Çalışma Soruları

1) $y\sqrt{1-\ln^2 y}\,\sin x\,dx + (1+\cos^2 x)\,dy = 0$ d.d çözünüz.

$$\underbrace{\int \frac{\sin x}{1+\cos^2 x}\,dx}_{} + \underbrace{\int \frac{dy}{y\sqrt{1-\ln^2 y}}}_{} = \int 0$$

$\cos x = u$ $\underbrace{-\sin x\,dx = du}_{}$ $\int \frac{-du}{1+u^2} = \operatorname{arccot} u$ $= \operatorname{arccot}(\cos x)$	$\left.\begin{array}{l}\ln y = t \\ \frac{dy}{y} = dt\end{array}\right\}$ $\int \frac{dt}{\sqrt{1-t^2}} = \arcsin t$ $= \arcsin(\ln y)$

sonuç:

$\operatorname{arccot}(\cos x) + \arcsin(\ln y) = c$

2) $2y\,dy + 4x^3\sqrt{4-y^4}\,dx = 0$ d.d çözünüz.

$$\underbrace{\int \frac{2y\,dy}{\sqrt{4-y^4}}}_{} + \int 4x^3\,dx = 0$$

$$\left.\begin{array}{l} y^2 = t \\ 2y\,dy = dt \end{array}\right\} \int \frac{dt}{\sqrt{4-t^2}} = \int \frac{\frac{1}{2}dt}{\sqrt{1-\left(\frac{t}{2}\right)^2}} = \frac{1}{2}\arcsin\left(\frac{t}{2}\right) = \frac{1}{2}\arcsin\left(\frac{y^2}{2}\right)$$

sonuç: $\frac{1}{2}\arcsin\left(\frac{y^2}{2}\right) + x^4 = c$

3) $y^2 y' - \sqrt{x^4 - x^4 y^6} = 0$

$$y^2\,dy - x^2\sqrt{1-y^6}\,dx = 0 \Rightarrow \int \frac{y^2\,dy}{\sqrt{1-y^6}} - \int x^2\,dx = \int 0$$

$$\left.\begin{array}{l} y^3 = t \\ 3y^2\,dy = dt \end{array}\right\} \int \frac{dt}{3\sqrt{1-t^2}} = \frac{1}{3}\arcsin t = \frac{1}{3}\arcsin(y^3)$$

sonuç: $\frac{1}{3}\arcsin(y^3) - \frac{x^3}{3} = c$

4) $y' = \dfrac{y}{x+\sqrt{xy}}$ d.d çözünüz.

$(x+\sqrt{xy})dy - y\,dx = 0$ Homojen D.D

$\left.\begin{aligned} y &= ux \\ dy &= u\,dx + x\,du \end{aligned}\right\}$ $(x+\sqrt{ux^2})(u\,dx+x\,du) - ux\,dx = 0$

$(\cancel{ux} + ux\sqrt{u} - \cancel{ux})dx + x(x+x\sqrt{u})du = 0$

$\dfrac{ux\sqrt{u}\,dx}{u\sqrt{u}\cdot x^2} + \dfrac{x^2(1+\sqrt{u})du}{u\sqrt{u}\cdot x^2} = 0 \Bigg\} \displaystyle\int\frac{dx}{x} + \int\frac{(1+\sqrt{u})}{u\sqrt{u}}du = \int 0$

$\ln x + \int u^{-\frac{3}{2}}du + \int \frac{1}{u}du = c \Rightarrow \ln x - \dfrac{2}{\sqrt{u}} + \ln u = c$

$\ln x - \dfrac{2}{\sqrt{\frac{y}{x}}} + \ln\left(\dfrac{y}{x}\right) = c$

5) $xy\,dy = (x^2+xy+y^2)dx$ diferansiyel denkleminin $x=1$ için $y=0$ koşuluna uyan özel çözümünü bulunuz.

$\left.\begin{aligned} y &= ux \\ dy &= u\,dx + x\,du \end{aligned}\right\}$ $ux^2(u\,dx + x\,du) = (x^2+ux^2+u^2x^2)dx$

$(\cancel{u^2x^2} - x^2 - ux^2 - \cancel{u^2x^2})dx + ux^3du = 0$

$\dfrac{-x^2(1+u)dx}{(1+u)x^3} + \dfrac{ux^3du}{(1+u)x^3} = 0 \Rightarrow -\displaystyle\int\frac{dx}{x} + \int\frac{u\,du}{1+u} = \int 0$

$-\ln x + \int\left(1 - \dfrac{1}{1+u}\right)du = c$

$-\ln x + u - \ln(u+1) = c \Rightarrow -\ln x + \dfrac{y}{x} - \ln\left(\dfrac{y}{x}+1\right) = c$

$x=1,\ y=0$ için $-\ln 1 + 0 - \ln 1 = c \Rightarrow c=0$

Sonuç: $-\ln x + \dfrac{y}{x} - \ln\left(\dfrac{y}{x}+1\right) = 0$

7) $dy + (y \cdot \cot x - 5e^{\cos x})dx = 0$ diferansiyel denklemini tam diferansiyel denklem haline getirerek, $x = \frac{\pi}{2}$ için $y = 0$ koşuluna uyan özel çözümünü bulunuz.

$$\frac{\partial m}{\partial y} = \cot x \neq \frac{\partial N}{\partial x} = 0$$

1°) $\mu = \mu(x)$ olsun.

$$\ln \mu = \int \frac{\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}}{N} dx = \int \frac{\cot x - 0}{1} dx = \ln(\sin x) \Rightarrow \boxed{\mu = \sin x}$$

$\sin x\, dy + (y \cos x - 5 \sin x\, e^{\cos x})dx = 0$ Tam D.D

$$\frac{\partial f}{\partial x} = y \cos x - 5 \sin x \cdot e^{\cos x}$$

$$\frac{\partial f}{\partial y} = \sin x$$

$$\frac{\partial f}{\partial y} = \sin x \Rightarrow f(x,y) = y \sin x + h(x)$$

$$\frac{\partial f}{\partial x} = \cancel{y \cos x} + \frac{dh}{dx} = \cancel{y \cos x} - 5 \sin x\, e^{\cos x}$$

$$\int dh = \underbrace{\int -5 \sin x\, e^{\cos x} dx}$$

$\cos x = u$

$-\sin x\, dx = du$

$$5 \int e^u du = 5e^u = 5e^{\cos x} + c$$

$$h(x) = 5e^{\cos x} + c$$

$$f(x,y) = y \sin x + 5e^{\cos x} + c = k$$

6) $y' = \dfrac{9x+2y-7}{2x-y-3}$ d.d çözünüz.

$(2x-y-3)dy - (9x+2y-7)dx = 0$

$$\left.\begin{array}{ll} x = x_1 + h & , dx = dx_1 \\ y = y_1 + k & , dy = dy_1 \end{array}\right\}$$

$(2x_1+2h-y_1-k-3)dy_1 - (9x_1+9h+2y_1+2k-7)dx_1 = 0$

$$\left.\begin{array}{l} 2/\ 2h-k=3 \\ 9h+2k=7 \end{array}\right\}$$

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$13h = 13$

$h = 1 \Rightarrow k = -1$

$$\left.\begin{array}{ll} x = x_1 + 1 & \Rightarrow \boxed{x_1 = x-1} \\ y = y_1 - 1 & \boxed{y_1 = y+1} \end{array}\right.$$

$(2x_1 - y_1)dy_1 - (9x_1+2y_1)dx_1 = 0$

$$\left.\begin{array}{l} y_1 = ux_1 \\ dy_1 = udx_1 + x_1 du \end{array}\right\} \begin{array}{l} (2x_1 - ux_1)(udx_1 + x_1 du) - (9x_1 + 2ux_1)dx_1 = 0 \\ (\cancel{2ux_1} - u^2x_1 - 9x_1 - \cancel{2ux_1})dx_1 + x_1^2(2-u)du = 0 \end{array}$$

$$\frac{-x_1\cancel{(u^2+9)}dx_1}{\cancel{(u^2+9)}x_1^2} + \frac{\cancel{x_1^2}(2-u)du}{(u^2+9)\cancel{x_1^2}} = 0$$

$$-\int\frac{dx_1}{x_1} + \int\frac{2du}{u^2+9} - \int\frac{udu}{u^2+9} = \int 0$$

$$-\ln x_1 + \frac{2}{3}\arctan\frac{u}{3} - \frac{1}{2}\ln(u^2+9) = c$$

$$-\ln x_1 + \frac{2}{3}\arctan\left(\frac{y_1}{3x_1}\right) - \frac{1}{2}\ln\left(\left(\frac{y_1}{x_1}\right)^2 + 9\right) = c$$

$$-\ln(x-1) + \frac{2}{3}\arctan\left(\frac{y+1}{3(x-1)}\right) - \frac{1}{2}\ln\left(\left(\frac{y+1}{x-1}\right)^2 + 9\right) = c$$

8) $y' = \frac{y}{x} + e^{\frac{y}{x}}$ diferansiyel denklemini çözünüz.

$$\left.\begin{aligned} \frac{y}{x} &= u \Rightarrow y = ux \\ dy &= u\,dx + x\,du \end{aligned}\right\} \qquad dy = \left(\frac{y}{x} + e^{\frac{y}{x}}\right)dx$$

$$u\,dx + x\,du = (u + e^u)dx \Rightarrow x\,du = (\cancel{u} + e^u - \cancel{u})dx$$

$$\frac{x\,du}{e^u \cdot x} = \frac{e^u\,dx}{e^u \cdot x} \Rightarrow e^{-u}du = \frac{dx}{x}$$

$$-e^{-u} = \ln x + c \Rightarrow -e^{-\frac{y}{x}} = \ln x + c$$

9) $\cos y\,dx + (x\sin y - 2)dy = 0$ diferansiyel denklemini çözünüz.

$$\frac{\partial m}{\partial y} = -\sin y \neq \frac{\partial N}{\partial x} = \sin y$$

1°) $\mu = \mu(x)$ olsun.

$$\ln \mu = \int \frac{m_y - N_x}{N}dx = \int \frac{-\sin y - \sin y}{(x\sin y - 2)}dx \neq \mu(x)$$

2°) $\mu = \mu(y)$ olsun.

$$\ln \mu = \int \frac{N_x - m_y}{m}dy = \int \frac{\sin y + \sin y}{\cos y}dy = 2\int \frac{\sin y}{\cos y}dy = -2\ln(\cos y)$$

$$\Rightarrow \mu = \frac{1}{\cos^2 y}$$

$$\frac{1}{\cos y}dx + \left(\frac{x\sin y}{\cos^2 y} - \frac{2}{\cos^2 y}\right)dy = 0 \qquad \text{Tam D.D}$$

$$\frac{\partial f}{\partial x} = \frac{1}{\cos y} \quad ; \quad \frac{\partial f}{\partial y} = \frac{x\sin y}{\cos^2 y} - \frac{2}{\cos^2 y}$$

$$f(x,y)=\frac{x}{\cos y}+h(y)$$

$$\frac{\partial f}{\partial y}=\cancel{\frac{+x\sin y}{\cos^2 y}}+\frac{dh}{dy}=\cancel{\frac{x\sin y}{\cos^2 y}}-\frac{2}{\cos^2 y}$$

$$\Rightarrow dh=-2\int\frac{dy}{\cos^2 y}=-2\int\sec^2 y\,dy=-2\tan y+c$$

$$f(x,y)=\frac{x}{\cos y}-2\tan y+c=k$$

10) $y^2dx+x(x-y)dy=0$ diferansiyel denkleminin $x=-1$ için $y=1$ koşuluna uyan özel çözümünü bulunuz.

$$\left.\begin{aligned} y&=ux\\ dy&=udx+xdu\end{aligned}\right\}\ u^2x^2dx+x(x-ux)(udx+xdu)=0$$

$$(\cancel{u^2x^2}+ux^2-\cancel{u^2x^2})\,dx+x^3(1-u)\,du=0$$

$$\frac{ux^2dx}{u.x^3}+\frac{x^3(1-u)\,du}{u.x^3}=0$$

$$\int\frac{dx}{x}+\int\left(\frac{1-u}{u}\right)du=\int 0$$

$$\ln x+\ln u-u=\ln c\Rightarrow\frac{ux}{c}=e^{u}\Rightarrow\frac{y}{x}\cdot\frac{x}{c}=e^{y/x}$$

$$y=ce^{y/x}$$

$x=-1,\ y=1$ için $\quad 1=c.e^{-1}\Rightarrow c=e$

$$\boxed{y=e^{\frac{y}{x}+1}}$$