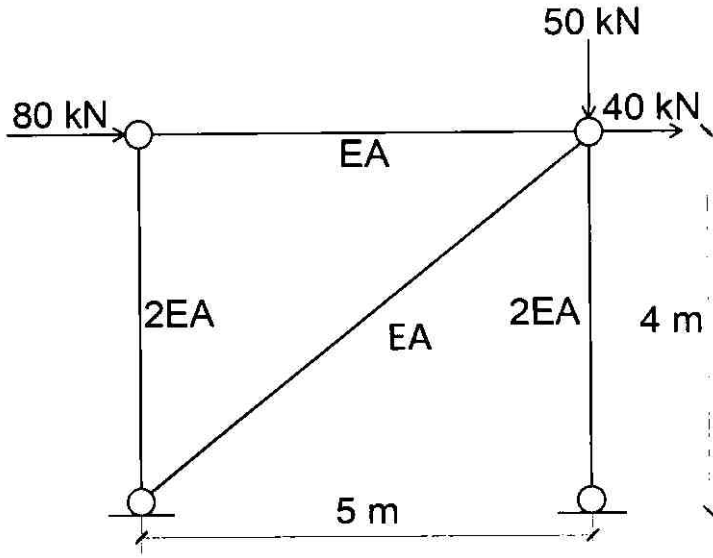


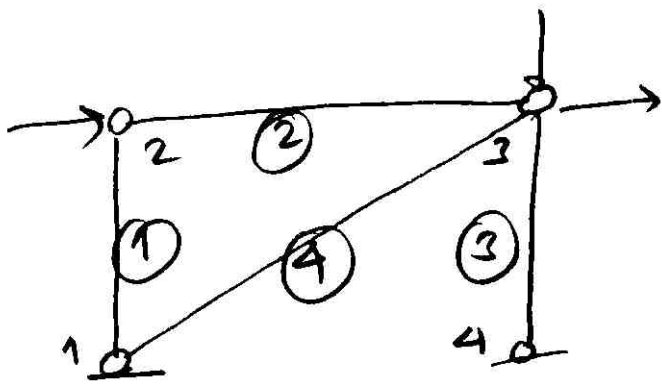
KAFES ÖRNEK ÇÖZÜMÜ: Şekilde verilen kafes sistemin düğüm noktalarının yerdeğişmelerini ve çubuk kuvvetlerini sonlu elemanlar yöntemi ile bulunuz ($EA=4 \cdot 10^6$ kN).



İstenenler

- Eleman rijitlik matrisleri
- Sistem rijitlik matrisi
- Sistem yük matrisi
- Yerdeğişmeler
- Çubuk kuvvetleri

Kafes Örneği
S1-4



D.N	X	Y
1	0	0
2	0	4
3	5	4
4	5	0

E.N	(L) (R)		L	C	S
	e	d			
1	1	2	4	0	1
2	2	3	5	1	0
3	3	4	4	0	-1
4	1	3	6,403	0,78	0,624

$$[k_1] = \begin{bmatrix} 2 \cdot 10^6 & -2 \cdot 10^6 \\ -2 \cdot 10^6 & 2 \cdot 10^6 \end{bmatrix} \quad [k_2] = 10^5 \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \quad [k_3] = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$[k_4] = \begin{bmatrix} 624 & -624 \\ -611 & +4 \end{bmatrix}$$

$$[t_1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad [t_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [t_3] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[t_4] = \begin{bmatrix} 0,78 & 0,624 & 0 & 0 \\ 0 & 0 & 0,78 & 0,624 \end{bmatrix}$$

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad [C_2] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[C_3] = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad [C_4] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[K_1] = 10^6$$

K2/A

$$\begin{bmatrix}
 0,3809 & 0,2047 & 0 & 0 & 0 & -0,3809 & -0,3047 & 0 & 0 & 0 \\
 0,3047 & 3,2438 & 0 & -2 & -0,3047 & -0,2438 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0,8 & 0 & -0,8 & 0 & 0 & 0 & 80 & 0 \\
 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -0,3809 & -0,2047 & -0,8 & 0 & 1,1809 & 0,3047 & 0 & 0 & 40 & -50 \\
 -0,3047 & -0,2438 & 0 & 0 & 0,3047 & 3,2438 & -2 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -2 & 2 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 D_1 \\
 D_2 \\
 D_3 \\
 D_4 \\
 D_5 \\
 D_6 \\
 D_7 \\
 D_8
 \end{bmatrix}$$

$$D_3 = 0,4734 \times 10^{-3}$$

$$D_4 = 0$$

$$D_5 = 0,3734 \times 10^{-3}$$

$$D_6 = -0,0730 \times 10^{-3}$$

Label yordagz tikmelik

$$\{d\}^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0,4734 \cdot 10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\{d\}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 0,4734 \cdot 10^{-3} \\ 0 \\ 0,3734 \cdot 10^{-3} \\ -0,0730 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 0,4734 \cdot 10^{-3} \\ 0,3734 \cdot 10^{-3} \end{Bmatrix}$$

$$\{d\}^3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} 0,3734 \cdot 10^{-3} \\ 0 \\ -0,0730 \cdot 10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} +0,0730 \cdot 10^{-3} \\ 0 \end{Bmatrix}$$

$$\{d\}^4 = \begin{bmatrix} 0,7809 & 0,6247 & 0 & 0 \\ 0 & 0 & 0,7809 & 0,6247 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0,3734 \cdot 10^{-3} \\ -0,0730 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0,246 \cdot 10^{-3} \end{Bmatrix}$$

Gesam KVV result

$$F_1 = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix}$$

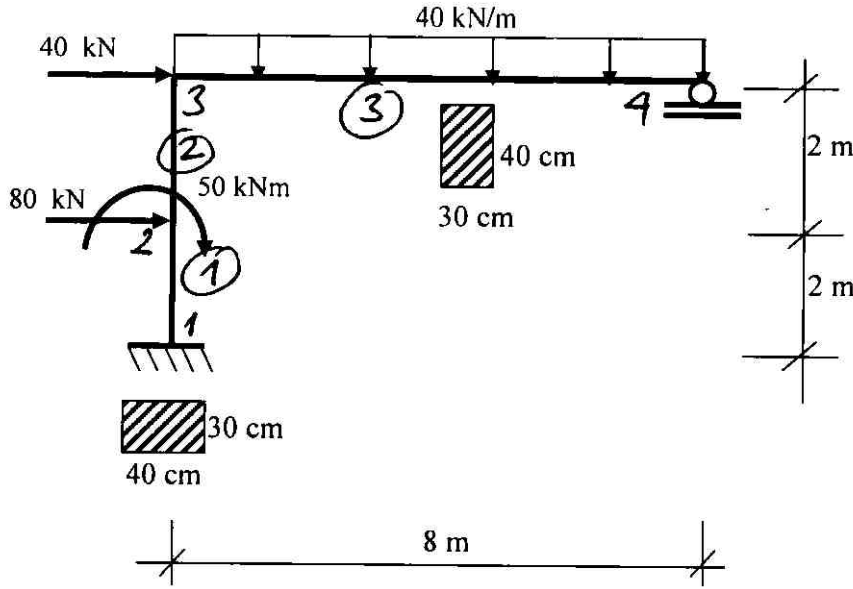
$$F_2 = 10^5 \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{Bmatrix} 0,4734 \cdot 10^{-3} \\ 0,3724 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 80 \\ -80 \end{Bmatrix}$$

$$F_3 = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} 0,0720 \cdot 10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 146 \\ -146 \end{Bmatrix}$$

$$F_4 = \begin{bmatrix} 62400 & -62400 \\ -62400 & 62400 \end{bmatrix} \begin{Bmatrix} 0 \\ 0,246 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} -153,678 \\ 153,678 \end{Bmatrix}$$

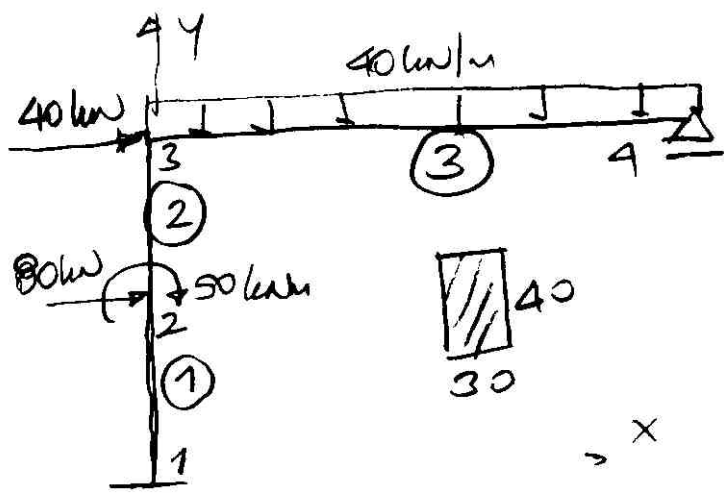
ÖRNEK: Şekilde verilen sistemin düğüm noktalarının yerdeğiřtirmelerini ve çubuk kuvvetlerini sonlu elemanlar yöntemi ile bulunuz.

($E: 21 \times 10^6 \text{ kN/m}^2$)



Not: Tekil yük bulunan çubuklarda,
'tekil yükün etki ettiği' noktaları düğüm
noktası olarak ele almak sistem
kolaylığı sağlayacaktır!

ÖRNEK
1-9 sayfa



DN	X	Y
1	0	0
2	0	2
3	0	4
4	8	4

$$E = 21 \times 10^6 \text{ kN/m}^2$$

$$I = \frac{0,3 \times 0,4^3}{12} = 1,6 \times 10^{-4} \text{ m}^4$$

$$EA = 252 \times 10^4 \text{ kN}$$

$$EI = 3,36 \times 10^4 \text{ kNm}^2$$

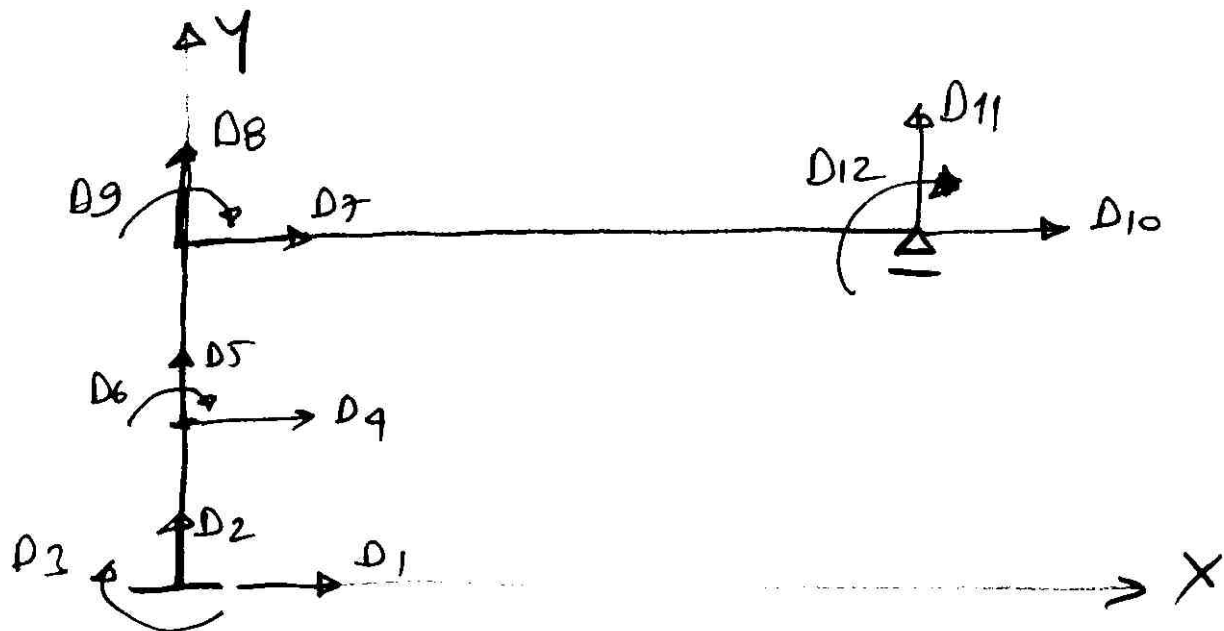
E.N	$\bar{i}$	$\bar{j}$	L	C	S
1	1	2	2	0	1
2	2	3	2	0	1
3	3	4	8	1	0



Eleman yedöğütükne
yönlee'

$$[k_e] = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix}$$

81/9



$$[k_1][k_2] = 10^4 \begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 5,04 & 5,04 & 0 & -5,04 & 5,04 \\ 0 & 5,04 & 6,72 & 0 & -5,04 & 3,36 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -5,04 & -5,04 & 0 & 5,04 & -5,04 \\ 0 & 5,04 & 3,36 & 0 & -5,04 & 6,72 \end{bmatrix}$$

$$[k_3] = 10^4 \begin{bmatrix} 31,5 & 0 & 0 & -31,5 & 0 & 0 \\ 0 & 0,07875 & 0,315 & 0 & -0,07875 & 0,315 \\ 0 & 0,315 & 1,68 & 0 & -0,315 & 0,84 \\ -31,5 & 0 & 0 & 31,5 & 0 & 0 \\ 0 & -0,07875 & -0,315 & 0 & 0,07875 & -0,315 \\ 0 & 0,315 & 0,84 & 0 & -0,315 & 1,68 \end{bmatrix}$$

8.2/9

$$[\tau_1] =$$

$$[T_2] =$$

$$[T_3] =$$

$$[c_1] =$$

$$[C_2] =$$

$$[C_3] =$$

$$[k_{e1}] = [T_1]^T [k_1] [T_1] \quad [k_{e2}] = [T_2]^T [k_2] [T_2]$$

$$[K_2] = [T_2]^T [K_1] [T_2] \quad [K_2] = [C_1]^T [K_1] [C_1]$$

$$[k_2] = [T_2]' [k_3] [T_2] ; [k_3] = 2 \times 10^6 \text{ lb/in}$$

$$[K] = [K_1] + [K_2] + [K_3]$$

0.3/9

Yerel Koordinatlarda Yerdeğiştirme

$$\{u_1\}' = [T_1]\{u\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0,0197 \\ -0,000131 \\ 0,0173 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0,000131 \\ 0,0197 \\ 0,0173 \end{Bmatrix}^{1e}$$

$$\{u_2\}^2 = [T_2]\{u\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0,0197 \\ -0,000131 \\ 0,0173 \\ 0,0599 \\ -0,000262 \\ 0,0221 \end{Bmatrix} = \begin{Bmatrix} -0,000131 \\ 0,0197 \\ 0,0173 \\ -0,000262 \\ 0,0599 \\ 0,0221 \end{Bmatrix}^{2e}$$

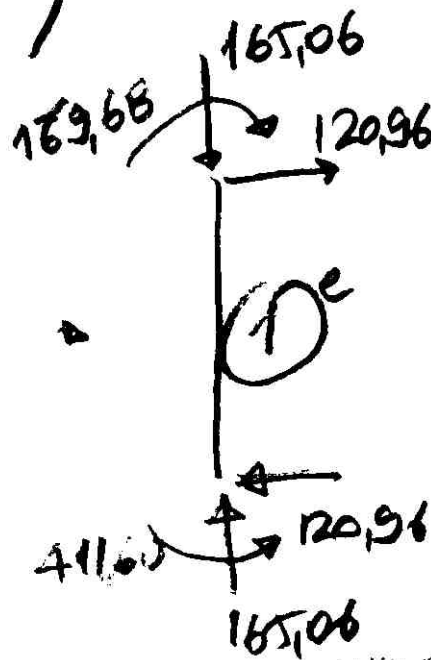
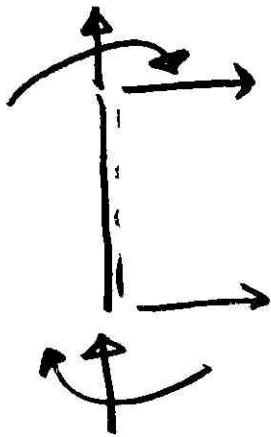
$$\{u_3\}^3 = [T_3]\{u\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0,0599 \\ -0,000262 \\ 0,0221 \\ 0,0599 \\ 0 \\ -0,0238 \end{Bmatrix} = \begin{Bmatrix} 0,0599 \\ 0,000262 \\ 0,0221 \\ 0,0599 \\ 0 \\ -0,0238 \end{Bmatrix}$$

Güçlük Kuvvetlerinin Hesabı

1. eleman

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = 10^4 \begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 5,04 & 5,04 & 0 & -5,04 & 5,04 \\ 0 & 5,04 & 6,72 & 0 & -5,04 & 3,36 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -5,04 & -5,04 & 0 & 5,04 & -5,04 \\ 0 & 5,04 & 3,36 & 0 & -5,04 & 6,72 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0,000131 \\ 0,0197 \\ 0,0133 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

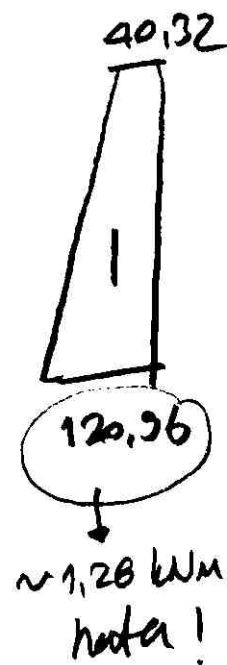
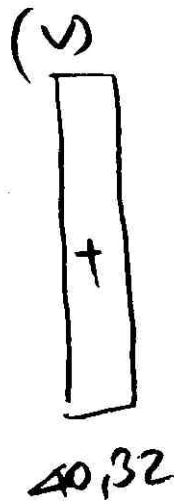
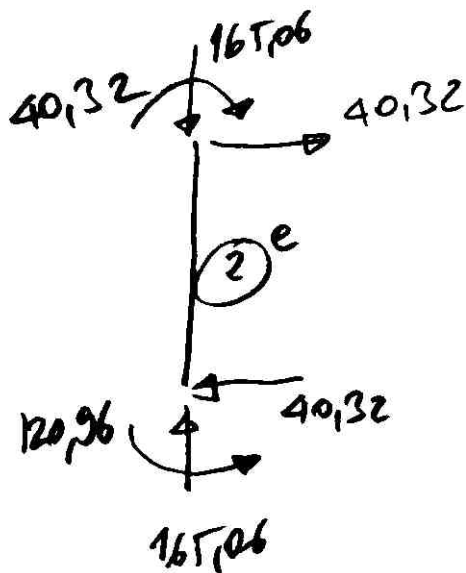
$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} 165,06 \\ -120,96 \\ -411,60 \\ -165,06 \\ 120,96 \\ 169,68 \end{Bmatrix}$$



* Not: $D_5 = -0,000131$ değeri hesaplarda $-0,0001$ olarak görülebilmek. Bu durumda N değeri 126 kN gibi ($N_{gersek} = 165 \text{ kN}$) farklı bir değer hesaplanabilmektedir.

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{2e} = 10^4 \begin{matrix} \text{2. element} \\ \begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 5,04 & 5,04 & 0 & -5,04 & 5,04 \\ 0 & 5,04 & 6,72 & 0 & -5,04 & 3,36 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -5,04 & -5,04 & 0 & 5,04 & -5,04 \\ 0 & 5,04 & 3,36 & 0 & -5,04 & 6,72 \end{bmatrix} \end{matrix} \begin{Bmatrix} -0,000131 \\ 0,0197 \\ 0,0173 \\ -0,000262 \\ 0,0599 \\ 0,0221 \end{Bmatrix} - \begin{Bmatrix} 0 \end{Bmatrix}$$

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{2e} = \begin{Bmatrix} 165,06 \\ -40,32 \\ -120,96 \\ -165,06 \\ 40,32 \\ 40,32 \end{Bmatrix}$$



3. eleman

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{3.e} = 10^4 \begin{bmatrix} 31,5 & 0 & 0 & -31,5 & 0 & 0 \\ 0 & 0,07875 & 0,315 & 0 & -0,07875 & 0,315 \\ 0 & 0,315 & 1,68 & 0 & -0,315 & 0,84 \\ -31,5 & 0 & 0 & 31,5 & 0 & 0 \\ 0 & -0,07875 & -0,315 & 0 & 0,07875 & -0,315 \\ 0 & 0,315 & 0,84 & 0 & -0,315 & 1,68 \end{bmatrix} \begin{Bmatrix} 0,0599 \\ 0,000262 \\ 0,0221 \\ 0,0599 \\ 0 \\ -0,0238 \end{Bmatrix} = \begin{Bmatrix} 0 \\ +160 \\ 213,333 \\ 0 \\ +160 \\ 213,333 \end{Bmatrix}$$

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{3.e} = \begin{Bmatrix} 0 \\ -165,15 \\ -41,14 \\ 0 \\ -154,85 \\ 0 \end{Bmatrix} \rightarrow 0,826 \text{ Nm hatır!}$$

elemanlar yönleri
azalışa doğru
pozitif alınmıştır!

