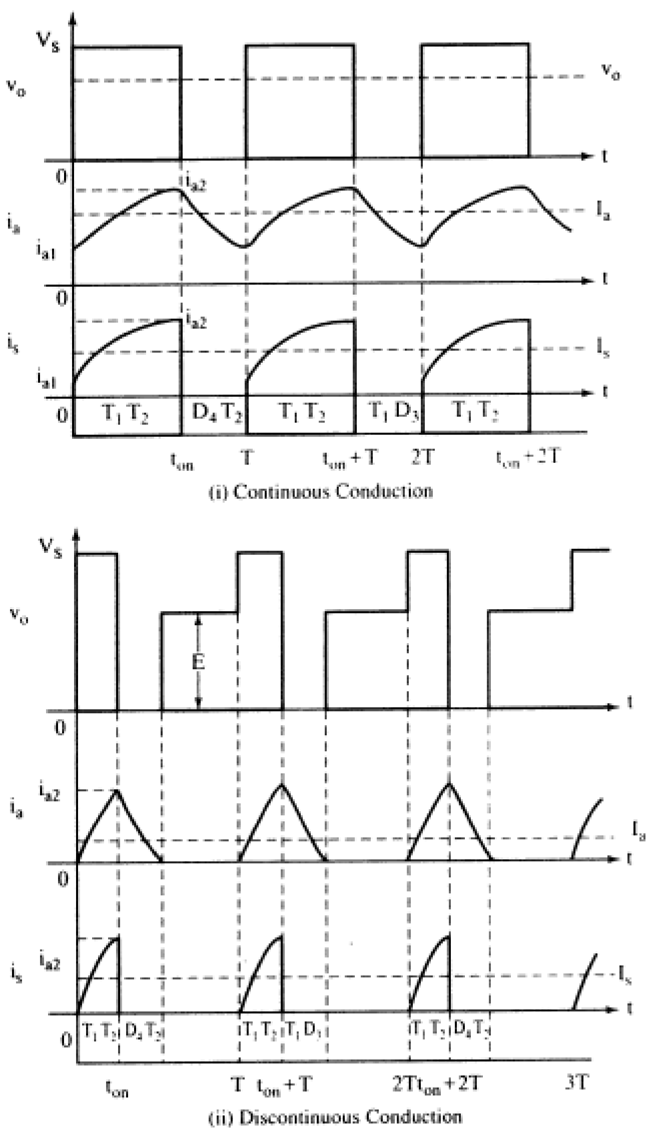


Figure 4.5 Voltage and current waveforms in first-quadrant operation

discontinuous current conduction are shown in Figure 4.5. Note that, in the discontinuous current-conduction mode; the induced emf of the load appears across the load when the current is zero. The load voltage, therefore, is a stepped waveform. The operation discussed here corresponds to motoring in the clockwise direction, or *forward motoring*. It can be observed that the average output voltage will vary from 0 to V_s; the duty cycle can be varied only from 0 to 1.

The output voltage can also be varied by another switching strategy. Armature current is assumed continuous. Instead of providing zero voltage during turn-off time to the load, consider that T1 and T2 are simultaneously turned off, to enable conduction by diodes D3 and D4. The voltage applied across the load then is equal to the negative source voltage, resulting in a reduction of the average output voltage. The disadvantages of this switching strategy are as follows:

(i) Switching losses double, because two power devices are turned off instead of one only.

(ii) The rate of change of voltage across the load is twice that of the other strategy. If the load is a dc machine, then it has the deleterious effect of causing higher dielectric losses in the insulation and therefore reduced life. Note that the dielectric is a capacitor with a resistor in series.

(iii) The rate of change of load current is high, contributing to vibration of the armature in the case of the dc machine.

(iv) Since a part of the energy is being circulated between the load and source in every switching cycle, the switching harmonic current is high, resulting in additional losses in the load and in the cables connecting the source and converter.

Therefore, this switching strategy is not considered any further in this chapter.

4.3.2 Second-Quadrant Operation

Second-quadrant operation corresponds to a positive current with a negative voltage across the load terminals. Assume that the load's emf is negative. Consider that T_1 or T_2 is conducting at a given time. The conducting transistor is turned off. The current in the inductive load has to continue to flow until the energy in it is depleted to zero. Hence, the diodes D_3 and D_4 will take over, maintaining the load current in the same direction, but the load voltage is negative in the new circuit configuration, as is shown in Figure 4.6. The voltage and current waveforms are shown in Figure 4.7. When diodes D_3 and D_4 are conducting, the source receives power from the load. If the source cannot absorb this power, provision has to be made to consume the power. In that case, the overcharge on the filter capacitor is periodically dumped into a resistor connected across the source by controlling the on-time of a transistor in series with a resistor. This form of recovering energy from the load is known as regenerative braking and is common in low-HP motor drives, where the saving in energy might not be considerable or cost-effective. When the current in the load is decreasing, T_2 is turned on. This allows the short-circuiting of the load through T_2 and D_4, resulting in an increase in the load current. Turning off T_2 results in a pulse of current flowing into the source via D_3 and D_4. This operation allows the priming up of the current and a building up of the energy in the inductor from the load's emf, thus enabling the transfer of energy from the load to the source. Note that it is possible to transfer energy from load to source even when E is lower in magnitude than V_s. This particular operational feature is sometimes referred to as *boost operation* in dc-to-dc power supplies. Priming up the load current can also be achieved alternatively, by using T_1 instead of T_2.

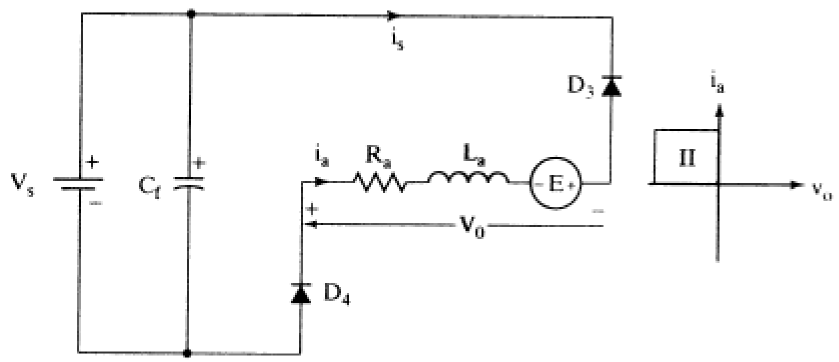


Figure 4.6 Second-quadrant operation, with negative load voltage and positive current

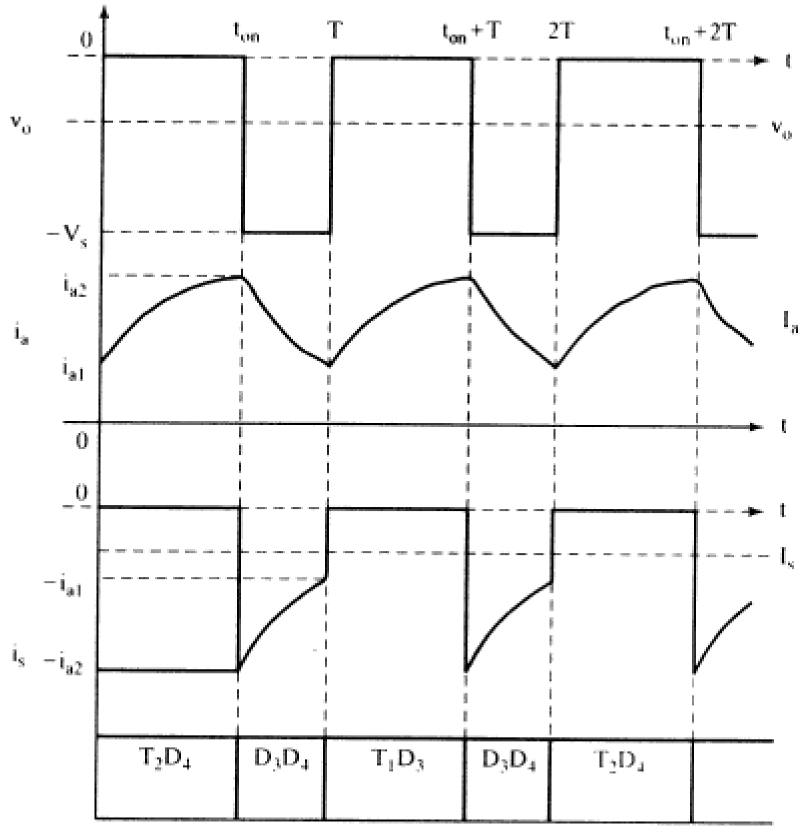
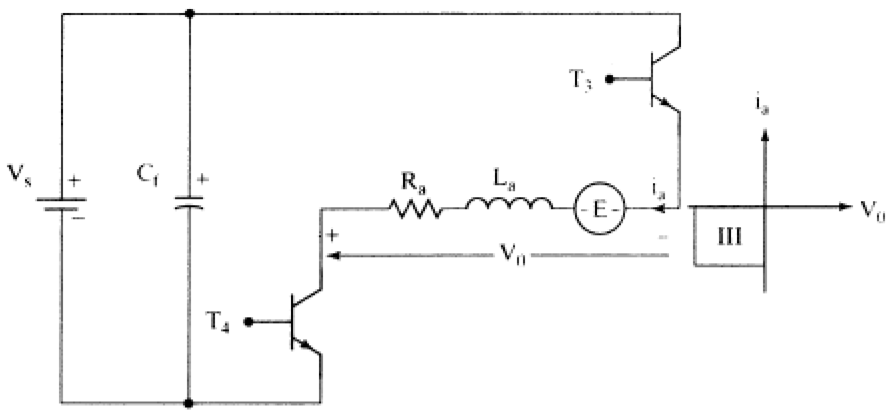


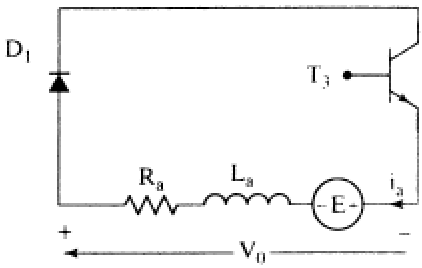
Figure 4.7 Second-quadrant operation of the chopper

4.3.3 Third-Quadrant Operation

Third-quadrant operation provides the load with negative current and voltage. A negative emf source, $-E$, is assumed in the load. Switching on T_3 and T_4 increases the current in the load, and turning off one of the transistors short-circuits the load, decreasing the load current. That way, the load current can be controlled within the externally set limits. The circuit configurations for the switching instants are shown in Figure 4.8. The voltage and current waveforms under continuous and discontinuous current-conduction modes are shown in Figure 4.9. Note the similarity between first- and third-quadrant operation.

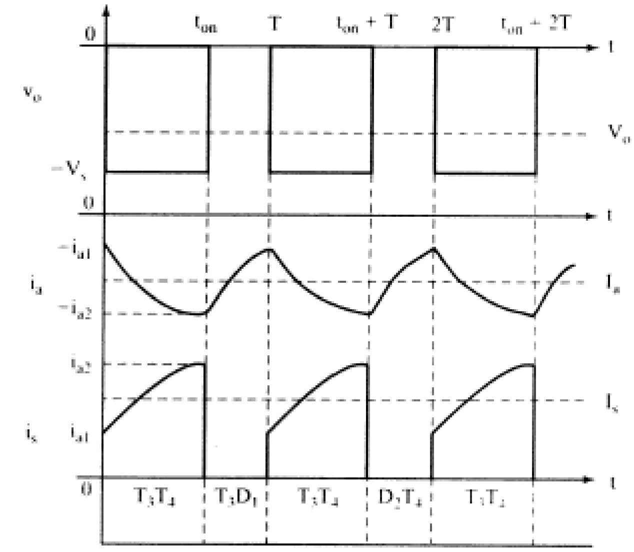


(i) Increasing load current

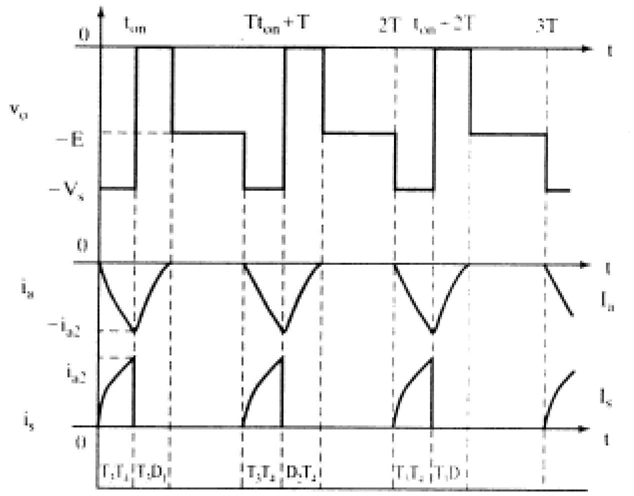


(ii) Decreasing load current

Figure 4.8 Modes of operation in the third quadrant



(i) Continuous Conduction



(ii) Discontinuous Conduction

Figure 4.9 Third-quadrant operation

4.3.4 Fourth-Quadrant Operation

Fourth-quadrant operation corresponds to a positive voltage and a negative current in the load. A positive load-emf source E is assumed. To send energy to the dc source from the load, note that the armature current has to be established to flow

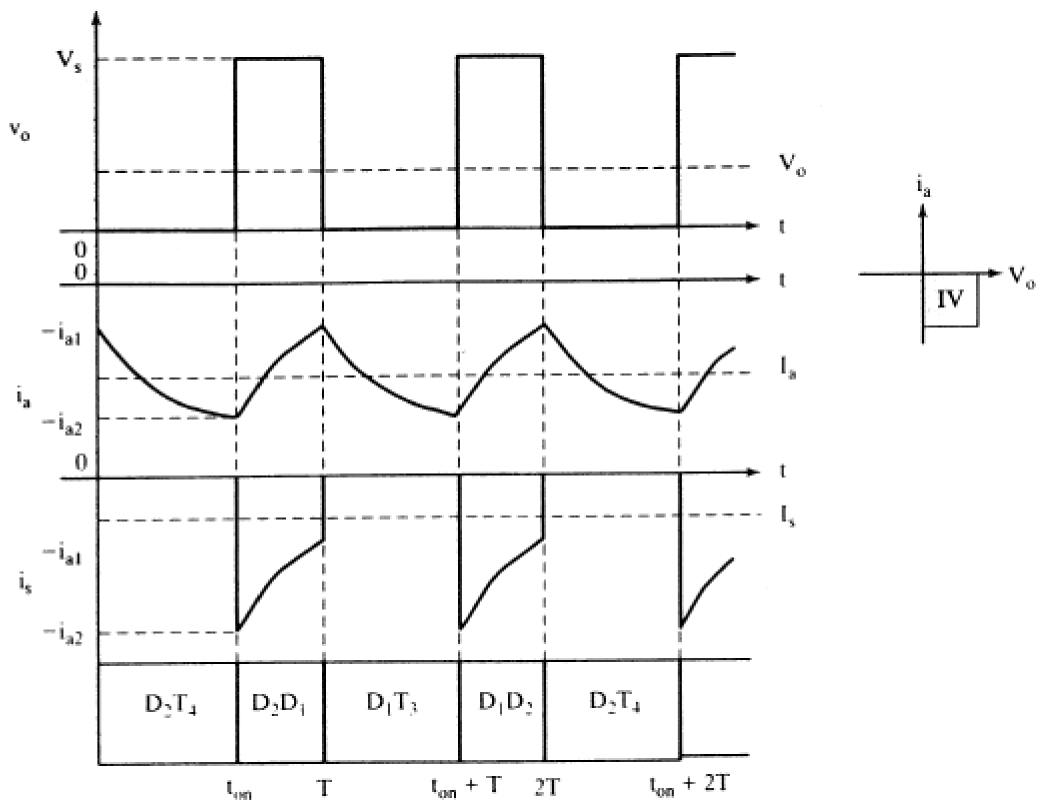


Figure 4.10 Fourth-quadrant operation of the chopper

from the right side to the left side as seen in Figure 4.2. By the convention adopted in this book, that direction of current is negative. Assume that the machine has been operating in quadrant I with a positive current in the armature. When a brake command is received, the torque and armature current command goes negative. The armature current can be driven negative from its positive value through zero. Opening T_1 and T_2 will enable D_3 and D_4 to allow current via the source, reducing the current magnitude rapidly to zero. To establish a negative current, T_4 is turned on. That will short-circuit the load, making the emf source build a current through T_4 and D_2. When the current has reached a desired peak, T_4 is turned off. That forces D_1 to become forward-biased and to carry the load current to the dc input source via D_2 and the load. When the current falls below a lower limit, T_4 is again turned on, to build up the current for subsequent transfer to the source. The voltage and current waveforms are shown in Figure 4.10. The average voltage across the load is positive, and the average load current is negative, indicating that power is transferred from the load to the source. The source power is the product of average source current and average source voltage, and it is negative, as is shown in Figure 4.10.

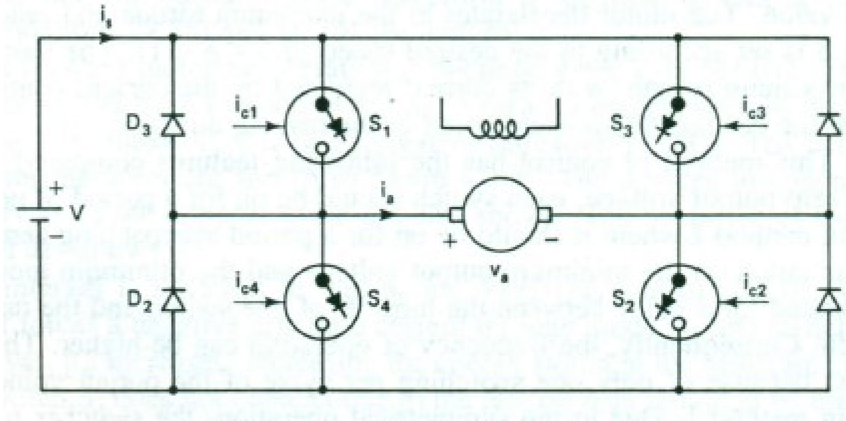


Figure 4.15 Class E four-quadrant chopper.

4.7.3 Four-Quadrant Control

The four-quadrant operation can be obtained by using the class E chopper shown in 4.15. The chopper can be controlled using the following methods.

Method I. If S_2 is kept closed continuously and S_1 and S_4 are controlled, one gets a two-quadrant chopper as shown in figure 4.12a. This provides a variable positive terminal voltage and the armature current in either direction, giving the motor control in quadrants I and II.

Now if S_3 is kept closed continuously and S_1 and S_4 are controlled, a two-quadrant chopper is obtained, which can supply a variable negative terminal voltage and the armature current in either direction, giving motor control in quadrants III and IV.

For the changeover from forward motoring to reverse motoring, the following sequence of steps is followed.

In the first quadrant S_2 is on continuously, and S_1 and S_4 are being controlled. For the changeover, δ is reduced to its minimum value. The motor current reverses [equation (4.33)] and reaches the maximum permissible value. The current control loop restricts it from exceeding the maximum permissible value. The motor decelerates at the maximum torque and reaches zero speed. Now S_2 is opened, S_3 is continuously closed and δ for the pair S_1, S_4 is adjusted corresponding to the desired speed. The motor now accelerates at the maximum torque in the reverse direction and its current is regulated by the current-control loop. Finally it settles at the desired speed.

This method of control has the following features: The utilization factor of the switches is low due to the asymmetry in the circuit operation. Switches S_3 and S_2 should remain on for a long period. This can create commutation problems when the switches are realized using thyristors. The minimum output voltage depends directly on the minimum time for which the switch can be closed. Since there is always a restriction on the minimum time for which the switch can be closed, particularly in thyristor choppers, the minimum available output voltage, and, therefore, the minimum available motor speed, is restricted.

To ensure that the switches S_1 and S_4, and S_3 and S_4 are not on at the same time, some fixed time interval must elapse between the turn-off of one switch and the turn-on of another switch. This restricts the maximum permissible frequency of operation. It also requires two switching operations during a cycle of the output voltage.

Method II. Switches S_1 and S_2 with diodes D_1 and D_2 provide a circuit identical to the chopper of figure 4.13. This chopper can provide a positive current and a variable voltage in either direction, thus allowing motor control in quadrants I and IV. Switches S_3 and S_4 with diodes D_3 and D_4 form another chopper, which can provide a negative current and a variable voltage in either direction, thus allowing the motor control in quadrants II and III.

The switch-over from quadrant I to quadrant III can be carried out using the following sequence of steps. In quadrant I, the switches S_1 and S_2 are controlled with $0.5 < \delta < 1.0$. The armature current has the direction shown in figure 4.15. For the changeover, S_1 and S_2 are turned off. The armature current now flows through diode D_1, source V, and diode D_2, and quickly falls to zero. The motor back emf has the polarity with the left terminal positive. Now the switches S_3 and S_4 are controlled with δ in the range $0 < \delta < 0.5$, but approaching 0.5. The motor current flows in the reverse direction and reaches the maximum value [equation (4.39)]. The current-control loop regulates δ to keep the current from exceeding the maximum permissible value. The motor decelerates at the maximum torque and reaches zero speed. Now δ is set according to the desired speed ($0.5 < \delta < 1$). The motor accelerates at the maximum torque, with its current regulated by the current-control loop and settles at the desired steady-state speed in the reverse direction.

This method of control has the following features compared to method I: At near-zero output voltage, each switch should be on for a period of nearly T sec., unlike in method I where it should be on for a period approaching zero. Thus, there is no limitation on the minimum output voltage and the minimum motor speed. There is no need for a delay between the turn-off of one switch and the turn-on of another switch. Consequently, the frequency of operation can be higher. The switching loss is less because of only one switching per cycle of the output voltage compared to two in method I. Due to the symmetrical operation, the switches have a better utilization factor.

Method III. This method is a modification of method II. In method II, switches S_1 and S_2 with diodes D_1 and D_2 form one chopper, which allows motor control in quadrants I and IV. The second chopper, providing operation in quadrants II and III is formed by switches S_3 and S_4, and diodes D_3 and D_4. In method II, these choppers are controlled separately. In the present method, these choppers are controlled simultaneously as follows.[9]

The control signals for the switches S_1–S_4 are denoted by i_{c1}, i_{c2}, i_{c3}, and i_{c4}, respectively. As with the convention adopted, a switch conducts if its control signal is present and it is forward biased; otherwise it remains open. The control signal i_{c1} to i_{c4}, and the waveform of v_a, i_a, and i_s for forward motoring and forward regeneration are shown in figure 4.16a and b, respectively. Switches S_1 and S_2 are given control signals with a phase difference of T secs. Switch S_1 receives a control signal from $t = 0$ to $t = 2\delta T$, where $\delta = t_{on}/2T$. The control signal for switch S_2 is present from $t = T$ to $t = T + 2\delta T$. Switches S_1 and S_4, and S_2 and S_3 form complementary pairs in the sense that the switches of the same pair receive control signals alternately. Usually some interval must elapse between the turn-off of one switch and the turn-on of another switch of the same pair to ensure that they are not on at the same time. This interval has been neglected in drawing the waveforms of figure 4.16.

In a duration of 2T seconds, which is also the time period of each switch, the chopper operates in four intervals, which are marked as I, II, III, and IV in figures 4.16a and b. The devices under conduction during these intervals are also shown. The operation of the machine in quadrant I can be explained as follows.

In interval I, switches S_1 and S_2 are conducting. The motor is subjected to a positive voltage equal to the source voltage and the armature current increases. At the end of interval I, S_2 is turned off. In interval II, switches S_1 and S_3 receive control signals. Since the motor is carrying a positive current, it flows through a path consisting of D_1 and S_1. Now v_a is zero and i_a is decreasing. Switch S_3 remains off as it is reverse biased by the voltage drop of the conducting diode D_1. At the beginning of interval III, S_2 is turned on again. Now $v_a = V$ and i_a is increasing. At the end of interval III, switch S_1 is turned off. In interval IV, switches S_2 and S_4 receive control signals. The positive motor current flows through S_2 and D_2, and S_4 does not conduct due to the reverse bias applied by the drop of diode D_2.

Note that the output voltage waveform is identical to that of figure 4.14a. Hence, equations (4.38) and (4.39) are applicable.

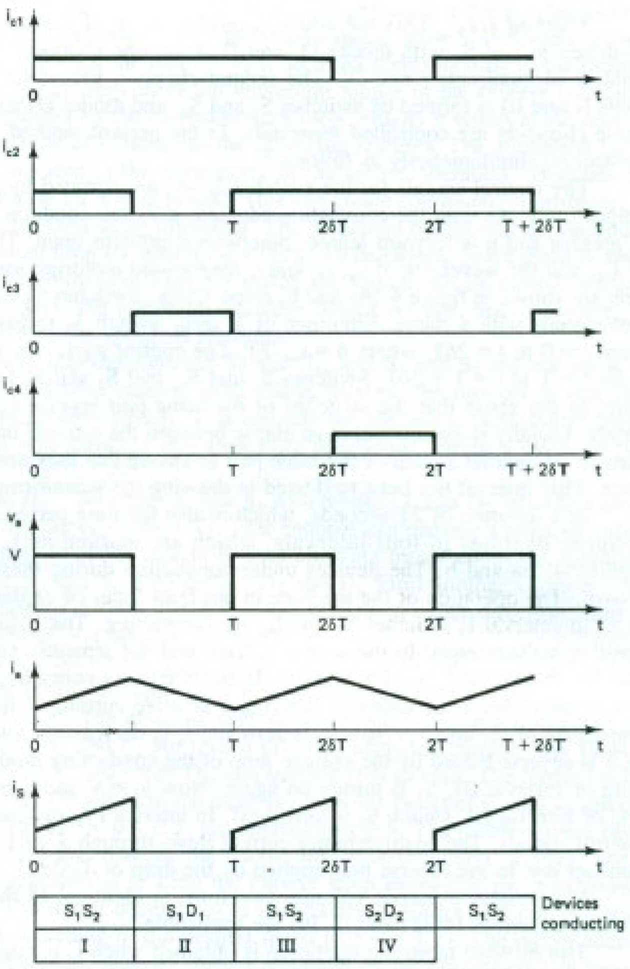
The forward motoring operation is obtained when I_a is positive. The operation can be transferred from forward motoring to forward regeneration by decreasing δ or increasing E to make $V_a < E$ or I_a negative [equation (4.39)]. The waveforms for forward regeneration are shown in figure 4.16b. The devices in conduction in the four intervals of the chopper cycle are also shown. The operation of the chopper is explained as follows.

In interval I, switches S_1 and S_3 are receiving control signals. The positive back emf forces a negative armature current through diode D_3 and switch S_3. During this interval, $|i_a|$ increases, increasing the energy stored in the armature circuit inductance. Switch S_1 does not conduct due to the reverse bias provided by the drop of the conducting diode D_3. Switch S_3 is opened at the end of interval I. The armature current is forced through diode D_3, source V, and diode D_4, and the energy is fed to the source. Although switches S_1 and S_2 are receiving the control signals, they remain open due to the reverse bias provided by the voltage drops of diodes D_3 and D_4. The motor terminal voltage is now V and $|i_a|$ is decreasing. S_4 is turned on in interval III. The armature current now flows through switch S_4 and diode D_4. Switch S_2 also receives a control signal; however, it does not conduct due to the reverse bias applied by diode D_4. The armature current magnitude again builds up. S_4 is turned off at the

end of interval III. The armature current is forced again through diode D_3, the source, and diode D_4, and the energy is fed to the source.

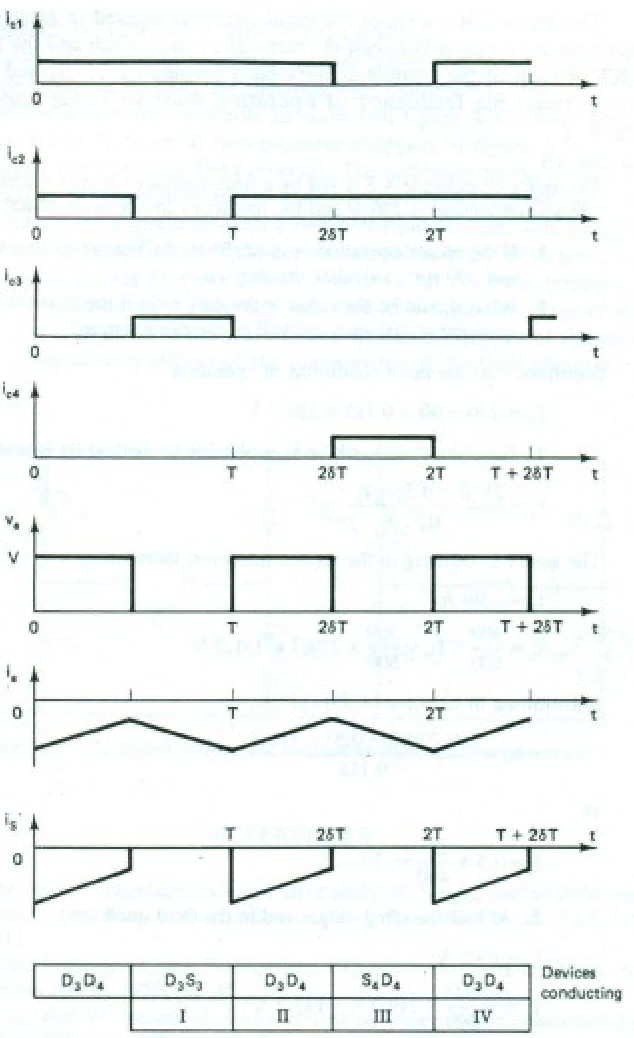
The motoring and regenerative braking operations in the reverse direction are obtained when $0 < \delta < 0.5$, for which V_a is negative. Reverse motoring is obtained by setting δ such that $|V_a| > |E|$ and reverse regeneration is realized when $|E| > |V_a|$.

This method has a simpler control circuit compared to methods I and II. Since some time must elapse between the turn-off of one switch and the turn-on of another switch of each of the complementary pairs formed by S_1, S_4 and S_2, S_3, the maximum permissible frequency of operation must be lower compared to that of method II.



(a) Forward motoring, $0.5 \leq \delta \leq 1.0$ and $V_a > E$

Figure 4.16 Waveforms of the four quadrant chopper of Fig. 4.15 using *method III* *(continued on next page)*.



(b) Forward regeneration, $0.5 \leq \delta \leq 1.0$ and $V_a < E$

Figure 4.16 (continued).

Example 4.5

The motor of example 4.3 is fed by a four-quadrant chopper controlled by method III. The source voltage is 230 V and the frequency of operation is 400 Hz.

1. If the motor operation is required in the second quadrant at the rated torque and 300 rpm, calculate the duty ratio.
2. What should be the value of the duty ratio if the motor is working in the third quadrant at 400 rpm and half of the rated torque?

Solution: At the rated conditions of operation

$$E_r = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. Equation (4.39), which is applicable to method III is reproduced here:

$$I_a = \frac{2V(\delta - 0.5) - E}{R_a} \tag{4.39}$$

The motor is working in the second quadrant, therefore,

$$I_a = -90 \text{ A}$$

$$E = \frac{300}{500} \times E_r = \frac{300}{500} \times 219.7 = 131.8 \text{ V}$$

Substituting in equation (4.39), gives

$$-90 = \frac{2 \times 230(\delta - 0.5) - 131.8}{0.115}$$

or

$$\delta = 0.5 + \frac{121}{460} = .76\,.$$

2. At half the rated torque and in the third quadrant

$$I_a = -45 \text{ A}$$

$$E = -\frac{400}{500} \times 219.7 = -175.7 \text{ V}$$

Substituting in equation (4.39), gives

$$-45 = \frac{2 \times 230(\delta - 0.5) + 175.7}{0.115}$$

or

$$\delta = 0.5 - \frac{181}{460} = 0.11\,.$$

Four-quadrant Operation with Field Control

When field control is required for getting speeds higher than base speed and the transient response need not be fast, the four-quadrant operation is obtained by a combination of field and armature controls as shown in figure 4.17. Both armature and field are supplied by the class D two-quadrant choppers of figure 4.13. The reversal switch RS is employed for the field reversal. The armature chopper provides operation in the first and the fourth quadrant with a positive field current and operation in the second and the third quadrant with a negative field current. When the field connection is to be reversed, first the field current should be reduced to zero. The use of the class D two-quadrant chopper allows a reversal of the field terminal voltage, which forces the field current to become zero fast. The main advantage of this circuit is the lower cost compared to the class E four-quadrant chopper of figure 4.13, because of the lower current ratings of the components of the field chopper.

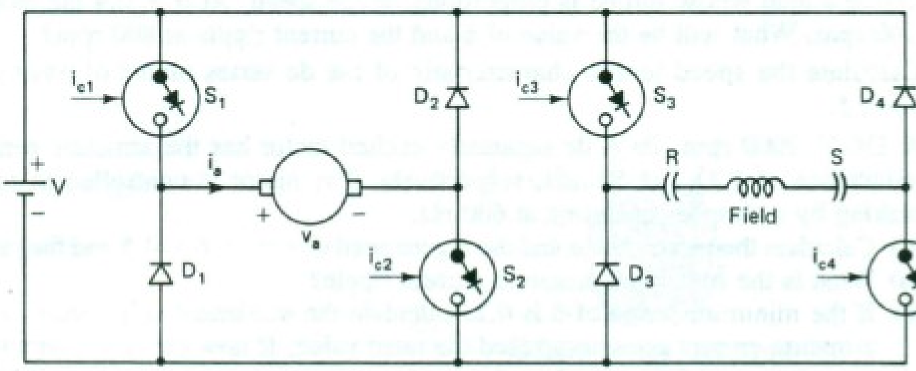


Figure 4.17 Combined armature and field control for four-quadrant operation.

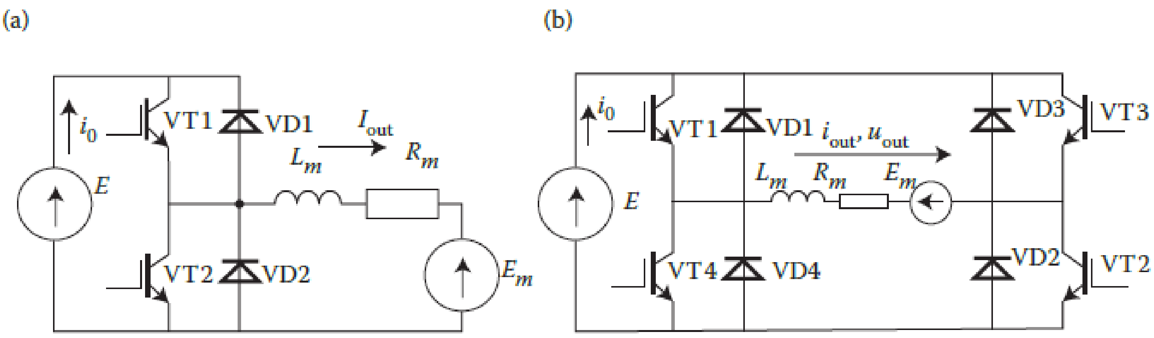


Figure 5.12 (a) Two-quadrant and (b) four-quadrant dc voltage converters.

5.4.2 *Four-quadrant converter*

The rotation of a dc electrical machine may be reversed by means of a four-quadrant dc converter (Figure 5.12b).

The converter may operate in four modes.

Quadrant I. The electrical machine operates in the motor mode, with forward rotation. An output-voltage pulse is formed at the motor input when transistors VT1 and VT2 are switched on simultaneously, and $u_{out} = E$. To create an inactive interval, it is sufficient to switch off one of the transistors—say, VT2. Then the motor current flows through VT1 and diode VD3; $U_{out} = 0$ and $i_0 = 0$. In this quadrant, the converter resembles a step-down dc/dc converter: $U_{out} = \gamma E$.

Quadrant II. The motor turns in the same direction, but with recuperative braking. Consequently, the machine operates in the generative mode, and the current i_{out} is reversed. Two modes alternate in the converter.

- An interval of length γT_{sw}, in which all the transistors are on; current passes through diodes VD1 and VD2; and the motor current flows through source E, to which energy is returned.
- An interval of length $(1 - \gamma)T_{sw}$, in which transistor VT3 is on; the load current passes through the circuit VT3–VD1, bypassing the source; and $i_0 = 0$. The same results may be obtained by switching on VT4, which forms a circuit with diode VD2.

In the second quadrant, the converter resembles a step-up dc/dc converter, in which the energy source is the emf E_m.

Quadrant III. The direction of rotation is reversed; the directions of the voltages and currents in the electrical machine are the opposite to those shown in Figure 5.12b. When transistors VT3 and VT4 are switched on simultaneously, $u_{out} = -E$; energy is sent from the source to the motor. When one of those transistors is switched off, current flows through the circuit consisting of a transistor and a diode, bypassing the source: $u_{out} = 0$; $i_0 = 0$; and $u_{out} = -\gamma E$.

Quadrant IV. Recuperative braking occurs. When all the transistors are switched off, the current in the electrical machine, whose direction is as in Figure 5.12b, passes through the circuit VD3–VD4, returning energy to source E. When transistor VT1 is turned on, current i_{out} flows through diode VD3, bypassing the source. The same result may be obtained by switching on transistor VT2, which forms a circuit with diode VD4.

We may note the similarities between multiquadrant voltage converters and voltage source inverters (Section 6.1). The circuit in Figure 5.12a corresponds to a half-bridge voltage inverter with asymmetric connection of the load and the circuit in Figure 5.12b to a single-phase bridge inverter.

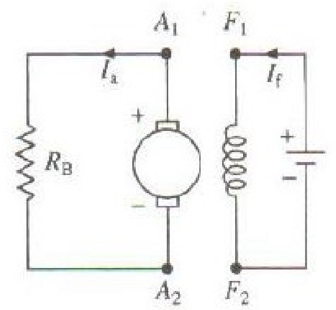
5.4.1 *Two-quadrant converter*

In Figure 5.12a, we show the circuit diagram of a two-quadrant dc converter. The load considered is a dc motor, which is replaced by an equivalent circuit consisting of the motor's counteremf E_m, its resistance R_m, and its inductance L_m.

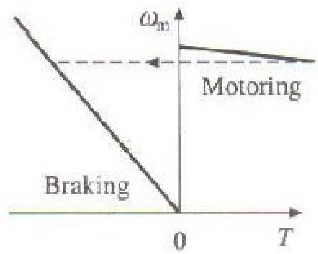
When an electrical machine operates in the motor mode, only transistor VT1 operates in the converter, and transistor VT2 is always off. When transistor VT1 is turned on, it connects source E to the motor; motor current $i_{out} = i_0$ passes through VT1. When transistor VT1 is turned off, the motor current passes through diode VD2, and the voltage applied to the motor is zero. It is readily evident that the conducting section of the circuit corresponds to a step-down dc/dc converter (Section 5.2.1). In that case, neglecting the losses, we write the output voltage in the form

$$U_{out} = E\frac{T_p}{T_{sw}} = \gamma E. \tag{5.32}$$

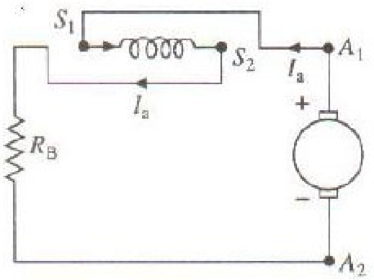
In recuperative braking, the motor continues to turn, and the polarity E_m is unchanged. However, on switching to the generator mode, the polarity of current i_{out} will be the opposite of that in Figure 5.12a. In this mode, no control pulses are sent to transistor VT1. When transistor VT2 is off, current flows through diode VD1; the polarity of current i_0 is reversed, and the motor energy is recuperated to source E. When transistor VT2 is turned on, it transmits the current i_{out}. Operation in the recuperation mode corresponds to a step-up dc/dc converter (Section 5.2.2), if we assume that the energy source is the motor's counteremf E_m.



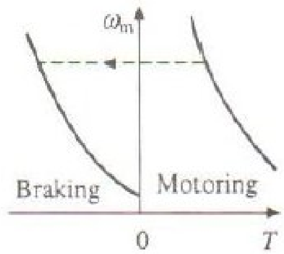
Separately excited motor



Separately excited motor



(b) Series motor



Series motor

6.5. THE FOUR-QUADRANT CHOPPER

A d.c. brush motor with separate excitation is fed through a four-quadrant chopper (Table 6.1e). Show the waveforms of voltage and current in the third and fourth quadrants.

Solution:

The basic circuit of a four-quadrant chopper is shown in Figure 6.9.

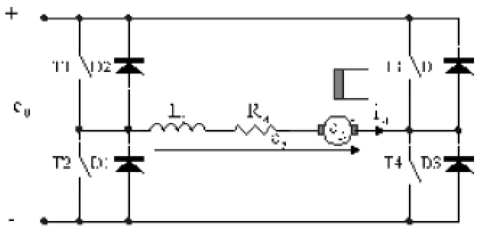


Figure 6.9. D.c. brush motor fed through a four-quadrant chopper

If T_4 is on all the time, T_1-D_1 and T2-D2 provide first- and (respectively) second-quadrant operations as shown in previous paragraphs. With T_2 on all the time and T_3-D_3 and, respectively, T_4-D_4 the third- and fourth-quadrant operations is obtained (Figure 6.10). So, in fact, we have 2 two-quadrant choppers acting in turns.

However, only 2 out of 4 main switches are turned on and off with the frequency f_{ch} while the third main switch is kept on all the time and the fourth one is off all the time.

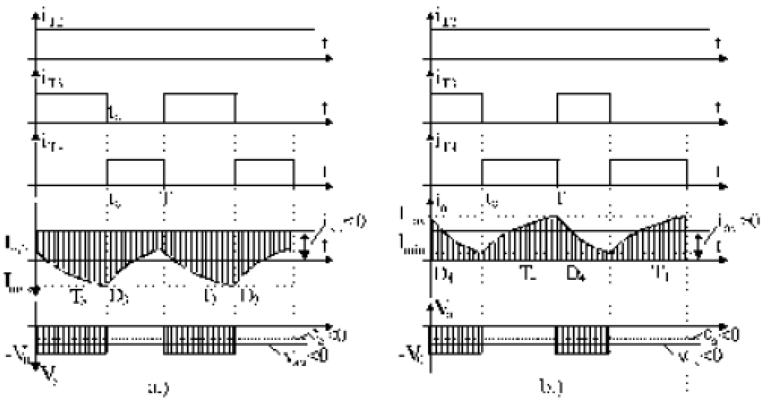


Figure 6.10. Four-quadrant chopper supplying a d.c. brush motor

a.) Third quadrant: iav<0, Vav<0; b.) Fourth quadrant: iav>0, Vav<0.

Four-quadrant operation is required for fast response reversible variable speed drives.

As expected, discontinuous current mode is also possible but it should be avoided by increasing the switching frequency f_{ch} or adding an inductance in series with the motor.

Let us assume that:

A d.c. brush motor, fed through a four-quadrant chopper, works as a motor in the third quadrant (reverse motion). The main data are $V_0 = 120V$, $R_a = 0.5\Omega$, $L_a = 2.5mH$, rated current $I_{an} = 20A$; rated speed $n_n = 3000$ rpm; separate excitation.

a. Calculate the rated e.m.f., e_g, and rated electromagnetic torque, T_e.

b. For $n = -1200$ rpm and rated average current ($i_{av} = -I_{an}$) determine the average voltage V_{av}, t_c / $T = \alpha_{on}$, and maximum and minimum values of motor current I_{max} and I_{min} for 1kHz switching frequency.

Solution:

a. The motor voltage equation for steady state is:

$$V_{av} = R_a i_a + e_g \tag{6.54}$$

for rated values $V_{av} = V_0 = 120V$, $i_a = i_{an} = 20A$, thus

$$e_{gn} = K_e \lambda_p n_n = V_{av} - R_a i_a = 120 - 20 \cdot 0.5 = 110 \text{ V} \tag{6.55}$$

$$K_e \lambda_p = \frac{e_{gn}}{n_n} = \frac{110}{50} = 2.2 \text{ Wb} \tag{6.56}$$

b. The motor equation in the third quadrant is

$$V_{av} = R_a i_a + e_g = 0.5 \cdot (-20) + 2.2 \cdot (-20) = -54 \text{ V}, \tag{6.57}$$

the conducting time t_c for T_3 (Figure 6.10a) is

$$\frac{t_c}{T} = \frac{V_{av}}{-V_0} = \frac{-54}{-120} = 0.45 \tag{6.58}$$

$$t_c = T \cdot 0.45 = \frac{1}{f_{ch}} \cdot 0.45 = \frac{1}{10^3} \cdot 0.45 = 0.45 \cdot 10^{-3} \text{ s} \tag{6.59}$$

Form ((6.40)-(6.41)) the motor current variation (Figure 6.10a) is described by

$$i_a = \frac{V_0' - e_g}{R_a} + A \cdot e^{-t\frac{R_a}{L_a}}; \quad 0 < t \le t_c \tag{6.60}$$

$$i_a' = -\frac{e_g}{R_a} + A' \cdot e^{-(t-t_c)\frac{R_a}{L_a}}; \quad t_c < t \le T \tag{6.61}$$

The current continuity condition ($i_a(t_c) = i_a'(t_c)$) provides

$$t_c = -\frac{L_a}{R_a} \cdot \ln\left[\left(A' - \frac{V_0'}{R_a}\right) / A\right] \tag{6.62}$$

The second condition is obtained from the average current expression

$$i_{av} = \frac{1}{T}\left[\int_0^{t_c} i_a dt + \int_{t_c}^{T} i_a' dt\right] =$$

$$\frac{1}{T}\left\{\frac{V_0' - e_g}{R_a} t_c - \frac{e_g}{R_a}(T - t_c) + \frac{L_a}{R_a}\left[\left(1 - e^{-t_c\frac{R_a}{L_a}}\right)A + A'\left(1 - e^{-(T-t_c)\frac{R_a}{L_a}}\right)\right]\right\} \tag{6.63}$$

From (6.62) and (6.63) we obtain:

$$\left(A' - \frac{V_0'}{R_a}\right) / A = e^{-t_c\frac{R_a}{L_a}};$$

$$V_0' = -V_0; \quad e_g = K_e \lambda_p n = 2.2 \cdot (-20) = -44 \text{ V}$$

$$\left(A' + \frac{(-120)}{0.5}\right) / A = e^{-0.45 \cdot 10^{-3} \frac{0.5}{2.5 \cdot 10^{-3}}} = 0.914 \tag{6.64}$$

$$-20 = 10^{3}\left\{\frac{-120-(-44)}{0.5}0.45\cdot 10^{-3} - \frac{(-44)}{0.5}0.55\cdot 10^{-3}\right.$$
$$\left. + \frac{2.5\cdot 10^{-3}}{0.5}\left[\left(1-e^{-0.45\cdot 10^{-3}\frac{0.5}{3.5\cdot 10^{-3}}}\right)A + A'\left(1-e^{-0.55\cdot 10^{-3}\frac{0.5}{3.5\cdot 10^{-3}}}\right)\right]\right\} \tag{6.65}$$

$$-20 = -20 + 0.43A + 0.5205A' \tag{6.66}$$

$$0.43A + 0.5205A' = 0 \tag{6.67}$$

$$A' + 240 = 0.914A \tag{6.68}$$

$$A = 137.62;\ A' = -113.92 \tag{6.69}$$

Now we may calculate $I_{min} = i_a(0)$

$$I_{min} = A + \frac{V_a' - e_g}{R_a} = 137.92 + \frac{-120-(-44)}{0.5} = -15.08 \text{ A} \tag{6.70}$$

Also $I_{max} = i_a'(t_c)$

$$I_{max} = A' - \frac{e_g}{R_a} = -113.92 + \frac{-(-44)}{0.5} = -25.92 \text{ A} \tag{6.71}$$

7.4 4-Quadrant Chopper

The circuit diagram to facilitate 4-quadrant operation is given in Fig. 7.5(a). This circuit contains four controlled switches and four diodes. Here, v_a and i_a are indicated with reference to motor terminals A and AA. The controlled switches are realized using IGBTs in this chapter. This circuit is capable of providing motoring, regenerative braking, operation of the motor in the reverse speed and regenerative braking in that direction corresponding to the four quadrants of V-I diagram, which is shown in Fig. 7.5(b). The following section explains the converter fed drive characteristics in four different quadrants.

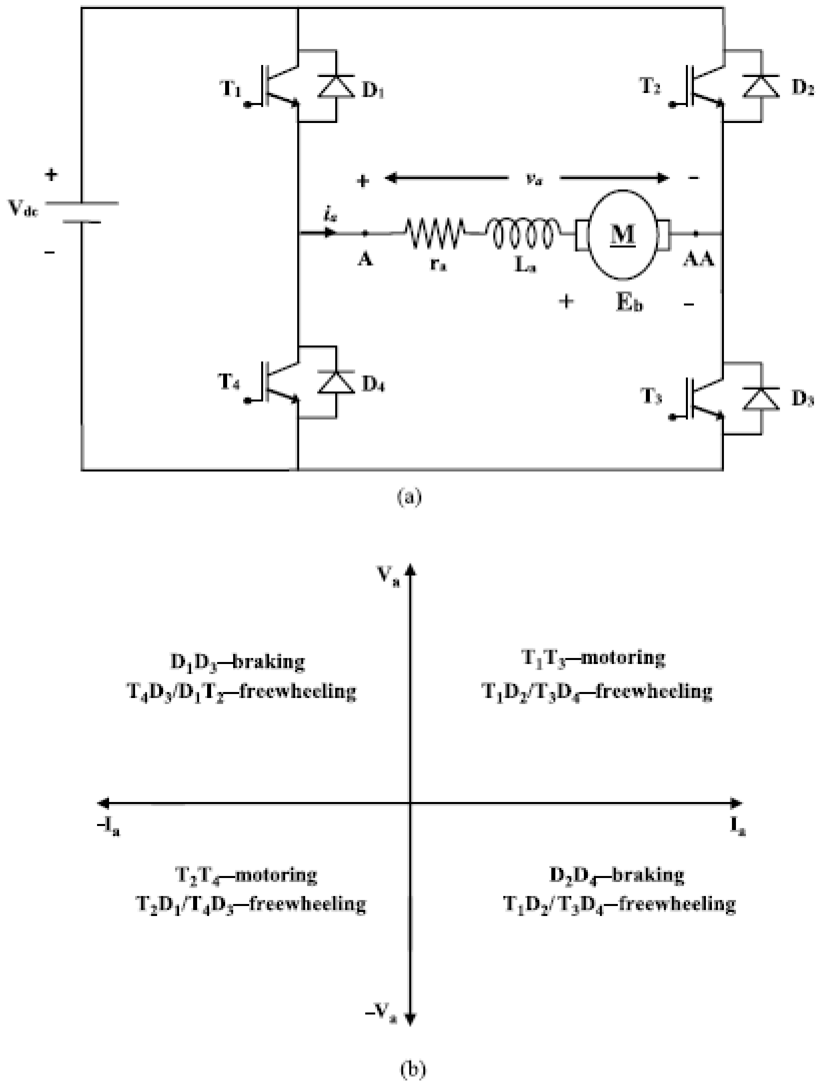


FIGURE 7.5
(a) 4-quadrant chopper circuit. (b) V-I diagram.

7.4.1 Motoring in the Forward Direction

Fig. 7.6(a) shows the devices conducting during the motoring of the drive in the forward direction. During the period T_{on} of the converter, the IGBTs T_1 and T_3 are simultaneously gated so that $v_a = V_{dc}$ and the armature current i_a is positive. During the OFF period of the dc/dc converter, T_1 alone is switched OFF, such that the armature current i_a now freewheels through T_3 and D_4, making the motor terminal voltage zero. The motor terminal voltage and current waveforms for continuous mode are shown in Fig. 7.6(b). As seen in this figure, the average values of voltage and current are positive, thus the average output power is always positive, leading to first-quadrant operation as indicated in Fig. 7.6(c). It may be noted that freewheeling of the armature current is also possible through D_2 and T_1, which is indicated by a dotted line.

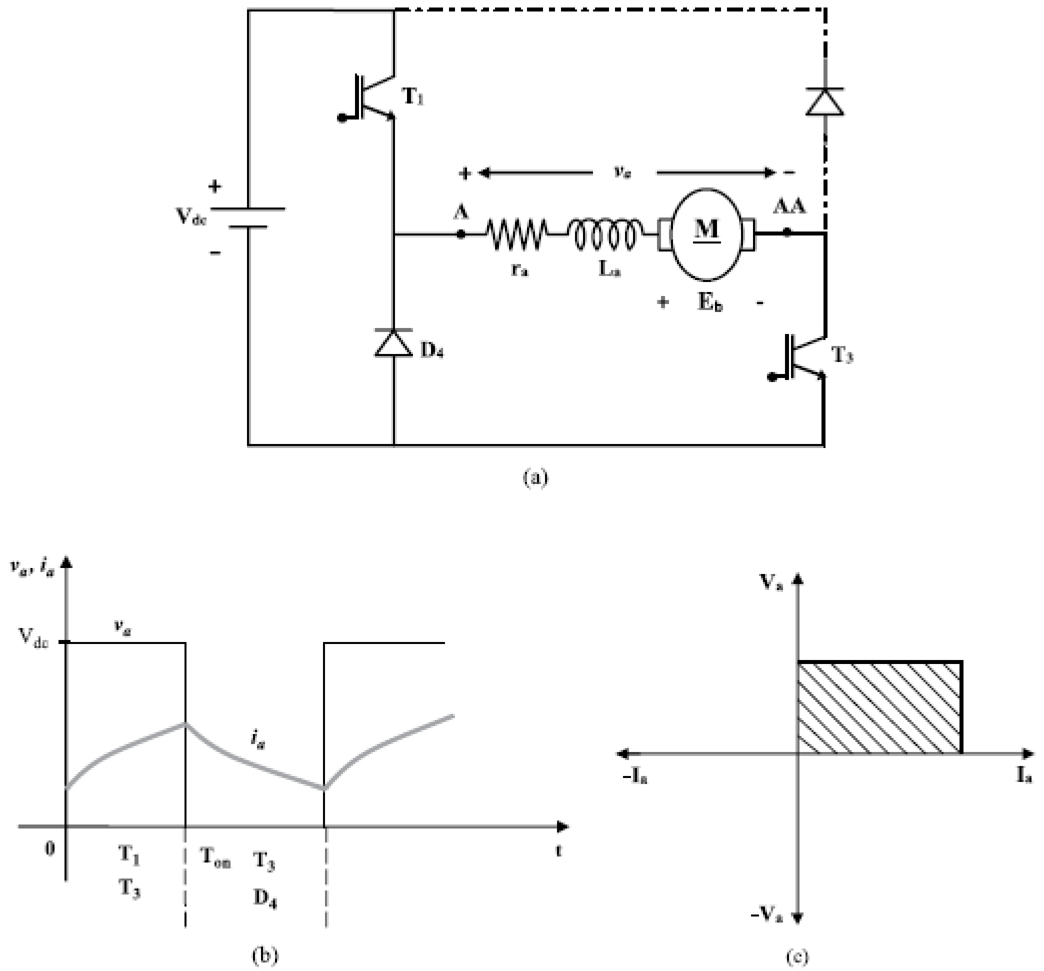


FIGURE 7.6
First-quadrant circuit. (a) Equivalent circuit. (b) Motor voltage and current waveforms. (c) V-I diagram.

7.4.2 Regenerative Braking after Forward Rotation

When regenerative braking is required, the IGBT T_1 is switched OFF. For the regenerative braking to take place, the electromagnetic torque (T_e) must be made negative. Because $T_e \propto (\Phi_m I_a)$, reversal of polarity of T_e is possible by changing the polarity of either Φ_m or I_a. We consider the case of reversal of I_a alone because reversal of Φ_m requires more time due to the increased field time constant.

Consider Fig. 7.7(a). Assume that freewheeling was taking place through T_3 and D_4 and that the regenerative braking command has come during the freewheeling action. To initiate the armature current reversal process during regenerative braking, T_4 is triggered and the gating signal to T_3 is continued till armature current goes to zero. Although T_4 is gated, the forward voltage drop across D_4 prevents T_4 from conducting and, as such, T_3-D_4 continues to cause freewheeling armature current. When this current drops to zero, the back-emf E_b causes reversal of armature current through T_4 and D_3. Armature current, i_a now flows from A to AA and is in the negative direction as shown in Fig. 7.7(b). This current rises exponentially, and when T_4 is turned OFF, the armature current maintains its direction

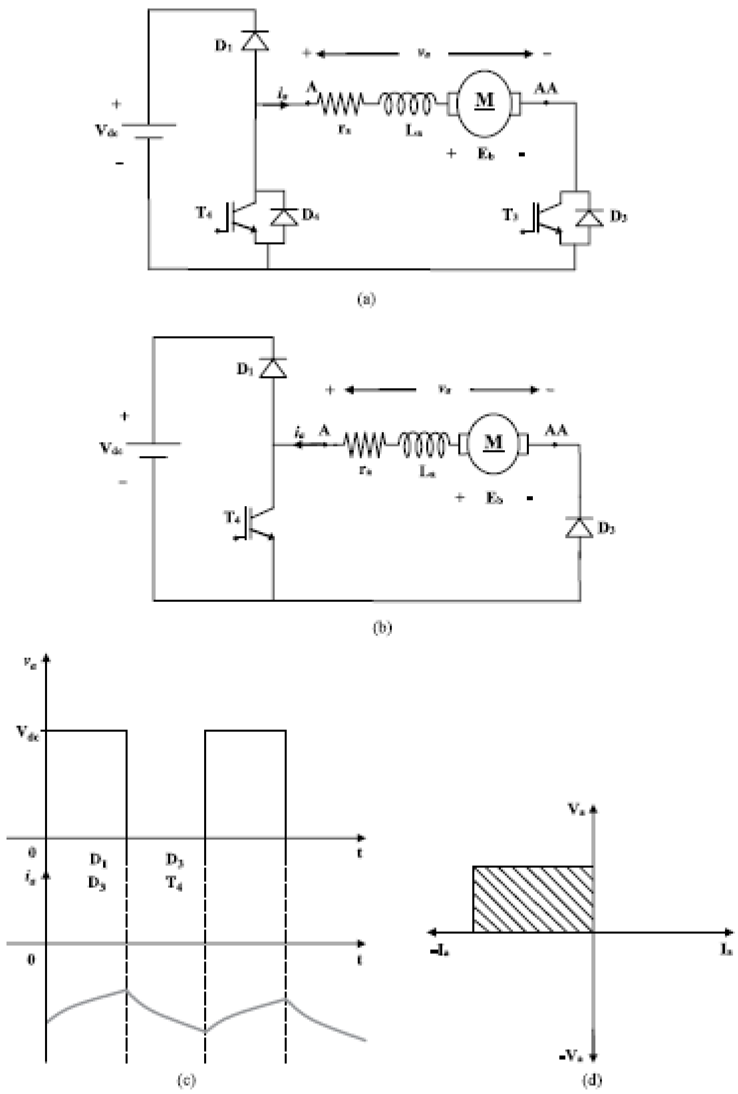


FIGURE 7.7
Second-quadrant operation. (a) Circuit during transition. (b) Circuit during regenerative braking. (c) Motor voltage and current waveforms. (d) V-I diagram.

through D_3-D_1 flowing to the source leading to regenerative braking. Pulse-width modulation of T_4 results in uniform braking. Armature voltage and current during regeneration are sketched in Fig. 7.7(c). This is a second-quadrant operation in the V-I diagram and is given in Fig. 7.7(d). It may be noted that free-wheeling of motor current can also take place through D_1-T_2.

7.4.3 Motoring in the Reverse Direction/Third-Quadrant Operation

Now the motor should be accelerated in the reverse direction. To achieve this, IGBTs T_2 and T_4 are switched ON, and both v_a and i_a get reversed. The product of v_a and i_a is positive, indicating that the process is in the motoring operation. Because i_a is reversed, the direction of electromagnetic torque is also reversed, so the motor is accelerated in the opposite direction. Freewheeling of the armature current can take place either through T_4-D_3 or D_1-T_2. The power circuit, steady-state voltage, current waveforms, and V-I diagram are given in Fig. 7.8.

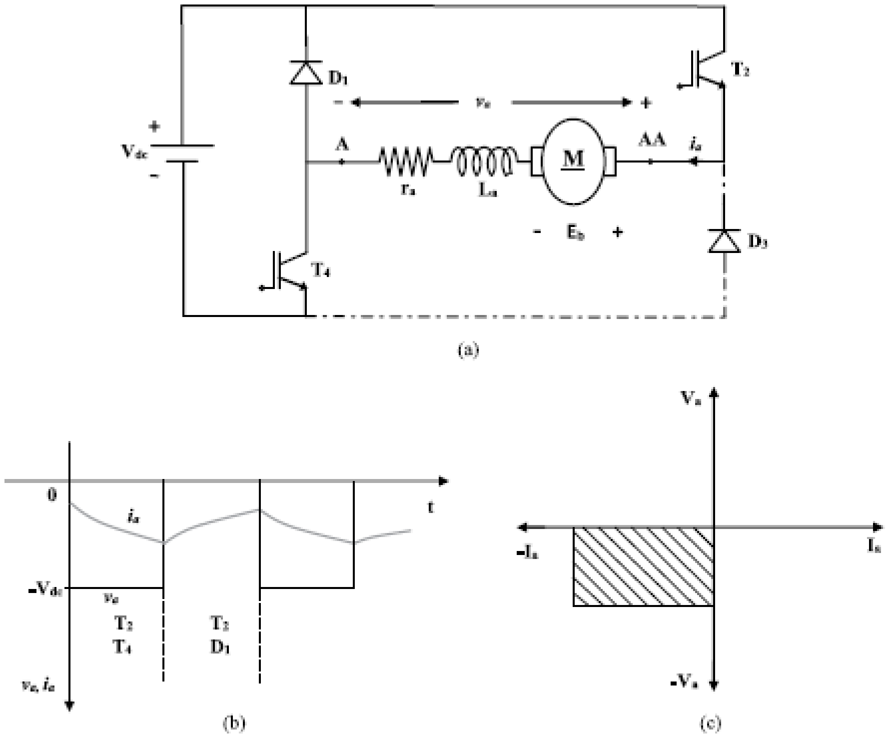


FIGURE 7.8
Motoring in reverse-braking. (a) Equivalent circuit. (b) Motor voltage and current waveforms. (c) V-I diagram.

7.4.4 Regenerative Braking after Speed Reversal

A process similar to that for second-quadrant operation takes place for the regenerative braking mode. Referring to Fig. 7.9(a), D_1-T_2 is the path of freewheeling, and when T_1 is triggered for regenerative braking, T_1 is prevented from conducting because of the forward voltage drop across D_1. The devices D_1-T_2 stop conducting when the armature current becomes zero, and the back-emf now drives the armature current through D_2 and T_1. Now armature current i_a flows from A to AA and hence is positive. When T_1 is turned OFF,

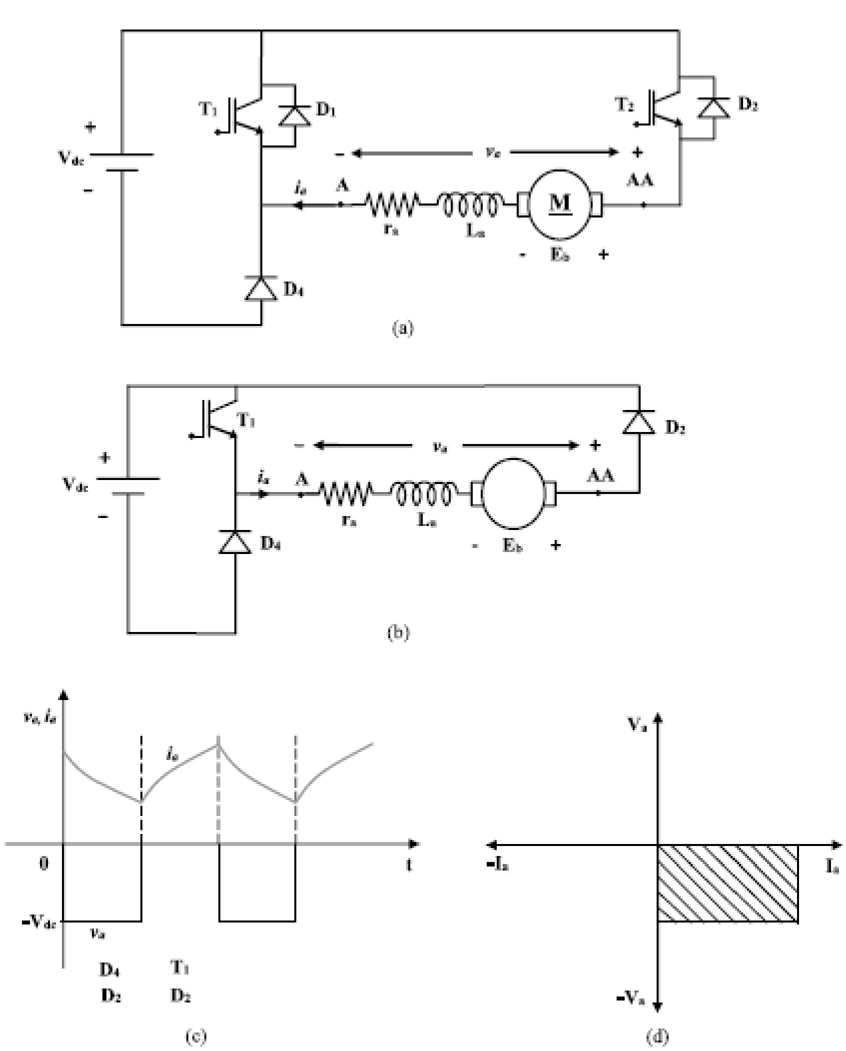


FIGURE 7.9
Fourth-quadrant operation. (a) Circuit during transition. (b) Circuit during regenerative braking. (c) Motor voltage and current waveforms. (d) V-I diagram.

regeneration takes place through D_4-D_2 and leads to fourth-quadrant operation. The power circuit, typical steady-state waveforms, and V-I diagram are given in Fig. 7.9.

7.4.5. Four-quadrant Chopper, or Type-E Chopper

The power circuit diagram for a four-quadrant chopper is shown in Fig. 7.10 (*a*). It consists of four semiconductor switches CH1 to CH4 and four diodes D1 to D4 in antiparallel. Working of this chopper in the four quadrants is explained as under :

First quadrant : For first-quadrant operation of Fig. 7.10 (*a*), CH4 is kept on, CH3 is kept off and *CH1 is operated.* With CH1, CH4 on, load voltage $v_0 = V_s$ (source voltage) and load current i_0 begins to flow. Here both v_0 and i_0 are positive giving first quadrant operation. When CH1 is turned off, positive current freewheels through CH4, D2. In this manner, both V_0, I_0 can be controlled in the first quadrant.

Second quadrant : *Here CH2 is operated* and CH1, CH3 and CH4 are kept off. With CH2 on, reverse (or negative) current flows through L, CH2, D4 and E. Inductance L stores energy during the time CH2 is on. When CH2 is turned off, current is fed back to source through diodes D1, D4. Note that here $\left(E + L\dfrac{di}{dt}\right)$ is more than the source voltage V_s. As load voltage V_0 is positive and I_0 is negative, it is second quadrant operation of chopper. Also, power is fed back from load to source.

Third quadrant : For third-quadrant operation of Fig. 7.10 (*a*), CH1 is kept off, CH2 is kept on and *CH3 is operated.* Polarity of load emf E must be reversed for this quadrant working. With CH3 on, load gets connected to source V_s so that both v_0, i_0 are negative leading to third quadrant operation. When CH3 is turned off, negative current freewheels through CH2, D4. In this manner, v_0 and i_0 can be controlled in the third quadrant.

Fourth quadrant : *Here CH4 is operated* and other devices are kept off. Load emf E must have its polarity reversed to that shown in Fig. 7.10 (*a*) for operation in the fourth quadrant. With CH4 on, positive current flows through CH4, D2, L and E. Inductance L stores energy during the time CH4 is on. When CH4 is turned off, current is fed back to source through diodes D2, D3. Here load voltage is negative, but load current is positive leading to the chopper operation in the fourth quadrant. Also power is fed back from load to source.

The devices conducting in the four quadrants are indicated in Fig. 7.10 (*b*).

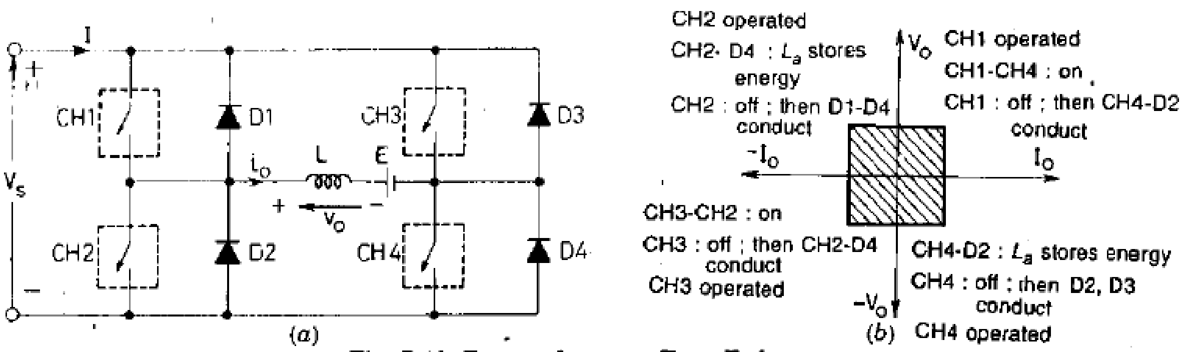


Fig. 7.10. Four-qudrant, or Type-E chopper
(*a*) circuit diagram and (*b*) operation of conducting devices.

Solved Examples

Example 8.16 A four-quadrant chopper is driving a separately excited dc motor load. The motor parameters are $R = 0.1$ ohm, $L = 10$ mH. The supply voltage is 200 V d.c. If the rated current of the motor is 10 A and if the motor is driving the rated torque. Determine:

(i) the duty cycle of the chopper if $E_b = 150$ V.
(ii) the duty cycle of the chopper if $E_b = -110$ V.

Solution:

For a four-quadrant chopper, the average voltage in all the four-modes is given by

$$E_0 = 2\,E_{dc} \cdot (\alpha - 0.5)$$

(i) The average current, $i_0 = \dfrac{E_0 - E_b}{R} = \dfrac{2\,E_{dc} \cdot (\alpha - 0.5) - E_b}{R}$

$$10 = \frac{2 \times 200\,(\alpha - 0.5) - 150}{0.1} \quad \therefore\ \alpha = 0.876$$

Since, $\alpha > 0.5$, this mode is forward-motoring

(ii) Now, $10 = \dfrac{2 \times 200\,(\alpha - 0.5) - 110}{0.1}, \quad \therefore\ \alpha = 0.228$

As $\alpha < 0.5$, this mode is reverse motoring mode.

8.5.5 Four-Quadrant Chopper (or Class E Chopper)

Figure 8.25(a) shows the basic power circuit of Type E chopper. From Fig. 8.25, it is observed that the four-quadrant chopper system can be considered as the parallel combination of two Type C choppers. In this chopper configuration, with motor load, the sense of rotation can be reversed without reversing the polarity of excitation. In Fig. 8.25, CH_1, CH_4 D_2 and D_3 constitute one Type C chopper and CH_2, CH_3, D_1 and D_4 form another Type C chopper circuit. Figure 8.25(b) shows Class-E with R–L load.

If chopper CH_4 is turned on continuously, the antiparallel connected pair of devices CH_4 and D_4 constitute a short-circuit. Chopper CH_3 may not be turned on at the same time as CH_4 because that would short circuit source E_{dc}.

With CH_4 continuously on, and CH_3 always off, operation of choppers CH_1 and CH_2 will make E_0 positive and I_0 reversible, and operation in the first and second quadrants is possible. On the other hand, with CH_2 continuously on and CH_1 always off, operation of CH_3 and CH_4 will make E_0 negative and I_0 reversible, and operation in the third and fourth quadrants is possible.

The operation of the four-quadrant chopper circuit is explained in detail as follows:

When choppers CH_1 and CH_4 are turned-on, current flows through the path, $E_{dc+} - CH_1$ – load – $CH_4 - E_{dc-}$. Since both E_0 and I_0 are positive, we get the first quadrant operation. When both the choppers CH_1 and CH_4 are turned-off, load dissipates its energy through the path load–$D_3 - E_{dc+} - E_{dc-} - D_2$ – load. In this case, E_0 is negative while I_0 is positive, and fourth-quadrant operation is possible.

When choppers CH_2 and CH_3 are turned-on, current flows through the path, $E_{dc+} - CH_3$ – load – $CH_2 - E_{dc-}$. Since both E_0 and I_0 are negative, we get the third-quadrant operation. When both choppers CH_2 and CH_3 are turned-off, load dissipates its energy through the path load– $D_1 - E_{dc+} - E_{dc-} - D_2$ – load. In this case, E_0 is positive and I_0 is negative, and second-quadrant operation is possible.

This four-quadrant chopper circuit consists of two bridges, forward bridge and reverse bridge. Chopper bridge CH_1 to CH_4 is the forward bridge which permits energy flow from source to load. Diode bridge D_1 to D_4 is the reverse bridge which permits the energy flow from load-to-source. This four-quadrant chopper configuration can be used for a reversible regenerative d.c. drive.

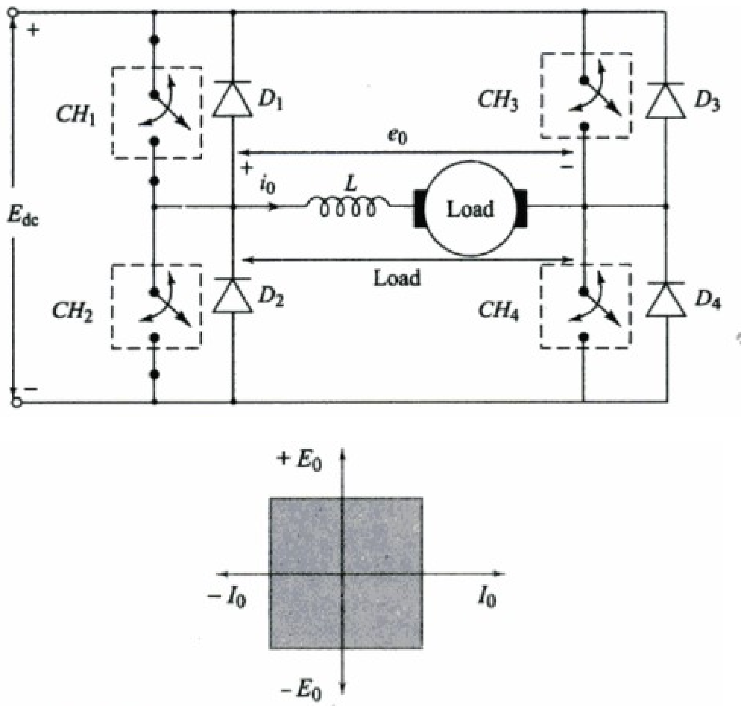


Fig. 8.25(a) *Type E chopper circuit and characteristic*

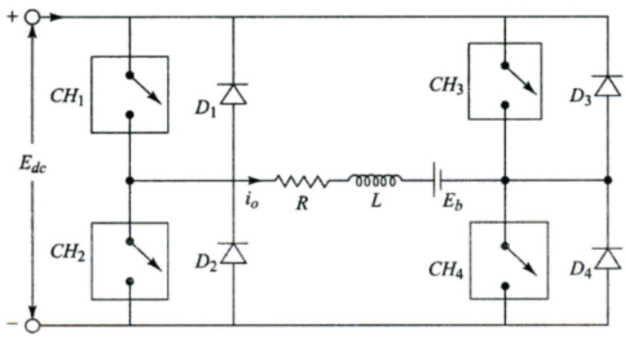


Fig. 8.25(b) *Class E chopper with R–L load*

The bridge type converter shown in Fig. 9.13 a is connected to the armature circuit of a DC motor; it may be supplied with constant voltage u_D from a DC bus or a battery. The converter contains four electronic switches where two in each half-bridge are drawn in the form of a transfer switch (at the same time excluding accidental short circuits of the DC bus); the diodes which can be part of the electronic switches allow an inductive load current to continue during the short protective intervals, when all contacts are open (similar to the red-light-overlap on a signal crossing).

By assigning logic symbols S_1, S_2 to the otherwise ideally assumed switches, the voltage equation of the load circuit is

$$L_a \frac{i_a}{dt} + R_a i_a + e = u_a \,, \tag{9.8}$$

where, depending on the switching state

$$u_a = \frac{1}{2}(S_1 - S_2)\, u_D \text{ and } i_D = \frac{1}{2}(S_1 - S_2)\, i_a \tag{9.9}$$

holds.

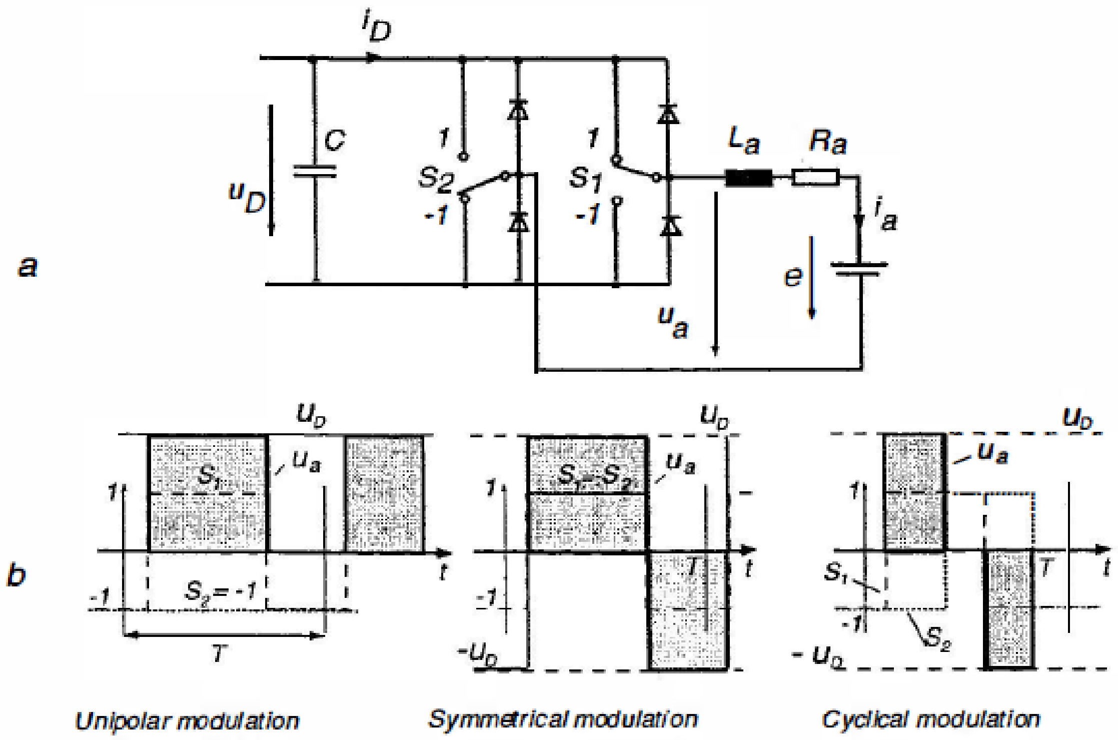


Fig. 9.13. Four-quadrant DC/DC converter with inductive load,
(a) Circuit, **(b)** Different modulation patterns

The pulse-width-modulation (PWM) of the converter at the frequency $f = 1/T$ can follow different switching strategies, as illustrated in Fig. 9.13 b with the output voltage u_a during a switching period:

- Unipolar modulation
  One of the switches is assumed to be stationary, e.g. S_2 = -1 = const., whereas the other half-bridge is pulse-width-modulated, $S_2 = \pm 1$, so that the output voltage u_a assumes the values u_D or zero; the same applies with $S_1 = -1 =$ const. for negative output voltages.
- Symmetrical modulation
  With this modulation pattern the switches are operated in diagonal pairs, S_1 = - S_2, so that the short circuit interval is omitted and the output voltage alternates between the values u_D and -u_D. During the unavoidable (but in Fig. 9.13 neglected) protective intervals the diodes are carrying the load current.
- Cyclical modulation
  With this modulation scheme the two transfer switches are operated sequentially, so that the output is alternatively short circuited at the upper or lower supply bus. Hence the output voltage u_a assumes a ternary waveform, $u_a = u_D$, 0, -u_D. Whereas with symmetrical switching only the mean of the output voltage can be controlled in steady state, the cyclical modulation offers an additional degree of freedom that may for instance be used for eliminating harmonics of the output voltage u_a.

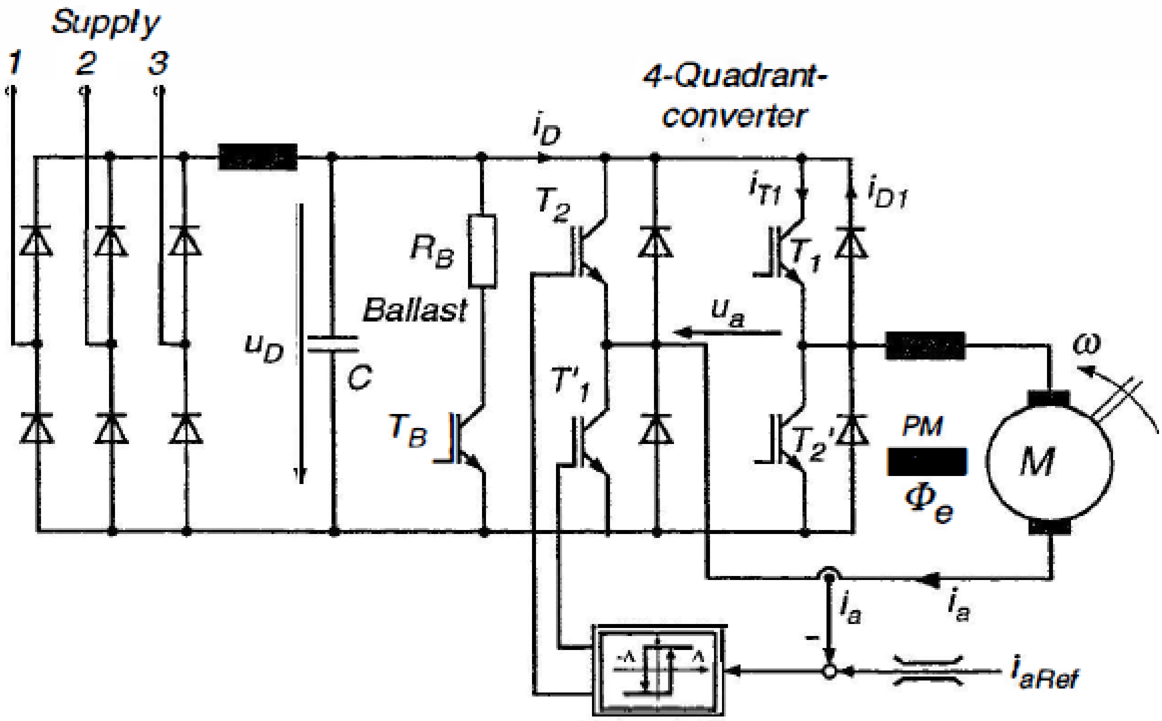


Fig. 9.14. DC Servo drive with voltage source DC/DC converter

A four-quadrant converter with IGBT-switches is depicted in Fig. 9.14 as frequently used for DC servo drives; protective circuitry is again omitted. For simplicity an On/Off device is drawn for current control but a linear current controller with constant frequency PWM would normally be preferred.

A diode rectifier followed by a smoothing filter, whose capacitor C absorbs also the modulation-induced ripple components of the link current i_D serves as the supply of the DC link with constant voltage u_D. For instance, when rapidly braking the drive, power released from the kinetic energy flows back into the DC-link causing negative current i_D and, because of the unidirectional line-side rectifiers, could result in an overcharge of the capacitor; this is prevented by dissipating the energy in a resistive ballast circuit that can also be pulse-width-modulated, depending on the link voltage. In view of the losses this is only practical with small drives or when it happens only occasionally; otherwise a reversible line-side supply (an active front-end converter) is preferable as will be shown in Fig. 9.18 and further discussed in Sect. 13.2.

Some of the steady state waveforms in a converter like the one in Fig. 9.14 are indicated in Fig. 9.15, showing the output voltage u_a alternating between u_D and $-u_D$ and the alternating current components of i_D, which must be absorbed by the capacitor. The control can be arranged as before; an inner current loop controlling the converter via a pulse-width modulator is important for safe operation. The current controller in Fig. 9.14 is again drawn as an On/Off switch, but this is only an illustrative example.

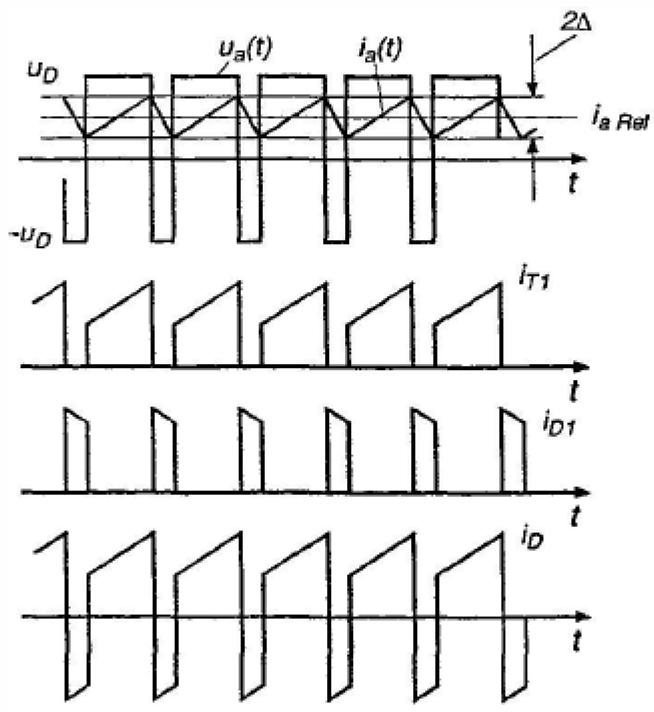
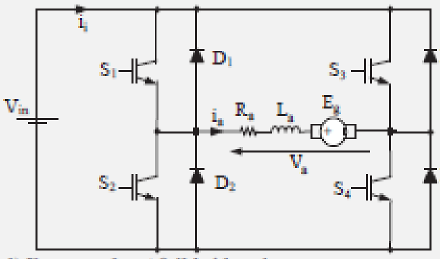
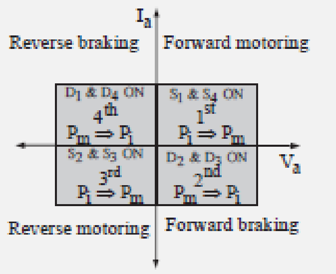
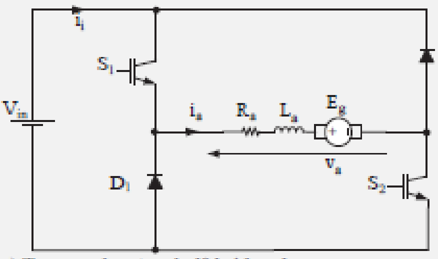
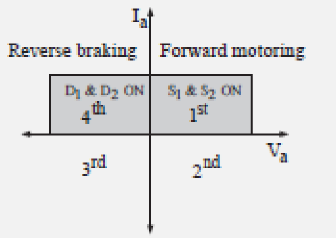


Fig. 9.15. Waveforms of DC/DC converter with symmetrical modulation

The typical response time of the current loop, employing a switched transistor converter in combination with a DC disk motor, is 1 or 2 ms. For many applications this justifies the assumption that the current control loop acts as controllable current source having instantaneous response. Current limit is achieved by limiting the current reference produced by the superimposed speed controller. The next higher level of control could be a position control loop as shown in Fig. 15.9, where the response may be further improved by feed-forward signals from a reference generator.

Transistor converters have the important advantage that they can be switched at frequencies > 5 kHz, thus enlarging the control bandwidth as compared to line-commutated converters. With field effect transistors or IGBT's, the frequency can even be increased beyond the audible threshold > 16 kHz, so that the drive is no longer emitting objectionable acoustic noise.

Table 12.3 DC motor drive systems employing dc–dc converters

dc–dc converter topology	Quadrant(s) of operation
 d) Four-quadrant full-bridge chopper	
 e) Two-quadrant or half-bridge chopper	

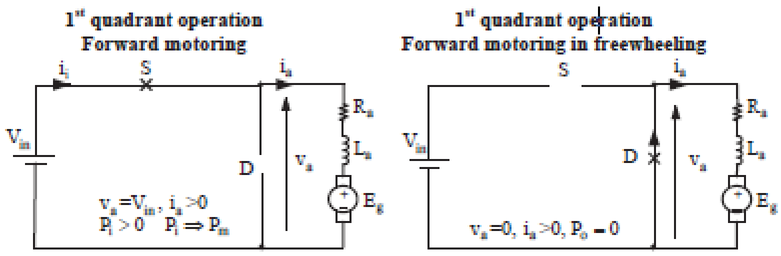


Figure 12.12 Operating modes of the step-down or first quadrant chopper presented in Table 12.3.

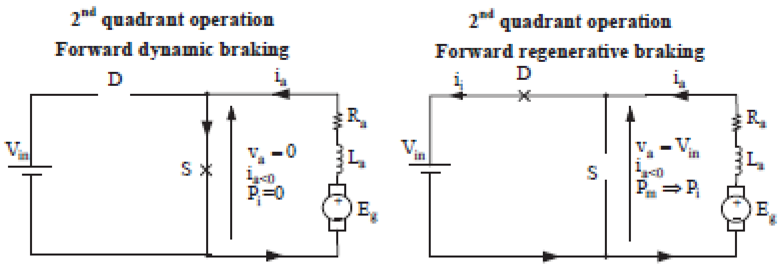


Figure 12.13 Operating modes of the step-up or second quadrant chopper presented in Table 12.3.

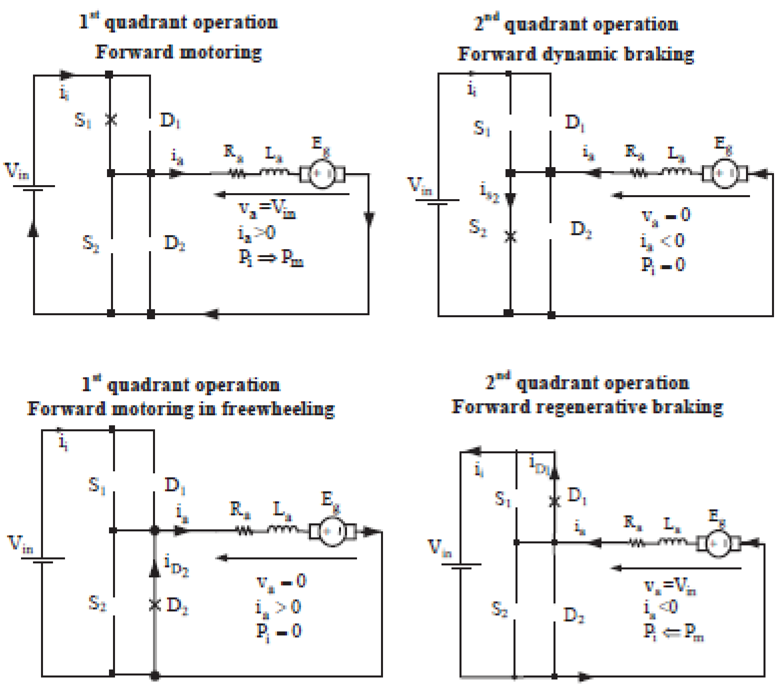
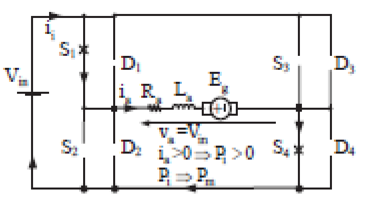


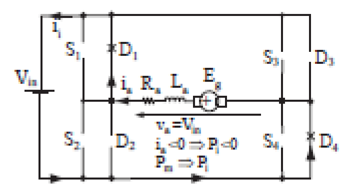
Figure 12.14 Operating modes of the two-quadrant converter presented in Table 12.3.

1st quadrant operation



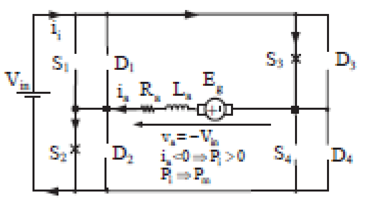
During this mode of operation only the switches S_1 and S_4 are conducting applying positive voltage and current to the armature. The machine is this mode operates as a motor with clockwise rotation. Keeping S_1 in conduction and applying PWM to S_4 speed control is achieved. Freewheeling current is achieved through S_1 and D_3.

2nd quadrant operation



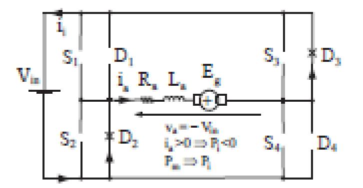
During this mode of operation only the diodes D_1 and D_4 are conducting applying positive voltage to the armature but creating a negative flow current. The machine in this mode operates as a generator with clockwise speed and, consequently, a regenerative braking mode is achieved.

3rd quadrant operation



During this mode of operation only the switches S_2 and S_3 are conducting applying negative voltage to the armature and creating a negative armature current. The machine is this mode operates as a motor with counter clockwise rotation. Keeping S_2 in conduction and applying PWM to S_3 speed control is achieved. Freewheeling current is achieved through S_2 and D_4.

4th quadrant operation



During this mode of operation only the diodes D_2 and D_3 are conducting applying negative voltage to the armature but creating a positive flow of armature current. The machine in this mode operates as a generator with counter clockwise speed and, consequently, a regenerative braking mode is achieved.

Figure 12.15 Operating modes of the full-bridge four-quadrant converter presented in Table 12.3.

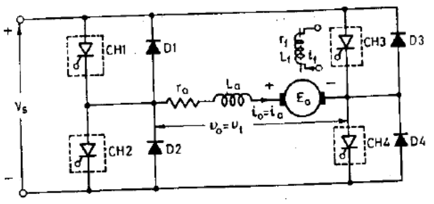
12.5.4. Four-quadrant Chopper Drives

In four-quadrant dc chopper drives, a motor can be made to work in forward-motoring mode (first quadrant), forward regenerative braking mode (second quadrant), reverse motoring mode (third quadrant) and reverse regenerative-braking mode (fourth quadrant). The circuit shown in Fig. 12.24 (*a*) offers four-quadrant operation of a separately-excited dc motor. This circuit consists of four choppers, four diodes and a separately-excited dc motor. Its operation in the four quadrants can be explained as under :

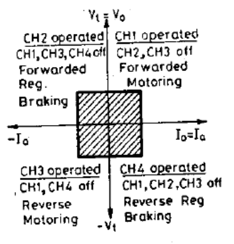
Forward motoring mode. During this mode or first-quadrant operation, choppers CH2, CH3 are kept off, CH4 is kept on whereas *CH1 is operated.* When CH1, CH4 are on, motor voltage is positive and positive armature current rises. When CH1 is turned off, positive armature current free-wheels and decreases as it flows through CH4, D2. In this manner, controlled motor operation in first quadrant is obtained.

Forward regenerative-braking mode. A dc motor can work in the regenerative-braking mode only if motor generated emf is made to exceed the dc source voltage. For obtaining this mode, CH1, CH3 and CH4 are kept off whereas *CH2 is operated*. When CH2 is turned on, negative armature current rises through CH2, D4, E_a, L_a, r_a. When CH2 is turned off, diodes D1, D2 are turned on and the motor acting as a generator returns energy to the dc source. This results in forward regenerative-braking mode in the second-quadrant.

Reverse motoring mode. This operating mode is opposite to forward motoring mode. Choppers CH1, CH4 are kept off, CH2 is kept on whereas *CH3 is operated*. When CH3 and CH2 are on, armature gets connected to source voltage V_s so that both armature voltage V_t and armature current i_a are negative. As armature current is reversed, motor torque is reversed and consequently motoring mode in third quadrant is obtained. When CH3 is turned off, negative armature current freewheels through CH2, D4, E_a, L_a, r_a; armature current decreases and thus speed control is obtained in third quadrant. Note that during this mode, polarity of E_a is opposite to that shown in Fig. 12.24 (a).



(a)

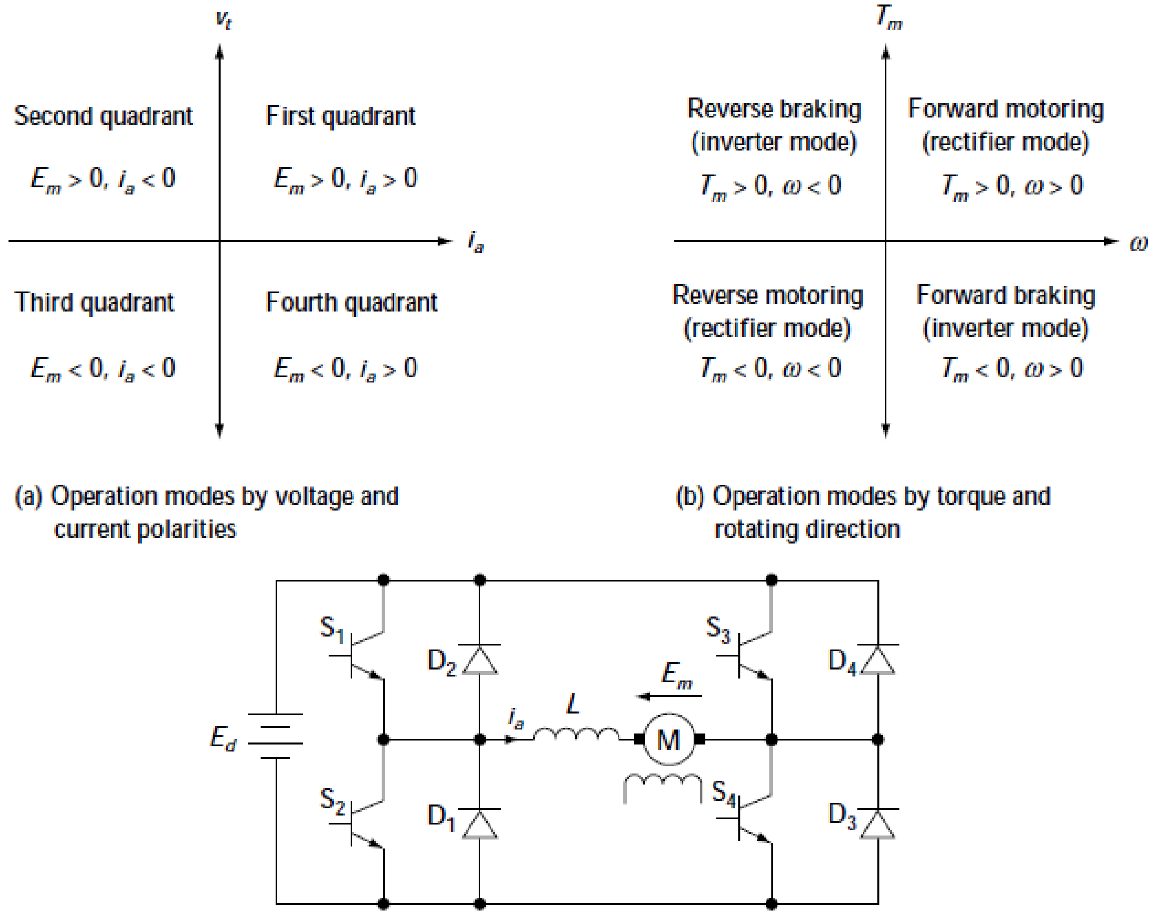


(b)

Fig. 12.24. Four-quadrant dc chopper drive (a) circuit diagram and (b) four-quadrant diagram.

Reverse Regenerative-braking mode. As in forward braking mode, reverse regenerative-braking mode is feasible only if motor generated emf is made to exceed the dc source voltage. For this operating mode, CH1, CH2 and CH3 are kept off whereas *CH4 is operated*. When CH4 is turned on, positive armature current i_a rises through CH4, D2, r_a, L_a, E_a. When CH4 is turned off, diodes D2, D3 begin to conduct and motor acting as a generator returns energy to the dc source. This leads to reverse regenerative-braking operation of the dc separately-excited motor in fourth quadrant.

Note that in Fig. 12,24 (a), the numbering of choppers is done to agree with the quadrants in which these are operated. For example, CH1 is operated for first quadrant, ..., CH4 for fourth quadrant etc.



(a) Operation modes by voltage and current polarities

(b) Operation modes by torque and rotating direction

(c) Circuit

12.6 FOUR-QUADRANT dc/dc CONVERTER DRIVE

A simplified schematic diagram of a four-quadrant chopper drive system is shown in Fig. 12.6-1. Typical steady-state waveforms that depict the operation of the converter are shown in Fig. 12.6-2. As the name implies, four-quadrant operation (current versus voltage) is possible. That is, the instantaneous armature current i_a and the instantaneous armature voltage may be positive or negative. In fact, four-quadrant operation is depicted in each switching period in Fig. 12.6-2. In particular, I_1 is negative and I_2 is positive and v_a is v_S during interval A and $-v_S$ during interval B; however, the average v_a and the average i_a are positive. Therefore, from an average-value

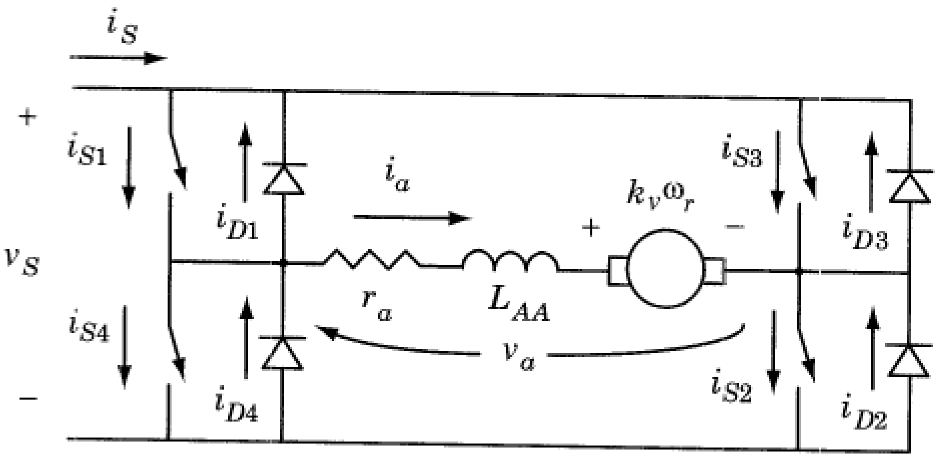


Figure 12.6-1 Four-quadrant chopper drive system.

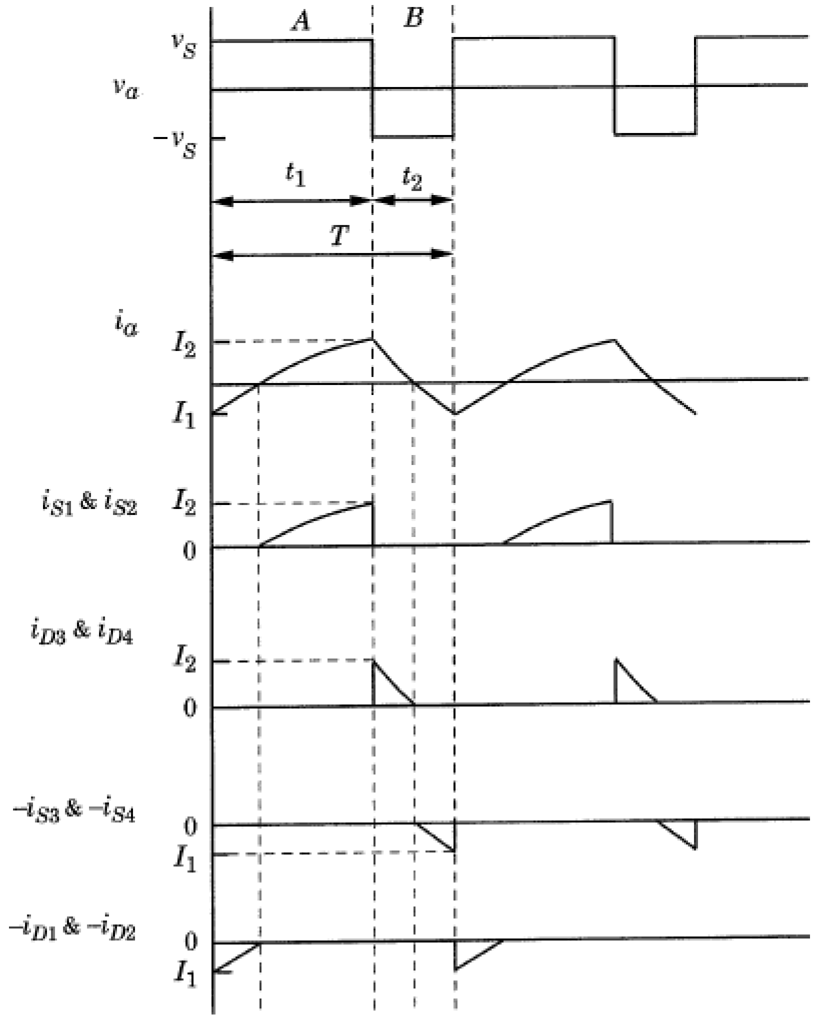


Figure 12.6-2 Typical waveforms for steady-state operation of a four-quadrant chopper drive.

point of view, the dc drive system depicted in Fig. 12.6-2 is operating as a motor if the rotor speed ω_r is positive (ccw). This is 1st quadrant operation of an average-current versus average-voltage plot even though four-quadrant operation of i_a versus v_a occurs each switching period. One must distinguish between four-quadrant operation during a period and four-quadrant average-value operation.

If v_a is positive and i_a is negative (average or instantaneous), then operation is in the 4th quadrant of an i_a versus v_a plot; and if ω_r is positive (ccw), the machine is operating as a generator. In the 2nd quadrant, v_a is negative and i_a is positive; and if ω_r is negative (cw), we have generator operation. In the 3rd quadrant, v_a is negative and i_a is negative; and if ω_r is negative (cw), we have motor operation. It is important to emphasize that one-, two-, and four-quadrant chopper operation has been defined from the plot of i_a versus v_a. However, the T_e versus ω_r plot is also used to define drive operation where the 1st and 3rd quadrants depict motor operation and the 2nd and 4th quadrants depict generator operation. At first glance, one might assume that the quadrants of the i_a versus v_a plot can be assigned the same modes of operation. Actually this is only the case if ω_r is positive (ccw) when v_a is positive and negative (ccw) when v_a is negative as stipulated above. In order to illustrate this, let us assume that v_a and i_a are both positive and ω_r is positive (ccw); the machine is operating as a motor, and operation is in the 1st quadrant of i_a versus v_a and in the 1st quadrant of T_e versus ω_r. If v_a and i_a are positive and ω_r is zero, the power $(v_a i_a)$ is being dissipated in r_a and the machine is neither a motor nor a generator. If, however, ω_r is made slightly negative by supplying an input torque, v_a and i_a can still both be positive (1st quadrant) and yet generator action is occurring because T_e is positive and ω_r is negative (2nd quadrant of T_e versus ω_r). Therefore, we must know the direction (sign) of ω_r when assigning motor or generator action to the four quadrants of the i_a versus v_a plot.

There are numerous switching strategies that might be used with a four-quadrant chopper. The switching depicted in Fig. 12.6-2 is perhaps one of the least involved. In this case, there are only two states. In the first state, which occurs over interval A, $S1$ and $S2$ are closed and $S3$ and $S4$ are open. The second state occurs over interval B, wherein the $S3$ and $S4$ are closed and $S1$ and $S2$ are open. As in the case of the previous dc/dc converters, we will consider the switches and diodes as being ideal.

During interval A, $S1$ and $S2$ are closed and $S3$ and $S4$ are open. At the beginning of the interval, i_a is negative (I_1) in Fig. 12.6-2. Because $S1$ and $S2$ cannot carry negative armature current, I_1 must flow through diodes $D1$ and $D2$. Note in Fig. 12.6-2 that $-i_{D1}$, $-i_{D2}$, $-i_{S3}$, and $-i_{S4}$ are plotted for the purpose of a direct comparison with i_a. During interval A, the armature voltage v_a is v_S; and because v_S is larger than the counter emf, the armature current increases from the negative value of I_1 toward zero. During this part of the interval, the source current is $-i_{D1}$, which is also $-i_{D2}$. When i_a reaches zero, $D1$ and $D2$ block positive armature current flow; however, $S1$ and $S2$ are closed ready to carry a positive i_a. Hence, the current increases from zero to I_2 through $S1$ and $S2$. During this part of the interval, the source current i_S is i_{S1}, which is also i_{S2}.

During interval B, v_a is $-v_S$ and are $S1$ and $S2$ are open with $S3$ and $S4$ closed. At the beginning of interval B, i_a is positive (I_2); however, $S3$ and $S4$ cannot conduct a positive armature current. Hence at the beginning of interval B the positive I_2 flows through diodes $D3$ and $D4$. This continues until i_a is driven to zero by $-v_S$. During this part of interval B, the source current i_S is $-i_{D3}$ or $-i_{D4}$. When i_a reaches zero, diodes $D3$ and $D4$ block negative i_a; thus, $S3$ and $S4$ carry the negative armature current to the end of interval B where $i_a = I_1$, which is negative. During this part of interval B, the source current i_S is i_{S3} or i_{S4}. We have completed a switching cycle.

Expressions for I_1 and I_2 can be derived by a procedure similar to that used in the case of the previous choppers. It can be shown that

$$I_1 = \frac{v_s}{r_a}\left[\frac{2e^{-(1-k)T/\tau_a} - e^{-T/\tau_a} - 1}{1 - e^{-T/\tau_a}}\right] - \frac{k_v\omega_r}{r_a} \tag{12.6-1}$$

$$I_2 = \frac{v_s}{r_a}\left[\frac{1 - 2e^{-kT/\tau_a} + e^{-T/\tau_a}}{1 - e^{-T/\tau_a}}\right] - \frac{k_v\omega_r}{r_a} \tag{12.6-2}$$

If k and v_S do not change significantly from one switching period to the next, the average armature voltage may be expressed as

$$\begin{aligned}\bar{v}_a &= \frac{1}{T}\left[\int_0^{kT} v_S\, d\xi + \int_{kT}^{T} -v_S\, d\xi\right]\\ &= \frac{1}{T}[kTv_S - (1-k)Tv_S]\\ &= (2k-1)v_S \end{aligned} \tag{12.6-3}$$

Note that when $k = 0$, $v_a = -v_S$ and when $k = 1$, $v_a = v_S$. It is clear that the time-domain block diagram for the four-quadrant chopper drive is the same as that shown in Fig. 12.5-3 for the two-quadrant chopper drive with $v_a = kv_S$ replaced with $v_a = (2k-1)v_S$ which is (12.6-3).

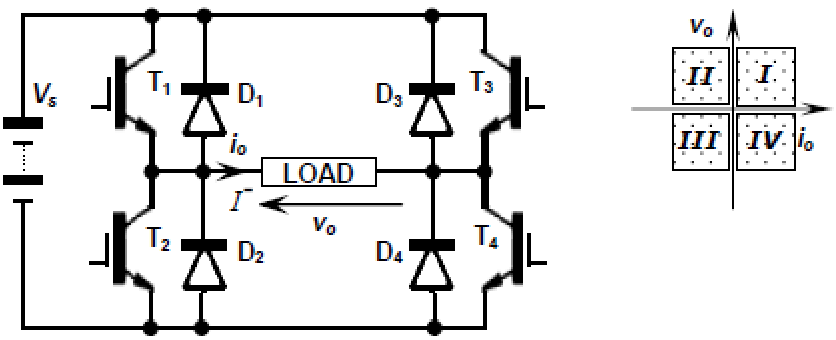


Figure 13.13. *Four-quadrant dc chopper circuit, showing first quadrant i_o and v_o references.*

13.6 Four-quadrant dc chopper

The four-quadrant H-bridge dc chopper is shown in figure 13.13 where the load current and voltage are referenced with respect to T_1, so that the quadrant of operation with respect to the switch number is persevered.

The H-bridge is a flexible basic configuration where its use to produce single-phase ac is considered in chapter 14.1.1, while its use in smps applications is considered in chapter 15.8.2. It can also be used as a dc chopper for the four-quadrant control of a dc machine.

With the flexibility of four switches, a number of different control methods can be used to produce four-quadrant output voltage and current (bidirectional voltage and current). All practical methods should employ complementary device switching in each leg (either T_1 or T_4 on but not both and either T_2 or T_3 on, but not both) so as to minimise distortion by ensuring current continuity around zero current output.

One control method involves controlling the H-bridge as two virtually independent two-quadrant choppers, with the over-riding restriction that no two switches in the same leg conduct simultaneously. One chopper is formed with T_1 and T_4 grouped with D_2 and D_3, which gives positive current i_o but bidirectional voltage $\pm v_o$ (QI and QIV operation). The second chopper is formed by grouping T_2 and T_3 with D_1 and D_4, which gives negative output current $-i_o$, but bi-direction voltage $\pm v_o$ (QII and QIII operation).

The second control method is to unify the operation of all four switches within a generalised control algorithm.

With both control methods, the chopper output voltage can be either multilevel or bipolar, depending on whether zero output voltage loops are employed or not. Bipolar output states increase the ripple current magnitude, but do facilitate faster current reversal, without crossover distortion. Operation is independent of the direction of the output current i_o.

Since the output voltage is reversible for each control method, a triangular based modulation control method, as used with the asymmetrical H-bridge dc chopper in figure 13.9, is applicable in each case. Two generalised unified H-bridge control approaches are considered.

13.6.1 *Unified four-quadrant dc chopper - bipolar voltage output switching*

The simpler output to generate is bipolar output voltages, which use one reference carrier triangle as shown in figure 13.14 parts (c) and (d). The output voltage switches between $+V_s$ and $-V_s$ and the relative duration of each state depends on the magnitude of the modulation index δ.

If $\delta = 0$ then T_1 and T_4 never turn-on since T_2 and T_3 conduct continuously which impresses $-V_s$ across the load.

At the other extreme, **if** $\delta = 1$ then T_1 and T_4 are on continuously and V_s is impressed across the load.

If $\delta = ½$ then T_1 and T_4 are turned on for half of the period T, while T_2 and T_3 are on for the remaining half of the period. The output voltage is $-V_s$ for half of the time and $+V_s$ for the remaining half of any period. The average output voltage is therefore zero, but disadvantageously, the output current needlessly ripples about zero (with an average value of zero).

The chopper output voltage is defined in terms of the triangle voltage reference level v_Δ by

- $v_\Delta > \delta,\ v_o = -V_s$
- $v_\Delta < \delta,\ v_o = +V_s$

From figure 13.14c and d, the average output voltage varies linearly with δ such that

$$\begin{aligned}\overline{V}_o &= \frac{1}{T}\left(\int_0^{t_T} +V_s\,dt + \int_{t_T}^{T} -V_s\,dt\right)\\ &= \frac{1}{T}(2t_T - T)V_s = \left(2\frac{t_T}{T} - 1\right)V_s\end{aligned} \tag{13.105}$$

Examination of figures 13.14c and d reveals that the relationship between t_T and δ must produce

$$\begin{aligned}&\text{when } \delta = 0,\ t_T = 0 \text{ and } v_o = -V_s\\ &\text{when } \delta = ½,\ t_T = ½T \text{ and } v_o = 0\\ &\text{when } \delta = 1,\ t_T = T \text{ and } v_o = +V_s\end{aligned}$$

that is

$$\delta = \frac{t_T}{T}$$

which on substituting for t_T/T in equation (13.105) gives

$$\overline{V}_o = \left(2\frac{t_T}{T} - 1\right) V_s \qquad (13.106)$$
$$= (2\delta - 1)\, V_s \quad \text{for} \quad 0 \le \delta \le 1$$

The average output voltage can be positive or negative, depending solely on δ. No current discontinuity occurs since the output voltage is never zero. Even when the average is zero, ripple current flows though the load.

The rms output voltage is independent of the duty cycle and is V_s.

The output ac ripple voltage is

$$V_r = \sqrt{V_{rms}^2 - V_o^2} \qquad (13.107)$$
$$= \sqrt{V_s^2 - (2\delta - 1)V_s^2} = 2\,V_s\sqrt{\delta(1-\delta)}$$

The ac ripple voltage is zero at $\delta = 0$ and $\delta = 1$, when the output voltage is pure dc, namely - V_s or V_s, respectively. The maximum ripple voltage occurs at $\delta = ½$, when $V_r = V_s$.

The output ripple factor is

$$RF = \frac{V_r}{V_o} = \frac{2\,V_s\sqrt{\delta(1-\delta)}}{(2\delta - 1)V_s} \qquad (13.108)$$
$$= \frac{2\sqrt{\delta(1-\delta)}}{(2\delta - 1)}$$

Circuit operation is characterized by two time domain equations

During the **on-period for T1 and T4**, when $v_o(t)=V_s$

$$L\frac{di_o}{dt} + R\,i_o + E = V_s$$

which yields

$$i_o(t) = \frac{V_s - E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le t_T \qquad (13.109)$$

During the **on-period for T2 and T3**, when $v_o(t)= - V_s$

$$L\frac{di_o}{dt} + R\,i_o + E = -V_s$$

which, after shifting the zero time reference to t_T, gives

$$i_o(t) = -\frac{V_s + E}{R}\left(1 - e^{\frac{-t}{\tau}}\right) + \hat{I}\,e^{\frac{-t}{\tau}} \qquad \text{for} \quad 0 \le t \le T - t_T \qquad (13.110)$$

The initial conditions $\hat{I}$ and $\check{I}$ are determined by using the usual steady-state boundary condition method.

$$\text{where} \quad \hat{I} = \frac{V_s}{R}\frac{1 - 2e^{\frac{-t_T}{\tau}} + e^{\frac{-T}{\tau}}}{1 - e^{\frac{-T}{\tau}}} - \frac{E}{R} \qquad \text{(A)}$$

$$\text{and} \quad \check{I} = \frac{V_s}{R}\frac{2e^{\frac{t_T}{\tau}} - 1 + e^{\frac{-T}{\tau}}}{1 - e^{\frac{-T}{\tau}}} - \frac{E}{R} \qquad \text{(A)} \qquad (13.111)$$

Example 13.7: *Four-quadrant dc chopper*

The H-bridge, dc-to-dc chopper in figure 13.13 feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated with a 200Hz multilevel carrier as in figure 13.14 a and b, with a modulation depth of δ = ¼, determine:

i. the average output voltage and switch T_1 on-time
ii. the rms output voltage and ac ripple voltage
iii. the average output current, hence quadrant of operation
iv. the electromagnetic power being extracted from the back emf E.

If the mean load current is to be halved, what is

v. the modulation depth, δ, requirement
vi. the average output voltage and the corresponding switch T_1 on-time
vii. the electromagnetic power being extracted from the back emf E?

Solution

The main circuit and operating parameters are

- modulation depth δ = ¼
- period $T_{carrier} = 1/f_{carrier} = 1/200\text{Hz} = 5\text{ms}$
- E=55V and V_s=340V dc
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

i. The average output voltage is given by equation (13.114), and for δ < ½,

$$\overline{V}_o = \left(\frac{t_T}{T} - 1\right)V_s = (2\delta - 1)V_s$$
$$= 340\text{V}\times(2\times¼ - 1) = -170\text{V}$$

where

$$t_T = 2\delta T = 2\times¼\times(½\times5\text{ms}) = 1.25\text{ms}$$

Figure 13.14 reveals that the carrier frequency is half the switching frequency, thus the 5ms in the above equation has been halved. The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 3.75ms.

ii. The rms load voltage, from equation (13.118), is

$$V_{rms} = \sqrt{1-2\delta}\ V_s$$
$$= 340\text{V}\times\sqrt{1 - 2\times¼} = 240\text{V rms}$$

From equation (13.119), the output ac ripple voltage is

$$V_r = \sqrt{2}\,V_s\sqrt{\delta(1-2\delta)}$$
$$=\sqrt{2}\times 340\text{V}\times\sqrt{¼\times(1 - 2\times¼)} = 170\text{V ac}$$

iii. The average output current is given by equation (13.117)

$$\overline{I}_o = \frac{\overline{V}_o - E}{R} = \frac{(2\delta-1)V_s - E}{R}$$
$$= \frac{340\text{V}\times(2\times¼ - 1) - 55\text{V}}{10\Omega} = -22.5\text{A}$$

Since both the average output current and voltage are negative (-170V and -22.5A) the chopper with a modulation depth of δ = ¼, is operating in the third quadrant.

iv. The electromagnetic power developed by the back emf E is given by

$$P_E = E\overline{I}_o = 55\text{V}\times(-22.5\text{A}) = -1237.5\text{W}$$

v. The average output current is given by

$$\overline{I}_o = \left(\overline{V}_o - E\right)\!\big/_R = \left((2\delta-1)V_s - E\right)\!\big/_R$$

when the mean current is -11.25A, δ = 0.415, as derived in part vi.

vi. Then, if the average current is halved to -11.25A

$$\overline{V}_o = E + \overline{I}_o R$$
$$=55\text{V} - 11.25\text{A}\times 10\Omega = -57.5\text{V}$$

The average output voltage rearranged in terms of the modulation depth δ gives

$$\delta = ½\left(1 + \frac{\overline{V}_o}{V_s}\right)$$
$$= ½\times\left(1 + \frac{-57.5\text{V}}{340\text{V}}\right) = 0.415$$

The switch on-time when δ < ½ is given by

$$t_T = 2\delta T = 2\times 0.415\times(½\times 5\text{ms}) = 2.07\text{ms}$$

From figure 13.14b both T_1 and T_4 are turned on for 2.07ms, although, from table 13.3B, for negative load current, $\overline{I}_o = -11.25\text{A}$, the parallel connected freewheel diodes D_1 and D_4 conduct alternately, rather than the switches (assuming $\hat{I}_o < 0$). The switches T_1 and T_4 are turned on for 1.25ms, while T_2 and T_3 are subsequently turned on for 2.93ms.

vii. The electromagnetic power developed by the back emf E is halved and is given by

$$P_E = E\overline{I}_o = 55\text{V}\times(-11.25\text{A}) = -618.75\text{W}$$

If the output current never goes positive, that is $\hat{I}$ is negative, then T_1, T_4, D_2, and D_3 do not conduct, thus do not appear in the output device sequence. The conducting sequence is as shown in table 13.3B for $\hat{I} < 0$.

Unlike the bipolar control method, the output sequence is affected by the average output voltage level, as well as the polarity of the output current swing. The transition between the six possible sequences due to load voltage and current polarity changes, is seamless. The only restriction is that devices in any leg do not conduct simultaneously. This is ensured by inserting a brief dead-time between a switch turning off and its leg complement being turned on.

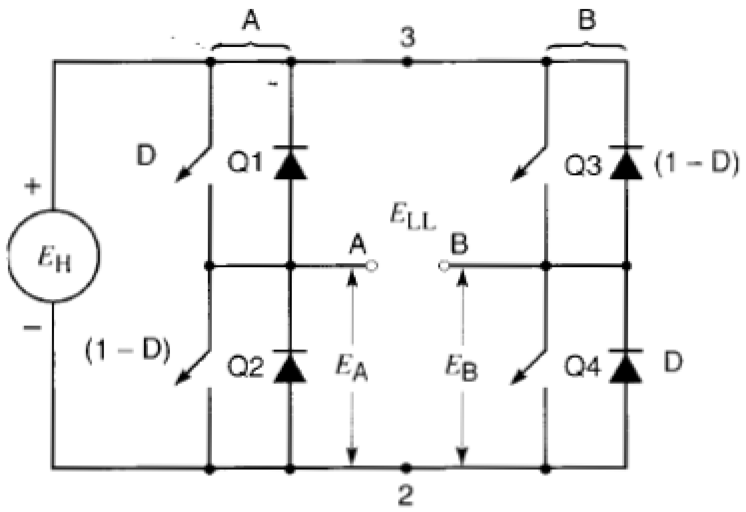


Figure 21.70
Four-quadrant dc-to-dc converter.

Example 22-5

A trolley-bus is driven by a 150 hp, 1500 r/min, 600 V series motor. The nominal full-load current is 200 A and the total resistance of the armature and field is 0.1 Ω. The bus is fed from a 700 V dc line.

A chopper controls the torque and speed. The chopper frequency varies from 50 Hz to 1600 Hz, but the *on* time T_a is fixed at 600 μs.

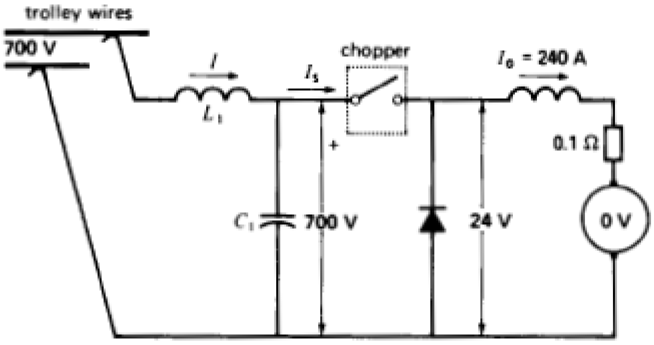


Figure 22.18a
See Example 22-5.

a. Calculate the chopper frequency and the current drawn from the line when the motor is at standstill and drawing a current of 240 A.
b. Calculate the chopper frequency when the motor delivers its rated output.

Solution

a. Referring to Fig. 22.18a, the armature *IR* drop is 240 A × 0.1 Ω = 24 V, and the cemf is zero because the motor is at standstill.

Consequently, E_0 = 24 V and E_s = 700 V. We can find the frequency from

$$E_0 = E_s f T_a \qquad (21.14)$$

$$24 = 700 f \times 600 \times 10^{-6}$$

$$f = 57.14 \text{ Hz}$$

$$T_a + T_b = 1/f = 1/57.14$$

$$= 17\,500 \text{ µs (Fig. 22.18b)}$$

The dc current drawn from the catenary is

$$I = I_s = P/E_s = 24 \times 240/700$$

$$= 8.23 \text{ A}$$

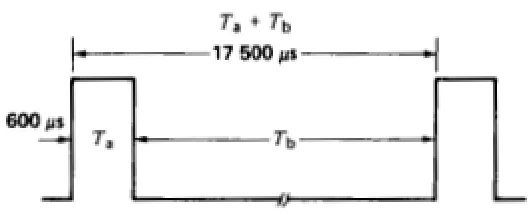


Figure 22.18b
Current pulses I_s drawn by the chopper from the 700 V source when the motor is stalled.

(Note the very low current drawn from the line during start-up)

b. At rated output the voltage across the motor terminals is 600 V (Fig. 22.19a). The required frequency is therefore given by:

$$E_0 = E_s f T_a$$

$$600 = 700 f \times 600 \times 10^{-6}$$

$$f = 1429 \text{ Hz}$$

$$T_a + T_b = 1/f = 1/1429$$

$$= 700 \ \mu\text{s (Fig. 22.19b)}$$

Line current I:

$$I = I_s = P/E_s$$

$$= 600 \times 200/700$$

$$= 171 \text{ A}$$

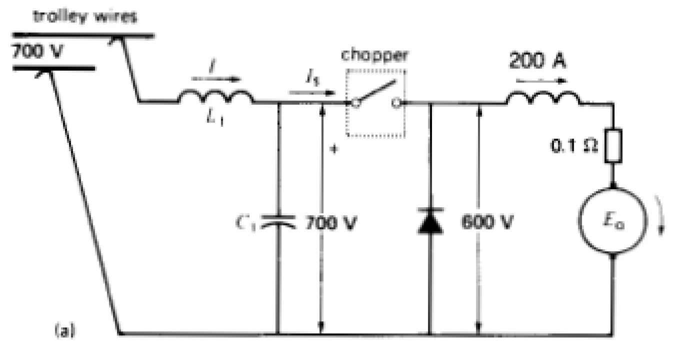


Figure 22.19a
Conditions when the motor is running at rated torque and speed.

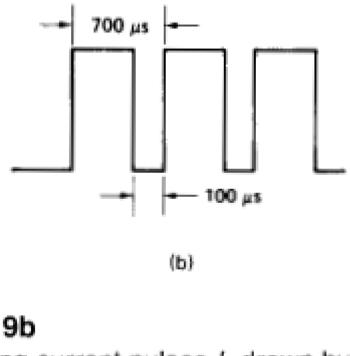


Figure 22.19b
Corresponding current pulses I_s drawn by the chopper from the 700 V source.

Example 22-6

Referring to Example 22-5 and Fig. 22.18a, calculate the peak value of currents I_s and I when the motor is at standstill.

Solution

a. Although the average value of I_s is 8.23 A, its peak value is 240 A. The current flows in a series of brief, sharp pulses. On the other hand, the armature current I_o is steady at 240 A.
b. The average value of line current I is 8.23 A. The voltage across the capacitor fluctuates and so current I will have a ripple because inductor L_1 does not have infinite inductance. Consequently, the peak value of I will be slightly greater than the average value.

21.42 Four-quadrant dc-to-dc converter

The 2-quadrant converter we have studied can only be used with a load whose voltage has a specific polarity. Thus, in Fig. 21.69, given the polarity of E_H, terminal 1 can only be (+) with respect to terminal 2. We can overcome this restriction by means of a *4-quadrant converter.* It consists of two identical 2-quadrant converters arranged as shown in Fig. 21.70. Switches Q1, Q2 in converter arm A open and close alternately, as do switches Q3, Q4 in converter arm B. The switching frequency (assumed to be 100 kHz) is the same for both. The switching sequence is such that Q1 and Q4 open and close simultaneously. Similarly, Q2 and Q3 open and close simultaneously. Consequently, if the duty cycle for Q1 is D, it will also be D for Q4. It follows that the duty cycle for Q2 and Q3 is $(1 - D)$.

The dc voltage E_A appearing between terminals A, 2 is given by

$$E_A = DE_H$$

The dc voltage E_B between terminals B, 2 is

$$E_B = (1 - D)E_H$$

The dc voltage E_{LL} between terminals A and B is the difference between E_A and E_B:

$$\begin{aligned} E_{LL} &= E_A - E_B \\ &= DE_H - (1 - D)E_H \end{aligned}$$

thus

$$E_{LL} = E_H\,(2\,D - 1) \qquad (21.24)$$

Equation 21.24 indicates that the dc voltage is zero when $D = 0.5$. Furthermore, the voltage changes linearly with D, becoming $+E_H$ when $D = 1$, and $-E_H$ when $D = 0$. The polarity of the output voltage can therefore be either positive or negative. Moreover, if a device is connected between terminals A, B, the direction of dc current flow can be either from A to B or from B to A. Consequently, the converter of Fig. 21.70 can function in all four quadrants.

The *instantaneous* voltages E_{A2} and E_{B2} oscillate constantly between zero and $+E_H$. Fig. 21.71 shows the respective waveshapes when $D = 0.5$. Similarly, Fig. 21.72 shows the waveshapes when $D = 0.8$. Note that the instantaneous voltage E_{AB} between the output terminals A, B oscillates between $+E_H$ and $-E_H$. In practice, the alternating components that appear between terminals A, B are filtered out. Consequently, only the dc component E_{LL} remains as the active driving emf across the external device connected to terminals A, B.

Consider, for example, the block diagram of a converter feeding dc power to a passive load R (Fig. 21.73). The power is provided by source E_H. As we have seen, the magnitude and polarity of E_{LL} can be varied by changing the duty cycle D. The switching frequency f of several kilohertz is assumed to be constant. Inductor L and capacitor C act as filters so that the dc current flowing in the resistance has negligible ripple. Because the switching frequency is high, the inductance and capacitance can be small, thus making for inexpensive filter components.

The dc currents and voltages are related by the power-balance equation $E_H I_H = E_{LL} I_L$. We neglect the switching losses and the small control power associated with the D and f input signals.

Fig. 21.74 shows the converter connected to an active device E_0, which could be either a source or a load. If need be, the polarity of E_0 could be the reverse of that shown.

In all these applications we can force power to flow from E_H to E_0, or vice versa, by simply adjusting the duty cycle D. This 4-quadrant dc-to-dc converter is therefore an extremely versatile device.

The inductor L is a crucially important part of the converter. It alone is able to absorb energy at one

voltage level (high or low) and release it at another voltage level (low or high). And it performs this duty automatically, in response to the electronic switches and their duty cycle.

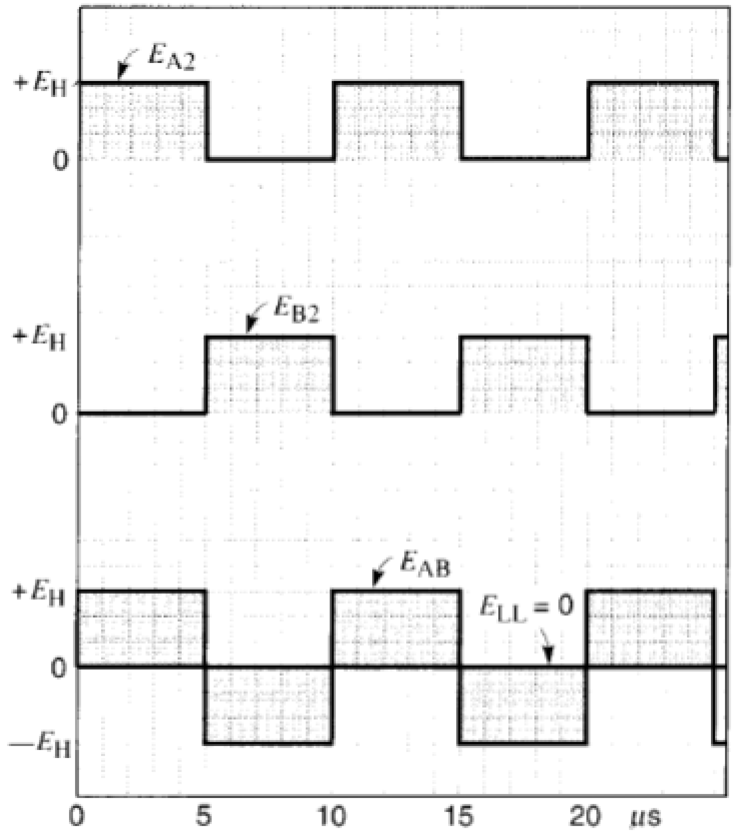


Figure 21.71
Voltage output when $D = 0.5$. The average voltage is zero.

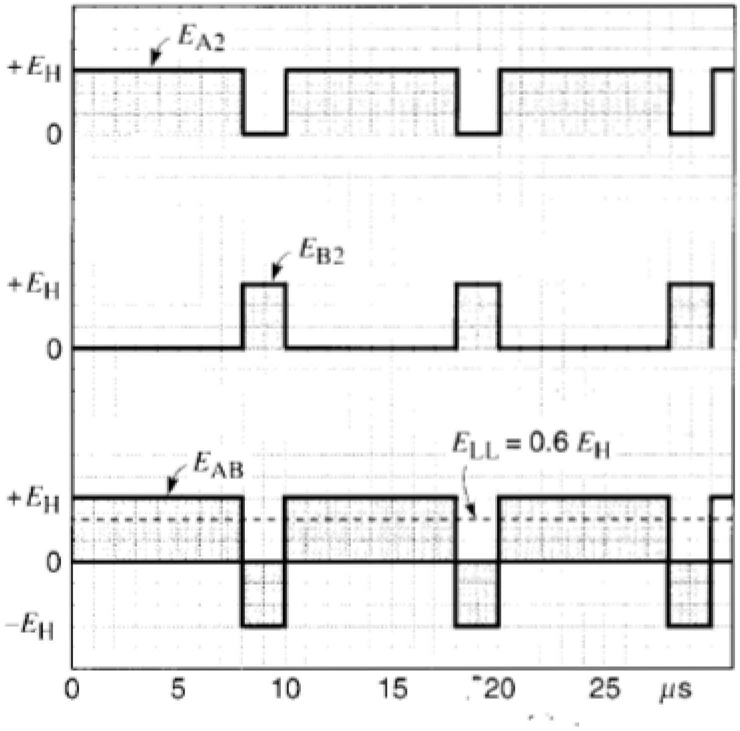


Figure 21.72
Voltage output when $D = 0.8$. The average voltage E_{LL} is 0.6 E_H.

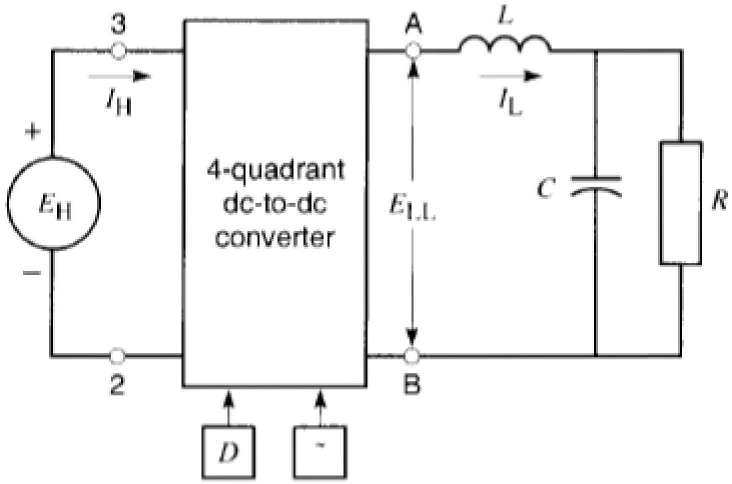


Figure 21.73
Four-quadrant dc-to-dc converter feeding a passive dc load R.

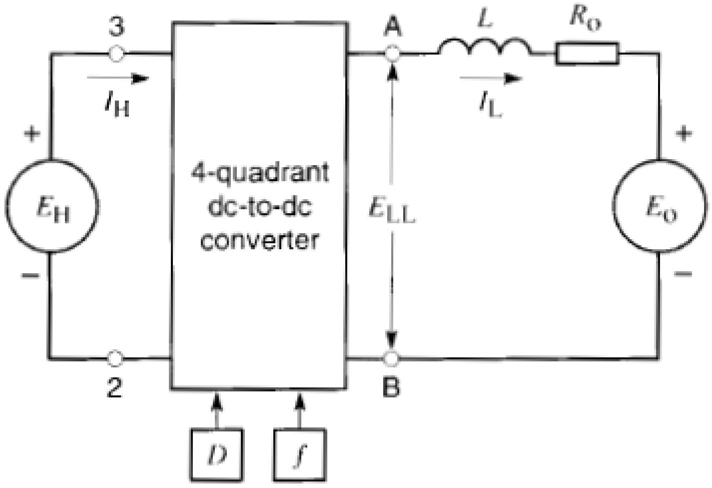


Figure 21.74
Four-quadrant dc-to-dc converter feeding an active dc source/sink E_o.

Example 22-7

A 25 hp, 250 V, 900 r/min dc motor is connected to a dc-to-dc converter that operates at a switching frequency of 2 kHz. The converter is fed by a 6-pulse rectifier connected to a 240 V, 3-phase, 60 Hz line (Fig. 22.21a). A 500 μF capacitor C and an inductor L_d act as filters. The armature resistance and inductance are respectively 150 mΩ and 4 mH. The rated dc armature current is 80 A. We wish to determine the following:

a. The required duty cycle when the motor develops its rated torque at rated speed
b. The waveshape of currents I_1, I_2, and I_a
c. The waveshape of voltages E_{12} and E_{AB}.

V

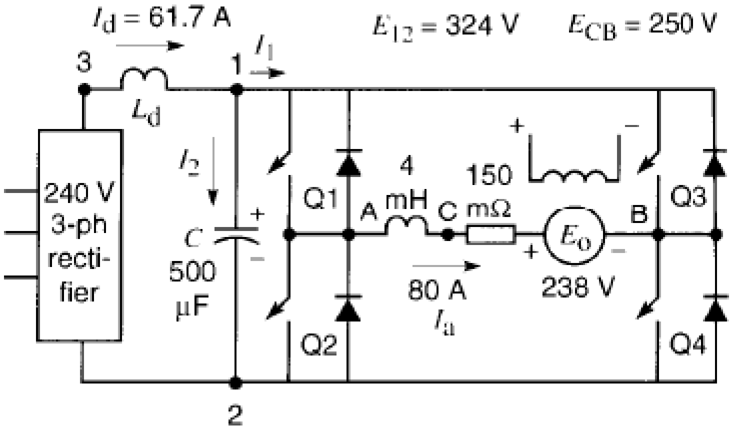


Figure 22.21a
See Example 22-7.

thus,

$$250 = 324\,(2\,D - 1)$$

and so,

$$D = 0.886$$

The 250 V appears between terminals A, B (Fig. 22.21a).

Because the motor develops rated torque, the armature draws its rated current, namely 80 A. The voltage drop in the armature resistance is

$$80\text{ A} \times 0.15\ \Omega = 12\text{ V}$$

The induced armature voltage, or counter emf, at 900 r/min is, therefore,

$$E_o = 250 - 12 = 238 \text{ V}$$

The dc power input to the motor is

$$P = 250 \text{ V} \times 80 \text{ A} = 20\,000 \text{ W}$$

Neglecting the losses in the converter, and recalling that the dc output of the rectifier is 324 V, it follows that current I_d is given by

$$324\, I_d = 20\,000$$
$$I_d = 61.7 \text{ A}$$

The frequency of the converter is 2 kHz and so the period of one cycle is

$$T = 1/f = 1/2000 = 500 \text{ µs}$$

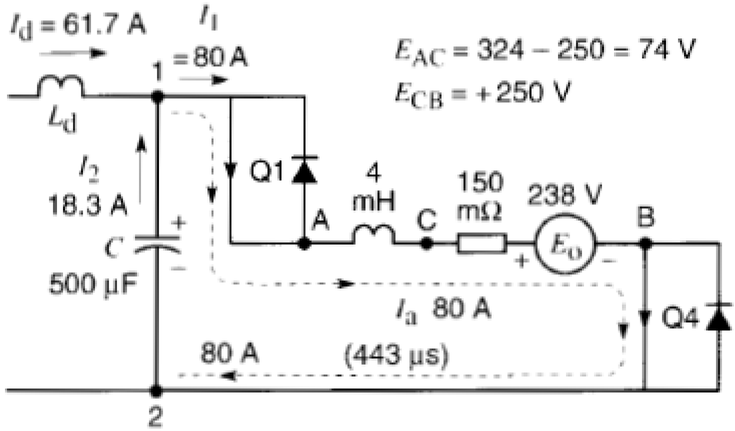


Figure 22.21b
Circuit when Q1 and Q4 are "on." Current I_a is increasing. $E_{CA} = -74$ V.

The *on* and *off* times of Q1 (and Q4) are, respectively,

$$T_a = DT = 0.886 \times 500 = 443 \text{ µs}$$
$$T_b = 500 - 443 = 57 \text{ µs}$$

It follows that the corresponding *on* and *off* times of Q2 (and Q3) are 57 µs and 443 µs.

We recall that Q1 and Q4 operate simultaneously, followed by Q2 and Q3, which also open and close simultaneously.

When Q1 and Q4 are conducting, the armature current follows the path shown in Fig. 22.21b. This lasts for 443 μs and during this time I_1 (= 80 A) flows in the positive direction. Note however, that the rectifier only furnishes 61.7 A, whereas the armature current is 80 A. It follows that the difference (80 − 61.7) = 18.3 A must come from the capacitor. The capacitor discharges, causing the voltage across it to drop by an amount ΔE given by

$$\Delta E = Q/C = 18.3 \text{ A} \times 443 \text{ μs}/500 \text{ μF} = 16 \text{ V}$$

Q1 and Q4 then open for 57 μs. During this interval Q2 and Q3 are closed (Fig. 22.21c), but they cannot carry the armature current because it is flowing opposite to the direction permitted by these IGBTs. However, the current *must* continue to flow because of the armature inductance. Fortunately, a path is offered by the diodes D2 and D3 associated with Q2 and Q3, as shown in the figure. Note that I_1

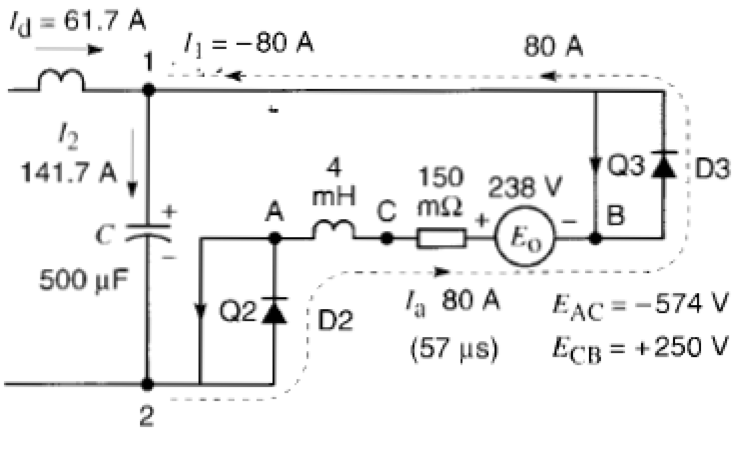


Figure 22.21c
Circuit when D2 and D3 are conducting. Current I_a is decreasing.

(= 80 A) now flows toward terminal 1, which is opposite to the direction it had in Fig. 22.21b.

Meanwhile, current I_d furnished by the rectifier continues to flow unchanged because of the presence of inductor L_d. As a result, by Kirchhoff's current law, the current I_2 must flow into the capacitor and its value is (80 + 61.7) = 141.7 A. This highlights the absolute necessity of having a capacitor in the circuit. Without it, the flow of armature current would be inhibited during this 57 μs interval. The capacitor charges up and the increase in voltage ΔE is given by

$$\Delta E = Q/C = 141.7 \text{ A} \times 57 \text{ μs}/500 \text{ μF} = 16 \text{ V}$$

Note that the increase in voltage across the capacitor during the 57 μs interval is exactly equal to the decrease during the 443 μs interval. The peak-to-peak ripple across the capacitor is, therefore, 16 V. Thus, the voltage between points **1** and **2** fluctuates between (324 + 8) = 332 V and (324 − 8) = 316 V. This 2.5 percent fluctuation does not affect the operation of the motor.

Let us now look more closely at the armature current, particularly as regards the ripple. In Fig. 22.21b the voltage across the armature inductance can be found by applying KVL:

$$E_{AC} + E_{CB} + E_{B2} + E_{21} + E_{1A} = 0$$
$$E_{AC} + 250 + 0 - 324 + 0 = 0$$

Hence

$$E_{AC} = 74 \text{ V}$$

Therefore, the volt seconds accumulated during this 443 μs interval is 74 × 443 = 32 782 μs·V. The resulting increase in armature current ΔI_a is

$$\Delta I_a = A/L_a = 32\,782 \times 10^{-6}/0.004 = 8 \text{ A} \quad (2.28)$$

Next, consider Fig. 22.21c. The voltage across the armature inductance can again be found by applying KVL:

$$E_{AC} + E_{CB} + E_{B1} + E_{12} + E_{2A} = 0$$
$$E_{AC} + 250 + 0 + 324 + 0 = 0$$

Hence, $E_{AC} = -574$ V. This negative voltage causes a very rapid decrease in the armature current. The decrease during the 57 μs interval is given by

$$\Delta I_a = 574 \times 57 \times 10^{-6}/0.004 = 8 \text{ A} \quad (2.28)$$

The 8 A decrease during the 57 μs interval is precisely equal to the increase during the previous 443 μs interval. The peak-to-peak ripple is, therefore, 8 A, which means that the armature current fluctuates between (80 + 4) = 84 A and (80 − 4) = 76 A. Figure 22.21d shows the waveshapes of the various voltages and currents.

Example 22-8

We now consider the question of dynamic braking. The same motor is used as in Example 22-7, and we assume it is running at 900 r/min at the moment that braking is applied. We further assume that the inertia of the motor and its load is very large. As a result, the speed cannot change quickly. The connection between the converter and the 6-pulse rectifier is removed and a braking resistance of 20 Ω is connected between terminals 1 and 2, along with the 500 μF capacitor (Fig. 22.22). We assume that a braking torque equal to 75 percent of nominal torque is sufficient. Consequently, the required armature current is 0.75 × 80 A = 60 A. The switching frequency remains unchanged at 2 kHz. We wish to determine the following:

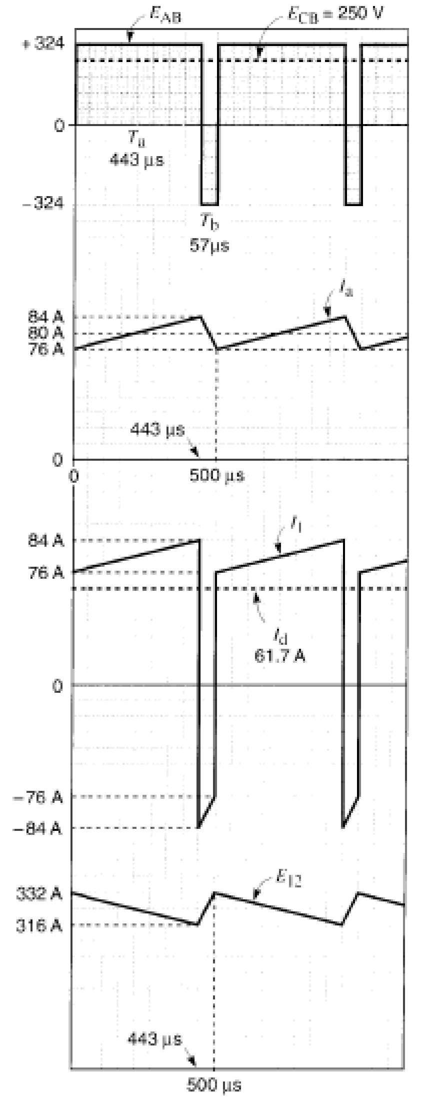


Figure 22.21d
Waveshapes of currents and voltages in Example 22-7.

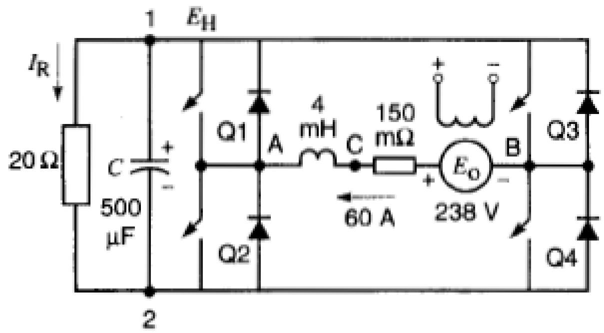


Figure 22.22
Dynamic braking. See Example 22-8.

a. The voltage across the resistor
b. The duty cycle required
c. The braking behavior of the system

Solution

a. Because the motor is turning at 900 r/min at the moment that braking is applied, the induced voltage E_0 remains at 238 V. However, the motor must now operate as a generator and so the 60 A braking current flows out of the (+) terminal, as shown in Fig. 22.22.

The voltage drop across the armature resistance is $0.15\ \Omega \times 60\ \text{A} = 9\ \text{V}$.

The dc voltage between terminals A, B is $(238 - 9) = 229$ V, which is the required *average* output voltage E_{LL} of the converter.

To calculate the dc input voltage E_H between terminals 1, 2 of the converter, we reason as follows:

Due to the large inertia, the speed will remain essentially constant at 900 r/min for, say, 10 cycles of the converter switching frequency.

The power output of the generator during this 10-cycle period is equal to the power absorbed by the 20 Ω braking resistor. Thus,

$$229\ \text{V} \times 60\ \text{A} = (E_H)^2/20\ \Omega$$

hence $$E_H = E_{12} = 524\ \text{V}$$

This voltage is much higher than the previous operating voltage of 324 V. It is actually an advantage because the higher voltage automatically prevents the input rectifier from continuing to feed power to the drive system. On the other hand, the voltage should not be too high, otherwise it could exceed the withstand capability of the switching IGBT devices.

The *average* current in the resistor is $524\ \text{V}/20\ \Omega = 26\ \text{A}$.

b. Knowing the input and output voltages of the converter, we can determine the value of the duty cycle:

$$E_{LL} = E_H(2D - 1) \qquad (21.24)$$

$$229 = 524(2D - 1)$$

Therefore

$$D = 0.72$$

The *on* and *off* times of Q1 (and Q4) are, therefore,

$$T_a = DT = 0.72 \times 500 = 360\ \mu s$$

$$T_b = 500 - 360 = 140\ \mu s$$

It follows that the corresponding *on* and *off* times of Q2 (and Q3) are 140 μs and 360 μs. Q1 and Q4 still operate simultaneously, as do Q2 and Q3.

When Q2 and Q3 are closed, the armature current follows the path shown in Fig. 22.23. This lasts for 140 μs and during this time I_1 (= 60 A) flows out of terminal 1. The current in the resistor is still 26 A. It follows that a current (60 + 26) = 86 A must come from the capacitor. The capacitor discharges, causing the voltage across it to drop by an amount ΔE given by

$$\Delta E = Q/C = 86\ \text{A} \times 140\ \mu\text{s}/500\ \mu\text{F} = 24\ \text{V}$$

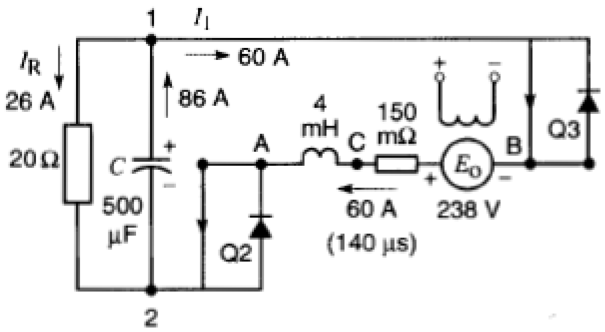


Figure 22.23
Current flows through IGBTs Q2 and Q3.

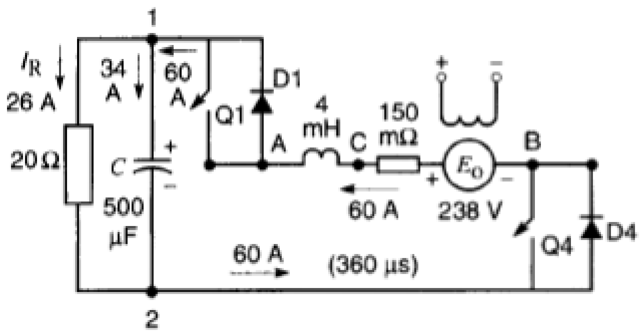


Figure 22.24
Current flows through diodes D1 and D4.

Next, when Q2, Q3 open and Q1, Q4 close, the current has to circulate via diodes D1 and D4 (Fig. 22.24). Applying KCL, a current of (60 − 26) = 34 A must flow into the capacitor during 360 μs. The resulting increase in voltage is

$$\Delta E = Q/C = 34\text{ A} \times 360\ \mu\text{s}/500\ \mu\text{F} = 24\text{ V}$$

Thus, the increase in voltage during 360 μs is exactly equal to the decrease during the remaining 140 μs of the switching cycle. The voltage across the resistor fluctuates between 524 + 12 = 536 V and 524 − 12 = 512 V.

This example shows that the converter can transfer power to the passive braking resistor. In so doing, the motor will slow down and the voltage between terminals A, B will decrease progressively. By continually adjusting the duty cycle during the deceleration period, it is possible to maintain the 60 A braking current until the speed is only a fraction of its rated value. This adjustment is of course done automatically by means of an electronic control circuit.

EXAMPLE 6.1

A dc motor with a rated terminal voltage of 110 V is to be fed from a 220 V dc mains with the help of a dc/dc converter. Find the duty ratio required for the converter.

SOLUTION:

$$V_{in} = 220\text{ V},\ V_o = 110\text{ V}$$

$$V_{o(av)} = D \cdot V_{in}$$

$$D = \frac{110}{220} \times 100 = 50\%$$

EXAMPLE 6.2

A separately excited dc motor fed from a 250-V mains through a dc/dc converter is running at 1,500 rpm and produces a back-emf of 195 V. Given an armature resistance 1 Ω, and the duty ratio of the chopper is 80%. Calculate the armature current.

SOLUTION:

$$V_{in} = 250\ \text{V} \quad E_b = 195\ \text{V} \quad D = 80\%$$

$$V_{o(av)} = D \cdot V_{in} = 0.8 \times 250 = 200\ \text{V}$$

$$I_a = \frac{V_0 - E_b}{R_a} = \frac{200 - 195}{1} = 5\ \text{A}$$

EXAMPLE 6.3

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 110$ V, $L_a = 1$ mH, $r_a = 0.25\ \Omega$, $E_b = 11$ V, $T = 4{,}500\ \mu s$, $T_{on} = 1{,}000\ \mu s$. Find I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$I_{min} = \frac{110}{0.25}\left[\frac{e^{\frac{0.25}{1\times10^{-3}}1000\times10^{-6}} - 1}{e^{\frac{0.25}{1\times10^{-3}}4500\times10^{-6}} - 1}\right] - \frac{11}{0.25}$$

$$I_{min} = 16.0759\ \text{A}$$

If I_{min} is positive, then the motor current is continuous.

$$I_{max} = \frac{V_{dc}}{r_a}\left[\frac{1 - e^{\frac{-r_a}{L_a}T_{on}}}{1 - e^{\frac{-r_a}{L_a}T}}\right] - \frac{E_b}{r_a}$$

$$I_{min} = \frac{110}{0.25}\left[\frac{e^{-\frac{0.25}{1\times10^{-3}}1000\times10^{-6}} - 1}{e^{-\frac{0.25}{1\times10^{-3}}4500\times10^{-6}} - 1}\right] - \frac{11}{0.25}$$

$$I_{max} = 100.11\ \text{A}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T}$$

$$= 110 \times \frac{1000 \times 10^{-6}}{4500 \times 10^{-6}}$$

$$V_{o(av)} = 24.44\ \text{V}$$

EXAMPLE 6.4

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 200$ V, $L_a = 2$ mH, $r_a = 1.0\ \Omega$, $E_b = 40$ V, $T = 10$ ms, $T_{on} = 7$ ms. Calculate I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$= \frac{200}{1}\left[\frac{e^{\frac{1}{2\times10^{-3}}\times7\times10^{-3}} - 1}{e^{\frac{1}{2\times10^{-3}}\times10\times10^{-3}} - 1}\right] - \frac{40}{1}$$

$$I_{min} = 3.572 \text{ A}$$

Positive sign of I_{min} indicates the motor current is continuous.

$$I_{max} = \frac{V_{dc}}{r_a}\left[\frac{1 - e^{\frac{-r_a}{L_a}T_{on}}}{1 - e^{\frac{-r_a}{L_a}T}}\right] - \frac{E_b}{r_a}$$

$$= \frac{200}{1}\left[\frac{1 - e^{-\frac{1}{2\times10^{-3}}\times7\times10^{-3}}}{1 - e^{-\frac{1}{2\times10^{-3}}\times10\times10^{-3}}}\right] - \frac{40}{1}$$

$$I_{max} = 155.276 \text{ A}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T}$$

$$= 200 \times \frac{7\times10^{-3}}{10\times10^{-3}}$$

$$V_{o(av)} = 140 \text{ V}$$

EXAMPLE 6.5

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 200$ V, $L_a = 0.8$ mH, $r_a = 0.65\ \Omega$, $E_b = 120$ V, $T = 3$ ms, $T_{on} = 0.25$ ms. Find I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$= \frac{200}{0.65}\left[\frac{e^{\frac{0.65}{0.8\times10^{-3}}0.25\times10^{-3}} - 1}{e^{\frac{0.65}{0.8\times10^{-3}}3\times10^{-3}} - 1}\right] - \frac{120}{0.65}$$

$$I_{min} = -177.9857 \text{ A}$$

A negative sign indicates that the motor current is discontinuous.

Hence, $I_{min} = 0$. From Equation (6.6),

$$I_{max} = \frac{V_{dc} - E_b}{r_a}\left(1 - e^{-\frac{r_a}{L_a}T_{on}}\right)$$

$$= \frac{200 - 120}{0.65}\left(1 - e^{-\frac{0.65}{0.8\times10^{-3}}0.25\times10^{-3}}\right)$$

$$I_{max} = 22.6245\text{ A}$$

$$t^x = -\frac{L_a}{r_a}\ln\left[\frac{E_b}{V_{dc}\left(1 - e^{-\frac{r_a}{L_a}T_{on}}\right) + E_b\left(e^{-\frac{r_a}{L_a}T_{on}}\right)}\right]$$

$$= -\frac{0.8\times10^{-3}}{0.65}\ln\left[\frac{120}{200\left(1 - e^{\frac{-0.65}{0.8\times10^{-3}}0.25\times10^{-3}}\right) + 120\left(e^{\frac{-0.65}{0.8\times10^{-3}}0.25\times10^{-3}}\right)}\right]$$

$$t^x = 0.1423\text{ ms}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T} + E_b\left(1 - \frac{T_{on}}{T} - \frac{t^x}{T}\right)$$

$$= 200\frac{0.25\times10^{-3}}{3\times10^{-3}} + 120\left(1 - \frac{0.25\times10^{-3}}{3\times10^{-3}} - \frac{0.1423\times10^{-3}}{3\times10^{-3}}\right)$$

$$V_{o(av)} = 120.9812\text{ V}$$

EXAMPLE 6.6

A first-quadrant chopper drives a separately excited dc motor with the following details: $V_{dc} = 110$ V, $L_a = 0.2$ mH, $r_a = 0.25\ \Omega$, $E_b = 40$ V, $T = 2{,}500\ \mu s$, $T_{on} = 1{,}250\ \mu s$. Find I_{min}, I_{max}, and $V_{o(av)}$.

SOLUTION:

$$I_{min} = \frac{V_{dc}}{r_a}\left[\frac{e^{\frac{r_a}{L_a}T_{on}} - 1}{e^{\frac{r_a}{L_a}T} - 1}\right] - \frac{E_b}{r_a}$$

$$I_{min} = \frac{110}{0.25}\left[\frac{e^{\frac{0.25}{0.2\times10^{-3}}1250\times10^{-6}} - 1}{e^{\frac{0.25}{0.2\times10^{-3}}2500\times10^{-6}} - 1}\right] - \frac{40}{0.25}$$

$$I_{min} = -83.754\text{ A}$$

A negative sign indicates that the motor current is discontinuous.

Hence $I_{min} = 0$. Now, using Equation (6.6),

$$I_{max} = \left(\frac{V_{dc} - E_b}{R}\right)\left(1 - e^{-\frac{R}{L}T_{on}}\right)$$

$$= \left(\frac{110 - 40}{025}\right)\left(1 - e^{-\frac{-0.25}{0.2\times10^{-3}}1250\times10^{-6}}\right)$$

$$I_{max} = 221.312 \text{ A}$$

$$t^x = -\frac{L_a}{r_a}\ln\left[\frac{E_b}{V_{dc}\left(1 - e^{-\frac{r_a}{L_a}T_{on}}\right) + E_b\left(e^{-\frac{r_a}{L_a}T_{on}}\right)}\right]$$

$$t^x = -\frac{0.2\times10^{-3}}{0.25}\ln\left[\frac{40}{110\left(1 - e^{\frac{-0.25}{0.2\times10^{-3}}1250\times10^{-6}}\right) + 40\left(e^{\frac{-0.25}{0.2\times10^{-3}}1250\times10^{-6}}\right)}\right]$$

$$t^x = 0.6948 \text{ ms}$$

$$V_{o(av)} = V_{dc}\frac{T_{on}}{T} + E_b\left(1 - \frac{T_{on}}{T} - \frac{t_x}{T}\right)$$

$$= 110\frac{1250\times10^{-6}}{2500\times10^{-6}} + 40\left(1 - \frac{1250\times10^{-6}}{2500\times10^{-6}} - \frac{0.6948\times10^{-3}}{2500\times10^{-6}}\right)$$

$$V_{o(av)} = 63.882 \text{ V}$$

EXAMPLE 6.7

A first-quadrant chopper is feeding a separately excited motor rated 200 V, 10 A, 1,500 rpm. The chopper is supplied from a constant bus bar voltage of 300 V. If the SCR turn-OFF time is 38 μs, compute the value of commutation components.

$$L = \frac{t_1}{F(x)} \frac{V_{dc}}{xI_a}$$

where t_1 = circuit turn-OFF time (and must be greater than SCR turn-OFF time) and V_{dc} = source voltage:

$$\omega_{LC} t_1 = \pi - 2\sin^{-1}\left(\frac{1}{x}\right) = F(x)$$

I_a: Maximum load current
I_m: Peak capacitor current
$V_{dc} = 300$ V
$t_1 = 38 + 38 = 76$ μs
$x = 1.5$
$I_a = 10$ A

$$L = \frac{76 \times 10^{-6} \times 300}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 1.5 \times 10}$$

$$L = 903.61\ \mu H$$

$$C = \frac{t_1}{F(x)} \frac{xI_a}{V_{dc}}$$

$$C = \frac{76 \times 10^{-6} \times 1.5 \times 10}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 300}$$

$$C = 2.259\ \mu F$$

EXAMPLE 6.8

First-quadrant chopper using current-commutated SCRs is used to drive a 230-V, 25-A, 1,450-rpm separately excited dc motor. If the motor is supplied from a 110-V busbar and the turn-OFF time of the SCR is 25 μs, design the commutation circuit.

SOLUTION:

$$L = \frac{t_1}{F(x)} \frac{V_{dc}}{xI_a}$$

$$\omega_{LC} t_1 = \pi - 2\sin^{-1}\left(\frac{1}{x}\right) = F(x)$$

$V_{dc} = 110$ V
$t_1 = 25 + 25 = 50$ µs
$x = 1.5$
$I_a = 25$ A

$$L = \frac{50 \times 10^{-6} \times 110}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 1.5 \times 25}$$

$L = 87.19$ µH

$$C = \frac{t_1}{F(x)} \frac{xI_a}{V_{dc}}$$

$$C = \frac{50 \times 10^{-6} \times 1.5 \times 25}{\left(\pi - 2\sin^{-1}\left(\frac{1}{1.5}\right)\right) \times 110}$$

$C = 10.13$ µF

EXAMPLE 6.9

The speed of a 230-V, 25-A, 150-rpm separately excited dc motor is controlled by connecting a current-commutated first-quadrant chopper in the armature. The input voltage to the chopper is 300 V. The circuit turn-OFF time is fixed at 25 µs and the peak capacitor current is twice the load current. Compute the values of commutating components.

SOLUTION:

$V_{dc} = 300$ V
$t_1 = 25$ ms
$x = 2$
$I_a = 25$ A

$$L = \frac{t_1}{F(x)} \frac{V_{dc}}{xI_a}$$

$$L = \frac{25 \times 10^{-6} \times 300}{\left(\pi - 2\sin^{-1}\left(\frac{1}{2}\right)\right) \times 2 \times 25}$$

L = 71.62 µH

$$C = \frac{t_1}{F(x)} \frac{xI_a}{V_{dc}}$$

$$C = \frac{25 \times 10^{-6} \times 2 \times 25}{\left(\pi - 2\sin^{-1}\left(\frac{1}{2}\right)\right) \times 300}$$

C = 1.989 µF

EXAMPLE 7.1

A 230-V, 6-A, 1,500-rpm separately excited dc motor has an armature resistance of 5.1 Ω. A 1-quadrant chopper supplied from a 300-V dc bus is operating at a duty ratio of 60% and supplies power to the motor armature at rated current. Compute the motor speed.

SOLUTION:

At rated condition,

$$\begin{aligned} E_{b\,rated} &= V_a - I_{a\,rated} \times r_a \\ &= 230 - (5.1 \times 6) \\ &= 199.4\ \mathrm{V} \end{aligned}$$

$$E_{b\,rated} \propto 1,500\ \mathrm{rpm}(N_{rated})$$

At 60% duty ratio, $E_{b1} \propto N_1$.

$$\begin{aligned} \text{Armature voltage} &= 300 \times (60/100) \\ &= 180\ \mathrm{V} \end{aligned}$$

$$\begin{aligned} \text{Back-emf, } E_{b1} &= 180 - (5.1 \times 6) \\ &= 149.4\ \mathrm{V} \end{aligned}$$

Therefore,

$$\begin{aligned} N_1 &= \frac{E_{b1}}{E_{b\,rated}} N_{rated} \\ &= \frac{149.4}{199.4} \cdot 1500 \approx 1124\ \mathrm{rpm} \end{aligned}$$

EXAMPLE 7.2

A first-quadrant dc/dc converter is fed from a 300-V dc bus. When the converter supplies power to a separately excited dc motor at 40% duty ratio, the average armature current is 5 A at 1,560 rpm. What is the duty ratio required to reduce the speed to 1,300 rpm for the same armature current? The armature resistance is 5.1 Ω.

SOLUTION:

At 40% duty ratio, armature voltage is

$$\begin{aligned} V_a &= 300 \times \frac{40}{100} \\ &= 120\ \mathrm{V} \end{aligned}$$

$$\text{Back-emf at 40\% duty ratio, } E_{b1} = 120 - (I_a r_a)$$
$$= 120 - (5 \times 5.1)$$
$$= 94.5\text{ V}$$

$$E_{b1} \propto 1{,}560\text{ rpm}$$

Now back-emf, E_{b2}, at 1,300 rpm can be related as $E_{b2} \propto 1{,}300$ rpm.

$$E_{b2} = \frac{N_2}{N_1} E_{b1} = 78.75\text{ V}$$

$$\text{Armature voltage, } V_a = E_{b2} + I_a r_a$$

$$V_a = 300 \times D$$

Therefore,

$$D = 34.75\%$$

EXAMPLE 7.3

A separately excited dc motor is fed from a 440-V dc source through a single-quadrant chopper, $r_a = 0.2\ \Omega$, and armature current is 175 A. The voltage and torque constants are equal at 1.2 V/rad/s. The field current is 1.5 A. The duty cycle of chopper is 0.5. Find (a) speed and (b) torque.

SOLUTION:

a.

$$E_b = 220 - 175 \times 0.2 = 185\text{ V}$$
$$= 1.2 \times \omega_r \times I_f$$
$$\omega_r = 185/(1.2 \times 1.5)$$
$$= 102.77\text{ rad/s}$$
$$= 981.38\text{ rpm}$$

b.

$$\text{Torque} = 1.2 \times 1.5 \times 175 = 315\text{ N-m}$$

EXAMPLE 7.4

A separately excited dc motor has the following name plate data: 220 V, 100 A, 2,200 rpm. The armature resistance is 0.1 Ω, and inductance is 5 mH. The motor is fed by a chopper that is operating from a dc supply of 250 V. Due to restrictions in the power circuit, the chopper can be operated over a duty cycle ranging from 30% to 70%. Determine the range of speeds over which the motor can be operated at rated torque.

SOLUTION:

Because e torque is constant, i_a is the same for all the values of D.

$$V_{o(av)} = DV_{dc}$$

At D = 0.3,

$$\begin{aligned} V_{o(av)} &= 0.3 \times 250 \\ &= 75\text{ V} \\ E_{b(0.3)} &= V_{o(av)} - I_a r_a \\ &= 75 - (100 \times 0.1) \\ &= 65\text{ V} \end{aligned}$$

At D = 0.7,

$$\begin{aligned} V_{o(av)} &= 0.7 \times 250 \\ &= 175\text{ V} \\ E_{b(0.7)} &= 175 - 10 \\ &= 165\text{ V} \end{aligned}$$

Under rated conditions, $V_a = 220$ V, $I_a = 100$ A, $r_a = 0.1\ \Omega$

$$\begin{aligned} E_{b(rated)} &= 220 - (100 \times 0.1) \\ &= 210\text{ V} \end{aligned}$$

$$N_r = 2200\text{ rpm}$$

$$\frac{N_{0.7}}{N_r} = \frac{E_{b(0.7)}}{E_{b(rated)}}$$

$$\begin{aligned} N_{(0.7)} &= \frac{165}{210} \times 2200 \\ &= 1{,}728.5714\text{ rpm} \end{aligned}$$

$$\begin{aligned} N_{(0.3)} &= \frac{65}{210} \times 2200 \\ &= 680.95\text{ rpm} \end{aligned}$$

Hence speed can be varied in the range $680.95 \le N \le 1728.5714$.

EXAMPLE 7.5

A separately excited dc motor has an armature resistance 2.3 Ω, and armature current is 100 A. (a) Find the voltage across the braking resistance for a duty ratio of 25%. (b) Find the power dissipated in braking resistance.

SOLUTION:

$$\text{Average current} = I_a(1 - D) = 100(1 - 0.25) = 75\text{ A}$$

$$\text{Average Voltage} = I_{b(avg)} \times R_b = 75 \times 2.3 = 172.5\text{ V}$$

$$P_b = I_a^2 R_b (1 - D)$$

$$P_b = 100^2 \times 2.3 \times (1 - 0.25) = 17250\text{ W}$$

EXAMPLE 7.6

A separately excited dc motor has the following name plate data: 200 V, 75 A, and 1,500 rpm. The armature resistance is 0.2 Ω. If dynamic baking takes place at 600 rpm at rated torque, compute the duty ratio. The braking resistance is 5 Ω.

SOLUTION:

E_b under rated condition,

$$E_{b(rated)} = V - I_a r_a$$
$$= 200 - (75 \times 0.2)$$
$$= 185 \text{ V}$$

$$\frac{E_{b(600)}}{E_{b(rated)}} = \frac{N}{N_{rated}}$$

$$E_{b(600)} = \left(\frac{N}{N_{rated}}\right) E_{b\,rated}$$

$$E_{b(600)} = \left(\frac{600}{1500}\right) \times 185 = 74 \text{ V}$$

Now,

$$E_{b(600)} = I_a \left(r_a + R_b (1 - D)\right)$$

Because braking takes place at rated torque, $I_a = I_{arated} = 75$ A,

$$\text{i.e., } E_{b(600)} = 75\left(0.2 + 5(1 - D)\right)$$
$$74 = 75 \times 0.2 + 75 \times 5(1 - D)$$
$$\therefore D = 0.84$$

EXAMPLE 7.7

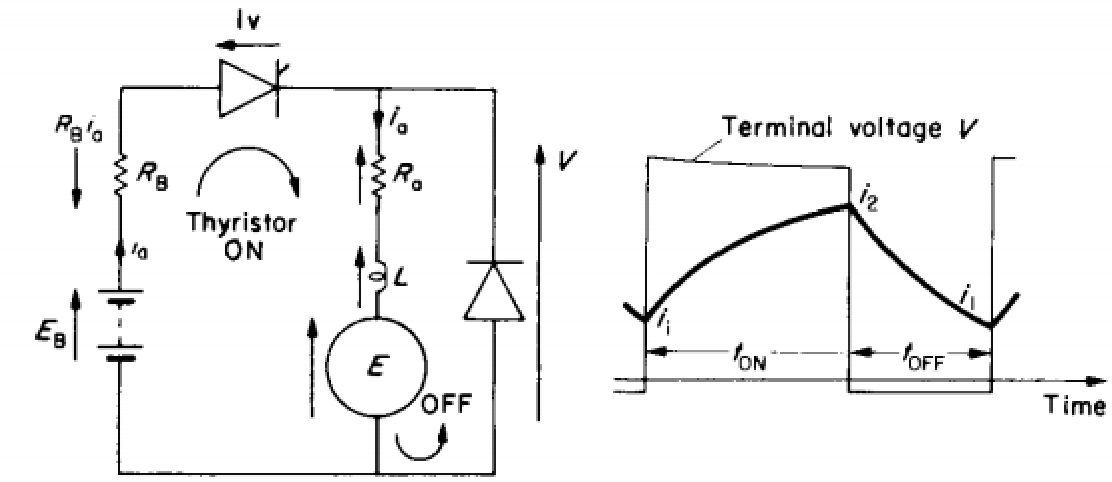
A dual-input dc/dc converter is supplied from two dc sources: 12-V and 24-V batteries. The duty ratio of the power switch connected to the first source is 40%, while that of the second source is 25%. Compute the average output voltage. The load consists of large inductance and resistance.

SOLUTION:

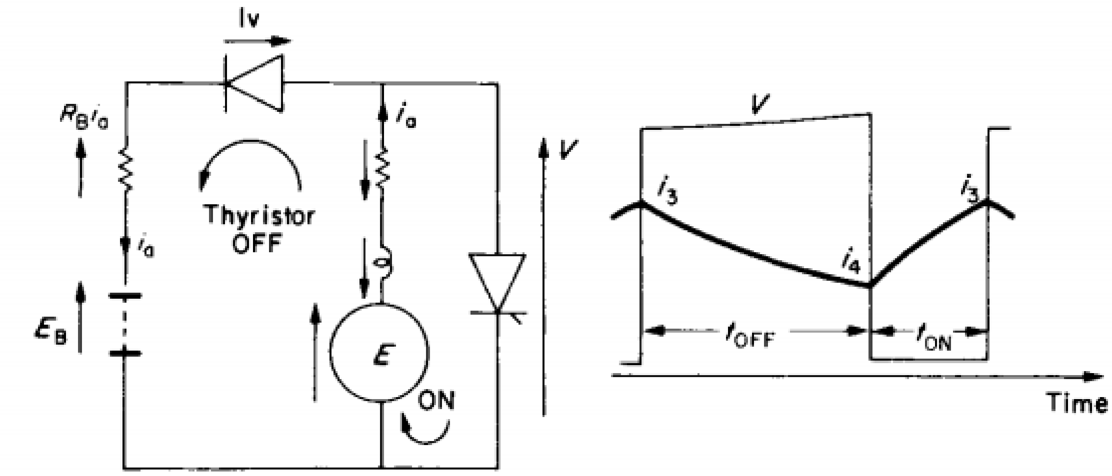
$$V_{o(av)} = 12 \times \frac{40}{100} + 24 \times \frac{25}{100}$$
$$= 4.8 + 6$$
$$= 10.8 \text{ V}$$

Example 7.1

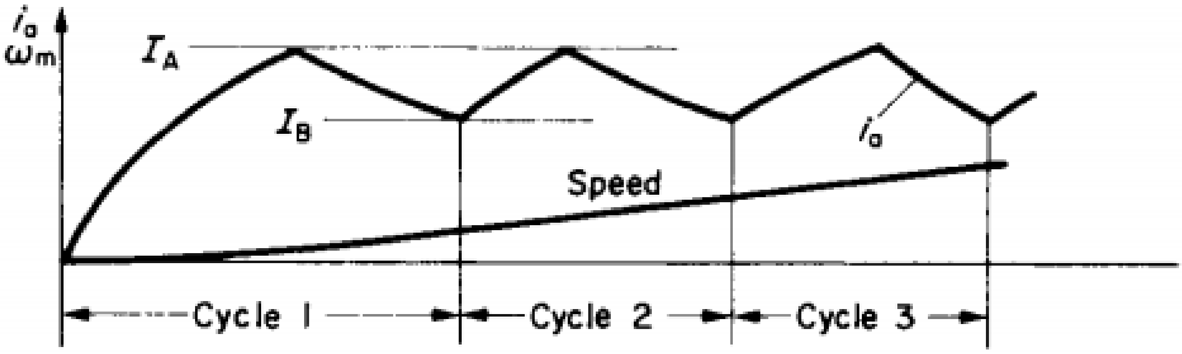
An electrically-driven automobile is powered by a d.c. series motor rated at 72 V, 200 A. The motor resistance and inductance are respectively 0.04 Ω and 6 milli-henrys. Power is supplied via an ON/OFF controller having a fixed frequency of 100 Hz. When the machine is running at 2500 rev/min the generated-e.m.f. per field-ampere, k_{fs}, is 0.32 V which may be taken as a mean "constant" value over the operating range of current. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor, when operating at this particular speed and with a duty-cycle ratio δ of 3/5. Mechanical, battery and semi-conductor losses may be neglected when considering the relevant diagrams of Fig. 7.1a.



(a) MOTORING (Motoring conventions)



(b) GENERATING (Generating conventions)



(c) Acceleration between limits

FIG. 7.1. Chopper-fed d.c. machine.

Chopping period = 1/100 = 10 msec and for $\delta = 3/5$; ON + OFF = 6 + 4 msec.

The equations are:

for ON period: $V = k_{fs}i + Ri + Lpi$—from eqn (7.2a).

Substituting: $72 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$.

For OFF period: $0 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$—from eqn (7.2b).

Rearranging:

ON $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 200 = I_{max}$,

OFF $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 0 = I_{min}$.

Current oscillates between a "low" of i_1 and a "high" of i_2, with $\tau = 0.0167$ second

ON $i_2 = i_1 + (200 - i_1)(1 - \mathrm{e}^{-0.006/0.0167})$,

$i_2 = 200 - (200 - i_1)\mathrm{e}^{-0.36} = \underline{60.46 + 0.698 i_1}$.

OFF $i_1 = i_2 + (0 - i_2)(1 - \mathrm{e}^{-0.004/0.0167})$

$i_1 = i_2\ \mathrm{e}^{-0.24} = \underline{0.787 i_2}$.

Hence, by substituting: $i_2 = 60.46 + 0.698 \times 0.787 i_2$,

from which $\underline{i_2 = 134.1\ \mathrm{A}}$ and $\underline{i_1 = 105.6\ \mathrm{A}}$.

$$\text{Torque} = k_\phi i = \frac{k_{fs} i}{\omega_m} \times i = \frac{k_{fs}}{\omega_m} i^2.$$

$$\text{Mean torque} = \frac{0.32}{2500 \times 2\pi/60}\left(\frac{134.1^2 + 105.6^2}{2}\right) = \underline{17.8\ \mathrm{Nm}}.$$

$$\text{Mean power} = \omega_m T_e = \frac{2\pi}{60} \times 2500 \times 17.8 = \underline{4.66\ \mathrm{kW}} = \underline{6.25\ \mathrm{hp}}.$$

Example 7.2

The chopper-controlled motor of the last question is to be separately excited at a flux corresponding to its full rating. During acceleration, the current pulsation is to be maintained as long as possible between 170 and 220 A. During deceleration the figures are to be 150 and 200 A. The total mechanical load referred to the motor shaft corresponds to an armature current of 100 A and rated flux. The total inertia referred to the motor shaft is

1.2 kg m^2. The battery resistance is 0.06 Ω and the semiconductor losses may be neglected. Determine the ON and OFF periods for both motoring and regenerating conditions and hence the chopping frequency when the speed is 1000 rev/min.

Calculate the accelerating and decelerating rates in rev/min per second and assuming these rates are maintained, determine the time to accelerate from zero to 1000 rev/min and to decelerate to zero from 1000 rev/min. Reference to all the diagrams of Fig. 7.1 will be helpful.

Rated flux at rated speed of 2500 rev/min corresponds to an e.m.f.:

$$E = V - RI_a = 72 - 0.04 \times 200 = 64\,\text{V}$$

At a speed of 1000 rev/min therefore, full flux corresponds to $64 \times 1000/2500 = 25.6$ V

Acceleration Total resistance $= R_a + R_B = 0.04 + 0.06 = 0.1\,\Omega$

For ON period $E_B = E + Ri_a + Lpi_a$,

$$72 = 25.6 + 0.1\,i_a + 0.006\,pi_a.$$

Rearranging: $0.06\,di_a/dt + i_a = 464 = I_{max}$.

Solution is: $i_2 = i_1 + (I_{max} - i_1)(1 - e^{-t_{ON}/\tau})$

and since i_1 and i_2 are known: $220 = 170 + (464 - 170)(1 - e^{-t_{ON}/0.06})$.

$$\frac{220-170}{464-170} = 1 - e^{-t_{ON}/0.06}$$

from which: $\underline{t_{ON} = 0.01118.}$

For OFF period $0 = 25.6 + 0.04i_a + 0.006pi_a$ (note resis. $= R_a$).

Rearranging: $0.15\,di_a/dt + i_a = -640 = I_{min}$.

Solution is: $i_1 = i_2 + (I_{min} - i_2)(1 - e^{-t_{OFF}/\tau})$.

Substituting i_1 and i_2: $170 = 220 + (-640 - 220)(1 - e^{-t_{OFF}/0.15})$,

$$\frac{170-220}{-640-220} = 1 - e^{-t_{OFF}/0.15},$$

from which: $\underline{t_{OFF} = 0.008985}$ $\underline{t_{ON} + t_{OFF} = 0.02017\text{ second.}}$

Duty cycle $\delta = 0.01118/0.02017 = \underline{0.554.}$ Chopping frequency $= 1/0.02017 =$ 49.58 Hz.

Deceleration

Thyristor ON $\quad 0 = E - R_a i_a - Lpi_a.$

Substituting: $\quad = 25.6 - 0.04 i_a - 0.006 p i_a.$

Rearranging: $\quad 0.15 di_a/dt + i_a = 640 = I_{max}.$

Solution is: $\quad i_3 = i_4 + (I_{max} - i_4)(1 - e^{-t_{ON}/\tau}).$

Substituting: $\quad 200 = 150 + (640 - 150)(1 - e^{-t_{ON}/0.15}),$

$$\frac{200 - 150}{640 - 150} = 1 - e^{-t_{ON}/0.15},$$

from which: $\quad \underline{t_{ON} = 0.01614.}$

Thyristor OFF $\quad E_B = E - Ri_a - Lpi_a,$

$$72 = 25.6 - 0.1 i_a - 0.006 Lpi_a.$$

Rearranging: $\quad 0.06 di_a/dt + i_a = -464 = I_{min}.$

Solution is: $\quad i_4 = i_3 + (I_{min} - i_3)(1 - e^{-t_{OFF}/\tau}).$

Substituting: $\quad 150 = 200 + (-464 - 200)(1 - e^{-t_{OFF}/0.06}),$

$$\frac{150 - 200}{-464 - 200} = 1 - e^{-t_{OFF}/0.06}$$

from which: $\quad \underline{t_{OFF} = 0.004697} \quad \underline{t_{ON} + t_{OFF} = 0.02084 \text{ second.}}$

Duty cycle $\delta = 0.01614/0.02084 = \underline{0.774.}$ Chopping frequency $= 1/0.02084 =$ $\underline{47.98 \text{ Hz.}}$

Accelerating time

Load torque $= k_\phi I_a = \dfrac{E}{\omega_m} I_a \quad = \dfrac{64}{2500 \times 2\pi/60} \times 100 = 24.45$ Nm.

During acceleration:

$$k_\phi I_{mean} = \frac{64}{2500 \times 2\pi/60} \times \frac{220 + 170}{2} = 0.2445 \times 195 = 47.67 \text{ Nm}.$$

Constant $d\omega_m/dt \quad = \dfrac{T_e - T_m'}{J} = \dfrac{47.67 - 24.45}{1.2} = 19.35$ rad/s per second

$$= 19.35 \times \frac{60}{2\pi} = \underline{184.8 \text{ rev/min per sec.}}$$

$$\text{Accelerating time to 1000 rev/min} = \frac{1000}{184.8} = \underline{5.41 \text{ seconds.}}$$

Decelerating time

During deceleration: $k_\phi I_{mean} = 0.2445(-200 - 150)/2 = -42.8$ Nm.

Note that this electromagnetic torque is now in the same sense as T_m, opposing rotation.

The mechanical equation is: $\quad T_e = T_m + J\, d\omega_m/dt,$

$$-42.8 = 24.45 + 1.2\, d\omega_m/dt.$$

from which:

$$\frac{d\omega_m}{dt} = \frac{-67.25}{1.2} = -56.04 \text{ rad/s per second} = \underline{-535.1 \text{ rev/min per second.}}$$

Time to stop from 1000 rev/min with this torque maintained $= 1000/535.1 =$ $\underline{1.87 \text{ seconds.}}$

Example 4

A separately excited d.c. motor with R_a = 1.2 ohms and L_a = 30 mH , is to be controlled using class-A thyristor chopper as shown in Fig.9.11 .The d.c. supply V_d = 120V . By ignoring the effect of the armature inductance L_a , it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle γ =1.

(b) Draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant $Ke\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to γ = 0.5.

Solution

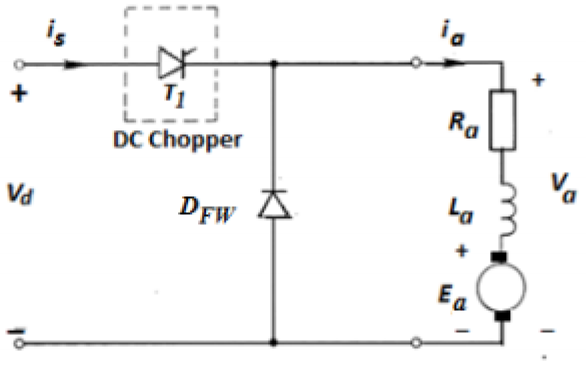


Fig. 9.11 Thyristor chopper drive.

The average armature voltage is

$$V_{av} = \gamma\, V_d \;= 1 \times\, 120 = 120\, V$$

The motor's speed:

$$n \;= \frac{V_{av}}{K_e\emptyset} - \;\frac{R_a}{K_T K_e \emptyset^2} T_d$$

At no load $T_d = 0$, hence

or $$n_o \;= \frac{\gamma V_d}{K_e\emptyset} = \frac{120}{0.042} = 2857\, rpm$$

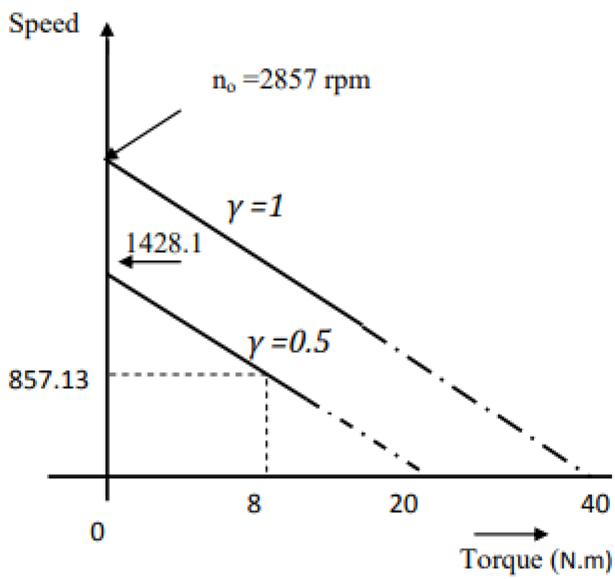
At starting, $n = 0$. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e\emptyset} - \;\frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$\therefore T_{st} \;= \frac{9.55\; \gamma V_d}{R_a} K_e\emptyset$$

$$T_{st} \;= \frac{9.55\; x120}{1.2} \times\; 0.042 = 40\; N.m$$

Fig.9.11 Speed-torque characteristics



(b) At $\gamma = 0.5$

$$V_a = \gamma V_d = 0.5 \, x \, 120 = 60 \, V$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5 \, rpm$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20 \text{ N.m}$$

At $\gamma = 0.5$, $T_L = 8$ N.m

$$n = \frac{60}{0.042} - \frac{1.2}{9.55(0.042)^2} \times 8 = 857.13 \text{ rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 5

In the microcomputer -controlled class –A IGBT transistor DC chopper shown in Fig.12.6, the input voltage V_d = 260V, the load is a separately excited d.c. motor with Ra = 0.28 Ω and L_a = 30 mH . The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor n (rpm) when the chopper is switched fully ON such that the duty cycle γ = 1.0.

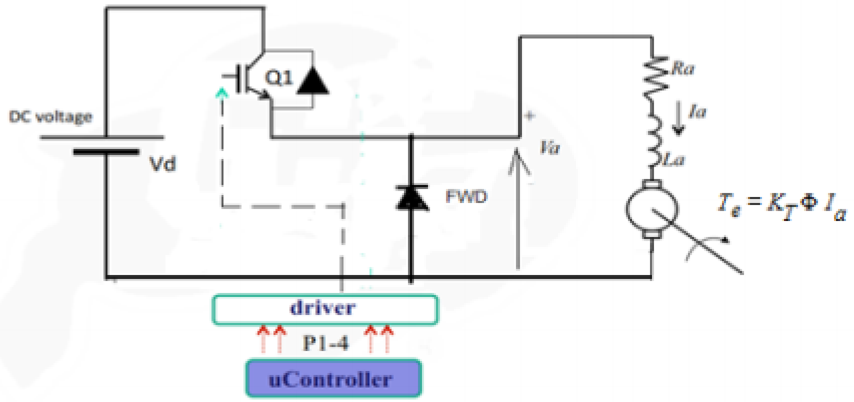


Fig.12.6 IGBT Chopper drive.

Solution

(a) With steady – state operation of the motor, the armature inductance L_a behaves like a short circuit and therefore has no effect at all.

At stand still n = 0 , and therefore E_a = 0 , hence from Eq.(12.22)

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30\ A$$

$$\therefore\ V_{a0} = 0.28\ \times 30 = 8.4\ V$$

At full speed n = 2500 rpm ,

$$E_{a2500} = K_e\ \phi\ n = 0.1\ \times\ 2500 = 250\ V$$

For separately excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4\,V$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. γ =1, then $V_a = V_d = 260$ V .

At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e\,\phi\,n + I_a R_a \ = 260\ \text{V}$$

$$0.1\,n + 30\ x\ 0.28\ = 260 \qquad \rightarrow \qquad n = 2516\ \text{rpm}$$

Example 1: A separately-excited d.c. motor with $R_a = 0.3\Omega$, and $L_a = 15\,mH$ is to be speed controlled over a range 0 - 2000 r.p.m. The d.c. supply is 220V. The load torque is constant and requires an average armature current of 25A.

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi = 0.1002$ V/rpm.

Solution: In the steady-state, the armature inductance has no effect. The required motor terminal voltages are:

At $n = 0$, $E_b = 0$, so that

$$V_{dc} = E_b + I_a R_a = I_a R_a = 25 \times 0.3 = 7.5\,V.$$

At $n = 2000$ rpm,

$$E_b = K_e\Phi n = 0.1002 \times 2000 = 200.4\,V.$$

$$\therefore V_{dc} = E_b + I_a R_a = 200.4 + 25 \times 0.3 = 207.9\,V.$$

$$V_o = \gamma V_d$$

To give $V_o = 7.5\,V$ $\therefore$ $7.5 = \gamma_o \times 220$ or $\gamma_o = \frac{7.5}{220} = 0.034$

To give $V_o = 207.9\,V$ $\therefore$ $207.9 = \gamma_{2000} \times 220$

or $\gamma_{2000} = \frac{207.9}{220} = 0.943$.

Range of γ: $0.034 \leq \gamma \leq 0.943$.

(b) If the chopper was to be switched fully on, what is the speed of the motor when $\gamma = 1$.

Sol. when $\gamma = 1$, $V_o = 220$.

$$\therefore n = \frac{E_b}{K_e\Phi}, \quad E_b = V_o - I_a R_a = 220 - 25 \times 0.3 = 212.5\,V$$

$$n = \frac{212.5}{0.1002} = \underline{\underline{2121}} \text{ rpm}$$

Example :

An electrically-driven automobile is powered by d.c. series motor rated at 100V, 200A. The motor resistance and inductance are respecrively 0.65 Ω and 6 mH. power is supplied from ideal battry of 120V via class-A d.c. chopper having a fixed frequency of 100 Hz. The machine constant $K_e\phi = 0.00025$ V/rpm and the motor speed is 2500 rpm. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor when running at 2500 rpm with duty cycle γ of 3/5.

Solution:

Chopping period $T = \frac{1}{f} = \frac{1}{100} = 10$ ms.

$$t_{on} = \gamma T = \frac{3}{5} \times 10 = 6 \text{ ms}$$

$$\therefore \; t_{off} = 10 - 6 = 4 \text{ ms}.$$

$$I_{max} = \frac{V_{av}}{R_a} + \frac{t_{off}}{2L_a} V_{av} = \frac{\gamma V_i}{R_a} + \frac{t_{off}}{2L_a}\gamma V_i$$

$$= \frac{\frac{3}{5} \times 120}{0.65} + \frac{4 \times 10^{-3}}{2 \times 6 \times 10^{-3}} \left(\frac{3}{5} \times 120\right)$$

$$= 110.7 + 24 = 134.7 \text{ A}$$

$$I_{min} = \frac{V_{av}}{R_a} - \frac{t_{off}}{2L_a} V_{av}$$

$$= 110.7 - 24 = 86.76 \text{ A}.$$

For series motor:

Mean torque: $T_e = K_T \phi I_{av}^2 = 9.55 K_e \Phi \left(\frac{I_{max} + I_{min}}{2}\right)^2$

$$= 9.55 \times 0.00025 \left(\frac{134.7 + 86.76}{2}\right)^2$$

$$= 30 \text{ N.m}.$$

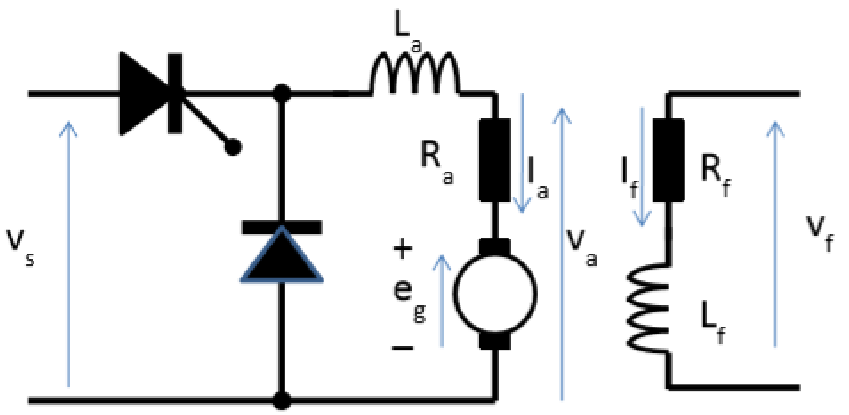
Mean power: $P_e = \omega T_e = \frac{2\pi}{60} \times 2500 \times 30 = 7.85$ kW

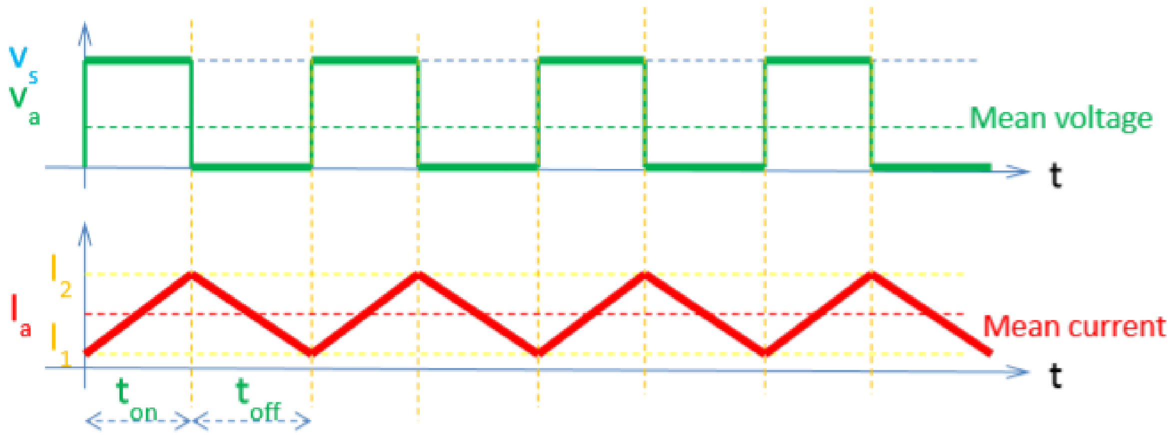
$= 10.5$ hp.

DC-DC MOTOR DRIVES (Choppers)

A chopper directly converts a fixed-voltage DC supply to a variable-voltage DC supply.

Step-Down Chopper (Motoring)





During t_{on} time

$$Vs = L\frac{di}{dt} + Va \qquad (1)$$

$$Vs - Va = L\frac{di}{dt} \qquad (2)$$

$$di = \frac{Vs-Va}{L}dt \qquad (3)$$

$$\Delta I = I_2 - I_1 = \frac{Vs-Va}{L}t_{on} \qquad (4)$$

During t_{off} time

$$0 = -L\frac{di}{dt} + Va \qquad (5)$$

$$Va = L\frac{di}{dt} \qquad (6)$$

$$di = \frac{Va}{L}dt \qquad (7)$$

$$\Delta I = \frac{Va}{L}t_{off} \qquad (8)$$

$$t_{on} = DT,\ t_{off} = (1-D)T \quad where D\ is\ the\ Duty\ cycle$$

Equating ΔIs

$$\Delta I = \frac{Vs-Va}{L}t_{on} = \frac{Va}{L}t_{off} = \frac{Vs-Va}{L}DT = \frac{Va}{L}(1-D)T => \boxed{Va = DVs}$$

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \qquad (9)$$

$$I_1 + I_2 = 2I_{mean} \quad (10)$$

adding (10) to (4) or (8)

$$2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{on} \quad or \quad 2I_2 = 2I_{mean} + \frac{Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{on}} \quad or \quad \boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{on}} \quad or \quad \boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{off}}$$

Example : A simple DC step-down chopper is operated at a frequency of 2KHz from a 120 V DC source to supply a motor load with Ra = 0.85 ohms, La = 0.32 mH. The required torque genrated by the motor is 20 Nm, at 1000 rpm, and field current is measured to be 1A. If Kv = 0.8345 V/A-rad/S, determine (a) the duty cycle for the switching pulse, (b) the mean load current, and (c) the max & min load currents.

(a) $V_a = DV_s = I_a R_a + E_g \qquad I_a = ? \qquad E_g = ?$

$$T = K_V I_a I_f \Rightarrow I_a = \frac{20}{0.8345 \ x \ 1} = 23.96 \ A$$

$$E_g = K_V \omega I_f = 0.8345 \ x \ 2\pi \ x \ \frac{1000}{60} = 87.38 \ V$$

$V_a = 23.96 \ x \ 0.85 + 87.38 = 107.75$ therefore $D = \frac{107.74}{120} = 0.89 = 89\%$

(b) $I_{mean} = I_a = 23.96 \ A$

(c) $\boldsymbol{I_{max} = I_{mean} + \frac{Vs - Va}{2L} t_{on}}$ **or** $\boldsymbol{I_{max} = I_{mean} + \frac{Va}{2L} t_{off}}$

We know $t_{on} = DT, \ t_{off} = (1 - D)T \quad and \quad T = 1/f$

$$I_{max} = 23.96A + \frac{120V - 107.75V}{2 * 0.32mH} * 0.89 * \frac{1}{2000} = 32.47A$$

$\boldsymbol{I_{min} = I_{mean} - \frac{Vs - Va}{2L} t_{on}}$ **or** $\boldsymbol{I_1 = I_{mean} - \frac{Va}{2L} t_{off}}$

$$I_{min} = 23.96A - \frac{120V - 107.75V}{2 * 0.32mH} * 0.89 * \frac{1}{2000} = 15.45 \ A$$

Example : A separately excited DC motor is powerd by a DC chopper from a 600 V dc source. The armature resistance Ra = 0.05 ohms. The back e.m.f.constant of the motor is kv=1.527 V/A-rads/s. The armature voltage is continuous and ripple free. If the duty cycle of the copper is 60%, determine (a) the input power from the source, (b) the equivalent input resistance of the chopper drive, (c) the motor speed, and (d) the developed torque.

(a) P_{input}= ?, $P_{input} = DV_s I_a = 0.6 \times 600 \times 250 = 90$ kw

(b) Req= ?, $Req = \frac{Vs}{Is} = \frac{Vs}{DIs} = \frac{600V}{0.6 \ x \ 250A} = 4 \ \Omega$

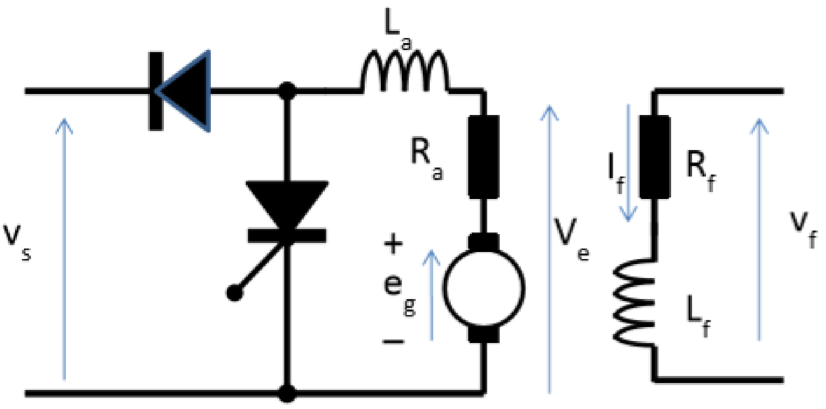
(c) ω = ?, $E_g = k_v \omega I$, E_g= ?, $V_a = I_a R_a + E_g$, V_a = ?, $V_a = DV_s = 0.6 \times 600 = 360$ V

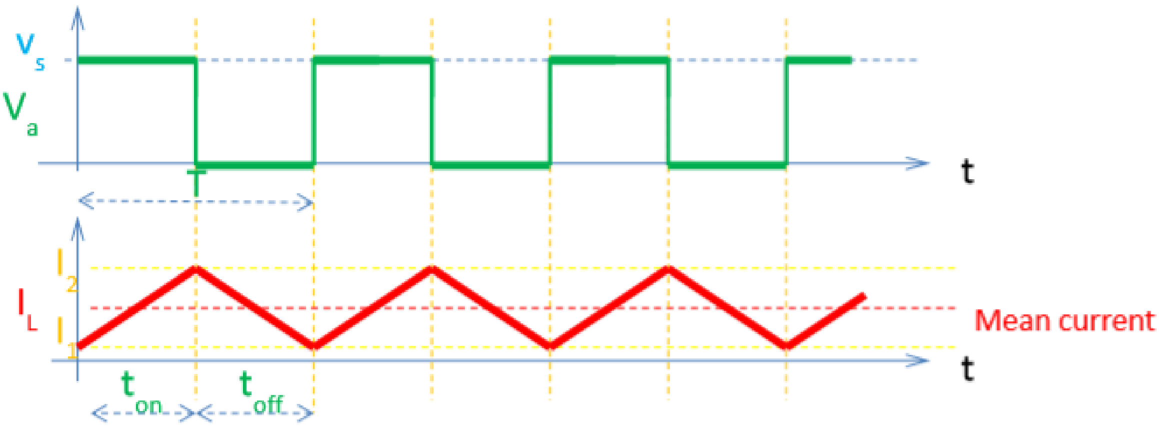
$E_g = 360 - 250 \times 0.05 = 347.5$ V

$\omega = \frac{347.5V}{1.525 \ x \ 2.5A} = 91.03$ rad/s or $91.03 \ x \ \frac{60}{2\pi}$=869.3 rpm

(d) T_D=?, $T_D = k_v I_f I_a = 1.527 \times 250 \times 2.5 = 954.38$ Nm

Step-Up Chopper – (Regenerative Braking)





During t_{on} time

$$Ve = L\frac{di}{dt} \qquad (1)$$

$$di = \frac{Ve}{L}dt \qquad (2)$$

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} \qquad (3)$$

During t_{off} time

$$Ve = -L\frac{di}{dt} + Vs \qquad (4)$$

$$di = \frac{Vs-Ve}{L}dt \qquad (5)$$

$$\Delta I = I_2 - I_1 = \frac{Vs-Ve}{L}t_{off} \qquad (6)$$

Equating ΔIs

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} = \frac{Vs-Ve}{L}t_{off} \;=>\; \frac{Ve}{L}DT = \frac{Vs-Ve}{L}(1-D)T \qquad (7)$$

$$VeD = Vs - Ve - VsD + VeD \;=>\; Vs = \frac{Ve}{1-D}$$

Since average voltage across L is zero, therefore Ve = Va and $\boxed{Vs = \frac{Va}{1-D}}$ (8)

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \qquad (9)$$

$$I_1 + I_2 = 2I_{mean} \qquad (10)$$

adding (10) to (3)or(6) remembering Ve = Va

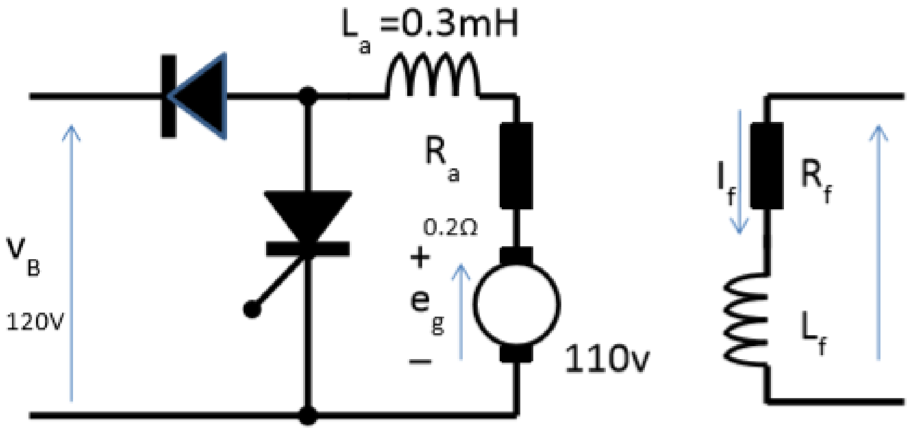
$$2I_2 = 2I_{mean} + \frac{Va}{L}t_{on} \qquad or \qquad 2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{on}} \qquad or \qquad \boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{on}} \qquad or \qquad \boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{off}}$$

Example : In a battery powered car, operating a frequency of 5 KHz, the battery voltage is 120 V. It is driven by a DC motor and employs chopper control. The resistance of the motor is 0.2 ohms and its inductance is 0.3 mH. During braking, the chopper configuration is changed to voltage step-up mode. While going down the hill at a certain speed, the back emf of the motor is 110 V and the breaking current is 10 A. Detrermine (a) the copper duty cycle, and (b) Max and Min values of the current.



(a) $D = ?, V_B = \frac{Va}{1-D} = \frac{IaRa+Eg}{1-D} > 1 - D = \frac{IaRa+Eg}{V_B} = \frac{110V+0.2x10}{120} => D = 6.67\%$

(b) $I_{max} = I_{mean} + \frac{Va}{2L}t_{on} = 10A + \frac{112x0.0667}{2x.03x10^{-3}x2000} = 16.22\ A$

$$I_{min} = I_{mean} - \frac{Va}{2L}t_{on} = 10A - \frac{112x0.0667}{2x.03x10^{-3}x2000} = 3.78\ A$$

Problem-Repeat above for 50V back emf.

Motoring and Regenerative Braking `Two-Quadrant Chopper (buck-boost`

Example 13.1: ***DC chopper with load back emf (first quadrant)***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine, with and without (rotor standstill) the back emf:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the average load output current, average switch current, and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, (and electromagnetic efficiency for $E > 0$);
viii. sketch the output current and voltage waveforms.

5. The speed of a separately excited DC motor is controlled by a chopper. The DC supply voltage is 120 V, armature circuit resistance is 0.5 Ω, armature circuit inductance is 20 mH, and back emf constant is 0.05 V/RPM. The motor drives a constant torque load requiring an average current of 20A. Assuming the motor current to be continuous, determine the range of speed control and the range of duty cycle.

Given Data:

V_s=120 volts, R_a=0.5 ohms, L_a=20mH, K=0.05 V/RPM. I_a=20A

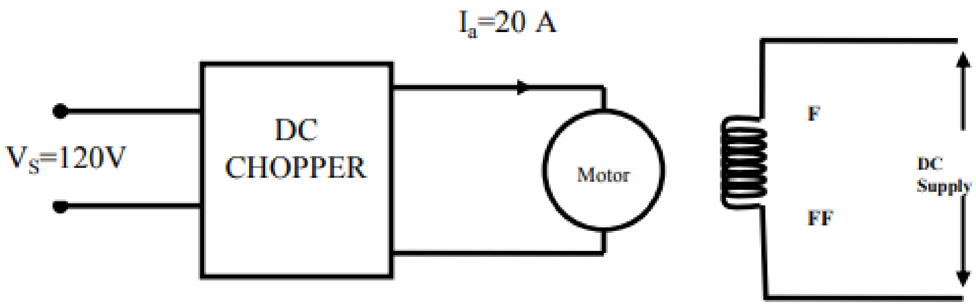
Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution



(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because V_a = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 20A and minimum possible speed is **ZERO**

$$\alpha x 120 = (0.05) x 0 + (20 x 0.05)$$

$$120\alpha = 10$$

$$\alpha = \frac{10}{120} = 0.08$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.08 \le \alpha \le 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x120 = 0.05xN + (20x0.5)$$
$$120 = 0.05N + 10$$
$$N = \frac{120 - 10}{0.05} = 2200rpm$$

The range of speed control is $0 \le N \le 2200RPM$

6. A 230 volts, 960 rpm, 200 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a dc source of 230 volts through a chopper. Assuming continuous conduction

   a) Calculate the duty ratio of chopper for monitoring operation at rated torque and 350 rpm

   b) If maximum duty ratio of chopper is limited to 0.95 and maximum permissible motor speed obtainable without field weakening

Given Data

V_s=230 volts, N=960 rpm, I_a=200 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =350rpm.

(ii) If $\alpha = 0.95$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (200x0.02) = 226 volts$$
$$E \text{ at } 350 \text{ rpm (ie) } \mathrm{E}_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$220 = Kx\omega_1$$
$$\omega_1 = \frac{960x2\pi}{60} = 100.53 rad/\sec$$
$$\therefore K = \frac{226}{100.53} = 2.24 Volts.\sec/rad$$

E_2 at 350 rpm is given by

$$\omega_2 = \frac{350x2\pi}{60} = 36.651$$
$$\therefore E_2 = 36.65x2.24 = 82.1 Volts$$

Motor terminal voltage at 350 rpm is

$$V_{350rpm} = 82.1 + (200x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{350\text{ rpm}}}{\mathrm{V}_{960\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.95\text{x}230 = 218.5 \text{ Volts}$$

$$\therefore E = V_a + I_a R_a = 218.5 + (200x0.02) = 222.5 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{222.5}{2.24} = 99.330 rad/\sec$$
$$N = \frac{99.330x60}{2\pi} = 948.53 rpm$$

7. A DC series motor is fed from a 600 volts source through a chopper. The DC motor has the following parameters armature resistance is equal to 0.04 Ω, field resistance is equal to 0.06 Ω, constant $k = 4 \text{ x } 10^{-3} Nm/Amp^2$. The average armature current of 300 Amps is ripple free. For a chopper duty cycle of 60% determine

i. Input power drawn from the source.

ii. Motor speed and

iii. Motor torque.

Given Data

V_s=600 volts, I_a=300 amps, R_a=0.04 ohms, R_f=0.06 ohms, K=$4x10^{-3} Nm/amp^2$ $\delta = 0.6$

DC SERIES motor.

Solution

a. Power input to the motor = P = $V_a I_a$

$$V_a = \delta V_s = 0.6x600 = 360 Volts$$
$$\therefore P = 360x300 = 108KW$$

b. For a DC series motor

$$E_a = K_a \phi \omega_m$$
$$= \mathrm{KI_a} \omega_m [\because \phi = I_a]$$
$$= 4x10^{-3} x300x\omega_m$$
$$\therefore V_a = E + I_a(R_a + R_s) = KI_a\omega_m + I_a(R_a + R_s)$$
$$\Rightarrow 0.6x600 = 4x10^{-3} x300x\omega_m + 300(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 27.5 rad/\sec(or) 2626 rpm$$
$$Motor \text{ Torque T} = \mathrm{Ka}\phi \mathrm{I_a} = KI_a^2$$
$$= 4x10^{-3} x300^2$$
$$= 360 \text{ N-M}$$

8. A 230 V, 1100 rpm, 220 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a chopper, which provides both motoring and braking operations. Calculate

   i. The duty ratio of chopper for motoring operation at rated torque and 400 rpm

   ii. The maximum permissible motor speed obtainable without field weakening, if the maximum duty ratio of the chopper is limited to 0.9 and the maximum permissible motor current is twice the rated current.

Given Data

V_s=230 volts, N=1100 rpm, I_a=220 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =400rpm.

(ii) If $\alpha = 0.9$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (220x0.02) = 225.6volts$$
$$E \text{ at 400 rpm (ie) } \mathrm{E}_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$\omega_1 = \frac{1110x2\pi}{60} = 115.192 rad/\sec$$
$$\therefore K = \frac{225.6}{115.192} = 1.95 Volts.\sec/rad$$

E_2 at 400 rpm is given by

$$\omega_2 = \frac{400x2\pi}{60} = 41.887 rad/\sec$$
$$\therefore E_2 = 41.887x1.95 = 81.68 Volts$$

Motor terminal voltage at 400 rpm is

$$V_{400rpm} = 81.68 + (220x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{400\text{ rpm}}}{\mathrm{V}_{1100\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.9\text{x}230 = 207 \text{ Volts}$$

$$\therefore E = V_a + I_a R_a = 207 + (2x220x0.02) = 215.8 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{215.8}{1.95} = 110.667 rad/\sec$$
$$N = \frac{110.667x60}{2\pi} = 1056.78 rpm$$

9. A DC chopper is used to control the speed of a separately excited dc motor. The DC voltage is 220 V, R_a= 0.2 Ω and motor constant $K_e\phi$=0.08 V/rpm. The motor drives a constant load requiring an average armature current of 25 A. Determine

   iv. The range of speed control

   v. The range of duty cycle. Assume continuous conduction

Given Data:

V_s=220 volts, R_a=0.2 ohms, L_a=20mH, K=0.08 V/RPM. I_a=25A

Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution

(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because V_a = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 25A and minimum possible speed is **ZERO**

$$\alpha x220 = (0.08)x0 + (25x0.2)$$

$$220\alpha = 10$$

$$\alpha = \frac{10}{220} = 0.04$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.04 \le \alpha \le 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x220 = 0.08xN + (25x0.2)$$

$$220 = 0.08N + 5$$

$$N = \frac{220-5}{0.08} = 2687.5rpm$$

The range of speed control is $0 \le N \le 2687.5RPM$

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

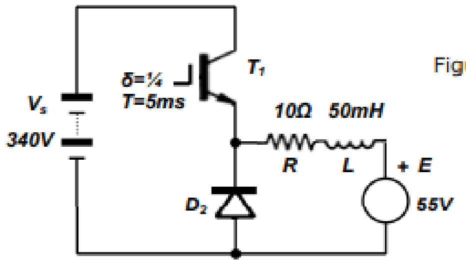


Figure Example 13.1. *Circuit diagram.*

i. From equations (13.2) and (13.3) the average and rms output voltages are both independent of the back emf, namely

$$\overline{V}_o = \frac{t_T}{T} V_s = \delta V_s$$

$$= ¼ \times 340\text{V} = 85\text{V}$$

$$V_r = \sqrt{\frac{t_T}{T}}\, V_s = \sqrt{\delta}\, V_s$$

$$= \sqrt{¼} \times 240\text{V} = 120\text{V rms}$$

ii. The rms ripple voltage hence ripple factor are given by equations (13.4) and (13.5), that is

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$

$$= 340\text{V}\sqrt{¼ \times (1 - ¼)} = 147.2\text{V ac}$$

and

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1}$$

$$= \sqrt{\frac{1}{¼} - 1} = \sqrt{3} = 1.732$$

No back emf, $E = 0$

iii. From equation (13.13), with E=0, the maximum and minimum currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} = 11.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} = 5.62\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{p-p} = \hat{I} - \check{I}$$
$$= 11.90\text{A} - 5.62\text{A} = 6.28\text{A}$$

Alternatively the ripple can be extracted from figure 13.4 using $T/\tau = 1$ and $\delta = ¼$.

iv. From equations (13.11) and (13.12), with $E = 0$, the time domain load current equations are

$$i_o = \frac{V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right)+\check{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 34\times\left(1-e^{\frac{-t}{5\text{ms}}}\right)+5.62\times e^{\frac{-t}{5\text{ms}}}$$

$$= 34-28.38\times e^{\frac{-t}{5\text{ms}}}\quad\text{(A)}\qquad\text{for}\quad 0\le t\le 1.25\text{ms}$$

$$i_o = \hat{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 11.90\times e^{\frac{-t}{5\text{ms}}}\quad\text{(A)}\qquad\text{for}\quad 0\le t\le 3.75\text{ms}$$

v. The average load current from equation (13.17), with $E = 0$, is

$$\overline{I}_o = \overline{V}_o/R = 85\text{V}/10\Omega = 8.5\text{A}$$

The average switch current, which is the average supply current, is

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I}-\check{I}\right)$$

$$= \frac{¼\times(340\text{V - 0})}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(11.90\text{A - }5.62\text{A}) = 2.22\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$
$$= 8.50\text{A} - 2.22\text{A} = 6.28\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s\overline{I}_i = 340\text{V}\times 2.22\text{A} = 754.8\text{W}$$
$$P_{out} = P_{in} = 754.8\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o_{rms}} = \sqrt{\frac{P_{out}}{R}}$$
$$= \sqrt{\frac{754.8\text{W}}{10\Omega}} = 8.7\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$
$$= \frac{340\text{V}}{2.22\text{A}} = 153.2\,\Omega$$

Load back emf, E = 55V

i. and ii. The average output voltage, rms output voltage, ac ripple voltage, and ripple factor are independent of back emf, provided the load current is continuous. The earlier answers for E = 0 are applicable.

iii. From equation (13.13), the maximum and minimum load currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-\delta T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} - \frac{55\text{V}}{10\Omega} = 6.40\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{\delta T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} - \frac{55\text{V}}{10\Omega} = 0.12\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{pp} = \hat{I} - \check{I}$$
$$= 6.4\text{A} - 0.12\text{A} = 6.28\text{A}$$

The ripple value is the same as the E = 0 case, which is as expected since ripple current is independent of back emf with continuous output current.
Alternatively the ripple can be extracted from figure 13.4 using T/τ =1 and δ=¼.

iv. The time domain load current is defined by

$$i_o = \frac{V_s - E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}e^{\frac{-t}{\tau}}$$

$$i_o(t) = 28.5\times\left(1-e^{\frac{-t}{5ms}}\right) + 0.12e^{\frac{-t}{5ms}}$$

$$= 28.5 - 28.38e^{\frac{-t}{5ms}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = -\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}e^{\frac{-t}{\tau}}$$

$$i_o(t) = -5.5\times\left(1-e^{\frac{-t}{5ms}}\right) + 6.4e^{\frac{-t}{5ms}}$$

$$= -5.5 + 11.9e^{\frac{-t}{5ms}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.37) is

$$\overline{I}_o = {V_o - E}\Big/{R}$$

$$= 85\text{V-}55\text{V}\Big/10\Omega = 3\text{A}$$

The average switch current is the average supply current,

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta\left(V_s - E\right)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{¼ \times \left(340\text{V} - 55\text{V}\right)}{10\Omega} - \frac{5\text{ms}}{5\text{ms}} \times \left(6.40\text{A} - 0.12\text{A}\right) = 0.845\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$

$$= 3\text{A} - 0.845\text{A} = 2.155\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s \overline{I}_i = 340\text{V} \times 0.845\text{A} = 287.3\text{W}$$

$$P_{out} = P_{in} = 287.3\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o,rms} = \sqrt{\frac{P_{out} - E\overline{I}_o}{R}}$$

$$= \sqrt{\frac{287.3\text{W} - 55\text{V} \times 3\text{A}}{10\Omega}} = 3.5\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$

$$= \frac{340\text{V}}{0.845\text{A}} = 402.4\ \Omega$$

The electromagnetic efficiency is given by equation (13.22), that is

$$\eta = \frac{E\overline{I}_o}{P_{in}}$$

$$= \frac{55\text{V} \times 3\text{A}}{287.3\text{W}} = 57.4\%$$

viii. The output voltage and current waveforms for the first-quadrant chopper, with and without back emf, are shown in the figure to follow.

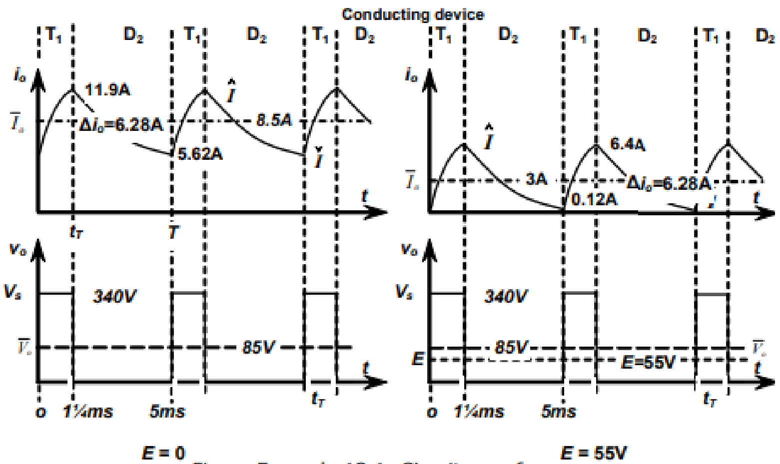


Figure Example 13.1. *Circuit waveforms.*

6. A step-down chopper supplies a separately excited dc motor with a supply voltage $E = 240$ V and back emf $E_b = 100$ V. Other data are total inductance L

$= 30$ mH, armature resistance $R_a = 2.5\ \Omega$, chopper frequency $= 200$, and duty cycle $= 50\%$. Assuming continuous current determine $I_{\max}$, $I_{\min}$, and the current ripple.

Solution

The circuit and waveforms are given in Fig. 3.3. From Eqn (3.17),

$$I_{\max} = \frac{E}{R_a}\left(\frac{1 - e^{-\tau_{\mathrm{ON}}/T_a}}{1 - e^{-\tau/T_a}}\right) - \frac{E_b}{R_a}$$

$$\frac{\tau_{\mathrm{ON}}}{\tau} = 0.5 \quad \text{and} \quad \tau = \frac{1}{200} = 0.005$$

Hence,

$$\tau_{\mathrm{ON}} = \frac{0.5}{200} = 0.0025$$

$$T_a = \frac{L_a}{R_a} = \frac{30 \times 10^{-3}}{2.5} = 0.012$$

$$\frac{\tau_{\mathrm{ON}}}{T_a} = \frac{0.0025}{0.012}$$

$$\frac{\tau}{T_a} = \frac{0.005}{0.012}$$

$$(1 - e^{-\tau_{\mathrm{ON}}/T_a}) = 0.188, \quad (e^{-\tau_{\mathrm{ON}}/T_a} - 1) = 0.2316$$

$$(1 - e^{-\tau/T_a}) = 0.34, \quad (e^{-\tau/T_a} - 1) = 0.5169$$

Hence,

$$I_{\max} = \frac{240}{2.5} \times \frac{0.188}{0.34} - \frac{100}{2.5} = 13.0 \text{ A}$$

From Eqn (3.18),

$$I_{\min} = \frac{E}{R_a}\left[\frac{e^{\tau_{\mathrm{ON}}/T_a} - 1}{e^{\tau/T_a} - 1}\right] - \frac{E_b}{R_a}$$

$$= \frac{240}{2.5} \times \frac{0.2316}{0.5169} - \frac{100}{2.5} = 3.0 \text{ A}$$

The current ripple is

$$\Delta i_{\mathrm{ld}} = \frac{I_{\max} - I_{\min}}{2} = \frac{13 - 3}{2} = 5 \text{ A}$$

7. A step-down chopper feeds a dc motor load. The data pertaining to this chopper-based drive is $E = 210$ V, $R_a = 7\ \Omega$, L (including armature inductance) = 12 mH. Chopper frequency = 1.5 kHz, duty cycle = 0.55, and E_b = 55 V. Assuming continuous conduction, determine the (a) average load current, (b) current ripple, (c) RMS value of current through chopper, (d) RMS value of current through D_{FW}, and (e) effective input resistance seen by the source, and (f) RMS value of load current.

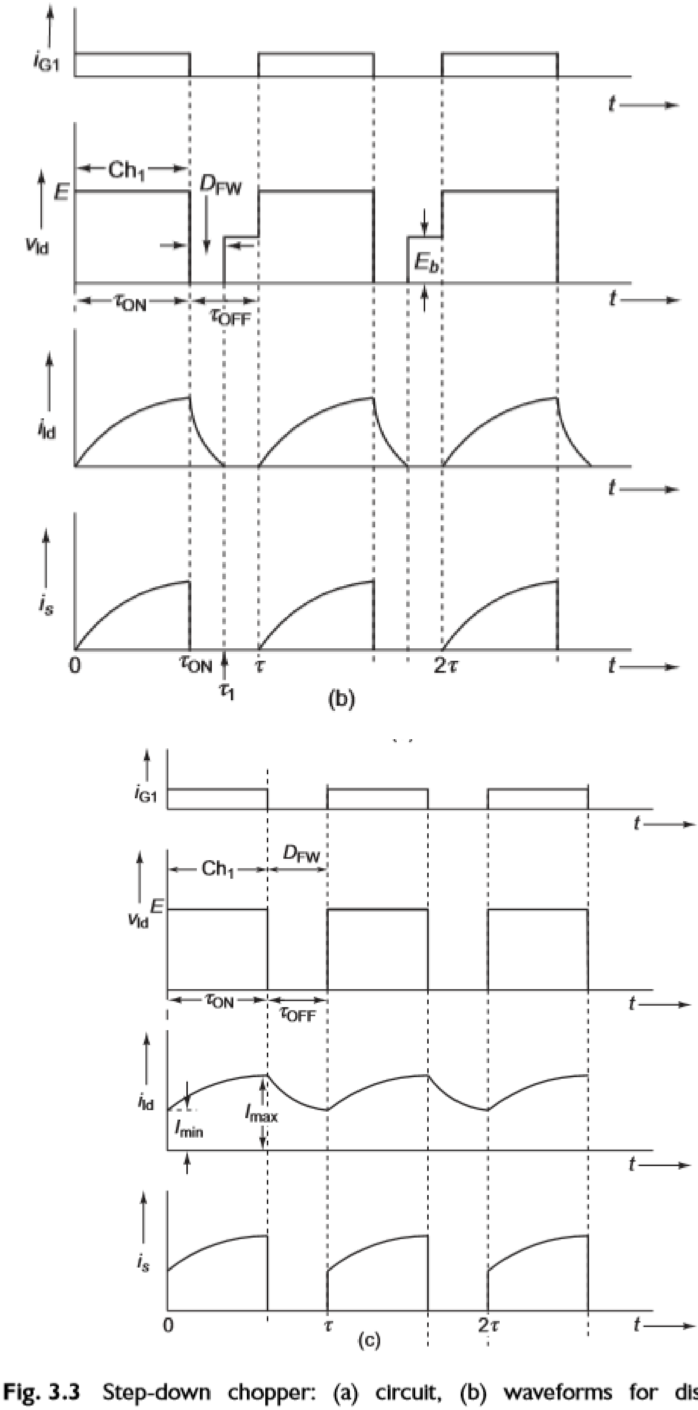


Fig. 3.3 Step-down chopper: (a) circuit, (b) waveforms for discontinuous conduction, (c) waveforms for continuous conduction

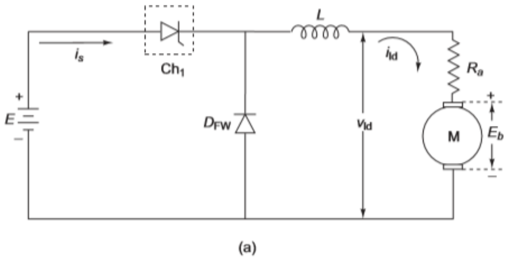


Fig. 3.3(a)

Solution

(a) The circuit and waveforms are given in Fig. 3.3. The average load current is given by Eqn (3.21) as

$$I_{ld} = \frac{E}{R_a}\frac{\tau_{ON}}{\tau} - \frac{E_b}{R_a}$$

Substitution of values gives

$$I_{ld} = \frac{210}{7} \times 0.55 - \frac{55}{7}$$
$$= 8.64 \text{ A}$$

$$I_{max} = \frac{E}{R_a}\frac{(1 - e^{-\tau_{ON}/T_a})}{(1 - e^{-\tau/T_a})} - \frac{E_b}{R_a} \qquad (3.17)$$

and

$$I_{min} = \frac{E}{R_a}\frac{(e^{\tau_{ON}/T_a} - 1)}{(e^{\tau/T_a} - 1)} - \frac{E_b}{R_a} \qquad (3.18)$$

The current ripple can now be obtained as

$$\Delta i_{ld} = \frac{I_{max} - I_{min}}{2} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - e^{\tau_{ON}/T_a} - e^{\tau_{OFF}/T_a}}{e^{\tau/T_a} - 1}\right] \qquad (3.19)$$

(b) The current ripple is given by Eqn (3.19) as

$$\Delta i_{ld} = \frac{E}{2R_a}\left\{\frac{1 + e^{\tau/T_a} - e^{\tau_{ON}/T_a} - e^{\tau_{OFF}/T_a}}{e^{\tau/T_a} - 1}\right\}$$

Here,

$$T_a = \frac{L_a}{R} = \frac{12 \times 10^{-3}}{7}$$

$$\tau = \frac{1}{1.5 \times 10^3} = \frac{1 \times 10^{-3}}{1.5}$$

$$\frac{\tau}{T_a} = \frac{7}{1.5 \times 12} = 0.389$$

$$\frac{\tau_{ON}}{T_a} = \frac{0.55 \times 10^{-3}}{1.5} \times \frac{7}{12} \times 10^{-3} = 0.214$$

$$\frac{\tau_{OFF}}{T_a} = \frac{0.45 \times 10^{-3}}{1.5} \times \frac{7}{12} \times 10^{-3} = 0.175$$

Substitution of values gives

$$\Delta i_{ld} = \frac{210}{2 \times 7} \times \frac{1 + 1.475 - 1.239 - 1.191}{1.475 - 1} = 1.42 \text{ A}$$

(c) It is assumed that the load current increases linearly from I_{min} to I_{max} during $(0, \tau_{ON})$. Thus the instantaneous current i_{ld} can be expressed as

$$i_{ld} = I_{min} + \frac{I_{max} - I_{min}}{\tau_{ON}} t, \quad 0 \le t \le \tau_{ON}$$

The RMS value of the current through the chopper can now be found as

$$I_{ch(RMS)} = \sqrt{\frac{1}{\tau} \int_0^{\tau_{ON}} (i_{ld})^2 dt}$$

Here,

$$I_{ch(RMS)} = \left\{ \sqrt{\frac{\tau_{ON}}{\tau} \left[I_{min}^2 + I_{min}(I_{max} - I_{min}) + \frac{(I_{max} - I_{min})^2}{3} \right]} \right\}$$

where

$$\begin{aligned}
I_{min} &= \frac{E}{R_a} \left[\frac{e^{\tau_{ON}/T_a} - 1}{e^{\tau/T_a} - 1} \right] - \frac{E_b}{R_a} \\
&= \frac{210}{7} \left[\frac{1.239 - 1}{1.475 - 1} \right] - \frac{55}{7} = 15.095 - 7.857 = 7.24 \text{ A} \\
I_{max} &= \frac{E}{R_a} \left[\frac{1 - e^{-\tau_{ON}/T_a}}{1 - e^{-\tau/T_a}} \right] - E_b/R_a \\
&= \frac{210}{7} \frac{(1 - 0.807)}{(1 - 0.678)} - \frac{55}{7} = 17.981 - 7.857 = 10.12 \text{ A}
\end{aligned}$$

Thus,

$$\begin{aligned}
I_{ch(RMS)} &= \sqrt{\frac{\tau_{ON}}{\tau} \left[7.24^2 + 7.24(10.12 - 7.24) + \frac{(10.12 - 7.24)^2}{3} \right]} \\
&= \sqrt{0.55 \times 76.03} \\
&= 6.46 \text{ A}
\end{aligned}$$

(d) The RMS value of current through D_{FW} can be found as

$$\begin{aligned}
I_{D_{FW}(RMS)} &= \sqrt{\frac{1}{\tau} \int_0^{\tau_{OFF}} (i_{ld})^2 dt'} \quad (t' = t - \tau_{ON}) \\
&= \sqrt{\frac{\tau_{OFF}}{\tau} \left[(10.12)^2 + \frac{(2.88)^2}{3} - 10.12 \times 2.88 \right]}
\end{aligned}$$

(where $I_{max} - I_{min} = 10.12 - 7.24 = 2.88$ A)

$$= \sqrt{0.45 \times 76.02} = 5.85 \text{ A}$$

(e) Average source current $= (\tau_{ON}/\tau) \times$ average load current

$$= 0.55 \times \frac{(7.24 + 10.12)}{2} = 0.55 \times 8.68 = 4.77 \text{ A}$$

Hence, the effective input resistance seen by the source = 210/4.77 = 44 Ω.

(f) RMS value of load current $= \sqrt{\frac{1}{\tau} \left[\int_0^{\tau_{ON}} \{i_{ld}(t)\}^2 dt + \int_0^{\tau_{OFF}} \{i_{ld}(t')\}^2 dt' \right]} = 8.72$ A

with $t' = t - \tau_{ON}$. This is seen to be nearly equal to the average load current, namely, 8.68 A.

8. A step-down chopper has the following data: $R_{ld} = 0.40\ \Omega$, $E = 420$ V, $E_b = 25$ V. The average load current is 175 A and the chopper frequency is 280 Hz. Assuming the load current to be continuous, and linearly rising to the maximum and then linearly falling, calculate the inductance L which would limit the maximum ripple in the load current to 12% of the average load current.

Solution

The circuit is given in Fig. 3.3(a). The expression for the current ripple in Eqn (3.19) can be written with substitutions $\tau_{ON} = \delta\tau$ and $\tau_{OFF} = (1-\delta)\tau$, where δ is in the range $0 < \delta < 1$. Thus,

$$\Delta i_{ld} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - e^{\delta\tau/T_a} - e^{(1-\delta)\tau/T_a}}{e^{\tau/T_a} - 1}\right]$$

with $\delta = \tau_{ON}/\tau$. Differentiating the ripple current with respect to δ and equating this to zero gives the value of δ for maximum ripple:

$$\frac{d(\Delta i_{ld})}{d\delta} = \left(-\frac{\tau}{T_a}e^{\delta\tau/T_a}\right) + \frac{\tau}{T_a}e^{(1-\delta)\tau/T_a} = 0$$

This yields

$$e^{\delta\tau/T_a} = e^{(1-\delta)\tau/T_a}$$

or

$$\delta = 1 - \delta$$

This gives $\delta = 0.5$. Substituting this value of δ in the expression for Δi_{ld} gives

$$\Delta i_{ld} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - 2e^{0.5\tau/T_a}}{e^{\tau/T_a} - 1}\right]$$

$$= E/2R_a\left[\frac{(e^{0.5\tau/T_a} - 1)^2}{(e^{0.5\tau/T_a} - 1)(e^{0.5\tau/T_a} + 1)}\right]$$

$$= \frac{E}{2R_a}\left[\frac{e^{0.5\tau/T_a} - 1}{e^{0.5\tau/T_a} + 1}\right]$$

$$= \frac{E}{2R_a}\tanh\frac{R_a}{4fL}$$

where $f = 1/\tau$ is the chopper frequency. If $4fL >> R_a$, then $\tanh(R_a/4fL) \approx R_a/4fL$. Thus,

$$\Delta I_{ld(max)} = \frac{E}{2R_a}\frac{R_a}{4fL} = \frac{E}{8fL}$$

The condition $\Delta I_{ld(max)} = 12\% I_{ld}$ gives

$$\frac{E}{8fL} = 0.12 \times 175$$

Hence,

$$L = \frac{E}{8f \times 0.12 \times 175} = \frac{420}{8 \times 280 \times 0.12 \times 175} = 0.0089\ \text{H}$$

$$= 8.9\ \text{mH}$$

9. A 240-V, separately excited dc motor has an armature resistance of 2.2 Ω and an inductance of 4 mH. It is operated at constant load torque. The initial speed is 600 rpm and the armature current is 28 A. Its speed is now controlled by a step-down chopper with a frequency of 1 kHz, the input voltage remaining at 240 V. (a) If the speed is reduced to 300 rpm, determine the duty cycle of the chopper. (b) Compute the current ripple with this duty cycle.

Solution

(a) The equation for the motor is

$$V = E_b + I_a R_a$$

Substitution of values gives

$$240 = E_b + 28 \times 2.2$$

Thus,

$$E_b = 240 - 28 \times 2.2 = 178.4 \text{ V}$$

Other quantities on the right-hand side of the expression for E_b remaining constant, it can be expressed as

$$E_b = kN$$

or

$$178.4 = k600$$

$$k = \frac{178.4}{600} = 0.297$$

The new speed is 300 rpm. Hence,

$$E_{b(\text{new})} = 0.297 \times 300 = 89.1 \text{ V}$$

The new applied voltage with the same load torque, that is, the same armature current, is

$$V_{\text{new}} = E_b + I_a R_a = 89.1 + 28 \times 2.2 = 150.7 \text{ V}$$

The duty cycle of the chopper (τ_{ON}/τ) can be determined from the relation

$$V_{\text{new}} = \frac{\tau_{\text{ON}}}{\tau} \times \text{input voltage}$$

This gives

$$\frac{\tau_{\text{ON}}}{\tau} = \frac{V_{\text{new}}}{\text{input voltage}} = \frac{150.7}{240} = 0.628$$

(b) $T_a = L/R_a = 4 \times 10^{-3}/2.2$; $\tau = 1/1000 = 10^{-3}$. Therefore,

$$\frac{\tau}{T_a} = \frac{2.2}{4} = 0.55$$

$$\frac{\tau_{\text{ON}}}{T_a} = \frac{\tau_{\text{ON}}}{\tau}\frac{\tau}{T_a} = 0.628 \times 0.55 = 0.345$$

$$\frac{\tau_{\text{OFF}}}{T_a} = \frac{\tau}{T_a} - \frac{\tau_{\text{ON}}}{T_a} = 0.55 - 0.345 = 0.205$$

The current ripple is given as

$$\Delta i_{\text{ld}} = \frac{E}{2R_a} \frac{(1 + e^{\tau/T_a} - e^{\tau_{\text{ON}}/T_a} - e^{\tau_{\text{OFF}}/T_a})}{e^{\tau/T_a} - 1}$$

$$= \frac{150.7}{2 \times 2.2} \times \frac{1 + 1.733 - 1.412 - 1.227}{1.733 - 1}$$

$$= 4.39 \text{ A}$$

10. A dc chopper is used for regenerative braking of a separately excited dc motor as shown in Fig. 7.24(b). The data are $E = 400$ V, $R = 0.2\ \Omega$, and $L_a = 0.2$ mH. The back emf constant K_b^1 $(= K_b\phi_f)$, assuming ϕ_f to be constant, is equal to 1.96 V/rad s, and the average load current is 200 A. The frequency of the chopper is 1 kHz and $(\tau_{ON}/\tau)_a$ is 0.5. Compute the (a) average load voltage, (b) motor speed, (c) power regenerated and fed back to the battery, (d) equivalent resistance viewed from the motor side when it is working as a generator, and (e) minimum and maximum permissible speeds for regenerative braking.

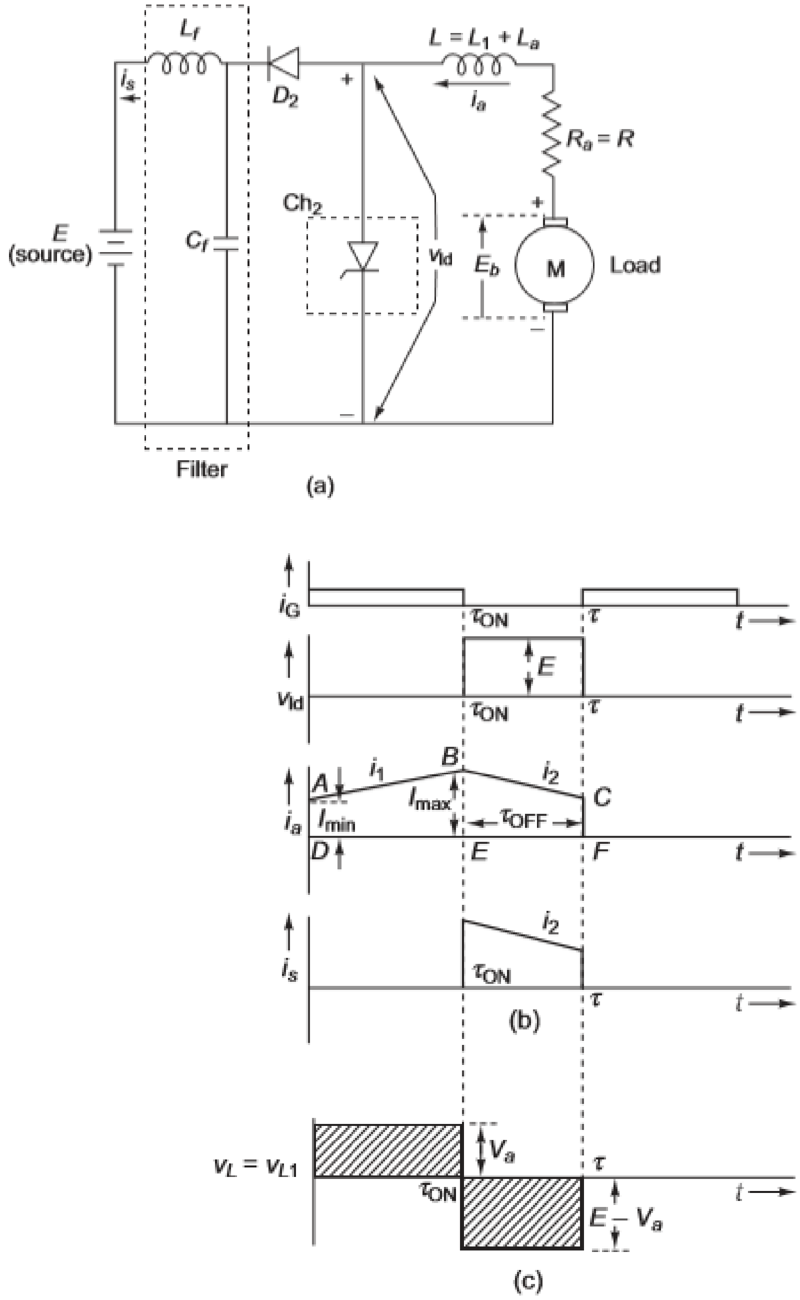


Fig. 7.26 Regenerative braking of a chopper-based dc drive: (a) circuit diagram, (b) waveforms, (c) voltage across inductor L for one period (τ)

Solution

(a) The circuit and waveforms are shown in Figs 7.26(a) and (b). The average load voltage is

$$V_a = E\left(1 - \frac{\tau_{ON}}{\tau}\right)_a = 400 \times 0.5 = 200 \text{ V}$$

(b)
$$I_a = \frac{I_{max} + I_{min}}{2} = 200 \text{ A}$$

or $I_{max} + I_{min} = 400$ A. From Eqns (7.64) and (7.65),

$$I_{max} + I_{min} = \frac{2E_b}{R} - \frac{E}{R}\left[\frac{1 - e^{-\tau_{OFF}/T_a}}{1 - e^{-\tau/T_a}} + \frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$

Here,

$$T_a = \frac{L_a}{R} = \frac{0.0002}{0.2} = 0.001$$

$$\tau = \frac{1}{1000} = 0.001$$

Hence,

$$\frac{\tau}{T_a} = 1 \text{ and } \frac{\tau_{ON}}{T_a} = \frac{0.0005}{0.001} = 0.5$$

Also

$$\tau_{OFF}/T_a = 0.5$$

Substituting the above values in the equation for $I_{max} + I_{min}$ gives

$$400 = \frac{2E_b}{0.2} - \frac{400}{0.2}\left[\frac{1 - 0.606}{1 - 0.368} + \frac{1.649 - 1}{2.718 - 1}\right]$$
$$= 10E_b - 2000(0.623 + 0.377)$$

This gives the value of the back emf as

$$E_b = 240$$

$$\text{Speed in rad/s} = \frac{240}{K_b^1} = \frac{240}{1.96} = 122.4$$

$$\text{Speed in rpm} = \frac{60}{2\pi} \times 122.4 = 1169 \text{ rpm}$$

(c) From Eqn (7.69), the power regenerated is

$$P_{reg} = \frac{E}{\tau}\left\{\frac{(E_b - E)}{R}[\tau_{OFF} + T_a(e^{-\tau_{OFF}/T_a} - 1)] - T_a I_{max}(e^{-\tau_{OFF}/T_a} - 1)\right\}$$

$$I_{max} = \frac{E_b}{R} - \frac{E}{R}\left[\frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$
$$= \frac{240}{0.2} - \frac{400}{0.2}\left[\frac{1.649 - 1}{2.718 - 1}\right]$$
$$= 1200 - 2000 \times \frac{0.649}{1.718} = 444 \text{ A}$$

Substitution of all values in the expression for P_{reg} gives

$$P_{\text{reg}} = 400\left\{\frac{240-400}{0.2}\left[\frac{\tau_{\text{OFF}}}{\tau} + \frac{T_a}{\tau}(e^{-0.5}-1) - \frac{T_a}{\tau}I_{\max}(e^{-0.5}-1)\right]\right\}$$

$$= 400\left\{\frac{-160}{0.2}[0.5 + 1(0.606-1)] - 1 \times 444(0.606-1)\right\}$$

$$= 400\left\{\frac{(-160 \times 0.106)}{0.2} + 444 \times 0.394\right\}$$

$$= 400 \times 90 = 36{,}000 \text{ W} \text{ or } 36 \text{ kW}$$

(d) Average generated voltage

$$E_b = E(1-\delta) + I_a R_a = 400(1-0.5) + 200 \times 0.2 = 240 \text{ V}$$

Average current through the motor, $I_a = 200$ A. Hence the equivalent resistance is

$$\frac{E_b}{I_a} = \frac{240}{200} = 1.2\ \Omega$$

(e) Minimum and maximum permissible speeds for regeneration are obtained from the inequality

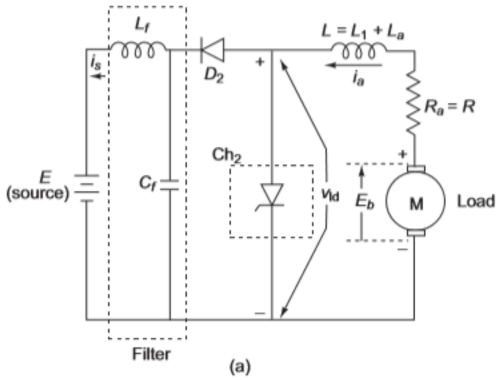
$$0 \le \omega_m \le \frac{E}{K_b^1}$$

or

$$0 \le \omega_m \le \frac{400}{1.96} = 204$$

Thus the maximum permissible speed is 204 rad/s or 1949 rpm. Also the minimum permissible speed is zero rpm.

13. A dc chopper is used for regenerative braking of a dc series motor as shown in Fig. 7.26(a). The data are $E = 400$ V, $R_a = 0.05\ \Omega$, $R_f = 0.04\ \Omega$, $L = L_a + L_f = 0.09$ mH. The product $K_b\phi_f$ may be assumed to be constant at 1.5 V/rad s and the average load current is 200 A. The frequency of the chopper is 1 kHz and $\tau_{ON}/\tau = 0.6$. Compute the (a) average load voltage, (b) motor speed, (c) power regenerated and fed back to the battery, (d) equivalent resistance viewed from the motor side when it is working as a generator, and (e) minimum and maximum permissible speeds.



(a)

Solution

(a) $$V_a = E\left(1 - \frac{\tau_{ON}}{\tau}\right) = 400(1 - 0.6) = 160 \text{ V}$$

(b) The correct value of E_b is arrived at as follows:

$$I_a = \frac{I_{max} + I_{min}}{2} = 200 \text{ A}$$

or

$$I_{max} + I_{min} = 400 \text{ A}$$

$$T_a = \frac{L_a + L_f}{R_a + R_f} = \frac{0.09 \times 10^{-3}}{0.09} = 0.001 \text{ s}$$

$$\tau = \frac{1}{f_{Ch}} = \frac{1}{1000} = 0.001 \text{ s}$$

From Eqns (7.64) and (7.65),

$$2I_a = 400 = I_{max} + I_{min} = \frac{2E_b}{R} - \frac{E}{R}\left[\frac{1 - e^{-\tau_{OFF}/T_a}}{1 - e^{-\tau/T_a}} + \frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$

Thus,

$$400 = \frac{2E_b}{0.09} - \frac{400}{0.09}\left[\frac{0.33}{0.632} + \frac{0.492}{1.718}\right]$$

Rearranging terms gives

$$\frac{2E_b}{0.09} = 400 + \frac{400}{0.09} \times 0.808$$

and we obtain

$$E_b = 179.6 \text{ V}$$

This gives

$$\text{Speed } N = 119.5 \times \frac{60}{2\pi} = 1141 \text{ rpm}$$

$$\begin{aligned} I_{\max} &= \frac{E_b}{R} - \frac{E}{R}\left[\frac{e^{\tau_{\mathrm{OFF}}/T_a}-1}{e^{\tau/T_a}-1}\right] \\ &= \frac{179.6}{0.09} - \frac{400}{0.09} \times \frac{0.492}{1.728} \\ &= 724.5 \text{ A} \end{aligned}$$

$$\begin{aligned} I_{\min} &= \frac{E_b}{R} - \frac{E}{R}\frac{(1-e^{-\tau_{\mathrm{OFF}}/T_a})}{(1-e^{-\tau/T_a})} \\ &= \frac{179.6}{0.09} - \frac{400}{0.09} \times \frac{0.33}{0.632} \\ &= -325 \text{ A} \end{aligned}$$

(c) $$P_{\mathrm{reg}} = \frac{E}{\tau}\left\{\frac{(E_b - E)}{R}\left[\frac{\tau_{\mathrm{OFF}}}{\tau} + \frac{T_a}{\tau}(e^{-\tau_{\mathrm{OFF}}/T_a}-1)\right] - \frac{T_a I_{\max}}{\tau}(e^{-\tau_{\mathrm{OFF}}/T_a}-1)\right\}$$

$$= 400\left\{\frac{(179.6-400)}{0.09}[0.4 + 1(0.67-1)] - 1 \times 724.5(0.67-1)\right\}$$

$$\begin{aligned} &= 400\left\{\frac{-220.4}{0.09}[0.4 - 0.33] - 724.5(-0.33)\right\} \\ &= 400\left\{\frac{-220.4}{0.09} \times 0.07 + 239\right\} \\ &= 400\{-171.4 + 239\} \\ &= 400 \times 67.6 = 27{,}031 \text{ W} \approx 27 \text{ kW} \end{aligned}$$

(d) Equivalent resistance = back emf/average current = 179.6/200 $\approx$ 0.9 Ω

(e) The minimum and maximum speeds are given as

$$0 \le N \le \frac{E}{K_b \phi_f}\frac{60}{2\pi}$$

or

$$0 \le N \le \frac{400}{1.5}\frac{60}{2\pi}$$

This gives

$$0 \le N \le 2546 \text{ rpm}$$

Hence the minimum and maximum speeds are 0 and 2546 rpm, respectively.

PROBLEMS

10.1 Determine the ripple factor (RF), defined as RF $= V_{rip}/V_{dc} = (V_{rms}^2 - V_{dc}^2)^{1/2}/V_{dc}$, for the following circuits.

(a) Fig. 10.17.

(b) Fig. 10.18, for $\alpha = 90°$.

What is the significance of the ripple factor?

CHAPTER: 10

$\boxed{10.1}$ (a) From equation 10.1

$$V_{dc} = \frac{\sqrt{2}V_p}{\pi}$$

From Fig 10.17

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(\sqrt{2}V_p \sin\theta)^2 d\theta} = \frac{V_p}{\sqrt{2}}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{\sqrt{2}} \times \frac{\pi}{\sqrt{2}V_p} = \frac{\pi}{2} = 1.5706$$

$$RF = \left\{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1\right\}^{\frac{1}{2}} = (1.5706^2 - 1)^{\frac{1}{2}} = 1.2114$$

(b) From equation 10.2

$$V_{dc} = \frac{V_p}{\sqrt{2}\pi}(1 + \cos 90°) = \frac{V_p}{\sqrt{2}\pi}$$

$$V_{rms} = \left\{\frac{1}{2\pi}\int_{\frac{\pi}{2}}^{\pi}(\sqrt{2}V_p \sin\theta)^2 d\theta\right\}^{\frac{1}{2}} = \frac{V_p}{2}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{2} \times \frac{\sqrt{2}\pi}{V_p} = \frac{\pi}{\sqrt{2}} = 2.2218$$

$$RF = (2.2218^2 - 1)^{\frac{1}{2}} = 1.984$$

RF is a measure of ripple content

Example 4.5

A chopper is used to control the speed of a dc motor as shown in Fig. 4.20*a*. The motor is accelerated under current control (i.e., constant torque). Assume that the armature current remains constant at I_a amperes during start-up.

1 Show that the maximum rms ripple current in the chopper current i_{CH} occurs at a duty cycle of one-half, that is, at $\alpha = 0.5$.

2 For the duty cycle $\alpha = 0.5$, determine the values of the input filter components L and C for the following conditions: Supply voltage = 120 V. Chopper frequency $f_{CH} = 400$ Hz. Start-up motor current $I_a = 100$ A. Rms fundamental current to be allowed in the supply is 10% of the dc component of the source current. Electrolytic capacitor of rating 1000 μF and 300 V dc can take 5 A rms ripple current. For the design $f_{CH} > 2f_r$.

3 For the values of L and C obtained in part 2, determine the average and first three harmonic currents (in rms) in the supply.

Solution

1 The chopper current i_{CH} is in the form of square pulses of magnitude I_a and width α as shown in Fig. 4.21. Therefore, the dc component $I_{CH_{dc}}$, rms current I_{rms}, and ripple current I_{ripple} are as follows:

$$I_{CH_{dc}} = I_a \alpha$$

$$I_{rms} = \left(\int_0^{\alpha} I_a^2 \, d\alpha \right)^{1/2} = I_a \sqrt{\alpha}$$

$$I_{ripple} = \left[\left(I_a \sqrt{\alpha} \right)^2 - (I_a \alpha)^2 \right]^{1/2}$$

$$= I_a(\alpha - \alpha^2)^{1/2}$$

For maximum ripple current

$$\frac{dI_{ripple}}{d\alpha} = 0$$

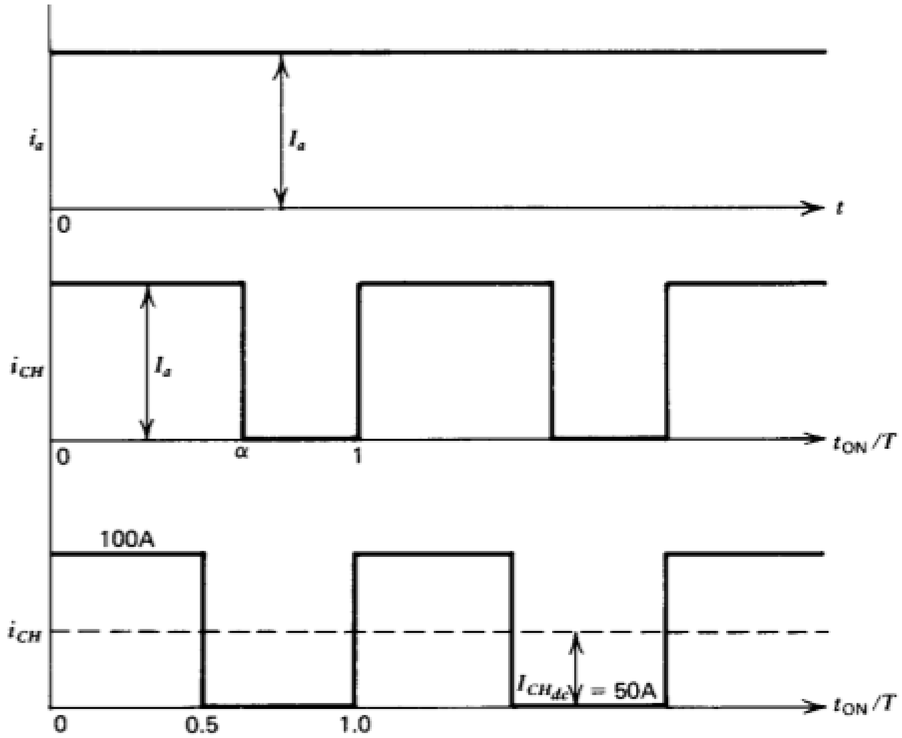


Fig. 4.21 Example 4.5

or

$$\frac{I_a(1-2\alpha)}{2(\alpha-\alpha^2)^{1/2}} = 0$$

from which

$$\alpha = 0.5$$

2 The L-C filter should be selected for the worst case, which corresponds to $\alpha = 0.5$. At this duty cycle (bottom wave form in Fig. 4.21) the Fourier series for the chopper current is

$$i_{CH} = I_{CH_{dc}} + \frac{4}{\pi}\frac{I_a}{2n}(\sin\omega t + \sin 3\omega t + \sin 5\omega t + \cdots)$$

Now,

$$I_{CH_{dc}} = \frac{100}{2} = 50 \text{ A}$$

The fundamental, third, and fifth harmonic currents in the chopper are

$$I_{CH_1} = \frac{4 \times 100}{\sqrt{2} \times \pi \times 2} = 45 \text{ A}$$

$$I_{CH_3} = 15 \text{ A}$$

$$I_{CH_5} = 9 \text{ A}$$

The dc component of the chopper current comes from the supply only. The capacitor cannot provide a dc current. Therefore, the dc component I_0 of the supply current is

$$I_0 = I_{CH_{dc}} = 50 \text{ A}$$

If the fundamental supply current I_1 is not to exceed 10% of the dc current I_0, then

$$I_1 = 5 \text{ A}$$

From equation 4.55

$$I_1 = \frac{X_C}{X_L - X_C} I_{CH_1}$$

$$5 = \frac{X_C}{X_L - X_C} \times 45$$

or

$$X_L = 10X_C$$

Fundamental capacitor current I_{C_1} is

$$I_{C_1} = \frac{X_L}{X_L - X_C} I_{CH_1}$$

$$= \frac{10X_C}{10X_C - X_C} \times 45$$

$$= 50 \text{ A}$$

Each electrolytic capacitor can take 5 A of current. Therefore 10 capacitors connected in parallel are required.

$$C = 10{,}000\ \mu\text{F}$$

$$X_C = \frac{1}{2\pi 400 \times 10^4 \times 10^{-6}}\ \Omega = 3.98 \times 10^{-2}\ \Omega$$

$$X_L = 10X_C = 3.98 \times 10^{-1}\ \Omega$$

$$L = \frac{0.398}{2\pi 400} = 158\ \mu\text{H}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = 127\ \text{Hz}$$

which makes

$$f_{CH} = \frac{400}{127} f_r = 3.15 f_r$$

3 From equation 4.58

$$I_1 = \frac{45}{(3.15)^2 - 1} = 5.0\ \text{A}$$

$$I_3 = \frac{15}{(3 \times 3.15)^2 - 1} = 0.17\ \text{A}$$

$$I_5 = \frac{9}{(5 \times 3.15)^2 - 1} = 0.036\ \text{A}$$

and

$$I_0 = 50\ \text{A}$$

Example 1: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle γ.

(b) The average value of the output voltage V_o.

(c) The rms value of the output voltage V_{orms}.

(d) The ripple factor RF.

(e) The output d.c. power.

Solution:

(a) $t_{on} = 1\text{ms}$, T=2.5 ms

$$\gamma = \frac{t_{on}}{T} = \frac{1\ ms}{2.5\ ms} = 0.4$$

(b) $V_{av} = V_o = \gamma\ V_d = 0.4 \text{ x } 100 = 40$ V.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.4} \text{ x} 100 = 63.2$ V.

(d) $RF = \sqrt{\dfrac{1-\gamma}{\gamma}} = \sqrt{\dfrac{1-0.4}{0.4}} = 1.225$

(e)

$$I_a = \frac{V_o}{R} = \frac{40}{10} = 4A$$

$P_{av} = I_a V_o = 4\text{x}40 = 160$ W

Example 2: An 80 V battery supplies RL load through a DC chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with 5Ω resistor. Load current, due to improper selection of frequency of chopping, varies widely between 9A and 10.2.

(a) Find the average load voltage, current and the duty cycle of the chopper.

(b) What is the operating frequency *f* ?

(c) Find the ripple current to maximum current ratio.

Solution:

(a) The average load voltage and current are:

$$V_{av} = V_o = \gamma V_d$$

$$I_{av} = \frac{1}{2}(I_2 + I_1) = \frac{9 + 10.2}{2} = 9.6A$$

$$I_{av} = \frac{V_{av}}{R} = \frac{\gamma V_d}{R} \qquad or \qquad \gamma = \frac{I_{av}\ R}{V_i} = \frac{9.6\ x\ 5}{80} = 0.6$$

$$V_{av} = 0.6 \text{ x } 80 = 48 \text{ V.}$$

(b) To find the operating (chopping) frequency:

During the ON period,

$$V_d = Ri + L\frac{di}{dt} \qquad \dots\dots\dots\dots\dots.(1)$$

Assuming $\frac{di}{dt} \cong constant$

$$\frac{di}{dt} \cong \frac{\Delta I}{t_{on}} = \frac{10.2 - 9}{\gamma T} = \frac{1.2}{\gamma T}$$

From eq.(1)

$$L\frac{di}{dt} \cong V_d - I_{av}\, R = 80 - 5 \times 9.6 = 32V$$

or

$$\frac{di}{dt} = \frac{32}{L} = \frac{32}{0.4} = 80\ A.s$$

but

$$\frac{di}{dt} = \frac{1.2}{\gamma T} = 80 = \frac{1.2}{0.6\,T}$$

$$\therefore T = \frac{1.2}{0.6x80} = 25\ ms$$

Hence

$$f = \frac{1}{T} = \frac{1}{25 \times 10^{-3}} = 40\ Hz$$

The maximum current I_m occurs at $\gamma = 1$,

$$\therefore\ I_m = \frac{\gamma V_d}{R} = \frac{1 \times 80}{5} = 16A$$

Ripple current $I_r = \Delta I = 10.2 - 9 = 1.2$ A

$$\therefore\ \frac{I_r}{I_m} = \frac{1.2}{16} = 0.075 \quad or\ \ 7.5\%\,.$$

Example 3: A DC Buck converter operates at frequency of 1 kHz from 100V DC source supplying a 10 Ω resistive load. The inductive component of the load is 50mH.For output average voltage of 50V volts, find:

(a) The duty cycle

(b) t_{on}

(c) The rms value of the output current

(d) The average value of the output current

(e) I_{max} and I_{min}

(f) The input power

(g) The peak-to-peak ripple current.

Solution:

(a) $V_{av} = V_o = \gamma V_d$

$$\gamma = \frac{V_{av}}{V_d} = \frac{50}{100} = 0.5$$

(b) $T = 1/f = 1 / 1000 = 1\text{ms}$

$$\gamma = \frac{t_{on}}{T}$$

$t_{on} = \gamma T = 05 \text{ x1ms} = 0.5 \text{ ms}$.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.5}\ x\,100 = 70.07\ V$

(d)

$$I_{av} = \frac{V_{av}}{R} = \frac{50}{10} = 5\ A$$

(e)

$$I_{max} = \frac{V_{av}}{R} + \frac{t_{off}}{2L} V_{av} = \frac{50}{10} + \frac{(1-0.5)\times 10^{-3}}{2\times 50\times 10^{-3}} x50$$

= 5 + 0.25 = 5,25 A

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av} = \frac{50}{10} - \frac{(1-0.5)\times 10^{-3}}{2\times 50\times 10^{-3}} x50$$

= 5 - 0.25 = 4.75 A

(f)

$$I_{s(av)} = \frac{\gamma}{2}\,(I_{min} + I_{max}) = \gamma I_{av} = 0.5\ \times 5 = 2.5A$$

$$P_{in}\ = I_{s(av)} V_d = 2.5\ \times 100 = 250\ W$$

(g) $$I_{p-p} = \Delta I = I_{max} -\ I_{min} = 5.25 - 4.75 = 0.5A$$

DC-DC CONVERTER

1. A class-A transistor chopper circuit shown in Fig.1 supplied with power from an ideal battery of terminal voltage 120 V .The load voltage waveforms consists of rectangular pulses of duration 1 ms in an overall cycle of 3 ms.

(a) Sketch the waveforms of v_L and i_L.
(b) Calculate the duty cycle γ,
(c) Calculate the average and r.m.s. values of the load voltage.
(d) Find the average value of the load current if R = 10 ohms.
(e) Calculate the input power and the ripple factor *RF*.

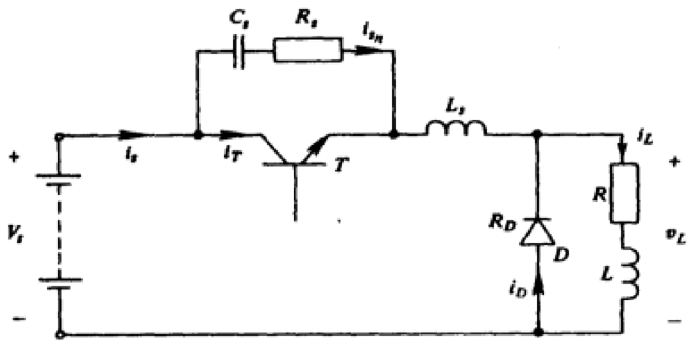


Fig.1

Question 1 .

(a) Waveforms of Voltage v_L and current i_L are shown in Fig.Q2

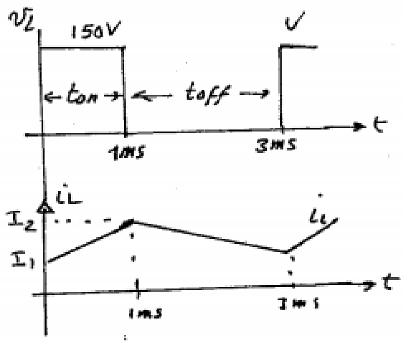


Fig.Q2.

(b) The duty cycle γ is

$$\gamma = \frac{t_{on}}{t_{on}+t_{off}} = \frac{1}{1+2} = \frac{1}{3}.$$

(c) $V_{av} = \gamma V_{in} = \frac{1}{3} \times 150 = 50V.$

$V_{L r.m.s} = \sqrt{\gamma}\, V_{in} = \sqrt{\frac{1}{3}} \times 150 = 86.6V.$

(d) $I_{av} = \frac{V_{av}}{R} = \frac{50}{20} = 2.5 A.$

(e) $I_{in} = I_{av}\gamma = 2.5 \times \frac{1}{3} = 0.833 A$

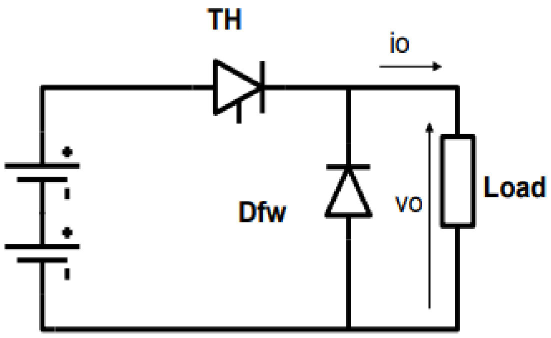
$P_{in} = V_s I_{in} = 0.833 \times 150 = 125 W.$

$RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-\frac{1}{3}}{\frac{1}{3}}} = \sqrt{2} = 1.414$

v

2. A class-A DC chopper shown in Fig.2 is operating at a frequency of 2kHz from 96V DC source to supply a load of resistance 8 ohms . The load time constant is 6 ms. If the mean load voltage is 57.6 V, find duty cycle (mark to space ratio), the mean load current, and the magnitude of the current ripple. Derive any formula used.

Fig.2



[Ans: γ =0.6 , Iav =7.2, Δ I = 0.24 A]

Solution:

$$T = \frac{1}{f} = \frac{1}{2000} = 0.5\ ms.$$

Load time constant $\tau = \frac{L}{R} = 6\ ms = 12T.$, hence the current variation is treated linear.

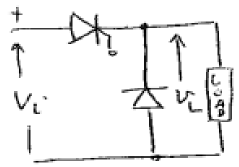
$$V_{av} = \gamma V_i$$

$$57.6 = \gamma \times 96 \qquad \therefore \gamma = \frac{57.6}{96} = 0.6 \ \equiv \text{mark-space ratio.}$$

$$V_{0r.m.s} = V_i\sqrt{\gamma} = 96 \times \sqrt{0.6} = 74.36V.$$

$$I_{av} = \frac{V_{av}}{R} = \frac{57.6}{8} = 7.2\,A.$$

Current ripple $\Delta I = (V_i - V_{av})\frac{\Delta t}{L}$.



This comes from:

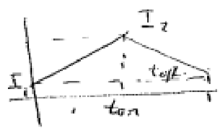
During conduction:

$$V_i - V_L = L\frac{di}{dt} \equiv L\frac{\Delta i}{\Delta t}.$$

$$\Delta i = (V_i - V_L)\frac{\Delta t}{L} = I_2 - I_1 \quad \text{---(1)}$$

$$\Delta t = t_{on} - 0 = t_{on}.$$

$$\therefore I_2 - I_1 = (V_i - V_L)\frac{t_{on}}{L}.$$



During off period (from eq.(1)) $V_i = 0$

$$I_1 - I_2 = (0 - V_L)\frac{(T - t_{on})}{L}$$

or $$I_2 - I_1 = V_L\frac{(T - t_{on})}{L} = V_L\frac{t_{off}}{L}$$

Also $$I_{av} = \frac{I_1 + I_2}{2}$$

Hence

Hence

$$I_1 = I_{av} - V_L \frac{t_{off}}{2L} = I_{min}$$

$$I_2 = I_{av} + V_L \frac{t_{off}}{2L} = I_{max} \qquad V_L = V_{av}$$

$$\tau = \frac{L}{R} = 6\times10^{-3} \qquad \therefore L = 6\times10^{-3}\times R = 6\times10^{-3}\times 8 = 48\,mH$$

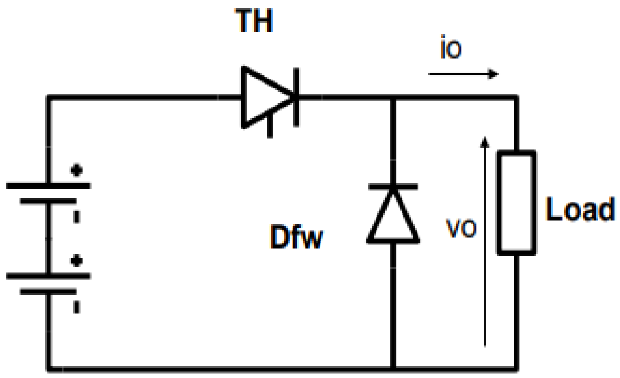
$$I_{max} = I_2 = 7.2 + \frac{57.6\times0.2\times10^{-3}}{2\times48\times10^{-3}} = 7.32\,A$$

$$I_{min} = I_1 = 7.2 - \frac{57.6\times0.2\times10^{-3}}{2\times48\times10^{-3}} = 7.08\,A$$

$$\Delta I = I_{max} - I_{min} = 7.32 - 7.08 = 0.24\,A$$

3. A DC Buck converter (class-A chopper) supplies power to a load having 6 ohms resistance and 20 mH inductance. The source voltage is 100V d.c. and the output load voltage is 60V. If the ON time is 1.5 ms, find:

(a) Chopper switching frequency.
(b) Imax and Imin (I_2 and I_1).
(c) The average diode current.
(d) The average input current.
(e) Peak- to- peak ripple current.



[Ans: fc = 40Hz , Imax = 1.5A,Imin =8.5A, Iav(D) = Iav= 10A , ΔI= 3A]

Solution :

(a) $\gamma = \frac{V_{av}}{V_i} = \frac{60}{100} = 0.6$

$\gamma = \frac{t_{on}}{T} \Longrightarrow T = \frac{1.5}{0.6} = 2.5\ ms.$

$f = \frac{1}{T} = \frac{1}{2.5} \times 10^{3} = 400\ Hz.$

(b) $I_{max} = \frac{V_{av}}{R} + V_{av}\left(\frac{t_{off}}{2L}\right) = \frac{60}{6} + 60\left(\frac{(2.5-1.5)\times 10^{-3}}{2\times 20\times 10^{-3}}\right)$

$= 10 + \frac{60}{40} = 10 + 1.5 = 11.5\ A.$

$I_{min} = \frac{V_{av}}{R} - V_{av}\left(\frac{t_{off}}{2L}\right) = 10 - 1.5 = 8.5\ A.$

(c) $I_{in(av)} = \gamma I_{av} = 0.6 \times \frac{V_{av}}{R} = 0.6 \times \frac{60}{60} = 6\ A.$

(d) $I_{Diode} = I_{av} = \frac{V_{av}}{R} = \frac{60}{6} = 10\ A.$

(e) P-P riple $\Delta I = I_{max} - I_{min} = 11.5 - 8.5 = 3.0\ A.$

4. In a class-A chopper circuit an ideal battery of terminal voltage 100V supplies a series load of resistance 0.5 Ohms and inductance of 1.0 mH . The thyristor is switched on for 1 ms in an overall period of 3 ms. Calculate the average values of the load voltage and current and the power taken from the battery. Assuming continuous current conduction .Also calculates the r.m.s value of the load current taking the firs two harmonics of the Fourier series.

[Ans: Vav = 33.3V , Iav = 66.7A, Pin= 2223W , $I_{Lr.m.s}$ = 69.1A]

Solition

$t_{on} = 1\ ms$, $T = 3\ ms$

$\therefore \gamma = \frac{t_{on}}{T} = \frac{1}{3}$

$\omega = 2\pi f = \frac{2\pi}{T}$ $\quad \therefore T = \frac{2\pi}{\omega} = \frac{3}{1000}$

Hence $\omega = \frac{2000\pi}{3} = 2094.4$ rad/s.

$\tau = \frac{L}{R} = \frac{1\times10^{-3}}{0.5} = 2\ mS$.

$V_{o\,av} = \gamma V_i = \frac{100}{3} = 33.33\ V$.

The Average load current $I_{av} = \frac{V_{o(av)}}{R} = \frac{33.33}{0.5} = 66.7A$.

The average supply (input current) $I_{in(av)} = \gamma I_{av} = \frac{1}{3}\times 66.7$
$= 22.23A$.

Input power $P_{in} = V_i\, I_{in(av)}$
$= 100 \times 22.23 = \underline{2223}\ W$.

The impedance of the load to current of fundamental frequency: $Z_L = \sqrt{R^2 + (\omega L)^2} = \sqrt{(0.5)^2 + (2094.4\times10^{-3})^2} = 2.153\,\Omega$

The fundamental component of the load voltage:

$V_{o1(r.m.s)} = \frac{V_i}{\sqrt{2}\pi}\sqrt{\sin^2 2\pi\gamma + (1-\cos 2\pi\gamma)^2} = \frac{100}{\sqrt{2}\pi}\sqrt{0.75 + 2.25} = \frac{55.13}{\sqrt{2}}\ V.$

$\therefore I_{o1} = \frac{V_{o1(r.m.s)}}{Z_L} = \frac{55.13}{\sqrt{2}\times 2.153} = 18.1\ A$ - (r.m.s value of the fundamental Load current).

5. In a class-A chopper circuit an ideal battery of terminal voltage 100V supplies a series load of resistance 10 Ohms. The chopping frequency is f =1 kHz and the duty cycle is set to be 0.5 .Determine:

(a) The average output voltage.
(b) The rms output voltage.
(c) The chopper efficiency.
(d) The ripple factor.
(e) The fundamental component of output harmonic voltage.

[As: Va=110V, $V_{Lr.m.s}$ = 155.56V, ή=100%,RF=1.0, $V_{L1\ r.m.s}$ =99V]

Solution:

(a) $V_o = \gamma V_i = 0.5 \times 220 = 110V.$

(b) $V_{o_{r.m.s}} = \sqrt{\gamma}\, V_i = \sqrt{0.5} \times 220 = 155.56\,V.$

(c) $I_{av} = \frac{V_o}{R} = \frac{110}{10} = 11\,A.$

$P_o = I_{av}^2 R = 11^2 \times 10 = 1210\,W.$

$I_{in(av)} = \gamma I_a = 0.5 \times 11 = 5.5A.$

$P_{in} = V_i \cdot I_{in(ave)} = 220 \times 5.5 = 1210\,W$

$\therefore \eta = \frac{P_o}{P_i} = \frac{1210}{1210} = 100\%.$

(d) $RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-0.5}{0.5}} = 1.$

(e) $V_{o_{1(r.m.s)}} = \frac{V_i}{\sqrt{2}\pi}\sqrt{\sin^2 2\pi\gamma + (1-\cos 2\pi\gamma)^2}$

$= \frac{220}{\sqrt{2}\pi}\sqrt{\sin^2(2\pi \times \frac{1}{2}) + (1-\cos(2\pi \times \frac{1}{2}))^2}$

$= \frac{220}{\sqrt{2}\pi}\sqrt{0+(2)^2} = \frac{220\sqrt{2}}{\pi} = 70.06\sqrt{2}$

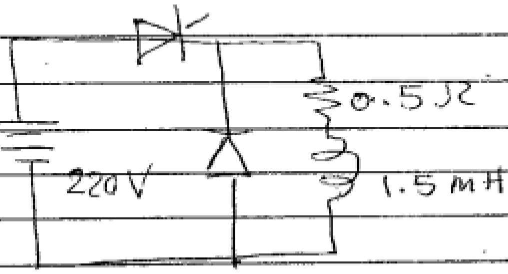
6. In the chopper circuit sown in Fig.1 (problem 1) Vi = 220V, L = 1.5 mH , R = 0.5 ohm and it operating with T = 3 ms and t_{on} = 1.5 ms.

(a) Determine the minimum, maximum and average values of load current.

(b) Express the load current variation in terms of ON and OFF periods.

[Ans: (a) Imin = 165A,Imax=275A, i_1= 165+36660 t , i_2=165-36660t]

problem 6 -



T = 3 ms

t_{on} = 1.5 ms.

Imin & Imax

sol. $t_{off} = T - t_{on} = 1.5$ ms.

$$\gamma = \frac{t_{on}}{T} = \frac{1.5}{3.} = 0.5$$

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av}$$

$V_{av} = \gamma V_i = 0.5 \times 220 = 110$ V.

7. A separately excited d.c. motor with Ra = 1.2 ohms and La = 30 mH , is to be controlled using class-A transistor chopper .The d.c. supply is 120V .

(a) It is required to draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant KeΦ has a value of 0.042 V/rpm.

(b) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle γ = 0.5.

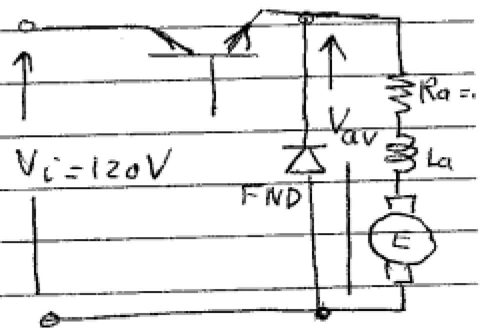
[Ans : n = 857 rpm]

Solution : In the steady state the armature inductance has no effect.

(a) At $\gamma = 1$

$$V_{av} = \gamma V_{in} = 1 \times 120 = 120\,V.$$

$$n = \frac{V_{av}}{Ke\phi} - \frac{\Sigma R_a\, T}{(Ke\phi)^2 \times 9.55}$$



when $T = 0$

$$n_o = \frac{120}{0.042} = 2857 \text{ rpm}$$

when $n = 0$, $T = T_{st}$

$$\frac{\Sigma R_a}{9.55(Ke\phi)^2} \cdot T_{st} = \frac{V_{av}}{Ke\phi}$$

$$\therefore T_{st} = \frac{V_{av}}{\Sigma R_a} \frac{(Ke\phi)}{\frac{1}{9.55}} = \frac{120}{1.2} \times \frac{(0.042)}{\frac{1}{9.55}} = 40\,N.m$$

Examlpe: **A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.**

(a) The duty cycle D.

(b) The average value of the output voltage V_{dc}.

(c) The *rms* value of the output voltage V_{rms}.

(d) The ripple factor *RF*.

(e) The output dc power.

(a) $$D = \frac{t_{ON}}{T} = \frac{1msec}{2.5msec} = 0.4$$

(b) $$V_{dc} = DV_s = 0.4 \times 100 = 40 \text{ V}$$

(c) $$V_{rms} = \sqrt{D}V_s = \sqrt{0.4} \times 100 = 63.2 \text{ V}$$

(d) $$RF = \sqrt{\frac{1-D}{D}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$$

(e) $$P_o = \frac{{V_{dc}}^2}{R} = \frac{40^2}{10} = 160 \text{ W}$$

Examlpe: **A dc chopper has a resistive load of 20Ω and input voltage V_S=220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.**

$$V_s = 220\text{V}$$

$$D = \frac{t_{ON}}{T} = 0.8$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T}(V_s - V_{CH}) = 0.8(220 - 1.5) = 174.8 \text{ V}$$

$$T = \frac{1}{f} = \frac{1}{10 \times 10^{-3}} = 0.1\text{m } sec$$

$$t_{ON} = DT = 0.8 \times 0.1 \times 10^{-3} = 80\mu\, sec$$

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Examlpe: buck dc-dc converter with Low Pass Filter has the following parameters:

$V_s = 50$ V $\quad L = 400\ \mu$H $\quad f = 20$ kHz
$D = 0.4 \quad C = 100\ \mu$F $\quad R = 20\ \Omega$

Assuming ideal components, calculate (a) the output voltage Vo, (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

(a) $V_o = V_s D = (50)(0.4) = 20$ V

(b)
$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$
$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$
$$= 1 + \frac{1.5}{2} = 1.75 \text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$
$$= 1 - \frac{1.5}{2} = 0.25 \text{ A}$$

The average inductor current is 1 A, and Δi_L=1.5 A.

(c)
$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$
$$= 0.00469 = 0.469\%$$

Example: Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50. and the switching frequency is 25kHz.

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let L=120 μH.

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5 \text{ A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2 \text{ A}$$

$$I_{\max} = 1.5 + 1.2 = 2.7 \text{ A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3 \text{ A}$$

$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Designing a Buck Converter

Assume:

V_{in} = 12 V

V_{OUT} = 5 volts

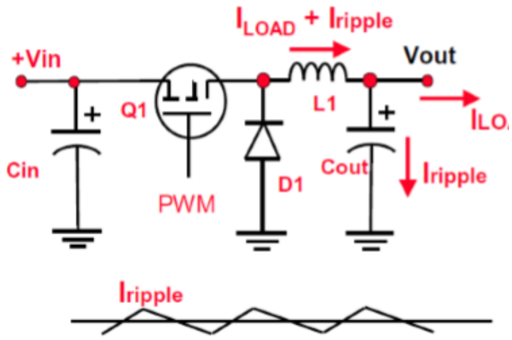
I_{LOAD} = 2 amps

F_{sw} = 400 KHz

D = V_{in} / V_{out} = 5V /12V = 0.416

Define Ripple current:

$I_{ripple} = 0.3 \bullet I_{LOAD}$ (typically 30%)



For an Inductor: $V = L \bullet \Delta I / \Delta T$

Rearrange and substitute:

L = (Vin – Vout) • (D / Fsw) / Iripple

Calculate:

L = 7 V • (0.416 / 400 kHz) / 0.6A

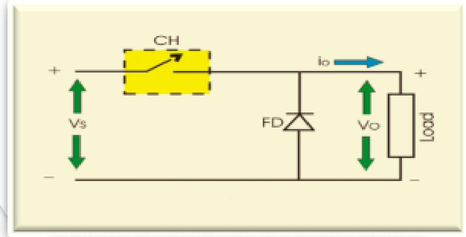
L = 12.12 uh

Select C, Diode (Schottky), and the MOSFET

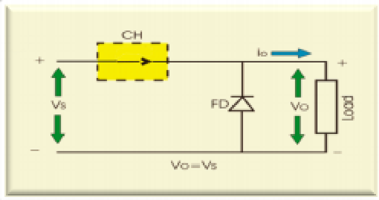
Calculate the Efficiency

The Buck (Step-Down) Converter

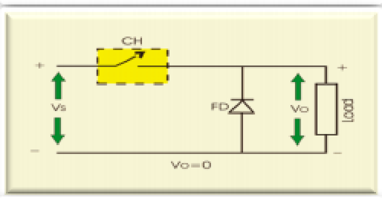
- Step down chopper as Buck converted is used to reduce the input voltage level at the output side. Circuit diagram of a step down chopper is shown in the figure.



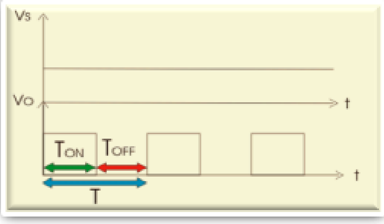
- When CH is turned ON, Vs directly appears across the load as shown in figure. So V_O=V_S.



- When CH is turned OFF, Vs is disconnected from the load. So output voltage $V_O = 0$.



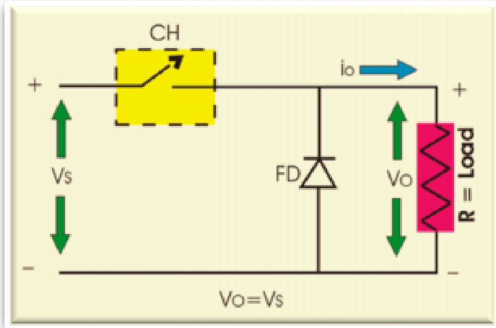
- The voltage waveform of step down chopper



- T_{ON} → It is the interval in which chopper is in ON state.
- T_{OFF} → It is the interval in which chopper is in OFF state.
- V_S → Source or input voltage.
- V_O → Output or load voltage.
- T → Chopping period = $T_{ON} + T_{OFF}$
- F=1/T is the frequency of chopper switching or chopping frequency

Operation of Step Down Chopper with Resistive Load

- **When CH is ON, $V_O = V_S$ When CH is OFF, $V_O = 0$**



The Average output voltage is

$$V_{dc} = V_o = \frac{1}{T}\int_0^{T_{ON}} V_s dt = \frac{V_s T_{ON}}{T} = DV_s$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{DV_s}{R}$$

$$D = \frac{T_{ON}}{T} \qquad\qquad T = T_{ON} + T_{OFF}$$

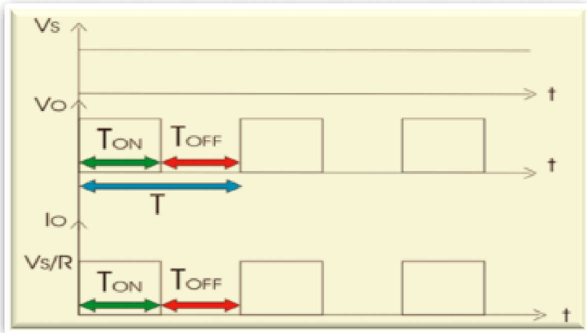
- ***Where,***
- ***D is <u>duty cycle</u> = T_{ON}/T. T_{ON} can be varied from 0 to T, so $0 \le D \le 1$.***
- ***The output voltage V_O can be varied from 0 to V_S.***

The *rms* output voltage is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T_{ON}} V_s^2 dt} = V_s\sqrt{\frac{T_{ON}}{T}} = \sqrt{D}V_s$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{D}V_s}{R}$$

$$P_o = V_{rms}I_{rms} = \frac{V_{rms}^2}{R} = D\frac{V_s^2}{R}$$



The output voltage is always less than the input voltage and hence the name step down chopper is justified.

Ripple factor (*RF*) can be found from

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{\frac{DV_s^2}{D^2V_s^2} - 1} = \sqrt{\frac{1}{D} - 1} = \sqrt{\frac{1-D}{D}}$$

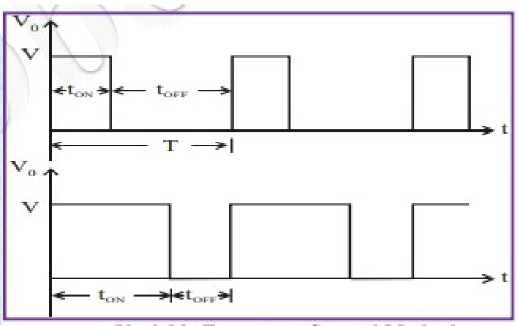
Methods of Control

1- Pulse Width Modulation

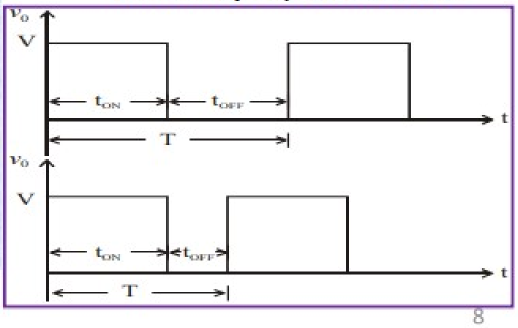
- t_{ON} is varied keeping chopping frequency '*f*' & chopping period 'T' constant.
- Output voltage is varied by varying the ON time t_{ON}

2- Variable Frequency Control

- Chopping frequency '*f*' is varied keeping either t_{ON} or t_{OFF} constant.
- To obtain full output voltage range, frequency has to be varied over a wide range.
- This method produces harmonics in the output and for large t_{OFF} load current may become discontinuous



Variable Frequency Control Method



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Examlpe: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle D.

(b) The average value of the output voltage V_{dc}.

(c) The *rms* value of the output voltage V_{rms}.

(d) The ripple factor *RF*.

(e) The output dc power.

(a) $$D = \frac{t_{ON}}{T} = \frac{1msec}{2.5msec} = 0.4$$

(b) $$V_{dc} = DV_s = 0.4 \times 100 = 40\text{ V}$$

(c) $$V_{rms} = \sqrt{D}V_s = \sqrt{0.4} \times 100 = 63.2\text{ V}$$

(d) $$RF = \sqrt{\frac{1-D}{D}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$$

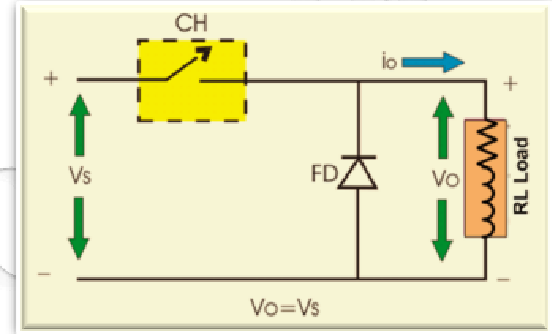
(e) $$P_o = \frac{{V_{dc}}^2}{R} = \frac{40^2}{10} = 160\text{ W}$$

Dr. Firas Obeidat

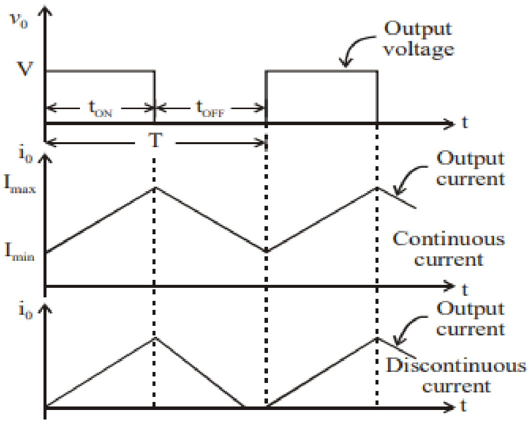
The Buck (Step-Down) Converter

Step Down Chopper with RL Load

- When chopper is ON, supply is connected across load. Current flows from supply to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in inductor 'L'.

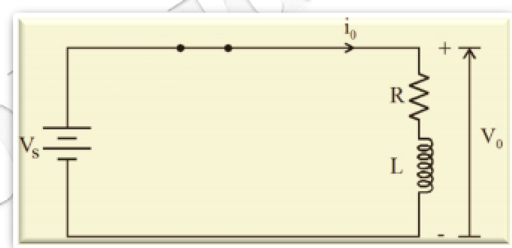


- Load current can be continuous or discontinuous depending on the values of '*L*' and duty cycle '*D*'
- For a continuous current operation, load current varies between two limits I_{max} and I_{min}.
- When current becomes equal to I_{max} the chopper is turned-off and it is turned-on when current reduces to I_{min}.



Continuous Current Operation When Chopper Is ON ($0 \le t \le t_{ON}$)

- When the switch is closed in the buck converter, the circuit will be as shown in the figure, the diode is reverse-biased.



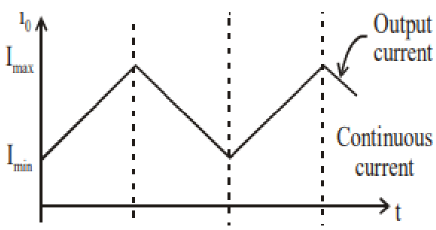
The voltage across the inductor is

$$V_s = V_R + V_L$$

$$V_s = V_R + L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{V_s - V_R}{L}$$

$$\Delta i = \int_0^{DT} \frac{V_s - V_R}{L} dt = \frac{V_s - V_R}{L} DT = \frac{V_s - V_R}{L} t_{ON} \qquad (1)$$

$$\frac{di}{dt} = \frac{\Delta i}{t_{ON}} = \frac{I_{max} - I_{min}}{t_{ON}} = \frac{V_s - V_R}{L}$$



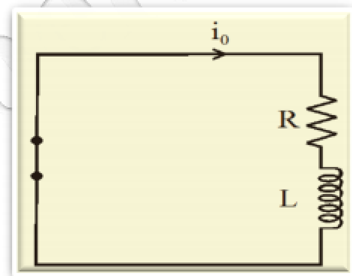
From straight line equation

$$i_{o1} = I_{min} + \frac{I_{max} - I_{min}}{t_{ON}} t = I_{min} + \frac{I_{max} - I_{min}}{DT} t = I_{min} + \frac{V_s - V_R}{L} t \qquad (2)$$

Continuous Current Operation When Chopper Is <u>OFF</u> ($t_{ON} \le t \le T$)

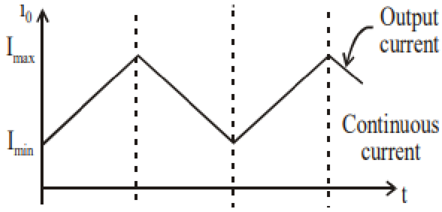
$$0 = V_R + V_L$$

$$0 = V_R + L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = -\frac{V_R}{L}$$



$$\Delta i = \int_0^{t_{OFF}} -\frac{V_R}{L}dt = -\frac{V_R}{L}t_{OFF} \qquad (3)$$

$$\frac{di}{dt} = \frac{\Delta i}{t_{OFF}} = \frac{I_{min} - I_{max}}{t_{OFF}} = -\frac{I_{max} - I_{min}}{t_{OFF}} = -\frac{V_R}{L}$$



From straight line equation

$$i_{o2} = I_{max} + \frac{I_{min} - I_{max}}{t_{OFF}}(t - t_{ON}) = I_{max} - \frac{V_R}{L}(t - t_{ON}) \qquad (4)$$

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s - V_R}{L}t_{ON} - \frac{V_R}{L}t_{OFF} = 0 \longrightarrow \frac{V_s - V_R}{V_R} = \frac{t_{OFF}}{t_{ON}}$$

$$\frac{V_s}{V_R} - 1 = \frac{t_{OFF}}{t_{ON}} \longrightarrow \frac{V_s}{V_R} = \frac{t_{OFF}}{t_{ON}} + 1$$

$$\frac{V_s}{V_R} = \frac{t_{OFF} + t_{ON}}{t_{ON}} = \frac{T}{t_{ON}} \longrightarrow V_R = DV_s$$

From equation (1)

since $D = \dfrac{t_{ON}}{T}$

$f = \dfrac{1}{T}$

$$\boxed{\Delta i = \frac{V_s - DV_s}{L}DT = \frac{V_s(1 - D)D}{Lf}}$$

Dr. Firas Obeidat

At steady state operation, the average inductor current must be the same as the average current in the load resistor.

$$I_L = I_R = \frac{V_R}{R}$$

The maximum and minimum values of the inductor current are computed as

$$I_{max} = I_L + \frac{\Delta i}{2}$$

$$\boxed{I_{max} = I_L + \frac{V_s(1-D)D}{2Lf} = I_L + \frac{V_R(1-D)}{2Lf}}$$

$$I_{min} = I_L - \frac{\Delta i}{2}$$

$$\boxed{I_{min} = I_L - \frac{V_s(1-D)D}{2Lf} = I_L - \frac{V_R(1-D)}{2Lf}}$$

The average dc output voltage and current can found as

$$V_{dc} = DV_s \qquad\qquad I_{dc} \cong \frac{I_{max} - I_{min}}{2}$$

Examlpe: A dc chopper has a resistive load of 20Ω and input voltage V_S=220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.

$$V_s = 220\text{V}$$

$$D = \frac{t_{ON}}{T} = 0.8$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T}(V_s - V_{CH}) = 0.8(220 - 1.5) = 174.8\text{ V}$$

$$T = \frac{1}{f} = \frac{1}{10 \times 10^{-3}} = 0.1\text{m } sec$$

$$t_{ON} = DT = 0.8 \times 0.1 \times 10^{-3} = 80\mu\ sec$$

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Step Down Chopper with RL Load

Examlpe: A Chopper circuit is operating at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

$$V_s = 460\text{V}$$

Chopping period

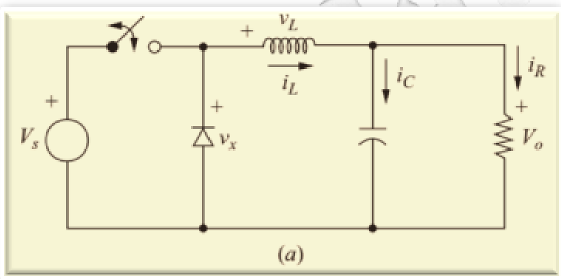
$$T = \frac{1}{f} = \frac{1}{2 \times 10^{-3}} = 0.5\text{m}\,sec$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T} V_s$$

$$t_{ON} = \frac{TV_{dc}}{V_s} = \frac{0.5 \times 10^{-3} \times 350}{460} = 0.38\text{m}\,sec$$

Step Down Chopper with Low Pass Filter

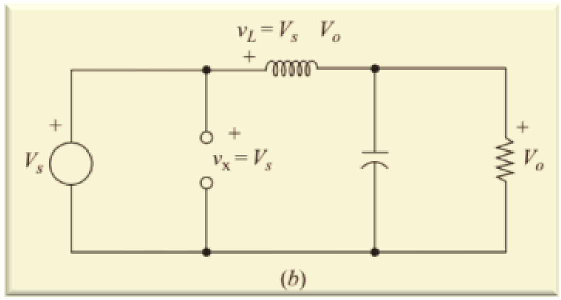
- **This converter is used if the objective is to produce an output that is purely DC.**
- **If the low-pass filter is ideal, the output voltage is the average of the input voltage to the filter.**



(a)

Analysis for the Switch Closed

When the switch is closed in the buck converter circuit of fig. a, the diode is reverse-biased and fig. b is an equivalent circuit. The voltage across the inductor is



(b)

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

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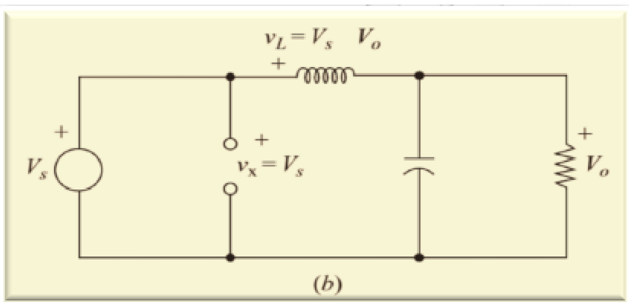
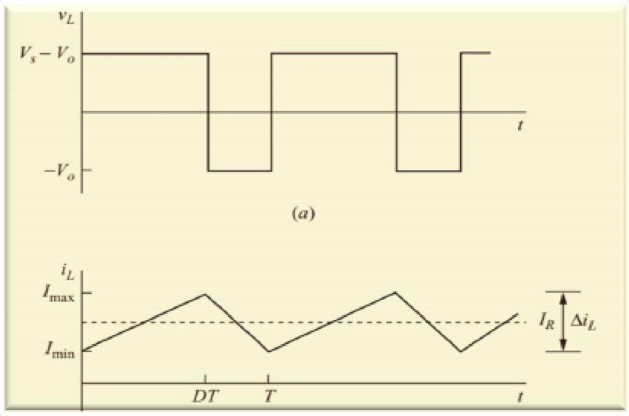
Analysis for the Switch Closed

Since the derivative of the current is a positive constant, the current increases linearly. The change in current while the switch is closed is computed by modifying the preceding equation.





$$(\Delta i_L)_{closed} = \int_0^{DT} \frac{V_s - V_o}{L} dt = \frac{V_s - V_o}{L} DT$$

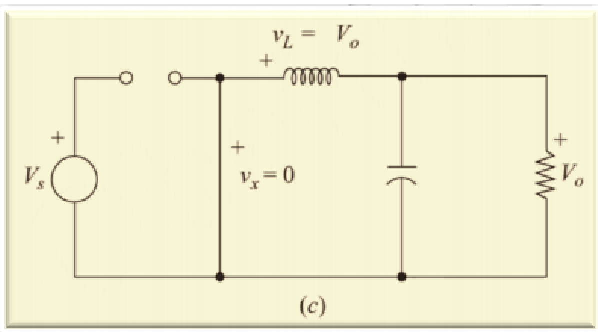
or

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L} \qquad (1)$$

$$(\Delta i_L)_{closed} = \left(\frac{V_s - V_o}{L}\right) DT$$

Analysis for the Switch Opened

When the switch is open, the diode becomes forward-biased to carry the inductor current and the equivalent circuit of fig. c applies. The voltage across the inductor when the switch is open is



$$v_L = -V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V_o}{L}$$

The derivative of current in the inductor is a negative constant, and the current decreases linearly. The change in inductor current when the switch is open is

$$(\Delta i_L)_{opened} = \int_0^{(1-D)T} \frac{-V_o}{L} dt = \frac{-V_o}{L}(1-D)T \quad \text{or}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

$$(\Delta i_L)_{open} = -\left(\frac{V_o}{L}\right)(1-D)T \qquad (2)$$

Dr. Firas

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

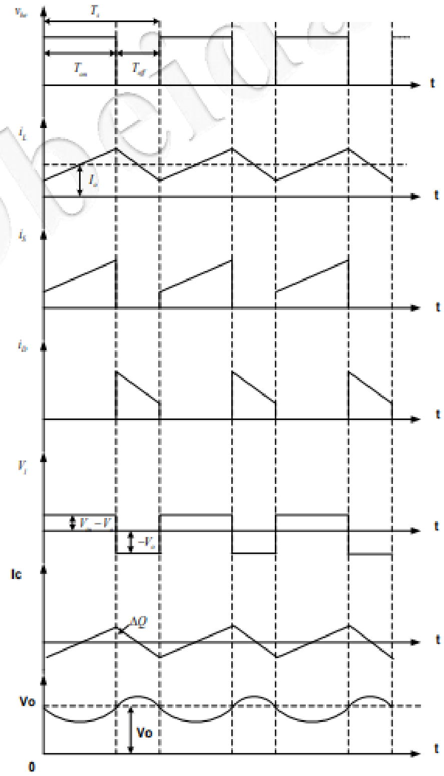
Using equations 1&2

$$\left(\frac{V_s - V_o}{L}\right)(DT) - \left(\frac{V_o}{L}\right)(1-D)T = 0$$

$$\boxed{V_o = V_s D}$$

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero for steady-state operation:

$$I_L = I_R = \frac{V_o}{R}$$



The maximum and minimum values of the inductor current are computed as

$$I_{\max} = I_L + \frac{\Delta i_L}{2}$$
$$= \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2}$$
$$= \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

Since I_{min}=0 is the boundary between continuous and discontinuous current,

$$I_{\min} = 0 = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$(Lf)_{\min} = \frac{(1-D)R}{2}$$

The minimum combination of inductance and switching frequency for continuous current in the buck converter is

$$L_{\min} = \frac{(1-D)R}{2f} \qquad \text{for continuous current}$$

Dr. Firas Obeidat

where L_{min} is the minimum inductance required for continuous current. In practice, a value of inductance greater than L_{min} is desirable to ensure continuous current.

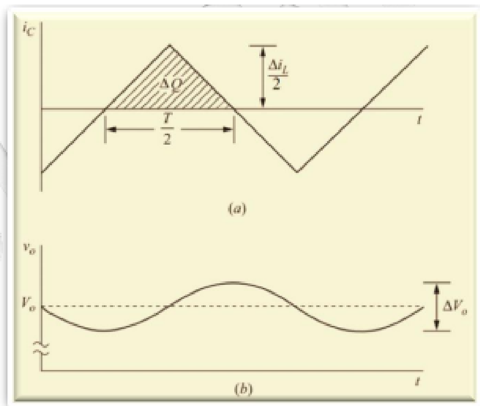
Since the converter components are assumed to be ideal, the power supplied by the source must be the same as the power absorbed by the load resistor.

$$P_s = P_o$$
$$V_s I_s = V_o I_o$$
$$\frac{V_o}{V_s} = \frac{I_s}{I_o}$$

This relationship is similar to the voltage-current relationship for a transformer in AC applications. Therefore, the buck converter circuit is equivalent to a DC transformer.

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$



While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,

$$Q = C V_o$$
$$\Delta Q = C \Delta V_o$$
$$\Delta V_o = \frac{\Delta Q}{C}$$

The change in charge ΔQ is the area of the triangle above the time axis

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T \Delta i_L}{8}$$

$$\Delta V_o = \frac{T \Delta i_L}{8C}$$ Substitute $(\Delta i_L)_{open}$ in the above equation yields

$$\Delta V_o = \frac{T V_o}{8CL}(1-D)T = \frac{V_o(1-D)}{8LCf^2}$$ ΔV_o is the peak-to-peak ripple voltage at the output

The required capacitance in terms of specified voltage ripple:

$$C = \frac{1-D}{8L(\Delta V_o / V_o) f^2}$$

Examlpe: buck dc-dc converter with Low Pass Filter has the following parameters:

$V_s = 50$ V $\quad L = 400\ \mu$H $\quad f = 20$ kHz

$D = 0.4 \quad C = 100\ \mu$F $\quad R = 20\ \Omega$

Assuming ideal components, calculate (a) the output voltage Vo, (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

(a) $V_o = V_s D = (50)(0.4) = 20$ V

(b)

$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75 \text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$= 1 - \frac{1.5}{2} = 0.25 \text{ A}$$

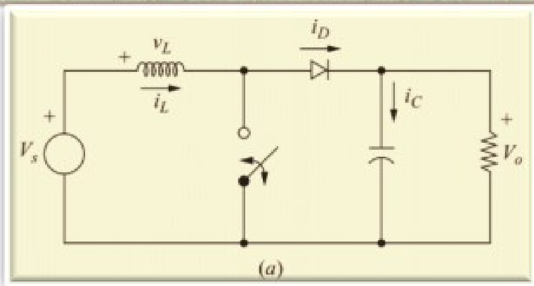
The average inductor current is 1 A, and Δi_L=1.5 A.

(c)

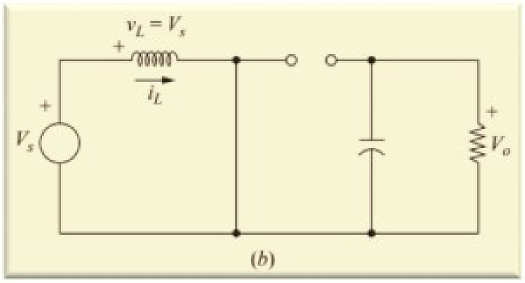
$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$

$$= 0.00469 = 0.469\%$$

The Boost (Step-Up) Converter



(a)



(b)

- **It is called a boost converter because the output voltage is larger than the input.**

Analysis for the Switch Closed

When the switch is closed, the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

$$v_L = V_s = L\frac{di_L}{dt} \quad \text{or} \quad \frac{di_L}{dt} = \frac{V_s}{L}$$

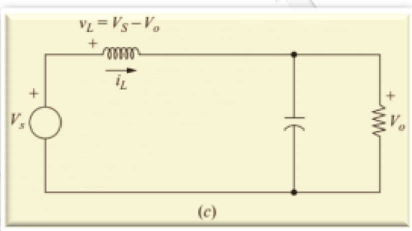
The rate of change of current is a constant, so the current increases linearly while the switch is closed. The change in inductor current is computed from

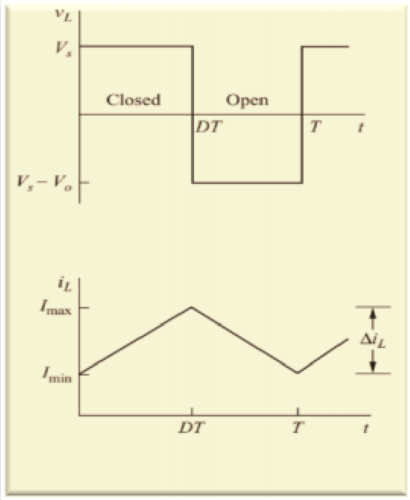
$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L} \quad \textbf{or}$$

$$(\Delta i_L)_{closed} = \int_0^{DT} \frac{V_s}{L}\,dt = \frac{V_s}{L}DT \qquad (1)$$

***

Analysis for the Switch opened





When the switch is opened, the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current. Assuming that the output voltage V_o is a constant, the voltage across the inductor is

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L} \qquad \textbf{or}$$

$$(\Delta i_L)_{opened} = \int_0^{(1-D)T} \frac{V_S - V_O}{L}\,dt = \frac{V_S - V_O}{L}(1-D)T \qquad (2)$$

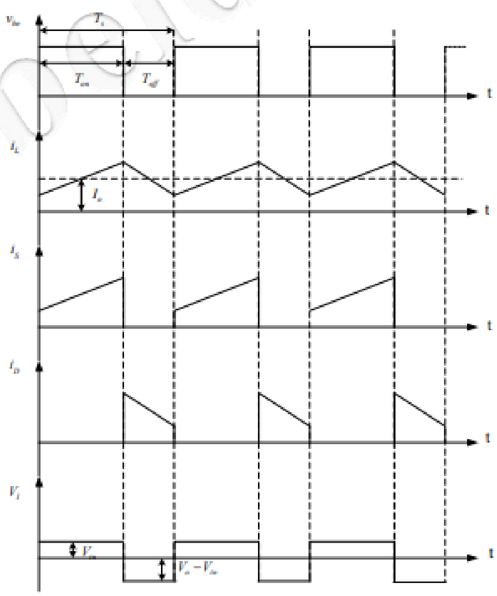
For steady-state operation, the net change in inductor current must be zero. Using equations 1&2

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s DT}{L} + \frac{(V_s - V_o)(1-D)T}{L} = 0$$

$$V_s(D + 1 - D) - V_o(1 - D) = 0$$

$$\boxed{V_o = \frac{V_s}{1-D}} \qquad (3)$$



If the switch is always open and D is zero, the output voltage is the same as the input. As the duty ratio is increased, the denominator of equation 3 becomes smaller, resulting in a larger output voltage. The boost converter produces an output voltage that is greater than or equal to the input voltage. However, the output voltage cannot be less than the input.

The average current in the inductor is determined by recognizing that the average power supplied by the source must be the same as the average power absorbed by the load resistor. Output power is

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$$P_o = \frac{V_o^2}{R} = V_o I_o$$

Input power is $V_s I_s = V_s I_L$. Equating input and output powers and using eq. 3

$$V_s I_L = \frac{V_o{}^2}{R} = \frac{[V_s/(1-D)]^2}{R} = \frac{V_s{}^2}{(1-D)^2 R}$$

$$I_L = \frac{V_s}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s} \qquad (4)$$

Maximum and minimum inductor currents are determined by using the average value and the change in current from eq. 1.

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L} \qquad (5)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \qquad (6)$$

Since I_{min}=0 is the boundary between continuous and discontinuous current,

$$I_{\min} = 0 = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

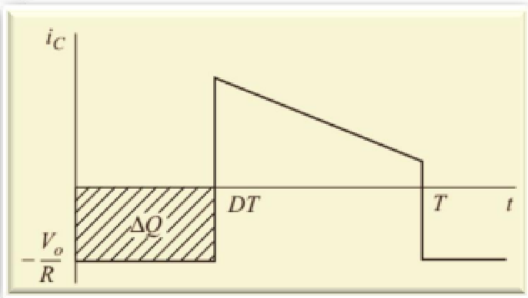
$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf}$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is

$$L_{\min} = \frac{D(1-D)^2 R}{2f} \qquad (7)$$

The peak-to-peak output voltage ripple can be calculated from the capacitor current waveform. The change in capacitor charge can be calculated from

$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C \Delta V_o$$



An expression for ripple voltage is then

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

expressing capacitance in terms of output voltage ripple yields

$$C = \frac{D}{R(\Delta V_o / V_o) f}$$

Example: Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50. and the switching frequency is 25kHz.

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let L=120 μH.

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5\,\text{A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2\,\text{A}$$

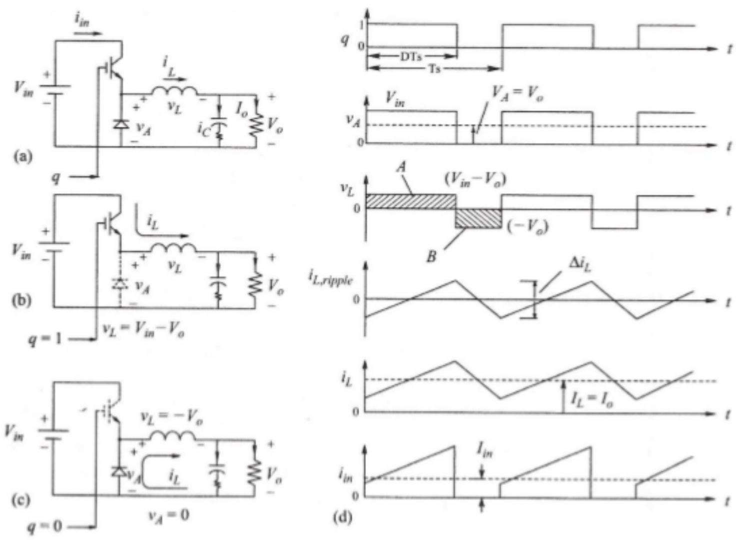
$$I_{\max} = 1.5 + 1.2 = 2.7\,\text{A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3\,\text{A}$$

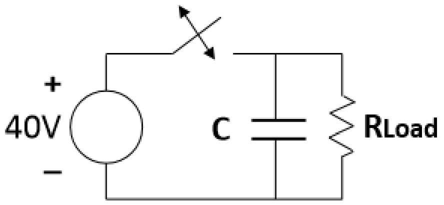
$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Buck Converter Analysis

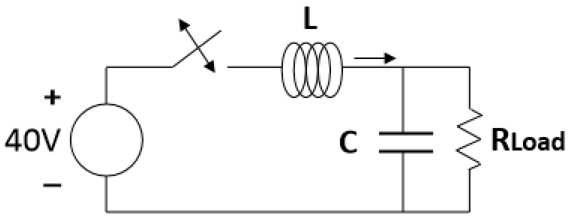
- $V_o = V_A = DV_{in}$; D = switch duty ratio
- $\Delta_{iL} = \frac{1}{L}(V_{in} - V_o)DT_s = \frac{1}{L}V_o(1-D)T_s$
- $I_L = Io = \frac{Vo}{R}$



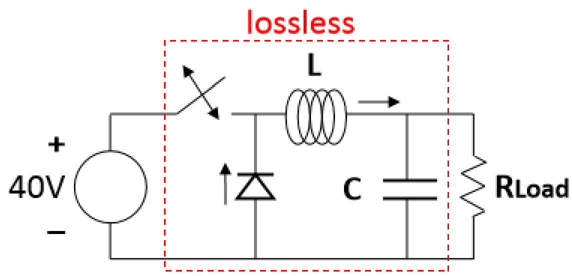
Examples of DC Conversion



Try adding a large C in parallel with the load to control ripple. But if the C has 13Vdc, then when the switch closes, the source current spikes to a huge value and burns out the switch.



Try adding an L to prevent the huge current spike. But now, if the L has current when the switch attempts to open, the inductor's current momentum and resulting *Ldi/dt* burns out the switch.



A DC-DC Buck Converter

By adding a "free wheeling" diode, the switch can open and the inductor current can continue to flow. With high-frequency switching, the load voltage ripple can be reduced to a small value.

Designing a Buck Converter

Assume:

V_{in} = 12 V

V_{OUT} = 5 volts

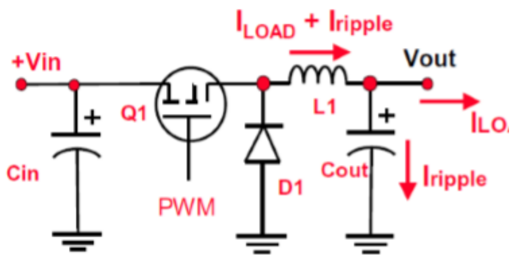
I_{LOAD} = 2 amps

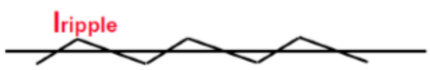
F_{sw} = 400 KHz

D = V_{in} / V_{out} = 5V /12V = 0.416

Define Ripple current:

$I_{ripple} = 0.3 \bullet I_{LOAD}$ (typically 30%)





For an Inductor: V = L • ΔI / ΔT

Rearrange and substitute:

L = (Vin – Vout) • (D / Fsw) / Iripple

Calculate:

L = 7 V • (0.416 / 400 kHz) / 0.6A

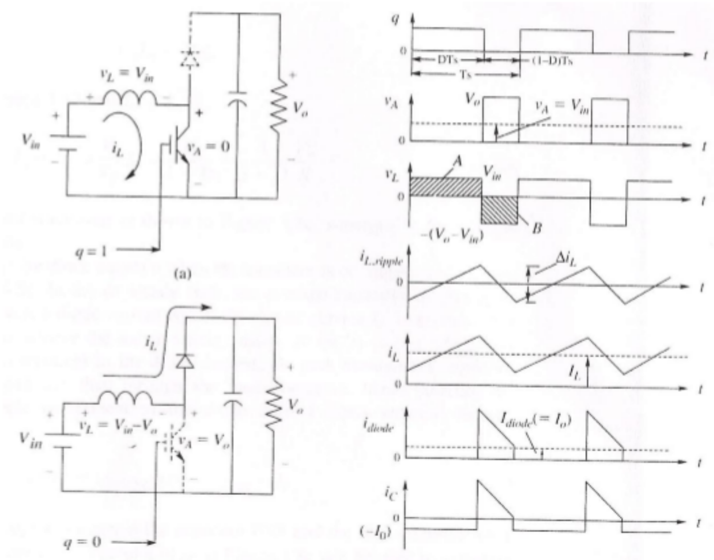
L = 12.12 uh

Select C, Diode (Schottky), and the MOSFET

Calculate the Efficiency

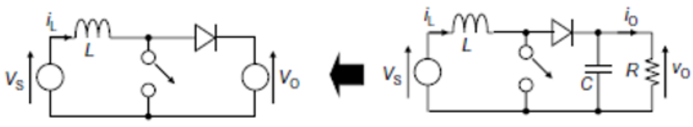
Boost Converter

- $\Delta_{iL} = \frac{1}{L}(V_{in})DT_s = \frac{1}{L}(V_o - V_{in})(1-D)T_s$
- $\frac{V_o}{V_{in}} = \frac{1}{1-D}$



Boost (Step Up) Converter

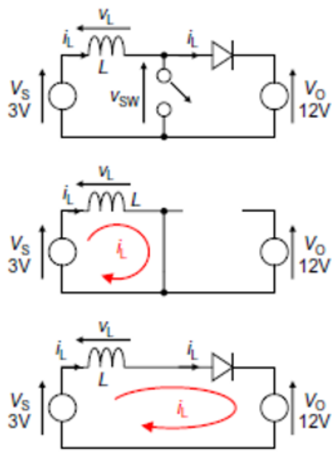
- Step-up
- Same components
- Different topology!



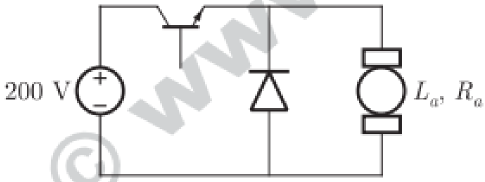
boost converter operating into a voltage source

boost converter with resistive load

- See stages of operation



Q. 2 The separately excited dc motor in the figure below has a rated armature current of 20 A and a rated armature voltage of 150 V. An ideal chopper switching at 5 kHz is used to control the armature voltage. If $L_a = 0.1$ mH, $R_a = 1\,\Omega$, neglecting armature reaction, the duty ratio of the chopper to obtain 50% of the rated torque at the rated speed and the rated field current is



(A) 0.4
(B) 0.5
(C) 0.6
(D) 0.7

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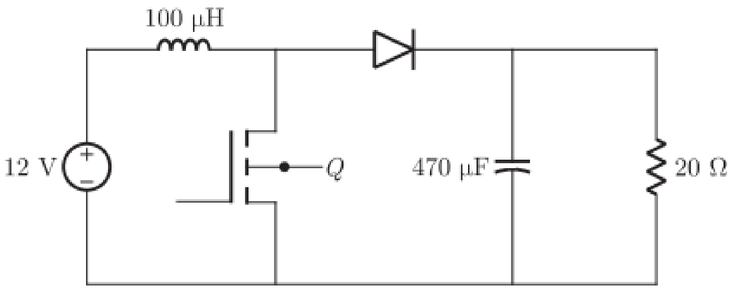
Common Data For Q. 3 and 4

In the figure shown below, the chopper feeds a resistive load from a battery source. MOSFET Q is switched at 250 kHz, with duty ratio of 0.4. All elements of the circuit are assumed to be ideal



Sol. 2 Option (D) is correct.

Given, the rated armature current

$$I_{a(rated)} = 20\text{ A}$$

as rated armature voltage

$$V_{a(rated)} = 150\text{ volt}$$

Also, for the armature, we have

$$L_a = 0.1\text{ mH},\ R_a = 1\,\Omega$$

and $T = 50\%\ \text{of}\ T_{rated}$ (T → Torque)

So, we get

$$I = [I_{a(\text{rotated})}](0.5) = 10\text{ A}$$

$$N = N_{\text{rated}},$$

$$I_f = I_{f\,rated} \rightarrow \text{rated field current}$$

At the rated conditions,

$$E = V - I_{a(\text{rated})}R_a$$

$$= 150 - 20(1) = 130\text{ volt}$$

For given torque,

$$V = E + I_a R_a = 130 + (10)(1) = 140\text{ V}$$

Therefore, chopper output $= 140$ V

or, $D(200) = 140$

or, $D = \frac{140}{200} = 0.7$ (D → duty cycle)

Q. 3 The Peak to Peak source current ripple in amps is

(A) 0.96 (B) 0.144

(C) 0.192 (D) 0.228

Sol. 3 Option (C) is correct.

Here, as the current from source of 12 V is the same as that pass through inductor. So, the peak to peak current ripple will be equal to peak to peak inductor current. Now, the peak to peak inductor current can be obtained as

$$I_L \text{ (Peak to Peak)} = \frac{V_s}{L} D T_s$$

where, $V_s \rightarrow$ source voltage $= 12$ volt,

$L \rightarrow$ inductance $= 100\mu\text{H} = 10^{-4}\text{H}$,

$D \rightarrow$ Duty ration $= 0.4$,

$T_S \rightarrow$ switching time period of MOSFET $= \frac{1}{f_S}$

and $f_s \rightarrow$ switching frequency $= 250$ kHz

Therefore, we get

$$I_{L(\text{Peak to Peak})} = \frac{12}{10^{-4}} \times 0.4 \times \frac{1}{250 \times 10^3} = 0.192 \text{ A}$$

This is the peak to peak source current ripple.

Sol. 4 Option (B) is correct.

Here, the average current through the capacitor will be zero. (since, it is a boost converter). We consider the two cases :

Case I : When MOSFET is ON

$$i_{c_1} = - i_0 \qquad (i_0 \text{ is output current})$$

(since, diode will be in cut off mode)

Case II : When MOSFET is OFF

Diode will be forward biased and so

$$i_{c_1} = I_s - i_0 \qquad (I_s \text{ is source current})$$

Therefore, average current through capacitor

$$I_{c,\,avg} = \frac{i_{c_1} + I_{c_2}}{2}$$

$$\Rightarrow \qquad 0 = \frac{DT_s(-i_o) + (1-D)T_s(I_s - i_o)}{2} \quad (D \text{ is duty ratio})$$

Solving the equation, we get

$$I_s = \frac{i_0}{(1-D)} \qquad \text{....(1)}$$

Since, the output load current can be given as

$$i_0 = \frac{V_0}{R} = \frac{V_s/_{1-D}}{R} = \frac{12/0.6}{20} = 1\text{A}$$

Hence, from Eq. (1)

$$I_s = \frac{i_0}{1-D} = \frac{1}{0.6} = \frac{5}{3}\text{A}$$

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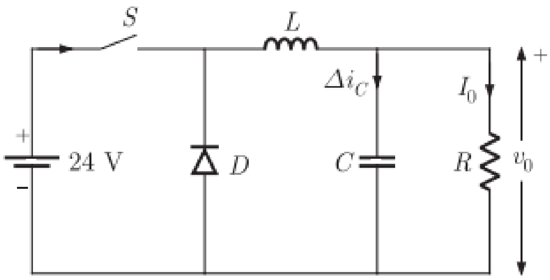
Q. 9 In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio of 50%. Given that Δi_c is 1.6 A peak-to-peak and I_0 is 5 A dc, the peak current in S, is

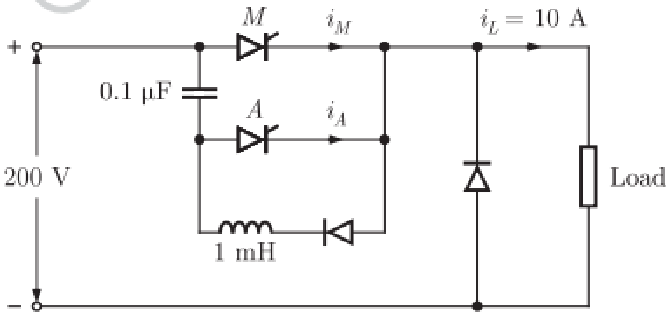


(A) 6.6 A
(B) 5.0 A
(C) 5.8 A
(D) 4.2 A

Sol. 9 Option (C) is correct.

$$I_S = I_0 + \frac{\Delta i_c}{2} = 5 + 0.8 = 5.8\text{ A}$$

Q. 14 A voltage commutated chopper circuit, operated at 500 Hz, is shown below.



If the maximum value of load current is 10 A, then the maximum current through the main (M) and auxiliary (A) thyristors will be

(A) $i_{M\max} = 12$ A and $i_{A\max} = 10$ A

(B) $i_{M\max} = 12$ A and $i_{A\max} = 2$ A

(C) $i_{M\max} = 10$ A and $i_{A\max} = 12$ A

(D) $i_{M\max} = 10$ A and $i_{A\max} = 8$ A

Sol. 14 Option (A) is correct.

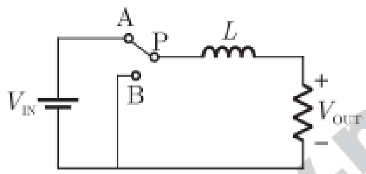
Maximum current through main thyristor

$$I_M(\max) = I_0 + V_s\sqrt{\frac{C}{L}} = 10 + 200\sqrt{\frac{0.1 \times 10^{-6}}{1 \times 10^{3}}} = 12 \text{ A}$$

Maximum current through auxiliary thyristor

$$I_A(\max) = I_0 = 10 \text{ A}$$

Q. 17 The power electronic converter shown in the figure has a single-pole double-throw switch. The pole P of the switch is connected alternately to throws A and B. The converter shown is a



(A) step down chopper (buck converter)

(B) half-wave rectifier

(C) step-up chopper (boost converter)

(D) full-wave rectifier

(c) www.nodia.co.in

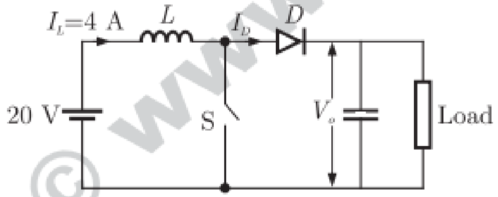
Sol. 17 Option (A) is correct.

The figure shows a step down chopper circuit.

$$\because \quad V_{out} = DV_{in}$$

where, $D =$ Duty cycle and $D < 1$

Q. 33 In the circuit shown in the figure, the switch is operated at a duty cycle of 0.5. A large capacitor is connected across the load. The inductor current is assumed to be continuous.

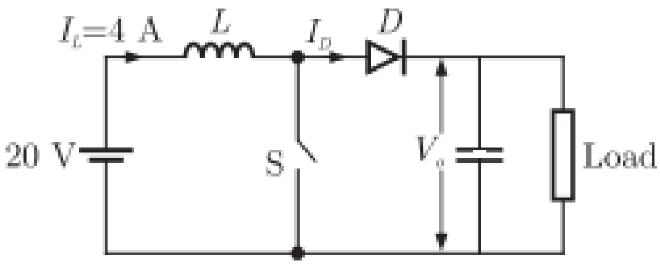


The average voltage across the load and the average current through the diode will respectively be

(A) 10 V, 2 A (B) 10 V, 8 A

(C) 40 V 2 A (D) 40 V, 8 A

Sol. 33 Option (C) is correct.



In the given diagram

when switch S is open $I_0 = I_L = 4$ A, $V_s = 20$ V

when switch S is closed $I_D = 0$, $V_0 = 0$ V

Duty cycle $= 0.5$ so average voltage is $\frac{V_s}{1-\delta}$

$$\text{Average current} = \frac{0+4}{2} = 2 \text{ amp}$$

$$\text{Average voltage} = \frac{20}{1-0.5} = 40 \text{ V}$$

Q. 44 The minimum approximate volt-second rating of pulse transformer suitable for triggering the SCR should be : (volt-second rating is the maximum of product of the voltage and the width of the pulse that may applied)

(A) 2000 μV-s (B) 200 μV-s

(C) 20 μV-s (D) 2 μV-s

Sol. 44 Option (A) is correct.

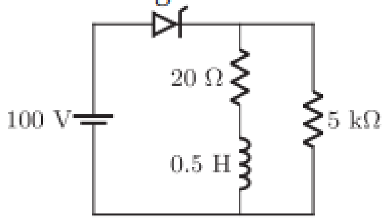
We know that the pulse width required is equal to the time taken by i_a to rise upto i_L

so,
$$V_s = L\frac{di}{dt} + R_i(V_T \approx 0)$$
$$i_a = \frac{200}{1}[1 - e^{-t/0.15}]$$

Here also $t = T$, $\quad i_a = i_L = 0.25$

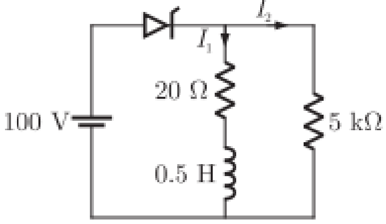
$$0.25 = 200[1 - e^{-T/0.5}]$$
$$T = 1.876 \times 10^{-4} = 187.6\ \mu\text{s}$$
$$\text{Width of pulse} = 187.6\ \mu\text{s}$$
$$\text{Magnitude of voltage} = 10\ \text{V}$$
$$V_{sec} \text{ rating of P.T.} = 10 \times 187.6\ \mu\text{s}$$
$$= 1867\ \mu\text{V-s is approx to } 2000\ \mu\text{V-s}$$

Q. 52 An SCR having a turn ON times of 5 μsec, latching current of 50 A and holding current of 40 mA is triggered by a short duration pulse and is used in the circuit shown in figure. The minimum pulse width required to turn the SCR ON will be



(A) 251 μsec
(B) 150 μsec
(C) 100 μsec
(D) 5 μsec

Sol. 52 Option (B) is correct.



In this given circuit minimum gate pulse width time= Time required by i_a rise up to i_L

$$i_2 = \frac{100}{5 \times 10^3} = 20 \text{ mA}$$

$$i_1 = \frac{100}{20}[1 - e^{-40t}]$$

$\therefore$ anode current $I = I_1 + I_2 = 0.02 + 5[1 - e^{-40t}]$

$$0.05 = 0.05 + 5[1 - e^{-40t}]$$

$$1 - e^{-40t} = \frac{0.03}{5}$$

$$T = 150 \ \mu\text{s}$$

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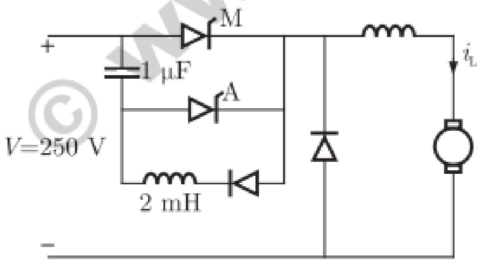
Common Data For Q. 53 and 54

A voltage commutated chopper operating at 1 kHz is used to control the speed of dc as shown in figure. The load current is assumed to be constant at 10 A



Q. 53 The minimum time in μsec for which the SCR M should be ON is.

(A) 280 (B) 140

(C) 70 (D) 0

Q. 54 The average output voltage of the chopper will be

(A) 70 V

(B) 47.5 V

(C) 35 V

(D) 0 V

Sol. 53 Option (B) is correct.

Given $I_L = 10$ A. So in the +ve half cycle, it will charge the capacitor, minimum time will be half the time for one cycle.

so min time required for charging

$$= \frac{\pi}{\omega_0} = \pi\sqrt{LC} = 3.14 \times \sqrt{2 \times 10^{-3} \times 10^{-6}} = 140\ \mu\text{sec}$$

Sol. 54 Option (C) is correct.

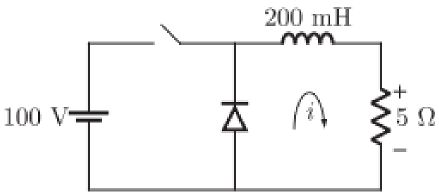
Given $T_{on} = 140\ \mu\text{sec}$

$$\text{Average output} = \frac{T_{on}}{T_{total}} \times V$$

$$T_{total} = 1/f = \frac{1}{103} = 1 \text{ msec}$$

$$\text{so average output} = \frac{140 \times 10^{-6}}{1 \times 10^{-3}} \times 250 = 35 \text{ V}$$

Q. 60 The given figure shows a step-down chopper switched at 1 kHz with a duty ratio $D = 0.5$. The peak-peak ripple in the load current is close to



(A) 10 A (B) 0.5 A

(C) 0.125 A (D) 0.25 A

Sol. 60 Option (C) is correct.

$$\text{Duty ratio } \alpha = 0.5$$

here $$T = \frac{1}{1 \times 10^{-3}} = 10^{-3} \text{ sec}$$

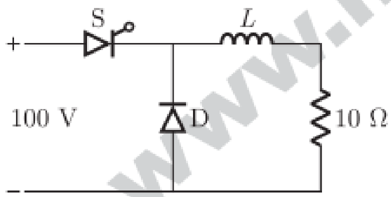
$$T_a = \frac{L}{R} = \frac{200 \text{ mH}}{5} = 40 \text{ msec}$$

$$\text{Ripple} = \frac{V_s}{R}\left[\frac{(1 - e^{-\alpha T/T_s})(1 - e^{-(1-\alpha)T/T_a})}{1 - e^{-T/T_s}}\right]$$

$$(\Delta I)_{\max} = \frac{V_s}{4fL} = \frac{100}{4 \times 10^3 \times 200 \times 10^{-3}}$$

$$= 0.125 \text{ A}$$

Q. 69 Figure shows a chopper operating from a 100 V dc input. The duty ratio of the main switch S is 0.8. The load is sufficiently inductive so that the load current is ripple free. The average current through the diode D under steady state is



(A) 1.6 A (B) 6.4 A

(B) 8.0 A (D) 10.0 A

Sol. 69 Option (C) is correct.

$V_s = 100$ V, duty ratio$= 0.8$, $R = 10\ \Omega$

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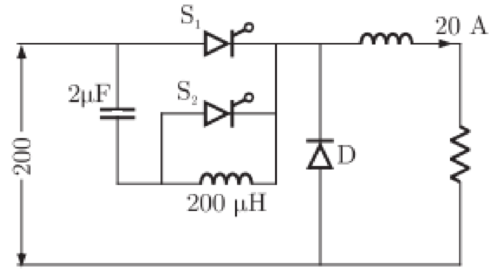
Q. 70 Figure shows a chopper. The device S_1 is the main switching device. S_2 is the auxiliary commutation device. S_1 is rated for 400 V, 60 A. S_2 is rated for 400 V, 30 A. The load current is 20 A. The main device operates with a duty ratio of 0.5. The peak current through S_1 is



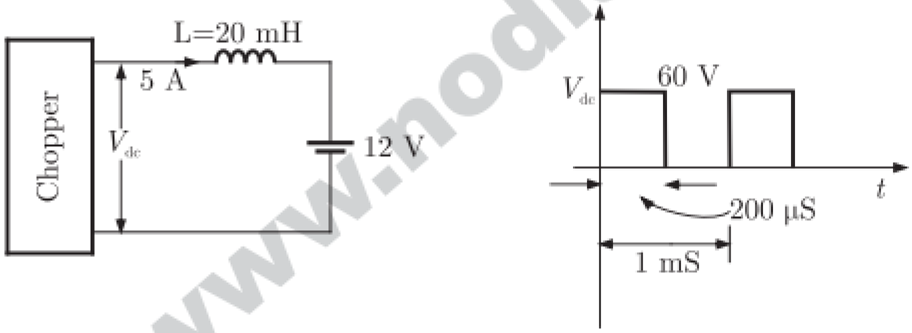
(A) 10 A
(B) 20 A
(C) 30 A
(D) 40 A

Sol. 70 Option (D) is correct.

Peak current through S_1

$$I = I_0 + V_S\sqrt{C/L} = 20 + 200\sqrt{\frac{2\times 10^{-6}}{200\times 10^{-6}}} = 40\text{ A}$$

Q. 78 A chopper is employed to charge a battery as shown in figure. The charging is 5 A. The duty ratio is 0.2. The chopper output voltage is also shown figure. The peak to peak ripple current in the charging current is



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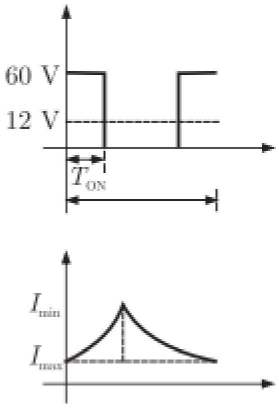
(A) 0.48 A
(B) 1.2 A
(C) 2.4 A
(D) 1 A

Sol. 78 Option (A) is correct.

In the chopper during turn on of chopper V-t area across L is,

$$\int_0^{T_{on}} V_L\,dt = \int_0^{T_{on}} L\left(\frac{di}{dt}\right)dt = \int_{i_{min}}^{i_{max}} L\,di = L\,(i_{max} - i_{min}) = L(\Delta I)$$



V-t are applied to 'L' is $= (60 - 12)\,T_{on} = 48\,T_{on}$