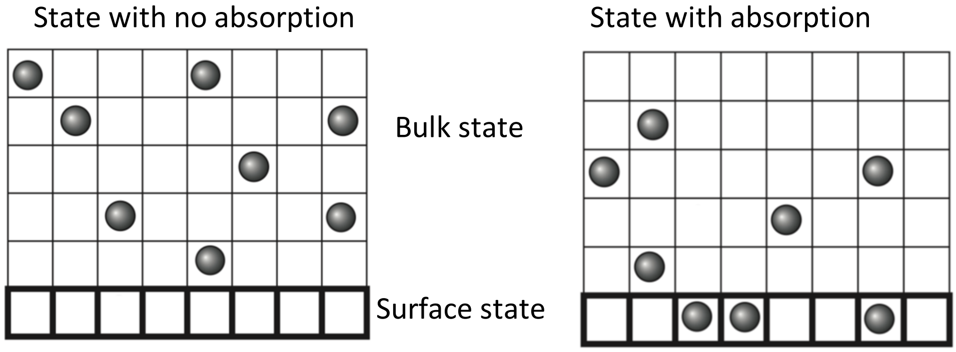
Question 1.

Consider a discrete model for the location of a gas in the bulk state and the surface state. The gas has N atoms in a container with M spatial cells. The surface of the container has N spatial cells where the atoms may be absorbed. An atom in the bulk state can have two magnetic moments $\vec{\mu} = \mp\mu_0\hat{\imath}$. An atom in the surface state can have only one magnetic moment $\vec{\mu} = +\mu_0\hat{\jmath}$. The system placed in a uniform magnetic field of $\vec{B} = +B_0\hat{\jmath}$. The energy of a magnetic moment in a magnetic field is $\varepsilon = -\vec{\mu}.\vec{B}$. Let n be the number of atoms absorbed by the surface.

a) Find the possible number of microstates in microcanonical ensemble.

b) Using the minimization of free energy, find a relation between the number of atoms absorbed by the surface n and temperature of the system T. **Don't try to express n as a function of T explicitly.**

c) What is the number of absorbed atoms n for the limiting case $T \to 0$?

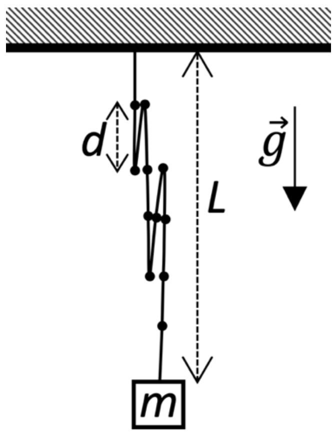


Question 2.

A simple model of a rubber band is described as a one-dimensional polymer, with N monomers of length d, that can point in either $+y$ or $-y$. A mass of m is suspended from the free end of the band as shown in the figure.

a) Find the extension of the rubber band L as a function of the temperature T, $L(T)$, in microcanonical ensemble.

b) Find the extension of the rubber band L as a function of the temperature T, $L(T)$, in canonical ensemble.



Question 3.

A two-dimensional gas confined in the(x, y) plane is characterized by N non interacting particles in equilibrium at the temperature T. The Hamiltonian (energy) of the single particle is

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{2}mw^2[a(x^2 + y^2) + bxy]$$

where p_x, p_y are the components of the momentum and m, w, a and b are constants. ($a > 0$ and $a^2 > b^2$).

$\int_{-\infty}^{+\infty} e^{-(Ax^2+Bx)}dx = e^{B^2/A}\left(\frac{\pi}{A}\right)^{1/2}$.

a) In canonical ensemble, find the energy of the system as a function of temperature $E(T)$.

b) Find the grand partition function for the system. (Leave you result in summation form, don't do further calculation)

Question 4.

Consider a system of three particles with three energy eigenstates of $0, \varepsilon, 3\varepsilon$ and 5ε. Write the partition function for three particle system.

a) If the particles are non-identical.

b) If the particles are obeying Bose-Einstein statistics.

c) If the particles are obeying Fermi-Dirac statistics