INS 3121
SOIL MECHANICS

Stress in a Soil Mass

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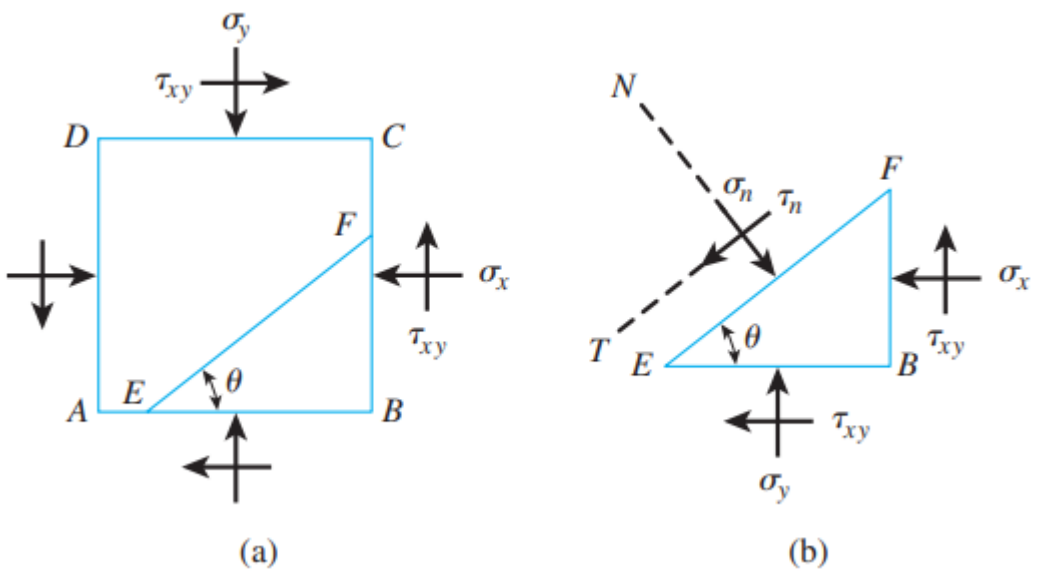
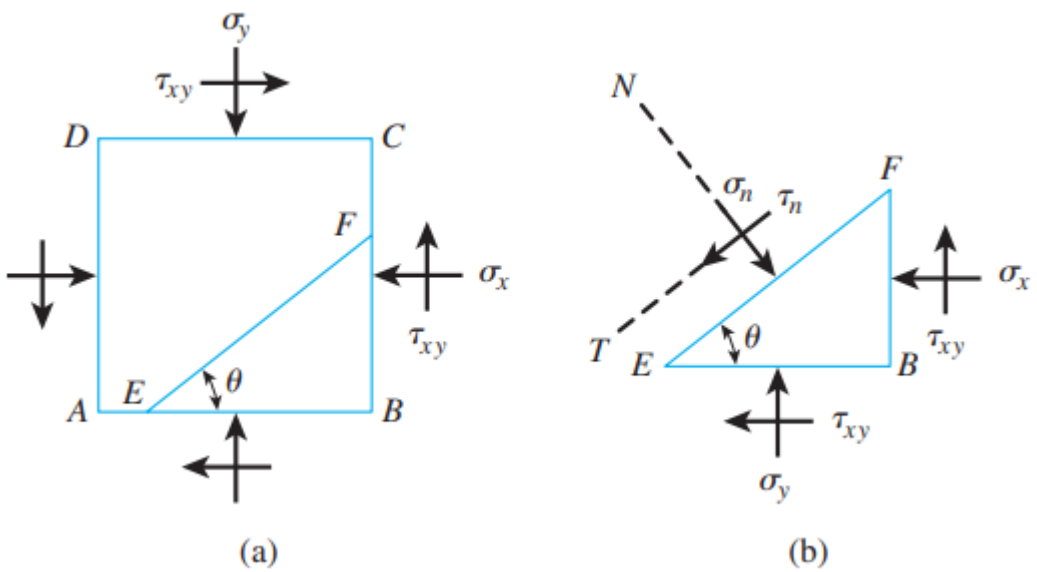
9.1 Normal and Shear Stresses on a Plane

- Construction of a foundation – net stress increase
- the principles of estimation of vertical stress increases in soil caused by various types of loading, based on the theory of elasticity
- Normal stress - σ_n
- Shear stress - τ_n
- $\overline{EB} = \overline{EF} cos\theta$
- $\overline{FB} = \overline{EF} sin\theta$



9.1 Normal and Shear Stresses on a Plane

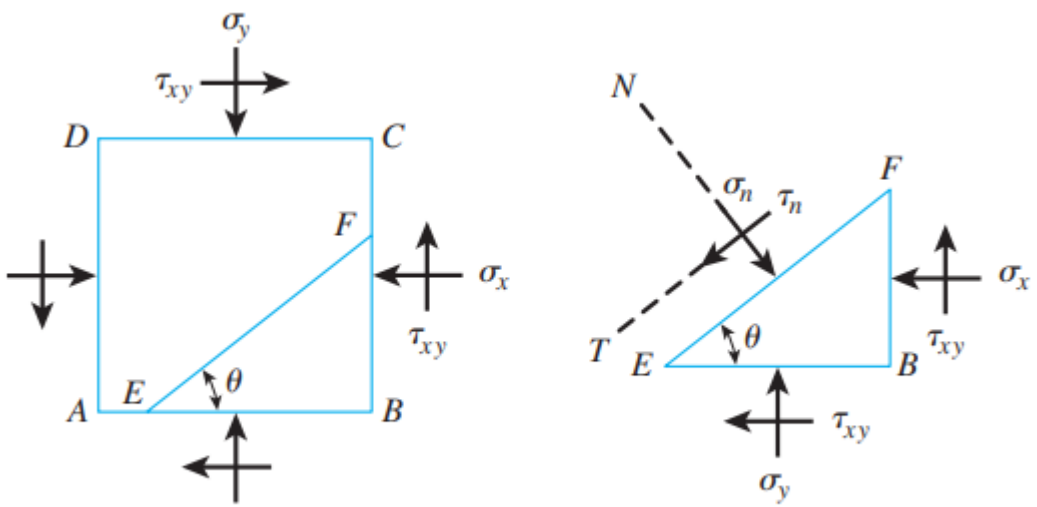
- Normal stress - σ_n
- Shear stress - τ_n
- $\overline{EB} = \overline{EF} cos\theta$
- $\overline{FB} = \overline{EF} sin\theta$
- $\boxed{\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2} cos2\theta + \tau_{xy} sin2\theta}$



- $tan2\theta = \frac{2\tau_{xy}}{\sigma_y - \sigma_x}$ for $\boxed{\tau_n = 0}$

9.1 Normal and Shear Stresses on a Plane

- **Major principal stress:**
- $\sigma_n = \sigma_1 = \frac{(\sigma_y+\sigma_x)}{2} + \sqrt{\left[\frac{\sigma_y-\sigma_x}{2}\right]^2 + \tau_{xy}^2}$
- **Minor principal stress:**
- $\sigma_n = \sigma_3 = \frac{(\sigma_y+\sigma_x)}{2} - \sqrt{\left[\frac{\sigma_y-\sigma_x}{2}\right]^2 + \tau_{xy}^2}$

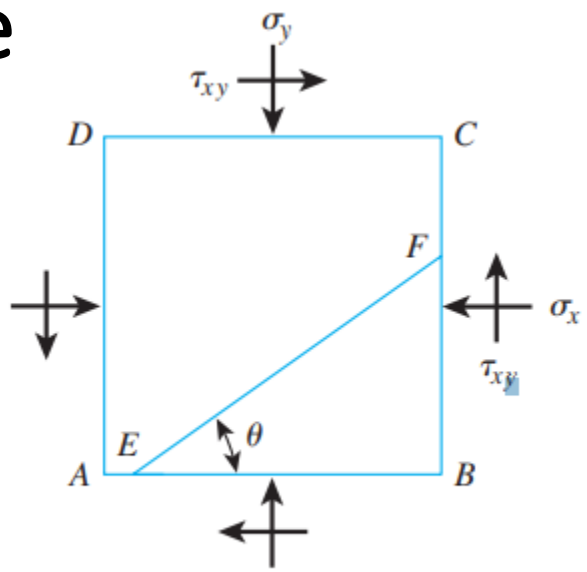


(a) (b)

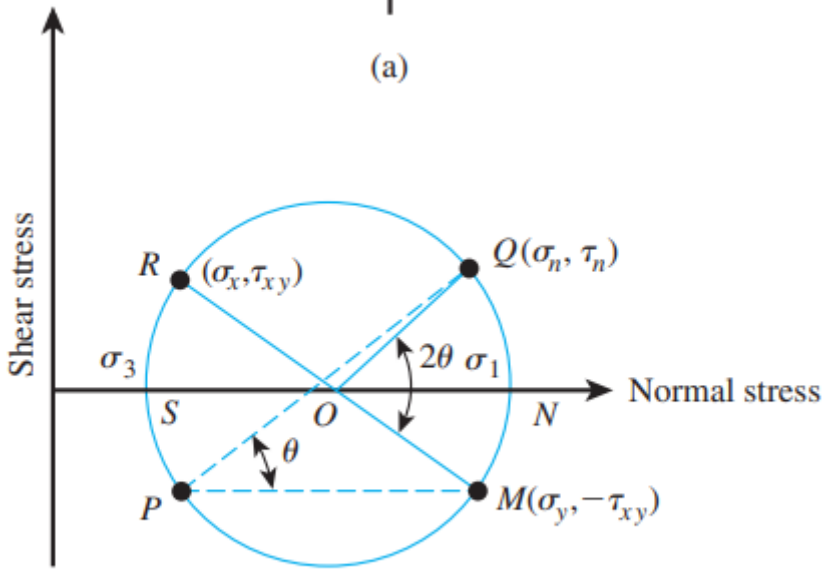
9.1 Normal and Shear Stresses on a Plane

- **the pole method, or the method of origin of planes**

1. Draw a line from a known point on the Mohr's circle parallel to the plane on which the state of stress acts.
2. The point of intersection of this line with the Mohr's circle is called the pole
3. If we need to find the stresses on a plane EF, we draw a line from the pole parallel to EF
4. The point of intersection of this line with the Mohr's circle is Q. The coordinates of Q give the stresses on the plane EF.



(a)



(b)

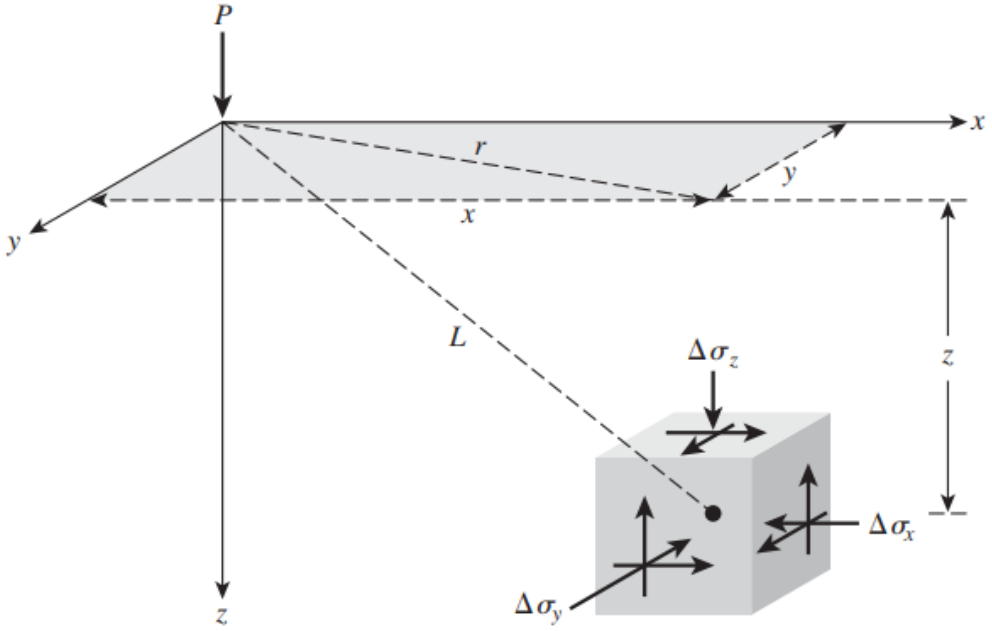
9.2 Stresses Caused by a Point Load

- Boussinesq (1883)
- Assumption : a homogeneous, elastic, and isotropic medium

$$\Delta\sigma_x = \frac{P}{2\pi}\left\{\frac{3x^2z}{L^5} - (1-2\mu)\left[\frac{x^2-y^2}{Lr^2(L+z)} + \frac{y^2z}{L^3r^2}\right]\right\}$$

$$\Delta\sigma_y = \frac{P}{2\pi}\left\{\frac{2y^2z}{L^5} - (1-2\mu)\left[\frac{y^2-x^2}{Lr^2(L+z)} + \frac{x^2z}{L^3r^2}\right]\right\}$$

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$



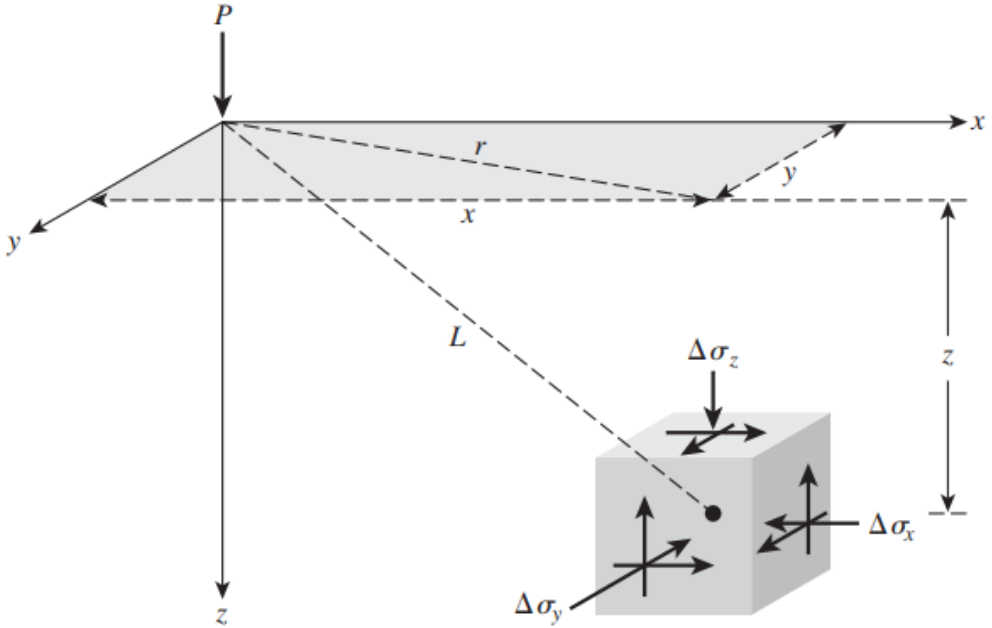
9.2 Stresses Caused by a Point Load

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2 + z^2)^{5/2}}$$

where $r = \sqrt{x^2 + y^2}$

$$L = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

μ = Poisson's ratio



$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2 + 1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

$$I_1 = \frac{3}{2\pi}\frac{1}{[(r/z)^2 + 1]^{5/2}}$$

9.2 Stresses Caused by a Point Load

Table 10.1 Variation of I_1 for Various Values of r/z [Eq. (10.14)]

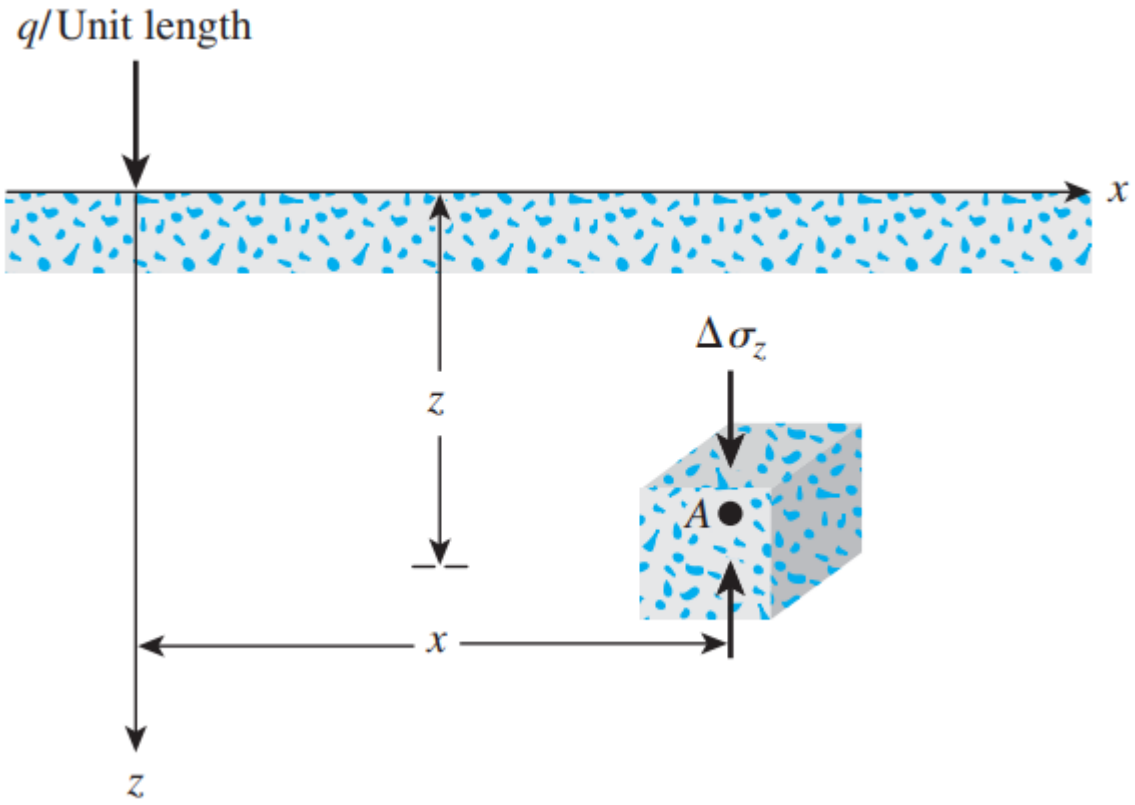
r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

9.3 Vertical Stress Caused by a Vertical Line Load

$$\Delta\sigma_z = \frac{2qz^3}{\pi(x^2 + z^2)^2}$$

$$\Delta\sigma_z = \frac{2q}{\pi z[(x/z)^2 + 1]^2}$$

$$\frac{\Delta\sigma_z}{(q/z)} = \frac{2}{\pi[(x/z)^2 + 1]^2}$$



9.3 Vertical Stress Caused by a Vertical Line Load

Table 10.2 Variation of $\Delta\sigma_z/(q/z)$ with x/z [Eq. (10.16)]

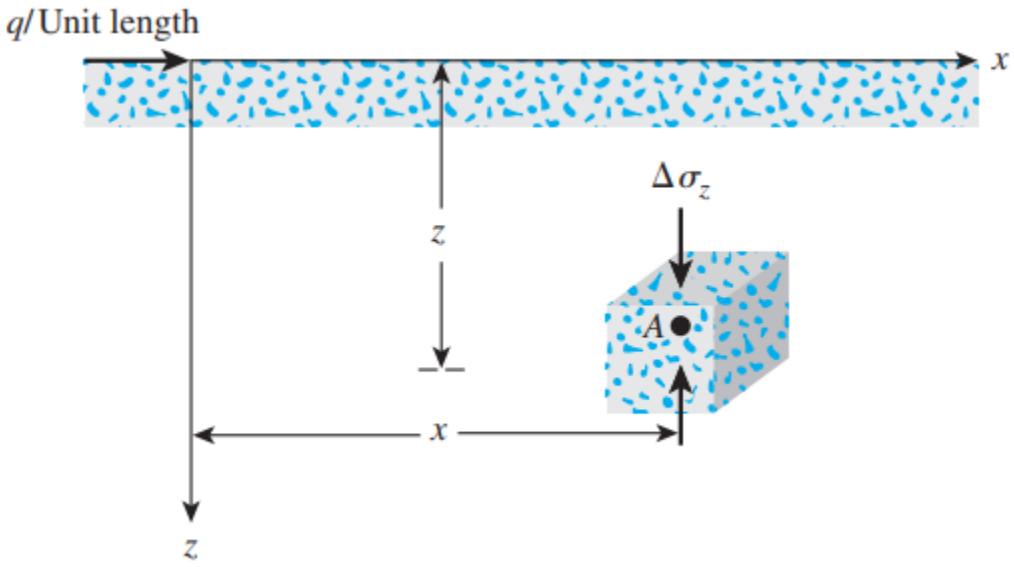
x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0.637	1.3	0.088
0.1	0.624	1.4	0.073
0.2	0.589	1.5	0.060
0.3	0.536	1.6	0.050
0.4	0.473	1.7	0.042
0.5	0.407	1.8	0.035
0.6	0.344	1.9	0.030
0.7	0.287	2.0	0.025
0.8	0.237	2.2	0.019
0.9	0.194	2.4	0.014
1.0	0.159	2.6	0.011
1.1	0.130	2.8	0.008
1.2	0.107	3.0	0.006

9.3 Vertical Stress Caused by a Horizontal Line Load

$$\Delta\sigma_z = \frac{2q}{\pi}\frac{xz^2}{(x^2+z^2)^2}$$

Table 10.3 Variation of $\Delta\sigma_z/(q/z)$ with x/z

x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0	0.7	0.201
0.1	0.062	0.8	0.189
0.2	0.118	0.9	0.175
0.3	0.161	1.0	0.159
0.4	0.189	1.5	0.090
0.5	0.204	2.0	0.051
0.6	0.207	3.0	0.019

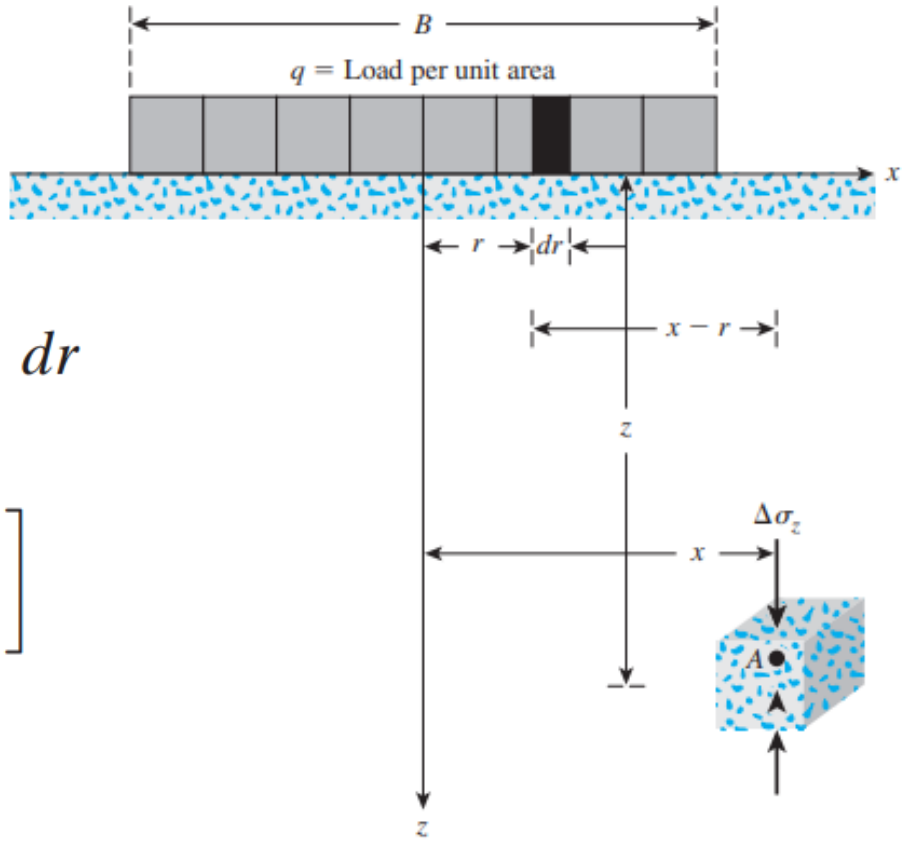


9.4 Vertical Stress Caused by a Vertical Strip Load (Finite Width and Infinite Length)

$$d\sigma_z = \frac{2(q\,dr)z^3}{\pi[(x-r)^2+z^2]^2}$$

$$\Delta\sigma_z = \int d\sigma_z = \int_{-B/2}^{+B/2}\left(\frac{2q}{\pi}\right)\left\{\frac{z^3}{[(x-r)^2+z^2]^2}\right\}dr$$

$$= \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2-z^2-(B^2/4)]}{[x^2+z^2-(B^2/4)]^2+B^2z^2}\right\}$$



9.4 Vertical Stress Due to Embankment Loading

$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right]$$

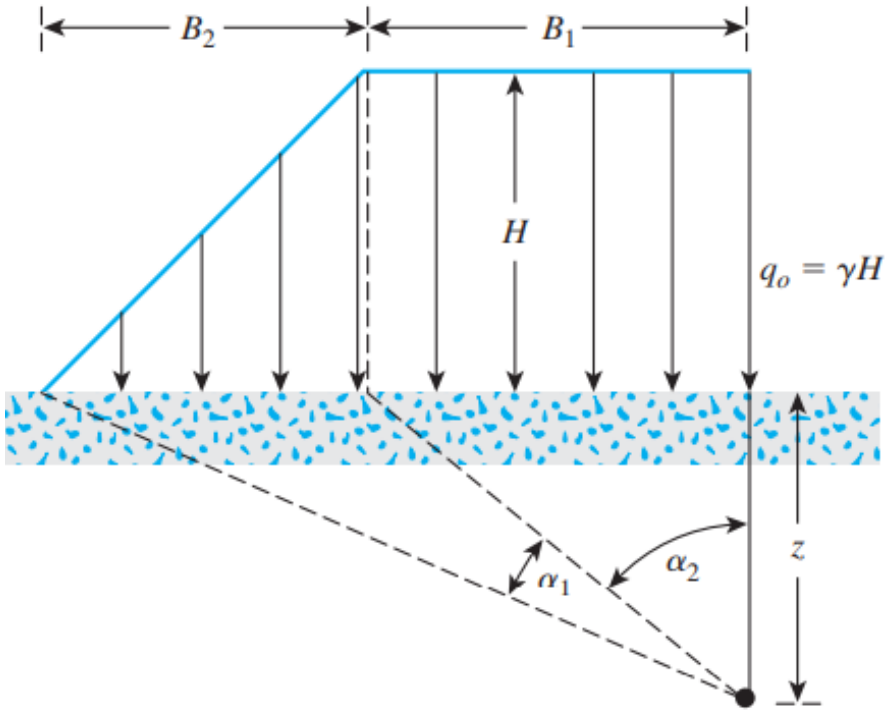
where $q_o = \gamma H$

γ = unit weight of the embankment soil

H = height of the embankment

$$\alpha_1\,(\text{radians}) = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right)$$

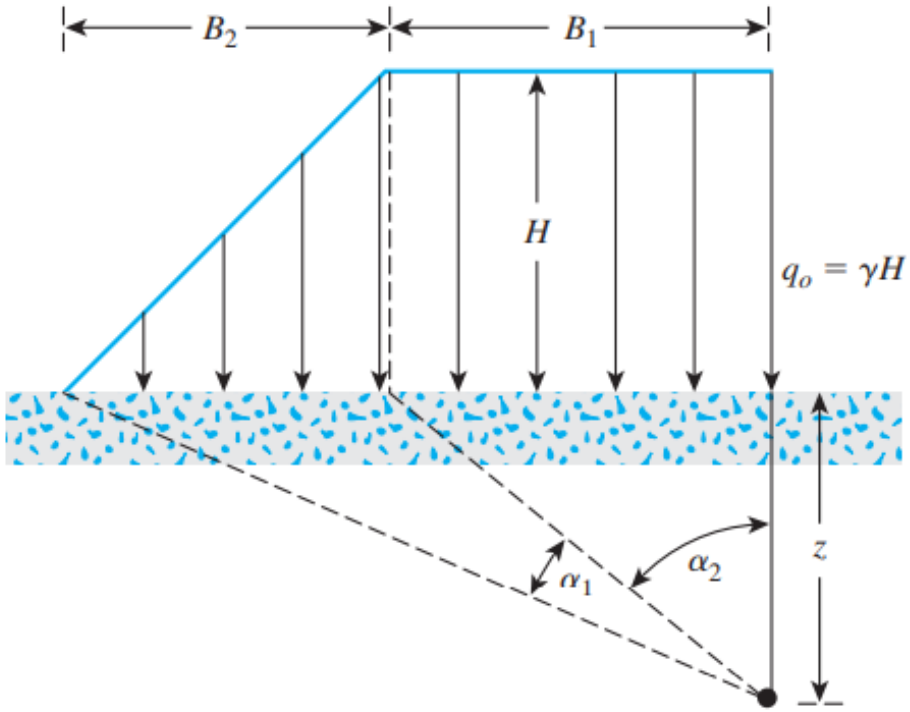
$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right)$$



9.4 Vertical Stress Due to Embankment Loading

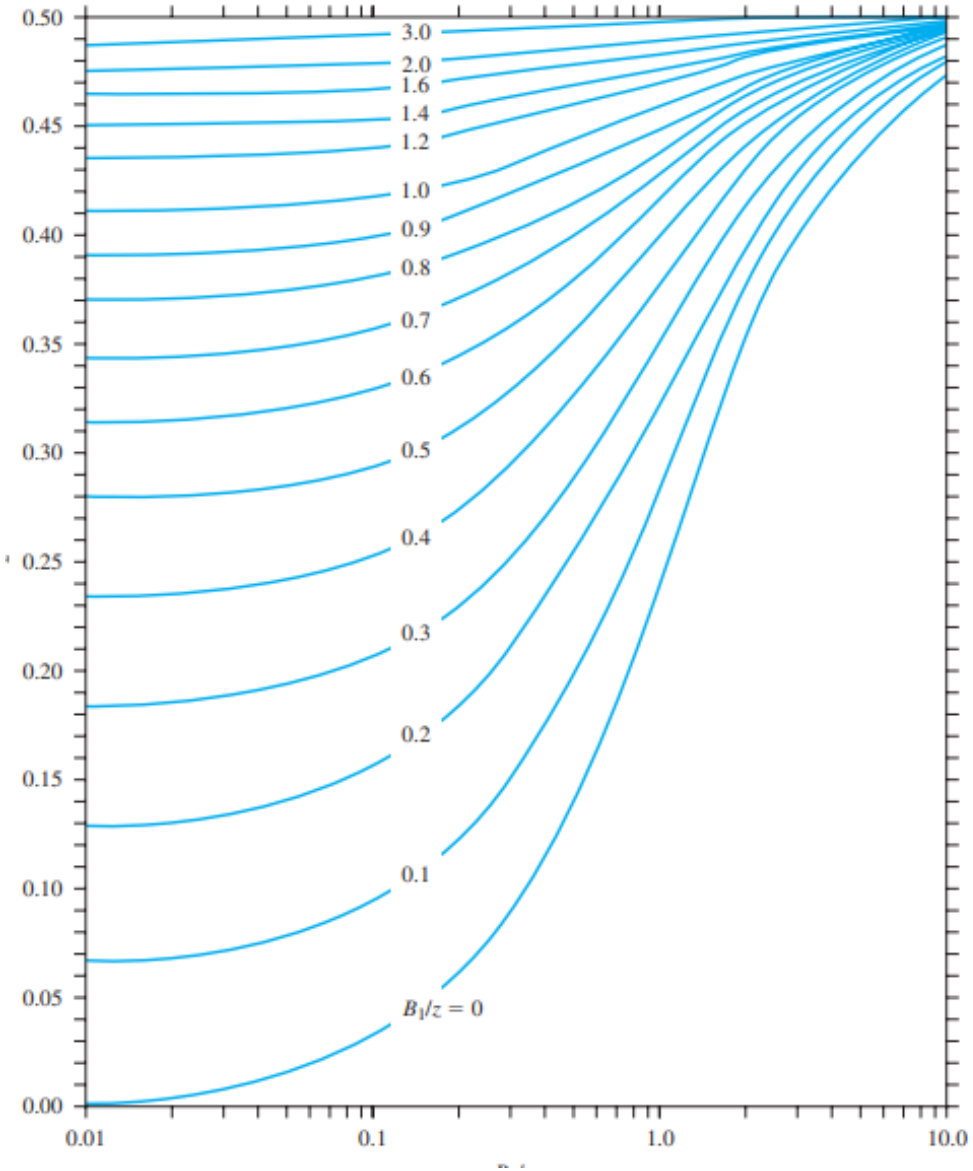
- $\Delta\sigma_z = q_0 I_2$

Where $I_2 =$
$a\ function\ of\ \frac{B_1}{z}\ and\ \frac{B_2}{z}$.



9.4 Vertical Stress Due to Embankment Loading

- The variation of I_2 with $\frac{B_1}{z}$ and $\frac{B_2}{z}$ is shown in the figure (Osterberg, 1957)

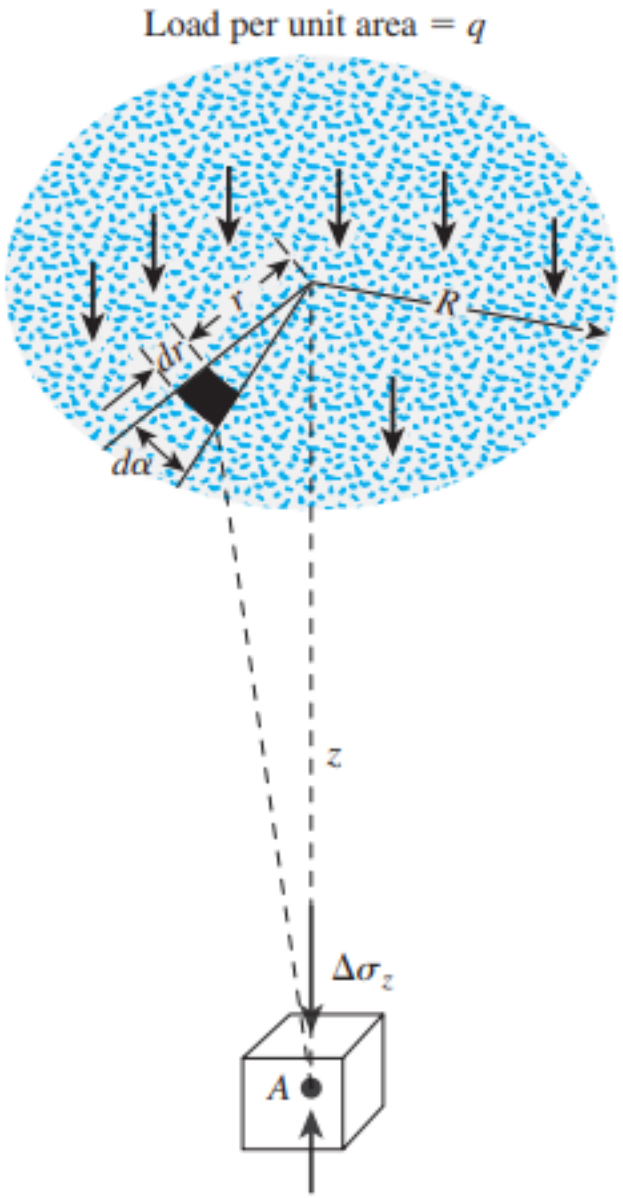


9.5 Vertical Stress Below the Center of a Uniformity Loaded Circular Area

$$d\sigma_z = \frac{3(qr\,dr\,d\alpha)}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$\Delta\sigma_z = \int d\sigma_z = \int_{\alpha=0}^{\alpha=2\pi}\int_{r=0}^{r=R}\frac{3q}{2\pi}\frac{z^3 r}{(r^2+z^2)^{5/2}}\,dr\,d\alpha$$

$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2+1]^{3/2}}\right\}$$



9.5 Vertical Stress Below the Center of a Uniformity Loaded Circular Area

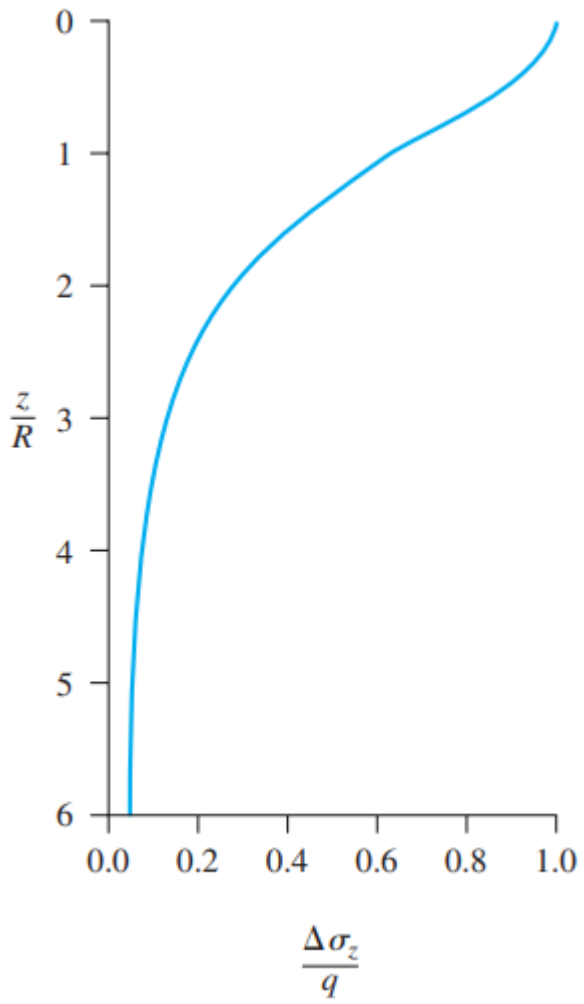
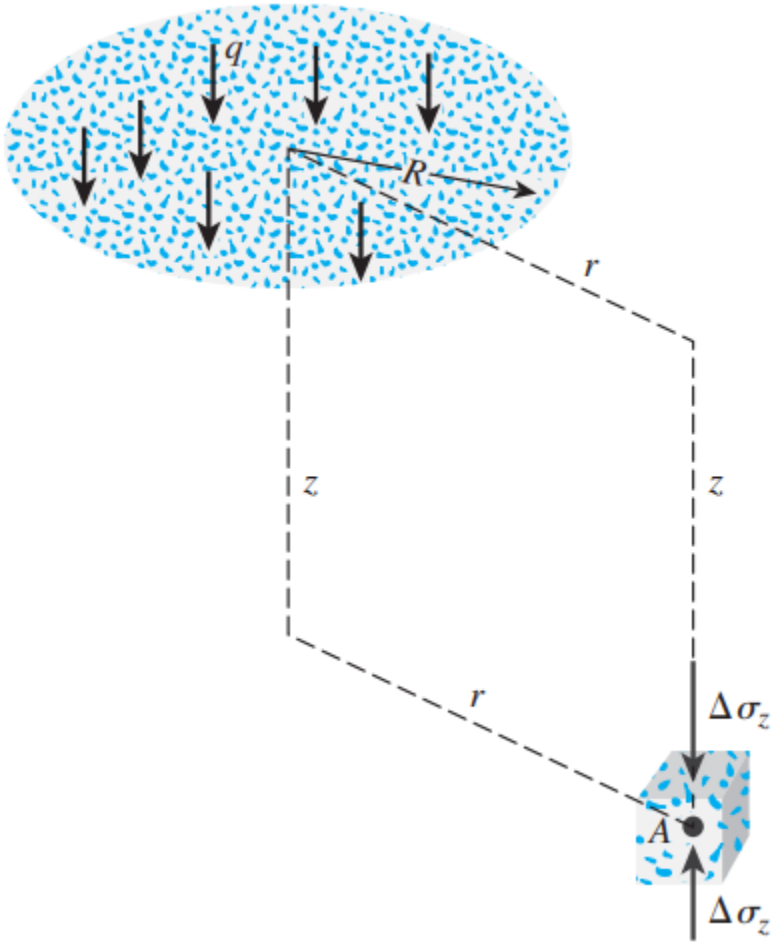


Table 10.5 Variation of $\Delta\sigma_z/q$ with z/R [Eq. (10.25)]

z/R	$\Delta\sigma_z/q$	z/R	$\Delta\sigma_z/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

9.6 Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

- $\Delta\sigma_z = q(A' + B')$



9.6 Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

- $\Delta\sigma_z = q(A' + B')$, where A' and B' are functions of z/R and r/R (Table)

Table 10.6 Variation of A' with z/R and r/R*

	r/R								
z/R	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.5**	**2**
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

*Source: From Ahlvin, R. G., and H. H. Ulery. Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space. In Highway Research Bulletin 342, Highway Research Board, National Research Council, Washington, D.C., 1962, Tables 1 and 2, p. 3. Reproduced with permission of the

9.6 Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

- $\Delta\sigma_z = q(A' + B')$, where A' and B' are functions of z/R and r/R (Table)

Table 10.7 Variation of B' with z/R and r/R*

	r/R								
z/R	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1**	**1.2**	**1.5**	**2**
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

*Source: From Ahlvin, R. G., and H. H. Ulery. Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space. In Highway Research Bulletin 342, Highway Research Board, National Research Council, Washington, D.C., 1962, Tables 1 and 2, p. 3. Reproduced with permission of the

9.6 Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

- $\Delta\sigma_z = q(A' + B')$, where A' and B' are functions of z/R and r/R (Table)

Table 10.6 *(continued)*

3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0
0.00211	0.00084	0.00042						
0.00419	0.00167	0.00083	0.00048	0.00030	0.00020			
0.00622	0.00250							
0.01013	0.00407	0.00209	0.00118	0.00071	0.00053	0.00025	0.00014	0.00009
0.01742	0.00761	0.00393	0.00226	0.00143	0.00097	0.00050	0.00029	0.00018
0.01935	0.00871	0.00459	0.00269	0.00171	0.00115			
0.02142	0.01013	0.00548	0.00325	0.00210	0.00141	0.00073	0.00043	0.00027
0.02221	0.01160	0.00659	0.00399	0.00264	0.00180	0.00094	0.00056	0.00036
0.02143	0.01221	0.00732	0.00463	0.00308	0.00214	0.00115	0.00068	0.00043
0.01980	0.01220	0.00770	0.00505	0.00346	0.00242	0.00132	0.00079	0.00051
0.01592	0.01109	0.00768	0.00536	0.00384	0.00282	0.00160	0.00099	0.00065
0.01249	0.00949	0.00708	0.00527	0.00394	0.00298	0.00179	0.00113	0.00075
0.00983	0.00795	0.00628	0.00492	0.00384	0.00299	0.00188	0.00124	0.00084
0.00784	0.00661	0.00548	0.00445	0.00360	0.00291	0.00193	0.00130	0.00091
0.00635	0.00554	0.00472	0.00398	0.00332	0.00276	0.00189	0.00134	0.00094
0.00520	0.00466	0.00409	0.00353	0.00301	0.00256	0.00184	0.00133	0.00096
0.00438	0.00397	0.00352	0.00326	0.00273	0.00241			

9.6 Vertical Stress at Any Point Below a Uniformly Loaded Circular Area

- $\Delta\sigma_z = q(A' + B')$, where A' and B' are functions of z/R and r/R (Table)

Table 10.7 *(continued)*

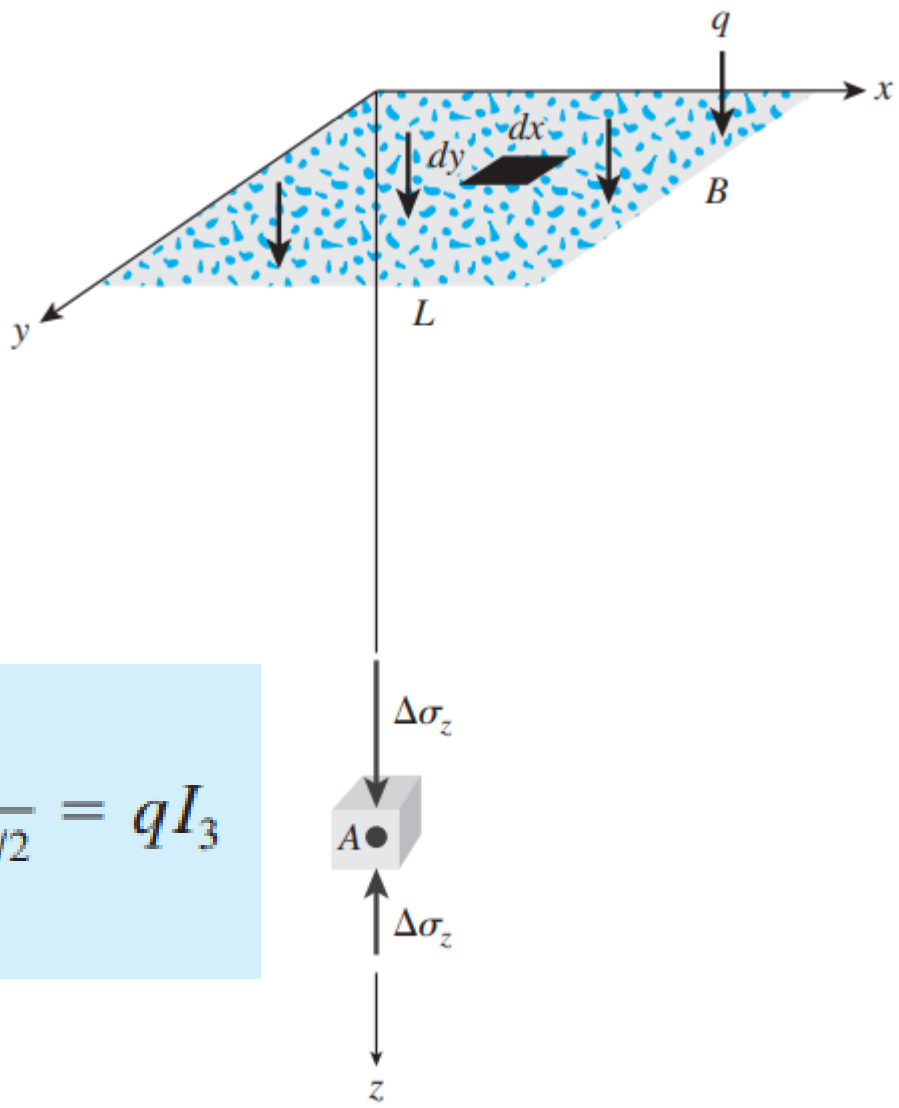
3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0
−0.00210	−0.00084	−0.00042						
−0.00412	−0.00166	−0.00083	−0.00024	−0.00015	−0.00010			
−0.00599	−0.00245							
−0.00991	−0.00388	−0.00199	−0.00116	−0.00073	−0.00049	−0.00025	−0.00014	−0.00009
−0.01115	−0.00608	−0.00344	−0.00210	−0.00135	−0.00092	−0.00048	−0.00028	−0.00018
−0.00995	−0.00632	−0.00378	−0.00236	−0.00156	−0.00107			
−0.00669	−0.00600	−0.00401	−0.00265	−0.00181	−0.00126	−0.00068	−0.00040	−0.00026
0.00028	−0.00410	−0.00371	−0.00278	−0.00202	−0.00148	−0.00084	−0.00050	−0.00033
0.00661	−0.00130	−0.00271	−0.00250	−0.00201	−0.00156	−0.00094	−0.00059	−0.00039
0.01112	0.00157	−0.00134	−0.00192	−0.00179	−0.00151	−0.00099	−0.00065	−0.00046
0.01515	0.00595	0.00155	−0.00029	−0.00094	−0.00109	−0.00094	−0.00068	−0.00050
0.01522	0.00810	0.00371	0.00132	0.00013	−0.00043	−0.00070	−0.00061	−0.00049
0.01380	0.00867	0.00496	0.00254	0.00110	0.00028	−0.00037	−0.00047	−0.00045
0.01204	0.00842	0.00547	0.00332	0.00185	0.00093	−0.00002	−0.00029	−0.00037
0.01034	0.00779	0.00554	0.00372	0.00236	0.00141	0.00035	−0.00008	−0.00025
0.00888	0.00705	0.00533	0.00386	0.00265	0.00178	0.00066	0.00012	−0.00012
0.00764	0.00631	0.00501	0.00382	0.00281	0.00199			

9.7 Vertical Stress Caused by a Rectangularly Loaded Area

$$dq = q\,dx\,dy$$

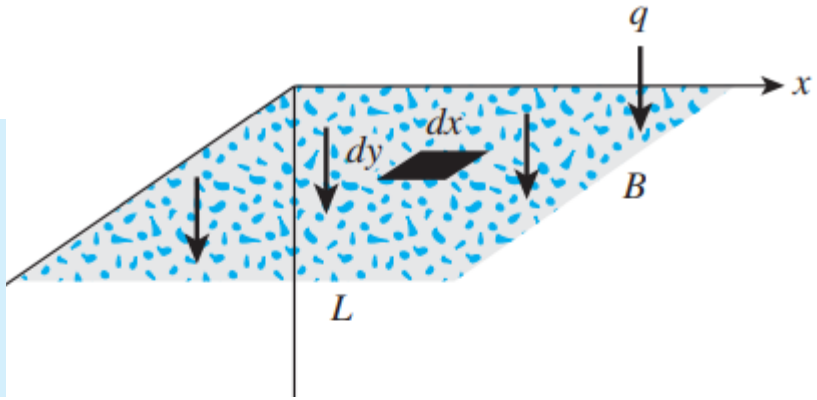
$$d\sigma_z = \frac{3q\,dx\,dy\,z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}}$$

$$\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_3$$



9.7 Vertical Stress Caused by a Rectangularly Loaded Area

$$\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B} \int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_3$$

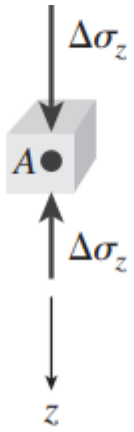


where

$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1}\left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1}\right)\right]$$

$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$



9.7 Vertical Stress Caused by a Rectangularly Loaded Area

$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$

- The variation of I_3 with m and n is shown in Table 10.8 and Figure 10.21.

9.7 Vertical Stress Caused by a Rectangularly Loaded Area

- The variation of I_3 with m and n is shown in Table 10.8 and Figure 10.21.

Table 10.8 Variation of I_3 with m and n [Eq. (10.30)]

										m
n	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

9.7 Vertical Stress Caused by a Rectangularly Loaded Area

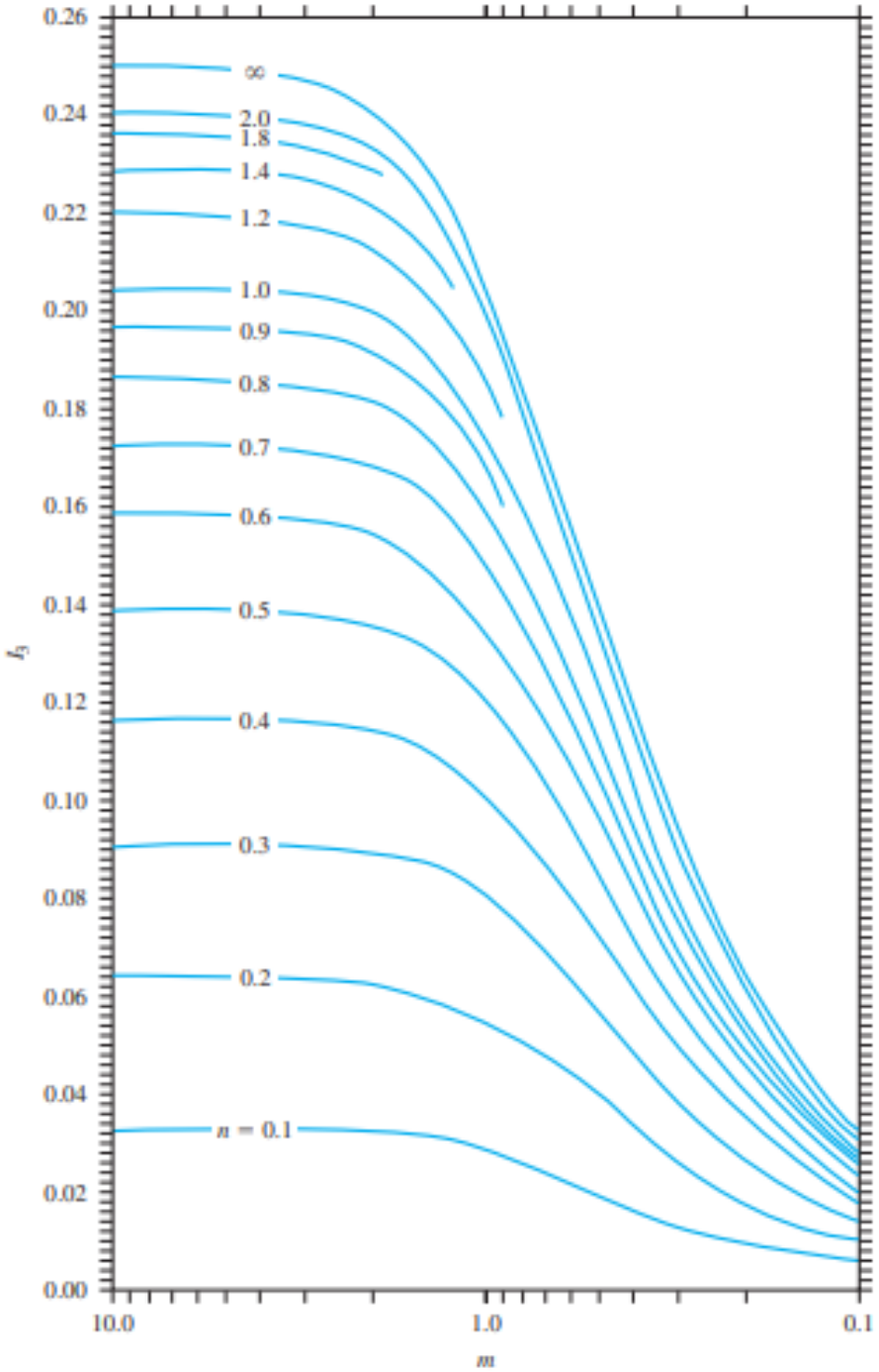
- The variation of I_3 with m and n is shown in Table 10.8 and Figure 10.21.

Table 10.8 *(continued)*

1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0
0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0619	0.0620	0.0620
0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1153	0.1154	0.1154
0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1372	0.1374	0.1374
0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1560	0.1561	0.1562
0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1719	0.1719
0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.1850
0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1954	0.1956	0.1957
0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2260	0.2263	0.2264
0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2320	0.2323	0.2325
0.2103	0.2183	0.2237	0.2274	0.2299	0.2333	0.2350	0.2362	0.2366	0.2367
0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2434	0.2439	0.2441
0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2455	0.2461	0.2463
0.2172	0.2260	0.2320	0.2362	0.2391	0.2434	0.2455	0.2472	0.2479	0.2481
0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2460	0.2479	0.2486	0.2489
0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492

9.7 Vertical Stress Caused by a Rectangularly Loaded Area

- The variation of I_3 with m and n is shown in Table 10.8 and Figure 10.21.



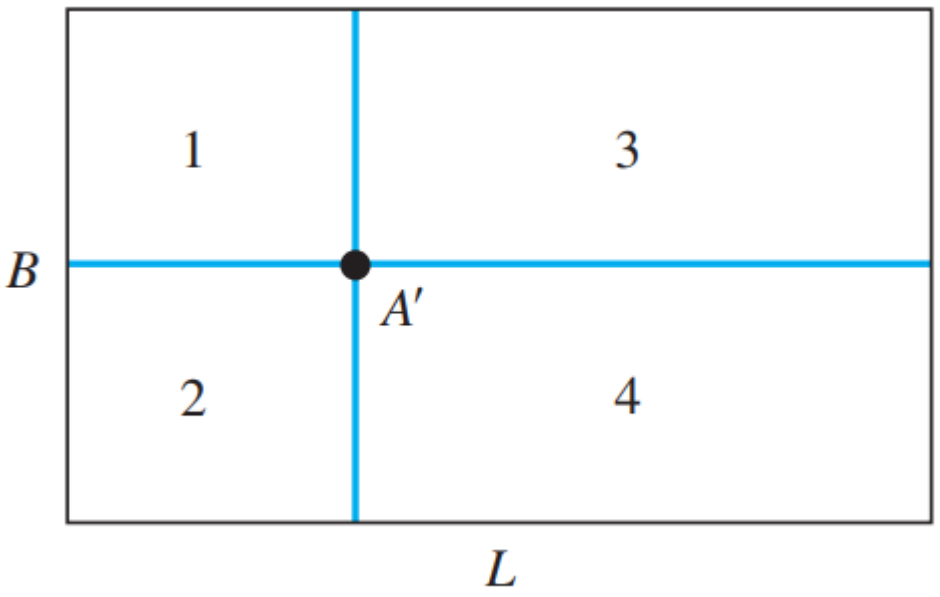
9.7 Vertical Stress Caused by a Rectangularly Loaded Area

- The loaded area can be divided into four rectangles as shown.
- $\Delta\sigma_Z = q\left[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}\right]$

Where

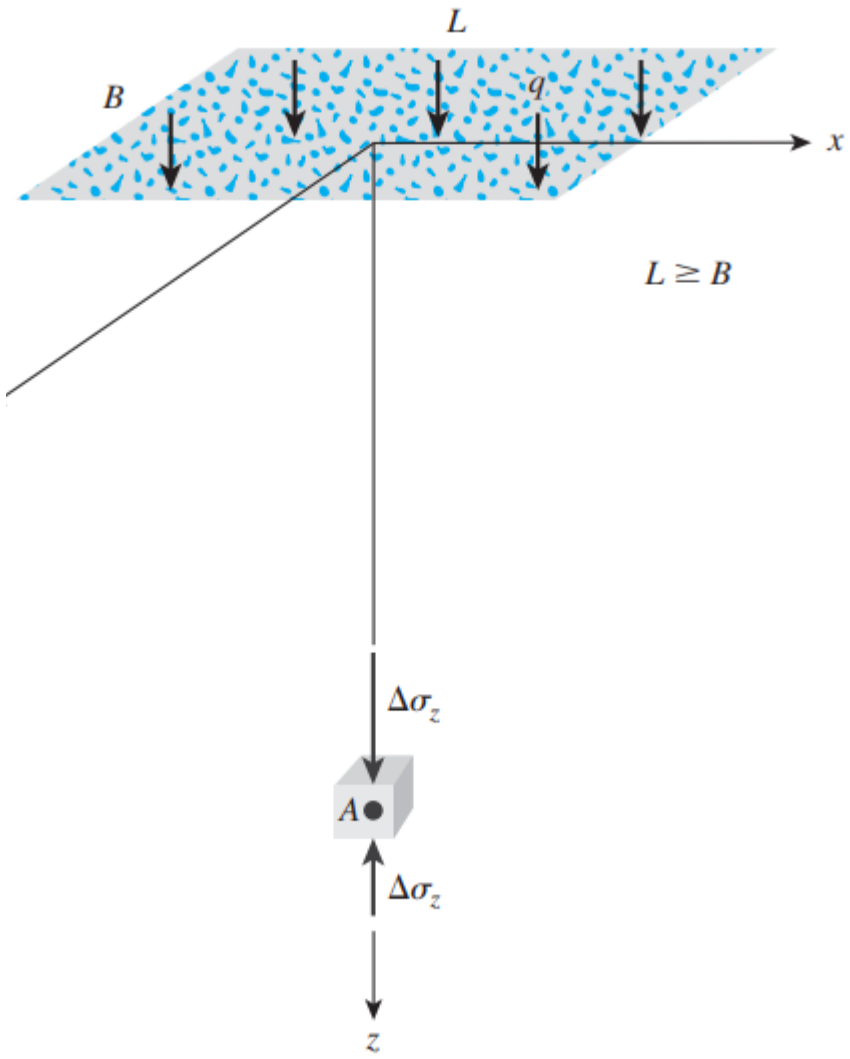
$I_{3(1)} + I_{3(2)} + I_{3(3)}, and\ I_{3(4)} =$
$values\ of\ I_3\ for\ rectangles\ 1,2,3\ and\ 4, respectively$

- $\boxed{\Delta\sigma_z = qI_4}$



9.7 Vertical Stress Caused by a Rectangularly Loaded Area

- $\boxed{\Delta\sigma_z = qI_4}$



where

$$I_4 = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}}\frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right] \quad (10.35)$$

$$m_1 = \frac{L}{B} \quad (10.36)$$

$$n_1 = \frac{z}{b} \quad (10.37)$$

$$b = \frac{B}{2} \quad (10.38)$$

The variation of I_4 with m_1 and n_1 is given in Table 10.9.

9.7 Vertical Stress Caused by a Rectangularly Loaded Area

- $\boxed{\Delta\sigma_z = qI_4}$

where

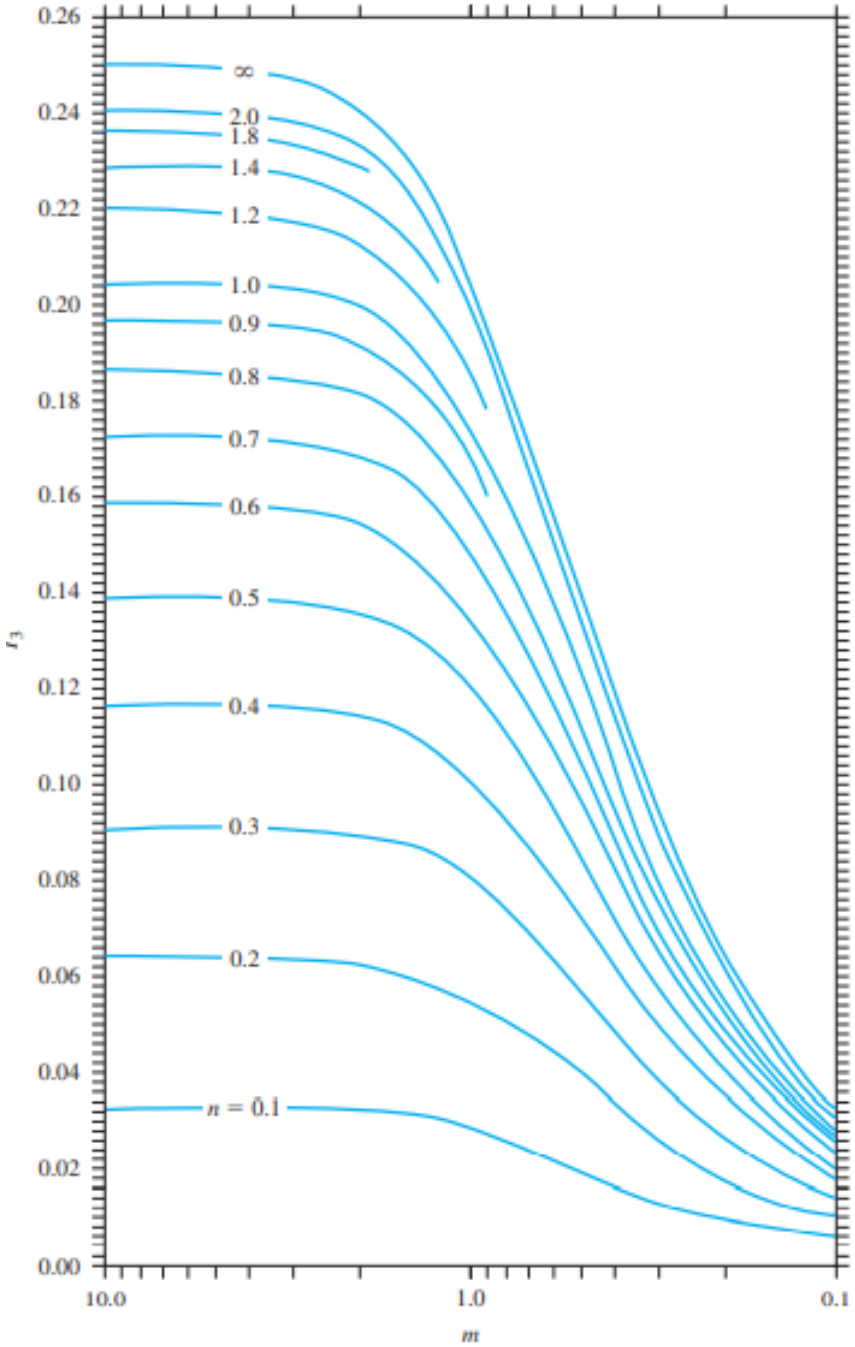
$$I_4 = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1+m_1^2+n_1^2}}\frac{1+m_1^2+2n_1^2}{(1+n_1^2)(m_1^2+n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2+n_1^2}\sqrt{1+n_1^2}}\right] \quad (10.35)$$

$$m_1 = \frac{L}{B} \quad (10.36)$$

$$n_1 = \frac{z}{b} \quad (10.37)$$

$$b = \frac{B}{2} \quad (10.38)$$

The variation of I_4 with m_1 and n_1 is given in Table 10.9.



9.7 Vertical Stress Caused by a Rectangularly Loaded Area

- $\boxed{\Delta\sigma_z = qI_4}$

Table 10.9 Variation of I_4 with m_1 and n_1 [Eq. (10.35)]

	m_1									
n_1	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

9.8 Stress Isobars

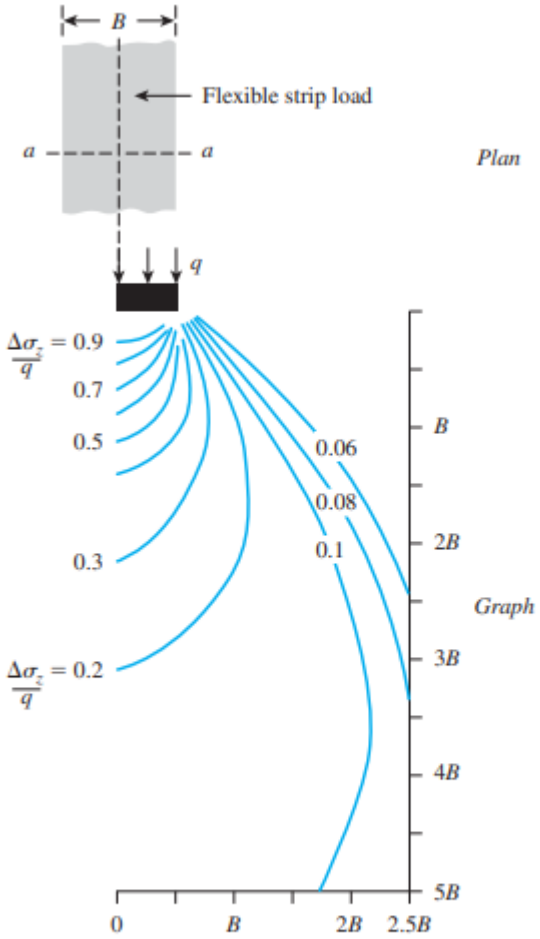


Figure 10.25 Vertical pressure isobars under a flexible strip load (*Note:* Isobars are for line a–a as shown on the plan)

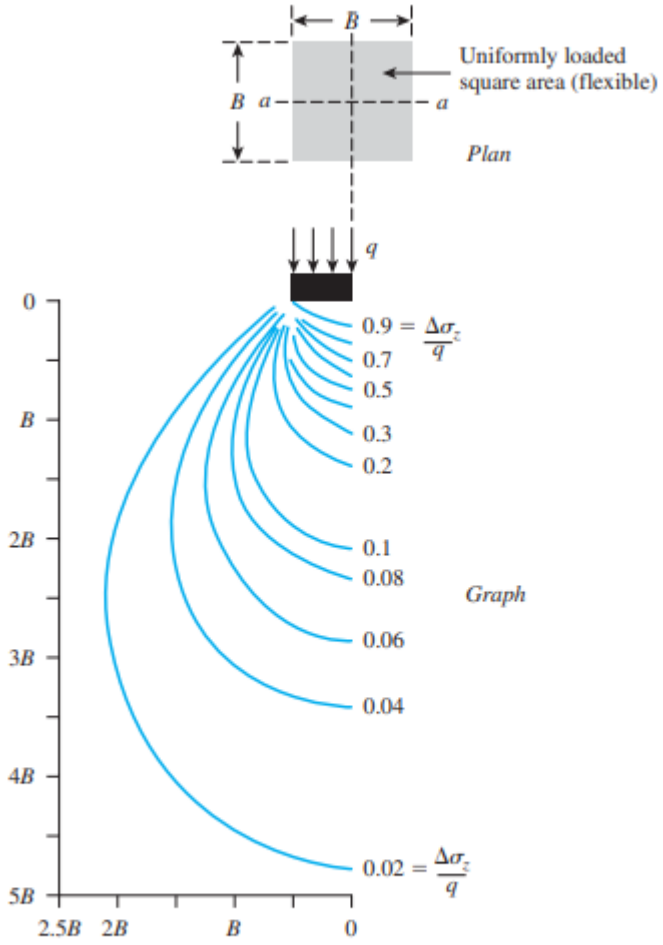


Figure 10.26 Vertical pressure isobars under a uniformly loaded square area (*Note:* Isobars are for line a–a as shown on the plan)

9.9 Influence Chart for Vertical Pressure

$$\frac{R}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3} - 1}$$

Table 10.10 Values of R/z for Various Pressure Ratios [Eq. (10.39)]

$\Delta\sigma_z/q$	R/z	$\Delta\sigma_z/q$	R/z
0	0	0.55	0.8384
0.05	0.1865	0.60	0.9176
0.10	0.2698	0.65	1.0067
0.15	0.3383	0.70	1.1097
0.20	0.4005	0.75	1.2328
0.25	0.4598	0.80	1.3871
0.30	0.5181	0.85	1.5943
0.35	0.5768	0.90	1.9084
0.40	0.6370	0.95	2.5232
0.45	0.6997	1.00	∞
0.50	0.7664		

9.9 Influence Chart for Vertical Pressure

Influence chart

1. Draw concentric circles. The radii of the circles are equal to the R/z values corresponding to Δp/q=0,0.1,0.2,…,1. (Note : For Δp/q=0, R/z=0. and for Δp/q=1, R/z=∞, so there are nine circles shown.)
2. The unit length for plotting the circles is $\overline{AB}$
3. The circles are divided by several equally spaced radial lines.
4. The influence value of the chart is given by 1/N, where N is equal to the number of elements in the chart. In Figure 10.27 there are 200 element; hence, the influence value is 0.005.

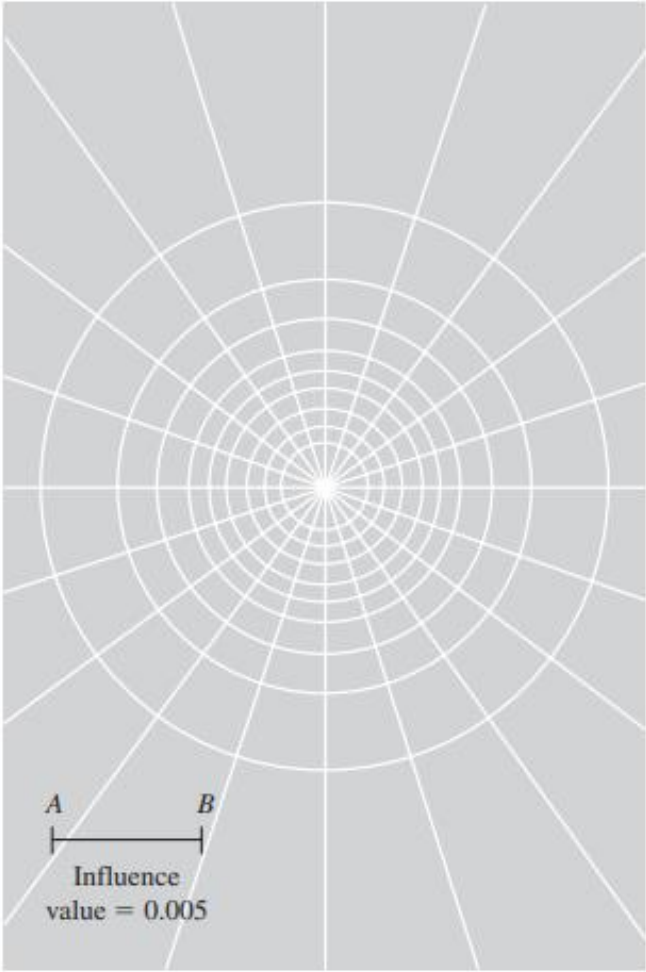


Figure 10.27
Influence chart for vertical pressure based on Boussinesq's theory (*Bulletin No. 338. Influence Charts for Computation of Stresses in Elastic. Foundations, by Nathan M. Newmark. University of Illinois, 1942.*)

9.9 Influence Chart for Vertical Pressure

- The procedure for obtaining vertical pressure at any point below a loaded are is as follows :
  1. Determine the depth z below the uniformly loaded area at which the stress increase is required.
  2. Plot the plan of the loaded area with a scale of z equal to the unit length of the chart ($\overline{AB}$)
  3. Place the plan (plotted in Step2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.
  4. Count the number of element (M) of the chart enclosed by the plan of the loaded area.

$$\Delta\sigma_z = (IV)qM$$

where IV = influence value
q = pressure on the loaded area

Summary & Essensial points

1. Determining vertical stress at a point due to the application of various types of loading on the surface of a soil mass.
2. The equations and graphs presented in this chapter are based entirely on theory of elasticity.
3. However, the limitations of these theories must be acknowledged. This is because soil deposits, in general, are not homogeneous, perfectly elastic and isotropic
4. The expected difference between the theoretical estimation and actual field value is ±25 to 30%.