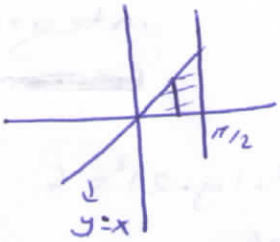
(*) $\displaystyle\int_0^{\pi/2}\int_y^{\pi/2} y\cos x^3\,dx\,dy = ?$ → integrasyon sırası değiştirilmeli

$D:\ y=\frac{\pi}{2} \quad y=0 \quad x=y \quad x=\frac{\pi}{2}$

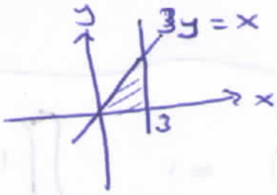


$$I=\int_0^{\pi/2}\int_0^{x} y\cos x^3\,dy\,dx=\int_0^{\pi/2}\frac{y^2}{2}\cos x^3\Big|_0^x\,dx=\int_0^{\pi/2}\frac{x^2}{2}\cos x^3\,dx$$

$$=\frac{1}{6}\sin x^3\Big|_0^{\pi/2}=\frac{1}{6}\sin\frac{\pi^3}{8}$$

(*) $\displaystyle\int_0^1\int_{3y}^{3} e^{x^2}\,dx\,dy$ integralini hesaplayın.

$D:\ y=0 \quad y=1 \quad x=3 \quad x=3y$

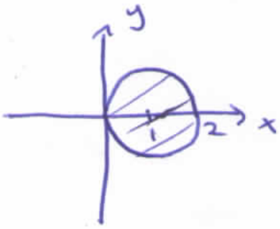


$$I=\int_0^3\int_0^{x/3} e^{x^2}\,dy\,dx=\int_0^3 y e^{x^2}\Big|_0^{x/3}\,dx$$

$$=\int_0^3\left(\frac{x}{3}e^{x^2}\right)dx=\frac{1}{6}e^{x^2}\Big|_0^3=\frac{1}{6}(e^9-1)$$

(*) $\displaystyle\iint_D (x^2+y^2)\,dx\,dy$ integralini $D:\ x^2+y^2=2x$ bölgesinde kutupsal koordinatlara dönüştürerek yazınız.

$x^2+y^2-2x=0 \Rightarrow (x-1)^2+y^2=1$



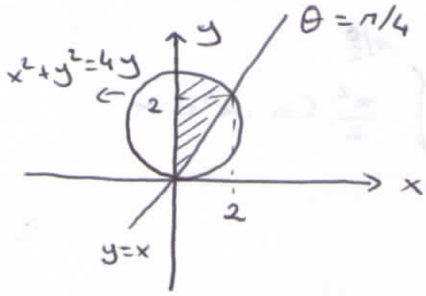
$$\left.\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \\ dx\,dy=r\,dr\,d\theta \\ x^2+y^2=r^2 \end{array}\right\} \quad I=\int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r^2\cdot r\,dr\,d\theta$$

$x^2+y^2-2x=0$

$r^2-2r\cos\theta=0 \quad r=0 \quad r=2\cos\theta$

(*) $D: \left\{ \begin{array}{l} x^2+y^2-4y=0 \\ y=x \\ x=0 \end{array} \right\}$ olmak üzere $I = \iint_D (3y-2)\,dx\,dy$ integralinin sınırlarını kutupsal dönüşümle yazınız.

$x^2+y^2-4y=0 \rightarrow x^2+(y-2)^2=4 \rightarrow$ Merkezi $(0,2)$ de Yarıçapı 2 olan çember



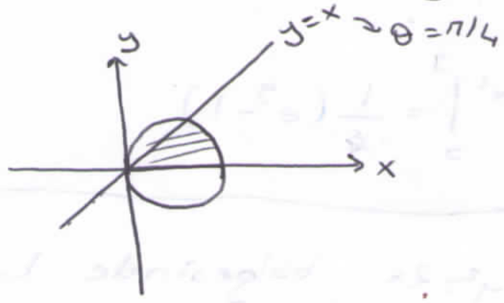
$\left. \begin{array}{l} x^2+y^2-4y=0 \\ y=x \end{array} \right\} \begin{array}{l} 2x^2-4x=0 \\ x=0 \quad x=2 \end{array}$

$\left. \begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \\ dxdy=r\,dr\,d\theta \end{array} \right\} \rightarrow I = \int_{\pi/4}^{\pi/2} \int_0^{4\sin\theta} (3r\sin\theta-2)\,r\,dr\,d\theta$

$\left. \begin{array}{l} x^2+y^2-4y=0 \\ r^2-4r\sin\theta=0 \\ r=0 \quad r=4\sin\theta \end{array} \right\}$

(*) $D: \left\{ \begin{array}{l} x^2+y^2-4x=0 \\ y=x \\ y=0 \end{array} \right\} \Rightarrow \iint_D (2x-3)\,dxdy \rightarrow$ kutupsal dönüşümle sınırları yazın.

$x^2+y^2-4x=0 \rightarrow (x-2)^2+y^2=4 \rightarrow$ Merkezi $(2,0)$ yarıçapı 2 olan çember



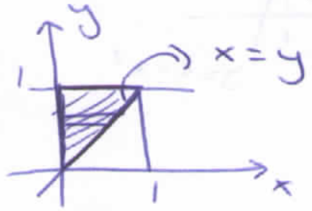
$\left. \begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \\ dxdy=r\,dr\,d\theta \end{array} \right\} \begin{array}{l} x^2+y^2-4x=0 \\ r^2-4r\cos\theta=0 \\ r=0 \quad r=4\cos\theta \end{array}$

$I = \int_0^{\pi/4} \int_0^{4\cos\theta} (2r\cos\theta-3)\,r\,dr\,d\theta$

* $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx = ?$

integral bu hali ile çözülemez. İntegrasyon sırasını değiştirmeliyiz.

$$\int_0^1 \int_x^1 e^{x/y}\, dy\, dx \Rightarrow \begin{matrix} x=1 & x=0 \\ y=1 & y=x \end{matrix} \Rightarrow$$



Sınırları bölgeyi y'ye göre düzgün olarak yazmalıyız.

$$I = \int_0^1 \int_0^y e^{x/y}\, dx\, dy = \int_0^1 \frac{e^{x/y}}{\frac{1}{y}} \Big|_0^y dy = \int_0^1 y e^{x/y} \Big|_0^y dy$$

$$= \int_0^1 (ye - y)\, dy = \frac{y^2}{2}e - \frac{y^2}{2} \Big|_0^1$$

$$= \frac{e}{2} - \frac{1}{2}$$

---

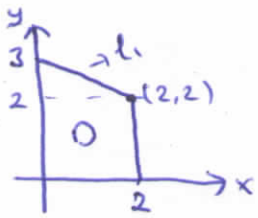
* $z = x+y$ yüzeyinin altında $R = [0,1] \times [0,2]$ dikdörtgeni üstünde bulunan cismin hacmi?

$$V = \iint_R (x+y)\, dx\, dy = \int_0^1 \int_0^2 (x+y)\, dy\, dx = \int_0^1 xy + \frac{y^2}{2} \Big|_0^2 dx = \int_0^1 (2x+2)\, dx$$

$$= x^2 + 2x \Big|_0^1 = \underline{\underline{3}}$$

---

* $f(x,y) = 2x - y$ fonksiyonunu köşeleri $(0,0), (2,0), (2,2), (0,3)$ koordinatları üzerinde olan yamuk üzerinde integre ediniz.



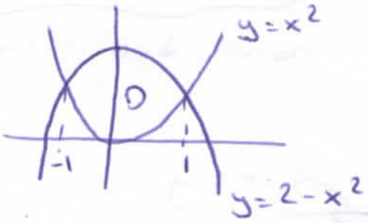
l_1 doğrusu: $(0,3), (2,2)$

$$\frac{x - x_1}{y - y_1} = \frac{x - x_2}{y_2 - y_2} \Rightarrow \frac{x - 0}{y - 3} = \frac{0 - 2}{3 - 2} \Rightarrow x + 2y = 6$$

$$\boxed{y = 3 - \frac{x}{2}}$$

$$I = \iint_O (2x - y)\, dx\, dy = \int_0^2 \int_0^{3 - x/2} (2x - y)\, dy\, dx = \underline{\underline{3}}$$

(*) $z=1+2xy$ nin altında ve $y=x^2$, $y=2-x^2$ parabolleri ile sınırlı bölgenin üstünde bulunan T cisminin hacmi?



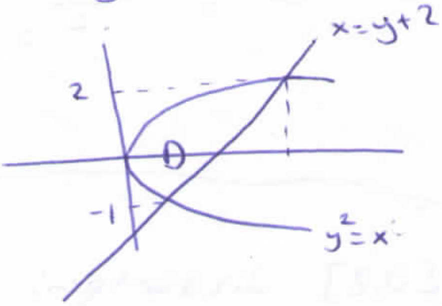
$$y=x^2 \ , \ y=2-x^2 \Rightarrow x^2=2-x^2 \Rightarrow x=\mp 1$$

$$V=\iint_D (1+2xy)\,dx\,dy=\int_{-1}^{1}\int_{x^2}^{2-x^2}(1+2xy)\,dy\,dx$$

$$=\int_{-1}^{1} y+xy^2 \Big|_{x^2}^{2-x^2} dx$$

$$=\int_{-1}^{1}\left(2-x^2+x(2-x^2)^2-x^2-x\cdot x^4\right)dx=\frac{8}{3}$$

(*) $x=y^2$, $x=y+2$ ile sınırlı bölgenin alanı?



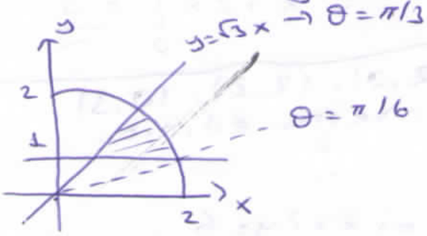
$$A=\iint_D dx\,dy=\int_{-1}^{2}\int_{y^2}^{y+2}dx\,dy=\int_{-1}^{2}(y+2-y^2)\,dy$$

$$=\frac{y^2}{2}+2y-\frac{y^3}{3}\Big|_{-1}^{2}$$

$$=\frac{9}{2}$$

$x=y+2$, $y^2=x \Rightarrow y^2=y+2 \Rightarrow y=-1$, $y=2$

(*) 2 katlı kutupsal integrasyon ile $y=1$ üzerinde, $y=\sqrt{3}x$ in altında kalan ve $y^2+x^2=4$ çemberi ile sınırlı alanı bulun.



$$A=\int_{\pi/6}^{\pi/3}\int_{1/\sin\theta}^{2} r\,dr\,d\theta=\int_{\pi/6}^{\pi/3}\frac{r^2}{2}\Big|_{\operatorname{Cosec}\theta}^{2}d\theta$$

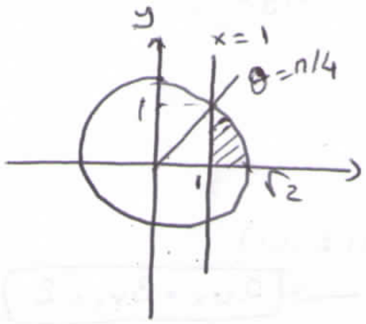
$y=1 \Rightarrow r\sin\theta=1$

$r=\frac{1}{\sin\theta}$

$$=\int_{\pi/6}^{\pi/3}\frac{1}{2}\left[4-\operatorname{Cosec}^2\theta\right]d\theta$$

$$=\frac{1}{2}\left(4\theta+\operatorname{Cot}\theta\right)\Big|_{\pi/6}^{\pi/3}=\frac{\pi-\sqrt{3}}{6}$$

⊛ $D: \left\{ \begin{array}{l} x^2+y^2 \le 2 \\ x \ge 1 \\ y \ge 0 \end{array} \right\} \Rightarrow I = \iint_D x \cdot dA$ integralinin sınırlarını kutupsal dönüşüm ile yazınız.

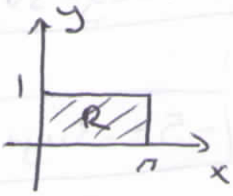


$$\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \\ dx\,dy = r\,dr\,d\theta \end{array}\right\} \begin{array}{l} x=1 \Rightarrow r\cos\theta = 1 \to \boxed{r=\dfrac{1}{\cos\theta}} \\ x^2+y^2=2 \Rightarrow r^2=2 \to \boxed{r=\sqrt{2}} \end{array}$$

$$I = \int_0^{\pi/4} \int_{1/\cos\theta}^{\sqrt{2}} r\cos\theta \cdot r \cdot dr\,d\theta$$

⊛ $f(x,y) = x\cos xy$ fonksiyonunun $R: \left\{ \begin{array}{l} 0 \le x \le \pi \\ 0 \le y \le 1 \end{array} \right\}$ bölgesindeki ortalama değerini bulunuz.

$$\bar{f} = \frac{1}{R\text{'nin Alanı}} \cdot \iint_R f(x,y)\,dA$$



R'nin Alanı $= \pi$

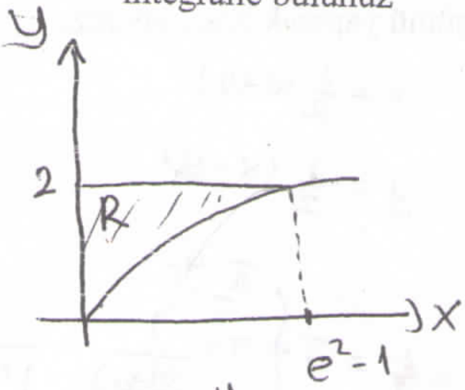
$$\iint_R f(x,y)\,dA = \int_0^{\pi}\int_0^1 x\cos xy\,dy\,dx$$

$$= \int_0^{\pi} \sin xy \Big|_0^1 dx = \int_0^{\pi} \sin x\,dx = (-\cos x)\Big|_0^{\pi} = -(-1-1) = 2$$

$$\bar{f} = \frac{1}{\pi} \cdot 2 = \frac{2}{\pi}$$

2016- yaz okulu

S.3-a) $y=\ln(1+x)$ eğrisi ve $x=0,\ y=2$ doğruları ile sınırlanmış bölgenin alanını **iki katlı** integralle bulunuz



$\ln(1+x)=2 \Rightarrow x=e^2-1$

$R=\{(x,y): 0\le x\le e^2-1,\ \ln(1+x)\le y\le 2\}$

$$A=\int_0^2\int_0^{e^y-1} dx\,dy$$

$$=\int_0^2 (e^y-1)\,dy$$

$$=(e^y-y)\Big|_0^2$$

$$=e^2-3 \text{ br}^2$$

$$A=\int_0^{e^2-1}\int_{\ln(1+x)}^{2} dy\,dx$$

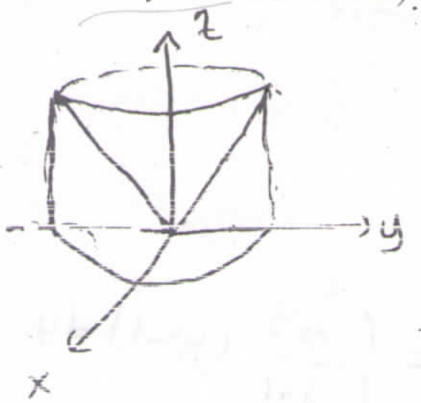
$$=\int_0^{e^2-1}[2-\ln(1+x)]\,dx$$

$$=\Big[2x-\big((1+x)\ln(1+x)-x\big)\Big|_0^{e^2-1}$$

$$=e^2-3 \text{ br}^2$$

$\left|\begin{array}{l} u=\ln(1+x) \\ dv=dx \end{array}\right|$

-b) Üstten $z=\sqrt{x^2+y^2}$ konisi, alttan $z=0$ düzlemi ve yandan $x^2+y^2=1$ silindiri ile sınırlanan cismin hacmini bulunuz. (Yol Gösterme: R bölgesi, xy-düzleminde $x^2+y^2=1$ dairesidir.)



$\left.\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \end{array}\right\} \Rightarrow$ $x^2+y^2=1$, $0\le r\le 1$, $0\le\theta\le 2\pi$

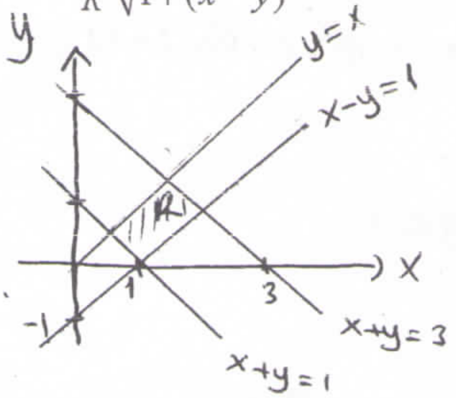
$$V=\iint_R f(x,y)\,dA=\int_0^{2\pi}\int_0^1 \sqrt{r^2}\,r\,dr\,d\theta$$

$$=\int_0^{2\pi}\frac{1}{3}r^3\Big|_0^1 d\theta=\frac{2\pi}{3}\text{ br}^3$$

2016- yaz okulu

S.4-a) Eğer R, $x+y=1$, $x+y=3$, $x-y=0$ ve $x-y=1$ doğruları ile sınırlanmış bölge ise,

$\iint\limits_R \frac{x^2-y^2}{\sqrt{1+(x-y)^2}}\,dA$ integralini $u=x-y, v=x+y$ dönüşümü yaparak hesaplayınız.



$x-y=0 \to u=0$

$x-y=1 \to u=1$

$x+y=1 \to v=1$

$x+y=3 \to v=3$

$x=\frac{1}{2}(u+v)$

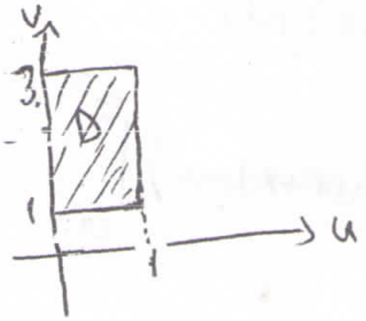
$y=\frac{1}{2}(v-u)$

I. Yol

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2} = J$

II. Yol

$J = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} = \frac{1}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = 1/2$

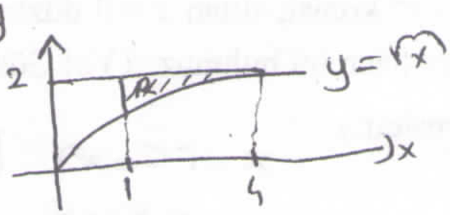


$$I = \int_0^1 \int_1^3 \frac{1}{2}\frac{uv}{\sqrt{1+u^2}}\,dv\,du = \frac{1}{2}\int_0^1 \frac{u}{\sqrt{1+u^2}} \frac{v^2}{2}\Big|_1^3 du$$

$$= 2\sqrt{1+u^2}\Big|_0^1 = 2 //$$

-b) $\int\limits_1^4 \int\limits_{\sqrt{x}}^2 \frac{e^y\,dy}{y+1}dx$ integralini hesaplayınız

$R: \begin{cases} y=\sqrt{x}, & y=2 \\ x=1, & x=4 \end{cases}$



$R = \{(x,y) : 1 \le x \le 4,\ \sqrt{x} \le y \le 2\}$

$R' = \{(x,y) : 1 \le y \le 2,\ 1 \le x \le y^2\}$

$$I = \int_1^4 \int_{\sqrt{x}}^2 \frac{e^y}{y+1}\,dy\,dx = \int_1^2 \int_1^{y^2} \frac{e^y}{y+1}\,dx\,dy = \int_1^2 \frac{e^y}{y+1}(y^2-1)\,dy$$

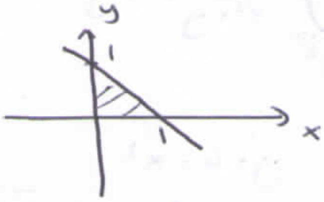
$$= \int_1^2 (y-1)e^y\,dy = (y-2)e^y\Big|_1^2 = e$$

$\begin{bmatrix} u=y-1 \\ dv=e^y dy \end{bmatrix} \to \begin{matrix} du=dy \\ v=e^y \end{matrix} \Rightarrow uv-\int v\,du = (y-1)e^y - \underbrace{\int e^y dy}_{e^y}$

$= (y-2)e^y$

(*) $I = \int_0^1 \int_0^{1-y} e^{\frac{x-y}{x+y}} \, dx\, dy = ?$ $\begin{matrix} u = x-y \\ v = x+y \end{matrix}$ dönüşümü yapılmalı

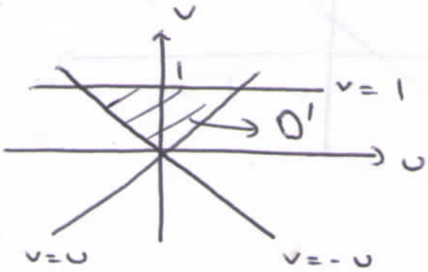
$D: \; y=1 \quad y=0 \quad x=0 \quad x=1-y \quad \rightarrow$



Bölgeyi sınırlayan: $x = 1-y, \; x=0, \; y=0$

D	$\longrightarrow$	D'
$x+y=1$	$\longrightarrow$	$\boxed{v=1}$
$x=0$	$\longrightarrow$	$\boxed{u+v=0}$
$y=0$	$\longrightarrow$	$\boxed{u-v=0}$

$\left.\begin{matrix} u = x-y \\ v = x+y \end{matrix}\right\} \begin{matrix} u+v = 2x \\ u-v = -2y \end{matrix}$



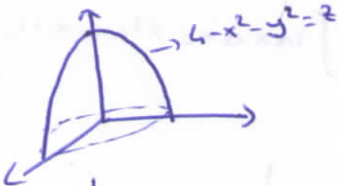
$$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$$

$$I = \iint_{D'} e^{u/v} \cdot \frac{1}{2}\, du\, dv = \int_0^1 \int_{-v}^{v} e^{u/v} \frac{1}{2}\, du\, dv = \frac{1}{2}\int_0 \left(\frac{e^{u/v}}{\frac{1}{v}} \Big|_{-v}^{v} \right) dv$$

$$= \frac{1}{2}\int_0^1 v.(e - e^{-1})\, dv = \frac{1}{2}\left(e - \frac{1}{e}\right) \cdot \frac{v^2}{2}\Big|_0^1$$

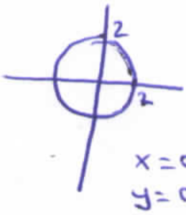
$$= \frac{1}{4}\left(e - \frac{1}{e}\right)$$

(*) $z = 4 - x^2 - y^2$ paraboloidi ve $z \geq 0$ ın sınırladığı bölgenin hacmini 2 katlı integral ile hesaplayın.



$\downarrow$

$D: \; x^2 + y^2 = 4$



$x = r\cos\theta$
$y = r\sin\theta$
$|J| = r$
$dx\,dy = r\,dr\,d\theta$

$$V = \iint_D (4 - x^2 - y^2)\, dx\, dy = \iint_D (4 - r^2)\, r\, dr\, d\theta$$

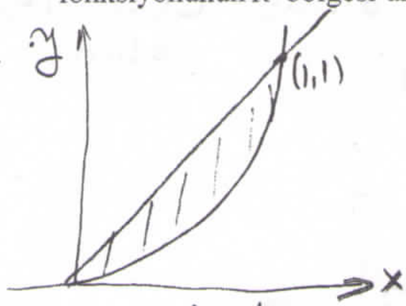
$$= \int_0^{2\pi} \int_0^2 (4r - r^3)\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(2r^2 - \frac{r^4}{4} \Big|_0^2 \right) d\theta$$

$$= \underline{\underline{8\pi}}$$

2016 - büt

4-a) R bölgesi, xy-düzleminde, $y=x$, $y=x^3$, $x\geq 0, y\geq 0$ ile sınırlı bir bölge olsun. $f(x,y)=e^{2x^2-x^4}$ fonksiyonunun R bölgesi üzerinde ortalama değerini hesaplayınız. (R bölgesini çiziniz) *(12 Puan)*



$$\text{Ort. Değer} = \frac{1}{R\text{nin Alanı}} \iint_R f\,dA$$

$$\iint_R dA = \int_0^1\int_{x^3}^{x} dy\,dx = \int_0^1\int_y^{\sqrt[3]{y}} dx\,dy = 1/4 \quad 5$$

$$\iint_R e^{2x^2-x^4}\,dA = \int_0^1\int_{x^3}^{x} e^{2x^2-x^4}\,dy\,dx = \int_0^1 e^{2x^2-x^4}(x-x^3)\,dx = \int_0^1 \frac{1}{4}e^u\,du = \frac{1}{4}(e-1) \quad 5$$

$$\text{Ortalama Değer} = \frac{1}{\cancel{1/4}}\cdot\cancel{\tfrac{1}{4}}(e-1) = e-1 \quad 2$$

4-b) R bölgesi, $2x^2+6xy+5y^2=1$ ile sınırlı bir bölge ise, $x=2u-v$ ve $y=-u+v$ dönüşümünü yaparak, $\iint_R \sqrt{2x^2+6xy+5y^2}\,dxdy$ integralini hesaplayınız. *(13 Puan)*

$$2x^2+6xy+5y^2=1$$
$$2(2u-v)^2+6(2u-v)(-u+v)+5(-u+v)^2=1$$
$$2(4u^2-4uv+v^2)+6(-2u^2+2uv+uv-v^2)+5(u^2-2uv+v^2)=1$$
$$8u^2-\cancel{8uv}+2v^2-12u^2+\cancel{18uv}-6v^2+5u^2-\cancel{10uv}+5v^2=1$$

R', $u^2+v^2=1$, R eliptik bölge yarıçapı 1 olan daireye dönüştü 2

$$J=\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}=1 \qquad dxdy=|J|\,dudv \Rightarrow dxdy=dudv \quad 2$$

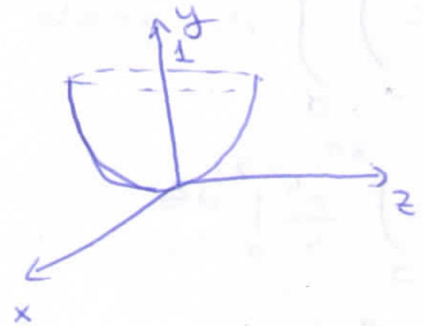
$$\underbrace{\iint_R \sqrt{2x^2+6xy+5y^2}\,dxdy = \iint_{R'}\sqrt{u^2+v^2}\,dudv}_{4} = \underbrace{\int_{\theta=0}^{2\pi}\int_{r=0}^{1} r\cdot r\,dr\,d\theta}_{4}$$

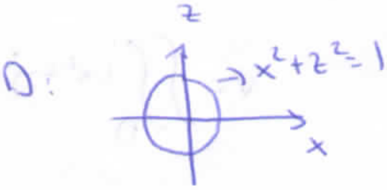
$u=r\cos\theta$
$v=r\sin\theta$

$$=\underbrace{\frac{2\pi}{3}}_{1}$$

10

(*) $y = x^2 + z^2$ paraboloidinin $y=1$ ile kesilmesi ile oluşan cismin hacmi?





$x = r\cos\theta$

$z = r\sin\theta$

$dx\,dz = r\,dr\,d\theta$

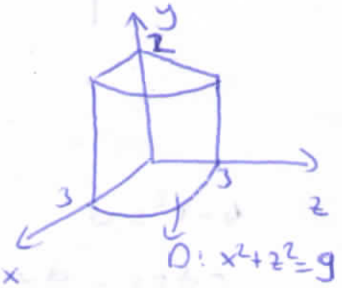
$x^2+z^2 = r^2$

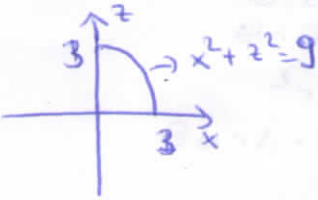
$$V = \iint_D (1-(x^2+z^2))\,dx\,dz$$

$$= \int_0^{2\pi}\int_0^1 (1-r^2)\,r\,dr\,d\theta = \int_0^{2\pi} \left(\frac{r^2}{2}-\frac{r^4}{4}\right)\Big|_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{1}{4}\,d\theta = \frac{\pi}{2}$$

(*) $x^2+z^2=9$, $y=2$, $y=0$, $z=0$, $x=0$ arasındaki cismin hacmi?





$x = r\cos\theta$

$z = r\sin\theta$

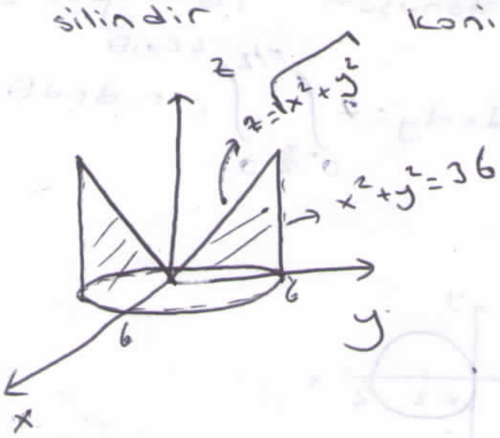
$dx\,dz = r\,dr\,d\theta$

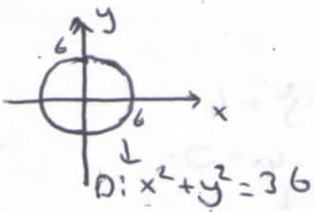
$$V = \iint_D 2\,dx\,dz = \int_0^{\pi/2}\int_0^3 2r\,dr\,d\theta$$

$$= \int_0^{\pi/2} r^2\Big|_0^3 d\theta = \int_0^{\pi/2} 9\,d\theta = \frac{9\pi}{2}$$

(*) $\underbrace{x^2+y^2=36}_{\text{silindir}}$, $\underbrace{z^2=x^2+y^2}_{\text{koni}}$, $z \geq 0$ arasında kalan hacim?

$$V = \iint_D \sqrt{x^2+y^2}\,dx\,dy = \int_0^{2\pi}\int_0^6 \sqrt{r^2}\cdot r\,dr\,d\theta$$





$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2=r^2$

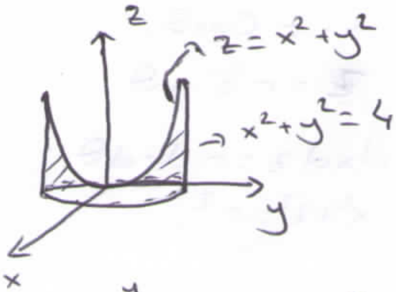
$dx\,dy = r\,dr\,d\theta$

$$= \int_0^{2\pi} \frac{r^3}{3}\Big|_0^6 d\theta$$

$$= \frac{6^3}{3}\cdot 2\pi$$

$$= 144\pi$$

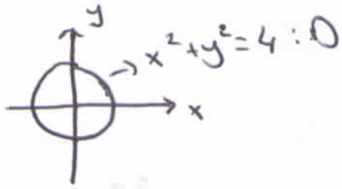
(*) xy düzleminin üstünde, $z = x^2+y^2$ paraboloidi ve $x^2+y^2=4$ silindiri arasındaki cismin hacmi?



$$V = \iint_D (x^2+y^2)\,dA = \int_0^{2\pi}\int_0^2 r^2 \cdot r\,dr\,d\theta$$

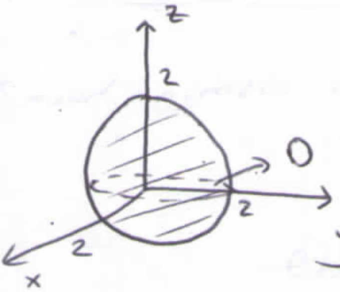
$$= \int_0^{2\pi} \frac{r^4}{4}\Big|_0^2 d\theta$$

$$= \underline{\underline{8\pi}}$$

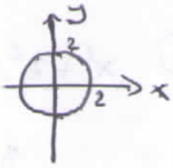


$$\left\{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \\ x^2+y^2=r^2 \\ dx\,dy = r\,dr\,d\theta \end{array}\right\}$$

(*) $x^2+y^2+z^2=4$ küresinin hacmini bulunuz.



izdüşüm bölgesi $D : x^2+y^2=4$



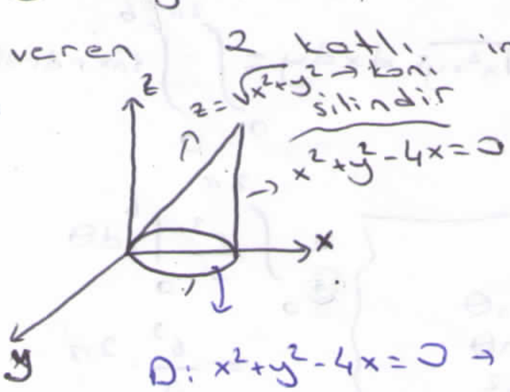
$x = r\cos\theta$
$y = r\sin\theta$
$x^2+y^2=r^2$
$dx\,dy = r\,dr\,d\theta$

$$\frac{V}{2} = \iint_D \sqrt{4-x^2-y^2}\,dx\,dy = \int_0^{2\pi}\int_0^2 \sqrt{4-r^2}\cdot r\,dr\,d\theta$$

$4-r^2=u$
$r\,dr = -\frac{du}{2}$
$r=2 \to u=0$
$r=0 \to u=4$

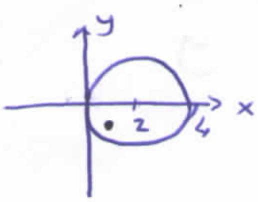
$$= \int_0^{2\pi}\int_0^4 -\frac{\sqrt{u}}{2}\,du\,d\theta = \int_0^{2\pi} \frac{8}{3}\,d\theta = \frac{16\pi}{3} \quad \Rightarrow V = \frac{32}{3}\pi$$

(*) $x^2+y^2-4x=0$, $z \geq 0$, $x^2+y^2=z^2$ arasında kalan hacimi veren 2 katlı integrali kutupsal dönüşüm ile yazınız.



$$V = \iint_D \sqrt{x^2+y^2}\,dx\,dy = \int_{-\pi/2}^{\pi/2}\int_0^{4\cos\theta} r \cdot r\,dr\,d\theta$$

$D: x^2+y^2-4x=0 \to (x-2)^2+y^2=4$



$x = r\cos\theta$
$y = r\sin\theta$
$dx\,dy = r\,dr\,d\theta$

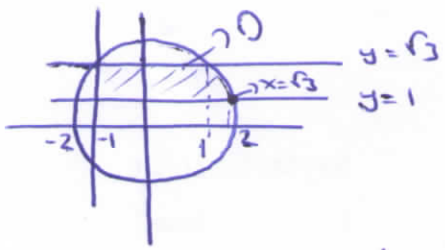
$x^2+y^2-4x=0$
$\downarrow$
$r^2 = 4r\cos\theta$
$r=0 \quad r=4\cos\theta$

$$\int_1^{\sqrt{3}} \int_{-1}^{\sqrt{4-y^2}} \frac{x}{y}\, dx\, dy$$

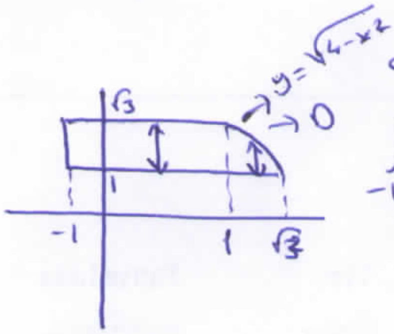
a) integrasyon sırasını değiştir

b) $\left.\begin{array}{l} x^2+y^2=u \\ y^2=v \end{array}\right\}$ dönüşümü ile çözün.

$\left.\begin{array}{ll} y=\sqrt{3} & y=1 \\ x=\sqrt{4-y^2} & x=-1 \end{array}\right\}$ D bölgesi



$\Rightarrow$

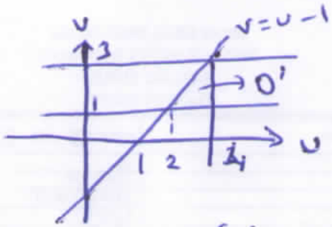


a) $$\int_{-1}^{1}\int_{1}^{\sqrt{3}} \frac{x}{y}\, dy\, dx + \int_{1}^{\sqrt{3}}\int_{1}^{\sqrt{4-x^2}} \frac{x}{y}\, dy\, dx$$

b)

D		D'
$y=\sqrt{3}$	$\rightarrow$	$v=3$
$y=1$	$\rightarrow$	$v=1$
$x^2+y^2=4$	$\rightarrow$	$u=4$
$x=-1$	$\rightarrow$	$u=v+1$

$$|J| = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{\left|\frac{\partial(u,v)}{\partial(x,y)}\right|} = \frac{1}{\begin{vmatrix} 2x & 2y \\ 0 & 2y \end{vmatrix}} = \frac{1}{4xy}$$



$$\iint_{D'} \frac{x}{y}\cdot\frac{1}{4xy}\, du\, dv = \iint_{D'} \frac{1}{4\frac{y^2}{v}}\, du\, dv = \iint_{D'} \frac{1}{4v}\, du\, dv = \frac{1}{4}\int_2^4 \int_1^{u-1} \frac{1}{v}\, dv\, du$$

$$= -\frac{1}{2}(1-3\ln\sqrt{3})$$

$\sqrt{(2{,}01)^3+(1{,}95)^2}$ sayısını 2. mertebe Taylor açılımı ile yaklaşık olarak hesaplayın.

$f(x,y)=\sqrt{x^3+y^2}$ $\quad (a,b)=(2,1)$ olsun.

$$f_x = \frac{3x^2}{2\sqrt{x^3+y^2}} \qquad f_y = \frac{2y}{2\sqrt{x^3+y^2}} \qquad f_{xx} = \frac{6x\cdot 2\sqrt{x^3+y^2} - 3x^2\cdot 2\cdot \frac{3x^2}{2\sqrt{x^3+y^2}}}{4(x^3+y^2)}$$

$$f_{yy} = \frac{\sqrt{x^3+y^2} - y\cdot\frac{2y}{2\sqrt{x^3+y^2}}}{(x^3+y^2)} \qquad f_{yx} = -\frac{y\cdot\frac{3x^2}{2\sqrt{x^3+y^2}}}{x^3+y^2}$$

$f_x(2,1)=2 \quad f_y(2,1)=\frac{1}{3} \quad f_{xx}(2,1)=\frac{2}{3} \quad f_{yx}(2,1)=-\frac{2}{9} \quad f_{yy}(2,1)=\frac{8}{27}$

$$f(x,y) = 3+2(x-2)+\frac{1}{3}(y-1)+\frac{1}{2}\left(\frac{2}{3}\cdot(x-2)^2+2\cdot\left(-\frac{2}{9}\right)\cdot(y-1)(x-2)+\frac{8}{27}(y-1)^2\right)$$

$$f(2{,}01,\ 1{,}95) \cong 3+2\cdot(0{,}01)+\frac{1}{3}\cdot(0{,}95)+\frac{1}{2}\left(\frac{2}{3}\cdot(0{,}01)^2-\frac{4}{9}(0{,}01)\cdot(0{,}95)+\frac{8}{27}(0{,}95)^2\right)$$

$$= 3{,}45$$

2016 - final

3- a) $\sin[\pi(0.01)(1.05)+\ln(1.05)]$ nin yaklaşık değerini toplam diferansiyel veya lineer yaklaşım kullanarak hesaplayınız. *(12 Puan)*

$f(x,y) = \sin(\pi xy + \ln y)$ $\quad f(0,1)=0$ $\quad h=\Delta x = 0.01$

$k = \Delta y = 0.05$

$\frac{\partial f}{\partial x} = \pi y \cos(\pi xy + \ln y)$, $\quad \frac{\partial f(0,1)}{\partial x} = \pi$

$\frac{\partial f}{\partial y} = \left(\pi x + \frac{1}{y}\right)\cos(\pi xy + \ln y)$ $\quad \frac{\partial f(0,1)}{\partial y} = 1$

$f(0{,}01 ; 1{,}05) \cong 0 + 0{,}01.\pi + 0{,}05.1 \cong 0.08141$

3-b) $f(x,y) = xy + \frac{1}{x} + \frac{8}{y}$ fonksiyonunun kritik noktalarını bulunuz ve sınıflandırınız. *(13 Puan)*

bulunuz.(13 Puan)

$f_x = y - \frac{1}{x^2} = 0 \Rightarrow y = \frac{1}{x^2}$

$f_y = x - \frac{8}{y^2} = 0 \Rightarrow x = \frac{8}{y^2}$

$\Rightarrow x = \frac{8}{\left(\frac{1}{x^4}\right)} \Rightarrow x - 8x^4 = 0$

$x=0 \quad x = 1/2$

Kritik N.

$x = 0 \notin D_f$

$P(1/2, 4)$ için

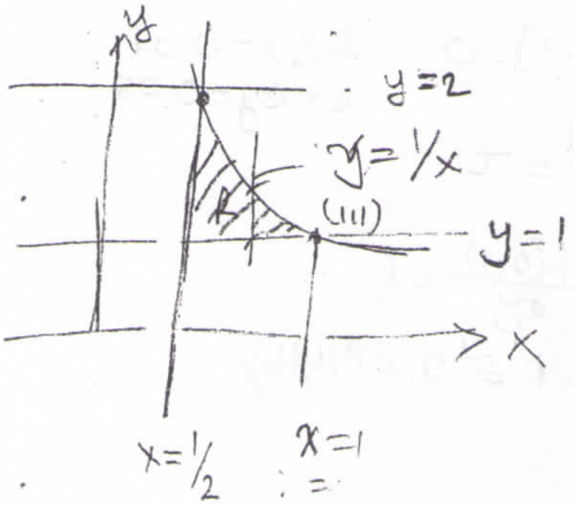
$f_{xx} = \frac{2}{x^3}$ $\quad f_{yy} = \frac{16}{y^3}$ $\quad f_{xy} = 1$

$f_{xx} \cdot f_{yy} - f_{xy}^2 = \frac{1}{\left(\frac{1}{2}\right)^3} \cdot \frac{16}{4^3} - 1 = 3 > 0$

$f_{xx} = 16 > 0$ $\quad \Rightarrow P(1/2, 4)$ lokal min noktadır.

2016-final

4-a) $\int_1^2 \int_{1/2}^{1/y} e^{\ln x - x}\,dxdy$ integralini, integral sırasını değiştirerek hesaplayınız. (12 Puan)



$$I = \int_{x=1/2}^{1} \left(\int_{y=1}^{1/x} e^{\ln x - x}\,dy \right) dx$$

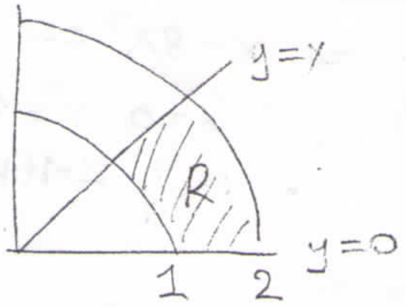
$$= \int_{1/2}^{1} e^{\ln x - x}\, y \Big|_1^{1/x} dx$$

$$= \int_{1/2}^{1} \left(\frac{1}{x} - 1\right) e^{\ln x - x}\,dx = e^{\ln x - x}\Big|_{1/2}^{1} = 1.e^{-1} - \frac{1}{2}e^{-1/2}$$

$$= \frac{1}{e} - \frac{1}{2\sqrt{e}}$$

4-b) R bölgesi $\{(x,y) \in \mathbb{R}^2 \,|\, 1 \le x^2 + y^2 \le 4, 0 \le y \le x\}$ olmak üzere $\iint_R \arctan\left(\frac{y}{x}\right) dA$ integralini hesaplayınız.

(R bölgesinin şeklini çiziniz).(13 Puan)



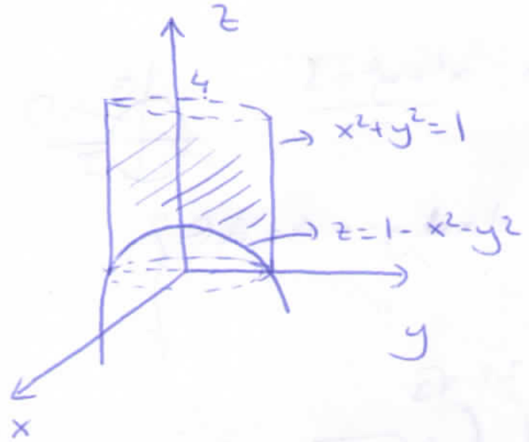
$$\iint_R \arctan\left(\frac{y}{x}\right) dx\,dy = \int_{\theta=0}^{\theta=\pi/4} \theta \, d\theta \int_{r=1}^{r=2} r\,dr$$

$$= \frac{3\pi^2}{64}$$

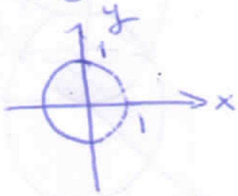
$$\left.\begin{matrix} x = r\cos\theta \\ y = r\sin\theta \end{matrix}\right\} \rightarrow r = \sqrt{x^2+y^2}$$

$\theta =$

⊛ E cismi $x^2+y^2=1$ silindirinin içinde, $z=4$ düzleminin altında, $z=1-x^2-y^2$ paraboloidinin üstündedir. Cismin hacmini bulunuz.



izdüşüm bölgesi: $x^2+y^2=1$



$D: x^2+y^2=1$

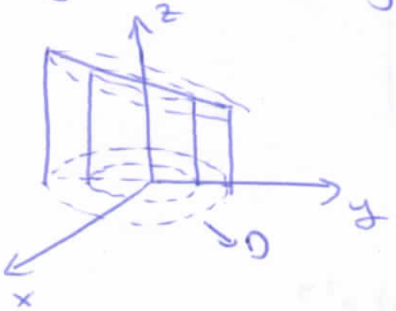
$$\left\{\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \\ dxdy=r\,dr\,d\theta \\ x^2+y^2=r^2 \end{array}\right\}$$

$$V=\iint_D (4-(1-x^2-y^2))\,dx\,dy=\iint_D (3+r^2)\,r\,dr\,d\theta$$

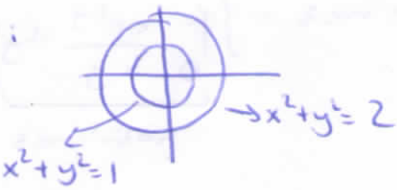
$$=\int_0^{2\pi}\int_0^1 (3r+r^3)\,dr\,d\theta=\int_0^{2\pi} \frac{3}{2}r^2+\frac{r^4}{4}\Big|_0^1 d\theta$$

$$=\int_0^{2\pi}\frac{7}{4}\,d\theta=\underline{\underline{\frac{7}{2}\pi}}$$

⊛ $5=z+y$ düzleminin altında, xy düzleminin üstünde, $x^2+y^2=2$ ve $x^2+y^2=1$ silindirleri arasında kalan cismin hacmi?



D:

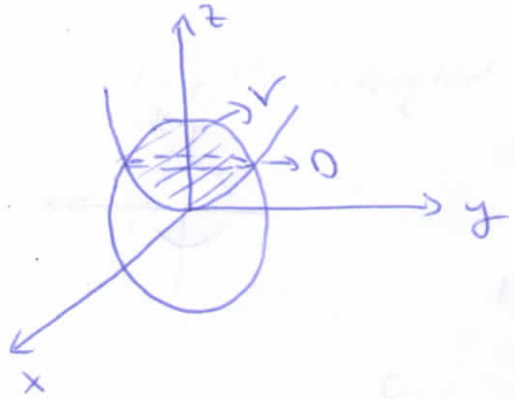


$$\left\{\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \\ dxdy=r\,dr\,d\theta \end{array}\right\}$$

$$V=\iint_D (5-y)\,dx\,dy=\int_0^{2\pi}\int_1^{\sqrt{2}}(5-r\sin\theta)\,r\,dr\,d\theta=\int_0^{2\pi}\left(\frac{5}{2}+\frac{1-2\sqrt{2}}{3}\sin\theta\right)d\theta$$

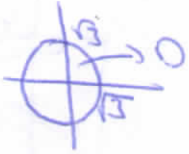
$$=\frac{5}{2}\theta-\frac{1-2\sqrt{2}}{3}\cos\theta\Big|_0^{2\pi}=\underline{\underline{5\pi}}$$

(*) $x^2+y^2+z^2 \leq 4$, $3z \geq x^2+y^2$ arasında kalan hacim?



$z^2+3z=4 \rightarrow \boxed{z=1}$
$z=-4$ X

$O: z=1 \rightarrow \underline{x^2+y^2=3}$

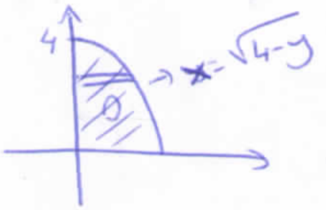


$$\left\{\begin{array}{l} x=r\cos\theta \\ y=r\sin\theta \\ x^2+y^2=r^2 \end{array}\right. \quad dx\,dy = r\,dr\,d\theta$$

$$V=\iint_D \left(\sqrt{4-x^2-y^2}-\frac{x^2+y^2}{3}\right)dx\,dy = \int_0^{2\pi}\int_0^{\sqrt{3}} \left(\sqrt{4-r^2}-\frac{r^2}{3}\right) r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[-\frac{(4-r^2)^{3/2}}{3}-\frac{r^4}{12}\right]\Bigg|_0^{\sqrt{3}} d\theta = \int_0^{2\pi}\left(-\frac{1}{3}-\frac{3}{4}+\frac{8}{3}\right)d\theta = \frac{19}{12}\cdot 2\pi = \underline{\frac{19}{6}\pi}$$

(*) D: 1. bölgede $y=4-x^2$ parabolü, $x=0$ ve $y=0$ doğruları arasında kalan bölge ise $I=\iint_D \frac{xe^{2y}}{4-y}\,dA=?$



$$I=\iint_D \frac{xe^{2y}}{4-y}\,dx\,dy \Rightarrow$$

(x'e göre düzgün bölge alınırsa integral çözülemez : $\underbrace{\iint_D \frac{xe^{2y}}{4-y}\,dy\,dx}_{\text{çözülemez}}$)

$$I=\int_0^4\int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\,dx\,dy = \int_0^4 \frac{x^2 e^{2y}}{2(4-y)}\Bigg|_0^{4-y} dy = \int_0^4 \frac{(4-y)e^{2y}}{2(4-y)}\,dy$$

$$= \frac{e^{2y}}{4}\Bigg|_0^4 = \frac{e^8-1}{4}$$

YTÜ - **Final** Sınav Soru ve Cevap Kağıdı			NOT TABLOSU				
			1. S	2. S	3. S	4. S	$\sum$
Adı Soyadı							
Öğrenci Numarası		Grup No					
Bölümü					Sınav Tarihi	05.06.2017	
Dersin Adı	MAT1072 MATEMATİK II	Sınav Süresi	90dk	Sınav Yeri			
Dersi veren Öğretim Üyesinin Adı Soyadı				İmza			

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

[redacted]

S. 1-a) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$ serisinin **toplamını** bulunuz. (12 P)

$$S_n = \sum_{k=1}^{n}\left(\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}\right)$$

$$= \left(1-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}\right)+\dots+\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$$

$$= 1-\frac{1}{\sqrt{n+1}}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right) = \lim_{n\to\infty}\left(1-\frac{1}{\sqrt{n+1}}\right) = 1 \ //$$

S. 1-b) Diferanşiyel yaklaşımı kullanarak, $\sqrt{(2.06)^2+5(0.97)^4}$ değerini yaklaşık olarak hesaplayınız. (13 P)

$f(x,y) = \sqrt{x^2+5y^4}$

$a=2, \quad \Delta x = 0.06$

$b=1, \quad \Delta y = -0.03$

$f(2,1) = \sqrt{4+5} = 3$

$f_x = \frac{x}{\sqrt{x^2+5y^4}}\Big|_{(2,1)} = \frac{2}{3}$

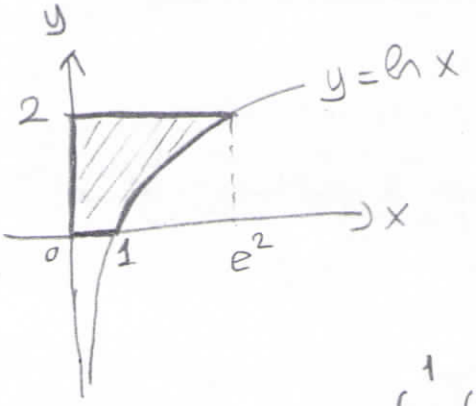
$f_y = \frac{10y^3}{\sqrt{x^2+5y^4}}\Big|_{(2,1)} = \frac{10}{3}$

$$f(a+\Delta x, b+\Delta y) \approx f(a,b) + f_x(a,b)\,\Delta x + f_y(a,b)\,\Delta y$$

$$\sqrt{(2.06)^2+5(0.97)^4} \approx 3 + \frac{2}{3}(0.06) + \frac{10}{3}(-0.03) = 2.94 \ //$$

2017 final

S. 4-a) $y=\ln x$ eğrisi, $y=0$, $y=2$ doğruları ve y-ekseni ile sınırlanmış bölgenin **alanını,** *iki katlı integral* ile hesaplayınız. (13 P)



1. YOL : $A=\int_0^2\int_0^{e^y} dx\,dy$

$A=\int_0^2 e^y dy = (e^2-1)\ \text{br}^2$ //

2. YOL : $A=\int_0^1\int_0^2 dy\,dx+\int_1^{e^2}\int_{\ln x}^2 dy\,dx = 2+\int_1^{e^2}(2-\ln x)\,dx$

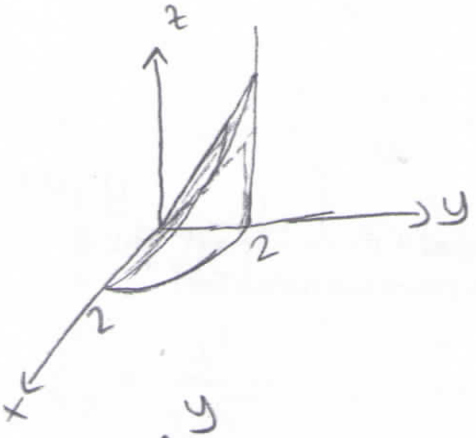
$A=2+\left[x(2-\ln x)+x\right]\Big|_1^{e^2}$

$A=2+\left[e^2(2-2)+e^2-2-1\right]=e^2-1$ //

$\int(2-\ln x)\,dx = x(2-\ln x)+\int dx$
$= x(2-\ln x)+x+C$

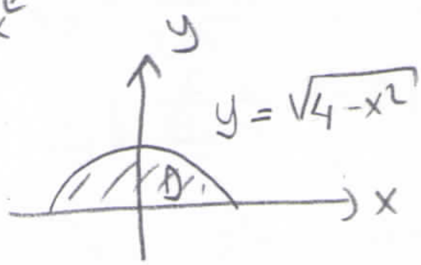
$2-\ln x=u$; $dv=dx$
$-\frac{1}{x}dx=du$; $v=x$

S. 4-b) $z=0$ ve $z=y$ düzlemleri ve $x^2+y^2=4$ silindiri ile oluşturulmuş cismin, xy-düzleminin üstünde kalan kısmının **hacmini** *iki katlı integral* ile bulunuz. (12 P)



$V=\iint_D y\,dA=\int_{-2}^{2}\int_0^{\sqrt{4-x^2}} y\,dy\,dx=\int_{-2}^{2}\frac{y^2}{2}\Big|_0^{\sqrt{4-x^2}}dx$

$V=\frac{1}{2}\int_{-2}^{2}(4-x^2)\,dx=\frac{1}{2}\left(4x-\frac{x^3}{3}\right)\Big|_{-2}^{2}=\frac{16}{3}\ \text{br}^3$ //



2. YOL $V=\iint_D y\,dA=\int_0^{\pi}\int_0^2 r\sin\theta\, r\,dr\,d\theta$

$V=\int_0^{\pi}\frac{r^3}{3}\Big|_0^2\sin\theta\,d\theta=\frac{8}{3}(-\cos\theta)\Big|_0^{\pi}$

$V=\frac{8}{3}(1+1)=\frac{16}{3}\ \text{br}^3$ //

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S. 3-a) $f(x,y)=e^{y}\sin(y^{2}-x)$ ile tanımlı f fonksiyonunun $(\pi,0)$ noktasındaki **yönlü** türevi, hangi yönlerde **sıfır** olur? (11 P)

$$\nabla f = \langle f_x, f_y \rangle$$

$$\nabla f = \langle -e^{y}\cos(y^2-x),\ e^{y}\sin(y^2-x^2) + 2ye^{y}\cos(y^2-x) \rangle$$

$$\nabla f(\pi,0) = \langle 1, 0 \rangle$$

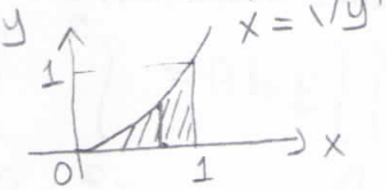
$\vec{u} = \langle a, b \rangle$ ve $a^2+b^2=1$ olsun.

$$D_{\vec{u}} f(\pi,0) = \nabla f(\pi,0)\cdot \vec{u} = \quad a=0 \Rightarrow b=\pm 1$$

$\vec{u} = \langle 0, \pm 1 \rangle$ veya $\vec{u} = \pm \vec{j}$

S. 3-b) $\int_0^1\int_{\sqrt{y}}^1 \frac{dx}{\sqrt{1+3x^3}}\,dy$ integralini hesaplayınız. (14 P)

$$\begin{cases} y=0\ ,\ y=1 \\ x=\sqrt{y}\ ,\ x=1 \end{cases}$$



$$\int_0^1\int_{\sqrt{y}}^1 \frac{dx}{\sqrt{1+3x^3}}\,dy = \int_0^1\int_0^{x^2} \frac{1}{\sqrt{1+3x^3}}\,dy\,dx = \int_0^1 \frac{1}{\sqrt{1+3x^3}}\, y\Big|_0^{x^2} dx$$

$$= \int_0^1 \frac{x^2}{\sqrt{1+3x^3}}\,dx = \frac{2}{9}\sqrt{1+3x^3}\Big|_0^1 = \frac{2}{9}(2-1) = \frac{2}{9}\ //$$

$$\left[\begin{array}{l} 1+3x^3 = t^2 \\ 9x^2dx = 2t\,dt \end{array} ;\ \int \frac{x^2}{\sqrt{1+3x^3}}\,dx = \frac{2}{9}\int \frac{t\,dt}{t} = \frac{2}{9}t + c = \frac{2}{9}\sqrt{1+3x^3} + c \right]$$