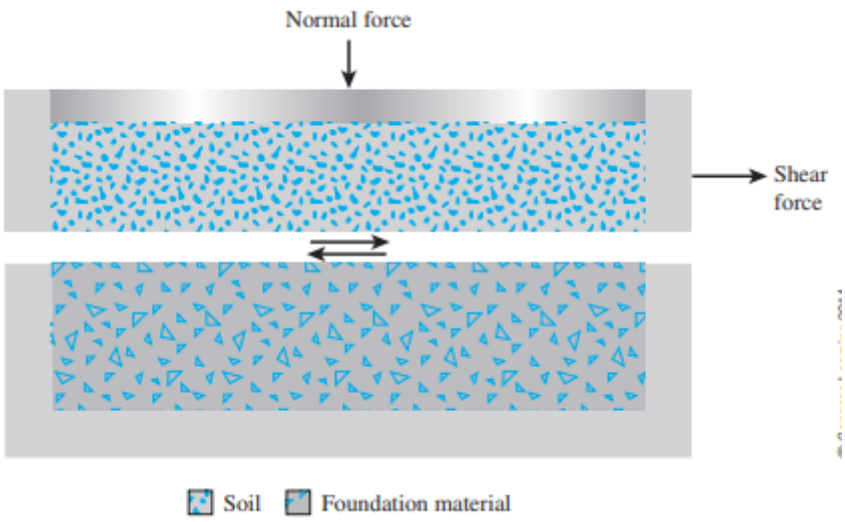
1.

Direct shear tests were performed on a dry, sandy soil. The size of the specimen was 60 mm. x 60 mm. x 19 mm. The results are as follows:

Test No	Normal Force (kN)	Maximum Shear Force (kN)	Normal Stress (kN/m^2))	Maximum Shear Stress (kN/m^2)
1	0.18	0.0936	50	26
2	0.36	0.1764	100	49
3	0.72	0.361	200	100

Determine the friction angle of sand.

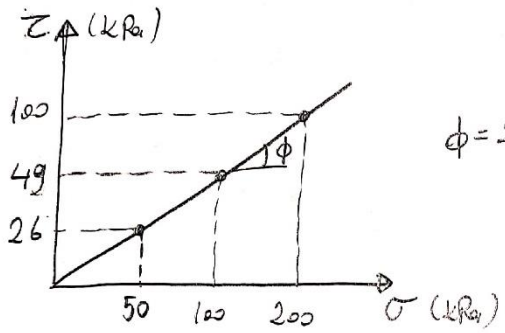


Solution (1)

* $\sigma_1 = \frac{P}{A} = \frac{0.18}{(0.06 \times 0.06)} = 50\ kN/m^2$, $\tau_1 = \frac{P}{A} = \frac{0.0936}{(0.06 \times 0.06)} = 26\ kN/m^2$

* $\sigma_2 = 100\ kN/m^2$, $\tau_2 = 49\ kN/m^2$

* $\sigma_3 = 200\ kN/m^2$, $\tau_3 = 100\ kN/m^2$



$\phi = 26.5°$

Question (2)

A consolidated-drained triaxial test was conducted on a normally consolidated clay. The results are as follows:

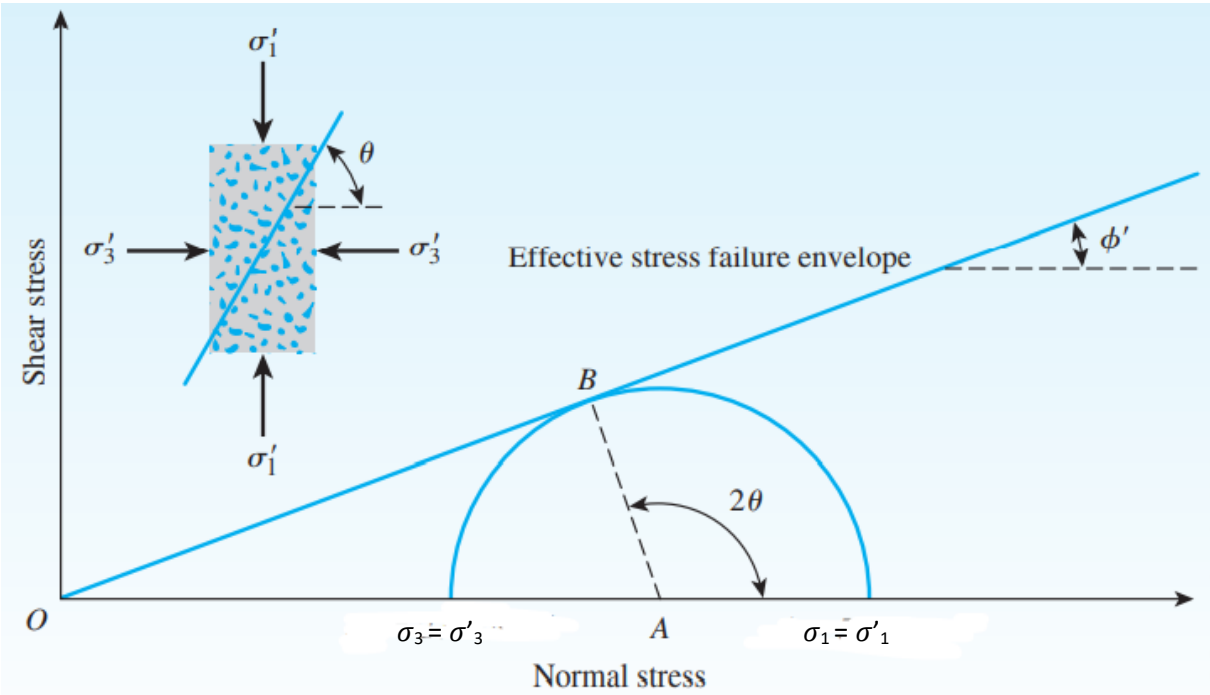
- $\sigma_3 = 112$ kN/m² (cell pressure/confining pressure)
- $(\Delta\sigma_d)_f = 175$ kN/m² (deviator stress at failure)

a) Find the angle of friction ϕ'

b) Determine the angle of (θ) that the failure plane makes with the major principal plane

c) Find the normal stress σ' and the shear stress τ_f on the failure plane.

d) Determine the effective normal stress on the plane of maximum shear stress.



Solution (2)

For normally consolidated soil, the failure envelope equation is

$$\tau_f = \sigma' \tan \phi' \quad (\text{because } c' = 0)$$

For the triaxial test, the effective major and minor principal stresses at failure are as:

$$\sigma_1' = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 112 + 175 = 287 \text{ kN/m}^2$$
$$\sigma_3' = \sigma_3 = 112 \text{ kN/m}^2$$

a)

$$\sin\phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{287 - 112}{287 + 112} = 0.44 \text{ or } \phi' \cong 26^\circ$$

b)

$$\theta = 45 + \frac{\phi'}{2} = 45' + \frac{26^\circ}{2} = 58^\circ$$

c)

$$\sigma' \text{ (on the failure plane)} = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$

$$\sigma' = \frac{287 + 112}{2} + \frac{287 - 112}{2} \cos(2 \times 58^\circ)$$

$$\sigma' = 161.14 \text{ kN/m}^2$$

$$\tau_f = \frac{287 - 112}{2} \sin(2 \times 58^\circ) = 78.64 \text{ kN/m}^2$$

d)

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$

$$\sigma' = \frac{287 - 112}{2} + \frac{287 - 112}{2} \underbrace{\cos 90^\circ}_{0} = 87.5 \text{ kN/m}^2$$

Question (3)

The equation of the effective stress failure envolepe for normally consolidated clayey soil is $\tau_f = \sigma' \tan 30°$. A drained triaxial test was conducted with the same soil at a chamber confining pressure of 70 kN/m². Calculate the deviator stress at failure.

Solution (3)

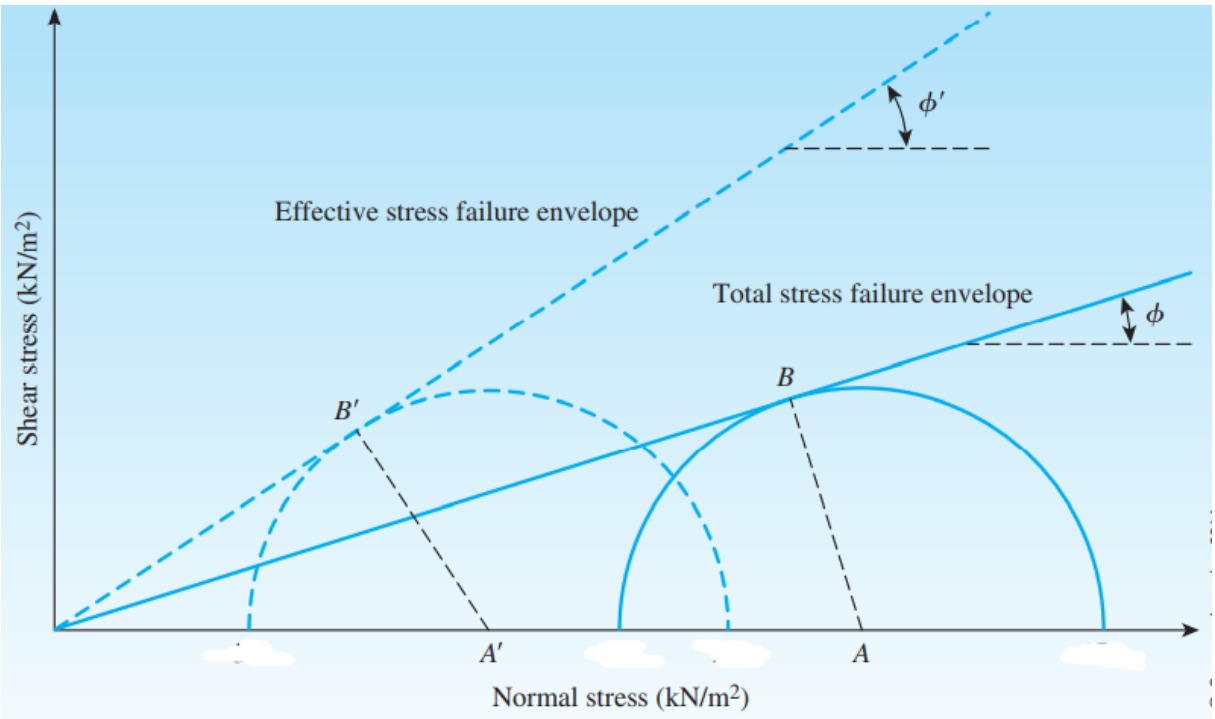
For normally consolidated clay $c' = 0$

$$\sigma_1' = \sigma_3' \tan^2(45 + \phi/2)$$

$$\phi' = 30°$$

$$\sigma_1' = 70 \cdot \tan^2(45 + 30/2) = 210 \text{ kN/m}^2$$

$$(\Delta\sigma_d)_f = \sigma_1' - \sigma_3' = 210 - 70 = 140 \text{ kN/m}^2$$



Question (4)

A consolidated - Undrained test on a normally consolidated clay yielded the following results

$\sigma_3 = 84\ kN/m^2$ (cell pressure)

deviator stress, $(\Delta\sigma)_f = 63.7\ kN/m^2$ $(\sigma_1 - \sigma_3)$

Pore pressure, $(\Delta u)_f = 47\ kN/m^2$

Calculate the consolidated - undrained friction angle and the drained friction angle.

Solution (4)

$\sigma_1 = \sigma_3 + (\Delta\sigma)_f = 84 + 63.7 = 147.7\ kN/m^2$

$\sigma_1 = \sigma_3 \cdot \tan^2(45 + \phi/2) + 2c \tan(45 + \phi/2)$ $\rightarrow 0$

$147.7 = 84 \cdot \tan(45 + \phi/2) \Rightarrow \underline{\phi \cong 16^\circ}$

Drained - friction angle

$\sigma_1' = 147.7 - 47.6 = 100\ kPa.$

$\sigma_3' = 84 - 47.6 = 36.4\ kPa.$

$\sigma_1' = \sigma_3' \cdot \tan^2(45 + \phi'/2) + 2c' \tan(45 + \phi'/2)$

$101.1 = 36.4 \times \tan^2(45 + \phi'/2)$

$\underline{\phi' = 27.8^\circ}$

Question (5)

A consolidated - Drained (CD) triaxial test was performed on a loose sandy soil. specimen with a cell pressure 150 kPa. The specimen failed at a deviator stress of 175 kPa.

a) Determine the effective shear strength parameters for sandy soil specimen

b) Determine the shear stress at failure (τ_f), and find the normal stress at failure (σ'_f).

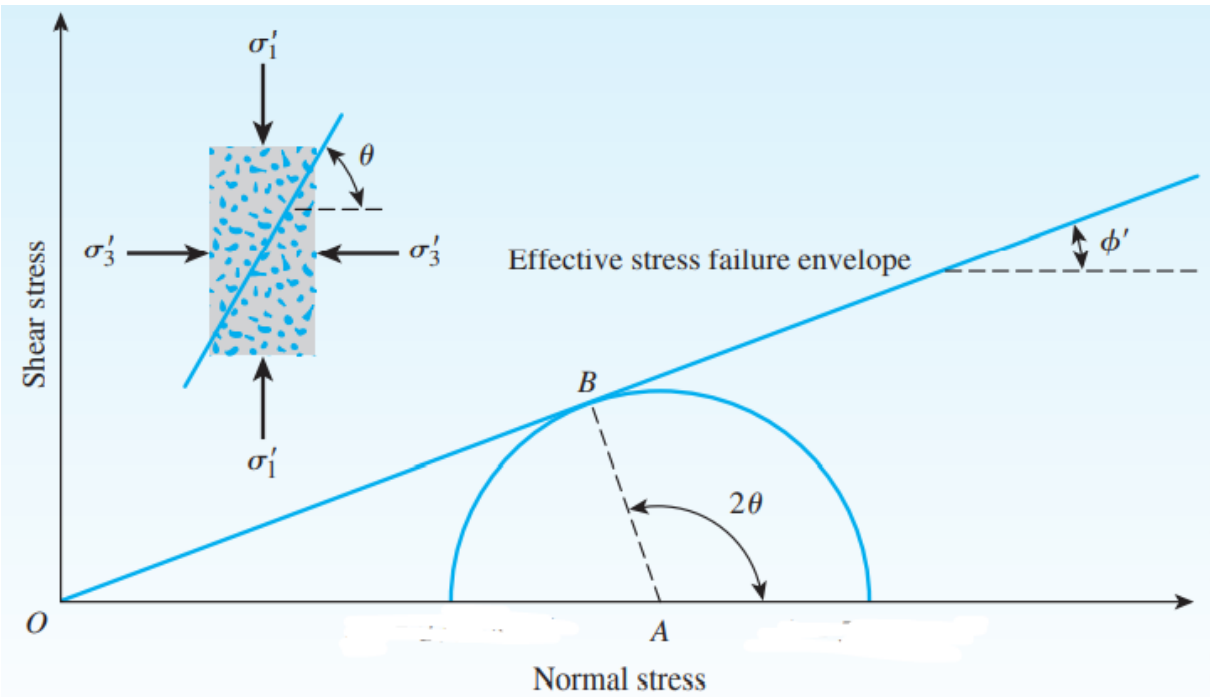
Solution (5)

$$\left.\begin{aligned} \sigma_3' &= 150 \text{ kPa} \\ \sigma_1' - \sigma_3' &= 175 \text{ kPa} \end{aligned}\right\} \sigma_1' = 175 + 150 = 325 \text{ kPa}$$

a) $\sigma_1' = \sigma_3' \cdot \tan^2(45 + \phi/2) + 2 \cdot c' \tan(\cancel{45} + \phi'/2)$ $\rightarrow 0$

$325 = 150 \cdot \tan^2(45 + \phi'/2)$

$\phi' = 21.62° \cong 22$, $c = 0$ } sheer strength parameters.



b) $\theta = 45 + \phi'/2 = 45 + 22/2 = 56°$

$$\sigma_f = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta = \frac{325+150}{2} + \frac{325+150}{2}\cdot \cos 112°$$

$\sigma_f = 204.72$ kPa.

$$\tau_f = \frac{\sigma_1-\sigma_3}{2}\cdot \sin 2\theta = \frac{325-150}{2}\cdot \sin 112$$

$\tau_f = 81.13$ kPa.

$$\{\tau_f = \sigma_f \cdot \tan(\phi)\}$$

$81.13 = \sigma_f \cdot \tan(21.62)$

$\sigma_f = 204.72$ kPa.

Question (6)

A consolidated-undrained (CU) triaxial test was conducted on a fully saturated and normally consolidated clay specimen and the sample failed when the deviator stress reached 166 kPa. If the cell pressure during the test was 200 kPa and equation of the effective stress envelope is expressed with $\tau_f' = \sigma' \tan(34°)$

a) Determine the total shear strength parameters for the specimen

b) Determine the pore water pressure developed in the clay specimen at failure.

Solution (6)

$\sigma_1 - \sigma_3 = 166$ kPa

$\sigma_3 = 200$ kPa

$\sigma_1 = 200 + 166$ kPa

6

a) $\sigma_1 = \sigma_3 \tan^2(45 + \phi/2) + 2c \cancel{\tan}^{\,0}(45 + \phi/2)$

$366 = 200 \cdot \tan^2(45 + \phi/2)$

$\phi = 17.05^\circ \quad c = 0$ } shear strength parameters (For total)

b) $u_f = ?$

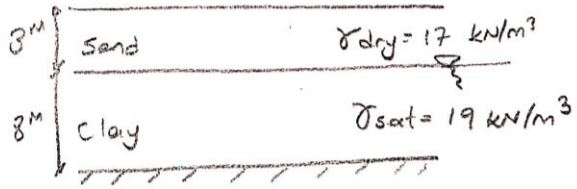
$(366 - u_f) = (200 - u_f) \cdot \tan^2(45 + 34/2)$ (for effective)

$u_f = 134.58$ kPa

	Cell Pressure (kPa)	Deviator Pressure (kPa)
CU	120	107
CU	200	179
CD	110	131.8
CD	200	240

Question (7)

A clay soil sample was recovered from the field at 7m depth and 4 triaxial test was conducted on these samples According to tests,



a) Determine the pore water pressure values at CU tests

b) Calculate the shear stress of the soil at 7m depth using effective stress parameters.

a) CD test 1 → $\sigma_3 = 110$ kPa, $\sigma_1' = 131.8$ kPa

CD test 2 → $\sigma_3 = 200$ kPa, $\sigma_1' = 440$ kPa

$$\sigma_3' = \sigma_1' \cdot \tan^2(45 - \phi'/2) - 2c \cancel{\tan(45 - \phi_d/2)}^{\,0}$$

$$\left.\begin{aligned} 110 &= 241.8 \tan^2(45 - \phi'/2) \\ 200 &= 440 \tan^2(45 - \phi'/2) \end{aligned}\right\} \phi_d = \phi_d' = 22^\circ$$

(7)

CU test 1; $\sigma_3 = 120$ kPa, $\sigma_1 = 227$ kPa

CU test 2; $\sigma_3 = 200$ kPa, $\sigma_1 = 379$ kPa

$$CU_1 \Rightarrow \overbrace{(120 - u_f)}^{\sigma_3'} = \overbrace{(227 - u_f)}^{\sigma_1'} \cdot \tan^2(45 - 22^\circ/2) \rightarrow u_f = 30.7 \text{ kPa}$$

$$CU_2 \Rightarrow (200 - u_f) = (379 - u_f) \cdot \tan^2(45 - 22^\circ/2) \rightarrow u_f = 50.5 \text{ kPa}$$

b) $\tau = \sigma' \tan(\phi') \rightarrow \sigma' = 17 \times 3 + (19 - 9.81) \times 4 = 87.76$ kPa

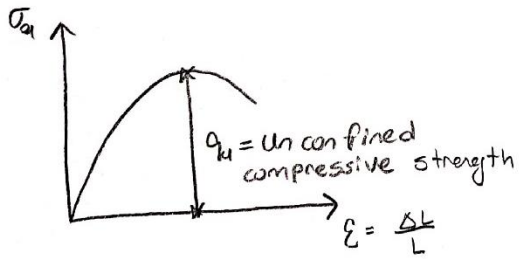
$\tau' = 87.76 \times \tan(22)$

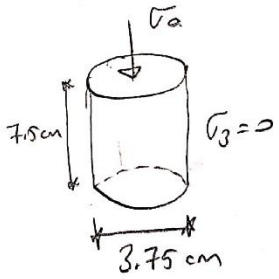
$\tau' = 35.46$ kPa.

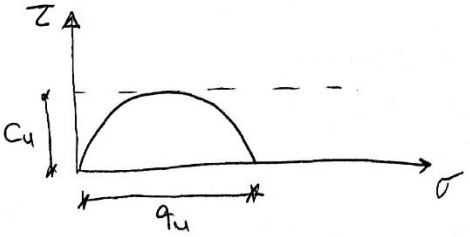
Question (8)

An unconfined compression test was conducted on a saturated clay soil. The applied vertical load at failure is $P_f = 19$ kg and the water content at the end of the test is 32 %. The sample diameter is 3.75 m and the height 7.5 cm calculate the unconfined compressive strength and cohesion of clay soil.

Solution (8)







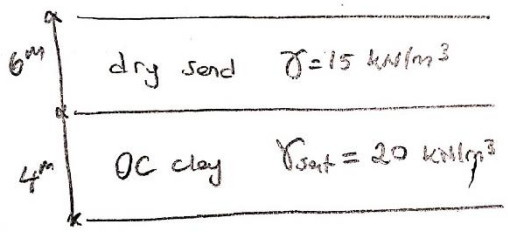
$$q_u = \frac{P_f}{A_f} = \frac{19}{\frac{\pi \cdot 3.75^2}{4}} = 1.72 \text{ kg/cm}^2 \cong 172 \text{ kPa}$$

$$C_u = q_u/2 = \frac{172}{2} = 86 \text{ kPa}$$

Question (9)

Direct shear test was conducted on the sand sample (recovered from 3 m depth) and failure observed at $\sigma' = 112$ kPa and $\tau_p = 70$ kPa. CD tests were conducted on the clay samples which were recovered from the middle of the clay layer and results are given below.

Stress (kPa)	1st Test	2nd Test
σ_3' (cell pressure)	110	225
$\Delta\sigma$ (Deviator stress at failure)	120	180



a) calculate the effective shear strength parameters for clay and sand soil.

b) Calculate the effective shear strength of soil at 5m and 8m depths.

Solution

a) For sand $\Rightarrow \tau_f = \sigma \tan\phi \rightarrow 70 = 112 \tan(\phi) \quad \phi = 32^\circ$

For clay $\Rightarrow \sigma_3' = \sigma_1' \tan^2(45 - \phi/2) - 2c_1' \tan(45 - \phi'/2)$

$110 = 230 \cdot \tan^2(45 - \phi'/2) - 2c' \tan(45 - \phi'/2) \rightarrow$ 1st test

$225 = 405 \cdot \tan^2(45 - \phi'/2) - 2c' \tan(45 - \phi'/2) \rightarrow$ 2nd test

$\Rightarrow \phi' = 12^\circ \quad c \simeq 25$ kPa.

b) at 3^m depth; $\sigma_v = 3 \times 15 = 45$ kPa

$\tau = \sigma \tan(\phi) \rightarrow \tau = 45 \tan(32) = 28.12$ kPa

c) at 8^m depth; $\sigma_u' = 6 \times 15 + 2(20 - 9.81) = 110.38$ kPa

$\tau = c' + \sigma' \tan(\phi')$

$\tau' = 25 + (110.38) \cdot \tan(12°)$

$c = 48.46$ kPa