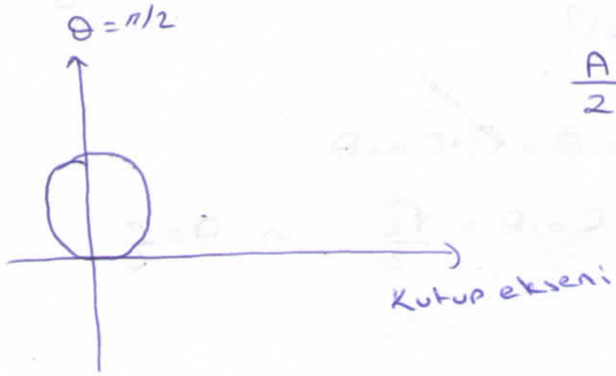
(*) $r=\sqrt{2}\sin\theta$ ile sınırlı bölgenin alanı?

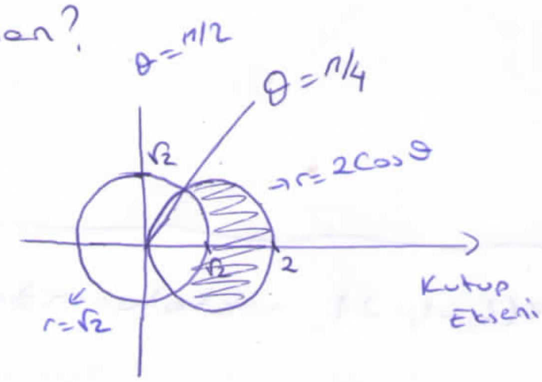


$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}(\sqrt{2}\sin\theta)^2\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi/2}2\sin^2\theta\,d\theta$$

$$=\int_0^{\pi/2}\frac{1-\cos 2\theta}{2}\,d\theta=\frac{\pi}{4}\qquad \boxed{A=\frac{\pi}{2}}$$

(*) a) $r=2\cos\theta$ eğrisinin içinde $r=\sqrt{2}$ nin dışında kalan alan?



$$2\cos\theta=\sqrt{2}\ \rightarrow\ \boxed{\theta=\frac{\pi}{4}}$$

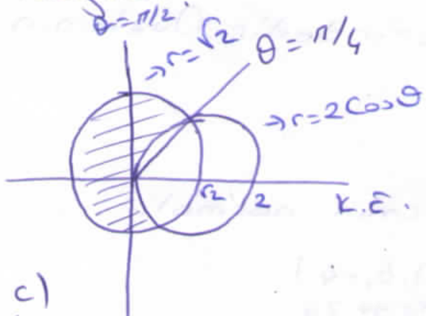
$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(2\cos\theta)^2\,d\theta-\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta$$

$$=\frac{1}{2}\left[\int_0^{\pi/4}(4\cos^2\theta-2)\,d\theta\right]$$

$$=\frac{1}{2}\int_0^{\pi/4}2\cos 2\theta\,d\theta=\frac{\sin 2\theta}{2}\Big|_0^{\pi/4}=\frac{1}{2}$$

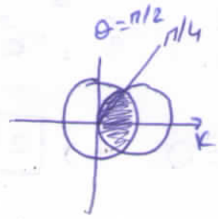
$$\boxed{A=1}$$

b) ✩ $r=2\cos\theta$ nın dışında, $r=\sqrt{2}$ nin içinde kalan alanı veren integral:



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/4}^{\pi}(\sqrt{2})^2\,d\theta-\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\theta)^2\,d\theta$$

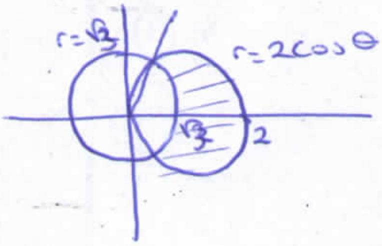
c) ✩ Ortak Alanı veren integral:



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/4}(\sqrt{2})^2\,d\theta+\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\theta)^2\,d\theta$$

(*)

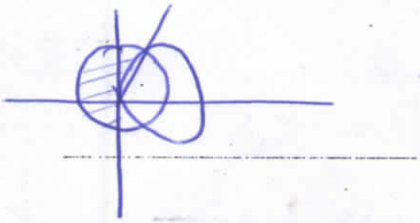
a) $r=2\cos\theta$ içinde, $r=\sqrt{3}$ dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/6}\left((2\cos\theta)^2-3\right)d\theta$$

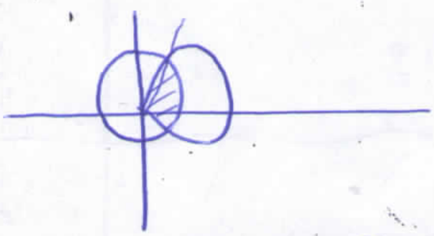
$2\cos\theta=\sqrt{3} \quad \theta=\frac{\pi}{6}$

b) $r=2\cos\theta$ dışında, $r=\sqrt{3}$ içindeki alan?



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/6}^{\pi}3\,d\theta-\int_{\pi/6}^{\pi/2}(2\cos\theta)^2d\theta$$

c) $r=2\cos\theta$, $r=\sqrt{3}$ içindeki ortak alan?



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/6}3d\theta+\frac{1}{2}\int_{\pi/6}^{\pi/2}(2\cos\theta)^2d\theta$$

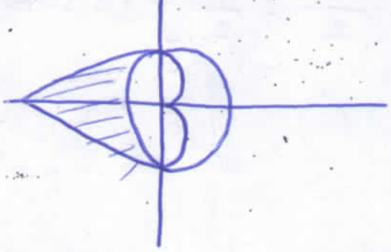
(*) $r=1$ içinde

$r=1-\cos\theta$ dışında.

$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}\left(1-(1-\cos\theta)^2\right)d\theta$$

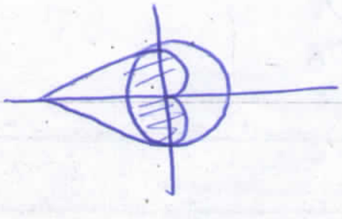
(*) $r=1$ dışında

$r=1-\cos\theta$ içinde



$$\frac{A}{2}=\frac{1}{2}\int_{\pi/2}^{\pi}\left((1-\cos\theta)^2-1^2\right)d\theta$$

(*) $r=1$, $r=1-\cos\theta$ ortak alan?

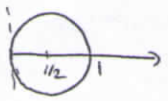


$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2d\theta+\cancel{\frac{\pi}{2}}$$

$$+\frac{1}{2}\int_{\pi/2}^{\pi}1^2\,d\theta$$

K. 9

(*) $r = \cos\theta$ çemberinin uzunluğu?



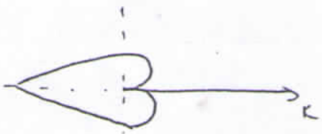
$$r = \cos\theta \qquad r' = -\sin\theta$$

$$r^2 + (r')^2 = \cos^2\theta + \sin^2\theta = 1$$

$$\sqrt{r^2+(r')^2} = 1$$

$$S = \int_{-\pi/2}^{\pi/2} d\theta = \theta \Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + \frac{\pi}{2} = \underline{\underline{\pi}}$$

(*) $r = 1-\cos\theta$ kardiyoidinin uzunluğu?



$$r = 1-\cos\theta \qquad r' = \sin\theta$$

$$r^2+(r')^2 = 1-2\cos\theta+\cos^2\theta+\sin^2\theta = 2-2\cos\theta$$

$$= 2-2\left[1-2\sin^2\frac{\theta}{2}\right] = 4\sin^2\frac{\theta}{2}$$

$$\sqrt{r^2+(r')^2} = \sqrt{4\sin^2\frac{\theta}{2}} = \left|2\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{2\pi} \left|2\sin\frac{\theta}{2}\right| d\theta = \int_0^{2\pi} 2\sin\frac{\theta}{2}\, d\theta = -4\cos\frac{\theta}{2}\Big|_0^{2\pi} = 4+4 = \underline{\underline{8}}$$

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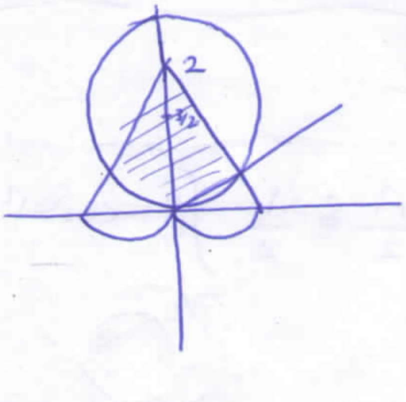
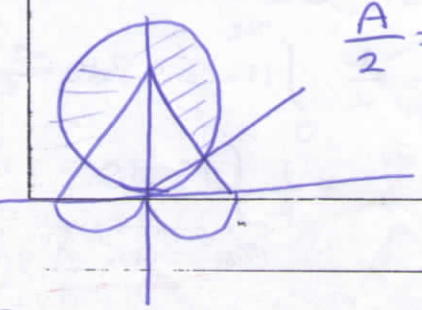
a) $r = 3\sin\theta$, $r = 1+\sin\theta$ ortak alan?



$$3\sin\theta = 1+\sin\theta \qquad \sin\theta = \frac{1}{2} \qquad \theta = \frac{\pi}{6}$$

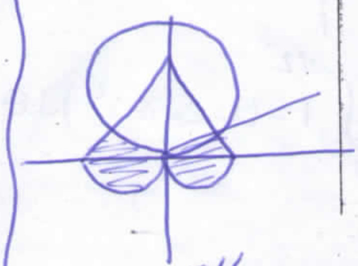
$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (3\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6} (1+\sin\theta)^2 d\theta$$

b) $r = 3\sin\theta$ içi
$r = 1+\sin\theta$ dışı



$$\frac{A}{2} = \frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2 - (1+\sin\theta)^2 d\theta$$

c) $r = 3\sin\theta$ dışı
$r = 1+\sin\theta$ içi



$$\frac{A}{2} = \frac{1}{2}\int_{-\pi/2}^{\pi/6} (1+\sin\theta)^2 d\theta - \frac{1}{2}\int_0^{\pi/6} (3\sin\theta)^2$$

YTÜ - Fen-Edebiyat Fakültesi **Sınav Soru ve Cevap Kağıdı**		**NOT TABLOSU**					
		1. S	**2. S**	**3.S**	**4.S**	**TOPLAM**	
Adı Soyadı							
Öğrenci Numarası		**Grup No**					
Bölümü		**Sınav Tarihi**	**06/04/2013**				
Dersin Adı	**MATEMATİK II**	Sınav Süresi	90 dk	**Sınav Yeri**			
Dersi veren Öğretim Üyesinin Adı Soyadı		**İmza**					

YÖK nun 2547 sayılı Kanunun ***Öğrenci Disiplin Yönetmeliğinin*** 9. Maddesi olan ***"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"*** fiili işleyenler **bir veya iki yarıyıl uzaklaştırma** cezası alırlar.

1) $\lim_{x\to\infty}[6x^5\sin(\frac{1}{x})-6x^4+x^2]$ limitini seri açılımlarından faydalanarak çözünüz.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\sin\frac{1}{x} = \frac{1}{x} - \frac{1}{x^3\,3!} + \frac{1}{5!\,x^5} - \cdots$$

$$\lim_{x\to\infty}\left[6x^5.\sin\frac{1}{x} - 6x^4 + x^2\right] = \lim_{x\to\infty}\left[6x^5\left(\frac{1}{x} - \frac{1}{3!x^3} + \frac{1}{5!x^5} - \frac{1}{7!x^7}\cdots\right] - 6x^4 + x^2\right.$$

$$= \lim_{x\to\infty}\left[\cancel{6x^4} - \cancel{x^2} + \frac{1}{20} - \frac{6}{7!}\cdot\frac{1}{x^2}\cdots\right] - \cancel{6x^4} + \cancel{x^2}$$

$$= \lim_{x\to\infty}\left[\frac{1}{20} - \underbrace{\frac{6}{7!}\cdot\frac{1}{x^2} + \cdots}_{0}\right] = \boxed{\frac{1}{20}}$$

⊗ $r=4$, $\theta=\frac{\pi}{2}$, $r=2\sec\theta$ arasında kalan alanı kutupsal integral ile hesaplayın.

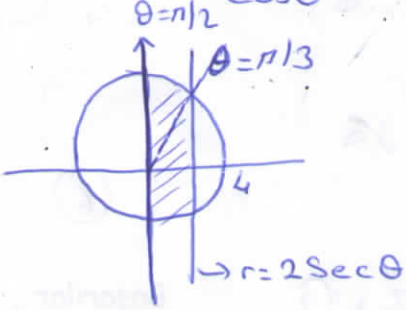
$r=4 \to$ 4 yarıçaplı merkezil çember

$r=2\sec\theta=\frac{2}{\cos\theta} \Rightarrow r\cos\theta=2$ $\boxed{x=2}$ doğrusu

$r\cos\theta=2 \Rightarrow r=\frac{2}{\cos\theta}$ $\quad r=4 \Rightarrow 4=\frac{2}{\cos\theta}$

$\cos\theta=\frac{1}{2}$

$\theta=\frac{\pi}{3}$

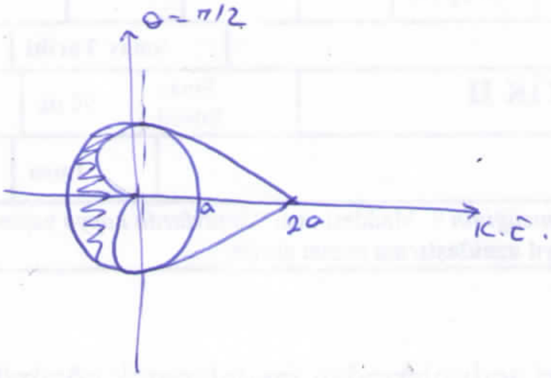


$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/3}(2\sec\theta)^2 d\theta + \frac{1}{2}\int_{\pi/3}^{\pi/2} 4^2\, d\theta$$

$$A = 4\tan\theta\Big|_0^{\pi/3} + 16\theta\Big|_{\pi/3}^{\pi/2} = 4\sqrt{3} + \frac{8\pi}{3}$$

Başarılar...

2) $a>0$ olmak üzere $r=a(1+\cos\theta)$ kardiyoidinin dışında, $r=a$ çemberinin içinde kalan bölgenin alanını hesaplayınız. (Şekil çiziniz)



$$\frac{A}{2}=\int_{\pi/2}^{\pi} a^2-(a+a\cos\theta)^2\,d\theta=\int_{\pi/2}^{\pi}\Big(-2a^2\cos\theta-a^2\underbrace{\cos^2\theta}_{\frac{1+\cos2\theta}{2}}\Big)\,d\theta$$

$$=-2a^2\sin\theta-a^2\frac{\theta}{2}-\frac{a^2}{4}\sin2\theta\Big|_{\pi/2}^{\pi}$$

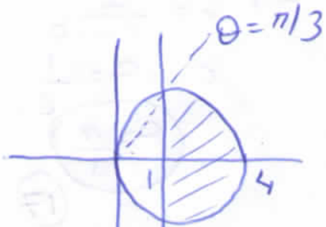
$$=-a^2\frac{\pi}{2}-\Big(-2a^2-a^2\frac{\pi}{4}\Big)=-\frac{a^2\pi}{2}+2a^2+\frac{a^2\pi}{4}$$

$$=2a^2-\frac{\pi}{4}a^2$$

⊛ $r=4\cos\theta$ ve $r\cos\theta\geq 1$ arasında kalan alanı veren integrali yazıp alanı bulunuz.

$r=4\cos\theta \Rightarrow$ Ötelenmiş Çember, $r\cos\theta\geq 1 \Rightarrow \boxed{x\geq 1}$

$r\cos\theta=1 \Rightarrow r=\frac{1}{\cos\theta}$ $\quad r=4\cos\theta \Rightarrow 4\cos\theta=\frac{1}{\cos\theta}$ $\quad \cos^2\theta=\frac{1}{4}$ $\boxed{\theta=\frac{\pi}{3}}$



$$\frac{A}{2}=\frac{1}{2}\int_0^{\pi/3}\underbrace{(4\cos\theta)^2}_{16\frac{(1+\cos2\theta)}{2}}d\theta-\frac{1}{2}\int_0^{\pi/3}\underbrace{\Big(\frac{1}{\cos\theta}\Big)^2}_{\sec^2\theta}d\theta$$

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$$A=8\Big[\theta+\frac{\sin2\theta}{2}\Big|_0^{\pi/3}\Big]-\tan\theta\Big|_0^{\pi/3}=\frac{8\pi}{3}+\sqrt{3}$$

Başarılar...

(*) $\begin{cases} x = 8\cos t + 8t\sin t \\ y = 8\sin t - 8t\cos t \\ 0 \le t \le \frac{\pi}{2} \end{cases}$ parametrizasyonu ile verilen eğrinin uzunluğu?

$$S = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = -\cancel{8\sin t} + \cancel{8\sin t} + 8t\cos t \Rightarrow \left(\frac{dx}{dt}\right)^2 = 64t^2\cos^2 t$$

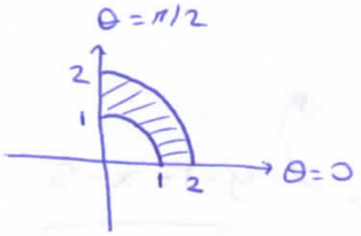
$$\frac{dy}{dt} = \cancel{8\cos t} - \cancel{8\cos t} + 8t\sin t \Rightarrow \left(\frac{dy}{dt}\right)^2 = 64t^2\sin^2 t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{64t^2} = 8t$$

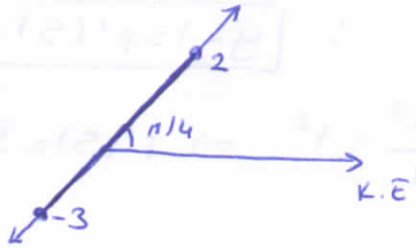
$$S = \int_0^{\pi/2} 8t\, dt = 4t^2 \Big|_0^{\pi/2} = \underline{\underline{\pi^2}}$$

(*) Kutupsal koordinatları aşağıdaki şartları sağlayan noktalar kümesinin grafiğini çiziniz.

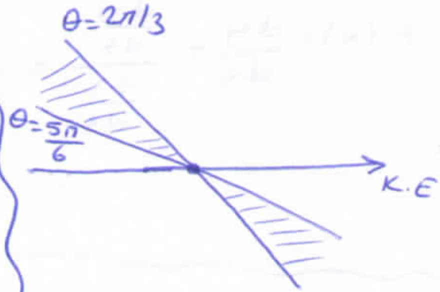
a) $1 \le r \le 2$ ve $0 \le \theta \le \pi/2$



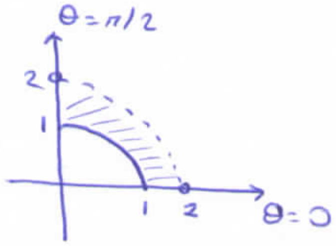
b) $-3 \le r \le 2$ ve $\theta = \frac{\pi}{4}$



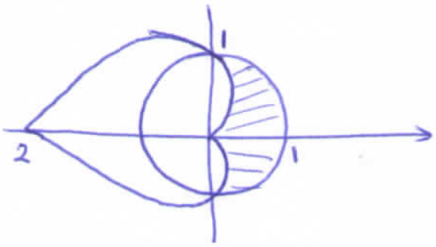
c) $\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$



d) $1 \le r < 2$, $0 \le \theta \le \frac{\pi}{2}$



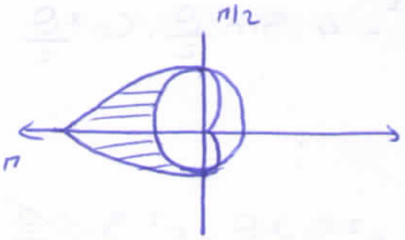
⊛ $r=1$ çemberinin içinde, $r=1-\cos\theta$ kardiyoidinin dışında kalan bölgenin alanını veren integral?



$$\frac{A}{2}=\int_0^{\pi/2}\frac{1}{2}\,d\theta-\frac{1}{2}\int_0^{\pi/2}(1-\cos\theta)^2\,d\theta$$

$$A=\int_0^{\pi/2}\left(1-(1-\cos\theta)^2\right)d\theta$$

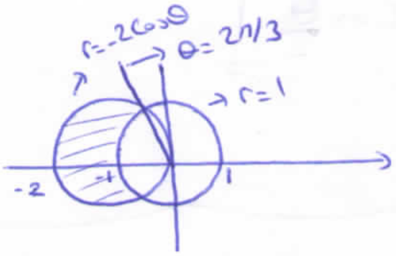
b) çemberin dışı, kardiyoidin içi:



$$\frac{A}{2}=\int_{\pi/2}^{\pi}\frac{1}{2}\cdot(1-\cos\theta)^2\,d\theta-\int_{\pi/2}^{\pi}\frac{1}{2}\,d\theta$$

$$A=\int_{\pi/2}^{\pi}\left((1-\cos\theta)^2-1\right)d\theta$$

⊛ $r=-2\cos\theta$ çemberinin içinde, $r=1$ çemberinin dışında kalan alan?



$$-2\cos\theta=1\quad\Rightarrow\quad\theta=\frac{2\pi}{3}$$

$$\frac{A}{2}=\frac{1}{2}\int_{2\pi/3}^{\pi}(-2\cos\theta)^2\,d\theta-\frac{1}{2}\int_{2\pi/3}^{\pi}1^2\,d\theta$$

$$A=\int_{2\pi/3}^{\pi}(4\overbrace{\cos^2\theta}^{\frac{1+\cos2\theta}{2}}-1)\,d\theta=\int_{\frac{2\pi}{3}}^{\pi}(1+2\cos2\theta)\,d\theta$$

$$=\theta+\sin2\theta\,\Big|_{2\pi/3}^{\pi}=\pi-\frac{2\pi}{3}-\sin\frac{\pi}{3}=\frac{\pi}{3}-\frac{\sqrt3}{2}$$

(*) $r = a\sin^2\frac{\theta}{2}$ eğrisinin $0 \le \theta \le \pi$ aralığındaki uzunluğu? $(a>0)$

$$S = \int_0^{\pi} \sqrt{r^2 + (r')^2}\, d\theta$$

$$r^2 = a^2 \sin^4\frac{\theta}{2}$$

$$r' = a.2.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\frac{1}{2} = a.\sin\frac{\theta}{2}.\cos\frac{\theta}{2}$$

$$(r')^2 = a^2.\sin^2\frac{\theta}{2}.\cos^2\frac{\theta}{2}$$

$$r^2 + (r')^2 = a^2.\sin^4\frac{\theta}{2} + a^2\sin^2\frac{\theta}{2}.\overbrace{\cos^2\frac{\theta}{2}}^{1-\sin^2\frac{\theta}{2}} = a^2\cancel{\sin^4\frac{\theta}{2}} + a^2\sin^2\frac{\theta}{2} - a^2\cancel{\sin^4\frac{\theta}{2}}$$

$$= a^2\sin^2\frac{\theta}{2}$$

$$\sqrt{r^2+(r')^2} = \sqrt{a^2.\sin^2\frac{\theta}{2}} = a.\left|\sin\frac{\theta}{2}\right|$$

$$S = \int_0^{\pi} a.\left|\sin\frac{\theta}{2}\right| d\theta = \int_0^{\pi} a.\sin\frac{\theta}{2}\, d\theta = -2a\cos\frac{\theta}{2}\Big|_0^{\pi}$$

$$= \boxed{2a}$$

(*) $r = \cos\theta$, $r = 2\cos\theta$, $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$ arasında kalan alan?

S.3 a) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ serisinin toplamını bulunuz.(13p)

$$\frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

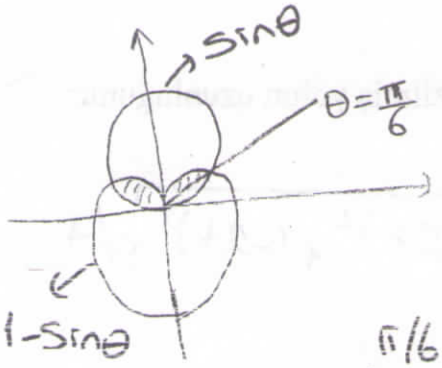
$$S_n = \sum_{k=1}^{n} \frac{2k+1}{k^2(k+1)^2} = \sum_{k=1}^{n} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right)$$

$$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \cdots + \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$S_n = 1 - \frac{1}{(n+1)^2}$$

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \lim_{n\to\infty} \left(1 - \frac{1}{(n+1)^2}\right) = 1 \,//$$

b) $\rho = 1 - sin\theta$ kardiyoidi ve $\rho = sin\theta$ çemberinin her ikisinin de içinde kalan bölgenin alanını bulunuz .(12p)



$$1 - \sin\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{6}, \ \frac{5\pi}{6}$$

$$\frac{A}{2} = \frac{1}{2}\int_0^{\pi/6} (\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} (1-\sin\theta)^2 d\theta$$

$$A = \int_0^{\pi/6} \frac{1-\cos 2\theta}{2} d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}(3 - 4\sin\theta - \cos 2\theta) d\theta$$

$$A = \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) + \left(\frac{\pi}{2} - \frac{7\sqrt{3}}{8}\right) = \frac{7\pi}{12} - \sqrt{3} \ br^2$$

Başarılar dilerim

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⊛ $\left.\begin{array}{l} x=4\sin\theta \\ y=2\cos\theta \end{array}\right\}$ eğrisinin $\theta=\frac{\pi}{4}$ deki teğeti?

$$m=\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\left.\frac{-2\sin\theta}{4\cos\theta}\right|_{\theta=\frac{\pi}{4}}=-\frac{1}{2}$$

$\theta=\frac{\pi}{4} \Rightarrow x_0=2\sqrt{2} \quad y_0=\sqrt{2}$

Teğet : $y-y_0=f'(x_0)(x-x_0) \Rightarrow \boxed{y-\sqrt{2}=-\frac{1}{2}(x-2\sqrt{2})}$

⊛ $\left.\begin{array}{l} x=2t^2+3 \\ y=t^4 \end{array}\right\}$ parametrik denklemi ile verilen eğrinin $t=-1$ deki normal doğrusunun denklemi?

$t=-1 \Rightarrow \begin{array}{l} x=5 \\ y=1 \end{array} \Rightarrow (5,1)$ den geçen normal : $y-1=m_N(x-5)$

$$m_T=\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4t^3}{4t}=t^2 \Rightarrow m_T=\left.t^2\right|_{t=-1}=1 \Rightarrow m_T.m_N=-1$$

$$\Downarrow$$

$$\boxed{m_N=-1}$$

$y-1=-(x-5) \Rightarrow \boxed{y=-x+6}$

⊛ $\left.\begin{array}{l} x(t)=t^2 \\ y(t)=1-t^2 \end{array}\right\}$ eğrisinin $-1\leq t\leq 0$ aralığındaki uzunluğu?

$$S=\int_{-1}^{0}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt=\int_{-1}^{0}\sqrt{(2t)^2+(-2t)^2}\,dt=\int_{-1}^{0}2\sqrt{2}.|t|\,dt$$

$$=-2\sqrt{2}\int_{-1}^{0}t\,dt=-2\sqrt{2}\left.\frac{t^2}{2}\right|_{-1}^{0}=\underline{\underline{\sqrt{2}}}$$