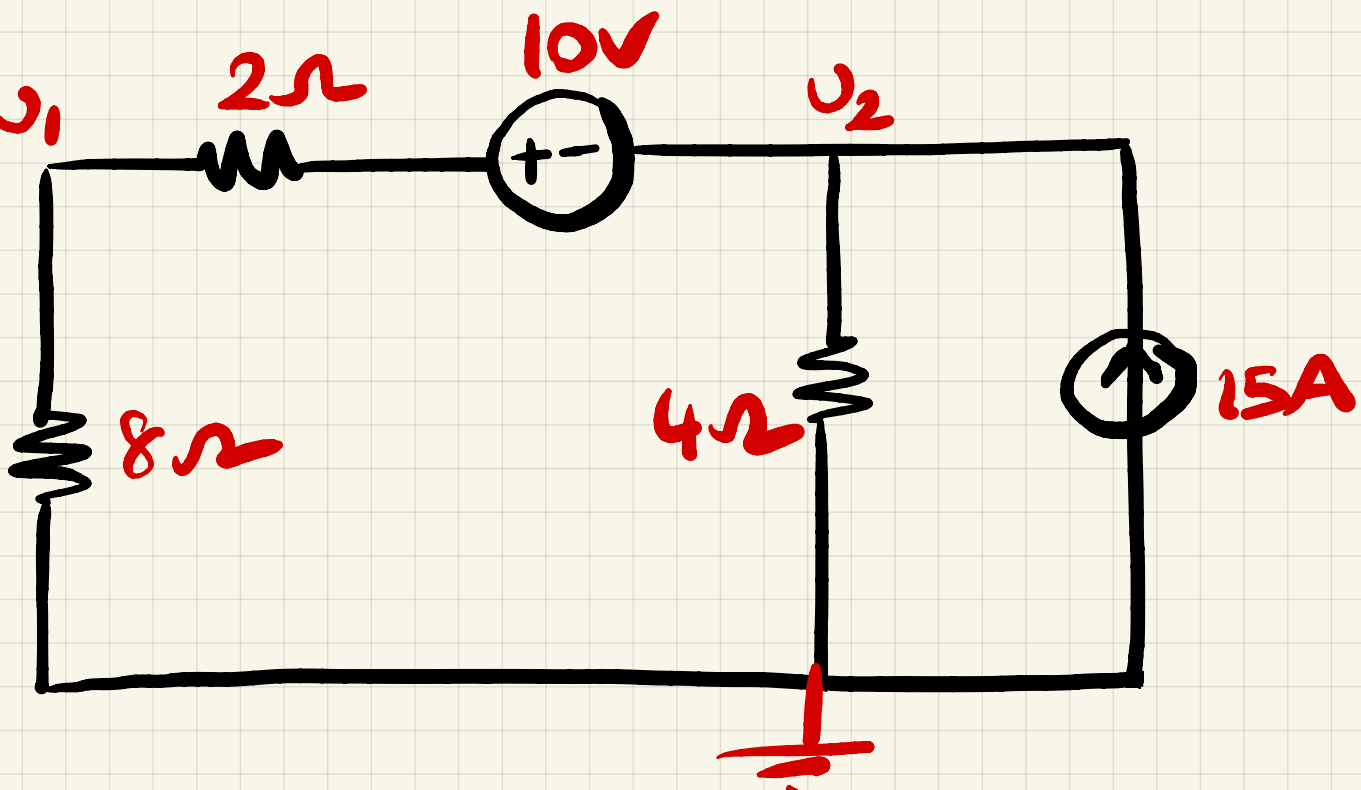
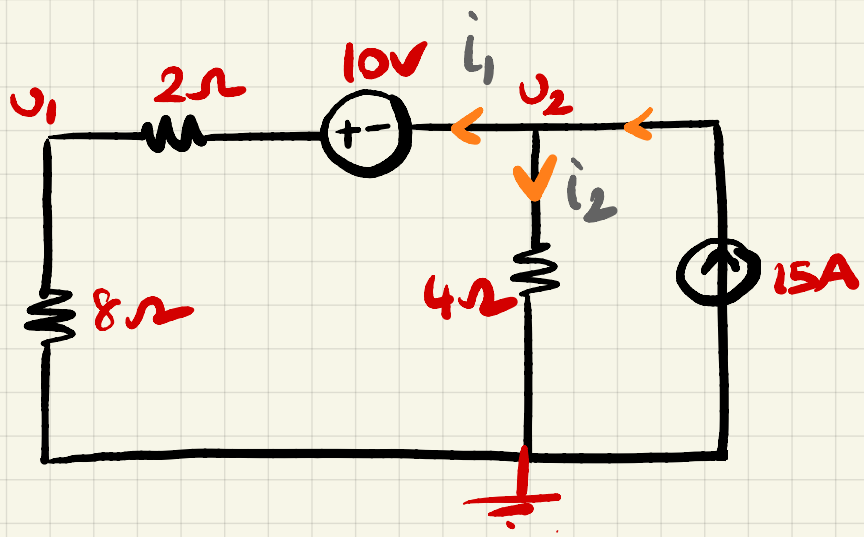
EXERCISES

Ex: v_1



Calculate v_1 and v_2.

Sol:



$$\frac{v_2+10}{10} + \frac{v_2}{4} - 15A = 0$$

$$2v_2 + 20 + 5v_2 = 300$$

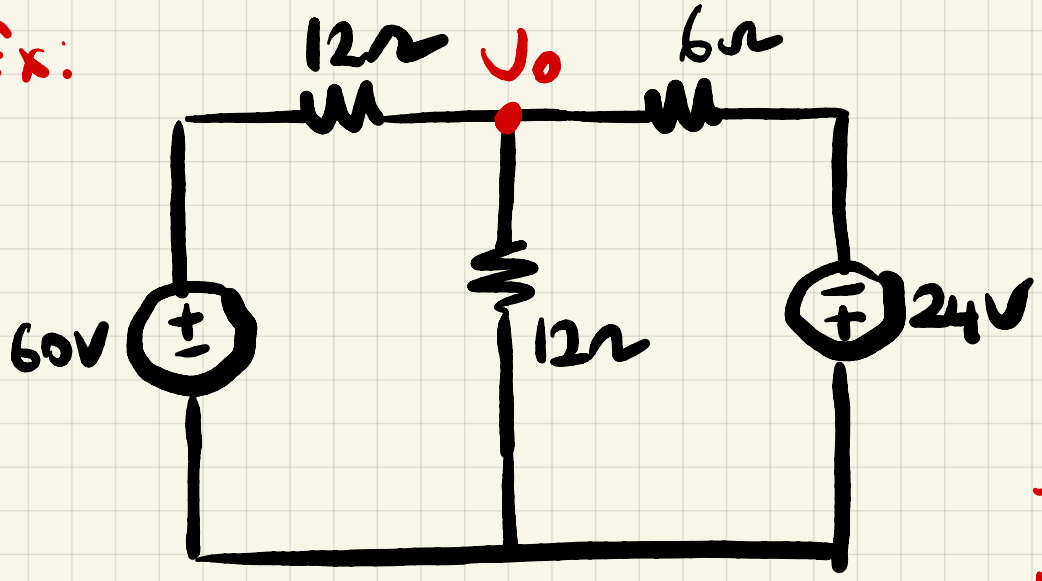
$$7v_2 = 280 \qquad v_2 = 40V$$

$$\frac{v_2+10}{10} = i_1$$

$$i_1 = 5A$$

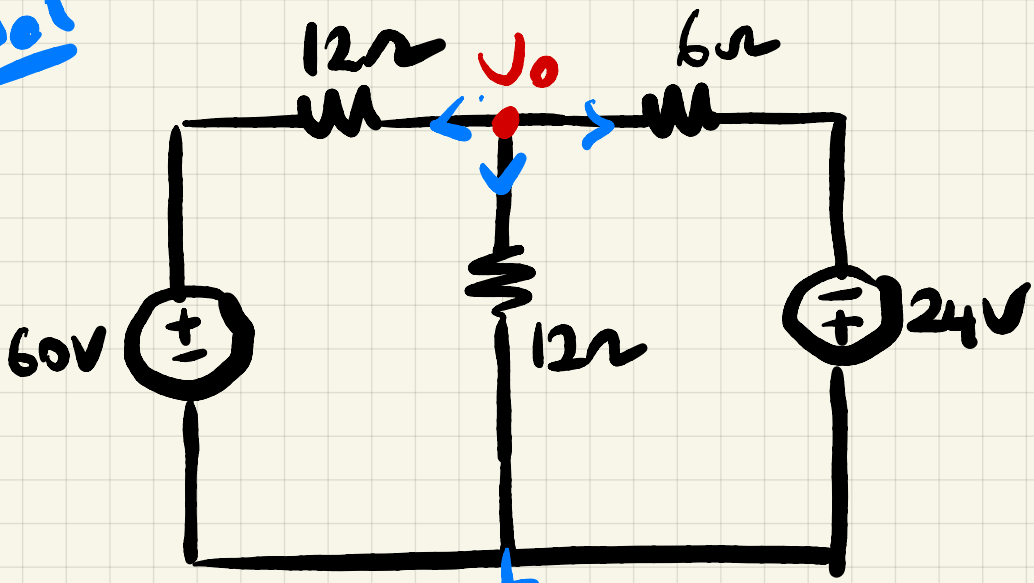
$$v_1 = 8 \times 5 = 40V \checkmark$$

Ex:



Find V_o and power absorbed by all resistors.

Sol



$$\frac{V_o - 60}{12} + \frac{V_o}{12} + \frac{V_o - (-24)}{6} = 0$$

$$V_o - 60 + V_o + 2V_o + 48 = 0$$

$$4V_o = 12V \qquad V_o = 3V$$

$$P_{12} = \frac{(60-3)^2}{R} =$$

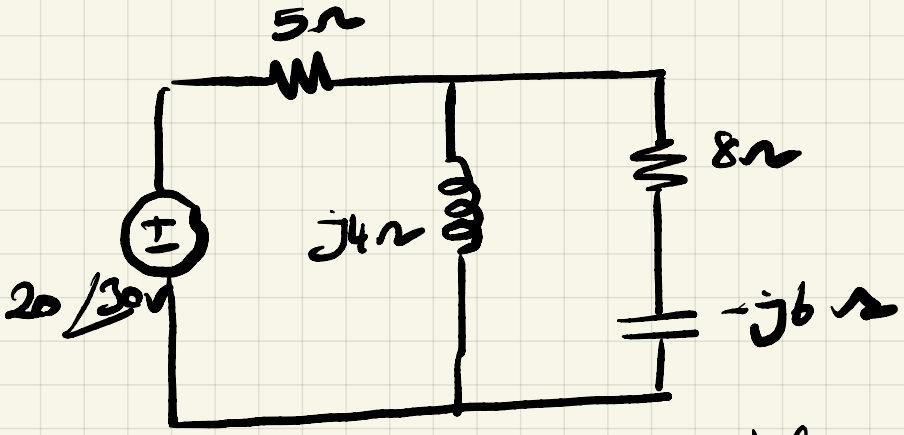
$$P_{12\Omega} = \frac{57^2}{12}$$

$$P_{12\Omega} = 270.75W$$

$$P_{12\Omega} = \frac{V_o^2}{R} = \frac{9}{12} = 0.75\,W$$

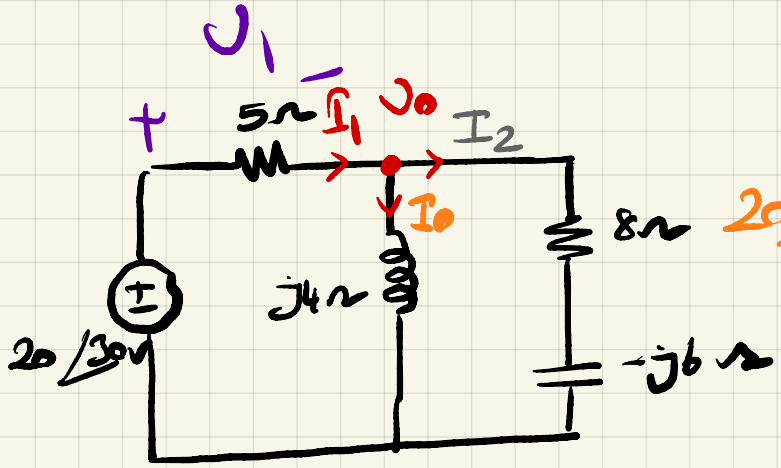
$$P_{6\Omega} = \frac{3-(-24)^2}{6} = 121.5W$$

Ex:



find the average power delivered by the 5Ω

Sol:



$20/30 = 20\cos 30 + j20\sin 30$

$= 17{,}32 + j10\ V$

$$\frac{20/30 - V_o}{5} = \frac{V_o}{4j} + \frac{V_o}{8-j6}$$

$$400/30 - 20V_o = -25jV_o + 8V_o + 6jV_o$$

$$400/30 = 28V_o - 19jV_o$$

$$V_o = \frac{400/30}{\sqrt{28^2+19^2}\ /\ \arctg\ 19/28}$$

$$V_o = \frac{400}{33{,}83} = 11{,}82/+64{,}16\ V$$

$$V_o = 11{,}82\cos 64{,}16 + j\,11{,}82\sin 64{,}16$$

$$V_o = 5{,}15 + j\,10{,}63\ V$$

$$I_1 = \frac{20/30 - 11{,}82/64}{5}$$

$$I_1 = \frac{17{,}32 + j10 - (5{,}15 - j\,11{,}82)}{5}$$

$$I_1 = \frac{12{,}17 - j\,1{,}82}{5}$$

$$I_1 = 2{,}43/-3^\circ\ A$$

$$V_1 = 12{,}17 - j\,1{,}82$$

$$V_1 = 12{,}30/-8{,}5\ V$$

$$P_1 = 29{,}89\ W$$

$$I_0 = \frac{V_0}{j4} = \frac{11{,}82 \angle 64{,}16}{4 \angle 90} = 2{,}95 \angle -25{,}84$$

$$I_2 = \frac{V_0}{8 - j6} = \frac{11{,}82 \angle 64{,}16}{10 \angle -36{,}86} = 1{,}18 \angle 101{,}02$$

$$S_{supply} = 20 \angle 30 \cdot (2{,}43 \angle +3) = 48{,}6 \angle 33 \text{ VA}$$

$$S_{supply} = 40{,}75 + j\,26{,}46$$

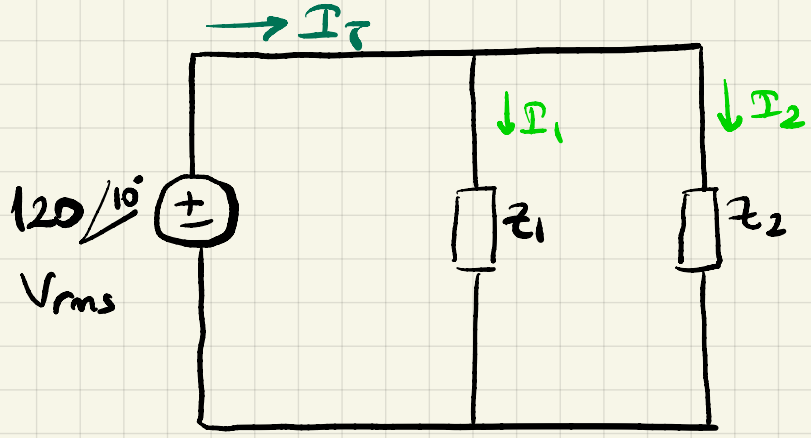
$$P_{8\Omega} = (1{,}18)^2 \cdot 8 = 11{,}13 \text{ W}$$

$$P_{rep} = 29{,}89 + 11{,}13 = 41{,}02 \text{ W}$$

$$\sim$$

$$40{,}75 \text{ W}$$

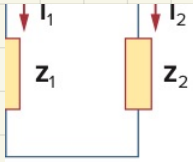
Ex:



$Z_1 = 60\angle -30^\circ\ \Omega$ and $Z_2 = 40\angle 45^\circ\ \Omega$

Calculate total apparent power, b) real power, c) reactive power, d) pf

Sol:



Solution:

The current through $\mathbf{Z}_1$ is

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1} = \frac{120\angle 10^\circ}{60\angle -30^\circ} = 2\angle 40^\circ \text{ A rms}$$

while the current through $\mathbf{Z}_2$ is

$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2} = \frac{120\angle 10^\circ}{40\angle 45^\circ} = 3\angle -35^\circ \text{ A rms}$$

The complex powers absorbed by the impedances are

$$\mathbf{S}_1 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_1^*} = \frac{(120)^2}{60\angle 30^\circ} = 240\angle -30^\circ = 207.85 - j120 \text{ VA}$$

$$\mathbf{S}_2 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_2^*} = \frac{(120)^2}{40\angle -45^\circ} = 360\angle 45^\circ = 254.6 + j254.6 \text{ VA}$$

The total complex power is

$$\mathbf{S}_t = \mathbf{S}_1 + \mathbf{S}_2 = 462.4 + j134.6 \text{ VA}$$

(a) The total apparent power is

$$|\mathbf{S}_t| = \sqrt{462.4^2 + 134.6^2} = 481.6 \text{ VA}.$$

(b) The total real power is

$$P_t = \text{Re}(\mathbf{S}_t) = 462.4 \text{ W or } P_t = P_1 + P_2.$$

(c) The total reactive power is

$$Q_t = \text{Im}(\mathbf{S}_t) = 134.6 \text{ VAR or } Q_t = Q_1 + Q_2.$$

(d) The pf $= P_t/|\mathbf{S}_t| = 462.4/481.6 = 0.96$ (lagging).

We may cross check the result by finding the complex power $\mathbf{S}_s$ supplied by the source.

$$\mathbf{I}_t = \mathbf{I}_1 + \mathbf{I}_2 = (1.532 + j1.286) + (2.457 - j1.721)$$
$$= 4 - j0.435 = 4.024\angle -6.21^\circ \text{ A rms}$$
$$\mathbf{S}_s = \mathbf{V}\mathbf{I}_t^* = (120\angle 10^\circ)(4.024\angle 6.21^\circ)$$
$$= 482.88\angle 16.21^\circ = 463 + j135 \text{ VA}$$

which is the same as before.