

Kutupsal Koor. Cevaplar

① $r = 1 + 2r \cos \theta$ kutupsal eğrisinin kartezyen denklemi?

a) $3x^2 - y^2 + 4x + 1 = 0$

b) $3x^2 + y^2 - 4x + 1 = 0$

c) $3x^2 - y^2 - 4x + 1 = 0$

d) $3x^2 + y^2 - 4x - 1 = 0$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$r = 1 + 2 \frac{r \cos \theta}{x} \rightarrow \text{her iki tarafı } r \text{ ile çarp}$$

$$r = 1 + 2x \rightarrow r^2 = (1 + 2x)^2$$

$$x^2 + y^2 = 1 + 4x + 4x^2$$

$\Downarrow$

$$3x^2 - y^2 + 4x + 1 = 0$$

Cevap
A

② $r = \frac{4}{2 \cos \theta - \sin \theta}$

kutupsal denklemi :

a) Doğrudur

b) Çemberdir

c) Kardiyoiddir

d) Paraboldür

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$r = \frac{4}{2 \cos \theta - \sin \theta}$$

$\Downarrow$

$$\frac{2r \cos \theta}{x} - \frac{r \sin \theta}{y} = 4$$

$\Downarrow$

$$2x - y = 4$$

Denklem bir doğru belirtir

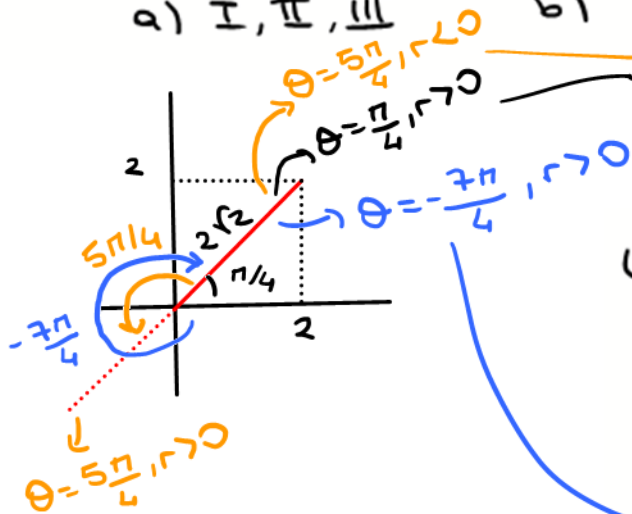
Cevap A

③ Kartezyen koordinatları $(x,y)=(2,2)$ noktasını aşağıdaki kutupsal koordinat çiftlerinden hangileri ile ifade edebiliriz?

I. $(2\sqrt{2}, \frac{\pi}{4})$ II. $(-2\sqrt{2}, \frac{5\pi}{4})$ III. $(2\sqrt{2}, -\frac{7\pi}{4})$

IV. $(-2\sqrt{2}, \frac{3\pi}{4})$

a) I, II, III b) I, III, IV c) I, II, III d) I, II, IV



$$(x, y) = (2, 2)$$

$\Downarrow$

$$(r, \theta) = (2\sqrt{2}, \frac{\pi}{4}) \rightarrow \text{I. doğru } \checkmark$$

$$= (-2\sqrt{2}, \frac{5\pi}{4}) \rightarrow \text{II. doğru } \checkmark$$

$$= (2\sqrt{2}, -\frac{7\pi}{4}) \rightarrow \text{III. doğru}$$

Daha kısa şekilde:

$$(r, \theta) = (-r, \pi + \theta) = (r, \theta + 2\pi) \rightarrow \text{kolayca kullanılarak:}$$

$$\downarrow$$

$$(2\sqrt{2}, \frac{\pi}{4}) = (-2\sqrt{2}, \pi + \frac{\pi}{4}) = (2\sqrt{2}, \frac{\pi}{4} - 2\pi)$$

$\downarrow$
 r

$\downarrow$
 θ

$$= (-2\sqrt{2}, \frac{5\pi}{4}) = (2\sqrt{2}, -\frac{7\pi}{4})$$

bulebiliriz



4. Soru: **Cevap E**

5. Soru: **Cevap A**

⑥ $(r, \theta) = (-3, \frac{5\pi}{6})$ noktasının kartezyen koordinatı?

a) $(-\frac{3\sqrt{3}}{2}, \frac{3}{2})$ b) $(\frac{3\sqrt{3}}{2}, -\frac{3}{2})$ c) $(-\frac{3\sqrt{3}}{2}, -\frac{3}{2})$

d) $(\frac{3\sqrt{3}}{2}, \frac{3}{2})$

$(r, \theta) = (-3, \frac{5\pi}{6})$

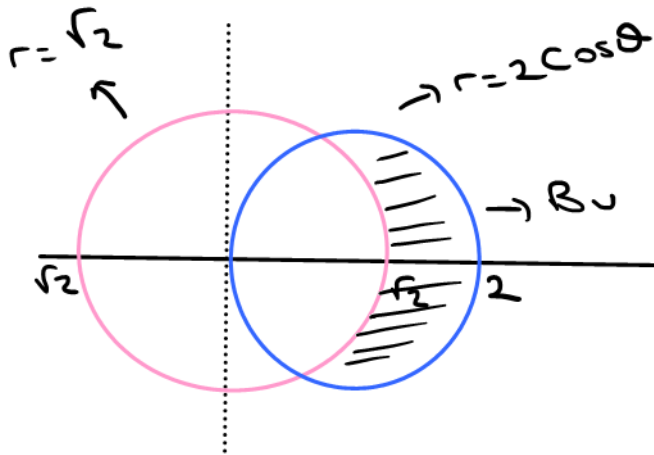
$$x = r \cos \theta = -3 \cdot \cos \frac{5\pi}{6} = (-3) \cdot (-\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}}{2}$$

$$y = r \sin \theta = -3 \cdot \sin \frac{5\pi}{6} = -3 \cdot \frac{1}{2} = -\frac{3}{2}$$

$\rightarrow (\frac{3\sqrt{3}}{2}, -\frac{3}{2})$

Cevap B

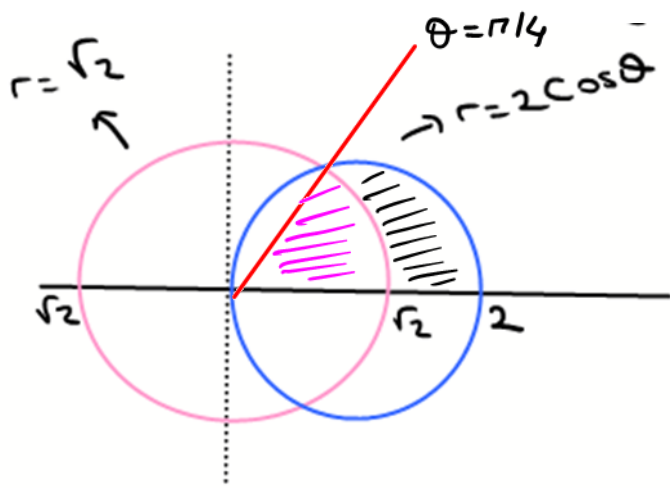
⑦ $r = 2 \cos \theta$ eğrisinin içinde $r = \sqrt{2}$ nin dışında kalan alanı veren integral?



$\rightarrow$ Bu alanı soruyor.
Üst yarımı hesaplayalım

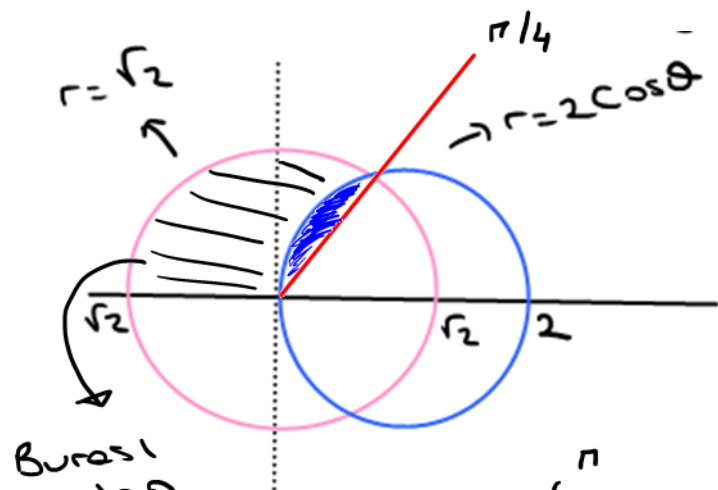
$$2 \cos \theta = \sqrt{2} \Rightarrow \cos \theta = \frac{\sqrt{2}}{2}$$

$\boxed{\theta = \frac{\pi}{4}}$ $\rightarrow$ Kesişim açısı



$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_0^{\pi/4} (2\cos\theta)^2 d\theta}_{\text{Siyah + Pembe}} - \underbrace{\frac{1}{2} \int_0^{\pi/4} (\sqrt{2})^2 d\theta}_{\text{Pembe}} = \underbrace{\frac{1}{2} \int_0^{\pi/4} (4\cos^2\theta - 2) d\theta}_{\text{Siyah}}$$

⑧ $r = 2\cos\theta$ dışında, $r = \sqrt{2}$ nin içinde kalan alanı veren integral?



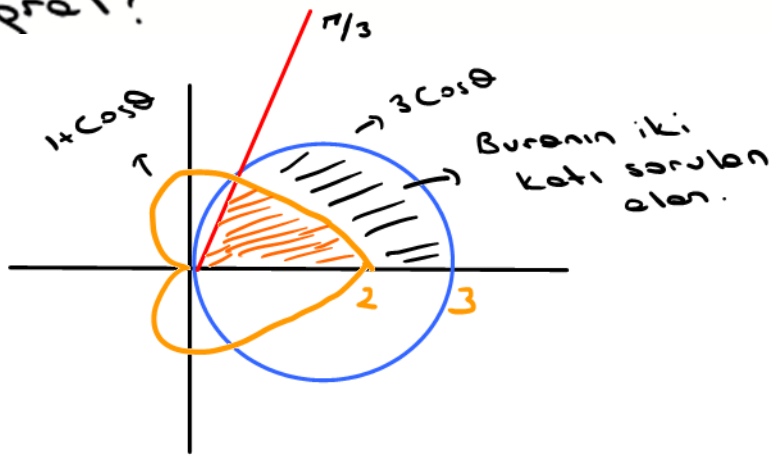
Burası sorulan alanın yarısı!

$$\Rightarrow \frac{A}{2} = \underbrace{\frac{1}{2} \int_{\pi/4}^{\pi} (\sqrt{2})^2 d\theta}_{\text{Siyah + Mavi}} - \underbrace{\frac{1}{2} \int_{\pi/4}^{\pi/2} (2\cos\theta)^2 d\theta}_{\text{Mavi}} \rightarrow \text{Siyah}$$

$$\Downarrow$$

$$A = \int_{\pi/4}^{\pi} 2 d\theta - \int_{\pi/4}^{\pi/2} 4\cos^2\theta d\theta$$

⑨ $r=3\cos\theta$ içinde $r=1+\cos\theta$ dışında kalan alanı veren integral?



$r=3\cos\theta \rightarrow x$ -ek. 6t. cember
 $r=1+\cos\theta \rightarrow$ Kalp
 Yatay
 Sivri x

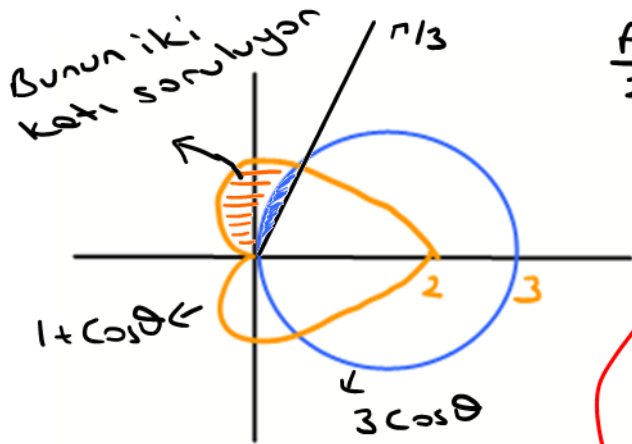
$$3\cos\theta = 1 + \cos\theta$$

$$2\cos\theta = 1 \rightarrow \theta = \frac{\pi}{3}$$

$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_0^{\pi/3} (3\cos\theta)^2 d\theta}_{\text{Siyah + Turuncu}} - \underbrace{\frac{1}{2} \int_0^{\pi/3} (1+\cos\theta)^2 d\theta}_{\text{Turuncu}}$$

$$A = \int_0^{\pi/3} ((3\cos\theta)^2 - (1+\cos\theta)^2) d\theta$$

⑩ $r=3\cos\theta$ dışında, $r=1+\cos\theta$ içinde kalan alanı veren integral?



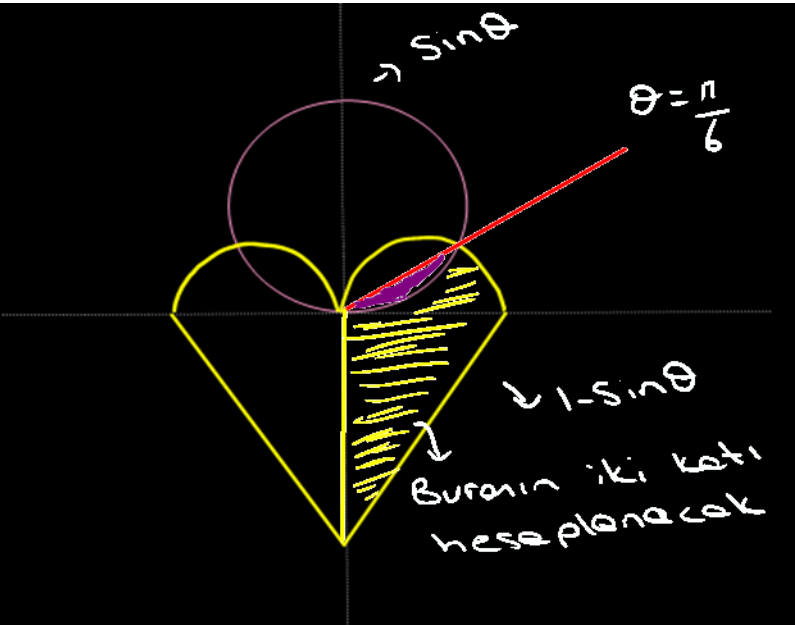
$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_{\pi/3}^{\pi} (1+\cos\theta)^2 d\theta}_{\text{Turuncu + Mavi}} - \underbrace{\frac{1}{2} \int_{\pi/3}^{\pi/2} (3\cos\theta)^2 d\theta}_{\text{Mavi}}$$

$$A = \int_{\pi/3}^{\pi} (1+\cos\theta)^2 d\theta - \int_{\pi/3}^{\pi/2} 9\cos^2\theta d\theta$$

⑪ $r = 1 - \sin \theta$ içinde, $r = \sin \theta$ dışında kalan alanı veren integral?

$$1 - \sin \theta = \sin \theta$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

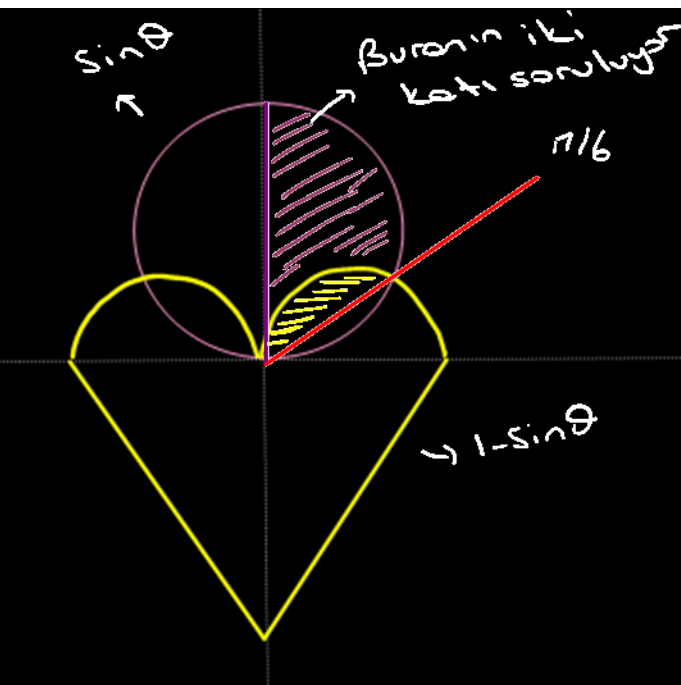


$$\frac{A}{2} = \frac{1}{2} \int_{-\pi/2}^{\pi/6} (1 - \sin \theta)^2 d\theta - \frac{1}{2} \int_0^{\pi/6} (\sin \theta)^2 d\theta$$

$\underbrace{\hspace{10em}}_{\text{Sarı + Mor}}$
 $\underbrace{\hspace{10em}}_{\text{Mor}}$

$$A = \int_{-\pi/2}^{\pi/6} (1 - \sin \theta)^2 d\theta - \int_0^{\pi/6} (\sin \theta)^2 d\theta$$

⑫ $r = 1 - \sin \theta$ dışında, $r = \sin \theta$ içinde kalan alanı veren integral?



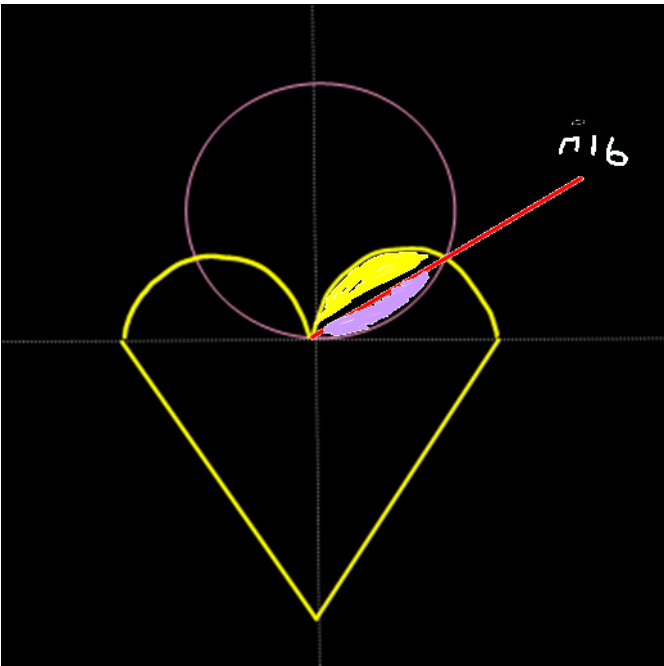
$$\frac{A}{2} = \frac{1}{2} \int_{\pi/6}^{\pi/2} (\sin \theta)^2 d\theta - \frac{1}{2} \int_{\pi/6}^{\pi/2} (1 - \sin \theta)^2 d\theta$$

$\underbrace{\hspace{10em}}_{\text{Mor + Sarı}}$
 $\underbrace{\hspace{10em}}_{\text{Sarı}}$

$$A = \int_{\pi/6}^{\pi/2} ((\sin \theta)^2 - (1 - \sin \theta)^2) d\theta$$

⑬ $r=1-\sin\theta$, $r=\sin\theta$ ortak alanı veren integral?

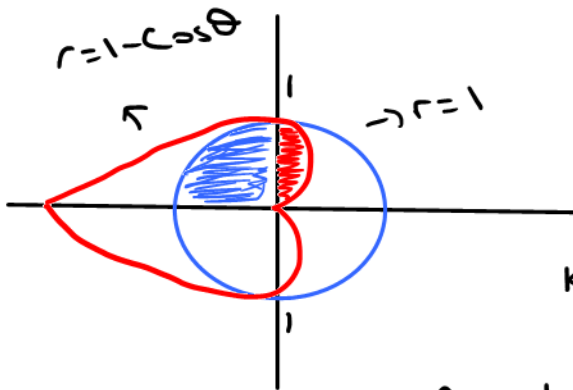
Sarı+Mor alanın iki katı soruluyor.



$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/6} (\sin\theta)^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} (1-\sin\theta)^2 d\theta$$

$$A = \int_0^{\pi/6} \sin^2\theta d\theta + \int_{\pi/6}^{\pi/2} (1-\sin\theta)^2 d\theta$$

⑭ $r=1$, $r=1-\cos\theta$ ortak alanı veren integral?



$r=1 \rightarrow$ Merkezli Dember

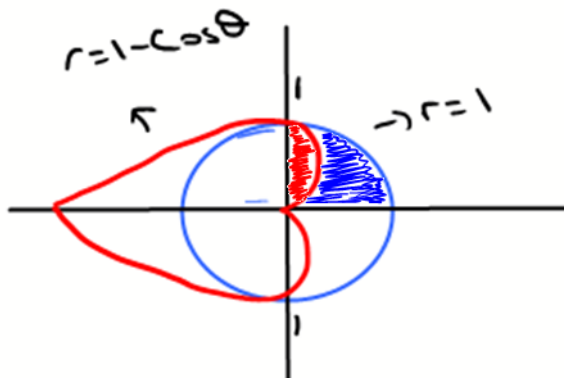
$r=1-\cos\theta \rightarrow$ Sırtı ve -x Yatay kalp

Kırmızı+Mavinin iki katı soruluyor.

$$\frac{A}{2} = \frac{1}{2} \int_0^{\pi/2} (1-\cos\theta)^2 d\theta + \frac{1}{2} \int_{\pi/2}^{\pi} 1^2 d\theta$$

$$A = \int_0^{\pi/2} (1-\cos\theta)^2 d\theta + \int_{\pi/2}^{\pi} d\theta$$

⑮ $r=1$ dışı, $r=1-\cos\theta$ dışı alanı veren integral?

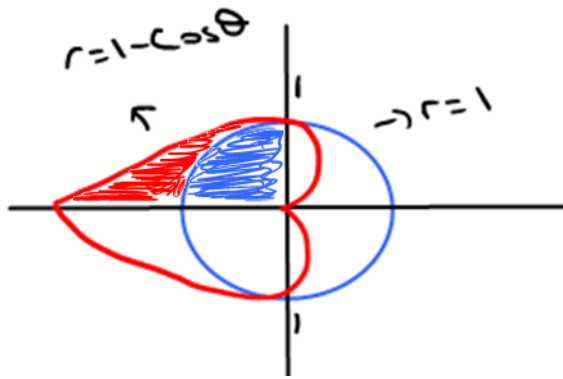


Mavi bölgenin iki katı soruluyor.

$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_0^{\pi/2} 1^2 d\theta}_{\text{Mavi + Kırmızı}} - \underbrace{\frac{1}{2} \int_0^{\pi/2} (1-\cos\theta)^2 d\theta}_{\text{Kırmızı}}$$

$$A = \int_0^{\pi/2} (1 - (1-\cos\theta)^2) d\theta$$

⑯ $r=1$ dışı, $r=1-\cos\theta$ dışı alanı veren integral?



Kırmızı bölgenin iki katı soruluyor.

$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_{\pi/2}^{\pi} (1-\cos\theta)^2 d\theta}_{\text{Kırmızı + Mavi}} - \underbrace{\frac{1}{2} \int_{\pi/2}^{\pi} 1^2 d\theta}_{\text{Mavi}}$$

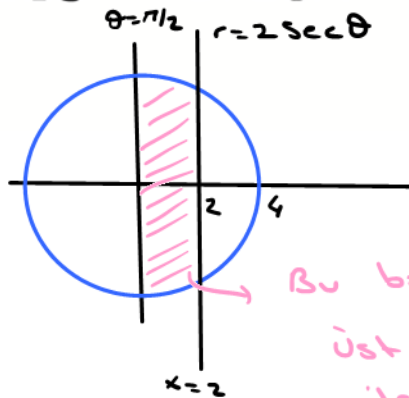
$$A = \int_{\pi/2}^{\pi} ((1-\cos\theta)^2 - 1) d\theta$$

⑰ $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan alanı veren integral?

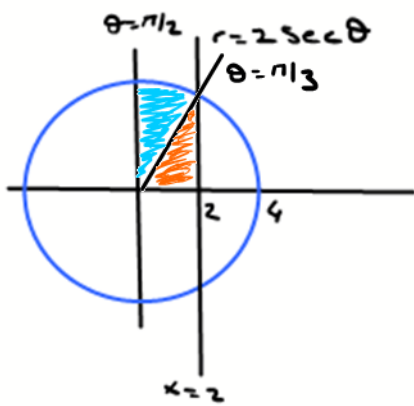
$r=4 \rightarrow$ Merkezli çember

$\theta=\frac{\pi}{2} \rightarrow$ doğru

$r=2\sec\theta \rightarrow \underbrace{r\cos\theta}_{x=2} = 2$
doğrusu



Bu bölge soruluyor.
Üst kısmı hesaplayıp iki ile çarpalım 😊



$$r=4 \quad r=2\sec\theta$$

↘
kesişimi

$$4 = 2\sec\theta \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$$

(Mavi + Turuncu) $\times 2 \rightarrow$ Aranan Alan

$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_0^{\pi/3} (2\sec\theta)^2 d\theta}_{\text{Turuncu}} + \underbrace{\frac{1}{2} \int_{\pi/3}^{\pi/2} 4^2 d\theta}_{\text{Mavi}}$$

$$A = \int_0^{\pi/3} (2\sec\theta)^2 d\theta + \int_{\pi/3}^{\pi/2} 16 d\theta$$

⑩ $r=2\sin\theta$, $r=\sqrt{2}$ integral?

ortak sınırlanan alanı veren

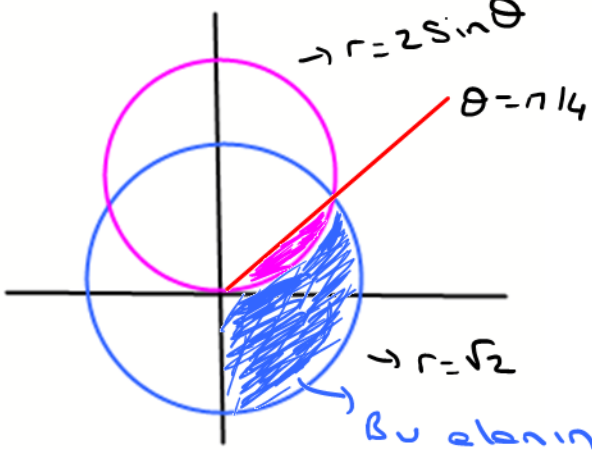
$$2\sin\theta = \sqrt{2} \Rightarrow \theta = \pi/4$$

Mavi + Pembe'nin iki katı soruluyor

$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_0^{\pi/4} (2\sin\theta)^2 d\theta}_{\text{Pembe}} + \underbrace{\frac{1}{2} \int_{\pi/4}^{\pi/2} (\sqrt{2})^2 d\theta}_{\text{Mavi}}$$

$$A = \int_0^{\pi/4} (2\sin\theta)^2 d\theta + \int_{\pi/4}^{\pi/2} 2 d\theta$$

19) $r = 2\sin\theta$ dışı, $r = \sqrt{2}$ içi kalan alanı veren
 mavimsi pembe integral? pembe



$\theta = \pi/4$

$r = \sqrt{2}$

Bu alanın iki katı soruluyor

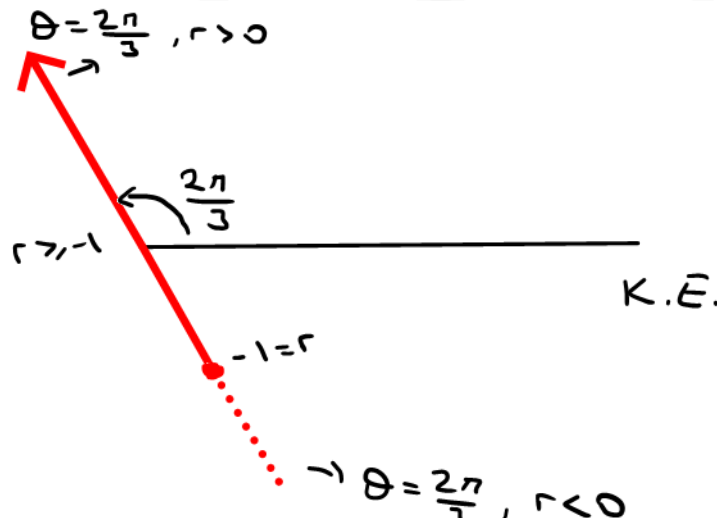
$$\frac{A}{2} = \frac{1}{2} \int_{-\pi/2}^{\pi/4} (\sqrt{2})^2 d\theta - \frac{1}{2} \int_0^{\pi/4} (2\sin\theta)^2 d\theta$$

$$A = \int_{-\pi/2}^{\pi/4} 2 d\theta - \int_0^{\pi/4} 4 \sin^2\theta d\theta$$

20) Cevabı en sonda 😊

21)

a) $\theta = \frac{2\pi}{3}$, $r \geq -1$



$\theta = \frac{2\pi}{3}$, $r > 0$

$r \geq -1$

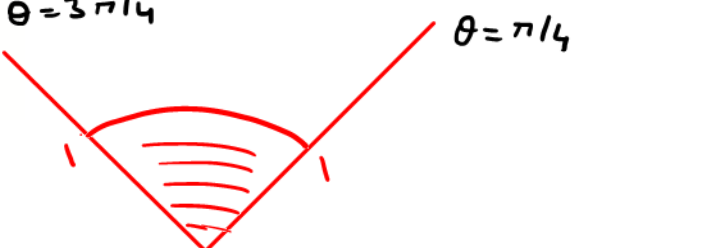
$\frac{2\pi}{3}$

$-1 = r$

$\theta = \frac{2\pi}{3}$, $r < 0$

K.E.

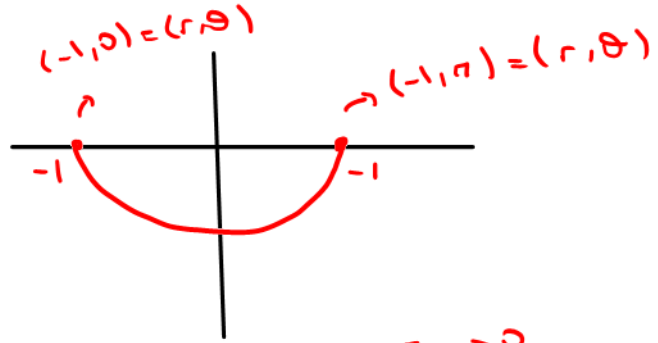
b) $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$, $0 \leq r \leq 1$



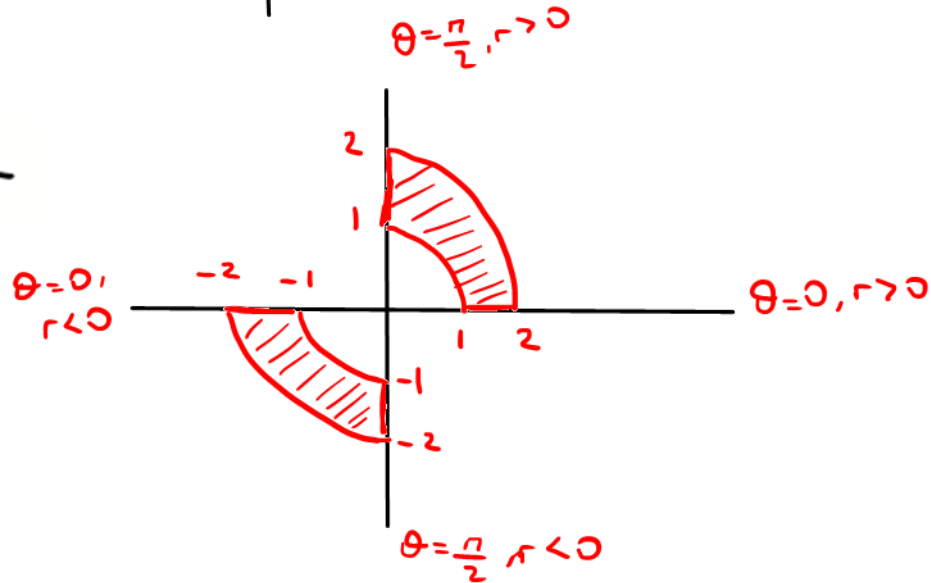
$\theta = 3\pi/4$

$\theta = \pi/4$

c) $0 \leq \theta \leq \pi$, $r = -1$



d) $0 \leq \theta \leq \frac{\pi}{2}$, $1 \leq |r| \leq 2$



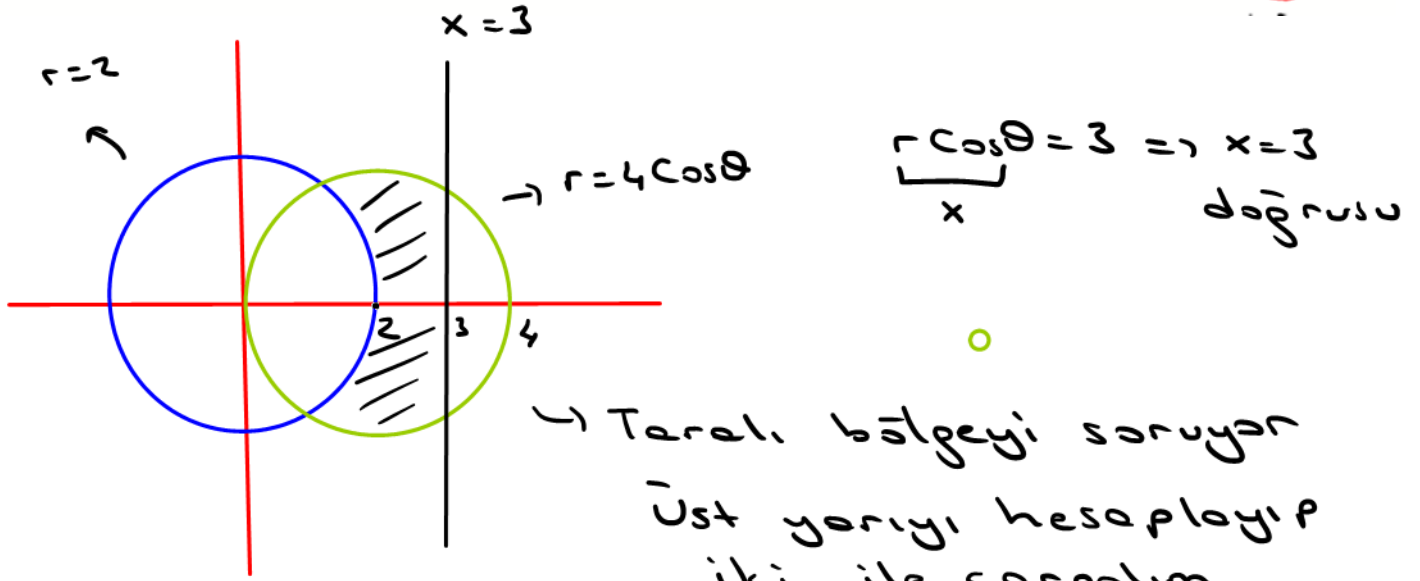
Not: Bazı özel trigonometrik değerler:

$$\cos \frac{\pi}{3} = \sin \frac{\pi}{6} = \frac{1}{2} \quad \cos \frac{\pi}{6} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \tan \frac{\pi}{3} = \cot \frac{\pi}{6} = \sqrt{3}$$

$$\cot \frac{\pi}{3} = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \quad \cot \frac{\pi}{4} = \tan \frac{\pi}{4} = 1$$

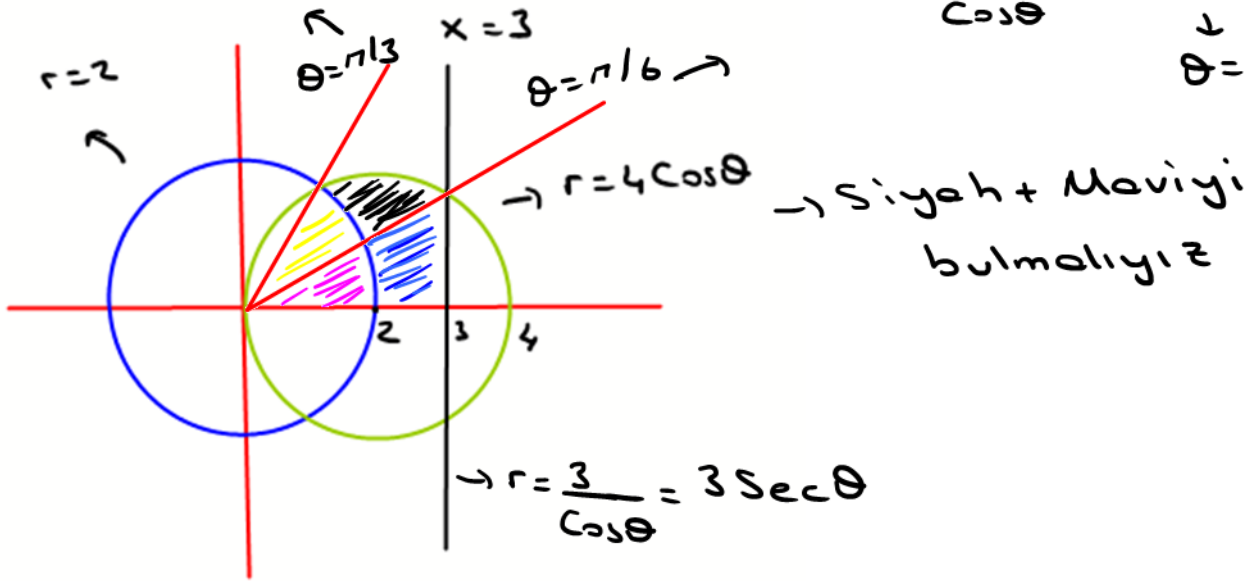
20) $r=2$, $r=4\cos\theta$, $r\cos\theta=3$ eğrilerinin için
 beraber sınırladığı bölgenin alanını veren
 integral? (Bölgeyi doğru belirleyip alanı veren
 integrali yazabilerseniz bu konu fazlasıyla
 tamamdır 😊)



$$2 = 4\cos\theta \rightarrow \theta = \pi/3$$

$$4\cos\theta = \frac{3}{\cos\theta} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$\downarrow$
 $\theta = \pi/6$

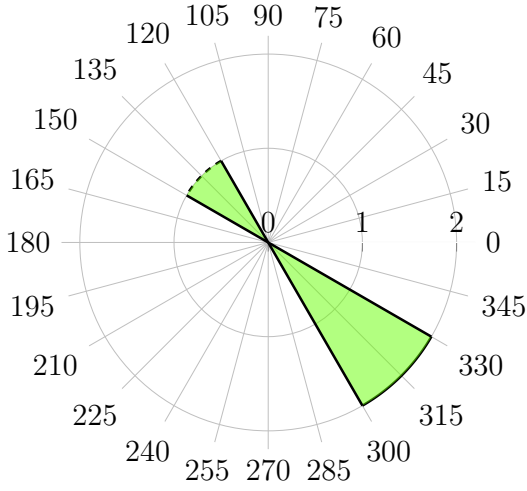


$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_0^{\pi/6} (3\sec\theta)^2 d\theta}_{\text{Mavi + Siyah}} + \underbrace{\frac{1}{2} \int_{\pi/6}^{\pi/3} (4\cos\theta)^2 d\theta}_{\text{Siyah + Sarı}} - \underbrace{\frac{1}{2} \int_0^{\pi/3} (2)^2 d\theta}_{\text{Sarı + Pembe}}$$

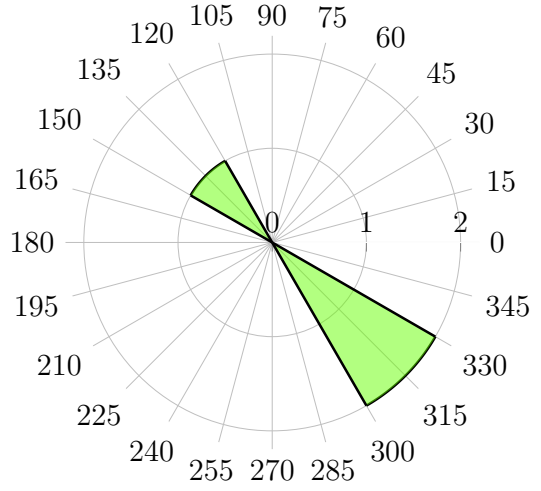
22

$-\frac{\pi}{3} \leq \theta \leq -\frac{\pi}{6}$ ve $-1 < r \leq 2$ eşitsizliklerinin temsil ettiği noktalar kümesinin kutupsal koordinatlardaki gösterimi aşağıdakilerden hangisidir?

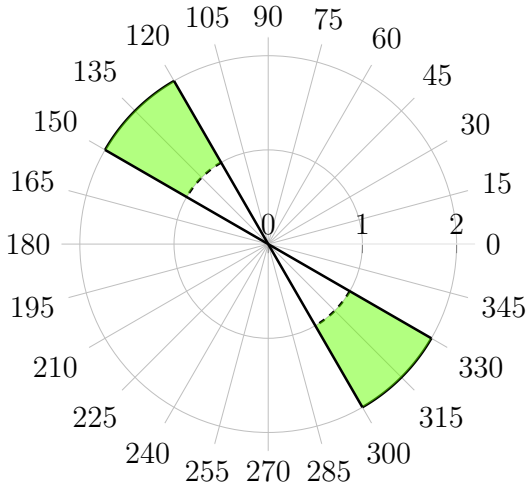
A)



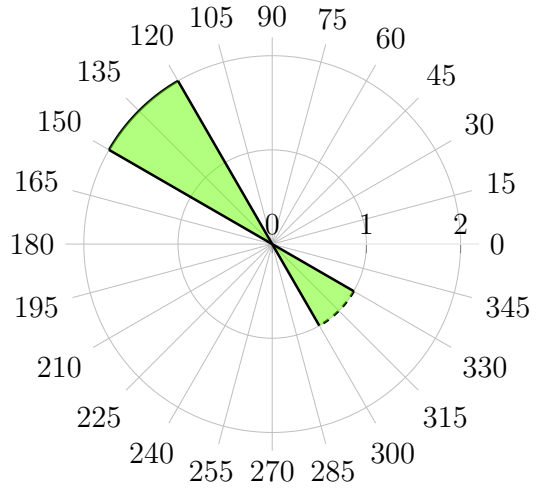
B)



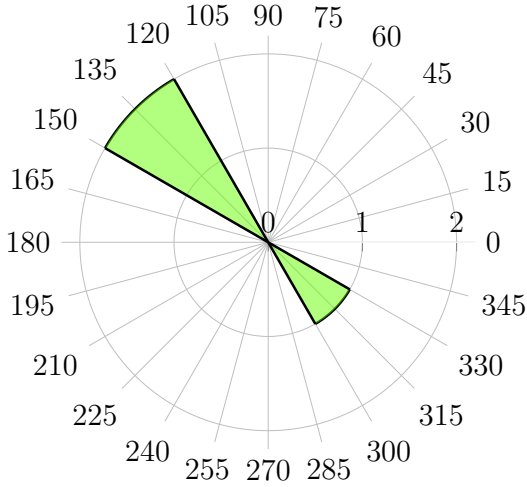
C)



D)



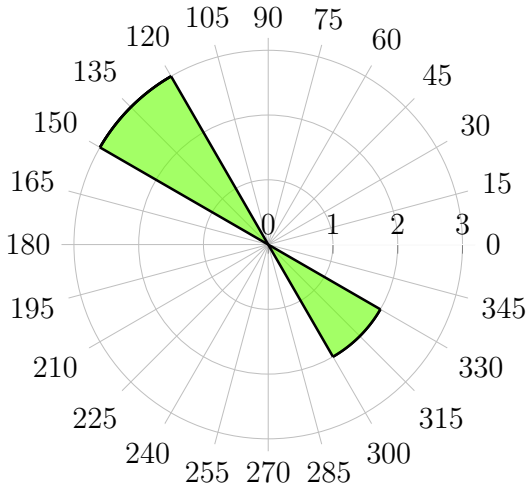
E)



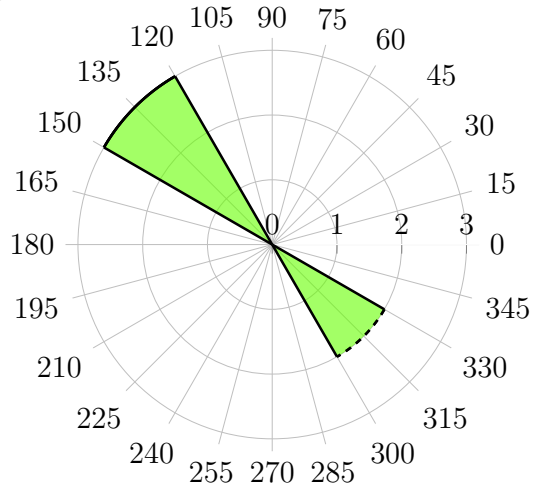
23

$-\frac{\pi}{3} \leq \theta \leq -\frac{\pi}{6}$ ve $-3 \leq r < 2$ eşitsizliklerinin temsil ettiği noktalar kümesinin kutupsal koordinatlardaki gösterimi aşağıdakilerden hangisidir?

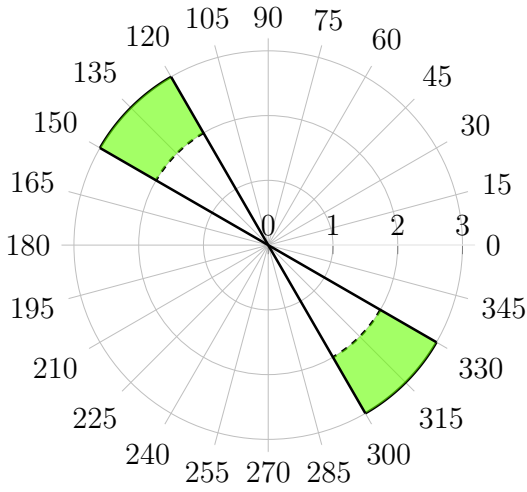
A)



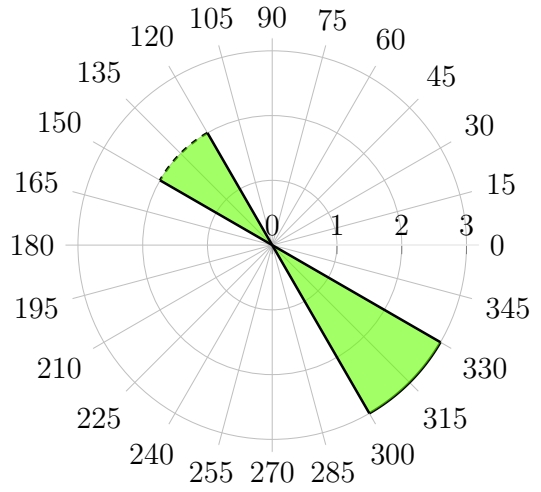
B)



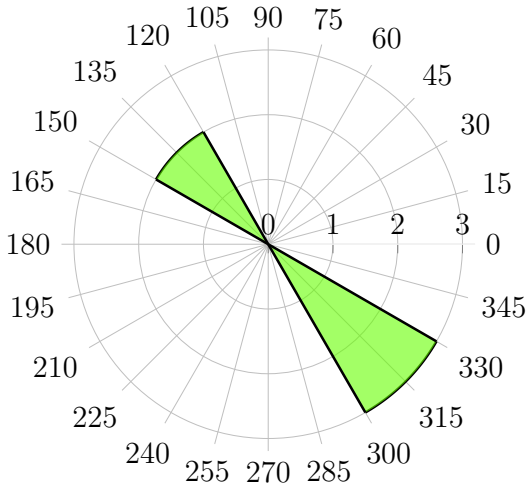
C)



D)



E)



$r = 2 + 2\cos\theta$ kardioidinin içinde, $r = 6\cos\theta$ çemberinin dışında kalan bölgenin alanını veren integral aşağıdakilerden hangisidir?

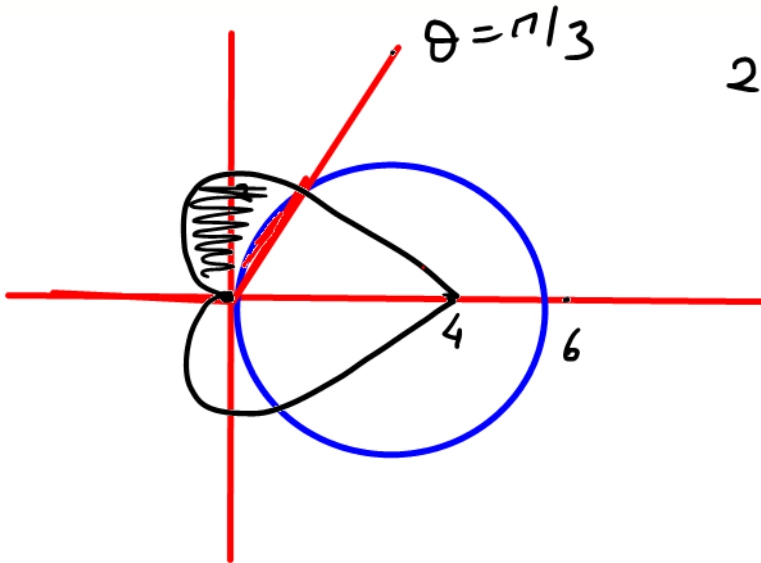
(a) $A = \int_0^{\frac{\pi}{3}} \left[(6\cos\theta)^2 - (2 + 2\cos\theta)^2 \right] d\theta$

(b) $A = \int_{\frac{\pi}{3}}^{\pi} \left[(2 + 2\cos\theta)^2 - (6\cos\theta)^2 \right] d\theta$

(c) $A = \int_{\frac{\pi}{3}}^{\pi} (2 + 2\cos\theta)^2 d\theta - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (6\cos\theta)^2 d\theta$

(d) $A = \int_{\frac{\pi}{3}}^{\pi} (2 + 2\cos\theta)^2 d\theta - \int_0^{\frac{\pi}{3}} (6\cos\theta)^2 d\theta$

(e) $A = \int_{\frac{\pi}{6}}^{\pi} (2 + 2\cos\theta)^2 d\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (6\cos\theta)^2 d\theta$



$$2 + 2\cos\theta = 6\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

$$\frac{A}{2} = \frac{1}{2} \int_{\frac{\pi}{3}}^{\pi} (2 + 2\cos\theta)^2 d\theta - \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (6\cos\theta)^2 d\theta$$

Siyah + Kırmızı

Kırmızı

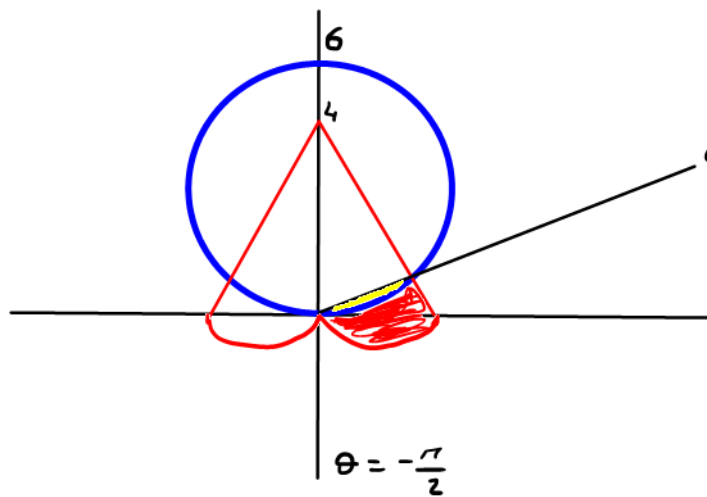
(25)

$r = 6\sin\theta$ çemberinin dışında, $r = 2 + 2\sin\theta$ kardioidinin içinde kalan bölgenin alanını veren integral aşağıdakilerden hangisidir?

(a) $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \left[(2 + 2\sin\theta)^2 - (6\sin\theta)^2 \right] d\theta$ (b) $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (2 + 2\sin\theta)^2 d\theta - \int_0^{\frac{\pi}{6}} (6\sin\theta)^2 d\theta$

(c) $A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left[(6\sin\theta)^2 - (2 + 2\sin\theta)^2 \right] d\theta$ (d) $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} (2 + 2\sin\theta)^2 d\theta - \int_0^{\frac{\pi}{3}} (6\sin\theta)^2 d\theta$

(e) $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (2 + 2\sin\theta)^2 d\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (6\sin\theta)^2 d\theta$



$$2 + 2\sin\theta = 6\sin\theta$$

$$\sin\theta = \frac{1}{2} \rightarrow \theta = \pi/6$$

$$\theta = \frac{\pi}{6}$$

$$\theta = -\frac{\pi}{2}$$

$$\frac{A}{2} = \underbrace{\frac{1}{2} \int_{-\pi/2}^{\pi/6} (2 + 2\sin\theta)^2 d\theta}_{\text{Kırmızı + Sarı}} - \underbrace{\frac{1}{2} \int_0^{\pi/6} (6\sin\theta)^2 d\theta}_{\text{Sarı}}$$