QUESTION 1

Consider the electrostatic potential in spherical coordinates $V(r,\theta,\phi) = r^2 Cos\theta$.

a) Find the corresponding electrostatic field vector $\vec{E}(r,\theta,\phi)$

b) Show that this electric field $\vec{E}(r,\theta,\phi)$ is a legitimate electric field.

c) Find the charge distribution $\rho(r,\theta,\phi)$ that produces this electric field $\vec{E}(r,\theta,\phi)$.

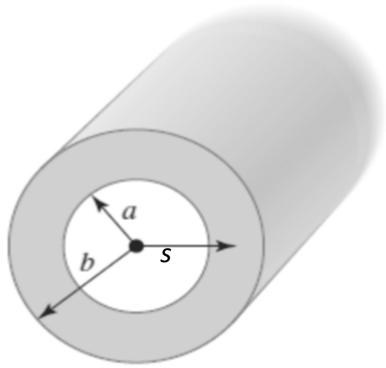
d) Show that this electric field $\vec{E}(r,\theta,\phi)$ satisfies the divergence theorem over an insulating solid sphere of radius R.

QUESTION 2

Consider an insulating long cylindrical shell of inner radius of a and outer radius of b. The electric field at the region $a \le s \le b$ is given as $\vec{E}(s,\phi,z) = \frac{k}{s^2}\hat{s}$.

a) Find the electric field vector $\vec{E}(s,\phi,z)$ at the regions $s \le a$ and $b \le s$.

b) Find the potential $V(r,\theta,\phi)$ at the regions $s \le a$, $a \le s \le b$ and $b \le s$. Take the potential $V(s = 2b) = V_0$ as reference.

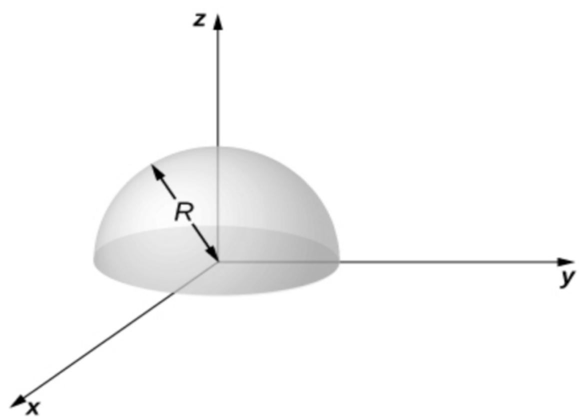


QUESTION 3

Consider a hemispherical shell of radius R with uniform surface charge density σ.

a) Find the electric field vector $\vec{E}$ at the center of the hemisphere.

b) Find the potential at the center of the hemisphere.



QUESTION 4

A solid sphere has radius R and carries a non-uniform charge density $\rho(r) = \alpha r^3$, where α is a constant. The potential formed by this sphere given as
$$V(r) = \begin{cases} \dfrac{\alpha R^6}{6\varepsilon_0 r} & r \ge R \\ \dfrac{\alpha R^5}{5\varepsilon_0} - \dfrac{\alpha}{30\varepsilon_0} r^5 & r \le R \end{cases}.$$

Find the energy stored in this solid sphere via following two ways

a) $W = \frac{\varepsilon_0}{2}\int E^2 d\tau$

b) $W = \frac{1}{2}\int V dq$