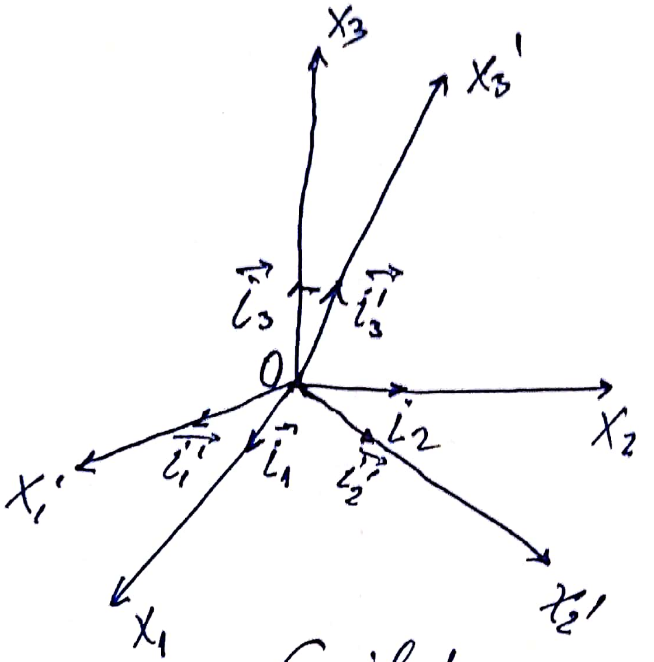
Tansörler hakda bilgiler.

O noktasına $OX_1X_2X_3$ ve $OX_1'X_2'X_3'$ koordinat sistemlerini birleştirelim. Bu sistemlerin ort-basis vektörlerini ise $\vec{i}_1, \vec{i}_2, \vec{i}_3$ ve $\vec{i}_1', \vec{i}_2', \vec{i}_3'$ gibi işaretleyelim (Şekil 1)



Şekil 1

$\vec{i}_1', \vec{i}_2'$ ve $\vec{i}_3'$ vektörlerinin her birinin $\vec{i}_1, \vec{i}_2$ ve $\vec{i}_3$ vektörlerle ifadesini, $\vec{i}_1, \vec{i}_2$ ve $\vec{i}_3$ vektörlerinden her birinde de $\vec{i}_1', \vec{i}_2'$ ve $\vec{i}_3'$ vektörlerle ifadelerini elde etmeye çalışalım.

farz edelim ki $\vec{i}_k'$ $(k=1,2,3)$ vektörünün $\vec{i}_1, \vec{i}_2$ ve $\vec{i}_3$ vektörlerinin skalar çarpımlarını α_{k1}, α_{k2} ve α_{k3} ile işaret etersek onda

$$\vec{i}_k' = \alpha_{k1}\vec{i}_1 + \alpha_{k2}\vec{i}_2 + \alpha_{k3}\vec{i}_3 \quad (1)$$

yaza biliriz

Burada $\alpha_{kj} = (\vec{i}_k' \cdot \vec{i}_j)$ - dir. (1)-in ifadesini

$$\begin{Bmatrix} \vec{i}_1' \\ \vec{i}_2' \\ \vec{i}_3' \end{Bmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \begin{pmatrix} \vec{i}_1 \\ \vec{i}_2 \\ \vec{i}_3 \end{pmatrix} \quad (2)$$

matris biçiminde de yaza biliriz

$$\vec{i}_k = \beta_{k1}\vec{i}'_1 + \beta_{k2}\vec{i}'_2 + \beta_{k3}\vec{i}'_3$$

$$\beta_{kj} = (\vec{i}_k \cdot \vec{i}'_j) \Rightarrow \beta_{kj} = \alpha_{jk}$$

$$\boxed{\vec{i}_k = \alpha_{1k}\vec{i}'_1 + \alpha_{2k}\vec{i}'_2 + \alpha_{3k}\vec{i}'_3}$$

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}^T \begin{pmatrix} \vec{i}'_1 \\ \vec{i}'_2 \\ \vec{i}'_3 \end{pmatrix}$$

$\underline{a_{ij}}$ $\quad \vec{a} = a_1\vec{i}_1 + a_2\vec{i}_2 + a_3\vec{i}_3 = a'_1\vec{i}'_1 + a'_2\vec{i}'_2 + a'_3\vec{i}'_3$

$$a'_k = \alpha_{k1}a_1 + \alpha_{k2}a_2 + \alpha_{k3}a_3$$

$$a_k = \alpha_{1k}a'_1 + \alpha_{2k}a'_2 + \alpha_{3k}a'_3$$

$$(\vec{i}_n \otimes \vec{i}_m) \; ; \; (\vec{i}_1 \otimes \vec{i}_2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(\vec{i}_2 \otimes \vec{i}_3) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{A} = a_{mn}(\vec{i}_m \otimes \vec{i}_n) =$$

$$= a_{11}(\vec{i}_1 \otimes \vec{i}_1) + a_{12}(\vec{i}_1 \otimes \vec{i}_2) + a_{13}(\vec{i}_1 \otimes \vec{i}_3) +$$

$$+\vec{i}'_1 \quad +\vec{i}'_1 \quad \vec{i}'_1 \quad \vec{i}'_1 \quad \vec{i}'_1 \quad \vec{i}'_1$$

$$a_{21}(\vec{i}_2 \otimes \vec{i}_1) + a_{22}(\vec{i}_2 \otimes \vec{i}_2) + a_{23}(\vec{i}_2 \otimes \vec{i}_3) +$$
$$a_{31}(\vec{i}_3 \otimes \vec{i}_1) + a_{32}(\vec{i}_3 \otimes \vec{i}_2) + a_{33}(\vec{i}_3 \otimes \vec{i}_3) =$$

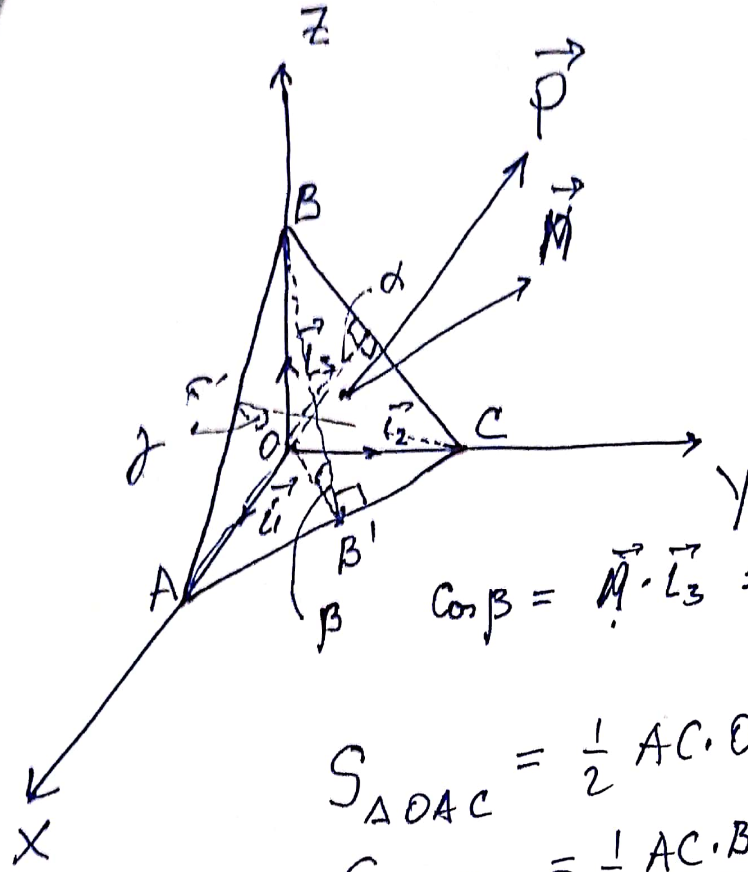
$$= a'_{11}(\vec{i}'_1 \otimes \vec{i}'_1) + a'_{12}(\vec{i}'_1 \otimes \vec{i}'_2) + a'_{13}(\vec{i}'_1 \otimes \vec{i}'_3) +$$
$$a'_{21}(\vec{i}'_2 \otimes \vec{i}'_1) + a'_{22}(\vec{i}'_2 \otimes \vec{i}'_2) + a'_{23}(\vec{i}'_2 \otimes \vec{i}'_3) +$$
$$a'_{31}(\vec{i}'_3 \otimes \vec{i}'_1) + a'_{32}(\vec{i}'_3 \otimes \vec{i}'_2) + a'_{33}(\vec{i}'_3 \otimes \vec{i}'_3)$$

$$a'_{11} = a_{11}\alpha_{11}\alpha_{11} + a_{12}\alpha_{11}\alpha_{12} + a_{13}\alpha_{11}\alpha_{13} +$$
$$+ a_{21}\alpha_{12}\alpha_{11} + a_{22}\alpha_{12}\alpha_{12} + a_{23}\alpha_{12}\alpha_{13} +$$
$$+ a_{31}\alpha_{13}\alpha_{11} + a_{32}\alpha_{13}\alpha_{12} + \alpha_{33}\alpha_{13}\alpha_{13}$$

$$a'_{11} = a_{nm}\alpha_{1n}\alpha_{1m} \; ; \; a'_{11} = \sum_{n;m=1}^{3} a_{nm}\alpha_{1n}\alpha_{1m}$$

$$a'_{pq} = \sum_{n;m=1}^{3} a_{nm}\alpha_{pn}\alpha_{qm} \; ; \; \underline{p;q = 1,2,3}$$

$$a'_{11} = a_{11}\alpha_{11}\alpha_{11} + a_{12}\alpha_{11}\alpha_{12} + a_{13}\alpha_{11}\alpha_{13} +$$
$$+ a_{21}\alpha_{12}\alpha_{11} + a_{22}\alpha_{12}\alpha_{12} + a_{22}\alpha_{12}\alpha_{13}$$
$$+ a_{31}\alpha_{13}\alpha_{11} + a_{32}\alpha_{13}\alpha_{12} + a_{33}\alpha_{13}\alpha_{13}$$



$|\vec n| = 1$

$\cos\beta = \vec n \cdot \vec l_3 = n_3 \rightsquigarrow n_z$

$OB' = BB' \cdot n_3$

$S_{\Delta OAC} = \frac{1}{2} AC \cdot OB' = \frac{1}{2} AC \cdot BB' \cdot n_3$

$S_{\Delta ABC} = \frac{1}{2} AC \cdot BB'$

$\boxed{S_{\Delta OAC} = n_3 \cdot S_{\Delta ABC}}$

$\cos\gamma = \vec n \cdot \vec l_2 = n_2$ (n_y) $\qquad OC' = CC' \cdot n_2$

$S_{\Delta OAB} = \frac{1}{2} BA \cdot OC' = \frac{1}{2} BA\, CC' \cdot n_2$

$S_{\Delta ABC} = \frac{1}{2} BA \cdot CC'$

$\boxed{S_{\Delta OAB} = n_2 S_{\Delta ABC}}$

$\boxed{S_{\Delta OBC} = n_1 S_{\Delta ABC}}$ $\qquad n_1 = (\vec n \cdot \vec l_1) = \cos\alpha$

$\Delta OAB \rightarrow \sigma_{yy}, \sigma_{yx}, \sigma_{yz}$

$\Delta OAC \rightarrow \sigma_{zz}, \sigma_{zx}, \sigma_{zy}$

$\Delta OBC \rightarrow \sigma_{xx}, \sigma_{xy}, \sigma_{xz}$

X ekseni yönünde kuvvetlerin izdüşümü

$$\overbrace{S_{\Delta OAB}}^{n_2 S_{\Delta ABC}} \sigma_{yx} + \overbrace{S_{\Delta OAC}}^{n_3 S_{\Delta ABC}} \sigma_{zx} + \overbrace{S_{\Delta OBC}}^{n_1 S_{\Delta ABC}} \sigma_{xx} =$$

$$\vec{P} \cdot \vec{i_1} \; \underline{S_{\Delta ABC}} \qquad \boxed{P_1 = \vec{P} \cdot \vec{i_1}}$$

$$\boxed{\begin{aligned} \sigma_{xx} n_1 + \sigma_{yx} n_2 + \sigma_{zx} n_3 &= P_1 . \\ \sigma_{xy} n_1 + \sigma_{yy} n_2 + \sigma_{zy} n_3 &= P_2 , \\ \sigma_{xz} n_1 + \sigma_{yz} n_2 + \sigma_{zz} n_3 &= P_3 , \end{aligned}} \quad (*)$$

$$P_2 = \vec{P} \cdot \vec{i_2} \; ; \; P_3 = \vec{P} \cdot \vec{i_3}$$

$$\vec{P} = P_1 \vec{i_1} + P_2 \vec{i_2} + P_3 \vec{i_3}$$

$$\sigma_n = \vec{P} \cdot \vec{n} = P_1 n_1 + P_2 n_2 + P_3 n_3$$

$$\boxed{\begin{aligned} \sigma_n = \; & \sigma_{xx} n_1^2 + \sigma_{yx} n_2 n_1 + \sigma_{zx} n_3 n_1 + \\ & + \sigma_{xy} n_1 n_2 + \sigma_{yy} n_2^2 + \sigma_{zy} n_3 n_2 + \\ & + \sigma_{xz} n_1 n_3 + \sigma_{yz} n_2 n_3 + \sigma_{zz} n_3^2 \end{aligned}}$$

$n_1 = \alpha_{11}$
$n_2 = \alpha_{12}$
$n_3 = \alpha_{13}$

$$\vec{n} \to O x_1' \; \vec{i_1'}$$

Kuvvetlere göre verilen sınır koşulları matematiksel olarak $(*)$ formulleri ile ifade edilir