Determine the order of each of the following equations; also state whether the equation is linear or nonlinear.

If it is nonlinear show explicitly the term(s) which violate linearity. Also determine whether the equation is homogeneous or nonhomogeneous.

① $y^{(5)} + y'' + 3\sin t^2 = 0$

② $x'' + x.x' - 1 = 0$

③ $\frac{d^4y}{dx^4} - y^2 = 0$

④ $\frac{d^3x}{dy^3} - \frac{d^2x}{dy^2} + y^4 - 3 = 0$

⑤ $y^2 y'' + \sin y = t$

⑥ $y''' + e^t y' - 2y = 1 + t^2$

⑦ $y'^2 + \sqrt{t}\, y = 0$

⑧ $(2-t)y'' + y.y' = \cos(t-y)$

Solution:

① 7th. order, linear, nonhom.

② 2nd ″ , nonlin. because of $x.x'$, nonhom

③ 4th ″ , ″ ″ ″ y^2, hom.

④ 3rd ″ , linear, nonhom.

⑤ 2nd ″ , nonlinear because of $y^2 y''$, nonhom.

⑥ 3rd ″ , linear, nonhom.

⑦ 1st ″ , nonlinear because of y'^2, hom.

⑧ 2nd ″ , ″ ″ ″ $y.y'$ and $\cos(t-y)$, nonhom

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Given $xdy = y(\ln y - \ln x)dx$ find gen. sol.

$x dy = y . \ln \frac{y}{x} dx$ can be written in the form $\frac{dy}{dx} = f(\frac{y}{x})$

$$\frac{dy}{dx} = \frac{y}{x} . \ln \frac{y}{x} \qquad \frac{dy}{dx} = f(\frac{y}{x})$$

Letting $y = ux$ or $\frac{y}{x} = u$;

differentiating we get $dy = udx + xdu$

$$u + x\frac{du}{dx} = u . \ln u \qquad \text{separable eq.}$$

$$x\frac{du}{dx} = u\ln u - u$$

$$\frac{du}{u(\ln u - 1)} = \frac{dx}{x}$$

Let $\ln u - 1 = t$, $\frac{du}{u} = dt$

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\ln|\ln u - 1| = \ln x + \ln c$$

$$\ln \frac{y}{x} = 1 + x.c$$

Replacing u by $\frac{y}{x}$

$$\boxed{y = x . e^{1+cx}}$$

$$\# \quad (x+x\ln x - x\ln y)dy - y(\ln x-\ln y)dx=0$$

$$\frac{1}{y}/ \quad \left(y+x.\ln\frac{x}{y}\right)dy - y.\ln\frac{x}{y}.dx=0$$

$$\left(1+\frac{x}{y}.\ln\frac{x}{y}\right)dy - \ln\frac{x}{y}\,dx=0$$

$$\frac{x}{y}=u \quad , \; dx=udy+ydu \;\rightarrow$$

$$(1+u.\ln u).\underline{\underline{dy}} - \ln u.(u\underline{\underline{dy}}+ydu)=0$$

collect all terms containing the term dy
simplify $u.\ln u dy$

$$dy - y.\ln u du=0$$

$$\int\frac{dy}{y}=\int\ln u du$$

$$\ln y = u\ln u - u + c$$

$$\ln y = \frac{x}{y}.\ln\frac{x}{y} - \frac{x}{y} + c$$

Solve the initial value problem

$$(2x^2+3y^2)\,dx - xy\,dy = 0 \qquad y(1)=0$$

Solution = Hom. D.E

$$\left(2+3\left(\frac{y}{x}\right)^2\right)dx - \frac{y}{x}\,dy = 0$$

$$\frac{y}{x}=u \Rightarrow dy = x\,du + u\,dx$$

$$(2+3u^2)\,dx - u\,(u\,dx + x\,du) = 0$$

$$2\,dx + 3u^2 dx - u^2 dx - ux\,du = 0$$

$$(2u^2+2)\,dx - ux\,du = 0$$

$$2\int \frac{dx}{x} - \int \frac{u}{u^2+1}\,du = 0$$

$$2\ln x - \frac{1}{2}\ln(u^2+1) = \ln c$$

$$x^2 = c.\sqrt{\left(\frac{y}{x}\right)^2+1} \Rightarrow x^4 = c.\left(\left(\frac{y}{x}\right)^2+1\right)$$

$$y(1)=0 \qquad 1 = c \Rightarrow x^6 = x^2+y^2$$

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find the general solution of the diff. eq

$$y' = -\frac{y}{x} - \frac{x}{y}$$

by using the substitution $y(x) = x \cdot z(x)$

Solution: $\left.\begin{array}{l} y = x \cdot z \\ y' = z + x \cdot z' \end{array}\right\}$ $z + xz' = \frac{-xz}{x} - \frac{x}{x \cdot z}$

$\Rightarrow \quad z + xz' = -z - \frac{1}{z}$

$\Rightarrow \quad \boxed{xz' + 2z = -\frac{1}{z}}$

Either $\quad z' + \frac{2}{x} z = -\frac{1}{x} \cdot z^{-1}$ Bernoulli

or $\quad xz' = -2z - \frac{1}{z}$ Seperable

$$\frac{4}{4}\int \frac{z}{2z^2+1}\, dz = -\int \frac{dx}{x}$$

$$\frac{1}{4} \cdot \ln(2z^2+1) = -\ln x + \ln c$$

$$(1+2z^2)^{1/4} = \frac{c}{x}$$

$$1 + 2z^2 = \frac{k}{x^4} \qquad (k = c^4)$$

$$z = \pm\sqrt{\frac{\frac{k}{x^4} - 1}{2}} \Rightarrow y = \pm \frac{x}{2}\sqrt{\frac{k}{x^4} - 1}$$

Find the integrating factor for the following diff-eq:

$$\underbrace{(2x^2+3y^2)}_{M}dx - \underbrace{xy}_{N}dy = 0$$

$$M_y = 6y \neq N_x = -y$$

$$\lambda(x) = e^{\int \frac{M_x - N_y}{N}dx} = e^{\int \frac{-y-6y}{-xy}dx} = e^{\int \frac{-7y}{-xy}dx}$$

$$= e^{\int \frac{7}{x}dx} = e^{7\ln x} = x^7 \Rightarrow \boxed{\lambda(x) = x^7}$$

Find the integrating factor for the following diff-eq:

$$\frac{dy}{dt} = t - y + 1$$

Linear dif. eq $=$ $y' + y = t + 1$

$p(t) = 1 \quad q(t) = t + 1$

$$\lambda(t) = e^{\int p(t)dt} = e^{\int dt} = e^t$$

Solve the equation $y' = \sec y \tan x$

$$\int \frac{dy}{\sec y} = \int \tan x \, dx$$

$$\int \cos y \, dy = \int \frac{\sin x}{\cos x} dx$$

$$\sin y = -\ln \cos x + C$$

Given that x^4 is integrating factor for the linear dif. eq $x^3y' + 4x^2y = 2 + \sin x$ and initial condition

$y(\pi/2) = \frac{4}{\pi^2}$ Solve

$$x^4 \Big/ \quad y' + \frac{4}{x} y = \frac{2 + \sin x}{x^3}$$

$$\underbrace{x^4y' + 4x^3y} = 2x + x \sin x$$

$$\frac{d}{dx}(x^4 . y) = 2x + x \sin x$$

$$x^4 y = \int (2 \cdot x + x \sin x) dx \qquad \begin{array}{ll} u = x & \sin x dx = dv \\ du = dx & -\cos x = v \end{array}$$

$$= x^2 - x\cos x + \int \cos x \, dx$$

G.S $x^4 y = x^2 - x \cos x + \sin x + C$

$x = \frac{\pi}{2}, \ y = \frac{4}{\pi^2} \Rightarrow C = -1$ $\qquad y = \dfrac{x^2 - x\cos x + \sin x - 1}{x^4}$ P.S

Initial condition

Given that $\mu = \frac{1}{x}$ is integrating factor for the d. eq $(x+3x^3\sin y)dx + (x^4\cos y)dy = 0$ and Initial Condition $y(1) = \pi/2$

Solve IVP

multiply the DE by $\frac{1}{x}$:

$$\underbrace{(1+3x^2\sin y)}_{M}dx + \underbrace{x^3\cos y}_{N}dy = 0$$

$$M_y = 3x^2\cos y = N_x = 3x^2\cos y \Rightarrow \text{exact DE.}$$

$$\frac{\partial\phi}{\partial x} = 1+3x^2\sin y \quad \text{---(1)}$$

$$\frac{\partial\phi}{\partial y} = x^3\cos y \quad \text{---(2)}$$

From (2). $\phi(x,y) = \int x^3\cos y\,dy + H(x)$

$$\boxed{\phi(x,y) = x^3\sin y + H(x)}$$

$$\frac{\partial\phi}{\partial x} = 3x^2\sin y + H'(x) \rightarrow H'(x) = 1 \Rightarrow$$

$$H(x) = x + k$$

$$\phi(x,y) = x^3\sin y + x + k$$

G.S $\phi(x,y) = c$

$$x^3\sin y + x + k = c$$

$$\boxed{x^3\sin y + x = C}$$

$y(1) = \pi/2 \Rightarrow c = 2$

P.S : $\boxed{x^3\sin y + x = 2}$

* Obtain the separable D.E from
$(x-2\sin y+3)dx + (2x-4\sin y-3)\cos y\, dy = 0$
by making substitution $z = \sin y$

$z = \sin y$ } differentiating this equality
$dz = \cos y . dy$ } substituting these in the equation gives

$(x-2z+3)dx + (2x-4z-3).dz = 0$

$(x-2z)$ and $(2x-4z)$ are linearly independent

So this DE can be covered separable DE. by

substitution $x-2z = u$
differentiating both sides, we find
$dx - 2dz = du \Rightarrow dx = 2dz + du$

$(u+3)(2dz + du) + (2u-3)dz = 0$
$2(u+3)dz + (u+3)du + (2u-3)dz = 0$
$(4u+3)dz + (u+3)du = 0$
$\int dz + \int \frac{u+3}{4u+3} du = \int 0$

II. ~~Find the~~ general solution

1°) $(x+y)^2 \cdot y' = 1$ d.d. nin genel çözümünü bulunuz.

Ç: $x+y=u \Rightarrow 1+y'=u' \Rightarrow y'=u'-1$

$u^2 \cdot (u'-1) = 1 \Rightarrow u'-1=\frac{1}{u^2} \Rightarrow u'=1+\frac{1}{u^2}$

$\Rightarrow u' = \frac{1+u^2}{u^2} \Rightarrow \frac{du}{dx} = \frac{1+u^2}{u^2} \Rightarrow \frac{u^2+1-1}{1+u^2} \cdot du = dx$

$\Rightarrow \int \left[1 - \frac{1}{1+u^2}\right] du = \int dx \Rightarrow u - tg^{-1}u = x+c$

$\Rightarrow \cancel{x}+y - tg^{-1}(x+y) = \cancel{x}+c \Rightarrow \boxed{y - tg^{-1}(x+y) = c}$ g.ç.

DE is in the form (3)

It can be transformed into separable DE

||

Solve

1) $(x+3y).y' = x+3y-2$ diferansiyel denkleminin genel çözümünü bulunuz. C

Çözüm: $x+3y=u \Rightarrow dx+3dy=du \Rightarrow \frac{du-dx}{3}=dy$

$(x+3y)\,dy-(x+3y-2)\,dx=0 \Rightarrow$

$u.\left(\frac{du-dx}{3}\right)-(u-2).dx=0 \Rightarrow$

$u\,(du-dx)-(3u-6)dx=0 \Rightarrow (-3u+6-u)dx+u\,du=0$

$u\,du+(6-4u)\,dx=0 \Rightarrow \int\frac{u}{6-4u}du+\int dx=0$

$x=\frac{1}{4}\int\frac{4u}{4u-6}du=\frac{1}{4}\int\frac{4u-6+6}{4u-6}dx=\frac{1}{4}\int\left(1+\frac{6}{4u-6}\right)du$

$x=\frac{1}{4}\left(u+\frac{6}{4}\ln|4u-6|\right)+c$

$x=\frac{1}{4}(x+3y)+\frac{3}{8}\ln|4(x+3y)-6|+c$

$\frac{\cancel{3}(x-y)}{\cancel{4}}=\frac{\cancel{3}}{\cancel{8}_{2}}\ln|4x+12y-6|+c$

$$\boxed{2(x-y)=\ln|4x+12y-6|+c}$$

DE is in the form (5)

It can be transformed into separable DE

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Soru 4 : $(x-y-2)dx + (4y-x-1)dy=0$

$\begin{vmatrix} 1 & -1 \\ -1 & 4 \end{vmatrix} \neq 0 \Rightarrow$ $(x-y)$ and $(4y-x)$ are lin. independent
It can be transformed into Homogeneous DE

$\left.\begin{array}{l} x=X+h \\ y=Y+k \end{array}\right\}$ $\left.\begin{array}{l} dx=dX \\ dy=dY \end{array}\right\}$

$[(X+h)-(Y+k)-2]dX + [4(Y+k)-(X+h)-1]dY=0$

$[X-Y+h-k-2]dX + [-X+4Y-h+4k-1]dY=0$

$\left.\begin{array}{l} h-k-2=0 \\ -h+4k-1=0 \end{array}\right\}$ $k=1,\ h=3$

$\left.\begin{array}{l} x=X+3 \\ y=Y+1 \end{array}\right\}$ $(X-Y)dX + [-X+4Y]dY=0$ HDD.

$$\frac{dY}{dX} = \frac{X-Y}{X-4Y}$$

$Y=uX$, $\frac{dY}{dX} = \frac{du}{dX}X + u \Rightarrow$

$$\frac{du}{dX}X + u = \frac{X-uX}{X-4uX} = \frac{1-u}{1-4u} \Rightarrow X\frac{du}{dX} = \frac{1-2u+4u^2}{1-4u}$$

$$\int \frac{dX}{X} = -\int \frac{4u-1}{4u^2-2u+1}du$$

$$\ln X = -\frac{1}{2}\ln(4u^2-2u+1) + \ln C$$

$X^2(4u^2-2u+1) = C^2$ $\qquad u = \frac{Y}{X} = \frac{y-1}{x-3}$

$$(x-3)^2\left[4\left(\frac{y-1}{x-3}\right)^2-2\left(\frac{y-1}{x-3}\right)+1\right]=K$$

$$4(y-1)^2-2(y-1)(x-3)+(x-3)^2=K$$

$$\boxed{x^2+4y^2-4x-2y-2xy=4}$$

Solve $t^2y' + 2yt - y^3 = 0$ $(t>0)$ Bernoulli

$$y' + \frac{2}{t}y = \frac{1}{t^2}y^3 \qquad n=3$$

$$y^{-3}y' + \frac{2}{t}y^{-2} = \frac{1}{t^2}$$

$$\boxed{y^{-2} = z}$$

$$-2y^{-3}y' = z'$$

$$-\frac{z'}{2} + \frac{2}{t}z = \frac{1}{t^2}$$

$$z' - \frac{4}{t}z = -\frac{2}{t^2} \quad \text{LDE}$$

$$\lambda = e^{\int -\frac{4}{t}dt} = e^{-4\ln t} = \frac{1}{t^4} \Rightarrow$$

$$\underbrace{\frac{1}{t^4}z' - \frac{4}{t^5}z}_{\frac{d}{dt}\left(\frac{1}{t^4}z\right)} = \frac{-2}{t^6}$$

$$\frac{d}{dt}\left(\frac{1}{t^4}z\right) = -\frac{2}{t^6} \Rightarrow \int d\left(\frac{1}{t^4}z\right) = \int -\frac{2}{t^6}dt$$

$$t^4 / \quad \frac{1}{t^4}z = \frac{2}{5t^5} + c \Rightarrow z = \frac{2}{5t} + ct^4 \Rightarrow \frac{1}{y^2} = \frac{2}{5t} + ct^4$$

Solve $y' = y\left(\frac{1}{x} - y\ln x\right)$

$$y^{-2} / \quad y' - \frac{1}{x}y = -\ln x \, y^2 \quad \text{Bernoulli}$$

$$y^{-2}y' - \frac{1}{x}y^{-1} = -\ln x$$

$$y^{-1} = z \qquad -y^{-2}y' = z'$$

$$-z' - \frac{1}{x}z = -\ln x$$

$$z' + \frac{1}{x}z = \ln x \quad \text{LDE}$$

$\lambda = x$ int. fac

$$\Rightarrow \quad x \cdot z = \int x\ln x\,dx = I \qquad \boxed{\begin{array}{ll} x = u & \ln x\,dx = dv \\ dx = du & x\ln x - x = v \end{array}}$$

$$I = x^2\ln x - x^2 - I + \frac{x^2}{2} + C \Rightarrow I = \frac{1}{2}\left(x^2\ln x - \frac{x^2}{2} + C\right) = xz$$

$$z = \frac{1}{y} = \frac{2x^2\ln x - x^2 + K}{4x}$$

G.S.

Find the general and singular solutions of the diff. eq:

$$y = xy' + \sqrt{y'^2+1}$$

This is a Clairaut Eq.

Introduce the parameter $\boxed{y'=p}$

$$\boxed{y = xp + \sqrt{p^2+1}} \quad (*)$$

Taking derivative both sides with respect to x, we get

$$\left.\begin{aligned} y' &= p + xp' + \frac{2pp'}{2\sqrt{p^2+1}} \\ \text{Since } y' &= p \end{aligned}\right\} \text{ we can write}$$

$$\cancel{p} = \cancel{p} + xp' + \frac{2pp'}{2\sqrt{p^2+1}} \Rightarrow p'\left(x + \frac{2p}{2\sqrt{p^2+1}}\right) = 0$$

Consider the case $p'=0 \Rightarrow$ then $p=c$

Substituting this in the equation $(*)$ we find the general solution $y = cx + \sqrt{c^2+1}$

The second case is described by the equation

$$x = \frac{-p}{\sqrt{p^2+1}}$$

Find the corresponding parametric expression for y: $y = \frac{1}{\sqrt{p^2+1}}$

The parameter p can be eliminated from the formulas for x and y,

Squaring $x^2 = \frac{p^2}{p^2+1}$

$+ \quad y^2 = \frac{1}{p^2+1}$

Adding

$x^2+y^2=1.$

Solve $\quad e^{y-xy'} = y'^2$

$$y - xy' = \ln y'^2$$

$$y = xy' + 2\ln p$$

$y' = p \qquad y = xp + 2\ln p$

$$\cancel{y'} = \cancel{p} + xp' + \frac{2p'}{p}$$

p

$$0 = p'\left(x + \frac{2}{p}\right)$$

① $p' = 0 \Rightarrow p = c \Rightarrow$ G.C $\quad y = xc + 2\ln c \quad$ general sol.

② $x + \frac{2}{p} = 0$

$$\boxed{\begin{aligned} x &= -\frac{2}{p} \\ y &= -2 + 2\ln p \end{aligned}}$$

parametric equations of singular sol.

$$y = -2 + 2.\ln\left(\frac{-2}{x}\right)$$

cartesian eq. of sing. sol.

1.v / 2012 $\quad y = 3xy' + 6y'^2$ Lagr.

$y' = p \qquad y = 3xp + 6p^2$

$\cancel{y'} = \cancel{3}p + 3xp' + 12pp'$

$0 = 2p + 3xp' + 12pp'$

$-2p = (3x + 12p) \cdot \frac{dp}{dx}$

$\frac{dx}{dp} = \frac{3x + 12p}{-2p}$

$x' + \frac{3}{2p} x = -6.$ l.d.d.

$\lambda = e^{\int \frac{3}{2p} dp} = e^{\frac{3}{2} \ln p} = p^{3/2}$

$\underbrace{p^{3/2} \cdot x' + \frac{3}{2p} \cdot p^{3/2} \cdot x}$ $= -6p^{3/2}$

$\int d(p^{3/2} \cdot x) = -\int 6p^{3/2} dp$

$p^{3/2} \cdot x = -6 \frac{p^{\frac{3}{2}+1}}{5/2} + k$

Parametric Eq. of General Solution
$$\begin{cases} x = -\frac{12}{5} p + k \cdot p^{-3/2} \\ y = 3p\left(-\frac{12}{5}p + k \cdot p^{-3/2}\right) + 6p^2 \end{cases}$$

Solve IVP

$$\underbrace{\mathrm{ArcSin}\,x}_{N}\, dy + \underbrace{\frac{y+2\sqrt{1-x^2}\,\mathrm{Cos}\,x}{\sqrt{1-x^2}}}_{M}\, dx = 0, \quad y(0)=0$$

$$M = \frac{y}{\sqrt{1-x^2}} + 2\mathrm{Cos}\,x \Rightarrow \frac{\partial M}{\partial y} = \frac{1}{\sqrt{1-x^2}}$$

$$N = \mathrm{ArcSin}\,x \Rightarrow \frac{\partial N}{\partial x} = \frac{1}{\sqrt{1-x^2}}$$

exact DE

There exist a $\phi(x,y)$

$$d\phi(x,y) = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = M dx + N dy$$

$$\boxed{d\phi(x,y) = 0 \quad \phi(x,y) = C}$$

the corresponding coefficients equal eachother

$$\frac{\partial \phi}{\partial x} = M \text{ --- } (1)$$

$$\frac{\partial \phi}{\partial y} = N \text{ -- } (2)$$

from (2) $\phi(x,y) = \int \mathrm{ArcSin}\,x\, dy$

$$\boxed{\phi(x,y) = y \cdot \mathrm{ArcSin}\,x + H(y)}$$

$$\frac{\partial \phi}{\partial x} = \frac{y}{\sqrt{1-x^2}} + H'(y) \underset{(1)}{=} \frac{y}{\sqrt{1-x^2}} + 2\mathrm{Cos}\,x$$

$$H'(y) = 2\mathrm{Cos}\,x \qquad H(y) = 2\mathrm{Sin}\,x + k$$

$$\phi(x,y) = y\,\mathrm{ArcSin}\,x + 2\mathrm{Sin}\,x + k$$

G.S. $\quad y\,\mathrm{ArSin}\,x + 2\mathrm{Sin}\,x + k = c \Rightarrow \boxed{y\,\mathrm{ArcSin}\,x + 2\mathrm{Sin}\,x = C}$

Solve $y = x(y'^{2} + 2y') - (y'^{2} + 2y' - 1)$ $(x>1)$

1st orde, high order D.E.

Lagrange.

$y' = p \Rightarrow \boxed{y = x(p^2+2p) - (p^2+2p-1)}$ ⊛

$\cancel{p=}y' = \cancel{\frac{p^2+2p}{p}} + x(2pp'+2p') - 2pp' - 2p'$

$0 = p(p+1) + x\underbrace{2p'(p+1) - 2p'(p+1)}$

$-p\cancel{(p+1)} = 2p'\cancel{(p+1)}(x-1)$ Separable DE

$-p = 2\frac{dp}{dx}(x-1)$

$-\frac{1}{2}\frac{dx}{x-1} = \frac{dp}{p}$

$-2/ \quad \ln p = -\frac{1}{2}\ln(x-1) + \ln c$

$-2\ln p = \ln(x-1) - 2\ln c = \ln(x-1) - \ln k$

$\frac{1}{p^2} = \frac{x-1}{k}$

Solve for $x = \frac{k}{p^2} + 1$

substitute x into ⊛ $y = (\frac{k}{p^2}+1)(p^2+2p) - (p^2+2p-1) = k+1+\frac{2k}{p}$

parametric equations of general solution.

(#) Obtain the DE that has the general solution $y = c_1 x + \frac{c_2}{x^2}$

This equation has two constants c_1 and c_2 So

It should be derived twice:

$-x / \quad y = c_1 x + \frac{c_2}{x}$

$x^2 / \quad y' = c_1 - \frac{c_2}{x^2}$

$x^3 / \quad y'' = \frac{2c_2}{x^3}$

$+$

By multiplying both side of the first eq by $-x$

the second eq by x^2

the third eq. by x^3

Adding we eliminate c_1, c_2

$$x^3 y'' + x^2 y' - xy = 0$$

(#) Obtain the DE that has the general solution

$$c_1(x+2) + c_2(x+2).\ln(x+2) = y$$

$-(x+2) / \quad c_1 + c_2 \ln(x+2) + c_2 \cancel{(x+2)} \frac{1}{\cancel{(x+2)}} = y'$

$(x+2)^2 / \quad \frac{c_2}{x+2} = y''$

$+$

$$(x+2)y'' - (x+2)y' + y = 0$$

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12-ek

$-1/ \ y = c_1 e^x + c_2 \sin x + c_3 \cos x$ eğri ailesinin d.d. bulunuz.

$y' = c_1 e^x + c_2 \cos x - c_3 \sin x$

$-1/ \ y'' = c_1 e^x - c_2 \sin x - c_3 \cos x$

$y''' = c_1 e^x - c_2 \cos x + c_3 \sin x$

+

$$y''' - y'' + y' - y = 0.$$

Ödev! $y = c_1 e^{-x} + c_2 \sin x + c_3 \cos x$

$$y''' + y'' + y' + y = 0.$$

2009 $\frac{y}{x} - \arcsin\left(\frac{\ln c}{x}\right) = 0$ eğri ail. d.d. bul.

Çözüm:

$(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$

$\frac{y}{x} = \arcsin\left(\frac{\ln c}{x}\right) \Rightarrow \frac{\ln c}{x} = \sin \frac{y}{x} \Rightarrow \ln c = x . \sin \frac{y}{x}$

$\rightarrow \ y = x \arcsin\left(\frac{\ln c}{x}\right)$

$y' = \arcsin\left(\frac{\ln c}{x}\right) + x \left(-\frac{\ln c}{x^2}\right) \frac{1}{\sqrt{1-\left(\frac{\ln c}{x}\right)^2}}$

$y' = \arcsin\left(\sin \frac{y}{x}\right) - \sin\left(\frac{y}{x}\right) \frac{1}{\sqrt{1-\sin^2 \frac{y}{x}}}$

$$\boxed{y' = \frac{y}{x} - \tan \frac{y}{x}}$$

ÖRN: c_1, c_2 parametre ol.üz.

$y = c_1 \cos x + c_2 \sin x$ eğri ailesini ç.ö.a.

$y' = -c_1 \sin x + c_2 \cos x$
$y'' = -c_1 \cos x - c_2 \sin x$
} olup bu bağıntılar fonk. ile birlikte

$$\Rightarrow + \begin{cases} y - c_1 \cos x - c_2 \sin x = 0 \\ y' + c_1 \sin x - c_2 \cos x = 0 \\ y'' + c_1 \cos x + c_2 \sin x = 0 \end{cases}$$

şeklinde bir sistem oluşturacak tarzda düzenlenirse c_1 ve c_2 parametreleri için

(Sistemdeki bağıntılar lin. b.lı olması için kat. det.i sıfır olmalıdır)

$$\begin{vmatrix} y & -\cos x & -\sin x \\ y' & \sin x & -\cos x \\ y'' & \cos x & \sin x \end{vmatrix} = 0$$

yazılır. Bu det. hesaplanırsa

$$\Rightarrow \boxed{y'' + y = 0}$$ dif. denk. elde edilir.

ÖRN 4/ $y = c_1 \cos 2x + c_2 \sin 2x$ eğri ailesinin dif. denk.ni bul.

$$+ \begin{cases} y' = -2c_1 \sin 2x + 2c_2 \cos 2x = 2[-c_1 \sin 2x + c_2 \cos 2x] \\ y'' = -4c_1 \cos 2x - 4c_2 \sin 2x = 4[-c_1 \cos 2x - c_2 \sin 2x] \end{cases}$$

$$\begin{vmatrix} y & -\cos 2x & -\sin 2x \\ y' & 2\sin 2x & -2\cos 2x \\ y'' & 4\cos 2x & 4\sin 2x \end{vmatrix} = 0 \rightarrow \boxed{y'' + 4y = 0}$$ dif. denk. elde edilir.

ÖRN: $y = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}$ eğri ailesinin dif denk. ?

ÖR) $y = Cx^2$ eğri ailesine ait dif. denk.i bulunuz.

$y' = 2Cx$ (Tek C var. 1 kez türev al)

$\frac{y}{y'} = \frac{x}{2} \Rightarrow \boxed{2y - xy' = 0}$

ÖR) $y = A\sin 3x + B\cos 3x$ genel çözümü verilen dif. denk.i bul.

$y' = 3A\cos 3x - 3B\sin 3x$ (2 parametre 2 türev, 3 denklem)

$y'' = -9A\sin 3x - 9B\cos 3x$

$y'' = -9\underbrace{(A\sin 3x + B\cos 3x)}_{y} \Rightarrow y'' = -9y \Rightarrow \boxed{y'' + 9y = 0}$

ÖR) $y = Ax^2 + Bx + C$

$y' = 2Ax + B$

3 parametre (A, B, C)
3 türev

$y'' = 2A$

$\boxed{y''' = 0}$

ÖR) $y = Ae^x + Bx + C$

$y' = Ae^x + B$

$\frac{y'}{y''} = \frac{Ae^x}{Ae^x} \Rightarrow \boxed{y''' = y''}$

ÖR) $y = A^x \Rightarrow \ln y = \ln A^x$

$\ln A = \frac{\ln y}{x}$

$y' = \underbrace{A^x}_{y} \underbrace{\ln A}_{\frac{\ln y}{x}}$

$y' = y \cdot \frac{\ln y}{x}$

$\boxed{xy' - y\ln y = 0}$

ÖR) $y = \tan(Ax + B)$

$y' = A(1 + \tan^2(Ax + B))$

$y' = A(1 + y^2)$ türevi $2y \cdot y'$

$y'' = A\, 2yy'$

$y' = A(1 + y^2)$

$\Rightarrow y''(1 + y^2) = y \cdot (y')^2$

CamScanner ile tarandı

Örneğin; $y = x^3 + A + B + C$ denklemi bir diferansiyel denklemin genel çözümü değildir. $A + B + C$ tek bir keyfi sabit olarak gözönüne alındığında $y = x^3 + D$ birinci mertebeden bir dif. denklemin genel çözümüdür.

Benzer biçimde, $y = \ln \frac{Ax}{B}$ denklemi de

$$y = \ln A + \ln x - \ln B = C + \ln x$$

şeklinde de tek bir keyfi sabitle ifade edilebilir ve birinci mertebeden bir dif. denklemin genel çözümü olur.

$$y = c_1 e^{c_2 + x} + c_3 e^{-x}$$
$$= \underbrace{c_1 e^{c_2}}_{A} e^{x} + c_3 e^{-x}$$
$$= A e^{x} + C e^{-x}$$

6 ek

ÖRNEK: $y''=e^x$ $\quad y(0)=0 \quad y'(0)=2$

başlangıç değer problemini çözünüz.

ÇÖZÜM: $y''=e^x \Rightarrow y'=e^x+c_1$

$$\Rightarrow y=e^x+c_1x+c_2$$

$y'(0)=1+c_1=2 \Rightarrow c_1=1$

$y(0)=0 \Rightarrow 1+0+c_2=0 \Rightarrow c_2=-1$

bulunur. Dolayısıyla istenilen çözüm

$$y=e^x+x-1$$

olarak elde edilir.

ÖRNEK: $y''=\operatorname{cosec}^2 x \quad y(\frac{\pi}{2})=1, \; y'(\frac{\pi}{2})=0$ başlangıç değer problemini çözünüz.

ÇÖZÜM:

$y''=\operatorname{cosec}^2 x \Rightarrow y'=-\operatorname{cotan} x+c_1$

$y=-\ln|\sin x|+c_1x+c_2$

$y'(\frac{\pi}{2})=0+c_1=0 \Rightarrow c_1=0$

$y(\frac{\pi}{2})=-\ln 1+0\cdot\frac{\pi}{2}+c_2=1 \Rightarrow c_2=1$

dir. İstenen özel çözüm

$$y=-\ln|\sin x|+1$$

olarak bulunur.

ueke