ÇOK DEĞİŞKENLİ FONKSİYONLAR (KISMİ TÜREV)

ALIŞTIRMALAR (Sayfa 16)

(1) Silindirik koordinatı $(3, \frac{\pi}{2}, 1)$ olan noktanın kartezyen koor. bulun.

$$x = r\cos\theta = 3.\cos\left(\frac{\pi}{2}\right) = 3.(0) = 0$$

$$y = r.\sin\theta = 3.\sin\left(\frac{\pi}{2}\right) = 3(1) = 3$$

$$z = 1$$

$*$ $(r, \theta, z) = (3, \frac{\pi}{2}, 1) \longleftrightarrow (x, y, z) = (0, 3, 1)$

(2) Silindirik koordinatı $(4, -\frac{\pi}{3}, 5)$ olan noktanın kartezyen koordinatını bulun.

$$x = r.\cos\theta = 4.\cos\left(-\frac{\pi}{3}\right) = 4.\left(\frac{1}{2}\right) = 2$$

$$y = r.\sin\theta = 4.\sin\left(-\frac{\pi}{3}\right) = 4.\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

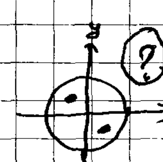
$$z = 5$$

$*$ $(r, \theta, z) = (4, -\frac{\pi}{3}, 5) \longleftrightarrow (x, y, z) = (2, -2\sqrt{3}, 5)$

(3) Kartezyen koordinatı $(1, -1, 4)$ olan noktanın silindirik koordinatını bulunuz.

$$r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-1}{1} = -1 \Longrightarrow$$



$? \rightarrow \left\{ \begin{matrix} (1,-1) \\ 4.\text{Bölgede} \end{matrix} \right\} \Rightarrow \theta = \frac{7\pi}{4}$

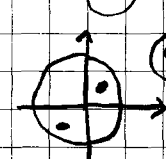
$$z = 4$$

$*$ $(x, y, z) = (1, -1, 4) \longleftrightarrow (r, \theta, z) = (\sqrt{2}, \frac{7\pi}{4}, 4)$

(4) Kartezyen koordinatı $(-1, -\sqrt{3}, 2)$ olan noktanın silindirik koordinatını bulunuz.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3} \Longrightarrow$$



$? \rightarrow \left\{ \begin{matrix} (-1,-\sqrt{3}) \\ 3.\text{Bölgede} \end{matrix} \right\} \rightarrow \theta = \frac{4\pi}{3}$

$$z = 2$$

$*$ $(x, y, z) = (-1, -\sqrt{3}, 2) \longleftrightarrow (r, \theta, z) = (2, \frac{4\pi}{3}, 2)$

(5) Küresel koordinatı $(2, \frac{\pi}{3}, \frac{\pi}{4})$ olan noktanın kartezyen koordinatını bulunuz.

$$x = \rho\cos\theta\sin\phi = 2.\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = 2.\left(\frac{1}{2}\right).\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$

$$y = \rho\sin\theta\sin\phi = 2.\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{2}$$

$$z = \rho\cos\phi = 2.\cos\left(\frac{\pi}{4}\right) = 2.\frac{\sqrt{2}}{2} = \sqrt{2}$$

$*$ $(\rho, \theta, \phi) = (2, \frac{\pi}{3}, \frac{\pi}{4}) \longleftrightarrow (x, y, z) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2}, \sqrt{2})$

⑥ Küresel koordinatı $(1,0,0)$ olan noktanın kartezyen koordinatını bulunuz.

$$x = \rho\cos\theta\sin\phi = 1.\cos(0).\sin(0) = 1.(1).(0) = 0$$

$$y = \rho\sin\theta\sin\phi = 1.\sin(0).\sin(0) = 1.(0).(0) = 0$$

$$z = \rho.\cos\phi = 1.\cos(0) = 1.(1) = 1$$

$$* \quad (\rho,\theta,\phi) = (1,0,0) \longleftrightarrow (x,y,z) = (0,0,1)$$

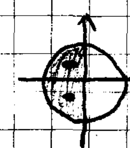
⑦ Kartezyen koordinatı $(-3,0,0)$ olan noktanın küresel koordinatını bulunuz.

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{(-3)^2+0^2+0^2} = \sqrt{9} = 3$$

$$z = \rho\cos\phi \to \cos\phi = \frac{z}{\rho} = \frac{0}{3} = 0 \Rightarrow \phi = \frac{\pi}{2} \qquad (0 \le \phi \le \pi)$$

$$x = \rho\cos\theta\sin\phi \Rightarrow -3 = 3.\cos\theta.\sin\left(\frac{\pi}{2}\right) \to \cos\theta = \frac{x}{\rho\sin\phi} = \frac{-3}{3.(1)} = -1$$

$\cos\theta = -1 \Rightarrow$



$? \to \left\{ \begin{array}{l} (-3,0) \\ \text{Negatif} \\ x\text{-ekseni} \\ \text{üzerinde} \end{array} \right\} \Rightarrow \theta = \pi$

$$* \quad (x,y,z) = (-3,0,0) \longleftrightarrow (\rho,\theta,\phi) = \left(3,\pi,\frac{\pi}{2}\right)$$

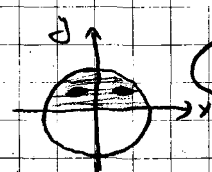
⑧ Kartezyen Koordinatı $(1,\sqrt{3},2)$ olan noktanın küresel koordinatını bulunuz

$$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{1^2+(\sqrt{3})^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

$$z = \rho.\cos\phi \Rightarrow \cos\phi = \frac{z}{\rho} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \phi = \frac{\pi}{4}$$

$$y = \rho\sin\theta\sin\phi \Rightarrow \sin\theta = \frac{y}{\rho\sin\phi} = \frac{\sqrt{3}}{2\sqrt{2}\sin\frac{\pi}{4}} = \frac{\sqrt{3}}{2\sqrt{2}.\left(\frac{1}{\sqrt{2}}\right)} = \frac{\sqrt{3}}{2}$$

$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow$



$? \to \left\{ \begin{array}{l} (1,\sqrt{3}) \\ 1.\text{ Bölgede} \end{array} \right\} \Rightarrow \theta = \frac{\pi}{3}$ olur

$$* \quad (x,y,z) = (1,\sqrt{3},2) \longleftrightarrow (\rho,\theta,\phi) = \left(2\sqrt{2},\frac{\pi}{3},\frac{\pi}{4}\right)$$

⑨ a) $r = 3 \to \sqrt{x^2+y^2} = 3 \longrightarrow x^2+y^2 = 3^2 \to$ x,y düzlemindeki yarıçapı 3 birim merkezi $(0,0)$ olan bu çemberin z-eksenine paralel olarak taradığı <u>dairesel silindirin</u> denklemidir.

b) $\rho = 3 \to \sqrt{x^2+y^2+z^2} = 3 \to x^2+y^2+z^2 = 3^2 \to$ Yarıçapı 3 birim merkezi $(0,0,0)$ noktası olan <u>Küre yüzeyinin</u> denklemidir.

c) $\phi = \frac{\pi}{3} \to z = \rho\cos\phi \to \frac{z}{\rho} = \cos\frac{\pi}{3} \to \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{1}{2}$

$(2z)^2 = x^2+y^2+z^2 \Rightarrow 3z^2 = x^2+y^2 \longrightarrow z^2 = \frac{x^2}{3}+\frac{y^2}{3} \to z = \sqrt{\frac{x^2}{3}+\frac{y^2}{3}}$

Pozitif z-ekseni tarafındaki <u>dairesel konidir</u>.

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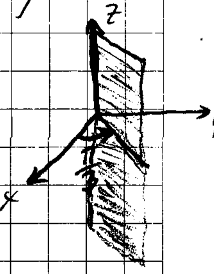
d) $\theta = \frac{\pi}{3} \longrightarrow$ Silindirik koordinatlarda düşününce r, z her türlü değeri alabileceği için, Pozitif x-ekseni ile $\theta = \frac{\pi}{3}$ açı yapan yarım düzlem denklemidir. (Bu düzlem z-ekseninin diğer tarafına geçmez)

$\theta = \frac{\pi}{3} \rightarrow$ Küresel koordinatlarda düşününce de ρ ve ϕ $(0 \leq \rho < \infty,\ 0 \leq \phi \leq \pi)$ her değeri alacağı için, Pozitif x-ekseni ile $\theta = \frac{\pi}{3}$ açı yapan yarım düzlem denklemidir.



(10) $z = r^2$ yüzeyinin cinsini belirleyiniz.

$z = r^2 \longrightarrow z = x^2 + y^2 \longrightarrow 0 \leq z < \infty$

pozitif z-ekseni tarafındaki <u>dairesel paraboloit</u>.

Not: $z = -x^2 - y^2 \longrightarrow$ negatif z-ekseni tarafındaki <u>dairesel paraboloit</u>.

(11) $\rho \sin\phi = 2$ yüzeyinin cinsini belirleyiniz.

$\rho \sin\phi = 2 \longrightarrow \sqrt{x^2+y^2+z^2} \cdot \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}} = 2 \Rightarrow \sqrt{x^2+y^2} = 2 \rightarrow \boxed{x^2+y^2 = 2^2}$

Not: $y = \rho\sin\theta \cdot \sin\phi \Rightarrow \sin\phi = \frac{y}{\rho\sin\theta} = \frac{y}{\sqrt{x^2+y^2+z^2} \cdot \frac{y}{\sqrt{x^2+y^2}}} = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$ $\left(\begin{array}{l} y = r.\sin\theta \\ \sin\theta = \frac{y}{\sqrt{x^2+y^2}} \end{array} \right)$

$\rho \sin\phi = 2 \longleftrightarrow x^2+y^2 = 2^2$

x, y düzleminde $(x^2+y^2 = 2^2)$ merkezi $(0,0)$ yarıçapı $= 2$ br dan çemberin z-eksenine paralel kaydırılmasıyla oluşan <u>dairesel silindir</u> denklemidir

(12) $r = 2.\cos\theta \longrightarrow \sqrt{x^2+y^2} = 2 \frac{x}{\sqrt{x^2+y^2}} \longrightarrow x^2+y^2 = 2x \rightarrow x^2-2x+1-1+y^2 = 0$

$\boxed{(x-1)^2 + y^2 = 1^2}$

Not: $x = r.\cos\theta \Rightarrow \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$

$r = 2\cos\theta \longleftrightarrow (x-1)^2 + y^2 = 1^2$

x, y düzleminde $\left((x-1)^2+y^2 = 1^2\right)$ merkezi $(1,0)$ ve yarıçapı $= 1$ br olan çemberin z-eksenine paralel kaydırılması ile oluşan dairesel silindir denklemidir.

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(13) $\rho = 2\cos\phi$ yüzeyinin cinsini belirleyiniz.

$$\rho = 2\cos\phi \rightarrow \sqrt{x^2+y^2+z^2} = 2\frac{z}{\sqrt{x^2+y^2+z^2}} \rightarrow x^2+y^2+z^2 = 2z$$

$$z = \rho.\cos\phi \Rightarrow \cos\phi = \frac{z}{\rho} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$x^2+y^2+z^2-2z+1-1=0$$

$$x^2+y^2+(z-1)^2 = 1^2$$

$$\rho = 2\cos\phi \longleftrightarrow x^2+y^2+(z-1)^2 = 1^2$$

* Merkezi $(0,0,1)$ ve yarıçapı $=1$ br olan küre denklemi.

(14) $r^2-2z^2=4$ yüzeyinin cinsini belirleyiniz.

$$r^2-2z^2=4 \rightarrow x^2+y^2-2z^2=4 \rightarrow \frac{x^2}{2^2}+\frac{y^2}{2^2}-\frac{z^2}{(\sqrt{2})^2}=1$$

$$r^2-2z^2=4 \longleftrightarrow \frac{x^2}{2^2}+\frac{y^2}{2^2}-\frac{z^2}{(\sqrt{2})^2}=1$$

* z-ekseni boyunca uzanan Tek Parçalı dairesel Hiperboloit.

(15) $x^2+y^2-z^2=16$ denklemini silindirik ve küresel koordinatlarda yaz.

Silindirik Koordinatlarda

$$(x^2+y^2)-z^2=16 \longleftrightarrow r^2-z^2=16 \text{ olur.}$$

Küresel koordinatlarda

$$x^2+y^2-z^2=16 \longrightarrow (x^2+y^2+z^2)-2z^2=16 \rightarrow \rho^2-2(\rho^2\cos^2\phi)=16$$

$$\rho^2(1-2\cos^2\phi)=16$$

$$x^2+y^2-z^2=16 \longleftrightarrow \rho^2(1-2\cos^2\phi)=16$$

(16) $x^2+y^2=2y$ denklemini silindirik ve küresel koordinatlarda yazınız.

Silindirik Koordinatlar

$$x^2+y^2=2y \longrightarrow r^2=2(r.\sin\theta) \longrightarrow r=2\sin\theta$$

Küresel Koordinatlar

$$x^2+y^2=2y \longrightarrow x^2+y^2-2y+1-1=0 \longrightarrow x^2+(y-1)^2=1$$

$$\rho^2\cos^2\theta\sin^2\phi+\rho^2\sin^2\theta\sin^2\phi = 2.\rho\sin\theta\sin\phi$$

$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta) = 2\rho\sin\theta\sin\phi$$

$$\rho\sin\phi = 2\sin\theta \longrightarrow \rho = \frac{2.\sin\theta}{\sin\phi} \text{ bulunur.}$$

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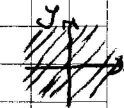
PROBLEMLER: Çok Değişkenli Fonksiyonlar (Kısmi Türev)

① Verilen f fonksiyonlarının En Geniş Tanım Kümesini bulunuz, Grafikle gösterin.

(1-1) $f(x,y) = 4-3x-2y$

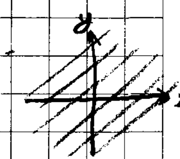


* Tanımsız olma durumu yoktur. E.G.T.K = xy-düzlemi

(1-2) $f(x,y)=\sqrt{x^2+2y^2}$

* Karekoklü ifadenin tanımlı olması için $\boxed{x^2+2y^2 \geq 0}$ olmalı

* $x^2 \geq 0$ ve $2y^2 \geq 0 \longrightarrow x^2+2y^2 \geq 0$ dır her zaman.

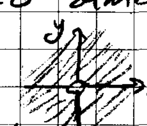


* Tanımsız olma durumu yoktur, E.G.T.K = xy-düzlemi

(1-3) $f(x,y) = \dfrac{1}{x^2+y^2}$

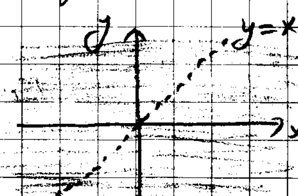
* kesirli ifadenin tanımlı olması için payda $x^2+y^2 \neq 0$ olmalı

$x^2+y^2=0 \Rightarrow x=0$ ve $y=0$ olmalıdır.



* E.G.T.K = xy-düzleminde $(0,0)$'dan farklı tüm noktalar

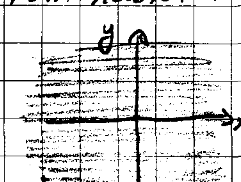
(1-4) $f(x,y)=\dfrac{1}{x-y}$



* $x-y \neq 0 \Rightarrow y \neq x$ olmalı.

* xy-düzleminde $y=x$ doğrusundan farklı tüm noktalar

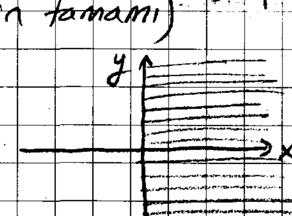
(1-5) $f(x,y)=\sqrt[3]{y-x^2}$



* Tek-kokl̈ü ifadeler her zaman tanımlıdır.

* E.G.T.K = $R \times R$ dir. (xy-düzleminin tamamı)

(1-6) $f(x,y)=\sqrt{2x}+\sqrt[3]{3y}$



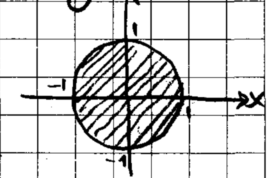
* $2x \geq 0 \rightarrow x \geq 0$ olmalı.

* E.G.T.K = $x \geq 0$ sağ yarı-düzlem

(1-7) $f(x,y)= \arcsin(x^2+y^2)$

* arcsin ve arccos'lu ifadelerin tanımlı olması için benzer şekilde düşünülür.

$-1 \leq x^2+y^2 \leq 1$ olmalı $\longrightarrow x^2+y^2 \leq 1$

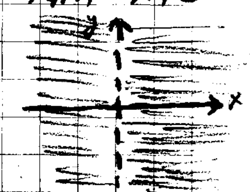


* E.G.T.K = $x^2+y^2 \leq 1$ çemberi ve iç bölgesi

(1-8) $f(x,y) = \arctan\left(\frac{y}{x}\right)$

* arctan ve arccot 'lı ifadeler her zaman tanımlıdır. Ancak buradaki kesirli ifadenin tanımlı olması için $x \neq 0$ olmalı.

* E.G.T.K = (x = 0 doğrusu haricindeki tüm noktalar)
(y-ekseni haricindeki tüm noktalar)

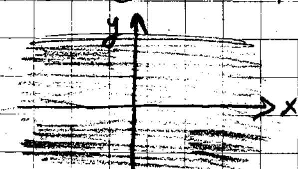


(1-9) $f(x,y) = e^{-x^2-y^2}$

$$f(x,y) = e^{-(x^2+y^2)} = \frac{1}{e^{x^2+y^2}}$$

* Tanımlı olması için payda 0 olmamalı. $e^{x^2+y^2}$ hiç bir zaman sıfır olamaz.

* E.G.T.K = xy-düzlemi dır.

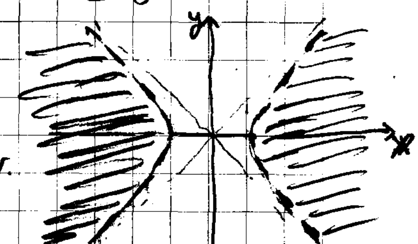


(1-10) $f(x,y) = \ln(x^2-y^2-1)$

* logaritmik bir ifadenin tanımlı olması için $\boxed{x^2-y^2-1>0}$ olmalı

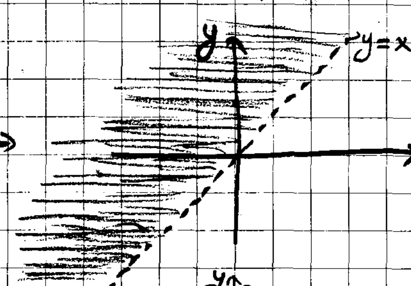
$x^2-y^2>1$ hiperbol grafiğidir.

Not: * $x^2-y^2=1$ hiperbolünde odaklar x-ekseni üzerinde
* $y^2-x^2=1$ hiperbolünde ise odaklar y-ekseni üzerindedir.



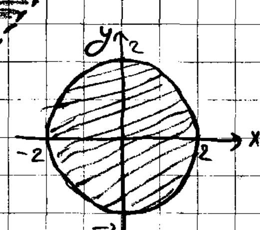
(1-11) $f(x,y) = \ln(y-x)$

* $y-x>0$ olmalı. $y>x \Rightarrow$



(1-12) $f(x,y) = \sqrt{4-x^2-y^2}$

* $4-x^2-y^2 \geq 0 \Rightarrow x^2+y^2 \leq 2^2 \Rightarrow$

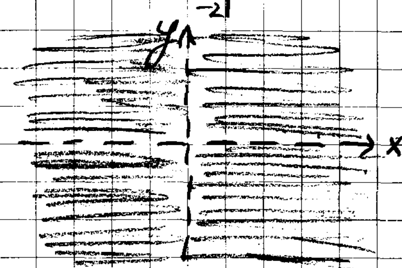


(1-13) $f(x,y) = \dfrac{1+\sin xy}{xy}$

* $x.y \neq 0$ olmalı

$x.y = 0 \Rightarrow x=0$ veya $y=0$

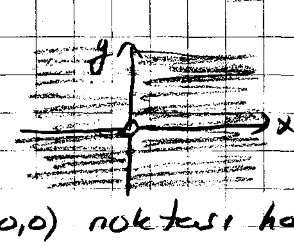
* E.G.T.K. = Eksenler haricindeki tüm noktalar,



(1-14) $f(x,y) = \dfrac{1+\sin xy}{x^2+y^2}$

* $x^2+y^2 \neq 0$ olmalı

$x^2+y^2=0 \Rightarrow x=0$ ve $y=0 \rightarrow (0,0)$ noktası hariç tüm noktalar.



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(1-15) $f(x,y)=\dfrac{x.y}{x^2-y^2}$

* $x^2-y^2\neq 0$ olmalı

$x^2-y^2=0 \Rightarrow x^2=y^2 \Rightarrow y=x$ veya $y=-x$

* E.G.T.K = $y=x$ ve $y=-x$ doğruları haricindeki tüm noktalar

(1-16) $f(x,y,z)=\dfrac{1}{\sqrt{z-x^2-y^2}}$

* $z-x^2-y^2>0$ olmalı

$z>x^2+y^2$

* E.G.T.K. = $z>x^2+y^2$ paraboloidinin iç bölgesindeki noktalar

(1-17) $f(x,y,z)=\arcsin\left(3-x^2-y^2-z^2\right)$

* $-1\leq 3-x^2-y^2-z^2\leq 1$ olmalı

$-1\leq -3+x^2+y^2+z^2\leq 1$

$2\leq x^2+y^2+z^2\leq 4$

* E.G.T.K. = Merkezleri orijinde ve yarıçapları $\sqrt{2}$, 2 olan küreler arasında kalan tüm noktaları

(1-18) $f(x,y,z)=\ln(x.y.z)$

* $x.y.z>0$ olmalı

* E.G.T.K. = Uzayda $x.y.z>0$ olan $\frac{1}{8}$ lik bölgenin 4 tanesi. x, y, z hepsinin pozitif olduğu bölge ile ikisinin negatif birinin de pozitif olduğu 3 bölge.

(1-19) $f(x,y,z)=\ln(z-x^2-y^2)$

* $z-x^2-y^2>0$ olmalı $\rightarrow$ $x^2+y^2<z$

* E.G.T.K = Uzayın $z>x^2+y^2$ paraboloidinin iç bölgesindeki tüm noktaları

(1-20) $f(x,y,z)=e^{\left(\frac{1}{x^2+y^2+z^2}\right)}$

* $x^2+y^2+z^2\neq 0$ olmalı

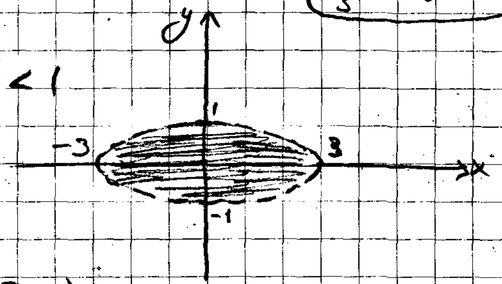
$x^2+y^2+z^2=0 \Rightarrow x=0 \wedge y=0 \wedge z=0$ olmalı

* E.G.T.K = Uzayın orijin (0,0,0) haricindeki tüm noktaları

(1-21) $f(x,y) = \ln(9-x^2-9y^2)$

* $9-x^2-9y^2>0 \Rightarrow x^2+9y^2<9 \Rightarrow \boxed{\frac{x^2}{3^2}+y^2<1}$ olmalı

* E.G.T.K $= \frac{x^2}{3^2}+y^2<1$
elipsinin iç bölgesi



(1-22) $f(x,y,z) = \arcsin(x^2+y^2-z)$

* $-1 \leq x^2+y^2-z \leq 1$ olmalı
$1 \leq x^2+y^2 \leq 3$

* E.G.T.K $= 1 \leq x^2+y^2 \leq 3$ orijin merkezli yarıçapı 1 ve $\sqrt{3}$ birim olan silindirler ve arasındaki tüm noktalar.

(1-23) $f(x,y,z) = \ln(16-4x^2-4y^2-z^2)$

* $16-4x^2-4y^2-z^2>0$ olmalı
$4x^2+4y^2+z^2<16 \longrightarrow \boxed{\frac{x^2}{2^2}+\frac{y^2}{2^2}+\frac{z^2}{4^2}<1}$

* E.G.T.k. = Orijin merkezli x-eksenini ∓ 2 de y-eksenini ∓ 2 de ve z-eksenini ∓ 4 te kesen elipsoidin iç bölgesi

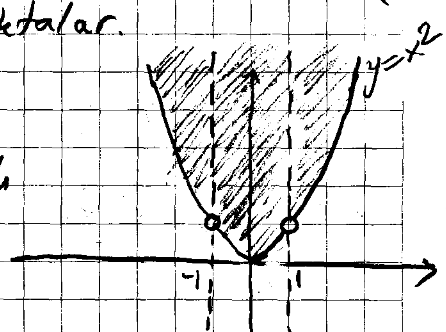
(1-24) $f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

* $1-x^2-y^2-z^2 \geq 0$ olmalı
$x^2+y^2+z^2 \leq 1$

* E.G.T.k. = Orijin merkezli yarıçapı 1 br olan küre ve iç bölgesindeki tüm noktalar.

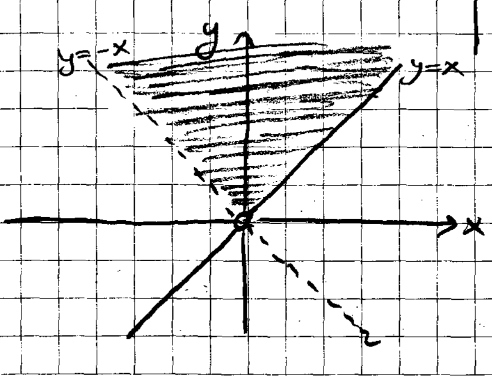
(1-25) $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$

* $y-x^2 \geq 0$ ve $1-x^2 \neq 0$ olmalı
$y \geq x^2 \wedge x^2 \neq 1$



(1-26) $f(x,y) = \sqrt{y-x} \cdot \ln(y+x)$

* $y-x \geq 0$ ve $y+x>0$
$y \geq x \wedge y>-x$



(2) Limitleri bulunuz veya mevcut olmadığını gösteriniz.

(2-1) $\lim\limits_{(x,y)\to(0,0)} \left(\ln\sqrt{1-x^2-y^2}\right) = \lim\limits_{r\to 0} \ln\sqrt{1-r^2} = \ln\sqrt{1} = \ln 1 = 0$ bulunur.

(2-2) $\lim\limits_{(x,y)\to(1,-1)} \left(\arcsin\left(\frac{xy}{\sqrt{x^2+y^2}}\right)\right) = \lim\limits_{r\to 0} \left(\arcsin\left(\frac{(1+r\cos\theta)(-1+r\sin\theta)}{\sqrt{(1+r\cos\theta)^2+(-1+r\sin\theta)^2}}\right)\right)$

$= \lim\limits_{r\to 0}\left[\arcsin\left(\frac{-1+r\sin\theta-r\cos\theta+r^2\sin\theta\cos\theta}{\sqrt{1+2r\cos\theta+r^2\cos^2\theta+1-2r\sin\theta+r^2\sin^2\theta}}\right)\right] = \lim\limits_{r\to 0}\left[\arcsin\left(\frac{-1}{\sqrt{2}}\right)\right] = \arcsin\left(-\frac{1}{\sqrt{2}}\right)$

$= -\frac{\pi}{4}$ olur

(2-3) $\lim\limits_{(x,y)\to(1,1)} \left(\frac{1-xy}{1+xy}\right) = \lim\limits_{r\to 0}\left[\frac{1-(1+r\cos\theta)(1+r\sin\theta)}{1+(1+r\cos\theta)(1+r\sin\theta)}\right] = \frac{1-1}{1+1} = \frac{0}{2} = 0$ olur

(2-4) $\lim\limits_{(x,y)\to(2,-2)} \left[\frac{4-xy}{4+xy}\right] = \lim\limits_{r\to 0}\left[\frac{4-(2+r\cos\theta)(-2+r\sin\theta)}{4+(2+r\cos\theta)(-2+r\sin\theta)}\right] =$

$= \lim\limits_{r\to 0}\left[\frac{4-(-4+2r\sin\theta-2r\cos\theta+r^2\sin\theta\cos\theta)}{4+(-4+2r\sin\theta-2r\cos\theta+r^2\sin\theta\cos\theta)}\right] = \lim\limits_{r\to 0}\left[\frac{8-2r\sin\theta+2r\cos\theta-r^2\sin\theta\cos\theta}{2r\sin\theta-2r\cos\theta+r^2\sin\theta\cos\theta}\right]$

$= \lim\limits_{r\to 0}\left[\frac{8}{r(2\sin\theta-2\cos\theta+r\sin\theta\cos\theta)}\right] = \frac{8}{0^+} \neq \frac{8}{0^-}$ Limiti Yoktur. ($+\infty$, $-\infty$ farklı iki limit)

(2-5) $\lim\limits_{(x,y)\to(0,0)} \left[\frac{\sin xy}{xy}\right] = \lim\limits_{r\to 0}\left[\frac{\sin(r^2\sin\theta\cos\theta)}{(r^2\sin\theta\cos\theta)}\right] = \lim\limits_{u\to 0}\frac{\sin u}{u} = 1$

(2-6) $\lim\limits_{(x,y)\to(0,0)} \left[\arctan\left(-\frac{1}{x^2+y^2}\right)\right] = \lim\limits_{r\to 0}\left[\arctan\left(-\frac{1}{r^2}\right)\right] = \arctan(-\infty) = -\frac{\pi}{2}$

(2-7) $\lim\limits_{(x,y)\to(0,0)} \left[\frac{y^4}{x^4+3y^4}\right] = \lim\limits_{r\to 0}\left[\frac{r^4\sin^4\theta}{r^4(\cos^4\theta+3\sin^4\theta)}\right] = \frac{\sin^4\theta}{\cos^4\theta+3\sin^4\theta}$

Açı değiştikçe farklı limit değerleri bulunur. Limit varsa tek olmak zorundadır. Bu nedenle Limit Yoktur.

(2-8) $\lim\limits_{(x,y)\to(0,0)} \left[\frac{x^2+\sin^2 y}{x^2+2y^2}\right] = \lim\limits_{r\to 0}\left[\frac{r^2\cos^2\theta+\sin^2(r\sin\theta)}{r^2(\cos^2\theta+2\sin^2\theta)}\right] = \left(\frac{0}{0}\right)$ L'hospital

$= \lim\limits_{r\to 0}\left[\frac{2r\cos^2\theta+\sin\theta.\sin^2(r.\sin\theta)}{2r(\cos^2\theta+2\sin^2\theta)}\right] = \left(\frac{0}{0}\right)$ L'hospital

$= \lim\limits_{r\to 0}\left[\frac{2.\cos^2\theta+\sin^2\theta.\sin^2(r.\sin\theta)}{2.(\cos^2\theta+2\sin^2\theta)}\right] = \frac{2\cos^2\theta}{2(\cos^2\theta+2\sin^2\theta)}$

Açı değiştikçe farklı limit değerleri bulunacağından Limit Yoktur.

(2-9) $\lim\limits_{(x,y)\to(0,0)} \left[\frac{xy\cos y}{3x^2+y^2}\right] = \lim\limits_{r\to 0}\left[\frac{r^2.\cos\theta\sin\theta.\cos(r.\sin\theta)}{r^2(3\cos^2\theta+\sin^2\theta)}\right] = \lim\limits_{r\to 0}\left[\frac{\cos\theta\sin\theta.\cos(r.\sin\theta)}{3.\cos^2\theta+\sin^2\theta}\right]$

$= \lim\limits_{r\to 0}\left[\frac{\sin\theta\cos\theta}{3\cos^2\theta+\sin^2\theta}\right] = \frac{\sin\theta.\cos\theta}{3\cos^2\theta+\sin^2\theta}$ (farklı limit değerlerine ulaşılır.) Limit Yoktur

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(2-10) $\lim_{(x,y)\to(0,0)}\left[\frac{6x^3y}{2x^4+y^4}\right]=\lim_{r\to 0}\left[\frac{6.r^4\cos^3\theta\sin\theta}{r^4(2\cos^4\theta+\sin^4\theta)}\right]=\lim_{r\to 0}\left[\frac{6.\cos^3\theta.\sin\theta}{2\cos^4\theta+\sin^4\theta}\right]$

$=\frac{6.\cos^3\theta\sin\theta}{2.\cos^4\theta+\sin^4\theta}$ θ açısına bağlı olarak limit değişecektir.

LiMiT YOK

(2-11) $\lim_{(x,y)\to(0,0)}\left[\frac{x.y}{\sqrt{x^2+y^2}}\right]=\lim_{r\to 0}\left[\frac{r^2.\cos\theta.\sin\theta}{r}\right]=\lim_{r\to 0}\left(\frac{r.\cos\theta.\sin\theta}{1}\right)=0$ bulunur.

(2-12) $\lim_{(x,y)\to(0,0)}\left[\frac{x^2.\sin^2y}{x^2+2y^2}\right]=\lim_{r\to 0}\left[\frac{r^2.\cos^2\theta.\sin^2(r.\sin\theta)}{r^2(\cos^2\theta+2\sin^2\theta)}\right]=\lim_{r\to 0}\left[\frac{\cos^2\theta.\sin^2(r.\sin\theta)}{\cos^2\theta+2\sin^2\theta}\right]$

$=\frac{0}{\cos^2\theta+2\sin^2\theta}=0$

$\cos^2\theta+2.\sin^2\theta=0$
$1+2\tan^2\theta=0\Rightarrow\tan^2\theta=-\frac{1}{2}\Rightarrow$ Çözüm $=\emptyset$ $\Rightarrow\left(\frac{0}{0}\right)$ belirsiz hali oluşmaz.

Bu nedenle bu fonksiyonun limiti vardır.

(2-13) $\lim_{(x,y)\to(0,0)}\left[\frac{x^2ye^y}{x^4+4y^2}\right]=\lim_{r\to 0}\left[\frac{r^3.\cos^2\theta\sin\theta\, e^{r.\sin\theta}}{r^2(r^2\cos^2\theta+4\sin^2\theta)}\right]=\lim_{r\to 0}\left[\frac{r\cos^2\theta\sin\theta.e^{r.\sin\theta}}{r^2\cos^2\theta+4\sin^2\theta}\right]$

$=\frac{0}{4.\sin^2\theta}=0$ ve $\left(\frac{0}{0}\right)$ belirsiz hali de olabilir.

Farklı iki durumla karşılaşılması sebebiyle Limit Yoktur.

(2-14) $\lim_{(x,y)\to(0,0)}\left[\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\right]=\lim_{r\to 0}\left[\frac{r^2}{\sqrt{r^2+1}-1}\right]=\left(\frac{0}{0}\right)$ L'hospital.

$=\lim_{r\to 0}\left[\frac{2r}{\frac{2r}{2\sqrt{r^2+1}}}\right]=\lim_{r\to 0}\left(2\sqrt{r^2+1}\right)=2.\sqrt{1}=2$ bulunur.

③ Aşağıdaki fonksiyonların kısmi türevlerini hesaplayınız.

(3-a) $z = x^3 + 4x^2y - 8y$

$z_x = 3x^2 + 4y.2x = 3x^2 + 8xy$ $\qquad$ $z_y = 0 + 4x^2 - 8 = 4x^2 - 8$

(3-b) $z = \dfrac{2x+y}{x}$

$z_x = \dfrac{(2)(x) - (2x+y)(1)}{x^2} = \dfrac{-y}{x^2}$ $\qquad$ $z_y = \dfrac{(1)(x) - (2x+y).(0)}{x^2} = \dfrac{1}{x}$

(3-c) $z = \ln\left(\tan\left(\frac{x}{y}\right)\right)$

$z_x = \dfrac{\frac{1}{y}.\sec^2\left(\frac{x}{y}\right)}{\tan\left(\frac{x}{y}\right)} = \dfrac{1}{y}.\dfrac{1}{\cos^2\left(\frac{x}{y}\right)}.\dfrac{\cos\left(\frac{x}{y}\right)}{\sin\left(\frac{x}{y}\right)} = \dfrac{1}{y}.\sec\left(\frac{x}{y}\right).\operatorname{cosec}\left(\frac{x}{y}\right)$

$z_y = \dfrac{x\left(-\frac{1}{y^2}\right).\sec^2\left(\frac{x}{y}\right)}{\tan\frac{x}{y}} = \dfrac{-x}{y}.\dfrac{1}{\cos^2\left(\frac{x}{y}\right)}.\dfrac{\cos\left(\frac{x}{y}\right)}{\sin\left(\frac{x}{y}\right)} = -\dfrac{x}{y}.\sec\left(\frac{x}{y}\right).\operatorname{cosec}\left(\frac{x}{y}\right)$

(3-d) $z = 10^{x^2+xy}$

$z_x = (2x+y)\left(10^{x^2+xy}\right)(\ln 10)$ $\qquad$ $z_y = (x).\left(10^{x^2+xy}\right).(\ln 10)$

(3-e) $z = e^{x^y}$

$z_x = \left(y\,x^{y-1}\right).\left(e^{x^y}\right)$ $\qquad$ $z_y = \left(1.x^y.\ln x\right).\left(e^{x^y}\right)$

(3-f) $z = \ln(xy^2 - 1)$

$z_x = \dfrac{y^2}{xy^2-1}$ $\qquad$ $z_y = \dfrac{2xy}{xy^2-1}$

(3-g) $z = \dfrac{\sin x}{y}$

$z_x = \dfrac{1}{y}.\cos x = \dfrac{\cos x}{y}$ $\qquad$ $z_y = \sin x\left(-\dfrac{1}{y^2}\right) = -\dfrac{\sin x}{y^2}$

(3-h) $z = x.e^y$

$z_x = e^y$ $\qquad$ $z_y = x.e^y$

(3-i) $z = \sqrt{(x^2-y^2)^3}$

$z_x = \dfrac{3(x^2-y^2)^2.(2x)}{2.\sqrt{(x^2-y^2)^3}} = \dfrac{3x.(x^2-y^2)^2}{\sqrt{(x^2-y^2)^3}}$ $\qquad$ $z_y = \dfrac{3(x^2-y^2)^2.(-2y)}{2.\sqrt{(x^2-y^2)^3}} = \dfrac{-3y(x^2-y^2)^2}{\sqrt{(x^2-y^2)^3}}$

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3-j

$$z = \ln\sqrt{x^2+y^2}$$

$$z_x = \frac{\frac{2x}{2\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} = \frac{x}{x^2+y^2} \qquad z_y = \frac{\frac{2y}{2\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} = \frac{y}{x^2+y^2}$$

3-k

$$z = \arctan\left[(\ln(xy))^2\right]$$

$$z_x = \frac{2.\ln(xy).\frac{y}{xy}}{1+(\ln(xy))^4} = \frac{2.\ln(xy)}{x\left(1+(\ln xy)^4\right)}$$

$$z_y = \frac{2.(\ln xy).\frac{x}{xy}}{1+(\ln xy)^4} = \frac{2.\ln(xy)}{y\left(1+(\ln xy)^4\right)}$$

3-l

$$z = e^{-2y}.\cos(4x)$$

$$z_x = e^{-2y}.4.(-\sin 4x) = -4.e^{-2y}.\sin(4x)$$

$$z_y = -2.e^{-2y}.\cos 4x$$

3-m

$$z = \arcsin\left(\frac{y}{x}\right)$$

$$z_x = \frac{y.\left(-\frac{1}{x^2}\right)}{\sqrt{1-\frac{y^2}{x^2}}} = \frac{-\frac{y}{x^2}}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \frac{-y}{x^2\sqrt{1-\left(\frac{y}{x}\right)^2}}$$

$$z_y = \frac{\frac{1}{x}}{\sqrt{1-\frac{y^2}{x^2}}} = \frac{1}{x.\sqrt{1-\left(\frac{y}{x}\right)^2}}$$

3-n

$$z = \frac{e^{x.y}}{e^x+e^y}$$

$$z_x = \frac{(y.e^{xy})(e^x+e^y)-(e^{xy})(e^x+e^y)}{(e^x+e^y)^2} = \frac{e^{xy}(e^x+e^y)(y-1)}{(e^x+e^y)^2}$$

$$z_y = \frac{x.e^{xy}(e^x+e^y)-(e^{xy})(e^x+e^y)}{(e^x+e^y)^2} = \frac{e^{xy}(e^x+e^y)(x-1)}{(e^x+e^y)^2}$$

④ Aşağıdaki fonksiyonların kısmi türevlerini hesaplayınız.

(4-a)
$u = xy^2z^2$

$u_x = y^2z^2 \qquad u_y = 2xyz^2 \qquad u_z = 2xy^2z$

(4-b)
$u = \ln(x^2+y^2+z^2)$

$u_x = \dfrac{2x}{x^2+y^2+z^2} \qquad u_y = \dfrac{2y}{x^2+y^2+z^2} \qquad u_z = \dfrac{2z}{x^2+y^2+z^2}$

(4-c)
$u = \dfrac{e^x}{z^2-y^2}$

$u_x = \dfrac{e^x}{z^2-y^2} \qquad u_y = \dfrac{(0).(z^2-y^2)-(e^x)(-2y)}{(z^2-y^2)^2} = \dfrac{2ye^x}{(z^2-y^2)^2}$

$u_z = \dfrac{(0).(z^2-y^2)-(e^x)(2z)}{(z^2-y^2)^2} = \dfrac{-2.z\,e^x}{(z^2-y^2)^2}$

(4-d)
$u = x.\sin(2y+z^2)$

$u_x = \sin(2y+z^2) \qquad u_y = x.2.\cos(2y+z^2) = 2x.\cos(2y+z^2)$

$u_z = x.(2z).\cos(2y+z^2) = 2xz.\cos(2y+z^2)$

(4-e)
$u = x.z.\ln y$

$u_x = z.\ln y \qquad u_y = \dfrac{x.z}{y} \qquad u_z = x.\ln y$

(4-f)
$u = e^z\cos(x.y)$

$u_x = e^z.y.(-\sin xy) = -y.e^z.\sin(xy)$

$u_y = e^z.x.(-\sin xy) = -xe^z.\sin(xy)$

$u_z = e^z.\cos(xy)$

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⑤ Aşağıdaki eşitlikleri gerçekleyiniz.

(5-a) $z=\dfrac{x^2-y^2}{xy} \Rightarrow x.z_x+y.z_y=0$ dir

$$z_x=\frac{(2x)(xy)-(x^2-y^2).y}{x^2y^2}=\frac{2x^2y-x^2y+y^3}{x^2y^2}=\frac{y(x^2+y^2)}{x^2y^2}=\frac{x^2+y^2}{x^2y}$$

$$z_y=\frac{(-2y)(xy)-(x^2-y^2).x}{x^2y^2}=\frac{-2xy^2-x^3+xy^2}{x^2y^2}=\frac{-x(x^2+y^2)}{x^2y^2}=-\frac{(x^2+y^2)}{xy^2}$$

$$x.z_x+y.z_y=x.\frac{(x^2+y^2)}{x^2y}+\frac{(-y)(x^2+y^2)}{xy^2}=\frac{x^2+y^2}{xy}-\frac{x^2+y^2}{xy}=0 \text{ olur.}$$

(5-b) $z=\dfrac{x^4-y^4}{xy} \Rightarrow x.z_x+y.z_y=2.z$ dir.

$$z_x=\frac{(4x^3)(xy)-(x^4-y^4)(y)}{x^2y^2}=\frac{3x^4y+y^5}{x^2y^2}=\frac{3x^4+y^4}{x^2y}$$

$$z_y=\frac{(-4y^3)(xy)-(x^4-y^4)(x)}{x^2y^2}=\frac{-3xy^4-x^5}{x^2y^2}=\frac{-3y^4-x^4}{xy^2}$$

$$x.z_x+y.z_y=\frac{3x^4+y^4}{xy}+\frac{-3y^4-x^4}{xy}=\frac{2x^4-2y^4}{xy}=2.\frac{x^4-y^4}{xy}=2.z$$

(5-c) $z=\dfrac{y^2}{x} \Rightarrow x.z_x+y.z_y=z$

$$z_x=y^2.\left(-\frac{1}{x^2}\right)=-\frac{y^2}{x^2} \qquad z_y=\frac{1}{x}.2y=\frac{2y}{x}$$

$$x.z_x+y.z_y=-\frac{y^2}{x}+\frac{2y^2}{x}=\frac{y^2}{x}=z$$

(5-d) $u=x^2-y^2-2xy+y+z \Rightarrow (x+y)u_x+(x-y)u_y+(y-x)u_z=0$

$u_x=2x-2y=2(x-y)$
$u_y=-2y-2x+1$
$u_z=1$

$$(x+y)(2(x-y))+(x-y)(-2x-2y+1)+(y-x)(1)=2x^2-2y^2-2x^2+2y^2+x-y+y-x=0$$

(5-e) $u=\dfrac{e^{xyz}}{e^x+e^y+e^z} \Rightarrow u_x+u_y+u_z=(xy+xz+yz-1)u$

$$u_x=\frac{(yz\, e^{xyz})(e^x+e^y+e^z)-(e^{xyz})(e^x)}{(e^x+e^y+e^z)^2}$$

$$u_z=\frac{xy(e^{xyz})(e^x+e^y+e^z)-(e^{xyz})(e^z)}{(e^x+e^y+e^z)^2}$$

$$u_y=\frac{(xz\, e^{xyz})(e^x+e^y+e^z)-(e^{xyz})(e^y)}{(e^x+e^y+e^z)^2}$$

$$u_x+u_y+u_z=\frac{e^{xyz}(e^x+e^y+e^z)(yz+xy+xz)-e^{xyz}(e^x+e^y+e^z)}{(e^x+e^y+e^z)^2}=\frac{e^{xyz}}{e^x+e^y+e^z}(yz+xy+xz-1)$$

$$=(xy+xz+yz-1).u$$

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(5-f) $u = \frac{z}{y} \cdot \ln\left(\frac{y}{x}\right) \Rightarrow x \cdot u_x + y \cdot u_y + z \cdot u_z = 0$

$$u_x = \frac{z}{y} \cdot \frac{y \cdot \left(\frac{-1}{x^2}\right)}{\frac{y}{x}} = \frac{z}{y} \cdot \left(\frac{-y}{x^2}\right) \cdot \left(\frac{x}{y}\right) = \frac{-z}{xy}$$

$$u_y = z \cdot \left(\frac{-1}{y^2}\right) \cdot \ln\left(\frac{y}{x}\right) + \frac{z}{y} \cdot \frac{\frac{1}{x}}{\frac{y}{x}} = -\frac{z}{y^2} \ln\left(\frac{y}{x}\right) + \frac{z}{y^2} = \frac{z}{y^2}\left(1 - \ln\left(\frac{y}{x}\right)\right)$$

$$u_z = \frac{1}{y} \cdot \ln\left(\frac{y}{x}\right)$$

$$* \; x \cdot u_x + y \cdot u_y + z \cdot u_z = \frac{-z}{y} + \frac{z}{y} - \frac{z}{y} \ln\left(\frac{y}{x}\right) + \frac{z}{y} \ln\left(\frac{y}{x}\right) = 0$$

(5-g) $u = \frac{z}{\sqrt[3]{x^2+y^2}} \Rightarrow 3\left(x u_x + y u_y + z \cdot u_z\right) = u$

$$u_x = \frac{-(z)\left(\frac{1}{3}(x^2+y^2)^{-\frac{2}{3}}(2x)\right)}{\sqrt[3]{(x^2+y^2)^2}} = \frac{-2xz}{3(x^2+y^2)\sqrt[3]{x^2+y^2}}$$

$$u_y = \frac{-(z)\left(\frac{1}{3}(x^2+y^2)^{-2/3}(2y)\right)}{\sqrt[3]{(x^2+y^2)^2}} = \frac{-2yz}{3 \cdot (x^2+y^2)\sqrt[3]{x^2+y^2}}$$

$$u_z = \frac{1}{\sqrt[3]{x^2+y^2}}$$

$$* \; 3\left(x u_x + y \cdot u_y + z \cdot u_z\right) = 3\left(\frac{-2x^2 z}{3(x^2+y^2)\sqrt[3]{x^2+y^2}} + \frac{-2y^2 z}{3(x^2+y^2)\sqrt[3]{x^2+y^2}} + \frac{z}{\sqrt[3]{x^2+y^2}}\right)$$

$$= \frac{-2z\,\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}\sqrt[3]{x^2+y^2}} + \frac{3z}{\sqrt[3]{x^2+y^2}} = \frac{z}{\sqrt[3]{x^2+y^2}} = u$$

(5-h) $z = \frac{2x+y}{x-y} \rightarrow x \cdot z_x + y \cdot z_y = 0$

$$z_x = \frac{(2)(x-y) - (2x+y)(1)}{(x-y)^2} = \frac{-3y}{(x-y)^2}$$

$$z_y = \frac{(1)(x-y) - (2x+y)(-1)}{(x-y)^2} = \frac{3x}{(x-y)^2}$$

$$* \; x \cdot z_x + y \cdot z_y = \frac{-3xy}{(x-y)^2} + \frac{3xy}{(x-y)^2} = 0$$

(5-i) $z = \frac{y}{x} \operatorname{arccot}\left(\frac{x}{y}\right) \Rightarrow x \cdot z_x + y \cdot z_y = 0$

$$z_x = y \cdot \left(\frac{-1}{x^2}\right) \cdot \operatorname{arccot}\left(\frac{x}{y}\right) + \left(\frac{y}{x}\right) \cdot \frac{\left(-\frac{1}{y}\right)}{1 + \frac{x^2}{y^2}} = \frac{-y}{x^2} \operatorname{arccot}\left(\frac{x}{y}\right) - \frac{y^2}{x(x^2+y^2)}$$

$$z_y = \left(\frac{1}{x}\right) \cdot \operatorname{arccot}\left(\frac{x}{y}\right) + \left(\frac{y}{x}\right) \cdot \frac{(-x)\left(-\frac{1}{y^2}\right)}{1 + \frac{x^2}{y^2}} = \frac{1}{x} \cdot \operatorname{arccot}\left(\frac{x}{y}\right) + \frac{y}{x^2+y^2}$$

$$* \; x \cdot z_x + y \cdot z_y = \frac{-y}{x} \operatorname{arccot}\left(\frac{x}{y}\right) - \frac{y^2}{x^2+y^2} + \frac{y}{x} \operatorname{arccot}\left(\frac{x}{y}\right) + \frac{y^2}{x^2+y^2} = 0$$

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(5-j) $z=\ln(2x-3y) \rightarrow 3.z_x+2.z_y=0$

$$z_x=\frac{2}{2x-3y} \qquad z_y=\frac{-3}{2x-3y}$$

$$* \quad 3.z_x+2.z_y=\frac{6}{2x-3y}-\frac{6}{2x-3y}=0$$

(5-k) $z=\ln\sqrt{x^2+y^2} \rightarrow x.z_x+y.z_y=1$

$$z_x=\frac{\frac{2x}{2\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}=\frac{x}{x^2+y^2} \qquad z_y=\frac{\frac{2y}{2\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}=\frac{y}{x^2+y^2}$$

$$* \quad x.z_x+y.z_y=\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}=\frac{x^2+y^2}{x^2+y^2}=1$$

(5-l) $z=\sqrt{x^2+y^2}\,\arcsin\left(\frac{y}{x}\right) \rightarrow x.z_x+y.z_y=z$

$$z_x=\frac{x}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)+\sqrt{x^2+y^2}.\frac{y\left(-\frac{1}{x^2}\right)}{\sqrt{1-\frac{y^2}{x^2}}}=\frac{x}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)-\frac{y}{x}.\frac{\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}}$$

$$z_y=\frac{y}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)+\sqrt{x^2+y^2}.\frac{\frac{1}{x}}{\sqrt{1-\frac{y^2}{x^2}}}=\frac{y}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)+\frac{\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}}$$

$$* \quad x.z_x+y.z_y=\frac{x^2}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)-y.\frac{\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}}+\frac{y^2}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)+y\frac{\sqrt{x^2+y^2}}{\sqrt{x^2-y^2}}$$

$$=\frac{x^2+y^2}{\sqrt{x^2+y^2}}\arcsin\left(\frac{y}{x}\right)=\sqrt{x^2+y^2}.\arcsin\left(\frac{y}{x}\right)=z$$

(5-m) ideal gaz kanunu $p.V=nRT$ (n=gazın mol sayısı, R=sabit) diğer ikisinin fonksiyonu olarak, (p=basınç), (V=hacim) ve (T=sıcaklık) üç değişkenin herbiri ile belirlenir. Bu takdirde $\frac{\partial p}{\partial V}.\frac{\partial V}{\partial T}.\frac{\partial T}{\partial p}=-1$ olduğunu gösteriniz.

$$p=\frac{nRT}{V} \rightarrow \frac{\partial p}{\partial V}=nRT\left(-\frac{1}{V^2}\right)=-\frac{nRT}{V^2}$$

$$V=\frac{nRT}{p} \rightarrow \frac{\partial V}{\partial T}=\frac{nR}{p}$$

$$T=\frac{p.V}{nR} \rightarrow \frac{\partial T}{\partial p}=\frac{V}{nR}$$

$$* \quad \frac{\partial p}{\partial V}.\frac{\partial V}{\partial T}.\frac{\partial T}{\partial p}=\frac{-nRT}{\cancel{V^2}}.\frac{\cancel{nR}}{p}.\frac{\cancel{V}}{\cancel{nR}}=\frac{-nRT}{V}.\frac{1}{p}=-p.\frac{1}{p}=-1$$

(5-n) Paralel bağlı R_1, R_2, R_3 dirençlerinin toplam R direnci $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$ denklemini sağlar. $\frac{\partial R}{\partial R_1}+\frac{\partial R}{\partial R_2}+\frac{\partial R}{\partial R_3}=\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}+\frac{1}{R_3^2}\right)\div\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^2$ olduğunu gösteriniz.

$$-\frac{1}{R^2}\frac{\partial R}{\partial R_1}=-\frac{1}{R_1^2} \rightarrow \frac{\partial R}{\partial R_1}=\frac{R^2}{R_1^2} \longleftrightarrow -\frac{1}{R^2}\frac{\partial R}{\partial R_2}=-\frac{1}{R_2^2} \Rightarrow \frac{\partial R}{\partial R_2}=\frac{R^2}{R_2^2}$$

$$-\frac{1}{R^2}\frac{\partial R}{\partial R_3}=-\frac{1}{R_3^2} \rightarrow \frac{\partial R}{\partial R_3}=\frac{R^2}{R_3^2}$$

$$* \quad \frac{\partial R}{\partial R_1}+\frac{\partial R}{\partial R_2}+\frac{\partial R}{\partial R_3}=\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}+\frac{1}{R_3^2}\right)\div\left(\frac{1}{R}\right)^2=\frac{R^2}{R_1^2}+\frac{R^2}{R_2^2}+\frac{R^2}{R_3^2}$$

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⑥ Aşağıdaki eşitlikleri gerçekleyiniz.

6-a

$z = \ln\left(\frac{x}{y}\right) \Rightarrow z_{xy} = z_{yx}$

$z_x = \dfrac{\frac{1}{y}}{\frac{x}{y}} = \frac{1}{y} \cdot \frac{y}{x} = \frac{1}{x} \Rightarrow z_{xy} = 0$

$z_y = \dfrac{x\cdot\left(-\frac{1}{y^2}\right)}{\frac{x}{y}} = \frac{-x}{y^2} \cdot \frac{y}{x} = -\frac{1}{y} \Rightarrow z_{yx} = 0$

$\Rightarrow z_{xy} = 0 = z_{yx}$

6-b

$z = e^x \cos y \Rightarrow z_{xx}\left(0, \frac{\pi}{3}\right) - z_{yy}\left(0, \frac{\pi}{3}\right) = 1$

$z_x = e^x \cdot \cos y \Rightarrow z_{xx} = e^x \cdot \cos y \Rightarrow z_{xx}\left(0, \frac{\pi}{3}\right) = e^0 \cdot \cos\frac{\pi}{3} = 1 \cdot \left(\frac{1}{2}\right) = \frac{1}{2}$

$z_y = -e^x \cdot \sin y \Rightarrow z_{yy} = -e^x \cdot \cos y \Rightarrow z_{yy}\left(0, \frac{\pi}{3}\right) = -e^0 \cos\frac{\pi}{3} = (-1)\left(\frac{1}{2}\right) = -\frac{1}{2}$

* $z_{xx}\left(0, \frac{\pi}{3}\right) - z_{yy}\left(0, \frac{\pi}{3}\right) = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$ bulunur.

6-c

$z = x \cdot \ln\left(\frac{y}{x}\right) \Rightarrow x^2 \cdot z_{xx} + 2xy \cdot z_{xy} + y^2 \cdot z_{yy} = 0$

$z_x = (1) \cdot \ln\left(\frac{y}{x}\right) + x \cdot \dfrac{y \cdot \left(-\frac{1}{x^2}\right)}{\frac{y}{x}} = \ln\left(\frac{y}{x}\right) + x \cdot \left(\frac{-y}{x^2}\right) \cdot \frac{x}{y} = \ln\left(\frac{y}{x}\right) - 1$

$z_{xx} = \dfrac{(-y/x^2)}{y/x} = -\frac{y}{x^2} \cdot \frac{x}{y} = -\frac{1}{x}$

$z_{xy} = \dfrac{\frac{1}{x}}{\frac{y}{x}} = \frac{1}{x} \cdot \frac{x}{y} = \frac{1}{y}$

$z_y = x \cdot \dfrac{\frac{1}{x}}{\frac{y}{x}} = x \cdot \frac{1}{x} \cdot \frac{x}{y} = \frac{x}{y} \Rightarrow z_{yy} = \frac{-x}{y^2}$

* $x^2 \cdot z_{xx} + 2xy \cdot z_{xy} + y^2 \cdot z_{yy} = x^2\left(-\frac{1}{x}\right) + 2xy\left(\frac{1}{y}\right) + y^2\left(\frac{-x}{y^2}\right) = -x + 2x - x = 0$

6-d

$u = \dfrac{1}{\sqrt{x^2+y^2+z^2}} \Rightarrow u_{xx} + u_{yy} + u_{zz} = 0$

$u = (x^2+y^2+z^2)^{-\frac{1}{2}} \Rightarrow u_x = -\frac{1}{2} \cdot (x^2+y^2+z^2)^{-3/2}(2x) = -x \cdot (x^2+y^2+z^2)^{-3/2}$

$u_{xx} = (-1)(x^2+y^2+z^2)^{-3/2} + (-x)\left[-\frac{3}{2} \cdot (x^2+y^2+z^2)^{-5/2}(2x)\right]$

$= -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2}$

$u_y = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y) = -y\,(x^2+y^2+z^2)^{-3/2}$

$u_{yy} = (-1)(x^2+y^2+z^2)^{-3/2} + (-y)\left[-\frac{3}{2} \cdot (x^2+y^2+z^2)^{-5/2} \cdot (2y)\right]$

$= -(x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}$

$u_z = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2z) = -z \cdot (x^2+y^2+z^2)^{-3/2}$

$u_{zz} = (-1)(x^2+y^2+z^2)^{-3/2} + (-z)\left[-\frac{3}{2} \cdot (x^2+y^2+z^2)^{-5/2}(2z)\right] = -(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2}$

* $u_{xx} + u_{yy} + u_{zz} = -3 \cdot (x^2+y^2+z^2)^{-3/2} + 3\,(x^2+y^2+z^2)\,(x^2+y^2+z^2)^{-5/2}$

$= -3\,(x^2+y^2+z^2)^{-3/2} + 3 \cdot (x^2+y^2+z^2)^{-3/2} = 0$ bulunur.

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6-e $z=(x-y)\ln(x-y) \Rightarrow z_{xx}+2z_{xy}+z_{yy}=0$

$z_x=(1)\ln(x-y)+(x-y)\dfrac{1}{x-y}=\ln(x-y)+1$

$z_{xx}=\dfrac{1}{x-y}$ $\qquad$ $z_{xy}=\dfrac{-1}{x-y}$

$z_y=(-1)\ln(x-y)+(x-y)\cdot\dfrac{-1}{x-y}=-\ln(x-y)-1$

$z_{yy}=-\dfrac{-1}{x-y}=\dfrac{1}{x-y}$

$\circledast$ $z_{xx}+2\cdot z_{xy}+z_{yy}=\dfrac{1}{x-y}+2\left(\dfrac{-1}{x-y}\right)+\dfrac{1}{x-y}=0$

6-f $z=x^4+\sin(x+y)-y\cdot\ln x \rightarrow z_{yxx}-2z_{yyx}+z_{yyy}=\dfrac{1}{x^2}$

$z_y=\cos(x+y)-\ln x$

$z_{yx}=-\sin(x+y)-\dfrac{1}{x}$ $\qquad$ $z_{yy}=-\sin(x+y)$

$z_{yxx}=-\cos(x+y)+\dfrac{1}{x^2}$ $\qquad$ $z_{yyx}=-\cos(x+y)$

$z_{yyy}=-\cos(x+y)$

$\circledast$ $z_{yxx}-2\cdot z_{yyx}+z_{yyy}=-\cos(x+y)+\dfrac{1}{x^2}+2\cos(x+y)-\cos(x+y)=\dfrac{1}{x^2}$

6-g Bir noktadaki muhtemel elektrik harcaması q (uygun birimlerle) $r=\sqrt{x^2+y^2+z^2}$ olmak üzere, $\phi(x,y,z)=\dfrac{q}{r}$ ile tanımlanır. ϕ'nin üç boyutlu $\dfrac{\partial^2\phi}{\partial x^2}+\dfrac{\partial^2\phi}{\partial y^2}+\dfrac{\partial^2\phi}{\partial z^2}=0$ Laplace denklemini sağladığını gösteriniz.

$$\phi(x,y,z)=q\cdot(x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial\phi}{\partial x}=q\cdot\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}\cdot(2x)=(-qx)(x^2+y^2+z^2)^{-3/2}=\frac{-qx}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial^2\phi}{\partial x^2}=(-q)(x^2+y^2+z^2)^{-3/2}+(-qx)\cdot\left[\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x)\right]=\frac{-q}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{3qx^2}{\sqrt{(x^2+y^2+z^2)^5}}$$

$$\frac{\partial\phi}{\partial y}=q\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2y)=(-qy)(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2\phi}{\partial y^2}=(-q)(x^2+y^2+z^2)^{-3/2}+(-qy)\left[\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2y)\right]=\frac{-q}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{3qy^2}{\sqrt{(x^2+y^2+z^2)^5}}$$

$$\frac{\partial\phi}{\partial z}=q\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}\cdot(2z)=(-qz)(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2\phi}{\partial z^2}=(-q)(x^2+y^2+z^2)^{-3/2}+(-qz)\left[\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2z)\right]=\frac{-q}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{3qz^2}{\sqrt{(x^2+y^2+z^2)^5}}$$

$\circledast$ $$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{-3q}{\sqrt{(x^2+y^2+z^2)^3}}+\frac{3q\cancel{(x^2+y^2+z^2)}}{\cancel{(x^2+y^2+z^2)}\sqrt{(x^2+y^2+z^2)^3}}=0$$ olur.

GIPTA

7) Aşağıdaki yüzeylerin verilen noktalardaki teğet düzlemlerinin denklemlerini yazınız.

7-a) $x^2+y^2+z^2=14 \qquad P(2,3,1)$

* $2x+2z.z_x=0 \Rightarrow z_x=-\frac{x}{z} \Rightarrow z_x(2,3,1)=-\frac{2}{1}=-2=m_1$

* $2y+2z.z_y=0 \Rightarrow z_y=\frac{-y}{z} \Rightarrow z_y(2,3,1)=-\frac{3}{1}=-3=m_2$

Teğet Düzleminin Denklemi $\rightarrow$ $z-z_0=m_1(x-x_0)+m_2(y-y_0)$

$z-1=(-2)(x-2)+(-3)(y-3)$

$z-1=-2x+4-3y+9$

$\boxed{2x+3y+z=14}$ olarak bulunur

Not: $P(2,3,1)$ düzlem üzerinde olduğundan düzlem denklemini sağlamalı...

$P(2,3,1) \rightarrow 2.(2)+3(3)+1\overset{?}{=}14 \rightarrow 14=14$ doğru olduğu görülür.

7-b) $4x^2+9y^2+z^2=29 \qquad P(1,-1,4)$

* $8x+2z.z_x=0 \Rightarrow z_x=-\frac{8x}{2z}=-\frac{4x}{z} \Rightarrow z_x(1,-1,4)=-\frac{4.(1)}{4}=-1=m_1$

* $18y+2z.z_y=0 \Rightarrow z_y=-\frac{18y}{2z}=-\frac{9y}{z} \Rightarrow z_y(1,-1,4)=-\frac{9(-1)}{4}=\frac{9}{4}=m_2$

Teğet düzleminin denklemi $\rightarrow$ $z-z_0=m_1(x-x_0)+m_2(y-y_0)$

$z-4=(-1)(x-1)+\frac{9}{4}(y-(-1))$

$z-4=-x+1+\frac{9}{4}y+\frac{9}{4} \Rightarrow 4z-16=-4x+4+9y+9$

$\boxed{4x-9y+4z=29}$ olarak bulunur.

Kontrol $P(1,-1,4) \leftrightarrow 4.(1)-9(-1)+4.(4)=4+9+16=29$ doğru

7-c) $x^2y+zy^2=10 \qquad P(1,-2,3)$

* $2xy+y^2.[(1).z_x]=0 \Rightarrow z_x=-\frac{2xy}{y^2}=-\frac{2x}{y} \Rightarrow z_x(1,-2)=-\frac{2.(1)}{(-2)}=1=m_1$

* $x^2+(z_y)(y^2)+(z)(2y)=0 \Rightarrow z_y=-\frac{x^2+2yz}{y^2} \Rightarrow z_y(1,-2,3)=-\frac{1^2+2(-2)(3)}{(-2)^2}=\frac{11}{4}=m_2$

Teğet Düzleminin Denklemi $\rightarrow$ $z-z_0=m_1(x-x_0)+m_2(y-y_0)$

$z-3=(1)(x-1)+\left(\frac{11}{4}\right)(y+2)$

$z-3=x-1+\frac{11y}{4}+\frac{22}{4}$

$4z-12=4x-4+11y+22$

$\boxed{4x+11y-4z=-30}$ olarak bulunur.

Kontrol: $P(1,-2,3) \leftrightarrow 4.(1)+11(-2)-4(3)=4-22-12=-30$ doğru.

GIPTA

(7-d) $2x^2-2xy-y^2-z=0 \quad P(1,-2,2)$

$z=2x^2-2xy-y^2 \quad P(1,-2,2)$ Kapalı formdan kurtarırsak

* $z_x = 4x-2y \Rightarrow z_x(1,-2) = 4.(1)-2(-2) = 4+4 = 8 = m_1$

* $z_y = -2x-2y \Rightarrow z_y(1,-2) = -2(1)-2(-2) = -2+4 = 2 = m_2$

Teğet Düzleminin denklemi $\rightarrow z-z_0 = m_1(x-x_0)+m_2(y-y_0)$

$$z-2 = 8(x-1)+2(y+2)$$

$$z-2 = 8x-8+2y+4$$

$\boxed{8x+2y-z=2}$ olarak bulunur.

Kontrol: $P(1,-2,2) \leftrightarrow 8.(1)+2(-2)-2 = 8-4-2 = 2$ doğru olduğu görülür.

(7-e) $z=3x^2-xy+y^2 \quad P(-1,-1,3)$

* $z_x = 6x-y \Rightarrow z_x(-1,-1) = 6(-1)-(-1) = -6+1 = -5 = m_1$

* $z_y = -x+2y \Rightarrow z_y(-1,-1) = -(-1)+2.(-1) = 1-2 = -1 = m_2$

Teğet Düzleminin Denklemi $\longrightarrow z-z_0 = m_1(x-x_0)+m_2(y-y_0)$

$$z-3 = (-5)(x+1)+(-1)(y+1)$$

$$z-3 = -5x-5-y-1$$

$\boxed{5x+y+z=-3}$ bulunur.

Kontrol: $P(-1,-1,3) \leftrightarrow 5(-1)+(-1)+(3) = -5-1+3 = -3$ düzlem denklemini sağladığı görülür.

⑧ Aşağıdaki fonksiyonların toplam diferansiyellerini hesaplayınız.

(8-a) $z = \sin x \cdot \cos y$

$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy = (\cos y \cos x)\, dx + (-\sin x \cdot \sin y)\, dy$

(8-b) $z = e^{y^2 - x^2}$

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \left(-2x\, e^{y^2-x^2}\right) dx + \left(2y\, e^{y^2-x^2}\right) dy$

(8-c) $z = \sin^3(x+y)$

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \left(3 \cdot \sin^2(x+y) \cdot \cos(x+y)\right) dx + \left(3 \sin^2(x+y) \cdot \cos(x+y)\right) dy$

(8-d) $z = y^{\ln x}$

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \left(\frac{1}{x} \cdot y^{\ln x} \cdot \ln y\right) dx + \left(\ln x \cdot y^{\ln x - 1}\right) dy$

(8-e) $z = y \cdot x^{\sin y}$

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \left(y \cdot \sin y \cdot x^{\sin y - 1}\right) dx + \left(x^{\sin y} + (y) \cdot (\cos y) \cdot x^{\sin y} \ln x\right) dy$

(8-f) $u = \tan x \cdot \tan y \cdot \tan z$

$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} \cdot dy + \frac{\partial u}{\partial z} \cdot dz = (\tan y \tan z \cdot \sec^2 x)\, dx + (\tan x \tan z \cdot \sec^2 y)\, dy + (\tan x \tan y \sec^2 z)\, dz$

(8-g) $u = 2^{x+y+z}$

$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz = \left(2^{x+y+z} \cdot \ln 2\right) dx + \left(2^{x+y+z} \ln 2\right) dy + \left(2^{x+y+z} \ln 2\right) dz$

(8-h) $u = x^{y \cdot z}$

$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz = \left(yz \cdot x^{yz-1}\right) dx + \left(z \cdot x^{yz} \ln x\right) dy + \left(y \cdot x^{yz} \ln x\right) dz$

(8-i) $z = \tan(x^2 + y^2)$

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \left(2x \cdot \sec^2(x^2+y^2)\right) dx + \left(2y \sec^2(x^2+y^2)\right) dy$

(8-j) $z = \ln \sqrt{x^2+y^2}$

$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} \cdot dy = \frac{\frac{2x}{2 \cdot \sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} dx + \frac{\frac{2y}{2 \cdot \sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} dy$

$= \left(\frac{x}{x^2+y^2}\right) dx + \left(\frac{y}{x^2+y^2}\right) dy$

GIPTA

(8-k) $z=\sqrt{(x^2+4y^2+16)^3}$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=\frac{3(x^2+4y^2+16).(2x)}{2\sqrt{(x^2+4y^2+16)^3}}dx+\frac{3(x^2+4y^2+16)(8y)}{2\sqrt{(x^2+4y^2+16)^3}}dy$$

$$=\frac{3x}{\sqrt{x^2+4y^2+16}}dx+\frac{12y}{\sqrt{x^2+4y^2+16}}dy$$

(8-l) $z=e^{(y/x)}$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=\left(\left(\frac{-y}{x^2}\right).e^{y/x}\right)dx+\left(\frac{1}{x}e^{y/x}\right)dy$$

(8-m) $z=\cos\left(\frac{y}{x}\right)$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=\left(\frac{-y}{x^2}\right)\left(-\sin\frac{y}{x}\right)dx+\left(\frac{1}{x}\right)\left(-\sin\frac{y}{x}\right)dy$$

$$=\frac{y}{x^2}\sin\left(\frac{y}{x}\right)dx-\frac{1}{x}\sin\left(\frac{y}{x}\right)dy$$

(8-n) $z=\arctan\left(\frac{y}{x}\right)$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=\frac{\left(\frac{-y}{x^2}\right)}{1+\frac{y^2}{x^2}}dx+\frac{\frac{1}{x}}{1+\frac{y^2}{x^2}}dy$$

$$=\frac{-y}{x^2}\cdot\frac{x^2}{x^2+y^2}dx+\frac{1}{x}\cdot\frac{x^2}{x^2+y^2}dy$$

$$=\frac{-y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dy$$

GIPTA

(9) Aşağıdaki ifadelerin yaklaşık değerlerini toplam diferansiyelden yararlanarak hesaplayın.

(9-a) $(3{,}02)^2.(1{,}91) + (3{,}02)(1{,}91)^2 = ?$ $(28{,}437226)$

$f(x,y) = x^2y + x.y^2 \Rightarrow df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = (2xy+y^2)dx + (x^2+2xy)dy$

$\Delta x = dx = (0{,}02)$ ve $x=3$

$\Delta y = dy = (-0{,}09)$ ve $y=2$

$\boxed{f(x+\Delta x, y+\Delta y) \approx df + f(x,y)}$

$(3{,}02)^2.(1{,}91) + (3{,}02)(1{,}91)^2 \approx (2xy+y^2)dx + (x^2+2xy)dy + f(x,y)$

$\approx (2.3.2+2^2)(0{,}02) + (3^2+2.3.2)(-0{,}09) + (3^2.2+3.2^2)$

$\approx 16.(0{,}02) + 21(-0{,}09) + (18+12)$

$\approx 0{,}32 - 1{,}89 + 30$

$\approx 30 - 1{,}57 \approx 28{,}43$

(9-b) $\dfrac{(2{,}06)^2 + (3{,}95)^2}{(1{,}92)^2} = ?$ $(5{,}3835991753)$

$f(x,y,z) = \dfrac{x^2+y^2}{z^2} \Rightarrow df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

$= \left(\frac{2x}{z^2}\right)dx + \left(\frac{2y}{z^2}\right)dy + \left(\frac{(x^2+y^2).(-2)}{z^3}\right)dz$

$\Delta x = dx = 0{,}06$ ve $x=2$
$\Delta y = dy = (-0{,}05)$ ve $y=4$
$\Delta z = dz = (-0{,}08)$ ve $z=2$

$= \left(\frac{2.2}{2^2}\right)(0{,}06) + \left(\frac{2.4}{2^2}\right).(-0{,}05) + \left(\frac{(2^2+4^2).(-2)}{2^3}\right)(-0{,}08)$

$= 0{,}06 + (-0{,}2) + (0{,}4) = 0{,}26$

$\dfrac{(2{,}06)^2 + (3{,}95)^2}{(1{,}92)^2} \approx df + f(x,y) \approx 0{,}26 + \dfrac{2^2+4^2}{2^2} \approx 5{,}26$

NOT: Değişimler mutlak değerce büyük olduğunda hatanın da arttığına dikkat ediniz. $(5{,}383599 \leftrightarrow 5{,}26)$

(*) (9-a) daki hata payının çok düşük olduğunu görünüz.
$(28{,}437226 \leftrightarrow 28{,}43)$

GIPTA

(9-c) $\sqrt{(2{,}97)^2+(2{,}05)^4} \approx ?$ $(5{,}14605735)$

$$f(x,y)=\sqrt{x^2+y^4} \rightarrow df=\frac{\partial f}{\partial x}.dx+\frac{\partial f}{\partial y}.dy$$

$$=\left(\frac{2x}{2\sqrt{x^2+y^4}}\right)dx+\left(\frac{4y^3}{2\sqrt{x^2+y^4}}\right)dy$$

$$\boxed{\begin{array}{l}\Delta x=dx=(-0{,}03) \quad \text{ve} \quad x=3\\ \Delta y=dy=(0{,}05) \quad \text{ve} \quad y=2\end{array}}$$

$$=\left(\frac{2.3}{2\sqrt{9+16}}\right)(-0{,}03)+\left(\frac{4.2^3}{2\sqrt{9+16}}\right).(0{,}05)$$

$$=\frac{3}{5}.(-0{,}03)+\frac{16}{5}.(0{,}05)=-0{,}018+0{,}16$$

$$=0{,}142$$

$$\sqrt{(2{,}97)^2+(2{,}05)^4}\approx df+f(x,y)\approx(0{,}142)+\sqrt{3^2+2^4}\approx 5{,}142$$

(9-d) $\sqrt{(4{,}97)^2+(4{,}01)^2+2.(1{,}96)^2}=?$ $(6{,}961623373897786)$

$$f(x,y,z)=\sqrt{x^2+y^2+2z^2} \rightarrow df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial z}.dz$$

$$=\left(\frac{x}{\sqrt{x^2+y^2+2z^2}}\right)dx+\left(\frac{y}{\sqrt{x^2+y^2+2z^2}}\right)dy+\left(\frac{2z}{\sqrt{x^2+y^2+2z^2}}\right)dz$$

$$\boxed{\begin{array}{l}\Delta x=dx=(-0{,}03) \quad \text{ve} \quad x=5\\ \Delta y=dy=(0{,}01) \quad \text{ve} \quad y=4\\ \Delta z=dz=(-0{,}04) \quad \text{ve} \quad z=2\end{array}}$$

$$=\left(\frac{5}{\sqrt{5^2+4^2+2.2^2}}\right)(-0{,}03)+\left(\frac{4}{\sqrt{5^2+4^2+2.2^2}}\right)(0{,}01)+\left(\frac{2.2}{\sqrt{5^2+4^2+2.2^2}}\right)(-0{,}04)$$

$$=\frac{5}{7}(-0{,}03)+\frac{4}{7}.(0{,}01)+\frac{4}{7}(-0{,}04)=$$

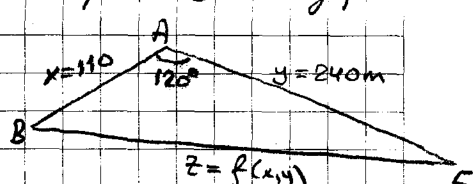
$$=(-0{,}02142857)+0{,}00571428+(-0{,}02285714)$$

$$=-0{,}03857143$$

$$\sqrt{(4.97)^2+(4{,}01)^2+2.(1.96)^6}\approx df+f(x,y,z)\approx(-0{,}03857143)+\sqrt{5^2+4^2+2.2^2}$$

$$\approx 6{,}96142857$$

(9-e) Üçgen şeklindeki bir arsanın bir açısı 120° ve bu açıya ait kenarların uzunlukları 110 m ve 240 m dir. Bu uzunlukların ölçülmesinde 3 cm eksik ölçüm hatası yapıldığına göre, üçüncü kenarın hesaplanmasında yapılacak hatayı yaklaşık olarak hesaplayınız.



$$\boxed{\begin{array}{l}\Delta x=dx=(-0{,}03)m \quad \text{ve} \quad x=110\,m\\ \Delta y=dy=(-0{,}03)m \quad \text{ve} \quad y=240\,m\end{array}}$$

$$z=f(x,y)=\sqrt{x^2+y^2-2xy.\cos 120}=\sqrt{x^2+y^2+xy}$$

$$df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy=\left(\frac{2x+y}{2\sqrt{x^2+y^2+xy}}\right)dx+\left(\frac{2y+x}{2\sqrt{x^2+y^2+xy}}\right)dy$$

$$=\left(\frac{2.110+240}{2.\sqrt{110^2+240^2+110.240}}\right)(-0{,}03)+\left(\frac{2.240+110}{2\sqrt{110^2+240^2+110.240}}\right)(-0{,}03)$$

$$=\left(\frac{460}{2.310}\right)(-0{,}03)+\left(\frac{590}{2.310}\right)(0{,}03)=(-0{,}022258)+(-0{,}028548)$$

$$=-0{,}050806\ m\cong 5cm$$

Üçüncü kenar $(-0{,}050806)$ metre kısa ölçüm hatası yapılacak.

(9-f) Bir dikdörtgenin kenarları 5 cm ve 12 cm dir. Her bir ölçüde yapılan hata $\frac{1}{12}$ cm dir. Bu dikdörtgenin hesaplanmış alanında ve köşegen uzunluğundaki hatayı hesaplayınız

* $\boxed{\text{ALAN}}$ $\quad A(x,y) = x.y \Rightarrow dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy$

$$\boxed{\begin{array}{l} \Delta x = dx = \mp 1/12 \quad \text{ve } x=5 \text{ cm} \\ \Delta y = dy = \mp 1/12 \quad \text{ve } y=12 \text{ cm} \end{array}}$$

$$= (y).dx + (x)dy$$
$$= 12\left(\mp\frac{1}{12}\right) + 5\left(\mp\frac{1}{12}\right) = \mp(1+0{,}4167)$$
$$= \mp(1{,}04167) \quad \text{(Alandaki hata)}$$

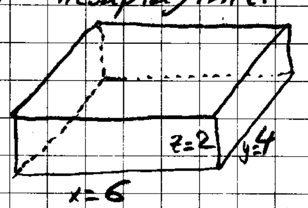
* $\boxed{\text{KÖŞEGEN UZUNLUĞU}}$ $\quad f(x,y) = \sqrt{x^2+y^2} \Rightarrow df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

$$df = \left(\frac{x}{\sqrt{x^2+y^2}}\right)dx + \left(\frac{y}{\sqrt{x^2+y^2}}\right)dy$$

$$\boxed{\begin{array}{l} \Delta x = dx = \mp 1/12 \quad \text{ve } x=5 \text{ cm} \\ \Delta y = dy = \mp 1/12 \quad \text{ve } y=12 \text{ cm} \end{array}}$$

$$= \left(\frac{5}{\sqrt{5^2+12^2}}\right)(\mp 1/12) + \left(\frac{12}{\sqrt{5^2+12^2}}\right)(\mp 1/12)$$
$$= \left(\frac{5}{13}\right)\left(\mp\frac{1}{12}\right) + \left(\frac{12}{13}\right)\left(\mp\frac{1}{12}\right)$$
$$= \mp 0{,}108974 \quad \text{(Köşegen uzun. hata)}$$

(9-g) Üstü açık bir metal kutunun boyutları 6, 4, 2 cm dir. Kutunun kalınlığı 0,1 cm artırılırsa hacminin değişimini hesaplayınız.



$$H(x,y,z) = x.y.z$$

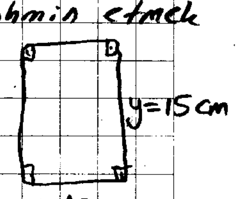
$$dH = \frac{\partial H}{\partial x}dx + \frac{\partial H}{\partial y}.dz + \frac{\partial H}{\partial z}.dz$$

$$\boxed{\begin{array}{l} \Delta x = dx = 0{,}2\text{cm} \quad \text{ve } x=6 \text{ cm} \\ \Delta y = dy = 0{,}2 \text{ cm} \quad \text{ve } y=4 \text{ cm} \\ \Delta z = dz = 0{,}1 \text{ cm} \quad \text{ve } z=2 \text{ cm} \end{array}}$$

$$= (yz)dx + (xz)dy + (xy)dz$$
$$= (4.2)(0{,}2) + (6.2)(0{,}2) + (6.4)(0{,}1)$$
$$= 1{,}6 + 2{,}4 + 2{,}4 = 6{,}4 \text{ cm}^3$$
(Hacimdeki değişim)

NOT: $\Delta z = 0{,}1$ dur çünkü üstü açık

(9-h) Bir dikdörtgenin taban ve yüksekliği, her bir ölçümde 0,1 cm'ye kadar hata payı ile sırasıyla 10 cm ve 15 cm olarak ölçülmüştür. Dikdörtgenin alanının hesaplanmasındaki maksimum hatayı tahmin etmek için diferansiyelleri kullanınız.



$$A(x,y) = x.y \rightarrow dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy$$

$$\boxed{\begin{array}{l} \Delta x = dx = +0{,}1 \quad \text{ve } x=10\text{cm} \\ \Delta y = dy = +0{,}1 \quad \text{ve } y=15 \text{ cm} \end{array}}$$

Maksimum hata

$$= (y)dx + (x)dy$$
$$= 15.(0{,}1) + (10).(0{,}1) = 1{,}5 + 1 = 2{,}5 \text{ cm}^2$$
(Maksimum hata)

GIPTA

(9-1) Dik çembersel silindirin taban yarıçapı r ve silindirin yüksekliği h sırasıyla 3 cm ve 9 cm olarak ölçülmüştür. Her bir ölçümde yaklaşık olarak 1 mm hata vardır.

a) Silindirin hacmi

b) Silindirin toplam yüzey alanı

hesaplanırken yapılan <u>maksimum</u> hatayı tahmin etmek için diferansiyelleri kullanınız.

a) Silindirin hacmi: $V(r,h) = \pi . r^2 . h$

$$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} . dh = (2\pi r h) . dr + (\pi r^2) . dh$$

$\boxed{\begin{array}{l} \Delta r = dr = 0{,}1 \text{ cm ve } r = 3 \text{ cm} \\ \Delta h = dh = 0{,}1 \text{ cm ve } h = 9 \text{ cm} \end{array}}$

$$= (2\pi . 3 . 9) . (0{,}1) + (\pi . 3^2) . (0{,}1)$$

$$= 5{,}4\pi + 0{,}9\pi = 6{,}3\pi \text{ cm}^3$$

(Maksimum hata)

b) Silindirin toplam yüzey Alanı: $A(r,h) = 2\pi r^2 + 2\pi r . h$

$$dA = \frac{\partial A}{\partial r} dr + \frac{\partial A}{\partial h} dh = (4\pi r + 2\pi h) dr + (2\pi r) dh$$

$$= (4\pi . 3 + 2\pi . 9)(0{,}1) + (2\pi . 3)(0{,}1)$$

$\boxed{\begin{array}{l} \Delta r = dr = 0{,}1 \text{ cm ve } r = 3 \text{ cm} \\ \Delta h = dh = 0{,}1 \text{ cm ve } h = 9 \text{ cm} \end{array}}$

$$= 2\pi + 0{,}6\pi = 2{,}6\pi \text{ cm}^2$$

(Maksimum hata)

(9-2) Yatay bir düzlemden α eğim açısı v_0 ilk hızıyla ateşlenen (boşlukta ateşlenen) merminin menzili $R = \frac{1}{32} . v_0^2 . \sin 2\alpha$ dır. Eğer v_0 hızı 400 ft/s den 410 ft/s 'ye ve α açısı 30° den 31° ye yükseltilirse, merminin menzilindeki değişime tahmini bir yaklaşımda bulunmak için diferansiyeli kullanınız.

$$R(v_0, \alpha) = \frac{1}{32} . v_0^2 . \sin 2\alpha$$

$$dR = \frac{\partial R}{\partial v_0} dv_0 + \frac{\partial R}{\partial \alpha} d\alpha = \left(\frac{1}{16} . v_0 . \sin 2\alpha\right) dv_0 + \left(\frac{1}{16} v_0^2 . \cos 2\alpha\right) d\alpha$$

$\boxed{\begin{array}{l} v_0 = 400 \text{ ve } dv_0 = 10 = \Delta v_0 \\ \alpha = 30° \text{ ve } d\alpha = 1° = \Delta\alpha \end{array}}$

$$\cong \left(\frac{1}{16} . 400 . \sin 60\right)(10) + \left(\frac{1}{16} (400)^2 . \cos 60\right) . (1)$$

$$= 216{,}50635 + 5000 = 5216{,}50635 \text{ ft}$$

menzildeki değişim

(9-3) Kapalı bir dikdörtgen kutunun boyutları ölçüldüğünde 0,1 cm lik muhtemel hata yapılmak üzere 10 cm, 15 cm ve 20 cm olarak bulunuyor. Kutunun toplam yüzey alanı hesabında <u>maksimum</u> hatayı bulmak için diferansiyeli kullanınız.

$$A(x,y,z) = 2xy + 2xz + 2yz$$

$\boxed{\begin{array}{l} \Delta x = dx = 0{,}1 \text{ cm ve } x = 10 \text{ cm} \\ \Delta y = dy = 0{,}1 \text{ cm ve } y = 15 \text{ cm} \\ \Delta z = dz = 0{,}1 \text{ cm ve } z = 20 \text{ cm} \end{array}}$

$$dA = \frac{\partial A}{\partial x} dx + \frac{\partial A}{\partial y} dy + \frac{\partial A}{\partial z} . dz$$

$$= (2y+2z) dx + (2x+2z) dy + (2x+2y) dz$$

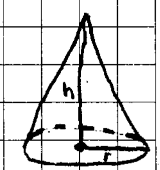
$$= (30+40)(0{,}1) + (20+40)(0{,}1) + (20+30)(0{,}1)$$

$$= 7 + 6 + 5 = 18 \text{ cm}^2$$

Alandaki Maksimum hata

9-k) Dik çembersel koninin taban yarıçapı r ve yüksekliği h, sırasıyla, 5 cm ve 10 cm olarak ölçülmüştür. Her bir ölçümde $\frac{1}{16}$ cm'ye kadar muhtemel hata vardır. Koninin hacminin hesaplanmasında ortaya çıkabilecek olan maksimum hatayı tahmin etmek için diferansiyeli kullanınız.

$$H(r,h) = \frac{1}{3}\cdot \pi r^2 \cdot h$$



$$\boxed{\begin{array}{l} r = 5 \text{ cm} \quad \text{ve} \quad \Delta r = dr = \frac{1}{16} \text{ cm} \\ h = 10 \text{ cm} \quad \text{ve} \quad \Delta h = dh = \frac{1}{16} \text{ cm} \end{array}}$$

$$\begin{aligned} dH &= \frac{\partial H}{\partial r} dr + \frac{\partial H}{\partial h}\cdot dh \\ &= \left(\frac{2}{3}\cdot \pi r h\right) dr + \left(\frac{1}{3}\pi r^2\right) dh \\ &= \left(\frac{2}{3}\cdot \pi \cdot 5\cdot 10\right)\left(\frac{1}{16}\right) + \left(\frac{1}{3}\pi\cdot 5^2\right)\cdot\left(\frac{1}{16}\right) \\ &= \frac{100\pi}{48} + \frac{25\pi}{48} = \frac{125}{48}\pi \text{ cm}^3 \end{aligned}$$

(Maksimum hata)

Not: $H(r,h) = H(5,10) = \frac{1}{3}\cdot \pi \cdot 5^2 \cdot 10 = \frac{250\pi}{3}$ cm^3 iken yapılacak olan $\frac{125\pi}{48}$ cm^3 maksimum hata ile yeni hacim

$$\frac{250\pi}{3}\cdot\frac{16}{16} + \frac{125\pi}{48} = \frac{4000\pi}{48} + \frac{125\pi}{48} = \frac{4125\pi}{48} \text{ dır.}$$

(10) Aşağıdaki bileşik fonksiyonların türevlerini hesaplayınız.

(10-a) $z = 2x^2y + 3xy^2 \quad ; \quad x = 2t \quad , y = 4t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = (4xy + 3y^2)\cdot(2) + (2x^2 + 6xy)(4)$$

$$= \left[4(2t)(4t) + 3.(4t)^2\right].(2) + \left[2.(2t)^2 + 6.(2t)(4t)\right](4)$$

$$= 64t^2 + 96t^2 + 32t^2 + 192t^2 = 384t^2$$

(10-b) $z = 2xy \quad ; \quad x = \sin t \quad , y = \cos t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} = (2y)\cdot(\cos t) + (2x)\cdot(-\sin t)$$

$$= 2.\cos^2 t - 2\sin^2 t = 2(\cos^2 t - \sin^2 t) = 2.\cos(2t)$$

(10-c) $z = y^3 - y.x^2 \quad ; \quad x = 3.\cos 2t \quad , y = \cos t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} = (-2xy)(-6\sin 2t) + (3y^2 - x^2)(-\sin t)$$

$$= 36.\cos(2t).\sin(2t).\cos t + \left[3\cos^2 t - 9\cos^2(2t)\right](-\sin t)$$

(10-d) $z = e^y \sin x \quad ; \quad x = 3\cos 2t \quad , y = 4.\sin t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = (e^y\cos x).(-6\sin 2t) + (e^y \sin x)(4\cos t)$$

$$= \left[e^{4.\sin t}.\cos(3.\cos 2t)\right](-6.\sin 2t) + e^{4\sin t}.\sin(3.\cos 2t)(4\cos t)$$

(10-e) $z = \arctan\left(\frac{y}{x}\right) \quad ; \quad x = e^t - e^{-t} \quad , y = e^t + e^{-t}$

$$\frac{dz}{dt} = \left(\frac{\partial z}{\partial x}\right)\left(\frac{dx}{dt}\right) + \left(\frac{\partial z}{\partial y}\right)\cdot\left(\frac{dy}{dt}\right) = \left(\frac{\frac{-y}{x^2}}{1 + \frac{y^2}{x^2}}\right)\cdot(e^t + e^{-t}) + \left(\frac{\frac{1}{x}}{1 + \frac{y^2}{x^2}}\right)(e^t - e^{-t})$$

$$= \left(\frac{-y}{x^2 + y^2}\right)(e^t + e^{-t}) + \left(\frac{x}{x^2 + y^2}\right)(e^t - e^{-t})$$

(10-f) $u = x.\cos z - y.\cos z \quad ; \quad x = \sin 2t \quad , y = \cos 2t \quad , z = 2t$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt}$$

$$= (\cos z)\cdot(2.\cos 2t) + (-\cos z)\cdot(-2.\sin 2t) + (-x.\sin z + y.\sin z)(2)$$

$$= 2.\cos^2(2t) + 2.\sin 2t\cos 2t - 2\sin^2 2t + 2\cos 2t\sin 2t$$

$$= 2.(\cos^2 2t - \sin^2 2t) + 4\sin 2t\cos 2t$$

$$= 2.\cos 4t + 2\sin 4t = 2(\cos 4t + \sin 4t)$$

GIPTA

(10-g) $y^2+z.y^2+xz^2=0$; $x=t^2$, $y=t^3$

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}=\left(-\frac{z^2}{y^2+2xz}\right).(2t)+\left(-\frac{2y+2yz}{y^2+2xz}\right).(3t^2)$$

$*\left(\frac{\partial z}{\partial x}\right)\longrightarrow y^2.z_x+z^2+x.2z.z_x=0 \Rightarrow z_x(y^2+2xz)=-z^2 \Rightarrow z_x=-\frac{z^2}{y^2+2xz}$

$*\left(\frac{\partial z}{\partial y}\right)\longrightarrow 2y+2yz+z_y y^2+2xz\,z_y=0 \Rightarrow z_y(y^2+2xz)=-(2y+2yz)$

$\Rightarrow z_y=-\frac{2y+2yz}{y^2+2xz}$

(10-h) $z=\frac{2x+y}{y-2x}$; $x=2u-3v$, $y=u+2v$; $(u,v)=(2,1)$

$\left.\begin{matrix}u=2\\v=1\end{matrix}\right\}\Rightarrow\left\{\begin{matrix}x=2.2-3.1=1\\y=2+2.(1)=4\end{matrix}\right\}\Rightarrow\left\{z=\frac{2(1)+4}{4-2.(1)}=\frac{6}{2}=3\right.$

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u}=\left(\frac{2(y-2x)+2.(2x+y)}{(y-2x)^2}\right).(2)+\left(\frac{(y-2x)-(2x+y)}{(y-2x)^2}\right).(1)$$

$=\left(\frac{4y}{(y-2x)^2}\right).(2)+\left(\frac{-4x}{(y-2x)^2}\right).(1)=$ (u, v ye bağlı yazılabilir)

$*\ \frac{\partial z}{\partial u}(2,1)=\left(\frac{4.4}{(4-2)^2}\right)(2)+\left(-\frac{4.1}{(4-2)^2}\right)(1)=8-1=7$ bulunur.

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}=\left(\frac{4y}{(y-2x)^2}\right)(-3)+\left(\frac{-4x}{(y-2x)^2}\right)(2)$$

$*\ \frac{\partial z}{\partial v}(2,1)=\left(\frac{4.4}{(4-2)^2}\right).(-3)+\left(\frac{-4.1}{(4-2)^2}\right).(2)=-24-2=-26$ bulunur.

(10-i) $z=x^2+y^2$; $x=u^2+v^2$, $y=u^2-v^2$

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u}=(2x)(2u)+(2y)(2u)$$
$$=(2u^2+2v^2)(2u)+(2u^2-2v^2).(2u)$$

$$\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}=(2x)(2v)+(2y)(-2v)$$
$$=(2u^2+2v^2)(2v)+(2u^2-2v^2)(-2v)$$

(10-j) $u=z.\sin\left(\frac{y}{x}\right)$; $x=3r^2+2s$, $y=4r-2s^3$, $z=2r^2-3s^2$

$$\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}+\frac{\partial u}{\partial z}\frac{\partial z}{\partial r}=\left(\frac{-yz}{x^2}.\cos\left(\frac{y}{x}\right)\right)(6r)+\left(\frac{z}{x}.\cos\left(\frac{y}{x}\right)\right)(4)+\left(\sin\left(\frac{y}{x}\right)\right)(4r)$$

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial s}+\frac{\partial u}{\partial z}\frac{\partial z}{\partial s}=\left(\frac{-yz}{x^2}\cos\left(\frac{y}{x}\right)\right)(2)+\left(\frac{z}{x}\cos\left(\frac{y}{x}\right)\right)(-6s^2)+\left(\sin\left(\frac{y}{x}\right)\right)(-6s)$$

(10-k) $z=x^3-xy+y^3$; $x=r\cos\theta$, $y=r\sin\theta$

$$\frac{\partial z}{\partial r}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial r}=(3x^2-y)(\cos\theta)+(-x+3y^2)(\sin\theta)$$

$$\frac{\partial z}{\partial \theta}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \theta}=(3x^2-y)(-r\sin\theta)+(-x+3y^2)(r\cos\theta)$$

GIPTA

(10-l) $w = \ln(x^2+y^2+z^2)$, $x = u.e^{v}\sin v$, $y = ue^{v}\cos v$, $z = u.e^{v}$; $(u,v) = (-2,0)$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$= \left(\frac{2x}{x^2+y^2+z^2}\right).\left(e^{v}\sin v\right) + \left(\frac{2y}{x^2+y^2+z^2}\right)\left(e^{v}\cos v\right) + \left(\frac{2z}{x^2+y^2+z^2}\right)\left(e^{v}\right)$$

$$\begin{Bmatrix} u=-2 \\ v=0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} x = -2\,e^0\sin 0 = 0 \\ y = -2\,e^0\cos 0 = -2 \\ z = -2.e^0 = -2 \end{Bmatrix} \Rightarrow \left\{ w = \ln\left(0^2+(-2)^2+(-2)^2\right) = \ln 2^3 = 3.\ln 2 \right\}$$

$$\frac{\partial w}{\partial u}(-2,0) = \left(\frac{2.0}{0+4+4}\right).\left(e^0\sin 0\right) + \left(\frac{2(-2)}{0+4+4}\right)\left(e^0.\cos 0\right) + \left(\frac{2(-2)}{0+4+4}\right)\left(e^0\right)$$

$$= 0 + \left(-\frac{1}{2}\right)(1) + \left(-\frac{1}{2}\right)(1) = -1 \text{ bulunur.}$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

$$= \left(\frac{2x}{x^2+y^2+z^2}\right)\left(ue^{v}.\sin v + ue^{v}\cos v\right) + \left(\frac{2y}{x^2+y^2+z^2}\right)\left(ue^{v}\cos v - ue^{v}\sin v\right) + \left(\frac{2z}{x^2+y^2+z^2}\right)\left(ue^{v}\right)$$

$$\frac{\partial w}{\partial v}(-2,0) = \left(\frac{2.0}{0+4+4}\right)\left(-2.e^0.\sin 0 - 2\,e^0\cos 0\right) + \left(\frac{2.(-2)}{0+4+4}\right)\left(-2e^0.\cos 0 + 2e^0\sin 0\right) + \left(\frac{2.(-2)}{0+4+4}\right)\left(-2e^0\right)$$

$$= 0 + \left(-\frac{1}{2}\right)(-2) + \left(-\frac{1}{2}\right).(-2) = 1+1 = 2 \text{ bulunur,}$$

(10-m) $w = xy + yz + xz$; $x = u+v$, $y = u-v$, $z = u.v$; $(u,v) = \left(\frac{1}{2},1\right)$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$= (y+z)(1) + (x+z)(1) + (y+x)(v)$$

$$\begin{Bmatrix} u=\frac{1}{2} \\ v=1 \end{Bmatrix} \Rightarrow \begin{Bmatrix} x = \frac{1}{2}+1 = \frac{3}{2} \\ y = \frac{1}{2}-1 = -\frac{1}{2} \\ z = \frac{1}{2}.1 = \frac{1}{2} \end{Bmatrix} \rightarrow \left\{ w = \left(-\frac{3}{4}\right)+\left(-\frac{1}{4}\right)+\left(\frac{3}{4}\right) = -\frac{1}{4} \right\}$$

$$\frac{\partial w}{\partial u}\left(\frac{1}{2},1\right) = \left(-\frac{1}{2}+\frac{1}{2}\right)(1) + \left(\frac{3}{2}+\frac{1}{2}\right)(1) + \left(\frac{3}{2}+\left(-\frac{1}{2}\right)\right)(1) = 0+2+1 = 3 \text{ olur.}$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}.\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

$$= (y+z)(1) + (x+z)(-1) + (y+x)(u)$$

$$\frac{\partial w}{\partial v}\left(\frac{1}{2},1\right) = \left(-\frac{1}{2}+\frac{1}{2}\right).(1) + \left(\frac{3}{2}+\frac{1}{2}\right)(-1) + \left(-\frac{1}{2}+\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$= 0 + (-2) + \frac{1}{2} = -\frac{3}{2}$$

(10-11) $u = e^{qr}\arcsin(p)$; $p=\sin x$, $q = z^2 \ln y$, $r = \frac{1}{z}$; $(x,y,z) = \left(\frac{\pi}{4}, \frac{1}{2}, -\frac{1}{2}\right)$

$$\begin{Bmatrix} x = \frac{\pi}{4} \\ y = \frac{1}{2} \\ z = -\frac{1}{2} \end{Bmatrix} \Rightarrow \begin{Bmatrix} p = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \\ q = (-\frac{1}{2})^2 \cdot \ln\frac{1}{2} = -\frac{1}{4}\ln 2 \\ r = \frac{1}{-1/2} = -2 \end{Bmatrix}$$

* $$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}$$

$$= \left(e^{qr}\frac{1}{\sqrt{1-p^2}}\right)(\cos x) + \left(r \cdot e^{qr}\arcsin p\right)(0) + \left(q \cdot e^{qr}\arcsin p\right)(0)$$

$$\frac{\partial u}{\partial x}\left(\frac{\pi}{4}, \frac{1}{2}, -\frac{1}{2}\right) = \left(e^{\frac{1}{2}\ln 2} \cdot \frac{1}{\sqrt{1-\frac{1}{2}}}\right) \cdot \cos\frac{\pi}{4} + 0 + 0$$

$$= \sqrt{2} \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$$ bulunur

* $$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial y} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial y} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial y}$$

$$= \left(e^{qr}\frac{1}{\sqrt{1-p^2}}\right) \cdot (0) + \left(re^{qr}\arcsin p\right) \cdot \left(\frac{z^2}{y}\right) + \left(qe^{qr}\arcsin p\right)(0)$$

$$\frac{\partial u}{\partial y}\left(\frac{\pi}{4}, \frac{1}{2}, -\frac{1}{2}\right) = 0 + \left(-2 \cdot e^{\frac{1}{2}\ln 2}\arcsin\frac{\sqrt{2}}{2}\right)\left(\frac{\frac{1}{4}}{1/2}\right) + 0$$

$$= \left[-2 \cdot (\sqrt{2}) \cdot \left(\frac{\pi}{4}\right)\right] \cdot \left(\frac{1}{2}\right) = -\frac{\sqrt{2}\pi}{4}$$ bulunur.

* $$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial z} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial z}$$

$$= \left(e^{qr}\frac{1}{\sqrt{1-p^2}}\right)(0) + \left(re^{qr}\arcsin p\right)(2z\ln y) + \left(qe^{qr}\arcsin p\right) \cdot \left(-\frac{1}{z^2}\right)$$

$$\frac{\partial u}{\partial z}\left(\frac{\pi}{4}, \frac{1}{2}, -\frac{1}{2}\right) = 0 + \left(-2e^{\frac{1}{2}\ln 2}\arcsin\frac{\sqrt{2}}{2}\right)\left(2 \cdot \left(-\frac{1}{2}\right)\ln\left(\frac{1}{2}\right)\right) + \left(-\frac{1}{4}\ln 2 \cdot e^{\frac{1}{2}\ln 2} \cdot \arcsin\frac{\sqrt{2}}{2}\right)(-4)$$

$$= 0 + \left[\left(-2 \cdot (\sqrt{2}) \cdot \left(\frac{\pi}{4}\right)\right)\right](\ln 2) + \left(\left(-\frac{1}{4}\ln 2\right) \cdot (\sqrt{2}) \left(\frac{\pi}{4}\right)\right)(-4)$$

$$= -\frac{\sqrt{2}\pi\ln 2}{2} + \frac{\sqrt{2}\pi\ln 2}{4} = \frac{\sqrt{2}\pi\ln 2}{4}$$ bulunur.

11) Aşağıdaki kapalı fonksiyonların türevlerini hesaplayınız.

11-a) $x+2y-e^{xy}=0$

$1+2y'-ye^{xy}-xe^{xy}y'=0 \Rightarrow y'(2-xe^{xy})=-(1-ye^{xy}) \Rightarrow y'=\frac{dy}{dx}=-\frac{1-ye^{xy}}{2-xe^{xy}}$

11-b) $4x-5y-\tan(xy)=0$

$4-5y'-(y+xy')\sec^2(xy)=0 \Rightarrow y'(-5-x\sec^2(xy))=-(4-y\sec^2(xy))$

$$y'=-\frac{4-y\sec^2(xy)}{-5-x\sec^2(xy)}=-\frac{F_x}{F_y}$$

11-c) $xy-e^x\cos y=0$

$y+xy'-(e^x\cos y+e^x(-\sin y)y')=0 \Rightarrow y'(x+e^x\sin y)=-(y-e^x\cos y)$

$$\frac{dy}{dx}=y'=-\frac{y-e^x\cos y}{x+e^x\sin y}=-\frac{F_x}{F_y}$$

11-d) $xy^2-\ln(2x+3y)=0$

$y^2+2xy\,y'-\frac{2+3y'}{2x+3y}=0 \Rightarrow y'\left(2xy-\frac{3}{2x+3y}\right)=-\left(y^2-\frac{2}{2x+3y}\right)$

$$\frac{dy}{dx}=y'=-\frac{y^2-\frac{2}{2x+3y}}{2xy-\frac{3}{2x+3y}}=-\frac{F_x}{F_y}$$

11-e) $\ln(x^2+y^2)=\arctan\left(\frac{y}{x}\right)$

$\frac{2x+2yy'}{x^2+y^2}=\frac{-\frac{y}{x^2}+\frac{1}{x}y'}{1+\frac{y^2}{x^2}} \Rightarrow \frac{2x}{x^2+y^2}+\frac{2y\,y'}{x^2+y^2}=\frac{-y}{x^2+y^2}+\frac{x\,y'}{x^2+y^2}$

$$y'\left(\frac{2y-x}{x^2+y^2}\right)=-\left(\frac{y}{x^2+y^2}+\frac{2x}{x^2+y^2}\right)$$

$$y'=-\frac{(y+2x)}{(2y-x)}$$

11-f) $x^2y+\sin(2x-y)=0$

$2xy+x^2y'+(2-y')\cos(2x-y)=0 \Rightarrow y'(x^2-\cos(2x-y))=-(2xy+2\cos(2x-y))$

$$\frac{dy}{dx}=y'=-\frac{2xy+2\cos(2x-y)}{x^2-\cos(2x-y)}=-\frac{F_x}{F_y}$$

11-g) $x^2+y^2+z^2-\ln(yz^2)=0$

* $2x+2z\cdot z_x-\frac{2yz\cdot z_x}{yz^2}=0 \Rightarrow z_x\left(2z-\frac{2}{z}\right)=-(2x) \Rightarrow z_x=-\frac{2x}{2z-\frac{2}{z}}$

* $2y+2z\cdot z_y-\frac{z^2+2yz\cdot z_y}{yz^2}=0 \Rightarrow z_y\cdot\left(2z-\frac{2}{z}\right)=-\left(2y-\frac{1}{y}\right) \Rightarrow z_y=-\frac{2y-\frac{1}{y}}{2z-\frac{2}{z}}$

(11-h) $3x^2-y^2+z^2-xy-4xz-1=0$; $P(0,0,1)$

* $6x+2z.z_x-y-4z-4xz_x=0 \Rightarrow z_x(2z-4x)=-(6x-y-4z) \Rightarrow z_x=-\frac{6x-y-4z}{2z-4x}$

$\Rightarrow z_x(0,0,1)=-\frac{6(0)-(0)-4(1)}{2(1)-4(0)}=-\frac{-4}{2}=2$ bulunur.

* $-2y+2z.z_y-x-4xz_y=0 \rightarrow z_y(2z-4x)=-(-2y-x) \Rightarrow z_y=-\frac{-2y-x}{2z-4x}$

$\Rightarrow z_y(0,0,1)=-\frac{-2(0)-(0)}{2(1)-4.(0)}=-\frac{0}{2}=0$ bulunur.

(11-i) $3z^3+(x^2+y^2)z-x-y-1=0$; $P(1,1,1)$

* $9z^2.z_x+(2x)z+(x^2+y^2).z_x-1=0 \Rightarrow z_x(9z^2+x^2+y^2)=-(2xz-1) \Rightarrow z_x=-\frac{2xz-1}{9z^2+x^2+y^2}$

$\Rightarrow z_x(1,1,1)=-\frac{2(1)(1)-1}{9.1^2+1^2+1^2}=-\frac{1}{11}$ bulunur.

* $9z^2.z_y+(2y)z+(x^2+y^2)z_y-1=0 \Rightarrow z_y(9z^2+x^2+y^2)=-(2yz-1) \Rightarrow z_y=\frac{1-2yz}{9z^2+x^2+y^2}$

$\Rightarrow z_y(1,1,1)=\frac{1-2.(1)(1)}{9.1^2+1^2+1^2}=\frac{-1}{11}$ bulunur.

(11-j) $z^3-6x^2z+4x^3-3y^2=1$; $P(1,-1,0)$

* $3z^2.z_x-12xz-6x^2z_x+12x^2=0 \Rightarrow z_x(3z^2-6x^2)=-(-12xz+12x^2) \Rightarrow z_x=-\frac{-12xz+12x^2}{3z^2-6x^2}$

$\rightarrow z_x(1,-1,0)=-\frac{-12(1)(0)+12(1)^2}{3(0)^2-6.(1)^2}=\frac{12}{-6}=-2$ bulunur.

* $3z^2.z_y-6x^2z_y-6y=0 \Rightarrow z_y(3z^2-6x^2)=-(-6y) \Rightarrow z_y=-\frac{-6y}{3z^2-6x^2}$

$z_y(1,-1,0)=-\frac{-6.(-1)}{3(0)^2-6.(1)^2}=-\frac{6}{-6}=1$ bulunur.

(11-k) $(x+y+z^2)z-x^2-y^2=10$; $P(1,1,2)$

* $(1+2zz_x).(z)+(x+y+z^2)(z_x)-2x=0 \Rightarrow z_x(3z^2+x+y)=-(z-2x) \Rightarrow z_x=-\frac{z-2x}{3z^2+x+y}$

$\Rightarrow z_x(1,1,2)=-\frac{(2)-2.(1)}{3.2^2+1+1}=-\frac{0}{14}=0$ bulunur.

* $(1+2zz_y)z+(x+y+z^2)z_y-2y=0 \rightarrow z_y(3z^2+x+y)=-(z-2y) \Rightarrow z_y=-\frac{z-2y}{3z^2+x+y}$

$\Rightarrow z_y(1,1,2)=-\frac{2-2.(1)}{3.(2)^2+1+1}=-\frac{0}{14}=0$ bulunur.

(11-l) $x^2y-z^2x+zy+1=0$; $P(1,0,1)$

$2xy-(2zxz_x+z^2)+yz_x=0 \Rightarrow z_x(-2xz+y)=-(2xy-z^2) \Rightarrow z_x=-\frac{2xy-z^2}{-2xz+y}$

$z_x(1,0,1)=-\frac{2(1)(0)-1^2}{-2(1)(1)+0}=\frac{1}{-2}=-\frac{1}{2}$ bulunur.

GIPTA

(11-m) $xe^y + \sin(xy) + y - \ln 2 = 0$; $P(0, \ln 2)$

$e^y + xe^y y' + (y + xy')\cos(xy) + y' = 0 \Rightarrow y'(xe^y + x\cos(xy) + 1) = -(e^y + y\cos(xy))$

$y' = -\dfrac{e^y + y.\cos(xy)}{xe^y + x\cos(xy) + 1} \Rightarrow y'(0, \ln 2) = -\dfrac{e^{\ln 2} + \ln 2.\cos(0.\ln 2)}{0.e^{\ln 2} + 0.\cos(0.\ln 2) + 1} = -\dfrac{2 + \ln 2}{1}$

$= -2 - \ln 2$ olur.

(11-n) $\sin(x+y) + \sin(y+z) + \sin(x+z) = 0$; $P(\pi, \pi, \pi)$

* $\cos(x+y) + z_x.\cos(y+z) + \cos(x+z) + z_x.\cos(x+z) = 0$

$\cos(\pi+\pi) + z_x.\cos(\pi+\pi) + \cos(\pi+\pi) + z_x.\cos(\pi+\pi) = 0$

$1 + z_x + 1 + z_x = 0 \Rightarrow 2.z_x = -2 \Rightarrow z_x = -1$ olur.

* $\cos(x+y) + (1 + z_y)\cos(y+z) + z_y.\cos(x+z) = 0$

$\cos(\pi+\pi) + (1 + z_y).\cos(\pi+\pi) + z_y\cos(\pi+\pi) = 0$

$1 + (1 + z_y).(1) + z_y = 0 \Rightarrow 2.z_y = -2 \Rightarrow z_y = -1$ olur.

GIPTA

(12) Aşağıdaki fonksiyonları verilen noktalarda Taylor Serisine açınız. (ikinci mertebe türev dahil)

(12-a) $z = 3xy - x^3 - y^3 \quad ; \quad P(1,1)$

$z = 3xy - x^3 - y^3 \Rightarrow f(1,1) = 3(1)(1) - 1^3 - 1^3 = 1$

$z_x = 3y - 3x^2 \Rightarrow z_x(1,1) = 3(1) - 3(1^2) = 0$

$z_y = 3x - 3y^2 \Rightarrow z_y(1,1) = 3(1) - 3(1^2) = 0$

$z_{xx} = -6x \Rightarrow z_{xx}(1,1) = -6(1) = -6$

$z_{xy} = 3 \Rightarrow z_{xy}(1,1) = 3$

$z_{yy} = -6y \Rightarrow z_{yy}(1,1) = -6.(1) = -6$

$z_{xxx} = -6 \Rightarrow z_{xxx}(1,1) = -6$
$z_{xxy} = 0 \Rightarrow z_{xxy}(1,1) = 0$
$z_{yyx} = 0 \Rightarrow z_{yyx}(1,1) = 0$
$z_{yyy} = -6 \Rightarrow z_{yyy}(1,1) = -6$

$$f(x,y) = f(1,1) + \frac{1}{1!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(1)}_{(1,1)} + \frac{1}{2!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial x}\right]^{(2)}_{(1,1)} + \frac{1}{3!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(3)}_{(1,1)} + \ldots$$

$$= 1 + 1\left[(x-1).0 + (y-1).0\right] + \frac{1}{2}\left[(x-1)^2.(-6) + 2(x-1)(y-1).(3) + (y-1)^2.(-6)\right] +$$
$$\frac{1}{6}\left[(x-1)^3.(-6) + 3(x-1)^2(y-1)(0) + 3(x-1)(y-1)^2.(0) + (y-1)^3.(-6)\right]$$

Açarsak, eşit olduklarını görebiliriz.

$$3xy - x^3 - y^3 \stackrel{?}{=} 1 + (-3)(x^2-2x+1) + 3(xy-x-y+1) + (-3)(y^2-2y+1) + (-1)(x^3-3x^2+3x-1) + (-1)(y^3-3y^2+3y-1)$$

$$= 1 - 3x^2 + 6x - 3 + 3xy - 3x - 3y + 3 - 3y^2 + 6y - 3 - x^3 + 3x^2 - 3x + 1 - y^3 + 3y^2 - 3y + 1$$

$= 3xy - x^3 - y^3$ olduğu görülür.

$$f(x,y) = 1 - 3(x-1)^2 + 3(x-1)(y-1) - 3(y-1)^2 - (x-1)^3 - (y-1)^3$$

(12-6) $z = x^3 + y^3 - 3x - 27y \quad ; \quad P(-1,-3)$

$z = f(x,y) = x^3 + y^3 - 3x - 27y \implies f(-1,-3) = (-1)^3 + (-3)^3 - 3.(-1) - 27(-3) = -1 - 27 + 3 + 3.27$
$= 56$

$z_x = 3x^2 - 3 \implies z_x(-1,-3) = 3(-1)^2 - 3 = 0$

$z_y = 3y^2 - 27 \to z_y(-1,-3) = 3(-3)^2 - 27 = 0$

$z_{xx} = 6x \implies z_{xx}(-1,-3) = 6(-1) = -6$

$z_{xy} = 0 \implies z_{xy}(-1,-3) = 0$

$z_{yy} = 6y \implies z_{yy}(-1,-3) = 6(-3) = -18$

$z_{xxx} = 6 \to z_{xxx}(-1,-3) = 6$

$z_{xxy} = 0 \to z_{xxy}(-1,-3) = 0$

$z_{yyx} = 0 \to z_{yyx}(-1,-3) = 0$

$z_{yyy} = 6 \implies z_{yyy}(-1,-3) = 6$

$$f(x,y) = f(-1,-3) + \frac{1}{1!}\left[(x+1)\frac{\partial z}{\partial x} + (y+3)\frac{\partial z}{\partial y}\right]^{(1)}_{(-1,-3)} + \frac{1}{2!}\left[(x+1)\frac{\partial z}{\partial x} + (y+3)\frac{\partial z}{\partial y}\right]^{(2)}_{(-1,-3)} + \frac{1}{3!}\left[(x+1)\frac{\partial z}{\partial x} + (y+3)\frac{\partial z}{\partial x}\right]^{(3)}_{(-1,-3)} + \ldots\ldots$$

$$= 56 + 1\left[(x+1)(0) + (y+3)(0)\right] + \frac{1}{2}\left[(x+1)^2.(-6) + 2.(x+1)(y+3).(0) + (y+3)^2.(-18)\right] +$$

$$\frac{1}{6}\left[(x+1)^3(6) + (x+1)^2(y+3)(0) + (x+1)(y+3)^2(0) + (y+3)^3.(6)\right]$$

$$= 56 - 3(x+1)^2 - 9(y+3)^2 + (x+1)^3 + (y+3)^3$$

Açarsak eşit olduğunu görebiliriz.

$$z = x^3 + y^3 - 3x - 27y \overset{?}{=} 56 - 3x^2 - 6x - 3 - 9y^2 - 54y - 81 + x^3 + 3x^2 + 3x + 1 + y^3 + 9y^2 + 27y + 27$$

$$= x^3 + y^3 - 3x - 27y$$

(12-C)

$z = 2x^3 - 2xy + y^2 \; ; \quad P\left(\frac{1}{3}, \frac{1}{3}\right)$

$z = f(x,y) = 2x^3 - 2xy + y^2 \Rightarrow f\left(\frac{1}{3}, \frac{1}{3}\right) = 2 \cdot \frac{1}{27} - 2 \cdot \frac{1}{9} + \frac{1}{9} = \frac{2-6+3}{27} = -\frac{1}{27}$

$z_x = 6x^2 - 2y \Rightarrow z_x\left(\frac{1}{3}, \frac{1}{3}\right) = 6 \cdot \left(\frac{1}{9}\right) - 2 \cdot \left(\frac{1}{3}\right) = \frac{2}{3} - \frac{2}{3} = 0$

$z_y = -2x + 2y \Rightarrow z_y\left(\frac{1}{3}, \frac{1}{3}\right) = -2 \cdot \left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right) = 0$

$z_{xx} = 12x \Rightarrow z_{xx}\left(\frac{1}{3}, \frac{1}{3}\right) = 12 \cdot \frac{1}{3} = 4$

$z_{xy} = -2 \Rightarrow z_{xy}\left(\frac{1}{3}, \frac{1}{3}\right) = -2$

$z_{yy} = 2 \rightarrow z_{yy}\left(\frac{1}{3}, \frac{1}{3}\right) = 2$

$z_{xxx} = 12 \Rightarrow z_{xxx}\left(\frac{1}{3}, \frac{1}{3}\right) = 12$

$z_{xxy} = 0 \Rightarrow z_{xxy}\left(\frac{1}{3}, \frac{1}{3}\right) = 0$

$z_{xyy} = 0 \Rightarrow z_{xyy}\left(\frac{1}{3}, \frac{1}{3}\right) = 0$

$z_{yyy} = 0 \Rightarrow z_{yyy}\left(\frac{1}{3}, \frac{1}{3}\right) = 0$

$$f(x,y) = f\left(\tfrac{1}{3}, \tfrac{1}{3}\right) + \frac{1}{1!}\left[\left(x-\tfrac{1}{3}\right)\frac{\partial z}{\partial x} + \left(y-\tfrac{1}{3}\right)\frac{\partial z}{\partial y}\right]^{(1)}_{\left(\frac{1}{3},\frac{1}{3}\right)} + \frac{1}{2!}\left[\left(x-\tfrac{1}{3}\right)\frac{\partial z}{\partial x} + \left(y-\tfrac{1}{3}\right)\frac{\partial z}{\partial x}\right]^{(2)}_{\left(\frac{1}{3},\frac{1}{3}\right)} + \frac{1}{3!}\left[\left(x-\tfrac{1}{3}\right)\frac{\partial z}{\partial x} + \left(y-\tfrac{1}{3}\right)\frac{\partial z}{\partial x}\right]^{(3)}_{\left(\frac{1}{3},\frac{1}{3}\right)} + \dots$$

$$= -\frac{1}{27} + (1)\left[\left(x-\tfrac{1}{3}\right)\cdot(0) + \left(y-\tfrac{1}{3}\right)(0)\right] + \frac{1}{2}\left[\left(x-\tfrac{1}{3}\right)^2(4) + 2\left(x-\tfrac{1}{3}\right)\left(y-\tfrac{1}{3}\right)(-2) + \left(y-\tfrac{1}{3}\right)^2(2)\right] +$$

$$\frac{1}{6}\cdot\left[\left(x-\tfrac{1}{3}\right)^3(12) + 3\left(x-\tfrac{1}{3}\right)^2\left(y-\tfrac{1}{3}\right)(0) + 3\left(x-\tfrac{1}{3}\right)\left(y-\tfrac{1}{3}\right)^2(0) + \left(y-\tfrac{1}{3}\right)^3(0)\right]$$

$$= -\frac{1}{27} + 2\left(x-\tfrac{1}{3}\right)^2 - 2\left(x-\tfrac{1}{3}\right)\left(y-\tfrac{1}{3}\right) + \left(y-\tfrac{1}{3}\right)^2 + 2\cdot\left(x-\tfrac{1}{3}\right)^3$$

Acıp eşit olduklarını görelim

$$2x^3 - 2xy + y^2 \stackrel{?}{=} -\frac{1}{27} + 2x^2 - \frac{4}{3}x + \frac{2}{9} - 2xy + \frac{2}{3}x + \frac{2}{3}y - \frac{2}{9} + y^2 - \frac{2}{3}y + \frac{1}{9} + 2x^3 - 2x^2 + \frac{2}{3}x - \frac{2}{27}$$

$= 2x^3 - 2xy + y^2$ eşit olduğu görülür.

GIPTA

(12-d) $z = \ln\left(\frac{x}{y}\right) \quad ; \quad P(1,1)$

$z = f(x,y) = \ln\left(\frac{x}{y}\right) \Rightarrow f(1,1) = \ln\left(\frac{1}{1}\right) = \ln 1 = 0$

$z_x = \frac{1/y}{x/y} = \frac{1}{x} \Rightarrow z_x(1,1) = \frac{1}{1} = 1$

$z_y = \frac{-x/y^2}{x/y} = -\frac{1}{y} \Rightarrow z_y(1,1) = -\frac{1}{1} = -1$

$z_{xx} = -\frac{1}{x^2} \Rightarrow z_{xx}(1,1) = -\frac{1}{1^2} = -1$

$z_{xy} = 0 \Rightarrow z_{xy}(1,1) = 0$

$z_{yy} = \frac{1}{y^2} \Rightarrow z_{yy}(1,1) = \frac{1}{1^2} = 1$

$z_{xxx} = \frac{2}{x^3} \Rightarrow z_{xxx}(1,1) = \frac{2}{1^3} = 2$

$z_{xxy} = 0 \Rightarrow z_{xxy}(1,1) = 0$

$z_{yyx} = 0 \Rightarrow z_{yyx}(1,1) = 0$

$z_{yyy} = \frac{-2}{y^3} \Rightarrow z_{yyy}(1,1) = \frac{-2}{1^3} = -2$

$$f(x,y) = f(1,1) + \frac{1}{1!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(1)}_{(1,1)} + \frac{1}{2!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(2)}_{(1,1)} + \frac{1}{3!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial x}\right]^{(3)}_{(1,1)} + \cdots$$

$$= 0 + (1)\left[(x-1)(1) + (y-1)(-1)\right] + \frac{1}{2}\left[(x-1)^2(-1) + 2(x-1)(y-1)(0) + (y-1)^2(1)\right] +$$
$$\frac{1}{6}\cdot\left[(x-1)^3\cdot 2 + 3(x-1)^2\cdot(y-1)(0) + 3(x-1)(y-1)^2(0) + (y-1)^3(-2)\right] + \cdots$$

$$= (x-1) - (y-1) - \frac{1}{2}(x-1)^2 + (y-1)^2 + \frac{1}{3}(x-1)^3 - 2(y-1)^3 + \cdots$$

(12-e) $z = e^y \cdot \cos x \quad ; \quad P\left(\frac{\pi}{2}, 0\right)$

$z = f(x,y) = e^y \cos x \Rightarrow f(\pi/2, 0) = e^0 \cdot \cos\frac{\pi}{2} = (1)(0) = 0$

$z_x = -e^y \sin x \Rightarrow z_x(\pi/2, 0) = -e^0 \cdot \sin(\pi/2) = (-1)(1) = -1$

$z_y = e^y \cos x \Rightarrow z_y(\pi/2, 0) = e^0 \cdot \cos(\pi/2) = (1)(0) = 0$

$z_{xx} = -e^y \cos x \Rightarrow z_{xx}(\pi/2, 0) = -e^0 \cdot \cos(\pi/2) = (-1)(1) = -1$

$z_{xy} = -e^y \sin x \Rightarrow z_{xy}(\pi/2, 0) = -e^0 \cdot \sin(\pi/2) = (-1)(1) = -1$

$z_{yy} = e^y \cos x \Rightarrow z_{yy}(\pi/2, 0) = e^0 \cdot \cos(\pi/2) = (1)\cdot(0) = 0$

$z_{xxx} = +e^y \sin x \Rightarrow z_{xxx}(\pi/2, 0) = e^0 \cdot \sin(\pi/2) = (1)(1) = 1$

$z_{xxy} = -e^y \cos x \Rightarrow z_{xxy}(\pi/2, 0) = -e^0 \cos(\pi/2) = (-1)\cdot(0) = 0$

$z_{yyx} = -e^y \sin x \Rightarrow z_{yyx}(\pi/2, 0) = -e^0 \cdot \sin(\pi/2) = (-1)(1) = -1$

$z_{yyy} = e^y \cos x \Rightarrow z_{yyy}(\pi/2, 0) = e^0 \cdot \cos(\pi/2) = (1)\cdot(0) = 0$

$$f(x,y) = f(x,y) + \frac{1}{1!}\left[(x-\tfrac{\pi}{2})\frac{\partial z}{\partial x} + y\cdot\frac{\partial z}{\partial y}\right]^{(1)}_{(\frac{\pi}{2},0)} + \frac{1}{2!}\left[(x-\tfrac{\pi}{2})\frac{\partial z}{\partial x} + y\cdot\frac{\partial z}{\partial y}\right]^{(2)}_{(\frac{\pi}{2},0)} + \frac{1}{3!}\left[(x-\tfrac{\pi}{2})\frac{\partial z}{\partial x} + y\cdot\frac{\partial z}{\partial y}\right]^{(3)}_{(\frac{\pi}{2},0)} + \cdots$$

$$= 0 + (1)\left[(x-\tfrac{\pi}{2})(-1) + y\cdot(0)\right] + \frac{1}{2}\left[(x-\tfrac{\pi}{2})^2\cdot(-1) + 2(x-\tfrac{\pi}{2})\cdot y\cdot(-1) + y^2(0)\right] +$$
$$\frac{1}{6}\left[(x-\tfrac{\pi}{2})^3(1) + (x-\tfrac{\pi}{2})^2\cdot y\cdot(0) + (x-\tfrac{\pi}{2})\cdot y^2\cdot(-1) + y^3(0)\right]$$

$$= -(x-\tfrac{\pi}{2}) - \frac{1}{2}(x-\tfrac{\pi}{2})^2 - 2y(x-\tfrac{\pi}{2}) + \frac{1}{6}(x-\tfrac{\pi}{2})^3 - y^2(x-\tfrac{\pi}{2}) + \cdots$$

GIPTA

(12-f) $2x^2z - yz - xy = 0 \quad ; \quad P(1,1,1)$ $\boxed{\text{KAPALI DURUM}}$

$z(2x^2-y^2) = xy \Rightarrow z = f(x,y) = \dfrac{xy}{2x^2-y^2} \Rightarrow f(1,1) = \dfrac{(1)(1)}{2.1^2-1^2} = \dfrac{1}{1} = 1$

Kapalı fonk. türevi

$z_x \rightarrow 4x.z + 2x^2.z_x - y.z_x - y = 0 \Rightarrow 4.1.1 + 2.1^2.z_x - 1.z_x - 1 = 0 \Rightarrow z_x(1,1,1) = -3$

$z_y \rightarrow 2x^2.z_y - z - y.z_y - x = 0 \Rightarrow 2.z_y - 1 - z_y - 1 = 0 \Rightarrow z_y(1,1,1) = 2$

$z_{xx} \rightarrow 4z + 4xz_x + 4x.z_x + 2x^2 z_{xx} - y z_{xx} = 0 \Rightarrow 4 + 4(-3) + 4.(-3) + 2.z_{xx} - z_{xx} = 0$

$z_{xx}(1,1,1) = 20$

$z_{xy} \rightarrow 4x z_y + 2x^2.z_{xy} - z_x - y z_{xy} - 1 = 0 \Rightarrow 4.(2) + 2z_{xy} - (-3) - z_{xy} - 1 = 0$

$z_{xy}(1,1,1) = -10$

$z_{yy} \rightarrow 2x^2.z_{yy} - z_y - z_y - y z_{yy} = 0 \Rightarrow 2z_{yy} - (2) - (2) - z_{yy} = 0 \Rightarrow z_{yy}(1,1,1) = 4$

$z_{xxx} \rightarrow 4.z_x + 4.z_x + 4x.z_{xx} + 4.z_x + 4x.z_{xx} + 4x.z_{xx} + 2x^2.z_{xxx} - y z_{xxx} = 0 \Rightarrow$

$4(-3) + 4(-3) + 4(20) + 4(-3) + 4.(20) + 4.(20) + 2.z_{xxx} - z_{xxx} = 0$

$z_{xxx} = -204$

$z_{xxy} \rightarrow 4.z_y + 4x z_{xy} + 4x z_{xy} + 2x^2 z_{xxy} - z_{xx} - y z_{xxy} = 0$

$4(2) + 4.(-10) + 4(-10) + 2z_{xxy} - (20) - z_{xxy} = 0 \Rightarrow z_{xxy}(1,1,1) = 92$

$z_{yyx} \rightarrow 4x.z_{yy} + 2x^2.z_{yyx} - z_{yx} - z_{yx} - y z_{yyx} = 0$

$4(4) + 2.z_{yyx} - (-10) - (-10) - z_{yyx} = 0 \Rightarrow z_{yyx}(1,1,1) = -36$

$z_{yyy} \rightarrow 2x^2.z_{yyy} - z_{yy} - z_{yy} - z_{yy} - y.z_{yyy} = 0$

$2.z_{yyy} - (4) - (4) - (4) - z_{yyy} = 0 \Rightarrow z_{yyy}(1,1,1) = 12$

$$f(x,y) = f(1,1) + \frac{1}{1!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(1)}_{(1,1)} + \frac{1}{2!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(2)}_{(1,1)} + \frac{1}{3!}\left[(x-1)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y}\right]^{(3)}_{(1,1)} + \dots$$

$$= 1 + (1)\left[(x-1).(-3) + (y-1)(2)\right] + \frac{1}{2}\left[(x-1)^2(20) + 2(x-1)(y-1)(-10) + (y-1)^2(4)\right] +$$

$$\frac{1}{6}\left[(x-1)^3.(-204) + 3(x-1)^2(y-1)(92) + 3(x-1)(y-1)^2(-36) + (y-1)^3(12)\right]$$

$$= 1 - 3(x-1) + 2(y-1) + 10(x-1)^2 - 10(x-1)(y-1) + 2(y-1)^2 - 34(x-1)^3 + 46(x-1)^2(y-1)$$

$$- 18(x-1)(y-1)^2 + 2(y-1)^3 + \dots\dots$$

(12-g) $z = x.e^y$; $P(0,0) \longrightarrow$ $\boxed{\text{Mac Laurin Seri Açılımı}}$

$z = f(x,y) = xe^y \to f(0,0) = 0.e^0 = 0(1) = 0$

$z_x = e^y \Rightarrow z_x(0,0) = e^0 = 1$

$z_y = x.e^y \Rightarrow z_y(0,0) = 0.e^0 = 0.(1) = 0$

$z_{xx} = 0 \Rightarrow z_{xx}(0,0) = 0$

$z_{xy} = e^y \Rightarrow z_{xy}(0,0) = e^0 = 1$

$z_{yy} = x.e^y \Rightarrow z_{yy}(0,0) = 0.e^0 = 0.(1) = 0$

$z_{xxx} = 0 \Rightarrow z_{xxx}(0,0) = 0$

$z_{xxy} = 0 \Rightarrow z_{xxy}(0,0) = 0$

$z_{yyx} = e^y \to z_{yyx}(0,0) = e^0 = 1$

$z_{yyy} = xe^y \Rightarrow z_{yyy}(0,0) = 0.e^0 = 0.(1) = 0$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} + \dots$$

$$= (0) + (1)\left[x(1) + y(0)\right] + \frac{1}{2}\left[x^2(0) + 2xy(1) + y^2(0)\right] +$$

$$\frac{1}{6}\left[x^3(0) + 3x^2y(0) + 3xy^2(1) + y^3(0)\right]$$

$$= x + xy + \frac{1}{2}.xy^2 + \dots\dots$$

(12-h) $z = e^x \cos y$; $P(0,0)$,

$z = f(x,y) = e^x.\cos x \Rightarrow f(0,0) = e^0.\cos(0) = (1)(1) = 1$

$z_x = e^x.\cos y \Rightarrow z_x(0,0) = e^0.\cos(0) = (1)(1) = 1$

$z_y = -e^x \sin y \to z_y(0,0) = -e^0.\sin(0) = (-1).(0) = 0$

$z_{xx} = e^x \cos y \Rightarrow z_{xx}(0,0) = e^0.\cos(0) = (1)(1) = 1$

$z_{xy} = -e^x \sin y \Rightarrow z_{xy}(0,0) = -e^0.\sin(0) = (-1).(0) = 0$

$z_{yy} = -e^x \cos y \Rightarrow z_{yy}(0,0) = -e^0.\cos(0) = (-1)(1) = -1$

$z_{xxx} = e^x \cos y \Rightarrow z_{xxx}(0,0) = e^0.\cos(0) = (1)(1) = 1$

$z_{xxy} = -e^x \sin y \Rightarrow z_{xxy}(0,0) = -e^0.\sin(0) = (-1)(0) = 0$

$z_{yyx} = -e^x \cos y \Rightarrow z_{yyx}(0,0) = -e^0.\cos(0) = (-1)(1) = -1$

$z_{yyy} = +e^x \sin y \Rightarrow z_{yyy}(0,0) = e^0.\sin(0) = (1)(0) = 0$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y.\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} + \dots\dots$$

$$= 1 + \left[x.(1) + y(0)\right] + \frac{1}{2}\left[x^2(1) + 2xy(0) + y^2(-1)\right] +$$

$$\frac{1}{6}\left[x^3(1) + 3x^2y(0) + 3xy^2(-1) + y^3(0)\right]$$

$$= 1 + x + \frac{1}{2}x^2 - y^2 + \frac{1}{6}x^3 - \frac{1}{2}xy^2 + \dots\dots$$

GIPTA

(12-L) $z = y \cdot \sin x \quad ; \quad P(0,0)$

$z = f(x,y) = y \cdot \sin x \Rightarrow f(0,0) = 0 \cdot \sin(0) = 0$

$z_x = y \cdot \cos x \Rightarrow z_x(0,0) = 0 \cdot \cos(0) = 0$

$z_y = \sin x \Rightarrow z_y(0,0) = \sin(0) = 0$

$z_{xx} = -y \sin x \Rightarrow z_{xx}(0,0) = -0 \cdot \sin(0) = 0$

$z_{xy} = \cos x \Rightarrow z_{xy}(0,0) = \cos(0) = 1$

$z_{yy} = 0 \Rightarrow z_{yy}(0,0) = 0$

$z_{xxx} = -y \cos x \Rightarrow z_{xxx}(0,0) = -0 \cdot \cos(0) = 0$

$z_{xxy} = -\sin x \Rightarrow z_{xxy}(0,0) = -\sin(0) = 0$

$z_{yyx} = 0 \Rightarrow z_{yyx}(0,0) = 0$

$z_{yyy} = 0 \Rightarrow z_{yyy}(0,0) = 0$

$z_{xxxx} = y \sin x \Rightarrow z_{xxxx}(0,0) = 0 \cdot \sin(0) = 0$
$z_{xxxy} = -\cos x \Rightarrow z_{xxxy}(0,0) = -\cos(0) = -1$
$z_{yyyx} = 0 \Rightarrow z_{yyyx}(0,0) = 0$
$z_{xxyy} = 0 \Rightarrow z_{xxyy}(0,0) = 0$
$z_{yyyy} = 0 \Rightarrow z_{yyyy}(0,0) = 0$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} +$$
$$\frac{1}{4!}\left[x\frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y}\right]^{(4)}_{(0,0)} + \cdots$$

$$= 0 + 0 + \frac{1}{2}\left[2xy\,(1)\right] + \frac{1}{6}\left[0\right] + \frac{1}{24}\left[4 \cdot x^3 y\,(-1)\right]$$

$$= xy - \frac{1}{6}x^3 y + \cdots$$

GIPTA

(12-i) $z = \sin x \cos y \quad ; \quad P(0,0)$

$z = f(x,y) = \sin x \cos y \Rightarrow f(0,0) = \sin(0)\cos(0) = (0).(1) = 0$

$z_x = \cos x \cos y \Rightarrow z_x(0,0) = \cos(0)\cos(0) = (1)(1) = 1$

$z_y = -\sin x \sin y \Rightarrow z_y(0,0) = -\sin(0)\sin(0) = -(0)(0) = 0$

$z_{xx} = -\sin x \cos y \Rightarrow z_{xx}(0,0) = -\sin(0)\cos(0) = -(0)(1) = 0$

$z_{xy} = -\cos x \sin y \Rightarrow z_{xy}(0,0) = -\cos(0)\sin(0) = -(1)(0) = 0$

$z_{yy} = -\sin x \cos y \Rightarrow z_{yy}(0,0) = -\sin(0)\cos(0) = -(0)(1) = 0$

$z_{xxx} = -\cos x \cos y \Rightarrow z_{xxx}(0,0) = -\cos(0).\cos(0) = -(1)(1) = -1$

$z_{xxy} = \sin x \sin y \Rightarrow z_{xxy}(0,0) = \sin(0)\sin(0) = (0)(0) = 0$

$z_{yyx} = -\cos x \cos y \Rightarrow z_{yyx}(0,0) = -\cos(0).\cos(0) = -(1).(1) = -1$

$z_{yyy} = \sin x \sin y \Rightarrow z_{yyy}(0,0) = \sin(0)\sin(0) = (0)(0) = 0$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} + \cdots\cdots$$

$$= 0 + [x(1) + y(0)] + \frac{1}{2}[x^2(0) + 2xy(0) + y^2(0)] +$$
$$\frac{1}{6}[x^3(-1) + 3x^2y(0) + 3xy^2(-1) + y^3(0)]$$

$$= x - \frac{1}{6}x^3 - \frac{1}{2}xy^2 + \cdots\cdots$$

(12-j) $z = e^x . \ln(1+y) \quad ; \quad P(0,0)$

$z = f(x,y) = e^x . \ln(1+y) \rightarrow f(0,0) = e^0 . \ln(1) = 1.(0) = 0$

$z_x = e^x . \ln(1+y) \Rightarrow z_x(0,0) = e^0 \ln(1) = 1.(0) = 0$

$z_y = e^x \frac{1}{1+y} \rightarrow z_y(0,0) = e^0 . \frac{1}{1} = (1)(1) = 1$

$z_{xx} = e^x . \ln(1+y) \Rightarrow z_{xx}(0,0) = e^0 . \ln(1) = 1.(0) = 0$

$z_{xy} = e^x . \frac{1}{1+y} \Rightarrow z_{xy}(0,0) = e^0 \frac{1}{1} = (1).(1) = 1$

$z_{yy} = e^x\left(\frac{-(1)(1)}{(1+y)^2}\right) \Rightarrow z_{yy}(0,0) = e^0 . \left(\frac{-1}{1^2}\right) = (1).(-1) = -1$

$z_{xxx} = e^x . \ln(1+y) \rightarrow z_{xxx}(0,0) = e^0 . \ln(1) = (1)(0) = 0$

$z_{xxy} = e^x . \left(\frac{-1}{(1+y)^2}\right) \rightarrow z_{xxy}(0,0) = e^0 . \left(\frac{-1}{1^2}\right) = (1)(-1) = -1$

$z_{yyx} = e^x . \left(\frac{-1}{(1+y)^2}\right) \Rightarrow z_{yyx}(0,0) = e^0\left(\frac{-1}{1^2}\right) = (1)(-1) = -1$

$z_{yyy} = e^x . \left(\frac{2}{(1+y)^2}\right) \Rightarrow z_{yyy}(0,0) = e^0\left(\frac{2}{1^2}\right) = (1)(2) = 2$

$$f(x,y) = 0 + [x(0) + y(1)] + \frac{1}{2}[x^2(0) + 2xy(1) + y^2(-1)]$$
$$+ \frac{1}{6}[x^3.(0) + 3x^2y(-1) + 3xy^2(-1) + y^3(2)]$$
$$= y + xy - y^2 - \frac{1}{2}x^2y - \frac{1}{2}xy^2 + \frac{1}{3}y^3 + \cdots\cdots$$

(12-k) $z = \ln(2x+y+1)$; $P(0,0)$

$z = f(x,y) = \ln(2x+y+1) \Rightarrow f(0,0) = \ln(1) = 0$

$z_x = \frac{2}{2x+y+1} \Rightarrow z_x(0,0) = \frac{2}{1} = 2$

$z_y = \frac{1}{2x+y+1} \Rightarrow z_y(0,0) = \frac{1}{1} = 1$

$z_{xx} = \frac{-(2)(2)}{(2x+y+1)^2} = \frac{-4}{(2x+y+1)^2} \Rightarrow z_{xx}(0,0) = \frac{-4}{1} = -4$

$z_{xy} = \frac{-(2)(1)}{(2x+y+1)^2} = \frac{-2}{(2x+y+1)^2} \Rightarrow z_{xy}(0,0) = \frac{-2}{1} = -2$

$z_{yy} = \frac{-(1)(1)}{(2x+y+1)^2} = \frac{-1}{(2x+y+1)^2} \Rightarrow z_{yy}(0,0) = \frac{-1}{1} = -1$

$z_{xxx} = \frac{-(-4)[(2)(2x+y+1)(2)]}{(2x+y+1)^4} = \frac{16}{(2x+y+1)^3} \Rightarrow z_{xxx}(0,0) = \frac{16}{1} = 16$

$z_{xxy} = \frac{-(-4)[2(2x+y+1)(1)]}{(2x+y+1)^4} = \frac{8}{(2x+y+1)^3} \Rightarrow z_{xxy}(0,0) = \frac{8}{1} = 8$

$z_{yyx} = \frac{-(-1)[2.(2x+y+1)(2)]}{(2x+y+1)^4} = \frac{4}{(2x+y+1)^3} \Rightarrow z_{yyx}(0,0) = \frac{4}{1} = 4$

$z_{yyy} = \frac{-(-1)[2(2x+y+1)(1)]}{(2x+y+1)^4} = \frac{2}{(2x+y+1)^3} \Rightarrow z_{yyy}(0,0) = \frac{2}{1} = 2$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} + \cdots$$

$$= 0 + [x(2) + y(1)] + \frac{1}{2}[x^2(-4) + 2xy(-2) + y^2(-1)]$$

$$+ \frac{1}{6}[x^3(16) + 3x^2y(8) + 3xy^2(4) + y^3(2)]$$

$$= 2x + y - 2x^2 - 2xy - \frac{1}{2}y^2 + \frac{8}{3}x^3 + 4x^2y + 2xy^2 + \frac{1}{3}y^3 + \cdots$$

GIPTA

(12-l) $z = \sin(x^2+y^2)$; $P(0,0)$

$z = f(x,y) = \sin(x^2+y^2) \Rightarrow f(0,0) = \sin(0) = 0$

$z_x = 2x \cdot \cos(x^2+y^2) \Rightarrow z_x(0,0) = 2.(0).\cos(0) = 0$

$z_y = 2y \cdot \cos(x^2+y^2) \Rightarrow z_y(0,0) = 2.(0).\cos(0) = 0$

$z_{xx} = 2.\cos(x^2+y^2) + (2x).(2x\cos(x^2+y^2)) \Rightarrow z_{xx}(0,0) = 2.\cos(0) + (2(0)).(2.(0).\cos 0)$

$z_{xx}(0,0) = 2$

$z_{xy} = 2x.(2y\sin(x^2+y^2)) = 4xy\sin(x^2+y^2) \Rightarrow z_{xy}(0,0) = 4(0)(0)\sin(0) = 0$

$z_{yy} = 2(\cos(x^2+y^2)) + (2y)(2y \cdot \sin(x^2+y^2)) \Rightarrow z_{yy}(0,0) = 2.\cos(0) + 0 = 2$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \ldots\ldots$$

$$= 0 + [x(0) + y(0)] + \frac{1}{2}[x^2(2) + 2xy(0) + y^2(2)] + \ldots$$

$$= x^2 + y^2 + \ldots\ldots$$

(12-m) $z = \cos(x^2+y^2)$; $P(0,0)$

$z = f(x,y) = \cos(x^2+y^2) \Rightarrow f(0,0) = \cos(0) = 1$

$z_x = -2x.\sin(x^2+y^2) \Rightarrow z_x(0,0) = -2.(0).\sin(0) = 0$

$z_y = -2y\sin(x^2+y^2) \Rightarrow z_y(0,0) = -2.(0).\sin(0) = 0$

$z_{xx} = (-2)(\sin(x^2+y^2)) - 2x(2x.\cos(x^2+y^2)) \Rightarrow z_{xx}(0,0) = 0$

$z_{xy} = -2x(2y\cos(x^2+y^2)) \Rightarrow z_{xy}(0,0) = 0$

$z_{yy} = -2(\sin(x^2+y^2)) - 2y(2y\cos(x^2+y^2)) \Rightarrow z_{yy}(0,0) = 0$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \ldots$$

$$= 1 + [x(0) + y(0)] + \frac{1}{2}[x^2(0) + 2xy(0) + y^2(0)] + \ldots$$

$$= 1 + \ldots\ldots$$

GIPTA

(12-n) $z = \dfrac{1}{1-x-y}$; $P(0,0)$

$$z = f(x,y) = \frac{1}{1-x-y} \Rightarrow f(0,0) = \frac{1}{1} = 1$$

$$z_x = \frac{-(1)(-1)}{(1-x-y)^2} = \frac{1}{(1-x-y)^2} \rightarrow z_x(0,0) = \frac{1}{1} = 1$$

$$z_y = \frac{-(1)(-1)}{(1-x-y)^2} = \frac{1}{(1-x-y)^2} \Rightarrow z_y(0,0) = \frac{1}{1} = 1$$

$$z_{xx} = \frac{-(1)(2(1-x-y)(-1))}{(1-x-y)^4} = \frac{2}{(1-x-y)^3} \Rightarrow z_{xx}(0,0) = \frac{2}{1} = 2$$

$$z_{xy} = \frac{-(1)(2(1-x-y)(-1))}{(1-x-y)^4} = \frac{2}{(1-x-y)^3} \Rightarrow z_{xy}(0,0) = \frac{2}{1} = 2$$

$$z_{yy} = \frac{-(1)(2.(1-x-y)(-1))}{(1-x-y)^4} = \frac{2}{(1-x-y)^3} \rightarrow z_{yy}(0,0) = \frac{2}{1} = 2$$

$$z_{xxx} = \frac{-(2)(3(1-x-y)^2(-1))}{(1-x-y)^6} = \frac{3!}{(1-x-y)^4} \Rightarrow z_{xxx}(0,0) = \frac{3!}{1} = 6$$

$$z_{xxy} = \frac{-(2)(3(1-x-y)^2(-1))}{(1-x-y)^6} = \frac{3!}{(1-x-y)^4} \Rightarrow z_{xxy}(0,0) = \frac{3!}{1} = 6$$

$$z_{yyx} = \frac{-(2)(3(1-x-y)^2(-1))}{(1-x-y)^6} = \frac{3!}{(1-x-y)^4} \Rightarrow z_{yyx}(0,0) = \frac{3!}{1} = 6$$

$$z_{yyy} = \frac{-(2)(3.(1-x-y)^2.(-1))}{(1-x-y)^6} = \frac{3!}{(1-x-y)^4} \Rightarrow z_{yyy}(0,0) = \frac{3!}{1} = 6$$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} + \ldots\ldots$$

$$= 1 + [x(1) + y(1)] + \frac{1}{2}[x^2(2) + 2xy(2) + y^2(2)] + \frac{1}{6}[x^3(6) + 3x^2y(6) + 3xy^2(6) + y^3(6)]$$

$$= 1 + x + y + x^2 + 2xy + y^2 + x^3 + 3x^2y + 3xy^2 + y^3 + \ldots\ldots$$

GIPTA

(12-0) $z = \dfrac{1}{1-x-y+xy}$; $P(0,0)$

$$z = f(x,y) = \frac{1}{1-x-y+xy} \Rightarrow f(0,0) = \frac{1}{1} = 1$$

$$z_x = \frac{-(1)(-1+y)}{(1-x-y+xy)^2} = \frac{1-y}{(1-x-y+xy)^2} = \frac{1}{(1-x)^2(1-y)} \Rightarrow z_x(0,0) = \frac{1}{1} = 1$$

$$z_y = \frac{-(1)(-1+x)}{(1-x-y+xy)^2} = \frac{1-x}{(1-x-y+xy)^2} = \frac{1}{(1-x)(1-y)^2} \Rightarrow z_y(0,0) = \frac{1}{1} = 1$$

$$z_{xx} = \frac{1}{1-y}\left(\frac{2}{(1-x)^3}\right) \Rightarrow z_{xx}(0,0) = \frac{1}{1}\cdot\left(\frac{2}{1}\right) = 2$$

$$z_{xy} = \frac{1}{(1-x)^2}\ \frac{1}{(1-y)^2} \Rightarrow z_{xy}(0,0) = \left(\frac{1}{1}\right)\left(\frac{1}{1}\right) = 1$$

$$z_{yy} = \left(\frac{1}{1-x}\right)\cdot\left(\frac{2}{(1-y)^3}\right) \Rightarrow z_{yy}(0,0) = \left(\frac{1}{1}\right)\left(\frac{2}{1}\right) = 2$$

$$z_{xxx} = \left(\frac{1}{1-y}\right)\left(\frac{3!}{(1-x)^4}\right) \Rightarrow z_{xxx}(0,0) = \left(\frac{1}{1}\right)\cdot\left(\frac{3!}{1}\right) = 6$$

$$z_{xxy} = \left(\frac{1}{(1-y)^2}\right)\left(\frac{2}{(1-x)^3}\right) \Rightarrow z_{xxy}(0,0) = \left(\frac{1}{1}\right)\cdot\left(\frac{2}{1}\right) = 2$$

$$z_{yyx} = \left(\frac{1}{(1-x)^2}\right)\cdot\left(\frac{2}{(1-y)^3}\right) \Rightarrow z_{yyx}(0,0) = \left(\frac{1}{1}\right)\left(\frac{2}{1}\right) = 2$$

$$z_{yyy} = \left(\frac{1}{1-x}\right)\left(\frac{3!}{(1-y)^4}\right) \Rightarrow z_{yyy}(0,0) = \left(\frac{1}{1}\right)\left(\frac{6}{1}\right) = 6$$

$$f(x,y) = f(0,0) + \frac{1}{1!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(1)}_{(0,0)} + \frac{1}{2!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(2)}_{(0,0)} + \frac{1}{3!}\left[x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right]^{(3)}_{(0,0)} + \ldots\ldots$$

$$= 1 + [x(1) + y(1)] + \frac{1}{2}[x^2(2) + 2xy(1) + y^2(2)] +$$
$$\frac{1}{6}[x^3(6) + 3x^2y(2) + 3xy^2(2) + y^3(6)]$$

$$= 1 + x + y + x^2 + xy + y^2 + x^3 + x^2y + xy^2 + y^3 + \ldots\ldots$$

(13) Aşağıdaki fonksiyonların ekstremumlarını bulunuz.

(13-a) $z = (3x^2 + y^2 - 6x) \cdot y = 3x^2y + y^3 - 6xy$

$z_x = 6xy - 6x = 6x(y-1)$

$z_y = 3x^2 + 3y^2 - 6x$

$$\begin{cases} 6x(y-1) = 0 \Rightarrow x = 0 \text{ veya } y = 1 \\ 3x^2 + 3y^2 - 6x = 0 \Rightarrow x^2 + y^2 - 2x = 0 \rightarrow y^2 = 2x - x^2 \end{cases}$$

$x = 0 \Rightarrow y^2 = 2.0 - 0^2 = 0 \rightarrow A(0,0)$ kritik nokta

$y = 1 \Rightarrow 1^2 = 2x - x^2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1 \Rightarrow B(1,1)$ kritik nokta

* $A = z_{xx} = 6y - 6 \Rightarrow \begin{cases} z_{xx}(0,0) = -6 \\ z_{xx}(1,1) = 0 \end{cases}$

* $C = z_{yy} = 6y \Rightarrow \begin{cases} z_{yy}(0,0) = 0 \\ z_{yy}(1,1) = 6 \end{cases}$

* $B = z_{xy} = 6x \Rightarrow \begin{cases} z_{xy}(0,0) = 0 \\ z_{xy}(1,1) = 6 \end{cases}$

$A(0,0)$ için $\{D = AC - B^2 = (-6)(0) - 0^2 = 0\} \Rightarrow$ {Şüpheli hal vardır.}

$B(1,1)$ için $\{D = AC - B^2 = (0)(6) - 6^2 = -36 < 0\} \Rightarrow$ {Yerel maksimum nok. yok. (Semer noktası var.)}

GIPTA

(13-6) $z = x^3 + xy^2 - x$

* $z_x = 3x^2 + y^2 - 1$

* $z_y = 2xy$

$2xy = 0 \Rightarrow x = 0$ veya $y = 0$

$3x^2 + y^2 - 1 = 0 \Rightarrow y^2 = 1 - 3x^2$

$x = 0 \Rightarrow y^2 = 1 - 3.0^2 \Rightarrow y^2 = 1 \Rightarrow \begin{cases} y_1 = 1 & \Rightarrow A(0,1) \\ y_2 = -1 & \Rightarrow B(0,-1) \end{cases}$

$y = 0 \Rightarrow 0^2 = 1 - 3x^2 \Rightarrow 3x^2 = 1 \Rightarrow x^2 = \frac{1}{3} \Rightarrow \begin{cases} x_1 = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} & \Rightarrow C(-\frac{\sqrt{3}}{3}, 0) \\ x_2 = +\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} & \Rightarrow D(+\frac{\sqrt{3}}{3}, 0) \end{cases}$

* $A = z_{xx} = 6x \Rightarrow \begin{cases} z_{xx}(0,1) = 6.0 = 0 \\ z_{xx}(0,-1) = 6.0 = 0 \\ z_{xx}(-\sqrt{3}/3, 0) = -2\sqrt{3} \\ z_{xx}(+\sqrt{3}/3, 0) = 2\sqrt{3} \end{cases}$

* $C = z_{yy} = 2x \Rightarrow \begin{cases} z_{yy}(0,1) = 2.(0) = 0 \\ z_{yy}(0,-1) = 2.(0) = 0 \\ z_{yy}(-\sqrt{3}/3, 0) = -\frac{2}{3}\sqrt{3} \\ z_{yy}(\sqrt{3}/3, 0) = \frac{2\sqrt{3}}{3} \end{cases}$

* $B = z_{xy} = 2y \Rightarrow \begin{cases} z_{xy}(0,1) = 2(1) = 2 \\ z_{xy}(0,-1) = 2(-1) = -2 \\ z_{xy}(-\sqrt{3}/3, 0) = 2(0) = 0 \\ z_{xy}(\sqrt{3}/3, 0) = 2(0) = 0 \end{cases}$

$A(0,1)$ için $\{ \Delta = AC - B^2 = (0)(0) - 2^2 = -4 < 0 \} \Rightarrow \{$ Yerel Maksimum yok SEMER NOKTASI $\}$

$B(0,-1)$ için $\{ \Delta = AC - B^2 = (0)(0) - (-2)^2 = -4 < 0 \} \Rightarrow \{$ Yerel Maksimum yok SEMER NOKTASI $\}$

$C(-\frac{\sqrt{3}}{3}, 0)$ için $\begin{cases} \Delta = AC - B^2 = (-2\sqrt{3})(-\frac{2\sqrt{3}}{3}) - 0^2 = 4 > 0 \\ A = z_{xx}(-\frac{\sqrt{3}}{3}, 0) = -2\sqrt{3} < 0 \end{cases} \Rightarrow \{$ Yerel Maksimum var $\}$

$D(\frac{\sqrt{3}}{3}, 0)$ için $\begin{cases} \Delta = AC - B^2 = (2\sqrt{3})(\frac{2\sqrt{3}}{3}) - 0^2 = 4 > 0 \\ A = z_{xx}(\frac{\sqrt{3}}{3}, 0) = 2\sqrt{3} > 0 \end{cases} \rightarrow \{$ Yerel Minimum var $\}$

Yerel Minimum Degeri bulmak gerekirse

$$z = \left(\frac{\sqrt{3}}{3}\right)^3 + \left(\frac{\sqrt{3}}{3}\right).0^2 - \frac{\sqrt{3}}{3} = \frac{3\sqrt{3}}{3^3} - \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{9} - \frac{3\sqrt{3}}{9} = -\frac{2\sqrt{3}}{9}$$ olur.

GIPTA

13-c

$z = y^3 + yx^2 - y - 1$

$z_x = 2xy$

$z_y = 3y^2 - x^2 - 1$

$\begin{cases} 2xy = 0 \Rightarrow x=0 \text{ veya } y=0 \\ 3y^2 - x^2 - 1 = 0 \Rightarrow y^2 = \frac{1}{3}(x^2+1) \end{cases}$

$x=0 \Rightarrow y^2 = \frac{1}{3}(0^2+1) = \frac{1}{3} \Rightarrow \begin{cases} y = \frac{\sqrt{3}}{3} \Rightarrow A\left(0, \frac{\sqrt{3}}{3}\right) \\ y = -\frac{\sqrt{3}}{3} \Rightarrow B\left(0, -\frac{\sqrt{3}}{3}\right) \end{cases}$

$y=0 \Rightarrow 0^2 = \frac{1}{3}(x^2+1) \Rightarrow x^2+1=0 \Rightarrow x^2 = -1 \Rightarrow$ real çözüm yok

* $A = z_{xx} = 2y \Rightarrow \begin{cases} z_{xx}\left(0, \frac{\sqrt{3}}{3}\right) = \frac{2\cdot\sqrt{3}}{3} \\ z_{xx}\left(0, -\frac{\sqrt{3}}{3}\right) = -\frac{2\sqrt{3}}{3} \end{cases}$

* $C = z_{yy} = 6y \Rightarrow \begin{cases} z_{yy}\left(0, \frac{\sqrt{3}}{3}\right) = 6\cdot\frac{\sqrt{3}}{3} = 2\sqrt{3} \\ z_{yy}\left(0, -\frac{\sqrt{3}}{2}\right) = 6\cdot\left(-\frac{\sqrt{3}}{3}\right) = -2\sqrt{3} \end{cases}$

* $B = z_{xy} = 2x \Rightarrow \begin{cases} z_{xy}\left(0, \frac{\sqrt{3}}{3}\right) = 2(0) = 0 \\ z_{xy}\left(0, -\frac{\sqrt{3}}{3}\right) = 2(0) = 0 \end{cases}$

$A\left(0, \frac{\sqrt{3}}{3}\right)$ için $\left\{ \begin{array}{l} \Delta = AC - B^2 = \left(\frac{2\sqrt{3}}{3}\right)(2\sqrt{3}) - 0^2 = 4 > 0 \\ A = z_{xx}\left(0, \frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3} > 0 \end{array} \right\} \Rightarrow$ {Yerel Minimum var}

$B\left(0, -\frac{\sqrt{3}}{3}\right)$ için $\left\{ \begin{array}{l} \Delta = AC - B^2 = \left(-\frac{2\sqrt{3}}{3}\right)(-2\sqrt{3}) - 0^2 = 4 > 0 \\ A = z_{xx}\left(0, -\frac{\sqrt{3}}{3}\right) = -\frac{2\sqrt{3}}{3} < 0 \end{array} \right\} \Rightarrow$ {Yerel Maksimum var}

Yerel Minimum değer

$z = f\left(0, \frac{\sqrt{3}}{3}\right) = \left(\frac{\sqrt{3}}{3}\right)^3 + \frac{\sqrt{3}}{3}\cdot(0)^2 - \frac{\sqrt{3}}{3} - 1 = \frac{3\sqrt{3}}{27} - \frac{\sqrt{3}}{3} - 1 = \frac{\sqrt{3}}{9} - \frac{3\sqrt{3}}{9} - \frac{9}{9} = \frac{-2\sqrt{3}-9}{9}$

Yerel Maksimum Değer

$z = f\left(0, -\frac{\sqrt{3}}{3}\right) = \left(-\frac{\sqrt{3}}{3}\right)^3 + \left(-\frac{\sqrt{3}}{3}\right)\cdot(0)^2 + \frac{\sqrt{3}}{3} - 1 = -\frac{\sqrt{3}}{9} + \frac{3\sqrt{3}}{9} - \frac{9}{9} = \frac{2\sqrt{3}-9}{9}$

GIPTA

(13-d) $z = x^3 + y^3 - xy$

$$\left\{\begin{array}{l} z_x = 3x^2 - y = 0 \\ z_y = 3y^2 - x = 0 \end{array}\right\}$$ Denklem sisteminin çözüm kümesi kritik noktalardır.

$$\left\{\begin{array}{l} 3x^2 - y = 0 \Rightarrow y = 3x^2 \\ 3y^2 - x = 0 \Rightarrow x = 3y^2 \end{array}\right\} \rightarrow \left\{\begin{array}{l} x = 3(3x^2)^2 \rightarrow 27x^4 - x = 0 \Rightarrow x\left[(3x)^3 - 1\right] = 0 \\ (x)(3x-1)(9x^2+3x+1) = 0 \Rightarrow \left\{\begin{array}{l} x_1 = 0 \\ x_2 = \frac{1}{3} \end{array}\right\} \end{array}\right.$$

$$x = 0 \Rightarrow 0 = 3y^2 \Rightarrow y^2 = 0 \Rightarrow y = 0 \Rightarrow A(0,0)$$

$$x = \frac{1}{3} \Rightarrow \frac{1}{3} = 3y^2 \Rightarrow y^2 = \frac{1}{9} \Rightarrow y = \mp\frac{1}{3} \Rightarrow \left\{\begin{array}{l} B\left(\frac{1}{3}, \frac{1}{3}\right) \\ C\left(\frac{1}{3}, -\frac{1}{3}\right) \end{array}\right.$$

$$* \; A = z_{xx} = 6x \Rightarrow \left\{\begin{array}{l} z_{xx}(0,0) = 6.(0) = 0 \\ z_{xx}\left(\frac{1}{3}, \frac{1}{3}\right) = 6.\frac{1}{3} = 2 \\ z_{xx}\left(\frac{1}{3}, -\frac{1}{3}\right) = 6.\frac{1}{3} = 2 \end{array}\right.$$

$$* \; C = z_{yy} = 6y \Rightarrow \left\{\begin{array}{l} z_{yy}(0,0) = 6(0) = 0 \\ z_{yy}\left(\frac{1}{3}, \frac{1}{3}\right) = 6\left(\frac{1}{3}\right) = 2 \\ z_{yy}\left(\frac{1}{3}, -\frac{1}{3}\right) = 6\left(-\frac{1}{3}\right) = -2 \end{array}\right.$$

$$* \; B = z_{xy} = -1 \Rightarrow \left\{\begin{array}{l} z_{xy}(0,0) = -1 \\ z_{xy}\left(\frac{1}{3}, \frac{1}{3}\right) = -1 \\ z_{xy}\left(\frac{1}{3}, -\frac{1}{3}\right) = -1 \end{array}\right.$$

$A(0,0)$ için $\left\{ \Delta = AC - B^2 = (0)(0) - (-1)^2 = -1 < 0 \right\} \Rightarrow \left\{\begin{array}{l}\text{Yerel ekstremum yok} \\ \text{SEMER NOKTASI}\end{array}\right\}$

$B\left(\frac{1}{3}, \frac{1}{3}\right)$ için $\left\{\begin{array}{l} \Delta = AC - B^2 = (2)(2) - (-1)^2 = 4 - 1 = 3 > 0 \\ z_{xx}\left(\frac{1}{3}, \frac{1}{3}\right) = 2 > 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l}\text{Yerel Minimum} \\ \text{var}\end{array}\right\}$

$C\left(\frac{1}{3}, -\frac{1}{3}\right)$ için $\left\{ D = AC - B^2 = (2)(-2) - (-1)^2 = -5 < 0 \right\} \Rightarrow \left\{\begin{array}{l}\text{Yerel ekstremum yok} \\ \text{SEMER NOKTASI}\end{array}\right\}$

Yerel Minimum Değer

$$z = f\left(\frac{1}{3}, \frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{27} + \frac{1}{27} - \frac{3}{27} = -\frac{1}{27}$$ dir.

GIPTA

(13-e)

$z = x^3 + y^3 - 3xy + 15$

$\left\{ \begin{array}{l} z_x = 3x^2 - 3y = 0 \\ z_y = 3y^2 - 3x = 0 \end{array} \right\}$ Denklem sisteminin çözümü kritik noktaları verir.

$3x^2 - 3y = 0 \Rightarrow y = x^2 \rightarrow 3(x^2)^2 - 3x = 0 \rightarrow x^4 - x = 0$

$x(x^3 - 1) = 0 \Rightarrow x(x-1)(x^2+x+1) = 0 \Rightarrow \left\{ \begin{array}{l} x = 0 \\ x = 1 \end{array} \right\}$ bulunur.

$x = 0 \Rightarrow 3y^2 - 3(0) = 0 \Rightarrow y^2 = 0 \Rightarrow y = 0 \Rightarrow A(0,0)$ kritik nokta

$x = 1 \Rightarrow 3y^2 - 3(1) = 0 \Rightarrow y^2 = 1 \Rightarrow y = \mp 1 \Rightarrow \left\{ \begin{array}{l} B(1,1) \\ C(1,-1) \end{array} \right\}$ kritik noktalar.

* $A = z_{xx} = 6x \Rightarrow \left\{ \begin{array}{l} z_{xx}(0,0) = 6(0) = 0 \\ z_{xx}(1,1) = 6(1) = 6 \\ z_{xx}(1,-1) = 6(1) = 6 \end{array} \right.$

* $C = z_{yy} = 6y \Rightarrow \left\{ \begin{array}{l} z_{yy}(0,0) = 6(0) = 0 \\ z_{yy}(1,1) = 6.(1) = 6 \\ z_{yy}(1,-1) = 6(-1) = -6 \end{array} \right.$

* $B = z_{xy} = -3 \Rightarrow \left\{ \begin{array}{l} z_{xy}(0,0) = -3 \\ z_{xy}(1,1) = -3 \\ z_{xy}(1,-1) = -3 \end{array} \right.$

$A(0,0)$ için $\Rightarrow \{ \Delta = AC - B^2 = (0)(0) - (-3)^2 = -9 < 0 \} \Rightarrow \{$ Yerel ekstremum yok SEMER NOKTASI $\}$

$B(1,1)$ için $\Rightarrow \left\{ \begin{array}{l} \Delta = AC - B^2 = (6)(6) - (-3)^2 = 27 > 0 \\ z_{xx}(1,1) = 6 > 0 \end{array} \right\} \Rightarrow \{$ Yerel Minimum Var $\}$

$C(1,-1)$ için $\Rightarrow \{ \Delta = AC - B^2 = (6)(-6) - (-3)^2 = -45 < 0 \} \Rightarrow \{$ Yerel Ekstremum yok SEMER NOKTASI $\}$

Yerel minimum değeri

$z = f(1,1) = 1^3 + 1^3 - 3(1)(1) + 15 = 14$ bulunur

(13-f) $z = x^2 + y^2 + 4x + 6y$

$$\begin{cases} z_x = 2x + 4 = 0 \\ z_y = 2y + 6 = 0 \end{cases}$$

$$\left.\begin{array}{l} 2x+4=0 \Rightarrow x = \frac{-4}{2} = -2 \\ 2y+6=0 \Rightarrow y = \frac{-6}{2} = -3 \end{array}\right\} \Rightarrow A(-2,-3) \text{ Kritik noktadır.}$$

* $A = z_{xx} = 2 \rightarrow \{ z_{xx}(-2,-3) = 2$

* $C = z_{yy} = 2 \rightarrow \{ z_{yy}(-2,-3) = 2$

* $B = z_{xy} = 0 \Rightarrow \{ z_{xy}(-2,-3) = 0$

$A(-2,-3)$ için $\left\{ \begin{array}{l} D = AC - B^2 = (2)(2) - 0^2 = 4 > 0 \\ z_{xx}(-2,-3) = 2 > 0 \end{array} \right\} \Rightarrow \{$ Yerel Minimum var$)$

Yerel Minimum Değer

$z = f(-2,-3) = (-2)^2 + (-3)^2 + 4(-2) + 6(-3) = 4 + 9 - 8 - 18 = -13$ bulunur.

$$\left.\begin{array}{l} z = \underline{x^2 + 4x + 4} - 4 \quad + \underline{y^2 + 6x + 9} - 9 \\ z = (x+2)^2 + (y+3)^2 - 13 \end{array}\right\} \text{ fonksiyonunun dairesel paraboloid olduğuna dikkat ediniz.}$$

* Paraboloidin Tepe noktasının $(-2,-3,-13)$ olduğuna dikkat ediniz. Ayrıca, bu noktanın yerel minimumunun olduğu noktadır.

(13-g) $z = x^3 - 3xy + y^3$

$\left\{\begin{array}{l} z_x = 3x^2 - 3y = 0 \\ z_y = -3x + 3y^2 = 0 \end{array}\right\}$ denklem sisteminin çözümü kritik noktalardır.

$3x^2 - 3y = 0 \Rightarrow y = x^2 \Rightarrow -3x + 3(x^2)^2 = 0 \Rightarrow -3x + 3x^4 = 0$

$\Rightarrow x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0 \Rightarrow x(x-1)(x^2 + x + 1) = 0 \Rightarrow \left\{\begin{array}{l} x = 0 \\ x = 1 \end{array}\right\}$

$x = 0 \Rightarrow -3(0) + 3y^2 = 0 \Rightarrow 3y^2 = 0 \Rightarrow y^2 = 0 \Rightarrow y = 0 \Rightarrow A(0,0)$ bulunur.

$x = 1 \Rightarrow -3(1) + 3y^2 = 0 \Rightarrow y^2 = 1 \Rightarrow y = \mp 1 \Rightarrow \left\{\begin{array}{l} B(1,1) \\ C(1,-1) \end{array}\right\}$ bulunur.

* $A = z_{xx} = 6x \Rightarrow \left\{\begin{array}{l} z_{xx}(0,0) = 6 \cdot 0 = 0 \\ z_{xx}(1,1) = 6 \cdot 1 = 6 \\ z_{xx}(1,-1) = 6(1) = 6 \end{array}\right.$

* $C = z_{yy} = 6y \Rightarrow \left\{\begin{array}{l} z_{yy}(0,0) = 6(0) = 0 \\ z_{yy}(1,1) = 6(1) = 6 \\ z_{yy}(1,-1) = 6(-1) = -6 \end{array}\right.$

* $B = z_{xy} = -3 \Rightarrow \left\{\begin{array}{l} z_{xy}(0,0) = -3 \\ z_{xy}(1,1) = -3 \\ z_{xy}(1,-1) = -3 \end{array}\right.$

$A(0,0)$ için $\Rightarrow \left\{ D = AC - B^2 = (0)(0) - (-3)^2 = -9 < 0 \right\} \Rightarrow \left\{\begin{array}{l} \text{Yerel ekstremum yok} \\ \text{SEMER NOKTASI} \end{array}\right\}$

$B(1,1)$ için $\Rightarrow \left\{\begin{array}{l} D = AC - B^2 = (6)(6) - (-3)^2 = 27 > 0 \\ z_{xx}(1,1) = 6 > 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \text{Yerel Minimum} \\ \text{Var} \end{array}\right\}$

$C(1,-1)$ için $\Rightarrow \left\{ \Delta = AC - B^2 = (6)(-6) - (-3)^2 = -45 < 0 \right\} \Rightarrow \left\{\begin{array}{l} \text{Yerel Ekstremum yok} \\ \text{SEMER NOKTASI} \end{array}\right\}$

Yerel Minimum Değer

$z = f(1,1) = 1^3 - 3(1)(1) + 1^3 = 1 - 3 + 1 = -1$ bulunur.

GIPTA

(13-h) $z = 4x - 2y - x^2 + xy - y^2$

$$\left\{\begin{array}{l} z_x = 4 - 2x + y = 0 \\ z_y = -2 + x - 2y = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 2x - y = 4 \\ x - 2y = 2 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y = 2x - 4 \\ x = 2y + 2 \end{array}\right\}$$

$y = 2x - 4 \Rightarrow x = 2(2x-4) + 2 \Rightarrow x = 4x - 6 \Rightarrow 3x = 6 \Rightarrow x = 2$

$x = 2 \Rightarrow 2 = 2y + 2 \Rightarrow 2y = 0 \Rightarrow y = 0 \Rightarrow A(2,0)$ kritik nokta

* $A = z_{xx} = -2 \Rightarrow \{ z_{xx}(2,0) = -2$
* $C = z_{yy} = -2 \Rightarrow \{ z_{yy}(2,0) = -2$
* $B = z_{xy} = 1 \Rightarrow \{ z_{xy}(2,0) = 1$

$A(2,0)$ için $\left\{\begin{array}{l} D = AC - B^2 = (-2)(-2) - 1^2 = 3 > 0 \\ z_{xx}(2,0) = -2 < 0 \end{array}\right\} \Rightarrow \{$ Yerel Maksimum var $\}$

Yerel Maksimum Değer

$z = f(2,0) = 4.(2) - 2(0) - 2^2 + 2.(0) - 0^2 = 8 - 4 = 4$ bulunur.

(13-i) $z = x^2 + 2xy$

$$\left\{\begin{array}{l} z_x = 2x + 2y = 0 \\ z_y = 2x = 0 \end{array}\right\} \Rightarrow 2x = 0 \Rightarrow x = 0 \Rightarrow 2.0 + 2y = 0 \Rightarrow y = 0 \Rightarrow A(0,0)$$

* $A = z_{xx} = 2 \Rightarrow z_{xx}(0,0) = 2$
* $C = z_{yy} = 0 \Rightarrow z_{yy}(0,0) = 0$
* $B = z_{xy} = 2 \Rightarrow z_{xy}(0,0) = 2$

$A(0,0)$ için $\{ D = AC - B^2 = (2)(0) - 2^2 = -4 < 0 \} \Rightarrow \{$ ekstremum yok SEMER NOKTASI $\}$

GIPTA

(13-i) $z = 6x^2 - 2x^3 + 3y^2 + 6xy$

$$\left\{\begin{array}{l} z_x = 12x - 6x^2 + 6y = 0 \\ z_y = 6y + 6x = 0 \end{array}\right\}$$ Denklem sisteminin çözümü kritik noktalardır.

$6y + 6x = 0 \Rightarrow x + y = 0 \Rightarrow y = -x \Rightarrow$

$12x - 6x^2 + 6(-x) = 0 \Rightarrow 6x^2 - 6x = 0 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow \left\{\begin{array}{l} x=0 \\ x=1 \end{array}\right\}$

$x = 0 \Rightarrow y = -x = -0 = 0 \Rightarrow A(0,0)$ kritik nokta

$x = 1 \Rightarrow y = -x = -1 \Rightarrow B(1,-1)$ kritik nokta

* $A = z_{xx} = 12 - 12x \Rightarrow \left\{\begin{array}{l} z_{xx}(0,0) = 12 \\ z_{xx}(1,-1) = 0 \end{array}\right.$

* $C = z_{yy} = 6 \Rightarrow \left\{\begin{array}{l} z_{yy}(0,0) = 6 \\ z_{yy}(1,-1) = 6 \end{array}\right.$

* $B = z_{xy} = 6 \Rightarrow \left\{\begin{array}{l} z_{xy}(0,0) = 6 \\ z_{xy}(1,-1) = 6 \end{array}\right.$

$A(0,0)$ için $\left\{\begin{array}{l} D = AC - B^2 = (12).(6) - 6^2 = 36 > 0 \\ z_{xx}(0,0) = 12 > 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{c} \text{Yerel Minimum} \\ \text{Var} \end{array}\right\}$

$B(1,-1)$ için $\left\{ D = AC - B^2 = (0)(6) - 6^2 = -36 < 0 \right\} \Rightarrow \left\{ \text{SEMER NOKTASI} \right\}$

Yerel Minimum Değer

$z = f(0,0) = 6.0^2 - 2.0^3 + 3.0^2 + 6.(0)(0) = 0$ bulunur.

(13-j) $z = x^2 + xy + 3x + 2y + 5$

$$\left\{\begin{array}{l} z_x = 2x + y + 3 = 0 \\ z_y = x + 2 = 0 \end{array}\right\}$$ Denklem Sisteminin çözümü kritik noktalardır.

$x + 2 = 0 \Rightarrow x = -2 \Rightarrow 2.(-2) + y + 3 = 0 \Rightarrow y = 1 \rightarrow A(-2,1)$

* $A = z_{xx} = 2 \Rightarrow z_{xx}(-2,1) = 2$
* $C = z_{yy} = 0 \Rightarrow z_{yy}(-2,1) = 0$
* $B = z_{xy} = 1 \Rightarrow z_{xy}(-2,1) = 1$

$A(-2,1)$ için $\{D = A.C - B^2 = (2)(0) - 1^2 = -1 < 0\} \Rightarrow \{$ Yerel ekstremum yok SEMER NOKTASI $\}$

(13-k) $z = 4xy - x^4 - y^4$

$$\left\{\begin{array}{l} z_x = 4y - 4x^3 = 0 \\ z_y = 4x - 4y^3 = 0 \end{array}\right\}$$ Denklem Sisteminin çözümü kritik noktalardır.

$4y - 4x^3 = 0 \Rightarrow y - x^3 = 0 \Rightarrow y = x^3 \Rightarrow$

$4x - 4(x^3)^3 = 0 \Rightarrow x - x^9 = 0 \Rightarrow x(1 - x^8) = 0 \Rightarrow$

$x(1 - x^4)(1 + x^4) = 0 \Rightarrow x(1 - x^2)(1 + x^2)(1 + x^4) = 0 \Rightarrow$

$x(1 - x)(1 + x)(1 + x^2)(1 + x^4) = 0 \Rightarrow \left\{\begin{array}{l} x = 0 \\ x = 1 \\ x = -1 \end{array}\right\}$ bulunur.

$x = 0 \Rightarrow y = x^3 = 0^3 = 0 \rightarrow A(0,0)$

$x = 1 \Rightarrow y = x^3 = 1^3 = 1 \rightarrow B(1,1)$

$x = -1 \Rightarrow y = (-1)^3 = -1 \rightarrow C(-1,-1)$

* $A = z_{xx} = -12x^2 \Rightarrow \left\{\begin{array}{l} z_{xx}(0,0) = 0 \\ z_{xx}(1,1) = -12 \\ z_{xx}(-1,-1) = -12 \end{array}\right.$
* $C = z_{yy} = -12y^2 \Rightarrow \left\{\begin{array}{l} z_{yy}(0,0) = 0 \\ z_{yy}(1,1) = -12 \\ z_{yy}(-1,-1) = -12 \end{array}\right.$
* $B = z_{xy} = 4 \rightarrow \left\{\begin{array}{l} z_{xy}(0,0) = 4 \\ z_{xy}(1,1) = 4 \\ z_{xy}(-1,-1) = 4 \end{array}\right.$

$A(0,0)$ için $\{D = AC - B^2 = (0)(0) - 4^2 = -16 < 0\} \rightarrow \{$ SEMER NOKTASI $\}$

$B(1,1)$ için $\left\{\begin{array}{l} D = AC - B^2 = (-12)(-12) - 4^2 = 128 > 0 \\ z_{xx}(1,1) = -12 < 0 \end{array}\right\} \rightarrow \{$ Yerel Maksimum var $\}$

$C(-1,-1)$ için $\left\{\begin{array}{l} D = AC - B^2 = (-12)(-12) - 4^2 = 128 > 0 \\ z_{xx}(-1,-1) = -12 < 0 \end{array}\right\} \Rightarrow \{$ Yerel Maksimum var $\}$

GIPTA

(13-l) $z = \frac{1}{x^2+y^2-1}$

$$\begin{cases} z_x = \frac{-(1)(2x)}{(x^2+y^2-1)^2} = \frac{-2x}{(x^2+y^2-1)^2} = 0 \\ z_y = \frac{-(1)(2y)}{(x^2+y^2-1)^2} = \frac{-2y}{(x^2+y^2-1)^2} = 0 \end{cases} \Rightarrow \begin{cases} -2x = 0 \Rightarrow x = 0 \\ -2y = 0 \Rightarrow y = 0 \end{cases} \Rightarrow A(0,0)$$

kritik nokta

* $A = z_{xx} = \frac{(-2)(x^2+y^2-1)^2 - 2(x^2+y^2-1)(2x)(-2x)}{(x^2+y^2-1)^4} = \frac{-2(x^2+y^2-1)+8x^2}{(x^2+y^2-1)^3} \Rightarrow$

$$z_{xx}(0,0) = \frac{2+0}{-1} = -2$$

* $C = z_{yy} = \frac{(-2)(x^2+y^2-1)^2 - 2y(2)(x^2+y^2-1)(2y)}{(x^2+y^2-1)^4} = \frac{-2(x^2+y^2-1)+8y^2}{(x^2+y^2-1)^3}$

$$z_{yy}(0,0) = \frac{2}{-1} = -2$$

* $B = z_{xy} = \frac{-(-2x)(2y)(2)(x^2+y^2-1)}{(x^2+y^2-1)^4} = \frac{8xy}{x^2+y^2-1} \Rightarrow z_{xy}(0,0) = \frac{0}{-1} = 0$

$A(0,0)$ için $\begin{cases} D = AC - B^2 = (-2)(-2) - 0^2 = 4 > 0 \\ z_{xx}(0,0) = -2 < 0 \end{cases} \Rightarrow$ {Yerel Maksimum var}

Yerel Maksimum Değer

$$z = f(0,0) = \frac{1}{0^2+0^2-1} = -1$$ bulunur.

(13-m) $z = y.\sin x$

$$\left.\begin{array}{l} z_x = y.\cos x = 0 \\ z_y = \sin x = 0 \end{array}\right\}$$ Denklem sisteminin çözümünü bulalım.

$\sin x = 0 \Rightarrow x_1 = 0 + 2k\pi$ ve $x_2 = \pi + 2k\pi$

$x=0 \Rightarrow y.\cos x = 0 \Rightarrow y = 0 \Rightarrow A(0,0)$ Kritik nokta

$x=\pi \Rightarrow y.\cos x = 0 \Rightarrow y = 0 \Rightarrow B(\pi,0)$ Kritik nokta

$x=2\pi \Rightarrow y.\cos x = 0 \Rightarrow y = 0 \Rightarrow C(2\pi,0)$ Kritik nokta

Bu kritik noktalar artırılabilir. (Biz $[0,2\pi]$ aralığı için bulduk)

* $A = z_{xx} = -y\sin x \Rightarrow \begin{cases} z_{xx}(0,0) = 0 \\ z_{xx}(\pi,0) = 0 \\ z_{xx}(2\pi,0) = 0 \end{cases}$

* $C = z_{yy} = 0 \Rightarrow \begin{cases} z_{yy}(0,0) = 0 \\ z_{yy}(\pi,0) = 0 \\ z_{yy}(2\pi,0) = 0 \end{cases}$

* $B = z_{xy} = \cos x \Rightarrow \begin{cases} z_{xy}(0,0) = \cos(0) = 1 \\ z_{xy}(\pi,0) = \cos(\pi) = -1 \\ z_{xy}(2\pi,0) = \cos(2\pi) = 1 \end{cases}$

$A(0,0)$ için $\{\Delta = AC - B^2 = (0)(0) - 1^2 = -1 < 0\} \Rightarrow \{$SEMER NOKTASI$\}$

$B(\pi,0)$ için $\{D = AC - B^2 = (0)(0) - (-1)^2 = -1 < 0\} \Rightarrow \{$SEMER NOKTASI$\}$

$C(2\pi,0)$ için $\{D = AC - B^2 = (0)(0) - 1^2 = -1 < 0\} \Rightarrow \{$SEMER NOKTASI$\}$

GIPTA

(13-n) $z = e^{2x} \cdot \cos y$

$$\left.\begin{aligned} z_x &= 2e^{2x} \cdot \cos y = 0 \\ z_y &= -e^{2x} \cdot \sin y = 0 \end{aligned}\right\}$$ Denklem sistminin çözümü bulunmalı

$$2 \cdot e^{2x} \cdot \cos y = 0 \Rightarrow \begin{cases} \cos y = 0 \Rightarrow y_1 = \frac{\pi}{2} + 2k\pi \\ \qquad\qquad\quad\; y_2 = \frac{3\pi}{2} + 2k\pi \\ e^{2x} = 0 \rightarrow x = -\infty \quad (\text{çözüm yok}) \end{cases}$$

$y = \frac{\pi}{2} \Rightarrow -e^{2x} \cdot \sin\frac{\pi}{2} = 0 \Rightarrow x = -\infty$ (çözüm yok)

$y = \frac{3\pi}{2} \Rightarrow -e^{2x} \cdot \sin\frac{3\pi}{2} = 0 \Rightarrow x = -\infty$ (çözüm yok)

Bu eğri için kritik nokta yoktur.

(13-o) $z = \frac{1}{x} + xy + \frac{1}{y}$

$$\left.\begin{aligned} z_x &= -\frac{1}{x^2} + y = 0 \\ z_y &= -\frac{1}{y^2} + x = 0 \end{aligned}\right\}$$ Denklem sisteminin çözümü kritik noktaları verir.

$$y = \frac{1}{x^2} \Rightarrow -\frac{1}{\left(\frac{1}{x^2}\right)^2} + x = 0 \Rightarrow -x^4 + x = 0 \Rightarrow x(1-x^3) = 0 \Rightarrow$$

$$x(1-x)(1+x+x^2) = 0 \Rightarrow \begin{Bmatrix} x_1 = 0 \\ x_2 = 1 \end{Bmatrix}$$ bulunur.

$x = 0 \Rightarrow y = \frac{1}{0^2} = \frac{1}{0^+} = \infty$ (çözüm yok)

$x = 1 \Rightarrow y = \frac{1}{1^2} = 1 \Rightarrow A(1,1)$ kritik noktadır.

* $A = z_{xx} = \frac{2}{x^3} \Rightarrow z_{xx}(1,1) = \frac{2}{1^3} = 2$
* $C = z_{yy} = \frac{2}{y^3} \Rightarrow z_{yy}(1,1) = \frac{2}{1^3} = 2$
* $B = z_{xy} = 1 \Rightarrow z_{xy}(1,1) = 1$

$A(1,1)$ için $\left\{\begin{aligned} D &= AC - B^2 = 2 \cdot 2 - 1^2 = 3 > 0 \\ z_{xx}(1,1) &= 2 > 0 \end{aligned}\right\} \Rightarrow \left\{ \text{Yerel Minimum var} \right\}$

Yerel Minimum Değer

$$z = f(1,1) = \frac{1}{1} + (1)\cdot(1) + \frac{1}{1} = 1+1+1 = 3$$ olur.

GIPTA