Example 12.1

The speed of a separately-excited d.c. motor with R_a = 1.2 Ω and L_a = 30 mH, is to be controlled using class-A thyristor chopper as shown in Fig.12.4. The d.c. supply V_d = 120 V. By ignoring the effect of the armature inductance L_a, it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle $\gamma = 1$.

(b) Draw the speed-torque characteristics for the motor when the duty cycle $\gamma = 1$. The motor design constant $Ke\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to $\gamma = 0.5$.

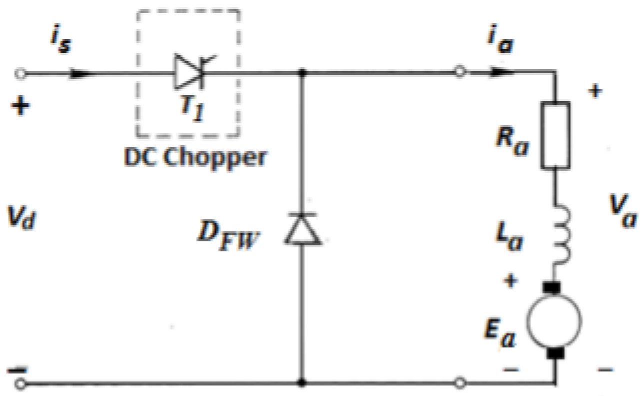


Fig. 12.4 Thyristor chopper drive.

Solution

The average armature voltage for $\gamma = 1$ is

$$V_{av} = \gamma V_d = 1 \times 120 = 120 \text{ V}$$

The motor's speed:

$$n = \frac{V_{av}}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

At no load $T_d = 0$, hence

$$\text{or} \quad n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{120}{0.042} = 2857 \text{ rpm}$$

At starting, $n = 0$. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$T_{st} = \frac{9.55\, \gamma V_d}{R_a} K_e \emptyset$$

$$\therefore \quad T_{st} = \frac{9.55 \times 120}{1.2} \times 0.042 = 40 \text{ Nm}$$

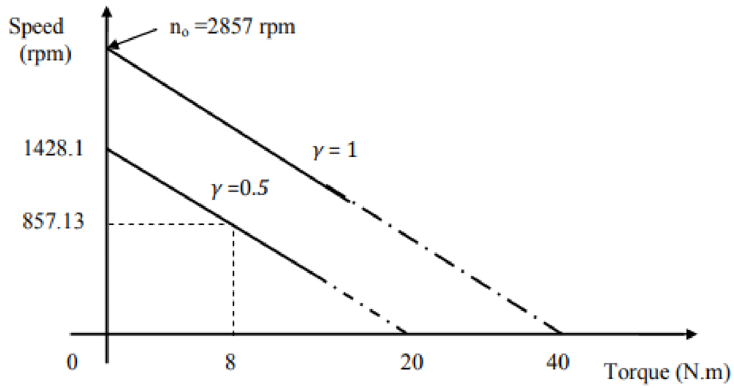


Fig. 12.5 Speed-torque characteristics.

(b) At $\gamma = 0.5$,

$$V_a = \gamma V_d = 0.5 \times 120 = 60 \text{ V}$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5 \text{ rpm}$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20 \text{ Nm}$$

At $\gamma = 0.5$, $T_L = 8$ Nm

$$n = \frac{60}{0.042} - \frac{1.2}{9.55 \times (0.042)^2} \times 8 = 857.13 \text{ rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 12.2

A d.c. motor is driven from a class-A d.c. chopper with source voltage of 220 V and at frequency of 1000 Hz. Determine the range of duty cycle to obtain a speed variation from 0 to 2000 rpm while the motor delivered a constant load of 70 Nm. The motor details as follows:
1kW, 200 V, 2000 rpm, 80% efficiency, R_a = 0.1 Ω, L_a = 0.02 H, and $K\emptyset$ = 0.54 V/rad /s.

Solution

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 2000}{60} = 209.3 \text{ rad/s}$$

$$I_{av} = \frac{P_{out}}{\text{Voltage } \times \eta} = \frac{1000}{200 \times 0.80} = 6.25 \text{ A}$$

$$I_{av} = \frac{\gamma V_d - E_a}{R_a} = \frac{\gamma V_d - K\emptyset\ \omega}{R_a}$$

$$T_{av} = K\emptyset\ I_{av}$$

$$T_{av} = K\emptyset\ \left(\frac{\gamma V_d - K\emptyset\ \omega}{R_a}\right) \text{Nm}$$

For $\omega_m = 0$

$$T_{av} = \frac{\gamma\ K\emptyset\ V_d}{R_a}$$

$$\gamma = \frac{T_{av}\ R_a}{K\emptyset\ V_d} = \frac{70 \times 0.1}{0.54 \times 220} = 0.058$$

$$\therefore\ \gamma_{min} = 0.058$$

and $$\gamma_{max} = \frac{T_{av} R_a}{K\emptyset V_d} + \frac{K\emptyset \omega_m}{V_d} = \frac{70 \times 0.1}{0.54 \times 220} + \frac{0.54 \times 209.3}{220} = 0.571$$

Hence the range of γ is $0.058 - 0.571$.

Example 12.3

In the microcomputer-controlled class-A IGBT transistor d.c. chopper shown in Fig.12.6, the input voltage $V_d = 260$ V, the load is a separately-excited d.c. motor with $R_a = 0.28\ \Omega$ and $L_a = 30$ mH. The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30 A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor n (rpm) when the chopper is switched fully ON such that the duty cycle $\gamma = 1.0$.

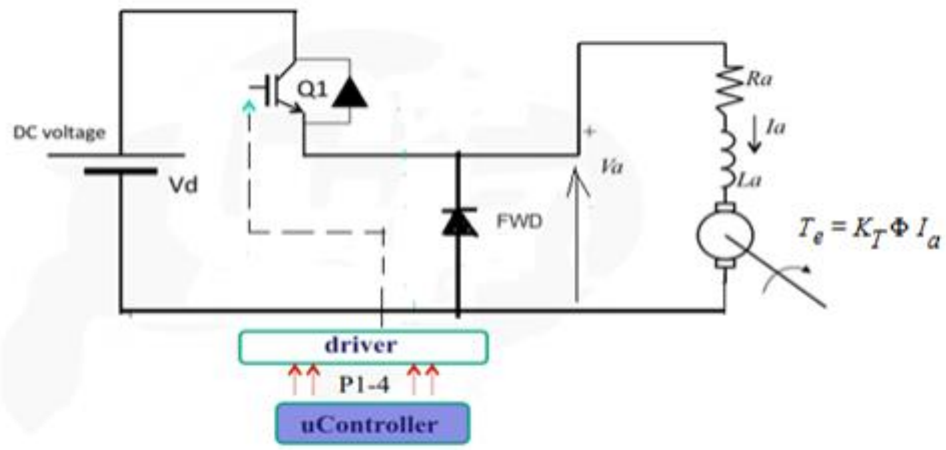


Fig.12.6 IGBT d.c. chopper drive.

Solution

(a) With steady-state operation of the motor, the armature induct behaves like a short circuit and therefore has no effect at all.

At stand still $n = 0$, and therefore , $E_a = 0$, hence from Eq.(12.22),

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30 \text{ A}$$

$$\therefore \ V_{a0} = 0.28 \times 30 = 8.4 \text{ V}$$

At full speed $n = 2500$ rpm ,

$$E_{a2500} = K_e \, \phi \, n = 0.1 \times 2500 = 250 \text{ V}$$

For separately-excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4 \text{ V}$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. $\gamma = 1$, then

$$V_a = V_d = 260 \text{ V} .$$

At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e \, \phi \, n + I_a R_a = 260\text{V}$$

$$0.1\, n + 30 \times 0.28 = 260 \qquad \rightarrow \qquad n = 2516 \text{ rpm}$$

Example 12.5

A separately-excited d.c. motor with R_a = 0.1 Ω and L_a = 20 mH, is to be controlled using class-A thyristor chopper. The d.c. supply is a battery with V_d = 400 V. The motor voltage constant is 5 V.s/rad. In the steady-state operation the average armature current I_a = 100 A and it is assumed to be continuous and ripple-free.

(a) For a duty cycle of 0.5, it is required to calculate (i) the input power to the motor, (ii) the speed of the motor, (iii) the developed torque. Mechanical, battery and semiconductor losses may be neglected.

(b) If the duty cycle of the chopper is varied between 20% and 80%, find the difference in speed resulting from this variation.

Solution

(a) Input power to the motor, speed of the motor and the developed torque are calculated as follows:

(i) For continuous current operation the input power is

$$P_{in} = V_a I_a = \gamma V_d I_a = 0.5 \times 400 \times 100 = 20 \text{ kW}$$

(ii) Speed of the motor can be calculated as,

The voltage across the armature circuit

$$V_a = \gamma V_d = 0.5 \times 400 = 200 \text{ V}$$

The induce voltage $E_a = K\emptyset\,\omega$

$$K\emptyset = 5 \text{ V.s/rad.}$$

$$E_a = V_a - I_a R_a = 200 - 100 \times 0.1 = 190 \text{ V}$$

$$\omega = \frac{E_a}{K\emptyset} = \frac{190}{5} = 38 \text{ rad/s}$$

To find the speed n in rpm

$$n = \frac{60}{2\pi}\omega = \frac{60}{2\pi} \times 38 = 363 \text{ rpm}$$

(iii) The torque produced by the motor,

$$T_m = \frac{E_a I_a}{\omega} = \frac{190 \times 100}{38} = 500 \text{ Nm}$$

(b) For duty cycle of 20%,

$$E_{a20\%} = \gamma V_d - I_a R_a = 0.2 \times 400 - 100 \times 0.1 = 70\ V$$

$$\omega_{20\%} = \frac{E_{a20\%}}{K\emptyset} = \frac{70}{5} = 14\ \frac{\text{rad}}{\text{s}} \quad \rightarrow \quad n_{20\%} = 14 \times \frac{60}{2\pi} = 133.7 \text{ rpm}$$

For duty cycle of 80%,

$$E_{a80\%} = \gamma V_d - I_a R_a = 0.8 \times 400 - 100 \times 0.1 = 310 \text{ V}$$

$$\omega_{80\%} = \frac{E_{a80\%}}{K\emptyset} = \frac{310}{5} = 62 \text{ rad/s} \rightarrow n_{80\%} = 62 \times \frac{60}{2\pi} = 592.3 \text{ rpm}$$

Hence the difference in speed is

$$n_{80\%} - n_{20\%} = 592.3 - 133.7 = 458.6 \text{ rpm}$$

Example 12.16. *A dc series motor is fed from 600 V dc source through a chopper. The dc motor has the following parameters :*

$$r_a = 0.04\ \Omega, \quad r_s = 0.06\ \Omega, \quad k = 4 \times 10^{-3}\ \text{Nm/amp}^2$$

The average armature current of 300 A is ripple free. For a chopper duty cycle of 60%, determine :

(a) input power from the source

(b) motor speed and (c) motor torque.

Solution. (*a*) Power input to motor

$$= V_t \cdot I_a = \alpha\, V_s \cdot I_a$$

$$= 0.6 \times 600 \times 300 = 108\ \text{kW}.$$

(*b*) For a dc series motor,

$$\alpha V_s = E_a + I_a R = k\, I_a\, \omega_m + I_a R$$

$$0.6 \times 600 = 4 \times 10^{-3} \times 300 \times \omega_m + 300\,(0.04 + 0.06)$$

$$\omega_m = \frac{360 - 30}{1.2} = 275\ \text{rad/sec or } 2626.1\ \text{rpm}$$

(*c*) Motor torque, $T_e = k\, I_a^2 = 4 \times 10^{-3} \times 300^2 = 360\ \text{Nm}.$

Example 12.17. *The chopper used for on-off control of a dc separately-excited motor has supply voltage of 230V dc, an on- time of 10 m sec and off-time of 15 m sec. Neglecting armature inductance and assuming continuous conduction of motor current, calculate the average load current when the motor speed is 1500 rpm and has a voltage constant of $K_v = 0.5$ V/rad per sec. The armature resistance is 3 Ω.* [*I.A.S., 1985*]

Solution. Chopper duty cycle

$$\alpha = \frac{T_{on}}{T_{on} + T_{off}} = \frac{10}{10 + 15} = 0.4$$

For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a = K_m \cdot \omega_m + I_a r_a$$

$$0.4 \times 230 = 0.5 \times \frac{2\pi \times 1500}{60} + I_a \times 3$$

$\therefore$ Motor load current, $I_a = \dfrac{92 - 25 \times \pi}{3} = 4.487$ A

Example 12.18. *A dc chopper is used to control the speed of a separately-excited dc motor. The dc supply voltage is 220 V, armature resistance r_a = 0.2 Ω and motor constant $K_a \phi$ = 0.08 V/rpm.*

This motor drives a constant torque load requiring an average armature current of 25 A. Determine (a) the range of speed control (b) the range of duty cycle α. Assumed the motor current to be continuous. [I.A.S., 1990]

Solution. For the motor armature circuit,

$$V_t = \alpha V_s = E_a + I_a r_a$$

As motor drives a constant torque load, motor torque T_e is constant and therefore armature current remains constant at 25 A.

Minimum possible motor speed is $N = 0$. Therefore,

$$\alpha \times 220 = 0.08 \times 0 + 25 \times 2.0 = 5$$

$$\alpha = \frac{5}{220} = \frac{1}{44}$$

Maximum possible motor speed corresponds to $\alpha = 1$, *i.e.* when 220 V dc is directly applied and no chopping is done.

∴ $$1 \times 220 = 0.08 \times N + 25 \times 0.2$$

or $$N = \frac{220 - 5}{0.08} = 2687.5 \text{ rpm}$$

∴ Range of speed control : $0 < N < 2687.5$ rpm and corresponding range of duty cycle : $\frac{1}{44} < \alpha < 1$.

Example 12.21. *A dc chopper is used for regenerative braking of a separately-excited dc motor. The dc supply voltage is 400V. The motor has $r_a = 0.2\ \Omega$, $K_m = 1.2$ V-s/rad. The average armature current during regenerative braking is kept constant at 300 A with negligible ripple.*

For a duty cycle of 60% for a chopper, determine

(a) power returned to the dc supply

(b) minimum and maximum permissible braking speeds and

(c) speed during regenerative braking.

Solution. (*a*) Average armature terminal voltage,

$$V_t = (1 - \alpha)\ V_s = (1 - 0.6) \times 400 = 160 \text{ V}.$$

Power returned to the dc supply

$$= V_t I_a = 160 \times 300 \text{ W} = 48 \text{ kW}$$

(*b*) From Eq. (12.41), minimum braking speed is

$$\omega_{mn} = \frac{I_a \cdot r_a}{K_m} = \frac{300 \times 0.2}{1.2} = 50 \text{ rad/s or } 477.46 \text{ rpm}$$

From Eq. (12.42), maximum braking speed is

$$\omega_{mx} = \frac{V_s + I_a \cdot r_a}{K_m} = \frac{400 + 300 \times 0.2}{1.2}$$

$$= 383.33 \text{ rad/s or } 3660.6 \text{ rpm}$$

(*c*) When working as a generator during regenerative braking, the generated emf is $E_a'' = K_m\ \omega_m = V_t + I_a r_a = 160 + 300 \times 0.2 = 220$ V

$\therefore$ Motor speed, $\omega_m = \dfrac{220}{1.2}$ rad/s or 1750.7 rpm

10. A 220-V, 80-A, separately excited dc motor operating at 800 rpm has an armature resistance of 0.18 Ω. The motor speed is controlled by a chopper operating at 1000 Hz. If the motor is regenerating, (a) determine the motor speed at full load current with a duty ratio of 0.7, this being the minimum permissible ratio (b) Repeat the calculation with a duty ratio of 0.1.

Solution

(a) When the machine is working as a motor, E_b is obtained from the equation

$$E_b = E - I_a R_a = 220 - 80 \times 0.18 = 220 - 14.4 = 205.6 \text{ V}$$

From the equation

$$E_b = kN$$

$$k = \frac{205.6}{800} = 0.257$$

When it is regenerating, the step-up configuration of Fig. 3.7(c) holds good. Thus,

$$E_b = E(1-\delta) + I_a R_a = 220(1-0.7) + 80 \times 0.18 = 66 + 14.4 = 80.4 \text{ V}$$

$$N = \frac{E_b}{k} = \frac{80.4}{0.257} = 313 \text{ rpm}$$

(b) The speed for $\delta = 0.1$ is obtained as follows:

$$E_b = 220(1-0.10) + 80 \times 0.18 = 198 + 14.4 = 212.4 \text{ V}$$

Therefore the speed is

$$N = \frac{212.4}{0.257} = 827 \text{ rpm}$$

11. A 250-V, 105-A, separately excited dc motor operating at 600 rpm has an armature resistance of 0.18 Ω. Its speed is controlled by a two-quadrant chopper with a chopping frequency of 550 Hz. Compute (a) the speed for motor operation with a duty ratio of 0.5 at 7/8 times the rated torque and (b) the motor speed if it regenerates at $\delta = 0.7$ with rated current.

Solution

(a) The initial back emf is to be determined from the equation

$$E_b = E - I_a R_a = 250 - 105 \times 0.18 = 231.1 \text{ V}$$

Hence the back emf constant $k = 231.1/600 = 0.385$. A fraction 7/8 of the rated current $I'_a = 7/8 \times 105 = 91.875$ A. The new E_b is obtained as

$$E'_b = E\delta - I'_a R_a$$

where $\delta = 0.5$ and $I'_a = 91.875$. Its numerical value is

$$E'_b = 250 \times 0.5 - 91.875 \times 0.18 = 108.46\text{V}$$

The new speed is

$$N = \frac{E'_b}{k} = \frac{108.46}{0.385} = 282 \text{ rpm}$$

(b) When it is regenerating, Eqn (7.54) is to be used. Thus,

$$I_a = \frac{E_b - E(1-\delta)}{R_a}$$

or

$$E_b = E(1-\delta) + I_a R_a$$

Substituting values gives

$$E_b = 250(1 - 0.7) + 105 \times 0.18 = 93.9 \text{ V}$$

From this, the speed $N = 93.9/0.385 = 244$ rpm

12. A 300-V, 100-A, separately excited dc motor operating at 600 rpm has an armature resistance and inductance of 0.25 Ω and 16 mH, respectively. It is controlled by a four-quadrant chopper with a chopper frequency of 1 kHz. (a) If the motor is to operate in the second quadrant at 4/5 times the rated current, at 450 rpm, calculate the duty ratio. (b) Compute the duty ratio if the motor is working in the third quadrant at 500 rpm and at 60% of the rated torque.

Solution

(a) $E_b - I_a R_a = 300 - 100 \times 0.25 = 275$ V. Back emf constant $k = E_b/N = 275/600 = 0.458$. Operation in the second quadrant implies that the motor works as a generator. Hence the motor terminal voltage V_a is written as

$$V_a = E(1 - \delta)$$

New current

$$I_a = \frac{4}{5} \times 100 = 80 \text{ A}$$

Hence,

$$E_b = V_a + I_a R_a$$
$$kN = E(1 - \delta) + I_a R_a$$

Substitution of values gives

$$0.458 \times 450 = 300(1 - \delta) + 80 \times 0.25$$

This yields $\delta = 0.38$.

(b) In the third quadrant, the machine works in the motoring mode but with reverse voltage and reverse current. The voltage equation relevant in this case is

$$E_b = V_a - I_a R_a$$

where

$$E_b = kN = 0.458 \times 500$$

$$V_a = E\delta = 300\delta$$

and

$$I_a = 0.6 \times 100 = 60 \text{ A}$$

By substituting numerical values, the equation becomes

$$0.458 \times 500 = 300 \times \delta - 60 \times 0.25$$

This gives $\delta = 0.813$.

8. A separately excited dc motor having a rating of 60 h.p. and running at 1200 rpm is supplied by a dc chopper whose source is a battery of 500 V. The field is also supplied by a chopper whose source is another battery of 300 V. The data pertaining to this chopper-based drive are as follows: $R_a = 0.18\ \Omega$, $K_b = 70$ V/(Wb rad/s), $\phi_f = 0.16 I_f$, $R_f = 120\ \Omega$, and $(\tau_{ON}/\tau)_f$ for the field chopper is 0.85. Assume that the load has sufficient inductance to make the load current continuous. If (τ_{ON}/τ) a for the armature is 0.65, compute the (a) mean armature current, (b) torque developed by the motor, (c) equivalent resistance for the armature circuit, and (d) total input power.

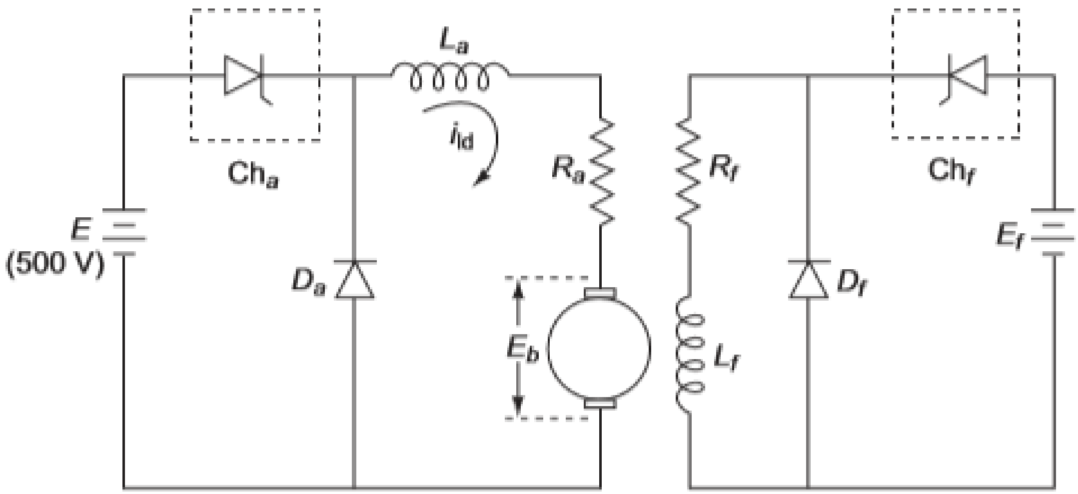


Fig. 7.33

Solution

(a) Let the source currents at the armature and field sides be denoted, respectively, as I_{sa} and I_{sf}. The circuit is shown in Fig. 7.33. Equation (3.23) gives the torque developed as

$$\mathcal{T}_d = K_t \phi_f I_{\text{ld}}(\omega) = K_b K_1 I_f I_a$$

Here,

$$K_b K_1 = 70 \times 0.016 = 1.12$$

$$I_f = E_f \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f \frac{1}{R_f} = \frac{300 \times 0.85}{120} = 2.125 \text{ A}$$

Hence the average torque is

$$\mathcal{T}_d = 1.12 \times 2.125 I_a = 2.38 I_a$$

$$\omega = \frac{2\pi \times 1200}{60} = 125.7$$

$$E_b = K_b K_1 I_f \omega = 2.38 \times 125.7 = 299 \text{ V}$$

$$V_a = E \left(\frac{\tau_{\text{ON}}}{\tau}\right)_a = 500 \times 0.65 = 325 \text{ V}$$

The mean armature current is

$$I_a = \frac{V_a - E_b}{R_a} = \frac{325 - 299}{0.18} = 144.4 \text{ A}$$

(b) Torque developed $\mathcal{T}_d = 2.38 \times 144.4 = 343.8$ N m.
(c) Input (or source) current is

$$I_{sa} = I_a \left(\frac{\tau_{\text{ON}}}{\tau}\right)_a = 144.4 \times 0.65 = 93.9 \text{ A}$$

Also,

$$I_{sf} = I_f \times \left(\frac{\tau_{\text{ON}}}{\tau}\right)_f = 2.125 \times 0.85 = 1.81 \text{ A}$$

$$\begin{aligned}\text{Armature source equivalent resistance} &= \frac{\text{armature source voltage}}{\text{armature source current}} \\ &= \frac{500}{93.9} \\ &= 5.32\ \Omega\end{aligned}$$

(d) Total input power $=$ power input to armature $+$ power input to field $= E_a I_{sa} + E_f I_{sf}$. Hence, it is given as $P_i = 500 \times 93.9 + 300 \times 1.81 = 46{,}950 + 543 = 47{,}493$ W ≈ 47.5 kW.

9. A separately excited dc motor has a rating of 50 h.p. and when supplied by a battery of 480 V through a chopper, it has a mean armature current of 120 A. The field is also supplied by a chopper whose source is a battery of 250 V. Other data for this chopper-based drive are $R_a = 0.2\ \Omega$, $R_f = 125\ \Omega$, $K_b = 72$ V/(Wb rad/s), $\phi_f = 0.015 I_f$, $(\tau_{ON}/\tau)_a = 0.7$, and $(\tau_{ON}/\tau)_f = 0.9$. The armature circuit has sufficient inductance to make the current continuous. Compute the (a) speed of the motor, (b) torque developed by the motor, (c) equivalent resistance, and (d) total input power.

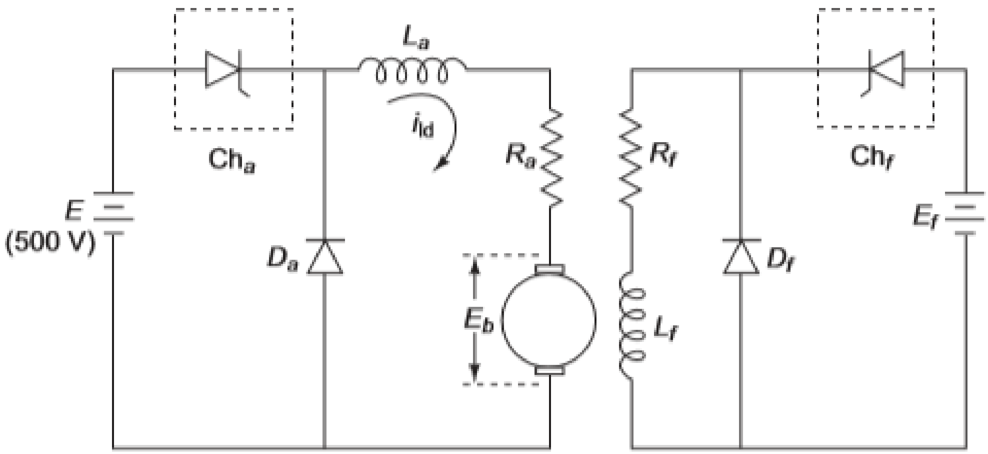


Fig. 7.33

Solution

(a) The circuit is the same as that given in Fig. 7.33.

$$V_a = 480\left(\frac{\tau_{ON}}{\tau}\right)_a = 480 \times 0.7 = 336 \text{ V}$$

$$E_b = V_a - I_a R_a = 336 - 120 \times 0.2 = 312 \text{ V}$$

$$\omega = \frac{E_b}{K_b \phi_f} = \frac{E_b}{K_b \times 0.015 I_f}$$

Here,

$$I_{sf} = 250\left(\frac{\tau_{ON}}{\tau}\right)_f \frac{1}{R_f} = \frac{250 \times 0.9}{125} = 1.8 \text{ A}$$

Hence,

$$\omega = \frac{E_b}{K_b \times 0.015 I_f} = \frac{312}{72 \times 0.015 \times 1.8} = 160 \text{ rad/s}$$

Also,

$$\text{speed} = \frac{60\omega}{2\pi} = \frac{60 \times 160}{2\pi} = 1528 \text{ rpm}$$

(b)

$$\begin{aligned}\text{Torque developed} &= K_b \times 0.015 I_f I_a \\ &= 72 \times 0.015 \times 1.8 \times 120 \\ &= 233.3 \text{ N m}\end{aligned}$$

The source current on the armature side is

$$I_{sa} = I_a \left(\frac{\tau_{ON}}{\tau}\right)_a = 120 \times 0.7 = 84 \text{ A}$$

(c)

$$\begin{aligned}\text{Armature source equivalent resistance} &= \frac{\text{armature source voltage}}{\text{armature source current}} \\ &= \frac{480}{84} = 5.7\ \Omega\end{aligned}$$

(d) Total input power

$$\begin{aligned}P_i &= \text{power input to armature} + \text{power input to field} \\ &= E_a I_{sa} + E_f I_{sf}\end{aligned}$$

where

$$I_{sf} = I_f \left(\frac{\tau_{ON}}{\tau}\right)_f = 1.8 \times 0.9 = 1.62 \text{ A}$$

Hence,

$$P_i = 480 \times 84 + 250 \times 1.62 = 40{,}320 + 405 = 40{,}725 \text{ W} \approx 40.7 \text{ kW}$$

12. A dc series motor is supplied by a battery of 420 V with a dc chopper interposed between the battery and the motor. It has a mean armature current of 120 A. Other data for this chopper-based drive are $R_a = 0.05\ \Omega$, $R_f = 0.06\ \Omega$, $K_b = 0.72$ V/(Wb rad/s), and $\phi_f = 0.016\ I_a$. The duty ratio τ_{ON}/τ is 0.65. Compute the (a) speed of the motor, (b) torque developed by the motor, (c) equivalent input resistance, and (d) total input power.

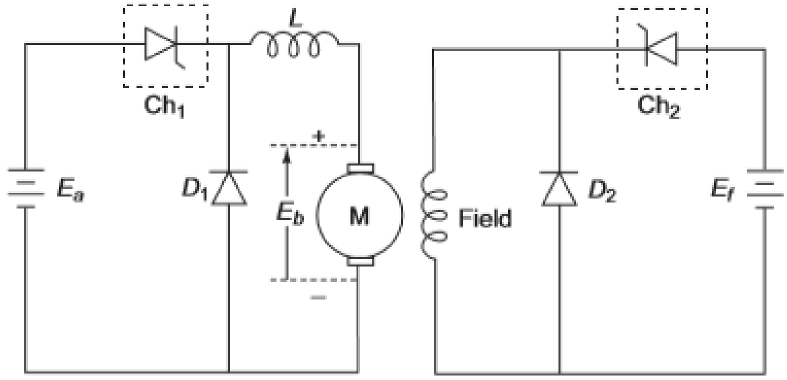


Fig. 7.25 Circuit diagram of a dc drive with the armature and field fed by separate choppers

Solution

The set-up is as shown in Fig. 7.25:

$$V_a = 420\frac{\tau_{ON}}{\tau} = 420 \times 0.65 = 273 \text{ V}$$

$$\begin{aligned} E_b &= V_a - I_a(R_a + R_f) \\ &= 273 - 120(0.05 + 0.06) \\ &= 259.8 \text{ V} \end{aligned}$$

$$\begin{aligned} \omega &= \frac{E_b}{K_b \times 0.016 I_a} \\ &= \frac{259.8}{0.72 \times 0.016 \times 120} \\ &= 188 \text{ rad/s} \end{aligned}$$

$$\begin{aligned} \text{Speed } N &= \frac{188 \times 60}{2\pi} \\ &= 1795 \text{ rpm} \end{aligned}$$

$$\begin{aligned} \text{Torque developed by the motor} &= K_b \times 0.016(I_a)^2 \\ &= 0.72 \times 0.016 \times (120)^2 \\ &= 165.9 \text{ N m} \end{aligned}$$

The source current is

$$I_s = I_a\frac{\tau_{ON}}{\tau} = 120 \times 0.65 = 78 \text{ A}$$

$$\text{Input power} = EI_s = 420 \times 78 = 32{,}760 \text{ W} = 32.76 \text{ kW}$$

Example 4

A separately excited d.c. motor with R_a = 1.2 ohms and L_a = 30 mH , is to be controlled using class-A thyristor chopper as shown in Fig.9.11 .The d.c. supply V_d = 120V . By ignoring the effect of the armature inductance L_a , it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle γ =1.

(b) Draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant $K e\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to γ = 0.5.

Solution

The average armature voltage is

$$V_{av} = \gamma\, V_d \;= 1\times\ 120 = 120\,V$$

The motor's speed:

$$n\ = \frac{V_{av}}{K_e\emptyset} - \ \frac{R_a}{K_T K_e \emptyset^2} T_d$$

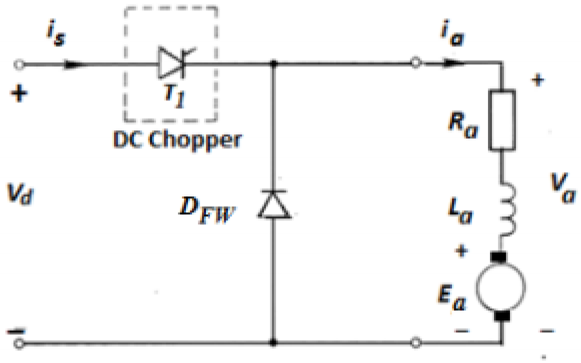


Fig. 9.11 Thyristor chopper drive.

At no load T_d = 0, hence

or $$n_o\ = \frac{\gamma V_d}{K_e\emptyset} = \frac{120}{0.042} = 2857\,rpm$$

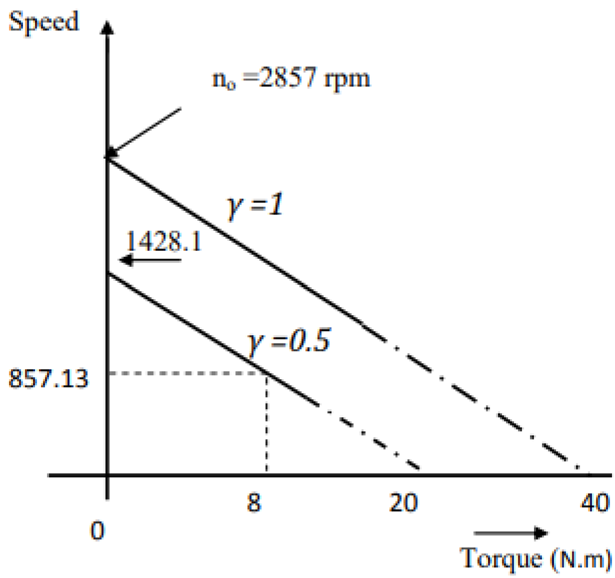
At starting, n = 0. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e\emptyset} - \ \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$\therefore T_{st}\ = \frac{9.55\ \gamma V_d}{R_a} K_e\emptyset$$

$$T_{st}\ = \frac{9.55\ x120}{1.2}\times\ 0.042 = 40\ N.m$$

Fig.9.11 Speed-torque characteristics



(b) At $\gamma = 0.5$

$$V_a = \gamma\, V_d = 0.5\ x\ 120 = 60\ V$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5\ rpm$$

$$T_{st} = \frac{9.55 \times 60}{1.2} \times 0.042 = 20\ \mathrm{N.m}$$

At $\gamma = 0.5$, $T_L = 8$ N.m

$$n = \frac{60}{0.042} - \frac{1.2}{9.55(0.042)^2} \times 8 = 857.13\ \mathrm{rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 5

In the microcomputer -controlled class –A IGBT transistor DC chopper shown in Fig.12.6, the input voltage $V_d = 260V$, the load is a separately excited d.c. motor with Ra = 0.28 Ω and L_a = 30 mH . The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor *n* (rpm) when the chopper is switched fully ON such that the duty cycle γ = 1.0.

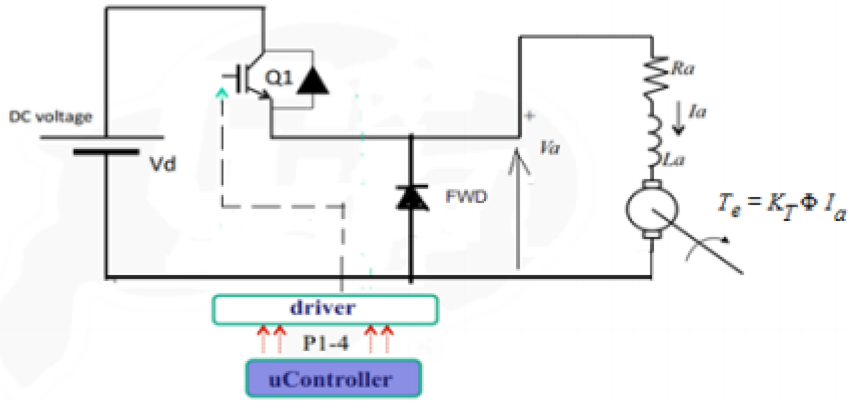


Fig.12.6 IGBT Chopper drive.

Solution

(a) With steady – state operation of the motor, the armature inductance L_a behaves like a short circuit and therefore has no effect at all.

At stand still n = 0 , and therefore $E_a = 0$, hence from Eq.(12.22)

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30\ A$$

$$\therefore\ V_{a0} = 0.28 \times 30 = 8.4\ V$$

At full speed n = 2500 rpm ,

$$E_{a2500} = K_e\,\phi\, n = 0.1 \times 2500 = 250\ V$$

For separately excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4\ V$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. $\gamma = 1$, then $V_a = V_d = 260$ V .

At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e\, \phi\, n + I_a R_a\ = 260\ \text{V}$$

$$0.1\, n + 30\ x\ 0.28\ = 260 \qquad \rightarrow \qquad n = 2516\ \text{rpm}$$

Example 1: A separately-excited d.c. motor with $R_a = 0.3\Omega$, and $L_a = 15\,mH$ is to be speed controlled over a range 0-2000 r.p.m. The d.c. supply is 220V. The load torque is constant and requires an average armature current of 25A.

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi = 0.1002$ V/rpm.

Solution: In the steady-state, the armature inductance has no effect. The required motor terminal voltages are:

At $n = 0$, $E_b = 0$, so that

$$V_{dc} = E_b + I_a R_a = I_a R_a = 25 \times 0.3 = 7.5\,V.$$

At $n = 2000$ rpm,

$$E_b = K_e\Phi n = 0.1002 \times 2000 = 200.4\,V.$$

$$\therefore V_{dc} = E_b + I_a R_a = 200.4 + 25 \times 0.3 = 207.9\,V.$$

$$V_o = \gamma V_d$$

To give $V_o = 7.5\,V$ $\therefore$ $7.5 = \gamma_o \times 220$ or $\gamma_o = \frac{7.5}{220} = 0.034$

To give $V_o = 207.9\,V$ $\therefore$ $207.9 = \gamma_{2000} \times 220$

or $\gamma_{2000} = \frac{207.9}{220} = 0.943$.

Range of γ: $0.034 \leq \gamma \leq 0.943$.

(b) If the chopper was to be switched fully on, what is the speed of the motor when $\gamma = 1$.

Sol. when $\gamma = 1$, $V_o = 220$.

$$\therefore n = \frac{E_b}{K_e\Phi}, \quad E_b = V_o - I_a R_a = 220 - 25 \times 0.3 = 212.5\,V$$

$$n = \frac{212.5}{0.1002} = \underline{\underline{2121}} \text{ rpm}$$

Example :

An electrically-driven automobile is powered by d.c. series motor rated at 100V, 200A. The motor resistance and inductance are respectively 0.65 Ω and 6 mH. power is supplied from ideal battry of 120V via class-A d.c. chopper having a fixed frequency of 100 Hz. The machine constant $K_e\phi = 0.00025$ V/rpm and the motor speed is 2500 rpm. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor when running at 2500 rpm with duty cycle γ of 3/5.

Solution:

Chopping period $T = \frac{1}{f} = \frac{1}{100} = 10$ ms.

$$t_{on} = \gamma T = \frac{3}{5} \times 10 = 6\text{ ms}$$

$$\therefore\ t_{off} = 10 - 6 = 4\text{ ms.}$$

$$I_{max} = \frac{V_{av}}{R_a} + \frac{t_{off}}{2L_a} V_{av} = \frac{\gamma V_i}{R_a} + \frac{t_{off}}{2L_a} \gamma V_i$$

$$= \frac{\frac{3}{5} \times 120}{0.65} + \frac{4 \times 10^{-3}}{2 \times 6 \times 10^{-3}} \left(\frac{3}{5} \times 120\right)$$

$$= 110.7 + 24 = 134.7\text{ A}$$

$$I_{min} = \frac{V_{av}}{R_a} - \frac{t_{off}}{2L_a} V_{av} = 86.76\text{ A.}$$

$$= 110.7 - 24$$

For series motor:

Mean torque: $T_e = K_T \phi I_{av}^2 = 9.55\, K_e \Phi \left(\frac{I_{max} + I_{min}}{2}\right)^2$

$$= 9.55 \times 0.00025 \left(\frac{134.7 + 86.76}{2}\right)^2$$

$$= 30\text{ N.m.}$$

Mean power: $P_e = \omega T_e = \frac{2\pi}{60} \times 2500 \times 30 = 7.85$ kW

$$= 10.5\text{ hp.}$$

EXAMPLE 7.1 A separately excited dc motor runs at 1000 rpm from a 200 V dc supply. What will be the speed of the motor when power is supplied from a single-phase full converter working at $\alpha = 60°$. The supply voltage is 230 V (rms).

Solution

Neglecting the $I_a R_a$ drop and assuming constant excitation. The ratio of speeds will be same as the ratio of the applied voltages at the given conditions. Therefore,

$$\frac{V_{a1}}{V_{a2}} = \frac{N_1}{N_2}$$

$$N_2 = \frac{V_{a2} N_1}{V_{a1}}$$

$$= \frac{2 \times 230 \times \sqrt{2} \times \cos 60° \times 1000}{\pi \times 200}$$

$$= 518 \text{ rpm}$$

EXAMPLE 7.2 A dc drive works at 1100 rpm when fed from a 220 V dc source. The same drive is supplied from a chopper connected to 220 V dc mains. What will be the duty ratio to obtain 900 rpm. Neglect the $I_a R_a$ drop.

Solution

The ratio of speeds can be equated to the ratio of applied voltages.

$$\frac{V_{a1}}{V_{a2}} = \frac{N_1}{N_2}$$

$$V_{a2} = V_{a1} \times \frac{N_2}{N_1}$$

$$= 220 \times \frac{900}{1100} = 180 \text{ volts}$$

Since the field is constant,

Hence,

$$\text{Duty ratio } \delta = \frac{180}{220} = 0.82$$

EXAMPLE 7.4 A single quadrant dc chopper is used to control the speed of a separately excited armature controlled dc motor. The chopper has a supply voltage of 230 V dc. The on time and off time of the chopper are 10 ms and 25 ms respectively. Also, $R_a = 2\,\Omega$. Assuming continuous conduction of the motor current, determine the average armature current and torque developed by the motor when it is running at speed of 1400 rpm. The back emf constant of the motor is 0.5 V/rad/s.

Solution

$$\text{Back emf, } E_b = 1400 \times 2\pi \times \frac{0.5}{60}$$
$$= 73.30 \text{ volts}$$

$$V_a = V_s \times \frac{T_{on}}{T} = 230 \times \frac{10}{25} = 92 \text{ volts}$$

$$I_a = \frac{(92 - 73.3)}{2} = 9.35\,\text{A}$$

$$T = \text{Torque constant} \times I_a$$

Also,

$$\text{Torque constant} = 0.5 \text{ Nm A}^{-1}$$

Therefore,

$$\text{Torque } T = 9.35 \times 0.5$$
$$= 4.675 \text{ Nm}$$

Describe speed control of DC series motor using step down chopper.

Ans:

Speed control of DC series motor with step down chopper:

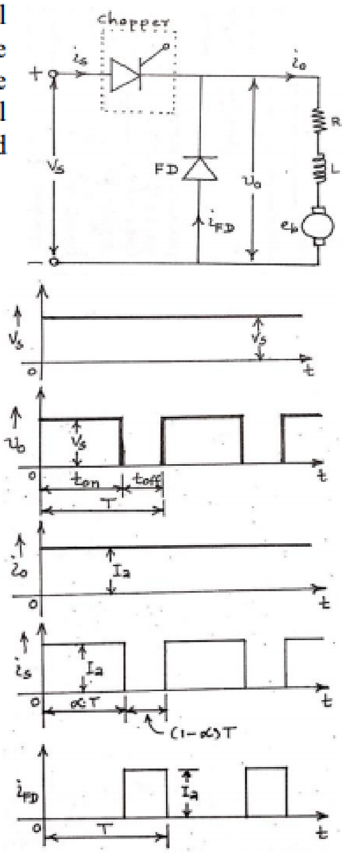


Figure shows the basic arrangement for speed control of DC series motor using step down chopper. Armature current is assumed continuous and ripple free. The waveforms for the source voltage V_s, Motor terminal voltage v_0, motor current i_0, dc source current i_s and freewheeling diode current i_{FD} are also shown.

Average motor voltage is given by,

$$V_0 = \frac{t_{on}}{T} V_s = \alpha V_s = f t_{on} V_s$$

where α = duty cycle = $\frac{t_{on}}{T}$

and f = Chopping frequency = $\frac{1}{T}$

Power delivered to motor is given by,

Power delivered to motor = Average motor voltage × Average motor current

$$= V_t I_a = \alpha\, V_s I_a$$

Motor voltage equation can be expressed as,

$$V_0 = \alpha\, V_s = E_b + I_a(R_a + R_{se})$$

The back emf is proportional to speed,

$$E_b \propto \omega_m \therefore E_b = K_m \omega_m$$

Thus voltage equation becomes,

$$V_0 = \alpha\, V_s = K_m \omega_m + I_a(R_a + R_{se})$$

The speed can be obtained as,

$$\omega_m = \frac{\alpha\, V_s - I_a(R_a + R_{se})}{K_m}$$

It is seen that by varying the duty cycle α of the chopper, armature terminal voltage can be controlled and thus speed of the dc series motor can be regulated.

EXAMPLE 7.1

A 230-V, 6-A, 1,500-rpm separately excited dc motor has an armature resistance of 5.1 Ω. A 1-quadrant chopper supplied from a 300-V dc bus is operating at a duty ratio of 60% and supplies power to the motor armature at rated current. Compute the motor speed.

SOLUTION:

At rated condition,

$$\begin{aligned} E_{b\ rated} &= V_a - I_{a\ rated} \times r_a \\ &= 230 - (5.1 \times 6) \\ &= 199.4\ V \end{aligned}$$

$$E_{b\ rated} \propto 1{,}500\ rpm (N_{rated})$$

At 60% duty ratio, $E_{b1} \propto N_1$.

$$\begin{aligned} \text{Armature voltage} &= 300 \times (60/100) \\ &= 180\ V \end{aligned}$$

$$\begin{aligned} \text{Back-emf, } E_{b1} &= 180 - (5.1 \times 6) \\ &= 149.4\ V \end{aligned}$$

Therefore,

$$\begin{aligned} N_1 &= \frac{E_{b1}}{E_{b\ rated}} N_{rated} \\ &= \frac{149.4}{199.4} \cdot 1500 \approx 1124\ rpm \end{aligned}$$

EXAMPLE 7.2

A first-quadrant dc/dc converter is fed from a 300-V dc bus. When the converter supplies power to a separately excited dc motor at 40% duty ratio, the average armature current is 5 A at 1,560 rpm. What is the duty ratio required to reduce the speed to 1,300 rpm for the same armature current? The armature resistance is 5.1 Ω.

SOLUTION:

At 40% duty ratio, armature voltage is

$$\begin{aligned} V_a &= 300 \times \frac{40}{100} \\ &= 120\ \text{V} \end{aligned}$$

$$\begin{aligned} \text{Back-emf at 40\% duty ratio, } E_{b1} &= 120 - (I_a r_a) \\ &= 120 - (5 \times 5.1) \\ &= 94.5\ \text{V} \end{aligned}$$

$$E_{b1} \propto 1{,}560\ \text{rpm}$$

Now back-emf, E_{b2}, at 1,300 rpm can be related as $E_{b2} \propto$ 1,300 rpm.

$$E_{b2} = \frac{N_2}{N_1} E_{b1} = 78.75\ \text{V}$$

$$\text{Armature voltage, } V_a = E_{b2} + I_a r_a$$

$$V_a = 300 \times D$$

Therefore,

$$D = 34.75\%$$

EXAMPLE 7.3

A separately excited dc motor is fed from a 440-V dc source through a single-quadrant chopper, $r_a = 0.2\ \Omega$, and armature current is 175 A. The voltage and torque constants are equal at 1.2 V/rad/s. The field current is 1.5 A. The duty cycle of chopper is 0.5. Find (a) speed and (b) torque.

SOLUTION:

a.

$$\begin{aligned} E_b &= 220 - 175 \times 0.2 = 185\ \text{V} \\ &= 1.2 \times \omega_r \times I_f \\ \omega_r &= 185/(1.2 \times 1.5) \\ &= 102.77\ \text{rad/s} \\ &= 981.38\ \text{rpm} \end{aligned}$$

b.

$$\text{Torque} = 1.2 \times 1.5 \times 175 = 315\ \text{N-m}$$

EXAMPLE 7.4

A separately excited dc motor has the following name plate data: 220 V, 100 A, 2,200 rpm. The armature resistance is 0.1 Ω, and inductance is 5 mH. The motor is fed by a chopper that is operating from a dc supply of 250 V. Due to restrictions in the power circuit, the chopper can be operated over a duty cycle ranging from 30% to 70%. Determine the range of speeds over which the motor can be operated at rated torque.

SOLUTION:

Because e torque is constant, i_a is the same for all the values of D.

$$V_{o(av)} = DV_{dc}$$

At $D = 0.3$,

$$\begin{aligned} V_{o(av)} &= 0.3 \times 250 \\ &= 75 \text{ V} \\ E_{b(0.3)} &= V_{o(av)} - I_a r_a \\ &= 75 - (100 \times 0.1) \end{aligned}$$

At $D = 0.7$,

$$\begin{aligned} V_{o(av)} &= 0.7 \times 250 \\ &= 175 \text{ V} \\ E_{b(0.7)} &= 175 - 10 \\ &= 165 \text{ V} \end{aligned}$$

Under rated conditions, $V_a = 220$ V, $I_a = 100$ A, $r_a = 0.1$ Ω

$$\begin{aligned} E_{b(rated)} &= 220 - (100 \times 0.1) \\ &= 210 \text{ V} \end{aligned}$$

$$N_r = 2200 \text{ rpm}$$

$$\frac{N_{0.7}}{N_r} = \frac{E_{b(0.7)}}{E_{b(rated)}}$$

$$\begin{aligned} N_{(0.7)} &= \frac{165}{210} \times 2200 \\ &= 1{,}728.5714 \text{ rpm} \end{aligned}$$

$$\begin{aligned} N_{(0.3)} &= \frac{65}{210} \times 2200 \\ &= 680.95 \text{ rpm} \end{aligned}$$

Hence speed can be varied in the range $680.95 \le N \le 1728.5714$.

EXAMPLE 7.5

A separately excited dc motor has an armature resistance 2.3 Ω, and armature current is 100 A. (a) Find the voltage across the braking resistance for a duty ratio of 25%. (b) Find the power dissipated in braking resistance.

SOLUTION:

$$\text{Average current} = I_a(1-D) = 100(1-0.25) = 75 \text{ A}$$

$$\text{Average Voltage} = I_{b(avg)} \times R_b = 75 \times 2.3 = 172.5 \text{ V}$$

$$P_b = I_a^2 R_b (1-D)$$

$$P_b = 100^2 \times 2.3 \times (1-0.25) = 17250 \text{ W}$$

EXAMPLE 7.6

A separately excited dc motor has the following name plate data: 200 V, 75 A, and 1,500 rpm. The armature resistance is 0.2 Ω. If dynamic baking takes place at 600 rpm at rated torque, compute the duty ratio. The braking resistance is 5 Ω.

SOLUTION:

E_b under rated condition,

$$E_{b(rated)} = V - I_a r_a$$
$$= 200 - (75 \times 0.2)$$
$$= 185 \text{ V}$$

$$\frac{E_{b(600)}}{E_{b(rated)}} = \frac{N}{N_{rated}}$$

$$E_{b(600)} = \left(\frac{N}{N_{rated}}\right) E_{b\,rated}$$

$$E_{b(600)} = \left(\frac{600}{1500}\right) \times 185 = 74 \text{ V}$$

Now,

$$E_{b(600)} = I_a\left(r_a + R_b(1-D)\right)$$

Because braking takes place at rated torque, $I_a = I_{arated} = 75$ A,

$$\text{i.e., } E_{b(600)} = 75\left(0.2 + 5(1-D)\right)$$

$$74 = 75 \times 0.2 + 75 \times 5(1-D)$$

$$\therefore D = 0.84$$

EXAMPLE 7.7

A dual-input dc/dc converter is supplied from two dc sources: 12-V and 24-V batteries. The duty ratio of the power switch connected to the first source is 40%, while that of the second source is 25%. Compute the average output voltage. The load consists of large inductance and resistance.

SOLUTION:

$$\begin{aligned} V_{o(av)} &= 12 \times \frac{40}{100} + 24 \times \frac{25}{100} \\ &= 4.8 + 6 \\ &= 10.8 \text{ V} \end{aligned}$$

EXAMPLE 10.5

The two-quadrant chopper shown in Fig. 10.38*a* is used to control the speed of the dc motor and also for regenerative braking of the motor. The motor constant is $K\Phi = 0.1$ V/rpm ($E_a = K\Phi n$). The chopping frequency is $f_c = 250$ Hz and the motor armature resistance is $R_a = 0.2\ \Omega$. The inductance L_a is sufficiently large and the motor current i_0 can be assumed to be ripple-free. The supply voltage is 120 V.

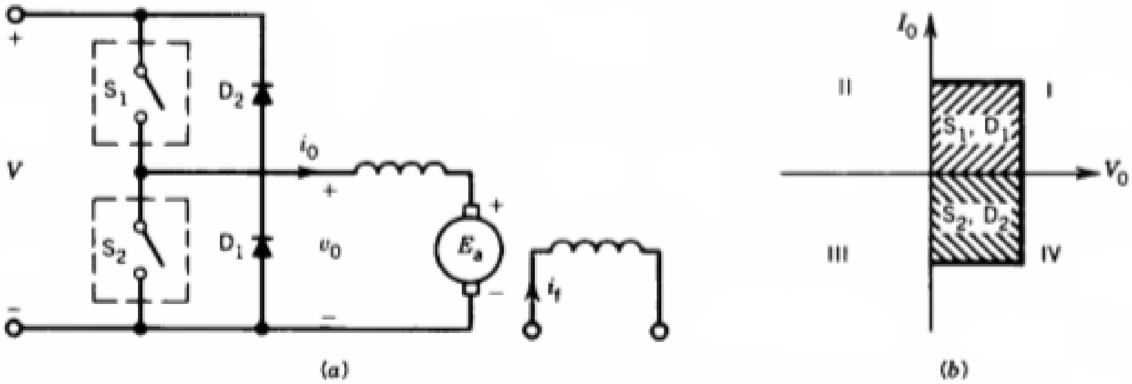


FIGURE 10.38 Two-quadrant chopper. (*a*) Circuit. (*b*) Quadrant operation.

(a) Chopper S_1 and diode D_1 are operated to control the speed of the motor. At $n = 400$ rpm and $i_0 = 100$ A (ripple-free),

  (i) Draw waveforms of v_0, i_0, and i_s.

  (ii) Determine the turn-on time (t_{on}) of the chopper.

  (iii) Determine the power developed by the motor, power absorbed by R_a, and power from the source.

(b) In the two-quadrant chopper S_2 and diode D_2 are operated for regenerative braking of the motor. At $n = 350$ rpm and $i_0 = -100$ A (ripple-free),

  (i) Draw waveforms of v_0, i_0, and i_s.

  (ii) Determine the turn-on time (t_{on}) of the chopper.

  (iii) Determine the power developed (and delivered) by the motor, power absorbed by R_a, and power to the source.

Solution

(a) **(i)** The waveforms are shown in Fig. E10.5*a*.

(ii) From Fig. 10.38*a*

$$V_0 = E_a + I_a R_a$$

$$= 0.1 \times 400 + 100 \times 0.2$$

$$= 60\text{ V}$$

$$60 = \frac{t_{on}}{T}\text{V} = \frac{t_{on}}{T}120$$

$$t_{on} = \frac{T}{2}$$

(iii)

$$P_{motor} = E_a I_0 = 0.1 \times 400 \times 100 = 4000\text{ W}$$

$$P_R = (i_0)^2_{rms} R_a = 100^2 \times 0.2 = 2000\text{ W}$$

$$P_s = V(i_s)_{avg} = 120 \times 100 \times \tfrac{2}{4} = 6000\text{ W}$$

(b) **(i)** The waveforms are shown in Fig. E10.5*b*.

(ii)

$$V_0 = E_a + (-I_0 R_a)$$

$$= 0.1 \times 350 - 100 \times 0.2$$

$$= 15\text{ V}$$

From Fig. E10.5*b*

$$V_0 = \frac{T - t_{on}}{T} V$$

$$15 = \left(1 - \frac{t_{on}}{T}\right) 120$$

$$\frac{t_{on}}{T} = \frac{7}{8}$$

$$t_{on} = \tfrac{7}{8} \times 4 = 3.5\text{ msec}$$

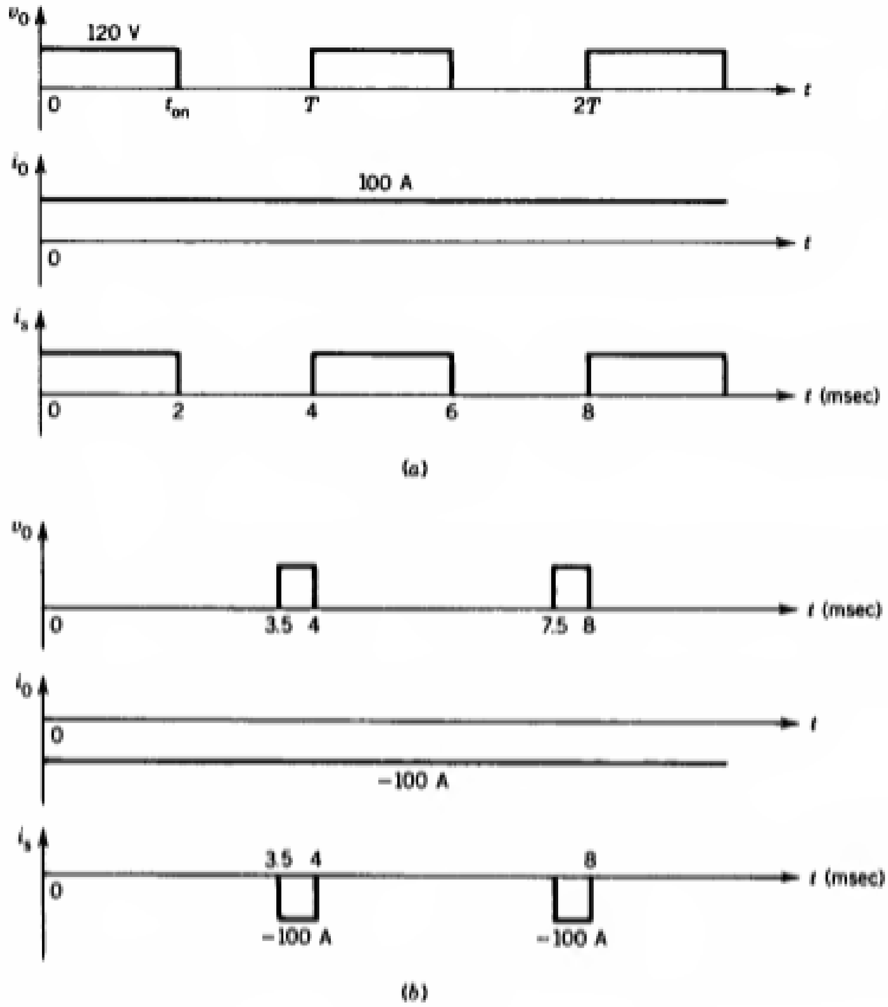


FIGURE E10.5

(iii)

$$P_{motor} = E_a I_0 = 0.1 \times 350(-100) = -3500 \text{ W}$$

$$P_R = 100^2 \times 0.2 = 2000 \text{ W}$$

$$P_s = V(i_s)_{avg} = 120(-100 \times \tfrac{1}{8}) = -1500 \text{ W}$$

10.21 A one-quadrant chopper, such as that shown in Fig. 10.34*a*, is used to control the speed of a dc motor.

Supply dc voltage = 120 V

$R_a = 0.15\ \Omega$

Motor back emf constant = 0.05 V/rpm

Chopper frequency = 250 Hz

At a speed of 1200 rpm, the motor current is 125 A. The motor current can be assumed to be ripple-free.

(a) Determine the duty ratio (α) of the chopper and the chopper on time t_{on}.

(b) Draw waveforms of v_0, i_0, and i_s.

(c) Determine the torque developed by the armature, power taken by the motor, and power drawn from the supply.

$\boxed{10.21}$ (a)

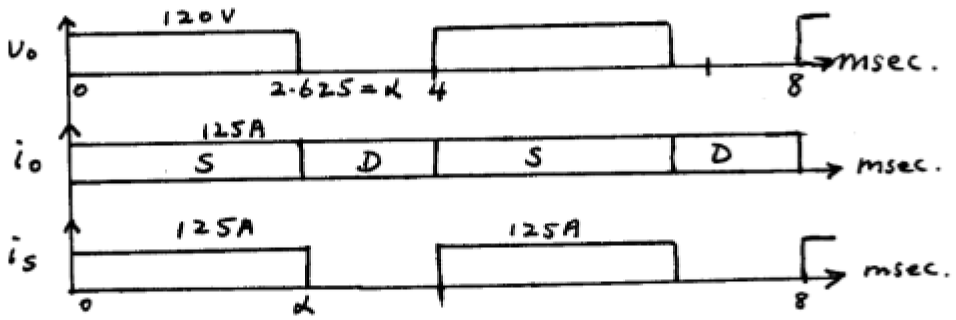
$$V_o = E_a + I_a R_a$$
$$= 0.05 \times 1200 + 125 \times 0.15$$
$$= 78.75\ V$$

$$T = \frac{10^3}{250}\ msec = 4\ msec$$

$$\alpha = \frac{78.75}{120} = 0.6563$$

$$t_{on} = \alpha T = 0.6563 \times 4 = 2.625\ msec.$$

(b)



(c)

$$E_a I_o = 60 \times 125 = 7500\ W$$

$$T = \frac{7500}{1200/60 \times 2\pi} = 59.683\ N \cdot m$$

$$P_o = V_o I_o = 78.75 \times 125 = 9844\ W$$

$$I_s = 125 \times 0.6563 = 82.03\ A$$

$$P_s = 120 \times 82.03 = 9844\ W$$

10.22 The power circuit configuration during regenerative braking of a subway car is shown in Fig. P10.22. The dc motor voltage constant is 0.3 V/rpm, and the dc bus voltage is 600 V. At a motor speed of 800 rpm and average motor current of 300 A,

(a) Draw the waveforms of v_0, i_a, and i_s for a particular value of the duty cycle $\alpha\ (= t_{on}/T)$.

(b) Determine the duty ratio α of the chopper for the operating condition.

(c) Determine the power fed back to the bus.

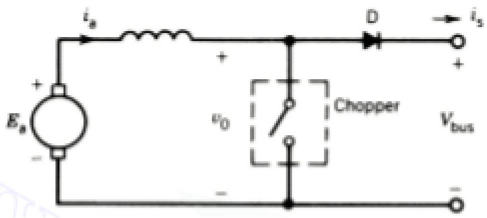
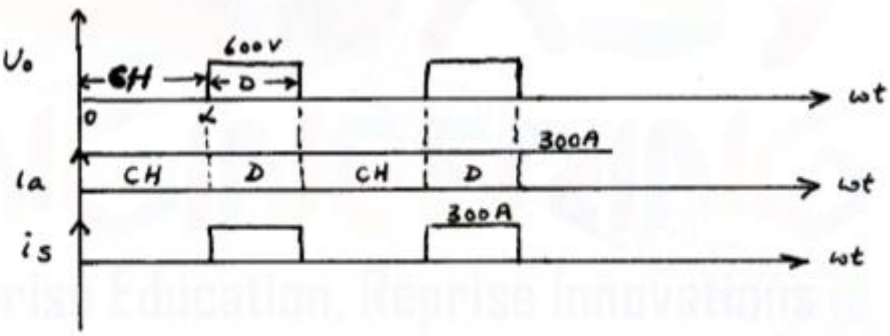


FIGURE P10.22

10.22 (a)



Apprise Education, Reprise Innovations

(b) $$V_o = (1-\alpha)\,600 = E_a = 0.3 \times 800 = 240\text{ V}$$

$$\alpha = 1 - \frac{240}{600} = 0.6$$

(c) $$I_s = (1-\alpha)\,300 = (1-0.6)\,300 = 120\text{ A}$$

$$P_s = 600 \times 120 = 72\text{ KW}$$

or $$P_s = P_a = E_a I_a = 240 \times 300 = 72\text{ kW}$$

10.23 In the chopper circuit shown in Fig. P10.23, the two switches are simultaneously turned on for time t_{on} and turned off for time $t_{off} = T - t_{on}$, where T is the chopping period. Assume voltage v_0 to be ripple-free and current i_L to be continuous.

(a) Derive an expression for V_0 as a function of the duty cycle $\alpha = t_{on}/T$ and the supply voltage V. Determine V_0 for $\alpha = 0, 0.5, 1.0$.

(b) Draw waveforms of v_0, v_L, i_L, i_0, and i for $\alpha = \frac{1}{2}$.

(c) What are the advantages and disadvantages of this circuit?

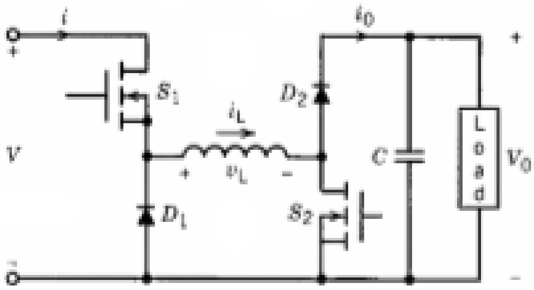


FIGURE P10.23

$\boxed{10.23}$ (a) during ton → $v_L = v = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}$

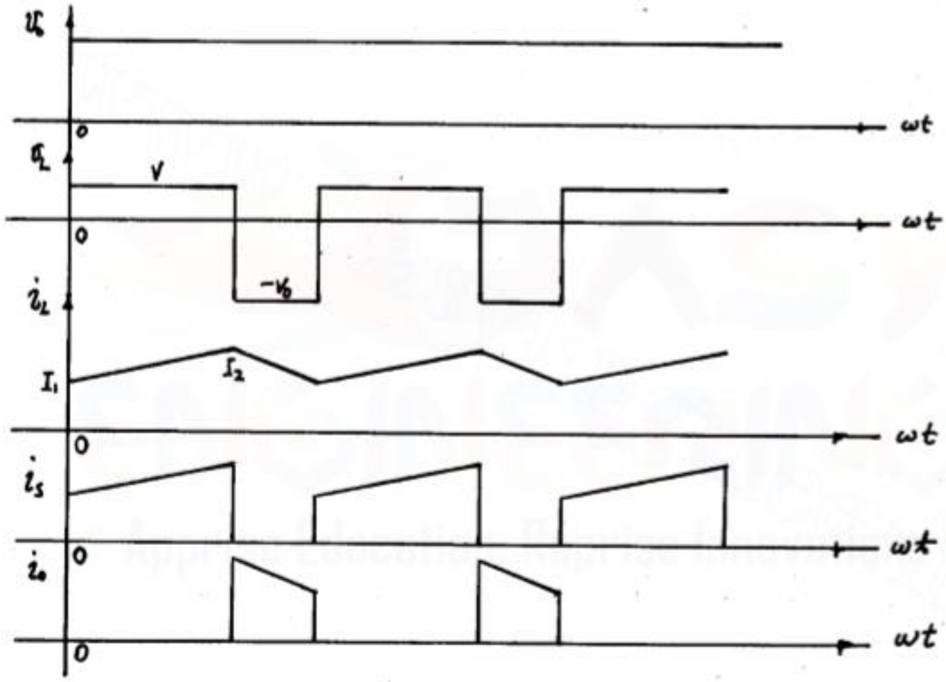
during toff → $v_L = -v_o = L\frac{I_1 - I_2}{t_{off}} = -L\frac{\Delta I}{t_{off}}$

$$\frac{V t_{on}}{L} = \frac{V_o t_{off}}{L}$$

$$V_o = \frac{t_{on}}{t_{off}} V = \frac{t_{on}}{T - t_{on}} V = \frac{\alpha}{1-\alpha} V$$

α	V_o
0	0
0.5	V
1.0	∞

(b)



(c) · This is a step-down, step-up chopper

· Polarity of V_o is the same as that of V

· Higher conduction losses → two devices conduct at a time

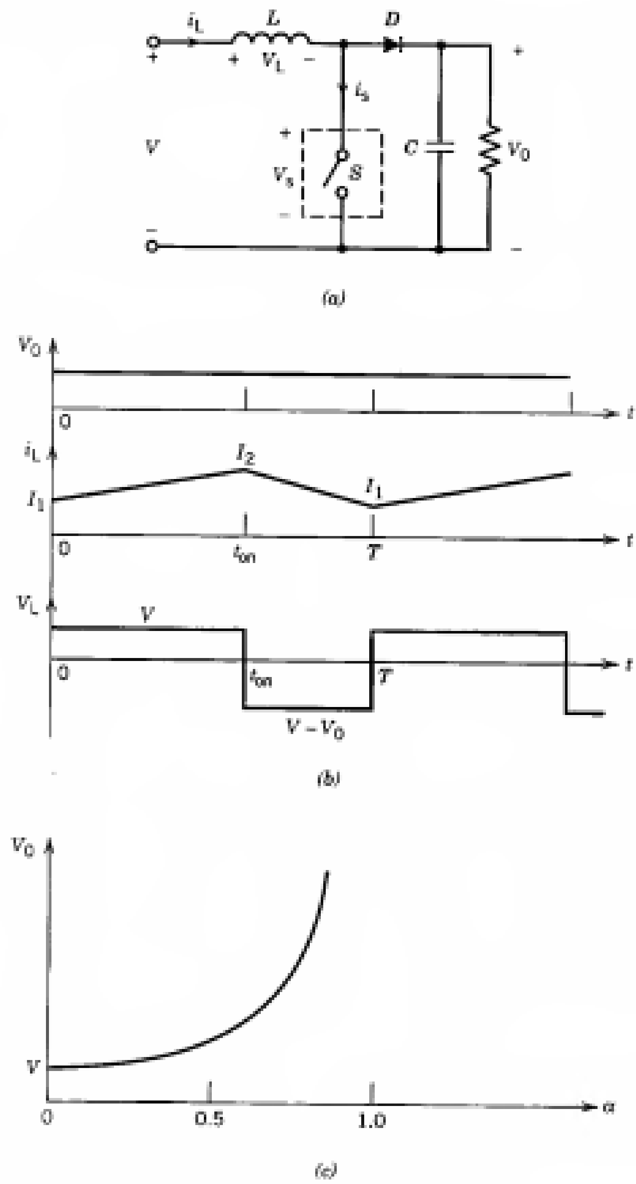


FIGURE 10.36 Boost converter. (*a*) Circuit. (*b*) Waveforms. (*c*) V_o versus α.

10.24 The boost converter of Fig. 10.36 is used to charge a battery bank from a dc voltage source with $V = 160$ V. Assume ideal switch and no-loss operation, and neglect the ripple at the output voltage. The battery bank consists of 100 identical batteries. Each battery has an internal resistance $R_b = 0.1\ \Omega$. At the beginning of the charging process, each battery voltage is $V_{b1} = 2.5$ V. When each battery is charged up to $V_{b2} = 3.2$ V, the charging process is completed. The average charging current is kept constant at 0.5 A.

(a) Calculate the variation of duty ratio α for the charging process.

(b) Draw qualitatively the waveforms of v_L, i_L, v_s, i_s, v_D, i_D for $V_{b1} = 2.5$ V.

$\boxed{10.24}$ (a) $I = 0.5A \quad R_1 = 100 \times R_D = 100 \times 0.1 = 10\Omega, \quad V_R = 0.5 \times 10 = 5V$

$$V_{o1} = 100 \times V_{D1} + V_R = 100 \times 2.5 + 5 = 255V$$

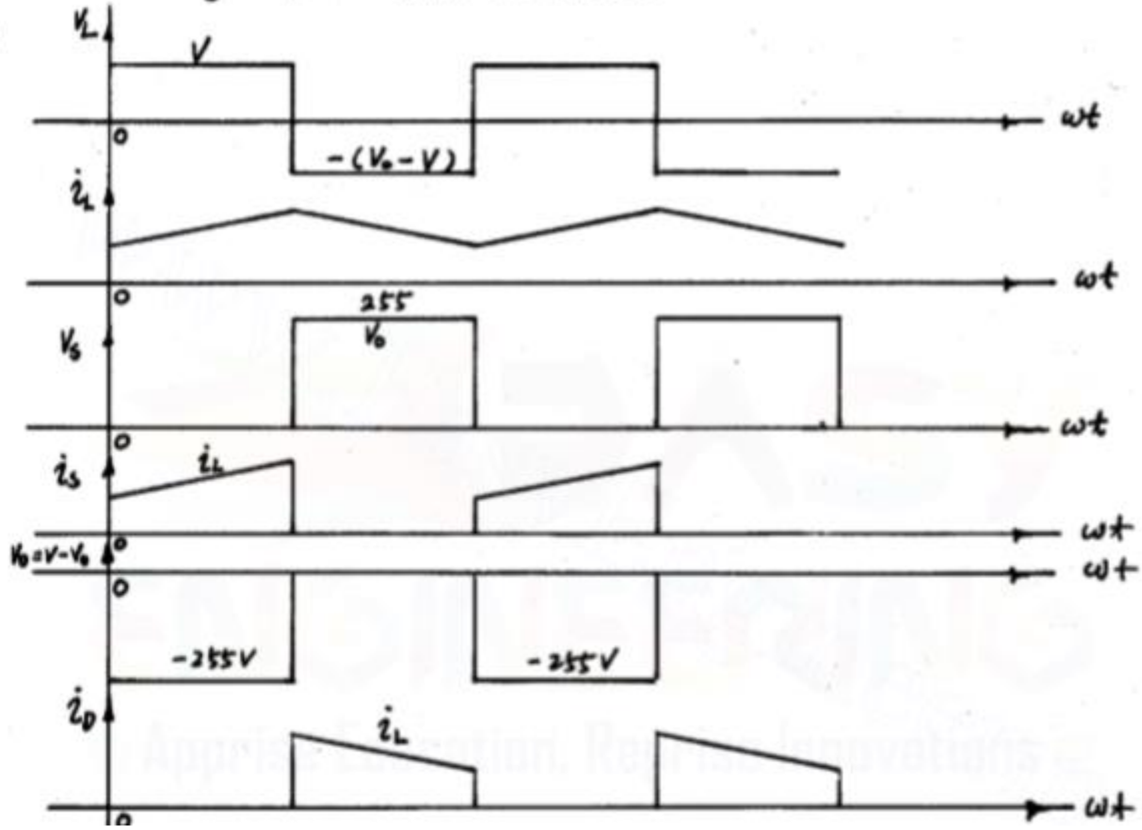
$$V_{o2} = 100 \times V_{D2} + V_R = 100 \times 3.2 + 5 = 325V$$

$$V_o = \frac{1}{1-\alpha} V \longrightarrow \alpha = \frac{V_o - V}{V_o}$$

$$\alpha_1 = \frac{255 - 150}{255} = 0.4118 \qquad \alpha_2 = \frac{325-150}{325} = 0.5385$$

α changes from 0.4118 to 0.5385

(b)



10.25 For the battery charging system of Problem 10.24:

(a) If the supply voltage available is $V = 150$ V (dc), which dc to dc converter would be used? Draw the circuit.

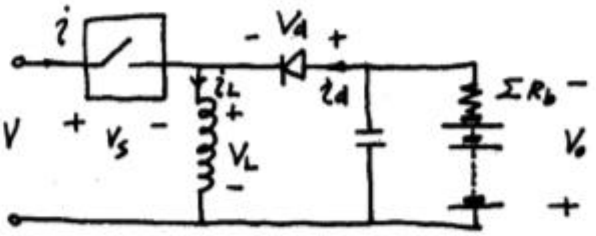
(b) Calculate the variation of the duty ratio α for the charging process.

(c) Draw qualitatively the waveforms of inductor voltage (v_L), inductor current (i_L), voltage across the chopper switch (v_s), current through the chopper switch (i_s), voltage across the diode (v_d), and current through the diode (i_d), for $v_{bl} = 1.2$ V.

$\boxed{10.25}$ (a) $V_{o1} = 100 V_{b1} + IR = 100 \times 1.2 + 100 \times 0.1 \times 0.5 = 125V$

$V_{o2} = 100 \times 3.2 + 5 = 325\ V$

$V = 150V$ Need a Buck-Boost Converter

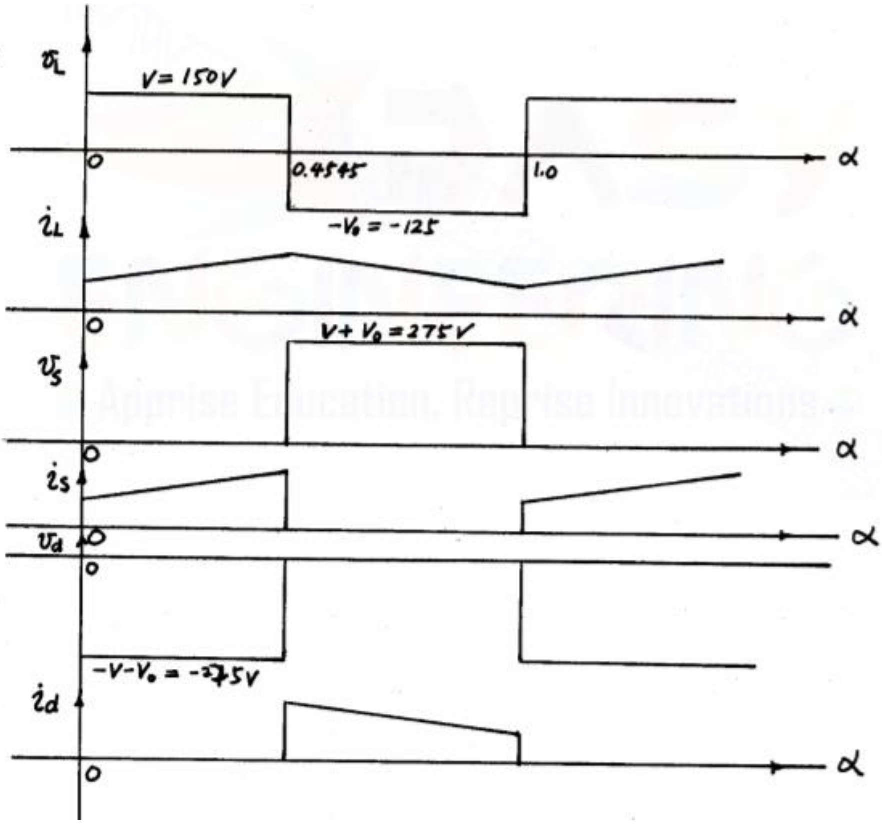


(b) $V_o = \dfrac{\alpha}{1-\alpha} V$

$\alpha = \dfrac{V_o}{V + V_o}$

$\alpha_1 = \dfrac{125}{150 + 125} = 0.4545 \qquad \alpha_2 = \dfrac{325}{150 + 325} = 0.6842$

(c)



10.26 Consider the two-quadrant chopper systems shown in Fig. P10.26. The two choppers S_1 and S_2 are turned on for time t_{on} and turned off for time $T - t_{on}$, where T is the chopping period.

(a) Draw the waveform of the output voltage v_0. Assume continuous output current i_0.

(b) Derive an expression for the average output voltage V_0 in terms of the supply voltage V and the duty ratio α (= t_{on}/T).

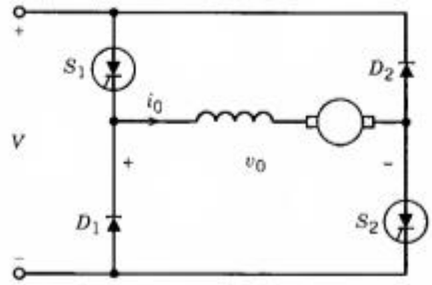
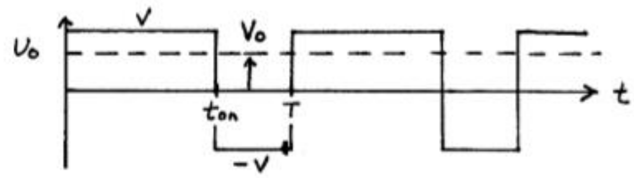


FIGURE P10.26

$\boxed{10.26}$ (a)



(b) $$V_o = \frac{1}{T}\int_0^T v_o\,dt = \frac{1}{T}\left[\int_0^{t_{on}} V\,dt + \int_{t_{on}}^{T} -V\,dt\right]$$

$$= V(2\alpha - 1)$$

Example 4.23 The armature voltage of a separately excited dc motor is controlled by a one-quadrant chopper with chopping frequency of 200 pulses per second from a 300 V dc source. The motor runs at a speed of 800 rpm when the chopper's time ratio is 0.8. Assume that the armature circuit resistance and inductance are 0.08 Ω and 15 mH, respectively, and that the motor develops a torque of 2.72 N·m per ampere of armature current.

Find the mode of operation of the chopper, the output torque, and horsepower under the specified conditions.

Solution From the problem specifications at 800 rpm, using Eq. (4.201), we get,

$$E_c = (2.72)\frac{2\pi}{60}(800) = 227.9 \text{ V}$$

$$E_c = K_1\phi_f\omega \tag{4.201}$$

$$\omega = \frac{t_{on}}{t_x}\frac{V_i}{K_1\phi_f} - \frac{R_a}{(K_1\phi_f)^2}\frac{T}{t_x}T_0 \tag{4.202}$$

In the continuous mode of operation, with $t_{on} > t^*_{on}$, we have $t_x = T$. As a result Eq. (4.202) reduces to

$$\omega = \frac{(t_{on}/T)V_i - (R_a/K_1\phi_f)T_o}{K_1\phi_f)} \tag{4.203}$$

The armature circuit time constant is obtained as

$$\tau = \frac{L_a}{R_a} = \frac{15 \times 10^{-3}}{0.08} = 187.5 \times 10^{-3} \text{ s}$$

The chopping period is given by

$$T = \frac{1}{200} = 5 \times 10^{-3}\text{s}$$

We obtain the critical on-time using Eq. (4.197) as

$$t^*_{on} = 187.5 \times 10^{-3}\, ln\left[1 + \frac{227.9}{300}(e^{5/187.5} - 1)\right]$$
$$= 3.8 \times 10^{-3}\text{s}$$

We know that $t_{on} = 0.8 \times 5 \times 10^{-3} = 4 \times 10^{-3}$. As a result, we conclude that the chopper output current is continuous.

To obtain the torque output, we use Eq. (4.203) rearranged as

$$T_o = \frac{K_1 \phi_f}{R_a} \left(\frac{t_{on}}{T} V_i - K_1 \phi_f \omega \right)$$

Thus we obtain

$$\begin{aligned} T_o &= \frac{2.72}{0.08} [0.8(300) - 227.9] \\ &= 411.4 \, \text{N} \cdot \text{m} \end{aligned}$$

The power output is obtained as

$$\begin{aligned} P_o &= (411.4) \frac{2\pi}{60} (800) = 34.5 \times 10^3 \text{ W} \\ &= 46.2 \, \text{hp} \end{aligned}$$

To illustrate the principle of field control, we have the following example.

Example 4.24 Assume for the motor of Example 4.23 that field chopper control is employed to run the motor at a speed of 1500 rpm while delivering the same power output as obtained at 800 rpm and drawing the same armature current.

Solution Although we can use Eq. (4.205), we use basic formulas instead,

$$E_c = \frac{P_a}{I_a} = \frac{34.5 \times 10^3}{151.3} = 227.9\ \text{V}$$

This is the same back EMF. Recall that

$$E_c = K_1 \phi_f \omega$$

Thus the required field flux is obtained as

$$\phi_{f_n} = \phi_{f_0} \frac{\omega_0}{\omega_n} = \frac{8}{15} \phi_{f_0}$$

where the subscript n denotes the present case, and the subscript 0 denotes the field flux for Example 4.23. Assume that ϕ_{f_0} corresponds to full applied field flux; then

$$\frac{\phi_{f_0}}{\phi_{f_n}} = \frac{V_i}{V_o} = \frac{15}{8}$$

The required chopped output voltage is V_o. Now we have

$$\frac{V_o}{V_i} = \frac{t_{on}}{T}$$

Thus

$$\frac{t_{on}}{T} = \frac{8}{15}$$

Assuming that $T = 5 \times 10^{-3}$ s, we get

$$t_{on} = 2.67 \times 10^{-3}\ \text{s}$$

Example 4.1

The speed of a separately excited dc motor is controlled by a chopper as shown in Fig. 4.8*a*. The dc supply voltage is 120 V, armature circuit resistance is $R_a = 0.5\ \Omega$, armature circuit inductance is $L_a = 20$ mH, and motor constant is $K_a\Phi = 0.05$ V/rpm. The motor drives a constant-torque load requiring an average armature current of 20 A. Assume that motor current is continuous.

Determine:

1 the range of speed control;
2 the range of the duty cycle α.

Solution

Minimum speed is zero at which $E_g = 0$. Therefore from equation 2.17

$$E_a = I_a R_a = 20 \times 0.5 = 10\,\text{V}$$

From equation 4.1

$$10 = 120\alpha$$

$$\alpha = \frac{1}{12}$$

Maximum speed corresponds to $\alpha = 1$ at which $E_a = E = 120$ V.

Therefore

$$E_g = E_a - I_a R_a$$

$$= 120 - (20 \times 0.5)$$

$$= 110\,\text{V}$$

From equation 2.13

$$N = \frac{E_g}{K_a\Phi} = \frac{110}{0.05} = 2200\,\text{rpm}$$

The range of speed is $0 < N < 2200$ rpm, and the range of the duty cycle is $1/12 < \alpha < 1$.

Example 4.1

A 250-V separately excited motor dc has an armature resistance of 2.5 Ω. When driving a load at 600 rpm with constant torque, the armature takes 20 A. This motor is controlled by a chopper circuit with a frequency of 400 Hz and an input voltage of 250 V.

1. What should be the value of the duty ratio if one desires to reduce the speed from 600 to 400 rpm, with the load torque maintained constant?
2. What should be the minimum value of the armature inductance, if the maximum armature current ripple expressed as a percentage of the rated current is not to exceed 10 percent?

Solution: With an input voltage of 250 V and at a constant torque, the motor will run at 600 rpm when $\delta = 1$.

1. At 600 rpm

$$E = V_a - I_a R_a = 250 - 20 \times 2.5 = 200 \text{ V}$$

At 400 rpm, the back emf

$$E_1 = 200 \times \frac{400}{600} = 133 \text{ V}$$

The average chopper output voltage

$$V_{a1} = E_1 + I_a R_a = 133 + 20 \times 2.5 = 183 \text{ V}.$$

Now $\delta V = V_{a1}$ or $\delta = V_{a1}/V = 183/250 = 0.73$.

2.

$$\Delta i_a = \frac{V}{2R_a}\left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1}\right] \tag{4.14}$$

$$\text{Per-unit current ripple} = (\Delta i_a)_p = \frac{\Delta i_a}{I_{rated}}$$

$$= \frac{V}{2R_a I_{rated}}\left[\frac{1 + e^{T/\tau_a} - e^{\delta T/\tau_a} - e^{(1-\delta)T/\tau_a}}{e^{T/\tau_a} - 1}\right] \tag{E4.1}$$

For the maximum value of the per-unit ripple

$$\frac{d(\Delta i_a)p}{d\delta} = 0\,,$$

therefore from equation (E4.1)

$$-\frac{T}{\tau_a}e^{\delta T/\tau_a} + \frac{T}{\tau_a}e^{(1-\delta)T/\tau_a} = 0 \quad \text{or} \quad \delta = 1 - \delta \quad \text{or} \quad \delta = 0.5\,.$$

Substituting in equation (E4.1), the maximum value of the per-unit ripple $(\Delta i_a)pm$ is given by the following equation:

$$(\Delta i_a)pm = \frac{V}{2R_a I_{rated}}\left[\frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1}\right] \tag{E4.2}$$

For $(\Delta i_a)pm = 0.1$

$$\frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1} = \frac{0.2R_a I_{rated}}{V} = \frac{0.2 \times 2.5 \times 20}{250} = 0.04$$

$$\text{or} \quad e^{0.5T/\tau_a} = 1.08 \quad \text{or} \quad 0.5T/\tau_a = \ell n(1.08) = 0.08 \quad \text{or} \quad \tau_a = \frac{T}{0.16}$$

$$\text{or} \quad L_a = \frac{R_a T}{0.16} = \frac{2.5}{400 \times 0.16} = 39.1 \text{ mH}\,.$$

Example 4.2

A 220-V, 100A dc series motor has an armature resistance and an inductance of 0.06 Ω and 2 mH, respectively. The field winding resistance and inductance are 0.04 Ω and 18 mH, respectively. Running on no load as a generator, with the field winding connected to a separate source, it gives the following magnetization characteristic at 700 rpm:

Field current	25	50	75	100	125	150	175	A
Terminal voltage	66.5	124	158.5	181	198.5	211	221.5	V

The motor is controlled by a chopper operating at 400 Hz and 220 V. Calculate the motor speed for a duty ratio of 0.7 and a load torque equal to 1.5 times the rated torque.

Solution: The speed at which the magnetization characteristic was measured = $700 \times 2\pi/60 = 73.3$ rad/sec.

voltage induced $E = K_e\Phi\omega_m$

$$K_e\Phi = K = \frac{E}{\omega_m}$$

$$\text{Torque } T_a = KI_a = \frac{EI_a}{\omega_m} \tag{E4.3}$$

From equation (E4.3) and the magnetization characteristic

I_a	25	50	75	100	125	150	175	A
T_a	22.7	84.6	162.2	246.9	338.5	431.8	528.8	N-m

The rated torque (torque at 100A) = 247 N-m

$1.5 \times$ Rated torque = $1.5 \times 247 = 370.5$ N-m

From the above T_a/I_a table the current at $1.5 \times$ Rated torque = 133 A

Also K at 133 A = 370.5/133 = 2.79

$$R_a = 0.06 + 0.04 = 0.1\ \Omega$$

$$\text{Now } \omega_m = \frac{\delta V - I_aR_a}{K} = \frac{0.7 \times 220 - 100 \times 0.1}{2.79} = 51.6 \text{ rad/sec.} = 492.7 \text{ rpm}$$

Example 4.3

A 230 V, 500 rpm, 90 A separately excited dc motor has the armature resistance and inductance of 0.115 Ω and 11 mH respectively. The motor is controlled by a chopper operating at 400 Hz. If the motor is regenerating,

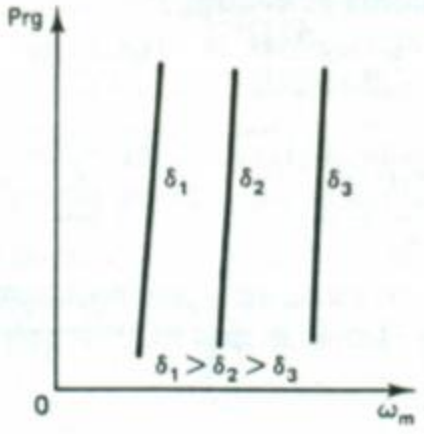


Figure 4.7 Regenerative braking performance curves of TRC chopper-fed dc separately excited motor.

1. Find the motor speed and the regenerated power at the rated current and a duty ratio of 0.5.
2. Calculate the maximum safe speed if the minimum value of the duty ratio is 0.1.

Solution: At rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. In regenerative braking

$$(1 - \delta)V = E - I_aR_a \quad \text{or} \quad E = (1 - \delta)V + I_aR_a \tag{E4.4}$$

At $\delta = 0.5$ and $I_a = 90$ A

$$E = 0.5 \times 230 + 90 \times 0.115 = 125 \text{ V}$$

Since $\dfrac{N}{N_r} = \dfrac{E}{E_r}$

where N_r = rated speed in rpm and N = speed to be calculated

Thus

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125}{219.7} = 284.5 \text{ rpm}$$

$$\tau_a = \frac{11 \times 10^{-3}}{0.115} = 95.65 \text{ mS}, \qquad T = \frac{1}{400} = 2.5 \text{ mS}$$

$$T/\tau_a = \frac{2.5 \times 10^{-3}}{95.65 \times 10^{-3}} = 0.026 \quad \text{and} \quad \tau_a/T = 38.3$$

Equation (4.30) is repeated here:

$$P_{rg} = \frac{V^2}{R_a}\left[\left(\frac{E}{V} - 1\right) \cdot (1 - \delta) + \frac{\tau_a}{T}\left\{\frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}}\right\}\right] \tag{4.30}$$

Now

$$\frac{e^{(1-\delta)T/\tau_a} + e^{\delta T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}} = x = \frac{e^{0.5T/\tau_a} + e^{0.5T/\tau_a} - e^{T/\tau_a} - 1}{1 - e^{T/\tau_a}}$$

$$= \frac{e^{0.5T/\tau_a} - 1}{e^{0.5T/\tau_a} + 1} = \frac{0.013}{2.013} = 0.0065$$

From equation (4.30),

$$P_{rg} = \frac{V^2}{R_a}\left[\left(\frac{E}{V} - 1\right) \cdot (1 - \delta) + \frac{\tau_a}{T}x\right]$$

$$= \frac{230^2}{0.115}\left[\left(\frac{125}{230} - 1\right)(1 - 0.5) + 38.3 \times 0.0065\right]$$

$$= 9.52 \text{ kW}$$

2. The maximum safe speed will be obtained at the minimum value of δ and the rated armature current. For higher speeds, the armature current will exceed the rated motor current and this operation will not be safe for the motor. At the maximum safe speed N_m, the back emf E_m is given by

$$E_m = (1 - \delta_{min})V + I_{ar}R_a = 0.9 \times 230 + 90 \times 0.115 = 217$$

$$N_m = \frac{N_r}{E_r} \times E_m = \frac{500}{219.7} \times 217 = 494 \text{ rpm}$$

Example 4.4

The motor of example 4.3 is controlled by a class C two-quadrant chopper operating with a source voltage of 230 V and a frequency of 400 Hz.

1. Calculate the motor speed for a motoring operation at $\delta = 0.5$ and half of rated torque.
2. What will be the motor speed when regenerating at $\delta = 0.5$ and rated torque?

Solution: At the rated conditions of operation,

$$E_r = V - I_{ar}R_a = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. From equation (4.33),

$$\delta V = E + I_a R_a \tag{E4.5}$$

At half the rated torque, $I_a = 45$ A
At $\delta = 0.5$

$$E = \delta V - I_a R_a = 0.5 \times 230 - 45 \times 0.115 = 109.8 \text{ V}$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 109.8}{219.7} = 250 \text{ rpm}$$

2. In the regenerative braking at the rated torque, $I_a = -90$ A
   From equation (E4.5),

$$E = \delta V - I_a R_a = 0.5 \times 230 + 90 \times 0.115 = 125.4$$

$$N = \frac{N_r E}{E_r} = \frac{500 \times 125.4}{219.7} = 285 \text{ rpm}$$

Example 4.5

The motor of example 4.3 is fed by a four-quadrant chopper controlled by method III. The source voltage is 230 V and the frequency of operation is 400 Hz.

1. If the motor operation is required in the second quadrant at the rated torque and 300 rpm, calculate the duty ratio.
2. What should be the value of the duty ratio if the motor is working in the third quadrant at 400 rpm and half of the rated torque?

Solution: At the rated conditions of operation

$$E_r = 230 - 90 \times 0.115 = 219.7 \text{ V}$$

1. Equation (4.39), which is applicable to method III is reproduced here:

$$I_a = \frac{2V(\delta - 0.5) - E}{R_a} \tag{4.39}$$

The motor is working in the second quadrant, therefore,

$$I_a = -90 \text{ A}$$

$$E = \frac{300}{500} \times E_r = \frac{300}{500} \times 219.7 = 131.8 \text{ V}$$

Substituting in equation (4.39), gives

$$-90 = \frac{2 \times 230(\delta - 0.5) - 131.8}{0.115}$$

or

$$\delta = 0.5 + \frac{121}{460} = .76 .$$

2. At half the rated torque and in the third quadrant

$$I_a = -45 \text{ A}$$

$$E = -\frac{400}{500} \times 219.7 = -175.7 \text{ V}$$

Substituting in equation (4.39), gives

$$-45 = \frac{2 \times 230(\delta - 0.5) + 175.7}{0.115}$$

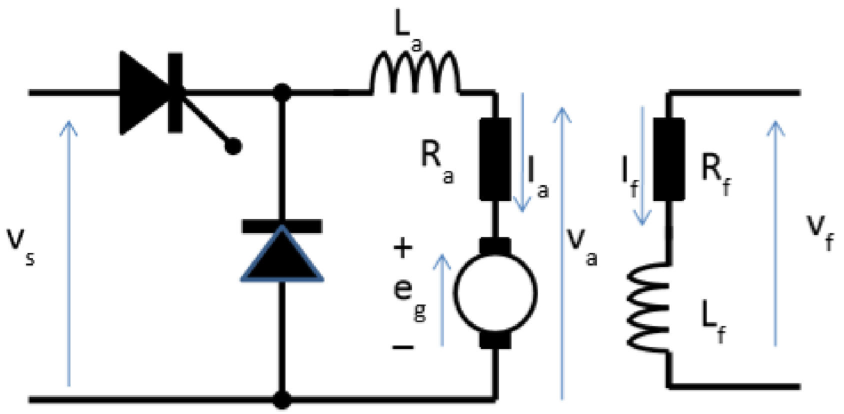
or

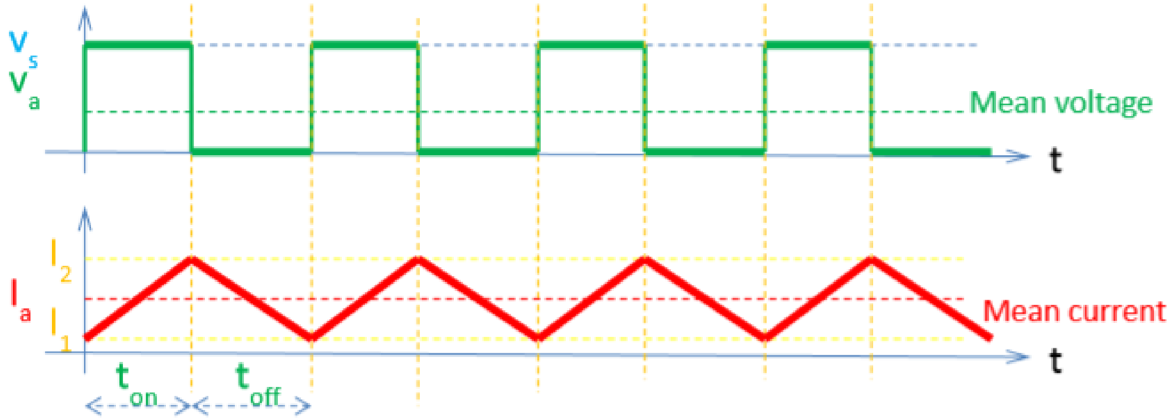
$$\delta = 0.5 - \frac{181}{460} = 0.11 .$$

DC-DC MOTOR DRIVES (Choppers)

A chopper directly converts a fixed-voltage DC supply to a variable-voltage DC supply.

Step-Down Chopper (Motoring)





During t_{on} time		*During t_{off} time*	
$Vs = L\frac{di}{dt} + Va$	(1)	$0 = -L\frac{di}{dt} + Va$	(5)
$Vs - Va = L\frac{di}{dt}$	(2)	$Va = L\frac{di}{dt}$	(6)
$di = \frac{Vs-Va}{L}dt$	(3)	$di = \frac{Va}{L}dt$	(7)
$\Delta I = I_2 - I_1 = \frac{Vs-Va}{L}t_{on}$	(4)	$\Delta I = \frac{Va}{L}t_{off}$	(8)

$$t_{on} = DT,\ t_{off} = (1-D)T \quad where D\ is\ the\ Duty\ cycle$$

Equating ΔIs

$$\Delta I = \frac{Vs-Va}{L}t_{on} = \frac{Va}{L}t_{off} = \frac{Vs-Va}{L}DT = \frac{Va}{L}(1-D)T \Rightarrow \boxed{Va = DVs}$$

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \qquad (9) \qquad\qquad I_1 + I_2 = 2I_{mean} \quad (10)$$

adding (10) to (4) or (8)

$$2I_2 = 2I_{mean} + \frac{Vs-Va}{L} t_{on} \qquad or \qquad 2I_2 = 2I_{mean} + \frac{Va}{L} t_{off}$$

$$I_2 = I_{mean} + \frac{Vs-Va}{2L} t_{on} \qquad or \qquad I_2 = I_{mean} + \frac{Va}{2L} t_{off}$$

similarily

$$I_1 = I_{mean} - \frac{Vs-Va}{2L} t_{on} \qquad or \qquad I_1 = I_{mean} - \frac{Va}{2L} t_{off}$$

Example : A simple DC step-down chopper is operated at a frequency of 2KHz from a 120 V DC source to supply a motor load with Ra = 0.85 ohms, La = 0.32 mH. The required torque genrated by the motor is 20 Nm, at 1000 rpm, and field current is measured to be 1A. If Kv = 0.8345 V/A-rad/S, determine (a) the duty cycle for the switching pulse, (b) the mean load current, and (c) the max & min load currents.

(a) $V_a = DV_s = I_aR_a + E_g \qquad I_a = ? \qquad E_g = ?$

$$T = K_V I_a I_f => I_a = \frac{20}{0.8345 \ x \ 1} = 23.96 \ A$$

$$E_g = K_V \omega I_f = 0.8345 \ x \ 2\pi \ x \ \frac{1000}{60} = 87.38 \ V$$

$V_a = 23.96 \ x \ 0.85 + 87.38 = 107.75$ therefore $D = \frac{107.74}{120} = 0.89 = 89\%$

(b) $I_{mean} = I_a = 23.96 \ A$

(c) $I_{max} = I_{mean} + \frac{Vs-Va}{2L} t_{on} \qquad or \qquad I_{max} = I_{mean} + \frac{Va}{2L} t_{off}$

We know $t_{on} = DT, \ \ t_{off} = (1-D)T \quad and \quad T = 1/f$

$$I_{max} = 23.96A + \frac{120V - 107.75V}{2*0.32mH} * 0.89 * \frac{1}{2000} = 32.47A$$

$$I_{min} = I_{mean} - \frac{Vs-Va}{2L} t_{on} \qquad or \qquad I_1 = I_{mean} - \frac{Va}{2L} t_{off}$$

$$I_{min} = 23.96A - \frac{120V - 107.75V}{2*0.32mH} * 0.89 * \frac{1}{2000} = 15.45 \ A$$

Example : A separately excited DC motor is powerd by a DC chopper from a 600 V dc source. The armature resistance Ra = 0.05 ohms. The back e.m.f.constant of the motor is kv=1.527 V/A-rads/s. The armature voltage is continuous and ripple free. If the duty cycle of the copper is 60%, determine (a) the input power from the source, (b) the equivalent input resistance of the chopper drive, (c) the motor speed, and (d) the developed torque.

(a) P_{input}= ?, $P_{input} = DV_sI_a = 0.6 \ x \ 600 \ x \ 250 = 90$ kw

(b) Req= ?, $Req = \frac{Vs}{Is} = \frac{Vs}{DIs} = \frac{600V}{0.6 \ x \ 250A} = 4\ \Omega$

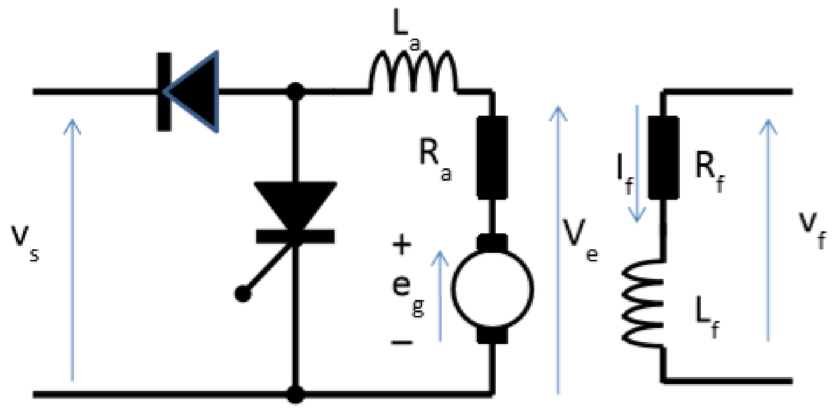
(c) ω = ?, $E_g = k_v \omega$ I, E_g= ?, $V_a = I_a R_a + E_g$, V_a = ?, $V_a = DV_s$ = 0.6 x 600 = 360 V

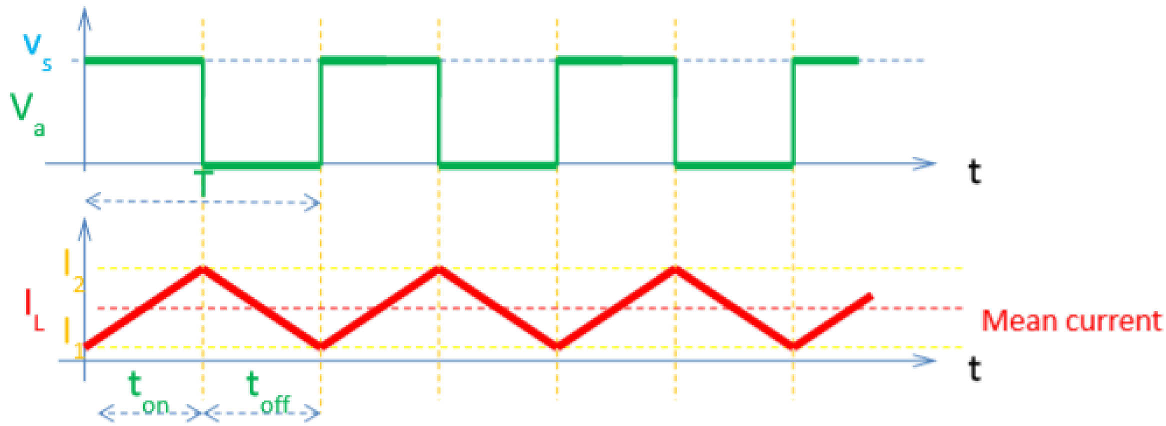
E_g = 360 – 250x0.05 = 347.5 V

$\omega = \frac{347.5V}{1.525 \ x \ 2.5A} = 91.03$ rad/s or $91.03 \ x \ \frac{60}{2\pi}$=869.3 rpm

(d) T_D=?, $T_D = k_v \ I_f \ I_a$ = 1.527 x 250 x 2.5 = 954.38 Nm

Step-Up Chopper – (Regenerative Braking)





During t_{on} time

$$Ve = L\frac{di}{dt} \quad (1)$$

$$di = \frac{Ve}{L}dt \quad (2)$$

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} \quad (3)$$

During t_{off} time

$$Ve = -L\frac{di}{dt} + Vs \quad (4)$$

$$di = \frac{Vs-Ve}{L}\,dt \quad (5)$$

$$\Delta I = I_2 - I_1 = \frac{Vs-Ve}{L}t_{off} \quad (6)$$

Equating ΔIs

$$\Delta I = I_2 - I_1 = \frac{Ve}{L}t_{on} = \frac{Vs-Ve}{L}t_{off} => \frac{Ve}{L}DT = \frac{Vs-Ve}{L}(1-D)T \quad (7)$$

$$VeD = Vs - Ve - VsD + VeD => Vs = \frac{Ve}{1-D}$$

Since average voltage across L is zero, therefore Ve = Va and $\boxed{Vs = \frac{Va}{1-D}}$ (8)

To find currents

$$I_{mean} = \frac{I_1+I_2}{2} \quad (9)$$

$$I_1 + I_2 = 2I_{mean} \quad (10)$$

adding (10) to (3)or(6) remembering Ve = Va

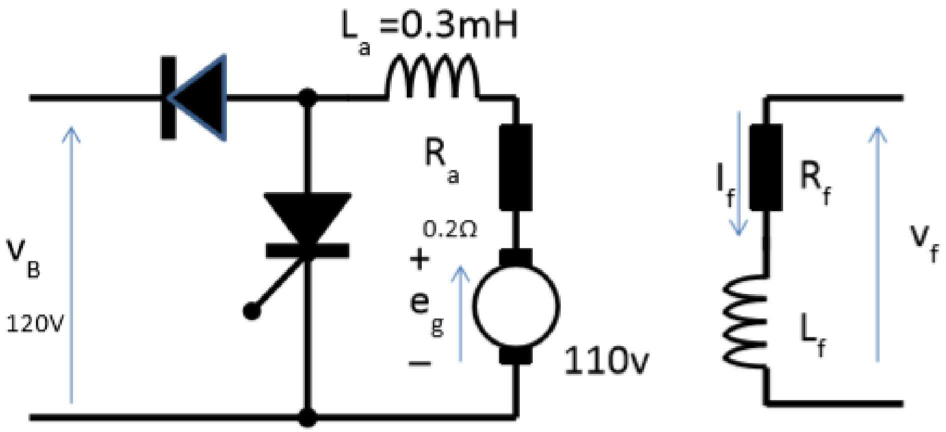
$$2I_2 = 2I_{mean} + \frac{Va}{L}t_{on} \quad or \quad 2I_2 = 2I_{mean} + \frac{Vs-Va}{L}t_{off}$$

$$\boxed{I_2 = I_{mean} + \frac{Va}{2L}t_{on}} \quad or \quad \boxed{I_2 = I_{mean} + \frac{Vs-Va}{2L}t_{off}}$$

similarily

$$\boxed{I_1 = I_{mean} - \frac{Va}{2L}t_{on}} \quad or \quad \boxed{I_1 = I_{mean} - \frac{Vs-Va}{2L}t_{off}}$$

Example : In a battery powered car, operating a frequency of 5 KHz, the battery voltage is 120 V. It is driven by a DC motor and employs chopper control. The resistance of the motor is 0.2 ohms and its inductance is 0.3 mH. During braking, the chopper configuration is changed to voltage step-up mode. While going down the hill at a certain speed, the back emf of the motor is 110 V and the breaking current is 10 A. Detrermine (a) the copper duty cycle, and (b) Max and Min values of the current.



(a) D = ?, $V_B = \frac{Va}{1-D} = \frac{IaRa+Eg}{1-D} > 1-D = \frac{IaRa+Eg}{V_B} = \frac{110V+0.2x10}{120} => D = 6.67\%$

(b) $I_{max} = I_{mean} + \frac{Va}{2L}t_{on} = 10A + \frac{112x0.0667}{2x.03x10^{-3}x2000} = 16.22\ A$

$I_{min} = I_{mean} - \frac{Va}{2L}t_{on} = 10A - \frac{112x0.0667}{2x.03x10^{-3}x2000} = 3.78\ A$

Problem-Repeat above for 50V back emf.

Motoring and Regenerative Braking Two-Quadrant Chopper (buck-boost

Example 13.1: ***DC chopper with load back emf (first quadrant)***

A first-quadrant dc-to-dc chopper feeds an inductive load of 10 ohms resistance, 50mH inductance, and back emf of 55V dc, from a 340V dc source. If the chopper is operated at 200Hz with a 25% on-state duty cycle, determine, with and without (rotor standstill) the back emf:

i. the load average and rms voltages;
ii. the rms ripple voltage, hence ripple factor;
iii. the maximum and minimum output current, hence the peak-to-peak output ripple in the current;
iv. the current in the time domain;
v. the average load output current, average switch current, and average diode current;
vi. the input power, hence output power and rms output current;
vii. effective input impedance, (and electromagnetic efficiency for $E > 0$);
viii. sketch the output current and voltage waveforms.

5. The speed of a separately excited DC motor is controlled by a chopper. The DC supply voltage is 120 V, armature circuit resistance is 0.5 Ω, armature circuit inductance is 20 mH, and back emf constant is 0.05 V/RPM. The motor drives a constant torque load requiring an average current of 20A. Assuming the motor current to be continuous, determine the range of speed control and the range of duty cycle.

Given Data:

V_s=120 volts, R_a=0.5 ohms, L_a=20mH, K=0.05 V/RPM. I_a=20A

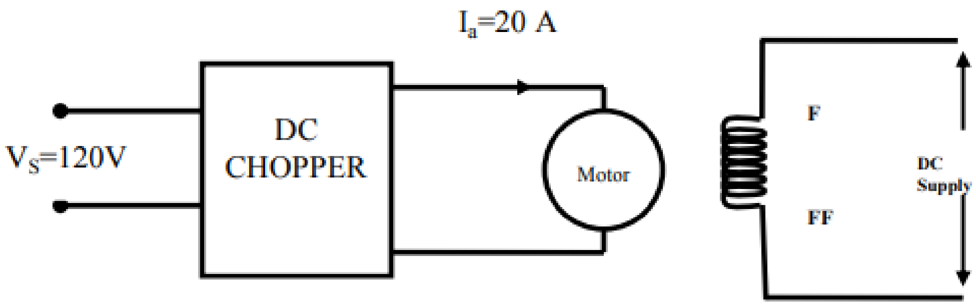
Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution



(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \left[\begin{array}{l} \because V_a = \alpha V_s \\ E_b = KN \end{array}\right]$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 20A and minimum possible speed is **ZERO**

$$\alpha x 120 = (0.05)x0 + (20x0.05)$$

$$120\alpha = 10$$

$$\alpha = \frac{10}{120} = 0.08$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.08 \le \alpha \le 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x120 = 0.05xN + (20x0.5)$$
$$120 = 0.05N + 10$$
$$N = \frac{120 - 10}{0.05} = 2200rpm$$

The range of speed control is $0 \le N \le 2200RPM$

6. A 230 volts, 960 rpm, 200 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a dc source of 230 volts through a chopper. Assuming continuous conduction

a) Calculate the duty ratio of chopper for monitoring operation at rated torque and 350 rpm

b) If maximum duty ratio of chopper is limited to 0.95 and maximum permissible motor speed obtainable without field weakening

Given Data

V_s=230 volts, N=960 rpm, I_a=200 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =350rpm.

(ii) If $\alpha = 0.95$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (200x0.02) = 226 volts$$
$$E \text{ at } 350 \text{ rpm (ie) } \mathrm{E}_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$220 = Kx\omega_1$$
$$\omega_1 = \frac{960x2\pi}{60} = 100.53 rad/\sec$$
$$\therefore K = \frac{226}{100.53} = 2.24 Volts.\sec/rad$$

E_2 at 350 rpm is given by

$$\omega_2 = \frac{350x2\pi}{60} = 36.651$$
$$\therefore E_2 = 36.65x2.24 = 82.1 Volts$$

Motor terminal voltage at 350 rpm is

$$V_{350rpm} = 82.1 + (200x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{350\text{ rpm}}}{\mathrm{V}_{960\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.95\text{x}230 = 218.5 \text{ Volts}$$

$$\therefore E = V_a + I_a R_a = 218.5 + (200x0.02) = 222.5 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{222.5}{2.24} = 99.330 rad/\sec$$
$$N = \frac{99.330x60}{2\pi} = 948.53 rpm$$

7. A DC series motor is fed from a 600 volts source through a chopper. The DC motor has the following parameters armature resistance is equal to 0.04 Ω, field resistance is equal to 0.06 Ω, constant $k = 4 \text{ x } 10^{-3} Nm/Amp^2$. The average armature current of 300 Amps is ripple free. For a chopper duty cycle of 60% determine

i. Input power drawn from the source.
ii. Motor speed and
iii. Motor torque.

Given Data

V_s=600 volts, I_a=300 amps, R_a=0.04 ohms, R_f=0.06 ohms, K=$4x10^{-3} Nm/amp^2$ $\delta = 0.6$

DC SERIES motor.

Solution

a. Power input to the motor = P =$V_a I_a$

$$V_a = \delta V_s = 0.6x600 = 360 Volts$$
$$\therefore P = 360x300 = 108KW$$

b. For a DC series motor

$$E_a = K_a \phi \omega_m$$
$$= \mathrm{KI_a} \omega_m [\because \phi = I_a]$$
$$= 4x10^{-3} x300x\omega_m$$
$$\therefore V_a = E + I_a(R_a + R_s) = KI_a\omega_m + I_a(R_a + R_s)$$
$$\Rightarrow 0.6x600 = 4x10^{-3}x300x\omega_m + 300(0.04 + 0.06)$$
$$\omega_m = \frac{360 - 30}{1.2} = 27.5 rad/\sec (or) 2626 rpm$$
$$Motor \text{ Torque } \mathrm{T = Ka\phi I_a} = KI_a^{\ 2}$$
$$= 4x10^{-3} x300^2$$
$$= 360 \text{ N-M}$$

8. A 230 V, 1100 rpm, 220 Amps separately excited DC motor has an armature resistance of 0.02 Ω. The motor is fed from a chopper, which provides both motoring and braking operations. Calculate

   i. The duty ratio of chopper for motoring operation at rated torque and 400 rpm

   ii. The maximum permissible motor speed obtainable without field weakening, if the maximum duty ratio of the chopper is limited to 0.9 and the maximum permissible motor current is twice the rated current.

Given Data

V_s=230 volts, N=1100 rpm, I_a=220 amps, R_a=0.02 ohms separately excited DC motor, chopper drive for both motoring and braking operation, Assume continuous conduction

Find

(i) $\alpha = ?$ at rated Torque and Speed =400rpm.

(ii) If $\alpha = 0.9$ and current is twice rated calculate speed

Solution

(i) At rated operation

$$E_1 = V_a - I_a R_a$$
$$\Rightarrow 230 - (220x0.02) = 225.6volts$$
$$E \text{ at 400 rpm (ie) } E_2 = ?$$

From rated condition

$$E_1 = K\omega_1$$
$$\omega_1 = \frac{1110x2\pi}{60} = 115.192 rad/\sec$$
$$\therefore K = \frac{225.6}{115.192} = 1.95 Volts.\sec/rad$$

E_2 at 400 rpm is given by

$$\omega_2 = \frac{400x2\pi}{60} = 41.887 rad/\sec$$
$$\therefore E_2 = 41.887x1.95 = 81.68 Volts$$

Motor terminal voltage at 400 rpm is

$$V_{400rpm} = 81.68 + (220x0.02) = 86.1 Volts$$
$$\alpha = \frac{V_{400\text{ rpm}}}{V_{1100\text{ rpm}}} = \frac{86.1}{230} = 0.37$$

(ii) Maximum available

$$V_a = \alpha V_s$$
$$= 0.9\text{x}230 = 207 \text{ Volts}$$

$$\therefore E = V_a + I_a R_a = 207 + (2x220x0.02) = 215.8 Volts$$

Speed at 222.5 volts E_b is

$$E_b = K\omega$$
$$\omega = \frac{215.8}{1.95} = 110.667 rad / \sec$$
$$N = \frac{110.667x60}{2\pi} = 1056.78 rpm$$

9. A DC chopper is used to control the speed of a separately excited dc motor. The DC voltage is 220 V, R_a= 0.2 Ω and motor constant $K_e\phi$=0.08 V/rpm. The motor drives a constant load requiring an average armature current of 25 A. Determine

   iv. The range of speed control

   v. The range of duty cycle. Assume continuous conduction

Given Data:

V_s=220 volts, R_a=0.2 ohms, L_a=20mH, K=0.08 V/RPM. I_a=25A

Constant Load, Separately excited DC motor

Find

- ✓ The range of Speed Control
- ✓ The range of duty cycle

Assume Continuous current mode

Solution

(i) Range of Duty cycle

Average output voltage of the motor

$$V_a = E_b + I_a R_a$$

$$\alpha V_s = E_b + I_a R_a \quad \begin{bmatrix} \because V_a = \alpha V_s \\ E_b = KN \end{bmatrix}$$

$$\alpha V_s = KN + I_a R_a$$

As motor drives a constant load, T is constant and I_a is 25A and minimum possible speed is **ZERO**

$$\alpha x220 = (0.08)x0 + (25x0.2)$$

$$220\alpha = 10$$

$$\alpha = \frac{10}{220} = 0.04$$

Maximum possible speed corresponds to $\alpha = 1$, i.e. when 120 volts is directly applied to the motor. Therefore the range of duty cycle is

$$0.04 \le \alpha \le 1$$

(ii) The range of Speed

$\alpha V_s = KN + I_a R_a$

Minimum speed N=0

Maximum speed at $\alpha = 1$

$$1x220 = 0.08xN + (25x0.2)$$

$$220 = 0.08N + 5$$

$$N = \frac{220-5}{0.08} = 2687.5rpm$$

The range of speed control is $0 \le N \le 2687.5RPM$

Solution

The main circuit and operating parameters are

- on-state duty cycle $\delta = ¼$
- period $T = 1/f_s = 1/200\text{Hz} = 5\text{ms}$
- on-period of the switch $t_T = 1.25\text{ms}$
- load time constant $\tau = L/R = 0.05\text{H}/10\Omega = 5\text{ms}$

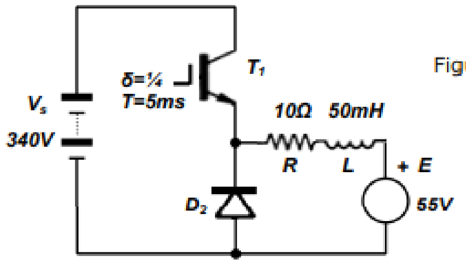


Figure Example 13.1. *Circuit diagram.*

i. From equations (13.2) and (13.3) the average and rms output voltages are both independent of the back emf, namely

$$\overline{V}_o = \frac{t_T}{T} V_s = \delta V_s$$
$$= ¼ \times 340\text{V} = 85\text{V}$$
$$V_r = \sqrt{\frac{t_T}{T}}\, V_s = \sqrt{\delta}\, V_s$$
$$= \sqrt{¼} \times 240\text{V} = 120\text{V rms}$$

ii. The rms ripple voltage hence ripple factor are given by equations (13.4) and (13.5), that is

$$V_r = \sqrt{V_{rms}^2 - V_o^2} = V_s\sqrt{\delta(1-\delta)}$$
$$= 340\text{V}\sqrt{¼ \times (1 - ¼)} = 147.2\text{V ac}$$

and

$$RF = \frac{V_r}{V_o} = \sqrt{\frac{1}{\delta} - 1}$$
$$= \sqrt{\frac{1}{¼} - 1} = \sqrt{3} = 1.732$$

No back emf, $E = 0$

iii. From equation (13.13), with E=0, the maximum and minimum currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-t_T}{\tau}}}{1-e^{\frac{-T}{\tau}}} = \frac{340\text{V}}{10\Omega}\times\frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} = 11.90\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{t_T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} = \frac{340\text{V}}{10\Omega}\times\frac{e^{1/4}-1}{e^{1}-1} = 5.62\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$I_{p-p} = \hat{I} - \check{I}$$
$$= 11.90\text{A} - 5.62\text{A} = 6.28\text{A}$$

Alternatively the ripple can be extracted from figure 13.4 using $T/\tau = 1$ and $\delta = ¼$.

iv. From equations (13.11) and (13.12), with $E = 0$, the time domain load current equations are

$$i_o = \frac{V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 34\times\left(1-e^{\frac{-t}{5\text{ms}}}\right) + 5.62\times e^{\frac{-t}{5\text{ms}}}$$

$$= 34 - 28.38\times e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = \hat{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 11.90\times e^{\frac{-t}{5\text{ms}}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.17), with $E = 0$, is

$$\overline{I}_o = \overline{V}_o/R = 85\text{V}/10\Omega = 8.5\text{A}$$

The average switch current, which is the average supply current, is

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{¼\times(340\text{V} - 0)}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(11.90\text{A} - 5.62\text{A}) = 2.22\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$
$$= 8.50\text{A} - 2.22\text{A} = 6.28\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s\overline{I}_i = 340\text{V}\times 2.22\text{A} = 754.8\text{W}$$
$$P_{out} = P_{in} = 754.8\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o_{rms}} = \sqrt{\frac{P_{out}}{R}}$$
$$= \sqrt{\frac{754.8\text{W}}{10\Omega}} = 8.7\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$
$$= \frac{340\text{V}}{2.22\text{A}} = 153.2\ \Omega$$

Load back emf, E = 55V

i. and ii. The average output voltage, rms output voltage, ac ripple voltage, and ripple factor are independent of back emf, provided the load current is continuous. The earlier answers for E = 0 are applicable.

iii. From equation (13.13), the maximum and minimum load currents are

$$\hat{I} = \frac{V_s}{R}\frac{1-e^{\frac{-\delta T}{\tau}}}{1-e^{\frac{-T}{\tau}}} - \frac{E}{R} = \frac{340\text{V}}{10\Omega} \times \frac{1-e^{\frac{-1.25\text{ms}}{5\text{ms}}}}{1-e^{\frac{-5\text{ms}}{5\text{ms}}}} - \frac{55\text{V}}{10\Omega} = 6.40\text{A}$$

$$\check{I} = \frac{V_s}{R}\frac{e^{\frac{\delta T}{\tau}}-1}{e^{\frac{T}{\tau}}-1} - \frac{E}{R} = \frac{340\text{V}}{10\Omega} \times \frac{e^{1/4}-1}{e^{1}-1} - \frac{55\text{V}}{10\Omega} = 0.12\text{A}$$

The peak-to-peak ripple in the output current is therefore

$$\begin{aligned} I_{pp} &= \hat{I} - \check{I} \\ &= 6.4\text{A} - 0.12\text{A} = 6.28\text{A} \end{aligned}$$

The ripple value is the same as the E = 0 case, which is as expected since ripple current is independent of back emf with continuous output current.
Alternatively the ripple can be extracted from figure 13.4 using T/τ =1 and δ=¼.

iv. The time domain load current is defined by

$$i_o = \frac{V_s - E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \check{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = 28.5\times\left(1-e^{\frac{-t}{5ms}}\right) + 0.12e^{\frac{-t}{5ms}}$$

$$= 28.5 - 28.38e^{\frac{-t}{5ms}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 1.25\text{ms}$$

$$i_o = -\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right) + \hat{I}\,e^{\frac{-t}{\tau}}$$

$$i_o(t) = -5.5\times\left(1-e^{\frac{-t}{5ms}}\right) + 6.4e^{\frac{-t}{5ms}}$$

$$= -5.5 + 11.9e^{\frac{-t}{5ms}} \quad \text{(A)} \qquad \text{for} \quad 0 \le t \le 3.75\text{ms}$$

v. The average load current from equation (13.37) is

$$\overline{I}_o = \frac{V_o - E}{R}$$

$$= \frac{85\text{V} - 55\text{V}}{10\Omega} = 3\text{A}$$

The average switch current is the average supply current,

$$\overline{I}_i = \overline{I}_{switch} = \frac{\delta(V_s - E)}{R} - \frac{\tau}{T}\left(\hat{I} - \check{I}\right)$$

$$= \frac{\frac{1}{4}\times(340\text{V} - 55\text{V})}{10\Omega} - \frac{5\text{ms}}{5\text{ms}}\times(6.40\text{A} - 0.12\text{A}) = 0.845\text{A}$$

The average diode current is the difference between the average load current and the average input current, that is

$$\overline{I}_{diode} = \overline{I}_o - \overline{I}_i$$

$$= 3\text{A} - 0.845\text{A} = 2.155\text{A}$$

vi. The input power is the dc supply multiplied by the average input current, that is

$$P_{in} = V_s \overline{I}_i = 340\text{V}\times 0.845\text{A} = 287.3\text{W}$$

$$P_{out} = P_{in} = 287.3\text{W}$$

From equation (13.18) the rms load current is given by

$$\overline{I}_{o,rms} = \sqrt{\frac{P_{out} - E\overline{I}_o}{R}}$$

$$= \sqrt{\frac{287.3\text{W} - 55\text{V}\times 3\text{A}}{10\Omega}} = 3.5\text{A rms}$$

vii. The chopper effective input impedance is

$$Z_{in} = \frac{V_s}{\overline{I}_i}$$

$$= \frac{340\text{V}}{0.845\text{A}} = 402.4\ \Omega$$

The electromagnetic efficiency is given by equation (13.22), that is

$$\eta = \frac{E\overline{I}_o}{P_{in}}$$

$$= \frac{55\text{V}\times 3\text{A}}{287.3\text{W}} = 57.4\%$$

viii. The output voltage and current waveforms for the first-quadrant chopper, with and without back emf, are shown in the figure to follow.

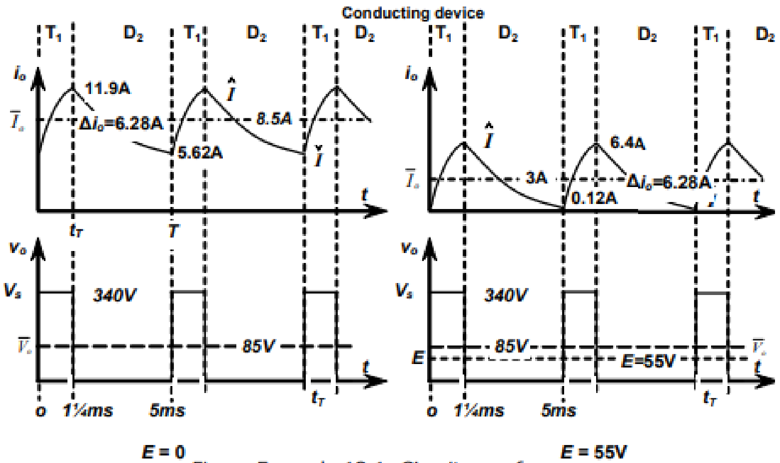


Figure Example 13.1. *Circuit waveforms.*

6. A step-down chopper supplies a separately excited dc motor with a supply voltage $E = 240$ V and back emf $E_b = 100$ V. Other data are total inductance L

$= 30$ mH, armature resistance $R_a = 2.5\ \Omega$, chopper frequency $= 200$, and duty cycle $= 50\%$. Assuming continuous current determine $I_{\max}$, $I_{\min}$, and the current ripple.

Solution

The circuit and waveforms are given in Fig. 3.3. From Eqn (3.17),

$$I_{\max} = \frac{E}{R_a}\left(\frac{1 - e^{-\tau_{ON}/T_a}}{1 - e^{-\tau/T_a}}\right) - \frac{E_b}{R_a}$$

$$\frac{\tau_{ON}}{\tau} = 0.5 \quad \text{and} \quad \tau = \frac{1}{200} = 0.005$$

Hence,

$$\tau_{ON} = \frac{0.5}{200} = 0.0025$$

$$T_a = \frac{L_a}{R_a} = \frac{30 \times 10^{-3}}{2.5} = 0.012$$

$$\frac{\tau_{ON}}{T_a} = \frac{0.0025}{0.012}$$

$$\frac{\tau}{T_a} = \frac{0.005}{0.012}$$

$$(1 - e^{-\tau_{ON}/T_a}) = 0.188, \quad (e^{-\tau_{ON}/T_a} - 1) = 0.2316$$

$$(1 - e^{-\tau/T_a}) = 0.34, \quad (e^{-\tau/T_a} - 1) = 0.5169$$

Hence,

$$I_{\max} = \frac{240}{2.5} \times \frac{0.188}{0.34} - \frac{100}{2.5} = 13.0 \text{ A}$$

From Eqn (3.18),

$$I_{\min} = \frac{E}{R_a}\left[\frac{e^{\tau_{ON}/T_a} - 1}{e^{\tau/T_a} - 1}\right] - \frac{E_b}{R_a}$$

$$= \frac{240}{2.5} \times \frac{0.2316}{0.5169} - \frac{100}{2.5} = 3.0 \text{ A}$$

The current ripple is

$$\Delta i_{ld} = \frac{I_{\max} - I_{\min}}{2} = \frac{13 - 3}{2} = 5 \text{ A}$$

7. A step-down chopper feeds a dc motor load. The data pertaining to this chopper-based drive is $E = 210$ V, $R_a = 7\ \Omega$, L (including armature inductance) = 12 mH. Chopper frequency = 1.5 kHz, duty cycle = 0.55, and E_b = 55 V. Assuming continuous conduction, determine the (a) average load current, (b) current ripple,

(c) RMS value of current through chopper, (d) RMS value of current through D_{FW}, and (e) effective input resistance seen by the source, and (f) RMS value of load current.

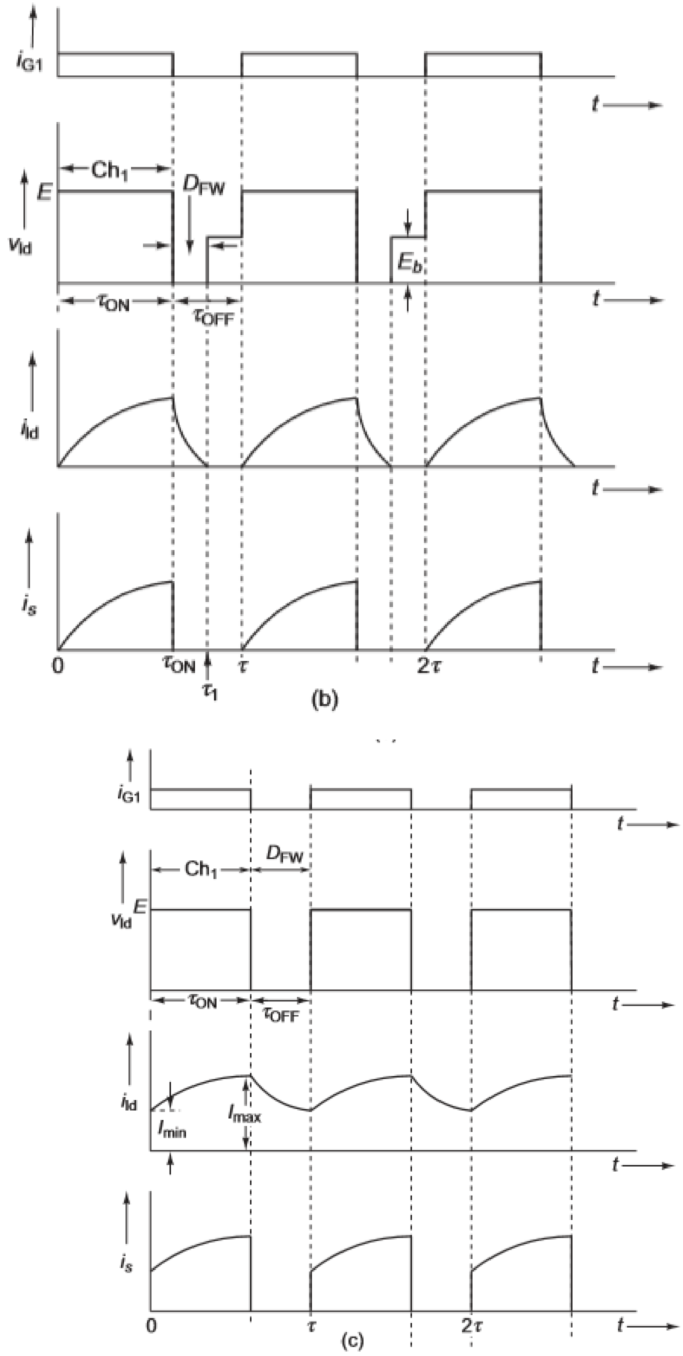


Fig. 3.3 Step-down chopper: (a) circuit, (b) waveforms for discontinuous conduction, (c) waveforms for continuous conduction

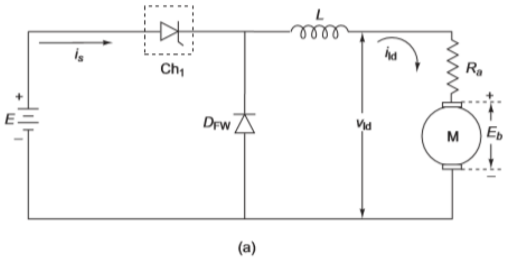


Fig. 3.3(a)

Solution

(a) The circuit and waveforms are given in Fig. 3.3. The average load current is given by Eqn (3.21) as

$$I_{\mathrm{ld}} = \frac{E}{R_a}\frac{\tau_{\mathrm{ON}}}{\tau} - \frac{E_b}{R_a}$$

Substitution of values gives

$$I_{\mathrm{ld}} = \frac{210}{7} \times 0.55 - \frac{55}{7}$$
$$= 8.64 \text{ A}$$

$$I_{\max} = \frac{E}{R_a}\frac{(1 - e^{-\tau_{\mathrm{ON}}/T_a})}{(1 - e^{-\tau/T_a})} - \frac{E_b}{R_a} \tag{3.17}$$

and

$$I_{\min} = \frac{E}{R_a}\frac{(e^{\tau_{\mathrm{ON}}/T_a} - 1)}{(e^{\tau/T_a} - 1)} - \frac{E_b}{R_a} \tag{3.18}$$

The current ripple can now be obtained as

$$\Delta i_{\mathrm{ld}} = \frac{I_{\max} - I_{\min}}{2} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - e^{\tau_{\mathrm{ON}}/T_a} - e^{\tau_{\mathrm{OFF}}/T_a}}{e^{\tau/T_a} - 1}\right] \tag{3.19}$$

(b) The current ripple is given by Eqn (3.19) as

$$\Delta i_{\mathrm{ld}} = \frac{E}{2R_a}\left\{\frac{1 + e^{\tau/T_a} - e^{\tau_{\mathrm{ON}}/T_a} - e^{\tau_{\mathrm{OFF}}/T_a}}{e^{\tau/T_a} - 1}\right\}$$

Here,

$$T_a = \frac{L_a}{R} = \frac{12 \times 10^{-3}}{7}$$

$$\tau = \frac{1}{1.5 \times 10^3} = \frac{1 \times 10^{-3}}{1.5}$$

$$\frac{\tau}{T_a} = \frac{7}{1.5 \times 12} = 0.389$$

$$\frac{\tau_{\mathrm{ON}}}{T_a} = \frac{0.55 \times 10^{-3}}{1.5} \times \frac{7}{12} \times 10^{-3} = 0.214$$

$$\frac{\tau_{\mathrm{OFF}}}{T_a} = \frac{0.45 \times 10^{-3}}{1.5} \times \frac{7}{12} \times 10^{-3} = 0.175$$

Substitution of values gives

$$\Delta i_{\mathrm{ld}} = \frac{210}{2 \times 7} \times \frac{1 + 1.475 - 1.239 - 1.191}{1.475 - 1} = 1.42 \text{ A}$$

(c) It is assumed that the load current increases linearly from $I_{\min}$ to $I_{\max}$ during $(0, \tau_{ON})$. Thus the instantaneous current i_{ld} can be expressed as

$$i_{ld} = I_{\min} + \frac{I_{\max} - I_{\min}}{\tau_{ON}} t, \quad 0 \leq t \leq \tau_{ON}$$

The RMS value of the current through the chopper can now be found as

$$I_{ch(RMS)} = \sqrt{\frac{1}{\tau} \int_0^{\tau_{ON}} (i_{ld})^2 dt}$$

Here,

$$I_{ch(RMS)} = \left\{ \sqrt{\frac{\tau_{ON}}{\tau} \left[I_{\min}^2 + I_{\min}(I_{\max} - I_{\min}) + \frac{(I_{\max} - I_{\min})^2}{3} \right]} \right\}$$

where

$$I_{\min} = \frac{E}{R_a} \left[\frac{e^{\tau_{ON}/T_a} - 1}{e^{\tau/T_a} - 1} \right] - \frac{E_b}{R_a}$$

$$= \frac{210}{7} \left[\frac{1.239 - 1}{1.475 - 1} \right] - \frac{55}{7} = 15.095 - 7.857 = 7.24 \text{ A}$$

$$I_{\max} = \frac{E}{R_a} \left[\frac{1 - e^{-\tau_{ON}/T_a}}{1 - e^{-\tau/T_a}} \right] - E_b/R_a$$

$$= \frac{210}{7} \frac{(1 - 0.807)}{(1 - 0.678)} - \frac{55}{7} = 17.981 - 7.857 = 10.12 \text{ A}$$

Thus,

$$I_{ch(RMS)} = \sqrt{\frac{\tau_{ON}}{\tau} \left[7.24^2 + 7.24(10.12 - 7.24) + \frac{(10.12 - 7.24)^2}{3} \right]}$$

$$= \sqrt{0.55 \times 76.03}$$

$$= 6.46 \text{ A}$$

(d) The RMS value of current through D_{FW} can be found as

$$I_{D_{FW}(RMS)} = \sqrt{\frac{1}{\tau} \int_0^{\tau_{OFF}} (i_{ld})^2 dt'} \quad (t' = t - \tau_{ON})$$

$$= \sqrt{\frac{\tau_{OFF}}{\tau} \left[(10.12)^2 + \frac{(2.88)^2}{3} - 10.12 \times 2.88 \right]}$$

(where $I_{\max} - I_{\min} = 10.12 - 7.24 = 2.88$ A)

$$= \sqrt{0.45 \times 76.02} = 5.85 \text{ A}$$

(e) Average source current $= (\tau_{ON}/\tau) \times$ average load current

$$= 0.55 \times \frac{(7.24 + 10.12)}{2} = 0.55 \times 8.68 = 4.77 \text{ A}$$

Hence, the effective input resistance seen by the source $= 210/4.77 = 44\ \Omega$.

(f) RMS value of load current $= \sqrt{\frac{1}{\tau} \left[\int_0^{\tau_{ON}} \{i_{ld}(t)\}^2 dt + \int_0^{\tau_{OFF}} \{i_{ld}(t')\}^2 dt' \right]} = 8.72$ A

with $t' = t - \tau_{ON}$. This is seen to be nearly equal to the average load current, namely, 8.68 A.

8. A step-down chopper has the following data: $R_{\mathrm{ld}} = 0.40\,\Omega$, $E = 420$ V, $E_b = 25$ V. The average load current is 175 A and the chopper frequency is 280 Hz. Assuming the load current to be continuous, and linearly rising to the maximum and then linearly falling, calculate the inductance L which would limit the maximum ripple in the load current to 12% of the average load current.

Solution

The circuit is given in Fig. 3.3(a). The expression for the current ripple in Eqn (3.19) can be written with substitutions $\tau_{\mathrm{ON}} = \delta\tau$ and $\tau_{\mathrm{OFF}} = (1-\delta)\tau$, where δ is in the range $0 < \delta < 1$. Thus,

$$\Delta i_{\mathrm{ld}} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - e^{\delta\tau/T_a} - e^{(1-\delta)\tau/T_a}}{e^{\tau/T_a} - 1}\right]$$

with $\delta = \tau_{\mathrm{ON}}/\tau$. Differentiating the ripple current with respect to δ and equating this to zero gives the value of δ for maximum ripple:

$$\frac{d(\Delta i_{\mathrm{ld}})}{d\delta} = \left(-\frac{\tau}{T_a}e^{\delta\tau/T_a}\right) + \frac{\tau}{T_a}e^{(1-\delta)\tau/T_a} = 0$$

This yields

$$e^{\delta\tau/T_a} = e^{(1-\delta)\tau/T_a}$$

or

$$\delta = 1 - \delta$$

This gives $\delta = 0.5$. Substituting this value of δ in the expression for Δi_{ld} gives

$$\Delta i_{\mathrm{ld}} = \frac{E}{2R_a}\left[\frac{1 + e^{\tau/T_a} - 2e^{0.5\tau/T_a}}{e^{\tau/T_a} - 1}\right]$$

$$= E/2R_a\left[\frac{(e^{0.5\tau/T_a} - 1)^2}{(e^{0.5\tau/T_a} - 1)(e^{0.5\tau/T_a} + 1)}\right]$$

$$= \frac{E}{2R_a}\left[\frac{e^{0.5\tau/T_a} - 1}{e^{0.5\tau/T_a} + 1}\right]$$

$$= \frac{E}{2R_a}\tanh\frac{R_a}{4fL}$$

where $f = 1/\tau$ is the chopper frequency. If $4fL >> R_a$, then $\tanh(R_a/4fL) \approx R_a/4fL$. Thus,

$$\Delta I_{\mathrm{ld(max)}} = \frac{E}{2R_a}\frac{R_a}{4fL} = \frac{E}{8fL}$$

The condition $\Delta I_{\mathrm{ld(max)}} = 12\% I_{\mathrm{ld}}$ gives

$$\frac{E}{8fL} = 0.12 \times 175$$

Hence,

$$L = \frac{E}{8f \times 0.12 \times 175} = \frac{420}{8 \times 280 \times 0.12 \times 175} = 0.0089 \text{ H}$$

$$= 8.9 \text{ mH}$$

9. A 240-V, separately excited dc motor has an armature resistance of 2.2 Ω and an inductance of 4 mH. It is operated at constant load torque. The initial speed is 600 rpm and the armature current is 28 A. Its speed is now controlled by a step-down chopper with a frequency of 1 kHz, the input voltage remaining at 240 V. (a) If the speed is reduced to 300 rpm, determine the duty cycle of the chopper. (b) Compute the current ripple with this duty cycle.

Solution

(a) The equation for the motor is

$$V = E_b + I_a R_a$$

Substitution of values gives

$$240 = E_b + 28 \times 2.2$$

Thus,

$$E_b = 240 - 28 \times 2.2 = 178.4 \text{ V}$$

Other quantities on the right-hand side of the expression for E_b remaining constant, it can be expressed as

$$E_b = kN$$

or

$$178.4 = k600$$

$$k = \frac{178.4}{600} = 0.297$$

The new speed is 300 rpm. Hence,

$$E_{b(\text{new})} = 0.297 \times 300 = 89.1 \text{ V}$$

The new applied voltage with the same load torque, that is, the same armature current, is

$$V_{\text{new}} = E_b + I_a R_a = 89.1 + 28 \times 2.2 = 150.7 \text{ V}$$

The duty cycle of the chopper (τ_{ON}/τ) can be determined from the relation

$$V_{\text{new}} = \frac{\tau_{\text{ON}}}{\tau} \times \text{input voltage}$$

This gives

$$\frac{\tau_{\text{ON}}}{\tau} = \frac{V_{\text{new}}}{\text{input voltage}} = \frac{150.7}{240} = 0.628$$

(b) $T_a = L/R_a = 4 \times 10^{-3}/2.2$; $\tau = 1/1000 = 10^{-3}$. Therefore,

$$\frac{\tau}{T_a} = \frac{2.2}{4} = 0.55$$

$$\frac{\tau_{\text{ON}}}{T_a} = \frac{\tau_{\text{ON}}}{\tau}\frac{\tau}{T_a} = 0.628 \times 0.55 = 0.345$$

$$\frac{\tau_{\text{OFF}}}{T_a} = \frac{\tau}{T_a} - \frac{\tau_{\text{ON}}}{T_a} = 0.55 - 0.345 = 0.205$$

The current ripple is given as

$$\begin{aligned}\Delta i_{\text{ld}} &= \frac{E}{2R_a}\frac{(1 + e^{\tau/T_a} - e^{\tau_{\text{ON}}/T_a} - e^{\tau_{\text{OFF}}/T_a})}{e^{\tau/T_a} - 1}\\ &= \frac{150.7}{2 \times 2.2} \times \frac{1 + 1.733 - 1.412 - 1.227}{1.733 - 1}\\ &= 4.39 \text{ A}\end{aligned}$$

10. A dc chopper is used for regenerative braking of a separately excited dc motor as shown in Fig. 7.24(b). The data are $E = 400$ V, $R = 0.2\ \Omega$, and $L_a = 0.2$ mH. The back emf constant $K_b^1\ (= K_b\phi_f)$, assuming ϕ_f to be constant, is equal to 1.96 V/rad s, and the average load current is 200 A. The frequency of the chopper is 1 kHz and $(\tau_{ON}/\tau)_a$ is 0.5. Compute the (a) average load voltage, (b) motor speed, (c) power regenerated and fed back to the battery, (d) equivalent resistance viewed from the motor side when it is working as a generator, and (e) minimum and maximum permissible speeds for regenerative braking.

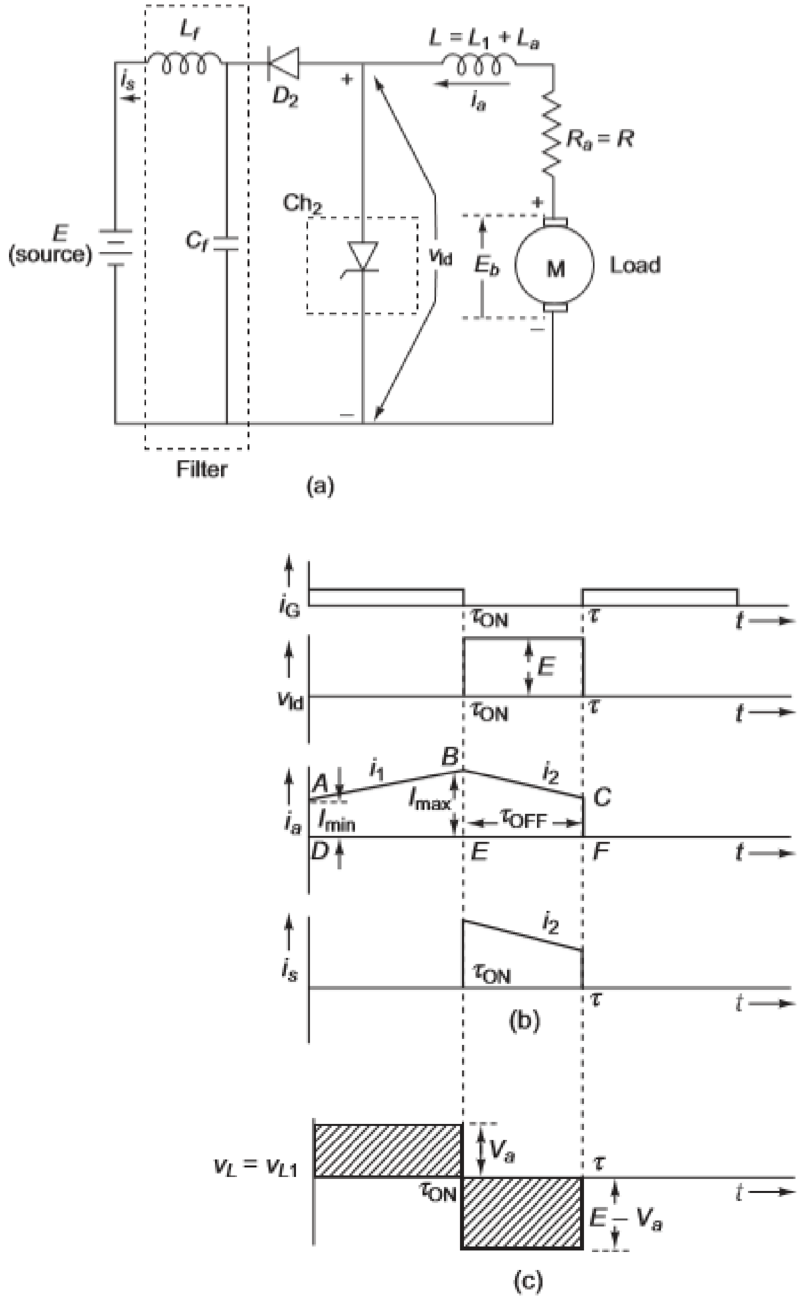


Fig. 7.26 Regenerative braking of a chopper-based dc drive: (a) circuit diagram, (b) waveforms, (c) voltage across inductor L for one period (τ)

Solution

(a) The circuit and waveforms are shown in Figs 7.26(a) and (b). The average load voltage is

$$V_a = E\left(1 - \frac{\tau_{ON}}{\tau}\right)_a = 400 \times 0.5 = 200 \text{ V}$$

(b)
$$I_a = \frac{I_{max} + I_{min}}{2} = 200 \text{ A}$$

or $I_{max} + I_{min} = 400$ A. From Eqns (7.64) and (7.65),

$$I_{max} + I_{min} = \frac{2E_b}{R} - \frac{E}{R}\left[\frac{1 - e^{-\tau_{OFF}/T_a}}{1 - e^{-\tau/T_a}} + \frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$

Here,

$$T_a = \frac{L_a}{R} = \frac{0.0002}{0.2} = 0.001$$

$$\tau = \frac{1}{1000} = 0.001$$

Hence,

$$\frac{\tau}{T_a} = 1 \text{ and } \frac{\tau_{ON}}{T_a} = \frac{0.0005}{0.001} = 0.5$$

Also

$$\tau_{OFF}/T_a = 0.5$$

Substituting the above values in the equation for $I_{max} + I_{min}$ gives

$$400 = \frac{2E_b}{0.2} - \frac{400}{0.2}\left[\frac{1 - 0.606}{1 - 0.368} + \frac{1.649 - 1}{2.718 - 1}\right]$$
$$= 10E_b - 2000(0.623 + 0.377)$$

This gives the value of the back emf as

$$E_b = 240$$

$$\text{Speed in rad/s} = \frac{240}{K_b^1} = \frac{240}{1.96} = 122.4$$

$$\text{Speed in rpm} = \frac{60}{2\pi} \times 122.4 = 1169 \text{ rpm}$$

(c) From Eqn (7.69), the power regenerated is

$$P_{reg} = \frac{E}{\tau}\left\{\frac{(E_b - E)}{R}[\tau_{OFF} + T_a(e^{-\tau_{OFF}/T_a} - 1)] - T_a I_{max}(e^{-\tau_{OFF}/T_a} - 1)\right\}$$

$$I_{max} = \frac{E_b}{R} - \frac{E}{R}\left[\frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$
$$= \frac{240}{0.2} - \frac{400}{0.2}\left[\frac{1.649 - 1}{2.718 - 1}\right]$$
$$= 1200 - 2000 \times \frac{0.649}{1.718} = 444 \text{ A}$$

Substitution of all values in the expression for P_{reg} gives

$$\begin{aligned}
P_{reg} &= 400\left\{\frac{240-400}{0.2}\left[\frac{\tau_{OFF}}{\tau}+\frac{T_a}{\tau}(e^{-0.5}-1) - \frac{T_a}{\tau}I_{max}(e^{-0.5}-1)\right]\right\}\\
&= 400\left\{\frac{-160}{0.2}[0.5+1(0.606-1)] - 1\times 444(0.606-1)\right\}\\
&= 400\left\{\frac{(-160\times 0.106)}{0.2} + 444\times 0.394\right\}\\
&= 400\times 90 = 36{,}000\text{ W or } 36\text{ kW}
\end{aligned}$$

(d) Average generated voltage

$$E_b = E(1-\delta)+I_aR_a = 400(1-0.5)+200\times 0.2 = 240\text{ V}$$

Average current through the motor, $I_a = 200$ A. Hence the equivalent resistance is

$$\frac{E_b}{I_a} = \frac{240}{200} = 1.2\ \Omega$$

(e) Minimum and maximum permissible speeds for regeneration are obtained from the inequality

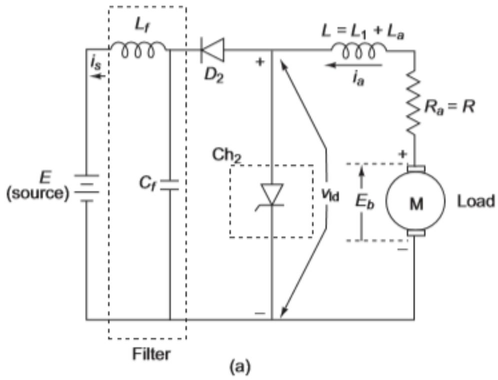
$$0 \le \omega_m \le \frac{E}{K_b^1}$$

or

$$0 \le \omega_m \le \frac{400}{1.96} = 204$$

Thus the maximum permissible speed is 204 rad/s or 1949 rpm. Also the minimum permissible speed is zero rpm.

13. A dc chopper is used for regenerative braking of a dc series motor as shown in Fig. 7.26(a). The data are $E = 400$ V, $R_a = 0.05\ \Omega$, $R_f = 0.04\ \Omega$, $L = L_a + L_f = 0.09$ mH. The product $K_b\phi_f$ may be assumed to be constant at 1.5 V/rad s and the average load current is 200 A. The frequency of the chopper is 1 kHz and $\tau_{ON}/\tau = 0.6$. Compute the (a) average load voltage, (b) motor speed, (c) power regenerated and fed back to the battery, (d) equivalent resistance viewed from the motor side when it is working as a generator, and (e) minimum and maximum permissible speeds.



(a)

Solution

(a) $$V_a = E\left(1 - \frac{\tau_{ON}}{\tau}\right) = 400(1 - 0.6) = 160 \text{ V}$$

(b) The correct value of E_b is arrived at as follows:

$$I_a = \frac{I_{max} + I_{min}}{2} = 200 \text{ A}$$

or

$$I_{max} + I_{min} = 400 \text{ A}$$

$$T_a = \frac{L_a + L_f}{R_a + R_f} = \frac{0.09 \times 10^{-3}}{0.09} = 0.001 \text{ s}$$

$$\tau = \frac{1}{f_{Ch}} = \frac{1}{1000} = 0.001 \text{ s}$$

From Eqns (7.64) and (7.65),

$$2I_a = 400 = I_{max} + I_{min} = \frac{2E_b}{R} - \frac{E}{R}\left[\frac{1 - e^{-\tau_{OFF}/T_a}}{1 - e^{-\tau/T_a}} + \frac{e^{\tau_{OFF}/T_a} - 1}{e^{\tau/T_a} - 1}\right]$$

Thus,

$$400 = \frac{2E_b}{0.09} - \frac{400}{0.09}\left[\frac{0.33}{0.632} + \frac{0.492}{1.718}\right]$$

Rearranging terms gives

$$\frac{2E_b}{0.09} = 400 + \frac{400}{0.09} \times 0.808$$

and we obtain

$$E_b = 179.6 \text{ V}$$

This gives

$$\text{Speed } N = 119.5 \times \frac{60}{2\pi} = 1141 \text{ rpm}$$

$$\begin{aligned} I_{\max} &= \frac{E_b}{R} - \frac{E}{R}\left[\frac{e^{\tau_{\mathrm{OFF}}/T_a} - 1}{e^{\tau/T_a} - 1}\right] \\ &= \frac{179.6}{0.09} - \frac{400}{0.09} \times \frac{0.492}{1.728} \\ &= 724.5 \text{ A} \end{aligned}$$

$$\begin{aligned} I_{\min} &= \frac{E_b}{R} - \frac{E}{R}\frac{(1 - e^{-\tau_{\mathrm{OFF}}/T_a})}{(1 - e^{-\tau/T_a})} \\ &= \frac{179.6}{0.09} - \frac{400}{0.09} \times \frac{0.33}{0.632} \\ &= -325 \text{ A} \end{aligned}$$

(c) $$P_{\mathrm{reg}} = \frac{E}{\tau}\left\{\frac{(E_b - E)}{R}\left[\frac{\tau_{\mathrm{OFF}}}{\tau} + \frac{T_a}{\tau}(e^{-\tau_{\mathrm{OFF}}/T_a} - 1)\right] - \frac{T_a I_{\max}}{\tau}(e^{-\tau_{\mathrm{OFF}}/T_a} - 1)\right\}$$

$$= 400\left\{\frac{(179.6 - 400)}{0.09}[0.4 + 1(0.67 - 1)] - 1 \times 724.5(0.67 - 1)\right\}$$

$$\begin{aligned} &= 400\left\{\frac{-220.4}{0.09}[0.4 - 0.33] - 724.5(-0.33)\right\} \\ &= 400\left\{\frac{-220.4}{0.09} \times 0.07 + 239\right\} \\ &= 400\{-171.4 + 239\} \\ &= 400 \times 67.6 = 27{,}031 \text{ W} \approx 27 \text{ kW} \end{aligned}$$

(d) Equivalent resistance = back emf/average current = 179.6/200 ≈ 0.9 Ω

(e) The minimum and maximum speeds are given as

$$0 \le N \le \frac{E}{K_b\phi_f}\frac{60}{2\pi}$$

or

$$0 \le N \le \frac{400}{1.5}\frac{60}{2\pi}$$

This gives

$$0 \le N \le 2546 \text{ rpm}$$

Hence the minimum and maximum speeds are 0 and 2546 rpm, respectively.

Example 12.4

A separately-excited d.c. motor has the following parameters:

$R_a = 0.5\ \Omega$, $L_a = 5.0$ mH , $K_e\ \Phi = 0.078$ V/rpm.

The motor speed is controlled by a class-A d.c. chopper fed from an ideal 200 V d.c. source. The motor is driven at a speed of 2180 rpm by switching on the thyristor for a period of 4 ms in each overall period of 6 ms .

(a) State whether the motor will operate in continuous or discontinuous current mode,
(b) Calculate the extinction angle of the current if it exist,
(c) Sketch the armature voltage and current waveforms,
(d) Calculate the maximum and minimum values of the armature current,
(e) Calculate the average armature voltage and current.

Solution

(a) To find whether the motor operates in continuous or discontinuous current modes ,we have to find the values of γ and γ' :

$$\gamma = \frac{t_{on}}{t_{on} - t_{off}} = \frac{t_{on}}{T} = \frac{4ms}{6ms} = 0.667$$

$$T = 6\ x\ 10^{-3}\ \text{s} \quad \rightarrow \quad \omega = 2\pi\,\frac{1}{T} = 1046.6\ \text{rad/s}$$

The armature circuit tim constant is $\tau = \frac{L_a}{R_a} = \frac{5}{0.5} = 10$ ms

Therefore, $\frac{2\pi}{\omega\tau} = \frac{2 \times 3.14}{1046.7 \times 10\ \times\ 10^{-3}} = 0.6$

At speed of 2180 rpm , $E_a = K_e\ \phi\ n = 0.078\ \times 2180 = 170$ V

The critical value of γ' will be , (using Eq.(12. 31))

$$\frac{E_a}{V_d} = \frac{e^{2\pi\gamma'/\omega\tau} - 1}{e^{2\pi/\omega\tau} - 1} = \frac{170}{200} = \frac{e^{0.6\,\gamma'} - 1}{e^{0.6} - 1}$$

From which $\gamma' = 0.0.8829$, therefore , $\gamma' > \gamma$, hence the motor is operating in discontinuous current mode.

(b) The extinction angle x of the current is calculated from Eq.(12.29) as,

$$x = \omega\tau \ln\left[e^{(2\pi\gamma)/\omega\tau}\left\{1 + (\frac{V_d}{E_a} - 1)(1 - e^{-2\pi\gamma/\omega\tau})\right\}\right]$$

$$\omega\tau = 1046.7x10\ x\ 10^{-3} = 10.467\ \text{rad}$$

$$x = 10.467 \ln\left[e^{(2\pi x0.6)/10.467}\left\{1 + (\frac{200}{170} - 1)(1 - e^{-2\pi x0.6/10.467})\right\}\right]$$

From which $x = 4.8$ rad $\rightarrow$ $x = 275.16°$

(c) The armature voltage and current waveforms are shown in Fig.12.7.

(d) The maximum and minimum values of the armature currents are:

$I_{min} = 0$, since it is discontinuous.
I_{maxD} is calculated from Eq.(12.26) as,

$$I_{max_D} = \frac{V_d - E_a}{R_a}(1 - e^{-2\pi\gamma/\omega\tau})$$

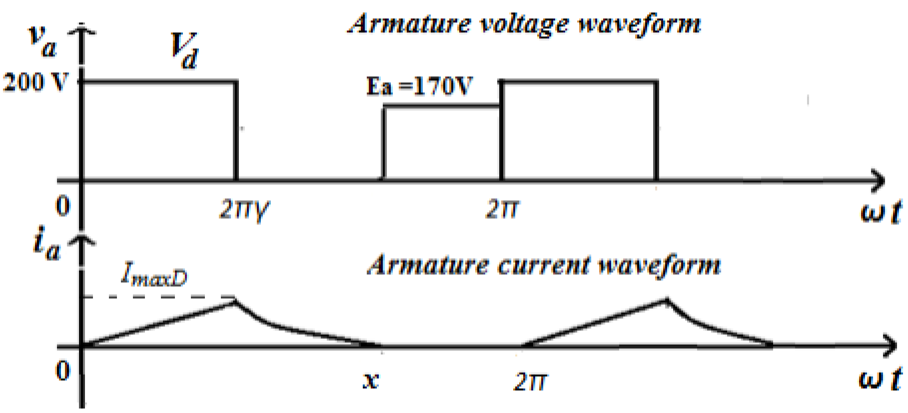


Fig.12.7 Armature voltage and current waveforms.

Example 12.6

A class-A d.c. chopper operating at a frequency of 500 Hz and feeding a separately-excited d.c. motor from 200 V d.c. source. The load torque is 35 Nm and speed is 1000 rpm. Motor resistance and inductance are 0.15 Ω and 1.0 mH respectively. The *emf* and torque constant of motor are 1.6 V/rad/s and 1.4 Nm /A respectively. Find (a) Maximum and minimum values of motor armature current, and (b) Variation of armature current. Neglect chopper losses.

Solution

(a) Let duty cycle = γ

$$V_d = 200 \text{ V}$$

$$V_{av} = \gamma V_d = \gamma \times 200$$

Average armature current $I_a = T / K\varphi = 35/1.4 = 25$ A

Back *emf* $E_a = K\,\varphi\,\omega = 1.6 \times (950 \times 2\pi/60) = 159.16$ V

$$V_{av} = E_a + I_a R_a$$

$$200\,\gamma = 159.16 + 25 \times 0.15 = 162.29 \text{ V}$$

$$\gamma = 0.8145$$

$$T = 1/500 = 2 \text{ ms}$$

$$t_{on} = \gamma T = 2 \times 0.8145 = 1.629 \text{ ms}$$

$$t_{off} = 2 - 1.629 = 0.371 \text{ ms}$$

From Eq.s (12.19) and (12.20) ,The maximum and minimum currents are calculated as

Let:

$$T = 2\pi\ , \quad t_{on} = 2\pi\gamma = \gamma T\ , \quad \tau = \frac{R_a}{L_a}\ \text{,and}\quad -t_{on}/\tau = \frac{-\gamma T R_a}{L_a}$$

Hence Eq.(12.19) and (12.20) can be re-written as

$$I_{max} = \frac{V_d}{R_a}\left(\frac{1-e^{-t_{on}/\tau}}{1-e^{-T/\tau}}\right) - \frac{E_a}{R_a}$$

and

$$I_{min} = \frac{V_d}{R_a}\left(\frac{e^{t_{on}/\tau}-1}{e^{T/\tau}-1}\right) - \frac{E_a}{R_a}$$

$$\frac{TR_a}{L_a} = \frac{2\times10^{-3}\times0.15}{1\times10^{-3}} = 0.30$$

$$e^{-\frac{\gamma TR_a}{L_a}} = e^{-0.8145\times0.3} = e^{-0.24435} = 0.7832$$

$$e^{-\frac{TR_a}{L_a}} = e^{-0.3} = 0.7408$$

$$I_{max} = \frac{200}{0.15}\left(\frac{1-0.7832}{1-0.7408}\right) - \frac{159.15}{0.15}$$

$$= 1333.34 \times \left(\frac{0.2168}{0.2592}\right) - 1061 = 54{,}2A$$

$$I_{min} = \frac{200}{0.15}\left(\frac{1.2767-1}{1.3498-1}\right) - \frac{159.15}{0.15}$$

$$= 1333.34 \times \left(\frac{0.2767}{0.3498}\right) - 1061 = 0$$

(b) Variation of armature current = $I_{max} - I_{min} = 54{,}2 - 0 = 54{,}2\text{A}$

Example 12.19. *A separately-excited dc motor is fed from 220 V dc source through a chopper operating at 400 Hz. The load torque is 30 Nm at a speed of 1000 rpm. The motor has $r_a = 0$, $L_a = 2$ mH and $K_m = 1.5$ V-sec/rad. Neglecting all motor and chopper losses, calculate*

(a) the minimum and maximum values of armature current and the armature current excursion,

(b) the armature current expressions during on and off periods.

Solution. As the armature resistance is neglected, armature current varies linearly between its minimum and maximum values.

(*a*) Average armature current, $I_a = \frac{T_e}{K_m} = \frac{30}{1.5} = 20$ A

Motor emf, $E_a = K_m \cdot \omega_m = 1.5 \times \frac{2\pi \times 1000}{60} = 157.08$ V

Motor input voltage, $\alpha V_s = V_t = E_a + I_a r_s = 157.08 + 0$

$\therefore$ $\alpha = \frac{157.08}{220} = 0.714$

Periodic time, $T = \frac{1}{f} = \frac{1}{400} = 2.5$ ms

On-period, $T_{on} = \alpha\, T = 0.714 \times 2.5 = 1.785$ ms

Off-period, $T_{off} = (1 - \alpha)\, T = 0.715$ ms

During on-period T_{on}, armature current will rise which is governed by the equation,

$$0 + L\frac{di_a}{dt} + E_a = V_s$$

or $$\frac{di_a}{dt} = \frac{V_s - E_a}{L} = \frac{220 - 157.08}{0.02} = 3146 \text{ A/s}$$

During off period, $$\frac{di_a}{dt} = -\frac{E_a}{L} = \frac{-157.08}{0.02} = -7854 \text{ A/s}$$

With current rising linearly, it is seen from Fig. 12.21 that

$$I_{mx} = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times T_{on}$$

$$= I_{mn} + 3146 \times 1.785 \times 10^{-3}$$

or $$I_{mx} = I_{mn} + 5.616 \qquad ...(i)$$

For linear variation between I_{mn} and I_{mx}, average value of armature current

$$I_a = \frac{I_{mx} + I_{mn}}{2} = 20 \text{ A}$$

or $$I_{mx} = 40 - I_{mn} \qquad ...(ii)$$

Solving Eqs. (*i*) and (*ii*), we get $I_{mx} = 22.808$ A

and $I_{mn} = 17.912$ A.

$\therefore$ Armature current excursion $= I_{mx} - I_{mn} = 22.808 - 17.912 = 5.616$ A (*b*) Armature current expression during turn-on,

$$i_a(t) = I_{mn} + \left(\frac{di_a}{dt} \text{ during } T_{on}\right) \times t$$

$$= 17.192 + 3146\,t \quad \text{for } 0 \le t \le T_{on}$$

Armature current expression during turn-off,

$$i_a(t) = I_{mx} + \left(\frac{di_a}{dt} \text{ during } T_{off}\right) \times t$$

$$= 22.808 - 7854\,t \quad \text{for } 0 \le t \le T_{off}$$

Example 12.20. *Repeat Example 12.19, in case motor has a resistance of 0.2 Ω for its armature circuit.*

Solution. (*a*) From Example 12.19, armature current, $I_a = 20$ A and motor emf, $E_a = 157.08$ V; source voltage, $V_s = 220$ V.

For armature circuit, $\alpha V_s = V_0 = V_t = E_a + I_a r_a = 157.08 + 20 \times 0.2 = 161.08$ V

$$\therefore \qquad \alpha = \frac{161.08}{220} = 0.7322$$

$$T_{on} = \alpha T = 0.7322 \times 2.5 = 1.831 \text{ ms}$$

$$T_{off} = T - T_{on} = 0.669 \text{ ms}, \frac{R}{L} = \frac{0.2}{0.02} = 10$$

During T_{on}, from Eq. (12.34), armature current is

$$i_a(t) = \frac{220 - 157.08}{0.2}(1 - e^{-10t}) + I_{mn} \cdot e^{-10t}$$

At $t = T_{on} = 1.831$ ms, current become I_{mx}. This gives

$$i_a(t) = I_{mx} = 5.7079 + 0.98187\, I_{mn} \qquad \ldots(i)$$

During T_{off}, from Eq. (12.35), armature current is

$$i_a(t) = \frac{-157.08}{0.2}(1 - e^{-10t}) + I_{mx} \cdot e^{-10t}$$

At $t = 0.669$ ms, $i_a(t) = I_{mn}$. This gives

$$i_a(t) = I_{mn} = -5.237 + 0.9933\, I_{mx} \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$I_{mx} = 5.7079 + 0.98187\,(-5.237 + 0.9933\,I_{mx})$$
$$= 0.5658 + 0.9753\,I_{mx}$$

or
$$I_{mx} = \frac{0.5658}{0.0247} = 22.907 \text{ A}$$
$$I_{mn} = -5.237 + 0.9933 \times 22.907 = 17.516 \text{ A}$$

$\therefore$ Armature current excursion

$$= I_{mx} - I_{mn} = 22.907 - 17.516 = 5.39 \text{ A}$$

(b) Armature current expression during turn-on period is

$$i_a(t) = 314.6\,(1 - e^{-10t}) + 17.516\,e^{-10t}$$

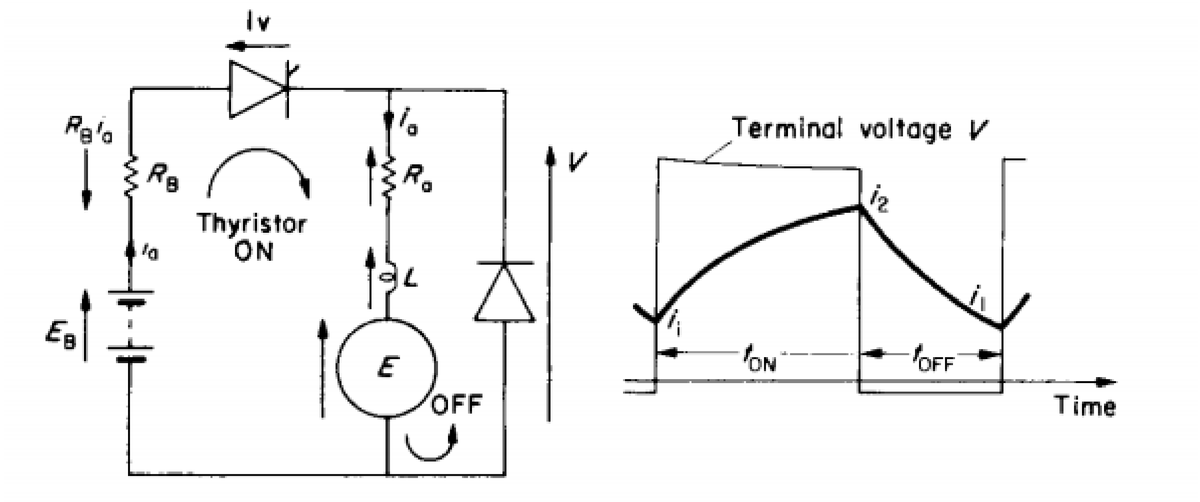
Armature current expression during turn-off period is

$$i_a(t) = -785.4\,(1 - e^{-10t}) + 22.907\,e^{-10t}$$

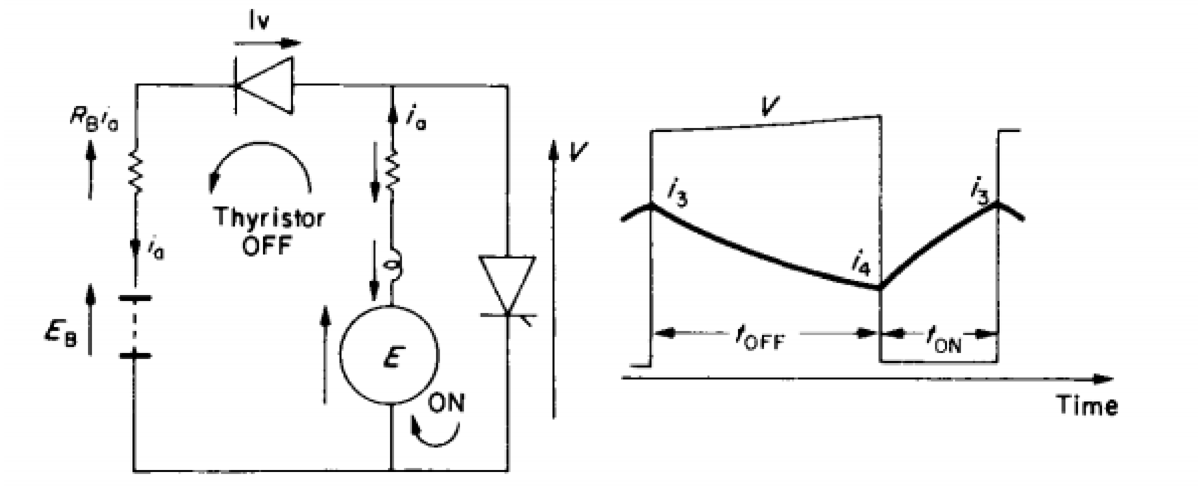
Example 7.1

An electrically-driven automobile is powered by a d.c. series motor rated at 72 V, 200 A. The motor resistance and inductance are respectively 0.04 Ω and 6 milli-henrys. Power is

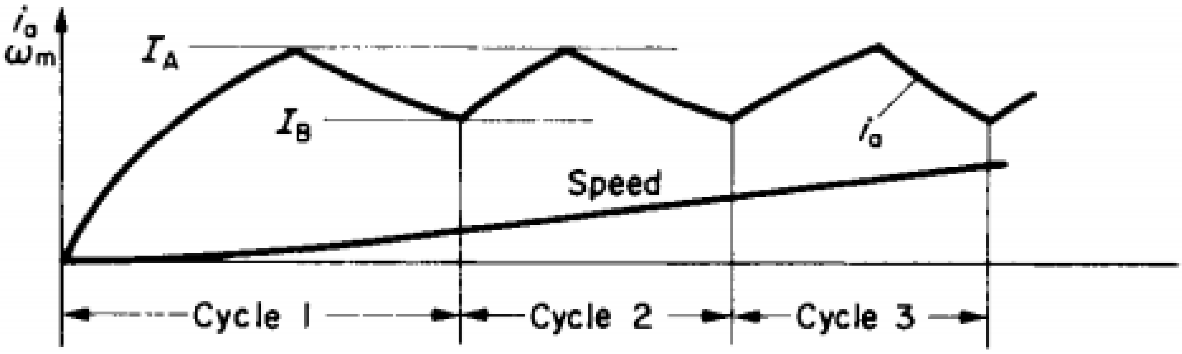
supplied via an ON/OFF controller having a fixed frequency of 100 Hz. When the machine is running at 2500 rev/min the generated-e.m.f. per field-ampere, k_{fs}, is 0.32 V which may be taken as a mean "constant" value over the operating range of current. Determine the maximum and minimum currents, the mean torque and the mean power produced by the motor, when operating at this particular speed and with a duty-cycle ratio δ of 3/5. Mechanical, battery and semi-conductor losses may be neglected when considering the relevant diagrams of Fig. 7.1a.



(a) MOTORING (Motoring conventions)



(b) GENERATING (Generating conventions)



(c) Acceleration between limits

FIG. 7.1. Chopper-fed d.c. machine.

Chopping period = 1/100 = 10 msec and for $\delta = 3/5$; ON + OFF = 6 + 4 msec.

The equations are:

for ON period: $V = k_{fs}i + Ri + Lpi$—from eqn (7.2a).

Substituting: $72 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$.

For OFF period: $0 = 0.32i + 0.04i + 0.006\,\mathrm{d}i/\mathrm{d}t$—from eqn (7.2b).

Rearranging:

ON $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 200 = I_{max}$,

OFF $0.0167\,\mathrm{d}i/\mathrm{d}t + i = 0 = I_{min}$.

Current oscillates between a "low" of i_1 and a "high" of i_2, with $\tau = 0.0167$ second

ON $i_2 = i_1 + (200 - i_1)(1 - e^{-0.006/0.0167})$,

$i_2 = 200 - (200 - i_1)e^{-0.36} = \underline{60.46 + 0.698 i_1}$.

OFF $i_1 = i_2 + (0 - i_2)(1 - e^{-0.004/0.0167})$

$i_1 = i_2\ e^{-0.24} = \underline{0.787 i_2}$.

Hence, by substituting: $i_2 = 60.46 + 0.698 \times 0.787 i_2$,

from which $\underline{i_2 = 134.1\text{ A}}$ and $\underline{i_1 = 105.6\text{ A}}$.

$$\text{Torque} = k_\phi i = \frac{k_{fs} i}{\omega_m} \times i = \frac{k_{fs}}{\omega_m} i^2.$$

$$\text{Mean torque} = \frac{0.32}{2500 \times 2\pi/60}\left(\frac{134.1^2 + 105.6^2}{2}\right) = \underline{17.8\text{ Nm}}.$$

$$\text{Mean power} = \omega_m T_e = \frac{2\pi}{60} \times 2500 \times 17.8 = \underline{4.66\text{ kW}} = \underline{6.25\text{ hp}}.$$

Example 7.2

The chopper-controlled motor of the last question is to be separately excited at a flux corresponding to its full rating. During acceleration, the current pulsation is to be maintained as long as possible between 170 and 220 A. During deceleration the figures are to be 150 and 200 A. The total mechanical load referred to the motor shaft corresponds to an armature current of 100 A and rated flux. The total inertia referred to the motor shaft is 1.2 kg m^2. The battery resistance is 0.06 Ω and the semiconductor losses may be neglected. Determine the ON and OFF periods for both motoring and regenerating conditions and hence the chopping frequency when the speed is 1000 rev/min.

Calculate the accelerating and decelerating rates in rev/min per second and assuming these rates are maintained, determine the time to accelerate from zero to 1000 rev/min and to decelerate to zero from 1000 rev/min. Reference to all the diagrams of Fig. 7.1 will be helpful.

Rated flux at rated speed of 2500 rev/min corresponds to an e.m.f.:

$$E = V - RI_a = 72 - 0.04 \times 200 = 64\ \text{V}$$

At a speed of 1000 rev/min therefore, full flux corresponds to $64 \times 1000/2500 = 25.6$ V

Acceleration Total resistance $= R_a + R_B = 0.04 + 0.06 = 0.1\ \Omega$

For ON period $E_B = E + Ri_a + Lpi_a$,

$$72 = 25.6 + 0.1\,i_a + 0.006\,pi_a.$$

Rearranging: $0.06\ di_a/dt + i_a = 464 = I_{max}$.

Solution is: $i_2 = i_1 + (I_{max} - i_1)(1 - e^{-t_{ON}/\tau})$

and since i_1 and i_2 are known: $220 = 170 + (464 - 170)(1 - e^{-t_{ON}/0.06})$.

$$\frac{220-170}{464-170} = 1 - e^{-t_{ON}/0.06}$$

from which: $\underline{t_{ON} = 0.01118.}$

For OFF period $0 = 25.6 + 0.04i_a + 0.006pi_a$ (note resis. $= R_a$).

Rearranging: $0.15\,di_a/dt + i_a = -640 = I_{min}$.

Solution is: $i_1 = i_2 + (I_{min} - i_2)(1 - e^{-t_{OFF}/\tau})$.

Substituting i_1 and i_2: $170 = 220 + (-640 - 220)(1 - e^{-t_{OFF}/0.15})$,

$$\frac{170-220}{-640-220} = 1 - e^{-t_{OFF}/0.15},$$

from which: $\underline{t_{OFF} = 0.008985}$ $\underline{t_{ON} + t_{OFF} = 0.02017\ \text{second.}}$

Duty cycle $\delta = 0.01118/0.02017 = \underline{0.554.}$ Chopping frequency $= 1/0.02017 =$ 49.58 Hz.

Deceleration

Thyristor ON $\qquad 0 = E - R_a i_a - Lpi_a$.

Substituting: $\qquad = 25.6 - 0.04 i_a - 0.006 pi_a$.

Rearranging: $\qquad 0.15\, di_a/dt + i_a = 640 = I_{max}$.

Solution is: $\qquad i_3 = i_4 + (I_{max} - i_4)(1 - e^{-t_{ON}/\tau})$.

Substituting: $\qquad 200 = 150 + (640 - 150)(1 - e^{-t_{ON}/0.15})$,

$$\frac{200-150}{640-150} = 1 - e^{-t_{ON}/0.15},$$

from which: $\qquad \underline{t_{ON} = 0.01614.}$

Thyristor OFF $\qquad E_B = E - Ri_a - Lpi_a$,

$$72 = 25.6 - 0.1 i_a - 0.006\, Lpi_a.$$

Rearranging: $\qquad 0.06\, di_a/dt + i_a = -464 = I_{min}$.

Solution is: $\qquad i_4 = i_3 + (I_{min} - i_3)(1 - e^{-t_{OFF}/\tau})$.

Substituting: $\qquad 150 = 200 + (-464 - 200)(1 - e^{-t_{OFF}/0.06})$,

$$\frac{150-200}{-464-200} = 1 - e^{-t_{OFF}/0.06}$$

from which: $\qquad \underline{t_{OFF} = 0.004697} \qquad \underline{t_{ON} + t_{OFF} = 0.02084 \text{ second.}}$

Duty cycle $\delta = 0.01614/0.02084 = \underline{0.774.}$ Chopping frequency $= 1/0.02084 =$ $\underline{47.98 \text{ Hz.}}$

Accelerating time

Load torque $= k_\phi I_a = \dfrac{E}{\omega_m} I_a \qquad = \dfrac{64}{2500 \times 2\pi/60} \times 100 = 24.45 \text{ Nm}$.

During acceleration:

$$k_\phi I_{mean} = \frac{64}{2500 \times 2\pi/60} \times \frac{220+170}{2} = 0.2445 \times 195 = 47.67 \text{ Nm}.$$

Constant $d\omega_m/dt \qquad = \dfrac{T_e - T_m'}{J} = \dfrac{47.67 - 24.45}{1.2} = 19.35 \text{ rad/s per second}$

$$= 19.35 \times \frac{60}{2\pi} = \underline{184.8 \text{ rev/min per sec.}}$$

$$\text{Accelerating time to 1000 rev/min} = \frac{1000}{184.8} = \underline{5.41 \text{ seconds.}}$$

Decelerating time

During deceleration: $k_\phi I_{mean} = 0.2445(-200 - 150)/2 = -42.8 \text{ Nm}$.

Note that this electromagnetic torque is now in the same sense as T_m, opposing rotation.

The mechanical equation is: $\qquad T_e = T_m + J\, d\omega_m/dt$,

$$-42.8 = 24.45 + 1.2\, d\omega_m/dt.$$

from which:

$$\frac{d\omega_m}{dt} = \frac{-67.25}{1.2} = -56.04 \text{ rad/s per second} = \underline{-535.1 \text{ rev/min per second.}}$$

Time to stop from 1000 rev/min with this torque maintained $= 1000/535.1 =$ $\underline{1.87 \text{ seconds.}}$

PROBLEMS

10.1 Determine the ripple factor (RF), defined as RF = $V_{rip}/V_{dc} = (V^2_{rms} - V^2_{dc})^{1/2}/V_{dc}$, for the following circuits.

(a) Fig. 10.17.

(b) Fig. 10.18, for $\alpha = 90°$.

What is the significance of the ripple factor?

CHAPTER: 10

$\boxed{10.1}$ (a) From equation 10.1

$$V_{dc} = \frac{\sqrt{2}V_p}{\pi}$$

From Fig 10.17

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(\sqrt{2}V_p \sin\theta)^2 d\theta} = \frac{V_p}{\sqrt{2}}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{\sqrt{2}} \times \frac{\pi}{\sqrt{2}V_p} = \frac{\pi}{2} = 1.5706$$

$$RF = \left\{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1\right\}^{\frac{1}{2}} = (1.5706^2 - 1)^{\frac{1}{2}} = 1.2114$$

(b) From equation 10.2

$$V_{dc} = \frac{V_p}{\sqrt{2}\pi}(1 + \cos 90°) = \frac{V_p}{\sqrt{2}\pi}$$

$$V_{rms} = \left\{\frac{1}{2\pi}\int_{\frac{\pi}{2}}^{\pi}(\sqrt{2}V_p \sin\theta)^2 d\theta\right\}^{\frac{1}{2}} = \frac{V_p}{2}$$

$$\frac{V_{rms}}{V_{dc}} = \frac{V_p}{2} \times \frac{\sqrt{2}\pi}{V_p} = \frac{\pi}{\sqrt{2}} = 2.2218$$

$$RF = (2.2218^2 - 1)^{\frac{1}{2}} = 1.984$$

RF is a measure of ripple content

Example 4.5

A chopper is used to control the speed of a dc motor as shown in Fig. 4.20*a*. The motor is accelerated under current control (i.e., constant torque). Assume that the armature current remains constant at I_a amperes during start-up.

1 Show that the maximum rms ripple current in the chopper current i_{CH} occurs at a duty cycle of one-half, that is, at $\alpha = 0.5$.

2 For the duty cycle $\alpha = 0.5$, determine the values of the input filter components L and C for the following conditions: Supply voltage = 120 V. Chopper frequency $f_{CH} = 400$ Hz. Start-up motor current $I_a = 100$ A. Rms fundamental current to be allowed in the supply is 10% of the dc component of the source current. Electrolytic capacitor of rating 1000 μF and 300 V dc can take 5 A rms ripple current. For the design $f_{CH} > 2f_r$.

3 For the values of L and C obtained in part 2, determine the average and first three harmonic currents (in rms) in the supply.

Solution

1 The chopper current i_{CH} is in the form of square pulses of magnitude I_a and width α as shown in Fig. 4.21. Therefore, the dc component $I_{CH_{dc}}$, rms current I_{rms}, and ripple current I_{ripple} are as follows:

$$I_{CH_{dc}} = I_a\alpha$$

$$I_{\text{rms}} = \left(\int_0^{\alpha} I_a^2\, d\alpha\right)^{1/2} = I_a\sqrt{\alpha}$$

$$I_{\text{ripple}} = \left[\left(I_a\sqrt{\alpha}\,\right)^2 - (I_a\alpha)^2\right]^{1/2}$$

$$= I_a(\alpha - \alpha^2)^{1/2}$$

For maximum ripple current

$$\frac{dI_{\text{ripple}}}{d\alpha} = 0$$

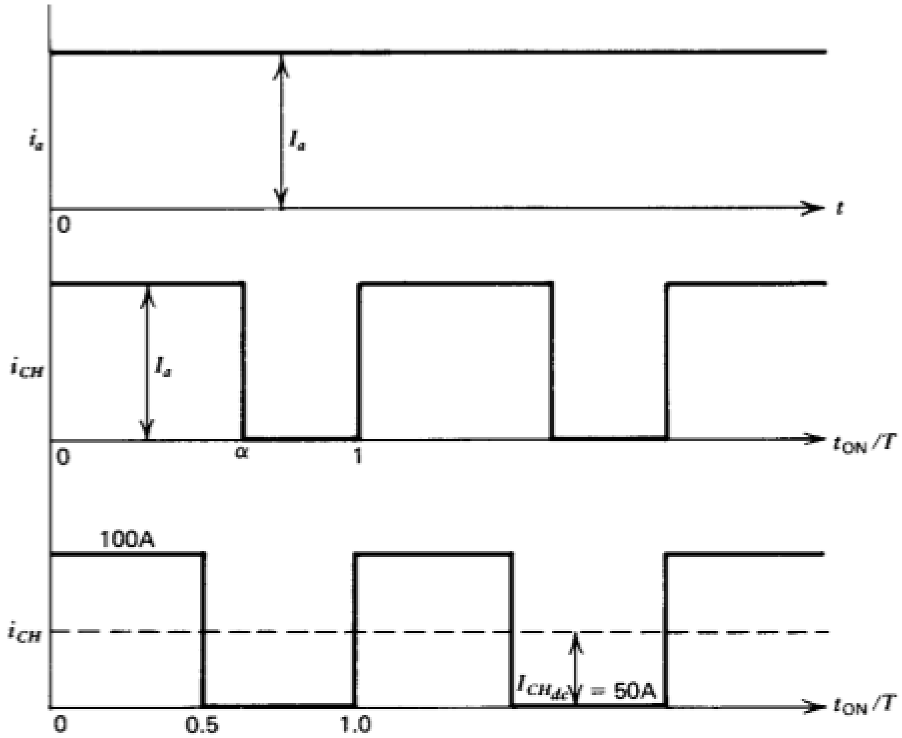


Fig. 4.21 Example 4.5

or

$$\frac{I_a(1-2\alpha)}{2(\alpha-\alpha^2)^{1/2}}=0$$

from which

$$\alpha = 0.5$$

2 The *L-C* filter should be selected for the worst case, which corresponds to $\alpha = 0.5$. At this duty cycle (bottom wave form in Fig. 4.21) the Fourier series for the chopper current is

$$i_{CH} = I_{CH_{dc}} + \frac{4}{\pi}\frac{I_a}{2n}(\sin\omega t + \sin 3\omega t + \sin 5\omega t + \cdots)$$

Now,

$$I_{CH_{dc}} = \frac{100}{2} = 50 \text{ A}$$

The fundamental, third, and fifth harmonic currents in the chopper are

$$I_{CH_1} = \frac{4 \times 100}{\sqrt{2} \times \pi \times 2} = 45 \text{ A}$$

$$I_{CH_3} = 15 \text{ A}$$

$$I_{CH_5} = 9 \text{ A}$$

The dc component of the chopper current comes from the supply only. The capacitor cannot provide a dc current. Therefore, the dc component I_0 of the supply current is

$$I_0 = I_{CH_{dc}} = 50 \text{ A}$$

If the fundamental supply current I_1 is not to exceed 10% of the dc current I_0, then

$$I_1 = 5 \text{ A}$$

From equation 4.55

$$I_1 = \frac{X_C}{X_L - X_C} I_{CH_1}$$

$$5 = \frac{X_C}{X_L - X_C} \times 45$$

or

$$X_L = 10X_C$$

Fundamental capacitor current I_{C_1} is

$$I_{C_1} = \frac{X_L}{X_L - X_C} I_{CH_1}$$

$$= \frac{10X_C}{10X_C - X_C} \times 45$$

$$= 50 \text{ A}$$

Each electrolytic capacitor can take 5 A of current. Therefore 10 capacitors connected in parallel are required.

$$C = 10{,}000\ \mu\text{F}$$

$$X_C = \frac{1}{2\pi 400 \times 10^4 \times 10^{-6}}\,\Omega = 3.98 \times 10^{-2}\ \Omega$$

$$X_L = 10X_C = 3.98 \times 10^{-1}\ \Omega$$

$$L = \frac{0.398}{2\pi 400} = 158\ \mu\text{H}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = 127\ \text{Hz}$$

which makes

$$f_{CH} = \frac{400}{127} f_r = 3.15 f_r$$

3 From equation 4.58

$$I_1 = \frac{45}{(3.15)^2 - 1} = 5.0\ \text{A}$$

$$I_3 = \frac{15}{(3 \times 3.15)^2 - 1} = 0.17\ \text{A}$$

$$I_5 = \frac{9}{(5 \times 3.15)^2 - 1} = 0.036\ \text{A}$$

and

$$I_0 = 50\ \text{A}$$

Example 1: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle γ.

(b) The average value of the output voltage V_o.

(c) The rms value of the output voltage V_{orms}.

(d) The ripple factor *RF*.

(e) The output d.c. power.

Solution:

(a) $t_{on} = 1\text{ms}$, T=2.5 ms

$$\gamma = \frac{t_{on}}{T} = \frac{1\ ms}{2.5\ ms} = 0.4$$

(b) $V_{av} = V_o = \gamma\ V_d = 0.4 \text{ x } 100 = 40$ V.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.4} \text{ x} 100 = 63.2$ V.

(d) $RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$

(e)

$$I_a = \frac{V_o}{R} = \frac{40}{10} = 4A$$

$P_{av} = I_a\ V_o = 4\text{x}40 = 160$ W

Example 2: An 80 V battery supplies RL load through a DC chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with 5Ω resistor. Load current, due to improper selection of frequency of chopping, varies widely between 9A and 10.2.

(a) Find the average load voltage, current and the duty cycle of the chopper.

(b) What is the operating frequency *f* ?

(c) Find the ripple current to maximum current ratio.

Solution:

(a) The average load voltage and current are:

$$V_{av} = V_o = \gamma V_d$$

$$I_{av} = \frac{1}{2}(I_2 + I_1) = \frac{9 + 10.2}{2} = 9.6A$$

$$I_{av} = \frac{V_{av}}{R} = \frac{\gamma V_d}{R} \qquad or \qquad \gamma = \frac{I_{av}\ R}{V_i} = \frac{9.6\ x\ 5}{80} = 0.6$$

$V_{av} = 0.6 \text{ x } 80 = 48$ V.

(b) To find the operating (chopping) frequency:

During the ON period,

$$V_d = Ri + L\frac{di}{dt} \qquad \dots\dots\dots\dots\dots(1)$$

Assuming $\frac{di}{dt} \cong constant$

$$\frac{di}{dt} \cong \frac{\Delta I}{t_{on}} = \frac{10.2 - 9}{\gamma T} = \frac{1.2}{\gamma T}$$

From eq.(1)

$$L\frac{di}{dt} \cong V_d - I_{av}\,R = 80 - 5 \times 9.6 = 32V$$

or

$$\frac{di}{dt} = \frac{32}{L} = \frac{32}{0.4} = 80\ A.s$$

but

$$\frac{di}{dt} = \frac{1.2}{\gamma T} = 80 = \frac{1.2}{0.6\ T}$$

$$\therefore T = \frac{1.2}{0.6x80} = 25\ ms$$

Hence

$$f = \frac{1}{T} = \frac{1}{25 \times 10^{-3}} = 40\ Hz$$

The maximum current I_m occurs at $\gamma = 1$,

$$\therefore\ I_m = \frac{\gamma V_d}{R} = \frac{1 \times 80}{5} = 16A$$

Ripple current $I_r = \Delta I = 10.2 - 9 = 1.2$ A

$$\therefore\ \frac{I_r}{I_m} = \frac{1.2}{16} = 0.075 \quad or\ \ 7.5\%\,.$$

Example 3: A DC Buck converter operates at frequency of 1 kHz from 100V DC source supplying a 10 Ω resistive load. The inductive component of the load is 50mH.For output average voltage of 50V volts, find:

(a) The duty cycle

(b) t_{on}

(c) The rms value of the output current

(d) The average value of the output current

(e) I_{max} and I_{min}

(f) The input power

(g) The peak-to-peak ripple current.

Solution:

(a) $V_{av} = V_o = \gamma V_d$

$$\gamma = \frac{V_{av}}{V_d} = \frac{50}{100} = 0.5$$

(b) $T = 1/f = 1 / 1000 = 1ms$

$$\gamma = \frac{t_{on}}{T}$$

$t_{on} = \gamma T = 05 \text{ x}1ms = 0.5 \text{ ms}$.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.5}\ x\ 100 = 70.07\ V$

(d)

$$I_{av} = \frac{V_{av}}{R} = \frac{50}{10} = 5\ A$$

(e)

$$I_{max} = \frac{V_{av}}{R} + \frac{t_{off}}{2L} V_{av} = \frac{50}{10} + \frac{(1-0.5)\times 10^{-3}}{2\times 50\times 10^{-3}} x50$$

= 5 + 0.25 = 5,25 A

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av} = \frac{50}{10} - \frac{(1-0.5)\times 10^{-3}}{2\times 50\times 10^{-3}} x50$$

= 5 - 0.25 = 4.75 A

(f)

$$I_{s(av)} = \frac{\gamma}{2}\,(I_{min} + I_{max}) = \gamma I_{av} = 0.5\ \times 5 = 2.5A$$

$$P_{in}\ = I_{s(av)} V_d = 2.5\ \times 100 = 250\ W$$

(g) $$I_{p-p} = \Delta I = I_{max} - \ I_{min} = 5.25 - 4.75 = 0.5A$$

DC-DC CONVERTER

1. A class-A transistor chopper circuit shown in Fig.1 supplied with power from an ideal battery of terminal voltage 120 V .The load voltage waveforms consists of rectangular pulses of duration 1 ms in an overall cycle of 3 ms.

(a) Sketch the waveforms of v_L and i_L.

(b) Calculate the duty cycle γ,

(c) Calculate the average and r.m.s. values of the load voltage.

(d) Find the average value of the load current if R = 10 ohms.

(e) Calculate the input power and the ripple factor *RF*.

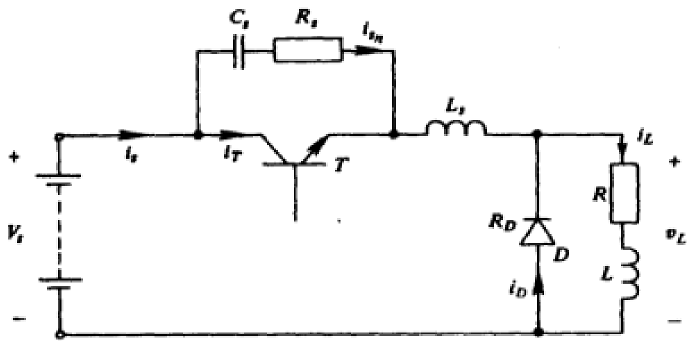


Fig.1

Question 1 .

(a) Waveforms of Voltage v_L and current i_L are shown in Fig. Q2

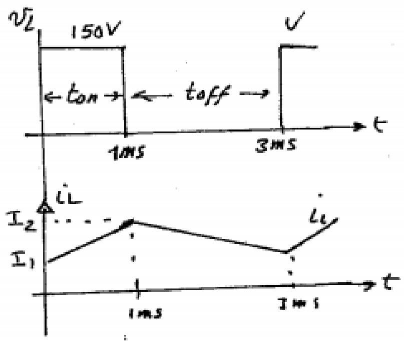


Fig. Q2.

(b) The duty cycle γ is

$$\gamma = \frac{t_{on}}{t_{on}+t_{off}} = \frac{1}{1+2}$$

$$= \frac{1}{3} .$$

(c) $V_{av} = \gamma V_{in} = \frac{1}{3} \times 150 = 50V$.

$V_{L r.m.s} = \sqrt{\gamma}\, V_{in} = \sqrt{\frac{1}{3}} \times 150 = 86.6 V$.

(d) $I_{av} = \frac{V_{av}}{R} = \frac{50}{20} = 2.5 A$.

(e) $I_{in} = I_{av}\gamma = 2.5 \times \frac{1}{3} = 0.833 A$

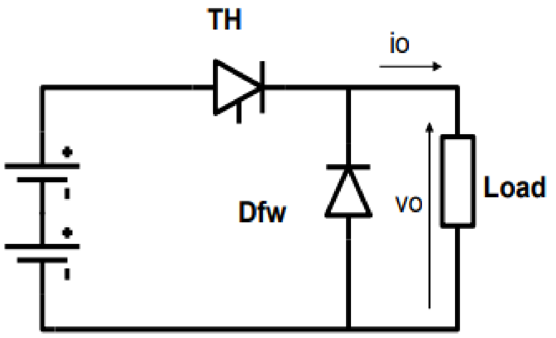
$P_{in} = V_s I_{in} = 0.833 \times 150 = 125 W$.

$RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-\frac{1}{3}}{\frac{1}{3}}} = \sqrt{2} = 1.414$

V

2. A class-A DC chopper shown in Fig.2 is operating at a frequency of 2kHz from 96V DC source to supply a load of resistance 8 ohms . The load time constant is 6 ms. If the mean load voltage is 57.6 V, find duty cycle (mark to space ratio), the mean load current, and the magnitude of the current ripple. Derive any formula used.

Fig.2



[Ans: γ =0.6 , Iav =7.2, Δ I = 0.24 A]

Solution :

$$T = \frac{1}{f} = \frac{1}{2000} = 0.5 \; ms .$$

Load time constant $\tau = \frac{L}{R} = 6 \; ms = 12T$. , hence the current variation is treated linear.

$$V_{av} = \gamma V_i$$

$$57.6 = \gamma \times 96 \qquad \therefore \gamma = \frac{57.6}{96} = 0.6 \; . \equiv \text{mark-space ratio.}$$

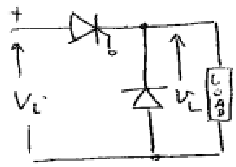
$$V_{0 r.m.s} = V_i \sqrt{\gamma} = 96 \times \sqrt{0.6} = 74.36 V .$$

$$I_{av} = \frac{V_{av}}{R} = \frac{57.6}{8} = 7.2 A .$$

Current ripple $\Delta I = (V_i - V_{av}) \frac{\Delta t}{L}$.

This comes from :

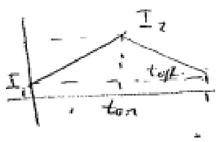
During conduction:



$$V_i - V_L = L \frac{di}{dt} \equiv L \frac{\Delta i}{\Delta t} .$$

$$\Delta i = (V_i - V_L) \frac{\Delta t}{L} = I_2 - I_1 \quad ---(1)$$

$$\Delta t = t_{on} - 0 = t_{on} .$$



$$\therefore I_2 - I_1 = (V_i - V_L) \frac{t_{on}}{L} .$$

During off period (from eq.(1)) $V_i = 0$

$$I_1 - I_2 = (0 - V_L) \frac{(T - t_{on})}{L}$$

or $$I_2 - I_1 = V_L \frac{(T - t_{on})}{L} = V_L \frac{t_{off}}{L}$$

Also $$I_{av} = \frac{I_1 + I_2}{2}$$

Hence

Hence

$$I_1 = I_{av} - V_L \frac{t_{off}}{2L} = I_{min}$$

$$I_2 = I_{av} + V_L \frac{t_{off}}{2L} = I_{max} \qquad V_L = V_{av}$$

$$\tau = \frac{L}{R} = 6\times10^{-3} \qquad \therefore L = 6\times10^{-3}\times R = 6\times10^{-3}\times 8 = 48\,mH$$

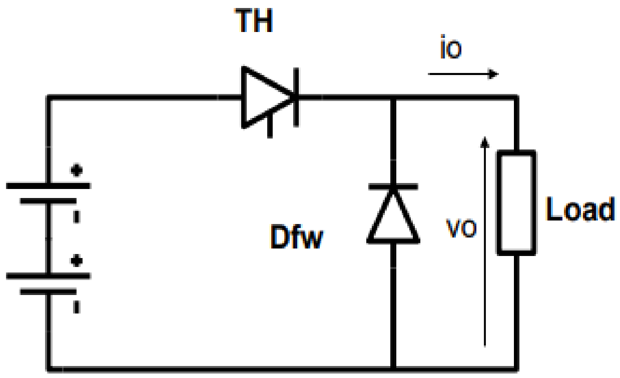
$$I_{max} = I_2 = 7.2 + \frac{57.6\times0.2\times10^{-3}}{2\times48\times10^{-3}} = 7.32\,A$$

$$I_{min} = I_1 = 7.2 - \frac{57.6\times0.2\times10^{-3}}{2\times48\times10^{-3}} = 7.08\,A$$

$$\Delta I = I_{max} - I_{min} = 7.32 - 7.08 = 0.24\,A$$

3. A DC Buck converter (class-A chopper) supplies power to a load having 6 ohms resistance and 20 mH inductance. The source voltage is 100V d.c. and the output load voltage is 60V. If the ON time is 1.5 ms, find:

(a) Chopper switching frequency.
(b) Imax and Imin (I_2 and I_1).
(c) The average diode current.
(d) The average input current.
(e) Peak- to- peak ripple current.



[Ans: fc = 40Hz , Imax = 1.5A,Imin =8.5A, Iav(D) = Iav= 10A , ΔI= 3A]

Solution :

(a) $\gamma = \frac{V_{av}}{V_i} = \frac{60}{100} = 0.6$

$\gamma = \frac{t_{on}}{T} \Longrightarrow T = \frac{1.5}{0.6} = 2.5\ ms.$

$f = \frac{1}{T} = \frac{1}{2.5} \times 10^{3} = 400\ Hz.$

(b) $I_{max} = \frac{V_{av}}{R} + V_{av}\left(\frac{t_{off}}{2L}\right) = \frac{60}{6} + 60\ \left(\frac{(2.5-1.5)\times 10^{-3}}{2\times 20\times 10^{-3}}\right)$

$= 10 + \frac{60}{40} = 10 + 1.5 = 11.5\ A.$

$I_{min} = \frac{V_{av}}{R} - V_{av}\left(\frac{t_{off}}{2L}\right) = 10 - 1.5 = 8.5\ A.$

(c) $I_{in(av)} = \gamma\ I_{av} = 0.6 \times \frac{V_{av}}{R} = 0.6 \times \frac{60}{60} = 6\ A.$

(d) $I_{Diode} = I_{av} = \frac{V_{av}}{R} = \frac{60}{6} = 10\ A.$

(e) p-p riple $\Delta I = I_{max} - I_{min} = 11.5 - 8.5 = 3.0\ A.$

4. In a class-A chopper circuit an ideal battery of terminal voltage 100V supplies a series load of resistance 0.5 Ohms and inductance of 1.0 mH . The thyristor is switched on for 1 ms in an overall period of 3 ms. Calculate the average values of the load voltage and current and the power taken from the battery. Assuming continuous current conduction .Also calculates the r.m.s value of the load current taking the firs two harmonics of the Fourier series.

[Ans: Vav = 33.3V , Iav = 66.7A, Pin= 2223W , $I_{Lr.m.s}$ = 69.1A]

Solition

$t_{on} = 1\ ms \quad , \quad T = 3\ ms$

$\therefore \gamma = \frac{t_{on}}{T} = \frac{1}{3}$

$\omega = 2\pi f = \frac{2\pi}{T} \qquad \therefore T = \frac{2\pi}{\omega} = \frac{3}{1000}$

Hence $\omega = \frac{2000\pi}{3} = 2094.4$ rad/s.

$\tau = \frac{L}{R} = \frac{1 \times 10^{-3}}{0.5} = 2\ mS.$

$V_{oav} = \gamma V_i = \frac{100}{3} = 33.33\ V.$

The Average load current $I_{av} = \frac{V_{o(av)}}{R} = \frac{33.33}{0.5} = 66.7A.$

The average supply (input Current) $I_{in(av)} = \gamma I_{av} = \frac{1}{3} \times 66.7$
$= 22.23A.$

Input power $P_{in} = V_i I_{in(av)}$
$= 100 \times 22.23 = \underline{2223}\ W.$

The impedance of the load to current of fundamental frequency: $Z_L = \sqrt{R^2 + (\omega L)^2} = \sqrt{(0.5)^2 + (2094.4 \times 10^{-3})^2} = 2.153\Omega$

The fundamental component of the load voltage:

$V_{o1(r.m.s)} = \frac{V_i}{\sqrt{2}\pi} \sqrt{\sin^2 2\pi\gamma + (1 - \cos 2\pi\gamma)^2} = \frac{100}{\sqrt{2}\pi} \sqrt{0.75 + 2.25} = \frac{55.13}{\sqrt{2}}\ V.$

$\therefore I_{o1} = \frac{V_{o1(r.m.s)}}{Z_L} = \frac{55.13}{\sqrt{2} \times 2.153} = 18.1\ A.$ (r.m.s value of the fundamental Load current).

5. In a class-A chopper circuit an ideal battery of terminal voltage 100V supplies a series load of resistance 10 Ohms. The chopping frequency is f =1 kHz and the duty cycle is set to be 0.5 .Determine:

(a) The average output voltage.
(b) The rms output voltage.
(c) The chopper efficiency.
(d) The ripple factor.
(e) The fundamental component of output harmonic voltage.

[As: Va=110V, $V_{Lr.m.s}$ = 155.56V, ή=100%,RF=1.0, $V_{L1\ r.m.s}$ =99V]

Solution:

(a) $V_o = \gamma V_i = 0.5 \times 220 = 110V.$

(b) $V_{o\,r.m.s} = \sqrt{\gamma}\, V_i = \sqrt{0.5} \times 220 = 155.56\,V.$

(c) $I_{av} = \frac{V_o}{R} = \frac{110}{10} = 11\,A.$

$P_o = I_{av}^2 R = 11^2 \times 10 = 1210\,W.$

$I_{in(av)} = \gamma I_a = 0.5 \times 11 = 5.5\,A.$

$P_{in} = V_i \cdot I_{in(ave)} = 220 \times 5.5 = 1210\,W$

$\therefore \eta = \frac{P_o}{P_i} = \frac{1210}{1210} = 100\%.$

(d) $RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-0.5}{0.5}} = 1.$

(e) $V_{o1(r.m.s)} = \frac{V_i}{\sqrt{2}\,\pi}\sqrt{\sin^2 2\pi\gamma + (1-\cos 2\pi\gamma)^2}$

$= \frac{220}{\sqrt{2}\pi}\sqrt{\sin^2(2\pi \times \frac{1}{2}) + (1-\cos(2\pi \times \frac{1}{2}))^2}$

$= \frac{220}{\sqrt{2}\,\pi}\sqrt{0+(2)^2} = \frac{220\sqrt{2}}{\pi} = 70.06\sqrt{2}$

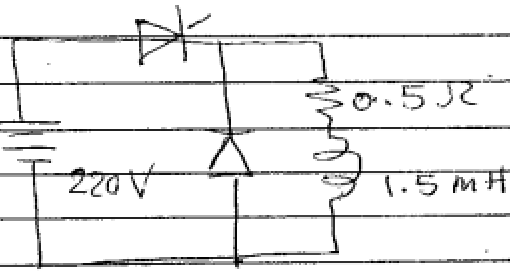
6. In the chopper circuit sown in Fig.1 (problem 1) Vi = 220V, L = 1.5 mH , R = 0.5 ohm and it operating with T = 3 ms and t_{on} = 1.5 ms.

(a) Determine the minimum, maximum and average values of load current.

(b) Express the load current variation in terms of ON and OFF periods.

[Ans: (a) Imin = 165A,Imax=275A, i_1= 165+36660 t , i_2 =165-36660t]

problem 6 -



T = 3 ms

t_{on} = 1.5 ms.

I_{min} & I_{max}

sol. $t_{off} = T - t_{on} = 1.5$ ms.

$$\gamma = \frac{t_{on}}{T} = \frac{1.5}{3} = 0.5$$

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av}$$

$V_{av} = \gamma V_i = 0.5 \times 220 = 110$ V.

7. A separately excited d.c. motor with Ra = 1.2 ohms and La = 30 mH , is to be controlled using class-A transistor chopper .The d.c. supply is 120V .

(a) It is required to draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant KeΦ has a value of 0.042 V/rpm.

(b) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle γ = 0.5.

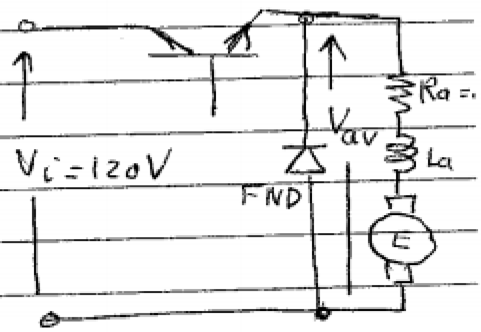
[Ans : n = 857 rpm]

Solution : In the steady state the armature inductance has no effect.

(a) At $\gamma = 1$

$V_{av} = \gamma V_{in} = 1 \times 120 = 120\,V.$

$$n = \frac{V_{av}}{K_e\phi} - \frac{\Sigma R_a\, T}{(K_e\phi)^2 \times 9.55}$$



when $T = 0$

$$n_o = \frac{120}{0.042} = 2857\ rpm$$

when $n = 0$, $T = T_{st}$

$$\frac{\Sigma R_a}{9.55(K_e\phi)^2} \cdot T_{st} = \frac{V_{av}}{K_e\phi}$$

$$\therefore T_{st} = \frac{V_{av}}{\Sigma R_a} \frac{(K_e\phi)}{\frac{1}{9.55}} = \frac{120}{1.2} \times \frac{(0.042)}{\frac{1}{9.55}} = 40\ N.m$$

Examlpe: **A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.**

(a) The duty cycle D.

(b) The average value of the output voltage V_{dc}.

(c) The *rms* value of the output voltage V_{rms}.

(d) The ripple factor *RF*.

(e) The output dc power.

(a) $$D = \frac{t_{ON}}{T} = \frac{1msec}{2.5msec} = 0.4$$

(b) $$V_{dc} = DV_s = 0.4 \times 100 = 40 \text{ V}$$

(c) $$V_{rms} = \sqrt{D}V_s = \sqrt{0.4} \times 100 = 63.2 \text{ V}$$

(d) $$RF = \sqrt{\frac{1-D}{D}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$$

(e) $$P_o = \frac{{V_{dc}}^2}{R} = \frac{40^2}{10} = 160 \text{ W}$$

Examlpe: **A dc chopper has a resistive load of 20Ω and input voltage V_S=220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.**

$$V_s = 220\text{V}$$

$$D = \frac{t_{ON}}{T} = 0.8$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T}(V_s - V_{CH}) = 0.8(220 - 1.5) = 174.8 \text{ V}$$

$$T = \frac{1}{f} = \frac{1}{10 \times 10^{-3}} = 0.1\text{m}\,sec$$

$$t_{ON} = DT = 0.8 \times 0.1 \times 10^{-3} = 80\mu\,sec$$

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Examlpe: buck dc-dc converter with Low Pass Filter has the following parameters:

$V_s = 50$ V $L = 400$ μH $f = 20$ kHz
$D = 0.4$ $C = 100$ μF $R = 20\ \Omega$

Assuming ideal components, calculate (a) the output voltage Vo, (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

(a) $V_o = V_s D = (50)(0.4) = 20$ V

(b)
$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$
$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$
$$= 1 + \frac{1.5}{2} = 1.75 \text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$
$$= 1 - \frac{1.5}{2} = 0.25 \text{ A}$$

The average inductor current is 1 A, and Δi_L=1.5 A.

(c)
$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$
$$= 0.00469 = 0.469\%$$

Example: Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50. and the switching frequency is 25kHz.

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let L=120 μH.

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5 \text{ A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2 \text{ A}$$

$$I_{\max} = 1.5 + 1.2 = 2.7 \text{ A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3 \text{ A}$$

$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Designing a Buck Converter

Assume:

V_{in} = 12 V

V_{OUT} = 5 volts

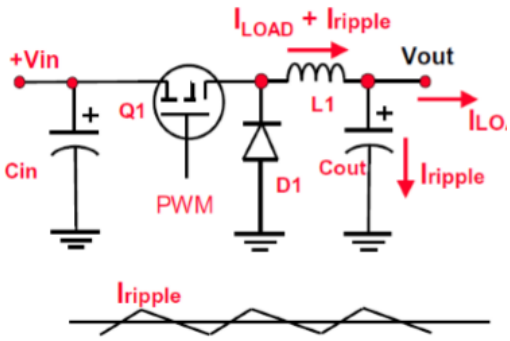
I_{LOAD} = 2 amps

F_{sw} = 400 KHz

D = V_{in} / $_{Vout}$ = 5V /12V = 0.416

Define Ripple current:

I_{ripple} = 0.3 • I_{LOAD} (typically 30%)



For an Inductor: V = L • ΔI / ΔT

Rearrange and substitute:

L = (Vin – Vout) • (D / Fsw) / Iripple

Calculate:

L = 7 V • (0.416 / 400 kHz) / 0.6A

L = 12.12 uh

Select C, Diode (Schottky),
and the MOSFET
Calculate the Efficiency

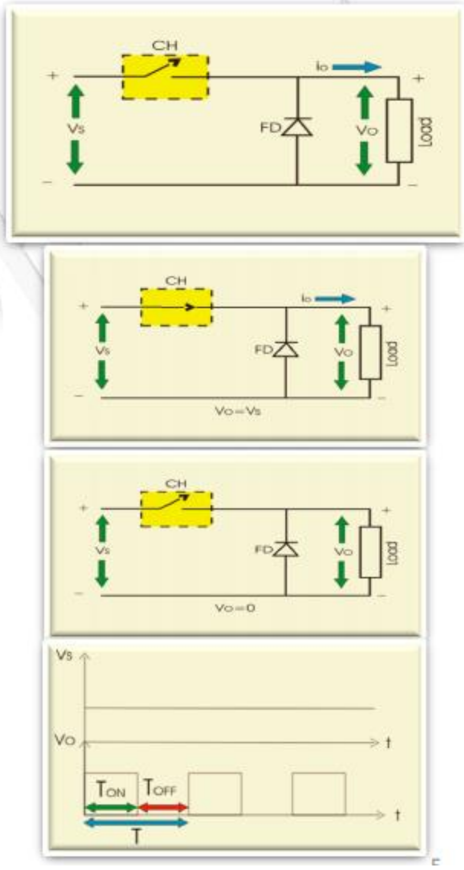
The Buck (Step-Down) Converter

- Step down chopper as Buck converted is used to reduce the input voltage level at the output side. Circuit diagram of a step down chopper is shown in the figure.

- When CH is turned ON, Vs directly appears across the load as shown in figure. So $V_O = V_S$.

- When CH is turned OFF, Vs is disconnected from the load. So output voltage $V_O = 0$.

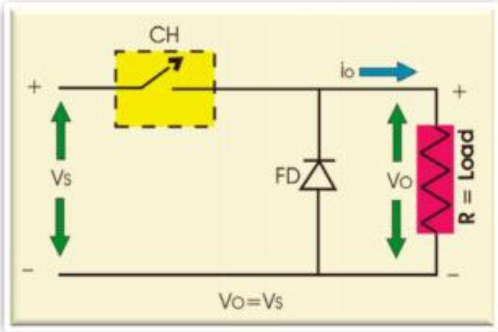
- The voltage waveform of step down chopper



- $T_{ON} \rightarrow$ It is the interval in which chopper is in ON state.
- $T_{OFF} \rightarrow$ It is the interval in which chopper is in OFF state.
- $V_S \rightarrow$ Source or input voltage.
- $V_O \rightarrow$ Output or load voltage.
- $T \rightarrow$ Chopping period $= T_{ON} + T_{OFF}$
- $F = 1/T$ is the frequency of chopper switching or chopping frequency

Operation of Step Down Chopper with Resistive Load

- When CH is ON, $V_O = V_S$ When CH is OFF, $V_O = 0$



The Average output voltage is

$$V_{dc} = V_o = \frac{1}{T}\int_0^{T_{ON}} V_s dt = \frac{V_s T_{ON}}{T} = DV_s$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{DV_s}{R}$$

$$D = \frac{T_{ON}}{T} \qquad\qquad T = T_{ON} + T_{OFF}$$

- ***Where,***
- ***D is <u>duty cycle</u> = T_{ON}/T. T_{ON} can be varied from 0 to T, so $0 \leq D \leq 1$.***
- ***The output voltage V_O can be varied from 0 to V_S.***

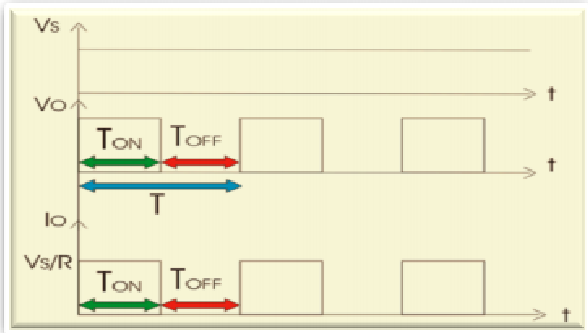
The *rms* output voltage is

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T_{ON}} {V_s}^2 dt} = V_s\sqrt{\frac{T_{ON}}{T}} = \sqrt{D}V_s$$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{\sqrt{D}V_s}{R}$$

$$P_o = V_{rms} I_{rms} = \frac{{V_{rms}}^2}{R} = D\frac{{V_s}^2}{R}$$

The output voltage is always less than the input voltage and hence the name step down chopper is justified.



Ripple factor (*RF*) can be found from

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{\frac{D{V_s}^2}{D^2{V_s}^2} - 1} = \sqrt{\frac{1}{D} - 1} = \sqrt{\frac{1-D}{D}}$$

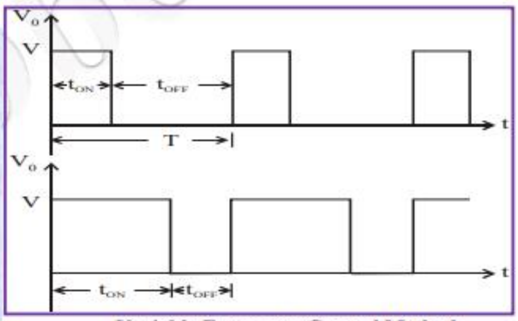
Methods of Control

1- Pulse Width Modulation

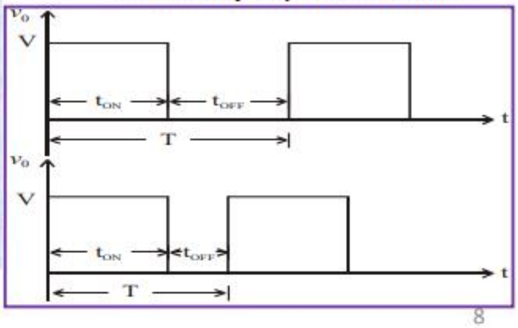
- **t_{ON} is varied keeping chopping frequency '*f*' & chopping period 'T' constant.**
- **Output voltage is varied by varying the ON time t_{ON}**

2- Variable Frequency Control

- **Chopping frequency '*f*' is varied keeping either t_{ON} or t_{OFF} constant.**
- **To obtain full output voltage range, frequency has to be varied over a wide range.**
- **This method produces harmonics in the output and for large t_{OFF} load current may become discontinuous**



Variable Frequency Control Method



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Examlpe: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle D.

(b) The average value of the output voltage V_{dc}.

(c) The *rms* value of the output voltage V_{rms}.

(d) The ripple factor *RF*.

(e) The output dc power.

(a) $$D = \frac{t_{ON}}{T} = \frac{1msec}{2.5msec} = 0.4$$

(b) $$V_{dc} = DV_s = 0.4 \times 100 = 40 \text{ V}$$

(c) $$V_{rms} = \sqrt{D}V_s = \sqrt{0.4} \times 100 = 63.2 \text{ V}$$

(d) $$RF = \sqrt{\frac{1-D}{D}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$$

(e) $$P_o = \frac{{V_{dc}}^2}{R} = \frac{40^2}{10} = 160 \text{ W}$$

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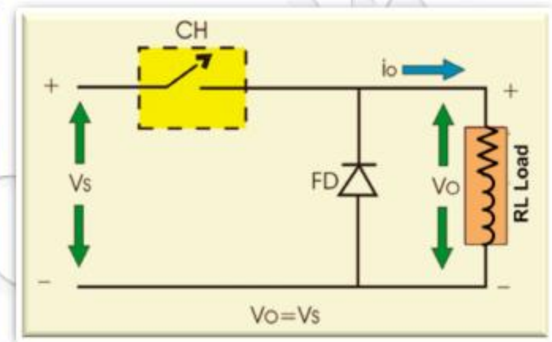
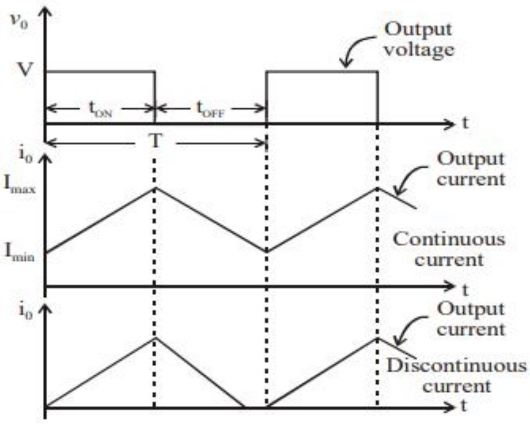
The Buck (Step-Down) Converter

Step Down Chopper with RL Load

- When chopper is ON, supply is connected across load. Current flows from supply to load.
- When chopper is OFF, load current continues to flow in the same direction through FWD due to energy stored in inductor 'L'.

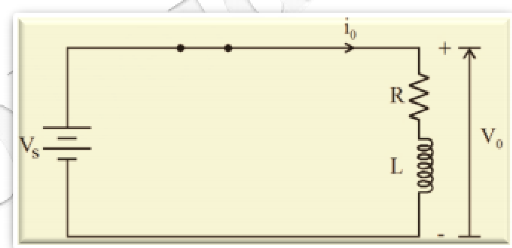


- Load current can be continuous or discontinuous depending on the values of '*L*' and duty cycle '*D*'
- For a continuous current operation, load current varies between two limits I_{max} and I_{min}.
- When current becomes equal to I_{max} the chopper is turned-off and it is turned-on when current reduces to I_{min}.



Continuous Current Operation When Chopper Is ON ($0 \leq t \leq t_{ON}$)

- When the switch is closed in the buck converter, the circuit will be as shown in the figure, the diode is reverse-biased.



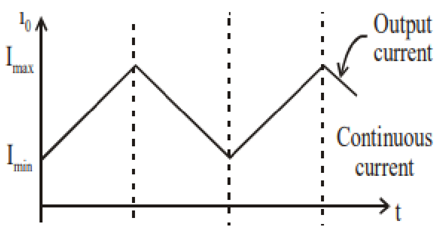
The voltage across the inductor is

$$V_s = V_R + V_L$$

$$V_s = V_R + L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{V_s - V_R}{L}$$

$$\Delta i = \int_0^{DT} \frac{V_s - V_R}{L} dt = \frac{V_s - V_R}{L} DT = \frac{V_s - V_R}{L} t_{ON} \qquad (1)$$

$$\frac{di}{dt} = \frac{\Delta i}{t_{ON}} = \frac{I_{max} - I_{min}}{t_{ON}} = \frac{V_s - V_R}{L}$$



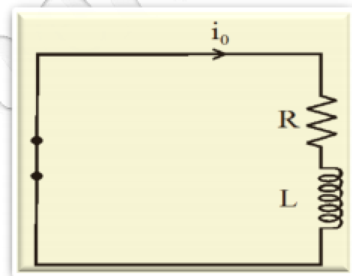
From straight line equation

$$i_{o1} = I_{min} + \frac{I_{max} - I_{min}}{t_{ON}} t = I_{min} + \frac{I_{max} - I_{min}}{DT} t = I_{min} + \frac{V_s - V_R}{L} t \qquad (2)$$

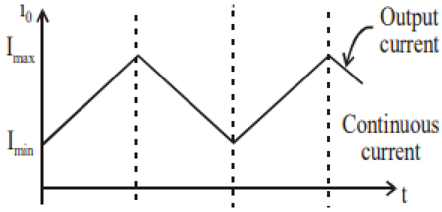
Continuous Current Operation When Chopper Is <u>OFF</u> ($t_{ON} \le t \le T$)

$$0 = V_R + V_L$$

$$0 = V_R + L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = -\frac{V_R}{L}$$



$$\Delta i = \int_0^{t_{OFF}} -\frac{V_R}{L}\,dt = -\frac{V_R}{L}t_{OFF} \qquad (3)$$



$$\frac{di}{dt} = \frac{\Delta i}{t_{OFF}} = \frac{I_{min} - I_{max}}{t_{OFF}} = -\frac{I_{max} - I_{min}}{t_{OFF}} = -\frac{V_R}{L}$$

From straight line equation

$$i_{o2} = I_{max} + \frac{I_{min} - I_{max}}{t_{OFF}}(t - t_{ON}) = I_{max} - \frac{V_R}{L}(t - t_{ON}) \qquad (4)$$

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s - V_R}{L}t_{ON} - \frac{V_R}{L}t_{OFF} = 0 \longrightarrow \frac{V_s - V_R}{V_R} = \frac{t_{OFF}}{t_{ON}}$$

$$\frac{V_s}{V_R} - 1 = \frac{t_{OFF}}{t_{ON}} \longrightarrow \frac{V_s}{V_R} = \frac{t_{OFF}}{t_{ON}} + 1$$

$$\frac{V_s}{V_R} = \frac{t_{OFF} + t_{ON}}{t_{ON}} = \frac{T}{t_{ON}} \longrightarrow V_R = DV_s$$

From equation (1)

$$\boxed{\Delta i = \frac{V_s - DV_s}{L}DT = \frac{V_s(1-D)D}{Lf}}$$

since $D = \frac{t_{ON}}{T}$

$$f = \frac{1}{T}$$

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At steady state operation, the average inductor current must be the same as the average current in the load resistor.

$$I_L = I_R = \frac{V_R}{R}$$

The maximum and minimum values of the inductor current are computed as

$$I_{max} = I_L + \frac{\Delta i}{2}$$

$$\boxed{I_{max} = I_L + \frac{V_s(1-D)D}{2Lf} = I_L + \frac{V_R(1-D)}{2Lf}}$$

$$I_{min} = I_L - \frac{\Delta i}{2}$$

$$\boxed{I_{min} = I_L - \frac{V_s(1-D)D}{2Lf} = I_L - \frac{V_R(1-D)}{2Lf}}$$

The average dc output voltage and current can found as

$$V_{dc} = DV_s \qquad\qquad I_{dc} \cong \frac{I_{max} - I_{min}}{2}$$

Examlpe: A dc chopper has a resistive load of 20Ω and input voltage V_S=220V. When chopper is ON, its voltage drop is 1.5 volts and chopping frequency is 10 kHz. If the duty cycle is 80%, determine the average output voltage and the chopper on time.

$$V_s = 220\text{V}$$

$$D = \frac{t_{ON}}{T} = 0.8$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T}(V_s - V_{CH}) = 0.8(220 - 1.5) = 174.8\text{ V}$$

$$T = \frac{1}{f} = \frac{1}{10 \times 10^{-3}} = 0.1\text{m } sec$$

$$t_{ON} = DT = 0.8 \times 0.1 \times 10^{-3} = 80\mu\ sec$$

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Step Down Chopper with RL Load

Examlpe: A Chopper circuit is operating at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.

$$V_s = 460\text{V}$$

Chopping period

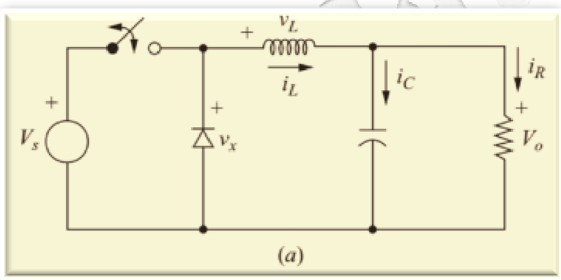
$$T = \frac{1}{f} = \frac{1}{2 \times 10^{-3}} = 0.5\text{m } sec$$

$$V_{dc} = DV_s = \frac{t_{ON}}{T} V_s$$

$$t_{ON} = \frac{TV_{dc}}{V_s} = \frac{0.5 \times 10^{-3} \times 350}{460} = 0.38\text{m } sec$$

Step Down Chopper with Low Pass Filter

- **This converter is used if the objective is to produce an output that is purely DC.**
- **If the low-pass filter is ideal, the output voltage is the average of the input voltage to the filter.**



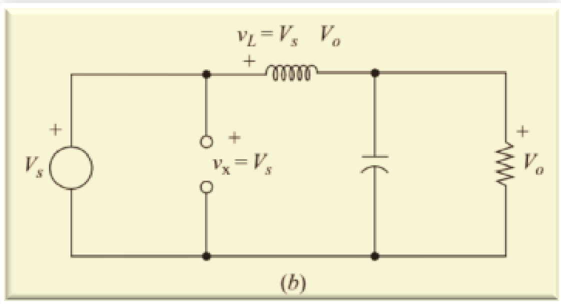
(a)

Analysis for the Switch Closed

When the switch is closed in the buck converter circuit of fig. a, the diode is reverse-biased and fig. b is an equivalent circuit. The voltage across the inductor is

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$



(b)

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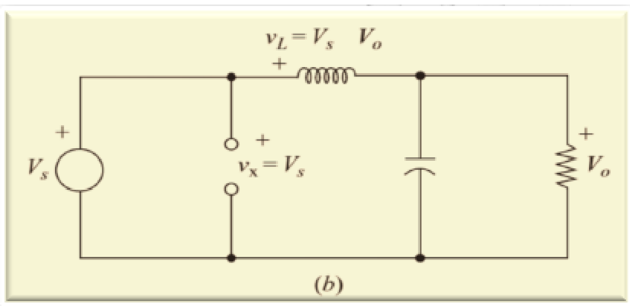
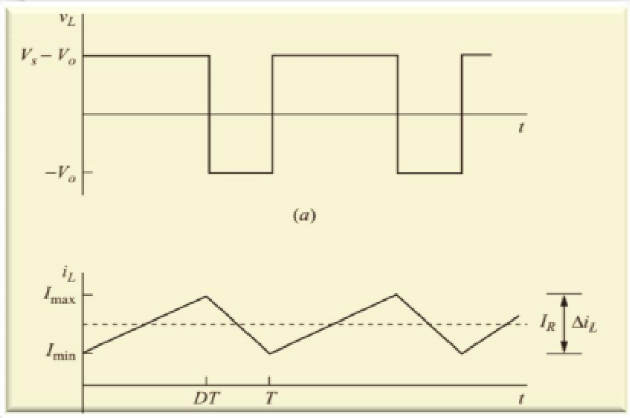
Analysis for the Switch Closed



(b)

Since the derivative of the current is a positive constant, the current increases linearly. The change in current while the switch is closed is computed by modifying the preceding equation.



(a)

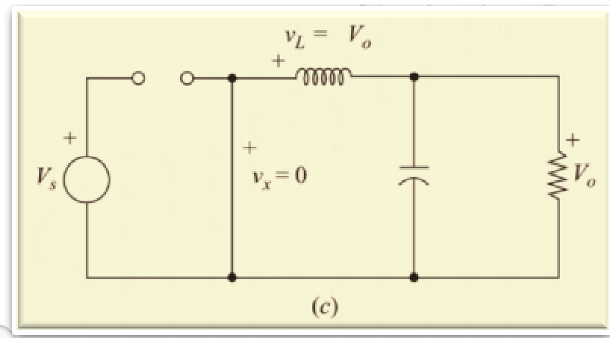
$$(\Delta i_L)_{closed} = \int_0^{DT} \frac{V_s - V_o}{L} dt = \frac{V_s - V_o}{L} DT$$

or

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L} \qquad (1)$$

$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L}\right) DT$$

Analysis for the Switch Opened



(c)

When the switch is open, the diode becomes forward-biased to carry the inductor current and the equivalent circuit of fig. c applies. The voltage across the inductor when the switch is open is

$$v_L = -V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{-V_o}{L}$$

The derivative of current in the inductor is a negative constant, and the current decreases linearly. The change in inductor current when the switch is open is

$$(\Delta i_L)_{opened} = \int_0^{(1-D)T} \frac{-V_o}{L} dt = \frac{-V_o}{L}(1-D)T$$

or

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T \qquad (2)$$

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

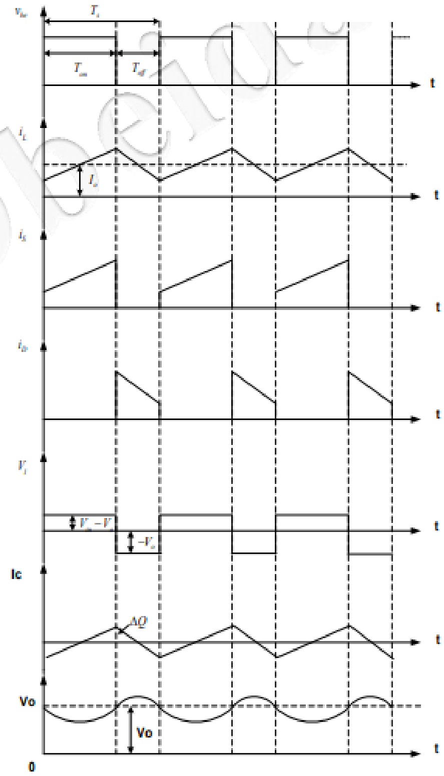
Using equations 1&2

$$\left(\frac{V_s - V_o}{L}\right)(DT) - \left(\frac{V_o}{L}\right)(1-D)T = 0$$

$$\boxed{V_o = V_s D}$$

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero for steady-state operation:

$$I_L = I_R = \frac{V_o}{R}$$



The maximum and minimum values of the inductor current are computed as

$$I_{\max} = I_L + \frac{\Delta i_L}{2}$$

$$= \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2}$$

$$= \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{L}(1-D)T\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

Since I_{min}=0 is the boundary between continuous and discontinuous current,

$$I_{\min} = 0 = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$(Lf)_{\min} = \frac{(1-D)R}{2}$$

The minimum combination of inductance and switching frequency for continuous current in the buck converter is

$$L_{\min} = \frac{(1-D)R}{2f} \qquad \text{for continuous current}$$

where L_{min} is the minimum inductance required for continuous current. In practice, a value of inductance greater than L_{min} is desirable to ensure continuous current.

Since the converter components are assumed to be ideal, the power supplied by the source must be the same as the power absorbed by the load resistor.

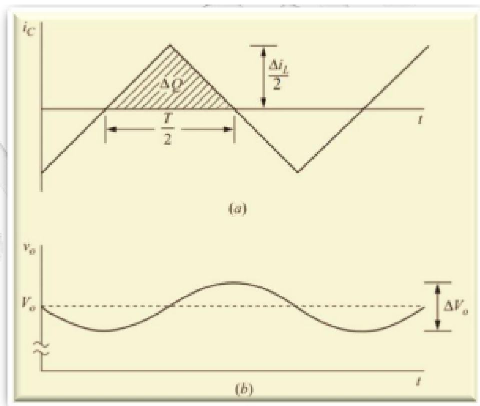
$$P_s = P_o$$
$$V_s I_s = V_o I_o$$
$$\frac{V_o}{V_s} = \frac{I_s}{I_o}$$

This relationship is similar to the voltage-current relationship for a transformer in AC applications. Therefore, the buck converter circuit is equivalent to a DC transformer.

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$

While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,



$$Q = CV_o$$
$$\Delta Q = C\Delta V_o$$
$$\Delta V_o = \frac{\Delta Q}{C}$$

The change in charge ΔQ is the area of the triangle above the time axis

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8}$$

$$\Delta V_o = \frac{T\Delta i_L}{8C}$$

Substitute $(\Delta i_L)_{open}$ in the above equation yields

$$\Delta V_o = \frac{TV_o}{8CL}(1-D)T = \frac{V_o(1-D)}{8LCf^2}$$

ΔV_o is the peak-to-peak ripple voltage at the output

The required capacitance in terms of specified voltage ripple:

$$C = \frac{1-D}{8L(\Delta V_o/V_o)f^2}$$

Examlpe: buck dc-dc converter with Low Pass Filter has the following parameters:

$V_s = 50$ V $\quad L = 400\ \mu$H $\quad f = 20$ kHz

$D = 0.4 \quad C = 100\ \mu$F $\quad R = 20\ \Omega$

Assuming ideal components, calculate (a) the output voltage Vo, (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

(a) $V_o = V_s D = (50)(0.4) = 20$ V

(b)

$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75\text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$= 1 - \frac{1.5}{2} = 0.25\text{ A}$$

The average inductor current is 1 A, and Δi_L=1.5 A.

(c)

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$

$$= 0.00469 = 0.469\%$$

The Boost (Step-Up) Converter

- It is called a boost converter because the output voltage is larger than the input.

Analysis for the Switch Closed

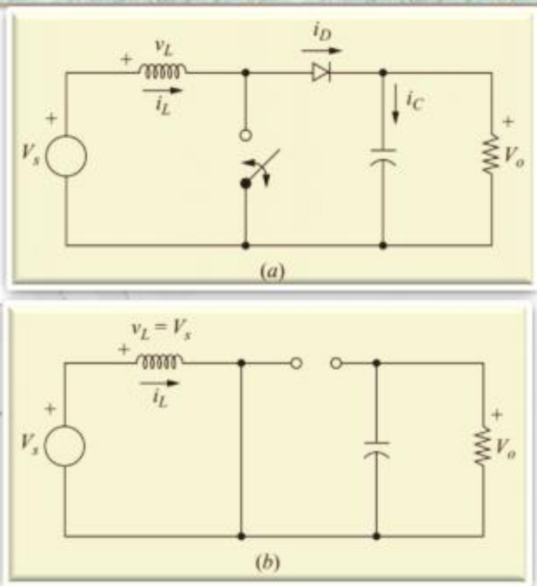
When the switch is closed, the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

$$v_L = V_s = L\frac{di_L}{dt} \quad \text{or} \quad \frac{di_L}{dt} = \frac{V_s}{L}$$

The rate of change of current is a constant, so the current increases linearly while the switch is closed. The change in inductor current is computed from

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L} \quad \text{or}$$

$$(\Delta i_L)_{closed} = \int_0^{DT} \frac{V_s}{L}dt = \frac{V_s}{L}DT \qquad (1)$$



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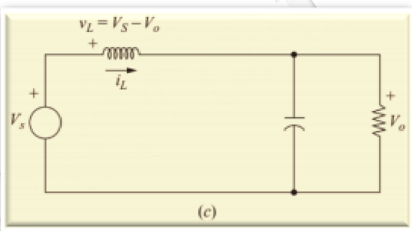
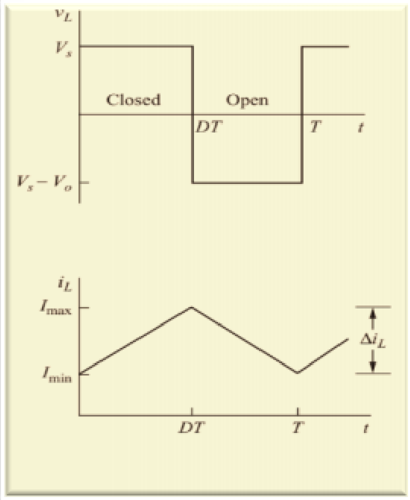
Analysis for the Switch opened

When the switch is opened, the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current. Assuming that the output voltage V_o is a constant, the voltage across the inductor is



$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$



The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L} \qquad \text{or}$$

$$(\Delta i_L)_{opened} = \int_0^{(1-D)T} \frac{V_S - V_O}{L}\, dt = \frac{V_S - V_O}{L}(1-D)T \qquad (2)$$

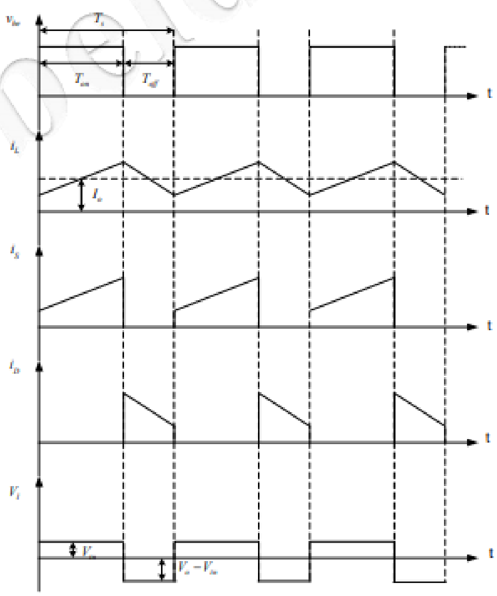
For steady-state operation, the net change in inductor current must be zero. Using equations 1&2

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s DT}{L} + \frac{(V_s - V_o)(1-D)T}{L} = 0$$

$$V_s(D + 1 - D) - V_o(1 - D) = 0$$

$$\boxed{V_o = \frac{V_s}{1-D}} \qquad (3)$$



If the switch is always open and D is zero, the output voltage is the same as the input. As the duty ratio is increased, the denominator of equation 3 becomes smaller, resulting in a larger output voltage. The boost converter produces an output voltage that is greater than or equal to the input voltage. However, the output voltage cannot be less than the input.

The average current in the inductor is determined by recognizing that the average power supplied by the source must be the same as the average power absorbed by the load resistor. Output power is

$$P_o = \frac{V_o^2}{R} = V_o I_o$$

Input power is $V_s I_s = V_s I_L$. Equating input and output powers and using eq. 3

$$V_s I_L = \frac{V_o{}^2}{R} = \frac{[V_s/(1-D)]^2}{R} = \frac{V_s{}^2}{(1-D)^2 R}$$

$$I_L = \frac{V_s}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s} \qquad (4)$$

Maximum and minimum inductor currents are determined by using the average value and the change in current from eq. 1.

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L} \qquad (5)$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L} \qquad (6)$$

Since I_{min}=0 is the boundary between continuous and discontinuous current,

$$I_{\min} = 0 = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

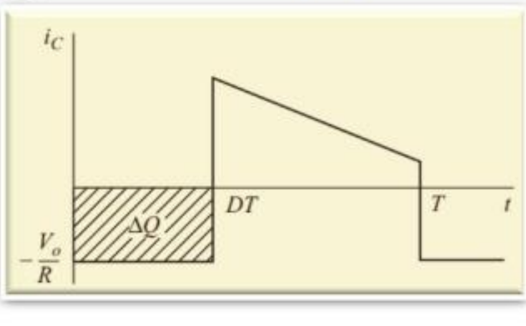
$$\frac{V_s}{(1-D)^2 R} = \frac{V_s DT}{2L} = \frac{V_s D}{2Lf}$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is

$$L_{\min} = \frac{D(1-D)^2 R}{2f} \qquad (7)$$

The peak-to-peak output voltage ripple can be calculated from the capacitor current waveform. The change in capacitor charge can be calculated from

$$|\Delta Q| = \left(\frac{V_o}{R}\right) DT = C\Delta V_o$$



An expression for ripple voltage is then

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf}$$

expressing capacitance in terms of output voltage ripple yields

$$C = \frac{D}{R(\Delta V_o/V_o) f}$$

Example: **Design a boost converter that will have an output of 30V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50. and the switching frequency is 25kHz.**

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let L=120 μH.

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5\,\text{A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2\,\text{A}$$

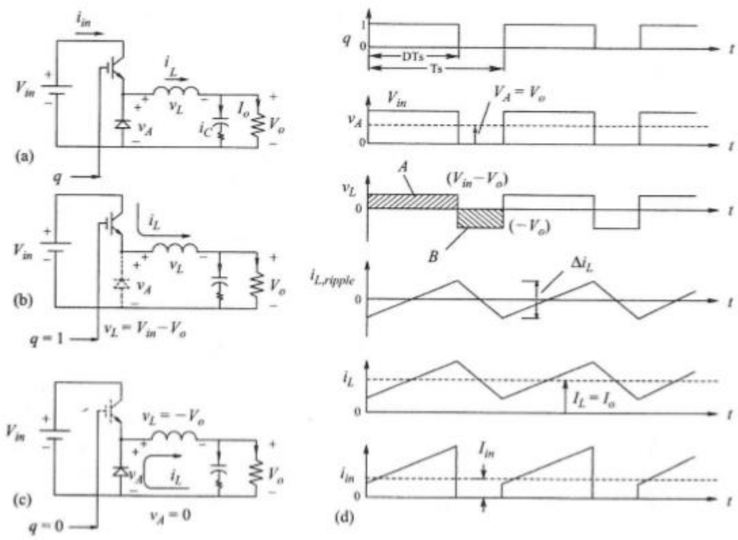
$$I_{\max} = 1.5 + 1.2 = 2.7\,\text{A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3\,\text{A}$$

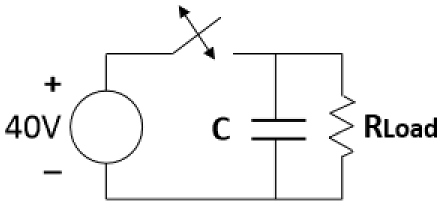
$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Buck Converter Analysis

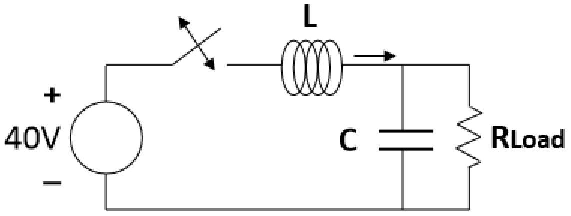
- $V_o = V_A = DV_{in}$; D = switch duty ratio
- $\Delta_{iL} = \frac{1}{L}(V_{in} - V_o)DT_s = \frac{1}{L}V_o(1-D)T_s$
- $I_L = Io = \frac{Vo}{R}$



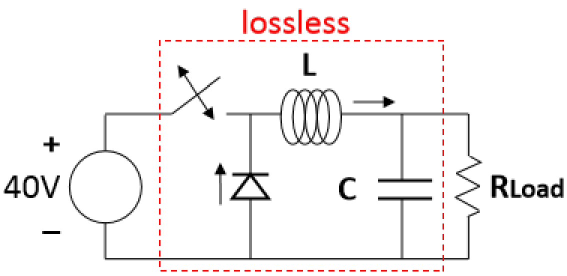
Examples of DC Conversion



Try adding a large C in parallel with the load to control ripple. But if the C has 13Vdc, then when the switch closes, the source current spikes to a huge value and burns out the switch.



Try adding an L to prevent the huge current spike. But now, if the L has current when the switch attempts to open, the inductor's current momentum and resulting *Ldi/dt* burns out the switch.



A DC-DC Buck Converter

By adding a "free wheeling" diode, the switch can open and the inductor current can continue to flow. With high-frequency switching, the load voltage ripple can be reduced to a small value.

Designing a Buck Converter

Assume:

V_{in} = 12 V

V_{OUT} = 5 volts

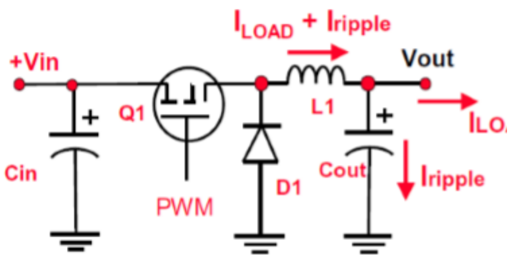
I_{LOAD} = 2 amps

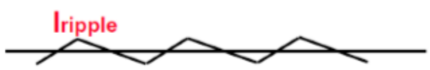
F_{sw} = 400 KHz

D = V_{in} / V_{out} = 5V /12V = 0.416

Define Ripple current:

$I_{ripple} = 0.3 \bullet I_{LOAD}$ (typically 30%)





For an Inductor: V = L • ΔI / ΔT

Rearrange and substitute:

L = (Vin – Vout) • (D / Fsw) / Iripple

Calculate:

L = 7 V • (0.416 / 400 kHz) / 0.6A

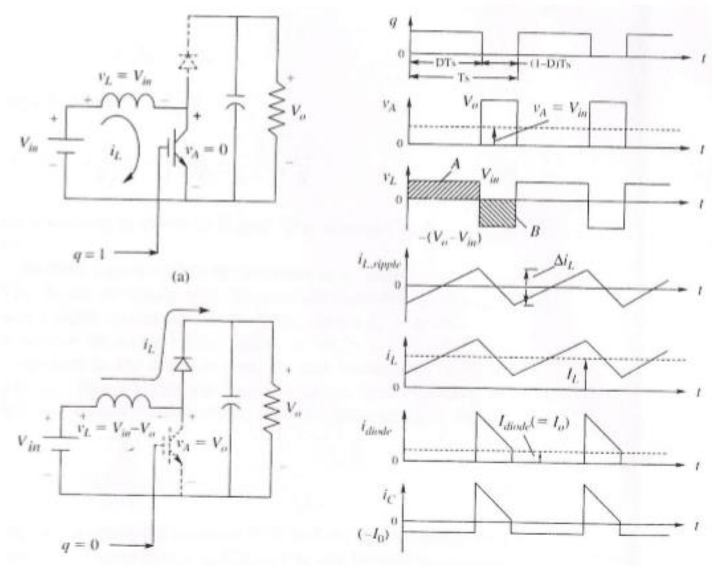
L = 12.12 uh

Select C, Diode (Schottky), and the MOSFET

Calculate the Efficiency

Boost Converter

- $\Delta_{iL} = \frac{1}{L}(V_{in})DT_s = \frac{1}{L}(V_o - V_{in})(1-D)T_s$
- $\frac{V_o}{V_{in}} = \frac{1}{1-D}$



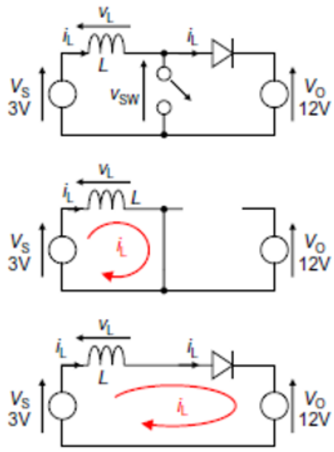
Boost (Step Up) Converter

- Step-up
- Same components
- Different topology!

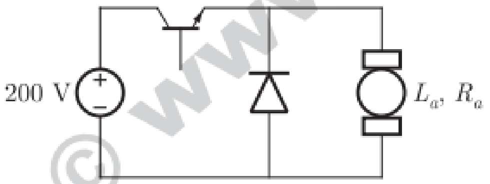
boost converter operating into a voltage source

boost converter with resistive load

- See stages of operation



Q. 2 The separately excited dc motor in the figure below has a rated armature current of 20 A and a rated armature voltage of 150 V. An ideal chopper switching at 5 kHz is used to control the armature voltage. If $L_a = 0.1$ mH, $R_a = 1\,\Omega$, neglecting armature reaction, the duty ratio of the chopper to obtain 50% of the rated torque at the rated speed and the rated field current is

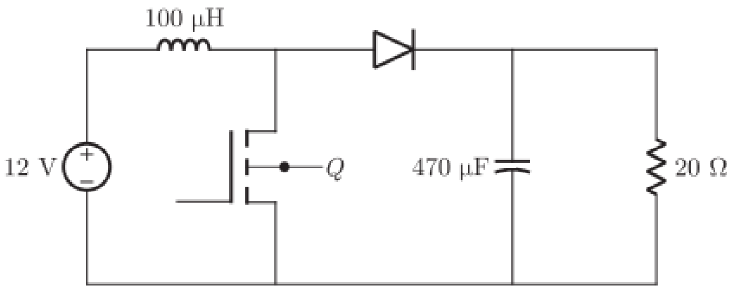


(A) 0.4 (B) 0.5

(C) 0.6 (D) 0.7

Common Data For Q. 3 and 4

In the figure shown below, the chopper feeds a resistive load from a battery source. MOSFET Q is switched at 250 kHz, with duty ratio of 0.4. All elements of the circuit are assumed to be ideal



Sol. 2 Option (D) is correct.

Given, the rated armature current

$$I_{a(rated)} = 20\text{ A}$$

as rated armature voltage

$$V_{a(rated)} = 150\text{ volt}$$

Also, for the armature, we have

$$L_a = 0.1\text{ mH},\ R_a = 1\,\Omega$$

and $T = 50\%$ of T_{rated} (T → Torque)

So, we get

$$I = [I_{a(\text{rotated})}](0.5) = 10\text{ A}$$

$$N = N_{\text{rated}},$$

$$I_f = I_{f\,rated} \rightarrow \text{rated field current}$$

At the rated conditions,

$$E = V - I_{a(\text{rated})}R_a$$

$$= 150 - 20(1) = 130\text{ volt}$$

For given torque,

$$V = E + I_a R_a = 130 + (10)(1) = 140\text{ V}$$

Therefore, chopper output $= 140$ V

or, $D(200) = 140$

or, $D = \frac{140}{200} = 0.7$ ($D \rightarrow$ duty cycle)

Q. 3 The Peak to Peak source current ripple in amps is

(A) 0.96 (B) 0.144

(C) 0.192 (D) 0.228

Sol. 3 Option (C) is correct.

Here, as the current from source of 12 V is the same as that pass through inductor. So, the peak to peak current ripple will be equal to peak to peak inductor current. Now, the peak to peak inductor current can be obtained as

$$I_L \text{ (Peak to Peak)} = \frac{V_s}{L} D T_s$$

where, $V_s \to$ source voltage $= 12$ volt,

$L \to$ inductance $= 100\mu\text{H} = 10^{-4}\text{H}$,

$D \to$ Duty ration $= 0.4$,

$T_S \to$ switching time period of MOSFET $= \frac{1}{f_S}$

and $f_s \to$ switching frequency $= 250$ kHz

Therefore, we get

$$I_{L(\text{Peak to Peak})} = \frac{12}{10^{-4}} \times 0.4 \times \frac{1}{250 \times 10^3} = 0.192 \text{ A}$$

This is the peak to peak source current ripple.

Sol. 4 Option (B) is correct.

Here, the average current through the capacitor will be zero. (since, it is a boost converter). We consider the two cases :

Case I : When MOSFET is ON

$$i_{c_1} = -i_0 \qquad (i_0 \text{ is output current})$$

(since, diode will be in cut off mode)

Case II : When MOSFET is OFF

Diode will be forward biased and so

$$i_{c_1} = I_s - i_0 \qquad (I_s \text{ is source current})$$

Therefore, average current through capacitor

$$I_{c,\,avg} = \frac{i_{c_1} + I_{c_2}}{2}$$

$$\Rightarrow \quad 0 = \frac{DT_s(-i_o) + (1-D)T_s(I_s - i_o)}{2} \quad (D \text{ is duty ratio})$$

Solving the equation, we get

$$I_s = \frac{i_0}{(1-D)} \qquad \text{....(1)}$$

Since, the output load current can be given as

$$i_0 = \frac{V_0}{R} = \frac{V_s/_{1-D}}{R} = \frac{12/0.6}{20} = 1\text{A}$$

Hence, from Eq. (1)

$$I_s = \frac{i_0}{1-D} = \frac{1}{0.6} = \frac{5}{3}\text{A}$$

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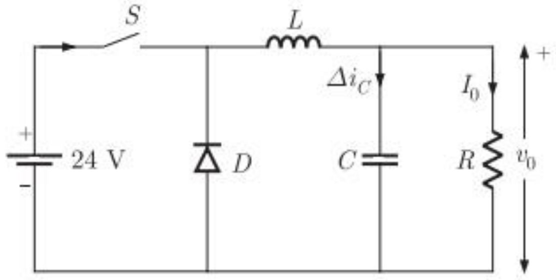
Q. 9

In the circuit shown, an ideal switch S is operated at 100 kHz with a duty ratio of 50%. Given that Δi_c is 1.6 A peak-to-peak and I_0 is 5 A dc, the peak current in S, is



(A) 6.6 A
(B) 5.0 A
(C) 5.8 A
(D) 4.2 A

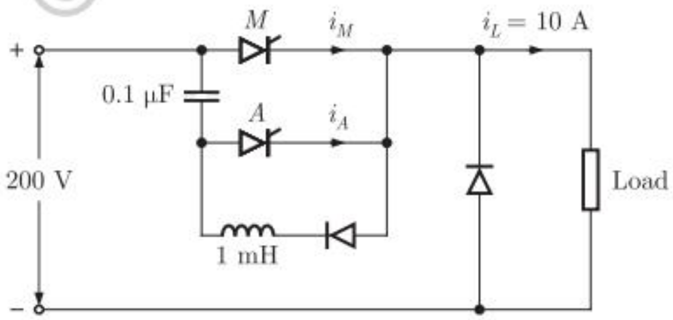
Sol. 9

Option (C) is correct.

$$I_S = I_0 + \frac{\Delta i_c}{2} = 5 + 0.8 = 5.8 \text{ A}$$

Q. 14

A voltage commutated chopper circuit, operated at 500 Hz, is shown below.



If the maximum value of load current is 10 A, then the maximum current through the main (M) and auxiliary (A) thyristors will be

(A) $i_{M\max} = 12$ A and $i_{A\max} = 10$ A

(B) $i_{M\max} = 12$ A and $i_{A\max} = 2$ A

(C) $i_{M\max} = 10$ A and $i_{A\max} = 12$ A

(D) $i_{M\max} = 10$ A and $i_{A\max} = 8$ A

Sol. 14

Option (A) is correct.

Maximum current through main thyristor

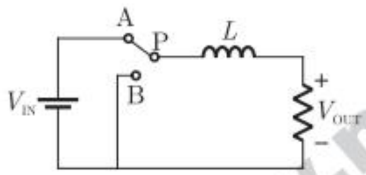
$$I_M(\text{max}) = I_0 + V_s\sqrt{\frac{C}{L}} = 10 + 200\sqrt{\frac{0.1\times10^{-6}}{1\times10^{3}}} = 12\text{ A}$$

Maximum current through auxiliary thyristor

$$I_A(\text{max}) = I_0 = 10\text{ A}$$

Q. 17

The power electronic converter shown in the figure has a single-pole double-throw switch. The pole P of the switch is connected alternately to throws A and B. The converter shown is a



(A) step down chopper (buck converter)

(B) half-wave rectifier

(C) step-up chopper (boost converter)

(D) full-wave rectifier

Sol. 17

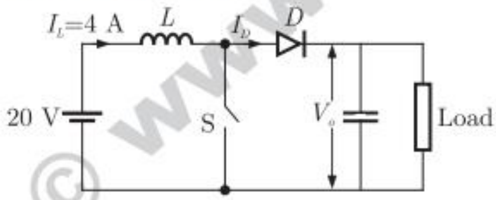
Option (A) is correct.

The figure shows a step down chopper circuit.

$$\because \qquad V_{out} = DV_{in}$$

where, $D =$ Duty cycle and $D < 1$

Q. 33 In the circuit shown in the figure, the switch is operated at a duty cycle of 0.5. A large capacitor is connected across the load. The inductor current is assumed to be continuous.

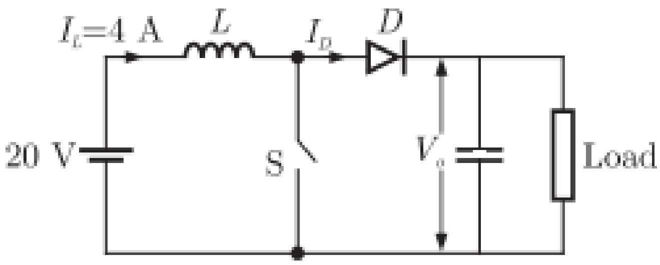


The average voltage across the load and the average current through the diode will respectively be

(A) 10 V, 2 A (B) 10 V, 8 A

(C) 40 V 2 A (D) 40 V, 8 A

Sol. 33 Option (C) is correct.



In the given diagram

when switch S is open $I_0 = I_L = 4$ A, $V_s = 20$ V

when switch S is closed $I_D = 0$, $V_0 = 0$ V

Duty cycle $= 0.5$ so average voltage is $\frac{V_s}{1-\delta}$

$$\text{Average current} = \frac{0+4}{2} = 2 \text{ amp}$$

$$\text{Average voltage} = \frac{20}{1-0.5} = 40 \text{ V}$$

Q. 44 The minimum approximate volt-second rating of pulse transformer suitable for triggering the SCR should be : (volt-second rating is the maximum of product of the voltage and the width of the pulse that may applied)

(A) 2000 μV-s (B) 200 μV-s

(C) 20 μV-s (D) 2 μV-s

Sol. 44 Option (A) is correct.

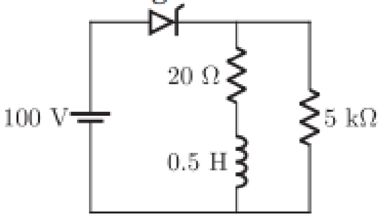
We know that the pulse width required is equal to the time taken by i_a to rise upto i_L

so,
$$V_s = L\frac{di}{dt} + R_i(V_T \approx 0)$$
$$i_a = \frac{200}{1}[1 - e^{-t/0.15}]$$

Here also $t = T$, $\quad i_a = i_L = 0.25$

$$0.25 = 200[1 - e^{-T/0.5}]$$
$$T = 1.876 \times 10^{-4} = 187.6\ \mu\text{s}$$
$$\text{Width of pulse} = 187.6\ \mu\text{s}$$
$$\text{Magnitude of voltage} = 10\ \text{V}$$
$$V_{sec}\ \text{rating of P.T.} = 10 \times 187.6\ \mu\text{s}$$
$$= 1867\ \mu\text{V-s is approx to } 2000\ \mu\text{V-s}$$

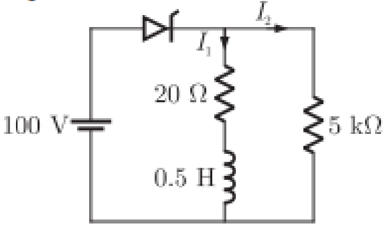
Q. 52 An SCR having a turn ON times of 5 μsec, latching current of 50 A and holding current of 40 mA is triggered by a short duration pulse and is used in the circuit shown in figure. The minimum pulse width required to turn the SCR ON will be



(A) 251 μsec (B) 150 μsec

(C) 100 μsec (D) 5 μsec

Sol. 52 Option (B) is correct.



In this given circuit minimum gate pulse width time= Time required by i_a rise up to i_L

$$i_2 = \frac{100}{5 \times 10^3} = 20 \text{ mA}$$

$$i_1 = \frac{100}{20}[1 - e^{-40t}]$$

$\therefore$ anode current $I = I_1 + I_2 = 0.02 + 5[1 - e^{-40t}]$

$$0.05 = 0.05 + 5[1 - e^{-40t}]$$

$$1 - e^{-40t} = \frac{0.03}{5}$$

$$T = 150 \ \mu\text{s}$$

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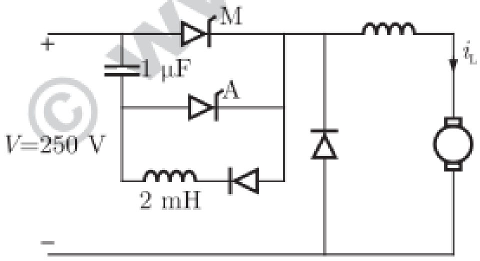
Common Data For Q. 53 and 54

A voltage commutated chopper operating at 1 kHz is used to control the speed of dc as shown in figure. The load current is assumed to be constant at 10 A



Q. 53 The minimum time in μsec for which the SCR M should be ON is.

(A) 280
(B) 140
(C) 70
(D) 0

Q. 54 The average output voltage of the chopper will be

(A) 70 V
(B) 47.5 V
(C) 35 V
(D) 0 V

Sol. 53 Option (B) is correct.

Given $I_L = 10$ A. So in the +ve half cycle, it will charge the capacitor, minimum time will be half the time for one cycle.

so min time required for charging

$$= \frac{\pi}{\omega_0} = \pi\sqrt{LC} = 3.14 \times \sqrt{2 \times 10^{-3} \times 10^{-6}} = 140\ \mu\text{sec}$$

Sol. 54 Option (C) is correct.

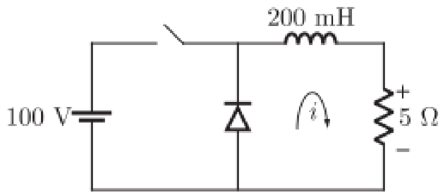
Given $$T_{on} = 140\ \mu\text{sec}$$

$$\text{Average output} = \frac{T_{on}}{T_{total}} \times V$$

$$T_{total} = 1/f = \frac{1}{103} = 1 \text{ msec}$$

$$\text{so average output} = \frac{140 \times 10^{-6}}{1 \times 10^{-3}} \times 250 = 35 \text{ V}$$

Q. 60 The given figure shows a step-down chopper switched at 1 kHz with a duty ratio $D = 0.5$. The peak-peak ripple in the load current is close to



(A) 10 A (B) 0.5 A

(C) 0.125 A (D) 0.25 A

Sol. 60 Option (C) is correct.

$$\text{Duty ratio } \alpha = 0.5$$

here

$$T = \frac{1}{1 \times 10^{-3}} = 10^{-3} \text{ sec}$$

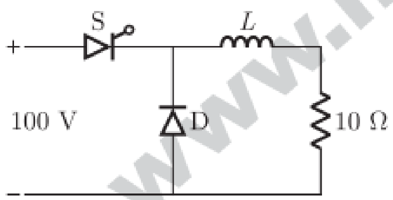
$$T_a = \frac{L}{R} = \frac{200 \text{ mH}}{5} = 40 \text{ msec}$$

$$\text{Ripple} = \frac{V_s}{R}\left[\frac{(1 - e^{-\alpha T/T_s})(1 - e^{-(1-\alpha)T/T_a})}{1 - e^{-T/T_s}}\right]$$

$$(\Delta I)_{\max} = \frac{V_s}{4fL} = \frac{100}{4 \times 10^3 \times 200 \times 10^{-3}}$$

$$= 0.125 \text{ A}$$

Q. 69 Figure shows a chopper operating from a 100 V dc input. The duty ratio of the main switch S is 0.8. The load is sufficiently inductive so that the load current is ripple free. The average current through the diode D under steady state is



(A) 1.6 A (B) 6.4 A

(B) 8.0 A (D) 10.0 A

Sol. 69 Option (C) is correct.

$V_s = 100$ V, duty ratio$= 0.8$, $R = 10\ \Omega$

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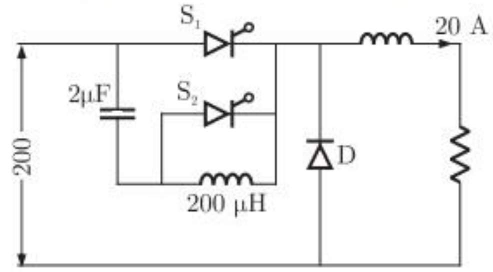
Q. 70 Figure shows a chopper. The device S_1 is the main switching device. S_2 is the auxiliary commutation device. S_1 is rated for 400 V, 60 A. S_2 is rated for 400 V, 30 A. The load current is 20 A. The main device operates with a duty ratio of 0.5. The peak current through S_1 is



(A) 10 A (B) 20 A

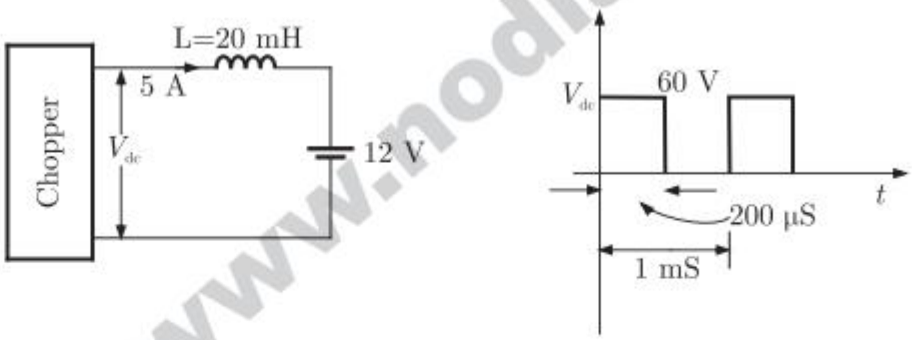
(C) 30 A (D) 40 A

Sol. 70 Option (D) is correct.

Peak current through S_1

$$I = I_0 + V_S\sqrt{C/L} = 20 + 200\sqrt{\frac{2\times10^{-6}}{200\times10^{-6}}} = 40 \text{ A}$$

Q. 78 A chopper is employed to charge a battery as shown in figure. The charging is 5 A. The duty ratio is 0.2. The chopper output voltage is also shown figure. The peak to peak ripple current in the charging current is



(A) 0.48 A (B) 1.2 A

(C) 2.4 A (D) 1 A

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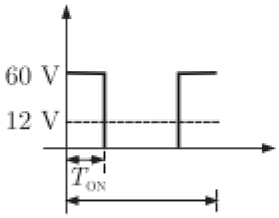
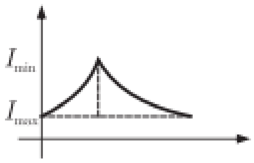
Sol. 78 Option (A) is correct.

In the chopper during turn on of chopper V-t area across L is,

$$\int_0^{T_{on}} V_L\, dt = \int_0^{T_{on}} L\left(\frac{di}{dt}\right) dt = \int_{i_{min}}^{i_{max}} L di = L(i_{max} - i_{min}) = L(\Delta I)$$





V-t are applied to 'L' is $= (60 - 12)\, T_{on} = 48\, T_{on}$