Çalışma Soruları 2

1) $x^2y+(y-x^3)y'=0$ diferansiyel denklemini $\mu=\mu(y)$ şeklinde y'ye bağlı bir integrasyon çarpanı bularak çözünüz.

$\mu=\mu(y)$ olsun.

$$\underbrace{(x^2y)}_{m}dx+\underbrace{(y-x^3)}_{N}dy=0$$

$$\frac{\partial m}{\partial y}=x^2, \quad \frac{\partial N}{\partial x}=-3x^2$$

$$\ln\mu=\int\frac{N_x-m_y}{m}dy=\int\frac{-3x^2-x^2}{x^2y}dy=\int-\frac{4dy}{y} \Rightarrow \mu=\frac{1}{y^4}$$

$$\frac{x^2}{y^3}dx+\left(\frac{1}{y^3}-\frac{x^3}{y^4}\right)dy=0$$

$$\left.\begin{array}{l}\dfrac{\partial f}{\partial x}=\dfrac{x^2}{y^3}\\[2ex] \dfrac{\partial f}{\partial y}=\boxed{\dfrac{1}{y^3}-\dfrac{x^3}{y^4}}\end{array}\right\} \begin{array}{l} f(x,y)=\dfrac{x^3}{3y^3}+h(y)\\[2ex] \dfrac{\partial f}{\partial y}=\cancel{-\dfrac{x^3}{y^4}}+\dfrac{dh}{dy}=\dfrac{1}{y^3}\cancel{-\dfrac{x^3}{y^4}}\end{array}$$

$$\frac{dh}{dy}=\frac{1}{y^3} \Rightarrow \int dh=\int\frac{dy}{y^3} \Rightarrow h=-\frac{1}{y^2}+c$$

$$f(x,y)=\frac{x^3}{3y^3}-\frac{1}{y^2}+c=k$$

2) $y'\sec x + y^2 = y$ diferansiyel denklemini çözünüz.

$$y'\sec x - y = -y^2 \quad \text{Bernoulli D.D}$$

$$\left.\begin{aligned} y^{1-2} &= y^{-1} = u \\ -y^{-2}y' &= u' \end{aligned}\right\} \quad \underbrace{y'y^{-2}}\sec x - y^{-1} = -1$$

$$-u'\sec x - u = -1$$

$$u' + \frac{u}{\sec x} = \frac{1}{\sec x} \Rightarrow \boxed{u' + u\cos x = \cos x}$$

Lineer D.D

$$\lambda(x) = e^{\int \cos x\, dx} = e^{\sin x}$$

$$u(x) = \frac{1}{e^{\sin x}}\left[\underbrace{\int e^{\sin x}\cdot \cos x\, dx} + c\right]$$

$$\left.\begin{aligned} \sin x &= t \\ \cos x\, dx &= dt \end{aligned}\right\} \int e^t dt = e^t = e^{\sin x}$$

$$u(x) = e^{-\sin x}\left[e^{\sin x} + c\right] = 1 + ce^{-\sin x}$$

$$y(x) = \frac{1}{u} = \frac{1}{1 + ce^{-\sin x}}$$

3) Özel bir çözümü $y_1 = x$ olan $y' = \frac{y^2}{x^2} - \frac{y}{x} + 1$ diferansiyel denklemini çözünüz.

$$\left.\begin{aligned} y &= y_1 + \frac{1}{u} = x + \frac{1}{u} \\ y' &= 1 - \frac{u'}{u^2} \end{aligned}\right\} \quad \cancel{1} - \frac{u'}{u^2} = \left(\cancel{1} + \frac{2}{ux} + \frac{1}{u^2x^2}\right) - \left(\cancel{1} + \frac{1}{ux}\right) + \cancel{1}$$

$$-\frac{u'}{u^2} = \frac{1}{ux} + \frac{1}{u^2x^2} \Rightarrow u' = -\frac{u}{x} - \frac{1}{x^2}$$

$$u' + \frac{u}{x} = -\frac{1}{x^2} \quad \text{Lineer D.D}$$

$$\lambda(x) = e^{\int \frac{dx}{x}} = e^{\ln x} = x \quad \left\{\begin{aligned} u(x) &= \frac{1}{x}\left[\int x\cdot\left(-\frac{1}{x^2}\right)dx + c\right] \\ u(x) &= \frac{1}{x}\left[-\ln x + c\right] \end{aligned}\right.$$

$$y(x) = x + \frac{x}{(c - \ln x)}$$

4) $y = xy' + y' - (y')^2$ d.d çözünüz.

$y' = p \Rightarrow y = xp + p - p^2$

$\underbrace{y'}_{\cancel{p}} = \cancel{p} + xp' + p' - 2pp'$

$p'\left[x + 1 - 2p\right] = 0$

a. $p' = 0 \Rightarrow p = c \Rightarrow y' = c \Rightarrow y = cx + g(c)$ Genel çözüm

b. $\boxed{x = 2p - 1}$

$y = xp + p - p^2 = (2p-1).p + p - p^2 = 2p^2 - p + p - p^2 = \boxed{p^2}$

$y = \left(\frac{x+1}{2}\right)^2$ Tekil çözüm

5) $y + xy' = (y')^4$ d.d çözünüz.

$y' = p \Rightarrow y + xp = p^4$

$\underbrace{y'}_{p} + p + xp' = 4p^3p'$

$2p = p'(4p^3 - x) \Rightarrow \frac{dx}{dp}.2p = \cancel{\frac{dp}{dx}}(4p^3 - x).\cancel{\frac{dx}{dp}}$

$2p\frac{dx}{dp} = 4p^3 - x \Rightarrow 2p\frac{dx}{dp} + x = 4p^3 \Rightarrow \frac{dx}{dp} + \frac{x}{2p} = 2p^2$ Lineer d.d

$\lambda(p) = e^{\int \frac{dp}{2p}} = e^{\frac{1}{2}\ln p} \Rightarrow \lambda = \sqrt{p}$

$x = \frac{1}{\sqrt{p}}\left[\int \sqrt{p}.2p^2 dp + c\right] = \frac{1}{\sqrt{p}}\left[\frac{4}{5}p^2\sqrt{p} + c\right]$

Çözümün parametrik denklemleri

$y + xp = p^4 \Rightarrow y = p^4 - p\left(\frac{1}{\sqrt{p}}\left[\frac{4}{5}p^2\sqrt{p} + c\right]\right) = p^4 - \frac{4}{5}p^3 - c\sqrt{p}$

6. $\frac{y'}{x} + \frac{2}{x^2} y = -x(\sec^2 x) y^2$ d.d çözünüz.

$y^{1-2} = y^{-1} = u$, $-y^{-2} y' = u'$ $\Bigg\}$ $\frac{y' y^{-2}}{x} + \frac{2 y^{-1}}{x^2} = -x(\sec^2 x)$

$-\frac{u'}{x} + \frac{2u}{x^2} = -x(\sec^2 x)$ Lineer D.D

$u' - \frac{2u}{x} = x^2 \sec^2 x$

$\lambda = e^{\int -\frac{2dx}{x}} = e^{-2\ln x} = \frac{1}{x^2}$

$u = x^2 \left[\int \frac{1}{\cancel{x^2}} \cancel{x^2} \sec^2 x \, dx + c \right] = x^2 (\tan x + c)$

$y = \frac{1}{u} = \frac{1}{x^2(\tan x + c)}$

7. $\frac{y}{x} + y'(2y + \ln x) = -x^2$ d.d çözünüz.

$\underbrace{\left(\frac{y}{x} + x^2\right)}_{m} dx + \underbrace{(2y + \ln x)}_{N} dy = 0$

$\frac{\partial m}{\partial y} = \frac{1}{x} = \frac{\partial N}{\partial x} = \frac{1}{x}$ Tam D.D

$\frac{\partial f}{\partial x} = \frac{y}{x} + x^2$ $\Big\}$ $f(x,y) = y \ln x + \frac{x^3}{3} + h(y)$

$\frac{\partial f}{\partial y} = 2y + \ln x$ $\Big\}$ $\frac{\partial f}{\partial y} = \ln x + \frac{dh}{dy} = 2y + \ln x$

$\frac{dh}{dy} = 2y \Rightarrow h = y^2 + c$

$f(x,y) = y \ln x + \frac{x^3}{3} + y^2 + c = k$

8) $\left(x+y\ln\left(\frac{x}{y}\right)\right)dx + x\ln\left(\frac{y}{x}\right)dy = 0$ d.d çözünüz.

$$\left.\begin{array}{l} y=ux \\ dy = u\,dx + x\,du \end{array}\right\} \left(x+ux\ln\left(\frac{1}{u}\right)\right)dx + x\ln u\,(u\,dx + x\,du) = 0$$

$$\left(x+ux\underbrace{\ln\left(\frac{1}{u}\right)}_{\ln u^{-1}} + ux\ln u\right)dx + x^2\ln u\,du = 0$$

$$\left(x-\cancel{ux\ln u}+\cancel{ux\ln u}\right)dx + x^2\ln u\,du = 0$$

$$\frac{x\,dx}{x^2} + \frac{x^2\ln u\,du}{x^2} = \frac{0}{x^2} \Rightarrow \int\frac{dx}{x} + \underbrace{\int\ln u\,du}_{I} = \int 0$$

$I : \int \ln u\,du$

$$\left.\begin{array}{ll} \ln u = \alpha & du = d\beta \\ \frac{du}{u} = d\alpha & u = \beta \end{array}\right\} I = \alpha\beta - \int\beta\,d\alpha = u\ln u - \int u\frac{du}{u} = u\ln u - u$$

$$\ln x + u\ln u - u = \ln c \Rightarrow \ln x + \frac{y}{x}\ln\left(\frac{y}{x}\right) - \frac{y}{x} = c$$

9. $y = c_1x + c_2x\ln x$ eğri ailesine ait olan d.d bulunuz.

$$\left.\begin{array}{l} y = c_1x + c_2x\ln x \\ y' = c_1 + c_2\ln x + c_2 \\ y'' = \frac{c_2}{x} \end{array}\right\} \begin{array}{l} \boxed{c_2 = xy''} \\ xy' - y = c_2x \Rightarrow \boxed{c_2 = y' - \frac{y}{x}} \end{array}$$

$$xy'' = y' - \frac{y}{x} \Rightarrow x^2y'' - xy' + y = 0$$

10. $y = -x(y')^2 + \ln y'$ d.d çözünüz.

$y' = p \Rightarrow y = -xp^2 + \ln p$

$$\underbrace{y'}_{p} = -p^2 - 2xpp' + \frac{p'}{p}$$

$$p^2 + p = p'\left(\frac{1}{p} - 2xp\right) \Rightarrow \frac{dx}{dp}(p^2+p) = \cancel{\frac{dp}{dx}}\left(\frac{1}{p} - 2xp\right) \cdot \cancel{\frac{dx}{dp}}$$

$(p^2+p)\frac{dx}{dp} + 2xp = \frac{1}{p}$ Lineer d.d

$$\frac{dx}{dp} + \frac{2xp}{p(p+1)} = \frac{1}{p^2(1+p)}$$

$$\lambda(p) = e^{\int \frac{2p\,dp}{p(p+1)}} = e^{\int \frac{2dp}{p+1}} = e^{2\ln(p+1)} = (p+1)^2$$

$$x(p) = \frac{1}{(p+1)^2}\left[\int (p+1)^{\cancel{2}} \cdot \frac{1}{p^2\cancel{(1+p)}}\,dp + c\right] = \frac{1}{(p+1)^2}\left[\ln p - \frac{1}{p} + c\right]$$

$$y(p) = -xp^2 + \ln p = \frac{-p^2}{(p+1)^2}\left[\ln p - \frac{1}{p} + c\right] + \ln p$$