Fluid Mechanics 1

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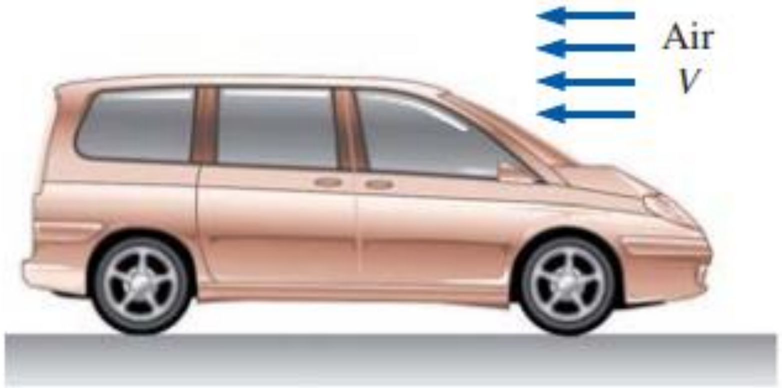
Chapter one INTRODUCTION AND BASIC CONCEPTS

Chapter two PROPERTIES OF FLUIDS

Chapter three PRESSURE AND FLUID STATICS

Q1-The drag force exerted on a car by air depends on a dimensionless drag coefficient, the density of air, the car velocity, and the frontal area of the car. That is, FD = function (CDrag, Afront, ro, V). Based on unit considerations alone, obtain a relation for the drag force.



Solution A relation for the air drag exerted on a car is to be obtained in terms of on the drag coefficient, the air density, the car velocity, and the frontal area of the car.

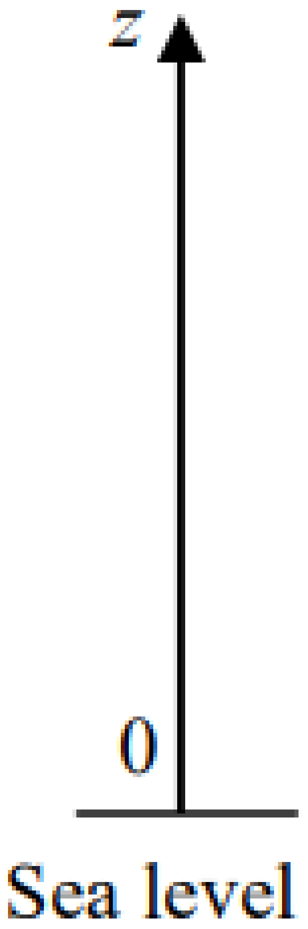
Analysis The drag force depends on a dimensionless drag coefficient, the air density, the car velocity, and the frontal area. Also, the unit of force F is newton N, which is equivalent to kg·m/s^2. Therefore, the independent quantities should be arranged such that we end up with the unit kg·m/s^2 for the drag force. Putting the given information into perspective, we have

$$F_D \text{ [kg·m/s}^2\text{]} \leftrightarrow C_{\text{Drag}} \text{ [ ]}, A_{\text{front}} \text{ [m}^2\text{]}, \rho \text{ [kg/m}^3\text{], and } V \text{ [m/s]}$$

It is obvious that the only way to end up with the unit "kg·m/s^2" for drag force is to multiply mass with the square of the velocity and the fontal area, with the drag coefficient serving as the constant of proportionality. Therefore, the desired relation is

$$F_D = C_{\text{Drag}} \rho A_{\text{front}} V^2$$

Discussion Note that this approach is not sensitive to dimensionless quantities, and thus a strong reasoning is required.



Q2- At 45° latitude, the gravitational acceleration as a function of elevation z above sea level is given by g =a - bz, where a =9.807 m/s^2 and b =3.32 x 10^{-6} s^{-2}. Determine the height above sea level where the weight of an object will decrease by 1 percent.

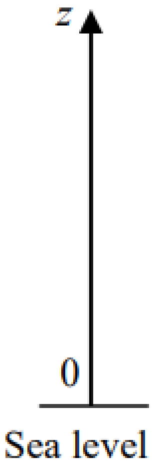
Solution The variation of gravitational acceleration above sea level is given as a function of altitude. The height at which the weight of a body decreases by 1% is to be determined.

Analysis The weight of a body at the elevation z can be expressed as

$$W = mg = m(a - bz)$$

where $a = g_s = 9.807$ m/s^2 is the value of gravitational acceleration at sea level and $b = 3.32\times10^{-6}$ s^{-2}. In our case,

$$W = m(a - bz) = 0.99W_s = 0.99mg_s$$



We cancel out mass from both sides of the equation and solve for z, yielding

$$z = \frac{a - 0.99g_s}{b}$$

Substituting,

$$z = \frac{9.807 \text{ m/s}^2 - 0.99\left(9.807 \text{ m/s}^2\right)}{3.32\times10^{-6} \text{ 1/s}^2} = 29{,}539 \text{ m} \cong \mathbf{29{,}500 \text{ m}}$$

where we have rounded off the final answer to three significant digits.

Discussion This is more than three times higher than the altitude at which a typical commercial jet flies, which is about 30,000 ft (9140 m). So, flying in a jet is not a good way to lose weight – diet and exercise are always the best bet.

FIGURE P2–13

Stockbyte/GettyImages

Q3-The pressure in an automobile tire depends on the temperature of the air in the tire. When the air temperature is 25°C, the pressure gage reads 210 kPa. If the volume of the tire is 0.025 m^3, determine the pressure rise in the tire when the air temperature in the tire rises to 50°C. Also, determine the amount of air that must be bled off to restore pressure to its original value at this temperature. Assume the atmospheric pressure to be 100 kPa.

Bir otomobil lastiğinin basıncı, lastik içerisindeki havanın sıcaklığına bağlıdır. Hava sıcaklığı 25°C iken etkin basınç 210 kPa'dır. Eğer lastiğin hacmi 0.025 m^3 ise, lastik içerisindeki sıcaklık 50°C'ye çıktığında meydana gelecek olan basınç artışını belirleyiniz. Ayrıca bu sıcaklıkta lastik içerisindeki basıncı başlangıçtaki değerine düşürmek için lastikten atılması gereken hava kütlesini belirleyiniz. Atmosfer basıncının 100 kPa olduğunu varsayınız.

Solution An automobile tire is inflated with air. The pressure rise of air in the tire when the tire is heated and the amount of air that must be bled off to reduce the temperature to the original value are to be determined.

Assumptions **1** At specified conditions, air behaves as an ideal gas. **2** The volume of the tire remains constant.

Properties The gas constant of air is $R = 0.287 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\left(\dfrac{\text{kPa}\cdot\text{m}^3}{\text{kJ}}\right) = 0.287 \dfrac{\text{kPa}\cdot\text{m}^3}{\text{kg}\cdot\text{K}}$.

Analysis Initially, the absolute pressure in the tire is

$$P_1 = P_g + P_{atm} = 210 + 100 = 310\ \text{kPa}$$

Treating air as an ideal gas and assuming the volume of the tire to remain constant, the final pressure in the tire is determined from

$$\frac{P_1 \mathbf{V}_1}{T_1} = \frac{P_2 \mathbf{V}_2}{T_2} \longrightarrow P_2 = \frac{T_2}{T_1} P_1 = \frac{323\text{K}}{298\text{K}}(310\text{kPa}) = 336\text{kPa}$$

Thus the pressure rise is

$$\Delta P = P_2 - P_1 = 336 - 310 = \mathbf{26.0\ kPa}$$

The amount of air that needs to be bled off to restore pressure to its original value is

$$m_1 = \frac{P_1 \mathbf{V}}{RT_1} = \frac{(336\ \text{kPa})(0.025\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323\text{K})} = 0.0906\text{kg}$$

$$m_2 = \frac{P_2 \mathbf{V}}{RT_2} = \frac{(310\text{kPa})(0.025\text{m}^3)}{(0.287\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(323\text{K})} = 0.0836\text{kg}$$

$$\Delta m = m_1 - m_2 = 0.0906 - 0.0836 = \mathbf{0.0070kg}$$

Discussion Notice that *absolute* rather than gage pressure must be used in calculations with the ideal gas law.

Q4-The ideal gas equation of state is very simple, but its range of applicability is limited. A more accurate but complicated equation is the Van der Waals equation of state given by

$$P = \frac{RT}{v - b} - \frac{a}{v^2}$$

where a and b are constants depending on critical pressure and temperatures of the gas. Predict the coefficient of compressibility of nitrogen gas at T =175 K and v= 0.00375 m^3 /kg, assuming the nitrogen to obey the Van der Waals equation of state. Compare your result with the ideal gas value. Take a = 0.175 m^6.kPa/ kg^2 and b = 0.00138 m^3 /kg for the given conditions. The experimentally measured pressure of nitrogen is 10,000 kPa.

Bir akışkanın üzerine basınç uygulandığında sıkıştığı, öte yandan akışkanın üzerindeki basınç düşürüldüğünde ise akışkanın genleştiği bilinen bir özelliktir. Diğer bir değişle akışkanlar basınca karşı elastik katılar gibi davranırlar. Dolayısıyla katılar için tanımlanan Young elastik modülüne benzer şekilde akışkanlar içinde **sıkıştırılabilirlik katsayısı K** (coefficient of compressibility) tanımlanabilir.

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \text{(Pa)}$$

Solution The coefficient of compressibility of nitrogen gas is to be estimated using Van der Waals equation of state. The result is to be compared to ideal gas and experimental values.

Assumptions 1 Nitrogen gas obeys the Van der Waals equation of state.

Analysis From the definition we have

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \frac{vRT}{(v-b)^2} - \frac{2a}{v^2}$$

since

$$\left(\frac{\partial P}{\partial v}\right)_T = \frac{2a}{v^3} - \frac{RT}{(v-b)^2}$$

The gas constant of nitrogen is $0.2968\ \text{kJ/kg}\cdot\text{K}$ (Table A–1). Substituting given data we obtain

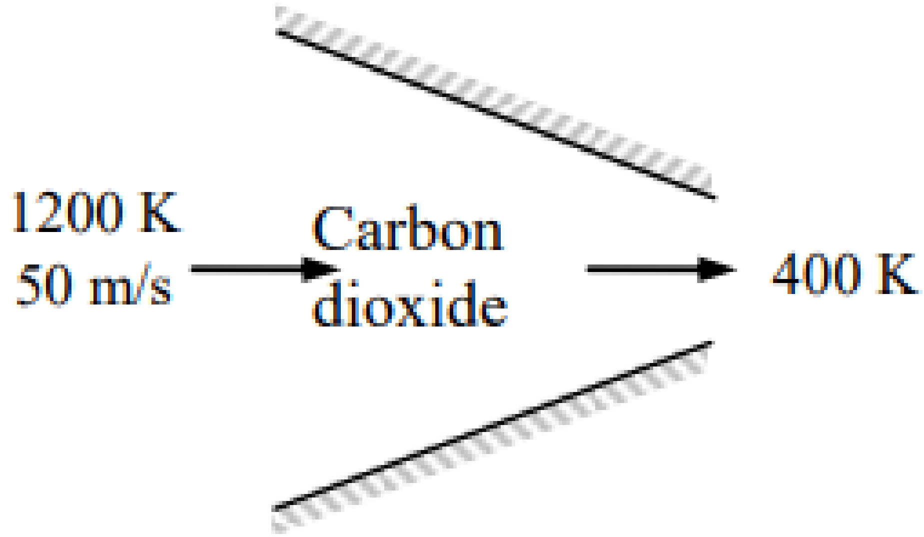
$$\kappa = \frac{\left(0.00375\,\frac{\text{m}^3}{\text{kg}}\right)\times\left(0.2968\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\times(175\ \text{K})}{\left(0.00375\,\frac{\text{m}^3}{\text{kg}} - 0.00138\,\frac{\text{m}^3}{\text{kg}}\right)^2} - \frac{2\times 0.175\ \text{m}^6\cdot\frac{\text{kPa}}{\text{kg}^2}}{\left(0.00375\,\frac{\text{m}^3}{\text{kg}}\right)^2} \cong 9788\ \text{kPa}$$

For the ideal gas behavior, the coefficient of compressibility is equal to the pressure (Eq. 2–15). Therefore we get

$$\kappa = P = \frac{RT}{v} = \frac{\left(0.2968\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\times(175\ \text{K})}{0.00375\ \text{m}^3/\text{kg}} \cong 13851\ \text{kPa}$$

whichis in error by 38.5% compared to experimentally measured pressure.

Q5-Carbon dioxide enters an adiabatic nozzle at 1200 K with a velocity of 50 m/s and leaves at 400 K. Assuming constant specific heats at room temperature, determine the Mach number **(a)** at the inlet and (**b)** at the exit of the nozzle. Assess the accuracy of the constant specific heat approximation.



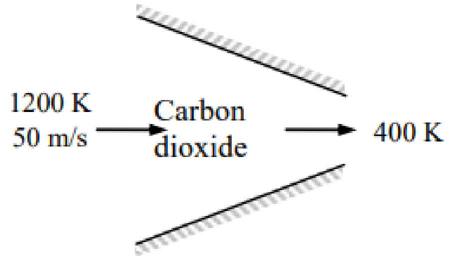
Solution Carbon dioxide flows through a nozzle. The inlet temperature and velocity and the exit temperature of CO_2 are specified. The Mach number is to be determined at the inlet and exit of the nozzle.

Assumptions **1** CO_2 is an ideal gas with constant specific heats at room temperature. **2** This is a steady-flow process.

Properties The gas constant of carbon dioxide is R = 0.1889 kJ/kg·K. Its constant pressure specific heat and specific heat ratio at room temperature are c_p = 0.8439 kJ/kg·K and k = 1.288.

Analysis (a) At the inlet

$$c_1 = \sqrt{k_1 RT_1} = \sqrt{(1.288)(0.1889\text{ kJ/kg}\cdot\text{K})(1200\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 540.3\text{ m/s}$$



Thus,

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{50\text{ m/s}}{540.3\text{ m/s}} = \mathbf{0.0925}$$

(b) At the exit,

$$c_2 = \sqrt{k_2 RT_2} = \sqrt{(1.288)(0.1889\text{ kJ/kg}\cdot\text{K})(400\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 312.0\text{ m/s}$$

The nozzle exit velocity is determined from the steady-flow energy balance relation,

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \quad \rightarrow \quad 0 = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$0 = (0.8439\text{ kJ/kg}\cdot\text{K})(400 - 1200\text{ K}) + \frac{V_2^2 - (50\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) \longrightarrow V_2 = 1163\text{ m/s}$$

Thus,

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{1163\text{ m/s}}{312\text{ m/s}} = \mathbf{3.73}$$

Discussion The specific heats and their ratio k change with temperature, and the accuracy of the results can be improved by accounting for this variation. Using EES (or another property database):

At 1200 K: c_p = 1.278 kJ/kg·K, k = 1.173 $\rightarrow$ c_1 = 516 m/s, V_1 = 50 m/s, Ma_1 = 0.0969

At 400 K: c_p = 0.9383 kJ/kg·K, k = 1.252 $\rightarrow$ c_2 = 308 m/s, V_2 = 1356 m/s, Ma_2 = 4.41

Therefore, the constant specific heat assumption results in an error of **4.5%** at the inlet and **15.5%** at the exit in the Mach number, which are significant.

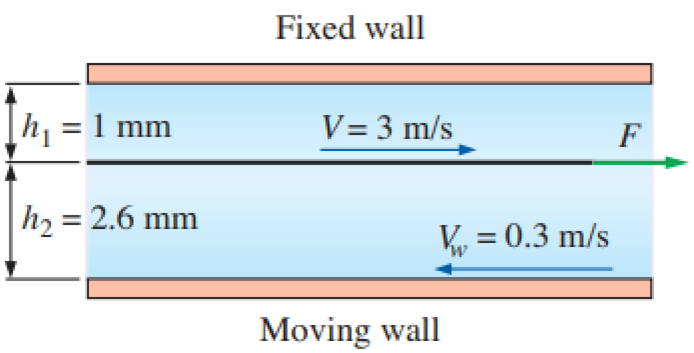


FIGURE P2–81

Q6-A thin 30-cm 3 30-cm flat plate is pulled at 3 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity of 0.3 m/s, as shown in Fig. P2–81. The dynamic viscosity of the oil is 0.027 Pa.s. Assuming the velocity in each oil layer to vary linearly, (a) plot the velocity profile and find the location where the oil velocity is zero and (b) determine the force that needs to be applied on the plate to maintain this motion.

20 cm × 20 cm boyutlarında ince düzlemsel levha, araları 3.6 mm kalınlığında bir yağ tabakası ile dolu paralel iki levha arasında 1 m/s hızla yatay olarak çekilmektedir. Levhalardan biri sabit, diğeri ise Şekil P2–81'de gösterildiği gibi 0.3 m/s sabit hızla sola doğru hareket etmektedir. Aradaki yağın dinamik viskozitesi 0.027 Pa·s olduğuna ve her bir yağ tabakasındaki hızın doğrusal değiştiği kabul edildiğine göre; (*a*) her bir tabakadaki hız profilini çizerek yağ hızının sıfır olduğu konumu bulunuz ve (*b*) bu hareketi sürdürebilmek için uygulanması gerekli olan *F* kuvvetini belirleyiniz.

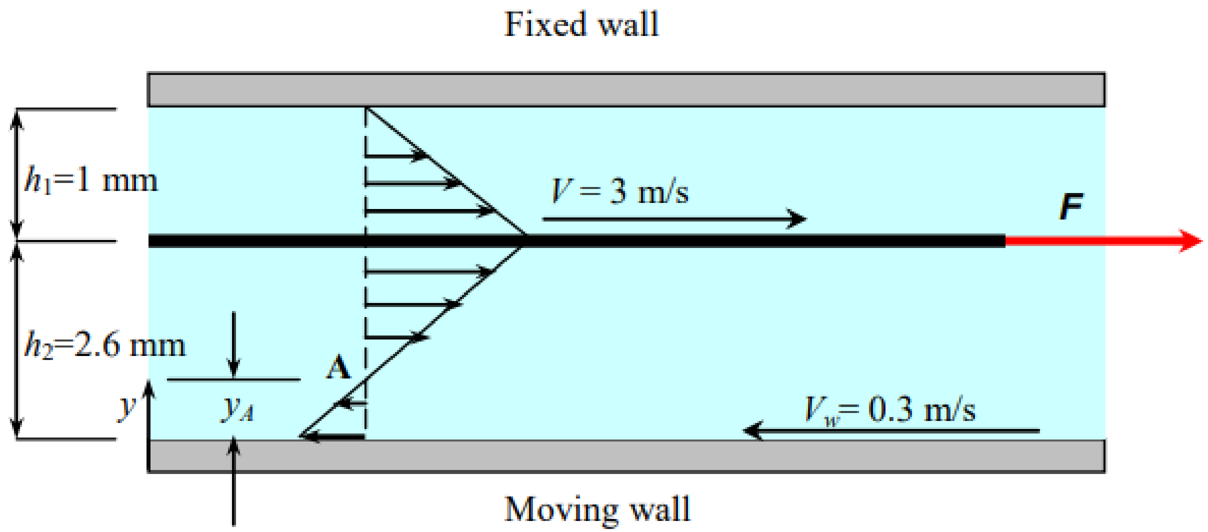
Solution A thin flat plate is pulled horizontally through an oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity. The location in oil where the velocity is zero and the force that needs to be applied on the plate are to be determined.

Assumptions **1** The thickness of the plate is negligible. **2** The velocity profile in each oil layer is linear.

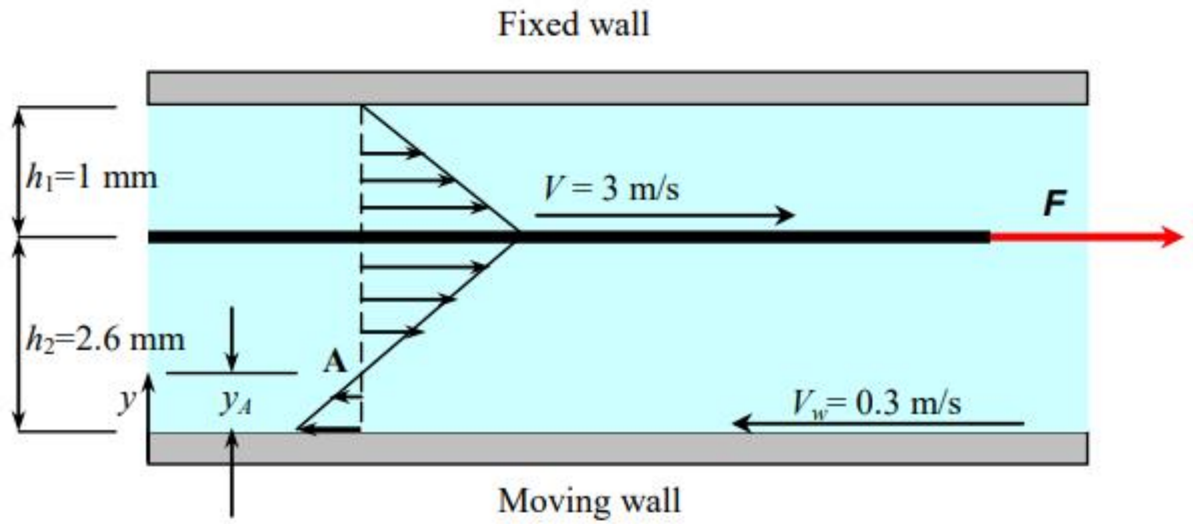
Properties The absolute viscosity of oil is given to be $\mu = 0.027$ Pa·s $= 0.027$ N·s/m^2.

Analysis (*a*) The velocity profile in each oil layer relative to the fixed wall is as shown in the figure below. The point of zero velocity is indicated by point *A*, and its distance from the lower plate is determined from geometric considerations (the similarity of the two triangles in the lower oil layer) to be

$$\frac{2.6 - y_A}{y_A} = \frac{3}{0.3} \quad \rightarrow \quad y_A = \mathbf{0.23636\ mm}$$



(*b*) The magnitudes of shear forces acting on the upper and lower surfaces of the plate are



(*b*) The magnitudes of shear forces acting on the upper and lower surfaces of the plate are

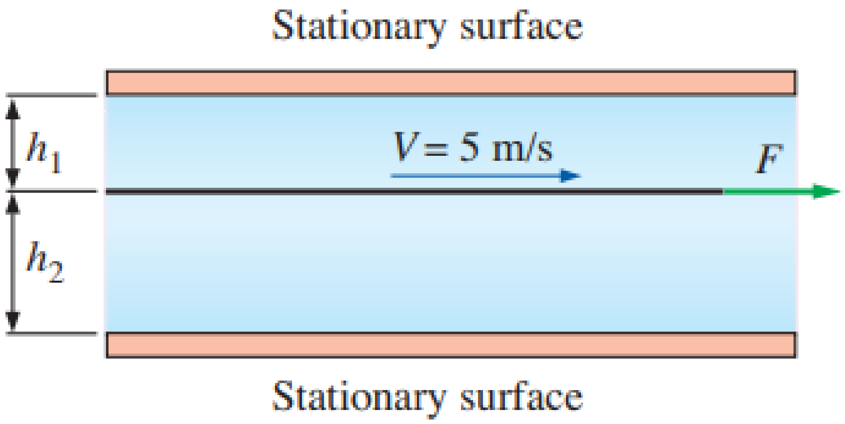
$$F_{\text{shear, upper}} = \tau_{w,\text{upper}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_1} = (0.027\ \text{N}\cdot\text{s/m}^2)(0.3\times 0.3\ \text{m}^2)\frac{3\ \text{m/s}}{1.0\times 10^{-3}\ \text{m}} = 7.29\ \text{N}$$

$$F_{\text{shear, lower}} = \tau_{w,\text{lower}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-V_w}{h_2} = (0.027\ \text{N}\cdot\text{s/m}^2)(0.3\times 0.3\ \text{m}^2)\frac{[3-(-0.3)]\ \text{m/s}}{2.6\times 10^{-3}\ \text{m}} = 3.08\ \text{N}$$

Noting that both shear forces are in the opposite direction of motion of the plate, the force F is determined from a force balance on the plate to be

$$F = F_{\text{shear, upper}} + F_{\text{shear, lower}} = 7.29 + 3.08 = \mathbf{10.4\,N}$$

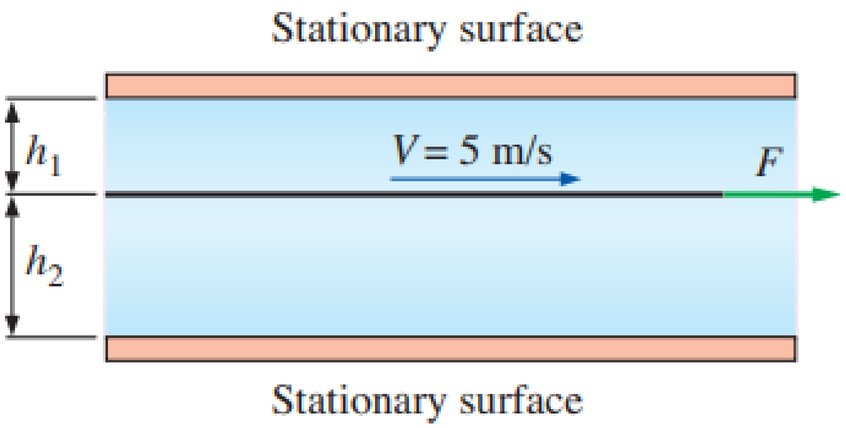
Discussion Note that wall shear is a friction force between a solid and a liquid, and it acts in the opposite direction of motion.



Q7-A thin plate moves between two parallel, horizontal, stationary flat surfaces at a constant velocity of 5 m/s. The two stationary surfaces are spaced 4 cm apart, and the medium between them is filled with oil whose viscosity is 0.9 N.s/ m^2 . The part of the plate immersed in oil at any given time is 2-m long and 0.5-m wide. If the plate moves through the mid-plane between the surfaces, determine the force required to maintain this motion.

- What would your response be if the plate was 1 cm from the bottom surface (h2) and 3 cm from the top surface (h1)?
- b) If the viscosity of the oil above the moving plate is 4 times that of the oil below the plate, determine the distance of the plate from the bottom surface (h2) that will minimize the force needed to pull the plate between the two oils at constant velocity.

İnce bir levha, birbirine paralel ve aralarında 4 cm kalınlığında dinamik viskozitesi 0.90 $N \cdot s/m^2$ olan bir yağ tabakası bulunan iki sabit levha arasında Şekil P2–94'te gösterildiği gibi 5 m/s sabit hızla çekilmektedir. Çekilen levhanın yağ içerisinde kalan kısmı 2 m uzunluğunda ve 0.5 m genişliğindedir. Bu levhanın yağ filminin tam ortasından çekilmesi durumunda gerekli kuvvet kaç N olur? Eğer levha üstteki levhadan $h_1 =$ 1 cm, alttaki levhadan ise $h_2 = 3$ cm uzakta olacak şekilde çekilmiş olsaydı cevabınız ne olurdu?



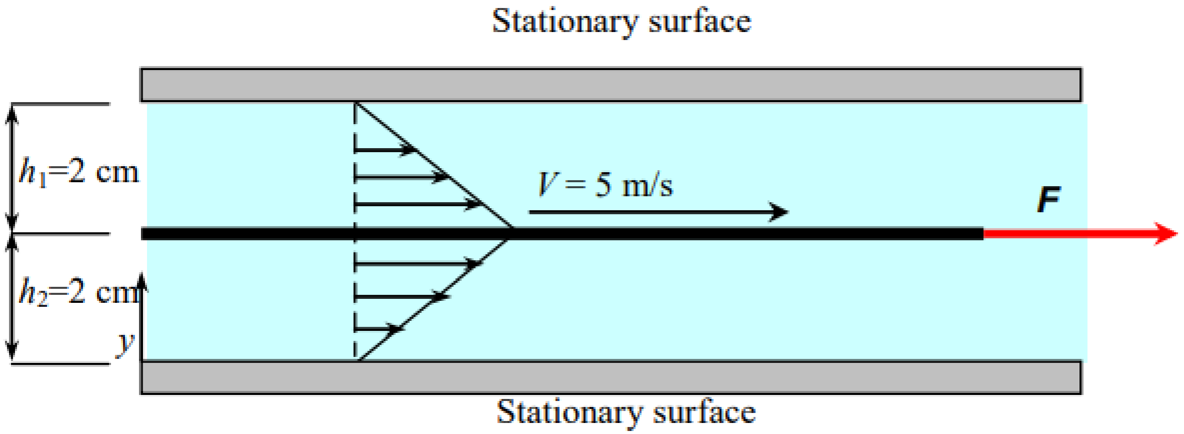
b) Problem 2–94'ü tekrar göz önüne alınız. Bu sefer çekilen levhanın üzerindeki akışkanın dinamik viskozitesi, levhanın altındaki akışkanın dinamik viskozitesinin 4 katı olsun. Bu durumda levhayı iki yağ arasında sabit bir hızla minimum kuvvetle çekebilmek için h_2 mesafesi ne olmalıdır?

Solution A thin flat plate is pulled horizontally through the mid plane of an oil layer sandwiched between two stationary plates. The force that needs to be applied on the plate to maintain this motion is to be determined for this case and for the case when the plate .

Assumptions **1** The thickness of the plate is negligible. **2** The velocity profile in each oil layer is linear.

Properties The absolute viscosity of oil is given to be $\mu = 0.9\ \text{N·s/m}^2$.

Analysis The velocity profile in each oil layer relative to the fixed wall is as shown in the figure.



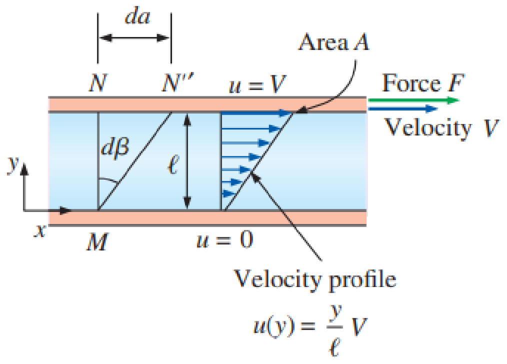


FIGURE 2–24
The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and V, and thus the velocity profile and the velocity gradient are

$$u(y) = \frac{y}{\ell}V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$

The magnitudes of shear forces acting on the upper and lower surfaces of the moving thin plate are

$$F_{\text{shear, upper}} = \tau_{w,\text{ upper}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_1} = (0.9\ \text{N}\cdot\text{s/m}^2)(0.5\times 2\ \text{m}^2)\frac{5\ \text{m/s}}{0.02\ \text{m}} = 225\ \text{N}$$

$$F_{\text{shear, lower}} = \tau_{w,\text{ lower}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-V_w}{h_2} = (0.9\ \text{N}\cdot\text{s/m}^2)(0.5\times 2\ \text{m}^2)\frac{5\ \text{m/s}}{0.02\ \text{m}} = 225\ \text{N}$$

Noting that both shear forces are in the opposite direction of motion of the plate, the force F is determined from a force balance on the plate to be

$$F = F_{\text{shear, upper}} + F_{\text{shear, lower}} = 225 + 225 = \mathbf{450N}$$

When the plate is 1 cm from the bottom surface and 3 cm from the top surface, the force F becomes

$$F_{\text{shear, upper}} = \tau_{w,\text{ upper}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_1} = (0.9\ \text{N}\cdot\text{s/m}^2)(0.5\times 2\ \text{m}^2)\frac{5\ \text{m/s}}{0.03\,\text{m}} = 150\ \text{N}$$

$$F_{\text{shear, lower}} = \tau_{w,\text{ lower}} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_2} = (0.9\ \text{N}\cdot\text{s/m}^2)(0.5\times 2\ \text{m}^2)\frac{5\ \text{m/s}}{0.01\,\text{m}} = 450\ \text{N}$$

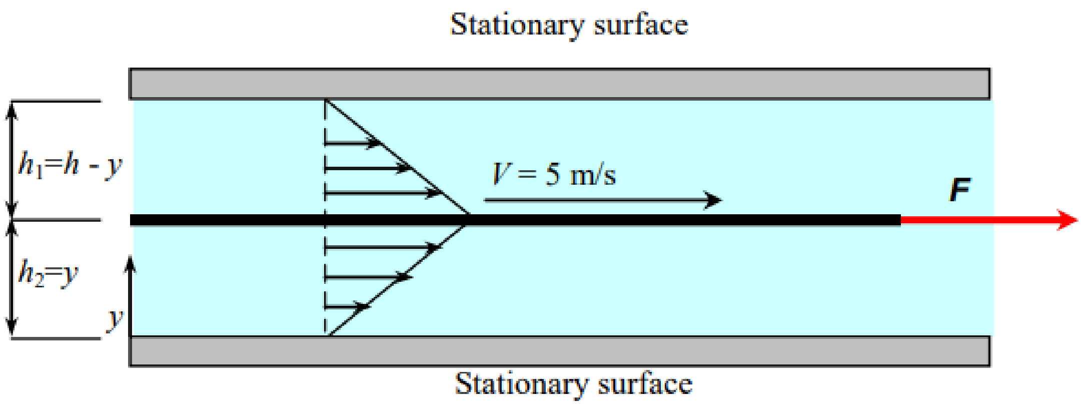
Noting that both shear forces are in the opposite direction of motion of the plate, the force F is determined from a force balance on the plate to be

$$F = F_{\text{shear, upper}} + F_{\text{shear, lower}} = 150 + 450 = \mathbf{600\,N}$$

Discussion Note that the relative location of the thin plate affects the required force significantly.

Properties The absolute viscosity of oil is $\mu = 0.9\ \text{N·s/m}^2$ in the lower part, and 4 times that in the upper part.

Analysis We measure vertical distance y from the lower plate. The total distance between the stationary plates is $h = h_1 + h_2 = 4\ \text{cm}$, which is constant. Then the distance of the moving plate is y from the lower plate and $h - y$ from the upper plate, where y is variable.



The shear forces acting on the upper and lower surfaces of the moving thin plate are

$$F_{\text{shear, upper}} = \tau_{w,\text{ upper}} A_s = \mu_{\text{upper}} A_s \left|\frac{du}{dy}\right| = \mu_{\text{upper}} A_s \frac{V}{h-y}$$

$$F_{\text{shear,lower}} = \tau_{w,\text{ lower}} A_s = \mu_{\text{lower}} A_s \left|\frac{du}{dy}\right| = \mu_{\text{lower}} A_s \frac{V}{y}$$

Then the total shear force acting on the plate becomes

$$F = F_{\text{shear, upper}} + F_{\text{shear,lower}} = \mu_{\text{upper}} A_s \frac{V}{h-y} + \mu_{\text{lower}} A_s \frac{V}{h-y} = A_s V \left(\frac{\mu_{\text{upper}}}{h-y} + \frac{\mu_{\text{lower}}}{y} \right)$$

The value of y that will minimize the force F is determined by setting $\dfrac{dF}{dy} = 0$:

$$\frac{\mu_{\text{upper}}}{(h-y)^2} - \frac{\mu_{\text{lower}}}{y^2} = 0 \quad \rightarrow \quad \frac{y}{h-y} = \sqrt{\frac{\mu_{\text{lower}}}{\mu_{\text{upper}}}}$$

Solving for y and substituting, the value of y that minimizes the shear force is determined to be

$$y = \frac{\sqrt{\mu_{\text{lower}} / \mu_{\text{upper}}}}{1 - \sqrt{\mu_{\text{lower}} / \mu_{\text{upper}}}} h = \frac{\sqrt{1/4}}{1 - \sqrt{1/4}} (4\ \text{cm}) = \mathbf{1\ cm}$$

Q8- Two immiscible Newtonian liquids flow steadily between two large parallel plates under the influence of an applied pressure gradient. The lower plate is fixed while the upper one is pulled with a constant velocity of $U = 10$ m/s. The thickness, h, of each layer of fluid is 0.5 m. The velocity profile for each layer is given by

$$V_1 = 6 + ay - 3y^2, \qquad -0.5 \le y \le 0$$

$$V_2 = b + cy - 9y^2, \qquad 0 \le y \le -0.5$$

where a, b, and c are constants.

(a) Determine the values of constants a, b, and c.

(b) Develop an expression for the viscosity ratio, e.g., $\mu_1/\mu_2 =$?

(c) Determine the forces and their directions exerted by the liquids on both plates if $\mu_1 = 10^{-3}$ Pa·s and each plate has a surface area of 4 m^2.

Birbirine karışmayan iki Newton tipi sıvı, paralel iki geniş levha arasında uygulanan bir basınç gradyeni nedeniyle daimi olarak akmaktadır. Alttaki levha sabit olup üstteki levha sabit bir $U = 10$ m/s hızla çekilmektedir. Her bir akışkan tabakasının kalınlığı 0.50 m'dir. Tabakalar içerisinde gelişen hız profilleri ise a, b ve c birer sabit olmak üzere aşağıdaki bağıntılarla verilmektedir:

$$V_1 = 6 + ay - 3y^2, \qquad 0 \le y \le 0.5 \text{ m}$$

$$V_2 = b + cy - 9y^2, \qquad -0.5 \text{ m} \le y \le 0$$

(a) Hız profillerinde yer alan a, b ve c sabitlerini belirleyiniz.

(b) Viskozite oranı μ_1/μ_2 için bir bağıntı geliştiriniz.

(c) Levhaların alanları 4 m^2 ve $\mu_1 = 10^{-3}$ Pa·s olduğuna göre, akışkanların levhalar üzerine uyguladığı kuvvetleri ve bu kuvvetlerin yönlerini belirleyiniz.

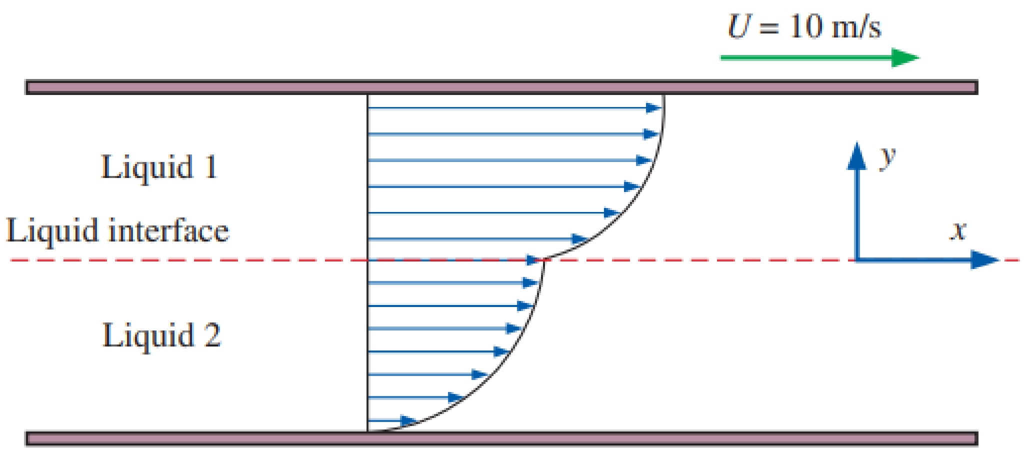


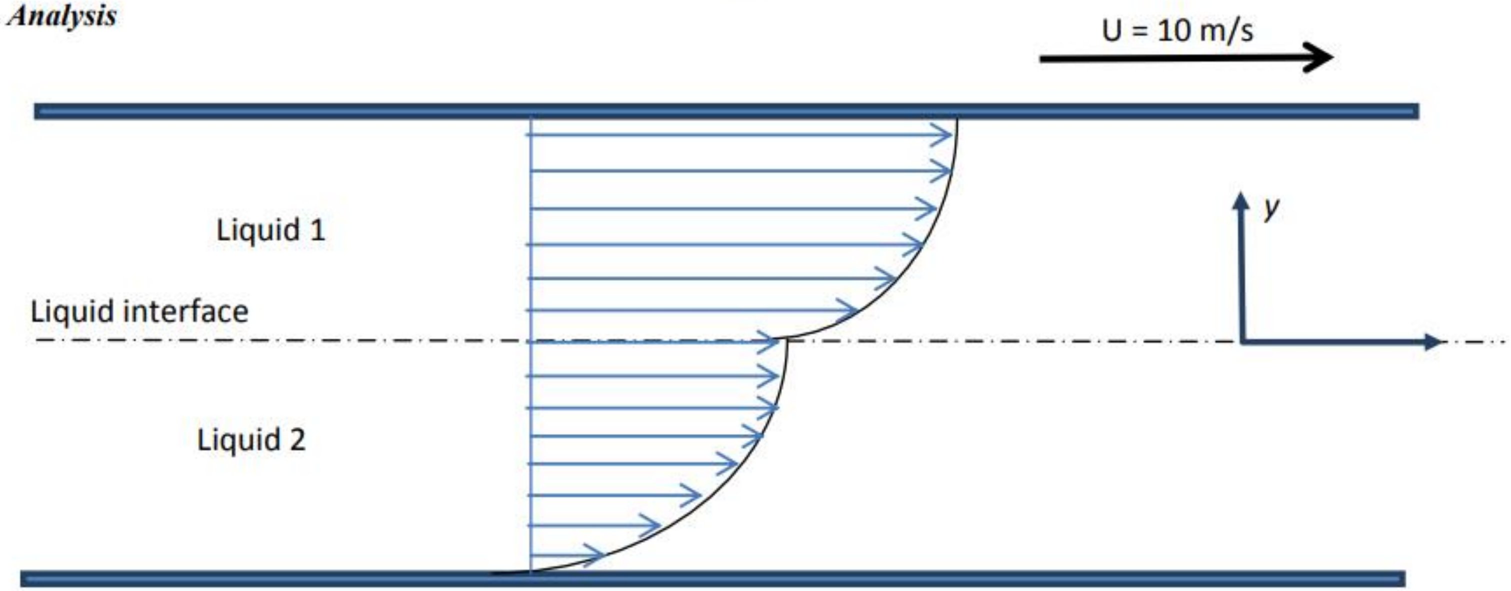
FIGURE P2–126

Solution Two immiscible Newtonian liquids flow steadily between two large parallel plates under the influence of an applied pressure gradient. The lower plate is fixed while the upper one is pulled with a constant velocity. The velocity profiles for each flow are given. The values of constants are to be determined. An expression for the viscosity ratio is to be developed. The forces and their directions exerted by liquids on both plates are to be determined.

Assumptions **1** The flow between the plates is one-dimensional. **2** The fluids are Newtonian.

Properties The viscosity of fluid one is given to be $\mu_1 = 10^{-3}\ \mathrm{Pa \cdot s}$.

Analysis



(a) The velocity profiles should satisfy the conditions $V_1(h) = 10,\ V_2(-h) = 0$ and $V_1(0) = V_2(0)$. It is clear that

$$V_1(0) = 6\ \mathrm{m/s}.$$

$$V_1(h) = 10:\ \ 10 = 6 + a \times 0.5 - 3 \times (0.5)^2 \rightarrow a = 9.5$$

$$V_2(0) = 6 = b + c \times 0 - 9(0)^2 \rightarrow b = 6$$

Finally,

$$V_2(-h) = 0 \rightarrow 0 = 6 + c \times (-0.5) - 9(-0.5)^2 \rightarrow c = 7.5$$

Therefore we have the velocity profiles as follows:

$$V_1 = 6 + 9.5y - 3y^2, \qquad -0.5 \le y \le 0$$
$$V_2 = 6 + 7.5y - 9y^2, \qquad 0 \le y \le -0.5$$

Therefore we have the velocity profiles as follows:

$$V_1 = 6 + 9.5y - 3y^2, \qquad -0.5 \le y \le 0$$
$$V_2 = 6 + 7.5y - 9y^2, \qquad 0 \le y \le -0.5$$

(b) The shear stress at the interface is unique, and then we have

$$\mu_1 \frac{dV_1}{dy}\bigg]_{y=0} = \mu_2 \frac{dV_2}{dy}\bigg]_{y=0} \rightarrow \frac{\mu_1}{\mu_2} = \frac{\left.\frac{dV_2}{dy}\right]_{y=0}}{\left.\frac{dV_1}{dy}\right]_{y=0}} = \frac{7.5 - 18y}{9.5 - 6y}\bigg]_{y=0} \cong 0.79$$

(c)

Lower plate:

$$F_L = \mu_2 \frac{dV_2}{dy}\bigg]_{y=-h} A = \left(\frac{10^{-3}\mathrm{N\cdot s/m^2}}{0.79}\right) \times \underbrace{[7.5 - 18y]_{y=-0.5}}_{16.5} \times (4\ \mathrm{m^2}) = 0.0835\ \mathrm{N}$$

it acts in the opposite direction of motion.

Upper plate:

$$F_U = \mu_1 \frac{dV_1}{dy}\bigg]_{y=h} A = (10^{-3}\mathrm{N\cdot s/m^2}) \times \underbrace{[9.5 - 6y]_{y=\ 0.5}} \times (4\ \mathrm{m^2}) = 0.026\ \mathrm{N}$$

Q9-The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. P3–12. Determine the gage pressure of air in the tank if h1 =0.4 m, h2 = 0.6 m, and h3 = 0.8 m. Take the densities of water, oil, and mercury to be 1000 kg/ m^3, 850 kg/ m^3, and 13,600 kg/ m^3, respectively.

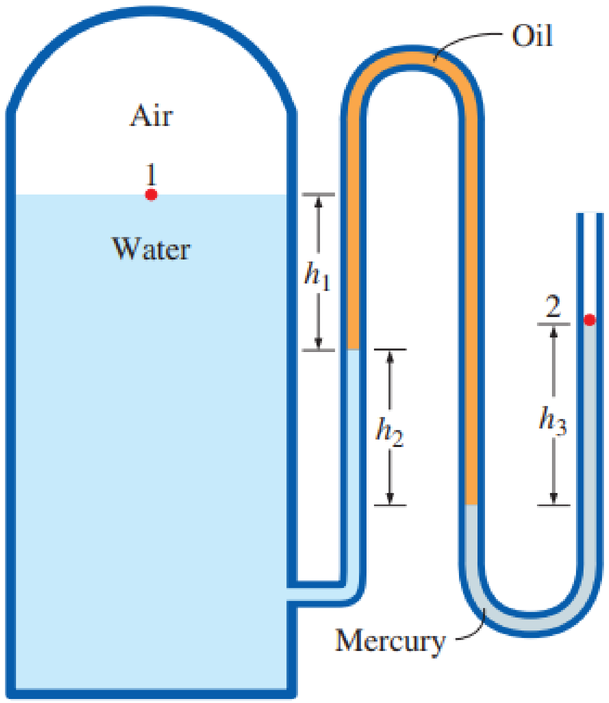


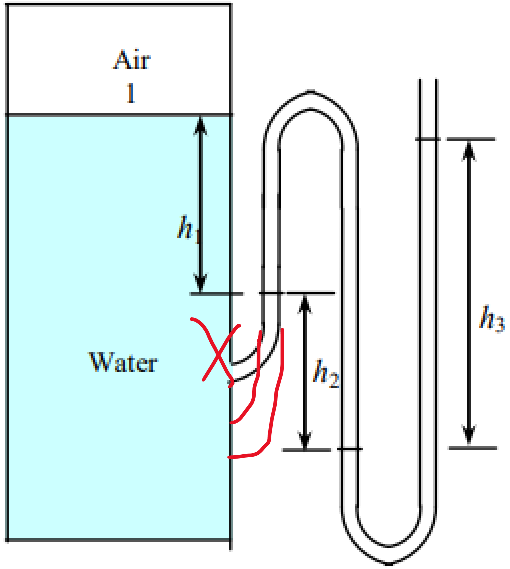
FIGURE P3–12

Solution The pressure in a pressurized water tank is measured by a multi-fluid manometer. The gage pressure of air in the tank is to be determined.

Assumptions The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air-water interface.

Properties The densities of mercury, water, and oil are given to be 13,600, 1000, and 850 kg/m^3, respectively.

Analysis Starting with the pressure at point 1 at the air-water interface, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach point 2, and setting the result equal to P_{atm} since the tube is open to the atmosphere gives



$$P_1 + \rho_{\text{water}} g h_1 + \rho_{\text{oil}} g h_2 - \rho_{\text{mercury}} g h_3 = P_{atm}$$

Solving for P_1,

$$P_1 = P_{\text{atm}} - \rho_{\text{water}} g h_1 - \rho_{\text{oil}} g h_2 + \rho_{\text{mercury}} g h_3$$

or,

$$P_1 - P_{\text{atm}} = g(\rho_{\text{mercury}} h_3 - \rho_{\text{water}} h_1 - \rho_{\text{oil}} h_2)$$

Noting that $P_{1,\text{gage}} = P_1 - P_{\text{atm}}$ and substituting,

$$P_{1,gage} = (9.81\ \text{m/s}^2)[(13{,}600\ \text{kg/m}^3)(0.8\ \text{m}) - (1000\ \text{kg/m}^3)(0.4\ \text{m})$$

$$- (850\ \text{kg/m}^3)(0.6\ \text{m})]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= \mathbf{97.8\,kPa}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis greatly.

Q10-The basic barometer can be used to measure the height of a building. If the barometric readings at the top and at the bottom of a building are 730 and 755 mmHg, respectively, determine the height of the building. Assume an average air density of 1.18 kg/ m^3.

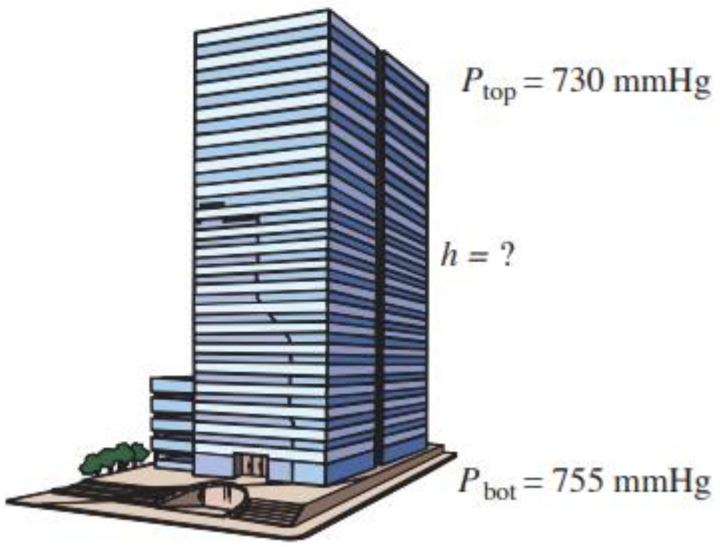


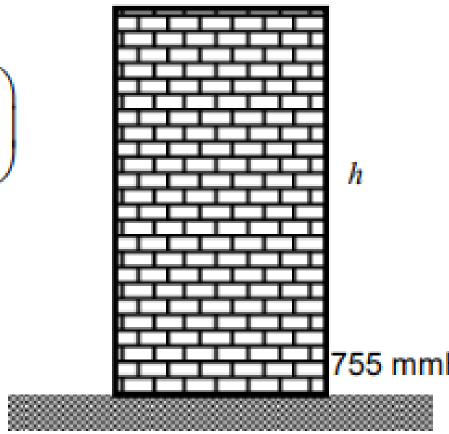
FIGURE P3–26

Solution A barometer is used to measure the height of a building by recording reading at the bottom and at the top of the building. The height of the building is to be determined.

Assumptions The variation of air density with altitude is negligible.

Properties The density of air is given to be $\rho = 1.18$ kg/m^3. The density of mercury is 13,600 kg/m^3.

Analysis Atmospheric pressures at the top and at the bottom of the building are



$$P_{top} = (\rho g h)_{top}$$
$$= (13{,}600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.730 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right)$$
$$= 97.36 \text{ kPa}$$

$$P_{bottom} = (\rho g h)_{bottom}$$
$$= (13{,}600 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(0.755 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right)$$
$$= 100.70 \text{ kPa}$$

Taking an air column between the top and the bottom of the building, we write a force balance per unit base area,

$$W_{air} / A = P_{bottom} - P_{top} \qquad \text{and} \qquad (\rho g h)_{air} = P_{bottom} - P_{top}$$

$$(1.18 \text{ kg/m}^3)(9.807 \text{ m/s}^2)(h)\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) = (100.70 - 97.36) \text{ kPa}$$

which yields $h = 288.6 \text{ m} \cong \mathbf{289\ m}$, which is also the height of the building.

Discussion There are more accurate ways to measure the height of a building, but this method is quite simple.

Q11-Two chambers with the same fluid at their base are separated by a 30-cm-diameter piston whose weight is 25 N, as shown in Fig. P3–54. Calculate the gage pressures in chambers A and B

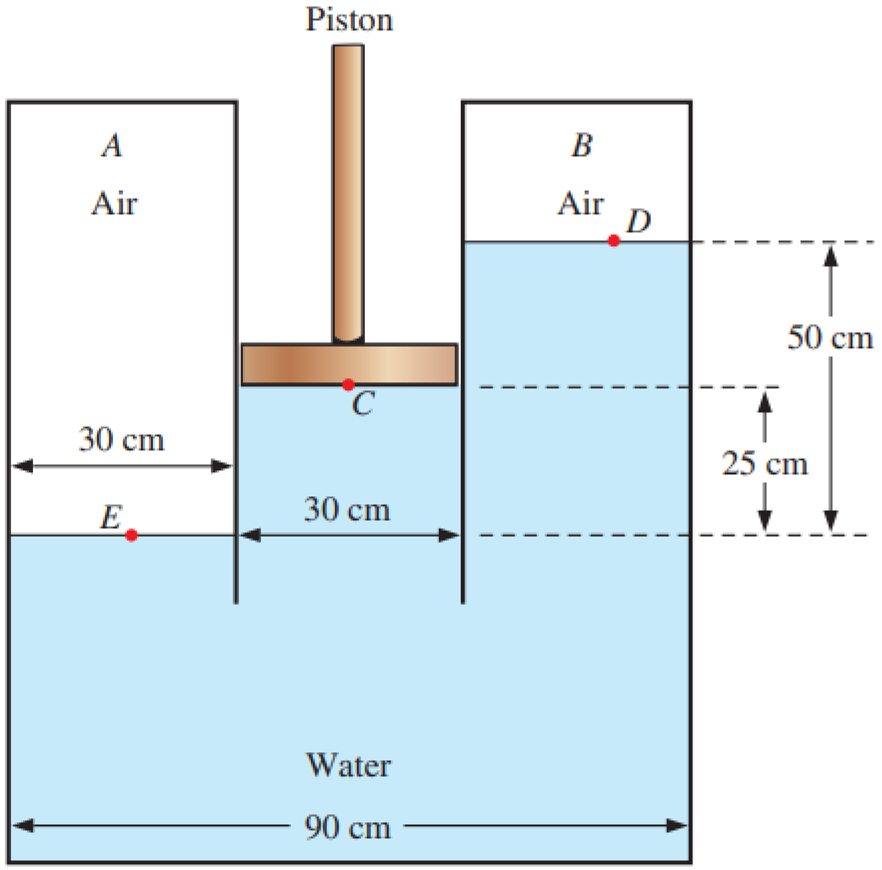
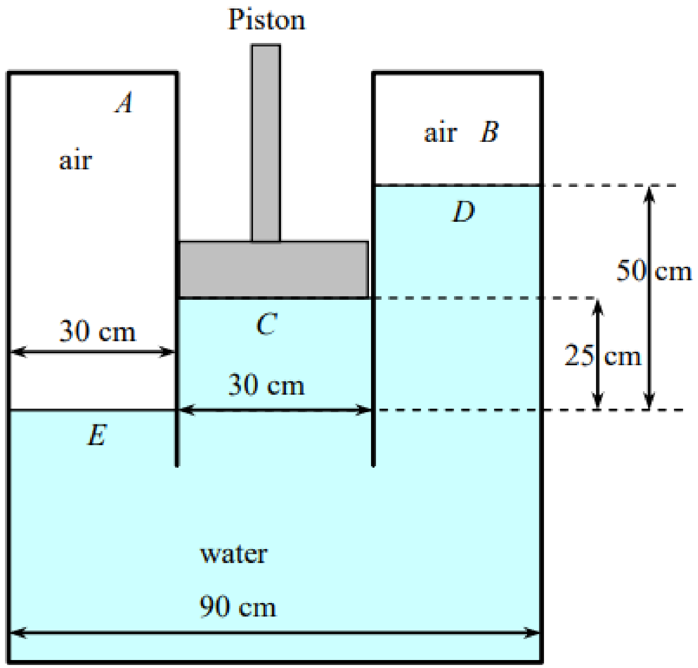


FIGURE P3–54

Solution Two chambers with the same fluid at their base are separated by a piston. The gage pressure in each air chamber is to be determined.



Assumptions **1** Water is an incompressible substance. **2** The variation of pressure with elevation in each air chamber is negligible because of the low density of air.

Properties We take the density of water to be ρ =1000 kg/m^3.

Analysis The piston is in equilibrium, and thus the net force acting on the piston must be zero. A vertical force balance on the piston involves the pressure force exerted by water on the piston face, the atmospheric pressure force, and the piston weight, and yields

$$P_C A_{\text{piston}} = P_{\text{atm}} A_{\text{piston}} + W_{\text{piston}} \quad \rightarrow \quad P_C = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}}$$

The pressure at the bottom of each air chamber is determined from the hydrostatic pressure relation to be

$$P_{\text{air A}} = P_E = P_C + \rho g \overline{CE} = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g \overline{CE} \quad \rightarrow \quad P_{\text{air A, gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} + \rho g \overline{CE}$$

$$P_{\text{air B}} = P_D = P_C - \rho g \overline{CD} = P_{\text{atm}} + \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g \overline{CD} \quad \rightarrow \quad P_{\text{air B,gage}} = \frac{W_{\text{piston}}}{A_{\text{piston}}} - \rho g \overline{CD}$$

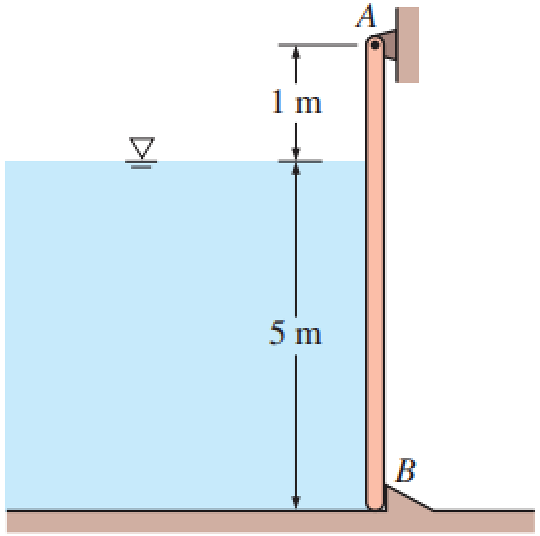
Substituting,

$$P_{\text{air A, gage}} = \frac{25 \text{ N}}{\pi (0.3 \text{ m})^2 / 4} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 2806 \text{ N/m}^2 = \mathbf{2.81 \text{ kPa}}$$

$$P_{\text{air B, gage}} = \frac{25 \text{ N}}{\pi (0.3 \text{ m})^2 / 4} - (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m})\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = -2099 \text{ N/m}^2 = \mathbf{-2.10 \text{ kPa}}$$

Discussion Note that there is a vacuum of about 2 kPa in tank B which pulls the water up.

Q12-A 6-m-high, 5-m-wide rectangular plate blocks the end of a 5-m-deep freshwater channel, as shown in Fig. P3–75. The plate is hinged about a horizontal axis along its upper edge through a point A and is restrained from opening by a fixed ridge at point B. Determine the force exerted on the plate by the ridge.



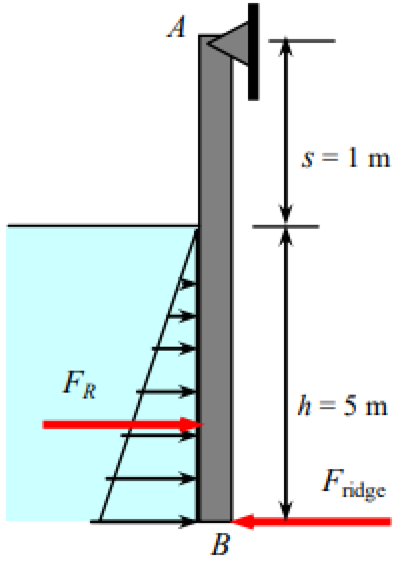
6 m yüksekliğinde ve 5 m genişliğindeki dikdörtgen levha Şekil P3–75'te gösterildiği gibi 5 m derinliğindeki tatlı su geçişini kapatmaktadır. Levha üst kenarındaki *A* noktasından geçen yatay bir eksen boyunca mafsallanmış olup açılması *B* noktasındaki sabit bir çıkıntı ile engellenmektedir. Çıkıntı tarafından levhaya uygulanan kuvveti hesaplayınız.

Solution A rectangular plate hinged about a horizontal axis along its upper edge blocks a fresh water channel. The plate is restrained from opening by a fixed ridge at a point *B*. The force exerted to the plate by the ridge is to be determined.

Assumptions Atmospheric pressure acts on both sides of the plate, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = \rho g(h/2)$$

$$= (1000\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(5/2\,\text{m})\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right) = 24.53\,\text{kN/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = (24.53\,\text{kN/m}^2)(6\,\text{m}\times 5\,\text{m}) = 735.9\,\text{m}$$

The line of action of the force passes through the pressure center, which is $2h/3$ from the free surface,

$$y_P = \frac{2h}{3} = \frac{2\times(5\,\text{m})}{3} = 3.333\,\text{m}$$

Taking the moment about point *A* and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R(s + y_P) = F_{\text{ridge}}\overline{AB}$$

Solving for F_{ridge} and substituting, the reaction force is determined to be

$$F_{\text{ridge}} = \frac{s + y_P}{\overline{AB}}F_R = \frac{(1+3.333)\,\text{m}}{5\,\text{m}}(735.9\,\text{kN}) = \mathbf{638\,kN}$$

Discussion The difference between F_R and F_{ridge} is the force acting on the hinge at point *A*.

Q13-A water trough of semicircular cross section of radius 0.6 m consists of two symmetric parts hinged to each other at the bottom, as shown in Fig. P3–79. The two parts are held together by a cable and turnbuckle placed every 3 m along the length of the trough. Calculate the tension in each cable when the trough is filled to the rim.

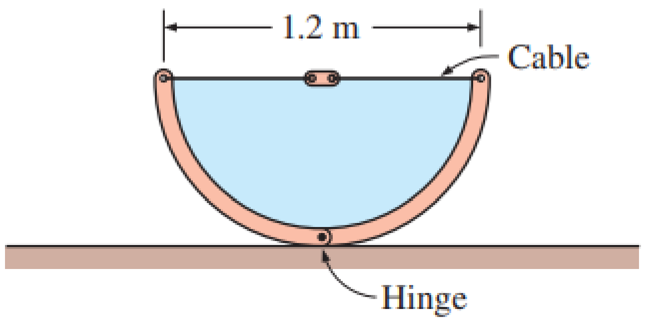


FIGURE P3–79

Yarım daire kesitli 0.6 m yarıçapındaki bir su yalağı Şekil P3–79'da görüldüğü gibi tabanda birbirlerine mafsallanmış iki simetrik parçadan oluşmaktadır. Bu iki parça, yalak boyunca 3 m aralıklarla yerleştirilen gergili bir kablo ile bir arada tutulmaktadır. Yalağın tam dolu olması halinde her bir kablodaki çekme kuvvetini hesaplayınız.

Solution Two parts of a water trough of semi-circular cross-section are held together by cables placed along the length of the trough. The tension T in each cable when the trough is full is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the trough wall, and thus it can be ignored in calculations for convenience. **2** The weight of the trough is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

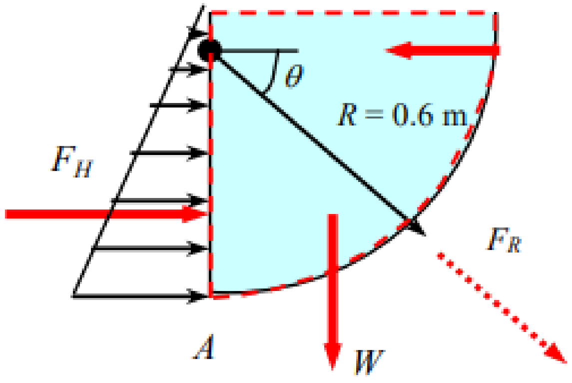
Analysis To expose the cable tension, we consider half of the trough whose cross-section is quarter-circle. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are:

Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g (R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.6/2\ \text{m})(0.6\ \text{m} \times 3\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 5297\ \text{N}$$



The vertical force on the horizontal surface is zero, since it coincides with the free surface of water. The weight of fluid block per 3-m length is

$$F_V = W = \rho g \mathbf{V} = \rho g[w \times \pi R^2/4]$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)[(3\ \text{m})\pi(0.6\ \text{m})^2/4]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 8321\ \text{N}$$

$$= 8321\ \mathrm{N}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 3-m long section of the trough become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(5297\ \mathrm{N})^2 + (8321\ \mathrm{N})^2} = 9864\ \mathrm{N}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{8321\ \mathrm{N}}{5297\ \mathrm{N}} = 1.571 \quad \rightarrow \quad \theta = 57.52°$$

Therefore, the line of action passes through the center of the curvature of the trough, making 57.52° downwards from the horizontal. Taking the moment about point A where the two parts are hinged and setting it equal to zero gives

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90 - 57.52)° = \mathbf{T} R$$

Solving for T and substituting, the tension in the cable is determined to be

$$\mathbf{T} = F_R \sin(90 - 57.52)° = (9864\ \mathrm{N}) \sin(90 - 57.52)° = \mathbf{5297 N}$$

Discussion This problem can also be solved without finding F_R by finding the lines of action of the horizontal hydrostatic force and the weight.

Q14-A cylindrical tank is fully filled with water (Fig. P3–80). In order to increase the flow from the tank, an additional pressure is applied to the water surface by a compressor. For P0 =0, P0 =3 bar, and P0 = 10 bar, calculate the hydrostatic force on the surface A exerted by water

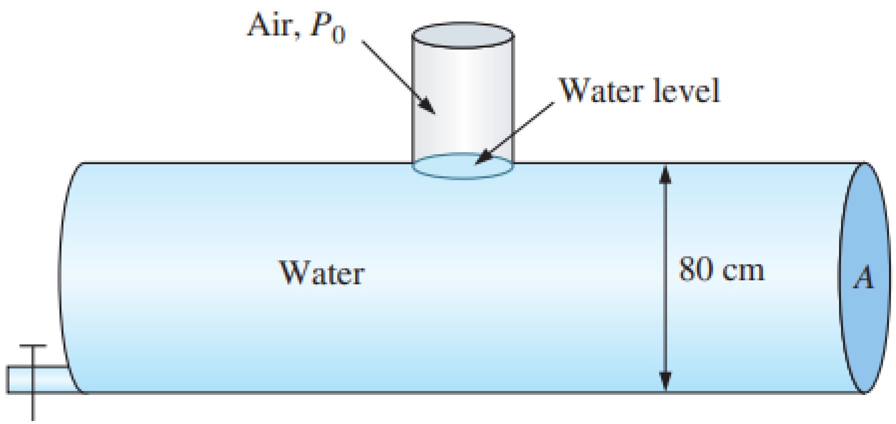


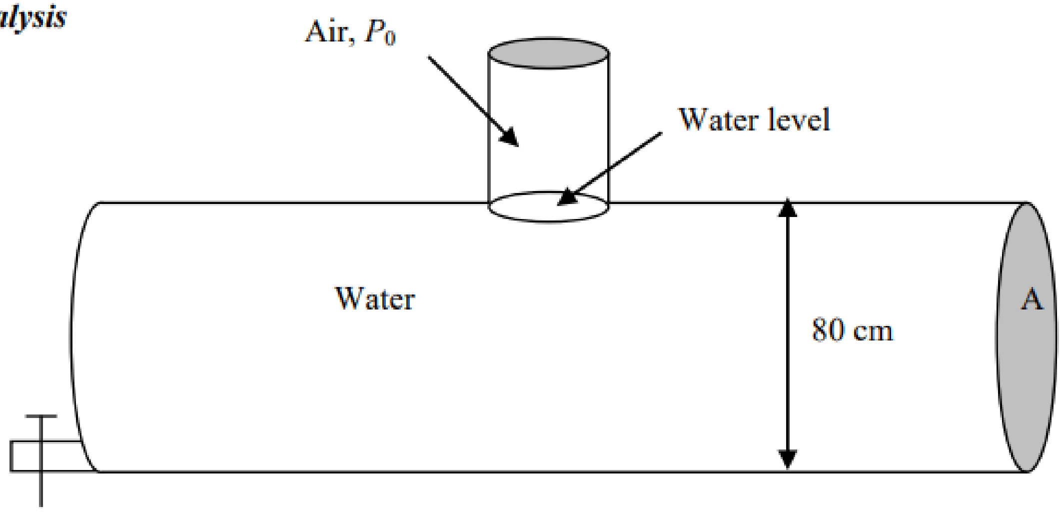
FIGURE P3–80

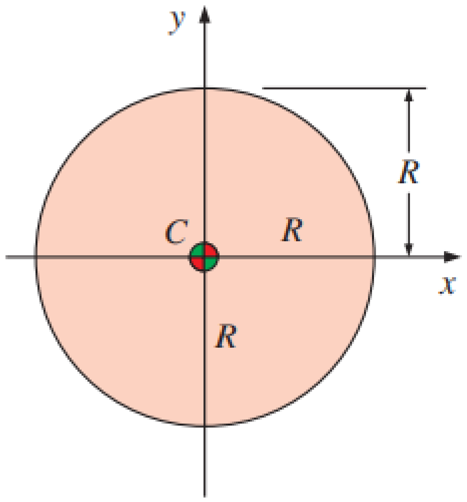
Solution A cylindrical tank is fully filled by water. The hydrostatic force on the surface A is to be determined for three different pressures on the water surface.

Assumptions Atmospheric pressure acts on both sides of the cylinder, and thus it can be ignored in calculations for convenience.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis





$A = \pi R^2,\ I_{xx,\,C} = \pi R^4/4$

(*b*) Circle

p=0 bar.

$$F_R = 9810 \times 0.4 \times \frac{\pi 0.8^2}{4} = \mathbf{1972\,N \cong 1.97\,kN}$$

$$y_{cp} = y_{cg} + \frac{I_{xc}}{y_{cg}A} = 0.4 + \frac{\pi 0.8^4/64}{0.4 \times \pi 0.8^2/4} = 0.5\ m$$

$$y_P = y_C + \frac{I_{xx,\,C}}{y_C A}$$

p=3 bar.

Additional imaginary water column

$$h = \frac{p_{air}}{\gamma_{water}} = \frac{3 \times 10^5\ Pa}{9810} = 30.58\,m$$

Therefore we can imagine the water level as if it were 30.58 m higher than its original level. In this case,

$$y_{cg} = h_{cg} = 0.4 + 30.58 = 30.98\ m$$

$$F_R = 9810 \times 30.58 \times \frac{\pi 0.8^2}{4} = \mathbf{150{,}791N \cong 151kN}$$

$$y_{cp} = y_{cg} + \frac{I_{xc}}{y_{cg} A} = 30.98 + \frac{\pi 0.8^4/64}{30.98 \times \pi 0.8^2/4} = 30.981\ m$$

p=10 bar.

Additional imaginary water column

$$h = \frac{p_{air}}{\gamma_{water}} = \frac{10 \times 10^5\ Pa}{9810} = 101.94\ m$$

$$F_R = 9810 \times 101.94 \times \frac{\pi 0.8^2}{4} = \mathbf{502671N \cong 503kN}$$

Q15-A 4-m-long quarter-circular gate of radius 3 m and of negligible weight is hinged about its upper edge A, as shown in Fig. P3–88. The gate controls the flow of water over the ledge at B, where the gate is pressed by a spring. Determine the minimum spring force required to keep the gate closed when the water level rises to A at the upper edge of the gate.

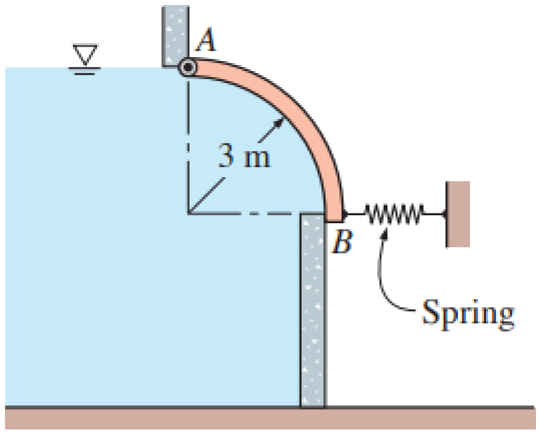


FIGURE P3–88

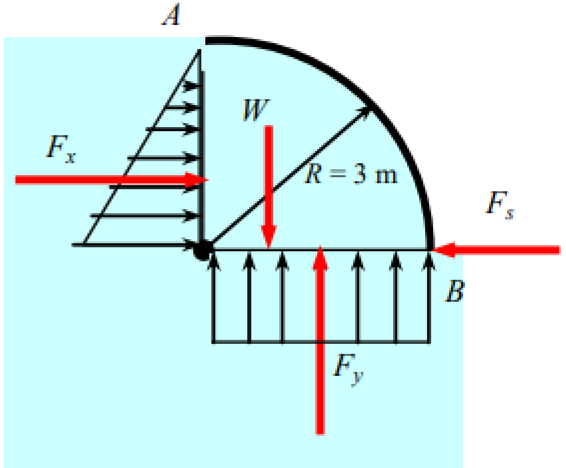
4 m boyunda ve 3 m yarıçapına sahip çeyrek daire şeklindeki ağırlıksız kapak üst kenarındaki *A* noktasından Şekil P3–88'de gösterildiği gibi mafsallanmıştır. Söz konusu kapak bir yay vasıtasıyla bastırılan *B* noktasındaki düz kesitte oluşan su akışını kontrol etmektedir. Su seviyesinin kapağın üst kenarındaki *A* noktasına ulaşması halinde kapağı kapalı biçimde tutabilmek için gerekli minimum yay kuvvetini hesaplayınız.

Solution A quarter-circular gate hinged about its upper edge controls the flow of water over the ledge at B where the gate is pressed by a spring. The minimum spring force required to keep the gate closed when the water level rises to A at the upper edge of the gate is to be determined.

Assumptions **1** The hinge is frictionless. **2** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **3** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout.

Analysis We consider the free body diagram of the liquid block enclosed by the circular surface of the gate and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as follows:



Horizontal force on vertical surface:

$$F_H = F_x = P_{ave}A = \rho g h_C A = \rho g (R/2)A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(3/2\text{ m})(4\text{ m}\times 3\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= 176.6\text{ kN}$$

Vertical force on horizontal surface (upward):

$$F_y = P_{avg}A = \rho g h_C A = \rho g h_{bottom} A$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(3\text{ m})(4\text{ m}\times 3\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 353.2\text{ kN}$$

The weight of fluid block per 4-m length (downwards):

$$W = \rho g V = \rho g\left[w\times\pi R^2/4\right]$$

$$= (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[(4\text{ m})\pi(3\text{ m})^2/4\right]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 277.4\text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 353.2 - 277.4 = 75.8\text{ kN}$$

Therefore, the net upward vertical force is

$$F_V = F_y - W = 353.2 - 277.4 = 75.8\ \text{kN}$$

Then the magnitude and direction of the hydrostatic force acting on the surface of the 4-m long quarter-circular section of the gate become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(176.6\ \text{kN})^2 + (75.8\ \text{kN})^2} = 192.2\ \text{kN}$$

$$\tan\theta = \frac{F_V}{F_H} = \frac{75.8\ \text{kN}}{176.6\ \text{kN}} = 0.429 \quad \rightarrow \quad \theta = 23.2°$$

Therefore, the magnitude of the hydrostatic force acting on the gate is 192.2 kN, and its line of action passes through the center of the quarter-circular gate making an angle 23.2° upwards from the horizontal.

The minimum spring force needed is determined by taking a moment about the point A where the hinge is, and setting it equal to zero,

$$\sum M_A = 0 \quad \rightarrow \quad F_R R \sin(90-\theta) - F_{\text{spring}} R = 0$$

Solving for F_{spring} and substituting, the spring force is determined to be

$$F_{\text{spring}} = F_R \sin(90-\theta) = (192.2\ \text{kN})\sin(90° - 23.2°) = \mathbf{177\ kN}$$

Discussion Several variations of this design are possible. Can you think of some of them?

Q16-Consider a 1-m wide inclined gate of negligible weight that separates water from another fluid. What would be the volume of the concrete block (SG = 2.4) immersed in water to keep the gate at the position shown? Disregard any frictional effects.

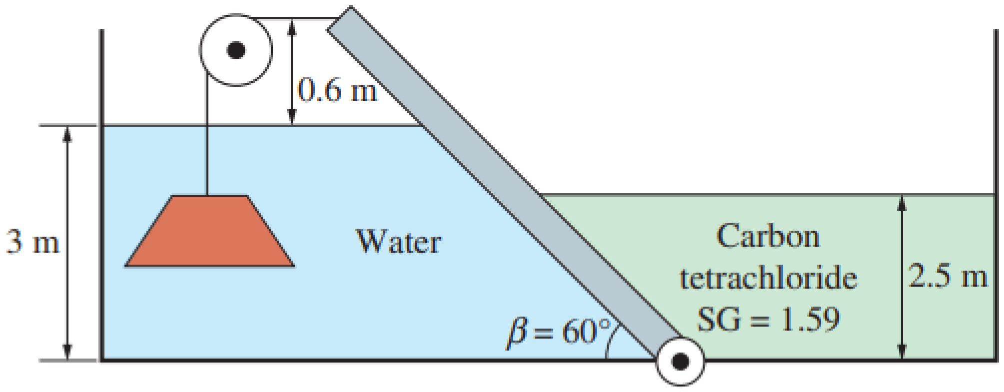


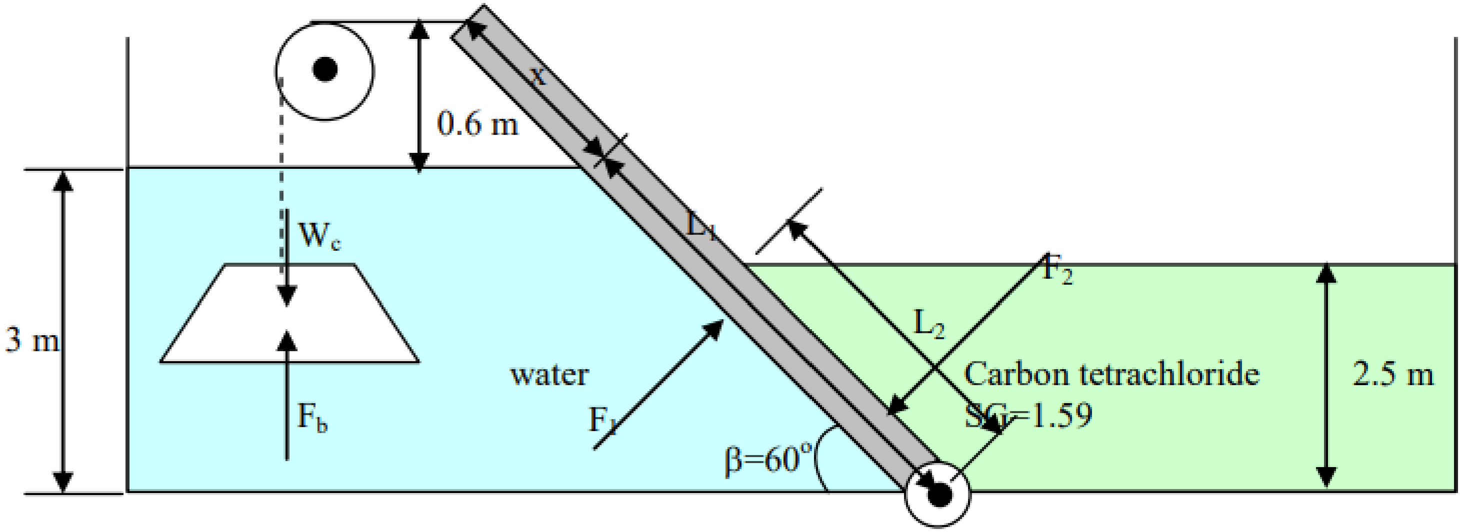
FIGURE P3–92

Solution An inclined gate separates water from another fluid. The volume of the concrete block to keep the gate at the given position is to be determined.

Assumptions **1** Atmospheric pressure acts on both sides of the gate, and thus it can be ignored in calculations for convenience. **2** The weight of the gate is negligible.

Properties We take the density of water to be 1000 kg/m^3 throughout. The specific gravities of concerete and carbon tetrachloride are 2.4 and 1.59, respectively.

Analysis



The force applied by water:

$$F_1 = \gamma h_{cg1} A_1 = 9810 \times \frac{3}{2} \times \left(1 \times \frac{3}{Sin\beta}\right) = 50974\ N$$

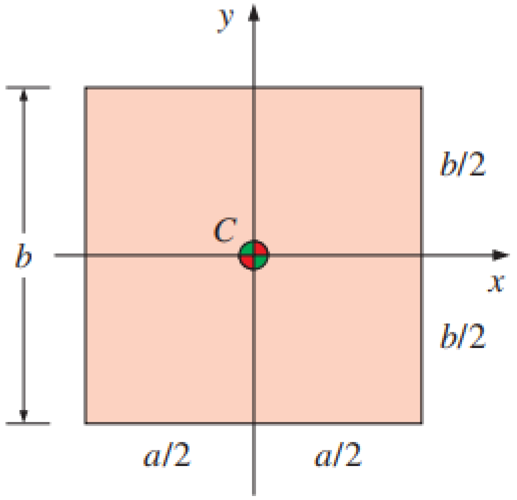
$$y_{cp1} = \frac{3}{2Sin60} + \frac{1 \times \left(\frac{3}{Sin60}\right)^3 \Big/ 12}{\frac{3}{2Sin60} \times \left(1 \times \frac{3}{Sin60}\right)} = 2.31\ m$$

$$y_P = y_C + \frac{I_{xx,\,C}}{y_C A}$$

The force applied by carbon tetrachloride:

$$F_2 = \gamma h_{cg2} A_2 = 1.59 \times 9810 \times \frac{2.5}{2} \times \left(1 \times \frac{2.5}{Sin60}\right) = 56284\ N$$

$$y_{cp2} = \frac{2.5}{2Sin60} + \frac{1 \times \left(\frac{2.5}{Sin60}\right)^3 \Big/ 12}{\frac{2.5}{2Sin60} \times \left(1 \times \frac{2.5}{Sin60}\right)} = 1.924\ m \qquad x = \frac{0.6}{Sin60} = 0.693\ m$$



$A = ab,\ I_{xx,\,C} = ab^3/12$

(a) Rectangle

Moment about hinge would give

$$(W_c - F_b) \times Sin\beta \times (x + L_1) + F_2 \times (L_2 - y_{cp2}) - F_1 \times (L_1 - y_{cp1}) = 0$$

Since $W_c - F_b = \forall_c(\gamma_c - \gamma_w)$, we obtain

$$\forall_c = \frac{F_1 \times (L_1 - y_{cp1}) - F_2 \times (L_2 - y_{cp2})}{(\gamma_c - \gamma_w) \times Sin\beta \times (L_1 + x)} = \frac{50974(3.464 - 2.41) - 56284(2.886 - 1.924)}{9810(2.4 - 1)Sin60(3.464 + 0.693)} = \mathbf{0.0946\,m^3}$$

Q17-An elastic air balloon having a diameter of 30 cm is attached to the base of a container partially filled with water at +4°C, as shown in Fig. P3–151. If the pressure of the air above the water is gradually increased from 100 kPa to 1.6 MPa, will the force on the cable change? If so, what is the percent change in the force? Assume the pressure on the free surface and the diameter of the balloon are related by P = C D^n , where C is a constant and n = -2. The weight of the balloon and the air in it is negligible.

30 cm çapındaki elastik bir hava balonu Şekil P3–151'de gösterildiği gibi +4°C'deki suyla kısmen dolu olan bir kabın tabanına bir ip vasıtasıyla tutturulmuştur. Eğer su yüzeyindeki hava basıncı kademeli olarak 100 kPa'dan 1.6 MPa'a artırılırsa ipteki kuvvet değişir mi? Değişir diyorsanız % kaç değişir? Serbest yüzeydeki basınç ile balon çapı arasında $P = C\,D^n$ bağıntısının bulunduğu varsayılmaktadır. Burada C bir sabit olup $n = -2$ alınabilir. Ayrıca balonun ve içerisindeki havanın ağırlığı ihmal edilmektedir. *Cevap:* %98.4

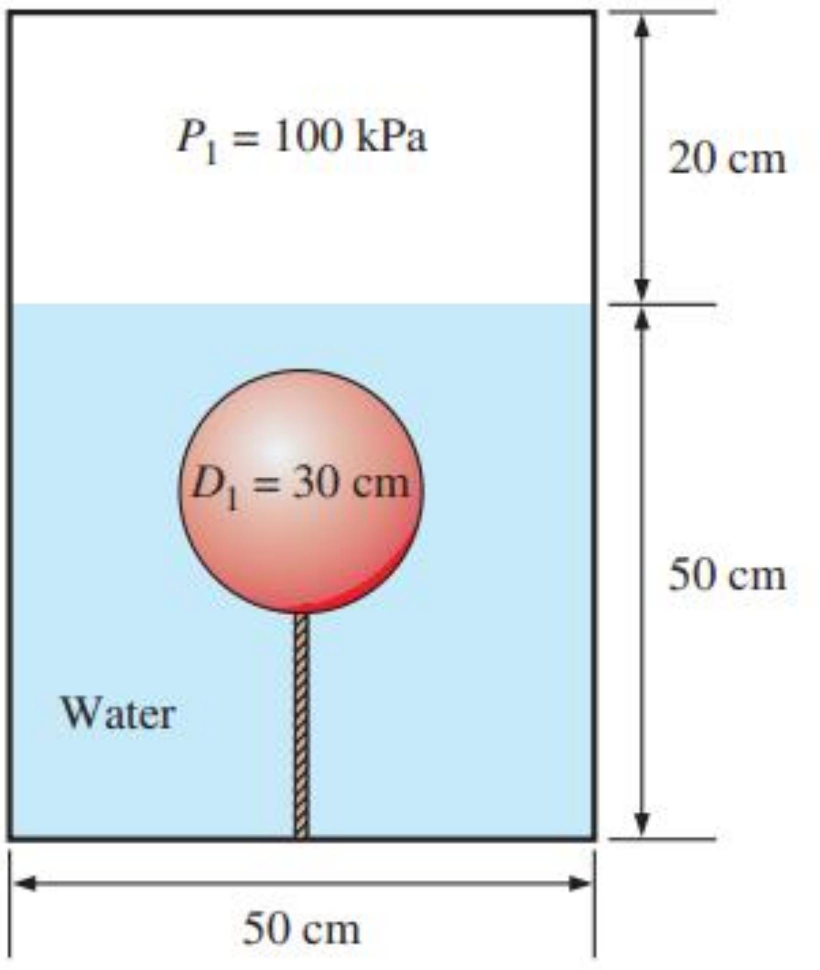
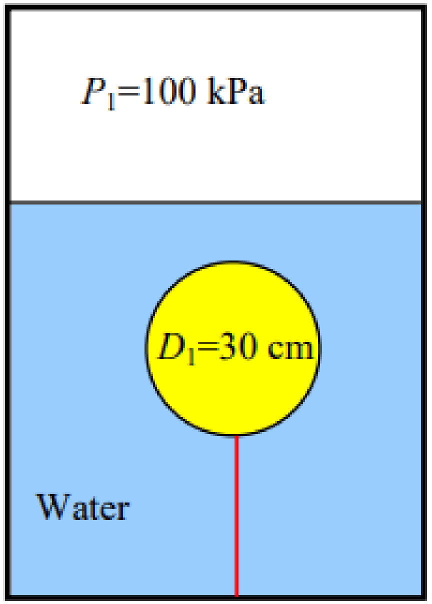


FIGURE P3–151

Solution An elastic air balloon submerged in water is attached to the base of the tank. The change in the tension force of the cable is to be determined when the tank pressure is increased and the balloon diameter is decreased in accordance with the relation $P = CD^{-2}$.



Assumptions **1** Atmospheric pressure acts on all surfaces, and thus it can be ignored in calculations for convenience. **2** Water is an incompressible fluid. **3** The weight of the balloon and the air in it is negligible.

Properties We take the density of water to be 1000 kg/m^3.

Analysis The tension force on the cable holding the balloon is determined from a force balance on the balloon to be

$$F_{cable} = F_B - W_{balloon} \cong F_B$$

The buoyancy force acting on the balloon initially is

$$F_{B,1} = \rho_w g V_{balloon,1} = \rho_w g \frac{\pi D_1^3}{6} = (1000\ \text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.30\,\text{m})^3}{6}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 138.7\,\text{N}$$

The variation of pressure with diameter is given as $P = CD^{-2}$, which is equivalent to $D = \sqrt{C/P}$. Then the final diameter of the ball becomes

$$\frac{D_2}{D_1} = \frac{\sqrt{C/P_2}}{\sqrt{C/P_1}} = \sqrt{\frac{P_1}{P_2}} \quad \rightarrow \quad D_2 = D_1\sqrt{\frac{P_1}{P_2}} = (0.30\,\text{m})\sqrt{\frac{0.1\,\text{MPa}}{1.6\,\text{MPa}}} = 0.075\,\text{m}$$

The buoyancy force acting on the balloon in this case is

$$F_{B,2} = \rho_w g V_{balloon2} = \rho_w g \frac{\pi D_2^3}{6} = (1000\ \text{kg/m}^3)(9.81\,\text{m/s}^2)\frac{\pi(0.075\,\text{m})^3}{6}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = 2.2\,\text{N}$$

Then the percent change in the cable for becomes

$$Change\% = \frac{F_{cable,1} - F_{cable,2}}{F_{cable,1}} * 100 = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} * 100 = \frac{138.7 - 2.2}{138.7} * 100 = \mathbf{98.4\%}.$$

Therefore, increasing the tank pressure in this case results in 98.4% reduction in cable tension.

Discussion We can obtain a relation for the change in cable tension as follows:

$$Change\% = \frac{F_{B,1} - F_{B,2}}{F_{B,1}} * 100 = \frac{\rho_w g V_{\text{balloon},1} - \rho_w g V_{\text{balloon},2}}{\rho_w g V_{\text{balloon},1}} * 100$$

$$= 100\left(1 - \frac{V_{\text{balloon},2}}{V_{\text{balloon},1}}\right) = 100\left(1 - \frac{D_2^3}{D_1^3}\right) = 100\left(1 - \left(\frac{P_1}{P_2}\right)^{3/2}\right)$$

Q18- A room in the lower level of a cruise ship has a 30-cm-diameter circular window. If the midpoint of the window is 4 m below the water surface, determine the hydrostatic force acting on the window, and the pressure center. Take the specific gravity of seawater to be 1.025.

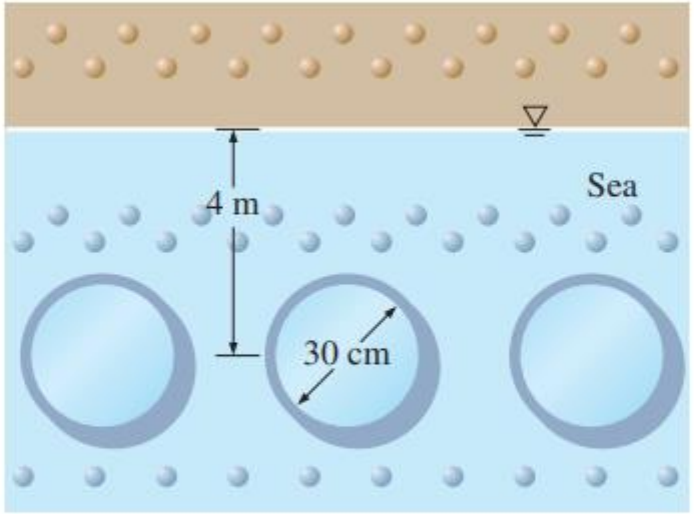


FIGURE P3–71

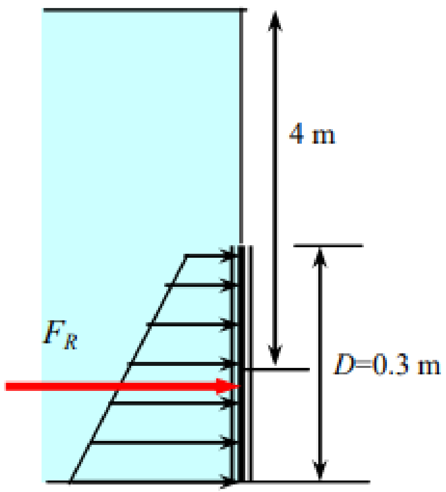
Bir yolcu gemisinin bodrum katındaki bir kamarası 30 cm çapında dairesel pencereye sahiptir. Pencerenin orta noktası su yüzeyinden 4 m aşağıda olduğuna göre, pencere üzerine etki eden hidrostatik kuvveti ve basınç merkezini belirleyiniz. Deniz suyunun bağıl yoğunluğunu 1.025 olarak alınız.

Solution A room in the lower level of a cruise ship is considered. The hydrostatic force acting on the window and the pressure center are to be determined.

Assumptions Atmospheric pressure acts on both sides of the window, and thus it can be ignored in calculations for convenience.

Properties The specific gravity of sea water is given to be 1.025, and thus its density is 1025 kg/m^3.

Analysis The average pressure on a surface is the pressure at the centroid (midpoint) of the surface, and is determined to be



$$P_{ave} = P_C = \rho g h_C = (1025\text{ kg/m}^3)(9.81\text{ m/s}^2)(4\text{ m})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 40{,}221\text{ N/m}^2$$

Then the resultant hydrostatic force on each wall becomes

$$F_R = P_{ave}A = P_{ave}[\pi D^2/4]$$
$$= (40{,}221\text{ N/m}^2)[\pi(0.3\text{ m})^2/4] = 2843\text{ N} \cong \mathbf{2840} \quad \text{(three significant digit)}$$

The line of action of the force passes through the pressure center, whose vertical distance from the free surface is determined from

$$y_P = y_C + \frac{I_{xx,C}}{y_C A} = y_C + \frac{\pi R^4/4}{y_C \pi R^2} = y_C + \frac{R^2}{4y_C} = 4 + \frac{(0.15\text{ m})^2}{4(5\text{ m})} = \mathbf{4.001\text{ m}}$$

Discussion For small surfaces deep in a liquid, the pressure center nearly coincides with the centroid of the surface. Here, in fact, to three significant digits in the final answer, the center of pressure and centroid are coincident. We give the answer to four significant digits to show that the center of pressure and the centroid are *not* coincident.