İkinci taraflı

$y'' - 2y' + y = xe^x$ → Homojen Olm. Sabit Kat. L.D.E

$y = (c_1 + c_2 x)e^x$ $\quad$ $c_2(x) = ?$

Lagr. Yönt. $\quad y = c_1(x)e^x + c_2(x)x.e^x$ arıyoruz.

$$-1 / \quad c_1' \cancel{e^x} + c_2' \cancel{x e^x} = 0$$
$$c_1' \cancel{e^x} + c_2'(1 + \cancel{x})e^x = xe^x$$
$$+$$

$$c_2' \cancel{e^x} = x\cancel{e^x}$$

$$c_2' = x \Rightarrow \boxed{c_2 = \frac{x^2}{2} + k}$$

L.D.E
- Hom = İkinci tarafsız
- Homd. Olm. = İkinci Taraflı

- y'yi içermeyen
- x'i içermeyen

2. mert. L.D.D.
Gen. çöz.
$y = c_1 y_1 + c_2 y_2$.

$\boxed{xy'' = xy'^2 + 4y''} \Rightarrow y' = ?\quad p = ?$

y'yi içermeyen

$\boxed{y' = p}$
$y'' = p'$

$\boxed{xp' = xp^2 + 4p'}$ $\quad$ p, x

$(x-4)p' = xp^2$

Euler
Değişken katsayılı
Lineer D.D.
$Ax^2y'' + Bxy' + Cy = f(x)$

$$\int \frac{dp}{p^2} = \int \frac{x-4+4}{x-4}\,dx$$

$$-\frac{1}{p} = \int \left(1 + \frac{4}{x-4}\right) dx$$

$$-\frac{1}{p} = x + 4\ln|x-4| + C$$

$$y' = p = \frac{-1}{x+4\ln|x-4|+C} \quad //$$

$y'' - 3xy' = 0$ $x=1$ civarında seri çöz?

Rekürans Bağıntısı

$a_3 = ?$

y_1 ve y_2

$$y = \sum_{n=0}^{\infty} a_n (x-1)^n \quad \text{seri çöz olsun:}$$

$$\sum_{n=2}^{\infty} a_n\, n(n-1)(x-1)^{n-2} - 3x \sum_{n=1}^{\infty} a_n n (x-1)^{n-1} = 0$$

$$x = (x-1) + 1$$

$$\sum_{n=2}^{\infty} a_n n(n-1)(x-1)^{n-2} - 3\left[(x-1)+1\right] \sum_{n=1}^{\infty} a_n \cdot n (x-1)^{n-1} = 0$$

x içermeyen

$f(y, y', y'') = 0$

$y' = p$

$y'' = p\frac{dp}{dy}$

$f(y, p, p\frac{dp}{dy}) = 0$

y bsız

p bağlı

y yi içermeyen

$f(x, y', y'') = 0$

$y' = p$

$y'' = \frac{dp}{dx}$

$f(x, p, \frac{dp}{dx}) = 0$

x bsız

p bağlı

$$\sum_{n=2-2}^{\infty} a_{n+2}(n+2)(n+1)(x-1)^{n-2+2} - 3\sum_{n=1}^{\infty} a_n n (x-1)^{n} - 3\sum_{n=1-1}^{\infty} a_{n+1}(n+1)(x-1)^{n-1+1} = 0$$

$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)(x-1)^{n} - 3\sum_{n=1}^{\infty} a_n n (x-1)^{n} - 3\sum_{n=0}^{\infty} a_{n+1}(n+1)(x-1)^{n} = 0$$

$n=0$ $\qquad\qquad$ $n=0$

$$\underbrace{2a_2 - 3a_1}_{0} + \sum_{n=1}^{\infty}\underbrace{\left[a_{n+2}(n+2)(n+1) - 3a_n n - 3a_{n+1}(n+1)\right]}_{0}(x-1)^n = 0$$

$$a_2 = \frac{3}{2}a_1$$

$$\boxed{a_{n+2} = \frac{3a_{n+1}(n+1) + 3na_n}{(n+1)(n+2)}}$$

$n \geq 1$ Rekürans Bağıntı

$$a_3 = \frac{3a_2 \cdot 2 + 3a_1}{2 \cdot 3}$$

$$\boxed{a_3 = a_2 + \frac{a_1}{2}}$$

A)
B)
C)

$y'' + \frac{y'}{y} = 0 \qquad y' = ?$

$$\left.\begin{aligned} y' &= p \\ y'' &= p\frac{dp}{dy} \end{aligned}\right\} \quad p\frac{dp}{dy} + \frac{p}{y} = 0 \qquad p\left(\frac{dp}{dy} + \frac{1}{y}\right) = 0$$

$p = 0$
$y' = 0$
$y = C$ T.Ç

$$\frac{dp}{dy} + \frac{1}{y} = 0$$

$$\int dp = -\int \frac{dy}{y} \Rightarrow p = -\ln y + \ln C \qquad \underbrace{p}_{y'} = \ln\frac{C}{y}$$

1) $y''' - 2y'' + y' - 2y = x$ Lagr Yönt Lineer Denk Sistemi nedir?

$r^3 - 2r^2 + r - 2 = 0$

$r^2(r-2) + r - 2 = 0$

$(r-2)(r^2+1) =$

$r_1 = 2 \quad r_{2,3} = \mp i \implies$

$y = c_1 e^{2x} + c_2 \cos x + c_3 \sin x$

$y = c_1(x) e^{2x} + c_2(x) \cos x + c_3(x) \sin x$

$$\left.\begin{aligned}
c_1' e^{2x} + c_2' \cos x + c_3' \sin x &= 0 \\
c_1' 2e^{2x} + c_2'(-\sin x) + c_3' \cos x &= 0 \\
c_1' 4e^{2x} + c_2'(-\cos x) + c_3'(-\sin x) &= x
\end{aligned}\right\}$$

$y'' + ay'^2 = 0 \quad$ $y = \frac{1}{2} \ln(2c_1 x + c_2)$ $a = ?$

$$\left.\begin{aligned}
y' &= \frac{1}{2} \cdot \frac{\cancel{2}c_1}{2c_1 x + c_2} = \frac{c_1}{2c_1 x + c_2} \\
y'' &= \frac{-2c_1^2}{(2c_1 x + c_2)^2}
\end{aligned}\right\}$ d.d. yer. yok.

$\frac{-2c_1^2}{(c_1x+c_2)^2} + a \cdot \frac{c_1^2}{(c_1x+c_2)^2} = 0 \implies a = 2$

$y' - 2xy = 0$ $\quad \underline{\underline{x=0}}$ civ. seri çöz? (Rek. Bağ)

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=1}^{\infty} a_n \cdot n\, x^{n-1} - 2x \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=1}^{\infty} a_n n \cdot x^{n-1} - 2 \sum_{n=0+2}^{\infty} a_{n-2} x^{n+1-2} = 0$$

$$\sum_{n=1}^{\infty} a_n n\, x^{n-1} - 2 \sum_{n=2}^{\infty} a_{n-2} x^{n-1}$$

$$\underbrace{a_1}_{0} + \sum_{n=2}^{\infty} \left[\underbrace{a_n n - 2a_{n-2}}_{0}\right] x^{n-1} = 0$$

$a_1 = 0$

$$\boxed{a_n = \frac{2a_{n-2}}{n} \quad n \geq 2}$$

$a_2 = a_0$ ✓

(Yineleme Bağıntısı)

A) $n \geq 0$ $\quad a_{n+2} = \dfrac{2a_n}{n+2}$

$a_2 = a_0$ ✓

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$= a_0 + a_0 x^2 + \frac{a_0}{2} x^4 + \dots$$

$$= a_0 \left(1 + x^2 + x^4 + \dots\right)$$

$$= a_0 \cdot y_1$$

$a_3 = \dfrac{2a_1}{3}$

$a_3 = 0$

$a_4 = \dfrac{2a_2}{4} = \dfrac{a_2}{2} = \dfrac{a_0}{2}$

$y'' - 4y' + 8y = 1 - e^{2x}\sin x$

$\int e^{x^2}dx = ?$

$r^2 - 4r + 8 = 0$

$r_{1,2} = \dfrac{4 \mp \sqrt{16-4\cdot 8}}{2} = 2 \mp 2i \Rightarrow y_H = e^{2x}(c_1\cos 2x + c_2\sin 2x)$

$e^{\alpha x} P_m(x)$

$e^{\alpha x}\cos\beta x$

$y = e^{\alpha x}\cdot z$

$f_1(x) = 1 \Longrightarrow y_{P_1} = K = 1/8$ Ö.D. Yok $r \neq 0$

$f_2(x) = -e^{2x}\sin x \Longrightarrow_{\text{1nd}} y_{P_2} = e^{2x}(A\cos x + B\sin x)$

2nd $8 /\ \boxed{y_{P_2} = e^{2x}\cdot z}$ değ. dön. $f(x) = e^{2x}\sin x$ fark $-\sin x$ fark indirger

$-4 /\ y' = (\cancel{z'} + \cancel{2z})e^{2x}$

$+\ y'' = (z'' + \cancel{2z'} + \cancel{2z'} + 4z)e^{2x}$

$(z'' + 4z)\cancel{e^{2x}} = -\cancel{e^{2x}}\sin x$

$\boxed{z'' + 4z = -\sin x}$ $r^2 + 4 = 0$ $r_{1,2} = \mp 2i$

$f(x) = -\sin x$ $4 / z_p = A\cos x + B\sin x$ Ö.D. $r \neq \mp i$ YOK.

$z' = -A\sin x + B\cos x$

$+\ z'' = -A\cos x - B\sin x$

$3A\cos x + 3B\sin x = -\sin x$

$y_{GU} = y_h + y_{p_1} + y_{p_2}$ //

$A = 0 \quad B = -1/3$

$z_p = -\frac{1}{3}\sin x$

$y_{p_2} = e^{2x}\left(-\frac{1}{3}\sin x\right)$

$x^3 y''' + 5x^2 y'' + 7xy' + 8y = 0$

$x = e^t$

$y' = e^{-t} Dy$

$y'' = e^{-2t} D(D-1)y$

$y''' = e^{-3t} D(D-1)(D-2)y$

$D = \frac{d}{dt}$

$Dy = \frac{dy}{dt} = y'$

$\cancel{e^{3t}} \cdot \cancel{e^{-3t}} D(D-1)(D-2)y + 5\cancel{e^{2t}}\cancel{e^{-2t}} D(D-1)y + 7\cancel{e^{t}}\cancel{e^{-t}} Dy + 8y = 0$

$\left[\underbrace{D(D-1)(D-2)}_{D^2-3D+2} + 5D(D-1) + 7D + 8\right] y = 0$

$(D^3 - 3D^2 + \cancel{2D} + 5D^2 - \cancel{5D} + 7D + 8)\, y = 0$

$(D^3 + 2D^2 + 4D + 8)\, y = 0$

$y''' + 2y'' + 4y' + 8y = 0$

$D \to r$

$r^3 + 2r^2 + 4r + 8 = 0$

$r^2(r+2) + 4(r+2) = 0$

$(r+2)(r^2+4) = 0$

$r = -2 \quad r_{2,3} = \mp 2i$

$y = c_1 e^{-2t} + c_2 \cos 2t + c_3 \sin 2t$

$= c_1 x^{-2} + c_2 \cos(2\ln x) + c_3 \sin(2\ln x)$

$x = e^t$

$\ln x = t$

$y'' + 2y' + y = e^{-x} \ln x$ ... Lagr. Yön. $c_1(x)$ $c_2(x)$ (Sabit ?)

Özel çözüm ?

$r(r-1)^2(r-2)=0$

$r=0 \quad r_{2,3}=1 \quad r_4=2$

$y_{GÇ}=?\quad c_1+\left[c_2e^{x}+c_3xe^{x}\right]+c_4e^{2x}$

D.D. belirleyiniz?

$r(r^2-2r+1)(r-2)=0$

$r(r^3-2r^2-2r+4r+r-2)=0$

$r^4-2r^3+3r^2-2r=0$

$\downarrow$

$y^{IV}-2y'''+3y''-2y'=0$

t^2e^{t}, e^{-t}, t, $\cos t$

$t^2e^t \Rightarrow r=1$ üç katlı kök: e^t, te^t, t^2e^t

$e^{-t} \Rightarrow r=-1$

$t \Rightarrow r=0$ iki katlı kök.

$\cos t \Rightarrow r=\pm i \Rightarrow \sin t$

$(r-1)^3(r+1)r^2(r^2+1)=0$

$(D-1)^3(D+1)D^2(D^2+1)y=0$

$y=(c_1+c_2t+c_3t^2)e^t+c_4e^{-t}+(c_5+c_6t)+c_7\cos t+c_8\sin t$

$e^t \cos t \longrightarrow \boxed{r_{1,2} = 1 \mp i \Rightarrow (r - r_1)(r - r_2) = 0 \quad r^2 - (r_1 + r_2) r + r_1 r_2 = 0}$

$(1-i)(1+i)$

$1 - i^2$

$r^2 - 2r + 1 = 0$

$y'' - 2y' + 2 = 0$

Sb Kats Lin Homo Adi DD → Belir Kats Yönt ← Basit $f(x)$

Sb Kats Lin Homo Adi DD → Lag. Yönt (Genel Yönt) ← Her $f(x)$

$y = c_1 e^{-3t} + c_2 t e^{-3t}$

$r_{1,2} = -3$

$(r+3)^2 = 0$

$y = (c_1 + c_2 x + c_3 x^2) e^{2x} + c_4 e^{-2x} + (c_5 \cos 3x + c_6 \sin 3x)$ $G_6 \Rightarrow$ DD ?

$y = e^x \left((c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x \right)$

$r_{1,2} = 1 \mp 2i = r_{3,4}$ $\left[(r - r_1)(r - r_2)\right]^2 = 0$