

solu $(3x-y+1)dx + (x-3y+2)dy = 0$

$\underbrace{3x-y}_{\text{lineer bagimsiz}} \quad \underbrace{x-3y}_{\text{lineer bagimsiz}} \quad \begin{matrix} x = x_1 + h \\ y = y_1 + k \end{matrix} \quad \begin{matrix} dx = dx_1 \\ dy = dy_1 \end{matrix}$

$$[3(x_1+h)-(y_1+k)+1]dx_1 + (x_1+h-3(y_1+k)+2)dy_1 = 0$$

$$[3x_1 - y_1 + 3h - k + 1]dx_1 + [x_1 - 3y_1 + h - 3k + 2]dy_1 = 0$$

$$\begin{cases} 3h - k + 1 = 0 \\ h - 3k + 2 = 0 \end{cases} \quad \begin{matrix} h = -\frac{1}{8} \\ k = \frac{5}{8} \end{matrix}$$

$$(3x_1 - y_1)dx_1 + (x_1 - 3y_1)dy_1 = 0 \quad \text{Homogen dif.}$$

$$\frac{dy_1}{dx_1} = \frac{y_1 - 3x_1}{x_1 - 3y_1}$$

$$\frac{y_1}{x_1} = u \rightarrow y_1 = u x_1$$

$$\frac{dy_1}{dx_1} = \frac{du}{dx_1} x_1 + u$$

$$\frac{du}{dx_1} x_1 + u = \frac{u x_1 - 3x_1}{x_1 - 3u x_1}$$

$$\frac{du}{dx_1} x_1 + u = \frac{u - 3}{1 - 3u}$$

$$\frac{du}{dx_1} x_1 = \frac{u - 3}{1 - 3u} - u$$

$$\frac{du}{dx_1} x_1 = \frac{\cancel{u} - 3 - \cancel{u} + 3u^2}{1 - 3u}$$

$$\frac{du}{dx_1} x_1 = \frac{3(u^2 - 1)}{1 - 3u}$$

$$\int \frac{1 - 3u}{u^2 - 1} du = \int 3 \frac{dx_1}{x_1}$$

$$\int \left(\frac{1}{u^2 - 1} - \frac{3u}{1 - u^2} \right) du = 3 \int \frac{dx_1}{x_1}$$

$$\frac{1}{2} \int \left(\frac{1}{u-1} - \frac{1}{u+1} \right) du - 3 \int \frac{u}{1-u^2} du = 3 \int \frac{dx_1}{x_1}$$

$$\frac{1}{2} [\ln(u-1) - \ln(u+1)] + \frac{3}{2} \ln(1-u^2) = 3 \ln x_1^3 + \ln C$$

$$\frac{1}{2} \ln \left(\frac{u-1}{u+1} \right) + \frac{3}{2} \ln(1-u^2) = \ln C x_1^3$$

$$\ln \left[(1-u^2)^{3/2} \left(\frac{u-1}{u+1} \right)^{1/2} \right] = \ln C x_1^3$$

$$\cancel{\ln \left[\left(1 - \left(\frac{y_1}{x_1} \right)^2 \right)^{3/2} \left(\frac{\frac{y_1}{x_1} - 1}{\frac{y_1}{x_1} + 1} \right)^{1/2} \right]} = \ln C x_1^3$$

$$\frac{1}{u^2 - 1} = \frac{A^{1/2}}{u-1} + \frac{B^{-1/2}}{u+1}$$

Basit kesirleri ayırma

$$x_1 = x + \frac{1}{8} ; y = y - \frac{5}{8} \Rightarrow \left[1 - \left(\frac{y - \frac{5}{8}}{x + \frac{1}{8}} \right)^2 \right]^{3/2} \left(\frac{\frac{y - \frac{5}{8}}{x + \frac{1}{8}} - 1}{\frac{y - \frac{5}{8}}{x + \frac{1}{8}} + 1} \right)^{1/2} = C \left(x + \frac{1}{8} \right)^3$$

(2)

SORU $y' - \frac{y}{x} = \frac{x^2}{y^2}$

$y' = \frac{y}{x} + \frac{1}{(\frac{y}{x})^2}$ Homojen drf.

$\frac{y}{x} = u \rightarrow y = ux$
 $y' = u'x + u$

$u'x + u = u + \frac{1}{u^2}$

$\frac{du}{dx} = \frac{1}{u^2} \rightarrow u^2 du = dx$
 $\frac{u^3}{3} = x + C$

$\boxed{\frac{1}{3} \left(\frac{y}{x}\right)^3 = x + C}$ Genel Çözüm

SORU $xy^2 \cdot dy = (x^3 + y^3) dx$

$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$ Homojen drf.

$\frac{y}{x} = u \rightarrow y = ux$
 $y' = u'x + u$

$u'x + u = \frac{x^3 + u^3 x^3}{u^2 x^3} \Rightarrow u'x + u = \frac{1 + u^3}{u^2}$

$\frac{du}{dx} x = \frac{1 + u^3}{u^2} - u$

$\frac{du}{dx} x = \frac{1 + \cancel{u^3} - \cancel{u^3}}{u^2} \Rightarrow u^2 du = \frac{dx}{x}$

$\frac{u^3}{3} = \ln x + C$

$\boxed{\frac{1}{3} \left(\frac{y}{x}\right)^3 = \ln x + C}$

Genel Çözüm

SORU $y' = \frac{1 - (x+y)}{1+x+y}$

$x+y = t$
 $1+y' = t' \Rightarrow y' = t' - 1$

$t' - 1 = \frac{1-t}{1+t}$

$\frac{dt}{dx} = \frac{1-t}{1+t} + 1$

$\frac{dt}{dx} = \frac{1 - \cancel{t} + 1 + \cancel{t}}{1+t} \Rightarrow$

$\frac{dt}{dx} = \frac{2}{1+t}$

$\int (1+t) dt = \int 2 dx$

$t + \frac{t^2}{2} = 2x + C$

$\boxed{(x+y) + \frac{(x+y)^2}{2} = 2x + C}$

Genel Çözüm

soru

(3)

$$y' = \frac{x+y-4}{x-y-6}$$

$\begin{matrix} x+y & x-y \\ \hline \text{lineer bağımız} \end{matrix}$

$$x = x_1 + h$$

$$dx = dx_1$$

$$y = y_1 + k$$

$$dy = dy_1$$

$$\frac{dy_1}{dx_1} = \frac{x_1 + y_1 + \overbrace{h+k-4}^{=0}}{x_1 - y_1 + \overbrace{h-k-6}^{=0}}$$

$$\begin{cases} h+k-4=0 \\ h-k-6=0 \end{cases} \Rightarrow \begin{cases} h=5 \\ k=-1 \end{cases}$$

$$\frac{dy_1}{dx_1} = \frac{x_1 + y_1}{x_1 - y_1}$$

$$\frac{y_1}{x_1} = u \rightarrow y_1 = u x_1$$

$$\frac{dy_1}{dx_1} = \frac{du}{dx_1} x_1 + u$$

$$\frac{du}{dx_1} x_1 + u = \frac{x_1 + u x_1}{x_1 - u x_1}$$

$$\frac{du}{dx_1} x_1 = \frac{1 + \cancel{u} - \cancel{u} + u^2}{1 - u}$$

$$\frac{du}{dx_1} x_1 = \frac{1+u}{1-u} - u$$

$$\frac{du}{dx_1} x_1 = \frac{1+u^2}{1-u}$$

$$\int \frac{1-u}{1+u^2} du = \int \frac{dx_1}{x_1}$$

$$\int \left(\frac{1}{1+u^2} - \frac{u}{1+u^2} \right) du = \int \frac{dx_1}{x_1}$$

$$\arctan u - \frac{1}{2} \ln(1+u^2) = \ln x_1 + C$$

$$\arctan \left(\frac{y_1}{x_1} \right) - \frac{1}{2} \ln \left(1 + \left(\frac{y_1}{x_1} \right)^2 \right) = \ln x_1 + C$$

$$\begin{cases} x_1 = x - 5 \\ y_1 = y + 1 \end{cases}$$

$$\boxed{\arctan \left(\frac{y+1}{x-5} \right) - \frac{1}{2} \ln \left(1 + \left(\frac{y+1}{x-5} \right)^2 \right) = \ln x_1 + C}$$

Genel Çözüm.

sonu

(4)

$$4xy dx + (x^2 + y^2 + 4) dy = 0$$

$$\frac{\partial M}{\partial y} = 4x \quad \frac{\partial N}{\partial x} = 2x \quad \text{Tam dif. değil}$$

$$\lambda = \lambda(y)$$

$$\ln \lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dy$$

$$\ln \lambda = \int \frac{2x - 4x}{4xy} dy = \int \frac{-2x}{4xy} dy = -\frac{1}{2} \ln y$$

$$\lambda = y^{-1/2} = \frac{1}{y^{1/2}} = \frac{1}{\sqrt{y}}$$

$$\frac{1}{\sqrt{y}} 4xy dx + \frac{1}{\sqrt{y}} (x^2 + y^2 + 4) dy = 0$$

$$4x\sqrt{y} dx + \left(\frac{x^2}{\sqrt{y}} + y\sqrt{y} + \frac{4}{\sqrt{y}} \right) dy = 0$$

$$\frac{\partial M_1}{\partial y} = \frac{2x}{\sqrt{y}} = \frac{\partial N_1}{\partial x} \quad \text{Tam dif.}$$

$$u(x, y) = C$$

$$\frac{\partial u}{\partial x} = 4x\sqrt{y}$$

$$\frac{\partial u}{\partial y} = \frac{x^2}{\sqrt{y}} + y\sqrt{y} + \frac{4}{\sqrt{y}}$$

$$u(x, y) = \int 4x\sqrt{y} dx$$

$$u(x, y) = 2x^2\sqrt{y} + h(y)$$

↓

$$\frac{\partial u}{\partial y} = 2x^2 \frac{1}{2\sqrt{y}} + \frac{dh}{dy}$$

$$\frac{x^2}{\sqrt{y}} + \frac{dh}{dy} = \frac{x^2}{\sqrt{y}} + y^{3/2} + 4y^{1/2}$$

$$\int dh = \int (y^{3/2} + 4y^{1/2}) dy$$

$$h(y) = \frac{2y^{5/2}}{5} + \frac{4 \cdot 2y^{3/2}}{3}$$

$$u(x, y) = C$$

$$2x^2\sqrt{y} + \frac{2}{5}\sqrt{y}^5 + \frac{8}{3}\sqrt{y}^3 = C \quad \text{Genel Çözüm}$$

sonu

$$(3x^2 - y^2) dy - 2xy dx = 0$$

1. yol Homojen dif.

$$\frac{dy}{dx} = \frac{2xy}{3x^2 - y^2}$$

$$\frac{y}{x} = v \rightarrow y = vx$$

$$y' = v'x + v$$

$$v'x + v = \frac{2vx^2}{3x^2 - v^2x^2}$$

$$\frac{dv}{dx} = \frac{2v}{3 - v^2} - v$$

$$\frac{dv}{dx} = \frac{2v - 3v + v^3}{3 - v^2}$$

$$\int \frac{3 - v^2}{v^3 - v} dv = \int \frac{dx}{x}$$

$$\int \frac{2 + 1 - v^2}{v(v^2 - 1)} dv$$

$$\int \left(\frac{2}{v(v^2 - 1)} - \frac{1}{v} \right) dv$$

$$\frac{2}{v(v-1)(v+1)} = \frac{A}{v} + \frac{B}{v-1} + \frac{C}{v+1}$$

$$A = -2 \quad B = 1 \quad C = 1$$

$$\int \left(\frac{-2}{v} + \frac{1}{v-1} + \frac{1}{v+1} \right) dv = \int \frac{dx}{x}$$

$$-2\ln v + \ln(v-1) + \ln(v+1) = \ln x + \ln C$$

$$\ln \frac{(v+1)(v-1)}{v^2} = \ln Cx$$

$$\frac{v^2 - 1}{v^2} = Cx \rightarrow 1 - \frac{1}{v^2} = Cx$$

$$v = \frac{y}{x} \Rightarrow \sqrt{1 - \frac{x^2}{y^2}} = Cx \quad \text{Genel Ç.}$$

SORU
2.501

$$\overbrace{(3x^2 - y^2)}^N dy - \overbrace{2xy}^M dx = 0$$

$$\frac{\partial M}{\partial y} = -2x \neq \frac{\partial N}{\partial x} = 6x$$

$$\lambda = \lambda(y)$$

$$\ln \lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} dy = \int \frac{6x - (-2x)}{-2xy} dy$$

$$\ln \lambda = \int \frac{8x}{-2xy} dy = -4 \int \frac{dy}{y}$$

$$\ln \lambda = -4 \ln y \Rightarrow \lambda = \frac{1}{y^4}$$

$$\frac{1}{y^4} (3x^2 - y^2) dy - 2xy \frac{1}{y^4} dx = 0$$

$$-2 \frac{x}{y^3} dx + \left(\frac{3x^2}{y^4} - \frac{1}{y^2} \right) dy = 0$$

$$\frac{\partial M_1}{\partial y} = \frac{6x}{y^4} = \frac{\partial N_1}{\partial x} \text{ Tam dif.}$$

$$u(x, y) = C$$

$$\frac{\partial u}{\partial x} = -\frac{2x}{y^3}$$

$$u(x, y) = \int -\frac{2x}{y^3} dx$$

$$u(x, y) = -\frac{x^2}{y^3} + h(y)$$

$$\downarrow$$

$$\frac{\partial u}{\partial y} = +\frac{3x^2}{y^4} + \frac{dh}{dy}$$

$$\frac{\partial u}{\partial y} = \frac{3x^2}{y^4} - \frac{1}{y^2}$$

$$\cancel{\frac{3x^2}{y^4}} + \frac{dh}{dy} = \cancel{\frac{3x^2}{y^4}} - \frac{1}{y^2}$$

$$\int dh = \int \frac{1}{y^2} dy$$

$$h(y) = -\frac{1}{y}$$

$$u(x, y) = C$$

$$\boxed{-\frac{x^2}{y^3} - \frac{1}{y} = C} \quad \text{Genel Çözüm}$$

SORU

$$(y + x^3 e^x) dx - x dy = 0$$

$$\frac{\partial M}{\partial y} = 1 \neq \frac{\partial N}{\partial x} = -1$$

$$\lambda = \lambda(x)$$

$$\ln \lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx = \int \frac{1 - (-1)}{-x} dx$$

$$\ln \lambda = -2 \ln x \rightarrow \lambda = \frac{1}{x^2}$$

$$\frac{1}{x^2} (y + x^3 e^x) dx - \frac{1}{x^2} x dy = 0$$

$$\left(\frac{y}{x^2} + x e^x \right) dx - \frac{1}{x} dy = 0$$

$$\frac{\partial M_1}{\partial y} = \frac{1}{x^2} = \frac{\partial N_1}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{y}{x^2} + x e^x$$

$$\frac{y}{x^2} + \frac{dk}{dx} = \frac{y}{x^2} + x e^x$$

$$\int dk = \int x e^x dx$$

$$k(x) = e^x (x - 1)$$

$$\frac{\partial u}{\partial y} = -\frac{1}{x}$$

$$u(x, y) = \int -\frac{1}{x} dy$$

$$u(x, y) = -\frac{y}{x} + k(x)$$

$$\downarrow$$

$$\frac{\partial u}{\partial x} = \frac{y}{x^2} + \frac{dk}{dx}$$

$$u(x, y) = C$$

$$\boxed{\frac{y}{x^2} + e^x (x - 1) = C} \quad \text{Genel Çözüm}$$

Soru $(x^2 + y^2 + 2x)dx + 2ydy = 0$
 $\frac{\partial M}{\partial y} = 2y \neq \frac{\partial N}{\partial x} = 0$

$\lambda = \lambda(x)$
 $\ln \lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx = \int \frac{2y - 0}{2y} dx$
 $\ln \lambda = \int dx = x \rightarrow \lambda = e^x$

$e^x(x^2 + y^2 + 2x)dx + 2ye^x dy = 0$
 $x^2e^x + y^2e^x + 2xe^x$

$\frac{\partial M_1}{\partial y} = 2ye^x = \frac{\partial N_1}{\partial x}$ Tam d.f.

$U(x, y) = C$

$\frac{\partial U}{\partial x} = x^2e^x + y^2e^x + 2xe^x$
 $y^2e^x + \frac{dk}{dx} = x^2e^x + y^2e^x + 2xe^x$

$dk = (x^2e^x + 2xe^x)dx$

$\int dk = \int (x^2 + 2x)e^x dx$
 2 kere kumi integrasyon
 yapilacak

$k(x) = x^2e^x$

$U(x, y) = C$

$y^2e^x + x^2e^x = C$

Genel Çözüm

Soru $(xy + x^2 + 1)dx + x^2dy = 0$

$\frac{\partial M}{\partial y} = x \neq \frac{\partial N}{\partial x} = 2x$

$\frac{1}{x}(xy + x^2 + 1)dx + \frac{1}{x}x^2dy = 0$

$(y + x + \frac{1}{x})dx + xdy = 0$

$\frac{\partial M_1}{\partial y} = 1 = \frac{\partial N_1}{\partial x}$ Tam d.f.

$U(x, y) = C$

$\frac{\partial U}{\partial x} = y + x + \frac{1}{x}$

$y + \frac{dk}{dx} = y + x + \frac{1}{x}$

$\int dk = \int (x + \frac{1}{x}) dx$

$k(x) = \frac{x^2}{2} + \ln x$

$\frac{\partial U}{\partial y} = x$

$U(x, y) = \int x dy$

$U(x, y) = xy + k(x)$

$\frac{\partial U}{\partial x} = y + \frac{dk}{dx}$

$U(x, y) = C$

$xy + \frac{x^2}{2} + \ln x = C$

$\lambda = \lambda(x)$

$\ln \lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx = \int \frac{x - 2x}{x^2} dx = \int \frac{-1}{x} dx$

$\ln \lambda = -\ln x \rightarrow \lambda = \frac{1}{x}$