WEEK - 4

- Root Mean Square (RMS Value)
-

- AC Cont

SUBJECT: RMS LECTURE: DATE: 22 / 11 / 23

IMPORTANT (EQUATIONS, LAWS, ETC.)

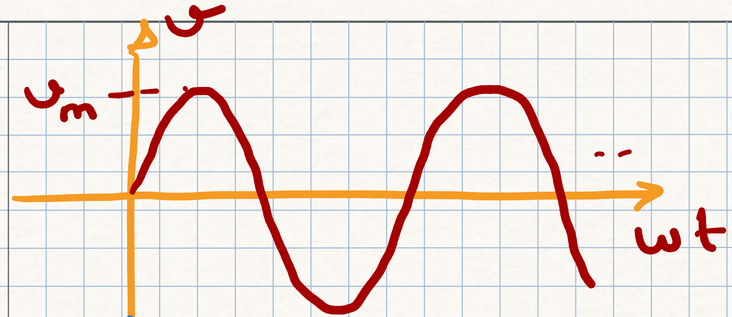
Root-Mean-Square (RMS) - Effective

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2}$$

NOTES

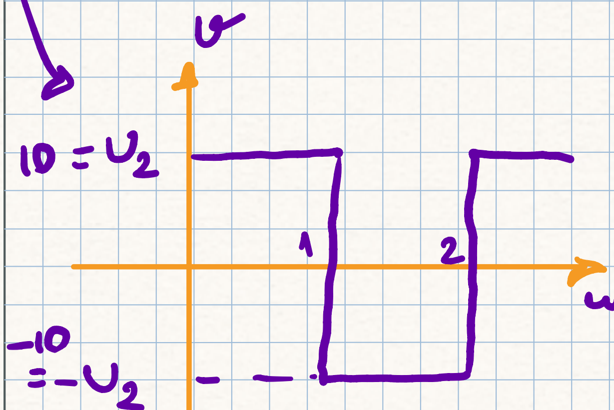
Root-Mean-Square

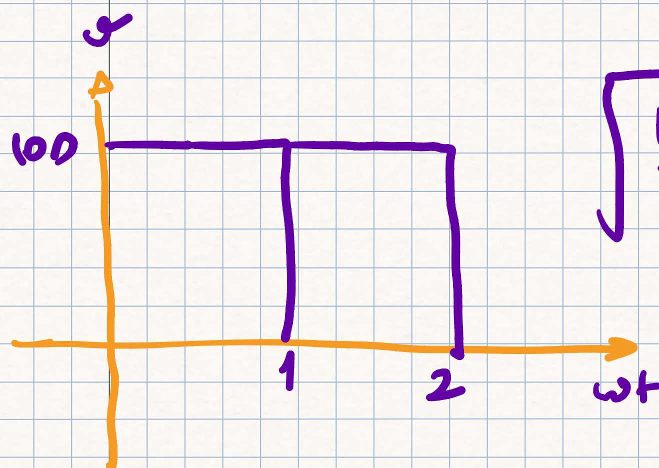
$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2}$$



Rms of Sinusoidal Signal is $\frac{V_m}{\sqrt{2}} = V_{rms}$

Rms of Square Signal is $\frac{V_m}{1} = V_{rms}$



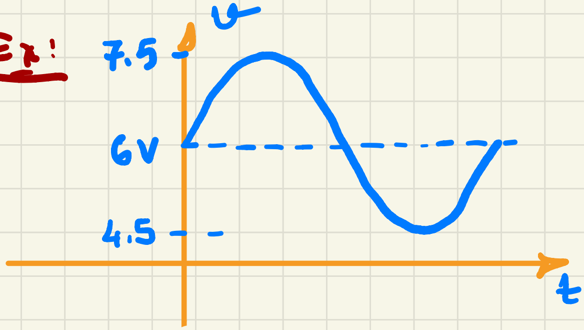


$$\sqrt{\frac{10^2 \cdot 2}{2}} = 10$$

Rms of Triangular Signal is $V_{rms} =$

V_m of 220 $V_{(rms)}$ =

Ex!



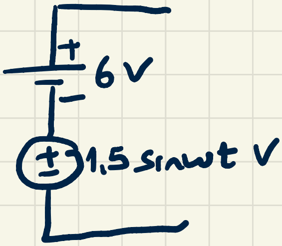
$V_{rms} = ?$

$V_{peak} = ?$ $7.5\,V$

$V_{peak\text{-}to\text{-}peak} = 3V$

$V_{AC} = ?$ $1{,}5 \sin \omega t\ V$

$V_{DC} = ?$ $6V$

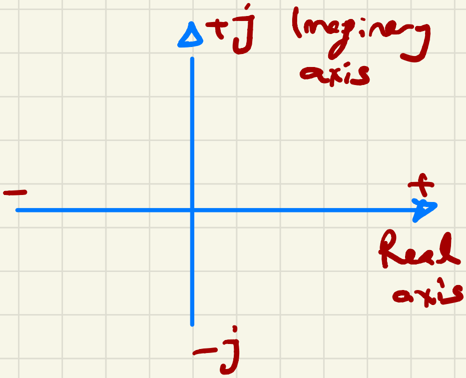


$\cancel{\frac{1{,}5}{\sqrt{2}} + 6 = 7{,}06\ V}$

$\frac{1{,}5}{\sqrt{2}} \approx 1{,}06V$

$$V_{rms} = \sqrt{V_{DC}^2 + V_{AC_{rms}}^2} = \sqrt{6^2 + (1{,}06)^2} \cong \underline{6.1\ V}$$

COMPLEX NUMBERS



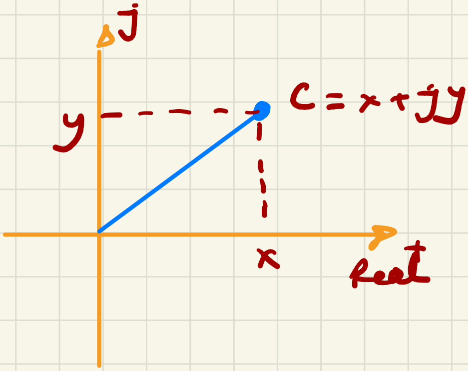
$i = j = \sqrt{-1}$

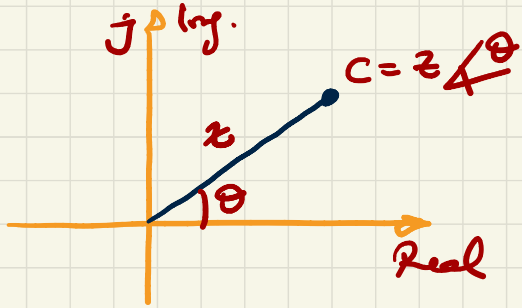
$j^2 = -1$

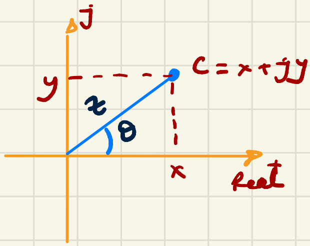
Rectangular form $C = X + jY$

Polar form $C = Z \angle \theta$

Exponential form $C = Ae^{j\theta}$







Rectangular to Polar

$C = x + jy$

$z = \sqrt{x^2 + y^2}$

$\theta = \arctan \dfrac{y}{x}$

$C = z \angle \theta$

Polar to Rectangular

$C = z \angle \theta$

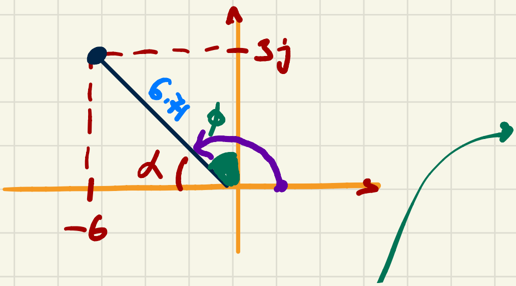
$x = z \cdot \cos\theta$

$y = z \cdot \sin\theta$

$C = x + jy$

Ex: Convert $C = 10\angle 45$ to rectangular

Ex: Convert $C = -6 + j3$ to polar form.



$$C_1 = x_1 + jy_1$$
$$C_2 = x_2 + jy_2$$

$+$ $-$

Summing + Substraction

$$C_1 \pm C_2 = (x_1 + x_2) \pm j(y_1 + y_2)$$

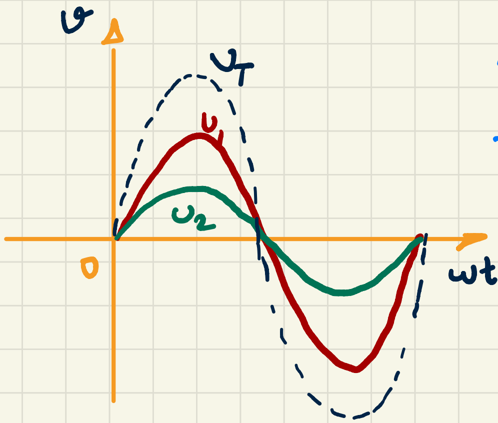
$$C_1 = z_1 \angle \theta_1$$
$$C_2 = z_2 \angle \theta_2$$

Multip and Divisions

$$C_1 \cdot C_2 = z_1 \cdot z_2 \angle \theta_1 + \theta_2$$

$$\frac{C_1}{C_2} = \frac{z_1}{z_2} \angle \theta_1 - \theta_2$$

PHASORS

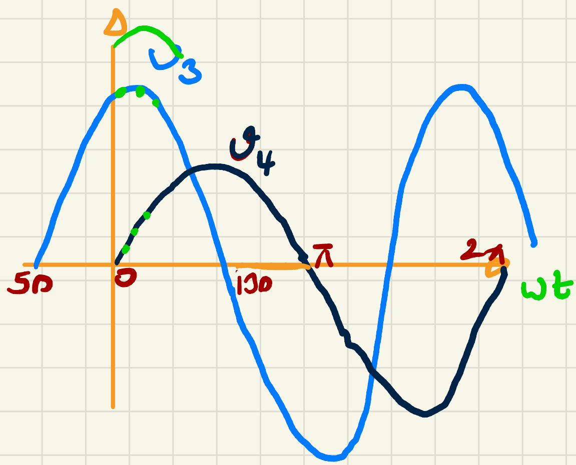


The addition (or substraction) of two sinusoidal voltages of the same frequency and phase angle is simply the sum (or difference) of the peak values of each with the sum (or difference) having the same phase angle.

$v_1 = 3\sin wt \longrightarrow$ Polar form $= \frac{3}{\sqrt{2}} \angle 0 = 2{,}12 \angle 0$

$v_2 = 2\sin wt \longrightarrow$ Polar form $= \frac{2}{\sqrt{2}} \angle 0 = 1{,}41 \angle 0$

$v_T = 5\sin wt \longrightarrow$ Polar form $= \frac{5}{\sqrt{2}} \angle 0 = 3{,}53 \angle 0$



$v_3 = 4\sin(wt + 50)$

$v_4 = 2\sin wt$

$v_3 = \frac{4}{\sqrt{2}} \angle 50°$

$v_4 = \frac{2}{\sqrt{2}} \angle 0°$

Time	(PHASOR) Polar-form	Rectangular
$5\sin\omega t$	$\frac{5}{\sqrt{2}}\angle 0^\circ$	$\frac{5}{\sqrt{2}}\cos 0 + j\frac{5}{\sqrt{2}}\sin 0$
$\downarrow$ $V_m \sin\omega t$!	$\downarrow$ $\frac{V_m}{\sqrt{2}} = V_{rms}$!	
$4\sin(\omega t+90)$	$\frac{4}{\sqrt{2}}\angle 90^\circ$	$\frac{4}{\sqrt{2}}\cos 90 + j\frac{4}{\sqrt{2}}\sin 90$
		$j2{,}82$

IMPEDANCE

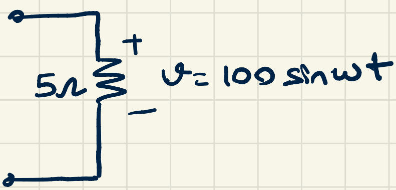
$V_m = V_{max}$, $I_m = I_{max}$!

Resistor

$v_R = V_m \sin\omega t \Rightarrow V_R = V_{rms}\angle 0 = \frac{V_m}{\sqrt{2}}\angle 0$

$i_R = I_m \sin\omega t \Rightarrow I_R = I_{rms}\angle 0 = \frac{I_m}{\sqrt{2}}\angle 0$

$$\boxed{\text{Impedance} \quad Z_R = \frac{V_{rms}\angle 0}{I_{rms}\angle 0} = R\angle 0}$$



$V = \frac{100}{\sqrt{2}}\angle 0 = 100 \sin\omega t$

$Z_R = 5\angle 0^\circ$

$I = \frac{V}{Z_R} = \frac{100/\sqrt{2}\ \angle 0}{5\angle 0} = 14{,}14\angle 0$

should be max !

$i = (14{,}14)\sqrt{2}\cos 0 + j(14.14)(\sqrt{2})\sin 0 = 20\sin\omega t$

(POLAR FORM)
PHASOR

$\frac{V_m}{\sqrt{2}} \angle 0 = V_{rms} \angle 0$ USE RMS

TIME $(V_m \sin \omega t)$ USE MAX. VALUE !

INDUCTANCE

$$Z_L = \frac{V \angle 0^\circ}{I \angle -90^\circ} = \underbrace{X_L}_{\text{Reactance}} \angle +90^\circ = \omega L \angle 90^\circ$$

$$= j\omega L = jX_L$$

Unit → Ω (OHM)

CAPACITANCE

$$Z_C = \frac{V \angle 0^\circ}{I \angle 90^\circ} = X_C \angle -90 = \frac{1}{\omega C} \angle -90$$

$$= -j\frac{1}{\omega C} = -jX_C$$

Unit → Ω (OHM)

RESISTANCE

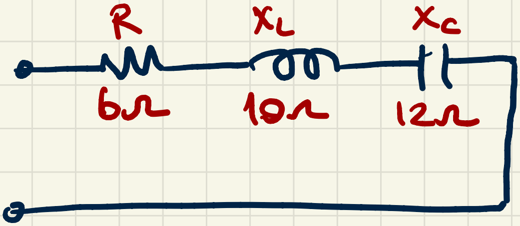
$$Z_R = \frac{V \angle 0}{I \angle 0} = R \angle 0^\circ$$

Unit → Ω (OHM)

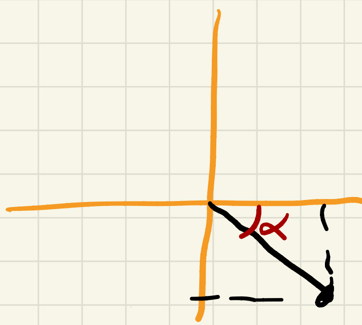
$$Z = R \mp jX$$

Impedance — Resistance — Reactance

Ex:



Z total impedance?



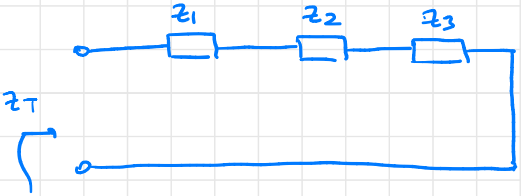
$$\alpha = \arctan \frac{2}{6} = 18^\circ$$

Δ Impedances

Δ Admittances

Δ Int. to AC Power

IMPEDANCES



$$Z_T = Z_1 + Z_2 + Z_3$$

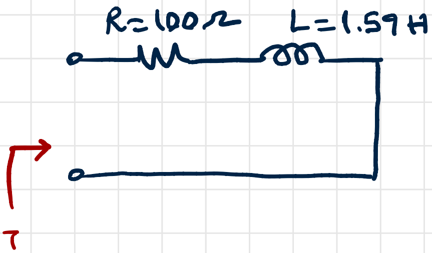
$$Z_1 = R_1 \pm jX_1$$
$$Z_2 = R_2 \pm jX_2$$
$$Z_3 = R_3 \pm jX_3$$

$$Z_T = R_T \pm jX_T$$

impedance — Resistance — Reactance

Unit is OHM (Ω)

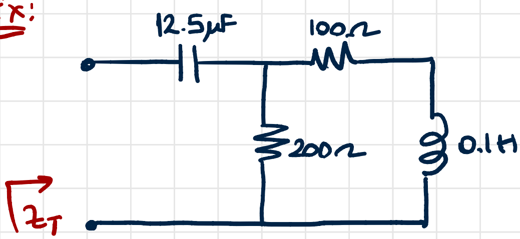
Ex:



If $f = 50Hz$ find $Z_T = ?$

Find rectangular and phasor form

Ex:

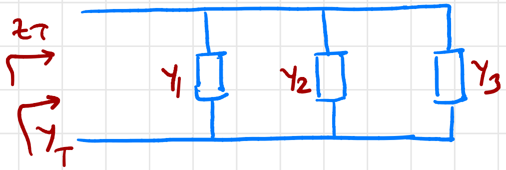


Determine the complex impedance between terminals shown in figure $\omega = 1000 \frac{rad}{s}$

ADMITTANCE (Y)

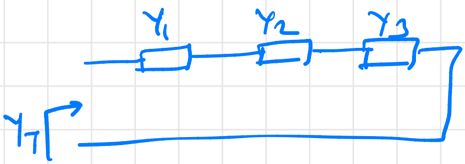
$$Y = \frac{1}{Z}$$

$$\underset{\text{Admittance}}{Y} = \underset{\text{Conductance}}{G} \pm j\underset{\text{Susceptance}}{B} \quad (\text{Siemens})(S)(\text{MHO's})$$



$$Y_T = Y_1 + Y_2 + Y_3$$

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$



$$\frac{1}{Y_T} = \frac{1}{Y_1} + \frac{1}{Y_2} + \frac{1}{Y_3}$$

$$Z_T = Z_1 + Z_2 + Z_3$$

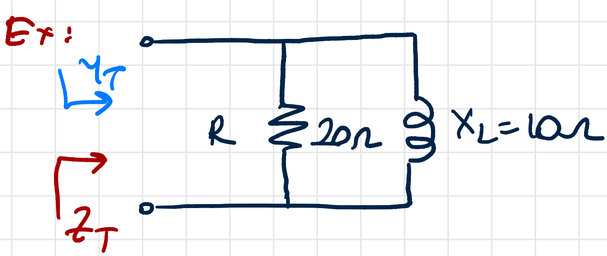
$$Y_R = \frac{1}{Z_R} = \frac{1}{R\angle 0} = G\angle 0 \quad (\text{Siemens}, S)$$

$$B = \frac{1}{X}$$

$$Y_L = \frac{1}{Z_L} = \frac{1}{X_L\angle 90} = \frac{1}{X_L}\angle -90 = B_L\angle -90 \ (S)$$

$$Y_C = \frac{1}{Z_C} = \frac{1}{X_C\angle -90} = \frac{1}{X_C}\angle 90 = B_C\angle 90^\circ \ (S)$$

Ex:



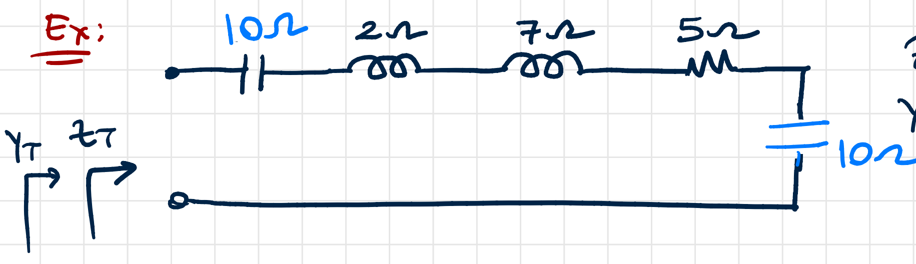
For the network:

a) Calculate input impedance

b) Find the admittance of each parallel branch.

c) Determine input admittance

Ex:



$Z_T = ?$

$Y_T = ?$

amplitude

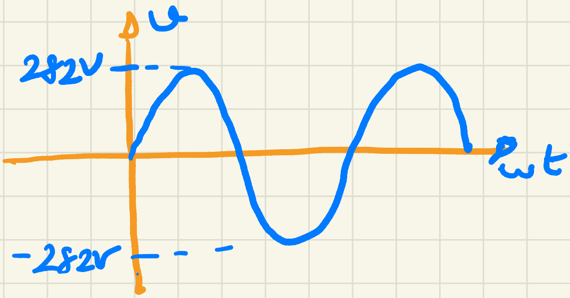
200 V (peak)

200 V (peak to peak)

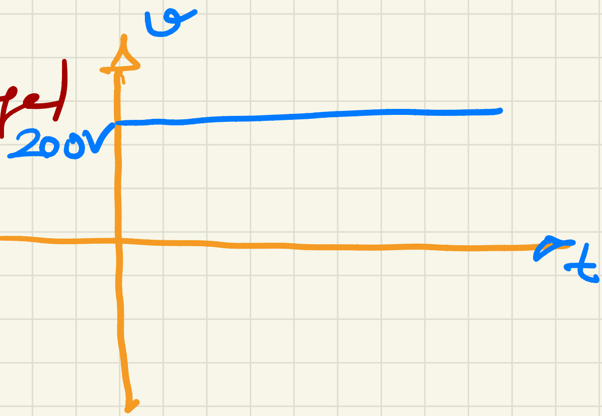
200V (rms)

effective

$200V \times \sqrt{2} =$

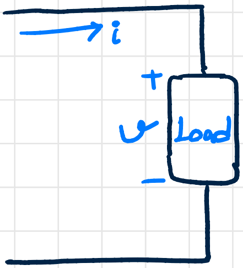


200 V (average)



POWER

POWER IN AC CIRCUITS



$$P = v \cdot i$$

$$v = V_m \sin(\omega t + \phi)$$

$$i = I_m \sin \omega t$$

$$P = V_m \sin(\omega t + \phi)\, I_m \sin \omega t$$

$$P = \frac{1}{2} V_m I_m \cos\phi \left(1 - \overset{0}{\cancel{\cos 2\omega t}}\right) + \frac{1}{2} V_m I_m \sin\phi \left(\overset{0}{\cancel{\sin 2\omega t}}\right)$$

$$P = \frac{1}{2} V_m I_m \cos\phi \quad [\text{Watt}, W]$$

$$\|$$

$$P = V_{rms} I_{rms} \cos\phi \quad [\text{Watt}, W]$$

REAL POWER
AVERAGE POWER
ACTIVE POWER

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

POWER FACTOR $\rightarrow \cos\phi = Fp, fp, pf$

$\phi \rightarrow$ power angle $\quad \phi = \phi_v - \phi_i$

$$\cos(\phi_v - \phi_i) = \cos(\phi_i - \phi_v)$$

$$\cos(-\alpha) = \cos(\alpha)$$

Reactive Power

Unit

$$Q = \frac{1}{2} V_m I_m \underbrace{\sin\phi} \quad [\text{VAr}] \rightarrow \text{Volt-Amper Reactive}$$

? $\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$

$$Q = V_{rms} I_{rms} \underbrace{\sin\phi} \quad [\text{VAR}]$$

Volt-Amper reactive

Reactive Power Factor

LAGGING POWER FACTOR ⇒ current lags voltage ⇒ INDUCTIVE LOAD

LEADING POWER FACTOR ⇒ current leads voltage ⇒ CAPACITIVE LOAD

ELI the ICE man

ELI → Inductive, ICE → Capacitive

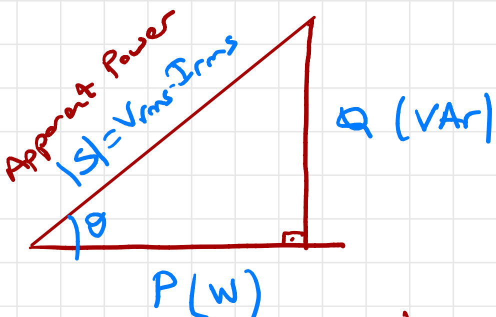
Complex Power

$$S = \mp P \pm jQ$$

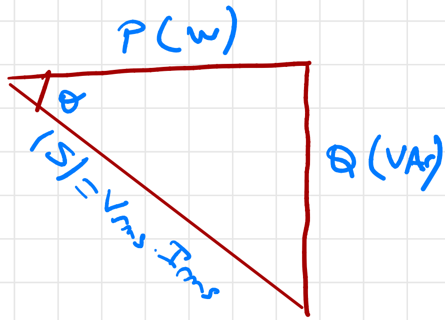
- S → VA
- − supplier, + consumer absorber
- P → Watts
- +jQ → inductive
- −jQ → capacitive
- Q → VAr

$$S = |S| \angle\theta$$

- S → Complex
- |S| → Apparent Power



Inductive (θ positive)



Capacitive (θ negative)

Complex Power

active P. → reactive power

$S = P \pm jQ$

$S = |S| \angle \theta$

complex power

Apparent Power

$\boxed{S = V_{rms} I^{*}_{rms}}$

I^{*}_{rms} → complex conjugate

$I_{rms} = 2\angle 30^\circ A$ $\quad I^{*}_{rms} = 2\angle -30^\circ A$

$I_{rms} = -4\angle -40^\circ A$ $\quad I^{*}_{rms} = -4\angle 40^\circ$

$S = V_{rms} \cdot I_{rms} \angle \theta_v - \theta_i$

$S = V_{rms} I_{rms} e^{j(\theta_v - \theta_i)}$

$S = V_{rms} e^{j\theta_v} \cdot I_{rms} e^{-j\theta_i}$

$\boxed{S = V_{rms} \cdot I^{*}_{rms}}$

Ex: find average power, reactive power, apparent power and complex power

if $\quad V = 100\angle 15^\circ V$, $I = 4\angle -105 A$

b) find average power, reactive power, apparent power and complex power

if $\quad v = 141,42 \sin(wt + 15)$ and $i = 5,66 \sin(wt - 105^\circ) A$

c) find average power, reactive power, apparent power and complex power

if $\quad V = 100(\cos 15 + j\sin 15)$ $\quad i = 4(\cos 105 - j\sin 105)$

$V = 96,59 + j25.88 V$ ve $\quad i = -1.03 - j3.86 A$

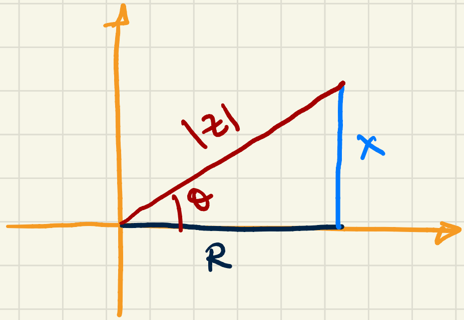
Solution

$P =$

Additional Power Relationships

Impedance is z

$$z = |z| \angle \theta = R + jX$$



$$\cos\theta = \frac{R}{|z|} \qquad \sin\theta = \frac{X}{|z|}$$

$$P = \frac{V_m \cdot I_m}{2} \cos\theta = \frac{V_m I_m}{2} \cdot \frac{R}{|z|}$$

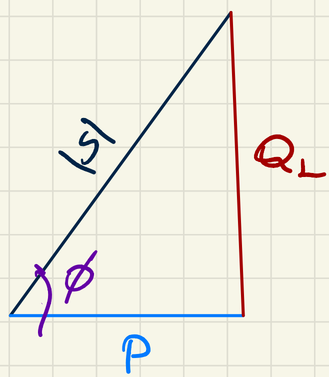
$$I_m = \frac{V_m}{|z|} \qquad P = \frac{I_m^2}{2} \cdot R = I_{rms}^2 \cdot R$$

$$Q = \frac{I_m^2}{2} \cdot X = I_{rms}^2 \cdot X = \frac{V_{rms}^2}{X}$$

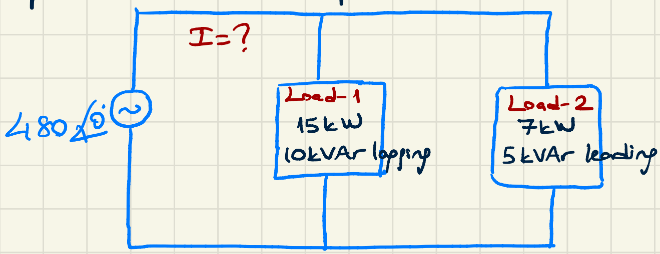
$$S = V_{rms} I_{rms} \cdot \cos\phi + j V_{rms} \cdot I_{rms} \cdot \sin\phi$$

$$P = |S| \cdot \cos\phi \qquad \cos\phi = \frac{P}{|S|}$$

$$pf = \frac{P}{|S|} = \cos\phi$$



Ex: Two parallel loads draw from a 480 V (rms) source as shown below; where Load-1 draws 15 kW and 10 kVAr lagging, and Load-2 draws 7 kW and 5 kVAr leading. Determined the combined power factor and total apparent power drawn from the source.



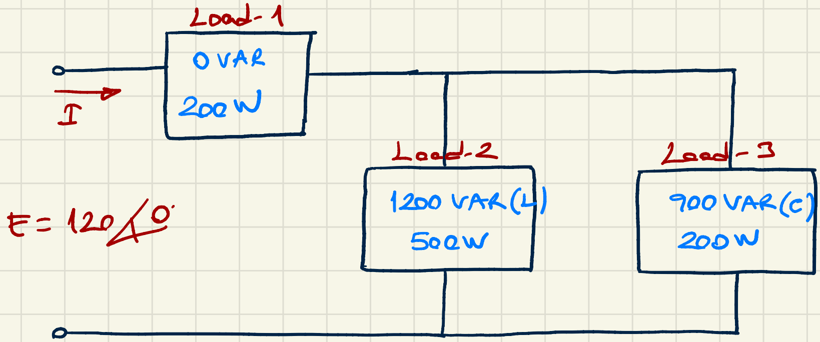
b) I = ?

$S_1 \rightarrow$ inductive (L) + VAR

$S_2 \rightarrow$ capacitive (C) -

Solution:

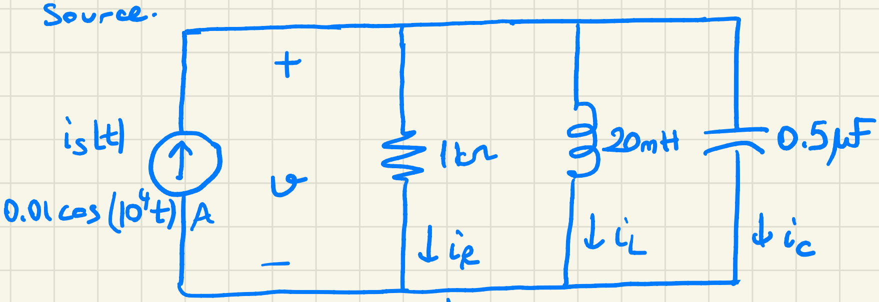
Ex:



Find the total number of Average Power, Apparent Power Reactive Power, and Power factor of the diagram.

Find I = ?

Ex: Consider the circuit shown in figure. Find the phasors I_s, V, I_R, I_L and I_c. Find the Apparent power for the source.



Solution:

$$i_s(t) = \overset{\text{Amplitude}}{10} \sin(10^4 t + 90)\ mA \qquad \omega = 10^4$$

$$I_s = \frac{10}{\sqrt{2}} \angle 90^\circ = 7.07 \angle 90^\circ\ mA$$

$$Z_1 = 1000\ \Omega$$

$$X_L = \omega L = 10^4 . 20.10^{-3} = 200\ \Omega$$

$$X_c = \frac{1}{\omega C} = \frac{1}{10^4 . 0,5\times10^{-6}} = 200\ \Omega$$

$$V = I . R \Rightarrow V = \overset{mA}{7.07 \angle 90} . \overset{\Omega}{1000} = 7,07 \angle 90\ V$$

$$I_L = \frac{7,07 \angle 90^\circ}{200 \angle 90^\circ} = 0,035 \angle 0^\circ\ A$$

$$I_c = \frac{7.07 \angle 90}{200 \angle -90} = 0.035 \angle 180^\circ\ A$$

$$S = V_{rms} . I_{rms}^*$$

$$= \overset{V}{7.07 \angle 90} . \overset{mA}{7.07 \angle -90}$$

$$S = 49,98 \angle 0\ mVA$$