Uygulama Soruları II (Working Questions II)

A. a) $y_h = c_1 e^{x} + c_2 e^{-x} + c_3 e^{\frac{2}{3}x} + c_4 e^{-\frac{2}{3}x} + c_5 e^{5x} + c_6 e^{-6x}$

b) $y_h = c_1 + c_2 x + (c_3 + c_4 x) e^{-2x} + c_5 \cos 3x + c_6 \sin 3x$

c) $y_h = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x + c_5 \cos x + c_6 \sin x$

d) $y_h = (c_1 + c_2 x + c_3 x^2) e^{2x} + c_4 e^{-2x} + e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

e) $y_h = (c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x$

f) $y_h = e^{-x}\left[(c_1 + c_2 x)\cos 3x + (c_3 + c_4 x)\sin 3x\right] + c_5 e^{2x} + c_6 e^{-2x}$

g) $y_h = e^{2x}\left[(c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x\right]$

h) $y_h = c_1 e^{\sqrt{2}x} + (c_2 + c_3 x)\cos 2x + (c_4 + c_5 x)\sin 2x + c_6 e^{\sqrt{3}x}$

i) $y_h = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x + c_5 \cos 3x + c_6 \sin 3x$

j) $y_h = c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x + e^{x}(c_5 \cos x + c_6 \sin x)$

k) $y_h = c_1 e^{2x} + (c_2 + c_3 x) e^{-x} + (c_4 + c_5 x + c_6 x^2) e^{-3x}$

l) $y_h = (c_1 + c_2 x + c_3 x^2)\cos \frac{3}{2}x + (c_4 + c_5 x + c_6 x^2)\sin \frac{3}{2}x$

m) $y_h = e^{-3x}\left[(c_1 + c_2 x + c_3 x^2)\cos 5x + (c_4 + c_5 x + c_6 x^2)\sin 5x\right]$

n) $y_h = e^{x}(c_1 \cos 2x + c_2 \sin 2x) + c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x + (c_5 + c_6 x) e^{-\sqrt{2}x}$

B. a. $y''' - 3y'' - y' + 3y = 2e^{3x} + 7x$

$\underbrace{r^3 - 3r^2 - r + 3 = 0} \Rightarrow r_1 = 1 \Rightarrow$

$$\begin{array}{l|l} r^3-3r^2-r+3 & r-1 \\ \hline \underline{-\ r^3-r^2} & r^2-2r-3 \\ -2r^2-r+3 & \\ \underline{-\ -2r^2+2r} & \\ -3r+3 & \\ \underline{-\ -3r+3} & \\ 0 & \end{array}$$

$\underbrace{(r-1)(r^2-2r-3) = 0}_{-3\ \ +1}$

$(r-1)(r-3)(r+1) = 0$

$\boxed{r_1 = 1,\ r_2 = 3,\ r_3 = -1}$

$\underline{y_{p1} = k.x.e^{3x}}$; $\underline{y_{p2} = (ax+b)}$

↳ rezonans

b. $y''' + 2y'' - 4y' - 8y = 4x + 7 + 2e^{-2x}$

$r^3 + 2r^2 - 4r - 8 = 0 \Rightarrow r^2(r+2) - 4(r+2) = 0 \Rightarrow (r^2-4)(r+2) = 0$

$r_1 = 2,\ r_2 = -2 = r_3$

$y_{p1} = (ax+b)$ $\quad y_{p2} = k.x^2.e^{-2x}$

↳ rezonans

c. $y''' - 3y'' + 3y' - y = 8xe^x$

$r^3 - 3r^2 + 3r - 1 = 0 \Rightarrow (r^3-1) - 3r(r-1) = 0 \Rightarrow (r-1)\underbrace{(r^2+r+1-3r)}_{(r^2-2r+1)} = 0$

$r_1 = r_2 = r_3 = 1$

$y_p = (ax+b).x^3.e^x$

↳ rezonans

d. $y^{v} + 9y''' = 11x \sin 3x + x + 2$

$r^5 + 9r^3 = 0 \Rightarrow r^3(r^2+9) = 0 \Rightarrow r_1 = r_2 = r_3 = 0,\ r_{4,5} = \mp 3i$

$y_{p1} = (ax+b).\underline{x}.\cos 3x + (cx+d).\underline{x}.\sin 3x + (ax+b).\underline{x^3}$

rezonans

e. $y^{IV} - 2y'' + y = 2x.e^{x} + e^{-x}$

$r^4 - 2r^2 + 1 = 0 \Rightarrow (r^2-1)^2 = 0 \Rightarrow (r-1)^2(r+1)^2 = 0 \Rightarrow r_1 = r_2 = 1,\ r_3 = r_4 = -1$

$y_{p1} = (ax+b).x^2.e^x,\ y_{p2} = k x^2.e^{-x}$

f. $y''' - y'' - y' + y = 9e^{-x} + 4e^{x}$

$r^3 - r^2 - r + 1 = 0 \Rightarrow r^2(r-1) - (r-1) = 0 \Rightarrow (r^2-1)(r-1) = 0 \Rightarrow r_1 = r_2 = 1,\ r_3 = -1$

$y_{p1} = k.e^{-x}.x \quad , \quad y_{p2} = k.x^2.e^x$

rezonans

g. $y^{IV} - y'' = 9x - 3 + 2e^{-x}$

$r^4 - r^2 = 0 \Rightarrow r^2(r^2-1) = 0 \Rightarrow r_1 = r_2 = 0,\ r_3 = 1,\ r_4 = -1$

$y_{p1} = (ax+b).x^2 \quad , \quad y_{p2} = k.x.e^{-x}$

h. $y'' + 4y = \underbrace{2\cos 2x - 4\sin 2x}_{y_{p1}} + \underbrace{3\sin 4x}_{y_{p2}}$

$r^2 + 4 = 0 \Rightarrow r_{1,2} = \mp 2i$

$y_{p1} = (A\cos 2x + B\sin 2x).x \quad , \quad y_{p2} = A\cos 4x + B\sin 4x$

C.1. $y'' - y' = (6-6x)e^x - 2$

$r^2 - r = 0 \Rightarrow r(r-1) = 0 \Rightarrow r_1 = 0,\ r_2 = 1 \Rightarrow \boxed{y_h = C_1 + C_2 e^x}$

$y_{p1} = (ax+b).e^x.\underbrace{x}_{\text{rezonans}} = (ax^2+bx)e^x$

$y_{p1}' = (2ax+b)e^x + (ax^2+bx)e^x$

$y_{p1}'' = 2ae^x + (2ax+b)e^x + (2ax+b)e^x + (ax^2+bx)e^x$

$[2a + 4ax + 2b + \cancel{ax^2} + \cancel{bx} - 2ax - b - \cancel{ax^2} - \cancel{bx}]e^x = (6-6x)e^x$

$2ax + 2a + b = 6 - 6x$

$2a = -6 \qquad 2a + b = 6$

$\underline{a = -3} \qquad b = 6 + 6 = 12$

$\boxed{y_{p1} = (-3x^2 + 12x)e^x}$

$y_{p2} = k.\underbrace{x}_{\text{rezonans}}$

$y_{p2}' = k$

$y_{p2}'' = 0$

$0 - k = -2$

$\boxed{k = 2}$

$\boxed{y_{p2} = 2x}$

Çözüm (solution) $\Rightarrow y = y_h + y_{p1} + y_{p2} = C_1 + C_2 e^x + (-3x^2 + 12x)e^x + 2x$

2. $y'' + 2y' + y = e^{-x}(2x+3)$

$r^2 + 2r + 1 = 0 \Rightarrow (r+1)^2 = 0 \Rightarrow r_1 = r_2 = -1 \Rightarrow \boxed{y_h = (c_1 + c_2 x)e^{-x}}$

$y_p = (ax+b)e^{-x}.x^2 = (ax^3 + bx^2)e^{-x}$

$y_p' = (3ax^2 + 2bx)e^{-x} - (ax^3 + bx^2)e^{-x}$

$y_p'' = (6ax + 2b)e^{-x} - (3ax^2 + 2bx)e^{-x} - (3ax^2 + 2bx)e^{-x} + (ax^3 + bx^2)e^{-x}$

$[(6ax + 2b) - 2\cancel{(3ax^2 + 2bx)} + \cancel{(ax^3 + bx^2)} + 2\cancel{(3ax^2 + 2bx)} - 2(ax^3 + bx^2) + \cancel{(ax^3 + bx^2)}]e^{-x}$

$= e^{-x}(2x+3)$

$$6ax+2b=2x+3 \Rightarrow \left.\begin{array}{ll} 6a=2 & 2b=3 \\ a=\frac{1}{3} & b=\frac{3}{2} \end{array}\right\} \boxed{y_p=\left(\frac{x^3}{3}+\frac{3}{2}x^2\right)e^{-x}}$$

$$y=(c_1+c_2x)e^{-x}+\left(\frac{x^3}{3}+\frac{3}{2}x^2\right)e^{-x}$$

3. $y''+4y'-4y=x^2e^{-2x}$

$$r^2+4r-4=0 \Rightarrow r_{1,2}=\frac{-4\mp\sqrt{16+4.4}}{2}=-2\mp2\sqrt{2}$$

$$y_h=c_1e^{(-2+2\sqrt{2})x}+c_2e^{(-2-2\sqrt{2})x}$$

$$y_p=(ax^2+bx+c)e^{-2x}$$

$$y_p'=(2ax+b)e^{-2x}-2(ax^2+bx+c)e^{-2x}$$

$$y_p''=2ae^{-2x}-2(2ax+b)e^{-2x}-2(2ax+b)e^{-2x}+4(ax^2+bx+c)e^{-2x}$$

$$\left[2a\cancel{-4(2ax+b)}\cancel{+4(ax^2+bx+c)}\cancel{+4(2ax+b)}-8(ax^2+bx+c)\cancel{-4(ax^2+bx+c)}\right]e^{-2x}$$
$$=x^2e^{-2x}$$

$$-8ax^2-8bx-8c+2a=x^2$$

$$-8a=1 \Rightarrow a=-\frac{1}{8} \qquad \begin{array}{c}-8b=0\\ b=0\end{array} \qquad \begin{array}{c}-8c+2a=0\\ c=-\frac{1}{32}\end{array}$$

$$y_p=\left(-\frac{1}{8}x^2-\frac{1}{32}\right)e^{-2x}$$

$$y=y_h+y_p=c_1e^{(-2+2\sqrt{2})x}+c_2e^{(-2-2\sqrt{2})x}+\left(-\frac{1}{8}x^2-\frac{1}{32}\right)e^{-2x}$$

4. $y''' - 3y'' - 4y' + 12y = -208(\sin x)^2 + 5e^{3x} + 104$

$$= 5e^{3x} + 104 - 208\left[\frac{1-\cos 2x}{2}\right] = 5e^{3x} + \cancel{104} - \cancel{104} + 104\cos 2x$$

$r^3 - 3r^2 - 4r + 12 = 0 \Rightarrow r^2(r-3) - 4(r-3) = 0 \Rightarrow (r^2-4)(r-3) = 0$

$r_1 = 2,\ r_2 = -2,\ r_3 = 3$

$y_h = c_1 e^{2x} + c_2 e^{-2x} + c_3 e^{3x}$

$y_{p1} = k.x.e^{3x}$

$y_{p1}' = ke^{3x} + 3kxe^{3x}$

$y_{p1}'' = 3ke^{3x} + 3ke^{3x} + 9kxe^{3x}$

$y_{p1}''' = 9ke^{3x} + 9ke^{3x} + 9ke^{3x} + 27kxe^{3x}$

$(27k + \cancel{27kx} - 18k - \cancel{27kx} - 4k - \cancel{12kx} + \cancel{12kx})e^{3x} = 5e^{3x}$

$5k = 5$

$k = 1$

$\boxed{y_{p1} = xe^{3x}}$

$y_{p2} = A\cos 2x + B\sin 2x$

$y_{p2}' = -2A\sin 2x + 2B\cos 2x$

$y_{p2}'' = -4A\cos 2x - 4B\sin 2x$

$y_{p2}''' = 8A\sin 2x - 8B\cos 2x$

$8A\sin 2x - 8B\cos 2x + 12A\cos 2x + 12B\sin 2x$

$+ 8A\sin 2x - 8B\cos 2x + 12A\cos 2x + 12B\sin 2x$

$= 104\cos 2x$

$(16A + 24B)\sin 2x + (24A - 16B)\cos 2x = 104\cos 2x$

$2/\ 16A + 24B = 0$

$3/\ 24A - 16B = 104$

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$104A = 104.3 \Rightarrow \boxed{A = 3}$

$B = -\dfrac{16A}{24} = -2$

$y_{p2} = 3\cos 2x - 2\sin 2x$

$y = c_1 e^{2x} + c_2 e^{-2x} + c_3 e^{3x} + xe^{3x} + 3\cos 2x - 2\sin 2x$

5. $y''' + 9y' = -36\sin 3x + 9$

$r^3 + 9r = 0 \Rightarrow r(r^2+9) = 0 \Rightarrow r_1 = 0\ ,\ r_{2,3} = \mp 3i$

$y_h = c_1 + c_2\cos 3x + c_3 \sin 3x$

$y_{p_1} = (A\cos 3x + B\sin 3x)\,x$

$y_{p_1}' = (-3A\sin 3x + 3B\cos 3x).x + (A\cos 3x + B\sin 3x)$

$y_{p_1}'' = (-9A\cos 3x - 9B\sin 3x).x + \overbrace{(-3A\sin 3x + 3B\cos 3x) + (-3A\sin 3x + 3B\cos 3x)}^{(-6A\sin 3x + 6B\cos 3x)}$

$y_{p_1}''' = (27A\sin 3x - 27B\cos 3x).x + (-9A\cos 3x - 9B\sin 3x) + (-18A\cos 3x - 18B\sin 3x)$

$\Big[\cancel{(27A\sin 3x - 27B\cos 3x).x} + (-27A\cos 3x - 27B\sin 3x) + \cancel{9(-3A\sin 3x + 3B\cos 3x)x}$

$+ 9(A\cos 3x + B\sin 3x)\Big] = -36\sin 3x$

$-18A\cos 3x - 18B\sin 3x = -36\sin 3x$

$\left.\begin{array}{l} -18A = 0 \Rightarrow A = 0 \\ -18B = -36 \Rightarrow B = 2 \end{array}\right\}$ $\boxed{y_{p_1} = 2x\sin 3x}$

$\left.\begin{array}{l} y_{p_2} = k.x \\ y_{p_2}' = k \\ y_{p_2}'' = 0 \\ y_{p_2}''' = 0 \end{array}\right\}$ $9k = 9$, $k = 1$, $\boxed{y_{p_2} = x}$

$y = c_1 + c_2\cos 3x + c_2 \sin 3x + 2x\sin 3x + x$

6. $(D^3 - 4D^2 + 3D)y = x^2 + \sin x$ $\quad (y''' - 4y'' + 3y' = x^2 + \sin x)$

$r^3 - 4r^2 + 3r = 0 \rightarrow r(r^2 - 4r + 3) = 0 \Rightarrow r_1 = 0,\ r_2 = 3,\ r_3 = 1$

$-3 \quad -1$

$$\boxed{y_h = c_1 + c_2 e^{3x} + c_3 e^{x}}$$

$y_{p1} = (ax^2 + bx + c).x = ax^3 + bx^2 + cx$

$y_{p1}' = 3ax^2 + 2bx + c$

$y_{p1}'' = 6ax + 2b$

$y_{p1}''' = 6a$

$6a - 24ax - 8b + 9ax^2 + 6bx + 3c = x^2$

$9a = 1 \qquad -24a + 6b = 0 \qquad 6a - 8b + 3c = 0$

$a = \frac{1}{9} \qquad b = 4a = \frac{4}{9} \qquad \frac{6}{9} - \frac{32}{9} + 3c = 0$

$c = \frac{26}{27}$

$$\boxed{y_{p1} = \frac{1}{9}x^3 + \frac{4}{9}x^2 + \frac{26}{27}x}$$

$y_{p2} = A\cos x + B\sin x$

$y_{p2}' = -A\sin x + B\cos x$

$y_{p2}'' = -A\cos x - B\sin x$

$y_{p2}''' = A\sin x - B\cos x$

$A\sin x - B\cos x + 4A\cos x + 4B\sin x - 3A\sin x$

$+ 3B\cos x = \sin x$

$2/\ -2A + 4B = 1$

$4A + 2B = 0$

$10B = 2 \Rightarrow B = \frac{1}{5}, \quad A = -\frac{B}{2} = -\frac{1}{10}$

$y_{p2} = -\frac{1}{10}\cos x + \frac{1}{5}\sin x$

$y = c_1 + c_2 e^{3x} + c_3 e^{x} + \frac{x^3}{9} + \frac{4}{9}x^2 + \frac{26}{27}x - \frac{1}{10}\cos x + \frac{1}{5}\sin x$

7. $y'' + y' - 2y = e^x \cdot x^2$

$r^2 + r - 2 = 0 \Rightarrow r_1 = -2,\ r_2 = 1 \Rightarrow \boxed{y_h = c_1 e^{-2x} + c_2 e^x}$

$+2 \quad -1$

$y_p = (ax^2 + bx + c)e^x \cdot x = (ax^3 + bx^2 + cx)e^x$

$y_p' = (3ax^2 + 2bx + c)e^x + (ax^3 + bx^2 + cx)e^x$

$y_p'' = (6ax + 2b)e^x + (3ax^2 + 2bx + c)e^x + (3ax^2 + 2bx + c)e^x + (ax^3 + bx^2 + cx)e^x$

$\Big[(6ax + 2b) + 2(3ax^2 + 2bx + c) + (\cancel{ax^3 + bx^2} + cx) + (3ax^2 + 2bx + c) + (\cancel{ax^3 + bx^2} + cx)$

$- 2(\cancel{ax^3 + bx^2} + cx)\Big] e^x = e^x \cdot x^2$

$9ax^2 + 6bx + 3c + 6ax + 2b = x^2$

$9a = 1 \qquad 6b + 6a = 0 \qquad 3c + 2b = 0$

$a = \frac{1}{9} \qquad b = -\frac{1}{9} \qquad c = -\frac{2}{3}b = \frac{2}{18} = \frac{1}{9}$

$y_p = \left(\frac{1}{9}x^3 - \frac{1}{9}x^2 + \frac{x}{9}\right)e^x$

$y = c_1 e^{-2x} + c_2 e^x + \left(\frac{x^3}{9} - \frac{1}{9}x^2 + \frac{x}{9}\right)e^x$

D. 1. $2yy'' = 3yy' + (y')^2 \quad (y \geq 0)$ x missing (x yok)

$\left.\begin{array}{l} y' = p \\ y'' = p\dfrac{dp}{dy} \end{array}\right\}$ $2yp\dfrac{dp}{dy} = 3yp + p^2 \Rightarrow p\left[2y\dfrac{dp}{dy} - 3y - p\right] = 0$

$1^\circ)\ p = 0 \Rightarrow y = c$ (not solution)

$2^\circ)\ 2y\dfrac{dp}{dy} - 3y - p = 0 \Rightarrow 2y\dfrac{dp}{dy} - p = 3y$

$\Rightarrow \dfrac{dp}{dy} - \dfrac{p}{2y} = \dfrac{3}{2}$ Lineer D.

$$\lambda(y) = e^{\int -\frac{dy}{2y}} = e^{-\frac{1}{2}\ln y} = \frac{1}{\sqrt{y}}$$

$$P = \sqrt{y}\left[\int \frac{1}{\sqrt{y}} \cdot \frac{3}{2}\, dy + c_1\right] = \sqrt{y}\left[3\sqrt{y} + c_1\right] = 3y + c_1\sqrt{y}$$

$$P = \frac{dy}{dx} = 3y + c_1\sqrt{y} \Rightarrow \int \frac{dy}{3y + c_1\sqrt{y}} = \int dx$$

$$\left.\begin{array}{l} y = t^2 \\ dy = 2t\,dt \end{array}\right\} \int \frac{2t\,dt}{3t^2 + c_1 t} = \int \frac{2\,dt}{3t + c_1} = \frac{2}{3}\ln(3t + c_1) = \frac{2}{3}\ln(3\sqrt{y} + c_1)$$

$$\boxed{\frac{2}{3}\ln(3\sqrt{y} + c_1) = x + c_2}$$

2. $x^2 y'' + x y' + 4y = 25x\ln x$ (Euler)

$$\left.\begin{array}{l} x = e^t, \ (t = \ln x) \\ y' = e^{-t} D y \\ y'' = e^{-2t} D(D-1) y \end{array}\right\} \begin{array}{l} \underbrace{e^{2t} \cdot e^{-2t}}_{1} D(D-1)y + \underbrace{e^{t} \cdot e^{-t}}_{1} Dy + 4y = 25e^t \cdot t \\ \left[D(D-1) + D + 4\right] y = 25te^t \end{array}$$

$$(D^2 + 4)y = 25te^t$$

$$r^2 + 4 = 0 \Rightarrow r_{1,2} = \mp 2i \rightarrow y_h = c_1\cos 2t + c_2 \sin 2t$$

$$\left.\begin{array}{l} y_p = (at+b)e^t \\ y_p' = ae^t + (at+b)e^t \\ y_p'' = ae^t + ae^t + (at+b)e^t \end{array}\right\} \begin{array}{l} (2a + (at+b) + 4(at+b))e^t = 25te^t \\ 5at + 5b + 2a = 25t \\ 5a = 25 \qquad 5b + 2a = 0 \\ a = 5 \qquad b = -2 \end{array}$$

$$y_p = (5t-2)e^t$$

$$y = c_1\cos 2(\ln x) + c_2 \sin 2(\ln x) + (5\ln x - 2)x$$

3. $2yy'' = (y')^2 + 1$ (x missing)

$$\left.\begin{aligned} y' &= p \\ y'' &= p\frac{dp}{dy} \end{aligned}\right\} \quad 2yp\frac{dp}{dy} = p^2+1 \Rightarrow 2yp\,dp = (p^2+1)dy$$

$$\frac{2p\,dp}{p^2+1} = \frac{dy}{y} \Rightarrow \ln(p^2+1) = \ln y + \ln c_1$$

$$p^2+1 = c_1 y$$

$$p = \sqrt{c_1 y - 1} \Rightarrow \frac{dy}{dx} = \sqrt{c_1 y - 1} \Rightarrow \int \frac{dy}{\sqrt{c_1 y - 1}} = \int dx$$

$$\left.\begin{aligned} c_1 y - 1 &= t^2 \\ c_1\,dy &= 2t\,dt \end{aligned}\right\} \quad \int \frac{2t\,dt}{c_1 t} = \frac{2}{c_1}t = \frac{2}{c_1}\sqrt{c_1 y - 1}$$

$$\frac{2}{c_1}\sqrt{c_1 y - 1} = x + c_2$$

4. $y'' + 4y = \tan 2x$

$$r^2 + 4 = 0 \Rightarrow r_{1,2} = \mp 2i \Rightarrow y_h = c_1\cos 2x + c_2 \sin 2x$$

$/2\sin 2x$
$$c_1'\cos 2x + c_2'\sin 2x = 0$$

$/\cos 2x$
$$-2c_1'\sin 2x + 2c_2'\cos 2x = \tan 2x$$

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$$c_2'\underbrace{(2\sin^2 2x + 2\cos^2 2x)}_{2} = \sin 2x \Rightarrow c_2' = \frac{1}{2}\sin 2x \Rightarrow c_2 = \frac{-1}{4}\cos 2x + k_2$$

$$c_1' = -c_2'\frac{\sin 2x}{\cos 2x} = -\frac{1}{2}\sin 2x \cdot \frac{\sin 2x}{\cos 2x} \Rightarrow c_1 = -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x}dx$$

$$c_1 = -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x}dx = -\frac{1}{2}\left[\int \sec 2x\,dx - \int \cos 2x\,dx\right]$$

$$c_1 = -\frac{1}{2}\left[\boxed{\frac{1}{2}}\ln|\sec 2x + \tan 2x| - \frac{1}{2}\sin 2x\right] + k_1$$

$$y = -\frac{1}{4}\ln|\sec 2x + \tan 2x| \cdot \cos 2x + \cancel{\frac{1}{4}\sin 2x \cdot \cos 2x} + k_1\cos 2x \cancel{- \frac{1}{4}\sin 2x\cos 2x}$$
$$+ k_2 \sin 2x$$

5. $y'' + 4y' + 4y = \dfrac{e^{-2x}}{x^2}$

$$r^2 + 4r + 4 = 0 \Rightarrow (r+2)^2 = 0 \Rightarrow r_1 = r_2 = -2 \Rightarrow y_h = c_1 e^{-2x} + c_2 x e^{-2x}$$

$$2/\quad \cancel{c_1' e^{-2x}} + c_2' x e^{-2x} = 0$$

$$-\cancel{2c_1' e^{-2x}} + c_2' e^{-2x} - 2c_2' x e^{-2x} = \frac{e^{-2x}}{x^2}$$

$$c_2' e^{-2x} = \frac{e^{-2x}}{x^2} \Rightarrow c_2' = \frac{1}{x^2} \Rightarrow c_2 = -\frac{1}{x} + k_2$$

$$c_1' = -c_2' x = -\frac{1}{x^2} \cdot x = \frac{-1}{x} \Rightarrow c_1 = -\ln x + k_1$$

$$y = -\ln x \cdot e^{-2x} + k_1 e^{-2x} - e^{-2x} + k_2 x e^{-2x}$$

6. $x y'' - 2y' = \dfrac{1}{x^2}$ (Euler or y missing)

$$\left.\begin{aligned} y' &= p \\ y'' &= \frac{dp}{dx} \end{aligned}\right\} \quad x\frac{dp}{dx} - 2p = \frac{1}{x^2} \Rightarrow \frac{dp}{dx} - \frac{2p}{x} = \frac{1}{x^3} \quad \text{Linear}$$

$$\lambda(x) = e^{\int -\frac{2dx}{x}} = e^{-2\ln x} = \frac{1}{x^2}$$

$$p = x^2\left[\int \frac{1}{x^2} \cdot \frac{1}{x^3}\,dx + c_1\right] = x^2\left[\frac{1}{4x^4} + c_1\right] = \frac{1}{4x^2} + \frac{c_1}{x^2}$$

$$p = \frac{dy}{dx} = \frac{1}{4x^2} + \frac{c_1}{x^2} \Rightarrow \int dy = \int \left(\frac{1}{4x^2} + \frac{c_1}{x^2}\right)dx + c_2$$

$$y = \frac{-1}{4x} - \frac{c_1}{x} + c_2$$

7. $x^2y''+xy'+4y=4\cos(2\ln x)$

$$x=e^t$$

$$y'=e^{-t}Dy$$

$$y''=e^{-2t}D(D-1)y$$

$$\underbrace{e^{2t}\cdot e^{-2t}}_{1}D(D-1)y+\underbrace{e^{t}\cdot e^{-t}}_{1}Dy+4y=4\cos 2t$$

$$[D(D-1)+D+4]y=4\cos 2t$$

$$(D^2+4)y=4\cos 2t$$

$$r^2+4=0 \Rightarrow r_{1,2}=\mp 2i \Rightarrow y_h=c_1\cos 2t+c_2\sin 2t$$

$2\sin 2t$

$\cos 2t$

$$c_1'\cos 2t+c_2'\sin 2t=0$$

$$-2c_1'\sin 2t+2c_2'\cos 2t=4\cos 2t$$

$+$

$$c_2'\underbrace{(2\sin^2 2t+2\cos^2 2t)}_{2}=4\cos^2 2t \Rightarrow c_2'=2\cos^2 2t$$

$$c_2'=(1+\cos 4t) \Rightarrow c_2=t+\frac{1}{4}\sin 4t+k_2$$

$$c_1'=-c_2'\frac{\sin 2t}{\cos 2t}=-2\cdot\cos^{\cancel{2}} 2t\cdot\frac{\sin 2t}{\cancel{\cos 2t}}=-\sin 4t$$

$$c_1=\frac{1}{4}\cos 4t+k_1$$

$$y=\underbrace{\frac{1}{4}\cos 4t\cdot\cos 2t}+k_1\cos 2t+t\sin 2t+\underbrace{\frac{1}{4}\sin 4t\sin 2t}+k_2\sin 2t$$

$$y=\frac{1}{4}\cos 2t+t\sin 2t+k_1\cos 2t+k_2\sin 2t$$

$$y=\frac{1}{4}\cos 2(\ln x)+\ln x\sin 2(\ln x)+k_1\cos 2(\ln x)+k_2\sin 2(\ln x)$$

8. $2yy'' - (y')^2 + 9 = 0 \quad$ (x missing)

$$\left.\begin{array}{l} y' = p \\ y'' = p\dfrac{dp}{dy} \end{array}\right\} \quad \begin{array}{l} 2yp\dfrac{dp}{dy} - p^2 + 9 = 0 \Rightarrow 2yp\dfrac{dp}{dy} = p^2 - 9 \\ \dfrac{2p\,dp}{p^2-9} = \dfrac{dy}{y} \Rightarrow \ln(p^2-9) = \ln y + \ln c_1 \end{array}$$

$$p^2 - 9 = c_1 y \Rightarrow p = \sqrt{c_1 y + 9}$$

$$\frac{dy}{dx} = \sqrt{c_1 y + 9} \Rightarrow \int \frac{dy}{\sqrt{c_1 y + 9}} = \int dx$$

$$\left.\begin{array}{l} c_1 y + 9 = t^2 \\ c_1 dy = 2t\,dt \end{array}\right\} \int \frac{2t\,dt}{c_1 \cdot t} = \frac{2}{c_1} t = \frac{2}{c_1}\sqrt{c_1 y + 9}$$

$$\frac{2}{c_1}\sqrt{c_1 y + 9} = x + c_2$$

9. $y^{IV} + y''' = \sin 2t$

$$r^4 + r^3 = 0 \Rightarrow r^3(r+1) = 0 \rightarrow r_1 = r_2 = r_3 = 0,\ r_4 = -1$$

$$y_h = c_1 + c_2 t + c_3 t^2 + c_4 e^{-t}$$

$$\left.\begin{array}{l} y_p = A\cos 2t + B\sin 2t \\ y_p' = -2A\sin 2t + 2B\cos 2t \\ y_p'' = -4A\cos 2t - 4B\sin 2t \\ y_p''' = 8A\sin 2t - 8B\cos 2t \\ y_p^{IV} = 16A\cos 2t + 16B\sin 2t \end{array}\right\} \quad \begin{array}{l} 8A\sin 2t - 8B\cos 2t + 16A\cos 2t + 16B\sin 2t \\ = \sin 2t \\ \left.\begin{array}{l} 8A + 16B = 1 \\ 2/\ 16A - 8B = 0 \\ + \\ \hline 40A = 1 \\ A = \dfrac{1}{40},\ B = \dfrac{1}{20} \end{array}\right\} y_p = \dfrac{1}{40}\cos 2t + \dfrac{1}{20}\sin 2t \end{array}$$

$$y = c_1 + c_2 t + c_3 t^2 + c_4 e^{-t} + \frac{1}{40}\cos 2t + \frac{1}{20}\sin 2t$$

10. $y''y^3 = 1$ (x missing)

$$\left.\begin{array}{l} y' = p \\ y'' = p\dfrac{dp}{dy} \end{array}\right\} \quad p\frac{dp}{dy} \cdot y^3 = 1 \Rightarrow \int p\,dp = \int \frac{dy}{y^3}$$

$$\frac{p^2}{2} = -\frac{1}{2y^2} + c_1$$

$$p = \sqrt{2c_1 - \frac{1}{y^2}} = \frac{\sqrt{2c_1y^2 - 1}}{y}$$

$$p = \frac{dy}{dx} = \frac{\sqrt{2c_1y^2-1}}{y} \Rightarrow \int \frac{y\,dy}{\sqrt{2c_1y^2-1}} = \int dx$$

$$2c_1y^2 - 1 = t^2$$
$$4c_1y\,dy = 2t\,dt$$

$$\int \frac{2t\,dt}{4c_1 \cdot t} = \frac{1}{2c_1}t = \frac{1}{2c_1}\sqrt{2c_1y^2-1}$$

$$\frac{1}{2c_1}\sqrt{2c_1y^2-1} = x + c_2$$

11. $y'' = -2x(y')^2$, $y(0) = 2$, $y'(0) = -1$ (y missing)

$$\left.\begin{array}{l} y' = p \\ y'' = \dfrac{dp}{dx} \end{array}\right\} \quad \frac{dp}{dx} = -2xp^2 \Rightarrow \int \frac{dp}{p^2} = -\int 2x\,dx$$

$$-\frac{1}{p} = -x^2 + c_1 \qquad (y'(0) = -1) \Rightarrow \frac{-1}{(-1)} = -0 + c_1 \Rightarrow \boxed{c_1 = 1}$$

$$-\frac{1}{p} = -x^2 + 1 \Rightarrow p = \frac{1}{x^2-1}$$

$$\frac{dy}{dx} = \frac{1}{x^2-1} \Rightarrow \int dy = \underbrace{\int \frac{dx}{x^2-1}}_{I} + c_2$$

$$I \Rightarrow \frac{A}{x-1} + \frac{B}{x+1} = \frac{1}{x^2-1}$$

$$\left.\begin{array}{l} A + B = 0 \\ A - B = 1 \end{array}\right\} \quad A = \frac{1}{2},\ B = -\frac{1}{2}$$

$$y = \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x+1) + c_2$$

12. $x^2 y'' + 2xy' - 1 = 0$ (Euler or y missing)

$$\left.\begin{aligned} y' &= p \\ y'' &= \frac{dp}{dx} \end{aligned}\right\} \quad x^2 \frac{dp}{dx} + 2xp = 1 \Rightarrow \frac{dp}{dx} + \frac{2p}{x} = \frac{1}{x^2} \quad \text{Linear}$$

$$\lambda(x) = e^{\int 2\frac{dx}{x}} = e^{2\ln x} = x^2$$

$$p = \frac{1}{x^2}\left[\int x^2 \cdot \frac{1}{x^2} dx + c_1\right] = \frac{1}{x^2}(x + c_1) = \frac{1}{x} + \frac{c_1}{x^2}$$

$$\int dy = \int \left(\frac{1}{x} + \frac{c_1}{x^2}\right) dx$$

$$y = \ln x - \frac{c_1}{x} + c_2$$

E. $y y'' + 4y^2 - \frac{1}{2}(y')^2 = 0$, $x = 0 \Rightarrow y = 1$, $y' = -\sqrt{8}$ (x missing)

$$\left.\begin{aligned} y' &= p \\ y'' &= p\frac{dp}{dy} \end{aligned}\right\} \quad yp\frac{dp}{dy} + 4y^2 - \frac{p^2}{2} = 0 \Rightarrow \frac{dp}{dy} - \frac{p}{2y} = -4yp^{-1} \quad \text{Bernoulli}$$

$$\left.\begin{aligned} p^{1-(-1)} &= p^2 = u \\ 2pp' &= u' \end{aligned}\right\} \quad \begin{aligned} &p'p - \frac{p^2}{2y} = -4y \\ &\frac{u'}{2} - \frac{u}{2y} = -4y \Rightarrow u' - \frac{u}{y} = -8y \quad \text{linear} \end{aligned}$$

$$\lambda(y) = e^{-\int \frac{1}{y} dy} = e^{-\ln y} = \frac{1}{y}$$

$$u = y\left[\int \frac{1}{y}(-8y)dy + c_1\right] = y(-8y + c_1)$$

$$p = \sqrt{u} = \sqrt{y(-8y + c_1)} \Rightarrow -\sqrt{8} = \sqrt{-8 + c_1} \Rightarrow 8 = -8 + c_1 \Rightarrow c_1 = 16$$

$$dy = \sqrt{16y - 8y^2}\, dx \Rightarrow \int \frac{dy}{\sqrt{16y - 8y^2}} = \int dx$$

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