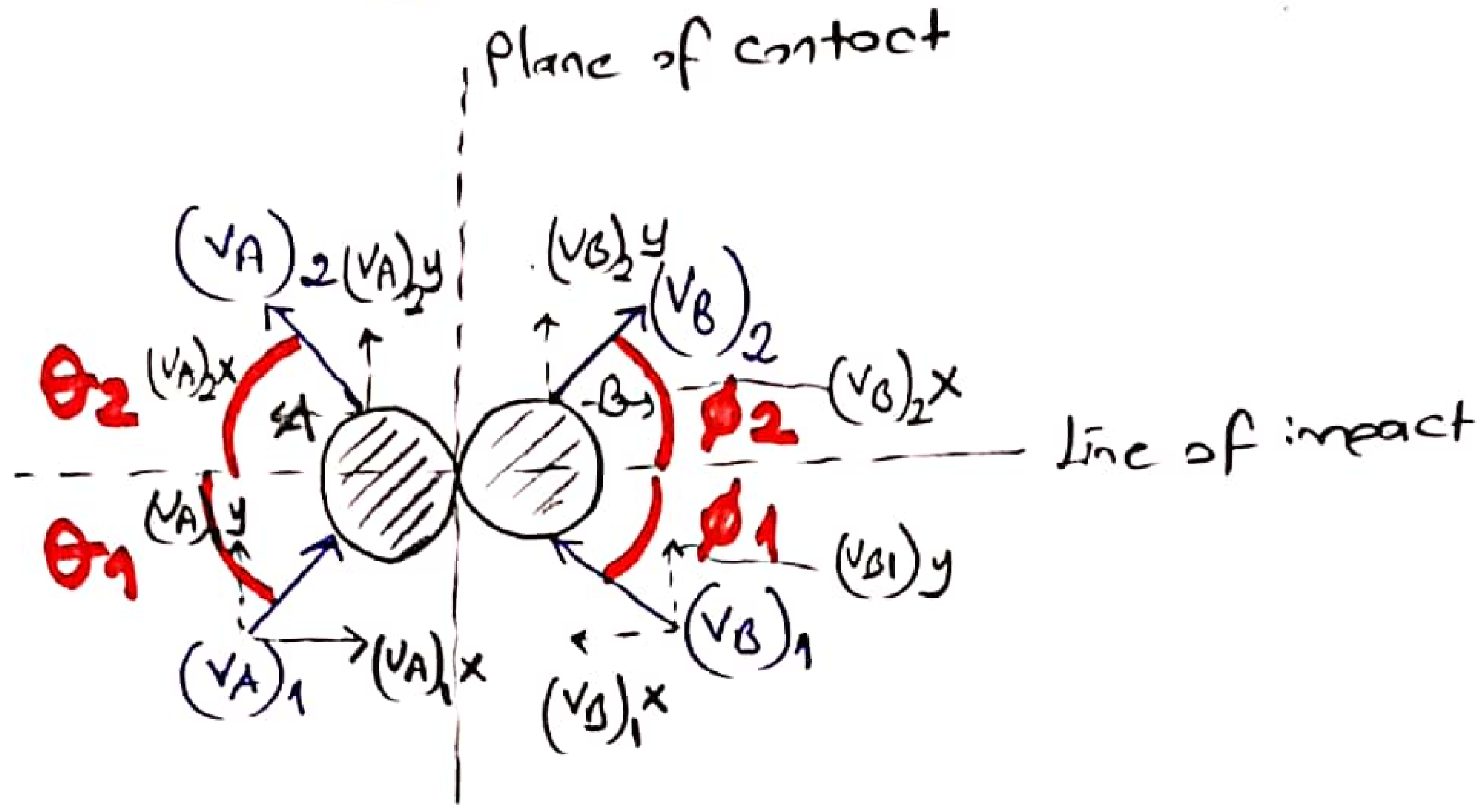
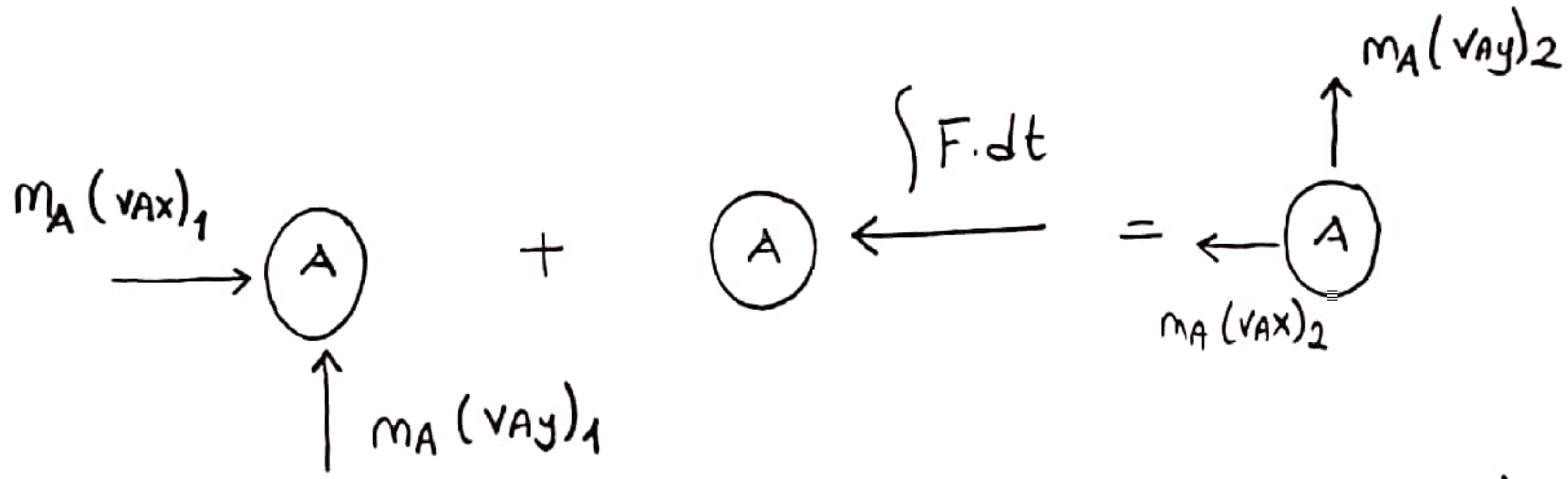
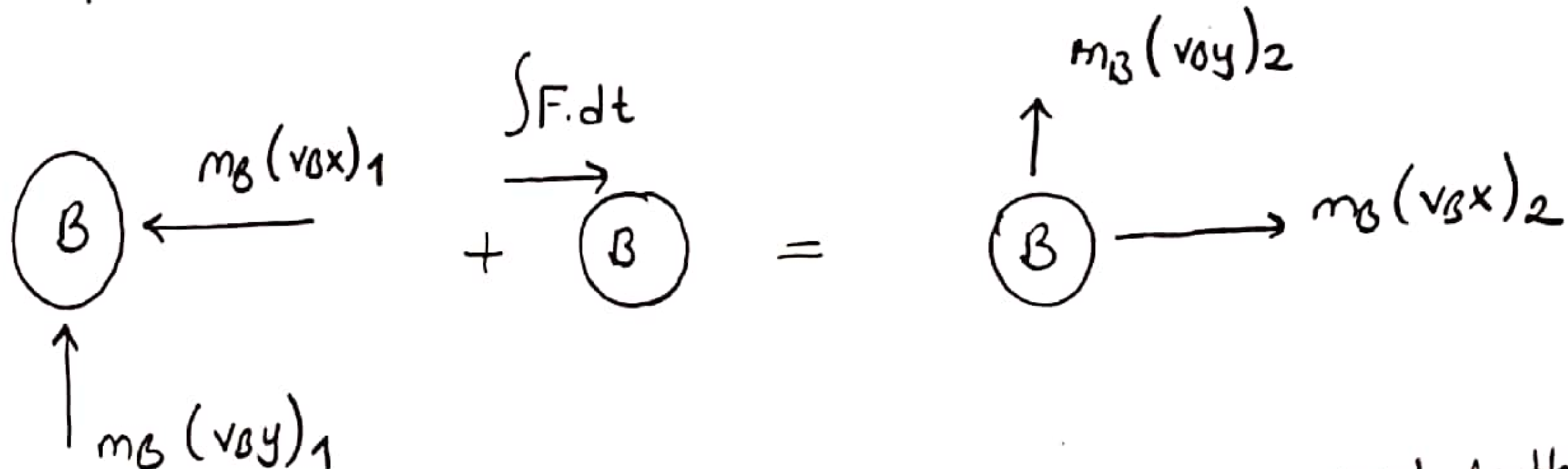
Oblique impact: When oblique impact occurs between two smooth particles, the particles move away from each other with velocities having unknown directions as well as unknown magnitudes.

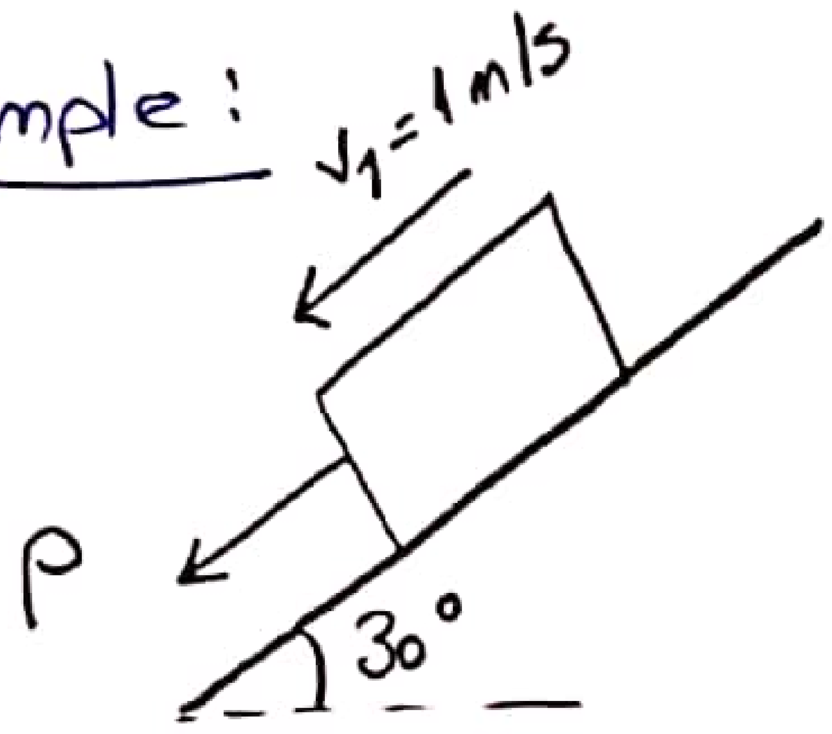




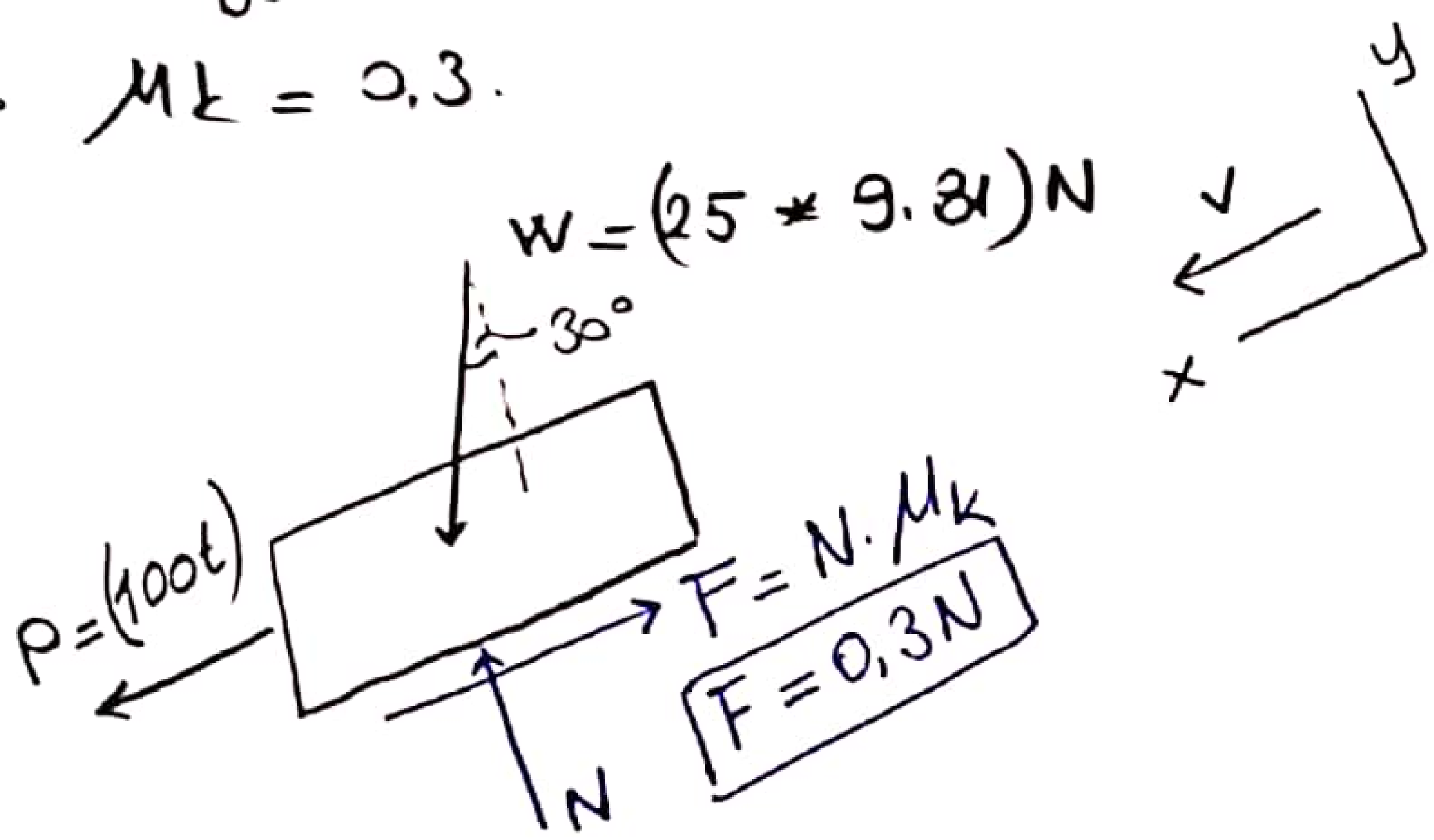


If the initial velocities are known, then 4 unknowns are present in the problem.

Example:



The 25 kg crate shown in Figure is acted upon by a force having a variable magnitude $P = (100t)$ N, where t is in seconds. Determine the crate's velocity 2s after P has been applied. The initial velocity is $v_1 = 1\,m/s$ down the plane and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0{,}3$.



Principle of Impulse and momentum:

(+↙) Apply the principle of impulse and momentum in the x direction, we'le

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \cdot dt = m(v_x)_2$$

$$\underbrace{25}_{kg}\,(\underbrace{1}_{m/s}) + \underbrace{\int_0^2 (100t)\,dt - 0{,}3N(2) + \underbrace{25 \cdot 9.81 \cdot \sin 30}\,(2)}_{\text{Impulse}}$$

$$= 25\,(v_2)$$

$$\boxed{25 + 200 - 0{,}6N + 245{,}25 = 25\,v_2} \quad \text{---} \quad (A)$$

In the y direction, the equation of equilibrium can be applied:

$+\uparrow \underline{\sum F_y = 0}$

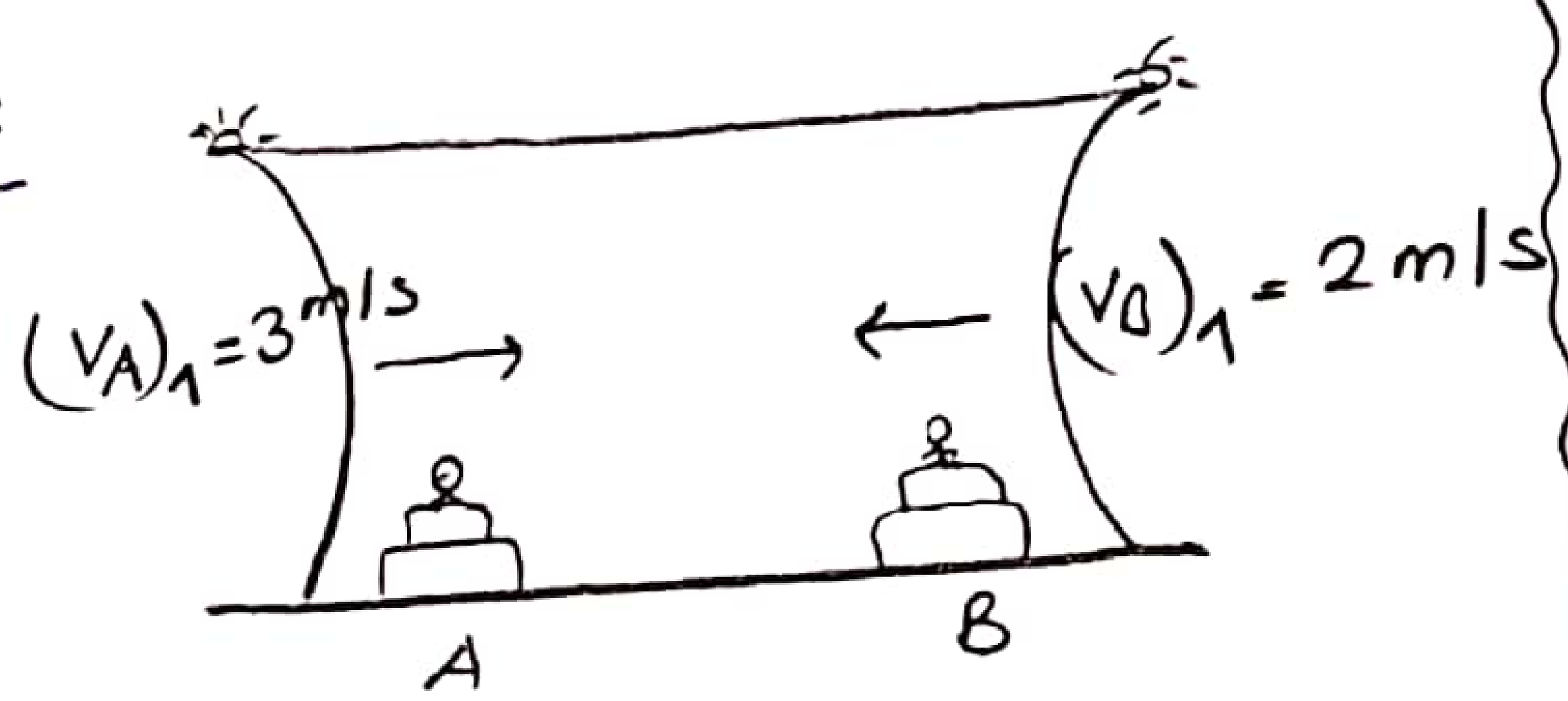
$$-25 \cdot 9{,}81 \cdot \cos 30 + N = 0$$

$$\boxed{N = 212{,}39\,N} \quad \cdots \text{(B)}$$

Let's substitute (B) into (A);

$$\boxed{V_2 = 13{,}7 \ m/s}$$

Example:



The bumper cars A and B in figure each have a mass of 150 kg and are coasting with the velocities shown before they freely collide head on. If no energy is lost during the collision, determine their velocities after collision.

** When particles collide or interact, conservation of linear momentum is often applied.

Conservation of momentum:

$$(\xrightarrow{+}) \quad m_A (V_A)_1 + m_B (V_B)_1 = m_A (V_A)_2 + m_B (V_B)_2$$

$$\underbrace{150}_{kg}\,\underbrace{(3)}_{m/s} + \underbrace{150}_{kg}\,\underbrace{(-2)}_{m/s} = \underbrace{150}_{kg}\,(V_A)_2 + \underbrace{150}_{kg}\,(V_B)_2$$

$$\boxed{(V_A)_2 = 1 - (V_B)_2} \quad \cdots \text{(1)}$$

Since no energy is lost, the conservation of energy theorem can be applied as follows:

$$T_1 + V_1 = T_2 + V_2$$

$$\underbrace{\frac{1}{2} m_A (v_A)_1^2 + \frac{1}{2} m_B (v_B)_1^2}_{T_1} + 0 = \overbrace{\frac{1}{2} m_A (v_A)_2^2 + \frac{1}{2} m_B (v_B)_2^2}^{T_2} + \underbrace{0}_{V_2}$$

m_A: 150 kg, $(v_A)_1^2$: $(3\ \text{m/s})^2$, m_B: 150 kg, $(v_B)_1^2$: $(2)^2$ 2 m/s

m_A: 150 kg, $(v_A)_2$: ?, m_B: 150 kg, $(v_B)_2$: ?

$$\boxed{(v_A)_2^2 + (v_B)_2^2 = 13} \quad \text{-- (2)}$$

Let's substitute Eq (1) into (2) and simplify;

$$(v_B)_2^2 - (v_B)_2 - 6 = 0$$

two roots:

$(v_B)_2 = 3$ m/s

$(v_B)_2 = -2$ m/s $\Longrightarrow$ the velocity of B just before the collision

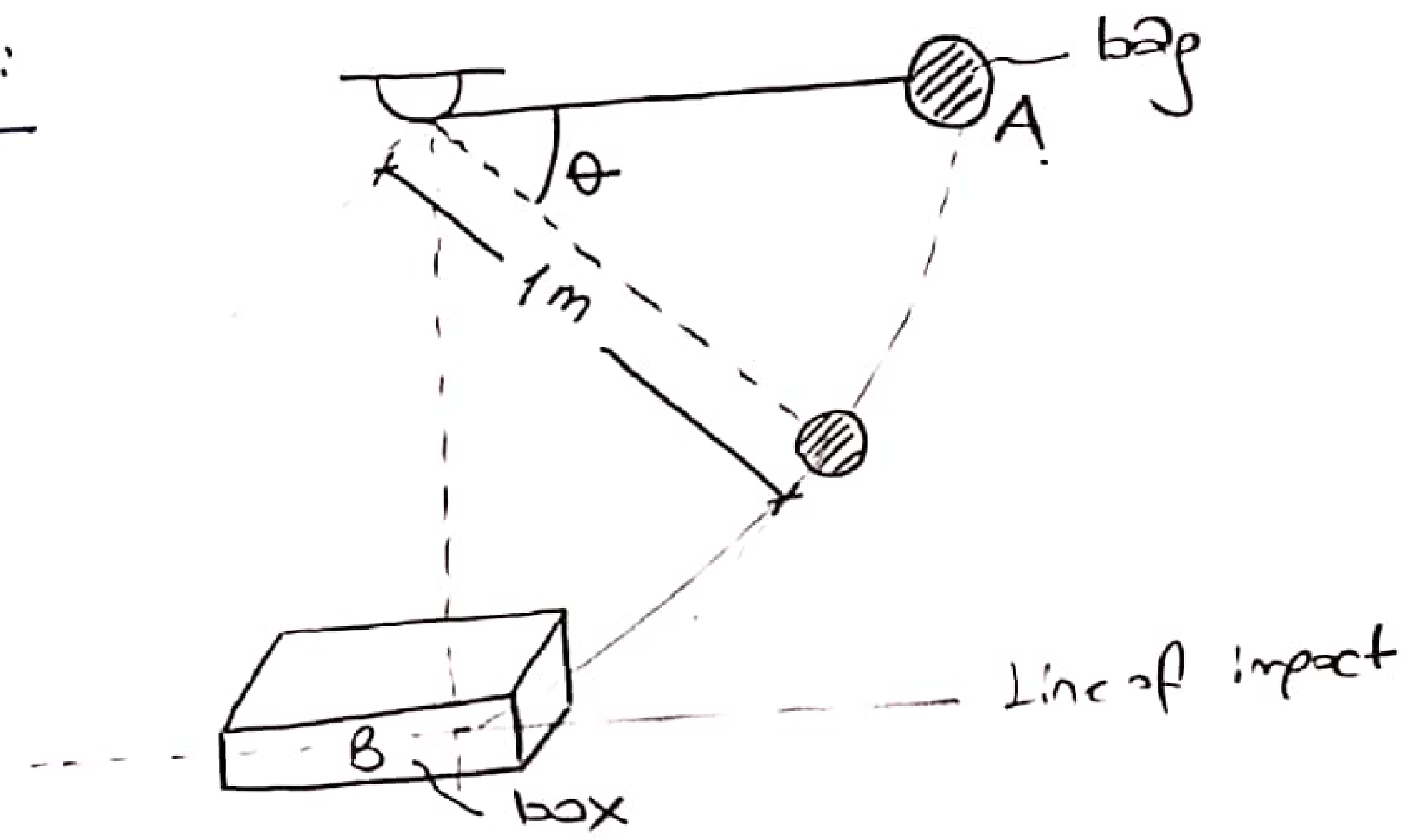
it must be the velocity of B just after collision

$$(v_B)_2 = 3\ \text{m/s} \longrightarrow$$

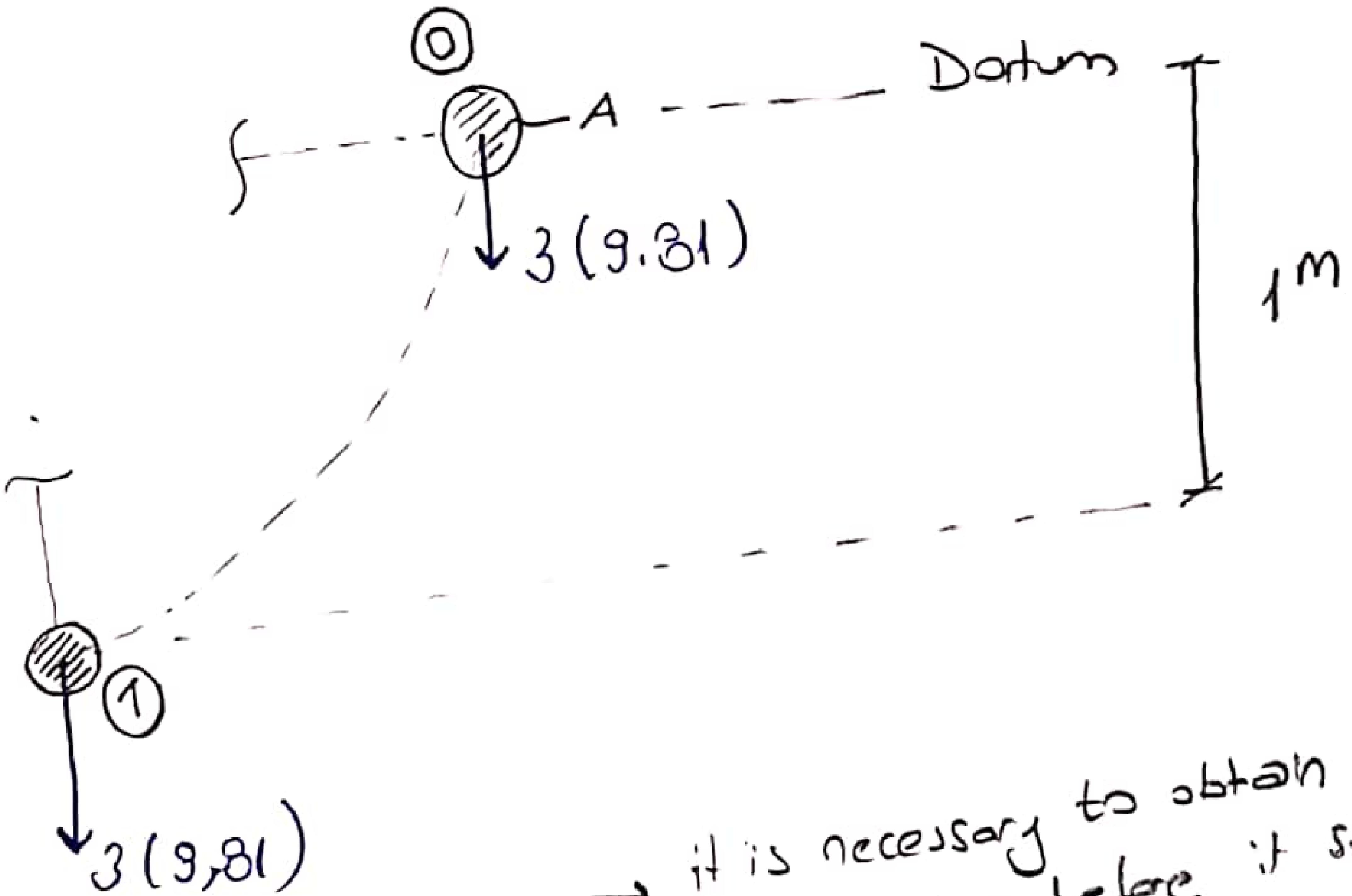
Substituting this result into Eq (1);

$$(v_A)_2 = 1 - 3 = -2\ \text{m/s} = 2\ \text{m/s} \longleftarrow$$

Example :



The bag A, having a mass of 3 kg, is released from rest at the position $\theta = 0°$, as shown in figure. After falling to $\theta = 90°$, it strikes on 9 kg box B. If the coefficient of restitution between the bag and box is $e = 0,5$, determine the velocities of the bag and box just after impact. What is the loss of energy during collision?



→ it is necessary to obtain the velocity of the bag just before it strikes to box.

Conservation of energy :

0 represents the datum ⇒ $\theta = 0°$

$$T_0 + V_0 = T_1 + V_1$$

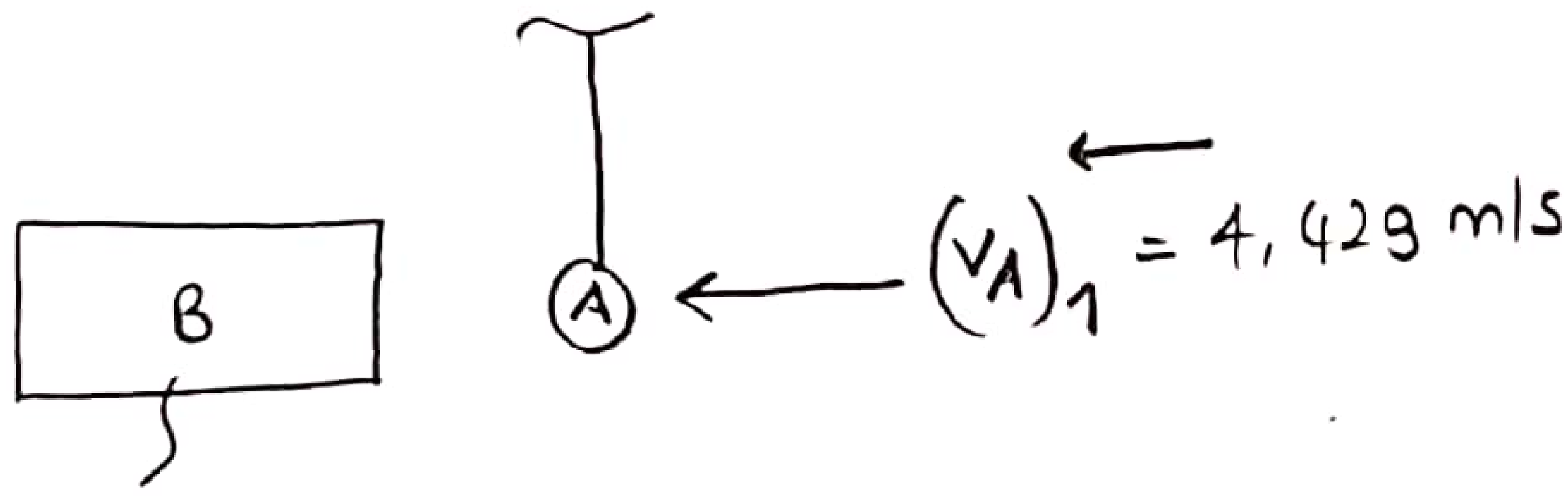
$$0 \quad 0 = \frac{1}{2}(3)(V_A)_1^2 - \underbrace{3 \cdot 9{,}81 \cdot (1)}_{mgh}$$

at datum

$$\boxed{(V_A)_1 = 4{,}429 \text{ m/s}}$$

<u>Conservation of momentum:</u> Applying the conservation of momentum to the system;

$$\overset{+}{\leftarrow} \quad m_B (v_B)_1 + m_A (v_A)_1 = m_B (v_B)_2 + m_A (v_A)_2$$



$(v_B)_1 = 0$

Just before impact.

$$0 + 3(4,429) = 9(v_B)_2 + 3(v_A)_2$$

$$(v_A)_2 = 4,429 - 3(v_B)_2 \quad -- \; (1)$$

<u>Coefficient of Resitution:</u> $\overset{+}{\leftarrow}$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0,5 = \frac{(v_B)_2 - (v_A)_2}{4,429 - 0}$$

$$(v_A)_2 = (v_B)_2 - 2,2145 \quad -- \; (2)$$

Solving Eqs (1) and (2);

$(v_A)_2 = -0,554$ m/s $= 0,554$ m/s $\rightarrow$

$(v_B)_2 = 1,66$ m/s $\leftarrow$

Loss of Energy: Applying the principle of work and energy to the bag and box just before and just after collision;

$$\boxed{\sum U_{1\to2} = T_2 - T_1}$$

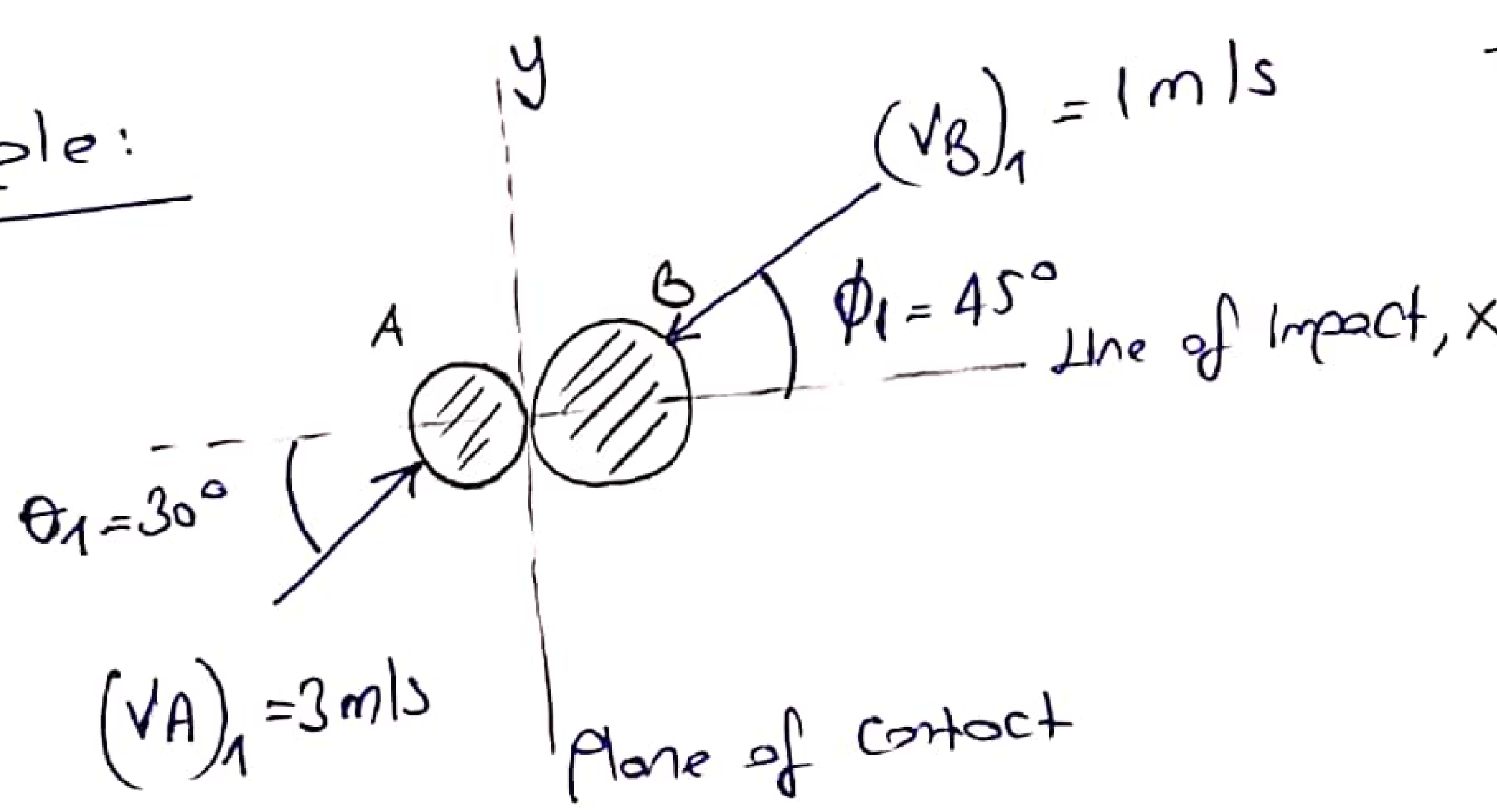
$$\sum U_{1\to2} = \underbrace{\left[\frac{1}{2}\,9\,(1{,}661)^2 + \frac{1}{2}\,3\,(0{,}554)^2\right]}_{T_2}$$

$$- \underbrace{\left[\frac{1}{2}\,3\,(4{,}429)^2\right]}_{T_1}$$

$$\boxed{\sum U_{1\to2} = -16.6\ \text{J}}$$

} the energy loss occurs due to inelastic deformation during the collision.

Example:



Two smooth disks A and B, having a mass of 1 kg and 2 kg, respectively collide with the velocities shown in Figure. If the coefficient of restitution for the disks is $e = 0{,}75$, determine the x and y components of the final velocity of each disk just after collision.

$(V_{Ax})_1 = 3 \cdot \cos 30 = 2{,}598 \, m/s$

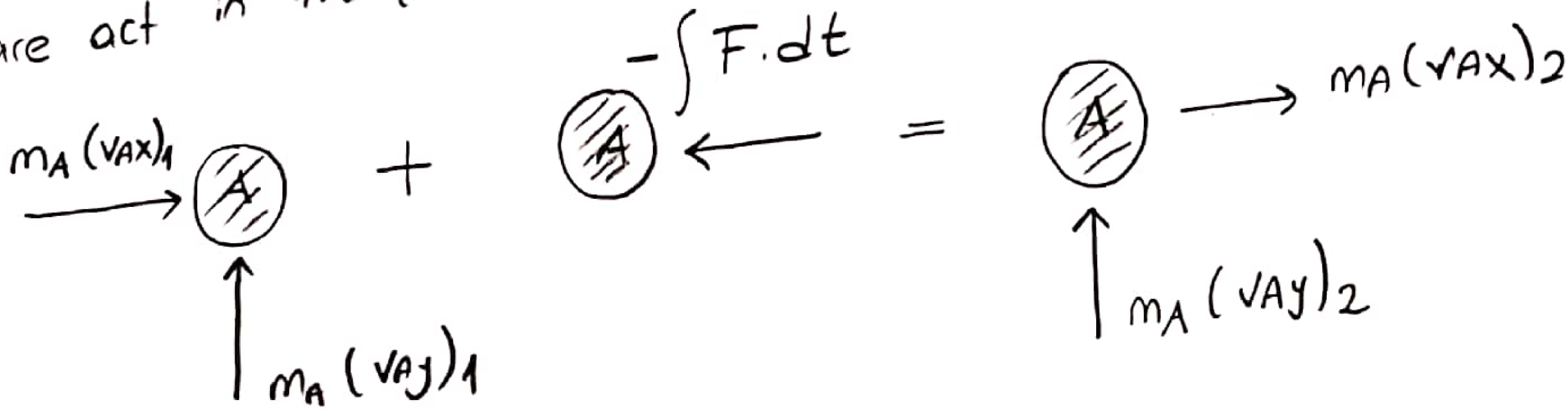
$(V_{Ay})_1 = 3 \cdot \sin 30 = 1{,}5 \, m/s$

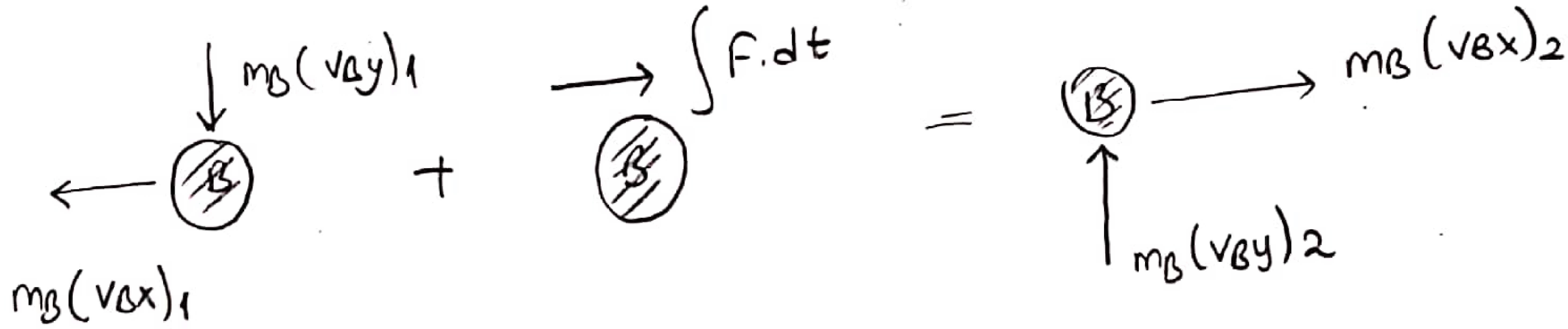
$(V_{Bx})_1 = -1 \cdot \cos 45 = -0{,}7071 \, m/s$

$(V_{By})_1 = -1 \cdot \sin 45 = -0{,}7071 \, m/s$

x and y components of initial velocities of A and B disks.

Let's assume that: 4 unknown velocity components after collision are act in the positive directions.





<u>Conservation of momentum in x direction:</u>

$$(\xrightarrow{+}) \quad \underbrace{m_A}_{1\,kg} \underbrace{(v_{Ax})_1}_{(2{,}598\, m/s)} + \underbrace{m_B}_{2\,kg} \underbrace{(v_{Bx})_1}_{(-0{,}7071\, m/s)} = \underbrace{m_A}_{1\,kg} (v_{Ax})_2 + \underbrace{m_B}_{2\,kg} (v_{Bx})_2$$

$$= 1 \, (v_{Ax})_2 + 2 \, (v_{Bx})_2$$

$$(V_{Ax})_2 + 2 \, (v_{Bx})_2 = 1{,}184 \quad \text{--- Eq 1}$$

$(\xrightarrow{+}) \quad e = \dfrac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$

$0{,}75 = \dfrac{(v_{Bx})_2 - (v_{Ax})_2}{2{,}598 - (-0{,}7071)}$

$\boxed{(v_{Bx})_2 - (v_{Ax})_2 = 2{,}482}$ --- Eq (2)

Solve Eq (1) and Eq (2);

$(v_{Ax})_2 = -1{,}26 \text{ m/s} = \leftarrow 1{,}26 \text{ m/s}$

$(v_{Bx})_2 = 1{,}22 \text{ m/s} \rightarrow$

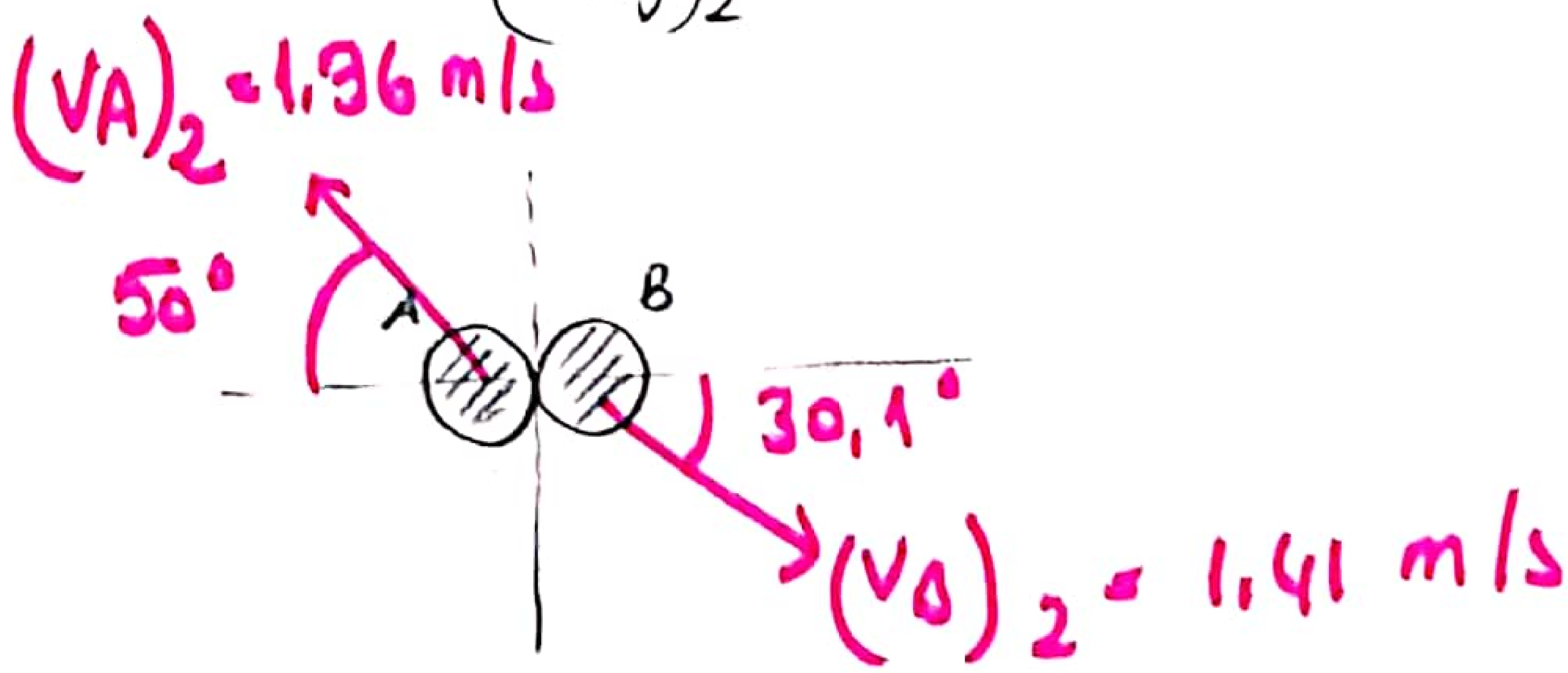
Conservation of momentum of each disk in y direction;

$(\uparrow +) \quad m_A (v_{Ay})_1 = m_A (v_{Ay})_2$

$(v_{Ay})_2 = 1{,}50 \text{ m/s} \uparrow$

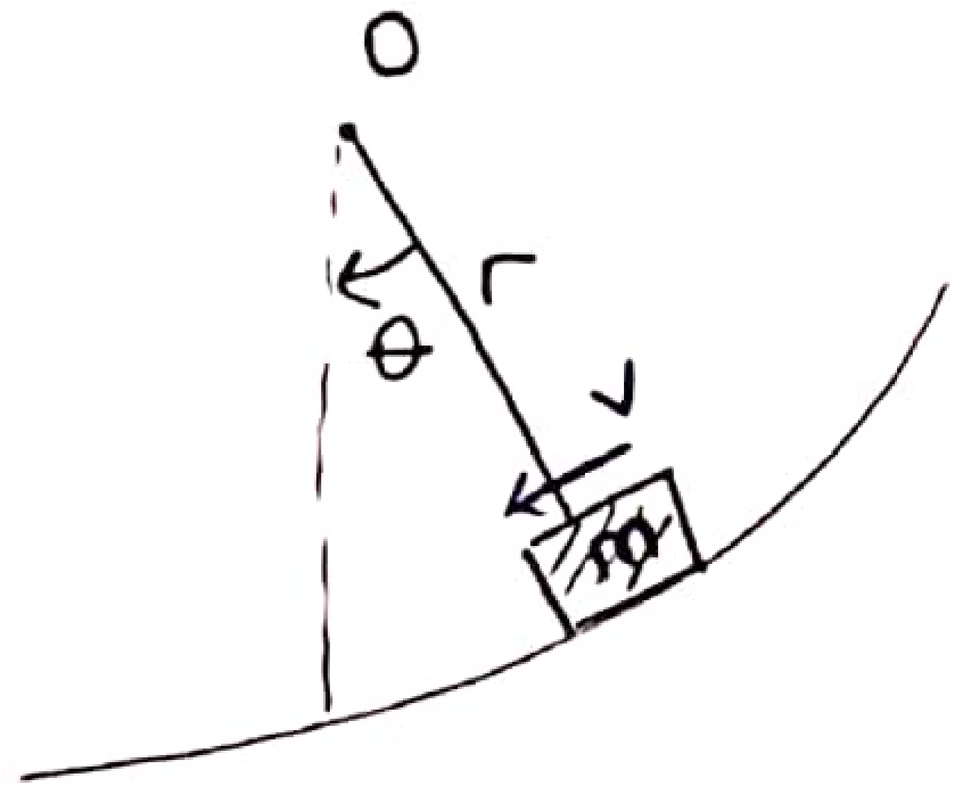
$(\uparrow +) \quad m_B (v_{By})_1 = m_B (v_{By})_2$

$(v_{By})_2 = -0{,}707 \text{ m/s} = 0{,}707 \text{ m/s} \downarrow$



Example: The box shown in Figure has a mass "m" and travels down the smooth circular ramp such that when it is at the angle θ it has a speed V. Determine its angular momentum about point "o" at this instant and the rate of increase in its speed.

a_t

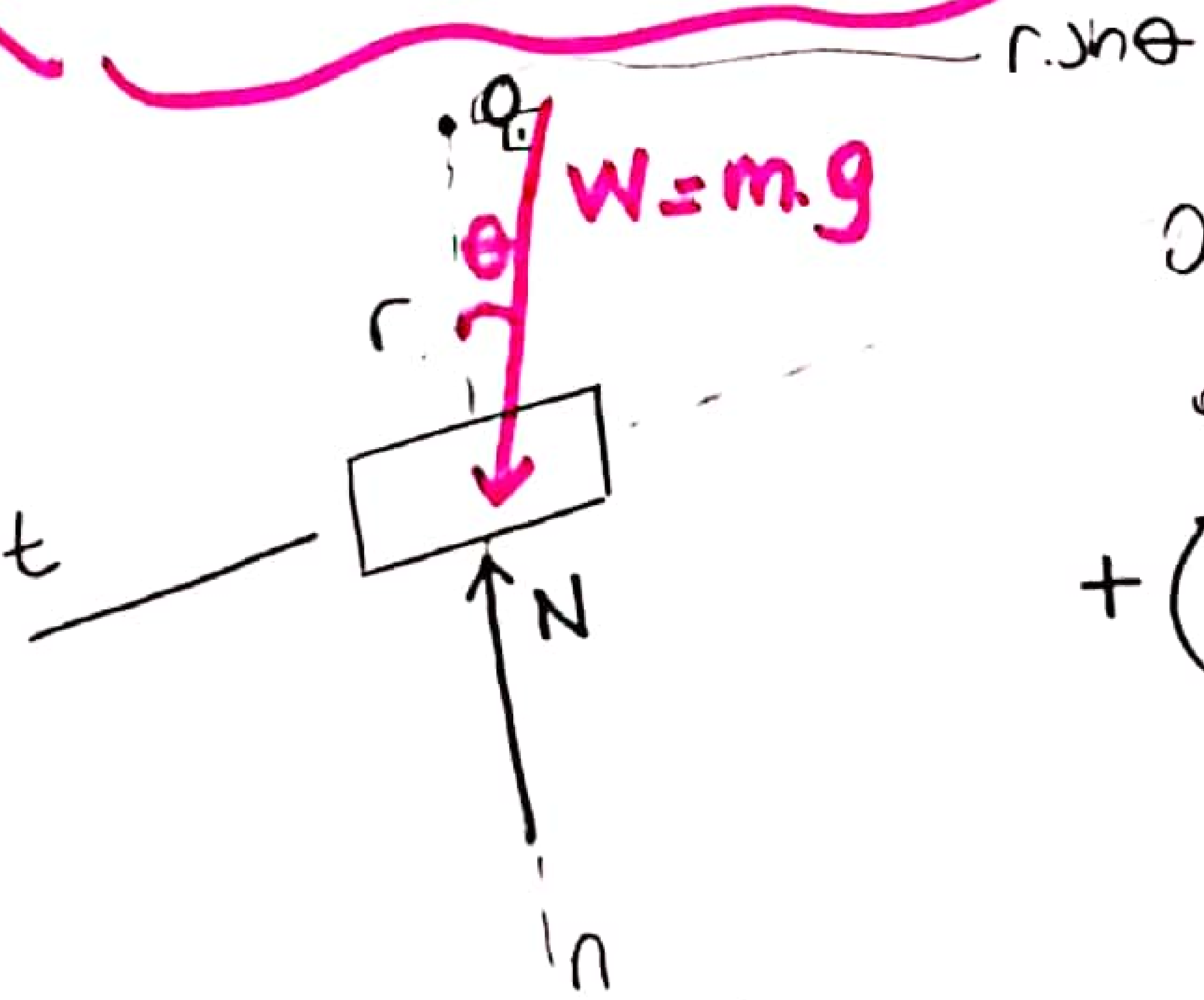


The angular momentum is ;

$$\boxed{H_o = r\,\underbrace{mv}_{\text{linear momentum}}}$$

The rate of increase in its speed $\left(\frac{dV}{dt} = a\right)$ can be found by applying ;

$$\boxed{+\circlearrowleft \sum M_o = \dot{H}_o}$$



Only the weight $W = mg$ contributes a moment about point "o".

$$+\circlearrowleft \sum M_o = \dot{H}_o$$

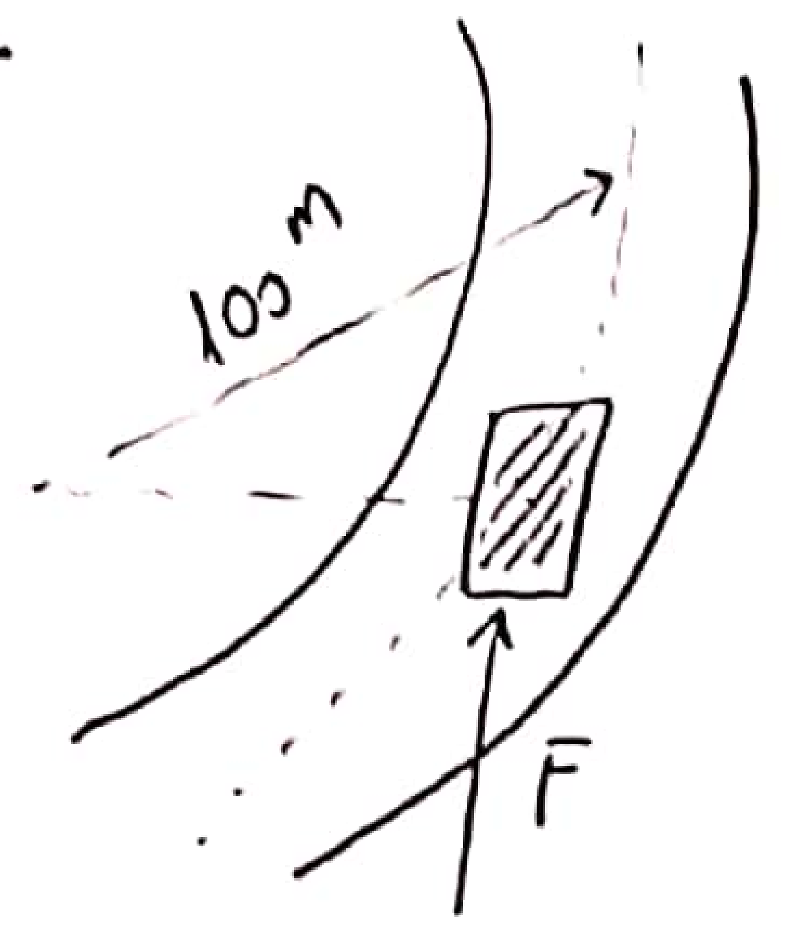
$$mg\,(r.\sin\theta) = \frac{d}{dt}(r.m.v)$$

r and m are constant ;

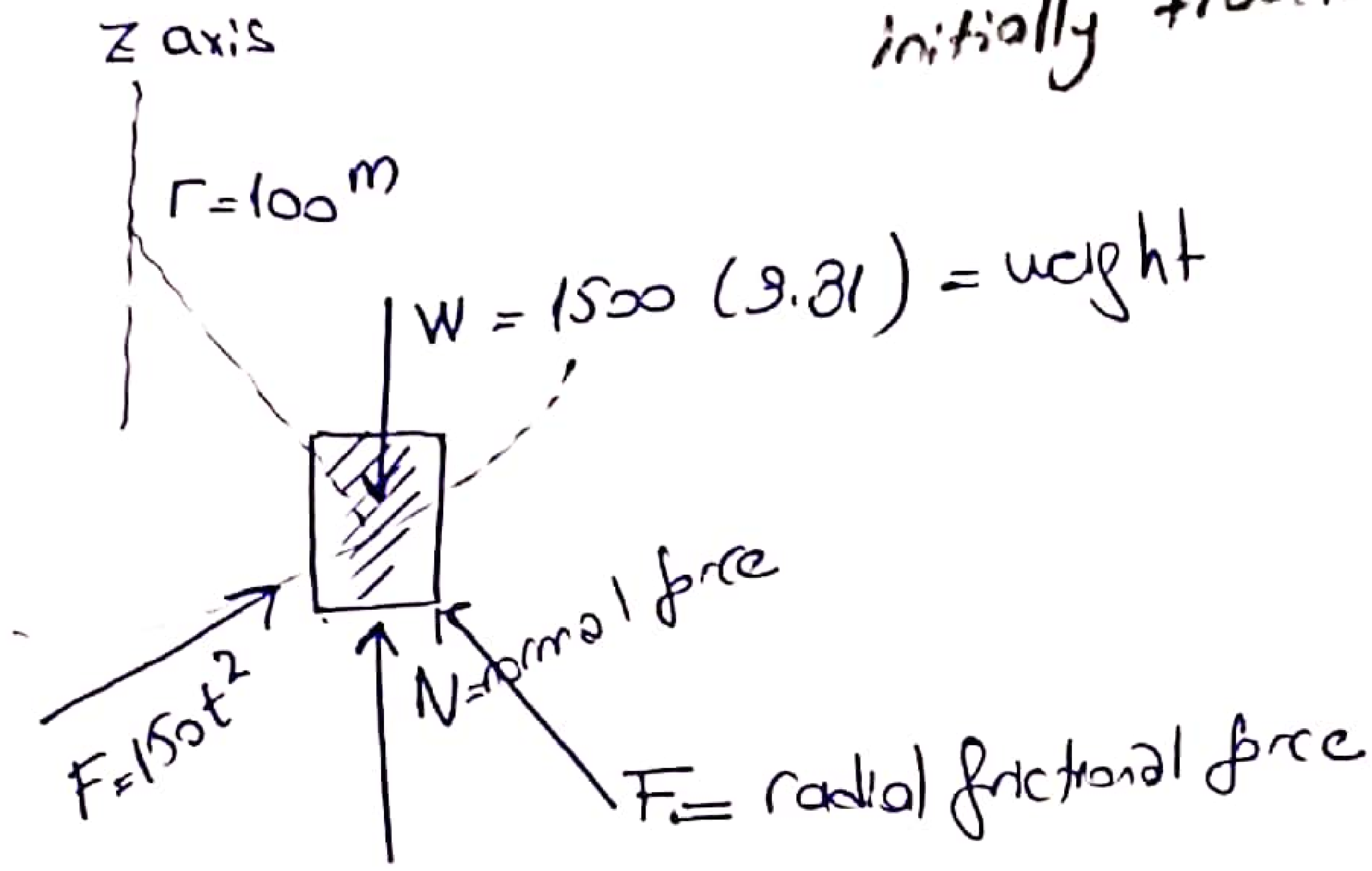
$$m g \cancel{r} \sin\theta = \cancel{r} m.\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = g.\sin\theta}$$

Example:



The 1500 kg car travels along the circular road shown in Figure. If the traction force of the wheels on the road is $F = (150t^2)$ N, where t is in seconds, determine the speed of the car when $t = 5$ s. The car initially travels with a speed of 5 m/s.



N, W and F will be eliminated since they act $\underbrace{\text{parallel}}^{W,\,N}$ to the Z axis or $\underbrace{\text{pass through}}_{F}$ it. Let's apply the principle of angular impulse and momentum about the "z" axis;

Principle of Angular impulse and momentum

$$(H_z)_1 + \sum \int_{t_1}^{t_2} \underbrace{M_z}_{rF} \cdot dt = (H_z)_2$$

$$\underbrace{r}_{100\,m} \cdot \underbrace{m}_{1500\,kg}\underbrace{(v)_1}_{5\,m/s} + \int_0^5 (150t^2)(\underbrace{r}_{100\,m})\,dt = \underbrace{r}_{100\,m} \cdot \underbrace{m}_{1500\,kg}\underbrace{(v)_2}_{?}$$

$$\boxed{(v)_2 = 9.17 \text{ m/s}}$$