

WEEK-6&7

THREE PHASE

AC POWER SYSTEM

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## THREE PHASE AC POWER SYSTEMS

An AC generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a single phase AC generator. If the number of coils on the rotor is increased in a specific manner, the result is a polyphase AC generator, which develop more than one AC phase voltage per rotation of the rotor.

In general, three phase systems are preferred over single-phase systems for the transmission of power for many reasons:

- 1- Thinner conductors can be used to transmit the same kVA at the same voltage which reduces the amount of copper required (typically about 25% less).
- 2- The lighter lines are easier to install, and supporting structures can be less massive

and farther apart.

3- 3-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.

4- In general, most larger motors are 3-phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

The frequency generated is determined by the number of poles on the rotor and the speed with which the shaft is turned. In the US, the line frequency is 60 Hz, whereas in Europe and Türkiye the chosen standard is 50 Hz.

On ships and aircrafts, the demand level permit the use of a 400 Hz line frequency

**Note:** Higher frequencies allow for lighter and more compact transformers, motors and power supplies.

Why not more than 400 Hz?

- ↳ EMI/EMC and efficiency
- ↳ Higher freq. requires complex shielding requirements!

The three-phase system is used by almost all commercial electric generators.

This doesn't mean that single phase and two phase generating systems are obsolete.

Most small emergency generators, such as gasoline type are one-phase generating systems. The two-phase system is commonly used in servomechanism,

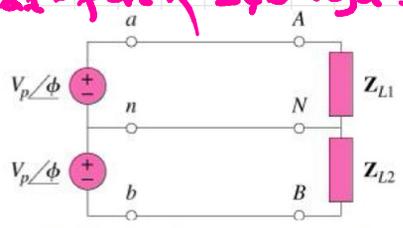
which are self-correcting control systems capable of detecting and adjusting their own operation. Servomechanisms are used in ships (and aircrafts) to keep them on course automatically or in simpler devices such as a thermostatic circuit, to regulate heat output.

Some electrical systems operate more efficiently if more than three phases are used. One such system involves the process of rectification. The greater the number of phases, the smoother is the DC output of the system.

Since all three phases are balanced and identical, the balanced 3-phase power circuit is generally shown by a 1-line diagram and analyzed on single-phase (per phase) basis. Then

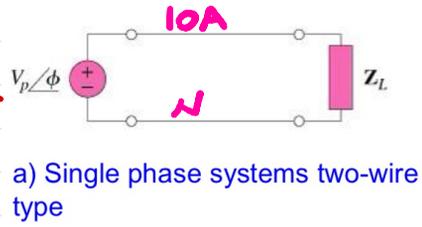
$$\text{Average 3-phase power} = 3 \times \text{Average 1-phase power}$$

$f_{a2} - n_{otr} \rightarrow 120/230$   
 $f_{a2} - f_{a2} \rightarrow 240 \text{ vya } 380$

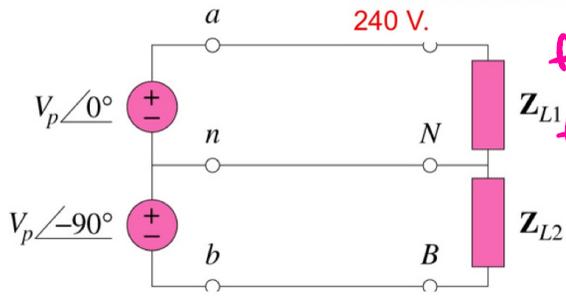


b) Single phase systems three-wire type.  
 Allows connection to both 120 V and 240 V.

Home  
 SINGLE PHASE



TWO PHASE



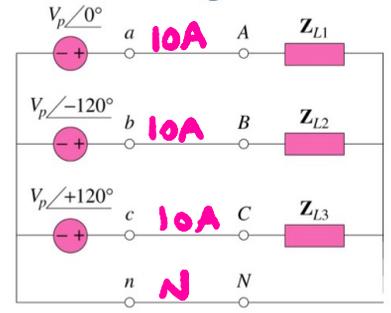
Two-phase three-wire system. The AC sources operate at different phases.

$f_{a2} \text{ notr} \rightarrow 120$   
 $f_{a2} - f_{a2} \rightarrow 208$   
 Denge prok 3-ph gire

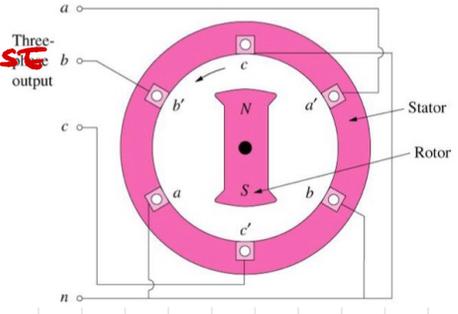
1 ph rangids

BALANCED THREE PHASE CIRCUIT

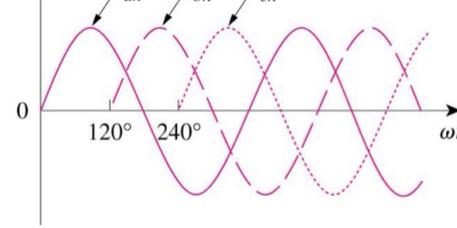
Three-phase four-wire system



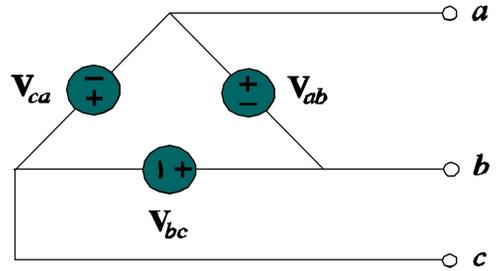
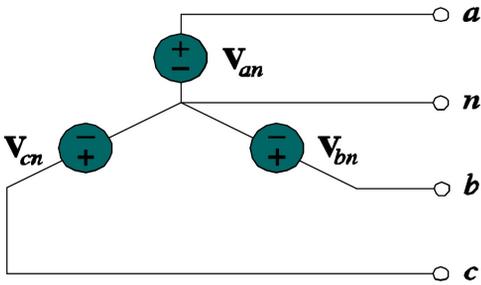
A Three-phase Generator



Voltages having 120° phase difference



# Balanced Three Phase Voltages



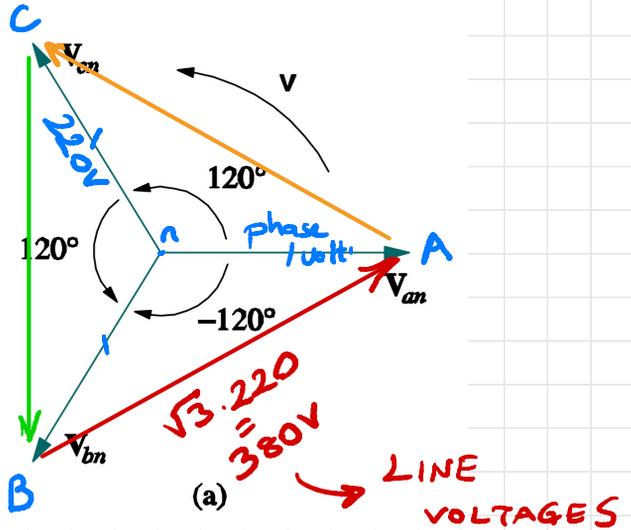
WYE (Y)

DELTA (Δ)

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ$$



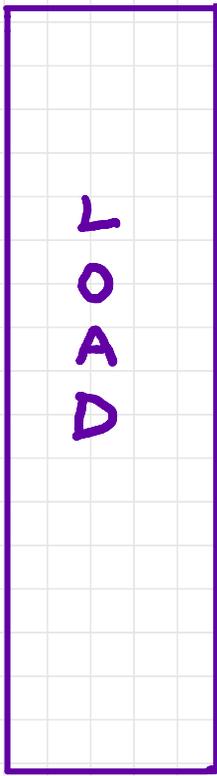
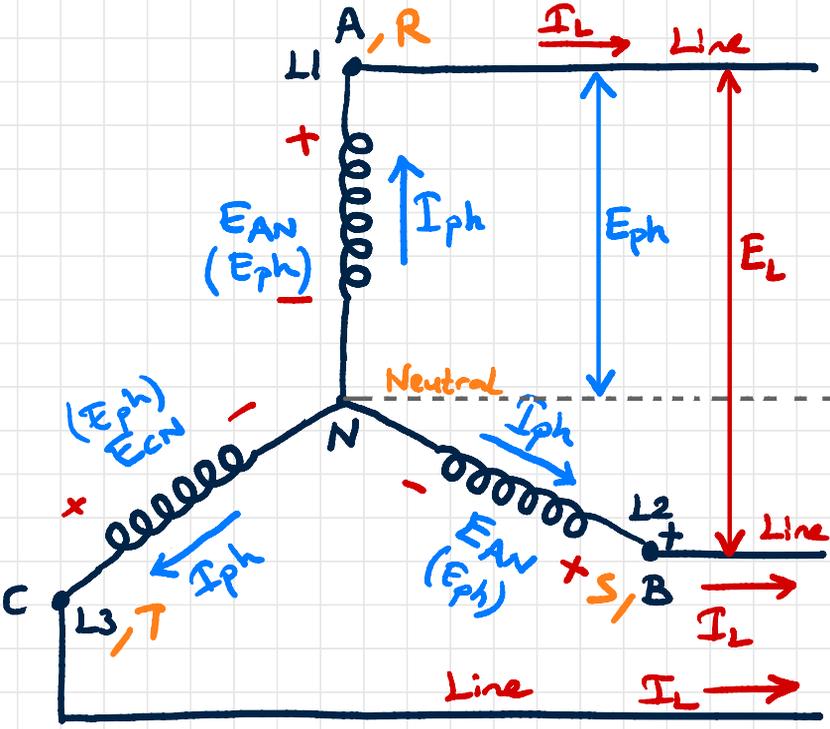
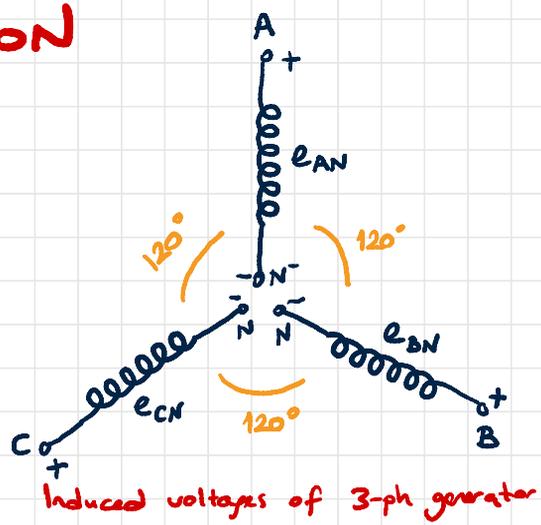
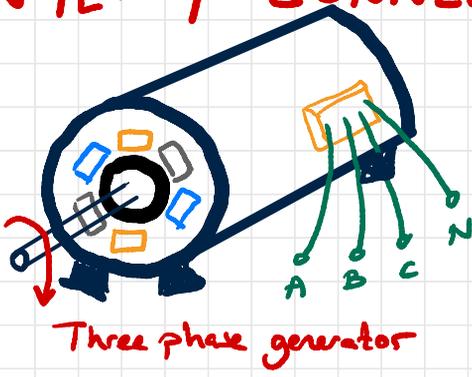
Sum of Line Voltages is ZERO!

USA L1 L2 L3 or 1 2 3

EU U V W or R S T

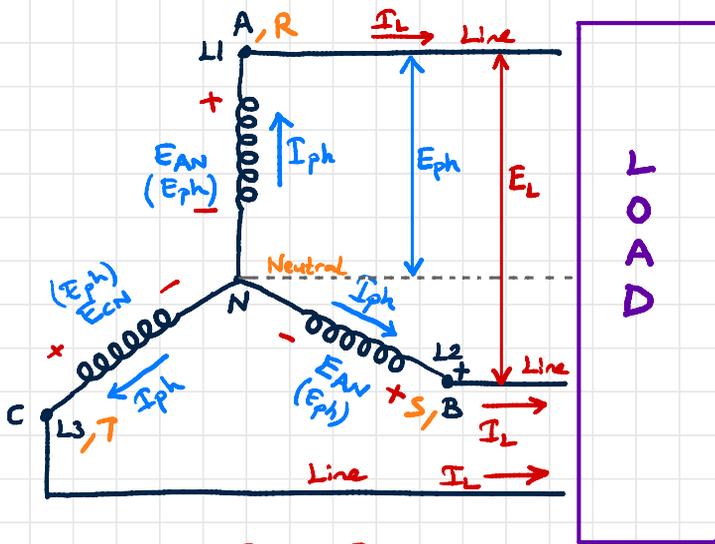
UK R Y B or A B C

# WYE-Y CONNECTION



$$I_L = I_{ph}$$

$$E_L = \sqrt{3} E_{ph}$$



$$I_L = I_{ph}$$

$$E_L = \sqrt{3} E_{ph}$$

$$P_1 = E_{ph} \cdot I_{ph} \cdot \cos \phi = P_2 = P_3$$

$$P_1 = \frac{E_L}{\sqrt{3}} \cdot I_L \cdot \cos \phi = P_2 = P_3$$

$$P_{sum} = 3 \times \frac{E_L}{\sqrt{3}} I_L \cdot \cos \phi$$



$$P_{sum} = \sqrt{3} \cdot E_L \cdot I_L \cdot \cos \phi$$

$$S_{sum} = \sqrt{3} E_L \cdot I_L$$

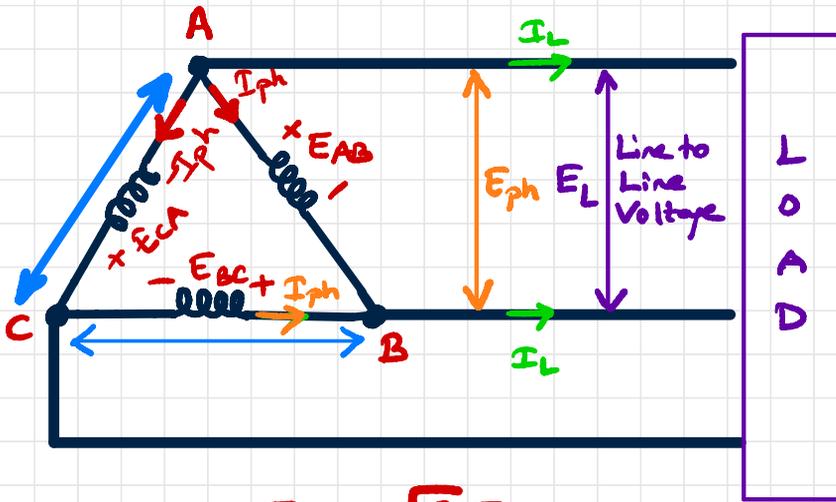
High voltage  $\rightarrow$   $\gamma$  connection

**NOTE:** phase voltage is smaller than Line Voltage



So all turnings can be isolated.

# Δ CONNECTED



$$I_L = \sqrt{3} I_{ph}$$

$$E_L = E_{ph}$$

$$P_1 = E_{ph} \cdot I_{ph} \cdot \cos \phi = P_2 = P_3$$

$$P_1 = E_L \cdot \frac{I_L}{\sqrt{3}} \cos \phi = P_2 = P_3$$

$$P_T = 3 \times E_L \cdot \frac{I_L}{\sqrt{3}} \cos \phi$$

$$P_T = \sqrt{3} \cdot E_L \cdot I_L \cdot \cos \phi \text{ (W)}$$

$$Q_T = \sqrt{3} \cdot E_L \cdot I_L \cdot \sin \phi \text{ (VAR)}$$

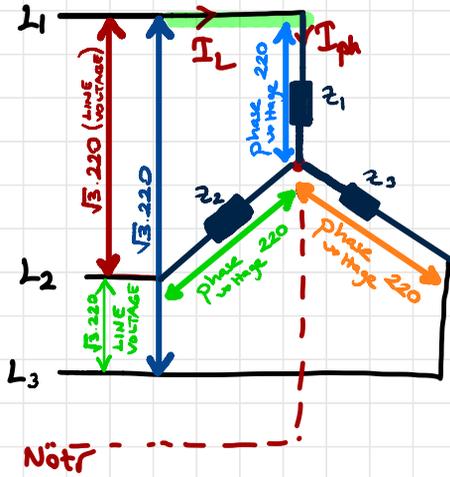
$$S_T = \sqrt{3} E_L \cdot I_L \text{ (VA)}$$

Note that the expression for 3 phase works out to be the same in both  $\gamma$  and  $\Delta$ .

In a 3-phase, 4 wire  $\gamma$  connected system the neutral current is the sum of three line currents. In a balanced  $\gamma$  system, the current in the neutral wire (if used) is always zero; because it is phasor sum of three equal line currents out of phase by  $120^\circ$  from each other. For this reason, a balanced 3-phase system performs exactly same with or without the neutral wire, except unbalanced load conditions.

In a 3-phase,  $\Delta$ -connected system, no wire exists for the return current, so the phasor sum of all line current is forced to be zero regardless of a balanced or unbalanced load. With an unbalanced 3-phase load, however, this imposed-zero on the return current produces unbalanced line-to-line voltages.

# WYE CONNECTION LOAD



$$I_L = I_{ph}$$

$$U_L = U_{ph} \cdot \sqrt{3} \Rightarrow U_{ph} = \frac{U_L}{\sqrt{3}}$$

$$P_1 = U_{ph} \cdot I_{ph} \cdot \cos \phi = P_2 = P_3$$

$$P_1 = \frac{U_L}{\sqrt{3}} I_L \cos \phi = P_2 = P_3$$

$$P_T = 3 \cdot P_1 = 3 \cdot \frac{U_L}{\sqrt{3}} \cdot I_L \cdot \cos \phi$$

$$P_T = \sqrt{3} U_L \cdot I_L \cdot \cos \phi \text{ Watts}$$

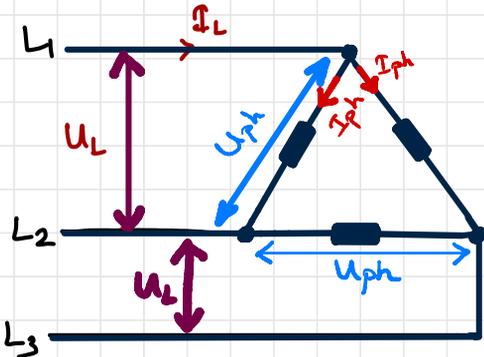
$$S_T = \sqrt{3} U_L \cdot I_L \text{ VA}$$

$$|S_T| = \sqrt{P_T^2 + Q_T^2}$$

$$S_T = P_T + j Q_T$$

$\downarrow$  VA       $\downarrow$  W       $\downarrow$  VAR

# DELTA ( $\Delta$ ) CONNECTION LOAD



$$U_L = U_{ph}$$

$$I_L = \sqrt{3} I_{ph}$$

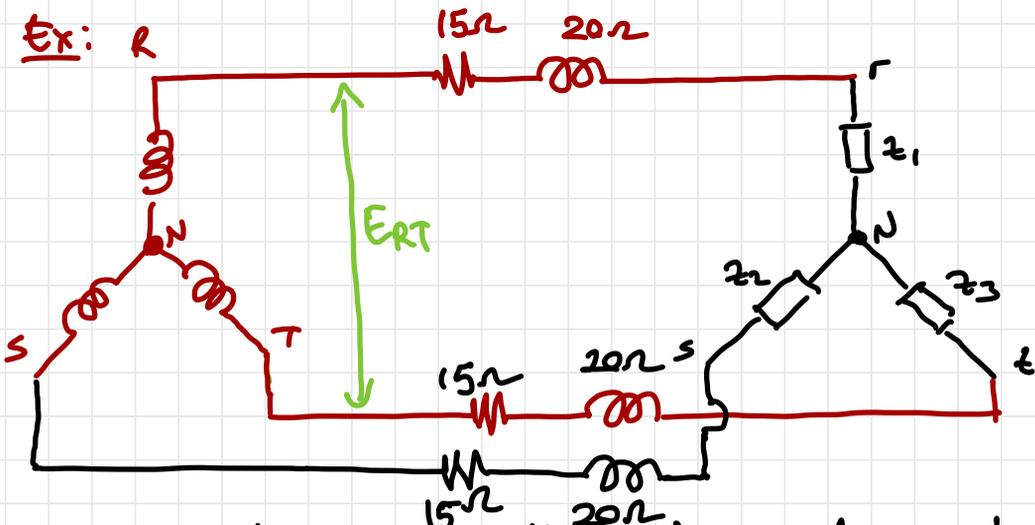
$$P_1 = U_{ph} \cdot I_{ph} \cdot \cos \phi = P_2 = P_3$$

$$P_1 = U_L \frac{I_L}{\sqrt{3}} \cdot \cos \phi = P_2 = P_3$$

$$P_T = 3 \cdot U_L \cdot \frac{I_L}{\sqrt{3}} \cdot \cos \phi$$

$$\boxed{P_T = \sqrt{3} U_L \cdot I_L \cdot \cos \phi} \text{ Watts}$$

$$S_T = \sqrt{3} U_L \cdot I_L \text{ VA}$$



This is three-wire, three-phase system, and the transmission line has an impedance of  $15\Omega + j20\Omega$ . The system delivers a total power of  $160\text{ kW}$  at  $12000\text{ V}$  to a balanced three phase load with a lagging pf of  $0.86$ .

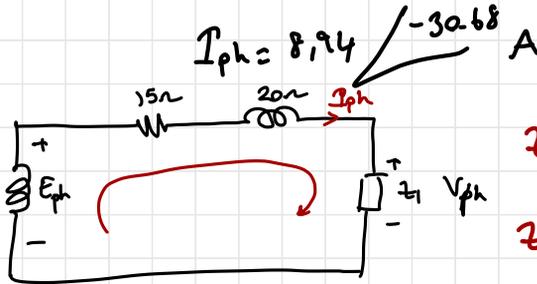
- Determine the magnitude of the line voltage of the generator. ( $E_{RT}$ )
- Find the pf of the total load applied to the generator.
- What is the efficiency of the system.

Solution:  $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{12000}{\sqrt{3}} = 6936,42V$

$P_{T(1,oad)} = 3 V_{ph} \cdot I_{ph} \cdot \cos \phi = 3 \cdot 6936,42 \cdot I_{ph} \cdot 0,86 = 160000W$

$I_{ph} = \frac{160000}{3(6936,42) \cdot 0,86} = \underline{\underline{8,94A}}$

$\theta = \cos^{-1}(0,86) \quad \theta = 30,68 \quad V_{ph} = V_{ph} \angle 0^\circ V$



$-E_{ph} + I_{ph}(15+j20) + V_{ph} = 0$

$E_{ph} = I_{ph} Z_{line} + V_{ph}$

$E_{ph} = (8,94 \angle -30,68^\circ) (25 \angle 53,13^\circ) + 6936,42 \angle 0^\circ$

$E_{ph} = 7142,98V + j85,35V$

$\underline{\underline{E_{ph}}} = 7143,5 \angle 0,68^\circ V$

$E_{RT} = \sqrt{3} \cdot E_{ph} = 12358,26V$

b)

$$\begin{aligned}
 P_T &= P_{\text{load}} + P_{\text{lines}} \\
 &= 160 \text{ kW} + 3 \cdot (I_L)^2 \cdot R_{\text{line}} \\
 &= 160 \text{ kW} + 3 (8,94)^2 \cdot 15 \Omega \\
 &= 162596,55 \text{ W}
 \end{aligned}$$

$$P_T = \sqrt{3} \cdot V_L \cdot I_L \cos \phi$$

$$\cos \phi = \frac{P_T}{\sqrt{3} V_L \cdot I_L}$$

$$\cos \phi = \frac{162596,55}{\sqrt{3} \cdot (2358) \cdot (8,94)}$$

$$P_T = \sqrt{3} \cdot V_L \cdot I_L \cos \phi$$

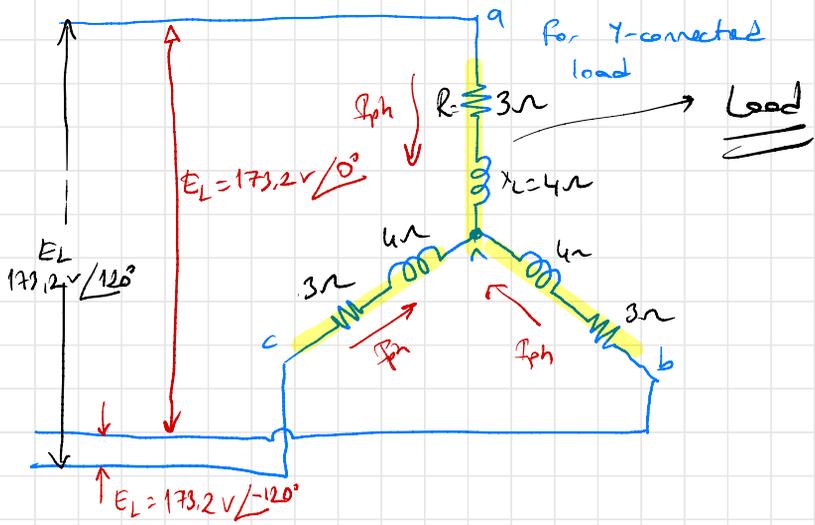
$$\cos \phi = \frac{P_T}{\sqrt{3} V_L \cdot I_L}$$

$$\cos \phi = \frac{162596,55}{\sqrt{3} \cdot (2358) \cdot (8,94)} = 0,856$$

$$f_p = 0,856 < 0,86 \text{ of load.}$$

$$c) \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{160 \text{ kW}}{160 \text{ kW} + 2596 \text{ W}} = 0,978$$

$$\eta = 97,8\%$$



- Find the average power to each phase and the total load
- Determine the reactive power to each phase and the total  $Q_T$
- Find the apparent power to each phase and the total apparent power  $S_T$
- Find the power factor of the load

Solution

a)  $P_{\phi} = V_{\phi} I_{\phi} \cos \theta$  ?

$P_T = \sqrt{3} V_L I_L \cos \theta$

$Z_1 = 3 + j4 (\Omega)$

$Z_1 = \sqrt{3^2 + 4^2} \text{ arctg } \frac{4}{3}$

$Z_1 = 5 / 53.3^\circ \Omega$

$\text{fp} = \cos(53.3) = 0.6$

$I_{\phi} = \frac{V_{\phi}}{Z} = \frac{100}{5} = 20 \text{ A}$

$V_{\phi} = \frac{V_L}{\sqrt{3}} = \frac{173.2}{\sqrt{3}} = 100 \text{ V}$

$I_{\phi} = I_L = 20 \text{ A}$

$P_T = \sqrt{3} 173.2 \cdot 20 \cdot (0.6) = 3600 \text{ W (total average power)}$

$P_{\phi} = V_{\phi} \cdot I_{\phi} \cdot \cos \theta = 100 \cdot 20 \cdot 0.6 = 1200 \text{ W}$

$3 P_{\phi} = 3600 \text{ W}$

$$b) Q_T = \sqrt{3} E_L I_L \sin 53,3$$
$$= \sqrt{3} (173,2) \cdot 20 \cdot (0,8) = 4800 \text{ VAR}$$

$$\frac{Q_T}{3} = Q_\phi = 1600 \text{ VAR (per line flow)}$$

$$Q_\phi = E_\phi I_\phi \sin 53,3 = 100 \cdot 20 \cdot 0,8 = 1600 \text{ VAR}$$

c) Apparent power.

$$S_\phi = V_\phi I_\phi = 100 \cdot 20 = 2000 \text{ VA}$$

$$S_T = 3 S_\phi = 6000 \text{ VA}$$

$$S_T = \sqrt{3} \cdot E_L \cdot I_L = \sqrt{3} (173,2) (20) = 6000 \text{ VA}$$

$$d) F_p = \frac{P_T}{S_T} = \frac{3600 \text{ W}}{6000 \text{ VA}} = 0,6 \text{ lagging}$$

**Ex 1:** Consider a balanced wye-connected load where each load impedance is  $Z_L = 50 + j50 \Omega$  and the phase voltages are 120V. Determine the total average power delivered to the load.

**Example 1:**

Consider a balanced wye-connected load where each load impedance is

$\hat{Z}_L = 50 + j50 \Omega$  and the phase voltages are 120 V. Determine the total average power delivered to the load.

**Solution:**

The line currents are

$$\hat{I}_a = \frac{120 \angle 0^\circ}{50 + j50} = 1.7 \angle -45^\circ \text{ A}$$

$$\hat{I}_b = \frac{120 \angle -120^\circ}{50 + j50} = 1.7 \angle -165^\circ \text{ A}$$

$$\hat{I}_c = \frac{120 \angle 120^\circ}{50 + j50} = 1.7 \angle 75^\circ \text{ A}$$

Hence, the average power delivered to each load is:

$$\begin{aligned} P_{av} &= 120 \times 1.7 \times \cos(45^\circ) \\ &= 144 \text{ W} \end{aligned}$$

The total average power delivered to the load is:

$$P_{avTot} = 3 \times 144 = 432 \text{ W}$$

**Ex 2:** If the line voltage of a balanced, wye-connected load is 208V and the total average power delivered to the load is 900W, determine each load if their power factors are 0,8 leading.

$$V_L = 208V$$

$$P_T = 900W$$

$$pf = 0,8 (C)$$

Line

$$P_T = \sqrt{3} V_L I_L \cos \theta$$

Phase

$$P_\phi = V_\phi I_\phi \cos \theta$$

$$P_\phi = \frac{\sqrt{3}}{3} V_L I_L \cos \theta$$

$$900 = \sqrt{3} \cdot 208 I_L \cdot 0,8$$

$$I_L = 3,12A = I$$

$$\cos \theta = 0,8$$

$$\cos^{-1} 0,8 = \theta$$

$$\theta = 37^\circ$$

$$Z = \frac{208}{3,12} \angle \theta$$

$$Z = \frac{120}{3,12} \angle -37^\circ = 38,46 \angle -37^\circ \Omega$$

**Ex 3:** A balanced  $\Delta$ -connected load is 208V, and the total average power delivered to the load is 600W. Determine the individual loads if they have a lagging power factor of 0.7.

$$V_L = 208V = V_\phi$$

$$P_T = 600W$$

$$pf = 0,7 \text{ (L)}$$

$$Z = ?$$

$$P_T = \sqrt{3} V_L I_L \cos \phi$$

$$600 = \sqrt{3} 208 I_L 0,7$$

$$I_L = \frac{600}{208\sqrt{3} \cdot 0,7} = \underline{\underline{2,38A}}$$

$$I_\phi = \frac{2,38}{\sqrt{3}} = 1,36A$$

$$Z_L = \frac{208}{1,36} = 153 \Omega$$

$$\cos \phi = 0,7 \text{ (L)}$$

$$\phi = \cos^{-1}(0,7)$$

$$\phi = 45,57^\circ$$

$$V = 208 \angle 0^\circ$$

$$I = 1,36 \angle -45,57^\circ$$

$$Z = \frac{208 \angle 0^\circ}{1,36 \angle -45,57^\circ} =$$

16

TOPLAM

VAR

200 VAR

$$Z = 153 \angle 45,57^\circ = 153(\cos 45,57^\circ) + j 153 \sin(45,57^\circ)$$

$$Z = 107 + j 109 \Omega$$

$$\text{FAR } Q_L = V_L I_L \sin(45,57^\circ) = 208 \cdot 1,36 \cdot 0,71 \text{ VAR}$$

$$Q_{\text{phase}} = V_{\phi} \cdot I_{\phi} \cdot \sin \phi$$

$$Q_T = 3Q_{\text{phase}} =$$

$$Q_T = \sqrt{3} V_L \cdot I_L \cdot \sin \phi$$

$$Q_{\text{phase}} = \frac{\sqrt{3}}{3} V_L \cdot I_L \cdot \sin \phi$$

Ex 3: A balanced  $\Delta$ -connected load is 208V, and the total average power delivered to the load is 600W. Determine the individual loads if they have a lagging power factor of 0.7.

$$V_L = 208 \quad V_\phi = 208$$

$$V_L = 208 \text{ V} = V_\phi \quad \left\{ \begin{array}{l} \frac{600}{3} = 200 \text{ W} \\ 200 \text{ W} = V_\phi \cdot I_\phi \cdot 0.7 \\ 200 \text{ W} = 208 \cdot I_\phi \cdot 0.7 \\ I_\phi = 1.36 \text{ A} \end{array} \right. \quad \text{pf}$$

$$P_T = 600 \text{ W}$$

$$\text{pf} = 0.7 \text{ (L)}$$

$$Z = ?$$

$$V_\phi = 208 \angle 0^\circ \checkmark$$

$$Z_L = \frac{208}{1.36} = 153 \Omega$$

$$\cos \phi = 0.7 \text{ (L)}$$

$$\phi = \cos^{-1}(0.7)$$

$$\phi = 45.57^\circ$$

$$Z = \frac{208 \angle 0^\circ}{1.36 \angle -45.57^\circ} =$$

16

TOPLAM  
KVAR  
200 VAR

$$Z = 153 \angle 45.57^\circ = 153(\cos 45.57^\circ) + j 153 \sin(45.57^\circ)$$

$$Z = 107 + j 109 \Omega$$

$$\text{KVAR } Q_L = V_\phi I_\phi \sin(45.57^\circ) = 208 \cdot 1.36 \cdot 0.71 \text{ VAR}$$

**Ex 4:** 3-phase power is supplied to a balanced  $\Delta$ -connected load. The line-to-line voltage is 208V, and the load consumes a total power of 15 kW at a lagging power factor of 0,6.

Determine the transmission-line currents and individual loads.

**Example 4:**

Three-phase power is supplied to a balanced,  $\Delta$ -connected load. The line-to-line voltage is 208 V, and the load consumes a total power of 15 kW at a lagging power factor of 0.6. Determine the transmission-line currents and the individual loads.

**Solution:**

From equation:

$$P_{avTot} = 3 \times \frac{I_L}{\sqrt{3}} V_L \cos \theta = \sqrt{3} V_L I_L \cos \theta \text{ then: } 15000 = \sqrt{3} V_L I_L \cos \theta$$

Therefore the transmission-line current is:

$$I_L = \frac{15000}{\sqrt{3}(208)(0.6)} = 69.39 \text{ A}$$

For delta connection,  $V_p = V_L$ , thus the magnitude of the individual load impedance is:

$$\begin{aligned} Z_L &= \frac{V_p}{I_L} \\ &= 5.19 \Omega \end{aligned}$$

Since the power factor is 0.6 lagging,

$\theta = \cos^{-1} 0.6 = 53.13^\circ$  Thus the individual loads are:

$$Z_L = 5.19 \angle 53.13^\circ = 3.12 + j4.15 \Omega$$

$$z_{\Delta} = 3 z_Y$$

$$z_{\Delta} = 3 z$$

