$y'' + 3y' + 2y = 0 \quad y(0) = 2 \quad y'(0) = -3$ BDP Lapl. Dön. a) $\mathcal{L}\{y(t)\} = Y(s) = ?$

b) $Y(s)$ veriliyor. $y(t) = ?$

$$\mathcal{L}\{y'' + 3y' + 2y\} = \mathcal{L}\{0\} \qquad \mathcal{L}\{y(t)\} = Y(s) = ?$$

$$\mathcal{L}\{y''\} + 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = 0 \qquad \mathcal{L}\{1\} = \frac{1}{s}$$

$$s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{-3} + 3\left[sY(s) - \underbrace{y(0)}_{2}\right] + 2Y(s) = 0 \qquad \mathcal{L}\{0\} = 0\,\mathcal{L}\{1\} = 0$$

$$s^2Y(s) - 2s + 3 + 3sY(s) - 6 + 2Y(s) = 0$$

$$(s^2 + 3s + 2)Y(s) = 2s + 3$$

$$Y(s) = \frac{2s+3}{s^2+3s+2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2s+3}{s^2+3s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-2t} + e^{-t} \;//$$

$$\frac{2s+3}{s^2+3s+2} = \frac{2s+3}{(s+2)(s+1)} = \frac{A}{\underset{(s+1)}{s+2}} + \frac{B}{\underset{(s+2)}{s+1}}$$

$$2s + 3 = (A+B)s + A + 2B$$

$$-\;\; A + B = 2$$
$$A + 2B = 3 \qquad B = 1 \quad A = 1$$

$y'' - 6y' + 9y = t^2e^{3t} \quad y(0) = 2 \quad y'(0) = 6$ BDP $\nearrow Y(s) = ?$ $\searrow y(t) = ?$

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 9\mathcal{L}\{y\} = \mathcal{L}\{t^2e^{3t}\}$$

$$s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{6} - 6\left[sY(s) - \underbrace{y(0)}_{2}\right] + 9Y(s) = \mathcal{L}\{t^2e^{3t}\}$$

$$(s^2 - 6s + 9)Y(s) - 2s - 6 + 12 = \frac{2}{(s-3)^3} \Rightarrow Y(s) = \frac{2}{(s-3)^5} + \frac{2s-6}{(s-3)^2}$$

$$\mathcal{L}\{t^2e^{3t}\} = F(s-3) = \frac{2}{(s-3)^3}$$

$$\mathcal{L}\{t^2\} = \frac{2}{s^3} = F(s)$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\Rightarrow \mathcal{L}\{e^{at}\cdot t^n\} = \frac{n!}{\ldots} \Rightarrow \mathcal{L}^{-1}\left\{\frac{n!}{\ldots}\right\} = e^{at}\cdot t^n$$

$$\left\{ \begin{array}{l} s^{n+1} \\ \mathcal{L}\{e^{at}f(t)\} = F(s-a) \end{array} \right\} \Rightarrow \mathcal{L}\{e^{at}\cdot t^n\} = \frac{n!}{(s-a)^{n+1}} \Rightarrow \mathcal{L}^{-1}\left\{\frac{n!}{(s-a)^{n+1}}\right\} = e^{at}\cdot t^n$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{4!}\mathcal{L}^{-1}\left\{\frac{2\cdot 4!}{(s-3)^5}\right\} + \mathcal{L}^{-1}\left\{\frac{2\cancel{(s-3)}}{(s-3)^{\cancel{2}}}\right\}$$

$$\mathcal{L}\{t^4\} = \frac{4!}{s^5}$$

$$y(t) = \frac{2}{4!}e^{3t}\cdot t^4 + 2e^{3t}$$

$$\mathcal{L}\{e^{3t}t^4\} = \frac{4!}{(s-3)^5}$$

$\mathcal{L}^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\} = \mathcal{L}^{-1}\left\{\frac{4(s+3)}{(s+4)^2}\right\} = 4\mathcal{L}^{-1}\left\{\frac{s+3+1-1}{(s+4)^2}\right\} = 4\mathcal{L}^{-1}\left\{\frac{s+4-1}{(s+4)^2}\right\}$

Paydayı çarp. ayır.

1.yol: Payı paydaya benzet

$$= 4\mathcal{L}^{-1}\left\{\frac{\overset{A}{1}}{(s+4)} - \frac{\overset{B}{1}}{(s+4)^2}\right\}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

$$= 4e^{-4t} - 4t\cdot e^{-4t}$$

$$\mathcal{L}\{e^{-4t}\cdot t\} = \frac{1}{(s+4)^2}$$

2.yol: $\frac{s+3}{(s+4)^2} = \frac{A}{s+4} + \frac{B}{(s+4)^2}$

$(s+4) \quad (1)$

$s+3 = As + 4A + B$

$1 \quad 1 \quad -1$

$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+s}\right\} = \mathcal{L}^{-1}\left\{\frac{2s+1}{s(s+1)}\right\}$

$$\frac{A}{s} + \frac{B}{s+1} = \frac{2s+1}{s(s+1)}$$

$(s+1) \quad (s)$

$(A+B)s + A = 2s+1$

$A+B=2 \quad B=1$

$A=1$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = 1 + e^{-t}$$

$\mathcal{L}^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\} = \mathcal{L}^{-1}\left\{\frac{6s-4}{(s-2)^2+16}\right\} = \mathcal{L}^{-1}\left\{\frac{6(s-2)+8}{(s-2)^2+16}\right\}$

$(s-2)^2 = s^2-4s+4$

$$\mathcal{L}\{\sin 4t\} = \frac{4}{s^2+16} \qquad \mathcal{L}\{e^{2t}\sin 4t\} = \frac{4}{(s-2)^2+16}$$

$$\mathcal{L}\{\cos 4t\} = \frac{s}{s^2+16} \qquad \mathcal{L}\{e^{2t}\cos 4t\} = \frac{s-2}{(s-2)^2+16}$$

$\mathcal{L}^{-1}\{ \quad s-2 \quad \} \ldots \mathcal{L}^{-1}\{ \quad 4 \quad \} \ldots e^{2t}\cos 4t \ldots e^{2t}\sin 4t$

$$6\,\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2+16}\right\}+2\,\mathcal{L}^{-1}\left\{\frac{4}{(s-2)^2+16}\right\}=6.e^{2t}\cos 4t+2.e^{2t}\sin 4t$$

Ödev $\mathcal{L}^{-1}\left\{\frac{s-6}{s^2-2s+5}\right\}$

reel kök yoksa tam kareye tamamla

Ödev: $\mathcal{L}^{-1}\left\{\frac{5}{(s+2)^4}\right\}$

$$\mathcal{L}\{t^3\}=\frac{3!}{s^4}\qquad \mathcal{L}\{e^{-2t}.t^3\}=\frac{3!}{(s+2)^4}$$

Ödev: $\mathcal{L}^{-1}\left\{\frac{7s^2+10s-1}{s^3+3s^2-s-3}\right\}=?$

$y''+2y'+y=t.e^{-t}$ $\quad y(0)=1 \quad y'(0)=-2$ $\quad\to Y(s)=?\quad \to y(t)=?$

$$\mathcal{L}\{y''\}+2\mathcal{L}\{y'\}+\mathcal{L}\{y\}=\mathcal{L}\{t.e^{-t}\}$$

$$s^2Y(s)-s\underbrace{y(0)}_{1}-\underbrace{y'(0)}_{-2}+2\left(sY(s)-\underbrace{y(0)}_{1}\right)+Y(s)=\frac{1}{(s+1)^2}$$

$$(s^2+2s+1)Y(s)-s+\cancel{2}-\cancel{2}=\frac{1}{(s+1)^2}$$

$$Y(s)=\frac{1}{(s+1)^4}+\frac{s}{(s+1)^2}$$

$$y(t)=\mathcal{L}^{-1}\{Y(s)\}=\frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{1.3!}{(s+1)^4}\right\}+\underbrace{\mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2}\right\}}$$

$$=\frac{1}{3!}.t^3.e^{-t}+\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}-\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}$$

$$y(t)=\frac{1}{3!}.t^3.e^{-t}+e^{-t}-e^{-t}t$$

$$\left(\frac{s}{(s+1)^2}=\frac{s+1-1}{(s+1)^2}=\frac{1}{s+1}-\frac{1}{(s+1)^2}\right)$$

$$\mathcal{L}\{t\}=\frac{1}{s^2}=F(s)$$

$$\mathcal{L}\{e^{at}t\}=F(s-a)$$

$$\mathcal{L}\{t^3\}=\frac{3!}{s^4}$$

$$\mathcal{L}\{e^{at}t^3\}=\frac{3!}{(s-a)^4}$$

$$\mathcal{L}\{t\}=\frac{1}{s^2}$$

$$\mathcal{L}\{e^{-t}t\}=\frac{1}{(s+1)^2}.$$

$y'' + y' - 2y = 3t \quad y(0) = 3 \quad y'(0) = 0 \qquad \mathcal{L}\{y\} = Y(s) = ?$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = 3\mathcal{L}\{t\}$$

$$s^2Y(s) - s\underbrace{y(0)}_{3} - \cancel{y'(0)} + sY(s) - \underbrace{y(0)}_{3} - 2Y(s) = 3\frac{1}{s^2}$$

$$(s^2+s-2)Y(s) - 3s - 3 = \frac{3}{s^2}$$

$$(s^2+s-2)Y(s) = 3s+3 + \frac{3}{s^2} = \frac{3s^3+3s^2+3}{s^2}$$

$$Y(s) = \frac{3s^3+3s^2+3}{s^2(s^2+s-2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{s+2}$$

$-1 \quad +2$

$(s^3+s^2-2s) \quad (s^2+s-2) \quad s^2(s+2) \quad s^2(s-1)$

$y(t) = \mathcal{L}^{-1}\{Y(s)\} = ?$

$$3s^3+3s^2+3 = \underbrace{(A+C+D)}_{3}s^3 + \underbrace{(A+B+2C-D)}_{3}s^2 + \underbrace{(-2A+B)}_{0}s - \underbrace{2B}_{3}$$

$B = -\frac{3}{2} \qquad 2A = B \qquad A = \frac{-3}{4}$

$$\boxed{y(t) = -\frac{3}{4} - \frac{3}{2}t + 3e^{t} + \frac{3}{4}\cdot e^{-2t}}$$

$A + C + D = 3$

$A + B + 2C - D = 3$

$\underbrace{A+B}_{3A}$, $B = 2A$

$$\begin{cases} C + \cancel{D} = 3 + \frac{3}{4} \\ 2C - \cancel{D} = 3 - 3A = 3 + \frac{9}{4} \end{cases}$$

$+$

$3C = 6 + 3 = 9 \quad C = 3$

$D = \frac{3}{4}$

Ödev: $y'' - y - 2y = 0 \quad y(0) = 1 \quad y'(0) = 0$

Ödev: $y'' + 2y' + y = 4e^{-t} \quad y(0) = 2 \quad y'(0) = -1$

$y'' + 16y = 5\sin x \qquad y(0) = y'(0) = 0$

$$s^2Y(s) - s\cancel{y(0)} - \cancel{y'(0)} + 16Y(s) = 5\frac{1}{s^2+1}$$

$$Y(s) = 5\frac{1}{(s^2+1)(s^2+16)} = \frac{1/3}{s^2+1} - \frac{1}{4}\frac{1/3 \cdot 4}{s^2+16}$$

$\mathcal{L}^{-1}\{Y(s)\} = ?$

$$\frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+16} = \frac{1}{(s^2+1)(s^2+16)}$$

$(s^2+16) \qquad (s^2+1)$

$$(A+C)s^3 + (16A+C)s + (B+D)s^2 + 16B + D = 1$$

$$(A+C)s^3 + (16A+C)s + (B+D)s^2 + 16B + D = 1$$

$$\underbrace{A=-C \quad 16A=C}_{A=C=0} \qquad B=-D \qquad 16B+\underset{-B}{D}=1$$

$B=1/15$

$D=-1/15$

$$y(x) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{3}\sin x - \frac{1}{12}\sin 4x$$

Ödev: $y'' + 4y' + 6y = 1 + e^{-t}$ $\quad y(0) = y'(0) = 0 \quad Y(s) = ?$

Ödev $\quad Y(s) = \dfrac{2s+1}{s^2+4s+6} \Rightarrow \mathcal{L}^{-1}\{Y(s)\} = y(t) = ?$

Ödev: $\quad y' + 3y = 13\sin 2t, \; y(0) = 6 \qquad Y(s) = ? \qquad y(t) = ?$

$$y'' + 4y = \begin{cases} t & , 0 \le t < 1 \\ 1 & , 1 \le t < \infty \end{cases} \qquad y(0) = y'(0) = 0 \quad \text{BDP} \quad Y(s) = ?$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \underbrace{\int_0^1 e^{-st} \cdot t\,dt}_{\text{Kısmi İnt.}} + \underbrace{\int_1^\infty e^{-st}\,dt}$$

Laplace Dönüşümünün tanımı

$u = t \qquad e^{-st}dt = dv$

$du = dt \qquad -\dfrac{e^{-st}}{s} = v$

$$\mathcal{L}\{f(t)\} = -\frac{1}{s} t e^{-st}\Big|_0^1 + \frac{1}{s}\int_0^1 e^{-st}\,dt + \lim_{k\to\infty}\left(\int_1^k e^{-st}\,dt\right)$$

$$= -\frac{1}{s} e^{-s} + \frac{1}{s}\left(-\frac{e^{-st}}{s}\Big|_0^1\right) + \lim_{k\to\infty}\left(\frac{e^{-st}}{-s}\Big|_1^k\right)$$

$$= -\frac{e^{-s}}{s} - \frac{1}{s^2}\left(e^{-s} - 1\right) - \frac{1}{s}\lim_{k\to\infty}\left(\underset{\downarrow\, s>0 \atop 0}{e^{-sk}} - e^{-s}\right)$$

$$= \cancel{-\frac{e^{-s}}{s}} - \frac{1}{s^2}(e^{-s} - 1) + \cancel{\frac{1}{s} e^{-s}}$$

$\mathcal{L}\{f(t)\} \quad \frac{1}{s^2}(1-e^{-s})$

$$\mathcal{L}\{f(t)\} = \frac{1}{s^2}\left(1-e^{-s}\right)$$

$$\mathcal{L}\{y''+4y\} = \mathcal{L}\{f(t)\}$$

$$s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + 4Y(s) = \frac{1}{s^2}\left(1-e^{-s}\right)$$

$$Y(s) = \frac{1-e^{-s}}{s^2(s^2+4)}$$

Ödev: $y''-y'+2y = \begin{cases} t & t<1 \\ t^2+1 & t\geq 1 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \int_0^1 e^{-st}\cdot t\,dt + \int_1^\infty (t^2+1)e^{-st}\,dt$$

\# $y''' - y = 1 \qquad y(0)=y'(0)=y''(0)=0 \qquad \to Y(s)=? \quad \to y(t)=?$

$$s^3Y(s) - s^2\underbrace{y(0)}_{0} - s\underbrace{y'(0)}_{0} - \underbrace{y''(0)}_{0} - Y(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s(s^3-1)}$$

ödev

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s^3-1)}\right\} = ?$$

$$\frac{1}{s(s^3-1)} = \frac{1}{s(s-1)(s^2+s+1)} = \frac{A}{s} + \frac{B}{s-1} + \frac{Cs+D}{s^2+s+1}$$

$$s^2+s+1 = (s+\tfrac{1}{2})^2+\tfrac{3}{4}$$

$A=-1 \quad B=\frac{1}{3}=D \quad C=\frac{2}{3}$

$$* \quad \mathcal{L}^{-1}\left\{\frac{2s+1}{s^2+s+1}\right\} = \mathcal{L}^{-1}\left\{\frac{2(s+\frac{1}{2})}{(s+\frac{1}{2})^2+\frac{3}{4}}\right\} = 2\cos\frac{\sqrt{3}}{2}\cdot e^{-\frac{1}{2}t}$$

Birinci Mertebeden Lineer Diferansiyel Denklemler Sistemi

Şimdiye kadar incelediğimiz dif. denklemler

$$f(x, y, y', y'', \ldots, y^{(n)}) = 0$$

Bu bölümde, birden fazla bağımlı değişken (bilinmeyen fonk), bir bağımsız değişken, bağımlı değişkenlerin 1. mert. türevlerini içeren lineer dif. denklemler den oluşan sistemin çözümünü bulacağız

t b.sız değişken

$$\left.\begin{aligned} x &= x(t) \\ y &= y(t) \end{aligned}\right\} \text{ bağımlı değ.}$$

$$\left.\begin{aligned} f\left(t, x, y, \frac{dx}{dt}, \frac{dy}{dt}\right) &= 0 \\ g\left(t, x, y, \frac{dx}{dt}, \frac{dy}{dt}\right) &= 0 \end{aligned}\right\}$$

iki bilinmeyenli iki denklemden oluşan dif. denk. sistemi

$$\left.\begin{aligned} a_1(t)\frac{dx}{dt} + a_2(t)\frac{dy}{dt} + a_3(t)x + a_4(t)y &= f_1(t) \\ b_1(t)\frac{dx}{dt} + b_2(t)\frac{dy}{dt} + b_3(t)x + b_4(t)y &= f_2(t) \end{aligned}\right\}$$

iki bilinmeyenli iki denklemden oluşan lineer denklem sistemi

$f_1(t) = 0$ ve $f_2(t) = 0 \Rightarrow$ lineer D. sistemine homojen sistem denir.

$a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ katsayılarının hepsi sabitler $\Rightarrow$ sb. kats. LDDS

Dif. denk sisteminin çözümü, sistemdeki dif. denklemleri sağlayan bağımlı değişkenlerin $\{x, y\}$ kümesidir.

$$\left.\begin{aligned} \frac{dx}{dt} &= A_1(t)x + A_2(t)y + F_1(t) \\ \frac{dy}{dt} &= B_1(t)x + B_2(t)y + F_2(t) \end{aligned}\right\}$$

b.ç. yazılabilen sisteme Normal sistem denir.

Örn: #

$$\left.\begin{aligned} 2\frac{dx}{dt} + 3\frac{dy}{dt} - 2x + y &= t^2 \\ \frac{dx}{dt} - 2\frac{dy}{dt} + 3x + 4y &= t \end{aligned}\right\}$$

t b.sız değ. x, y b.lı değ (bilinmeyen f.)

İki bilinmeyen iki 1. mert. l. dif. denk. oluşan sistem sb. kat. lineer homojen olmayan DDS

$$\frac{dx}{dt} - 2\frac{dy}{dt} + 3x + 4y = t$$

İki bilinmeyen iki 1. mert. Lif. denk. oluşan sistem sbt kat. lineer homojen olm. DDS

\# $$\frac{dx}{dt} - 5x - 7y = t^2$$
$$\frac{dy}{dt} - 2x + 3y = t$$

Normal formda yazılabilir.
Sb. kats DDS

\# $$\frac{dx}{dt} = 3y + t$$
$$\frac{dy}{dt} = 2x + t^2$$

Normal Hom. olmayan sb kats DDS

\# $$\frac{dx}{dt} = 3x + 2y$$
$$\frac{dy}{dt} = 2x - 4y$$

Normal Homojen sb kats DDS

\# $$\frac{dx}{dt} - 4x + t^2 = 0$$
$$\frac{dy}{dt} + \frac{dx}{dt} + x = 0$$

sb kats Lineer Hom. olmayan DDS

\# $$\frac{dx}{dt} = t^2x + (t+1)y + t^3$$
$$\frac{dy}{dt} = t \cdot e^t x + t^3 y - e^t$$

Normal Değ. kats Hom. olmayan lineer DDS

\# $$\frac{dx}{dt} = 3x + 2y + 2z + t$$
$$\frac{dy}{dt} = 2x - 4y + 5z - t^2$$
$$\frac{dz}{dt} = 4x + y - 3z + 2t + 1$$

Normal (lineer) sb kats hom. olm. 3 bilinmeyenli 3 denklemden oluşan DDS

LDDS çözümü için iki yöntem inceleyeceğiz:

1) Türetme Yoketme Yönt

2) Determinant Yönt (Operatör Yönt)

① Türetme Yoketme Yöntemi :

$$\left.\begin{aligned} a_1 x' + a_2 y' + a_3 x + a_4 y &= f(t) \\ b_1 x' + b_2 y' + b_3 x + b_4 y &= g(t) \end{aligned}\right\}$$

LDDS nin x ve y bilinmeyen fonksiyonları bulmak için aş. prosedür uygulanır:

Önce x bilinmeyenini bulmak istiyorsak ;
sistemde yer alan dif. denklem(lerin) t'ye göre türevi alınıp ; tüm dif. denklemlerden yararlanarak diğer bilinmeyen fonksiyon y yok edilerek sadece x'e bağlı sb. kats. lineer dif. denk. bulunur. Bu dif. denk Belirsiz Kats Yönt yada Parametrelerin Değ. Yönt ile çözülür.

x fonksiyonu sistemde yer alan dif. denklemlerden birinde yerine yazılarak y bilinmeyen fonksiyonu da bulunur.

\# $$\left.\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= x \end{aligned}\right\} \qquad x = ? \qquad y = ?$$

(I) $\dfrac{dx}{dt} = y \xrightarrow{t \text{ ye göre tür. al}} \dfrac{d^2x}{dt^2} = \dfrac{dy}{dt}$ (III)

(II) $\dfrac{dy}{dt} = x$ (II) ve (III)'den $\dfrac{d^2x}{dt^2} = x$

(y içermeyen) x'e bağlı Sb Kats 2. Hom DD

$\leftarrow x'' - x = 0 \qquad r^2 - 1 = 0 \qquad r_{1,2} = \mp 1 \Rightarrow x = c_1 e^t + c_2 e^{-t}$

y için 1. denklem kullanılır. $\dfrac{dx}{dt} = y \Rightarrow y = c_1 e^t - c_2 e^{-t}$

Soru: $\dfrac{dx}{dt} = y$ { D.D.S çözümü = { $x = c_1 e^t + c_2 e^{-t}$ $\Rightarrow$ d_1, d_2 ?, c_1, c_2

Soru: $\left.\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= x \end{aligned}\right\}$ D.D.S çözümü $= \begin{cases} x = c_1 e^{t} + c_2 e^{-t} \\ y = d_1 e^{t} + d_2 e^{-t} \end{cases}$ $\Rightarrow$ d_1, d_2'yi c_1, c_2 cinsinden bulunuz.

(2.mert 2 DD Sisteminin çözümü iki keyfi sb. içermelidir)

x ve y I. denklemde yazılırsa

$x = c_1 e^{t} + c_2 e^{-t} \Rightarrow \frac{dx}{dt} = c_1 e^{t} - c_2 e^{-t} = d_1 e^{t} + d_2 e^{-t}$ $\left.\begin{aligned} d_1 &= c_1 \\ d_2 &= -c_2 \end{aligned}\right\}$ olmalı

Örnek: $\left.\begin{aligned} \frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= 4x - 2y \end{aligned}\right\}$ $x = ?$ $y = ?$

(I) $\frac{dx}{dt} = 2x + 3y$ $\xrightarrow{t\text{'ye göre türev al}}$ $\frac{d^2x}{dt^2} = 2\frac{dx}{dt} + 3\frac{dy}{dt}$ .... (III)

(II) $\frac{dy}{dt} = 4x - 2y$

II, III te yer. yazılırsa

$\frac{d^2x}{dt^2} = 2\frac{dx}{dt} + 3(4x - 2y) \Rightarrow \frac{d^2x}{dt^2} = 2\frac{dx}{dt} + 12x - 6y$ IV

I den $\frac{dx}{dt} - 2x = 3y$ V

V, IV 'te yer. yazılırsa

$$\frac{d^2x}{dt^2} = 2\frac{dx}{dt} + 12x - 2\left(\frac{dx}{dt} - 2x\right)$$

$$x'' - 16x = 0$$

$r^2 - 16 = 0 \quad r_{1,2} = \mp 4 \Rightarrow$ $x = c_1 e^{4t} + c_2 e^{-4t}$

x, I' de yer. yazılırsa $\frac{dx}{dt} = 2x + 3y$

$$4c_1 e^{4t} - 4c_2 e^{-4t} = 2c_1 e^{4t} + 2c_2 e^{-4t} + 3y$$

$$\frac{2}{3} c_1 e^{4t} - 2c_2 e^{-4t} = y$$

$$\frac{2}{3}c_1 e^{4t} - 2c_2 e^{-4t} = y$$

Ödev: $\left.\begin{aligned} \frac{dx}{dt} &= 2x+3y \\ \frac{dy}{dt} &= 4x-2y \end{aligned}\right\}$ sisteminin çözümü $\left.\begin{aligned} x &= c_1 e^{4t} + c_2 e^{-4t} \\ y &= d_1 e^{4t} + d_2 e^{-4t} \end{aligned}\right\} \Rightarrow$ d_1, d_2 yi c_1, c_2 cinsinden yazınız.

2. Yöntem: Determinant (Operatör) Yöntemi

Sb. Kats Lin. Dif. Denk. Sistemi, t bsız değ. olsun $\frac{d}{dt} = D$ türev op. yazılarak sistem D türev op cinsinden ifade edilir.

$$\begin{cases} a_1 x' + a_2 y' + a_3 x + a_4 y = f(t) \\ b_1 x' + b_2 y' + b_3 x + b_4 y = g(t) \end{cases} \qquad \frac{dx}{dt} = Dx \qquad \frac{dy}{dt} = Dy$$

$$\begin{cases} a_1 Dx + a_2 Dy + a_3 x + a_4 y = f(t) \\ b_1 Dx + b_2 Dy + b_3 x + b_4 y = g(t) \end{cases}$$

D tür op lineer özelliğinden

$$\begin{cases} (a_1 D + a_3)x + (a_2 D + a_4)y = f(t) \\ (b_1 D + b_3)x + (b_2 D + b_4)y = g(t) \end{cases}$$

Homojen olmayan Cebirsel Lineer Denklem Sistemi elde edilir.

(Cramer Yöntemi) Katsayılar determinantı

$$\Delta = \begin{vmatrix} a_1 D + a_3 & a_2 D + a_4 \\ b_1 D + b_3 & b_2 D + b_4 \end{vmatrix} = a_1 b_2 D^2 + \text{-----}$$

Δ D nin bir polinomu olup bu polinomun derecesi, Dif. denk sisteminin mertebesini verir.

$$\Delta_1 = \begin{vmatrix} f(t) & a_2D+a_4 \\ g(t) & b_2D+b_4 \end{vmatrix} = (b_2D+b_4)\,f(t) - (a_2D+a_4)\,g(t)$$

sb.kats lineer

$x = \dfrac{\Delta_1}{\Delta} \Rightarrow \Delta x = \Delta_1$ x in diferansiyel denklemi bulunmuş olur.

$$\Delta_2 = \begin{vmatrix} a_1D+a_3 & f(t) \\ b_1D+b_3 & g(t) \end{vmatrix} = (a_1D+a_3)g(t) - (b_1D+b_3)f(t)$$

$y = \dfrac{\Delta_2}{\Delta} \Rightarrow \Delta y = \Delta_2$ y nin Sb kats L Dif Denk bulunur.

\# $\dfrac{dx}{dt} = -\dfrac{dy}{dt} + 2y - 3e^t$, $\dfrac{dy}{dt} = -x-y$ } $y(t)=?$ Ödev: $x(t)=?$

$\frac{d}{dt}=D$ Determinant yöntemi:

$$\left.\begin{aligned} Dx &= -Dy + 2y - 3e^t \\ Dy &= -x-y \end{aligned}\right\} \qquad \left.\begin{aligned} Dx + (D-2)y &= -3e^t \\ x + (D+1)y &= 0 \end{aligned}\right\}$$

$$\Delta = \begin{vmatrix} D & D-2 \\ 1 & 1+D \end{vmatrix} = D+D^2-D+2 = D^2+2$$

2.der. pol $\Rightarrow$ DD Sistemi 2.mertebedendir

Sistemin çözümü iki keyfi sb. içerir

$y = \dfrac{\Delta_2}{\Delta} \qquad \Delta y = \Delta_2 \qquad (D^2+2)y = \Delta_2$

$$\Delta_2 = \begin{vmatrix} D & -3e^t \\ 1 & 0 \end{vmatrix} = 3e^t \Rightarrow (D^2+2)y = 3e^t$$

$y'' + 2y = 3e^t$ sb. kats 2.mert homojen olmayan LDD.

$y''+2y=3e^t$

$y'' + 2y = 3e^t$

$r^2+2=0 \Rightarrow r_{1,2} = \pm\sqrt{2}\,i \Rightarrow y_H = c_1\cos\sqrt{2}t + c_2\sin\sqrt{2}t$

$f(t) = 3e^t \Rightarrow y_p = Ae^t \Rightarrow y'' = Ae^t \qquad 3Ae^t = 3e^t \Rightarrow A=1 \quad y_p = e^t$

$r \neq 1$

$y = c_1\cos\sqrt{2}t + c_2\sin\sqrt{2}t + e^t$

Türetme Yoketme Yöntemi

$(I) \quad \boxed{\frac{dx}{dt}} = -\frac{dy}{dt} + 2y - 3e^t$

$(II) \quad \frac{dy}{dt} = -x-y \xRightarrow{t \text{ ye göre türev al}} \frac{d^2y}{dt^2} = -\boxed{\frac{dx}{dt}} - \frac{dy}{dt} \quad \ldots (III)$

(I), III de yerine yazılırsa

$$\frac{d^2y}{dt^2} = +\cancel{\frac{dy}{dt}} - 2y + 3e^t - \cancel{\frac{dy}{dt}} \Rightarrow \boxed{\frac{d^2y}{dt^2} + 2y = 3e^t}$$

$\left.\begin{aligned} \frac{dx}{dt} &= x+y+a \\ \frac{dy}{dt} &= 4x+y+b \end{aligned}\right\}$ çözümü $\left\{\begin{aligned} x &= c_1e^{-t} + c_2e^{3t} - \frac{1}{3} \\ y &= -2c_1e^{-t} + 2c_2e^{3t} - \frac{5}{3} \end{aligned}\right\} \Rightarrow a+b=?$

I. denklemden $\quad -\cancel{c_1e^{-t}} + \cancel{3c_2e^{3t}} = -\cancel{c_1e^{-t}} + \cancel{3c_2e^{3t}} - 2 + a \qquad a=2$

II. denklemden $\quad \cancel{2c_1e^{-t}} + \cancel{6c_2e^{3t}} = \cancel{2c_1e^{-t}} + \cancel{6c_2e^{3t}} - 3 + b \qquad \Rightarrow b=3$

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$\left.\begin{aligned} \frac{dy}{dx} + \frac{dz}{dx} - 2y - 4z &= e^x \\ \frac{dy}{dx} + \frac{dz}{dx} - z &= e^{4x} \end{aligned}\right\} \quad y=? \qquad$ Ödev $\quad z=?$

Op. Yönt: $\quad Dy + Dz - 2y - 4z = e^x$

$Dy + Dz - z = e^{4x}$

$$(D-2)y + (D-4)z = e^x$$

$$Dy + (D-1)z = e^{4x}$$

$$\Delta = \begin{array}{cc} y & z \\ \hline \end{array}$$
$$\Delta = \begin{vmatrix} D-2 & D-4 \\ D & D-1 \end{vmatrix} = D^2 - 3D + 2 - (D^2 - 4D) = D+2$$

$\Delta = D+2$ 1.der. pol $\Rightarrow$ DDS mertebesi $= 1$ $\Rightarrow$ y ve z çözümleri 1 keyfi sb. içerecek

$$y = \frac{\Delta_1}{\Delta} \Rightarrow \Delta y = \Delta_1 \Rightarrow (D+2)y = \Delta_1$$

$$\Delta_1 = \begin{vmatrix} e^x & D-4 \\ e^{4x} & D-1 \end{vmatrix} = (D-1)e^x - (D-4)e^{4x} = De^x - e^x - De^{4x} + 4e^{4x}$$
$$= e^x - e^x - 4e^4 + 4e^x = 0$$

$$(D+2)y = 0 \Rightarrow y' + 2y = 0 \qquad r+2=0 \Rightarrow r=-2$$

$$\boxed{y = c_1 e^{-2x}}$$