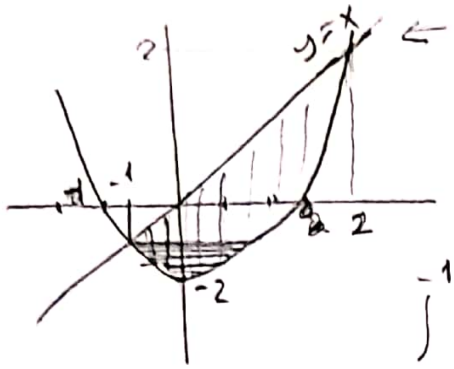
1.) $I = \int_{-1}^{2} \int_{x^2-2}^{x} f(x,y)\,dy\,dx$ int. sırasını değiştirerek çöz.



$y = x^2 - 2$

$x = x^2 - 2$

$x^2 - x - 2 = 0$

$-2 \quad 1$

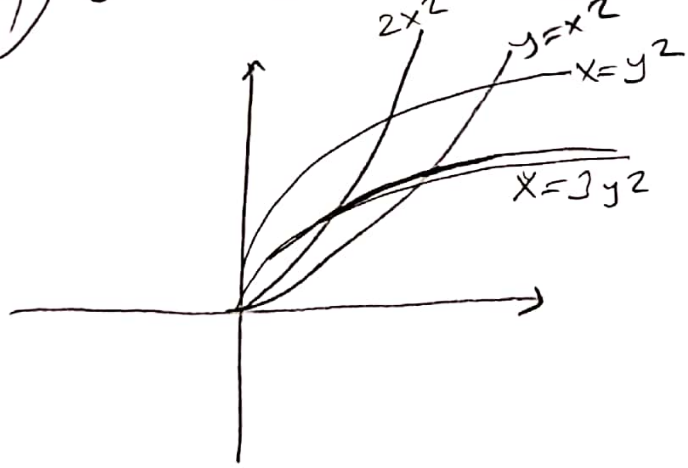
$$\int_{y=-2}^{-1} \int_{-\sqrt{y+2}}^{\sqrt{y+2}} f(x,y)\,dx\,dy + \int_{y=-1}^{2} \int_{x=y}^{\sqrt{y+2}} f(x,y)\,dx\,dy$$

$y = x^2 - 2$

$y + 2 = x^2$

$\pm\sqrt{y+2}$

2.) $y = x^2$, $y = 2x^2$, $x = y^2$ ve $x = 3y^2$ eğrileri arasındaki alan?



$u = \frac{y}{x^2}$ ve $v = \frac{x}{y^2}$

uv-düzleminde

$1 \leq u \leq 2 \qquad 1 \leq v \leq 3$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} -2y/x^3 & 1/x^2 \\ 1/y^2 & -2x/y^3 \end{vmatrix} = \frac{3}{x^2y^2} = 3u^2v^2$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{3u^2v^2}$$

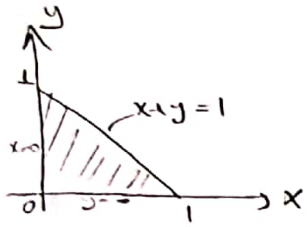
$$\iint_D dx\,dy = \iint_R \frac{1}{3u^2v^2}\,du\,dv$$

$$= \frac{1}{3} \int_1^2 \frac{du}{u^2} \int_1^3 \frac{dv}{v^2} = \frac{1}{9}$$

CamScanner ile tarandı

R bölgesi $\left.\begin{matrix} x+y=1 \\ x=0 \\ y=0 \end{matrix}\right\}$ eğrileri ile sınırlı bölge ise

$$\iint_R \cos\left(\frac{x-y}{x+y}\right)dx\,dy = ?$$



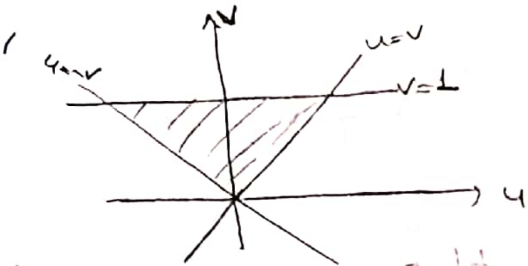
Bu int. hesaplamak için uygun dönüşüm

$$x-y=u$$
$$x+y=v$$
$$2x=u+v$$
$$x=\frac{u+v}{2} \qquad y=\frac{v-u}{2}$$

$x=0 \Rightarrow u+v=0 \quad u=-v$

$y=0 \Rightarrow v-u=0 \quad u=v$

$x+y=1 \Rightarrow v=1$



$$J=\begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4}+\frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\iint_R \cos\left(\frac{x-y}{x+y}\right)dx\,dy = \iint_{\Gamma} \cos\frac{u}{v}|J|\,du\,dv$$

$$\iint_{\Gamma} \cos\frac{u}{v}|J|\,du\,dv = \frac{1}{2}\int_0^1\int_{-v}^{v} \cos\frac{u}{v}\,du\,dv$$

$$\frac{\sin 3x}{4}\,dx = \frac{\cos 3x}{\frac{3}{4}} \quad \frac{1}{3}\cos\frac{3x}{4}$$

$$= \frac{1}{2}\int_0^1 \left(v\sin\frac{u}{v}\Big|_{-v}^{v}\right)dv = \frac{1}{2}\int_0^1 v(\underbrace{\sin 1 - \sin(-1)}_{2\sin 1})\,dv$$

$$\int \cos\frac{x}{v}\,dx \qquad \frac{\sin\frac{x}{v}}{\frac{1}{v}} = v\sin\frac{x}{v}$$

$$= \frac{1}{\cancel{2}}\cancel{2}\sin 1\int_0^1 v\,dv = \frac{\sin 1}{2}$$

Soru $y^2=x \quad y^2=8x \quad x^2=y \quad x^2=8y$ ~~eğrileriyle sınırlı~~ parabollerin $\frac{1}{v}\cos\frac{x}{v}$

1.bölgede sınırladığı alanı bulun.

Yol: $\left.\begin{matrix} y^2=ux \\ x^2=vy \end{matrix}\right\}$ Dönüşümü yapın. $\iint_D dA = \iint_D dx\,dy = \frac{1}{8}$

$u=1 \quad u=8$

$v=1 \quad v=8$ Dik. bölg. dönüşür.

$$\int_1^8\int_1^8 \frac{dA}{du\,dv}$$

$x=u^{1/3}v^{2/3} \qquad y=u^{2/3}v^{1/3}$

$|J|=3.$

$$I=\int_{x=0}^{1}\int_{y=0}^{1-x} e^{y/x+y}\,dy\,dx=?\qquad \left.\begin{array}{l}x+y=u\\ y=v\end{array}\right\}\text{dön. ile.}$$

$x=u-y=u(1-v)$

$\cancel{x}=0 \rightarrow u=0 \vee v=1$

$x=1 \rightarrow u(1-v)=1$

$$J=\begin{vmatrix}1 & -1\\ 0 & 1\end{vmatrix}=1$$

$y=0 \rightarrow uv=0 \Rightarrow u=0 \vee v=0$

$x+y=1 \rightarrow u=1$

$x=0 \qquad y=0 \qquad \left.\begin{array}{l}x+y=u\\ y=v\end{array}\right\} \Rightarrow \begin{array}{|l}x=u-v\\ y=v\\ \hline\end{array}$

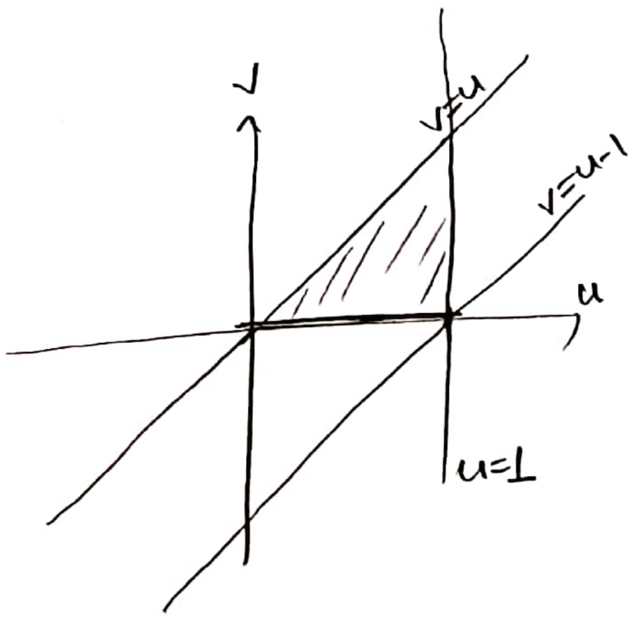
$x=1 \qquad y+x=1$

$x=0 \Rightarrow u=v$

$x=1 \Rightarrow v=u-1$

$x+y=1 \Rightarrow u=1$

$y=0 \rightarrow v=0$



$$I=\int_{u=0}^{1}\int_{v=0}^{u} e^{\frac{v}{u}}\,dv\,du$$

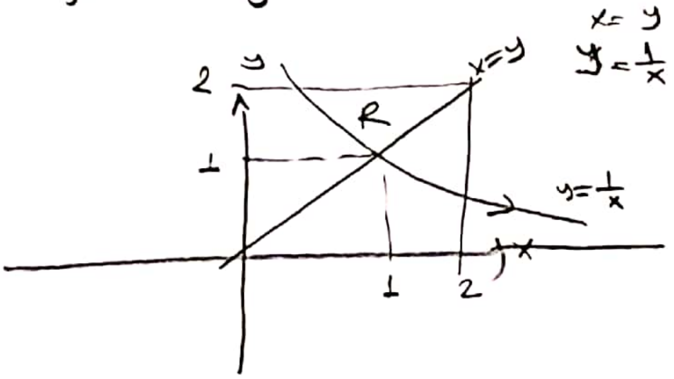
$$=\int_{u=0}^{1}\left(\frac{e^{\frac{v}{u}}}{\frac{1}{u}}\right|_{0}^{u} du$$

$$=\int_{0}^{1} u(e-1)\,du$$

$$=(e-1)\left(\frac{u^2}{2}\right|_{0}^{1}=\frac{e-1}{2}$$

CamScanner ile tarandı

$$I = \int_{y=1}^{2}\int_{x=\frac{1}{y}}^{y} \sqrt{\frac{y}{x}}\; e^{\sqrt{xy}}\, dx\, dy$$



$\sqrt{xy} = u \longrightarrow xy = u^2$

$\sqrt{\frac{y}{x}} = v \longrightarrow \frac{y}{x} = v^2$

$\frac{u^2}{v^2} = x^2$ ve $u^2v^2 = y^2$

$x = \frac{u}{v}$ ve $y = uv$

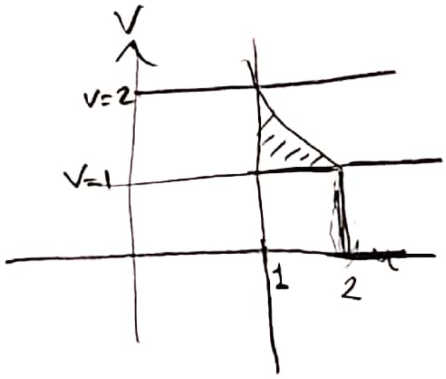
$J = \frac{2u}{v}$ $\quad \left.\begin{matrix} 1=uv \\ 2=uv \end{matrix}\right] x=\frac{1}{y} \Rightarrow u=1$

$xy=1$ iken D de $u=1$, $1 \leq v \leq 2$ dikey doğru parçası

$y=x$ sep sınırı D de $v=1$ $\quad 1 \leq u \leq 2$

$y=2$ yatay üst sınırı " $uv=2$, $1 \leq v \leq 2$

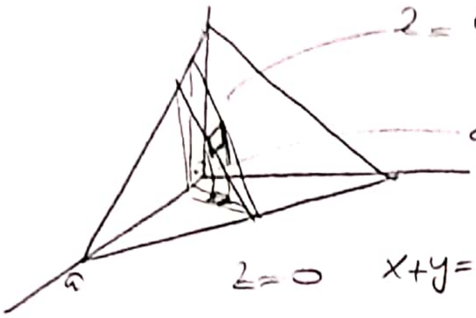
$$I = \int_{u=1}^{2}\int_{v=1}^{2/u} 2u\, e^{u}\, dv\, du = 2e(e-2)$$



Soru 4 a.) $x+y+z=a$ $\quad a>0$, $x=0,\ y=0,\ z=0$

tarafından sınırlanan 3boy. R bölg. çizin

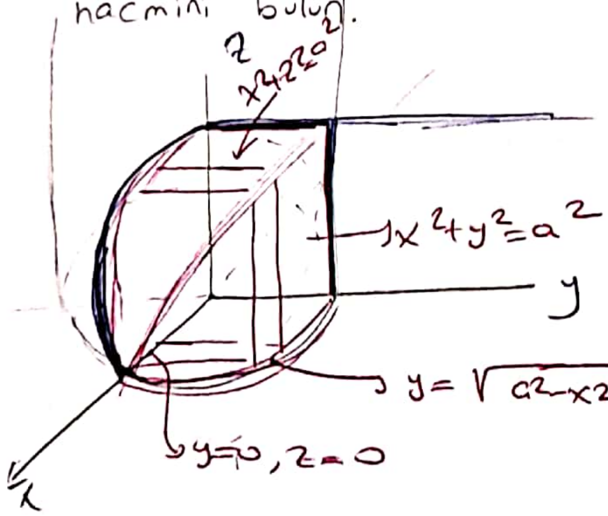
b.) $\iiint_R (x^2+y^2+z^2)\,dx\,dy\,dz$ nin int. hesaplayın.



$$\int_{x=0}^{a}\int_{y=0}^{a-x}\int_{z=0}^{a-x-y}(x^2+y^2+z^2)\,dz\,dy\,dx$$

S.187 de. $\dfrac{a^5}{20}$ Cevap.

$x^2+y^2=a^2$ ve $x^2+z^2=a^2$ silindirlerinin sınırladığı ortak bölgenin hacmini bulun.



$$V=8\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}} z\,dy\,dx$$

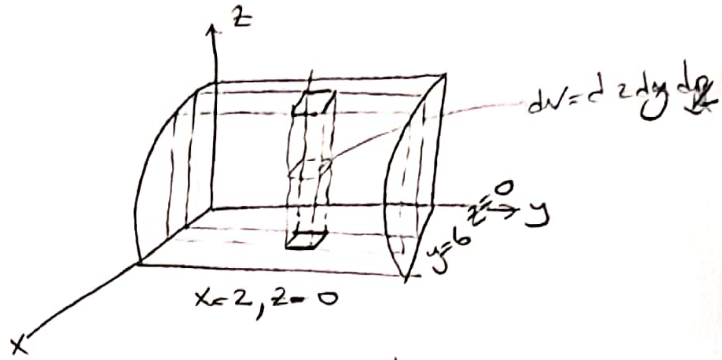
$$=8\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}}\sqrt{a^2-x^2}\,dy\,dx$$

$$=8\int_{x=0}^{a}(a^2-x^2)\,dx=\frac{16a^3}{3}$$

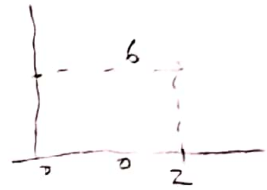
(4)

$z=4-x^2$ parabolik silindir ve $x=0,\ y=0,\ y=6,\ z=0$ düzlemleri ile sınırlanmış R bölgesinde

$$\iiint_R dx\,dy\,dz = ?$$



$$\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=0}^{4-x^2} dz\,dy\,dx$$

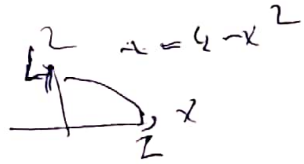


$$= \int_0^2\int_0^6 (4-x^2)\,dy\,dx$$

$$= \int_0^2 (4-x^2)y\Big|_0^6 dx = \int_0^2 (24-6x^2)\,dx = 32$$

şöyle de yazılabilir

$$\iint_{\Omega xz}\int_{y=0}^{6} dy\,dz\,dx$$



$$6\iint_{\Omega xz} dz\,dx$$

$$6\int_0^2\int_0^{4-x^2} dz\,dx$$

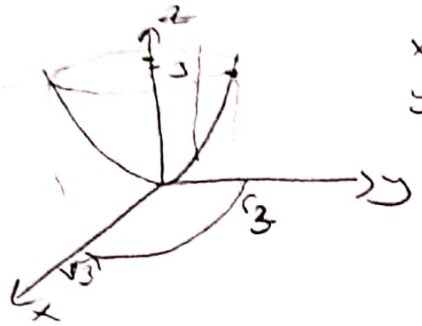
$$\int_0^2 (4-x^2)\,dx$$

$$4x - \frac{x^3}{3}\Big|^2$$

$$6\left(8-\frac{8}{3}\right) \quad \frac{16}{3}\cdot 6$$

Ü-4

R bölgesi : $z=x^2+y^2$ paraboloidi ve $z=3$ düzlemi ile sınırlı bölge old. göre silindirik koordinatlarda.



$x=r\cos\theta$

$y=r\sin\theta$

$x^2+y^2=r^2$

$x^2+y^2=3$ kesişim.

Xy'e düz. izdüşürsek.

$$V=\iint\int_{z=x^2+y^2}^{3} dz\,dA \qquad V=\int_0^{2\pi}\int_0^{\sqrt{3}}\int_{r^2}^{3} r\,dz\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\sqrt{3}}\left(r\,z\Big|_{r^2}^{3}\right)dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\sqrt{3}} r(3-r^2)\,dr\,d\theta$$

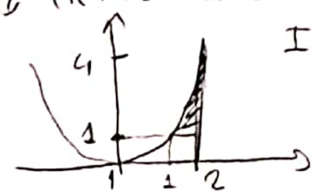
$$=\int_0^{2\pi}\left(3\frac{r^2}{2}-\frac{r^4}{4}\Big|_0^{\sqrt{3}}\right)d\theta=\int_0^{2\pi}\left(\frac{3\cdot 3}{2}-\frac{9}{4}\right)d\theta$$

$$\frac{9}{4}$$

$$=\frac{9}{\cancel{4}}\cancel{2}\pi=\frac{9}{2}\pi$$

4.) $I=\displaystyle\int_1^4\int_{\sqrt{y}}^{2}\left(\sin\frac{x^3}{3}-x\right)dx\,dy$

int. sırasını değiştirerek



$$=\quad I=\int_1^2\int_1^{x^2}\sin\left(\frac{x^3}{3}-x\right)dy\,dx$$

$$=\int_1^2\sin\left(\frac{x^3}{3}-x\right)(x^2-1)\,dx$$

$$=-\cos\left(\frac{x^3}{3}-x\right)\Big|_1^2$$

$$=-\cos\left(\frac{8}{3}-2\right)+\cos\left(\frac{1}{3}-1\right)$$

$$=0$$

b.) $\displaystyle\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sqrt{12-(x^2+y^2)}\,dy\,dx$

polar koordinatları kullanarak

$$=\int_0^{\pi}\int_0^{3} r\sqrt{12-r^2}\,dr\,d\theta$$

$$=\int_0^{\pi}d\theta\int_0^{3} r\sqrt{12-r^2}\,dr$$

$12-r^2=u \Rightarrow r\,dr=\frac{-1}{2}du$

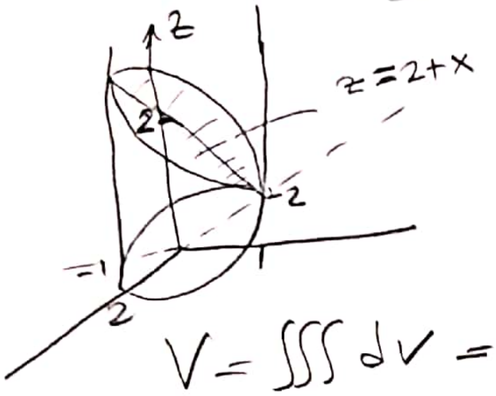
$$=\pi\cdot -\frac{1}{2}\int_{12}^{3}u^{1/2}\,du$$

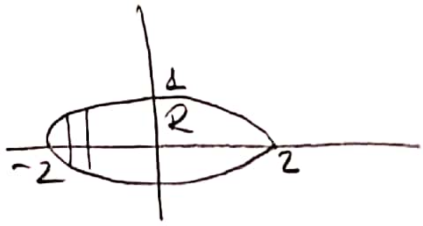
$$=\pi\left(\frac{1}{3}u^{3/2}\Big|_3^{12}\right.$$

$$= \ldots 7\sqrt{3}\pi$$

xy- düzleminin üstünde, $z=2+x$ düzleminin altında ve $x^2+4y^2=4$ silindirin iç bölgesinin hacmi ?

$$x^2+4y^2=4 \longrightarrow \frac{x^2}{2^2}+\frac{y^2}{1^2}=1$$





$$V=\iiint dV=\iint_{R_{xy}}\int_{z=0}^{z=2+x} dz\, dA$$

$$=\int_{x=-2}^{2}\int_{y=-\frac{\sqrt{4-x^2}}{2}}^{\frac{\sqrt{4-x^2}}{2}}\int_{z=0}^{2+x} dz\, dy\, dx$$

$$=\iint_{R_{xy}}\left[(2+x)-0\right]dA$$

$x=2r\cos\theta$
$y=(1)r\sin\theta$

$J=2r$ $\qquad dA=|J(r,\theta)|\,dr\,d\theta$

$0\leq\theta\leq 2\pi$
$0\leq r\leq 1$?

$$=\int_0^{2\pi}\underbrace{\int_0^{1}(2+2r\cos\theta)\,2r\,dr}_{2+\frac{4}{3}\cos\theta}\,d\theta$$

$$=\int_0^{2\pi}\left(2+\frac{4}{3}\cos\theta\right)d\theta=2\theta+\cancel{\frac{4}{3}\sin\theta}^{\,0}$$

$$=4\pi$$