n. mertebeden sabit katsayılı lineer diferansiyel denklemlerde 2. taraflı diferansiyel denklemin çözümü

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = f(x)$$

$f(x) \neq 0$

$f(x) = 0$ homojen kısmın çözümü $\rightarrow y_h$

$f(x) \neq 0$ 2. taraflı dif denk çözümü $y_ö$ (y özel)

$\boxed{y = y_h + y_ö}$ Genel Çözüm

3 tane yöntem vardır

Belirsiz katsayılar Yöntemi

Sabitlerin değişimi //

Türev Operatörü //

Belirsiz Katsayılar Yöntemi

x^n, $e^{\alpha x}$, $\sin\beta x$, $\cos\beta x$ bu fonksiyonlar varsa bu yöntem kullanılır.

$n \geq 0$ $n \in \mathbb{Z}$, $\alpha \neq 0$ $\alpha, \beta \in \mathbb{R}$

ÇÖZÜM TABLOSU

	$f(x)$	Karakteristik denklemin kökü değilse	$y_ö$
1)	k (sabit)	0	A
2)	x^n	0	$A_0 x^n + A_1 x^{n-1} + \dots + A_n$
3)	$e^{\alpha x}$	α	$A e^{\alpha x}$
4)	$x^n e^{\alpha x}$	α	$(A_0 x^n + \dots + A_n) e^{\alpha x}$
5)	$\sin\beta x$, $\cos\beta x$	$\mp i\beta$	$A\cos\beta x + B\sin\beta x$
6)	$e^{\alpha x}\sin\beta x$, $e^{\alpha x}\cos\beta x$	$\alpha \mp i\beta$	$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$
7)	$x^n \sin\beta x$, $x^n\cos\beta x$	$\mp i\beta$	$(A_0 x^n + \dots + A_n)\cos\beta x + (B_0 x^n + \dots + B_n)\sin\beta x$

Not: Tablodaki 2. sütunda yer alan $0, \alpha, \mp i\beta$, $\alpha \mp i\beta$ değerleri karakteristik denklemin k katlı kökü ise 3. sütundaki ifadeler x^k ile çarpılır.

ÖR/ $y''' - y'' - y' + y = \underbrace{5e^x}_{y_{ö_1}} + \underbrace{6x^2 e^{-x}}_{y_{ö_2}} - \underbrace{e^x \cos x}_{y_{ö_3}}$

$r^3 - r^2 - r + 1 = 0$

$r^2(r-1) - (r-1) = 0$

$(r^2 - 1)(r-1) = 0$

$\underbrace{r_{1,2} = 1}_{2 \text{ katlı kök}} \quad r_3 = -1$

$y_h = e^x (c_1 x + c_2) + c_3 e^{-x}$

$y_h = (c_1 + c_2 x) e^x + c_3 e^{-x}$

$f_1(x) = 5e^x$ için $(\alpha = 1)$ (2 katlı kök)

$y_{ö_1} = x^2 \cdot A e^x$ (özel durum)

$f_2(x) = 6x^2 e^{-x}$ $\alpha = -1$ (tek katlı kök)

$y_{ö_2} = x \cdot (a_0 x^2 + a_1 x + a_2) e^{-x}$ (özel durum)

$f_3(x) = -e^x \cos x$

$y_{ö_3} = e^x (A \cos x + B \sin x)$

ÖR/ $y''' - 3y'' + 4y' - 12y = 3e^{-2x} - 5\sin 3x + 7x \cos 2x$

$r^3 - 3r^2 + 4r - 12 = 0$

$r^2(r-3) + 4(r-3) = 0$

$(r^2 + 4)(r-3) = 0$

$r^2 = -4 \quad r_{1,2} = \mp 2i$

$r_3 = 3$

$y = (c_1 \cos 2x + c_2 \sin 2x) + c_3 e^{3x}$

$y_{ö_1} = A e^{-2x}$

$y_{ö_2} = A \sin 3x + B \cos 3x$

$y_{ö_3} = [(a_0 x + b_0)\cos 2x + (a_1 x + b_1)\sin 2x] \cdot x$

özel durum

ÖR/ $y'' + 4y' + 29y = 4e^{-2x} - 4e^{-2x}\sin 5x + x^2 - 5$

$r^2 + 4r + 29 = 0$

$r_{1,2} = \dfrac{-4 \mp \sqrt{16 - 4.1.29}}{2}$

$r_{1,2} = \dfrac{-4 \mp \sqrt{-100}}{2} = \dfrac{-4 \mp 10i}{2} \Rightarrow \begin{matrix} -2+5i \\ -2-5i \end{matrix}$

$\left.\begin{matrix} r_1 = -2+5i \\ r_2 = -2-5i \end{matrix}\right\} y_h = e^{-2x}(c_1 \sin 5x + c_2 \cos 5x)$

$y_{ö_1} = Ae^{-2x}$

$y_{ö_2} = e^{-2x}(A_0 \sin 5x + B_0 \cos 5x) \cdot x$

$y_{ö_3} = ax^2 + bx + c$

ÖR/ $y'' + y' - 2y = x^2 - 1$ dif denklemini çözünüz.

$r^2 + r - 2 = 0$

$-2 \quad 1$

$r_1 = -2 \quad r_2 = 1$

$\underline{y_h = c_1 e^{-2x} + c_2 e^x}$

$\left.\begin{matrix} y_ö = ax^2 + bx + c \\ y_ö' = 2ax + b \\ y_ö'' = 2a \end{matrix}\right\}$ $2a + 2ax + b - 2ax^2 - 2bx - 2c = x^2 - 1$

$-2a = 1 \quad a = -1/2$

$2a - 2b = 0 \quad 2a = 2b$

$b = -1/2$

$2a - 2c + b = -1$

$2\left(-1/2\right) - 2c - 1/2 = -1 \quad c = -\dfrac{1}{4}$

$-1 - \frac{1}{2} + 1 = 2c$

$-2 - 1 + 2 = 2c$

$c = -1/4$

$y_ö = \dfrac{-1}{2}x^2 - \dfrac{x}{2} - \dfrac{1}{4}$

$y = y_h + y_ö$

$y = c_1 e^{-2x} + c_2 e^x - \dfrac{x^2}{2} - \dfrac{x}{2} - \dfrac{1}{4}$

ÖR/ $y'' - y' = 2x+1$

$r^2 - r = 0 \quad r(r-1) = 0 \quad r_1 = 0 \quad r_2 = 1$

$y_h = c_1 + c_2 e^x$

$y_ö = (ax+b)x = ax^2 + bx$

$y_ö' = 2ax + b$

$y_ö'' = 2a$

$2a - 2ax - b = 2x + 1$

$-2ax + 2a - b = 2x + 1$

$-2a = 2 \quad a = -1$

$2a - b = 1 \Rightarrow -b = 1 - 2a$

$-b = 1 - 2.(-1)$

$-b = 3 \quad b = -3$

$y_ö = -x^2 - 3x$

$y = y_h + y_ö = c_1 + c_2 e^x - x^2 - 3x$

ÖR/ $y'' - 4y' + 3y = 4e^{2x}$ dif denk çözünüz.

$r^2 - 4r + 3 = 0 \quad r_1 = 3 \quad y_h = c_1 e^{3x} + c_2 e^x$

3 1 $\quad r_2 = 1$

$y_ö = Ae^{2x} \quad y_ö' = 2Ae^{2x} \quad y_ö'' = 4Ae^{2x}$

$4Ae^{2x} - 8Ae^{2x} + 3Ae^{2x} = 4e^{2x}$

$-A = 4 \quad A = -4$

$y_ö = -4e^{2x}$

$y = c_1 e^{3x} + c_2 e^x - 4e^{2x}$

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ÖR/ $y'' - 5y' + 6y = 5e^{2x}$

$r^2 - 5r + 6 = 0$ $\quad r_1 = 2$

2 3 $\quad r_2 = 3$

$y_h = c_1 e^{2x} + c_2 e^{3x}$

$\alpha = 2$ karakteristik denklemin kökü (tek katlı kök)

$y_ö = A e^{2x} \cdot x$ (özel durum)

$y_ö' = 2Ae^{2x} \cdot x + Ae^{2x}$

$y_ö'' = 4Ae^{2x} \cdot x + 2Ae^{2x} + 2Ae^{2x}$

$y_ö'' = 4Ae^{2x} \cdot x + 4Ae^{2x}$

$\cancel{4Ae^{2x} \cdot x} + 4Ae^{2x} - \cancel{10Ae^{2x} \cdot x} - 5Ae^{2x} + \cancel{6Ae^{2x} \cdot x} = 5e^{2x}$

$-Ae^{2x} = 5e^{2x} \quad A = -5$

$y_ö = -5xe^{2x}$

$y = c_1 e^{2x} + c_2 e^{3x} - 5xe^{2x}$

* Karakteristik polinomun kökleri şöyle olsaydı

$r_1 = r_2 = 2 \quad r_3 = 3$

$y_ö = Ae^{2x} \cdot x^2$ ($\alpha = 2$ karakteristik denklemin 2 katlı kökü)

ÖR /

$y'' + 5y' + 6y = 2\cos x$ dif denk çöz.

$r^2 + 5r + 6 = 0$ $r_1 = -3$ $y_h = c_1 e^{-3x} + c_2 e^{-2x}$

$-3 \quad -2$ $r_2 = -2$

$y_ö = A\cos x + B\sin x$

$y_ö' = -A\sin x + B\cos x$

$y_ö'' = -A\cos x - B\sin x$

$$-A\cos x - B\sin x + 5(-A\sin x + B\cos x) + 6(A\cos x + B\sin x) = 2\cos x$$

$$-A\cos x - B\sin x - 5A\sin x + 5B\cos x + 6A\cos x + 6B\sin x = 2\cos x$$

$$(5A + 5B)\cos x + (5B - 5A)\sin x = 2\cos x$$

$$\left.\begin{aligned} 5A + 5B &= 2 \\ 5B - 5A &= 0 \end{aligned}\right\} \quad 10B = 2 \quad B = 1/5 \quad A = 1/5$$

$$y_ö = \frac{1}{5}\cos x + \frac{1}{5}\sin x$$

$$y = y_h + y_ö = c_1 e^{-3x} + c_2 e^{-2x} + \frac{1}{5}\cos x + \frac{1}{5}\sin x$$

ÖR/ $y'' + 4y = \sin 2x$ dif denk çöz.

$r^2 + 4 = 0 \quad r_{1,2} = \mp 2i \quad \alpha = 0 \quad \beta = 2$

$y_h = \underline{c_1 \sin 2x + c_2 \cos 2x}$

$y_ö = (A \sin 2x + B \cos 2x) \cdot x$

özel durum

$y_ö = A x \sin 2x + B x \cos 2x$

$y_ö' = A \sin 2x + 2Ax \cos 2x + B \cos 2x - 2Bx \sin 2x$

$y_ö'' = \underbrace{2A \cos 2x + 2A \cos 2x} - 4Ax \sin 2x - \underbrace{2B \sin 2x - 2B \sin 2x} - 4Bx \cos 2x$

$y_ö'' = 4A \cos 2x - 4B \sin 2x - 4Ax \sin 2x - 4Bx \cos 2x$

$4A \cos 2x - 4B \sin 2x - \cancel{4Ax \sin 2x} - \cancel{4Bx \cos 2x} + \cancel{4Ax \sin 2x} + \cancel{4Bx \cos 2x} = \sin 2x$

$4A = 0 \qquad -4B = 1$

$A = 0 \qquad B = -1/4$

$y_ö = \underline{-\frac{1}{4} x \cos 2x}$

$y = c_1 \sin 2x + c_2 \cos 2x - \frac{x}{4} \cos 2x$

ÖR/ $y'' - 2y' + 2y = xe^x$ dif denk çöz.

$r^2 - 2r + 2 = 0$

$r_{1,2} = 1 \mp i \qquad r_1 = 1 + i \qquad r_2 = 1 - i$

$y_h = \underline{e^x (c_1 \cos x + c_2 \sin x)}$

$y_ö = (ax + b)e^x = axe^x + be^x$

$y_ö' = ae^x + axe^x + be^x$

$y_ö'' = ae^x + ae^x + axe^x + be^x = 2ae^x + axe^x + be^x$

$\cancel{2ae^x} + axe^x + be^x - \cancel{2ae^x} - \cancel{2axe^x} - \cancel{2be^x} + \cancel{2axe^x} + \cancel{2be^x} = xe^x$

$axe^x + be^x = xe^x \qquad a = 1 \qquad b = 0$

$$y_{\ddot{o}} = xe^{x}$$

$$y = y_h + y_{\ddot{o}} \qquad y = e^{x}(c_1 \cos x + c_2 \sin x) + xe^{x}$$

ÖR/ $y'' + y = e^{-x}\cos x$ dif denk çöz.

$$r^2 + 1 = 0 \qquad r_{1,2} = \mp i$$

$$y = c_1 \cos x + c_2 \sin x$$

$$y_{\ddot{o}} = e^{-x}(A\cos x + B\sin x)$$

$$y_{\ddot{o}} = e^{-x}A\cos x + e^{-x}B\sin x$$

$$y_{\ddot{o}}' = -e^{-x}A\cos x - e^{-x}A\sin x - e^{-x}B\sin x + e^{-x}B\cos x$$

$$y_{\ddot{o}}'' = \cancel{e^{-x}A\cos x} + e^{-x}A\sin x + e^{-x}A\sin x - \cancel{e^{-x}A\cos x} + \cancel{e^{-x}B\sin x}$$
$$- e^{-x}B\cos x - e^{-x}B\cos x - \cancel{e^{-x}B\sin x}$$

$$y_{\ddot{o}}'' = 2Ae^{-x}\sin x - 2Be^{-x}\cos x$$

$$2Ae^{-x}\sin x - 2Be^{-x}\cos x + e^{-x}A\cos x + e^{-x}B\sin x = e^{-x}\cos x$$

$$(2A+B)e^{-x}\sin x + (-2B+A)e^{-x}\cos x = e^{-x}\cos x$$

$$\left.\begin{aligned} 2A + B &= 0 \\ -2B + A &= 1 \end{aligned}\right\} \quad A = 1/5 \quad B = -2/5$$

$$y_{\ddot{o}} = e^{-x}\left(\frac{1}{5}\cos x - \frac{2}{5}\sin x\right)$$

$$y = y_h + y_{\ddot{o}} = c_1\cos x + c_2 \sin x + e^{-x}\left(\frac{1}{5}\cos x - \frac{2}{5}\sin x\right)$$

ÖR/ $y'' + y = \sin^2 x$ dif denk çöz.

$r^2 + 1 = 0 \qquad r_{1,2} = \mp i$

$y = c_1 \cos x + c_2 \sin x$

$\sin^2 x = \dfrac{1 - \cos 2x}{2} = \dfrac{1}{2} - \dfrac{1}{2} \cos 2x$

$$y'' + y = \underbrace{\frac{1}{2}}_{y_{ö_1}} - \underbrace{\frac{1}{2} \cos 2x}_{y_{ö_2}}$$

$y_{ö_1} = A$

$y_{ö_2} = A_0 \cos 2x + B \sin 2x$

ÖR/ $y'' + 4y = x \sin x$ dif denk çöz.

$r^2 + 4 = 0 \qquad r_{1,2} = \mp 2i \qquad y_h = c_1 \cos 2x + c_2 \sin 2x$

$y_ö = (ax + b) \cos x + (cx + d) \sin x$

$y_ö' = a \cos x - (ax + b) \sin x + c \sin x + (cx + d) \cos x$

$y_ö'' = -2a \sin x - (ax + b) \cos x + 2c \cos x - (cx + d) \sin x$

$-2a \sin x - (ax + b) \cos x + 2c \cos x - (cx + d) \sin x + 4(ax + b) \cos x + 4(cx + d) \sin x = x \sin x$

$(3ax + 3b + 2c) \cos x + (-2a + 3cx + 3d) \sin x = x \sin x$

$3ax \cos x + (3b + 2c) \cos x + (-2a + 3d) \sin x + 3cx \sin x = x \sin x$

$3c = 1 \qquad c = 1/3$

$3a = 0 \qquad a = 0$

$3b + 2c = 0 \qquad b = -2/9$

$-2a + 3d = 0 \qquad d = 0$

$$y_ö = -\frac{2}{9} \cos x + \frac{1}{3} x \sin x$$

$$y = c_1 \cos 2x + c_2 \sin 2x - \frac{2}{9} \cos x + \frac{x}{3} \sin x$$

ÖR/ $y''' - 2y'' = xe^{2x}$

$r^3 - 2r^2 = 0$

$r^2(r-2) = 0 \qquad r_{1,2} = 0 \qquad r_3 = 2$

$y_h = \underline{c_1 x + c_2 + c_3 e^{2x}}$

$y_ö = (ax+b)e^{2x} \cdot \underline{x}$

özel durum

$y_ö = (ax^2+bx)e^{2x}$

$y_ö' = (2ax+b)e^{2x} + 2(ax^2+bx)e^{2x}$

$y_ö'' = 2ae^{2x} + 2(2ax+b)e^{2x} + 2(2ax+b)e^{2x} + 2 \cdot (ax^2+bx) \cdot 2e^{2x}$

$y_ö'' = 2ae^{2x} + 4(2ax+b)e^{2x} + 4e^{2x}(ax^2+bx)$

$y_ö''' = 4ae^{2x} + 8a \cdot e^{2x} + 8(2ax+b)e^{2x} + 8e^{2x}(ax^2+bx) + 4e^{2x}(2ax+b)$

$y_ö''' = 4ae^{2x} + 8ae^{2x} + 12(2ax+b)e^{2x} + 8e^{2x}(ax^2+bx)$

$4ae^{2x} + 8ae^{2x} + 12(2ax+b)e^{2x} + 8e^{2x}(ax^2+bx) - 4ae^{2x} - 8(2ax+b)e^{2x}$

$- 8e^{2x}(ax^2+bx) = xe^{2x}$

$e^{2x}(12a + 24ax + 12b + 8\cancel{a}x^2 + 8\cancel{b}x - 4a - 16ax - 8b - 8a\cancel{x^2} - 8\cancel{b}x) = xe^{2x}$

$8ax + 8a + 4b = x \qquad 8a = 1 \quad a = 1/8$

$8a + 4b = 0 \quad b = -1/4$

$y_ö = \left(\frac{1}{8}x^2 - \frac{x}{4}\right)e^{2x}$

$y = c_1 x + c_2 + c_3 e^{2x} + \left(\frac{1}{8}x^2 - \frac{x}{4}\right)e^{2x}$