INTRODUCTION TO COASTAL HYDRAULICS
Solved Problems 1

1 a) A wave with a period of T = 10s propagates from deep water. Calculate the wave celerity (C_0) and the wave length (L_0) of this wave.

$$L_0 = 1.56T^2 = 1.56 \times 10^2 = 156m.$$

$$C_0 = \frac{L_0}{T} = \frac{156}{10} = 15.6\, m/s$$

b) What are the celerity and length of the same wave when it has propagated into 40 m depth? (L_{40},C_{40}=?).

This problem can be solved in three different ways.

1st Method :

L_0=156m

$$L_{40} = 156 \times \tanh\left(\frac{2\pi}{L_{40}} \times 40\right) \rightarrow L_{40} \cong 146m$$

2nd Method :

$$\frac{d}{L_0} = \frac{40}{156} = 0.256 \xrightarrow{GWT} \frac{d}{L_{40}} = 0.2731 \rightarrow \frac{40}{L_{40}} = 0.2731 \Rightarrow L_{40} = 146.5m$$

3rd Method :

$$\frac{d}{L_0} = 0.256 \xrightarrow{GWT} \tanh kd = 0.9373 \rightarrow L = L_0 \tanh kd = 156 \times 0.9373 \Rightarrow L_{40} = 146.2m$$

$$C_{40} = \frac{L_{40}}{T} = \frac{146.5}{10} = 14.65\, m/s$$

c) What should be the water depth if the wave is shallow water wave?

Shallow-water wave $\frac{d}{L} \leq 0.05 \xrightarrow{GWT} \frac{d}{L_0} = 0.0152 \rightarrow \frac{d}{156} = 0.0152 \Rightarrow d = 2.38m$

2 a) A wave with a period of T = 10s and a height of H = 2 m propagates from deep water. Calculate the wave energy (ρ=1000 kg/m³).

$$\overline{E}_0 = \frac{1}{8}\rho g H_0^2 = \frac{1}{8} \times 1000 \times 9.81 \times 2^2 = 4905\, N/m^2$$

b) Calculate the energy flux.

$$R_0 = \overline{E}_0 \times n_0 \times c_0 = 4905 \times \frac{1}{2} \times 15.6 = 38259\, N/ms$$

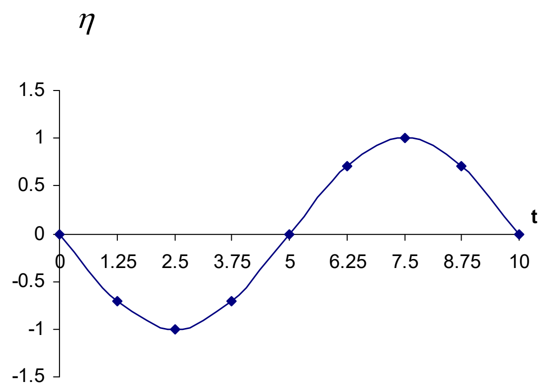
3) A wave with a period of T = 10s and a height of H = 2 m propagates from deep water into shallow water. Calculate the wave profile. Draw the wave profile for x=0 and for the time intervals at $\Delta t = 1.25s$.

$$\eta = a \sin(kx - \omega t) \qquad a = \frac{H}{2} = 1m$$

$$k_0 = \frac{2\pi}{L_0} = \frac{2\pi}{1.56T^2} = \frac{2\pi}{156} = 0.04 \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{10} = 0.628$$

$$\eta_0 = \sin(0.04x - 0.628t)$$

x=0 t=0 $\eta = 0$
x=0 t=1.25s $\eta = -0.71$
x=0 t=2.5s $\eta = -1$
x=0 t=3.75s $\eta = -0.71$
x=0 t=5s $\eta = 0$
x=0 t=6.25s $\eta = 0.71$
x=0 t=7.5s $\eta = 1$
x=0 t=8.75s $\eta = 0.71$
x=0 t=10 $\eta = 0$



4) If the wave profile is given by $\eta = 2\cos(0.13x - 1.03t)$

a) Determine the wave height, wave length, wave period and water depth.
b) η (x=100m, t=15s) = ?

a) a=2m k=0.13 $\omega = 1.03$

$$a = \frac{H}{2} \rightarrow 2 = \frac{H}{2} \Rightarrow H = 4m \qquad k = \frac{2\pi}{L} \rightarrow 0.13 = \frac{2\pi}{L} \Rightarrow L = 48.33m$$

$$\omega = \frac{2\pi}{T} \rightarrow 1.03 = \frac{2\pi}{T} \Rightarrow T = 6.1s$$

$$L_0 = 1.56T^2 = 1.56 \times 6.1^2 = 58m.$$

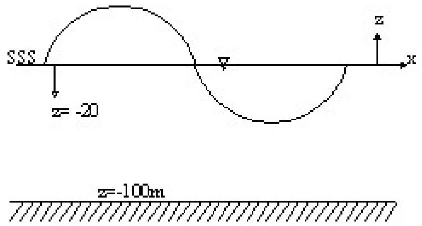
$$L = L_0 \tanh kd \rightarrow 48.33 = 58 \times \tanh(0.13d) \rightarrow 0.83 = \tanh(0.13d) \rightarrow 1.19 = 0.13d \Rightarrow d = 9.14m$$

b) $\eta = 2\cos(0.13 \times 100 - 1.03 \times 15) = -1.54$ m

5) A wave with a period of T = 10 s and a height of H = 2 m propagates into a depth of d = 100 m.

a) at Still Water Level (SWL),
b) at z = -20 m,
c) at bottom (z = -100m)

$u_{max}, w_{max}, A, B = ?$



$$\frac{d}{L_0} = \frac{100}{156} = 0.64 > 0.5 \rightarrow \text{deep water wave} \Rightarrow \text{u=w, A=B}$$

$u_{max} \Rightarrow \sin\theta = 1,\ \theta = \pi/2$

$w_{max} \Rightarrow \cos\theta = 1,\ \theta = 0$

$$u = \frac{a\omega\cosh\left[k\left(z+d\right)\right]}{\sinh kd} \times \sin(kx-\omega t)$$

$$w = \frac{-a\omega\sinh\left[k\left(z+d\right)\right]}{\sinh kd} \times \cos(kx-\omega t)$$

a) z=0,

$$u_{max} = \frac{a\omega\cosh\left[kd\right]}{\sinh kd} = a\omega\frac{e^{kd}/2}{e^{kd}/2} = a\omega = 1\times 0.628 = 0.628\,m/s$$

$$w_{max} = \frac{-a\omega\sinh\left(kd\right)}{\sinh kd} = -a\omega = -0.628\ m/s$$

$$A = \frac{a\cosh\left[k\left(z+d\right)\right]}{\sinh kd} = a\frac{\cosh kd}{\sinh kd} = a\frac{e^{kd}/2}{e^{kd}/2} = a = A = 1m$$

$$B = \frac{a\sinh\left[k\left(z+d\right)\right]}{\sinh kd} = a\frac{\sinh kd}{\sinh kd} = a = B = 1\ m$$

a) z= -20m

$$u_{max} = a\omega\frac{e^{k(z+d)}/2}{e^{kd}/2} = 0.628\times\frac{e^{k(z+d)}}{e^{kd}} = 0.628\times\frac{e^{kz}\,e^{kd}}{e^{kd}} = 0.628\times e^{-20\times\frac{2\pi}{156}} = 0.281m/s$$

$$w_{max} = -0.628\times e^{-20\times\frac{2\pi}{156}} = -0.281m/s$$

$$A = a\times e^{-20\times\frac{2\pi}{156}} = 0.447m = B$$

b) z=-d, At the bottom in deep water $\begin{array}{l} u = w = 0 \\ A = B = 0 \end{array}$

6 a) In deep water a wave has a height of 2 m and a period of 10 s. The wave moves into a water depth of 10 m. Calculate the wave height at the new depth.

$$H_{10} = H_0 \times K_{s_{10}}$$

$$\frac{d}{L_0} = \frac{10}{156} = 0.064 \xrightarrow{GWT} K_{s_{10}} = 0.9838 \rightarrow H_{10} = 2\times 0.9838 = 1.97m$$

b) What are the maximum orbital particle velocities at the bottom of the same wave at 10 m depth?

$$u = \frac{a\omega\cosh\left[k\left(z+d\right)\right]}{\sinh kd} \times \sin(kx-\omega t)$$

$$H_{10} = 1.97m \quad a = \frac{1.97}{2} = 0.985m \quad \omega = \frac{2\pi}{T} = 0.628s$$

$$\frac{d}{L_0} = \frac{10}{156} = 0.064 \xrightarrow{GWT} \sinh(kd) = 0.7335$$

$$u = \frac{a\omega\cosh\left[k\left(z+d\right)\right]}{\sinh kd} \times \sin(kx-\omega t)$$

At bottom; $z = -d \Rightarrow \quad \cosh\left[k\left(z+d\right)\right] = \cosh(0) = 1$

$\sin(kx-\omega t) = 1$ for u_{max}

$$u_{max} = \frac{0.985 \times 0.628}{0.7335} = 0.843 \text{ m/s}$$

$$w_{maks} = 0 \text{ at bottom}$$

7) In deep water a wave has a height of 2 m and a period of 11 s. The wave moves into a water depth of 2 m. Calculate the energy flux at the new depth.

$$L_0 = 1.56T^2 = 1.56 \times 11^2 = 188.76\text{m}$$

$$\frac{d}{L_0} = \frac{2}{188.76} = 0.0106 \xrightarrow{\text{GWT}} \frac{d}{L_2} = 0.041 < 0.05 \text{ Shallow water} \rightarrow \frac{2}{L_2} = 0.041 \Rightarrow L_2 = 48.78\text{m}$$

$$K_{s2} = 1.4196$$

$$H_2 = H_0 \times K_{s_2} = 2 \times 1.4196 = 2.84\,\text{m}$$

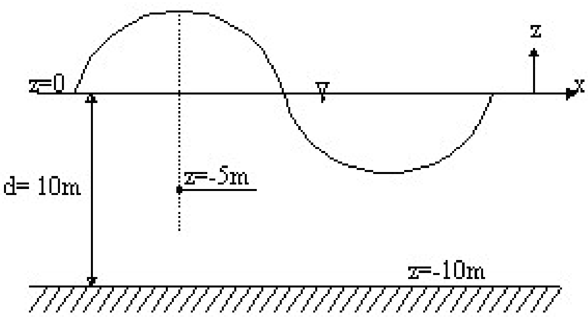
$$c_2 = \frac{L_2}{T} = \frac{48.78}{11} = 4.43\,\text{m/s} \quad \text{or} \quad c = \sqrt{gd}$$

$$c_2 = \sqrt{9.81 \times 2} = 4.43\,\text{m/s}$$

$$\bar{E}_2 = \frac{1}{8}\rho g H_2^2 = \frac{1}{8} \times 1000 \times 9.81 \times 2.84^2 = 9950\,\text{N/m}^2$$

$$R_2 = \bar{E}_2 \times n_2 \times c_2 = 9950 \times 4.43 \times 1 = 44080\,\text{N/ms}$$

8) A wave with a period of T = 6 s and a height of H = 4 m propagates into a nearshore depth of d = 10 m. Determine the total pressure under the wave crest at z=-5 m.



$$L_0 = 1.56T^2 = 1.56 \times 6^2 = 56.16\text{m}.$$

$$\frac{d_{10}}{L_0} = \frac{10}{56.16} = 0.178 \xrightarrow{\text{gwt}} \frac{d_{10}}{L_{10}} = 0.2066 \Rightarrow L_{10} = 48.4\text{m}$$

$$\frac{p}{\rho g} = \frac{\cosh\left(\frac{2\pi(d+z)}{L}\right)}{\cosh\left(\frac{2\pi d}{L}\right)} \times \eta - z$$

Passing wave crest ; $\eta = +a = 2\text{m}$

$$\frac{p}{1000 \times 9.81} = \frac{\cosh\left(\frac{2\pi(10+(-5))}{48.4}\right)}{\cosh\left(\frac{2\pi \times 10}{48.4}\right)} \times 2 - (-5) = 61200\text{N/m}^2$$

$$p^+ = 12150 \text{ N/m}^2 \qquad p_{hyd} = 49050 \text{ N/m}^2$$

$$p = p^{+} + p_{hyd} = 12150 + 49050 = 61200\ N/m^2$$

9) In deep water a wave has a height of 1.5 m and a period of 8 s.

a) What are the celerity, length and height of the wave at 5 m depth? ($H_5, C_5, L_5 = ?$). Is this wave propagating in the deep, shallow or intermediate water?

b) In deep water, determine the horizontal orbital particle velocity at a depth of 2 m below the surface when x=100 m and t=2 s.

c) At a depth of d = 5 m, determine the horizontal orbital particle velocity at a depth of 2 m below the surface when x=100 m and t=2 s.

d) Same wave is propagating through straight and parallel bottom contours at an angle of 30° from deep water to a depth of 5 m. Calculate the wave height and angle at 8 m depth. ($H_5 = ?, \alpha_5 = ?$)

a) $L_0 = 1.56T^2 = 1.56 \times 8^2 = 100\,m$

$$\frac{d_5}{L_0} = \frac{5}{100} = 0.05 \xrightarrow{GWT} \frac{d}{L_5} = \frac{5}{L_5} = 0.09416 \Rightarrow L_5 = 53.1m$$

$0.5 > 0.09416 > 0.05$ intermediate water wave

$$C_5 = \frac{L_5}{T} = \frac{53.1}{8} = 6.64\,m/s$$

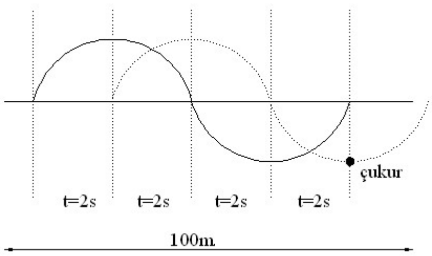
$$\frac{d}{L_0} = 0.05 \xrightarrow{GWT} K_{s5} = 1.023 \rightarrow H_5 = H_0 \times K_{s5} = 1.5 \times 1.023 = 1.53m$$

b) $$u = \frac{a\omega \cosh\left[k(z+d)\right]}{\sinh kd} \times \sin(kx - \omega t)$$

$$a_0 = \frac{H_0}{2} = \frac{1.5}{2} = 0.75m \qquad k_0 = \frac{2\pi}{L_0} = \frac{2\pi}{100} = 0.063 \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{8} = 0.785$$

$$u = 0.75 \times 0.785 \times \frac{e^{k(z+d)}/2}{e^{kd}/2} \times \underbrace{\sin(0.063 \times 100 - 0.785 \times 2)}_{-1}$$

$$u = -0.589 \times \frac{e^{kz}\ \cancel{e^{kd}}}{\cancel{e^{kd}}} = -0.589 \times e^{0.063 \times (-2)} = -0.52m/s$$



c) $$u = \frac{a\omega \cosh\left[k(z+d)\right]}{\sinh kd} \times \sin(kx - \omega t)$$

$$a_5 = \frac{H_5}{2} = \frac{1.53}{2} = 0.765\,m \qquad k_5 = \frac{2\pi}{L_5} = \frac{2\pi}{53.1} = 0.118$$

$$u=\frac{0.765\times 0.785\cosh\left[0.118\left(-2+5\right)\right]}{\sinh\left(0.118\times 5\right)}\times\underbrace{\sin(0.118\times 100-0.785\times 2)}_{-0.72}$$

$$u=-0.736\ m/s \qquad u_{\max}=1.02\,m/s$$

d) $$\frac{L_5}{L_0}=\frac{\sin\alpha_5}{\sin\alpha_0}\Rightarrow\frac{53.1}{100}=\frac{\sin\alpha_5}{\sin 30}\Rightarrow\sin\alpha_5=0.2655\Rightarrow\alpha_5=15.4^{\circ}$$

$$H_5=H_0\times K_{s5}\times K_{r5}$$

$$K_{r5}=\sqrt{\frac{\cos\alpha_0}{\cos\alpha_5}}=\sqrt{\frac{\cos 30}{\cos 15.4}}=0.9478$$

$$H_5=1.5\times 1.023\times 0.9478=1.45\,m$$

10) A wave in water 20 m deep has a period of T= 8 sec, a height of H_{20}=2 m and an incident wave angle of α_{20}=18°. Calculate the wave height at d=7 m.

$$L_0=1.56T^2=1.56\times 8^2=100\,m$$

$$\frac{d_{20}}{L_0}=\frac{20}{100}=0.2\xrightarrow{GWT}K_{s_{20}}=0.9181\ ,\quad \frac{d}{L_{20}}=\frac{20}{L_{20}}=0.2251\Rightarrow L_{20}=88.85m$$

$$\frac{L_{20}}{L_0}=\frac{\sin\alpha_{20}}{\sin\alpha_0}\Rightarrow\frac{88.85}{100}=\frac{\sin 18}{\sin\alpha_0}\Rightarrow\sin\alpha_0=0.348\Rightarrow\alpha_0=20.35^{\circ}$$

$$K_{r20}=\sqrt{\frac{\cos\alpha_0}{\cos\alpha_{20}}}=\sqrt{\frac{\cos 20.35}{\cos 18}}=0.99$$

$$H_{20}=H_0\times K_{s20}\times K_{r20}\Rightarrow 2=H_0\times 0.9181\times 0.99\Rightarrow H_0=2.2\,m$$

$$\frac{d_7}{L_0}=\frac{7}{100}=0.07\xrightarrow{GWT}K_{s7}=0.9713\ ,\quad \frac{d}{L_7}=\frac{7}{L_7}=0.1139\Rightarrow L_7=61.45\,m$$

$$\frac{L_7}{L_0}=\frac{\sin\alpha_7}{\sin\alpha_0}\Rightarrow\frac{61.45}{100}=\frac{\sin\alpha_7}{\sin 20.35}\Rightarrow\alpha_7=12.33^{\circ}$$

$$K_{r7}=\sqrt{\frac{\cos\alpha_0}{\cos\alpha_7}}=\sqrt{\frac{\cos 20.35}{\cos 12.33}}=0.9797$$

$$H_7=H_0\times K_{s7}\times K_{r7}\Rightarrow 2.2\times 0.9713\times 0.9797=2.09\,m$$