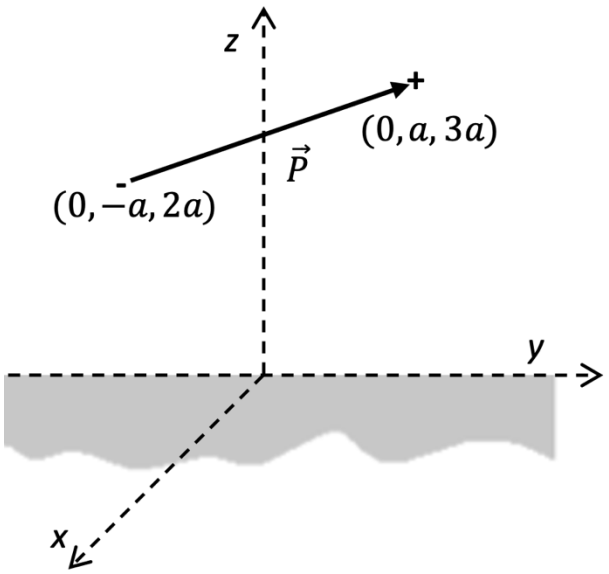
Question 1 (20P)

A (perfect) dipole $\vec{P}$ is situated above an infinite grounded conducting plane as shown in the figure. The dipole's positive end is at $(0, a, 3a)$ and negative end is at $(0, -a, 2a)$.

a) Find the dipole moment vector $\vec{P}$ and image dipole moment vector $\overrightarrow{P'}$ in cartesian coordinates.

b) Find the potential in cartesian coordinates $V(x, y, z)$ above the plane using method of images.

c) Find the charge distribution $\sigma(x, y, z)$ on the plane in cartesian coordinates.



Question 2 (20P)

A specified charge density $\sigma_0(\theta) = \frac{\sigma_0}{2}(1 - 3sin^2\theta)$ is glued over the surface of a spherical shell of radius *R*. Find the resulting potential inside and outside the sphere.

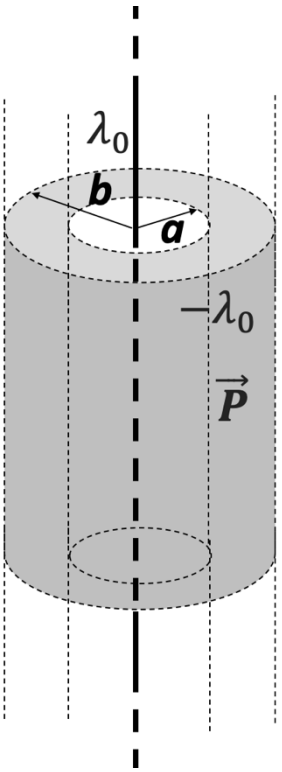
Question 3 (20P)

A long straight wire, carrying uniform line charge λ_0 is placed coaxially with rubber insulation of long cylindrical shell of inner radius a and outer radius b with uniform line charge density -λ_0 and varying polarization $\vec{P} = 3P_0 s\hat{s}$.

a) Find the bound charges.

b) Find the electric displacement vector $\vec{D}$ at $s \leq a,\ a \leq s \leq b$ and $s \geq b$.

c) Find the electric field vector $\vec{E}$ at $s \leq a,\ a \leq s \leq b$ and $s \geq b$.



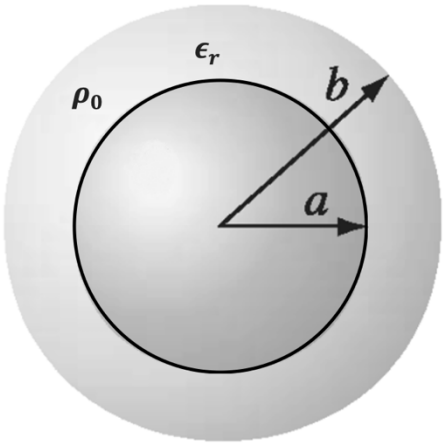
Question 4 (20P)

A spherical neutral (uncharged) conductor, of radius a is surrounded by linear dielectric material of relative permittivity (dielectric constant) ϵ_r and uniform embeded charge density ρ_0 out to radius b.

a) Find the electric displacement vector $\vec{D}$ at $r \leq a,\ a \leq r \leq b$ and $r \geq b$.

b) Find the electric field vector $\vec{E}$ at $r \leq a,\ a \leq r \leq b$ and $r \geq b$.

c) Find the energy of this configuration.



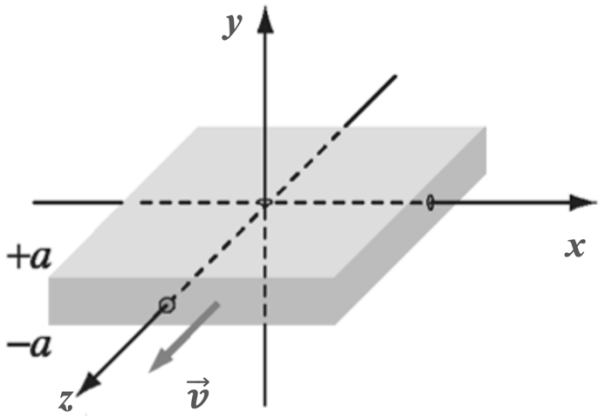
Question 5 (20P)

A thick slab extending from $y = -a$ to $y = +a$ (and infinite in the x and z directions) is pulled with velocity $\vec{v} = v_0\hat{z}$.

a) Find the magnetic field $\vec{B}$, as a function of y, at $a \leq y \leq -a$, $y \geq a$ and $y \leq a$.

b) Find the magnetic vector potential $\vec{A}$ above the plane.

Hint: $\vec{A}$ must be parallel to the current and depends only y.



Question 6 (20P)

A current wire is bent into a loop as shown in the figure.

a) Find the magnetic field vector $\vec{B}$ at the origin.

You can use the ready equations for the magnetic field of a straight wire and a circular arc.

b) Find the total magnetic dipole moment $\vec{m}$ of the loop.

c) Find the magnetic vector potential of the loop $\vec{A}(x, y, z)$.

d) Find the force acting on wire 2, if it is placed in a varying external magnetic field of $\vec{B} = \frac{B_0}{a} y\hat{z}$.