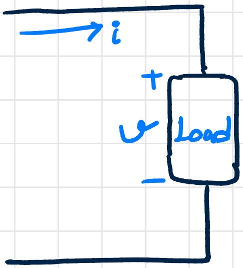
WEEK-5

AC POWER

POWER IN AC CIRCUITS



$$P = v \cdot i$$

$$v = V_m \sin(\omega t + \phi)$$

$$i = I_m \sin \omega t$$

$$P = V_m \sin(\omega t + \phi)\, I_m \sin \omega t$$

$$P = \frac{1}{2} V_m I_m \cos\phi \left(1 - \cancelto{0}{\cos 2\omega t}\right) + \frac{1}{2} V_m I_m \sin\phi \left(\cancelto{0}{\sin 2\omega t}\right)$$

$$P = \frac{1}{2} V_m I_m \cos\phi \quad [\text{Watt}, W]$$

REAL POWER
AVERAGE POWER
ACTIVE POWER

$$\parallel$$

$$P = V_{rms} I_{rms} \cos\phi \quad [\text{Watt}, W]$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

POWER FACTOR $\rightarrow$ $\cos\phi$ = Fp, fp, pf

$\phi \rightarrow$ power angle $\quad \phi = \phi_v - \phi_i$

$$\cos(\phi_v - \phi_i) = \cos(\phi_i - \phi_v)$$

$$\cos(-\alpha) = \cos(\alpha)$$

REACTIVE POWER

$$Q = \frac{1}{2} V_m I_m \underbrace{\sin\phi}_{}$$ Unit $[VAr]$ → Volt-Amper Reactive

? $\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$

$$Q = V_{rms} I_{rms} \underbrace{\sin\phi}_{}$$ $[VAR]$ Volt-Amper reactive

Reactive Power Factor

LAGGING POWER FACTOR ⇒ current lags voltage ⇒ INDUCTIVE LOAD

LEADING POWER FACTOR ⇒ current leads voltage ⇒ CAPACITIVE LOAD

ELI the ICE man

ELI → Inductive

ICE → Capacitive

COMPLEX POWER

$$S = \pm P \pm jQ$$

S → VA

+P → supplier

−P → consumer absorber

P → Watts

+jQ → inductive

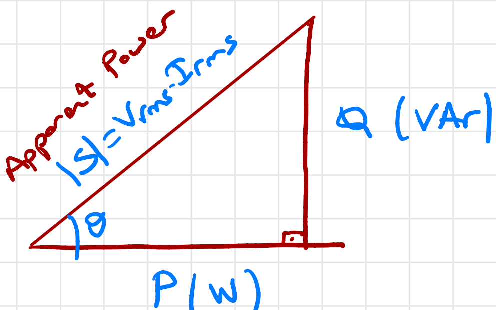
−jQ → capacitive

Q → VAr

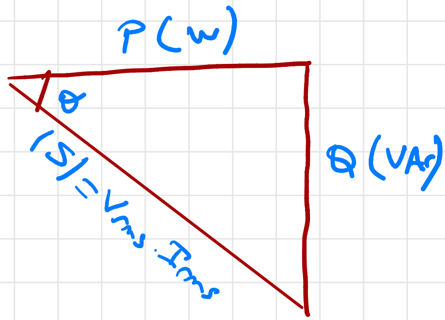
$$S = |S| \angle\theta$$

S → Complex

|S| → Apparent Power



Inductive (θ positive)



Capacitive (θ negative)

Complex Power

$$S = \underset{\text{active P.}}{P} \pm j \underset{\text{reactive power}}{Q}$$

$$\underbrace{S}_{\text{complex power}} = \underbrace{|S|}_{\text{Apparent Power}} \angle \theta$$

$$\boxed{S = V_{rms} I^{*}_{rms}}$$

$I^{*}_{rms} \rightarrow$ complex conjugate

$I_{rms} = 2\angle 30° A \quad I^{*}_{rms} = 2\angle -30° A$

$I_{rms} = -4\angle -40° A \quad I^{*}_{rms} = -4\angle 40°$

$$S = V_{rms} \cdot I_{rms} \angle \theta_v - \theta_i$$

$$S = V_{rms} I_{rms} e^{j(\theta_v - \theta_i)}$$

$$S = V_{rms} e^{j\theta_v} \cdot I_{rms} e^{-j\theta_i}$$

$$\boxed{S = V_{rms} \cdot I^{*}_{rms}}$$

Ex: find average power, reactive power, apparent power and complex power

if $\quad V = 100\angle 15° V, \; I = 4\angle -105 A$

b) find average power, reactive power, apparent power and complex power

if $\quad v = 141{,}42 \sin(wt + 15)$ and $i = 5{,}66 \sin(wt - 105°) A$

c) find average power, reactive power, apparent power and complex power

if $\quad V = 100(\cos 15 + j\sin 15) \qquad i = 4(\cos 105 - j\sin 105)$

$\quad V = 96{,}59 + j25.88 \; V$ ve $\qquad i = -1.03 - j3.86 \; A$

Solution

$P = V_{rms} I_{rms} \cos\phi \qquad Q = V_{rms} I_{rms} \sin\phi$

$V = 100\angle 15^\circ$ V , $I = 4\angle{+105}$ A

$S = V_{rms} \cdot I^{*}_{rms}$

$S = 100\angle 15 \cdot 4\angle{+105}$

$S = 100 \cdot 4\angle 120 = 400\angle 120$ VA $= |S|\angle\theta$ VA

Apparent Power (VA)

$S = P \pm jQ$ $\quad S = 400(\cos 120 + j\sin 120)$

$S = -200 + j\,346{,}4$ VA

complex power

P
Average Power
Active Power
(Watt)

Q
Reactive Power
(VAr)

$S = -200 + j\,346.4$ VA Complex Power

$P = -200$ Watts Average Power (Active Power)

$Q = 346.4$ VAr (Reactive Power – (Inductive))

$|S| = 400$ VA (Apparent Power)

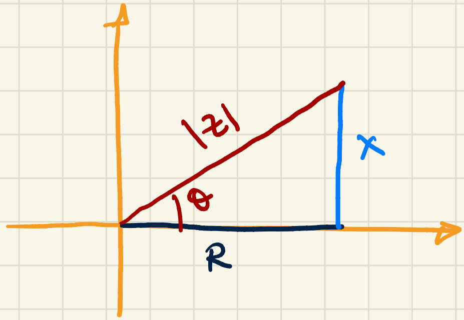
power factor → $\theta_v - \theta_i = \theta_i - \theta_v = |120|$

$\cos 120 = 0.5$ lagging

Additional Power Relationships

Impedance is Z

$$Z = |Z| \angle\theta = R + jX$$



$$\cos\theta = \frac{R}{|Z|} \qquad \sin\theta = \frac{X}{|Z|}$$

$$P = \frac{V_m \cdot I_m}{2} \cos\theta = \frac{V_m I_m}{2} \cdot \frac{R}{|Z|}$$

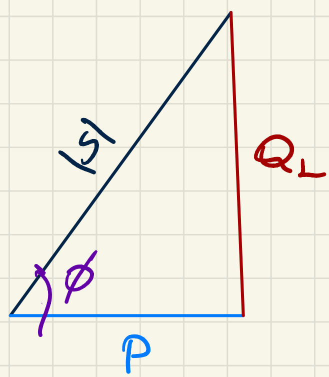
$$I_m = \frac{V_m}{|Z|} \qquad P = \frac{I_m^2}{2} \cdot R = I_{rms}^2 \cdot R$$

$$Q = \frac{I_m^2}{2} \cdot X = I_{rms}^2 \cdot X = \frac{V_{rms}^2}{X}$$

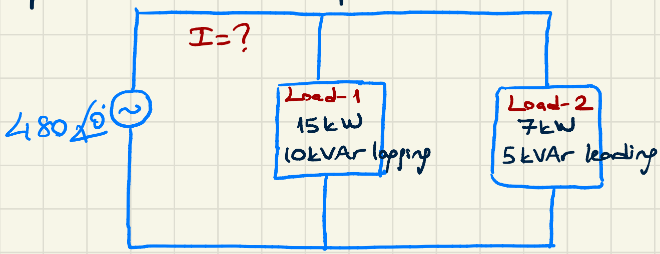
$$S = V_{rms} I_{rms} \cdot \cos\phi + j V_{rms} \cdot I_{rms} \cdot \sin\phi$$

$$P = |S| \cdot \cos\phi \qquad \cos\phi = \frac{P}{|S|}$$

$$pf = \frac{P}{|S|} = \cos\phi$$



Ex: Two parallel loads draw from a 480 V (rms) source as shown below; where Load-1 draws 15 kW and 10 kVAr lagging, and Load-2 draws 7 kW and 5 kVAr leading. Determined the combined power factor and total apparent power drawn from the source.

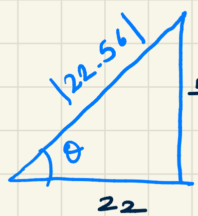


b) I=?

Solution:

$S_1 \rightarrow$ inductive (L) + VAR $\qquad S_2 \rightarrow$ capacitive (C) −

$$S_1 = 15 + j10 \text{ kVA}$$
$$S_2 = 7 - j5 \text{ kVA}$$
$$+$$
$$S_T = 22 + j5 \text{ kVA (lagging)}$$



$$|S| = \sqrt{22^2 + 5^2} = 22{,}56 \text{ kVA}$$

↳ Apparent Power

$$\text{arctg} \frac{5}{22} = 12.8^\circ \qquad S = |22.56| \angle 12.8^\circ \text{ kVA}$$

$$pf = \cos(12.8) = 0{,}975 \text{ lagging (L)}$$

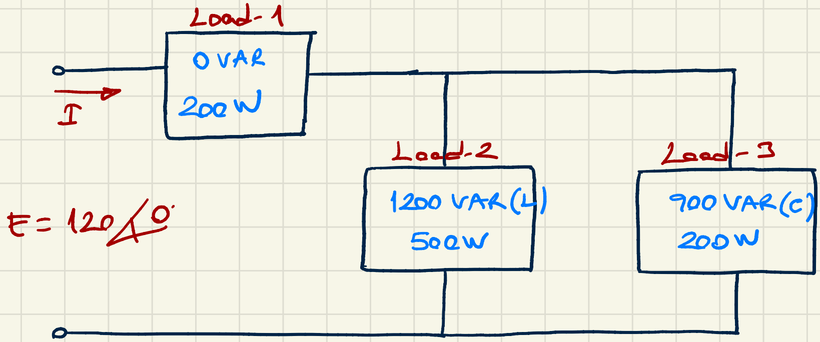
b) $S = V_{rms} \cdot I_{rms}^*$

$$22.56 \angle 12.8^\circ = 480 \angle 0 \cdot I_{rms}^* \qquad I_{rms}^* = \frac{22.56 \angle 12.8}{480 \angle 0^\circ}$$

$$I_{rms}^* = 0{,}047 \angle 12.8$$

$$I_{rms} = 0.047 \angle -12.8 \text{ kA}$$

Ex:



Find the total number of Average Power, Apparent Power, Reactive Power, and Power factor of the diagram.

Find $I = ?$

Loads	W	VAR	VA
1	200 W	0 VAR	$\sqrt{200^2+0^2} = 200\,VA$
2	500 W	1200 VAR (L)	$\sqrt{500^2+1200^2} = 1300\,VA$
+ 3	200 W	900 VAR (C)	$\sqrt{200^2+900^2} \cong 922\,VA$
Total →	900 W	300 VAR (L)	$\sqrt{900^2+300^2} = 948.68\,VA$

Total Average Power = 900 W

Total Reactive Power = 300 VAR (L)

Total Apparent Power = 948.68 VA

$S_T = 948.68\,VA$

$\cos\theta = \dfrac{P}{|S|} = \dfrac{900}{948.68}$

$\cos\theta = 0{,}94\,(L)$

$E = 120\angle 0$ $\quad S_T = 948.68$ $\quad I = \dfrac{948.68}{120} = 7.9\,A$

$\theta = ?$ $\quad \arccos(0.94) = \underline{19.94°}$ (inductive) $\longrightarrow$ $120\angle 0°$, $7.9\angle \theta\, 19.94°\,A$

$I = 7.9\,A \angle -19.94°\,A$

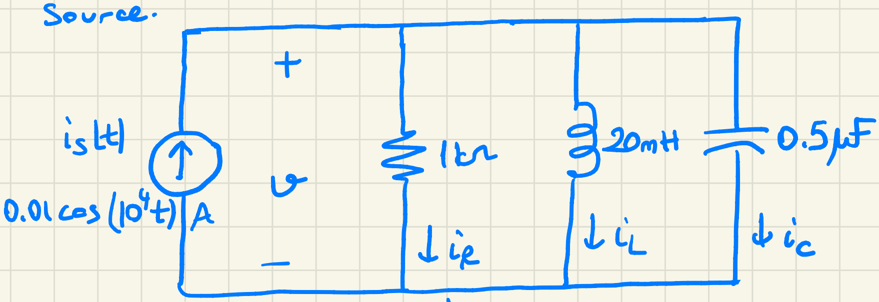
$S = V_{rms} \cdot I_{rms}^*$

$948.68\angle 19.94 = 120\angle 0 \cdot I_{rms}^*$

$I_{rms}^* = 7.9\angle 19.94$

$I_{rms} = 7.9\angle -19.94$

Ex: Consider the circuit shown in figure. Find the phasors I_s, V, I_R, I_L and I_C. Find the Apparent power for the Source.



Solution:

$$i_s(t) = \overset{\text{Amplitude}}{10} \sin(10^4 t + 90) \text{ mA} \qquad \omega = 10^4$$

$$I_s = \frac{10}{\sqrt{2}} \angle 90^\circ = 7.07 \angle 90^\circ \text{ mA}$$

$$Z_1 = 1000\ \Omega$$

$$X_L = \omega L = 10^4 \cdot 20 \cdot 10^{-3} = 200\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^4 \cdot 0.5 \times 10^{-6}} = 200\ \Omega$$

$$V = I \cdot R \Rightarrow V = \overset{mA}{7.07 \angle 90} \cdot \overset{\Omega}{1000} = 7.07 \angle 90\ V$$

$$I_2 = \frac{7.07 \angle 90^\circ}{200 \angle 90^\circ} = 0.035 \angle 0^\circ \text{ A}$$

$$I_C = \frac{7.07 \angle 90}{200 \angle -90} = 0.035 \angle 180^\circ \text{ A}$$

$$S = V_{rms} \cdot I_{rms}^*$$

$$= \overset{V}{7.07 \angle 90} \cdot \overset{mA}{7.07 \angle -90}$$

$$S = 49.98 \angle 0 \text{ mVA}$$