**STATISTICAL PHYSICS MIDTERM 1 2023** **25.04.2023**

**1) (30p)** The surface tension $\gamma(T,A)$ and heat capacity at constant area $C_A(T)$ for a water surface of area $A$ covered by a thin film containing $N$ organic molecules are given by

$$\gamma(T,A) = \gamma_0 - \frac{Nk_BT}{A-bN} + \frac{aN^2}{A^2} \text{ and } C_A(T) = Nk_B + Nk_B\left(\frac{T}{T_0}\right)^2.$$

Where $a$, $b$, $\gamma_0$ and $T_0$ are constants and $k_B$ is Boltzmann's constant. Expression for differential Helmholtz free energy is given by $dF = -SdT + \gamma dA$.

**a)** Find the entropy change $\Delta S(T,A)$ of the surface film, from the state $(T_0, A_0)$ to the state $(T, A)$.

**b)** The temperature of the surface increased from $T_i = 300.0K$ to $T_f = 300.1K$ keeping the surface area $A$ constant. What is the probability of the surface returning back to its initial state due to the fluctuations in temperature?

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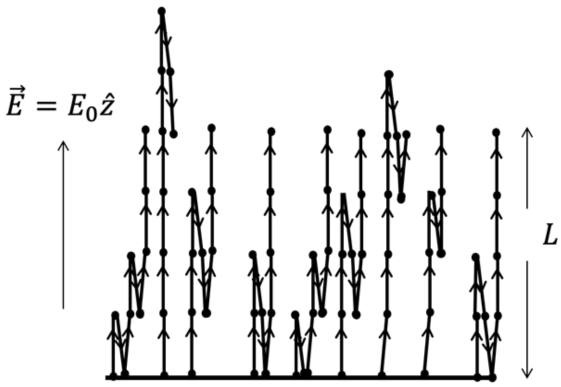
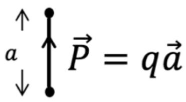
Consider a surface that is covered by $N$ dielectric polymer chains placed in a uniform electric field given by $\vec{E} = E_0\hat{z}$. Each chain consists of $n$ electric dipole monomer of which $n_\uparrow$ dipole oriented along $+z$ direction and $n_\downarrow$ dipole oriented along $-z$ direction. The energy of a dipole in an electric field is given by $\varepsilon = -\vec{P}.\vec{E}$. Assume that all chains on the surface have the same extension $L$ under constant and uniform electric field. The length of single dipole monomer is $a$. A model picture of this dielectric polymer surface in two dimensions is given.




**2) (30p)** In microcanonical ensemble, find the energy of the surface and the extension of dielectric polymers as a function of temperature, $E_N(T)$ and $L(T)$.

Discuss the extension of dielectric polymers at the limits of $T \to 0$, $T \to \infty$, $E_0 \to 0$ and $E_0 \to \infty$ in the context of order-disorder and entropy.

**3) (20p)** In canonical ensemble, write the partition function for a single dipole monomer. Find the energy of the surface and the extension of dielectric polymers as a function of temperature, $E_N(T)$ and $L(T)$.

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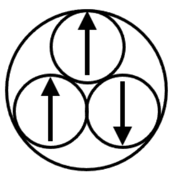
**4) (30p)** Consider a gas consists of $N$ non-interacting atoms in volume $V$. Each atom is formed by three fixed subatomic particles each with two possible energy state: spin up $+\frac{1}{3}\varepsilon$ and spin down $-\frac{1}{3}\varepsilon$. One of the possible states of a single atom is given in the figure.

**a)** Write the number of quantum states in microcanonical ensemble for the gas. Give the reasoning to your answer.

**b)** Write the single particle partition function and find the probability of an atom being at the lowest energy state.



**c)** Find the average energy of an atom and the total energy of the gas as a function of temperature using Boltzmann probability function in canonical ensemble.