INS 3121
SOIL MECHANICS

Flow of Water in Soils: Seepage

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7.1 Introduction

- In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow.

- In such cases, the ground water flow is generally calculated by the use of graphs referred to as flow nets.

- Flow net : based on Laplace's equation of continuity

- Steady state flow is two-dimensional flow

7.1 Introduction

- the rate of flow of water into the elemental block

$= v_x dzdy$ in the horizontal direction

$= v_z dxdy$ in the vertical direction

- the rates of outflow from the block in the horizontal and vertical directions are

$$\left(v_x + \frac{\delta v_x}{\delta x} dx\right) dzdy$$

$$\left(v_z + \frac{\delta v_z}{\delta z} dz\right) dxdy$$

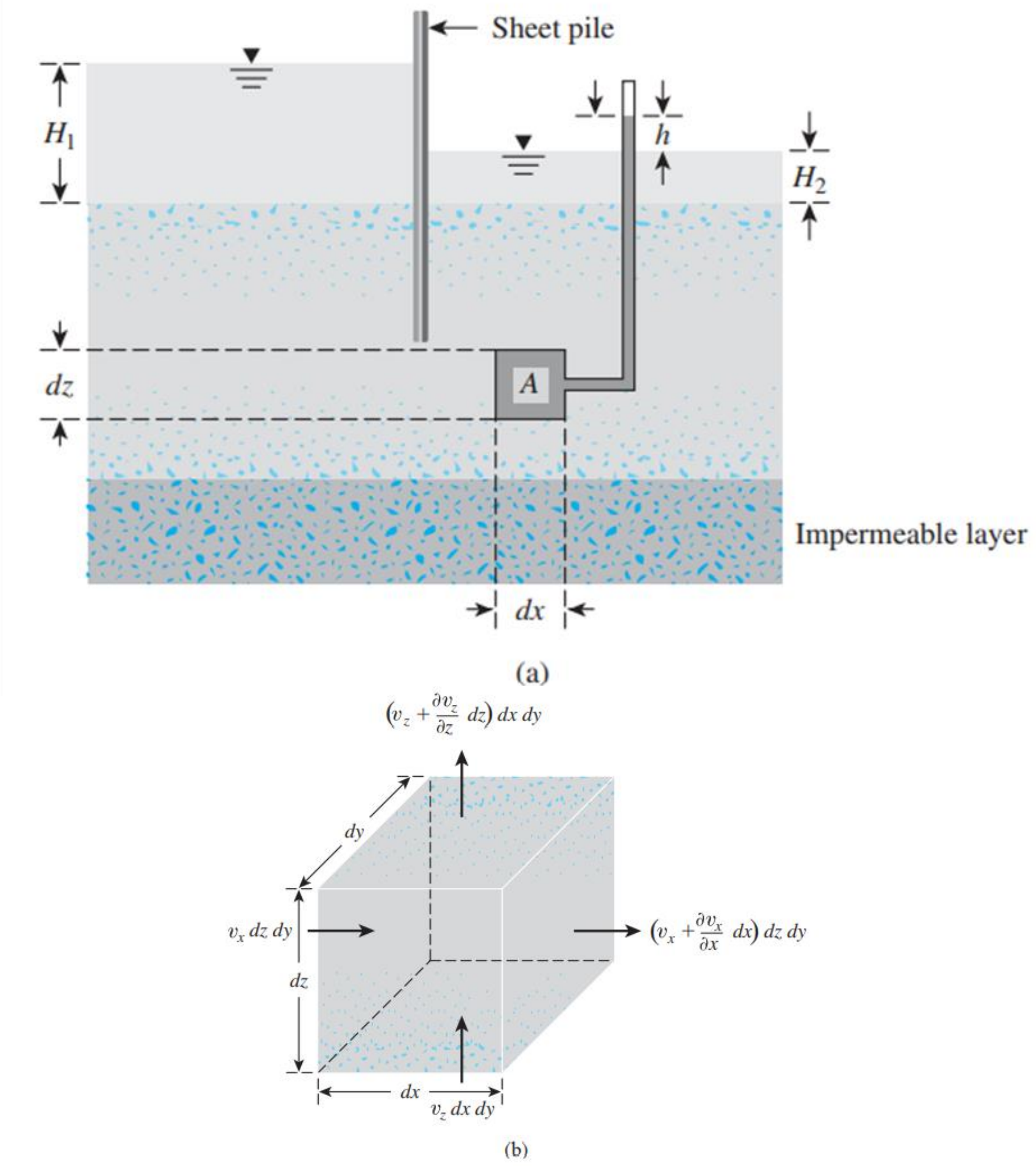


Figure 8.1 (a) Single-row sheet piles driven into permeable layer; (b) flow at A

7.1 Introduction

- Assuming water is incompressible and no volume change in the soil mass occurs, the total rate of inflow should be equal to the total rate of outflow.

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dz\, dy + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx\, dy\right] - [v_x\, dz\, dy + v_z\, dx\, dy] = 0$$

$$\boxed{\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0}$$: Continuity Eq.

$$v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \qquad v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \qquad \boxed{k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0}$$

- If the soil is isotropic with respect to the permeability coefficients-that is, $kx = kz$

$$\boxed{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$: Laplace EQ

7.2 Continuity Equation for Solution of Simple Flow Problems

- The head difference between the top of soil layer no. 1 and the bottom of soil layer no. 2 is h1
- Because the flow is in only the z direction,

$$\frac{\partial^2 h}{\partial z^2} = 0$$

$$h = A_1 z + A_2 \qquad (8.7)$$

- flow through soil layer no.1, boundary conditions

- Condition 1 : At $z = 0$, $h = h1$
- Condition 2 : At $z = H1$, $h = h2$

- Combining Eq.(8.7) and condition 1 gives

$$A_2 = h_1$$

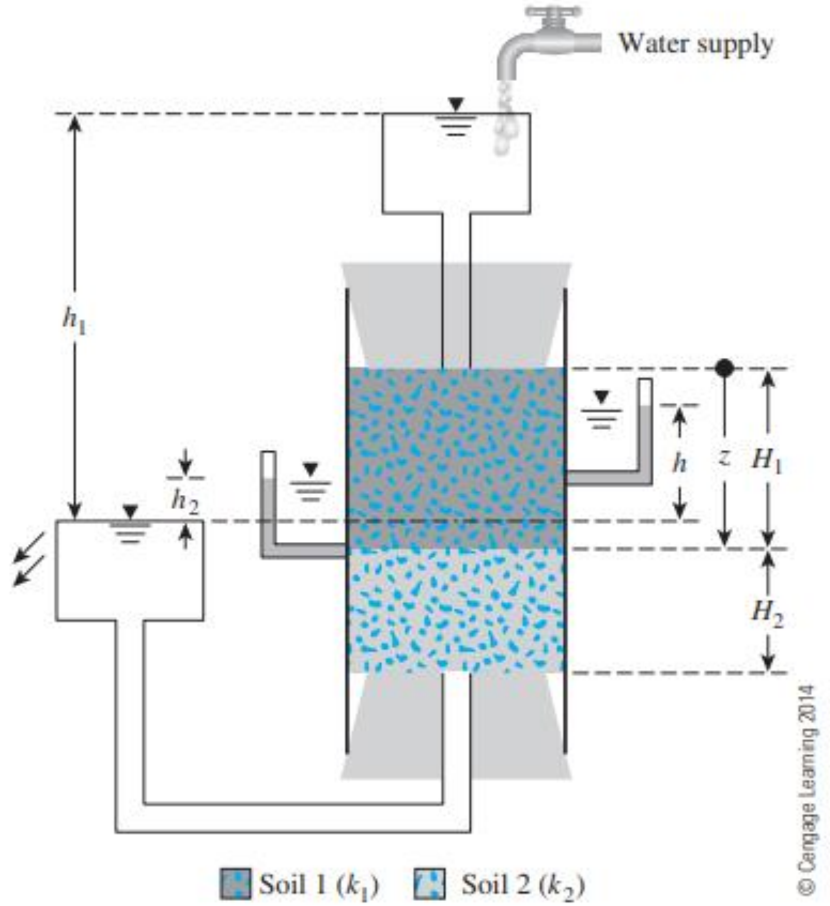


Figure 8.2 Flow through a two-layered soil

7.2 Continuity Equation for Solution of Simple Flow Problems

- $h = A_1 z + A_2$ (8.7)
- Combining Eq.(8.7) and condition gives

$$h_2 = A_1 H_1 + h_1$$

$$A_1 = -\left(\frac{h_1 - h_2}{H_1}\right)$$

  - $h = -\left(\frac{h_1 - h_2}{H_1}\right) z + h_1$ (for $0 \le z \le H_1$)
- Flow through soil layer no.2, boundary conditions

- Condition 1 : At $z = H_1$, $h = h_2$
- Condition 2 : At $z = H_1 + H_2$, $h = 0$

$$A_2 = h_2 - A_1 H_1$$

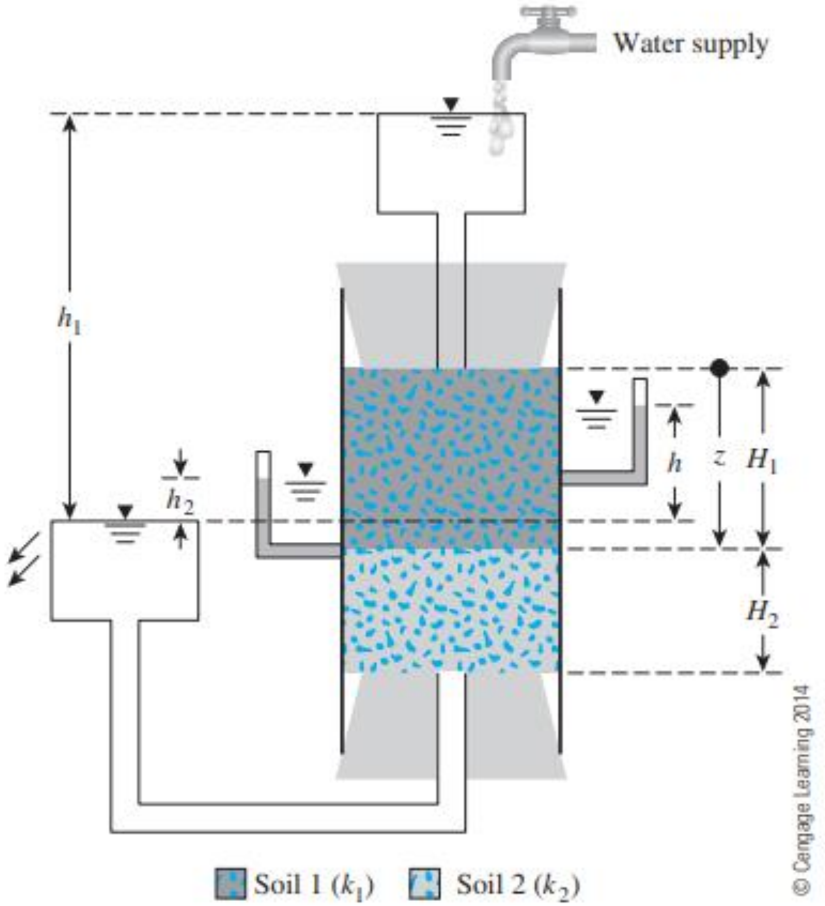


Figure 8.2 Flow through a two-layered soil

7.2 Continuity Equation for Solution of Simple Flow Problems

- $0 = A_1(H_1+H_2)+(h_2-A_1H_1)$
- $A_1H_1 + A_1H_2 + h_2 - A_1H_1 = 0$

$$A_1 = -\frac{h_2}{H_2}$$

- $h = -\left(\frac{h_2}{H_2}\right)z + h_2\left(1 + \frac{H_1}{H_2}\right)$ (for $H_1 \le z \le H_1 + H_2$)
- $q = k_1\left(\frac{h_1-h_2}{H_1}\right)A = k_2\left(\frac{h_2}{H_2}\right)A$

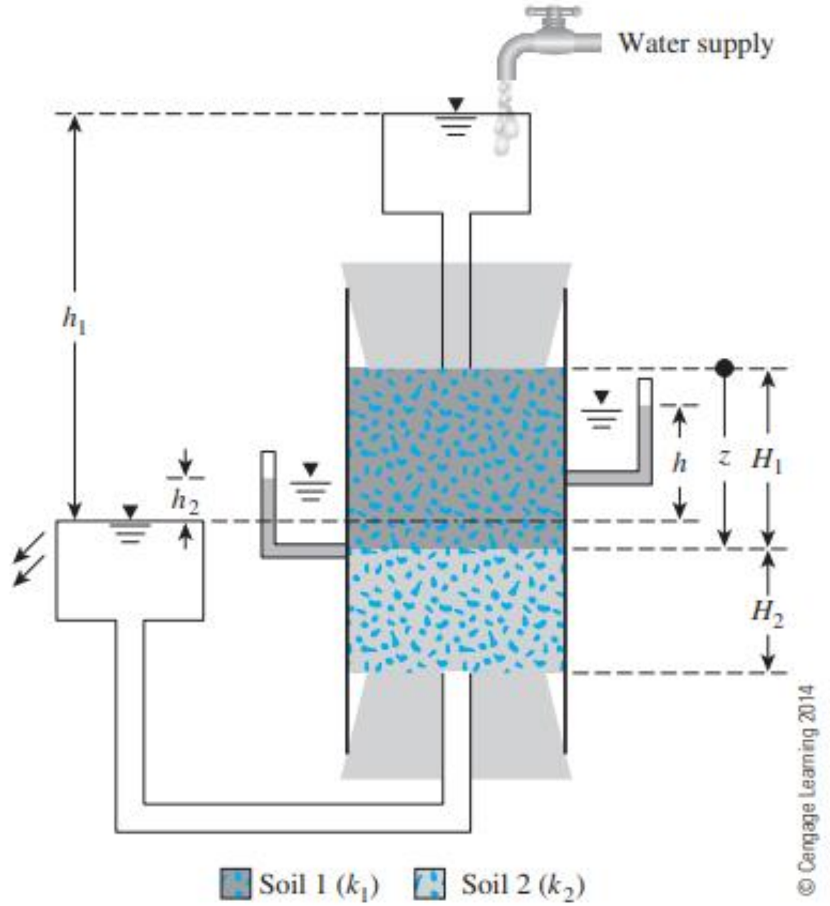


Figure 8.2 Flow through a two-layered soil

7.2 Continuity Equation for Solution of Simple Flow Problems

- $h_2 = \frac{h_1 k_1}{H_1(\frac{k_1}{H_1}+\frac{k_2}{H_2})}$
- $h = h_1(1 - \frac{k_2 z}{k_1 H_2 + k_2 H_1})$ (for $0 \le z \le H_1$)
- $h = h_1\left[\left(\frac{k_2 z}{k_1 H_2 + k_2 H_1}\right)(H_1 + H_2 - z)\right]$

(for $H_1 \le z \le H_1 + H_2$)

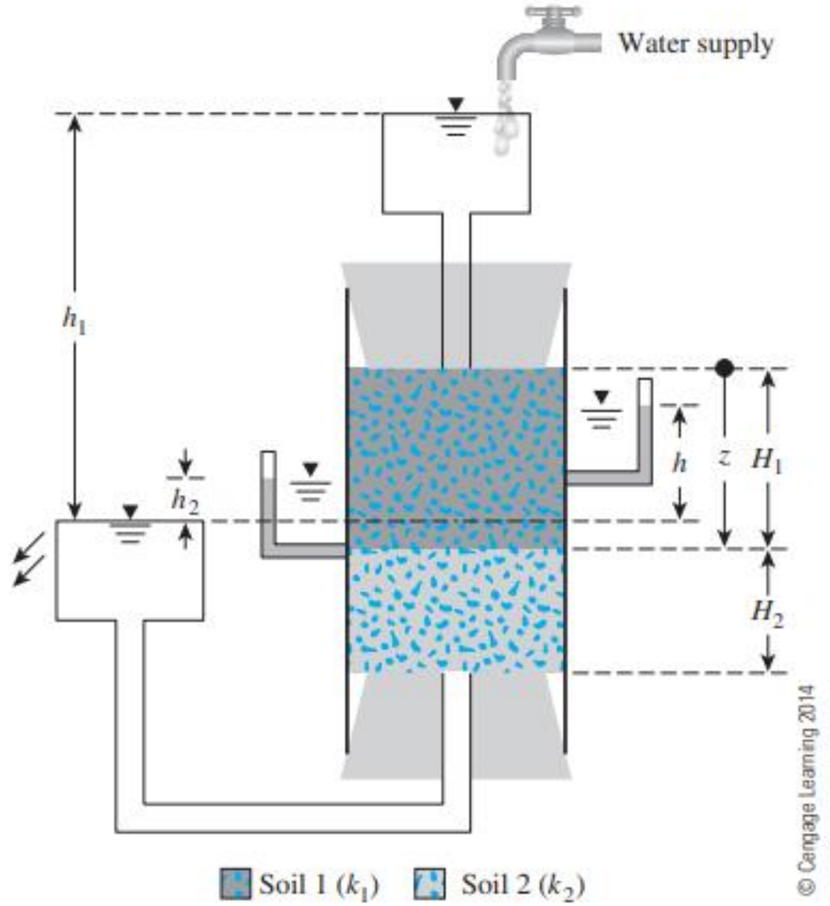


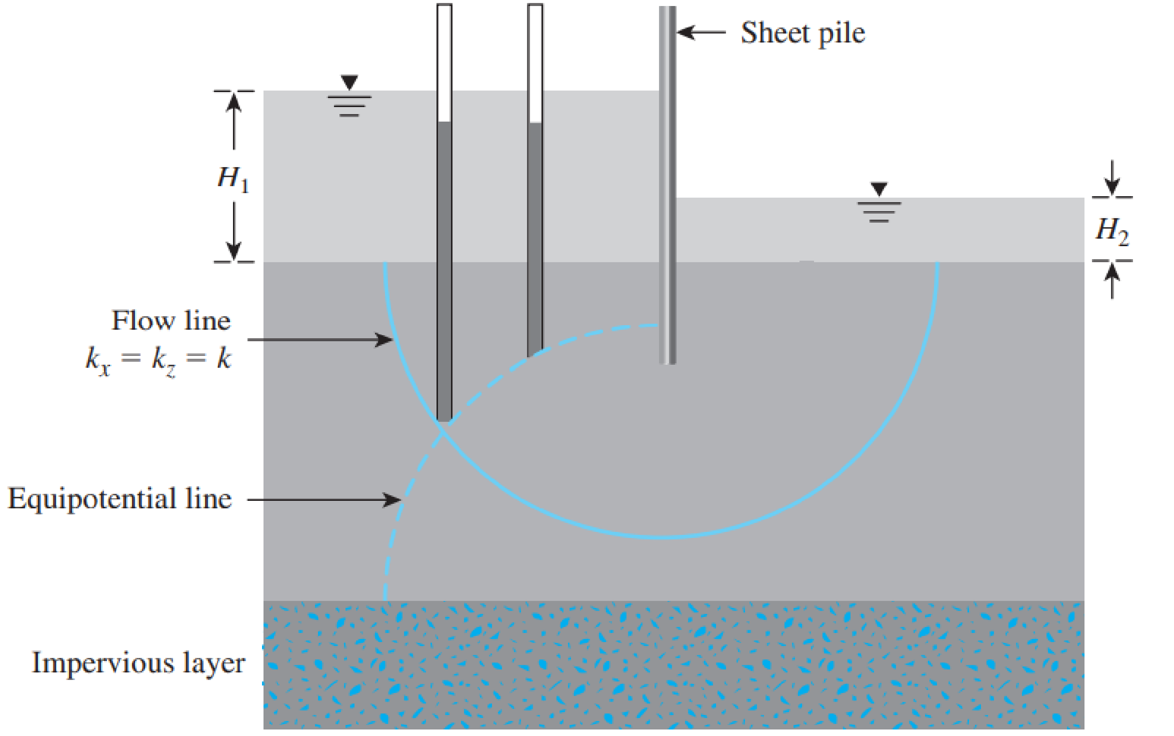
Figure 8.2 Flow through a two-layered soil

7.3 Flow Nets

- The continuity equation (Eq. 8.5) represents two orthogonal families of curves—that are

flow lines, equipotential lines

- flow line : a line along which a water particle will travel from upstream to the downstream side
- Equipotential line : a line along which the potential head at all points is equaL,
- If piezometers are placed at different points along an equipotential line, the level of water will rise to the same elevation in all of them.

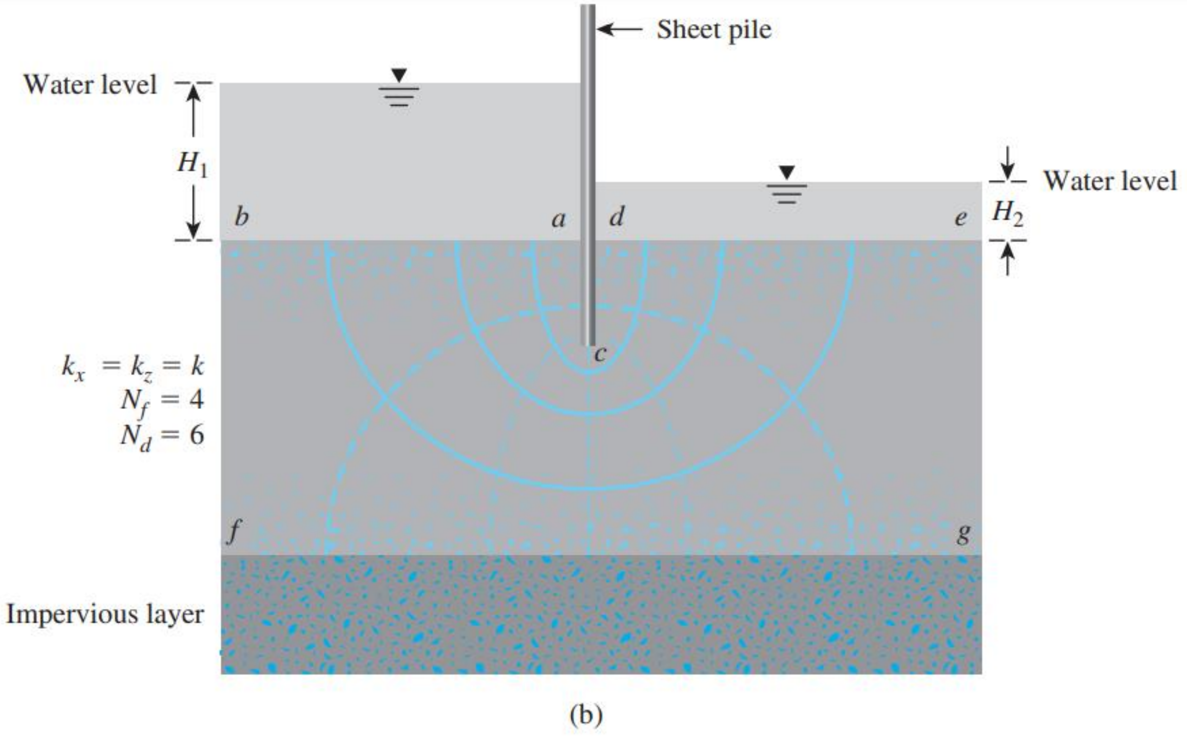


(a)

7.3 Flow Nets

- flow and equipotential lines are drawn in such a way that :

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximate squares.



igure 8.3 (a) Definition of flow lines and equipotential lines; (b) completed flow net

7.3 Flow Nets

- Drawing a flow net

1. Math solution
2. Numerical solution (FDM, FEM)
3. Electrical analogy and actual model
4. Trial sketch

- Key points in Drawing Flow nets

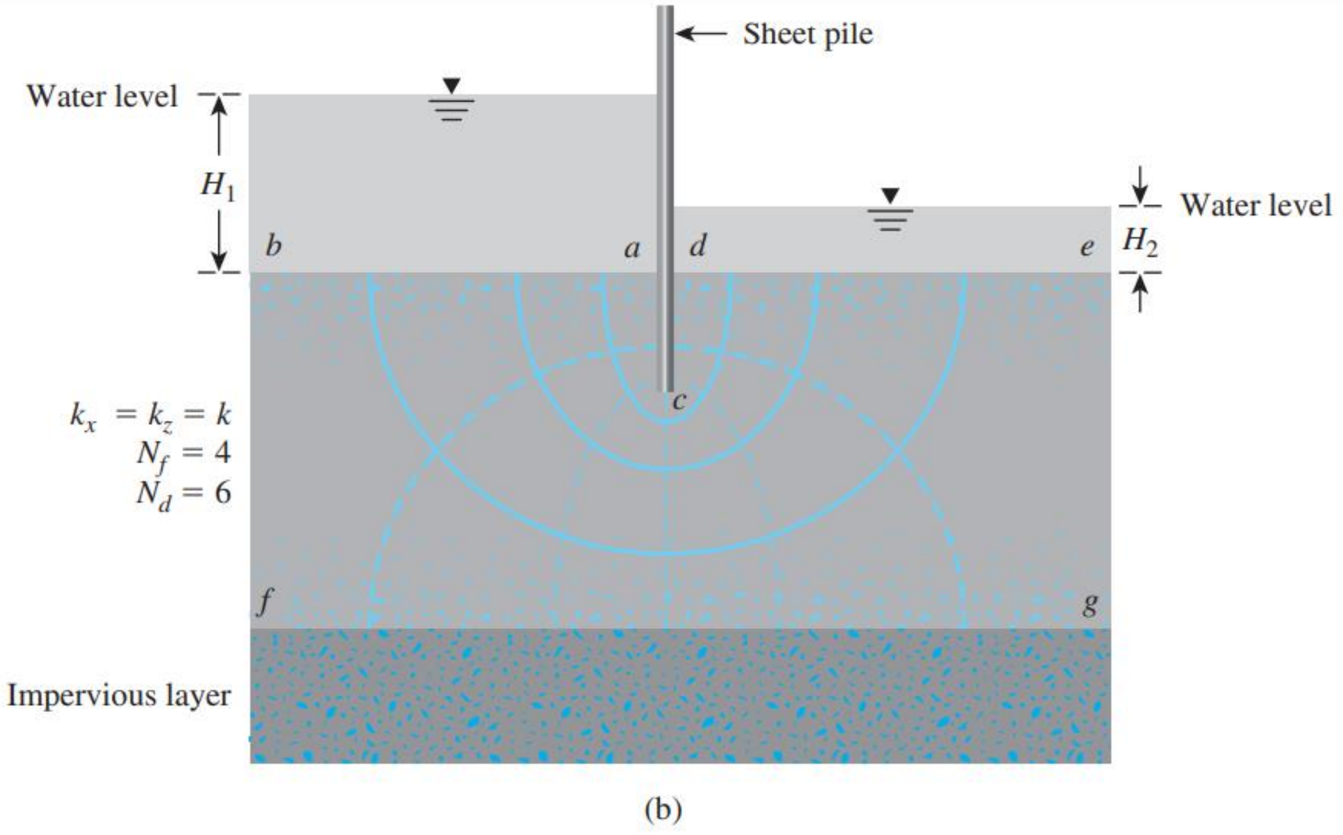
  Flow line (   )equipotential line

  Curvilinear square

  Boundary flow line

  Boundary equipotential line

  Partial flow line may be accepted



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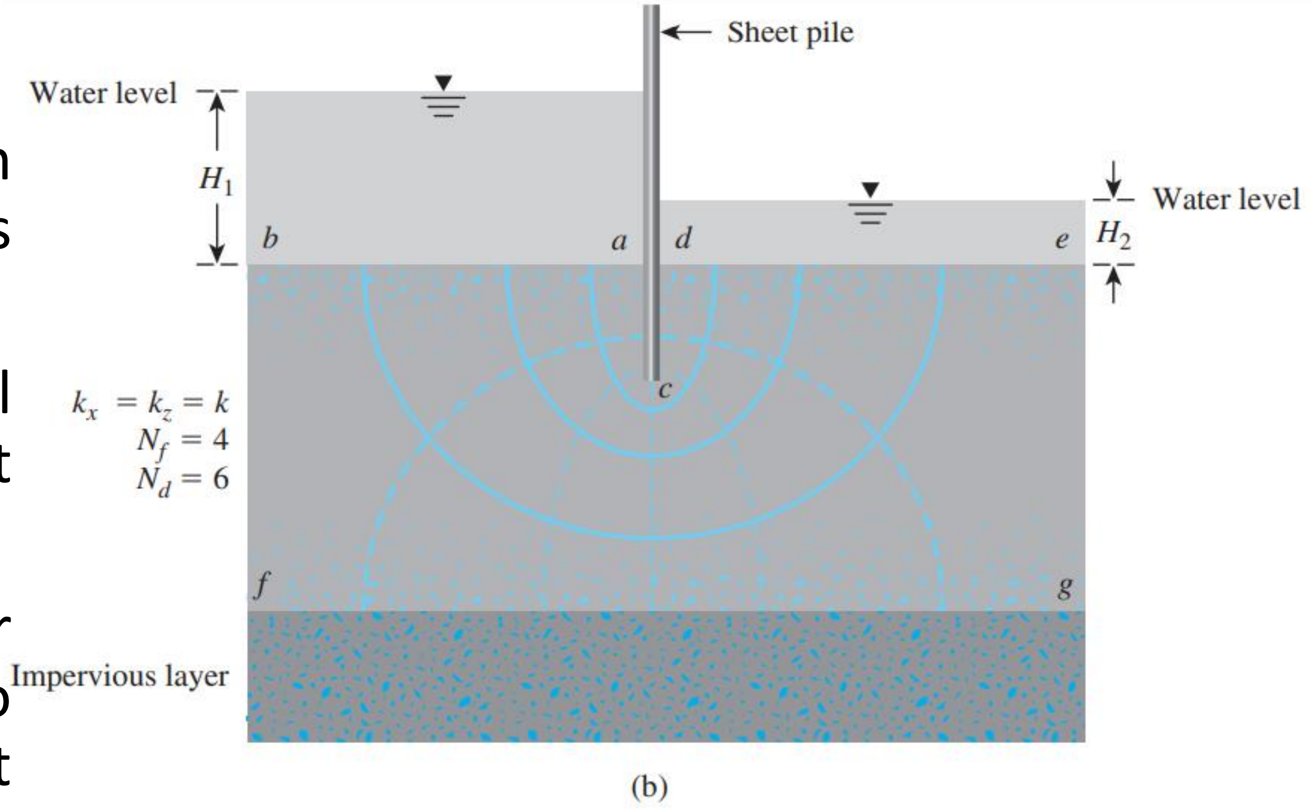
7.3 Flow Nets

Boundary conditions:

1. The upstream and downstream surfaces of the permeable layer (lines ab and de) are equipotential lines.
2. Because ab and de are equipotential lines, all of the flow lines intersect them at right angles.
3. The boundary of the impervious layer – that is, line f_g – is a flow line, and so is the surface of the impervious sheet pile, line acd.
4. The equipotential lines intersect acd and f_g at right angles.



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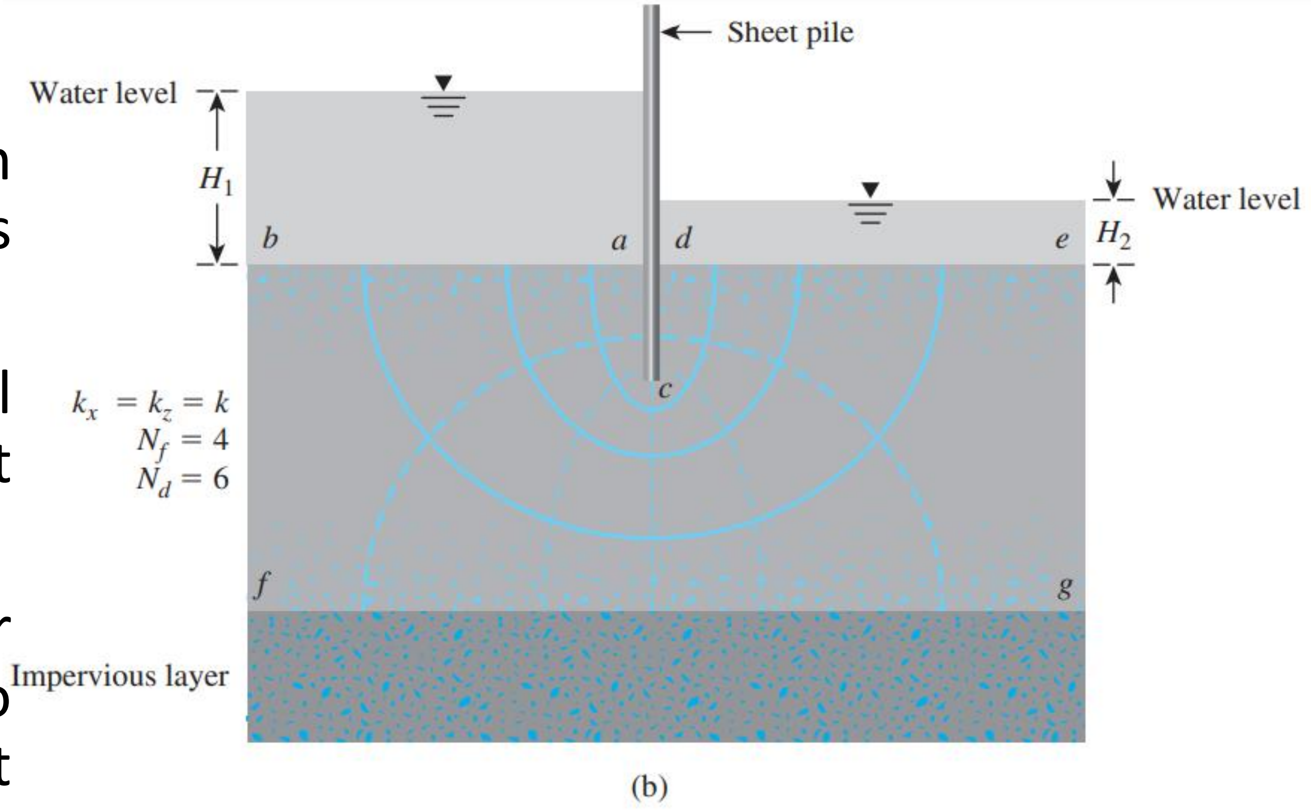

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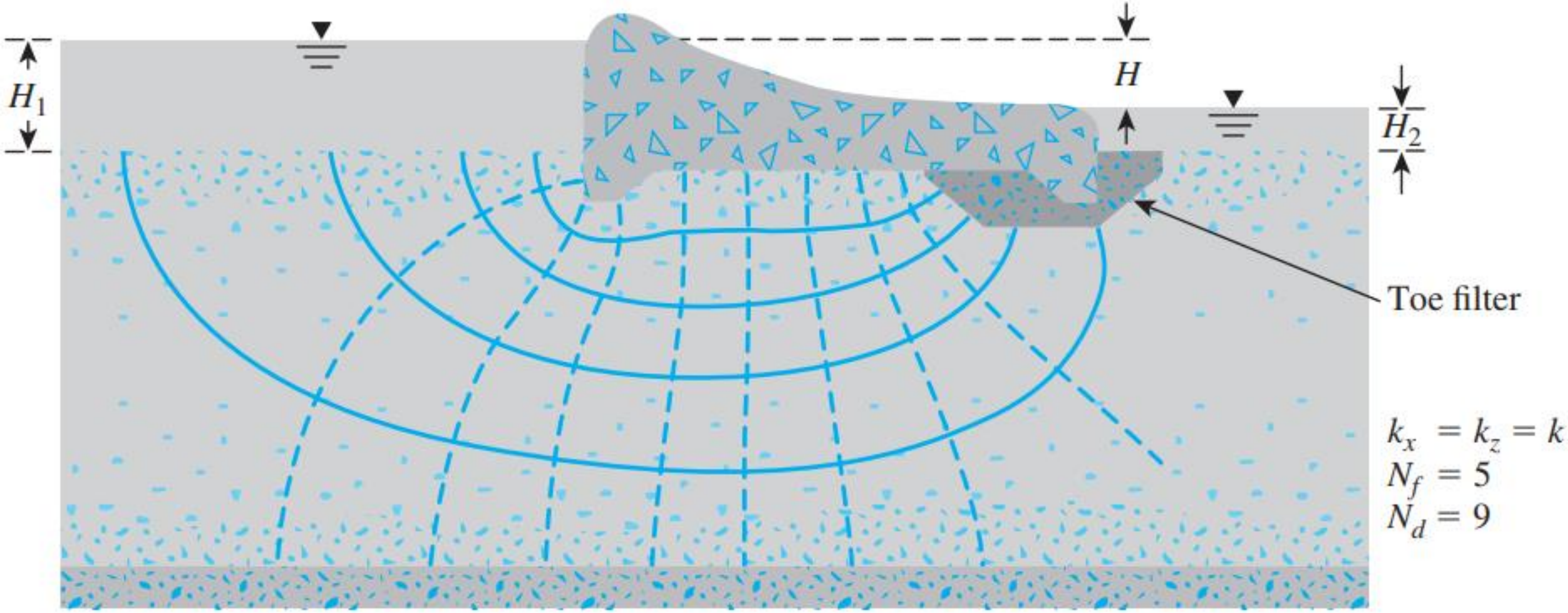


Figure 8.4 Flow net under a dam with toe filter

7.3 Seepage Calculation from a Flow Net

- flow channel - the strip between any two adjacent flow lines.
- Since there is no flow across the flow lines

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q$$

- $q = kiA$

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)l_1 = k\left(\frac{h_2 - h_3}{l_2}\right)l_2 = k\left(\frac{h_3 - h_4}{l_3}\right)l_3 = \cdots \qquad (8.18)$$

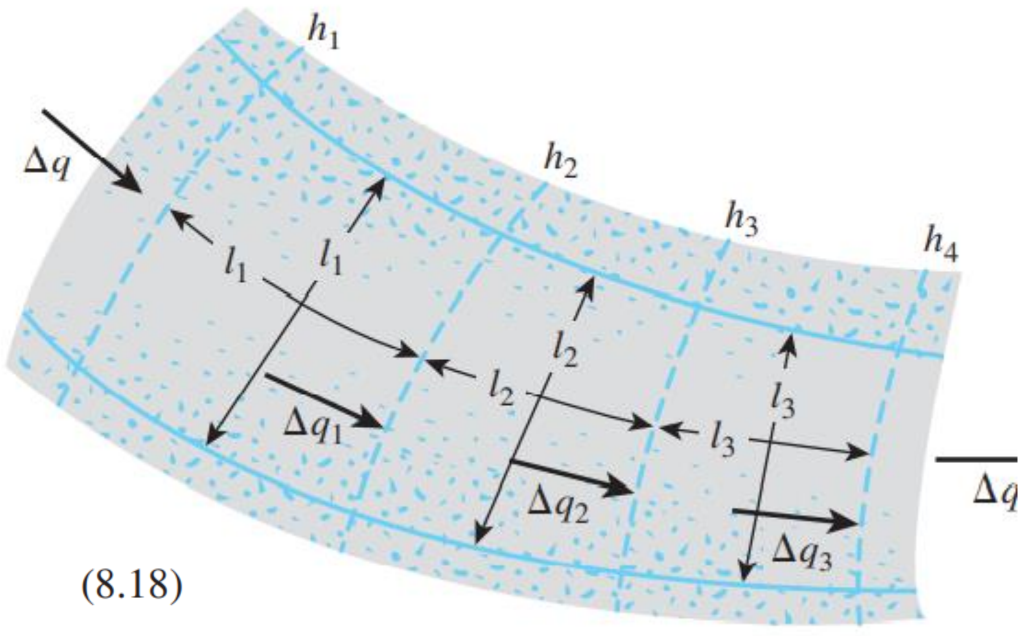


Figure 8.5 Seepage through a flow channel with square elements

7.3 Seepage Calculation from a Flow Net

- Eq.(8.18) shows that, if the flow elements are drawn as approximate squares, the drop in the piezometric level between any two adjacent equipotential lines is the same. $\Longrightarrow$ potential drop

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \frac{H}{N_d}$$

$$\Delta q = k\frac{H}{N_d}$$

H = head difference bet/ the up -and down-stream sides,

Nd = number of potential drops

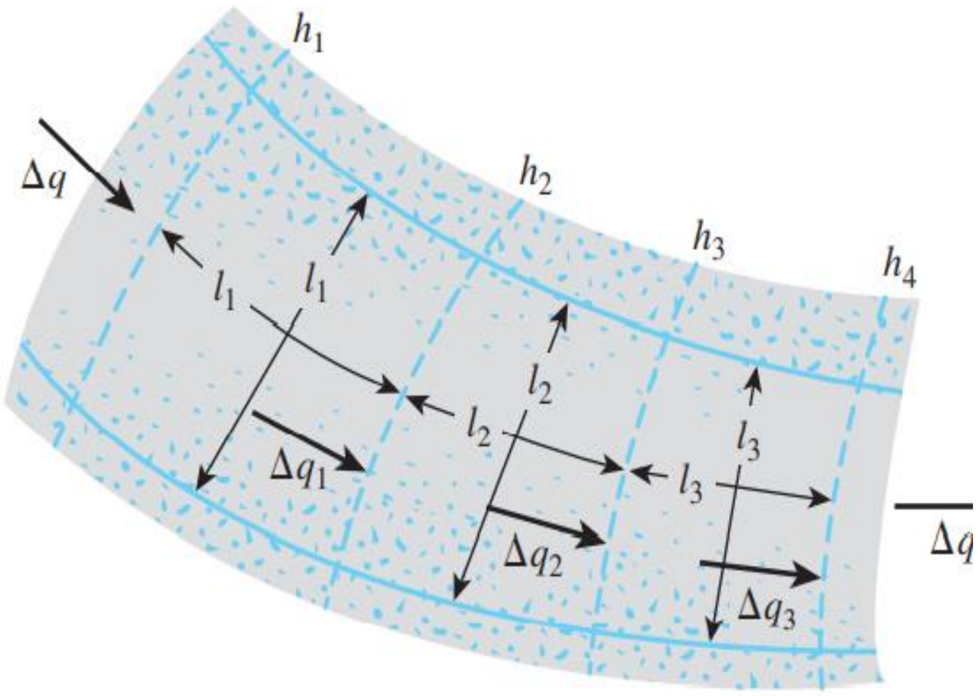


Figure 8.5 Seepage through a flow channel with square elements

7.3 Seepage Calculation from a Flow Net

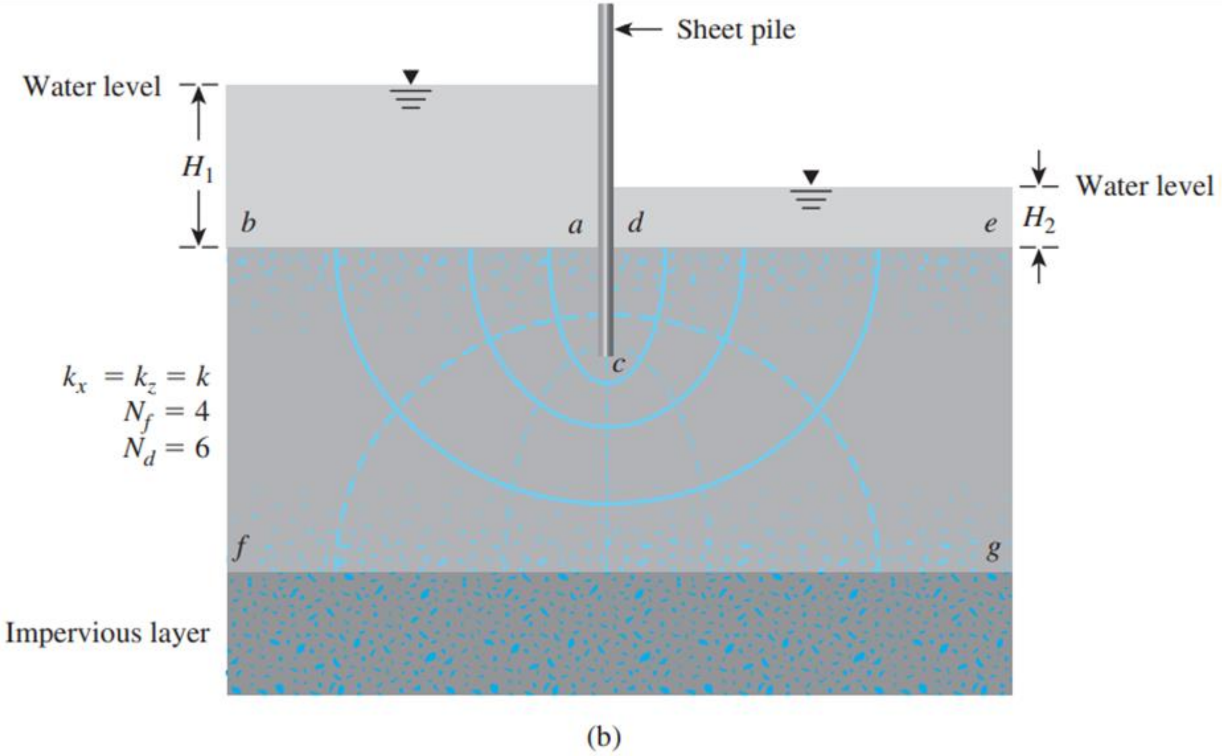
- In the Figure, for any flow channel,

$H = H_1 - H_2$ and $N_d = 6$

- If the number of flow channels in a flow net is equal to N_f

$$q = k\frac{HN_f}{N_d}$$

- Although convenient, it is not always necessary to draw square elements for a flow net.
- It is also possible to draw a rectangular mesh for a flow channel as shown in Figure



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7.3 Seepage Calculation from a Flow Net

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)b_1 = k\left(\frac{h_2 - h_3}{l_2}\right)b_2 = k\left(\frac{h_3 - h_4}{l_3}\right)b_3 = \cdots$$

If $b_1/l_1 = b_2/l_2 = b_3/l_3 = \cdots = n$

$$\Delta q = kh\frac{n}{N_d}$$

$$q = kH(\frac{N_f}{N_d})n$$

$$\Delta q_1 + \Delta q_2 = \frac{k}{N_d}H + \frac{k}{N_d}H = \frac{2kH}{N_d}$$

$$\Delta q_3 = \frac{k}{N_d}H(0.38)$$

The total rate of seepage can be given as

$$q = \Delta q_1 + \Delta q_2 + \Delta q_3 = 2.38\frac{kH}{N_d}$$

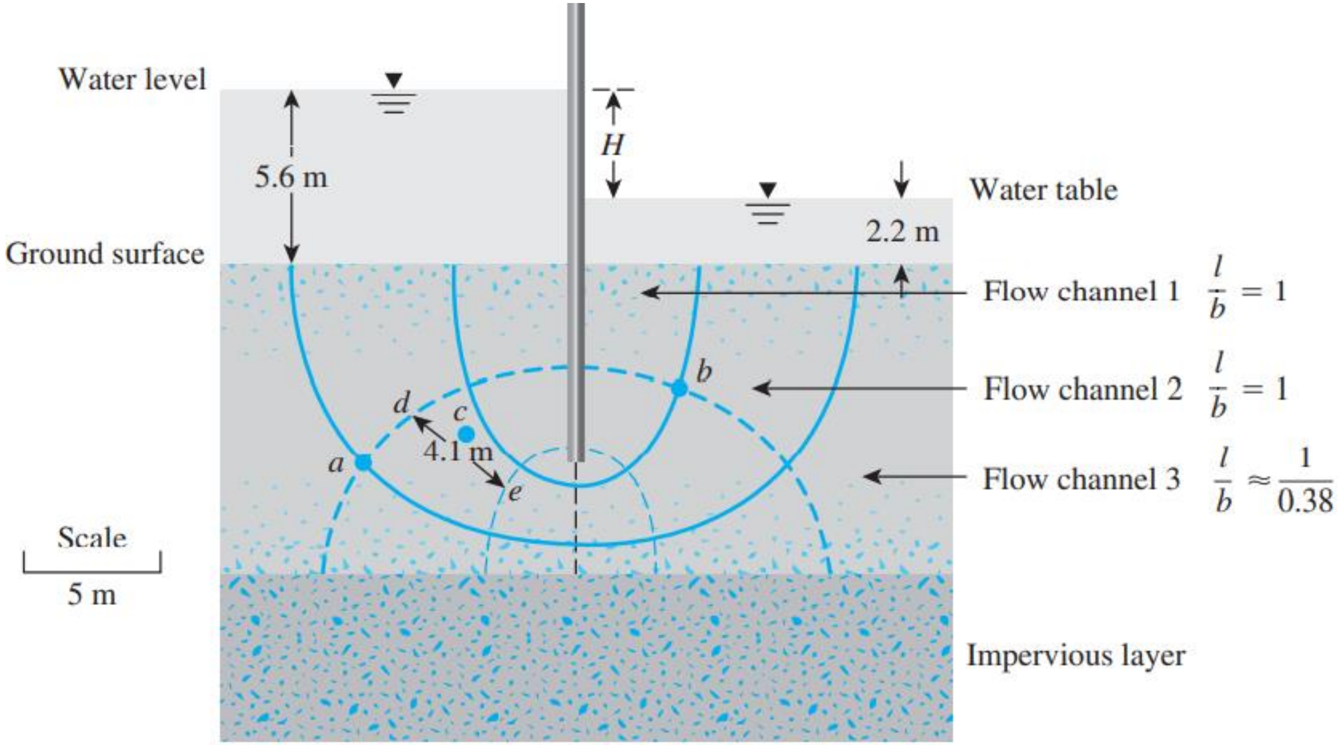
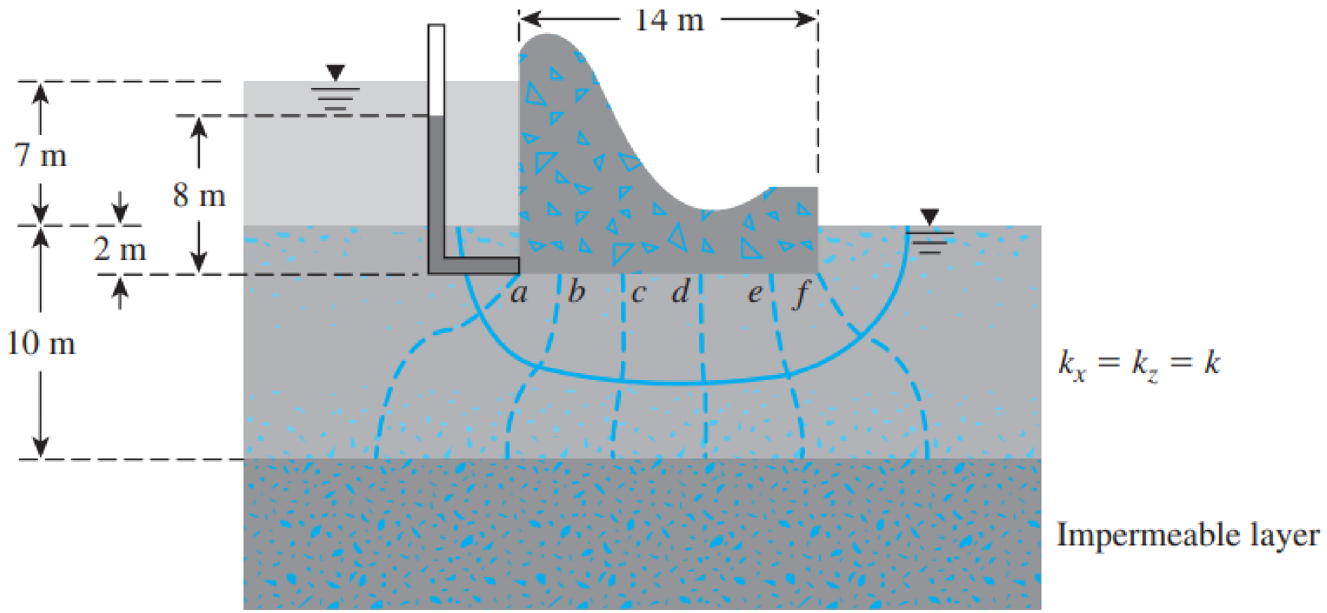


Figure 8.7 Flow net for seepage around a single row of sheet piles

7.4 Uplift Pressure under Hydraulic Structures

- $N_d = 7$, $H = 7$
- The loss of head for each potential drop;

$$\frac{H}{7} = \frac{7}{7} = 1\ m$$



(a)

7.4 Uplift Pressure under Hydraulic Structures

The uplift pressure at

$$a = (left\ corner\ of\ the\ base)$$
$$= (pressure\ head\ at\ a) \times \gamma_w$$
$$= [(7+2)-1]\gamma_w = 8\gamma_w$$

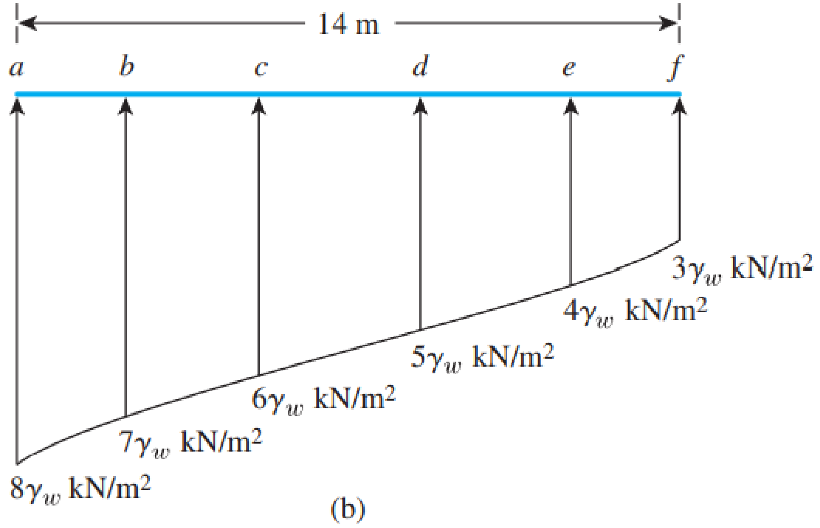
Similarly, the uplift pressure at

$$b = [9-(2)(1)]\gamma_w = 7\gamma_w$$

And at

$$f = [9-(6)(1)]\gamma_w = 3\gamma_w$$

The uplift force per unit length can be calculated by finding the area of the pressure diagram



ıure 8.12 (a) A weir; (b) uplift force under a hydraulic structure

7.5 Seepage through an Earth Dam on an Impervious Base

- homogeneous k
- The free surface of the water passing through the dam is given by abcd.

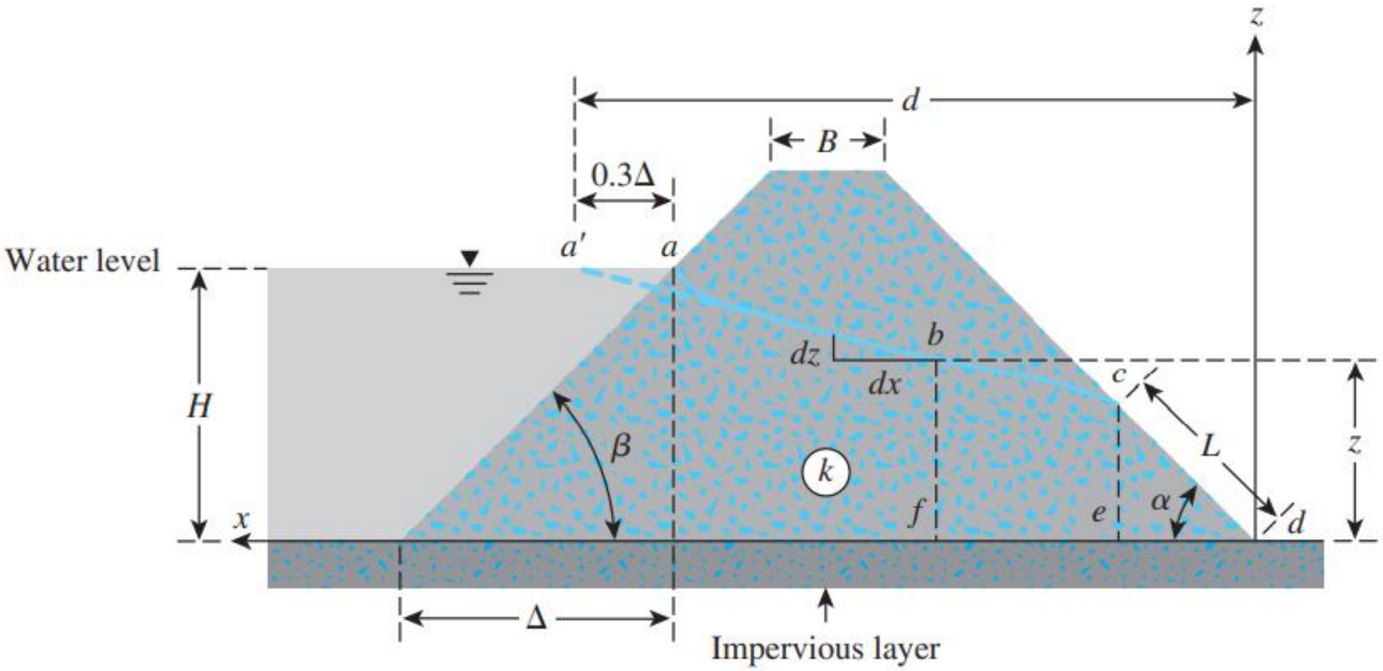


Figure 8.13 Flow through an earth dam constructed over an impervious base

7.5 Seepage through an Earth Dam on an Impervious Base

- Assumption
  - $a'\ bc$ is parabolic.
  - the slope of the free surface : equal to the hydraulic gradient.
  - This hydraulic gradient is constant with depth (Dupuit, 1863), so $i \cong \frac{dz}{dx}$
- Considering the triangle cde, we can give the rate of seepage per unit length of the dam.

$q = k(tana)(L\ sina) = kLtana\ sina$ (8.30)

the rate of seepage (per unit length of the dam) through the section bf

$q = kiA = k\left(\frac{dz}{dx}\right)(z \times 1) = kz\frac{dz}{dx}$ (8.31)

7.5 Seepage through an Earth Dam on an Impervious Base

For continuous flow

- $q_{Eq.(8.30)} = q_{Eq.(8.31)}$

Or

$$kz\frac{dz}{dx} = kLtana\ sina$$

Or

$$\int_{z=L\sin\alpha}^{z=H} kz\,dz = \int_{x=L\cos\alpha}^{x=d} (kL\tan\alpha\sin\alpha)\,dx$$

$$\tfrac{1}{2}(H^2 - L^2\sin^2\alpha) = L\tan\alpha\sin\alpha(d - L\cos\alpha)$$

$$\frac{H^2}{2} - \frac{L^2\sin^2\alpha}{2} = Ld\left(\frac{\sin^2\alpha}{\cos\alpha}\right) - L^2\sin^2\alpha$$

$$\frac{H^2\cos\alpha}{2\sin^2\alpha} - \frac{L^2\cos\alpha}{2} = Ld - L^2\cos\alpha$$

7.5 Seepage through an Earth Dam on an Impervious Base

Or

$$L^2 \cos\alpha - 2Ld + \frac{L^2 \cos\alpha}{sin^2\alpha} = 0$$

So,

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{cos^2\alpha} - \frac{H^2}{sin^2\alpha}} \qquad (8.32)$$

7.5 Seepage through an Earth Dam on an Impervious Base

- a step-by-step procedure to obtain the seepage rate q
  1. Obtain α.
  2. Calculate Δ (Figure 8.13) and then 0.3Δ.
  3. Calculate d.
  4. With known values of α and d, calculate L from Eq.(8.32)
  5. With known values of L, calculate q from Eq.(8.30)