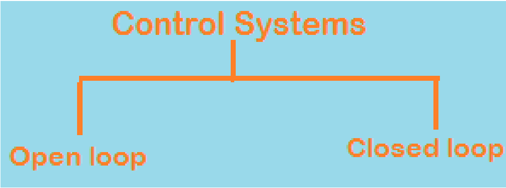
What is Control System?

Let us study about a new type of engineering study which is called as **Control Systems Engineering**.It's very interesting subject and has a lot of calculation part.Control system theory evolved as an engineering discipline and due to the universality of the principles involved, it is extended to various fields like economy, sociology, biology, medicine etc.In this, you will learn about open and **closed loop control system** and also their **differences**.

Control System theory has played a vital role in the advance of engineering and science.The automatic control has become an integral part of modern manufacturing and industrial processes.For example, numerical control of machine tools in manufacturing industries, controlling pressure, temperature, humidity, viscosity and flow in the process industry.

When a number of elements or components are connected in a sequence to perform a specific function, the group thus formed is called a system.In a system when the output quantity is controlled by varying the input quantity, the system is called **control system**.The output quantity is called controlled variable or response and input quantity is called command signal or excitation.



Types of Control systems:

Control systems are of two types.They are

1)**Open Loop System**
2)**Closed Loop System**

1)Open loop control system:

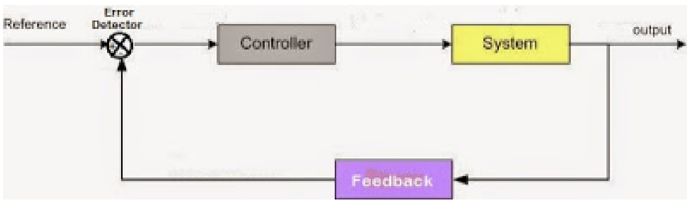
Any physical system which does not automatically correct the variation in its output is called an **open loop system** or control system in which the output quantity has no effect upon the input quantity are called **open loop control system**.This means that the output is not feedback to the input for correction.

In open loop control system, the output can be varied by varying the input.But due to external disturbances, the system output may change.When the output changes due to disturbances, it is not followed by changes in input to correct the output.In **open loop systems**, the changes in output are corrected by changing the input manually.

2)Closed loop control system:

Control systems in which the output has an effect upon the input quantity in order to maintain the desired output value are called **closed loop systems**.

The **open loop system** can be modified as closed loop system by providing a feedback.The provision of feedback automatically corrects the changes in output due to disturbances.Hence the closed loop system is also called automatic control system.The general **block diagram of an automatic control system** is shown in the figure below.It consists of an error detector, a controller, plant (open loop system) and feedback path elements.



The reference signal (or input signal) corresponds to the desired output.The feedback path elements sample the output and convert it to the same type as that of the reference signal.The feedback signal is proportional to the output signal and it is fed to the error detector.The error signal generated by the error detector is the difference between the reference signal and the feedback signal.The controller modifies and amplifies the error signal to produce better control action.The modified error signal is fed to the plant to correct its output.

Advantages of Open loop control system:

1.The **open loop systems** are simple and economical.

2.The open loop systems are easier to construct.

3.Generally, the **open loop systems** are stable.

Disadvantages of open loop systems:

1.The **open loop systems** are inaccurate and unreliable.

2..The changes in the output due to external disturbances are not corrected automatically.

Advantages of closed loop systems:

1.The **closed loop systems** are accurate.

2.The closed loop systems are accurate even in the presence of non-linearities.

3.The sensitivity of the systems may be made small to make the system more stable.

4.The closed loop systems are less affected by noise.

Disadvantages of closed loop systems:

1.The **closed loop systems** are complex and costly.

2.The feedback in closed loop system may lead to an oscillatory response.

3.The feedback reduces the overall gain of the system.

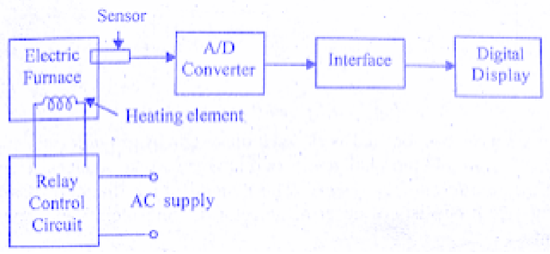
4.Stability is a major problem in closed loop system and more care is needed to design a stable **closed loop system**.

Open & Closed Loop Control Systems Examples:

Example 1 : Electrical Furnace

Open Loop System:

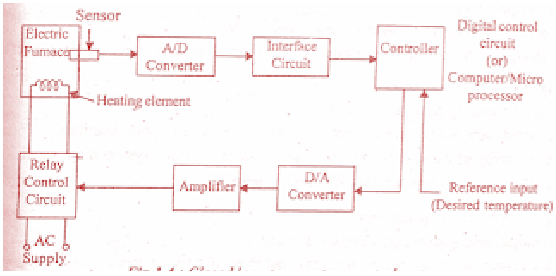
The electric furnace shown in the below is an **open loop system**.The output in the system is the desired temperature.The temperature of the system is raised by heat generated by the heating element.The output temperature depends on the time during which the supply to heater remains ON.The ON and OFF of the supply is governed by the time setting of the relay.



The temperature is measured by a sensor, which gives art analog voltage corresponding to the temperature of the furnace.The analog signal is converted to digital signal by an Analog to digital converter (AD converter).The digital signal is given to the digital display device to display the temperature.In this **open loop system**, if there is any change in output temperature then the time setting of the relay is not altered automatically.

Closed Loop System:

The electric furnace shown in the below figure is a **closed loop system**. The output of the *closed loop system* is the desired temperature and it depends on the time during which the supply to heater remains ON.



The switching ON and OFF of the relay is controlled by a controller which is a digital system or computer. The desired temperature is input to the system through the keyboard or as a signal corresponding to the desired temperature via ports. The actual temperature is sensed by Sensor and converted to digital signal by the A/D converter.

The computer reads the actual temperature and compares with the desired temperature. If it finds any difference then it sends the signal to switch ON or OFF the relay through D/A converter and amplifier.Thus the system automatically corrects any changes in output.Hence it is a **closed loop system**.

Example 2: Traffic Control System

Open Loop System:

Traffic control by means of traffic signals operated on a time basis constitutes an **open loop control system**.The sequence of control signals are based on a time slot given for each signal.The time slots are decided based on a traffic study.The system will not measure the density of the traffic before giving the signals.Since the time slot does not change according to traffic density, the system is **open loop system**.

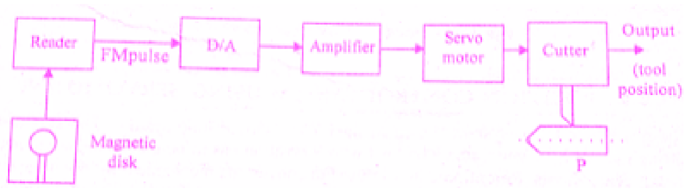
Closed Loop System:

Traffic control system can be made as a **closed loop system** if the time slots of the signals are decided based on the density of traffic.In **closed loop traffic control system**, the density of the traffic is measured on all the sides and the information is fed to a computer.The timings of the control signals are decided by the computer based on the density of traffic Since the **closed loop system** dynamically changes the timings, the flow of vehicles will be better than **open loop system**.

Example 3: Numerical Control System

Open Loop System:

Numerical control is a method of controlling the motion of machine components using of numbers.Here, the position of work head tool is controlled by the binary information contained in a disk.A magnetic disk is prepared in binary form representing the desired part P (P is the metal part to be machined). The tool will operate on the desired part P.To start the **open loop system**, the disk is fed through the reader to the D/A converter.



The D/A converter converts the FM(frequency modulated) output of the reader to a analog signal. It is amplified and fed to a servomotor which positions the cutter on the desired part P.The position of the cutter head is controlled by the angular motion of the servomotor.

This is an **open loop system** since no feedback path exists between the output and input.The system positions the tool for a given input command.Any deviation in the desired position is not checked and corrected automatically.

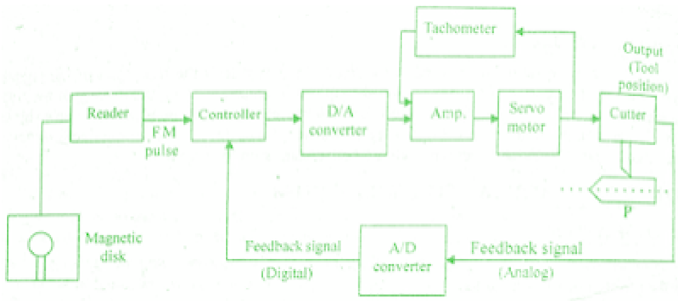
The position of the cutterhead is controlled according to the input of the servomotor.The transducer attached to the cutterhead converts the motion into an electrical signal.The analog electrical signal is converted to the digital pulse signal by the A/D converter.Then this signal is compared with the input pulse signal.

If there is any difference between these two, the controller sends a signal to the servomotor to reduce it.Thus the **closed loop system** automatically corrects any deviation in the desired output tool position.An advantage of numerical control is that complex parts can be produced with uniform tolerances at the maximum milling speed.

Closed Loop System:

A magnetic disk is prepared in binary form representing the desired part P (P is the metal part to be machined).To start the **closed loop system**, the disk is loaded in the reader.The controller compares the frequency modulated input pulse signal with the feedback pulse signal.The controller is a computer or microprocessor system.

The controller carries out mathematical operations on the difference in the pulse signals and generates an error signal.The D/A converter converts the controller output pulse (error signal) into an analog signal.The amplified analog signal rotates the servomotor to position the tool on the job.



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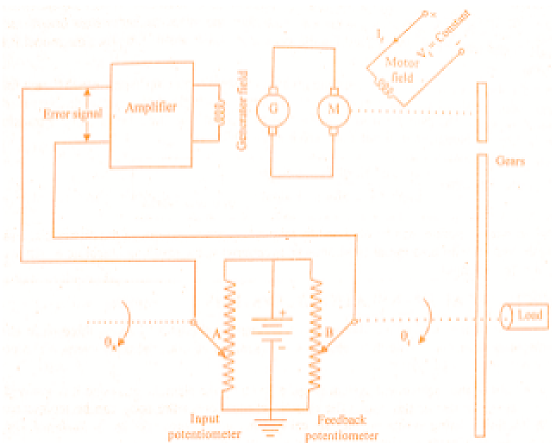
Example 4: Position Control System Using Servomotor

The **position control system** shown in the below is a **closed loop system**.The system consists of a Servomotor powered by a generator.The load whose position has to be controlled is connected to motor shaft through gear wheels.Potentiometers are used to convert the mechanical motion to electrical signals.The desired load position (θ_R) is set on the input potentiometer and the actual load position (θ_c) is fed to feedback potentiometer.



The difference between the two angular positions generates an error signal which is amplified and fed to generator field circuit.The induced emf of the generator drives the motor.The rotation of the motor stops when the error signal is zero, i.e. when the desired load position is reached.

This type of control systems are called servomechanisms.The servo or servomechanisms are **feedback control systems** in which the output is the mechanical position (or time derivatives of position e.g. velocity and acceleration).

https://www.electrical4u.com/speed-control-of-dc-motor/

Direct current (dc) motors have variable characteristics and are used extensively in variable-speed drives.

DC motors can provide a high starting torque and it is also possible to obtain speed control over a wide range.

The methods of speed control are normally simpler and less expensive than those of AC drives.

Both series and separately excited DC motors are normally used in variable-speed drives, but series motors are traditionally employed for traction applications.

Due to commutator, DC motors are not suitable for very high speed applications and require more maintenance than do AC motors.

With the recent advancements in power conversions, control techniques, and microcomputers, the ac motor drives are becoming increasingly competitive with DC motor drives.

Controlled rectifiers are generally used for the speed control of dc motors.

DC drives can be classified, in general, into three types:

- 1. Single-phase drives
- 2. Three-phase drives
- 3. DC-DC converter drives

- When supply is given to the stator winding of dc motor ,a flux produced in stator winding which is linked to the rotor winding & rotor starts rotating.
- Let consider a single coil stator having inductance Ls & resistance Rs.
- Then electrical equation for dc motor is given as

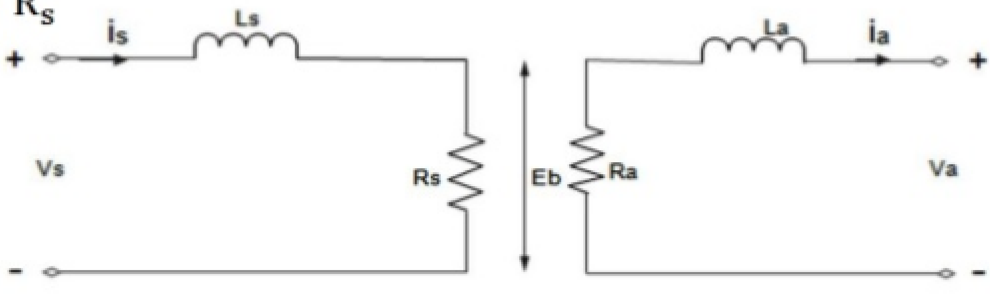
$$v_s(t) = L_s \frac{di_s}{dt} + R_s i_s$$

- Where i_S is current flowing through the stator, and V_S be the generated voltage. By using Laplace transform.

$$\frac{i_s(s)}{v_s(s)} = \frac{K_s}{1 + Z_s s}$$

$K_s = \frac{1}{R_s}$ is called the stator gain and

$Z_s = \frac{L_e}{R_s}$ is called as the stator time constant



- Fig. shows the generalized electrical diagram for dc motor. Let La and Ra be the inductance and resistance of the rotor winding with back emf (E_b).

$$V_a(t) = L_a\frac{dia}{dt} + R_a i_a + E_b \qquad \text{...(5.5.3)}$$

By taking Laplace transform of Equation (5.5.3)

$$\frac{i_a(s)}{V_a(s) - E_b(s)} = \frac{K_a}{1 + Z_a s} \qquad \text{...(5.5.4)}$$

Where $K_a = \frac{1}{R_a}$ is called as rotor gain and

$Z_a = \frac{L_a}{R_a}$ is called as rotor time constant

We know that

$$\text{Motor torque } (T_m) = K_\phi \phi i_a \qquad \text{...(5.5.5)}$$

$$E_b = K_\phi \phi\, w \qquad \text{...(5.5.6)}$$

Where ϕ is the flux generated in motor while K_ϕ is the proportionality constant.

- Since flux is proportional to the current i_s then

$$T_m = K i_s i_a$$

$$E_b = K i_s w$$

Where $$K = K_\phi K_o N$$

- We know that if motor is provided with supply it produces torque whose equa given as

$$Tm - T_L = J\frac{dw}{dt} + Fw \qquad \ldots$$

Where J is the rotor inertia and F is friction coefficient

By taking Laplace transform of equation (5.5.9) we get

$$\frac{W(s)}{T_m(s) - T_L(s)} = \frac{K_m}{1 + Z_m s} \qquad \ldots(5$$

Where, $K_m = \frac{1}{F}$ is the mechanical gain and

$Z_m = \frac{J}{F}$ is the mechanical time constant

The power given by the motor is the same at the input and the output of the gear. Let T'_m and ω' be the torque and the speed at the output of the gear it can be given as

$$T_m \omega = T'_m \omega' \qquad \text{...(5.5.11)}$$

and since $\omega' = \omega/n$

and $T'_m = n \times T_m$

Substituting the above equations in equation (5.5.9) with damping and inertia control, can be written as

$$T'_m - T_L = (J_a + n^2 J)\frac{d\omega'}{dt} + (F_G + n^2 F)\,\omega' \qquad \text{...(5.5.12)}$$

Where J_G and J are inertia constant, F_G and F are friction coefficient.

If G is for gear we compare the equation (5.5.9) and equation (5.5.12), we see that wit presence of gear, motor inertia and the damping of motor increases.

Then dynamic model of dc motor (linear) is given as

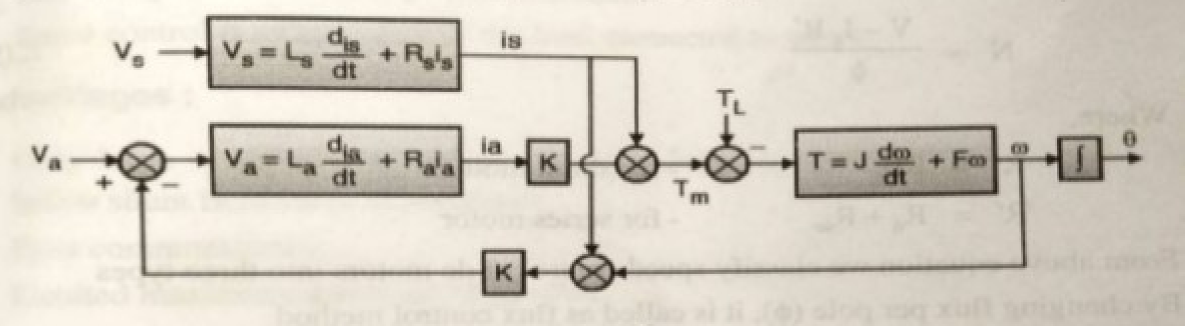


Fig. 5.5.2

Where θ is the rotor position.

Also dynamic model of dc motor (non-linear) is given as

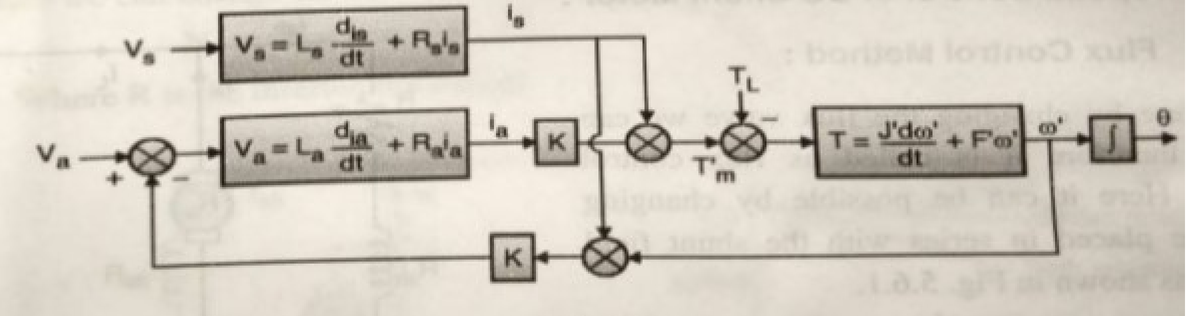


Fig. 5.5.3

J′ is given as

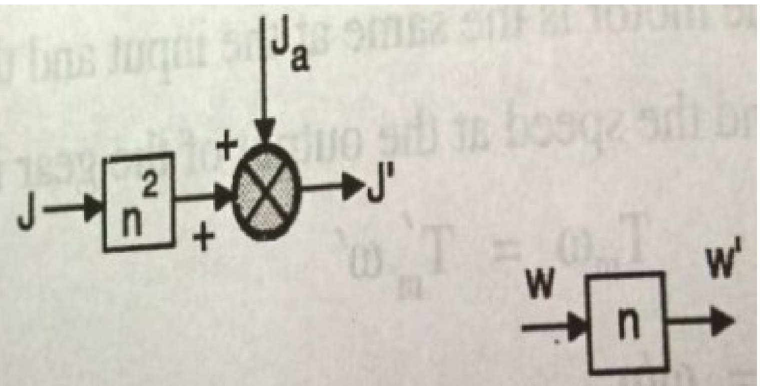


Fig. 5.5.4

F′ is given as

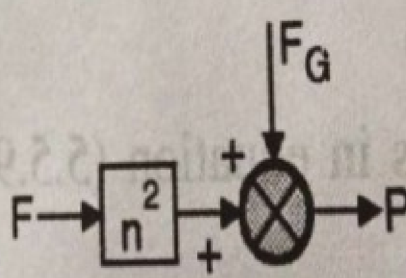


Fig. 5.5.5

7.21 DC MACHINE DYNAMICS

The DC machines are quite versatile and are capable of giving a variety of V-A and speed-torque characteristics by suitable combinations of various field windings. With solid-state controls their speeds and outputs can be controlled easily over a wide range for both dynamic and steady-state operation. By addition of the feedback circuit, the machine characteristics can be further modified. The aim of this section is to study dc machines with reference to their dynamic characteristics.

For illustration, let us consider the separately excited dc machine shown schematically in Fig. 7.106. For ease of analysis, the following assumptions are made:

(i) The axis of armature mmf is fixed in space, along the q-axis.

(ii) The demagnetizing effect of armature reaction is neglected.

(iii) Magnetic circuit is assumed linear (no hysteresis and saturation). As a result all inductances (which came into play in dynamic analysis) are regarded as constant.

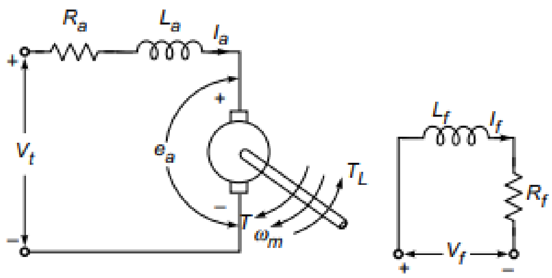


Fig. 7.106 *Schematic representation of a separately excited dc motor for dynamic analysis*

The two inductance parameters appearing in Fig. 7.106 are defined below:

L_a = armature self-inductance caused by armature flux; this is quite small* and may be neglected without causing serious error in dynamic analysis

L_f = self-inductance of field winding; it is quite large for shunt field and must be accounted for

Mutual inductance (between field and armature) = 0; because the two are in space quadrature.

Further for dynamic analysis it is convenient to use speed in rad/s rather than rpm.

Applying Kirchhoff's law to the armature circuit,

$$V_t = e_a(t) + R_a i_a(t) + L_a \frac{d}{dt} i_a(t) \tag{7.134}$$

where

$$e_a(t) = K_e i_f(t)\omega_m;\ K_e = \text{constant } (\phi(t) \propto i_f(t)) \tag{7.135}$$

Similarly for the field circuit,

$$v_f(t) = R_f i_f(t) + L_f \frac{d}{dt} i_f(t) \tag{7.136}$$

For motoring operation, the dynamic equation for the mechanical system is

$$T(t) = K_t i_f(t) i_a(t) = J\frac{d}{dt}\omega_m(t) + D\omega_m(t) + T_L(t) \tag{7.137}$$

* The armature mmf is directed along the low permeance q-axis.

where J = moment of inertia of motor and load in Nms^2

D = viscous damping coefficient representing rotational torque loss, Nm rad/s

Energy storage is associated with the magnetic fields produced by i_f and i_a and with the kinetic energy of the rotating parts. The above equations are a set of *nonlinear** (because of products $i_f(t)\,\omega_m$ and $i_f(t)i_a(t)$) *state equations* with *state variables* i_f, i_a and ω_m. The solution has to be obtained numerically.

Transfer Functions and Block Diagrams

In the simple linear case of motor response to changes in armature voltage, it is assumed that the field voltage is constant and steady-state is existing on the field circuit, i.e. I_f = constant. Equations (7.134), (7.136) and (7.137) now become linear as given below

$$v(t) = K'_e\,\omega_m(t) + R_a i_a(t) + L_a \frac{d}{dt} i_a(t) \tag{7.138}$$

$$T(t) = K'_t\, i_a(t) = J\frac{d}{dt}\,\omega_m(t) + D\omega_m(t) + T_L(t) \tag{7.139}$$

Laplace transforming Eqs (7.138) and (7.139)

$$V(s) = K'_e\omega_m(s) + (R_a + sL_a)\, I_a(s) \tag{7.140}$$

$$T(s) = K'_t I_a(s) = (sJ + D)\omega_m(s) + T_L(s) \tag{7.141}$$

These equations can be reorganized as

$$I_a(s) = \frac{V(s) - K'_e\omega_m(s)}{(R_a + sL_a)}$$

$$= [V(s) - K'_e\omega_m(s)] \times \frac{1/R_a}{(1+s\tau_a)} \tag{7.142}$$

where $\tau_a = L_a/R_a$ = armature circuit time-constant

Also

$$\omega_m(s) = [T(s) - T_L(s)] \times \frac{1/D}{(1+s\tau_m)} \tag{7.143}$$

where $\tau_m = J/D$ = mechanical time-constant

$$T(s) = K'_t I_a(s) \tag{7.144}$$

From Eqs (7.142) – (7.144), the block diagram of the motor can be drawn as in Fig. 7.107. It is a second-order feedback system with an oscillatory response in general. It is reduced to simple first-order system, if L_a and therefore τ_a is neglected

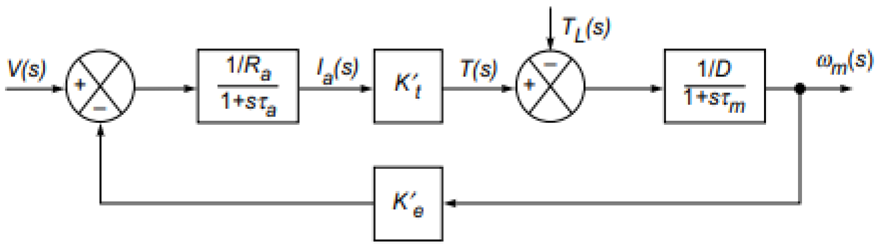


Fig. 7.107 *Block diagram of separately-excited dc motor; inputs V(s) and ω_m(s)*

* This is inspite of the fact that the magnetic circuit has been regarded as linear.

Shunt Generator Voltage Build-up

The qualitative explanation for the voltage build-up process in a shunt generator has already been advanced in Sec. 7.11. Here the mathematical treatment of this problem will be given, which in fact boils down to the solution of a nonlinear differential equation.

Referring to Fig. 7.108 it is seen that for any field current the intercept *ab*, between the OCC and the R_f-line gives the voltage drop caused by the rate of change of Φ_f and the intercept *bc* gives the drop in the field resistance. The two together balance out the generated emf e_a (neglecting $i_f R_a$, the armature drop). Thus

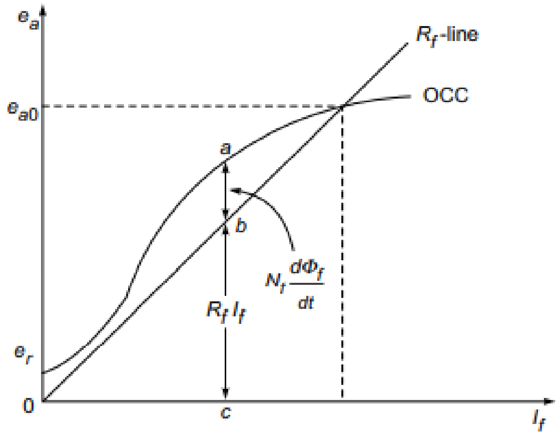


Fig. 7.108 *Magnetization curve and R_f-line*

$$N_f \frac{d\Phi_f}{dt} = e_a - R_f i_f \tag{7.145}$$

where Φ_f = field flux/pole

N_f = number of turns of field winding

The field flux Φ_f is greater than the direct axis air-gap flux Φ_d because of leakage. Taking this into account

$$\Phi_f = \sigma \Phi_d \tag{7.146}$$

Here σ is known as the *coefficient of dispersion.*

Recalling Eq. (7.3),

$$\Phi_d = \frac{e_a}{K_a \omega_m} \tag{7.147}$$

Substituting Eqs (7.146) and (7.147) in Eq. (7.148),

$$\frac{N_f \sigma}{K_a \omega_m} \cdot \frac{de_a}{dt} = e_a - i_f R_f \tag{7.148}$$

Multiplying numerator and denominator by $N_f P_{ag}$

where P_{ag} is the permeance of the air-gap/pole

$$\frac{N_f \sigma}{K_a \omega_m} = \frac{N_f^2 \sigma \mathcal{P}_{ag}}{K_a \omega_m \mathcal{P}_{ag} N_f}$$

It is easily recognized that the numerator is the unsaturated value of field inductance, L_f, and the denominator is the slope of the air-gap line. Both are constants. Hence,

$$\frac{L_f}{K_g} \frac{de_a}{dt} = e_a - R_f i_f \tag{7.149}$$

Rewriting Eq. (7.145)

$$dt = \frac{L_f}{K_g} \left(\frac{de_a}{e_a - R_f i_f} \right)$$

or

$$t = \frac{L_f}{K_g} \int_{e_r}^{e_a} \frac{de_a}{e_a - R_f i_f} \tag{7.150}$$

where the limits of integration,

e_r = residual voltage

e_a = instantaneous generated voltage

This integral can be evaluated graphically by summing up the areas on a plot of $1/(e_a - R_f i_f)$ against e_a. This approach is employed to plot e_a against time. The theoretical time needed for the generated emf to attain the no-load value, e_{a0} would be infinite; hence in practice the time needed to reach 0.95 e_{a0} is taken as the time needed to reach e_{a0}. The variation of e_a with time is plotted in Fig. 7.109.

The response is rather sluggish since only small voltage differences ($= e_a - R_f i_f$) contribute to the flux build-up (Φ_f).

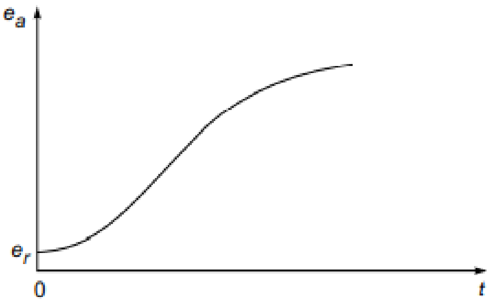


Fig. 7.109 *Voltage build-up of a shunt generator*

4

Direct Current Motor Modeling and Control Aspects

4.1 Introduction

Direct current (dc) motors have been used in the industry for the past several years. There are several classifications among dc motors, and separately excited dc motors and series motors present excellent speed-torque characteristics, suitable for many industrial utilizations. As such, dc motors are suitable for a wide range of variable-speed operation, braking, and speed reversal. For successful implementation of closed-loop speed control, a dc motor needs to be modeled either in state-space or in the transfer-function form. This chapter introduces of power conversion, state-space, and transfer-function models. Measurement of various motor parameters is also included.

4.2 Voltage Equation

A simple representation of a dc motor is shown in Fig. 4.1. The field system consists of a pair of electromagnets excited from the field voltage V_f. The field current is indicated as I_f. The armature conductors are assumed to carry the current as given in Fig. 4.1. The field flux is constant and stationary in space; furthermore, this flux is perpendicular to the armature current at any instant. This is one of the most interesting features of dc motors because such a position produces the maximum torque. Applying Fleming's left-hand rule, the armature flux can be obtained, and it can be seen that the introduction of field flux with armature flux gives clockwise rotational torque to the motor.

The induced voltage in the armature of a dc motor is equivalent to the generated voltage in a dc generator. The induced emf, generally termed back-emf, is labeled as E_b and is given by

$$E_b = \frac{\Phi_m Z_a N_r}{60} \cdot P_a / A \tag{4.1}$$

where,

Φ_m = Field flux
Z_a = Total number of armature conductors
N_r = Rotor speed in rpm
P_a = Number of poles
A = 2 for wave winding
= P_a for lap winding

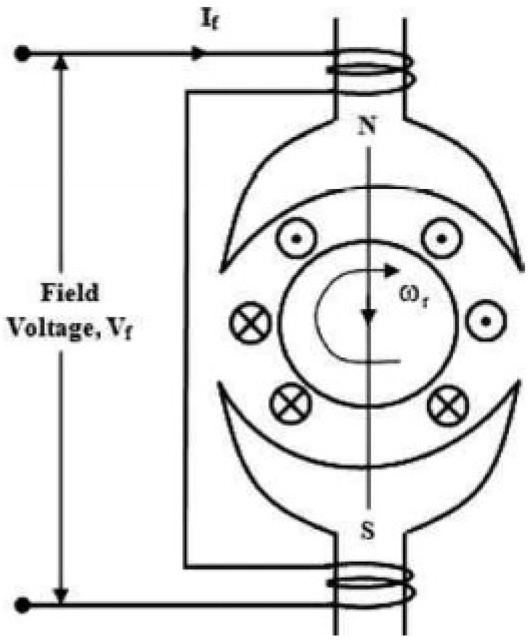


FIGURE 4.1
Sketch of a dc motor structure.

Putting

$$\omega_r = \frac{2\pi N_r}{60},$$

$$E_b = \frac{\Phi_m Z_a}{60} \cdot \frac{60\,\omega_r}{2\pi} \cdot P_a/A$$

$$= \frac{\Phi_m Z_a \omega_r}{2\pi} \cdot P_a/A$$

Making

$$K = \frac{Z_a}{2\pi} \cdot P_a/A, \text{ then}$$

$$E_b = K\Phi_m \omega_r$$

In a separately excited dc motor, field flux Φ_m is kept constant, and hence

$$E_b = K_b \omega_r \tag{4.2}$$

where $K_b = K\Phi_m$ and is called the back-emf constant. The unit of K_b is V/(rad/s).

4.3 Torque Equation

A separately excited dc motor can be represented in R-L-back emf form and is shown in Fig. 4.2. Here, r_a and L_a represent armature resistance and inductance, respectively. Armature voltage and current are shown as v_a and i_a in Fig. 4.2. Writing Kirchhoff's voltage law (KVL),

$$v_a = e_b + r_a i_a + L_a \frac{di_a}{dt} \tag{4.3}$$

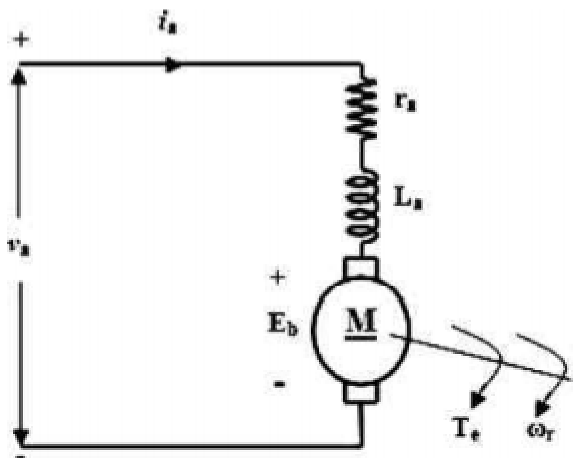


FIGURE 4.2
Sketch of a dc motor equivalent circuit.

Multiplying both sides using i_a, the power equation can be written as

$$v_a i_a = r_a i_a^2 + e_b i_a + L_a i_a \frac{di_a}{dt}$$

Under steady-state conditions, $\frac{di_a}{dt} \approx 0$ and hence

$$v_a I_a = r_a I_a^2 + E_b I_a$$

where $I_a^2 r_a$ represents armature copper loss. Hence, the air gap power is $E_b I_a$. If T_e represents the electromagnetic torque, then

$$\text{Output power} = T_e \omega_r$$

Neglecting friction and windage losses, this should be equal to air gap power.

$$\text{i.e., } E_b I_a = T_e \omega_r$$

$$\text{i.e., } T_e = \frac{E_b I_a}{\omega_r} \tag{4.4}$$

From Equation (4.2),

$$K_b = \frac{E_b}{\omega_r}$$

and substituting in Equation (4.4), we get

$$T_e = K_b I_a \tag{4.5}$$

It is evident that the torque constant is equivalent to the back-emf constant for a fixed field excited dc motor.

EXAMPLE 4.1

A permanent magnet dc commutator motor has no load speed of 5,000 rpm when connected to a 115-V dc supply. The armature resistance is 2.8 Ω, and other losses may be neglected. Find the speed of the motor at supply voltage of 80 V and developing a torque of 0.7 N-m.

SOLUTION:

As an assumption, under no-load, armature current $I_a = 0$.

$$\therefore E_b = V_a = 115 \text{ V}$$

$$K = \frac{115}{\left(\frac{2\pi \times 5,000}{60}\right)} \Rightarrow 0.22 \text{ V/rad/s}$$

Electromagnetic torque, $T_e = K_b I_a$

$$I_a = \frac{0.7}{0.22} \Rightarrow 3.18 \text{ A}$$

$$E_{b2} = V - I_a R_a \Rightarrow 80 - (3.18 \times 2.8)$$
$$= 71.09 \text{ V}$$

Now, $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$N_2 = 5000 \times \left(\frac{71.09}{115}\right) = 3090 \text{ rpm}$$

EXAMPLE 4.2

A separately excited dc motor runs at 1,500 rpm at no load with 220-V supply at the armature. The voltage is maintained at its rated value. The speed of the motor when it delivers a torque of 5 N-m is 1,400 rpm. The rotational losses and armature reaction losses are neglected. Find (a) armature resistance of the motor and (b) voltage applied to the armature for the motor to deliver a torque of 2.5 N-m at 1,350 rpm.

SOLUTION:

(a)

$$I_a = 0 \text{ (at no load)}$$
$$E_{b1} = V_a = 220 \text{ V}$$
$$E_{b1} = K\omega \Rightarrow 220 = K\left(\frac{2\pi \times 1500}{60}\right) \Rightarrow K = 1.401$$

Torque, $T_e = K_b I_a$

$$5 = 1.401 \times I_a \Rightarrow I_a = 3.568 \text{ A}$$
$$E_{b2} = V_a - I_a r_a$$
$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \Rightarrow E_{b2} = \frac{1400 \times 220}{1500} = 205.33 \text{ V}$$
$$r_a = \frac{V - E_{b2}}{I_a} = \frac{220 - 205.33}{3.568}$$
$$\therefore r_a = 4.1\ \Omega$$

(b) $T = K_b I_a \Rightarrow 2.5 = 1.401 \times I_a$

$$\Rightarrow I_a = 1.784 \text{ A}$$

$$E_b = K_b \omega_r \Rightarrow 1.401 \times \left(\frac{2\pi \times 1350}{60}\right) \Rightarrow 198.06 \text{ V}$$

$$V_a = E_b + I_a r_a$$
$$= 198.03 + (1.784 \times 4.1) = 205.34 \text{ V}$$

EXAMPLE 4.3

A separately excited dc motor has the parameters 220 V, 25 A, 1,500 rpm, $J = 0.6$ kg-m^2, $K_b = 0.567$ V/rad/s, and friction is negligible. If the motor starts from rest, find the time taken by the motor to reach a speed of 1,000 rpm with no load. The armature current is maintained constant at its rated value during starting.

SOLUTION:

$$\begin{aligned} T_e &= K_b I_a \\ &= 0.567 \times 25 \\ &= 14.175 \text{ N-m} \end{aligned}$$

During starting, armature current is constant, and hence torque remains constant at rated value.

$$J\frac{d\omega}{dt} = 14.175$$

Integrating on either side,

$$\int_{\omega_1}^{\omega_2} d\omega = \frac{14.175}{J}\int_{t_1}^{t_2} dt = 23.625\int_{t_1}^{t_2} dt$$
$$\omega_2 - \omega_1 = 23.625(t_2 - t_1)$$
$$\omega_1 = 0,\ \omega_2 = 104.71 \text{ rad/s}$$

And hence

$$t_1 - t_2 = 4.432 \text{ s}$$

The time taken by the motor to reach a speed of 1,000 rpm = 4.432 s.

EXAMPLE 4.4

A variable speed drive rated for 1,500 rpm, 60N-m is reversing to 1,000 rpm under no load. The motor torque is 20 N-m and reversing time is 0.5 s. Find the moment of inertia of the drive.

SOLUTION:

$$T_e - T_L = J\frac{d\omega}{dt}$$

$$T_L = 0 \text{ (Given)}$$

$$T_e = 20 \text{ N-m(Given)}$$

$$\Delta t = 0.5 \text{ sec}$$

$$\Delta\omega = (1500-(-1000)\times\frac{2\pi}{60}$$

$$= 261.799 \text{ rad/s}$$

$$J = \frac{20\times 0.5}{261.799}$$

$$J = 0.03819 \text{ kg-m}^2$$

EXAMPLE 4.5

In a speed-controlled dc motor drive, the load torque is 30 N-m. At time t = 0, the motor is running at 500 rpm and the generated torque is 90 N-m. The inertia of the drive is 0.01 N-ms^2/rad. The friction is negligible. Evaluate the time taken for the speed to reach 1,000 rpm.

SOLUTION:

$$J\frac{d\omega}{dt} = T_e - T_L$$

$$\Delta\omega = 1{,}000 - 500 = 500 \text{ rpm} = 52.35 \text{ rad/s}$$

$$T_e - T_L = 90 - 30 = 60 \text{ N-m}$$

$$J = 0.01$$

$$\Delta t = \frac{J\Delta\omega}{T_e - T_L}$$

$$= \frac{0.01\times 52.35}{60}$$

$$\Delta t = 8.726 \text{ ms}$$

EXAMPLE 4.6

An electric motor is developing a starting torque of 20 N-m, and starts with a load torque of 8 N-m on its shaft. If the acceleration at start is 100 rad/sec^2, what is the value of moment of inertia?

SOLUTION:

$$T_e - T_L = J\frac{d\omega}{dt}$$

$$20 - 8 = J\frac{d\omega}{dt}$$

$$\because \frac{\Delta\omega}{\Delta t} = 100 \text{ rad/sec}^2 \text{(Given)}$$

$$12 = 100\,J$$

$$J = 0.12 \text{ kg-m}^2$$

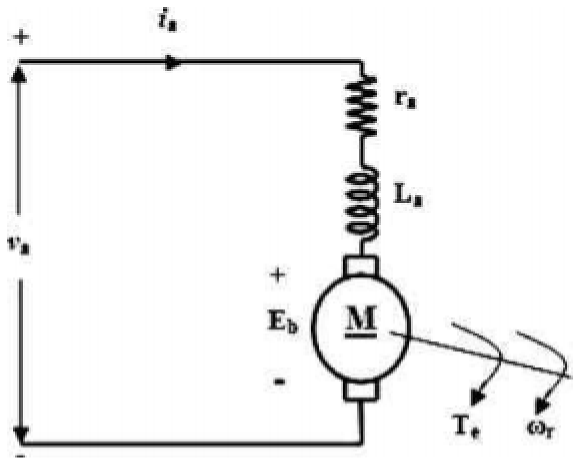


FIGURE 4.2
Sketch of a dc motor equivalent circuit.

4.4 State-Space Model

Referring to Fig. 4.2, the voltage equation of a separately excited dc motor is rewritten as

$$v_a = L_a\frac{di_a}{dt} + r_a i_a + e_b$$

Re-arranging the terms and setting $e_b = K_b\omega_r$,

$$\frac{di_a}{dt} = -\frac{r_a i_a}{L_a} - \frac{K_b\omega_r}{L_a} + \frac{v_a}{L_a} \tag{4.6}$$

Let J represent the moment of inertia in kg-m^2 and B the friction coefficient in N-m/(rad/s). Now

$$T_e = B\omega_r + J\frac{d\omega_r}{dt} + T_L$$

$$\text{i.e., } K_b i_a = B\omega_r + J\frac{d\omega_r}{dt} + T_L \tag{4.7}$$

$$\text{i.e., } \frac{d\omega_r}{dt} = -\frac{B\omega_r}{J} - \frac{T_L}{J} + \frac{K_b i_a}{J}$$

Equations (4.6) and (4.7) completely describe the dynamics of a dc motor and can be put in matrix form as

$$\begin{bmatrix} \dfrac{di_a}{dt} \\ \dfrac{d\omega_r}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-r_a}{L_a} & \dfrac{-K_b}{L_a} \\ \dfrac{K_b}{J} & \dfrac{-B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \dfrac{+1}{L_a} & 0 \\ 0 & \dfrac{-1}{J} \end{bmatrix} \begin{bmatrix} v_a \\ T_L \end{bmatrix} \tag{4.8}$$

This is equivalent to state-space form

$$\frac{dx}{dt} = Ax + Bu$$

where

$$x = \begin{bmatrix} i_a \\ \omega_r \end{bmatrix}$$

$$A = \begin{bmatrix} \dfrac{-r_a}{L_a} & \dfrac{-K_b}{L_a} \\ \dfrac{K_b}{J} & \dfrac{-B}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{1}{L_a} & 0 \\ 0 & \dfrac{-1}{J} \end{bmatrix}$$

where u is the input vector and is $\begin{bmatrix} v_a \\ T_L \end{bmatrix}$.

The Eigenvalues of the drive system can be found from A matrix by writing:

$$|sI - A| = 0 \tag{4.9}$$

$$\text{i.e.,}\quad \left| s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{-r_a}{L_a} & \frac{-K_b}{L_a} \\ \frac{K_b}{J} & \frac{-B}{J} \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} s + \frac{r_a}{L_a} & \frac{K_b}{L_a} \\ \frac{-K_b}{J} & s + \frac{B}{J} \end{bmatrix} \right| = 0$$

$$\text{i.e.,}\left(s + \frac{r_a}{L_a}\right)\left(s + \frac{B}{J}\right) + \frac{K_b^2}{JL_a} = 0$$

$$s^2 + \frac{sB}{J} + \frac{sr_a}{L_a} + \frac{r_aB}{L_aJ} + \frac{K_b^2}{JL_a} = 0$$

$$s^2 + s\left(\frac{B}{J} + \frac{r_a}{L_a}\right) + \frac{K_b^2}{JL_a} + \frac{r_aB}{L_aJ} = 0 \tag{4.10}$$

$$s = \frac{-\left(\frac{r_a}{L_a} + \frac{B}{J}\right) \pm \sqrt{\left(\frac{r_a}{L_a} + \frac{B}{J}\right)^2 - 4\left(\frac{K_b^2 + r_aB}{JL_a}\right)}}{2}$$

EXAMPLE 4.7

A separately excited dc motor has the following parameters:

$$r_a = 0.5\ \Omega,\quad L_a = 0.003\ \text{H},\quad k_b = 0.8\ \text{v/rad/sec}$$

$$J = 0.0167\ \text{kg-m}^2,\quad B = 0.01\ \text{N-m/rad/sec}$$

Find the Eigen value and asses the stability of the system.

SOLUTION:

$$s = \frac{-\left(\frac{r_a}{L_a} + \frac{B}{J}\right) \pm \sqrt{\left(\frac{r_a}{L_a} + \frac{B}{J}\right)^2 - 4\times\left(\frac{Br_a}{JL_a} + \frac{k_b^2}{JL_a}\right)}}{2}$$

$$s = \frac{-167.27 \pm \sqrt{-23519.28}}{2}$$

$$= -83.635 \pm i76.68$$

The system is stable because the roots are placed in the left side of the s-plane.

EXAMPLE 4.8

A separately excited dc motor has the following parameters:

$$r_a = 0.34\,\Omega, \quad L_a = 1.13\,\text{mH}, \quad k_b = 1.061\ \text{v/rad/sec}$$
$$J = 0.035\ \text{kg-m}^2, \quad B = 0$$

Find the Eigen value and assess the stability of the system.

SOLUTION:

$$s = \frac{-\left(\frac{r_a}{L_a}+\frac{B}{J}\right) \pm \sqrt{\left(\frac{r_a}{L_a}+\frac{B}{J}\right)^2 - 4\times\left(\frac{Br_a}{JL_a}+\frac{k_b^2}{JL_a}\right)}}{2}$$

$$s = \frac{-300.89 \pm \sqrt{-23317.29}}{2}$$

$$s = -150.45 \pm i76.35$$

The system is stable because the roots are placed in the left side of the s-plane.

EXAMPLE 4.9

A separately excited dc motor has the following parameters:

$$r_a = 1.39\,\Omega, \quad L_a = 0.00182\ \text{H}, \quad k_b = 0.331\ \text{v/rad/sec}$$
$$J = 0.002\ \text{kg-m}^2, \quad B = 0.005\ \text{N-m/rad/sec}$$

Find the Eigen value and assess the stability of the systems.

SOLUTION:

$$s = \frac{-\left(\frac{r_a}{L_a}+\frac{B}{J}\right) \pm \sqrt{\left(\frac{r_a}{L_a}+\frac{B}{J}\right)^2 - 4\times\left(\frac{Br_a}{JL_a}+\frac{k_b^2}{JL_a}\right)}}{2}$$

$$s^2 + 766.24s + 32008.52 = 0$$

$$s = \frac{-766.24 \pm \sqrt{766.24^2 - 4\times 1\times 32008.52}}{2}$$

$$s = -44.4, \quad -721.9$$

The system is stable because the roots are placed in the left side of the s-plane.

4.5 Transfer Function Model

The voltage equation of a dc motor is

$$v_a = \frac{L_a di_a}{dt} + r_a i_a + K_b \omega_r$$

Taking Laplace transform,

$$\begin{aligned} V_a(s) &= L_a s I_a(s) + r_a I_a(s) + K_b \omega_r(s) \\ &= I_a(s)\{L_a s + r_a\} + K_b \omega_r(s) \\ \therefore I_a(s) &= \frac{V_a(s) - K_b \omega_r(s)}{r_a + L_a s} \end{aligned} \tag{4.11}$$

The speed-torque equation is rewritten from Equation (4.7) as

$$\frac{d\omega_r}{dt} + \frac{B}{J}\omega_r = \frac{-T_L}{J} + \frac{K_b i_a}{J}$$

Taking Laplace transform,

$$\begin{aligned} s\omega_r(s) + \omega_r(s)\cdot\frac{B}{J} &= -\frac{T_L(s)}{J} + \frac{K_b I_a(s)}{J} \\ \omega_r(s) &= \frac{K_b I_a(s) - T_L(s)}{Js + B} \end{aligned} \tag{4.12}$$

Equations (4.11) and (4.12) can be rearranged to obtain the block diagram in Fig. 4.3. Neglecting the load torque $T_L(s)$, the no-load transfer function is obtained as

$$\frac{\omega_r(s)}{V_a(s)} = \frac{K_b}{JL_a s^2 + (BL_a + r_a J)s + Br_a + K_b^2} \tag{4.13}$$

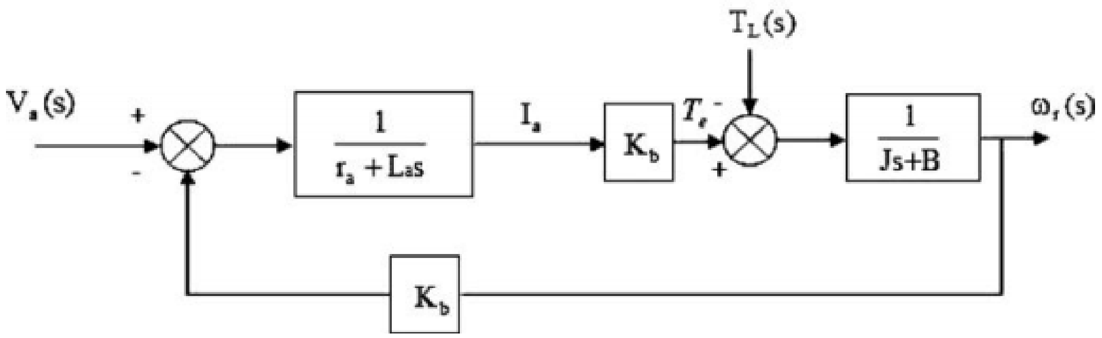


FIGURE 4.3
Transfer function of a dc motor.

4.6 Closed-Loop Control Design

To explain the closed-loop control of a dc motor, consider a proportional-integral (PI) controller be incorporated as given in Fig. 4.4. There are several methods to find the values of PI constants, and here the Routh-Hurwitz method is employed.

The characteristic equation for the above closed loop control system is

$$1 + G_c(s) \cdot G(s) = 0 \tag{4.14}$$

where $G_c(s)$ is the transfer function of PI controller and hence

$$G_c(s) = K_P + \frac{K_I}{S} \tag{4.15}$$

The dc motor is represented by G(s) and is

$$G(s) = \frac{K_b}{JL_a s^2 + (BL_a + r_a J)s + Br_a + K_b^2}$$

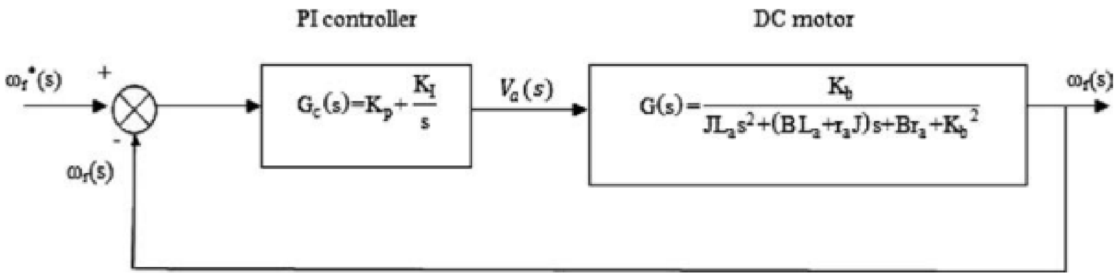


FIGURE 4.4
Closed-loop speed control using a proportional-integral (PI) controller.

Substituting $G_c(s)$ and $G(s)$ in the characteristic equation, we get

$$1 + \left[K_P + \frac{K_I}{S}\right] \times \left[\frac{K_b}{JL_a S^2 + (BL_a + r_a J)S + Br_a + K_b^2}\right] = 0$$

$$1 + \left[\frac{(K_p s + K_I)K_b}{JL_a s^3 + (BL_a + r_a J)s^2 + (Br_a + K_b^2)s}\right] = 0 \tag{4.16}$$

$$JL_a s^3 + (BL_a + r_a J)s^2 + (Br_a + K_b^2)s + (K_p s + K_I)K_b = 0$$

$$JL_a s^3 + (BL_a + r_a J)s^2 + (Br_a + K_b^2 + K_P K_b)s + K_I K_b = 0$$

This is the final characteristic equation for the above control system.

EXAMPLE 4.10

Consider a dc motor with the following parameters: L_a = 1.13 H, J = 0.035 kg-m^2, B = 0.1 N-m/rad/s, r_a = 0.3 Ω, K_b = 1.061 V/rad/s. Calculate the range of K_p and K_I values for the closed-loop operation of the motor.

SOLUTION:

The characteristic equation for the above case is obtained using Equation (4.16):

$$0.039\, s^3 + 0.123\, s^2 + (1.156 + 1.061\, K_P)s + 1.061\, K_I = 0.$$

To find the range of controller parameters (proportional gain K_P and integral gain K_I), apply the Routh-Hurwitz criteria.

$$\begin{array}{lll} s^3 & 0.039 & 1.156 + 1.061\, K_P \\ s^2 & 0.123 & 1.061\, K_I \\ s^1 & \dfrac{0.123(1.156 + 1.061\, K_P) - 0.0395{*}1.061\, K_I}{0.123} & \\ s^0 & 1.061\, K_I & \end{array}$$

To make the system stable, the first column of the Routh array should not contain any sign changes, which implies

$$\begin{aligned} 1.061\, K_I &> 0 \\ \frac{0.123(1.156 + 1.061\, K_P) - 0.0395 \times 1.061\, K_I}{0.123} &> 0 \\ 0.1421 + 0.130\, K_P - 0.041\, K_I &> 0 \\ 1.1 + K_P - 0.315\, K_I &> 0 \end{aligned}$$

As an illustration, let $K_I = 5$ (which is greater than zero). Substituting this value in the above equation,

$$\begin{aligned} 1.1 + K_P - 0.315 \times 5 > 0 \\ \therefore K_P > 0.4769 \end{aligned}$$

Here for $K_I = 5$, K_p should be greater than 0.4769.

It is important to mention that the above values of controller constants always guarantee stability, but they need not provide optimal dynamic stability. Improved dynamic response can be obtained by fine-tuning the values of K_p and K_I. In several cases, an inner current loop is also added, which helps limit the armature current to permissible values.

4.7 The dc Series Motor

In the dc series motor, the field circuit is connected in series with the armature coil as shown in Fig. 4.5. The back emf, E_b, is

$$E_b = K_b \Phi_m \omega_r$$

Armature current produces the flux, Φ_m and hence,

$$\Phi_m \alpha i_a$$
$$\Phi_m = k_f i_a$$

Hence,

$$E_b = K_b \cdot k_f i_a \omega_r \tag{4.17}$$

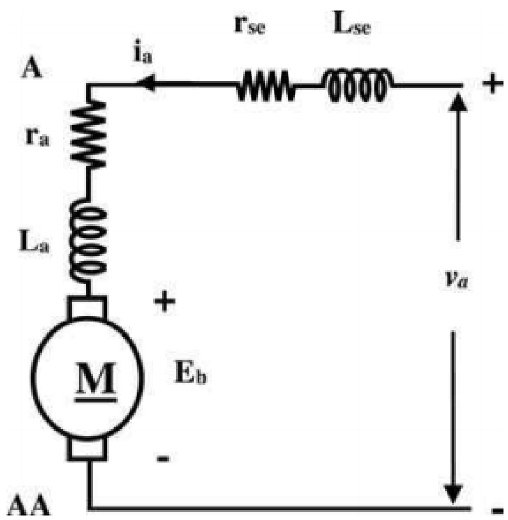


FIGURE 4.5
Sketch of a dc series motor.

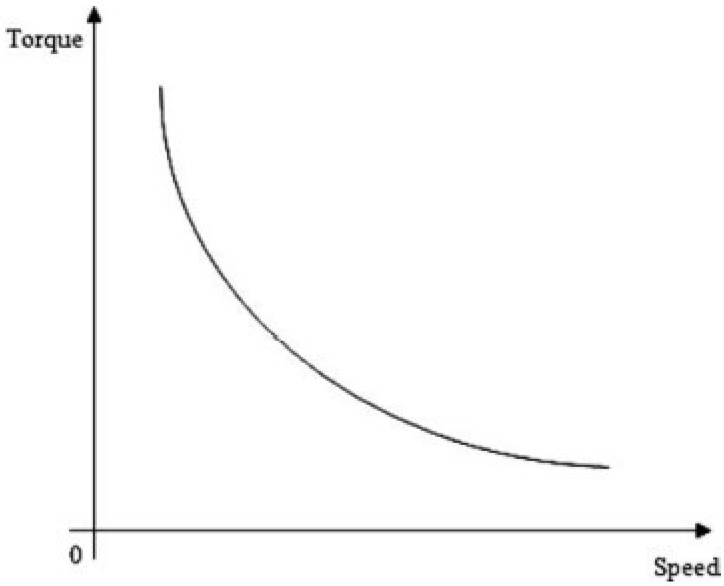


FIGURE 4.6
Torque-speed characteristics of dc series motor.

In the above equation, k_f is the series field constant. Reproducing Equation (4.4) yields

$$\begin{aligned} T_e &= \frac{E_b i_a}{\omega_r} = \frac{K_b \Phi_m \omega_r i_a}{\omega_r} \\ &= K_b \Phi_m i_a \\ &= K_b \cdot k_f i_a^2 \end{aligned} \tag{4.18}$$

Let $K_b k_f = k_T$, be as the torque constant, and its unit is N-m/A^2.
Thus,

$$T_e = k_T i_a^2$$

Furthermore, Equation (4.17) becomes

$$E_b = k_T i_a \omega_r \tag{4.19}$$

$$v_a = i_a r_a + k_T i_a \omega_r \tag{4.20}$$

In the above equation, r_a includes armature resistance together with series field resistance. The torque speed characteristic curve is shown in Fig. 4.6.

4.8 Determination of r_a and L_a

The armature resistance r_a of the motor is measured by applying a low dc voltage to armature terminals. The value of r_a can be taken as the ratio of applied armature voltage to the armature current. In case, the exact value is required, the brush voltage drop is to be subtracted from applied voltage.

To measure L_a, a low ac voltage (probably through a variac) is applied to the armature terminals, and the ratio of voltage to current is taken as Z_a. Then the armature inductance is computed as

$$L_a = \frac{\sqrt{Z_a^2 - r_a^2}}{2\pi f_1}$$

where f_1 is the frequency of ac supply in Hz.

4.9 Determination of K_b

The field current is adjusted to the rated value and the motor is rotated by a prime mover (another motor usually) at its rated speed, ω_r. The armature terminals are open circuited and the induced voltage across the armature is measured as E_b. The back-emf constant is then determined as

$$K_b = \frac{E_b}{\omega_r} \tag{4.21}$$

4.10 Determination of the Moment of Inertia of a Drive System

The moment of inertia of the dc drive system can be determined by retardation or a running-down test. The dc motor is started under no load and, once the no-load speed is reached, the input power is noted. Then the motor is switched OFF; as the armature

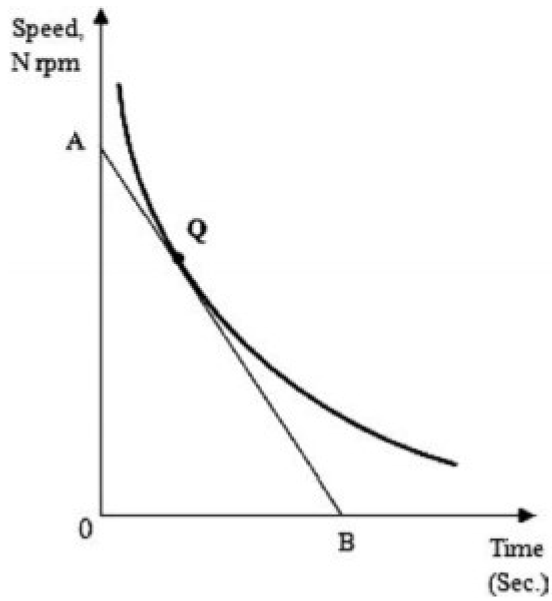


FIGURE 4.7
Speed versus time curve of dc motor.

slows down, its kinetic energy is drawn up to supply the various losses produced by rotation. Now, the variation of motor speed against time t is plotted as shown in Fig. 4.7.

If J is the moment of inertia of the armature and ω_r its angular velocity at any instant, then the kinetic energy of the armature is

$$K \cdot E = \frac{1}{2} J \omega_r^2$$

Rotational losses P_r = rate of loss of kinetic energy:

$$P_r = \frac{d}{dt}\left[\frac{1}{2} J \omega_r^2\right]$$

$$P_r = J \omega_r \frac{d\omega_r}{dt} \tag{4.22}$$

To calculate P_r it is therefore necessary to determine the curve of ω_r against time t.

Referring to Fig. 4.7, to find the gradient $\frac{dN}{dt}$ at any point Q, it is usual to draw the tangent to the curve and to measure the intercepts OA and OB.

$$\frac{dN}{dt} = \frac{OA}{OB}$$

Now, if the moment of inertia, J, is expressed in kg-m^2 and ω_r in rad/s, then the losses will be given by Equation (4.22). However,

$$\omega_r = \frac{2\pi N_r}{60}$$

$$\frac{d\omega_r}{dt} = \frac{2\pi}{60} \frac{dN_r}{dt}$$

Hence,

$$P_r = \left(\frac{2\pi}{60}\right)^2 J N_r \frac{dN_r}{dt} = 0.0109 \, J N_r \frac{dN_r}{dt}$$

Thus,

$$J = \frac{P_r}{0.0109 \times N_r \times \left(\frac{dN_r}{dt}\right)}$$

$$J = \frac{91.74 \times P_r}{N_r \times \left(\frac{dN_r}{dt}\right)} \tag{4.23}$$

EXAMPLE 4.11

A dc series motor has the following parameters: 220 V, 2.5 hp, 5,000 rpm, armature resistance = 1.26 Ω (includes armature resistance and series field resistance). Torque constant is $k_T = 0.035$ N-m/A^2. Under rated condition, find (a) motor current and (b) torque.

SOLUTION:

(a)

$$\omega_r = \frac{2\pi N_r}{60} = \frac{2 \times 3.14 \times 5,000}{60} = 523.3 \text{ rad/s}$$

$$v_a = i_a r_a + k_T i_a \omega_r$$

$$220 = i_a \times 1.26 + 0.035 \times i_a \times 523.3$$

$$\therefore i_a = 11.24 \text{ A}$$

(b)

$$T_e = k_T i_a^2$$

$$= 0.035 \times 11.24^2$$

$$= 4.4213 \text{ N-m}$$

EXAMPLE 11.1

A 240-V, permanent magnet, dc motor takes 2 A whenever it operates at no load. Its armature winding resistance and inductance are 1.43 Ω and 10.4 mH, respectively. The flux per pole is 5 mWb and the motor constant K_a is 360. The moment of inertia is 0.068 kg·m^2. If the motor is suddenly connected to a 240-V dc source while operating at no load, determine its speed and armature current as a function of time.

- **SOLUTION**

The no-load data of the motor helps us determine the friction coefficient, D, as outlined below

Since the motor requires 2 A at no load, its no-load speed is

$$\omega_m = \frac{240 - 1.43 \times 2}{360 \times 5 \times 10^{-3}} = 131.74 \text{ rad/s}$$

At no load, the developed torque

$$T_d = 360 \times 5 \times 10^{-3} \times 2 = 3.6 \text{ N·m}$$

is primarily for the rotational losses. The friction coefficient D can then be calculated as

$$D = \frac{3.6}{131.74} = 0.027 \text{ N·m·s}$$

Prior to the application of armature voltage the motor speed and armature current are zero. That is, at $t = 0$, $\omega_m(0) = 0$ and $i_a(0) = 0$. In addition, the load torque is zero because the motor operates at no load.

From Eq. (11.8), we get

$$\Omega_m(s) = \frac{1.8 \dfrac{240}{s}}{(0.027 + 0.068s)(1.43 + 10.4 \times 10^{-3}s) + 1.8^2}$$

or

$$\Omega_m(s) = \frac{610859.72}{s(s^2 + 137.89s + 4636.04)}$$

In order to determine the inverse Laplace transform of $\Omega_m(s)$, we expand $\Omega_m(s)$ into partial fractions as

$$\Omega_m(s) = \frac{A}{s} + \frac{B}{s + 79.84} + \frac{C}{s + 58.10}$$

where A, B, and C can now be determined by the root-substitution method. Thus,

$$A = s \left. \frac{610{,}859.72}{s(s + 79.84)\,(s + 58.10)} \right|_{s=0} = 131.74$$

$$B = (s + 58.10) \left. \frac{610{,}859.72}{(s + 58.10)s(s + 79.84)} \right|_{s=-58.10} = 351.87$$

$$C = (s + 79.84) \left. \frac{610{,}859.72}{(s + 79.84)s(s + 58.10)} \right|_{s=-79.84} = -483.56$$

Finally, we can take the inverse Laplace transform of

$$\Omega_m(s) = \frac{131.74}{s} + \frac{351.87}{s + 79.84} - \frac{483.56}{s + 58.10}$$

and get the angular velocity in rad/s as

$$\omega_m(t) = 131.74 + 351.87e^{-79.84t} - 483.56e^{-58.10t} \qquad \text{for } t > 0$$

The graph of $\omega_m(t)$ is given in Figure 11.2.
From Eq. (11.9), the Laplace transform of the armature current is

$$I_a(s) = \frac{23{,}076.92s + 9162.9}{s(s^2 + 137.89s + 4636.04)}$$

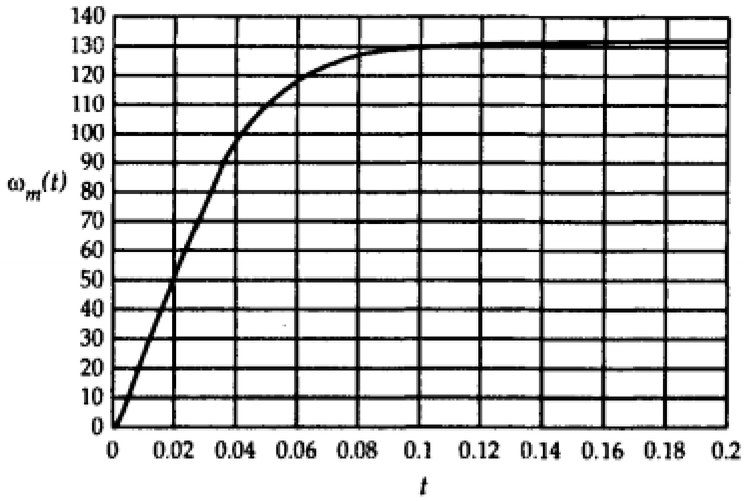


Figure 11.2 The motor speed as a function of time.

In terms of its partial fraction expansion, $I_a(s)$ can be written as

$$I_a(s) = \frac{2}{s} + \frac{1054}{s + 58.10} - \frac{1056}{s + 79.84}$$

Finally, we obtain the armature current as

$$i_a(t) = 2 + 1054e^{-58.10t} - 1056e^{-79.84t} \qquad \text{for } t > 0$$

which is shown graphically in Figure 11.3.

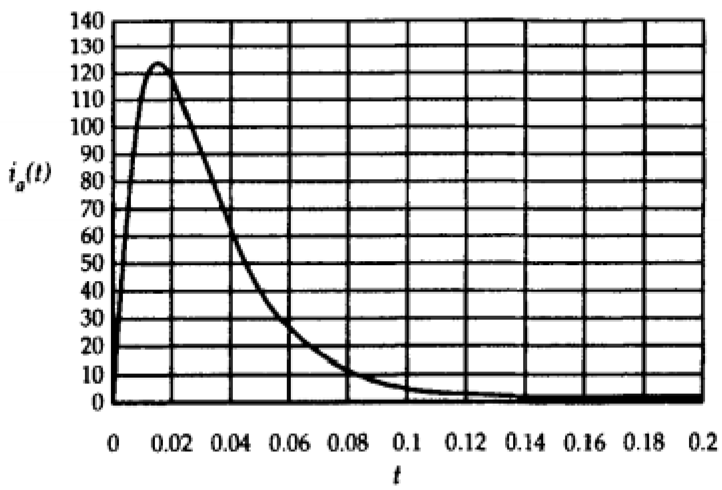


Figure 11.3 The armature current as a function of time.

EXAMPLE 11.2

The motor given in Example 11.1 is coupled to a load having a torque of 18.58 N·m. Determine the variation of the motor speed as a function of time after the motor is suddenly energized at its rated voltage at $t = 0$.

● SOLUTION

Since the voltage is applied to the armature circuit at $t = 0$, the initial speed and the armature current are both zero.
From Eq. (11.8),

$$\Omega_m(s) = \frac{1.8\,\dfrac{240}{s} - \dfrac{18.58}{s}\,(1.43 + 10.4 \times 10^{-3}s)}{(0.027 + 0.068s)\,(1.43 + 10.4 \times 10^{-3}s) + 1.8^2}$$

or

$$\Omega_m(s) = \frac{573{,}289.87 + s274.02}{s(s^2 + 137.89s + 4636.04)}$$

$$= \frac{123.66}{s} + \frac{342.29}{s + 79.84} - \frac{465.90}{s + 58.10}$$

The inverse Laplace transform of $\Omega_m(s)$ yields

$$\omega_m(t) = 123.66 + 342.29\,e^{-79.843t} - 465.90e^{-58.10t} \qquad \text{for } t > 0$$

The Laplace transform of the armature current is calculated from Eq. (11.9) as

$$I_a\,(s) = \frac{56{,}447.963 + s23{,}076.923}{s(s^2 + 137.89s + 4636.04)}$$

or in terms of its partial-fraction expansion as

$$= \frac{12.18}{s} - \frac{1027}{s + 79.84} + \frac{1015}{s + 58.10}$$

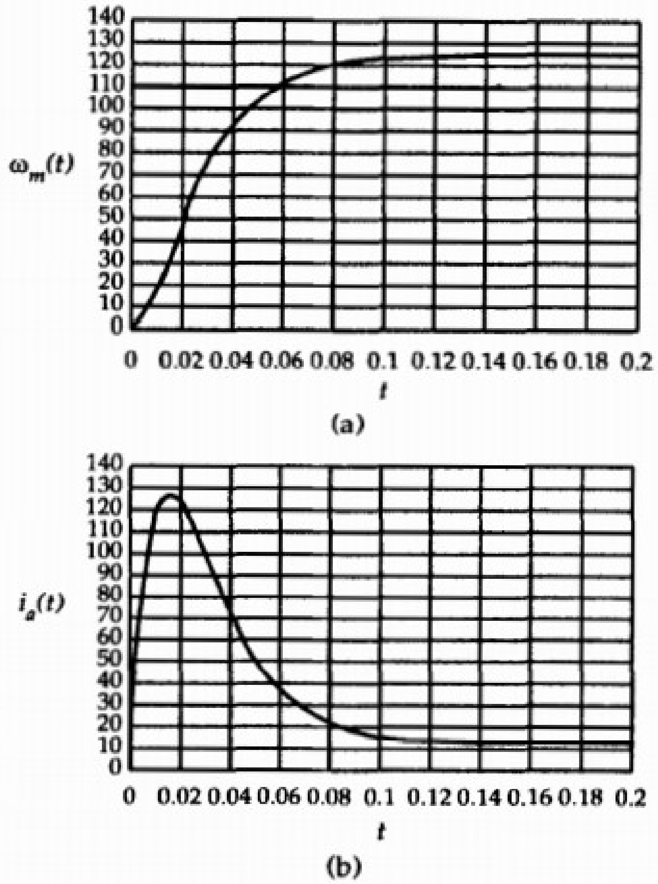


Figure 11.4 The variations in (**a**) the motor speed and (**b**) armature current as a function of time.

The inverse Laplace transform of $I_a(s)$ yields

$$i_a(t) = 12.18 - 1027e^{-79.84t} + 1015e^{-58.10t} \qquad \text{for } t > 0$$

The variations in motor speed and armature current as a function of time are given in Figure 11.4.

For all practical purposes, the motor attains its steady-state operation after five time constants. Thus, this motor takes approximately 86.06 ms (based upon the largest time constant) to achieve its steady state.

EXAMPLE 11.3

The motor studied in Example 11.1 is suddenly energized with its rated voltage at $t = 0$ when it was at rest and coupled to a linear load of $T_L = 0.1\ \omega_m$.

(a) Determine the variation of the motor speed as a function of time for $t \geq 0$.

(b) Calculate the time needed to achieve the steady state.

• SOLUTION

(a) Since the motor was at rest before it was energized at $t = 0$, the initial speed and the initial armature current are zero. Thus, from Eq. (11.8) with $T_L(s) = 0.1\,\Omega_m(s)$, we get

$$\Omega_m(s) = \frac{1.8\,(240/s) - 0.1\Omega_m(1.43 + 10.4 \times 10^{-3}s)}{(0.027 + 0.068s)\,(1.43 + 10.4 \times 10^{-3}s) + 1.8^2}$$

and after grouping the terms, we obtain

$$\Omega_m(s) = \frac{610{,}859.729}{s(s^2 + 139.37s + 4838.24)}$$

The roots of the polynomial in the denominator are

$$s_1 = 0, \qquad s_2 = -73.946, \qquad \text{and} \qquad s_3 = -65.392$$

Thus, in terms of partial-fraction expansion, $\Omega_m(s)$ can be written as

$$\Omega_m(s) = \frac{126.256}{s} + \frac{956.776}{s + 73.946} - \frac{1092}{s + 65.392}$$

The inverse Laplace transform yields, for $t > 0$

$$\omega_m(t) = 126.256 + 956.776e^{-73.946t} - 1092e^{-65.392t} \text{ rad/s}$$

The variation of the speed with time is also given in graphic form in Figure 11.5**a**. From Eq. (11.9), we can calculate the Laplace transform of the armature current as

$$I_a(s) = \frac{23{,}076.92s + 43{,}099.55}{s(s^2 + 139.37s + 4838.24)}$$

or in partial-fraction expansion form as

$$I_a(s) = \frac{8.908}{s} - \frac{2615}{s + 73.946} + \frac{2606}{s + 65.392}$$

Finally, we can obtain $i_a(t)$ from the inverse Laplace transform as

$$i_a(t) = 8.908 - 2615e^{-73.946t} + 2606e^{-65.392t} \text{ rad/s} \qquad \text{for } t>0$$

Figure 11.5**b** illustrates the variation of the armature current as a function of time.

(b) The largest time constant of the above exponential terms is

$$\tau = \frac{1}{65.392} = 0.0153 \text{ s}$$

To achieve a steady state, the time taken should, at least, be 5τ. Hence,

$$t = 5 \times 0.0153 = 0.0765 \text{ s} = 76.5 \text{ ms}$$

is the time needed to reach the steady state.

In Examples 11.1, 11.2, and 11.3, we deliberately used the same motor to give you the opportunity to observe the responses of the same motor under different operating conditions.

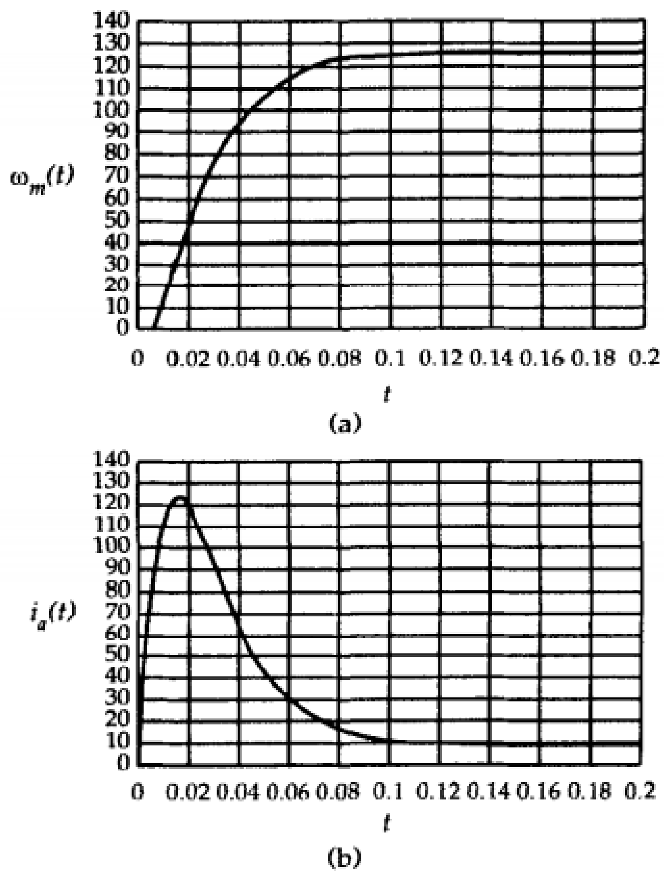


Figure 11.5 The variations in **(a)** the motor speed and **(b)** armature current as a function of time.

EXAMPLE 11.4

A 240-V, 12-hp, separately excited dc motor operating on a load of 15 N·m in the linear region of its magnetization characteristic has the following parameters $R_a = 0.28\ \Omega$, $L_a = 2.81$ mH, $R_f = 320\ \Omega$, $L_f = 2$ H, $J = 0.087$ kg·m^2, $D = 0.02$ N·m·s, and $K_T = 1.03$. Determine the variation of the motor speed, armature current, and field current as a function of time when the field voltage is suddenly reduced from 240 V to 192 V a $t = 0$.

• SOLUTION

Since the motor has been operating at steady state on a load of $T_L = 15$ N·m before the field voltage is suddenly changed, we have to evaluate first the initial conditions on $\omega_m(t)$, $i_a(t)$, and $i_f(t)$ from Eqs. (11.11), (11.12a), and (11.12b) as applied at steady state for $t < 0$,

$$240 = 320 i_f(0)$$

$$1.03 i_f(0)\omega_m(0) = 240 - 0.28 i_a(0)$$

$$1.03 i_f(0) i_a(0) = 15 + 0.02\omega_m(0)$$

Simultaneous solution of the above equations yields

$$\omega_m(0) = 300.79 \text{ rad/s}, \qquad i_a(0) = 27.2 \text{ A}, \qquad \text{and} \qquad i_f(0) = 0.75 \text{ A}$$

When the field voltage is reduced to 192 V suddenly, the field current will drop from 0.75 A to a steady-state value of $I_f = 192/320 = 0.6$ A after a short duration. Using Eqs. (11.14a), (11.14b), and (11.14c), we can determine the Laplace transform of the angular velocity, armature current, and field current as

$$\Omega_m(s) = \frac{300(s^2 + 99.80s + 1960.82)}{s(s^2 + 100.03s + 1581.73)}$$

$$I_a(s) = \frac{27.14(s^2 + 708.58s + 2116.01)}{s(s^2 + 100.03s + 1581.73)}$$

$$I_f(s) = \frac{0.75(s + 128)}{s(s + 160)}$$

or

$$\Omega_m(s) = \frac{371.9}{s} + \frac{24.16}{s + 80.079} - \frac{94.868}{s + 19.816}$$

$$I_a(s) = \frac{36.307}{s} - \frac{271.582}{s + 80.079} + \frac{262.513}{s + 19.816}$$

$$I_f(s) = \frac{0.6}{s} + \frac{0.15}{s + 160}$$

in partial fraction expansion form. Taking the inverse Laplace transform yields

$$\omega_m(t) = 371.9 + 24.16e^{-80.079t} - 94.868e^{-19.816t}$$

$$i_a(t) = 36.307 - 271.582e^{-80.079t} + 262.513e^{-19.816t}$$

$$i_f(t) = 0.6 + 0.15e^{-160t}$$

as the variation of the angular velocity, armature current, and field current, respectively. The waveforms are given in Figures 11.7, 11.8**a**, and 11.8**b**. From Figure 11.7, it is clear that the field current attains its steady state at about 30 ms whereas it takes about 300 ms for the speed and thereby the armature current to do so. This is in accordance with our assumption that the mechanical response is sluggish in comparison with the electrical response that resulted in the change of states. Another important fact that can be observed from this problem is that the armature current increases to a peak, which is well over the motor rating. Again this is mainly due to the mechanical time constant of the motor that does not allow a rapid change in the back emf of the motor. Therefore, it is recommended that the field current be gradually varied so that high currents will not take place in the armature circuit. Lastly, you can notice that the final value of the armature current is higher than what we had before the field current was reduced. The reason is that the mechanical losses increased significantly with increasing motor speed leading to a higher torque demand. Consequently, the increase in developed torque caused an increase in armature current.

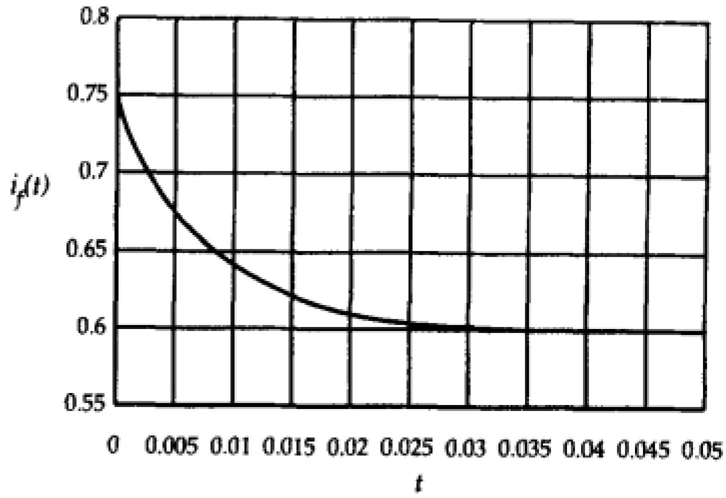


Figure 11.7 The field current as a function of time.

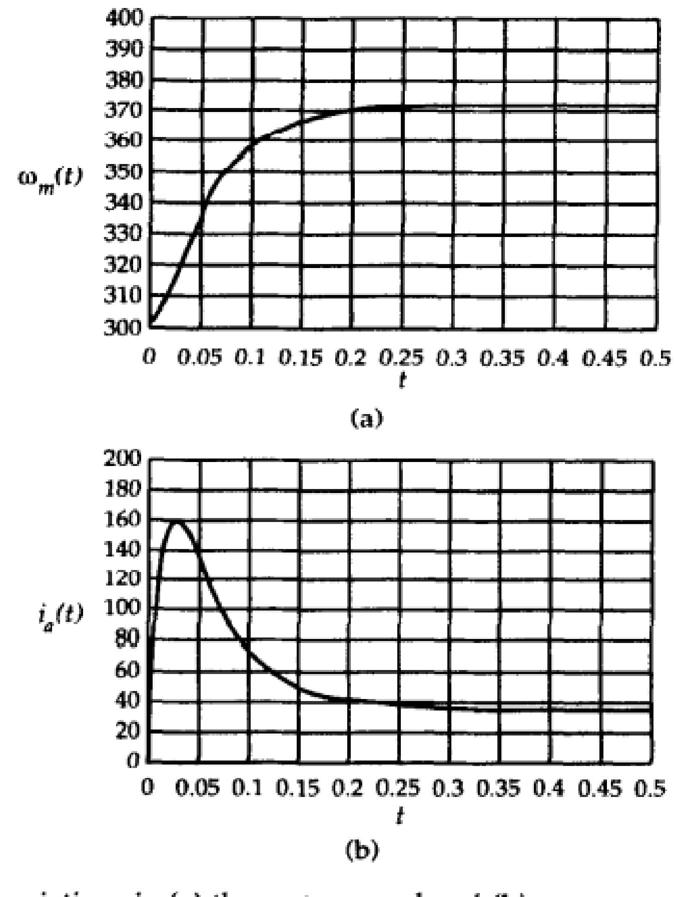


Figure 11.8 The variations in **(a)** the motor speed and **(b)** armature current as a function of time.

EXAMPLE 11.5

A separately excited dc generator operating at 1500 rpm has the following parameters: $R_f = 3\ \Omega$, $L_f = 25$ mH, and $K_e = 30$ V/A. If a dc voltage of 120 V is suddenly applied to the field winding under no load, determine (a) the field current and the generated voltage as a function of time, (b) the approximate time to reach the steady-state condition, and (c) the steady-state values of the field current and induced voltage.

● SOLUTION

(a) From Eq. (11.25)

$$I_f(s) = \frac{\frac{120}{s}}{3 + 0.025s}$$

$$= \frac{40}{s} - \frac{40}{s + 120}$$

Therefore, the field current is

$$i_f(t) = 40(1 - e^{-120t})\ \text{A} \qquad \text{for } t \geq 0$$

and the generated voltage is

$$e_a(t) = K_e i_f(t) = 1200(1 - e^{-120t})\ \text{V} \qquad \text{for } t \geq 0$$

The graphs of i_f and e_a are shown in Figures 11.10 and 11.11, respectively.

(b) For all practical purposes, the field current attains its steady-state value

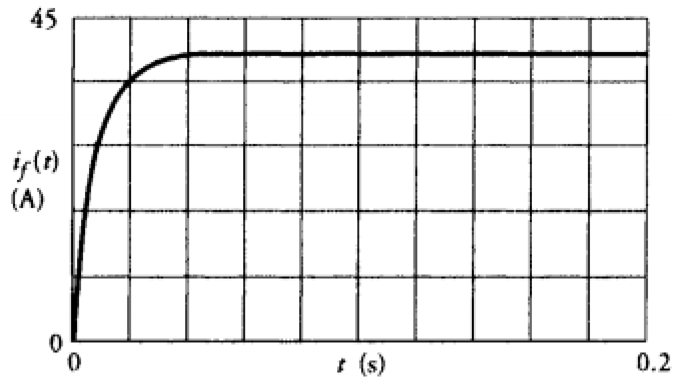


Figure 11.10 The field current as a function of time.

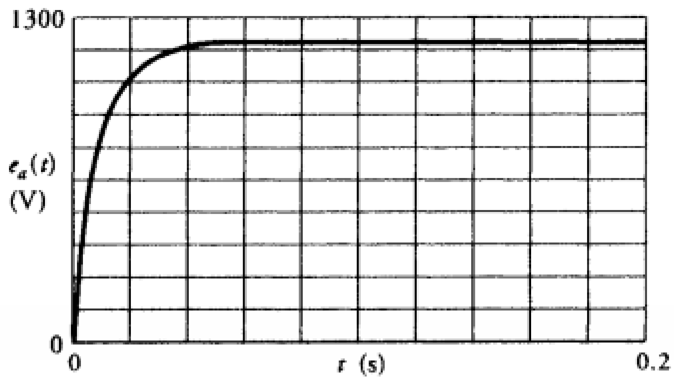


Figure 11.11 The induced voltage as a function of time.

after five time constants. Thus, the time required to reach the steady state is

$$T = \frac{5}{120} = 0.0417\ s \qquad \text{or} \qquad 41.7\ \text{ms}$$

(c) The final values of the field current and the induced (no-load) voltage are $I_f = 40$ A and $E_a = 1200$ V, respectively.

■

EXAMPLE 11.6

The parameters of a 240-V, PM motor are $R_a = 0.3\ \Omega$, $L_a = 2$ mH, $K = 0.8$, $J = 0.0678$ kg-m^2. Determine the motor speed and the armature current as a function of time when the motor is subjected to a torque of 100 N-m after 200 ms of starting at no load. Consider a step length of 0.01 s and observe the response for a period of 0.5 s. Neglect the frictional losses and assume that the motor operates in the linear region.

● **SOLUTION**

From Eq. (11.3), for $t < 200$ ms, we have

$$\underline{x}(t) = \begin{bmatrix} \omega_m(t) \\ i_a(t) \end{bmatrix}, \qquad \underline{u}(t) = \begin{bmatrix} 0 \\ 240 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 11.8 \\ -400 & -150 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -14.75 & 0 \\ 0 & 500 \end{bmatrix}$$

with initial values of $\omega_m(0) = 0$ and $i_a(t) = 0$.
For $t > 200$ ms,

$$\underline{u}(t) = \begin{bmatrix} 100 \\ 240 \end{bmatrix} \quad \text{and} \quad \underline{x}(0.2) = \begin{bmatrix} 299.93 \\ 0.25 \end{bmatrix}$$

The computed speed and armature current waveforms are shown in Figures 11.14 and 11.15, respectively.

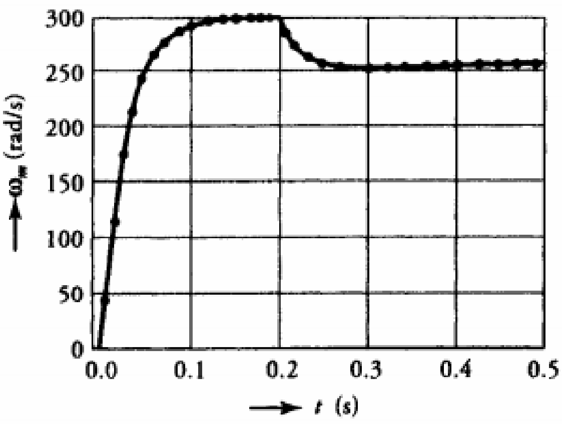


Figure 11.14 The motor speed as a function of time.

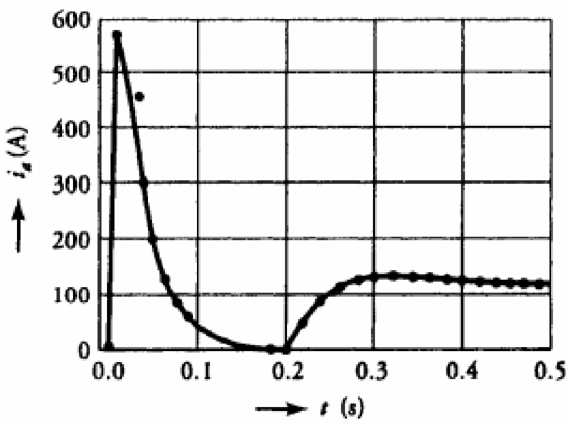


Figure 11.15 The armature current as a function of time.

EXAMPLE 9.1

A separately excited dc generator has the following parameters:

$$R_f = 100\,\Omega, \qquad L_f = 25\text{ H}$$

$$R_a = 0.25\,\Omega, \qquad L_{aq} = 0.02\text{ H}$$

$$K_g = 100\text{ V} \quad \text{per field ampere at rated speed}$$

(a) The generator is driven at rated speed and a field circuit voltage $V_f = 200$ V is suddenly applied to the field winding.

(i) Determine the armature-generated voltage as a function of time.

(ii) Determine the steady-state armature voltage.

(iii) Determine the time required for the armature voltage to rise to 90 percent of its steady-state value.

(b) The generator is driven at rated speed and a load consisting of $R_L = 1\,\Omega$ and $L_L = 0.15$ H in series is connected to the armature terminals. A field circuit voltage $V_f = 200$ V is suddenly applied to the field winding. Determine the armature current as a function of time.

Solution

(a) Field circuit time constant $\tau_f = 25/100 = 0.25$ sec.

(i) From Eq. 9.11,

$$e_a(t) = \frac{100 \times 200}{100}\left(1 - e^{-t/0.25}\right)$$

$$= 200(1 - e^{-4t})$$

(ii)

$$e_a(\infty) = 200\text{ V}$$

(iii)

$$0.9 \times 200 = 200\left(1 - e^{-4t}\right)$$

$$t = 0.575\text{ sec}$$

(b)

$$\tau_f = 0.25 \text{ sec}$$

$$\tau_{at} = \frac{0.15 + 0.02}{1 + 0.25} = 0.136 \text{ sec}$$

From Eq. 9.22,

$$I_a(s) = \frac{100 \times 200}{100 \times 1.25 \times 0.25 \times 0.136 s(s+4)(s+7.35)}$$

$$= \frac{4705.88}{s(s+4)(s+7.35)}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+7.35}$$

where $$A_1 = \left. \frac{4705.88}{(s+4)(s+7.35)} \right|_{s=0} = 160$$

$$A_2 = \left. \frac{4705.88}{s(s+7.35)} \right|_{s=-4} = -351$$

$$A_3 = \left. \frac{4705.88}{s(s+4)} \right|_{s=-7.35} = 191$$

From Eq. 9.25,

$$i_a(t) = 160 - 351e^{-4t} + 191e^{-7.35t}$$

EXAMPLE 9.2

A separately excited dc motor has the following parameters:

$$R_a = 0.5\,\Omega, \qquad L_{aq} \simeq 0, \qquad B \simeq 0$$

The motor generates an open-circuit armature voltage of 220 V at 2000 rpm and with a field current of 1.0 ampere.

The motor drives a constant load torque $T_L = 25\,\text{N} \cdot \text{m}$. The combined inertia of motor and load is $J = 2.5\,\text{kg} \cdot \text{m}^2$. With field current $I_f = 1.0$ A, the armature terminals are connected to a 220 V dc source.

(a) Derive expressions for speed (ω_m) and armature current (i_a) as a function of time.

(b) Determine the steady-state values of the speed and armature current.

Solution

(a)

$$E_a = K_m \omega_m$$

$$K_m = \frac{220}{(2000/60) \times 2\pi} = 1.05\,\text{V/rad/sec}$$

$$V_t = e_a + i_a R_a = K_m \omega_m + i_a R_a$$

$$T = K_m i_a = J \frac{d\omega_m}{dt} + T_L$$

From the last two equations,

$$V_t = K_m \omega_m + R_a \left(\frac{J}{K_m} \frac{d\omega_m}{dt} + \frac{T_L}{K_m} \right)$$

$$= K_m \omega_m + \frac{R_a J}{K_m} \frac{d\omega_m}{dt} + \frac{R_a T_L}{K_m}$$

$$= 1.05 \omega_m + \frac{0.5 \times 2.5}{1.05} \frac{d\omega_m}{dt} + \frac{0.5 \times 25}{1.05}$$

$$= 1.05 \omega_m + 1.19 \frac{d\omega_m}{dt} + 11.9$$

$$V_t(s) = \frac{220}{s} = 1.05 \omega_m(s) + 1.19 s \omega_m(s) + \frac{11.9}{s}$$

$$\omega_m(s) = \frac{220 - 11.9}{s(1.05 + 1.19s)}$$

$$= \frac{174.874}{s(s + 0.8824)}$$

$$= \frac{198.2}{s} - \frac{198.2}{s + 0.8824}$$

$$\omega_m(t) = 198.2(1 - e^{-0.8824t})$$

$$i_a = \frac{V_t - K_m\omega_m}{R_a}$$

$$= \frac{220 - 1.05\omega_m}{0.5}$$

$$= 440 - 2.1 \times 198.2(1 - e^{-0.8824t})$$

$$= 23.8 + 416.2e^{-0.8824t}$$

(b) Steady-state speed is $\omega_m(\infty) = 198.2$ rad/sec.
Steady-state current is $I_a = i_a(\infty) = 23.8$ A. ■

PROBLEMS

9.1 A separately excited dc generator has the following parameters:

$$R_f = 100\ \Omega, \quad L_f = 40\ \text{H}, \quad R_a = 0.2\ \Omega, \quad L_{aq} = 10\ \text{mH}$$

$$K_g = 100\ \text{V/field ampere at 1000 rpm}$$

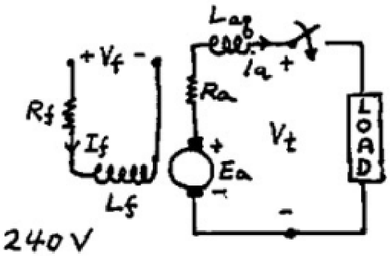
The generator is driven at the rated speed of 1200 rpm, and the field current is adjusted at 2 A. The armature is then suddenly connected to a load consisting of a resistance of 1.8 ohms and an inductance of 10 mH connected in series.

(a) Determine the load terminal voltage as a function of time.

(b) Determine the steady-state value of the load terminal voltage.

(c) Determine the torque as a function of time.

CHAPTER 9



$\boxed{9.1}$ (a) $K_g\Big|_{1200rpm} = 100 \times \frac{1200}{1000} = 120\ V/A$

@ 1200 rpm $I_f = 2A$

$\therefore E_a\Big|_{1200} = K_g\Big|_{1200} \times I_f = 120 \times 2 = 240V$

$$E_a(t) = \underbrace{(R_a + R_L)}_{R_T} i_a(t) + \underbrace{(L_{ag} + L_L)}_{L_T} \frac{d}{dt} i_a(t)$$

Take Laplace Transform

$$I_a(s) = \frac{E_a|_{1200}}{R_T} \times \frac{1}{s(1 + \frac{L_T}{R_T}s)}$$

$$i_a(t) = \mathcal{L}^{-1}\{I_a(s)\} = \frac{240}{2}(1 - e^{-t/\tau_{at}})$$

where $\tau_{at} = L_T/R_T = \frac{(10+10)10^{-3}}{0.2 + 1.8} = 0.01$ sec

$$V_t(t) = R_L i_a + L_L \frac{di_a}{dt}$$

$$= 1.8 \times 120(1 - e^{-100t}) + 10 \times 10^{-3}(120 \times 100 e^{-100t})$$

$$= 216 - 96 e^{-100t}\ V$$

(b) $V_t(\infty) = 216\ V$

(c) $T = K_f I_f i_a$

$E_a = K_f I_f \omega_m = K_g I_f$

$K_f \omega_m = K_g$

$\therefore K_f = \frac{K_g}{\omega_m} = \frac{100}{1000 \times 2\pi/60} = \frac{3}{\pi}$

$\therefore T = \frac{3}{\pi}(2)\, i_a$

$= 229.2(1 - e^{-100t})\ N \cdot m$

or $T = \frac{E_a i_a(t)}{\omega_m} = \frac{240}{1200 \times 2\pi/60} \times 120(1 - e^{-100t})$

$= 229.2(1 - e^{-100t})\ N \cdot m$

9.2 A separately excited dc motor has the following parameters:

$$R_a = 0.4\,\Omega, \qquad L_{aq} \simeq 0, \qquad K_m = 2\ \text{V/rad/sec}$$

The motor is connected to a load whose torque is proportional to the speed.

$$J = J_{motor} + J_{load} = 2.5\ \text{kg}\cdot\text{m}^2$$
$$B = B_{motor} + B_{load} = 0.25\ \text{kg}\cdot\text{m}^2/\text{sec}$$

The field current is maintained constant at its rated value. A voltage $V_t = 200$ V is suddenly applied across the motor armature terminals.

(a) Obtain an expression for the motor speed as a function of time.

(b) Determine the steady-state speed.

(c) Determine the time required for the motor to reach 95 percent of the steady-state speed.

$\boxed{9.2}$ (a) $V_t = Ri + K_m\omega_m$

$$V_t(s) = RI(s) + K_m\omega_m(s) \quad \cdots\cdots\cdots \quad (1)$$

$$T = K_m i = J\frac{d\omega_m}{dt} + B_m\omega_m + B_L\omega_m \ (= T_L)$$

$$K_m i = J\frac{d\omega_m}{dt} + B\omega_m$$

$$K_m I(s) = JS\omega_m(s) + B\omega_m(s) \quad \cdots\cdots\cdots \quad (2)$$

From ① & ②

$$V_t(s) = K_m\omega_m(s) + \frac{R(JS+B)}{K_m}\omega_m(s)$$

$$\frac{\omega_m(s)}{V_t(s)} = \frac{K_m}{K_m^2 + RB + RBS\tau_m} \qquad \left(\tau_m = \frac{J}{B} = \frac{2.5}{0.25} = 10\,\text{sec}\right)$$

$$= \frac{K_m}{(K_m^2+RB)(1+\frac{RB}{K_m^2+RB}S\tau_m)} = \frac{2}{2^2+0.4\times0.25}\times\frac{1}{(1+\frac{0.4\times0.25S}{2^2+0.4\times0.25}\times 10)}$$

$$= 0.488 \times \frac{1}{1+0.244S}$$

$$\omega_m(s) = \frac{200}{s}\times 0.488 \times \frac{1}{1+0.244S} = \frac{97.6}{S}\times\frac{1}{1+0.244S}$$

$$\omega_m(t) = 97.6\,(1 - e^{-t/0.244})$$

(b) $\omega_m(\infty) = 97.6$ rad/sec

(c) $0.95 \times 97.6 = 97.6(1 - e^{-t/0.244}) \rightarrow e^{-t/0.244} = 0.05 \rightarrow t = 0.732$ sec.

(d) $i_a(t) = \dfrac{V_t - K_m\omega_m}{R_a}$

$$= \frac{200 - 2(97.6 - 97.6e^{-t/0.244})}{0.4} = 500 - 488 + 488e^{-t/0.244}$$

$$= 12 + 488e^{-t/0.244}$$

$$i_a|_{ss} = 12\text{A}$$

9.3 A separately excited dc motor has the following parameters:

$$R_a = 0.5\,\Omega, \qquad L_{aq} = 0, \qquad B = 0, \qquad J = 0.1\text{ kg}\cdot\text{m}^2$$

The rotational loss is negligible.

The motor is used to drive an inertia load of 1.0 $\text{kg}\cdot\text{m}^2$. With the rated field current and an armature terminal voltage of 100 V, the motor and the load have a steady-state speed of 1500 rpm. At a certain time the armature terminal voltage is suddenly increased to 120 V.

(a) Obtain an expression for the speed of the motor–load system as a function of time.

(b) Determine the speed 1 second after the step increase in the terminal voltage.

(c) Determine the final steady-state speed of the motor.

$\boxed{9.3}$ a) Because rotational losses are neglected, in steady state motor does not produce any torque. Therefore, before the voltage was changed, $I_a = 0$

$E_a = V_t$

$K_m \omega_m = 100\text{ V}$

$$K_m = \frac{100}{\frac{1500}{60} \times 2\pi} \doteq 0.637 \text{ V/rad/sec.}$$

After the voltage was changed,

$$V_t = e_a + R_a i_a = K_m \omega_m + R_a i_a$$

Also $T = K_m i_a = J \frac{d\omega_m}{dt}$

$$\text{or } V_t = K_m \omega_m + R_a \frac{J}{K_m} \frac{d\omega_m}{dt}$$
$$= 0.637\,\omega_m + 0.5 \times 1.1/0.637 \frac{d\omega_m}{dt}$$

$$V_t = 0.64\,\omega_m + 0.86 \frac{d\omega_m}{dt}$$

$$V_t(s) = 0.64\,\omega_m(s) + 0.86\,(s\,\omega_m(s) - \omega_{m0})$$

where $\omega_{mo} = \frac{1500}{60} \times 2\pi = 157.1$ rad./sec.

$$\frac{120}{s} = 0.64\,\omega_m(s) + 0.86\,s\,\omega_m(s) - 0.86 \times 157.1$$

$$\Rightarrow \omega_m(s) = \frac{120 + 135.11s}{s(0.64 + 0.86s)}$$

$$= \frac{120 + 135.11s}{s\,0.64\left(1 + \frac{0.86}{0.64}s\right)} = \frac{139.1 + 157.1s}{s(s + 0.744)}$$

$$= \frac{A}{s} + \frac{B}{s + 0.744}$$

$$A = 187\,, \quad B = -29.9$$

$$\therefore \omega_m(t) = \mathcal{L}^{-1}\{\omega_m(s)\} = 187 - 29.9\,e^{-0.744t} \text{ rad./sec.}$$

(b) $$\omega_m\big|_{t=1} = 187 - 29.9\,e^{-0.744}$$
$$= 172.95 \text{ rad./sec.}$$

(c) $$\omega_m(\infty) = 187 \text{ rad./sec.}$$

9.4 A separately excited dc motor and its load have the following parameters:

$$R_a = 0.5\ \Omega, \quad L_{aq} \approx 0, \quad K_m = 2\ \text{V/rad/sec at rated field current}$$
$$J = 2.0\ \text{kg}\cdot\text{m}^2$$
$$B = 0.2\ \text{kg}\cdot\text{m}^2/\text{sec}$$

For rated field current and a terminal voltage $V_t = 200$ V:

(a) Determine the steady-state speed in radians per second (ω_m) and rpm (n). (You do not need to derive an expression for the motor speed as a function of time for this part.)

(b) The motor is running at the steady-state speed obtained in part (a). Now, suddenly, the terminal voltage is decreased to 100 V.

(i) Obtain an expression for the speed of the motor–load system as a function of time.

(ii) Determine the final steady-state speed of the motor in rpm.

$\boxed{9.4}$ (a) steady state

$T = B\omega_m = K_m I_a$

$I_a = \frac{B\omega_m}{K_m}$

$V_t - I_a R_a = E_a = K_m \omega_m$

$200 = I_a R_a + K_m\omega_m = \frac{B\omega_m}{K_m} R_a + K_m\omega_m = \omega_m\left(K_m + \frac{BR_a}{K_m}\right)$

$= \omega_m\left(2 + \frac{0.2 \times 0.5}{2}\right) = \omega_m(2 + 0.05)$

$\omega_m = \frac{200}{2.05} = 97.56$ rad/sec. $\quad n = \frac{97.56}{2\pi} \times 60 = 931.64$ rpm

(b)(i) After the voltage changes

$V_t = e_a + i_a R_a = K_m\omega_m + i_a R_a$

Also, $T = K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m$

$V_t = K_m\omega_m + \frac{R_a}{K_m}\left(J\frac{d\omega_m}{dt} + B\omega_m\right) = \omega_m\left(K_m + \frac{R_a B}{K_m}\right) + \frac{R_a J}{K_m}\frac{d\omega_m}{dt}$

$= \omega_m\left(2 + \frac{0.5 \times 0.2}{2}\right) + \frac{0.5 \times 2}{2}\frac{d\omega_m}{dt} = \omega_m \times 2.05 + 0.5\frac{d\omega_m}{dt}$

$V_t(s) = 2.05\,\omega_m(s) + 0.5\left[s\omega_m(s) - \omega_{m0}\right]$

$\frac{100}{s} = 2.05\,\omega_m(s) + 0.5\left[s\omega_m(s) - 97.56\right]$

$\omega_m(s) = \frac{\frac{100}{s} + 48.78}{2.05 + 0.5s} = \frac{100 + 48.78s}{s(2.05 + 0.5s)} = \frac{100 + 48.78s}{0.5s\left(s + \frac{2.05}{0.5}\right)} = \frac{200 + 0.9756s}{s(s + 4.1)}$

$= \frac{A}{s} + \frac{B}{s + 4.1}$

$A = \left.\frac{200 + 97.56s}{s + 4.1}\right|_{s=0} = \frac{200}{4.1} = 48.78$

$B = \left.\frac{200 + 97.56s}{s}\right|_{s=-4.1} = \frac{200 - 97.56 \times 4.1}{-4.1} = 48.78$

$\omega_m = 48.78 + 48.78\,e^{-4.1t}$

(ii) $\omega_m(\infty) = 48.78$ rad/sec $\rightarrow \frac{48.78}{2\pi} \times 60 = 465.82$ rpm

9.5 A separately excited dc motor has the following parameters:

$$R_a = 0.4\,\Omega, \qquad L_{aq} = 0, \qquad K_f = 1$$
$$B = 0, \qquad J = 4.5\ \mathrm{kg \cdot m^2}$$

The motor operates at no load with $V_t = 220$ V and $I_f = 2$ A. Rotational losses are negligible.

The motor is intended to be stopped by plugging, that is, by reversal of its armature terminal voltage w ($V_t = -220\ V$).

(a) Determine the no-load speed of the motor.

(b) Obtain an expression for the motor speed after plugging.

(c) Determine the time taken for the motor to reach zero speed.

9.5 (a) $K_m = K_f i_f = 1 \times 2 = 2$

No-load $\rightarrow$ $V_t = E_a = 220\ V = K_m \omega_{mo}$

$$\omega_{mo} = \frac{220}{2} = 110 \text{ rad./sec.}$$

(b) After voltage reversal $\rightarrow$ $V_t = -220\ V$

$$V_t = e_a + R_a i_a = K_m \omega_m + R_a i_a$$

$$T = K_m i_a = J \frac{d\omega_m}{dt}$$

$$V_t = K_m \omega_m + R_a \frac{J}{K_m} \frac{d\omega_m}{dt}$$

$$= 2\omega_m + \frac{0.4 \times 4.5}{2} \frac{d\omega_m}{dt}$$

$$= 2\omega_m + 0.9 \frac{d\omega_m}{dt}$$

$$V_t(s) = 2\,\omega_m(s) + 0.9\,(s\,\omega_m(s) - \omega_{mo})$$

$$-\frac{220}{s} = 2\,\omega_m(s) + 0.9\,s\,\omega_m(s) - 0.9 \times 110$$

$$\omega_m(s) = \frac{-244.44 + 110s}{s(s + 2.222)}$$

$$= \frac{A}{s} + \frac{B}{s + 2.222}$$

where $A = -110$ $\quad B = 220$

$$\omega_m(t) = \mathcal{L}^{-1}\{\omega_m(s)\} = -110 + 220e^{-2.222t} \text{ rad./sec.}$$

(c) $0 = -110 + 220e^{-2.222t}$

$t = 0.315$ sec.

9.6 A separately excited dc motor has the following parameters:

$$R_a = 0.4\,\Omega, \qquad K_f = 1$$
$$B = 0.1\ \text{kg}\cdot\text{m}^2/\text{sec}, \qquad J = 2.0\ \text{kg}\cdot\text{m}^2$$

The motor drives a constant load torque. With field current $I_f = 2$ A and armature terminals connected to a 100 V dc source, the motor rotates at 450 rpm.

(a) Determine the motor current I_a.

(b) Determine the friction torque ($B\omega_m$) and the load torque (T_L).

(c) The motor is now disconnected from the dc supply. Obtain an expression for speed as a function of time. The load torque remains on the motor shaft after the motor is disconnected from the supply. What is the new steady-state speed?

$\boxed{9.6}$ (a) $K_m = K_f i_f = 1 \times 2 = 2\,V/rad/sec$

$$\omega_{mo} = \frac{450}{60} \times 2\pi = 47.124\ rad/sec.$$

$$E_a = K_m\omega_m = 2 \times 47.124 = 94.248\,V \qquad I_a = \frac{100-94.248}{0.5} = 11.504A$$

(b) $T = K_m I_a = 2 \times 11.504 = 23\ N.m \qquad T_B = 0.1 \times 47.124 = 4.7124\ N.m.$

$$T_L = 23 - 4.7124 = 18.2876\ N.m$$

(c) $T = K_m i_a = 0 = J\frac{d\omega_m}{dt} + B_m\omega_m + T_L$

$$J(s\omega_m(s) - \omega_{mo}) + B\omega_m(s) + \frac{T_L}{s} = 0$$

$$JS\omega_m(s) - J\omega_{mo} + B\omega_m(s) + \frac{18.2876}{s} = 0$$

$$\omega_m(s) = \frac{J\omega_{mo} - \frac{18.2876}{s}}{B+JS} = \frac{94.248s - 18.2876}{2(s+0.05)s}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+0.05}$$

$$A_1 = s\omega_m(s)\Big|_{s=0} = \frac{94.248s - 18.2876}{2(s+0.05)}\Big|_{s=0} = -182.876$$

$$A_2 = (s+0.5)\omega_m(s)\Big|_{s=-0.05} = \frac{94.248s - 18.2876}{2s}\Big|_{s=-0.05} = 230$$

$$\omega_m(s) = -\frac{182.876}{s} + \frac{230}{s+0.05}$$

$$\omega_m(t) = -182.876 + 230e^{-0.05t}$$

$$\omega_m(\infty) = -182.876\ rad/sec$$

CHECK: $\omega_m = -182.876 + 230 = 47.124 = \omega_{mo}$

$$T_B = 0.1 \times 182.876 = 18.2876 = T_L$$

9.7 The motor in Problem 9.6 runs at 450 rpm, with $V_t = 100$ V dc, and the load torque as obtained in part (b). If the load is removed, find expressions for $\omega_m(t)$ and $i_a(t)$. What are the steady-state values for motor speed and motor current with load removed?

$\boxed{9.7}$ T_L removed

$$T = K_m i_a = J\frac{d\omega_m}{dt} + B_m\omega_m$$

$$V_t = R_a i_a + K_m\omega_m = K_m\omega_m + R_a\left(\frac{J}{K_m}\frac{d\omega_m}{dt} + \frac{B_m}{K_m}\omega_m\right)$$

$$= 2\omega_m + 0.5 \times \left(\frac{2}{2}\frac{d\omega_m}{dt} + \frac{0.1}{2}\omega_m\right) = 2\omega_m + 0.5\frac{d\omega_m}{dt} + 0.025\omega_m$$

$$100 = 2.025\omega_m + 0.5\frac{d\omega_m}{dt}$$

$$\frac{100}{s} = 2.025\omega_m(s) + 0.5\,[s\omega_m(s) - 47.124]$$

$$= 2.025\omega_m(s) + 0.5s\,\omega_m(s) - 23.562$$

$$\omega_m(s) = \frac{\frac{100}{s} + 23.562}{2.025 + 0.5s}$$

$$\omega_m(s) = \frac{100 + 23.562s}{0.5s(s+4.05)} = \frac{200 + 47.124s}{s(s+4.05)} = \frac{A_1}{s} + \frac{A_2}{s+4.05}$$

$$A_1 = s\omega_m(s)\big|_{s=0} = \frac{200}{4.05} = 49.38$$

$$A_2 = (s+1.0125)\omega_m(s)\big|_{s=-4.05} = \frac{200 - 47.124 \times 4.05}{-4.05} = \frac{9.15}{-4.05} = -2.26$$

$$\omega_m(s) = \frac{49.38}{s} - \frac{2.26}{s+4.05}$$

$$\omega_m(t) = 49.38 - 2.26\,e^{-4.05t}$$

$$\omega_m(\infty) = 49.38$$

$$\omega_{mo} = 49.38 - 2.26 = 47.12$$

$$i(t) = \frac{V_t - K_m\omega_m}{R_a} = \frac{100 - 2\omega_m}{0.5} = 200 - 4\omega_m$$

$$= 200 - 4(49.38 - 2.26e^{-4.05t}) = 2.48 + 9.04e^{-4.05t}$$

$i|_0 = 2.48 + 9.04 = 11.52\text{ A}$ $\quad$ $i|_\infty = 2.48\text{ A} \rightarrow$ No load.

9.8 A separately excited dc motor has the following parameters:

$$R_a = 0.5\ \Omega, \qquad K_f = 1$$
$$B = 0.1\ \text{kg}\cdot\text{m/sec}, \qquad J = 2.0\ \text{kg}\cdot\text{m}^2$$

With field current $I_f = 2$ A and the motor terminals connected to a 100 V dc supply, the motor rotates (with no load) and draws an armature current $I_a = 2.469$ A.

(a) Determine the motor speed and the developed torque.

(b) A load of constant torque $T_L = 10\ \text{N}\cdot\text{m}$ is now applied. Obtain an expression for speed as a function of time. What are the new steady-state speed, the motor current, and the developed torque?

$\boxed{9.8}$ (a) $K_f I_f = 1 \times 2 = 2$

$100 = 2\omega_o + 0.5 \times 2.469 = E_a + I_a R_a$

$\omega_o = \frac{100 - 1.24}{2} = 49.38 \text{ radian/sec} = \frac{49.38}{2\pi} \times 60 = 471.54 \text{ rpm}$

$T = K_f I_f I_a = 2 \times 2.469 = 4.938 \text{ N.m}$

(b) $100 = K_m \omega_m + R_a i_a$

$K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m + T_L$

$100 = K_m \omega_m + R_a \left[\frac{J}{K_m}\frac{d\omega_m}{dt} + B\frac{\omega_m}{K_m} + \frac{T_L}{K_m}\right]$

$= 2\omega_m + 0.5\left[\frac{2}{2}\frac{d\omega_m}{dt} + \frac{0.1}{2}\omega_m + \frac{10}{2}\right] = 2\omega_m + 0.5\frac{d\omega_m}{dt} + 0.025\omega_m + 2.5$

$97.5 = 0.5\frac{d\omega_m}{dt} + 2.025\omega_m$

In S domain

$\frac{97.5}{s} = 0.5[s\omega_m(s) - \omega_o] + 2.025\omega_m(s) = 0.5[s\omega_m(s) - 49.38] + 2.025\omega_m(s)$

$\omega_m(s) = \frac{\frac{97.5}{s} + 24.69}{0.5s + 2.025} = \frac{195 + 49.38s}{s(s + 4.05)} = \frac{A_1}{s} + \frac{A_2}{s + 4.05} = \frac{48.15}{s} + \frac{1.2319}{s + 4.05}$

$\omega_m(t) = 48.15 + 1.2319 e^{-4.05t}$

$\omega_m|_{ss} = \omega_m(\infty) = 48.15 \text{ rad/sec} \rightarrow 459.8 \text{ rpm}$

$I_a|_{ss} = \frac{V_t - K_m\omega_m}{R_a} = \frac{100 - 2 \times 48.15}{0.5} = 7.4A$

$T|_{ss} = K_m I_a = 2 \times 7.4 = 14.8 \text{ N.m}$

9.9 A separately excited dc motor has the following parameters:

$$R_a = 0.5\,\Omega, \qquad K_f = 1$$
$$B = 0.1\,\text{kg}\cdot\text{m/sec}, \qquad J = 2.0\,\text{kg}\cdot\text{m}^2$$

With field current $I_f = 2$ A and the motor terminals connected to a 100 V dc supply, the motor rotates (with no load) at a speed of 471.569 rpm.

(a) Determine the motor current and the developed torque.

(b) The field current is now reduced to 1.0 A. Obtain an expression for speed as a function of time. What are the new steady-state speed, the motor current, and the developed torque?

$\boxed{9.9}$ (a) $\omega_{mo} = \frac{471.569}{60} \times 2\pi = 49.3827 \text{ rad/sec}$

$I_a = \frac{100 - 2 \times 49.3827}{0.5} = 2.48A$

$K_m = K_f I_f = 2$

$T = K_m I_a = 2 \times 2.48 = 4.96 \text{ N.m}$

(b) If reduced to 1A Find $\omega_m(t)$

$K_m = K_f i_f = 1 \times 1 = 1$

$100 = K_m \omega_m + R_a i_a$

$K_m i_a = J \frac{d\omega_m}{dt} + B\omega_m$

$100 = K_m \omega_m + R_a \left(\frac{J}{K_m}\frac{d\omega_m}{dt} + \frac{B}{K_m}\omega_m\right) = K_m\omega_m + \frac{R_a J}{K_m}\frac{d\omega_m}{dt} + \frac{R_a B}{K_m}\omega_m$

$= 1\times\omega_m + \frac{0.5\times 2}{1}\frac{d\omega_m}{dt} + \frac{0.5\times 0.1}{1}\omega_m = \omega_m + \frac{d\omega_m}{dt} + 0.05\omega_m$

$\frac{100}{s} = \omega_m(s) + s\,\omega_m(s) - \omega_{m0} + 0.05\omega_m(s)$

$\frac{100}{s} + 49.38 = \omega_m(s)(1.05 + s)$

$\omega_m(s) = \frac{100 + 49.38s}{s(s+1.05)} = \frac{A_1}{s} + \frac{A_2}{s+1.05}$

$A_1 = \frac{100}{1.05} = 95.24$

$A_2 = \frac{100 + 49.38(-1.05)}{-(1.05)} = \frac{100 - 51.85}{-1.05} = -45.86$

$\omega_m(t) = 95.24 - 45.86\, e^{-1.05t}$

$\omega_m\big|_0 = 95.24 - 45.86 = 49.38$ rad/sec

CHECK : $\omega_m\big|_\infty = 95.24$ rad/sec $= 909.4729$ rpm

$I_a\big|_\infty = \frac{100 - 1\times 95.24}{0.5} = 9.52A$

$T = K_m I_a = 1\times 9.52$ N.m

9.10 A motor–generator set consists of a dc generator and a dc motor whose armatures are connected in series. The generator is driven at the rated speed and the motor field current is kept constant at its rated value. The machines have the following parameters:

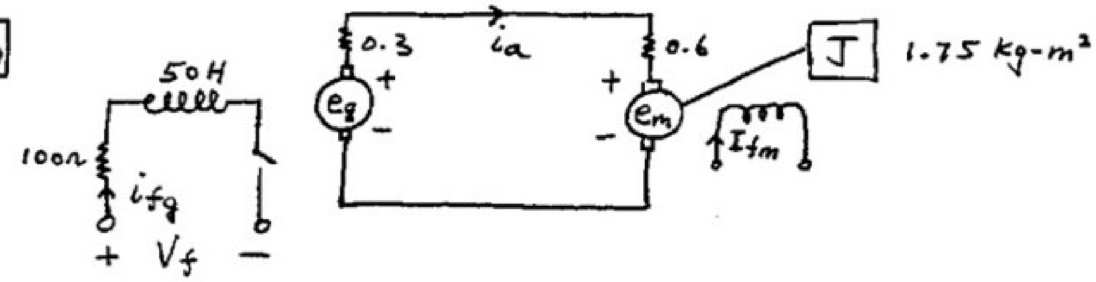
Generator	Motor
$R_a = 0.3\ \Omega$	$R_a = 0.6\ \Omega$
$K_g = K_f\omega_m = 100$ V/A	$K_m = K_f I_f = 1.1$ N · m/A

$$R_f = 100\ \Omega$$
$$L_f = 50\ \text{H}$$

The rotational losses and the armature inductances are negligible. The motor is coupled to an inertia load and the combined inertia of the motor and load is $J = 1.75$ kg · m^2.

Derive an expression for the motor speed subsequent to the application of a step voltage of 50 V to the generator field circuit.

$\boxed{9\text{-}10}$



$$V_f = R_f\, i_{fg} + L_f \frac{di_{fg}}{dt}$$

$$V_f(s) = R_f\, I_{fg}(s) + L_f\, s\, I_{fg}(s)$$

$$\frac{50}{s} = 100\, I_{fg}(s) + 50 s\, I_{fg}(s)$$

$$I_{fg}(s) = \frac{50}{s(100 + 50s)}$$

$$e_g = k_g\, i_{fg}$$

$$E_g(s) = k_g\, I_{fg}(s) = \frac{5000}{s(100 + 50s)}$$

$$i_a = \frac{e_g - e_m}{0.3 + 0.6} = \frac{e_g - k_m\omega_m}{0.9}$$

$$K_m i_a = J \frac{d\omega_m}{dt}$$

$$K_m \frac{e_g - K_m\omega_m}{0.9} = J \frac{d\omega_m}{dt}$$

$$\frac{1.1 e_g}{0.9} - \frac{(1.1)^2}{0.9}\omega_m = 1.75 \frac{d\omega_m}{dt}$$

$$1.2222 e_g - 1.3444 \omega_m = 1.75 \frac{d\omega}{dt}$$

$$1.2222 E_g(s) - 1.3444 \omega_m(s) = 1.75 s \omega_m(s)$$

$$K_m i_a = J \frac{d\omega_m}{dt}$$

$$K_m \frac{e_g - K_m\omega_m}{0.9} = J \frac{d\omega_m}{dt}$$

$$\frac{1.1 e_g}{0.9} - \frac{(1.1)^2}{0.9}\omega_m = 1.75 \frac{d\omega_m}{dt}$$

$$1.2222 e_g - 1.3444 \omega_m = 1.75 \frac{d\omega}{dt}$$

$$1.2222 E_g(s) - 1.3444 \omega_m(s) = 1.75 s \omega_m(s)$$

$$\omega_m(s) = \frac{1.2222\, E_g(s)}{1.3444 + 1.75s}$$

$$= \frac{1.2222 \times 5000}{s(100 + 50s)(1.3444 + 1.75s)}$$

$$= \frac{69.84}{s(2+s)(0.7682+s)}$$

$$= \frac{A}{s} + \frac{B}{2+s} + \frac{C}{0.7682+s}$$

$$= \frac{45.457}{s} + \frac{28.349}{2+s} - \frac{73.8057}{0.7682+s}$$

$$\omega_m(t) = 45.457 + 28.349e^{-2t} - 73.806\, e^{-0.7682t}$$

Example 2A A permanent-magnet dc motor similar to that shown in Fig. 2.2-6 is rated at 6 V with the following parameters: $r_a = 7\,\Omega$, $L_{AA} = 120$ mH, $k_T = 2$ oz · in./A, $J = 150\ \mu$oz · in. · s^2. According to the motor information sheet, the no-load speed is approximately 3350 r/min and the no-load armature current is approximately 0.15 A. Let us attempt to interpret this information.

First, let us convert k_T and J to units that we have been using in this text. In this regard, we will convert the inertia to kg · m^2, which is the same as N · m · s^2. To do this, we must convert ounces to newtons and inches to meters (Appendix A). Thus,

$$J = \frac{150 \times 10^{-6}}{(3.6)(39.37)} = 1.06 \times 10^{-6}\ \text{kg} \cdot \text{m}^2 \tag{2A-1}$$

We have not seen k_T before. It is the torque constant and, if expressed in the appropriate units, it is numerically equal to k_v. When k_v is used in the expression for $T_e (T_e = k_v i_a)$, it is often referred to as the *torque constant* and denoted k_T. When used in the voltage equation, it is always denoted k_v. Now we must convert oz · in. to N · m, whereupon k_T equals our k_v; hence,

$$k_v - \frac{2}{(16)(0.225)(39.37)} = 1.41 \times 10^{-2}\text{N} \cdot \text{m/A} = 1.41 \times 10^{-2}\text{V} \cdot \text{s/rad} \tag{2A-2}$$

What do we do about the no-load armature current? What does it represent? Well, probably it is a measure of the friction and windage losses. We could neglect it, but we will not. Instead, let us include it as B_m. First, however, we must calculate the no-load speed. We can solve for the no-load rotor speed from the steady-state armature voltage equation for the shunt machine, (2.4-2), with $L_{AF}i_f$ replaced by k_v:

$$\begin{aligned}\omega_r &= \frac{V_a - r_a I_a}{k_v} = \frac{6-(7)(0.15)}{1.41\times 10^{-2}} = 351.1 \text{ rad/s}\\ &= \frac{(351.1)(60)}{2\pi} = 3353 \text{ r/min} \qquad \text{(2A-3)}\end{aligned}$$

Now at this no-load speed,

$$T_e = k_v i_a = (1.41\times 10^{-2})(0.15) = 2.12\times 10^{-3}\text{N}\cdot\text{m} \qquad \text{(2A-4)}$$

Because T_L and $J(d\omega_r/dt)$ are zero for this steady-state no-load condition, (2.3-6) tells us that (2A-4) is equal to $B_m\omega_r$; hence,

$$B_m = \frac{2.12\times 10^{-3}}{\omega_r} = \frac{2.12\times 10^{-3}}{351.1} = 6.04\times 10^{-6}\,\text{N}\cdot\text{m}\cdot\text{s} \qquad \text{(2A-5)}$$

Example 2B The permanent-magnet dc machine described in Example 2A is operating with rated applied armature voltage and load torque T_L of 0.5 oz·in. Our task is to determine the efficiency where percent eff = (power output/power input) 100.

First let us convert oz·in. into N·m:

$$T_L = \frac{0.5}{(16)(0.225)(39.37)} = 3.53\times 10^{-3}\,\text{N}\cdot\text{m} \qquad \text{(2B-1)}$$

In Example 2A we determined k_v to be $1.41\times 10^{-2}\text{V}\cdot\text{s/rad}$ and determined B_m to be $6.04\times 10^{-6}\,\text{N}\cdot\text{m}\cdot\text{s}$.

During steady-state operation, (2.3-6) becomes

$$T_e = B_m\omega_r + T_L \qquad \text{(2B-2)}$$

From (2.3-5), with $L_{AF}i_f$ replaced by k_v, the steady-state electromagnetic torque is

$$T_e = k_v I_a \qquad \text{(2B-3)}$$

Substituting (2B-3) into (2B-2) and solving for ω_r yields

$$\omega_r = \frac{k_v}{B_m}I_a - \frac{1}{B_m}T_L \qquad \text{(2B-4)}$$

From (2.4-2) with $L_{AF}i_f = k_v$, we obtain

$$V_a = r_a I_a + k_v\omega_r \qquad \text{(2B-5)}$$

EXAMPLE 11.1

A 240-V, permanent magnet, dc motor takes 2 A whenever it operates at no load. Its armature winding resistance and inductance are 1.43 Ω and 10.4 mH, respectively. The flux per pole is 5 mWb and the motor constant K_a is 360. The moment of inertia is 0.068 kg·m^2. If the motor is suddenly connected to a 240-V dc source while operating at no load, determine its speed and armature current as a function of time.

• SOLUTION

The no-load data of the motor helps us determine the friction coefficient, D, as outlined below
Since the motor requires 2 A at no load, its no-load speed is

$$\omega_m = \frac{240 - 1.43 \times 2}{360 \times 5 \times 10^{-3}} = 131.74 \text{ rad/s}$$

At no load, the developed torque

$$T_d = 360 \times 5 \times 10^{-3} \times 2 = 3.6 \text{ N·m}$$

is primarily for the rotational losses. The friction coefficient D can then be calculated as

$$D = \frac{3.6}{131.74} = 0.027 \text{ N·m·s}$$

Prior to the application of armature voltage the motor speed and armature current are zero. That is, at $t = 0$, $\omega_m(0) = 0$ and $i_a(0) = 0$. In addition, the load torque is zero because the motor operates at no load.

From Eq. (11.8), we get

$$\Omega_m(s) = \frac{1.8\,\dfrac{240}{s}}{(0.027 + 0.068s)\,(1.43 + 10.4 \times 10^{-3}s) + 1.8^2}$$

or

$$\Omega_m(s) = \frac{610859.72}{s(s^2 + 137.89s + 4636.04)}$$

In order to determine the inverse Laplace transform of $\Omega_m(s)$, we expand $\Omega_m(s)$ into partial fractions as

$$\Omega_m(s) = \frac{A}{s} + \frac{B}{s + 79.84} + \frac{C}{s + 58.10}$$

where A, B, and C can now be determined by the root-substitution method. Thus,

$$A = s \left. \frac{610{,}859.72}{s(s + 79.84)(s + 58.10)} \right|_{s=0} = 131.74$$

$$B = (s + 58.10) \left. \frac{610{,}859.72}{(s + 58.10)s(s + 79.84)} \right|_{s=-58.10} = 351.87$$

$$C = (s + 79.84) \left. \frac{610{,}859.72}{(s + 79.84)s(s + 58.10)} \right|_{s=-79.84} = -483.56$$

Finally, we can take the inverse Laplace transform of

$$\Omega_m(s) = \frac{131.74}{s} + \frac{351.87}{s + 79.84} - \frac{483.56}{s + 58.10}$$

and get the angular velocity in rad/s as

$$\omega_m(t) = 131.74 + 351.87e^{-79.84t} - 483.56e^{-58.10t} \qquad \text{for } t > 0$$

The graph of $\omega_m(t)$ is given in Figure 11.2.
From Eq. (11.9), the Laplace transform of the armature current is

$$I_a(s) = \frac{23{,}076.92s + 9162.9}{s(s^2 + 137.89s + 4636.04)}$$

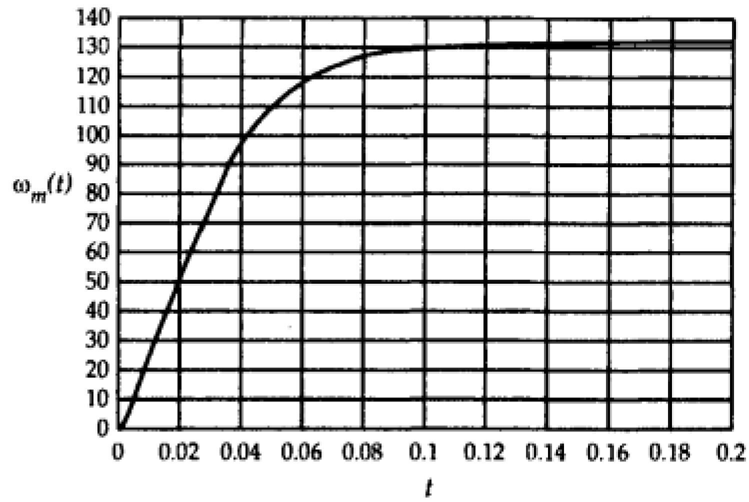


Figure 11.2 The motor speed as a function of time.

In terms of its partial fraction expansion, $I_a(s)$ can be written as

$$I_a(s) = \frac{2}{s} + \frac{1054}{s + 58.10} - \frac{1056}{s + 79.84}$$

Finally, we obtain the armature current as

$$i_a(t) = 2 + 1054e^{-58.10t} - 1056e^{-79.84t} \qquad \text{for } t > 0$$

which is shown graphically in Figure 11.3.

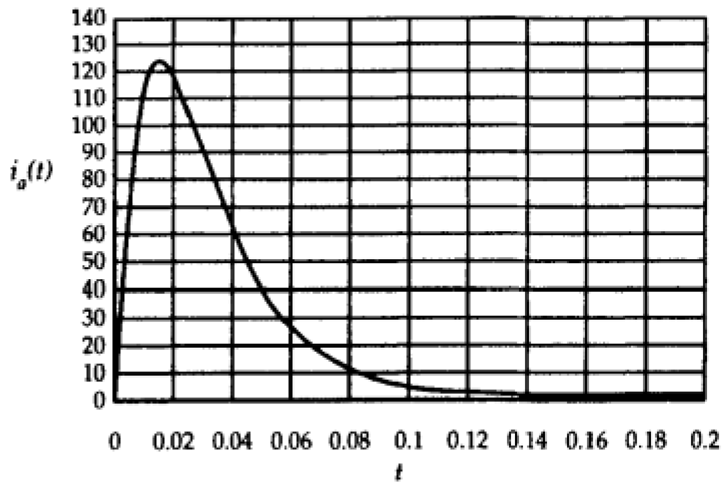


Figure 11.3 The armature current as a function of time.

EXAMPLE 11.2

The motor given in Example 11.1 is coupled to a load having a torque of 18.58 N·m. Determine the variation of the motor speed as a function of time after the motor is suddenly energized at its rated voltage at $t = 0$.

• SOLUTION

Since the voltage is applied to the armature circuit at $t = 0$, the initial speed and the armature current are both zero.
From Eq. (11.8),

$$\Omega_m(s) = \frac{1.8\,\dfrac{240}{s} - \dfrac{18.58}{s}\,(1.43 + 10.4 \times 10^{-3}s)}{(0.027 + 0.068s)\,(1.43 + 10.4 \times 10^{-3}s) + 1.8^2}$$

or

$$\Omega_m(s) = \frac{573{,}289.87 + s274.02}{s(s^2 + 137.89s + 4636.04)}$$

$$= \frac{123.66}{s} + \frac{342.29}{s + 79.84} - \frac{465.90}{s + 58.10}$$

The inverse Laplace transform of $\Omega_m(s)$ yields

$$\omega_m(t) = 123.66 + 342.29\,e^{-79.843t} - 465.90e^{-58.10t} \qquad \text{for } t > 0$$

The Laplace transform of the armature current is calculated from Eq. (11.9) as

$$I_a\,(s) = \frac{56{,}447.963 + s23{,}076.923}{s(s^2 + 137.89s + 4636.04)}$$

or in terms of its partial-fraction expansion as

$$= \frac{12.18}{s} - \frac{1027}{s + 79.84} + \frac{1015}{s + 58.10}$$

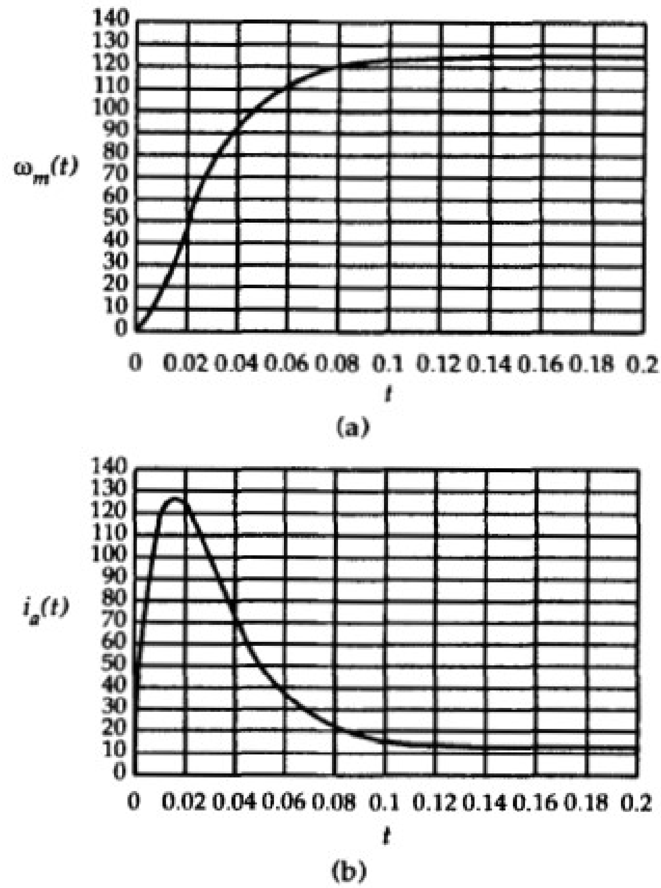


Figure 11.4 The variations in (a) the motor speed and (b) armature current as a function of time.

The inverse Laplace transform of $I_a(s)$ yields

$$i_a(t) = 12.18 - 1027e^{-79.84t} + 1015e^{-58.10t} \qquad \text{for } t > 0$$

The variations in motor speed and armature current as a function of time are given in Figure 11.4.

For all practical purposes, the motor attains its steady-state operation after five time constants. Thus, this motor takes approximately 86.06 ms (based upon the largest time constant) to achieve its steady state.

EXAMPLE 11.3

The motor studied in Example 11.1 is suddenly energized with its rated voltage at $t = 0$ when it was at rest and coupled to a linear load of $T_L = 0.1\ \omega_m$.

(a) Determine the variation of the motor speed as a function of time for $t \geq 0$.

(b) Calculate the time needed to achieve the steady state.

• SOLUTION

(a) Since the motor was at rest before it was energized at $t = 0$, the initial speed and the initial armature current are zero. Thus, from Eq. (11.8) with $T_L(s) = 0.1\ \Omega_m(s)$, we get

$$\Omega_m(s) = \frac{1.8\,(240/s) - 0.1\Omega_m(1.43 + 10.4 \times 10^{-3}s)}{(0.027 + 0.068s)\,(1.43 + 10.4 \times 10^{-3}s) + 1.8^2}$$

and after grouping the terms, we obtain

$$\Omega_m(s) = \frac{610{,}859.729}{s(s^2 + 139.37s + 4838.24)}$$

The roots of the polynomial in the denominator are

$$s_1 = 0, \qquad s_2 = -73.946, \qquad \text{and} \qquad s_3 = -65.392$$

Thus, in terms of partial-fraction expansion, $\Omega_m(s)$ can be written as

$$\Omega_m(s) = \frac{126.256}{s} + \frac{956.776}{s + 73.946} - \frac{1092}{s + 65.392}$$

The inverse Laplace transform yields, for $t > 0$

$$\omega_m(t) = 126.256 + 956.776e^{-73.946t} - 1092e^{-65.392t} \text{ rad/s}$$

The variation of the speed with time is also given in graphic form in Figure 11.5**a**. From Eq. (11.9), we can calculate the Laplace transform of the armature current as

$$I_a(s) = \frac{23{,}076.92s + 43{,}099.55}{s(s^2 + 139.37s + 4838.24)}$$

or in partial-fraction expansion form as

Finally, we can obtain $i_a(t)$ from the inverse Laplace transform as

$$i_a(t) = 8.908 - 2615e^{-73.946t} + 2606e^{-65.392t} \text{ rad/s} \qquad \text{for } t>0$$

Figure 11.5**b** illustrates the variation of the armature current as a function of time.

(b) The largest time constant of the above exponential terms is

$$\tau = \frac{1}{65.392} = 0.0153 \text{ s}$$

To achieve a steady state, the time taken should, at least, be 5τ. Hence,

$$t = 5 \times 0.0153 = 0.0765 \text{ s} = 76.5 \text{ ms}$$

is the time needed to reach the steady state.

In Examples 11.1, 11.2, and 11.3, we deliberately used the same motor to give you the opportunity to observe the responses of the same motor under different operating conditions.

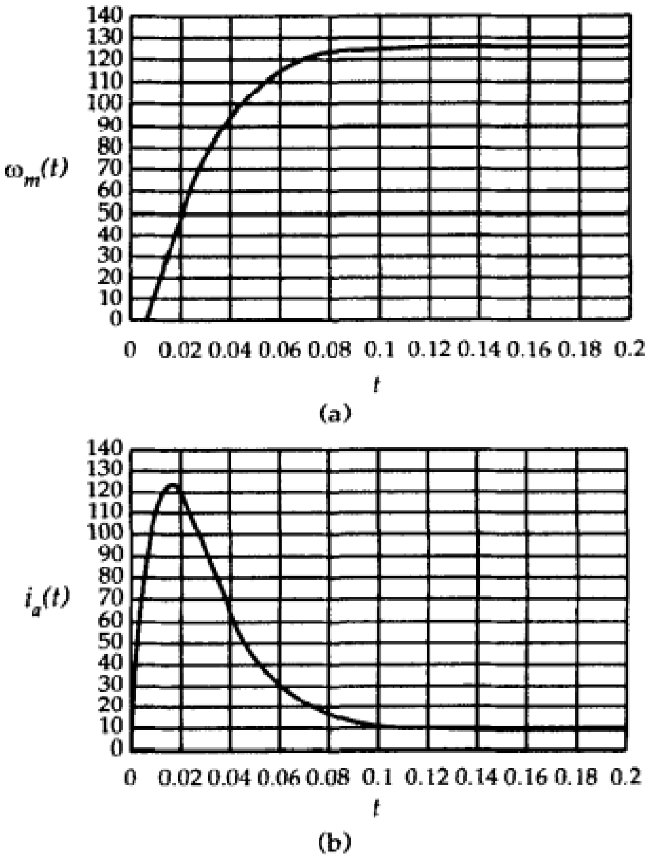


Figure 11.5 The variations in (**a**) the motor speed and (**b**) armature current as a function of time.

EXAMPLE 11.4

A 240-V, 12-hp, separately excited dc motor operating on a load of 15 N·m in the linear region of its magnetization characteristic has the following parameters $R_a = 0.28\ \Omega$, $L_a = 2.81$ mH, $R_f = 320\ \Omega$, $L_f = 2$ H, $J = 0.087$ kg·m², $D = 0.02$ N·m·s, and $K_T = 1.03$. Determine the variation of the motor speed, armature current, and field current as a function of time when the field voltage is suddenly reduced from 240 V to 192 V a $t = 0$.

● SOLUTION

Since the motor has been operating at steady state on a load of $T_L = 15$ N·m before the field voltage is suddenly changed, we have to evaluate first the initial conditions on $\omega_m(t)$, $i_a(t)$, and $i_f(t)$ from Eqs. (11.11), (11.12a), and (11.12b) as applied at steady state for $t < 0$,

$$240 = 320 i_f(0)$$

$$1.03 i_f(0)\omega_m(0) = 240 - 0.28 i_a(0)$$

$$1.03 i_f(0) i_a(0) = 15 + 0.02\omega_m(0)$$

Simultaneous solution of the above equations yields

$$\omega_m(0) = 300.79 \text{ rad/s}, \qquad i_a(0) = 27.2 \text{ A}, \qquad \text{and} \qquad i_f(0) = 0.75 \text{ A}$$

When the field voltage is reduced to 192 V suddenly, the field current will drop from 0.75 A to a steady-state value of $I_f = 192/320 = 0.6$ A after a short duration. Using Eqs. (11.14a), (11.14b), and (11.14c), we can determine the Laplace transform of the angular velocity, armature current, and field current as

$$\Omega_m(s) = \frac{300(s^2 + 99.80s + 1960.82)}{s(s^2 + 100.03s + 1581.73)}$$

$$I_a(s) = \frac{27.14(s^2 + 708.58s + 2116.01)}{s(s^2 + 100.03s + 1581.73)}$$

$$I_f(s) = \frac{0.75(s + 128)}{s(s + 160)}$$

or

$$\Omega_m(s) = \frac{371.9}{s} + \frac{24.16}{s + 80.079} - \frac{94.868}{s + 19.816}$$

$$I_a(s) = \frac{36.307}{s} - \frac{271.582}{s + 80.079} + \frac{262.513}{s + 19.816}$$

$$I_f(s) = \frac{0.6}{s} + \frac{0.15}{s + 160}$$

in partial fraction expansion form. Taking the inverse Laplace transform yields

$$\omega_m(t) = 371.9 + 24.16e^{-80.079t} - 94.868e^{-19.816t}$$
$$i_a(t) = 36.307 - 271.582e^{-80.079t} + 262.513e^{-19.816t}$$
$$i_f(t) = 0.6 + 0.15e^{-160t}$$

as the variation of the angular velocity, armature current, and field current, respectively. The waveforms are given in Figures 11.7, 11.8**a**, and 11.8**b**. From Figure 11.7, it is clear that the field current attains its steady state at about 30 ms whereas it takes about 300 ms for the speed and thereby the armature current to do so. This is in accordance with our assumption that the mechanical response is sluggish in comparison with the electrical response that resulted in the change of states. Another important fact that can be observed from this problem is that the armature current increases to a peak, which is well over the motor rating. Again this is mainly due to the mechanical time constant of the motor that does not allow a rapid change in the back emf of the motor. Therefore, it is recommended that the field current be gradually varied so that high currents will not take place in the armature circuit. Lastly, you can notice that the final value of the armature current is higher than what we had before the field current was reduced. The reason is that the mechanical losses increased significantly with increasing motor speed leading to a higher torque demand. Consequently, the increase in developed torque caused an increase in armature current.

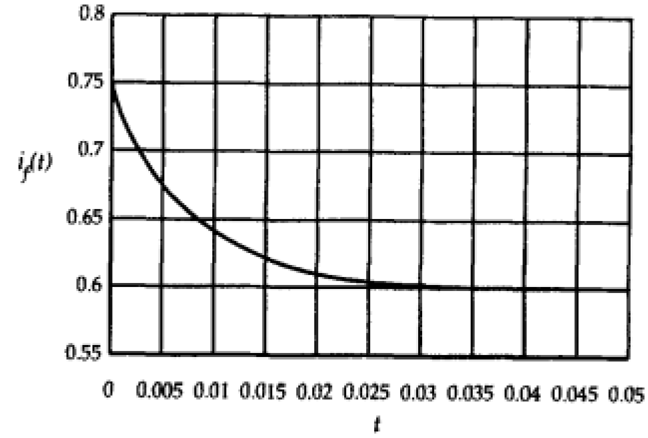


Figure 11.7 The field current as a function of time.

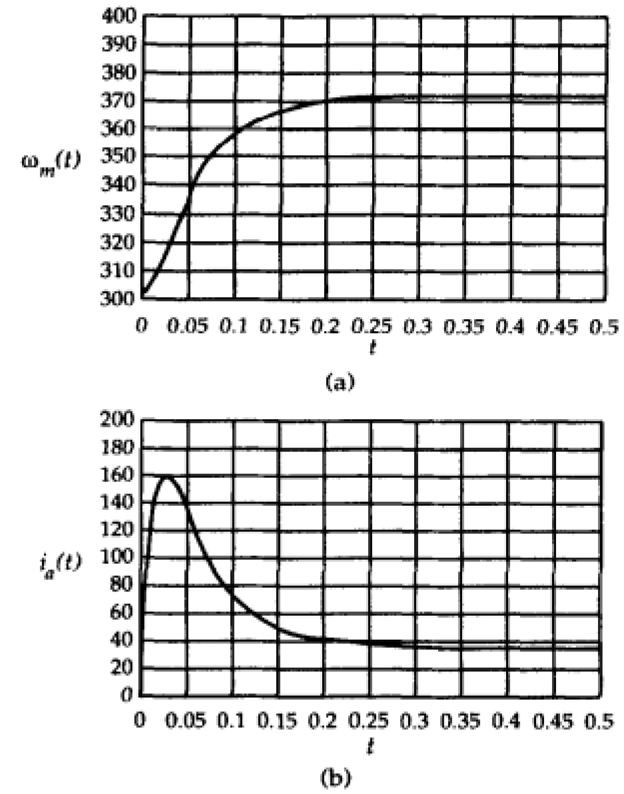


Figure 11.8 The variations in **(a)** the motor speed and **(b)** armature current as a function of time.

EXAMPLE 11.5

A separately excited dc generator operating at 1500 rpm has the following parameters: $R_f = 3\ \Omega$, $L_f = 25$ mH, and $K_e = 30$ V/A. If a dc voltage of 120 V is suddenly applied to the field winding under no load, determine (a) the field current and the generated voltage as a function of time, (b) the approximate time to reach the steady-state condition, and (c) the steady-state values of the field current and induced voltage.

● SOLUTION

(a) From Eq. (11.25)

$$I_f(s) = \frac{\frac{120}{s}}{3 + 0.025s} = \frac{40}{s} - \frac{40}{s + 120}$$

Therefore, the field current is

$$i_f(t) = 40(1 - e^{-120t})\ \text{A} \qquad \text{for } t \geq 0$$

and the generated voltage is

$$e_a(t) = K_e i_f(t) = 1200(1 - e^{-120t})\ \text{V} \qquad \text{for } t \geq 0$$

The graphs of i_f and e_a are shown in Figures 11.10 and 11.11, respectively.

(b) For all practical purposes, the field current attains its steady-state value

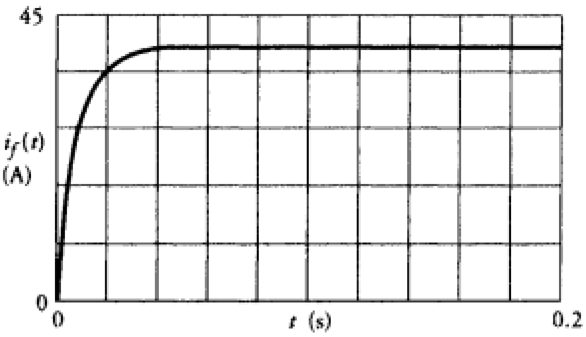


Figure 11.10 The field current as a function of time.

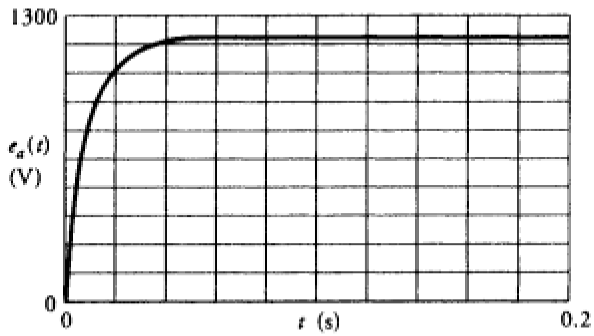


Figure 11.11 The induced voltage as a function of time.

after five time constants. Thus, the time required to reach the steady state is

$$T = \frac{5}{120} = 0.0417\ s \qquad \text{or} \qquad 41.7\ \text{ms}$$

(c) The final values of the field current and the induced (no-load) voltage are $I_f = 40$ A and $E_a = 1200$ V, respectively.

EXAMPLE 11.6

The parameters of a 240-V, PM motor are $R_a = 0.3\ \Omega$, $L_a = 2$ mH, $K = 0.8$, $J = 0.0678$ kg-m^2. Determine the motor speed and the armature current as a function of time when the motor is subjected to a torque of 100 N-m after 200 ms of starting at no load. Consider a step length of 0.01 s and observe the response for a period of 0.5 s. Neglect the frictional losses and assume that the motor operates in the linear region.

● **SOLUTION**

From Eq. (11.3), for $t < 200$ ms, we have

$$\underline{x}(t) = \begin{bmatrix} \omega_m(t) \\ i_a(t) \end{bmatrix}, \qquad \underline{u}(t) = \begin{bmatrix} 0 \\ 240 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 11.8 \\ -400 & -150 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -14.75 & 0 \\ 0 & 500 \end{bmatrix}$$

with initial values of $\omega_m(0) = 0$ and $i_a(t) = 0$.
For $t > 200$ ms,

$$\underline{u}(t) = \begin{bmatrix} 100 \\ 240 \end{bmatrix} \quad \text{and} \quad \underline{x}(0.2) = \begin{bmatrix} 299.93 \\ 0.25 \end{bmatrix}$$

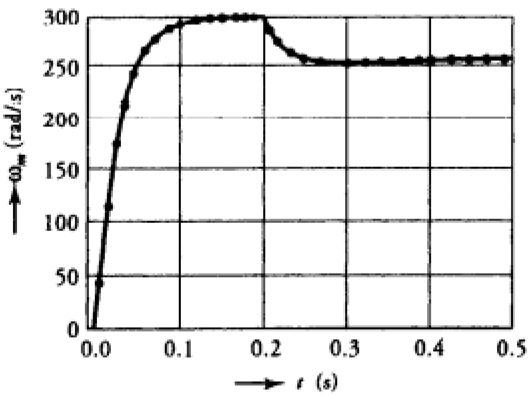


Figure 11.14 The motor speed as a function of time.

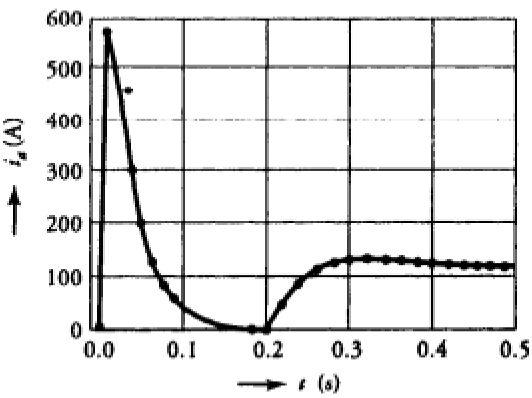


Figure 11.15 The armature current as a function of time.

EXAMPLE 10A A permanent-magnet dc motor is rated at 6 V with the following parameters: $r_a = 7\,\Omega$, $L_{AA} = 120\,\text{mH}$, $k_T = 2\,\text{oz·in/A}$, $J = 150\,\mu\text{oz·in·s}^2$. According to the motor information sheet, the no-load speed is approximately 3350 r/min, and the no-load armature current is approximately 0.15 A. Let us attempt to interpret this information.

First, let us convert k_T and J to units that we have been using in this book. In this regard, we will convert the inertia to kg·m^2, which is the same as N·m·s^2. To do this, we must convert ounces to newtons and inches to meters (Appendix A). Thus,

$$J = \frac{150 \times 10^{-6}}{(3.6)(39.37)} = 1.06 \times 10^{-6}\ \text{kg} \cdot \text{m}^2 \tag{10A-1}$$

We have not seen k_T before. It is the torque constant and, if expressed in the appropriate units, it is numerically equal to k_v. When k_v is used in the expression for T_e ($T_e = k_v i_a$), it is often referred to as the *torque constant* and denoted as k_T. When used in the voltage equation, it is always denoted as k_v. Now we must convert ounce·in into newton·meter, whereupon k_T equals our k_v; hence,

$$k_v = \frac{2}{(16)(0.225)(39.37)} = 1.41 \times 10^{-2}\ \text{N} \cdot \text{m/A} = 1.41 \times 10^{-2}\ \text{V} \cdot \text{s/rad} \tag{10A-2}$$

What do we do about the no-load armature current? What does it represent? Well, probably it is a measure of the friction and windage losses. We could neglect it, but we will not. Instead, let us include it as B_m. First, however, we must calculate the no-load speed. We can solve for the no-load rotor speed from the steady-state armature voltage equation for the shunt machine, (10.4-2), with $L_{AF}i_f$ replaced by k_v:

$$\begin{aligned} \omega_r &= \frac{V_a - r_a I_a}{k_v} = \frac{6 - (7)(0.15)}{1.41 \times 10^{-2}} = 351.1\ \text{rad/s} \\ &= \frac{(351.1)(60)}{2\pi} = 3353\ \text{r/min} \end{aligned} \tag{10A-3}$$

Now at this no-load speed,

$$T_e = k_v i_a = (1.41 \times 10^{-2})(0.15) = 2.12 \times 10^{-3}\ \text{N} \cdot \text{m} \tag{10A-4}$$

Since T_L and $J(d\omega_r/dt)$ are zero for this steady-state no-load condition, (10.3-4) tells us that (10A-4) is equal to $B_m\omega_r$; hence,

$$B_m = \frac{2.12 \times 10^{-3}}{\omega_r} = \frac{2.12 \times 10^{-3}}{351.1} = 6.04 \times 10^{-6}\ \text{N} \cdot \text{m} \cdot \text{s} \tag{10A-5}$$

EXAMPLE 10B The permanent-magnet dc machine described in Example 10A is operating with rated applied armature voltage and load torque T_L of 0.5 oz·in. Our task is to determine the efficiency where percent eff = (power output/power input) 100.

First let us convert ounce·in into newton·meter:

$$T_L = \frac{0.5}{(16)(0.225)(39.37)} = 3.53 \times 10^{-3}\ \mathrm{N \cdot m} \tag{10B-1}$$

In Example 10A, we determined k_v to be 1.41×10^{-2} V s/rad and B_m to be 6.04×10^{-6} N·m·s.

During steady-state operation, (10.3-6) becomes

$$T_e = B_m \omega_r + T_L \tag{10B-2}$$

From (10.3-5), with $L_{AF} i_f$ replaced by k_v, the steady-state electromagnetic torque is

$$T_e = k_v I_a \tag{10B-3}$$

Substituting (10B-3) into (10B-2) and solving for ω_r yields

$$\omega_r = \frac{k_v}{B_m} I_a - \frac{1}{B_m} T_L \tag{10B-4}$$

From (10.4-2) with $L_{AF}i_f = k_v$,

$$V_a = r_a I_a + k_v \omega_r \tag{10B-5}$$

Substituting (10B-4) into (10B-5) and solving for I_a yields

$$\begin{aligned} I_a &= \frac{V_a + (k_v / B_m) T_L}{r_a + (k_v^2 / B_m)} \\ &= \frac{6 + [(1.41\times10^{-2})/(6.04\times10^{-6})](3.53\times10^{-3})}{7 + (1.41\times10^{-2})^2/(6.04\times10^{-6})} = 0.357\ \text{A} \end{aligned} \tag{10B-6}$$

From (10B-4),

$$\begin{aligned} \omega_r &= \frac{1.41\times10^{-2}}{6.04\times10^{-6}} 0.357 - \frac{1}{6.04\times10^{-6}}(3.53\times10^{-3}) \\ &= 249\ \text{rad/s} \end{aligned} \tag{10B-7}$$

The power input is

$$P_{\text{in}} = V_a I_a = (6)(0.357) = 2.14\ \text{W} \tag{10B-8}$$

The power output is

$$P_{\text{out}} = T_L \omega_r = (3.53\times10^{-3})(249) = 0.8\ \text{W} \tag{10B-9}$$

The efficiency is

$$\begin{aligned} \eta &= \frac{P_{\text{out}}}{P_{\text{in}}} 100 \\ &= \frac{0.88}{2.14} 100 = 41.1\% \end{aligned} \tag{10B-10}$$

The low efficiency is characteristic of low-power dc motors due to the relatively large armature resistance. In this regard, it is interesting to determine the losses due to $i^2 r$, friction, and windage.

$$P_{i^2 r} = r_a I_a^2 = (7)(0.357)^2 = 0.89\ \text{W} \tag{10B-11}$$

$$P_{fw} = (B_m \omega_r)\omega_r = (6.04\times10^{-6})(249)^2 = 0.37\ \text{W} \tag{10B-12}$$

Let us check our work:

$$P_{\text{in}} = P_{i^2 r} + P_{fw} + P_{\text{out}} = 0.89 + 0.37 + 0.88 = 2.14\ \text{W} \tag{10B-13}$$

which is equal to (10B-8).

Consider the 100-hp dc motor of Examples 11.3 and 11.4 to be driving a load whose torque varies linearly with speed such that it equals rated full-load torque (285 N·m) at a speed of 2500 r/min. We will assume the combined moment of inertia of the motor and load to equal 0.92 kg·m^2. The field voltage is to be held constant at 300 V.

a. Calculate the armature voltage and current required to achieve speeds of 2000 and 2500 r/min.
b. Assume that the motor is operated from an armature-voltage controller and that the armature voltage is suddenly switched from its 2000 r/min to its 2500 r/min value. Calculate the resultant motor speed and armature current as a function of time.

c. Assume that the motor is operated from an armature-current controller and that the armature current is suddenly switched from its 2000 r/min to its 2500 r/min value. Calculate the resultant motor speed as a function of time.

■ Solution

a. Neglecting any rotational losses, the armature current can be found from Eq. 11.4 by setting $T_{mech} = T_{load}$

$$I_a = \frac{T_{load}}{K_f I_f}$$

Substituting

$$T_{load} = \left(\frac{n}{n_f}\right) T_{fl}$$

where n is the motor speed in r/min, $n_f = 2500$ r/min and $T_{fl} = 285$ N·m gives

$$I_a = \frac{n T_{fl}}{n_f K_f I_f}$$

Solving for $V_a = E_a + I_a R_a$ then allows us to complete the following table:

r/min	ω_m [rad/sec]	V_a [V]	I_a [A]	T_{load} [N·m]
2000	209	410	119	228
2500	262	513	149	285

b. The dynamic equation governing the speed of the motor is

$$J\frac{d\omega_{\rm m}}{dt} = T_{\rm mech} - T_{\rm load}$$

Substituting $\omega_{\rm m} = (\pi/30)n$ and $\omega_{\rm r} = (\pi/30)n_{\rm f}$ we can write

$$T_{\rm load} = \left(\frac{T_{\rm fl}}{\omega_{\rm f}}\right)\omega_{\rm m}$$

Under armature-voltage control,

$$T_{\rm mech} = K_{\rm f}I_{\rm f}I_{\rm a} = K_{\rm f}I_{\rm f}\left(\frac{V_{\rm a} - E_{\rm a}}{R_{\rm a}}\right)$$

$$= K_{\rm f}I_{\rm f}\left(\frac{V_{\rm a} - K_{\rm f}I_{\rm f}\omega_{\rm m}}{R_{\rm a}}\right)$$

and thus the governing differential equation is

$$J\frac{d\omega_{\rm m}}{dt} = K_{\rm f}I_{\rm f}\left(\frac{V_{\rm a} - K_{\rm f}I_{\rm f}\omega_{\rm m}}{R_{\rm a}}\right) - \left(\frac{T_{\rm fl}}{\omega_{\rm f}}\right)\omega_{\rm m}$$

or

$$\frac{d\omega_{\rm m}}{dt} + \frac{1}{J}\left(\frac{T_{\rm fl}}{\omega_{\rm f}} + \frac{(K_{\rm f}I_{\rm f})^2}{R_{\rm a}}\right)\omega_{\rm m} - \frac{K_{\rm f}I_{\rm f}V_{\rm a}}{J\,R_{\rm a}}$$

$$= \frac{d\omega_{\rm m}}{dt} + 48.4\,\omega_{\rm m} - 24.7V_{\rm a} = 0$$

From this differential equation, we see that with the motor initially at $\omega_m = \omega_i = 209$ rad/sec, if the armature voltage V_a is suddenly switched from $V_i = 413$ V to $V_f = 513$ V, the speed will rise exponentially to $\omega_m = \omega_f = 262$ rad/sec as

$$\omega_m = \omega_f + (\omega_i - \omega_f)e^{-t/\tau}$$
$$= 262 - 53e^{-t/\tau} \text{ rad/sec}$$

where $\tau = 1/48.4 = 20.7$ msec. Expressed in terms of r/min

$$n = 2500 - 50e^{-t/\tau} \text{ r/min}$$

The armature current will decrease exponentially with the same 20.7 msec time constant from an initial value of $(V_f - V_i)/R_a = 1190$ A to its final value of 149 A. Thus,

$$I_a = 149 + 1041e^{-t/\tau} \text{ A}$$

Notice that it is unlikely that the supply to the dc motor can supply this large initial current (eight times the rated full-load armature current) and, in addition, the high current and corresponding high torque could potentially cause damage to the dc motor commutator, brushes, and armature winding. Hence, as a practical matter, a practical controller would undoubtedly limit the rate of change of the armature voltage to avoid such sudden steps in voltage, with the result that the speed change would not occur as rapidly as calculated here.

c. The dynamic equation governing the speed of the motor remains the same as that in part (b) as does the equation for the load torque. However, in this case, because the motor is being operated from a current controller, the electromagnetic torque will remain constant at $T_{mech} = T_f = 285$ N·m after the current is switched from its initial value of 119 A to its final value of 149 A.

Thus

$$J\frac{d\omega_m}{dt} = T_{mech} - T_{load} = T_f - \left(\frac{T_f}{\omega_f}\right)\omega_m$$

or

$$\frac{d\omega_m}{dt} + \left(\frac{T_{f1}}{J\omega_f}\right)\omega_m - \frac{T_f}{J}$$

$$= \frac{d\omega_m}{dt} + 1.18\omega_m - 310 = 0$$

In this case, the speed will rise exponentially to $\omega_m = \omega_f = 262$ rad/sec as

$$\omega_m = \omega_f + (\omega_i - \omega_f)e^{-t/\tau}$$

$$= 262 - 53e^{-t/\tau} \text{ rad/sec}$$

where now the time constant $\tau = 1/1.18 = 845$ msec.

Clearly, the change in motor speed under the current controller is much slower. However, at no point during this transient do either the motor current or the motor torque exceed their rated value. In addition, should faster response be desired, the armature current (and hence motor torque) could be set temporarily to a fixed value higher than the rated value (e.g., two or three times rated as compared to the factor of 8 found in part (b)), thus limiting the potential for damage to the motor.

Practice Problem 11.6

Consider the dc motor/load combination of Example 11.6 operating under current (torque) control to be operating in the steady-state at a speed of 2000 r/min at an armature current of 119 A. If the armature current is suddenly switched to 250 A, calculate the time required for the motor to reach a speed of 2500 r/min.

Solution

0.22 sec

Example 2.1

A dc motor whose parameters are given in example 2.3 is started directly from a 220-V dc supply with no load. Find its starting speed response and the time taken to reach 100 rad/sec.

Solution

$$\frac{\omega(s)}{V(s)} = G_{\omega v}(s) = \frac{K_b}{s^2(JL_a) + s(B_lL_a + JR_a) + (B_lR_a + K_b^2)} = \frac{15{,}968}{s^2 + 167s + 12874}$$

$$V(s) = \frac{220}{s}$$

$$\omega(s) = \frac{3.512 \times 10^6}{s(s^2 + 167s + 12874)}$$

$$\omega(t) = 272.8(1 - 1.47e^{-83.5t}\sin(76.02t + 0.744))$$

The time to reach 100 rad/sec is evaluated by equating the left-hand side of the above equation to 100 and solving for t, giving an approximate value of 10 rad/ms.

Example 2.2

A separately-excited dc motor is delivering rated torque at rated speed. Find the efficiency of the motor at this operating point. The details of the machine are as follows: 1500 kW, 600V, rated current = 2650 A, 600 rpm, Brush voltage drop = 2 V, Field power input = 50 kW, Ra = 0.003645 Ω, L_a = 0.1 mH, Machine frictional torque coefficient = 15 N·m/(rad/sec). Field current is constant and the armature voltage is variable.

Solution To find the input power, the applied voltage to the armature to support a rated torque and rated speed has to be determined. In steady state, the armature voltage is given by

$$V_a = R_aI_{ar} + K_b\omega_{mr} + V_{br}$$

where I_{ar} is the rated armature current, given as 2650A, ω_{mr} is the rated speed in rad/sec, and V_{br} is the voltage drop across the brushes in the armature circuit and is equal to 2V (given in the problem). To solve this equation, the emf constant has to be solved for from the available data. Recalling that the torque and emf constants are equal, the torque constant can be computed from the rated electromagnetic torque and the rated current as

$$K_t = \frac{T_{er}}{I_{ar}} = \frac{T_s + T_f}{I_{ar}}$$

where the rated electromagnetic torque generated in the machine, T_{er}, is the sum of the rated shaft torque T_s and friction torque T_f. The rated shaft or output torque is obtained from the output power and rated speed as follows:

$$\text{Rated speed, } \omega_{mr} = \frac{2\pi * 600}{60} = 62.83 \text{ rad/sec}$$

$$\text{Rated shaft torque, } T_s = \frac{P_m}{\omega_{mr}} = \frac{1500 * 10^3}{62.83} = 23{,}873 \text{ N·m}$$

$$\text{Friction torque, } T_f = B_1\omega_{mr} = 15 * 62.83 = 942.45 \text{ N·m}$$

$$\text{The electromagnetic torque, } T_{er} = T_s + T_f = 23{,}873 + 942.45 = 24{,}815.45$$

Therefore, the torque constant is

$$K_t = \frac{T_{er}}{I_{ar}} = \frac{24{,}815.45}{2650} = 9.364 \text{ N·m/A}$$

$$K_b = 9.364 \text{ V/(rad/sec)}$$

Hence the input armature voltage is computed as

$$V_a = 0.003645 * 2650 + 9.364 * 62.83 + 2 = 600 \text{ V}$$

Armature and field power inputs = $V_a I_{ar}$ + Field power input = 600 * 2650 + 50,000 = 1640 kW

$$\text{Output power, } P_m = 1500 \text{ kW}$$

$$\text{Efficiency, } \eta = \frac{P_m}{P_i} = \frac{1500 * 10^3}{1640 * 10^3} * 100 = 91.46\%$$

Example 2.3

A separately-excited dc motor with the following parameters: $R_a = 0.5\ \Omega$, $L_a = 0.003$H, and $K_b = 0.8$ V/rad/sec, is driving a load of $J = 0.0167 \text{kg-m}^2$, $B_1 = 0.01$ N·m/rad/sec with a load torque of 100 N·m. Its armature is connected to a dc supply voltage of 220 V and is given the rated field current. Find the speed of the motor.

Solution The electromagnetic torque balance is given by

$$T_e = T_l + B_1\omega_m + J\frac{d\omega_m}{dt}$$

In steady state, $\frac{d\omega_m}{dt} = 0$

$$T_e = T_l + B_1\omega_m = 100 + 0.01\omega_m$$

$$T_e = K_b i_a = 100 + 0.01\omega_m$$

$$i_a = \frac{(100 + 0.01\omega_m)}{K_b} = (125 + 0.0125\omega_m)$$

$$e = V - R_a i_a = 220 - 0.5 \times (125 + 0.0125\omega_m) = 157.5 - 0.00625\omega_m = K_b\omega_m$$

Rearranging in terms of ω_m,

$$\omega_m(0.8 + 0.00625) = 157.5$$

Hence $\omega_m = \frac{157.5}{0.80625} = 195.35$ rad/sec

Example 2.4

A series-excited dc machine designed for a variable-speed application has the following name-plate details and parameters:

3 hp, 230 V, 2000 rpm

$R_a = 1.5\ \Omega$, $R_{se} = 0.7\ \Omega$, $L_a = 0.12$ H, $L_{se} = 0.03$ H, $M = 0.0675$ H, $B_1 = 0.0025$ N·m/(rad/sec)

Calculate (i) the input voltage required in steady state to deliver rated torque at rated speed and (ii) the efficiency at this operating point. Assume that a variable voltage source is available for this machine.

Solution (i) The name-plate details give the rated speed and rated power output of the machine, from which the rated torque is evaluated as follows:

$$\text{Rated speed, } \omega_{mr} = \frac{2\pi N_r}{60} = \frac{2\pi * 2000}{60} = 209.52 \text{ rad/sec}$$

$$\text{Rated output torque, } T_s = \frac{P_m}{\omega_{mr}} = \frac{3 * 745.6}{209.52} = 10.675 \text{ N·m}$$

$$\text{Friction torque of the machine, } T_f = B_1\omega_{mr} = 0.0025 * 209.52 = 0.52 \text{ N·m}$$

$$\text{Air gap torque, } T_e = T_s + T_f = 10.675 + 0.52 = 11.195 \text{ N·m}$$

The voltage equation of the dc series machine from the equivalent circuit is derived as

$$v = R_a i_a + R_{se} i_f + M i_f \omega_m + L_a \frac{di_a}{dt} + L_{se}\frac{di_f}{dt}$$

where the armature and field current are equal to one another in the series dc machine ($I_f = I_a$) and, in steady state, the derivatives of the currents are zero, resulting in the following expression:

$$V = (R_a + R_{se} + M\omega_m)I_a$$

and air gap torque is given by

$$T_e = M i_f i_a = M i_a^2 = M I_a^2 \quad (\text{N·m})$$

The air gap torque is computed as 11.195 N·m, and the steady-state armature current is found from the expression above as

$$I_a = \sqrt{\frac{T_e}{M}} = \sqrt{\frac{11.195}{0.0675}} = 12.88 \text{ A}$$

which, upon substitution in the steady-state input voltage equation at the rated speed, gives

$$V = (1.5 + 0.7 + 0.0675 * 209.52)\ 12.88 = 210.46\text{V}$$

(ii) The input power is $P_i = VI_a = 210.46 * 12.88 = 2{,}710.45$ W
The output power is $P_m = 3 * 745.6 = 2236.8$ W

$$\text{Efficiency is } \eta = \frac{P_m}{P_i} = \frac{2236.8}{2710.45} * 100 = 82.5\%$$

Example 3.1

A separately-excited dc motor has the following ratings and constants:

2.625 hp., 120V, 1313 rpm, $R_a = 0.8\ \Omega$, $R_f = 100\ \Omega$, $K_b = 0.764$ V.s / rad, $L_a = 0.003$ H, $L_f = 2.2$ H

The dc supply voltage is variable from 0 to 120 V both to the field and armature, independently. Draw the torque–speed characteristics of the dc motor if the armature and field currents are not allowed to exceed their rated values. The rated flux is obtained when the field voltage is 120 V. Assume that the field voltage can be safely taken to a minimum of 12 V only.

Solution **(i)** Calculation of rated values:

$$\text{Rated speed, } \omega_{mr} = \frac{2\pi N}{60} = \frac{2\pi \times 1313}{60} = 137.56 \text{ rad/sec}$$

$$\text{Rated torque, } T_{er} = \frac{\text{Output power}}{\text{Rated speed}} = \frac{2.625 \times 745.6}{137.56} = 14.23 \text{ N·m}$$

$$\text{Rated armature current, } I_{ar} = \frac{\text{Rated torque}}{K_b} = \frac{14.23}{0.764} = 18.63 \text{ A}$$

$$\text{Rated field current, } I_{fr} = \frac{V_{fr}}{R_f} = \frac{120}{100} = 1.2 \text{ A}$$

(ii) Calculation of torque–speed characteristics:

Case (a) Constant-flux/torque region:

$$e_1 = V_{max} - I_{ar}R_a = 120 - 18.63 \times 0.8 = 105.1$$

$$\omega_{m1} = \frac{e_{m1}}{K_b} = \frac{105.1}{0.764} = 137.56 \text{ rad/sec.}$$

$$\omega_{m1n} = \frac{\omega_{m1}}{\omega_{mr}} = \frac{137.56}{137.56} = 1.0 \text{ p.u.}$$

Hence, constant rated torque is available from 0 to 1.0 p.u. speed.

Case (b) Field-weakening region:

For 1 p.u. armature current, the maximum induced emf is

$$e_n = \frac{e_1}{e_r} = \frac{105.1}{105.1} = 1.0 \text{ p.u.}$$

To maintain this induced emf in the field-weakening region,

$$\phi_{fn} = \frac{e_n}{\omega_{mn}} = \frac{1.0}{\omega_{mn}} \text{ p.u.}$$

If the range of field variation is known, the maximum speed can be computed as follows:

$$I_{f\,min} = \frac{V_{f(min)}}{R_f} = \frac{12}{100} = 0.12\ \text{A}$$

1.2 A of field current corresponds to rated field flux and hence 0.12 A corresponds to $0.1\phi_{fr}$, and hence

$$0.1\text{p.u.} < \phi_{fn} < 1\ \text{p.u.}$$

$$\omega_{max} = \frac{1}{0.1} = 10\ \text{p.u.}$$

For various speeds between 1 and 10 p.u., the field flux is evaluated from the equation as

$$\phi_{fn} = \frac{1}{\omega_{mn}}\ \text{in p.u.}$$

$$T_{en} = \phi_{fn}\ \text{for}\ I_{an} = 1\ \text{p.u.}$$

The torque, power, and flux-vs.-speed plots are shown in Figure 3.2.

Example 3.2

Consider the dc motor given in Example 3.1, and draw the intermittent characteristics if the armature current is allowed to be 300% of rated value.

Solution **(i)** Constant-flux/torque region

$$I_{max} = 3I_{ar}$$

$$T_{em} = \text{maximum torque} = K_b I_{max} = 0.764 \times 3 \times 18.63 = 42.7\ \text{N·m.}$$

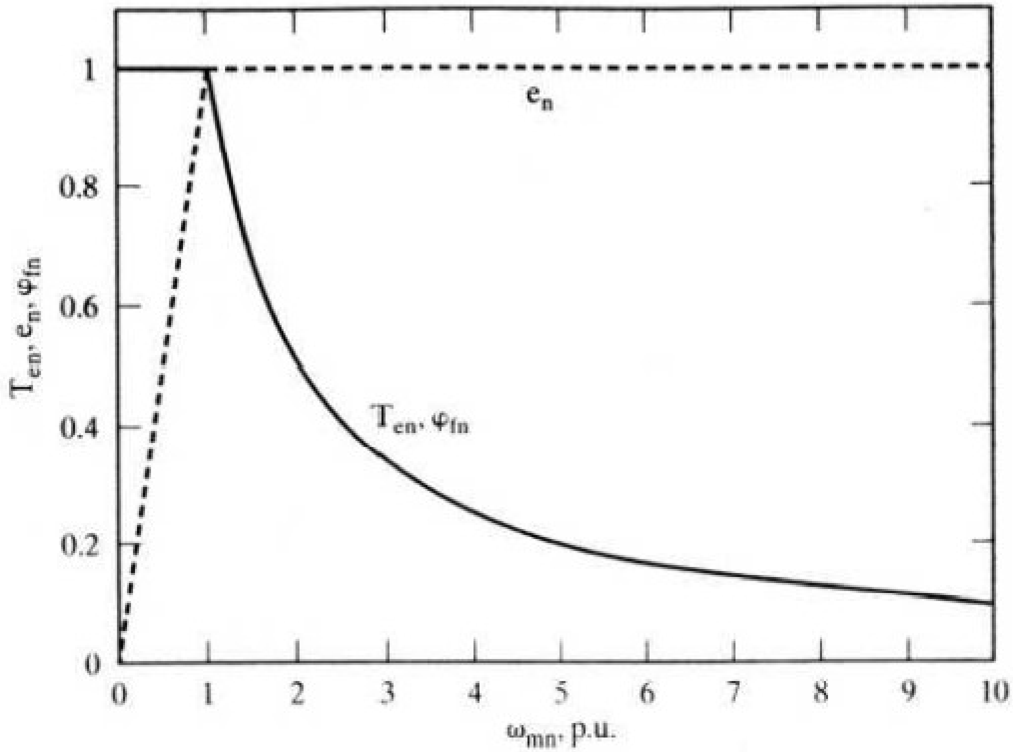


Figure 3.2 Continuous rating of the dc motor

$$T_{en} = \frac{T_{em}}{T_{er}} = \frac{42.7}{14.25} = 3 \text{ p.u.}$$

The maximum induced emf is

$$e_m = V_{max} - I_{max}R_a = 120 - (3 \times 18.63) \times 0.8 = 75.29 \text{ V}$$

Speed corresponding to this induced emf is

$$\omega_{ml} = \frac{e_m}{K_b} = \frac{75.29}{0.764} = 98.54 \text{ rad/sec}$$

$$\omega_{mln} = \frac{98.54}{137.56} = 0.716 \text{ p.u.}$$

Beyond this speed, field weakening is performed.

(ii) Field-weakening region:

$$I_{max} = 3I_{ar}$$

$$e_m = 75.29 \text{ V}$$

$$e_n = \frac{e_m}{105.1} = \frac{75.29}{105.1} = 0.716 \text{ p.u.}$$

$$\omega_{mn} = \frac{e_n}{\phi_{fn}} = \frac{0.716}{\phi_{fn}} \text{ p.u.}$$

The range of the normalized field flux is

$$0.1 < \phi_{fn} < 1$$

The maximum normalized speed is $\omega_{mn} = \dfrac{0.716}{\phi_{fn(min)}} = \dfrac{0.716}{0.1} = 7.16$ p.u.

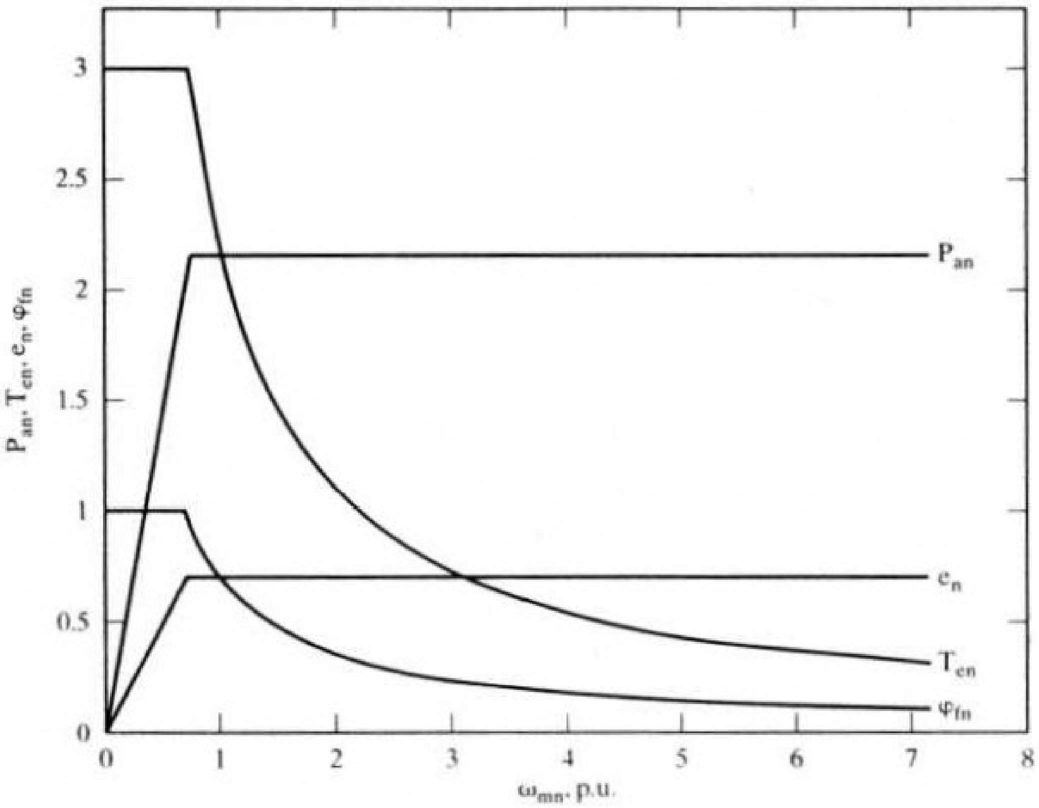


Figure 3.3 Normalized motor characteristics for 3 p.u. armature current

$$T_{en} = \phi_{fn} \text{ per rated current} = 3\phi_{fn} \text{ in the present case.}$$

The intermittent characteristics are drawn from the above equations and are shown in Figure 3.3.

Example 3.3

Consider a motor drive with $R_{an} = 0.1$ p.u., $\phi_{fn} = 1$ p.u., $V_n = 1.1$ p.u. and extreme load operating points $T_{e1(min)} = 0.1$ p.u., $\omega_{mn(min)} = \omega_{mn1} = 0.1$ p.u., $T_{e2(max)} = 1$ p.u., and $\omega_{mn(max)} = \omega_{mn2} = 1$ p.u.

(i) Find the normalized control voltages to meet these operating points.

(ii) Compute the change in control voltages required for a simultaneous change of ΔT_{en} = 0.02 p.u. and $\Delta\omega_{mn}$ = 0.01 p.u. for both the extreme operating points. From this, calculate the resolution required for the control voltage.

Solution Assume that the controller is linearized.

$$\therefore \alpha = \cos^{-1}\left\{\frac{v_c}{V_{cm}}\right\} = \cos^{-1}\{V_{cn}\}$$

from which the electromagnetic torque is,

$$T_{en} = \left[\frac{1.35V_n V_{cn} - \Phi_{fn}\omega_{mn}}{R_{an}}\right]\Phi_{fn}, \text{ p.u.}$$

where V_{cn} is the normalized control voltage for a given steady-state operating point and is obtained as

$$V_{cn} = \frac{T_{en}R_{an} + \phi_{fn}\omega_{mn}}{1.35V_n}$$

Since ϕ_{fn} = 1 p.u., the control voltage for minimum torque and speed is

$$V_{cn1} = \frac{T_{en1}R_{an} + \phi_{fn}\omega_{mn1}}{1.35V_n} = \frac{0.1 * 0.1 + 0.1}{1.35 * 1.1} = 0.074 \text{ p.u.}$$

Similarly for maximum torque and speed, the control voltage is

$$V_{cn2} = \frac{T_{en2}R_{an} + \phi_{fn}\omega_{mn2}}{1.35V_n} = \frac{1 * 0.1 + 1}{1.35 * 1.1} = 0.74 \text{ p.u.}$$

Incremental control voltage generates incremental torque and speed as

$$V_{cn} + \delta v_{cn} = \frac{R_{an}(T_{en} + \delta T_{en}) + \omega_{mn} + \delta\omega_{mn}}{1.35V_n}$$

For both changes, $\delta v_{cn} = \dfrac{R_{an}\Delta T_{en} + \delta\omega_{mn}}{1.35V_n}$

Dividing δV_{cn} by V_{cn} gives an expression in terms of steady-state operating points as

$$\frac{\delta v_{cn}}{V_{cn}} = \frac{R_{an}\delta T_{en} + \delta\omega_{mn}}{R_{an}T_{en} + \omega_{mn}}$$

δT_{en} = 0.02 p.u., $\delta\omega_{mn}$ = 0.01 p.u., T_{en1} = 0.1 p.u., ω_{mn1} = 0.1 p.u., T_{en2} = 1 p.u., ω_{mn2} = 1 p.u.

For T_{en1}, ω_{mn1}: $\dfrac{\delta v_{cn}}{V_{cn}} = \dfrac{0.1 * 0.02 + 0.01}{0.1 * 0.1 + 0.1} = 0.109$

For T_{en2}, ω_{mn2}: $\dfrac{\delta v_{cn}}{V_{cn}} = \dfrac{0.1 * 0.02 + 0.01}{0.1 * 1 + 1} = 0.0109$

Therefore, the resolution required in control voltage is

$$\delta V_{cn} = 0.109 * V_{cn} = 0.109 * 0.074 = 0.008066 \text{ p.u.}$$

Example 3.4

A separately-excited dc motor has 0.05 p.u. resistance and is fed from a three-phase converter. The normalized voltage and field flux are 1 p.u. Draw the torque–speed characteristics in the first quadrant for constant delay angles of 0, 30, 45, and 60 degrees. Indicate the safe operating region if the maximum torque limit is 2.5 p.u.

Solution

$$T_{en} = \frac{[1.35V_n \cos\alpha - \Phi_{fn}\omega_{mn}]}{R_{an}}\Phi_{fn}, \text{ p.u.}$$

Substituting the given values yields

$$T_{en} = 20[1.35\cos\alpha - \omega_{mn}], \text{ p.u.}$$

The torque–speed characteristics for various angles of delay are shown in Figure 3.21. The safe operating region is shaded in the figure.

Example 3.5

The details and parameters of a separately-excited dc machine are

100 hp, 500 V, 1750 rpm, 153.7 A, $R_a = 0.088\ \Omega$, $L_a = 0.00183$ H, $K_b = 2.646$ V/(rad/sec)

The machine is supplied from a three-phase controlled converter whose ac input is from a three-phase 415 V, 60 Hz utility supply. Assume that the machine is operating at 100 rpm with a triggering angle delay of 65°. Find the maximum air gap torque ripple at this operating point.

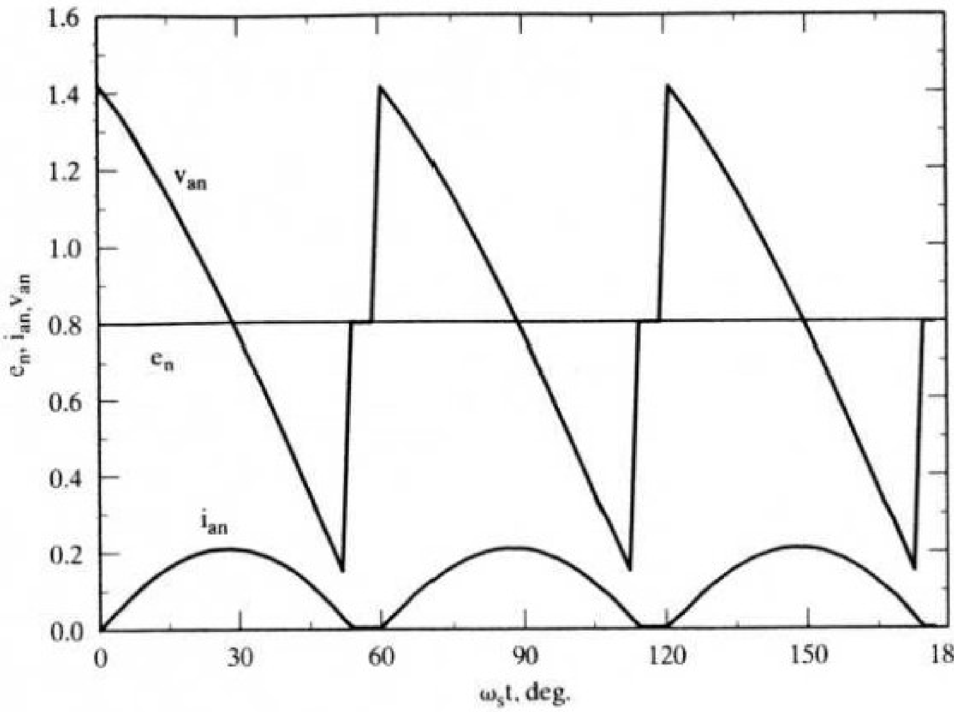


Figure 3.25 The armature current, applied voltage, and induced emf for a typical discontinuous conduction

Solution To find the current ripple, it is essential to determine whether the current is continuous for the given triggering delay by evaluating the critical triggering angle. It is found as follows:

Rotor speed, $\omega_m = \dfrac{2\pi N_r}{60} = \dfrac{2\pi * 100}{60} = 10.48$ rad/sec

Induced emf, $E = K_b\omega_m = 2.646 * 10.48 = 27.7$ V

Peak input voltage, $V_m = \sqrt{2} * 415 = 586.9$ V

Input angular frequency, $\omega_s = 2\pi f_s = 2\pi * 60 = 376.99$ rad/sec

Armature time constant, $T_a = \dfrac{L_a}{R_a} = \dfrac{0.00183}{0.088} = 0.0208$ sec

Machine impedance, $Z_a = \sqrt{R_a^2 + \omega_s^2 L_a^2} = 0.6955\Omega$

Machine impedance angle, $\beta = \tan^{-1}\left(\dfrac{\omega_s L_a}{R_a}\right) = 1.444$ rad

The critical triggering angle is

$$\alpha_c = \beta + \cos^{-1}\left\{\frac{E/V_m}{c_1} \cdot \frac{1}{\cos\beta} \cdot \left(1 - e^{-(\pi/3\tan\beta)}\right)\right\} - \frac{\pi}{3} + \theta_1$$

where

$$a_1 = \frac{\sqrt{3}}{2} = 0.866$$

$$b_1 = \frac{1}{2} - e^{-\left(\frac{\pi}{3\tan\beta}\right)} = -0.375$$

$$c_1 = \sqrt{a_1^2 + b_1^2} = 0.9437$$

$$\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right) = -0.4086 \text{ rad}$$

from which the critical angle is obtained as $\alpha_c = 1.5095$ rad $= 86.48°$. The triggering angle α is 65°, which is less than the critical triggering angle; therefore, the armature current is continuous. Having determined that the drive system is in continuous mode of conduction, we use the relevant equations to calculate the initial current, given by

$$i_{a1} = \frac{\left(\frac{V_m}{|Z_a|}\right)\left\{\sin\left(\frac{2\pi}{3} + \alpha - \beta\right) - \sin\left(\frac{\pi}{3} + \alpha - \beta\right)e^{-\left(\frac{\pi}{3\omega_s T_a}\right)}\right\} - \frac{E}{R_a}\left(1 - e^{-\left(\frac{\pi}{3\omega_s T_a}\right)}\right)}{1 - e^{-\frac{\pi}{3\omega_s T_a}}} = 2308.1 \text{ A}$$

The peak armature current is found by having $\omega_s t = \pi/6$, i.e.. at the midpoint of the cycle. This is usually the case, but the operating point can shift it beyond 30°; therefore, it is necessary to verify graphically or analytically where the maximum current occurs and then substitute that instant to get the peak armature current from the following equation:

$$i_a(t) = \left(\frac{V_m}{|Z_a|}\right)\{\sin(\omega_s t + \pi/3 + \alpha - \beta) - \sin(\pi/3 + \alpha - \beta)e^{-t/T_a}\} - \left(\frac{E}{R_a}\right)(1 - e^{-t/T_a})$$

$$+ i_{a1}e^{-t/T_a} = 2411.5 \text{ A}$$

The armature current ripple magnitude, $\Delta i_a = 2411.5 - 2308.1 = 103.4$ A

The ripple torque magnitude, $\Delta T_e = K_b \Delta i_a = 2.646 * 103.4 = 273.86$ N·m

Average air gap torque, $T_{e(av)} = I_{av}K_b \cong [\{2411.5 + 2308.1\} * 0.5] * 2.646 = 6244$ N·m

Note that the ripple current magnitude is less than 5% and therefore is approximated as a straight line between its minimum and maximum values in each part of its cycle.

Torque ripple as a percent of the operating average torque is

$$\Delta T_{en} = \frac{\Delta T_e}{T_{e(av)}} * 100 = \frac{273.86}{6244} * 100 = 4.4\%$$

Example 3.6

Design a speed-controlled dc motor drive maintaining the field flux constant. The motor parameters and ratings are as follows:

220 V, 8.3 A, 1470 rpm, $R_a = 4\ \Omega$, $J = 0.0607$ kg − m_2, $L_a = 0.072$ H, $B_t = 0.0869$ N·m / rad/sec, $K_b = 1.26$ V/rad/sec.

The converter is supplied from 230V, 3-phase ac at 60 Hz. The converter is linear, and its maximum control input voltage is ±10 V. The tachogenerator has the transfer function $G_\omega(s) = \frac{0.065}{(1 + 0.002s)}$. The speed reference voltage has a maximum of 10V. The maximum current permitted in the motor is 20 A.

Solution (i) Converter transfer function:

$$K_r = \frac{1.35\ V}{V_{cm}} = \frac{1.35 \times 230}{10} = 31.05 \text{ V/V}$$

$$V_{dc}(\max) = 310.5 \text{ V}$$

The rated dc voltage required is 220 V, which corresponds to a control voltage of 7.09 V. The transfer function of the converter is

$$G_r(s) = \frac{31.05}{(1 + 0.00138s)} \text{ V/V}$$

(ii) Current transducer gain: The maximum safe control voltage is 7.09 V, and this has to correspond to the maximum current error:

$$i_{max} = 20\ A$$

$$H_c = \frac{7.09}{I_{max}} = \frac{7.09}{20} = 0.355\ V/A$$

(iii) Motor transfer function:

$$K_1 = \frac{B_t}{K_b^2 + R_a B_t} = \frac{0.0869}{1.26^2 + 4 \times 0.0869} = 0.0449$$

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B_t}{J} + \frac{R_a}{L_a}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B_t}{J} + \frac{R_a}{L_a}\right)^2 - \left(\frac{K_b^2 + R_a B_t}{JL_a}\right)}$$

$$T_1 = 0.1077\ \text{sec}$$

$$T_2 = 0.0208\ \text{sec}$$

$$T_m = \frac{J}{B_t} = 0.7\ \text{sec}$$

The subsystem transfer functions are

$$\frac{I_a(s)}{V_a(s)} = K_1\frac{(1 + sT_m)}{(1 + sT_1)(1 + sT_2)} = \frac{0.0449(1 + 0.7s)}{(1 + 0.0208s)(1 + 0.1077s)}$$

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b/B_t}{(1 + sT_m)} = \frac{14.5}{(1 + 0.7s)}$$

(iv) Design of current controller:

$$T_c = T_2 = 0.0208\ \text{sec}$$

$$K = \frac{T_1}{2T_r} = \frac{0.1077}{2 \times 0.001388} = 38.8$$

$$K_c = \frac{KT_c}{K_1 H_c K_r T_m} = \frac{38.8 \times 0.0208}{0.0449 \times 0.355 \times 31.05 \times 0.7} = 2.33$$

(v) Current-loop approximation:

$$\frac{I_a(s)}{I_a^*(s)} = \frac{K_i}{(1 + sT_i)}$$

where

$$K_i = \frac{K_{fi}}{H_c} \cdot \frac{1}{(1 + K_{fi})}$$

$$K_{fi} = \frac{K_c K_r K_1 T_m H_c}{T_c} = 38.8$$

$$\therefore K_i = \frac{27.15}{28.09} \cdot \frac{1}{0.355} = 2.75$$

$$T_i = \frac{T_3}{1 + K_{fi}} = \frac{0.109}{1 + 38.8} = 0.0027 \text{ sec}$$

The validity of the approximations is evaluated by plotting the frequency response of the closed-loop current to its command, with and without the approximations. This is shown in Figure 3.34. From this figure, it is evident that the approximations are quite valid in the frequency range of interest.

(vi) Speed-controller design:

$$T_4 = T_i + T_\omega = 0.0027 + 0.002 = 0.0047 \text{ sec}$$

$$K_2 = \frac{K_i K_b H_\omega}{B_t T_m} = \frac{2.75 \times 1.26 \times 0.065}{0.0869 \times 0.7} = 3.70$$

$$K_s = \frac{1}{2K_2 T_4} = \frac{1}{2 \times 3.70 \times 0.0047} = 28.73$$

$$T_s = 4T_4 = 4 \times 0.0047 = 0.0188 = \text{sec}$$

The frequency responses of the speed to its command are shown in Figure 3.35 for cases with and without approximations. That the model reduction with the approximations has given a transfer function very close to the original is obvious from this figure. Further, the

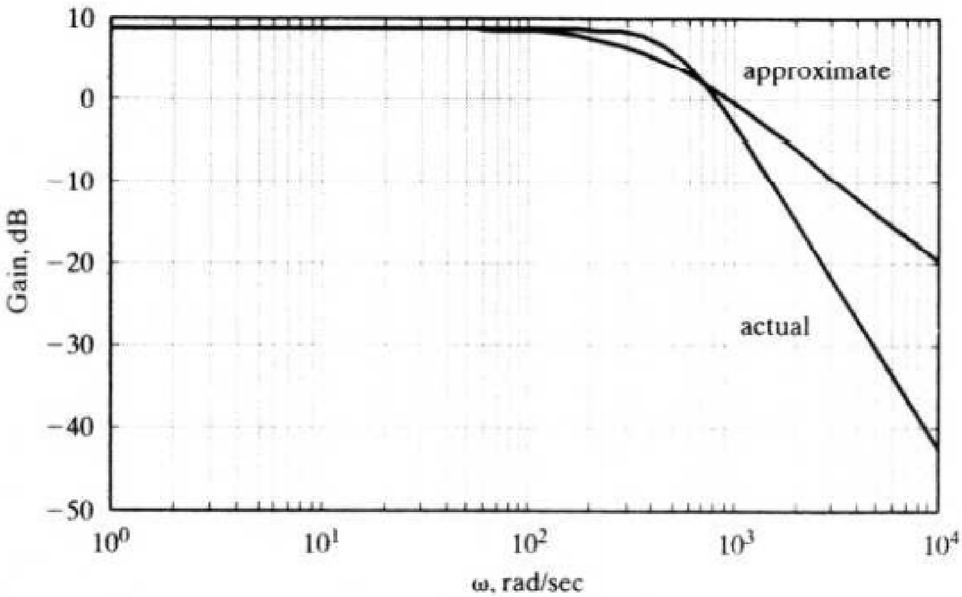


Figure 3.34 Frequency response of the current-transfer functions with and without approximation

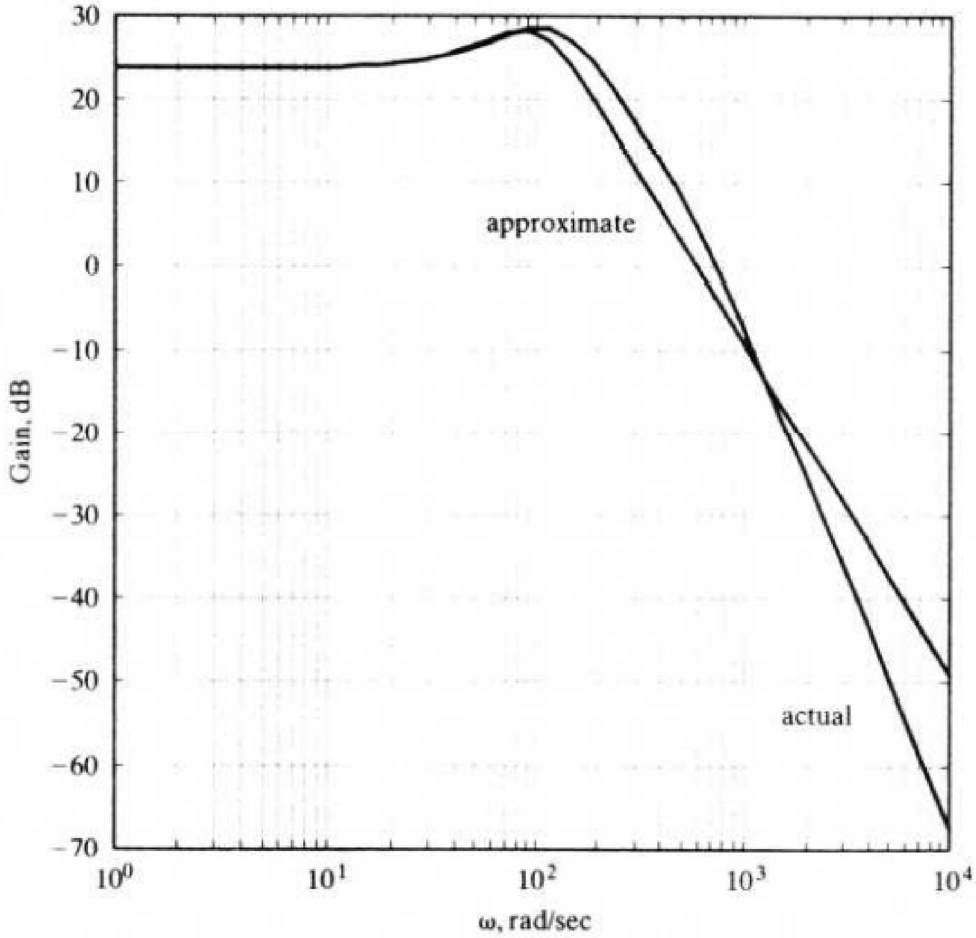


Figure 3.35 Frequency response of the speed-transfer functions with and without approximation

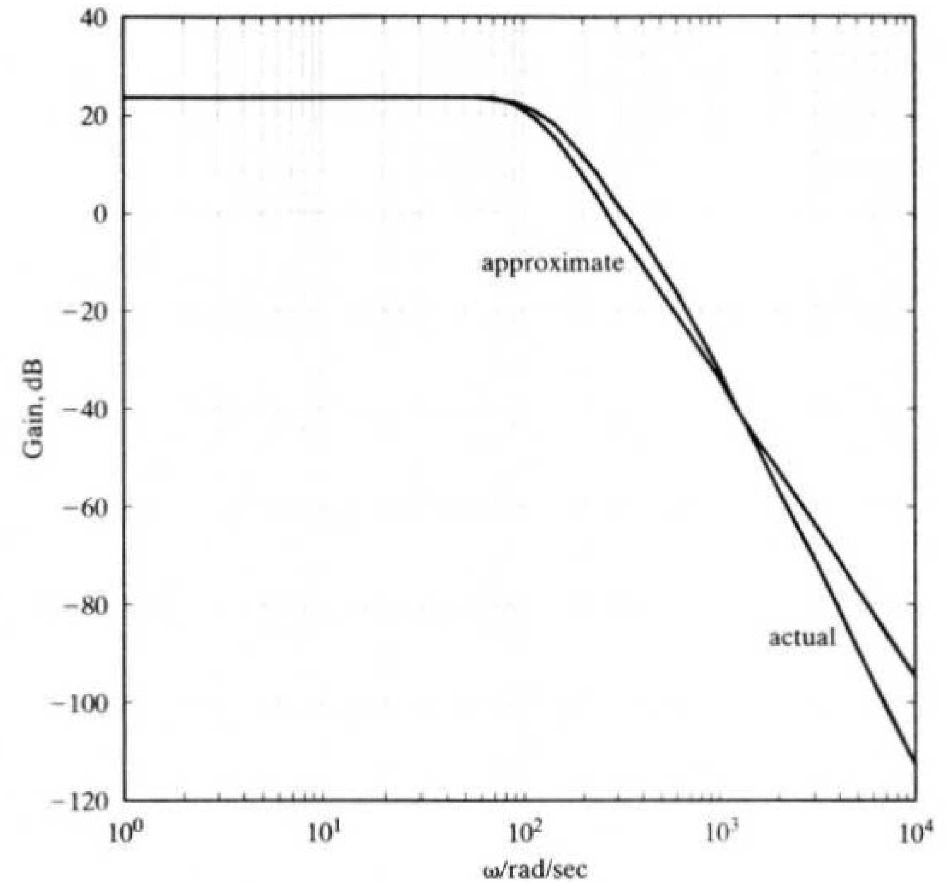


Figure 3.36 Frequency response of the speed-transfer function with smoothing for the cases with and without approximation

smoothing of the overshoot by the cancellation of the zero with a pole at $-1/4T_4$ is shown in Figure 3.36. This figure contains the approximated transfer function of third order for the speed to its command-transfer function and the one without any approximations. Again, the closeness of these two solutions justifies the approximations.

The time responses are important to verify the design of the controllers, and they are shown in Figure 3.37 for the case without smoothing and with smoothing. The case without any approximation is included here for the comparison of all responses.

Example 3.7

Assume motor poles are complex. Develop a design procedure for the current controller.

Solution If motor poles are complex, then the procedure outlined above is not applicable for the design of the current and speed controllers. One alternative is as follows: the current controller is designed by using the symmetric optimum criterion that was applied in the earlier speed controller design. The steps are given below.

Assuming $(1 + sT_m) \cong sT_m$ leads to the following current-loop transfer function:

$$\frac{i_a(s)}{i_a^*(s)} = \frac{K_2 \frac{K_c}{T_c}(1 + sT_c)}{b_0 + b_1 s + b_2 s^2 + b_3 s^3}$$

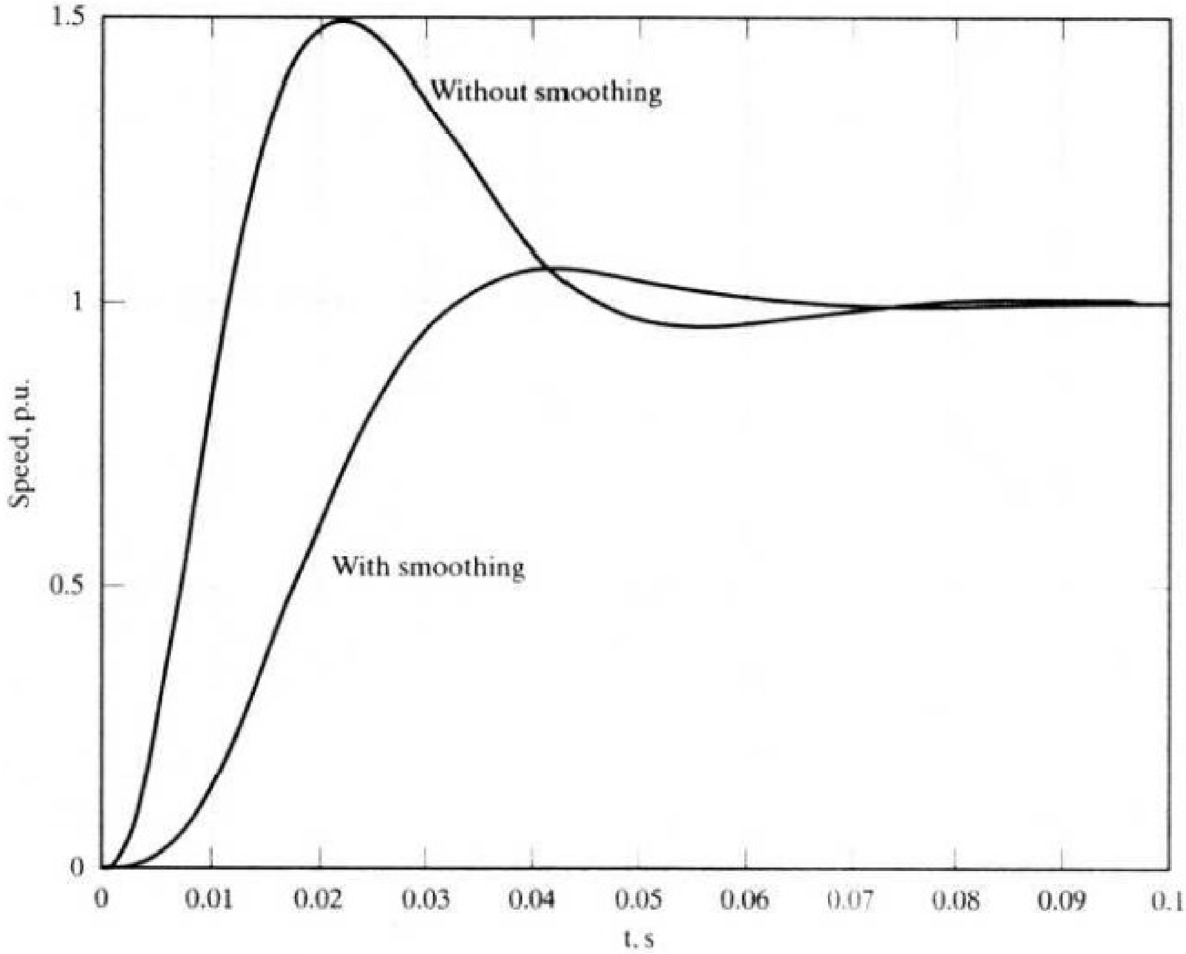


Figure 3.37 Time response of the speed controller

where

$$K_2 = K_1 K_r T_m$$

$$b_0 = 1 + K_2 \frac{K_c}{T_c} H_c$$

$$b_1 = T_1 + T_2 + T_r + K_2 K_c H_c$$

$$b_2 = (T_1 + T_2)T_r + T_1 T_2$$

$$b_3 = T_1 T_2 T_r$$

Applying symmetric-optimum conditions,

$$b_1^2 = 2b_0 b_2$$

$$(T_1 + T_2 + T_r + K_2 K_c H_c)^2 = 2\left(1 + K_2 \frac{K_c}{T_c} H_c\right)((T_1 + T_2)T_r + T_1 T_2)$$

$$b_2^2 = 2b_1 b_3$$

$$((T_1 + T_2)T_r + T_1 T_2)^2 = 2(T_1 + T_2 + T_r + K_2 K_c H_c)(T_1 T_2 T_r)$$

but $T_r << T_1, T_2$,

$$\therefore (T_1 + T_2)T_r << T_1 T_2$$

$$\frac{(T_1 T_2)^2}{2(T_1 T_2 T_r)} = T_1 + T_2 + T_r + K_2 K_c H_c$$

$$\frac{T_1 T_2}{2T_r} \cong K_2 K_c H_c \left[\because T_1 + T_2 + T_r << \frac{T_1 T_2}{2T_r}\right]$$

$$\therefore K_c = \frac{T_1 T_2}{2T_r} \frac{1}{K_2 H_c}.$$

Also,

$$\left(T_1 + T_2 + T_r + \frac{T_1 T_2}{2T_r}\right)^2 = 2\left(1 + \frac{T_1 T_2}{2T_r T_c}\right)((T_1 + T_2)T_r + T_1 T_2)$$

$$\frac{(T_1 T_2)^2}{4T_r^2} \cong 2T_1 T_2\left(1 + \frac{T_1 T_2}{2T_r T_c}\right)$$

$$\frac{1}{4T_r^2} \cong \frac{1}{T_r T_c}$$

$$T_c \cong \frac{4T_r^2}{T_r} \cong 4T_r.$$

The next step is to obtain the first-order approximation of the current-loop transfer function for the synthesis of the speed controller. Since the time constant T_c is known, the first-order approximation of the current loop is written as

$$\frac{i_a(s)}{i_a^*(s)} \cong \frac{K_i}{1 + sT_i}$$

where the steady-state gain is obtained from the exact transfer function, by setting $s = 0$, as

$$K_i = \frac{\dfrac{K_2 K_c}{T_c}}{1 + \dfrac{K_2 K_c H_c}{T_c}}$$

and $T_i = T_c$.

From this point, the speed-controller design follows the symmetric-optimum procedure outlined earlier for the case with the real motor poles.

Example 4.1 A voltage of 230 V applied to the armature of a dc motor results in a full-load armature current of 205 A. Assume that the armature resistance is 0.2 Ω. Find the back EMF E_c, the net output power, and the torque, assuming that the rotational losses are 1445 W at a full-load speed of 1750 rpm.

Solution The armature voltage and current are specified as

$$V_a = 230\,\text{V}$$
$$I_a = 205\,\text{A}$$

The back EMF is obtained as

$$\begin{aligned} E_c &= V_a - I_a R_a \\ &= 230 - 205(0.2) \\ &= 189\,\text{V} \end{aligned}$$

The power developed by the armature is thus

$$\begin{aligned} P_a &= E_c I_a \\ &= 189(205) = 38{,}745\,\text{W} \end{aligned}$$

The net output power is thus obtained by subtracting the rotational losses from the armature developed power:

$$\begin{aligned} P_\text{o} &= P_a - P_\text{rot} \\ &= 38{,}745 - 1445 \\ &= 37{,}300\,\text{W} \end{aligned}$$

The net output torque is now calculated as

$$\begin{aligned} T_\text{o} &= \frac{P_\text{o}}{\omega} \\ &= \frac{37{,}300}{\frac{2\pi}{60}(1750)} \\ &= 203.536\,\text{N}\cdot\text{m} \end{aligned}$$

Example 4.2 The combined armature and series field resistance of a 10-hp 240-V dc series motor is $0.6\,\Omega$. Neglecting rotational losses, it is required to calculate:

(a) Armature current at full rated load
(b) The value of maximum developed armature power and the corresponding armature current
(c) The armature current at half-rated load using Eqs. (4.25) and (4.26)

(a) At full-rated load, neglecting rotational losses, the armature developed power is

$$P_a = 10 \times 746 = 7460\,\mathrm{W}$$

The armature current is obtained as

$$I_a = \frac{240 - \sqrt{(240)^2 - 4(7460)(0.6)}}{2(0.6)}$$
$$= 33.968\,\mathrm{A}$$

Note that the second solution for I_a is 366.032 A if we take the positive signs in Eq. (4.21). The approximate solution of Eq. (4.26) is 31.083 A with an error of 2.885 A.

(b) For maximum armature power we have

$$I_{a_m} = \frac{240}{2(0.6)} = 200\,\mathrm{A}$$

The value of P_a at this current is obtained using Eq. (4.22) as

$$P_{a_m} = \frac{(240)^2}{4(0.6)} = 24{,}000\,\mathrm{W}$$

(c) At half-rated load we have

$$P_a = 5 \times 746 = 3730\,\mathrm{W}$$

The corresponding armature current is

$$I_a = \frac{240 - \sqrt{(240)^2 - 4(3730)(0.6)}}{2(0.6)}$$
$$= 16.198\,\mathrm{A}$$

The approximate equation [Eq. (4.43)] yields

$$I_a \cong \frac{3730}{240} = 15.542\,\mathrm{A}$$

The error in this case is 0.656 A.

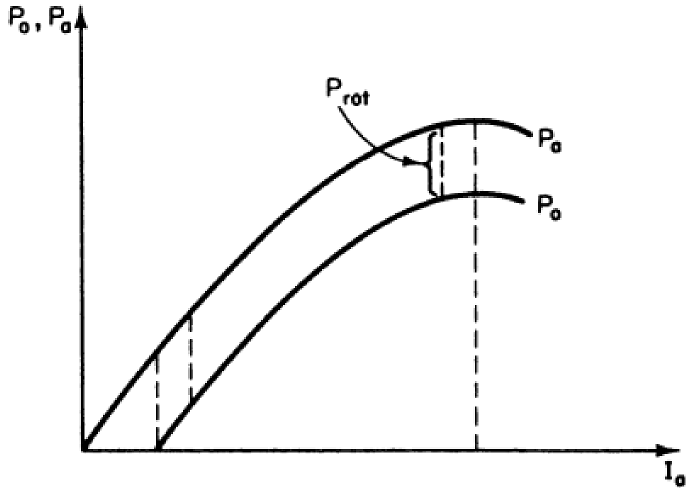


Figure 4.9 Obtaining the variation of output power with armature current for a dc series motor.

The output (or shaft) power of the motor is obtained by subtracting the rotational power losses form the armature-developed power. The resulting characteristic is shown in Fig. 4.9.

Example 4.3 A 600-V, 150-hp dc series motor operates at its full-rated load at 600 rpm. The armature resistance is $0.12\,\Omega$ and the series field resistance is

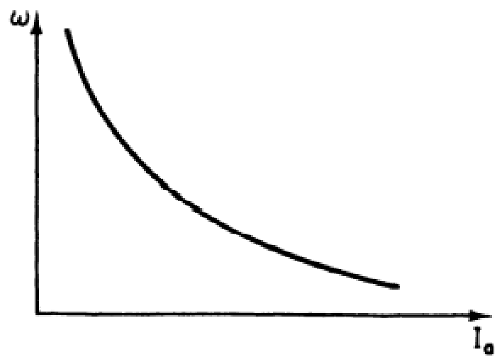


Figure 4.11 Variation of speed with armature current for a dc series motor.

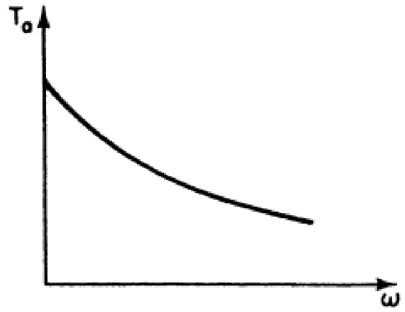


Figure 4.12 Torque speed characteristic of a dc series motor.

$0.04\,\Omega$. The motor draws 200 A at full load.

(a) Find the armature back EMF at full load
(b) Find the armature-developed power and internal-developed torque
(c) Assume that a change in load results in the line current dropping to 150 A. Find the new speed in rpm and the new developed torque

Solution

(a) The armature back EMF is obtained from

$$E_{c_1} = V_t - I_{a_1}(R_a + R_s)$$
$$= 600 - 200(0.12 + 0.04)$$
$$= 568\text{ V}$$

(b) The power developed by the armature is

$$P_{a_1} = E_{c_1} I_{a_1}$$
$$= 568(200) = 113.6 \times 10^3\text{ W}$$

As a result, we calculate the internal developed torque as

$$T_{a_1} = \frac{P_{a_1}}{\omega_1} = \frac{113.6 \times 10^3}{(2\pi/60)(600)} = 1808\text{ N} \cdot \text{m}$$

(c) For the new line current, we can calculate a new value for the back EMF:

$$E_{c_2} = 600 - 150(0.16) = 576 \text{ V}$$

From Eq. (4.18) we have

$$E_{c_1} = K_1 K_2 I_{a_1} \omega_1$$
$$E_{c_2} = K_1 K_2 I_{a_2} \omega_1$$

As a result,

$$\frac{\omega_2}{\omega_1} = \frac{I_{a_1}}{I_{a_2}} \frac{E_{c_2}}{E_{c_1}}$$

Thus

$$\omega_2 = \frac{200}{150}\left(\frac{576}{568}\right)\omega_1$$

or

$$n_2 = \frac{200}{150}\left(\frac{576}{568}\right)(600)$$
$$= 811.268 \text{ r/min}$$

In a similar manner, from Eq. (4.28), we can write

$$\frac{T_2}{T_1} = \left(\frac{I_2}{I_1}\right)^2$$

Thus

$$T_2 = 1808\left(\frac{150}{200}\right)^2 = 1017 \text{ N} \cdot \text{m}$$

Example 4.4 The series motor of Example 4.3 is required to be started using a starting resistor R_{st} such that the starting current is limited to 150% of rated value:

(a) Find the required value of R_{st} and the starting torque

(b) If the starting resistor is left in the armature circuit and the motor line current drops to its rated value of 200 A, find the armature back EMF and the speed of the motor

Solution

(a) At starting $E_c = 0$:

$$V_t = I_a(R_a + R_s + R_{st})$$

$$600 = 300(0.16 + R_{st})$$

Thus

$$R_{st} = 1.84\,\Omega$$

We know from Example 4.3 that for an armature current of 200 A, the corresponding torque is 1808 N·m. Thus for a starting current of 300 A, we have

$$T_{st} = T_{fld}\left(\frac{I_{st}}{I_{fld}}\right)^2$$

$$= 1808\left(\frac{300}{200}\right)^2 = 4068\ \mathrm{N\cdot m}$$

(b) With the starting resistor left in the armature circuit, we have

$$\begin{aligned} E_c &= V_t - I_a(R_a + R_s + R_{\text{st}}) \\ &= 600 - 200(2) \end{aligned}$$

Thus

$$E_c = 200\,\text{V}$$

We now have

$$\frac{E_{c_2}}{E_{c_1}} = \frac{n_2}{n_1}$$

or

$$\frac{200}{568} = \frac{n_2}{600}$$

As a result,

$$n_2 = 211.268\ \text{rpm}$$

Example 4.5 Consider the dc series motor of Example 4.3. The full-load output is 150 hp with full-load current given by 200 A.

(a) Find the rotational losses
(b) Find the armature current and power output at maximum efficiency as well as the value of maximum efficiency
(c) Find the full-load efficiency

Assume the rotational losses are fixed.

Solution

(a) The output power at full load is

$$P_o = 150 \times 746 = 111.9 \times 10^3 \text{ W}$$

From Example 4.3, the armature developed power at full load

$$P_a = 113.6 \times 10^3 \text{ W}$$

As a result,

$$\begin{aligned} P_{\text{rot}} &= P_a - P_o \\ &= 1.7 \times 10^3 \text{ W} \end{aligned}$$

(b) For maximum efficiency, the armature current is obtained by

$$I_a = \sqrt{\frac{1.7 \times 10^3}{0.16}} = 103.1\ \text{A}$$

The back EMF for this current is

$$\begin{aligned} E_c &= 600 - 103.1(0.16) \\ &= 583.5\ \text{V} \end{aligned}$$

The power developed by the armature is thus

$$P_a = 583.5 \times 103.1 = 60.16 \times 10^3\ \text{W}$$

As a result,

$$\begin{aligned} P_o &= 60.16 \times 10^3 - 1.7 \times 10^3 \\ &= 58.46 \times 10^3\ \text{W} \end{aligned}$$

The corresponding input power is

$$\begin{aligned} P_{\text{in}} &= 600 \times 103.1 \\ &= 61.86 \times 10^3\ \text{W} \end{aligned}$$

As a result, the maximum efficiency is

$$\eta_{\max} = \frac{58.46 \times 10^3}{61.86 \times 10^3} = 0.945$$

(c) The full-load input power is

$$P_{in} = 600 \times 200 = 120 \times 10^3 \text{ W}$$

The output power at full load is

$$P_o = 150 \times 746 = 111.9 \times 10^3 \text{ W}$$

Thus the full-load efficiency is

$$\eta = \frac{111.9 \times 10^3}{120 \times 10^3} = 0.9325$$

Note that at full load, the motor is operating beyond the point of maximum efficiency.

Example 4.6 The armature resistance of a 10-hp, 230-V dc shunt motor is 0.3 Ω. The field resistance is 160 Ω. The motor draws a line current of 3.938 A on no load at a speed of 1200 rpm. At full load, the armature current is 40 A.

(a) Find the armature current at no load
(b) Find the power developed by the armature on no load
(c) Find the full-load efficiency of the motor
(d) Find the full-load speed of the motor

Solution

(a) The field current is obtained as

$$I_f = \frac{230}{160} = 1.438 \text{ A}$$

Thus the armature current is obtained at

$$I_a = I_L - I_f = 3.938 - 1.438 = 2.5 \text{ A}$$

(b) The back EMF on no load is calculated as

$$\begin{aligned} E_c &= V_t - I_a R_a \\ &= 230 - 2.5(0.3) = 229.25 \text{ V} \end{aligned}$$

Thus we obtain the no-load armature power as

$$\begin{aligned} P_{\text{no load}} &= E_c I_a \\ &= 229.25(2.5) = 572.125 \text{ W} \end{aligned}$$

(c) At full load, we have

$$E_c = 230 - 40(0.3) = 218 \text{ V}$$

The armature power is thus calculated as

$$P_a = E_c I_a = 218(40) = 8720 \text{ W}$$

The net power output is obtained by subtracting the no-load power (rotational losses) from part (b) from the armature power:

$$P_o = 8720 - 573.125 = 8146.875 \text{ W}$$

The power input is found as

$$\begin{aligned} P_{in} &= V_t I_L \\ &= 230(41.438) = 9530.625 \text{ W} \end{aligned}$$

As a result, we calculate the efficiency:

$$\begin{aligned} \eta &= \frac{8146.875}{9530.625} \\ &= 0.855 \end{aligned}$$

(d) From part (c), the full-load back EMF is given by

$$E_{c_2} = 218 \text{ V}$$

The no-load back EMF from part (b) is

$$E_{c_1} = 229.25 \text{ V}$$

Thus we can find the full-load speed as

$$\begin{aligned} n_{fld} &= n_{\text{no load}} \frac{E_{c_2}}{E_{c_1}} \\ &= 1200\left(\frac{218}{229.25}\right) = 1141.112 \text{ rpm} \end{aligned}$$

Example 4.7 A 230-V, 25-hp dc shunt motor draws an armature current of 90 A at full rated load. Assume that the armature resistance is 0.2 Ω and that the shunt field resistance is 216 Ω. Find the rotational losses at full load and the motor efficiency in this case.

Solution At a full rated armature current of 90 A, the power developed by the armature is given by

$$\begin{aligned} P_a &= (V_t - I_a R_a) I_a \\ &= [230 - 90(0.2)](90) = 19{,}080 \text{ W} \end{aligned}$$

The output power at full load is

$$P_o = 25 \times 746 = 18{,}650 \text{ W}$$

As a result, the rotational losses are obtained from

$$\begin{aligned} P_{rot} &= P_a - P_o \\ &= 19{,}080 - 18{,}650 = 430 \text{ W} \end{aligned}$$

The line current is given by

$$\begin{aligned} I_L &= I_a + \frac{V_t}{R_f} \\ &= 90 + \frac{230}{216} = 91.065 \end{aligned}$$

The input power is thus

$$\begin{aligned} P_{in} &= V_t I_L = 230(91.065) \\ &= 20{,}944.91 \text{ W} \end{aligned}$$

The efficiency can thus be computed as

$$\eta = \frac{P_o}{P_{in}} = \frac{18{,}650}{20{,}944.91} = 0.89$$

Example 4.8 The rotational losses for a 230-V, 25-hp dc shunt motor are found to be 430 W. The armature resistance is 0.2 Ω and the shunt-field resistance is 216 Ω. Obtain the armature current corresponding to maximum efficiency using both the exact and approximate expressions. Calculate the output power in both cases and the corresponding maximum efficiency.

Solution The exact formula is given by Eq. (4.65) as

$$R_a I_a^2 + 2I_a I_f R_a - (P_{\text{rot}} + P_f) = 0$$

We have

$$\begin{aligned}
R_a &= 0.2\ \Omega \\
I_f &= \frac{230}{216} = 1.065\ \text{A} \\
P_f &= \frac{(230)^2}{216} = 244.907\ \text{W} \\
P_{\text{rot}} &= 430\ \text{W}
\end{aligned}$$

Thus we have

$$0.2I_a^2 + 2I_a(1.065)(0.2) - (430 + 244.907) = 0$$

Alternatively,

$$I_a^2 + 2.13I_a - 3374.537 = 0$$

The solution for I_a is obtained as

$$I_a = \frac{-2.13 \pm 116.201}{2}$$

We take

$$I_{a_{\max \eta_1}} = 57.036\,\text{A}$$

The output power is given by Eq. (4.64) as

$$\begin{aligned} P_o &= V_t I_a - I_a^2 R_a - P_{\text{rot}} \\ &= 230(57.036) - (57.036)^2(0.2) - 430 \\ &= 12{,}037.66\,\text{W} \end{aligned}$$

The input power is given by Eq. (4.62) as

$$\begin{aligned} P_{\text{in}} &= V_t I_a + P_f \\ &= 230(57.036) + 244.907 = 13{,}363.187\ \text{W} \end{aligned}$$

As a result, the maximum efficiency on the basis of the exact calculation is

$$\eta_{\max_1} = \frac{12{,}037.66}{13{,}363.187} = 0.901$$

The approximate formula [Eq. (4.66)] provides the value of armature current for maximum efficiency as

$$\begin{aligned} I_{a_{\max \eta_2}} &= \sqrt{\frac{P_{\text{rot}} + P_f}{R_a}} \\ &= 58.091\,\text{A} \end{aligned}$$

The output power is calculated as

$$\begin{aligned}
P_o &= V_t I_a - I_a^2 R_a - P_{rot} \\
&= 230(58.091) - (58.091)^2(0.2) - 430 \\
&= 12{,}256.017 \text{ W}
\end{aligned}$$

The input power is calculated as

$$\begin{aligned}
P_{in} &= V_t I_a + P_f \\
&= 230(58.09) + 244.907 \\
&= 13{,}605.837 \text{ W}
\end{aligned}$$

The corresponding maximum efficiency is

$$\eta_{max_2} = \frac{12{,}256.017}{13{,}605.837} = 0.901$$

The final answers are the same to the assumed accuracy.

Example 4.9 A cumulative compound motor is operated as a shunt motor and develops a torque of 2000 N·m when the armature current is 140 A. When reconnected as a cumulative compound motor at the same current, it develops a torque of 2400 N·m. Find the torque when the compound motor load is increased such that the armature current is increased by 10%.

Solution We use Eq. (4.88) to obtain

$$\beta I_{a_1} = \frac{2400}{2000} - 1 = 0.2$$

We now have

$$I_{a_2} = I_{a_1}(1.1)$$

As a result, we use Eq. (4.82) to obtain

$$\begin{aligned}\frac{T_{a_1}}{T_{a_2}} &= \frac{(1+0.2)140}{[1+1.1(0.2)](1.1\times 140)}\\ &= \frac{1.2}{1.1(1.22)} = 0.8942\end{aligned}$$

But

$$T_{a_1} = 2400\,\mathrm{N\cdot m}$$

Thus

$$T_{a_2} = \frac{2400}{0.8942} = 2684\ \mathrm{N\cdot m}$$

Clearly, the developed torque increases with an increase in armature current. The following example deals with the speed variations for the motor considered in this example.

Example 4.10 Assume that the combined armature and series field resistance of the motor of Example 4.9 is 0.16 Ω. Assume that the terminal voltage is 600 V.

Let the motor speed be 1200 rpm when operating as a cumulative compound motor with an armature current of 140 A. Find the speed corresponding to an armature current of 110% of 140 A.

Solution A direct application of Eq. (4.84) is all that we need:

$$\frac{\omega_1}{\omega_2} = \frac{600 - 0.16(140)}{600 - 0.16(140 \times 1.1)}\left[\frac{1 + 0.2(1.1)}{1 + 0.2}\right]$$
$$= 1.004$$

With

$$n_1 = 1200 \text{ rpm}$$

then

$$n_2 = \frac{n_1}{1.004} = 1195.346 \text{ rpm}$$

Clearly, the motor speed drops with an increase in armature current.

Example 4.11 The combined armature and series-field resistance of a dc series motor is 0.15 Ω. The motor operates from a 250-V supply, drawing an armature current of 85 A at a speed of 62.83 rad/s. Establish the torque–speed characteristic of the motor. Find the torque and speed when driving a constant power load with the characteristic

$$T_l = \frac{21.334 \times 10^3}{\omega}$$

Solution We note at the outset that we can solve the problem by assuming that armature-developed power is 21.334×10^3 W and apply techniques already known to us. We opt, however, for the following solution procedure.

We have

$$V_t = 250 \text{ V} \qquad I_a = 85 \text{ A} \quad \text{at } 62.83 \text{ rad/s}$$

Thus

$$E_c = 250 - (85)(0.15) = 237.25 \text{ V}$$
$$K_1 K_2 = \frac{237.25}{85(62.83)} = 44.424 \times 10^{-3}$$

As a result,

$$T_a = \frac{2.7765 \times 10^3}{(44.424 \times 10^{-3}\omega + 0.15)^2}$$

To match the load, we require that

$$\frac{2.7765 \times 10^3}{(44.424 \times 10^{-3}\omega + 0.15)^2} = \frac{21.334 \times 10^3}{\omega}$$

Thus

$$1.9735 \times 10^{-3}\omega^2 - 116.82 \times 10^{-3}\omega + 22.5 \times 10^{-3} = 0$$

The solution for ω is

$$\omega^* = 59.00 \text{ rad/s}$$

The corresponding torque is

$$T^* = 361.59 \text{ N} \cdot \text{m}$$

Example 4.12 A 120-V dc shunt motor has an armature resistance of 0.1 Ω and develops a torque of 111.16 N·m when running at 1100 rpm. Neglecting rotational losses, it is required to:

(a) Establish the torque–speed characteristic of this motor
(b) Find the developed torque at a speed of 1150 rpm
(c) Find the motor speed when the developed torque is 30 N·m
(d) Find the motor speed when it drives a load with the following torque–speed characteristic:

$$T_l = 6.2778\sqrt{\omega}$$

(e) Calculate the torque, armature-developed power, and armature current for the conditions of part (d)
(f) Assuming that the field resistance is 120 Ω, find the line current and the power input for the load conditions of part (d)

Solution

(a) We have

$$V_t = 120\,\text{V} \qquad T = 111.16\ \text{N}\cdot\text{m} \quad \text{at } 1100\ \text{rpm}$$
$$R_a = 0.1\,\Omega$$

The power developed by the armature can be calculated as

$$P_a = 111.16\left(\frac{2\pi}{60}1100\right) = 12.805 \times 10^3\ \text{W}$$

But

$$P_a = (V_t - I_a R_a) I_a$$

Thus

$$12.805 \times 10^3 = (120 - 0.1\,I_a) I_a$$

As a result,

$$
\begin{aligned}
0.1 I_a^2 - 120 I_a + 12.805 \times 10^3 &= 0 \\
I_a &= 118.39\,\text{A} \\
E_c &= 120 - 0.1(118.39) = 108.16\,\text{V} \\
K_{\text{sh}} &= \frac{E_c}{\omega} = \frac{108.16}{(2\pi/60)(1100)} = 938.97 \times 10^{-3} \\
\omega_0 &= \frac{V_t}{K_{\text{sh}}} = 127.8\ \text{rad/s} \\
\frac{K_{\text{sh}}^2}{R_a} &= 8.8166
\end{aligned}
$$

We can conclude that

$$T_a = 8.8166(127.8 - \omega)$$

(b) For $n = 1150$ rpm, we get

$$T_a = 8.8166\left[127.8 - \frac{2\pi}{60}(1150)\right] = 64.998\,\text{N}\cdot\text{m}$$

(c) For $T_a = 30\ \text{N}\cdot\text{m}$,

$$30 = 8.8166(127.8 - \omega)$$

As a result,

$$\omega = 124.4\ \text{rad/s}$$

(d) For a load with the torque–speed characteristic

$$T_l = 6.2778\sqrt{\omega}$$

to match, we have

$$6.2778\sqrt{\omega} = 8.8166(127.8 - \omega)$$

Thus

$$0.507 \times 10^{-3}\omega = (127.8 - \omega)^2$$

or

$$\omega^2 - 256.11\omega + 16.333 \times 10^3 = 0$$

The solution is

$$\omega = 119.95 \text{ rad/s}$$

(e) The torque is obtained from

$$T_l = 6.2778\sqrt{119.98} = 68.764\ \mathrm{N \cdot m}$$

The armature-developed power is

$$P_a = T_l\omega = 8.2503 \times 10^3\ \mathrm{W}$$

Thus we use

$$P_a = (V_t - I_a R_a)I_a$$

To obtain I_a:

$$8.2503 \times 10^3 = (120 - 0.1I_a)I_a$$

or

$$0.1I_a^2 - 120I_a + 8.250 \times 10^3 = 0$$

As a result,

$$I_a = 73.22\ \mathrm{A}$$

(f) The field current is calculated as

$$I_f = \frac{120}{120} = 1 \text{ A}$$

Thus

$$\begin{aligned} I_L &= I_a + I_f \\ &= 74.22 \text{ A} \end{aligned}$$

The power input is

$$\begin{aligned} P_{\text{in}} &= V_t I_L \\ &= 120(74.22) \\ &= 8.9064 \times 10^3 \text{ W} \end{aligned}$$

Example 4.13 A dc series motor is rated at 250 V and has a combined armature and series-field resistance of 0.15 Ω. The constant K_1K_2 is found to be

$$K_1 K_2 = 44.424 \times 10^{-3} \text{ V/A} \cdot \text{rads/s}$$

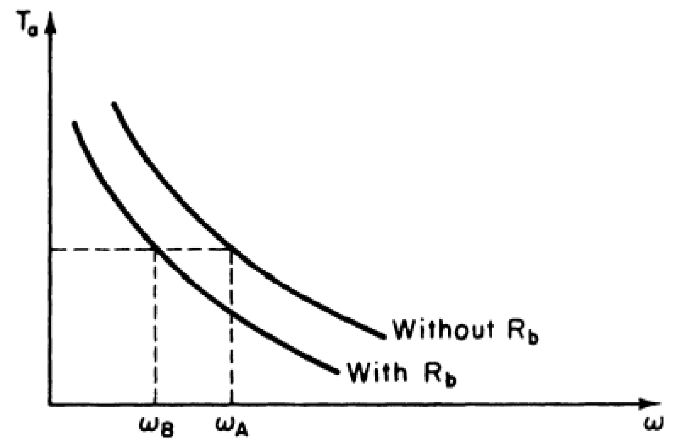


Figure 4.23 Effect of series resistance on torque–speed characteristic of series motor.

The motor drives a load with characteristic

$$T_l = \frac{21.334 \times 10^3}{\omega}$$

It is desired to drive the load at 55 rad/s. Find the required value of a series resistance, R_b, to achieve this requirement.

Solution The motor torque with R_b inserted in series with the field is

$$T_a = \frac{44.424 \times 10^{-3}(250)^2}{(0.15 + R_b + 44.424 \times 10^{-3}\omega)^2}$$

With $\omega = 55$ rad/s, the load torque is

$$T_l = \frac{21.334 \times 10^3}{55} = 387.89 \text{ N} \cdot \text{m}$$

To match the load, we have

$$387.89 = \frac{2.7765 \times 10^3}{(R_b + 2.5933)^2}$$

Thus we get

$$R_b = 0.0821\ \Omega$$

Example 4.14 Assume that the speed of the series motor of Example 4.13 is controlled to the same specifications, but now we use an armature shunt resistance control scheme. Find the required value of R_c, given that $R_a = 0.1\ \Omega$.

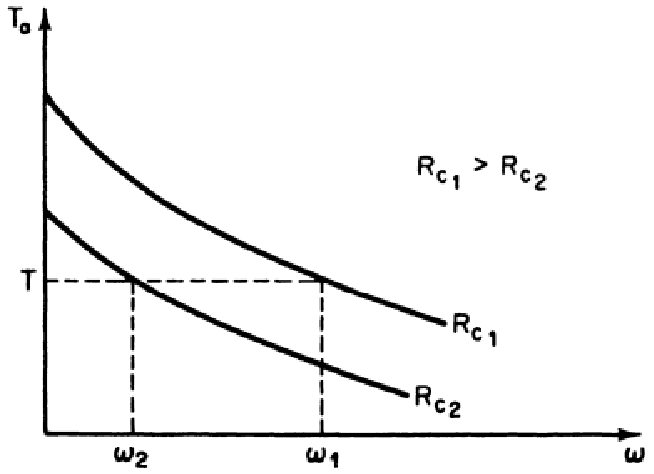


Figure 4.25 Effect of armature shunt resistance R_c on torque–speed characteristic of dc series motor.

Solution Our specifications call for

$$T_l = 387.89 \text{ N} \cdot \text{m}$$
$$\omega = 55 \text{ rad/s}$$

We are given that

$$R_a = 0.1\,\Omega$$

Thus

$$R_s = 0.05\,\Omega$$

We use the torque expression

$$T_a = \frac{K_1 K_2 V_t^2 (R_c - K_1 K_2 \omega)}{(R_a + R_c)\{R_s + [R_c/(R_a + R_c)](R_a + K_1 K_2 \omega)\}^2}$$

We thus have

$$387.89 = \frac{2.7765 \times 10^3 (R_c - 2.4433)}{(0.1 + R_c)\{0.5 + [R_c/(0.1 + R_c)](2.5433)\}^2}$$

After a few manipulations, we obtain

$$R_c^2 - 38.799 R_c - 4.0393 = 0$$

The required answer is obtained as

$$R_c = 38.903\,\Omega$$

Example 4.15 For the motor of Example 4.13, suppose that a resistance $R_c = 50\,\Omega$ is available. It is now decided to use series and shunt armature resistance control to achieve the required specifications. Find the value of the required R_b.

Solution With $R_c = 50\ \Omega$, our torque equation becomes

$$387.89 = \frac{2.7765 \times 10^3 (50 - 2.4433)}{(50.1)[0.05 + R_b + (50/50.1)(2.5433)]^2}$$

We solve for R_b to obtain

$$R_b = 18.419 \times 10^{-3}\,\Omega$$

Example 4.16 Assume that a diverter is used with the motor of Example 4.13, such that $\alpha = 0.9$. For the speed of 55 rad/s, find the resulting torque.

Solution The torque developed is obtained as

$$T_a = \frac{2.7765 \times 10^3(0.9)}{[0.1 + 0.9(0.05 + 2.5433)]^2}$$

As a result, we conclude that

$$T_a = 421.8 \text{ N} \cdot \text{m}$$

Note that this torque is higher than that without the diverter.

Example 4.17 The armature resistance of a dc shunt motor is $0.1\,\Omega$ and its shunt-field resistance is $120\,\Omega$. The value of ω_0 is 127.8 rad/s and the constant K_{sh} is 0.93897. The motor torque is 65 N·m. Find the value of R_e for the motor to run at 1100 rpm carrying this load.

Solution We use

$$\omega = \omega_0 x - x^2 \left(\frac{R_a}{K_{sh}^2} T_a \right)$$

where

$$x = 1 + \frac{R_e}{R_f}$$

Thus

$$\frac{2\pi(1100)}{60} = 127.8x - \frac{0.1(65)}{(0.93897)^2} x^2$$

As a result,

$$x = 16.381 \qquad \text{or} \qquad 0.954$$

We take

$$x = 1 + \frac{R_e}{R_f} = 16.381$$

Thus

$$\frac{R_e}{R_f} = 15.381$$

As a result,

$$R_e = 15.381(120) = 1.846 \times 10^3\,\Omega$$

Example 4.18 For the motor of Example 4.17, suppose that $R_e = 1.5 \times 10^3\,\Omega$. Now we want to achieve the same results, but with shunt-field and series armature resistance control. Find the required value of R_b.

Solution We use

$$\omega = \omega_0\left(1 + \frac{R_e}{R_f}\right) - \left(1 + \frac{R_e}{R_f}\right)^2 \frac{R_a + R_b}{K_{\text{sh}}^2} T_a$$

Thus

$$115.19 = 127.8\left(1 + \frac{1500}{120}\right) - \left(1 + \frac{1500}{120}\right)^2 \frac{(R_a + R_b)65}{(0.93897)^2}$$

As a result,

$$R_a + R_b = 0.11983$$

The required value of R_b is then

$$R_b \cong 0.02\,\Omega$$

Example 4.19 For the motor of Example 4.17, assume that series and shunt armature resistance control is used with $R_b = 0.03\,\Omega$. Find the value of R_c to obtain a torque of 65 N·m at a speed of 1100 rpm.

Solution Let

$$y = \frac{1}{1 + (R_b/R_c)}$$

Thus

$$\omega = \omega_0 y - \frac{R_a + R_b y}{K_{\text{sh}}^2} T_a$$

As a result,

$$115.19 = 127.8y - \frac{(0.1 + 0.3y)65}{(0.93897)^2}$$

The solution for y is

$$y = 0.97591$$

Thus

$$R_c = 1.2152\,\Omega$$

Example 4.20 A starter for a 220-V dc series motor is required such that the maximum current be 270 A and the minimum current is not be less than 162 A. The armature and series field resistances add up to $0.12\,\Omega$. The magnetization characteristic of the motor is as follows:

Motor Current (A)	162	180	198	216	234	252	270
EMF at 750 rpm	193	200	206	211.2	215.8	220	224

Find the required number of resistance steps, n.

Solution With the given information, we have

$$I_{\max} = 270 \text{ A}$$
$$I_{\min} = 163 \text{ A}$$

Thus

$$K = \frac{I_{\max}}{I_{\min}} = \frac{270}{162} = 1.66667$$

From the magnetization characteristic

$$E_{\max} = 224$$
$$E_{\min} = 193$$

Thus

$$a = \frac{E_{\max}}{E_{\min}} = \frac{224}{193} = 1.16062$$

We can now find b:

$$b = \frac{a}{K} = \frac{1.16062}{1.66667} = 0.696373$$

The resistance R_1 is obtained from

$$R_1 = \frac{V_t}{I_{\max}} = \frac{220}{270} = 0.814815\,\Omega$$

We also have

$$R_m = 0.12\,\Omega$$

All the necessary ingredients of Eq. (4.170) are now available and we thus have

$$n = 1 + \frac{\log\{1 - [(1 - 0.696)/0.696(1.667 - 1)](1 - 0.12/0.8148)\}}{\log 0.696373}$$
$$= 3.2543$$

We note that n is noninteger. We therefore take the closest integer of higher value than the calculated value. Thus we take

$$n = 4$$

Example 4.20 (Continued) The task here is to find $I_{\min}$. We start by noting that

$$y_0 = 1 - \frac{R_m}{R_1} = 1 - \frac{0.12}{0.8148} = 0.8527$$

Since we had $I_{\min} = 162$ A already, we move up to the next point on the magnetization characteristic. Thus take

$$I_{\min} = 180\,\text{A}$$
$$E_{\min} = 200\,\text{V}$$

Thus

$$K = \frac{270}{180} = 1.5$$
$$a = \frac{224}{200} = 1.12$$
$$b = \frac{a}{K} = 0.74667$$

We now calculate

$$y = b(K - 1)\frac{1 - b^3}{1 - b}$$
$$= 0.86023$$

The process is repeated for $I_{\text{min}} = 198$ A to obtain

$$y = 0.70558$$

In Fig. 4.39 we plot the two points in the $y - I_{\text{min}}$ plane. A simple linear approximation to solve for I^*_{min} requires solving

$$\frac{198 - I^*_{\text{min}}}{I^*_{\text{min}} - 180} = \frac{0.8527 - 0.70}{0.86 - 0.8527}$$

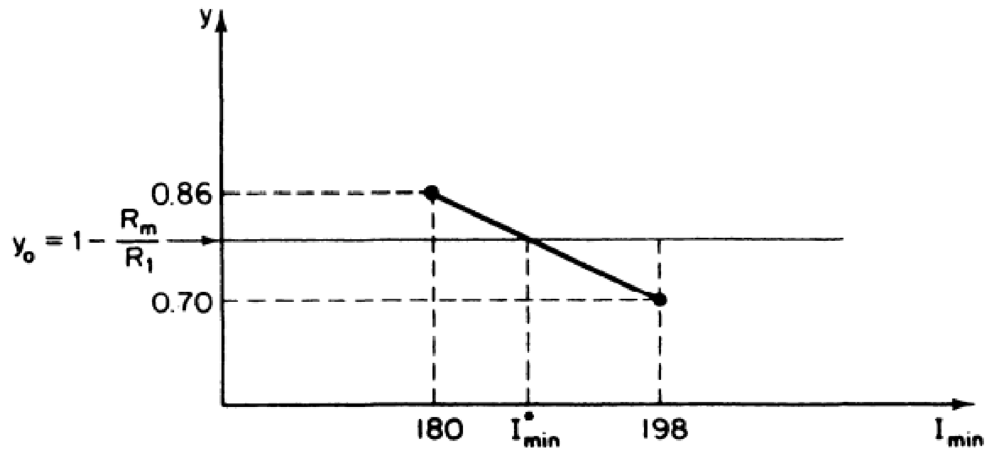


Figure 4.39 Finding I^*_{min}.

As a result, we conclude that

$$I^*_{\min} = 180.8\,\text{A}$$

Having obtained the proper values of n and $I_{\min}$, we can now conclude our design example by finding the resistance steps.

Example 4.20 (Conclusion) For $I_{\min} = 180.8$ A, we obtain

$$K = \frac{270}{180.8} = 1.4934$$
$$a = \frac{224}{200.3} = 1.1183$$
$$b = \frac{1.1183}{1.4934} = 0.7488$$

Now we have

$$R_1 = 0.8148\,\Omega$$
$$r_1 = b(K-1)R_1 = 0.7488(1.4934-1)(0.8148) = 0.301\,\Omega$$
$$r_2 = br_1 = 0.22544\,\Omega$$
$$r_3 = br_2 = 0.16882\,\Omega$$

This concludes the starter design.

Example 4.21 A 230-V three-phase supply is available to drive a separately excited dc motor through a three-phase full-wave bridge rectifier circuit connected to the armature terminals. The armature resistance is 0.2 Ω, and the motor draws a current of 205 A when running at 1750 rpm with an armature voltage of 230 V dc.

(a) Find the firing angle α under the specified conditions
(b) Find the firing angle α required for the motor to run at 875 rpm
(c) Find the motor's speed for a firing angle of 75 degrees
(d) Find the motor's speed for a firing angle of zero

Solution

(a) We apply Eq. (4.186):

$$230 = \frac{3\sqrt{2}}{\pi}(230)\cos\alpha$$

As a result,

$$\alpha = 42.2^\circ$$

(b) For a speed of 1750 rpm, we have

$$E_{c_1} = V_{a_1} - I_a R_a = 230 - 205(0.2) = 189\,\text{V}$$

For a speed of 875 rpm, we require

$$E_{c_2} = E_{c_1}\frac{\omega_2}{\omega_1} = \frac{189}{2} = 94.5\ \text{V}$$

As a result,

$$V_{a_2} = E_{c_2} + I_a R_a = 94.5 + 205(0.2) = 135.5\,\text{V}$$

We can now obtain the required firing angle:

$$V_{a_2} = 135.5 = \frac{3\sqrt{2}}{\pi}(230)\cos\alpha_2$$

$$\alpha_2 = 64.14^\circ$$

(c) With $\alpha_3 = 75$ degrees, we obtain

$$V_{a_3} = \frac{3\sqrt{2}}{\pi}(230)\cos 75^\circ = 80.39\text{ V}$$

Thus

$$\begin{aligned} E_{c_3} &= V_{a_3} - I_a R_a \\ &= 80.39 - 205(0.2) = 39.39\text{ V} \\ n_3 &= n_1 \frac{E_{c_3}}{E_{c_1}} = (1750)\frac{39.39}{189} = 364.74\text{ rpm} \end{aligned}$$

(d) With $\alpha_4 = 0$ degrees, we have

$$V_{a_4} = \frac{3\sqrt{2}}{\pi}(230) = 310.61\text{ V}$$

As a result,

$$\begin{aligned} E_{c_4} &= 310.61 - 41 = 269.61\text{ V} \\ n_4 &= 1750\left(\frac{269.61}{189}\right) = 2496.4\text{ rpm} \end{aligned}$$

Example 4.22 For the motor of Example 4.21, assume that the speed corresponding to $\alpha = 0$ is the base speed. Find the firing angle of the field rectifier circuit corresponding to a speed of 3000 rpm. What would be the firing angle for a speed that is twice base speed?

Solution Operation in region II should satisfy Eq. (4.178), requiring that

$$I_{f_b}\omega_b = I_f \omega$$

Invoking Eq. (4.191), we have

$$I_{f_b}\omega_b = I_{f_b}\omega\cos\alpha$$

Thus

$$\cos\alpha = \frac{\omega_b}{\omega} = \frac{2496.38}{3000}$$
$$\alpha = \cos^{-1} 0.83 = 33.7^\circ$$

For $\omega = 2\omega_b$,

$$\cos\alpha = 0.5$$

As a result,

$$\alpha = 60^\circ$$

Example 4.23 The armature voltage of a separately excited dc motor is controlled by a one-quadrant chopper with chopping frequency of 200 pulses per second from a 300 V dc source. The motor runs at a speed of 800 rpm when the chopper's time ratio is 0.8. Assume that the armature circuit resistance and inductance are 0.08 Ω and 15 mH, respectively, and that the motor develops a torque of 2.72 N·m per ampere of armature current.

Find the mode of operation of the chopper, the output torque, and horsepower under the specified conditions.

Solution From the problem specifications at 800 rpm, using Eq. (4.201), we get,

$$E_c = (2.72)\frac{2\pi}{60}(800) = 227.9 \text{ V}$$

The armature circuit time constant is obtained as

$$\tau = \frac{L_a}{R_a} = \frac{15 \times 10^{-3}}{0.08} = 187.5 \times 10^{-3}\ \mathrm{s}$$

The chopping period is given by

$$T = \frac{1}{200} = 5 \times 10^{-3}\mathrm{s}$$

We obtain the critical on-time using Eq. (4.197) as

$$\begin{aligned} t^*_{\text{on}} &= 187.5 \times 10^{-3}\, ln\left[1 + \frac{227.9}{300}(e^{5/187.5} - 1)\right] \\ &= 3.8 \times 10^{-3}\mathrm{s} \end{aligned}$$

We know that $t_{\text{on}} = 0.8 \times 5 \times 10^{-3} = 4 \times 10^{-3}$. As a result, we conclude that the chopper output current is continuous.

To obtain the torque output, we use Eq. (4.203) rearranged as

$$T_o = \frac{K_1\phi_f}{R_a}\left(\frac{t_{\text{on}}}{T}V_i - K_1\phi_f\omega\right)$$

Thus we obtain

$$\begin{aligned} T_o &= \frac{2.72}{0.08}[0.8(300) - 227.9] \\ &= 411.4\,\mathrm{N \cdot m} \end{aligned}$$

The power output is obtained as

$$\begin{aligned} P_o &= (411.4)\frac{2\pi}{60}(800) = 34.5 \times 10^3\ \mathrm{W} \\ &= 46.2\,\mathrm{hp} \end{aligned}$$

To illustrate the principle of field control, we have the following example.

Example 4.24 Assume for the motor of Example 4.23 that field chopper control is employed to run the motor at a speed of 1500 rpm while delivering the same power output as obtained at 800 rpm and drawing the same armature current.

Solution Although we can use Eq. (4.205), we use basic formulas instead,

$$E_c = \frac{P_a}{I_a} = \frac{34.5 \times 10^3}{151.3} = 227.9\,\text{V}$$

This is the same back EMF. Recall that

$$E_c = K_1 \phi_f \omega$$

Thus the required field flux is obtained as

$$\phi_{f_n} = \phi_{f_0} \frac{\omega_0}{\omega_n} = \frac{8}{15} \phi_{f_0}$$

where the subscript n denotes the present case, and the subscript 0 denotes the field flux for Example 4.23. Assume that ϕ_{f_0} corresponds to full applied field flux; then

$$\frac{\phi_{f_0}}{\phi_{f_n}} = \frac{V_i}{V_o} = \frac{15}{8}$$

The required chopped output voltage is V_o. Now we have

$$\frac{V_o}{V_i} = \frac{t_{on}}{T}$$

Thus

$$\frac{t_{on}}{T} = \frac{8}{15}$$

Assuming that $T = 5 \times 10^{-3}$ s, we get

$$t_{on} = 2.67 \times 10^{-3}\,\text{s}$$

EXAMPLE 9.1

A separately excited dc generator has the following parameters:

$$R_f = 100\,\Omega, \qquad L_f = 25\,\text{H}$$

$$R_a = 0.25\,\Omega, \qquad L_{aq} = 0.02\,\text{H}$$

$$K_g = 100\,\text{V} \quad \text{per field ampere at rated speed}$$

(a) The generator is driven at rated speed and a field circuit voltage $V_f = 200\,\text{V}$ is suddenly applied to the field winding.

(i) Determine the armature-generated voltage as a function of time.

(ii) Determine the steady-state armature voltage.

(iii) Determine the time required for the armature voltage to rise to 90 percent of its steady-state value.

(b) The generator is driven at rated speed and a load consisting of $R_L = 1\,\Omega$ and $L_L = 0.15\,\text{H}$ in series is connected to the armature terminals. A field circuit voltage $V_f = 200\,\text{V}$ is suddenly applied to the field winding. Determine the armature current as a function of time.

Solution

(a) Field circuit time constant $\tau_f = 25/100 = 0.25\,\text{sec}$.

(i) From Eq. 9.11,

$$e_a(t) = \frac{100 \times 200}{100}\left(1 - e^{-t/0.25}\right)$$

$$= 200(1 - e^{-4t})$$

(ii)

$$e_a(\infty) = 200\ \text{V}$$

(iii)

$$0.9 \times 200 = 200\left(1 - e^{-4t}\right)$$

$$t = 0.575\ \text{sec}$$

(b)

$$\tau_f = 0.25\ \text{sec}$$

$$\tau_{at} = \frac{0.15 + 0.02}{1 + 0.25} = 0.136\ \text{sec}$$

From Eq. 9.22,

$$I_a(s) = \frac{100 \times 200}{100 \times 1.25 \times 0.25 \times 0.136 s(s+4)(s+7.35)}$$

$$= \frac{4705.88}{s(s+4)(s+7.35)}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+7.35}$$

where $A_1 = \left.\dfrac{4705.88}{(s+4)(s+7.35)}\right|_{s=0} = 160$

$$A_2 = \left.\frac{4705.88}{s(s+7.35)}\right|_{s=-4} = -351$$

$$A_3 = \left.\frac{4705.88}{s(s+4)}\right|_{s=-7.35} = 191$$

From Eq. 9.25,

$$i_a(t) = 160 - 351e^{-4t} + 191e^{-7.35t} \quad \blacksquare$$

EXAMPLE 9.2

A separately excited dc motor has the following parameters:

$$R_a = 0.5\,\Omega, \qquad L_{aq} \simeq 0, \qquad B \simeq 0$$

The motor generates an open-circuit armature voltage of 220 V at 2000 rpm and with a field current of 1.0 ampere.

The motor drives a constant load torque $T_L = 25\,\text{N}\cdot\text{m}$. The combined inertia of motor and load is $J = 2.5\,\text{kg}\cdot\text{m}^2$. With field current $I_f = 1.0\,\text{A}$, the armature terminals are connected to a 220 V dc source.

(a) Derive expressions for speed (ω_m) and armature current (i_a) as a function of time.

(b) Determine the steady-state values of the speed and armature current.

Solution

(a)

$$E_a = K_m \omega_m$$

$$K_m = \frac{220}{(2000/60) \times 2\pi} = 1.05 \text{ V/rad/sec}$$

$$V_t = e_a + i_a R_a = K_m \omega_m + i_a R_a$$

$$T = K_m i_a = J\frac{d\omega_m}{dt} + T_L$$

From the last two equations,

$$V_t = K_m \omega_m + R_a \left(\frac{J}{K_m}\frac{d\omega_m}{dt} + \frac{T_L}{K_m} \right)$$

$$= K_m \omega_m + \frac{R_a J}{K_m}\frac{d\omega_m}{dt} + \frac{R_a T_L}{K_m}$$

$$= 1.05\omega_m + \frac{0.5 \times 2.5}{1.05}\frac{d\omega_m}{dt} + \frac{0.5 \times 25}{1.05}$$

$$= 1.05\omega_m + 1.19\frac{d\omega_m}{dt} + 11.9$$

$$V_t(s) = \frac{220}{s} = 1.05\omega_m(s) + 1.19 s\omega_m(s) + \frac{11.9}{s}$$

$$\omega_m(s) = \frac{220 - 11.9}{s(1.05 + 1.19s)}$$

$$= \frac{174.874}{s(s + 0.8824)}$$

$$= \frac{198.2}{s} - \frac{198.2}{s + 0.8824}$$

$$\omega_m(t) = 198.2(1 - e^{-0.8824t})$$

$$i_a = \frac{V_t - K_m \omega_m}{R_a}$$

$$= \frac{220 - 1.05\omega_m}{0.5}$$

$$= 440 - 2.1 \times 198.2(1 - e^{-0.8824t})$$

$$= 23.8 + 416.2e^{-0.8824t}$$

(b) Steady-state speed is $\omega_m(\infty) = 198.2$ rad/sec.
Steady-state current is $I_a = i_a(\infty) = 23.8$ A. ■

4.27. Figure 4-24 depicts the *Ward-Leonard system* for controlling the speed of the motor M. The generator field voltage, v_{fg}, is the input and the motor speed, ω_m, is the output. Obtain an expression for the transfer function for the system, assuming idealized machines. The load on the motor is given by $J\dot{\omega}_m + b\omega_m$, and the generator runs at constant angular velocity ω_g.

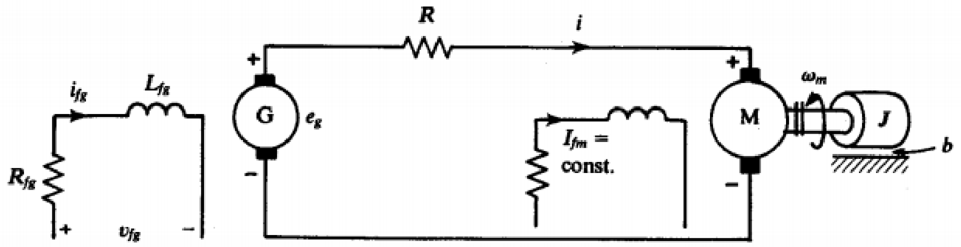


Fig. 4-24

From Fig. 4-24, (*4.16*), and (*4.19*), the equations of motion are:

$$v_{fg} = R_{fg}i_{fg} + L_{fg}\frac{di_{fg}}{dt} \quad \text{or} \quad V_{fg} = (R_{fg} + L_{fg}s)\, I_{fg}$$

$$e_g = k_g\omega_g i_{fg} = Ri + k_m I_{fm}\omega_m \quad \text{or} \quad k_g\omega_g I_{fg} = RI + k_m I_{fm}\Omega_m$$

$$T_m = k_m I_{fm} i = J\dot{\omega}_m + b\omega_m \quad \text{or} \quad k_m I_{fm} I = (b + Js)\Omega_m$$

Hence,

$$G(s) \equiv \frac{\Omega_m(s)}{V_{fg}(s)} = \frac{k_g\omega_g k_m I_{fm}}{(R_{fg} + L_{fg}s)(k_m^2 I_{fm}^2 + bR + JRs)}$$

4.28. A separately excited generator can be treated as a power amplifier when driven at a constant angular velocity ω_m. Derive an expression for the voltage gain, $V_L(s)/V_f(s)$, in terms of the parameters given in Fig. 4-25 and the proportionality constant $k\omega_m$ in (*4.8*).

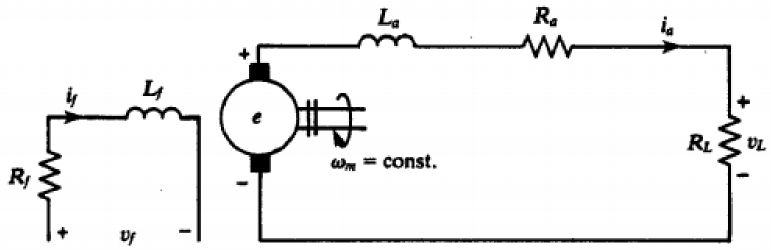


Fig. 4-25

In the transform domain, the equations are, from Fig. 4-25,

$$V_f = (R_f + L_f s)I_f \qquad \mathscr{E} = k\omega_m I_f$$

Also, $\mathscr{E} = (R_a + R_L + L_a s)I_a$ and $V_L + R_L I_a$. Consequently,

$$\frac{V_L(s)}{V_f(s)} = \frac{R_L k\omega_m}{R_f(R_a + R_L)} \cdot \frac{1}{(1 + \tau_f s)(1 + \tau_a s)}$$

where $\tau_f \equiv L_f/R_f$ and $\tau_a \equiv L_a/(R_a + R_L)$.

4.29. In Problem 4.28, $R_a = 0.1\ \Omega$, $R_f = 10\ \Omega$, $R_L = 0.5\ \Omega$, and $k\omega_m = 65$ V/A. What are (*a*) the voltage gain, and (*b*) the power gain if the generator is operating under steady state with 25 V applied across the field?

Notice that under steady state the d/dt terms go to zero, i.e., $s \to 0$. Therefore:

(*a*)
$$\text{voltage gain} = \frac{R_L k\omega_m}{R_f(R_a + R_L)} = \frac{(0.5)(65)}{(10)(0.1 + 0.5)} = 5.42$$

(*b*)
$$\text{input power to the field} = \frac{(25)^2}{10} = 62.5\ \text{W}$$

$$E = k\omega_m I_f = (65)\left(\frac{25}{20}\right) = 162.5\ \text{V}$$

$$I_a = \frac{162.5}{0.1 + 0.5} = 270.8\ \text{A}$$

$$\text{output power} = (270.8)^2(0.5) = 36\,675\ \text{W}$$

$$\text{power gain} = \frac{36\,675}{62.5} = 587$$

4.30. A separately excited dc motor, having a constant field current, accelerates a pure inertia load from rest. Represent the system by an electrical equivalent circuit. The various symbols are defined in Fig. 4-26(*a*).

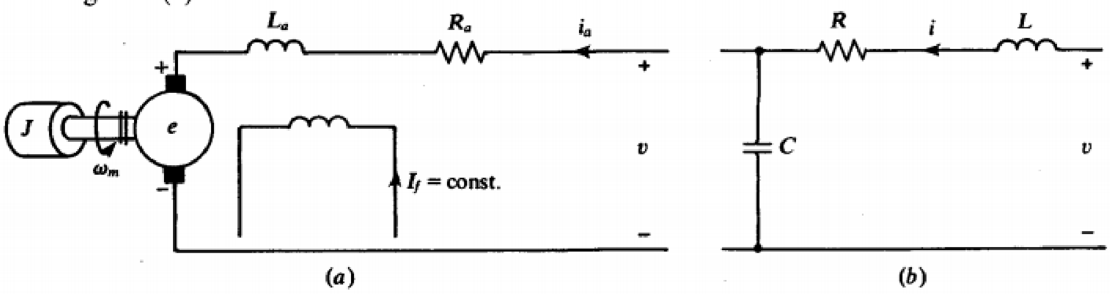


Fig. 4-26

The equations of motion are:

$$v = R_a i_a + L_a \frac{di_a}{dt} + e$$

$$e = kI_f \omega_m$$

$$T_e = kI_f i_a = J\dot{\omega}_m$$

These equations yield

$$v = R_a i_a + L_a \frac{di_a}{dt} + \frac{(kI_f)^2}{J} \int i_a \, dt$$

which is similar to

$$v = Ri + L \frac{di}{dt} + \frac{1}{C} \int i \, dt$$

corresponding to the circuit of Fig. 4-26(*b*). For equivalence: $R \leftrightarrow R_a$, $L \leftrightarrow L_a$, and $C \leftrightarrow J/(kI_f)^2$.