SONLU ELEMANLAR METODU
DERS NOTLARI

DOÇ. DR. ALİ KOÇAK
YILDIZ TEKNİK ÜNİVERSİTESİ

2015

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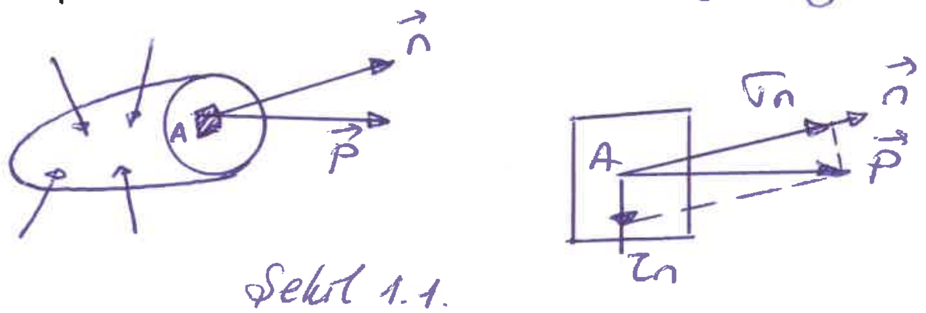
BÖLÜM I

ELASTİSİTE BAĞINTILARI

1) GERİLME HALİ

1.1) Giriş: Denge durumundaki bir sistemde dış kuvvetler ile iç kuvvetler denge halindedir. Dış yüklerin etkisi altındaki bir sistemin enine kesiti üzerinde iç kuvvetler kesit yüzeyi boyunca sürekli bir tarzda yayılı olarak bulunduğu düşünülür. Bu kesit yüzeyi üzerinde birim alana gelen şiddete gerilme denir.

Sistemin gelişigüzel enine yönde kesilmesi sonucunda ayrılan parçalardaki gerilme vektörü yüzeye dik değildir (Şekil 1.1). Bu gerilmeye eğik gerilme vektörü denir.

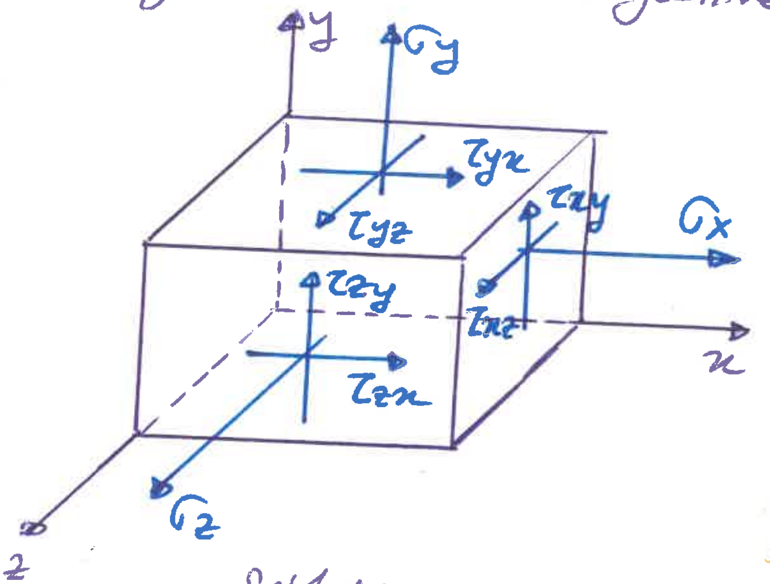


Şekil 1.1.

Eğik gerilme vektörünün, yüzeyin normali doğrultusundaki bileşenine, başka bir deyişle yüzeye dik bileşenine normal gerilme (σ_n), yüzeyin teğet doğrultusundaki bileşenine de kayma gerilmesi (τ_n) denir (Şekil 1.1)

Şekil 1.1 'de verildiği gibi doğrultu normali ($\vec{n}$) değiştikçe $\vec{p}$ gerilme vektörü de değişecektir. Dolayısıyla $\vec{n}$ ile $\vec{p}$ arasında bir bağıntı mümkündür ve bu da tek bir büyüklük olarak tanımlanabilir.

Buna gerilme hali, matematiksel olarak da adına gerilme tansörü denir. Buradan da anlaşılacağı gibi bu gerilme vektörü veya gerilme tansörü birbirlerinden bağımsız değildir. Şekil 1.1'de verilen A noktasının gerilme hali için üç vektörel büyüklüğe veya dokuz skaler büyüklük gerekmektedir. Şekil 1.2'de de verildiği gibi prizmatik elemanın gerilme tansörü 9 gerilme bileşenlidir.



$$[T_g] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Şekil 1.2.

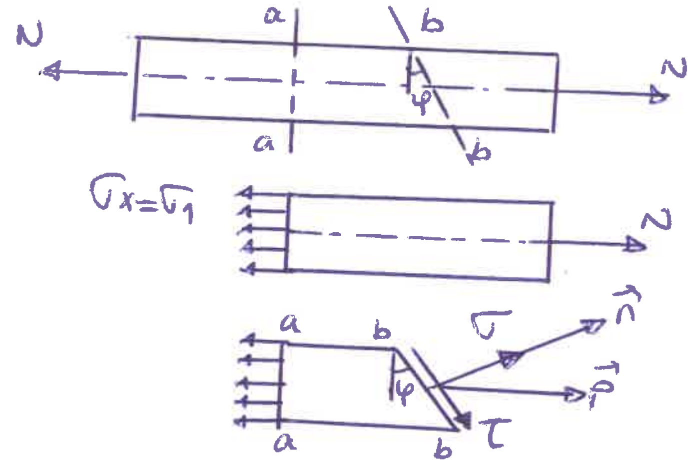
Gerilme tansöründe normal gerilmeler doğrultularına göre tek indisle belirtilir. Kayma gerilmeleri ise birinci indisi kayma gerilmelerinin normalini, ikinci indisi kayma gerilmelerinin yönünü olmak üzere iki indisle belirtilir. Dokuz skaler büyüklükten oluşan gerilme tansörü diyagonale göre simetriktir ve altı bağımsız skalerle belirtilebilir.

$$|\tau_{xy}| = |\tau_{yx}| \quad ; \quad |\tau_{xz}| = |\tau_{zx}| \quad ; \quad |\tau_{yz}| = |\tau_{zy}|$$

Özel durumlarda bu altı skaler büyüklük daha az sayı ile belirtilebilir. Bu gerilme halleri 1, 2 ve 3 eksenli gerilme halleridir.

1.2) Bir Eksenli Gerilme Hali

Bir elemanın ayırma kesitindeki gerilme vektörlerinin doğrultularının aynı olduğu gerilme halidir. Bir başka tanıma göre; eksenel çekme veya basınca maruz bir çubuğun bir kesitinde meydana gelen normal gerilmeler (σ) eksen doğrultusuna paraleldir. Bu doğrultuya dik kayma gerilmeleri (τ) ise sıfırdır (Şekil 1.3).

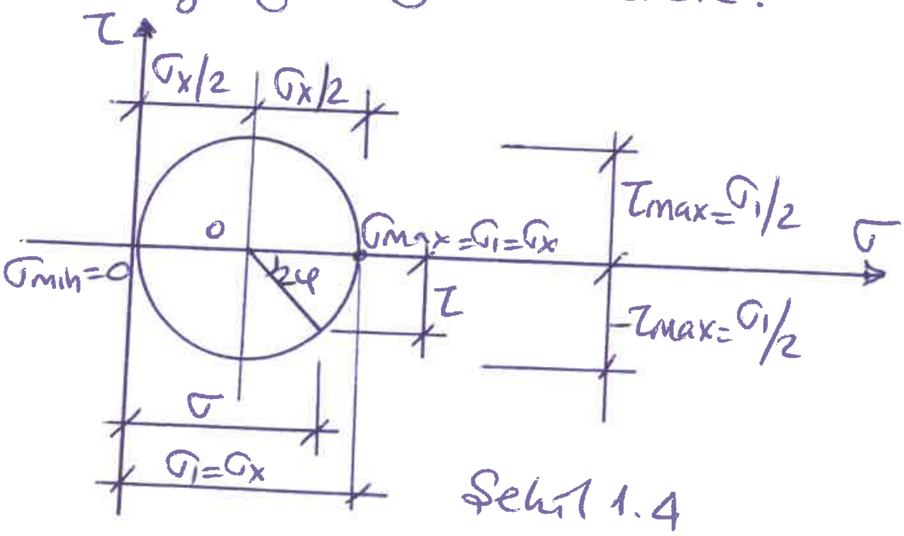


$$\sigma_x = \sigma_1 = \frac{N}{A}$$

$$[T_g] = [\sigma_x] = [\sigma_1] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Şekil 1.3

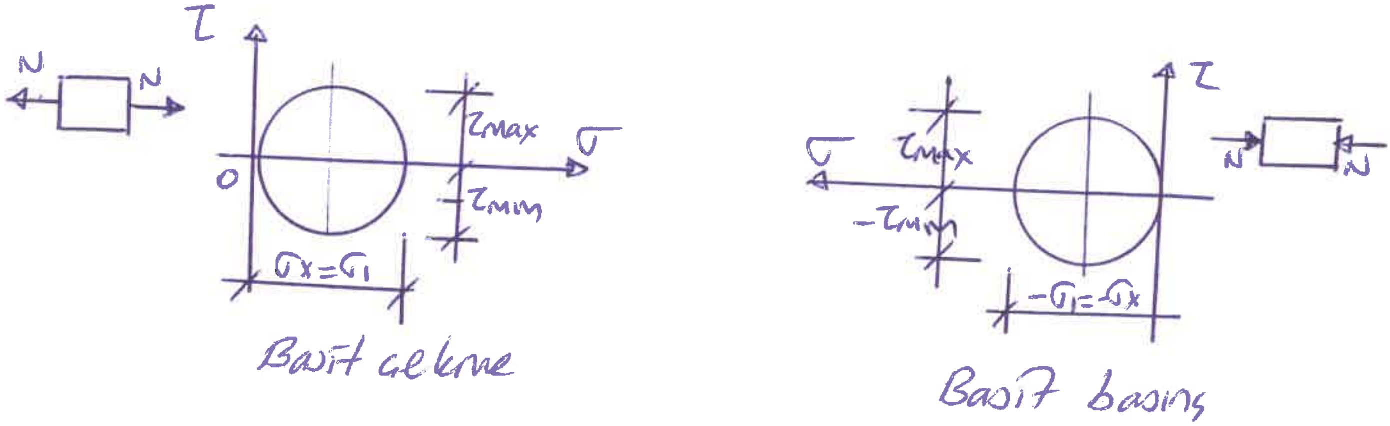
Şekilde'de görüleceği gibi aynı eleman üzerinde b-b kesitinde oluşacak gerilmeler ise σ ve τ'dur. Burada $\sigma = \sigma_1 \cdot \cos^2\varphi$, $\tau = -\sigma_1 \cdot \sin\varphi\cos\varphi$ 'dir. Bir eksenli gerilme halindeki durumu mohr dairesinde Şekil 1.4'de verildiği gibi gösterebiliriz.



$$\sigma_{max} = \sigma_1 = \sigma_x$$
$$\sigma_{min} = 0$$
$$\tau_{max} = \frac{\sigma_1}{2}$$
$$\tau_{min} = -\frac{\sigma_1}{2}$$

Şekil 1.4

Görüldüğü gibi bir eksenli gerilme halinde mohr dairesi başlangıç noktasından geçmektedir. Ayrıca bu gerilme haline basit çekme veya basit basınç durumu da denilmektedir (Şekil 1.5).

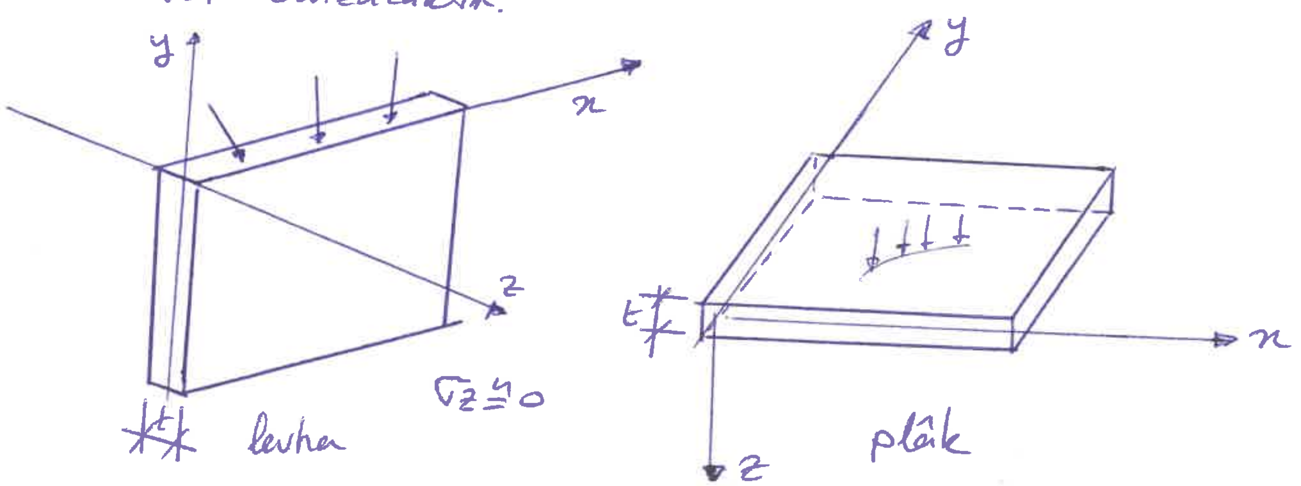


Şekil 1.5.

1.3) İki Eksenli Gerilme Hali (Düzlem Gerilme Hali)

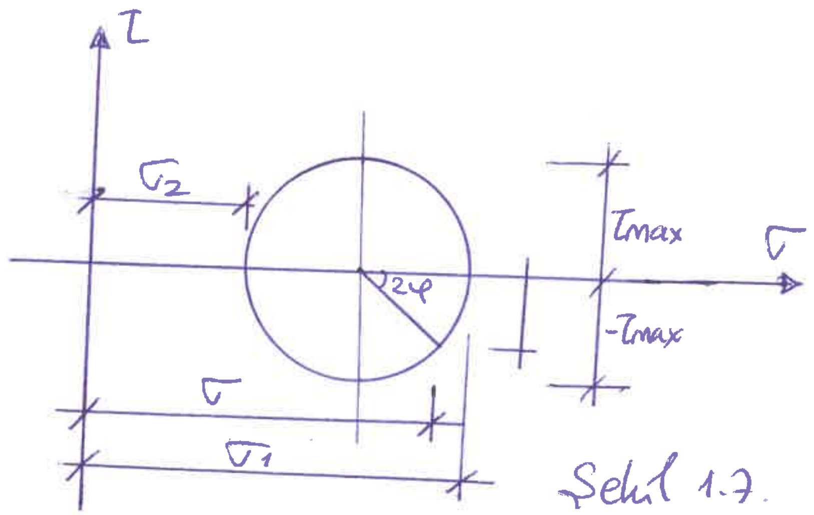
Bir elemanın ayırma kesitindeki gerilme vektörlerinin doğrultuları aynı düzlem içinde ise buna iki eksenli gerilme hali veya düzlem gerilme hali denir. Plâk veya levha gibi elemanlar bu gerilme hali ile ifade edilirler. Düzlem gerilme halinde eleman kalınlığı boyunca gerilmeler sıfırdır (Şekil 1.6) ($\sigma_z \cong 0$). Buna bağlı olarak kayma gerilmeleri de;

$\tau_{zx} = \tau_{xz} = 0, \quad \tau_{zy} = \tau_{yz} = 0$ 'dır. Bu durumda gerilme tansörü üçü bağımsız olmak üzere 4 skaler büyüklükten oluşacaktır.



Şekil 1.6.

İki eksenli gerilme hali için mohr çemberi şekil 1.7'de verilmiştir.



$$[T_g]=\begin{bmatrix}\sigma_x & \tau_{xy} & 0\\ \tau_{yx} & \sigma_y & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$|\tau_{xy}|=|\tau_{yx}|$$

Şekil 1.7.

Asal gerilme değerleri ise;

$$\sigma_{\substack{max\\min}}=\sigma_{1,2}=\frac{\sigma_x+\sigma_y}{2}\mp\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\tau_{\substack{max\\min}}=\mp\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}=\frac{\sigma_1-\sigma_2}{2}$$ şeklindedir.

1.4.) Üç Eksenli Gerilme Hali

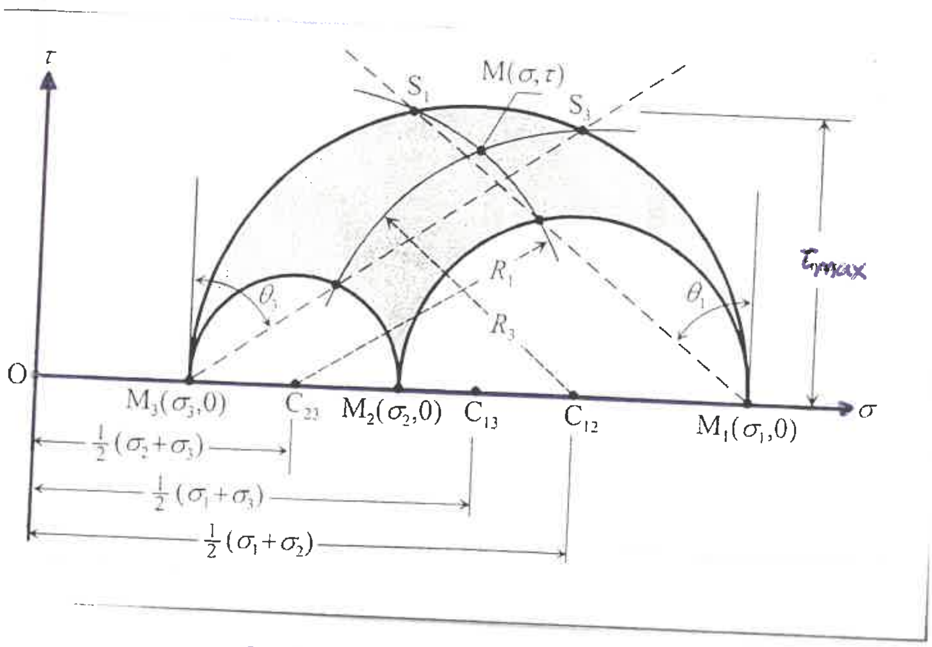
Bir elemanın ayırma kesitindeki gerilme vektörlerinin doğrultuları farklı ise buna üç eksenli gerilme hali denir. Gerilme tansörü altı bağımsız gerilme bileşeni olmak üzere toplam 9 skaler büyüklükten oluşmaktadır.

$$[T_g]=[\sigma]=\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_z\end{bmatrix}$$

Burada $|\tau_{xy}|=|\tau_{yx}|$, $|\tau_{xz}|=|\tau_{zx}|$ ve

$|\tau_{yz}|=|\tau_{zy}|$ 'dir.

Üç eksenli gerilme hali için mohr çemberi Şekil 1.8'de verildiği gibidir.

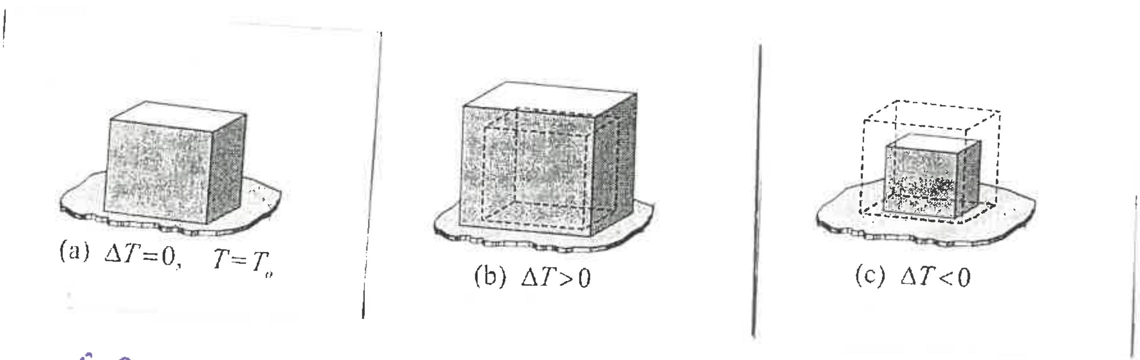


Şekil 1.8.

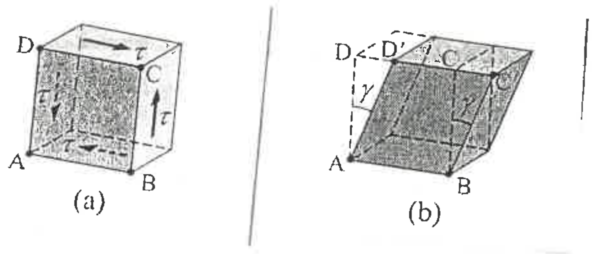
2) ŞEKİL DEĞİŞTİRME HALİ

2.1) Giriş : Şekil değiştirme, bir cismin veya bir elemanın noktalarının birbirlerine göre konumlarının veya rölatif durumunun değişmesidir.

Cismin genel olarak geometrik tipinin değişmeden kenarlarının uzaması veya kısalması suretiyle yalnızca kenarlarının değişmesi (Şekil 2.1) veya kenarlarının açısal olarak değişmesi (Şekil 2.2) şekil değiştirmedir.



(Şekil 2.1)



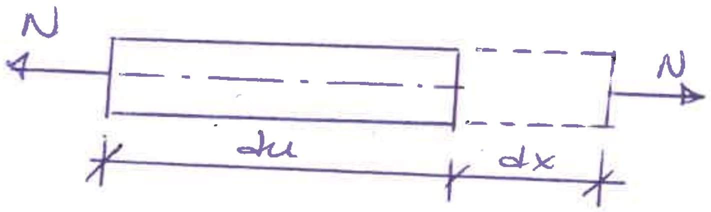
Şekil 2.2

Sözü edilen bu şekil değiştirmeler dış yükler, ısı değişimi ve yapı malzemesinin visko-elastik özelliğinden (sünme-rötre) dolayı meydana gelmektedir. Bu şekil değiştirme boy değişimi ve açı değişimi şeklindedir ve şekil değişimi sonucunda cismin şekli ve boyutları değişmektedir.

Cismin şekil değiştirme hali de tıpkı gerilme halinde ki gibi 1, 2 ve 3 eksenli şekil değiştirme hali şeklinde de incelenmektedir.

2.2) Bir Eksenli Şekil Değiştirme Hali

Bir eksenli sistemlerde şekil değiştirme uzama şekil değiştirmesidir (Şekil 2.3).



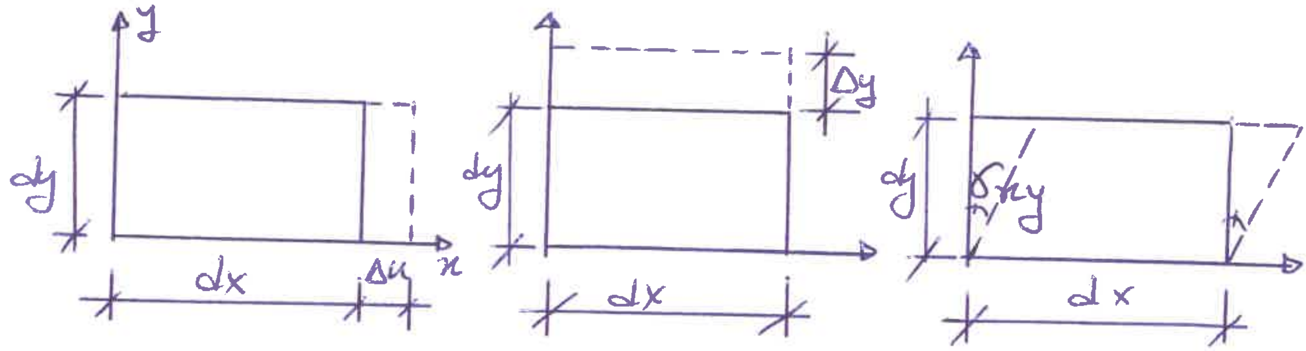
$\sigma = \frac{N}{A}$ $\quad \varepsilon = \frac{\sigma}{E} = \frac{\partial u}{\partial x}$

Şekil 2.3

Böyle bir sistemin şekil değiştirme tansörü $[\varepsilon] = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ şeklindedir.

2.3) İki Eksenli Şekil Değiştirme Hali (Düzlem Şekil Değiştirme Hali)

Şekil değiştirmiş bir cismin yer değiştirme vektörleri düzlemde kalıyorsa ve diğer z doğrultusundan etkilenmiyorsa bu şekil değiştirme haline düzlem şekil değiştirme hali denir. Tıpkı bir istinat duvarının veya betonarme perde elemanının uzun doğrultusu gibidir. Bu durumda z doğrultusundaki şekil değiştirmeler sıfır olacaktır ($\varepsilon_z \cong 0$, $\gamma_{xz} = \gamma_{yz} = 0$). Düzlemsel şekil değiştirmeler Şekil 2.3'de verildiği gibi olacaktır.



Şekil 2.3

Düzlem şekil değiştirme halinde, şekil değiştirme tansörü 4 skaler büyüklükten oluşmaktadır;

$$[\varepsilon] = \begin{bmatrix} \varepsilon_x & \gamma_{xy} \\ \gamma_{yx} & \varepsilon_y \end{bmatrix}$$

Burada $\gamma_{xy} = \gamma_{yx}$ 'dir. Asal doğrultudaki maksimum şekil değiştirmeler;

$$\varepsilon_{\substack{max \\ min}} = \frac{\varepsilon_x+\varepsilon_y}{2} \mp \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$

2.4) Üç Eksenli Şekil Değiştirme Hali

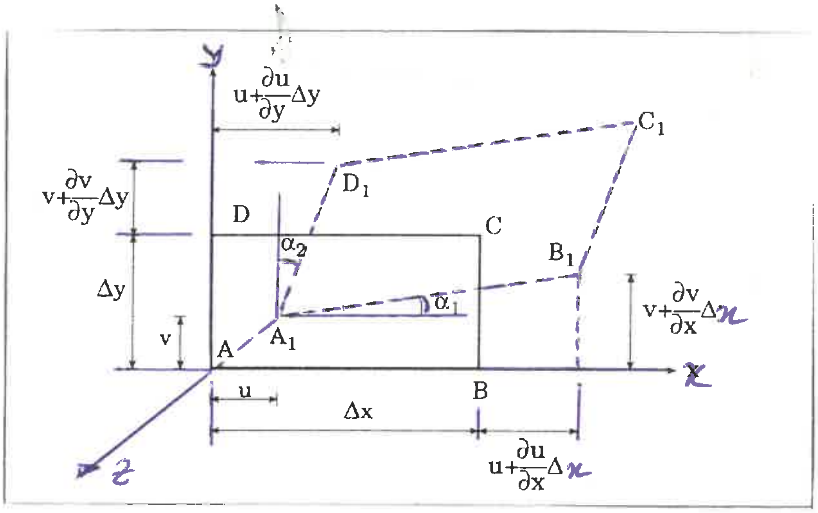
Prizmatik veya üç eksenli bir cismin şekil değiştirmesinde, şekil değiştirme tansörü 9 adettir.

$$[\varepsilon] = \begin{bmatrix} \varepsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_z \end{bmatrix}$$

Burada $\gamma_{xy}=\gamma_{yx}$, $\gamma_{zx}=\gamma_{xz}$ ve $\gamma_{yz}=\gamma_{zy}$ 'dir.

3) ŞEKİL DEĞİŞTİRME - YER DEĞİŞTİRME ARASINDAKİ BAĞINTILAR

Bir cismin şekil değiştiren noktalarının durumunu gösteren değerlerini, aynı noktaların yer değiştirme cinsinden verilmesi mümkündür. Tanımlanan noktanın yer değiştirmeleri x, y, z koordinatlarına göre sırasıyla u, v ve w ile dikdörtgen bir elemanın buna göre yer değiştirmeleri şekil 3.1 'de gösterildiği gibidir.



Şekil 3.1

Birim şekil değiştirmeleri yer değiştirmeleri cinsinden aşağıda verildiği gibi yazabiliriz.

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

$$\varepsilon_z = \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left[\frac{\partial u}{\partial x}\,\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\,\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\,\frac{\partial w}{\partial y}\right]$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \left[\frac{\partial u}{\partial x}\,\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\,\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\,\frac{\partial w}{\partial z}\right]$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \left[\frac{\partial u}{\partial y}\,\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y}\,\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\,\frac{\partial w}{\partial z}\right]$$

Bağıntılarda ilk terimler lineer durumu, parantez içi terimler ise lineer olmayan (non-lineer) durumu göstermektedir. Parantez içindeki ilk iki terim son terim yanında ihmal edilebilecek kadar küçük olduğundan analizlerde;

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\varepsilon_z = \frac{\partial w}{\partial z} + \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left[\frac{\partial w}{\partial x}\,\frac{\partial w}{\partial y}\right]$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \left[\frac{\partial w}{\partial x}\,\frac{\partial w}{\partial z}\right]$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\left[\frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}\right]$$

bağıntılarda verilen kısımları almak yeterlidir.

Matris formda yazılırsa;

$$\{\varepsilon\} = \underbrace{\{\varepsilon_o\}}_{\text{lineer}} + \underbrace{\{\varepsilon_n\}}_{\text{non-lineer}}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{o,m} \\ \varepsilon_{o,e} \end{Bmatrix} + \begin{Bmatrix} \varepsilon_{n,m} \\ \varepsilon_{n,e} \end{Bmatrix}$$

$\varepsilon_{o,m}$ = Lineer membran teriml

$\varepsilon_{o,e}$ = " eğilme terimleri

$\varepsilon_{n,m}$ = non-lineer membran ter

$\varepsilon_{n,e}$ = " eğilme ter.

Düzlem durumda;

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \\ -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{\partial^2 w}{\partial y^2} \\ -2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} + \begin{Bmatrix} \frac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \frac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial w}{\partial x} \cdot \dfrac{\partial w}{\partial y} \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{o,m} \\ \varepsilon_{oe} \end{Bmatrix} + \begin{Bmatrix} \varepsilon_{n,m} \\ 0 \end{Bmatrix}$$

$$\{\varepsilon_m\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \frac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \frac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix}$$

4) MALZEMELERİN MEKANİK ÖZELLİKLERİ VE GERİLME ŞEKİL DEĞİŞTİRME BAĞINTILARI

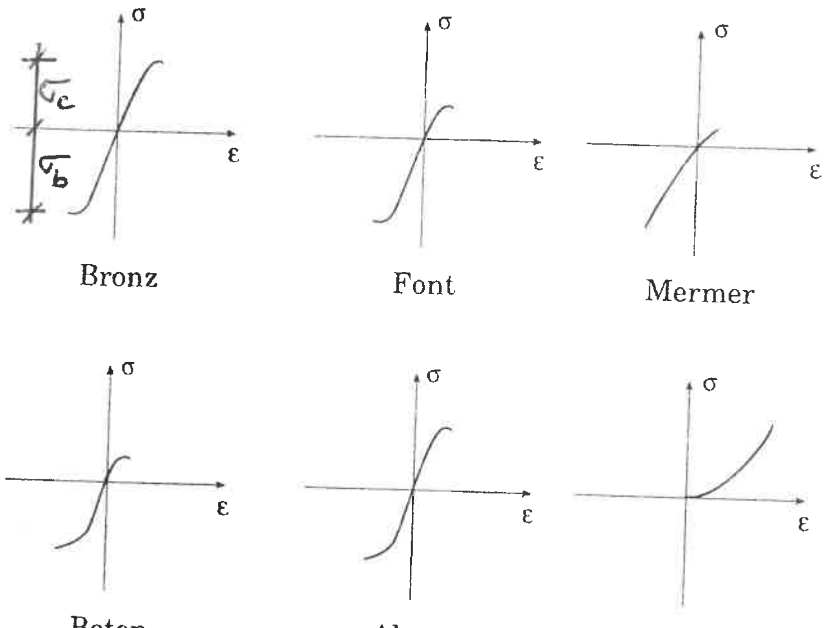
4.1) Giriş : İnşaat mühendisliğinde kullanılan malzemelerin, dış etkiler altındaki şekil değiştirmesinin, kırılmasının veya çatlama özelliklerinin incelenmesi ve tespiti malzemenin mekanik özelliklerinin en önemli konusudur. Bu mekanik özelliklerin tespitinde statik ve dinamik deneysel çalışmalar, sürekli ortamlar mekaniğinden yola çıkılarak yapılan araştırmalar ve malzemenin makroskopik yapısı ile atom kavramından yola çıkan çalışmalar rol oynamaktadır. Bu çalışmaların sonucunda elemanın malzemesi ile bunun dış etkiler altında tepki ve davranışlarını gösteren yani gerilme ile şekil değiştirme bileşenleri arasındaki ilişkiyi veren elastisite bağıntıları, bir başka deyişle bünye denklemleri bulunmaktadır.

Mühendislikte kullanılan malzemeler;

a) Metaller ve alaşımlar b) Polimerler

c) Seramik ve camlar 4) Kompozitler 'dir.

Bunun dışında ayrıca malzemeler kırılıncaya veya son limit duruma gelinceye kadar yaptıkları şekil değiştirme davranışları göz önüne alınarak sünek (düktil), gevrek ve plastik olarak üç grupta toplanabilirler. Sünek cisimler, kopmadan veya son limit duruma gelmeden önce büyük şekil değiştirme gösteren cisimlerdir. Buna en iyi örnek çeliktir. Gevrek cisimler, akma sınırları olmayan, kopmadan önce büyük uzama göstermeyen cisimlerdir. Buna örnek olarak beton, taş ve font'u gösterebiliriz. Plastik cisimler ise elastik özelliği az olan asfalt, ahşap ve deri gibi malzemelerin oluşturduğu cisimlerdir. Aşağıdaki şekilde bazı malzemelerin gerilme-şekil değiştirme eğrileri verilmiştir (Şekil 4.1)



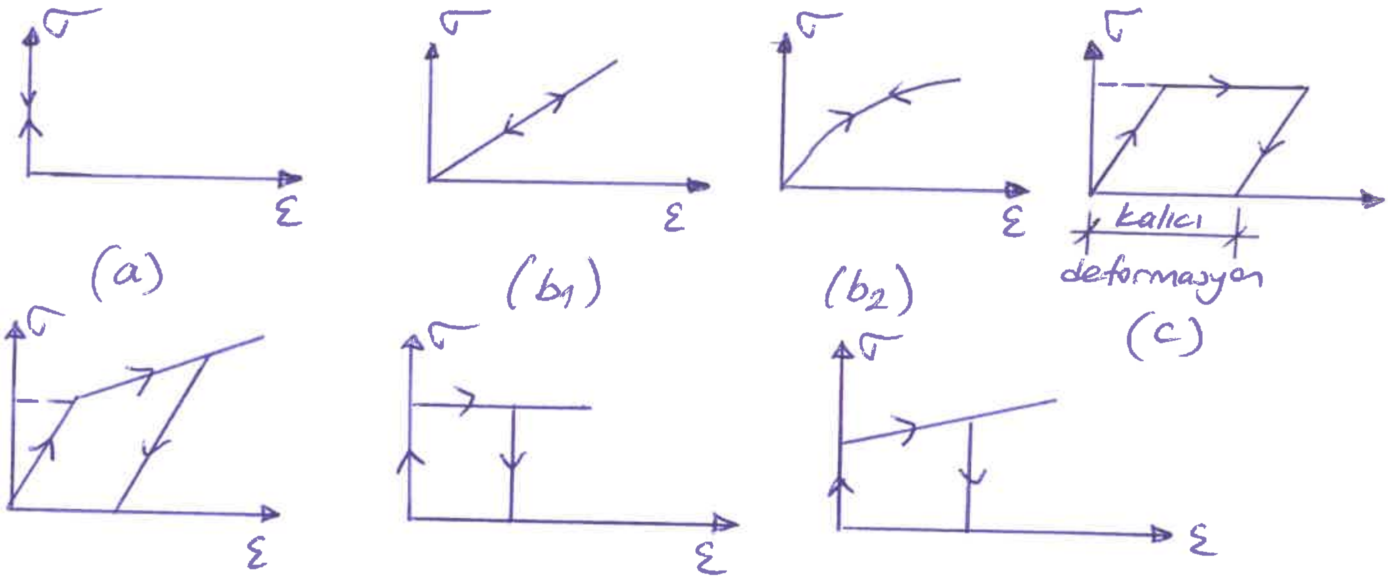
Şekil 4.1.

4.2) Gerilme-Şekil Değiştirme Diyagramlarının İdealleştirilmesi (İdeal Cisimler)

Cisimlerin mekanik özellikleri çok karışık ve birbirlerinden çok farklıdır. Herhangi bir mühendislik problemninde bunların tamamının birden göz önünde bulundurulması mümkün değildir. Basit olduğu kadar gerçeğe uygun düşecek bir teori kurulmak isteniyorsa, problemde en esaslı rolü oynayan cismin özellikleri dikkate alınmalı, etkileri az olan faktörler ihmal edilmelidir. Bu nedenle şekil değiştiren cisimlerin mekaniğinde konuya uygun düşen bir takım ideal cisimler kabul edilir ve bütün teori bunlar üzerine kurulur.

Cisimlerin elastik ve plastik özellikleri göz önüne alınarak tek eksenli gerilme halinde yapılan idealleştirmelerden bazıları;

a) Rijit cisim, b) Elastik Hooke Cismi, c) Elasto Plastik Cisim
d) Pekleşen Elasto Plastik Cisim, e) Rijit Plastik Cisim
f) Pekleşen Rijit Plastik Cisim (Şekil 4.2)



Şekil 4.2.

4.3) Hooke Kanunu (Bünye Bağıntıları)

Bir noktanın yakın civarında oluşan gerilme ve şekil değiştirme bileşenlerini ilişkilendiren ifadelere bünye bağıntıları (Hooke Yasaları) denir. Tek eksenli gerilme halinde Hooke Yasası;

$$\varepsilon_1 = \frac{\sigma_1}{E}$$

şeklindedir. Tek eksenli gerilme halinde cisim boyu gerilme vektörü doğrultusunda artarken buna dik doğrultuda kısalır veya uzar. Bu durumda;

$\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1 = -\nu\frac{\sigma_1}{E}$ olur. Buradaki poisson oranı boyuna ve enine şekil değiştirme oranı olup ($\nu = -\varepsilon_b/\varepsilon_e$) $0 \le \nu \le \frac{1}{2}$ arasındadır.

Basit kayma etkisi altındaki bir cisimde açı değişimi ile kayma gerilmeleri arasında doğrusal bir bağıntı bulunmaktadır. Bu da $\gamma = \frac{\tau}{G}$ ile tanımlanmaktadır.

4.4) Genel Hooke Kanunu

En genel durumda (x,y,z) takımında bir noktadaki gerilme tansörü $\sigma = \sigma(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz})$ ile şekil değiştirme tansörü $\varepsilon = \varepsilon(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{xz}, \gamma_{yz})$'nin bileşenleri arasındaki ilişkiyi kuran bünye bağıntıları süperpozisyon kuralı ile elde edilir. Yalnız normal ve kayma gerilmelerinin etkileri birbirlerinden bağımsızdır. Başka bir deyişle 1. mertebe teorisine göre üç eksenli gerilme halinde normal gerilmeler açı değişimi, kayma gerilmeleri de boy değişimi yapmayacaklarından gerilme şekil değiştirme bağıntıları iki ayrı grup halinde incelenebilir. Sonuç olarak normal gerilmelerin doğurduğu uzama oranları;

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\varepsilon_z \; \cancel{\varepsilon_y} = \frac{\sigma_z}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

Kayma gerilmelerinin sebep olduğu kayma açıları;

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad ; \quad \gamma_{zx} = \frac{\tau_{zx}}{G} \quad ; \quad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

olur.

Gerilmeleri şekil değiştirmeler cinsinden yazarsak;

$$\sigma_x = \frac{E.\nu}{(1+\nu)(1-2\nu)} (\varepsilon_x + \varepsilon_y + \varepsilon_z) + 2G\varepsilon_x$$

$$\sigma_y = \frac{E\nu}{(1+\nu)(1-2\nu)} (\varepsilon_x + \varepsilon_y + \varepsilon_z) + 2G\varepsilon_y$$

$$\sigma_z = \frac{E\nu}{(1+\nu)(1-2\nu)} (\varepsilon_x + \varepsilon_y + \varepsilon_z) + 2G\varepsilon_z$$

$$\tau_{xy} = G.\gamma_{xy} \; ; \; \tau_{xz} = G.\gamma_{xz} \; ; \; \tau_{yz} = G.\gamma_{yz}$$

olacaktır. Burada verilen G kayma modülü olup ve homojen ve izotrop malzemelerde $G = \frac{E}{2(1+\nu)}$ şeklindedir. Diğer yandan gerilme bağıntısındaki;

$\frac{E\nu}{(1+\nu)(1-2\nu)} = \lambda$ olarak tariflenir. G'yi de genel durum için μ şeklinde yazarsak her iki terime (μ, λ) lame sabiti adı verilir. Şekil değiştirmelerin gerilmeler cinsinden yazıldığı bağıntıyı matris formunda yazarsak;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}$$

Gerilmelerin şekil değiştirmeler cinsinden yazılımı ise;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} (\lambda+2\mu) & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & (\lambda+2\mu) & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & (\lambda+2\mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

şeklinde elde edilir. Her iki durumun kapalı şekilde yazılımı;

$$\{\varepsilon\} = [H]\{\sigma\}$$

$$\{\sigma\} = [D]\{\varepsilon\} \text{ 'dir.}$$

matrisleri yeniden yazarsak;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & 0 & 0 & 0 \\ H_{21} & H_{22} & H_{23} & 0 & 0 & 0 \\ H_{31} & H_{32} & H_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & H_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & H_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}$$

Burada; $H_{11} = 1/E$; $H_{12} = -\nu/E$; $H_{13} = -\nu/E$

şeklindedir.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

Burada $D_{11} = \lambda + 2\mu$; $D_{12} = \lambda$; $D_{13} = \lambda$ şeklindedir.

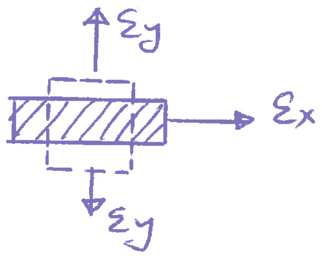
5- ELASTİSİTE SABİTLERİ

5.1) İZOTROP DURUM

Malzemedeki elastik özellikler koordinat takımından bağımsız ise bu malzemeye izotrop, aksi halde anizotrop denir.

5.1.1) Bir Boyutlu Gerilme ve Lineer Elastik İzotrop Durum

Gerilme:
$$\sigma_x = E.\varepsilon_x$$
$$\sigma_y = 0$$
$$\tau_{xy} = 0$$



Şekil Değiştirme:
$$\varepsilon_x = \frac{\sigma_x}{E}$$
$$\varepsilon_y = -\nu.\varepsilon_x$$
$$\gamma_{xy} = 0$$

$$\tau_{max} = \frac{\sigma_x}{2}$$

$$\gamma_{max} = \frac{\tau_{max}}{G}$$

Tanımlar

E : Elastisite Modülü

ν : Poisson Oranı

G : Kayma Modülü

5.1.2) Bir Boyutlu Şekil Değiştirme ve Lineer Elastik İzotrop Durum

Gerilme

$$\sigma_x = \frac{E \cdot \nu}{(1+\nu)(1-2\nu)} \varepsilon_x$$

$$\sigma_y = \sigma_z = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \varepsilon_x$$

$$\tau_{xy} = 0$$

Şekil Değiştirme

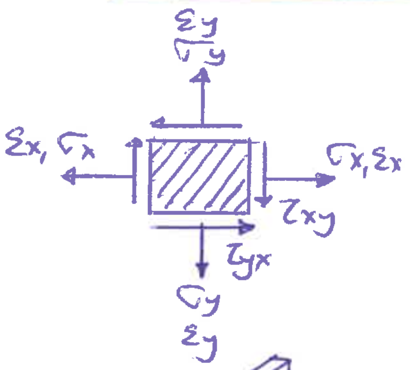
$$\varepsilon_x = \frac{\sigma_x}{E}$$

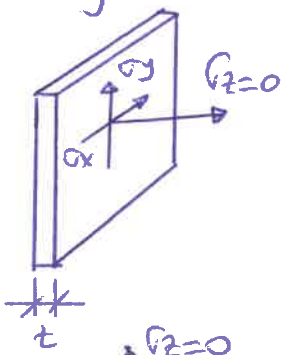
$$\varepsilon_y = 0$$

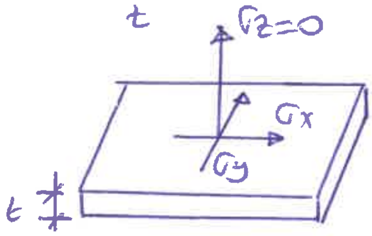
$$\gamma_{xy} = 0$$

Burada verilen $\frac{E \cdot \nu}{(1+\nu)(1-2\nu)} = \lambda$ olarak tarif edilen Lame sabitidir.

5.1.3) İki Boyutlu Gerilme ve Lineer Elastik İzotrop Durum (Düzlem Gerilme Hali)







$$\sigma_z = 0$$

$$\varepsilon_z \neq 0$$

Gerilmeler

$$\sigma_x = \frac{E\nu}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y) + 2G\varepsilon_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E\nu}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y) + 2G\varepsilon_y = \frac{E}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y)$$

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

Matris formda $\{\sigma\} = [D]\{\varepsilon\}$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Şekil Değiştirmeler

$$\varepsilon_z = -\frac{\nu}{(1-\nu)}(\varepsilon_x + \varepsilon_y)$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

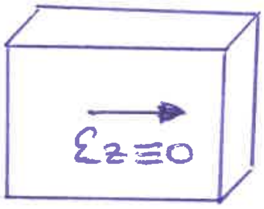
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\nu)}{E}\tau_{xy}$$

Bağıntılar matris formda yazılırsa; $\{\varepsilon\} = [H]\{\sigma\}$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix}\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

5.1.4) İki Boyutlu Şekil Değiştirme ve Lineer Elastik İzotrop Durum (Düzlem Şekil Değiştirme Hali)



$\sigma_z \neq 0$

$\sigma_z = \nu(\sigma_x + \sigma_y)$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} = 0$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu^2}{E}(\sigma_x + \sigma_y)$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu^2}{E}(\sigma_x + \sigma_y)$$

İfadeleri elde edilir ki buradan gerilme ve yer değiştirme bağıntıları aşağıda verildiği şekilde bulunacaktır.

Gerilmeler

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu\varepsilon_x\right]$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = G\gamma_{xy}$$

Matris formda yazılırsa;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

Şekil Değiştirmeler

$\varepsilon_z = 0$

$$\varepsilon_x = \frac{(1+\nu)}{E}\left[(1-\nu)\sigma_x - \nu\sigma_y\right]$$

$$\varepsilon_y = \frac{(1+\nu)}{E}\left[(1-\nu)\sigma_y - \nu\sigma_x\right]$$

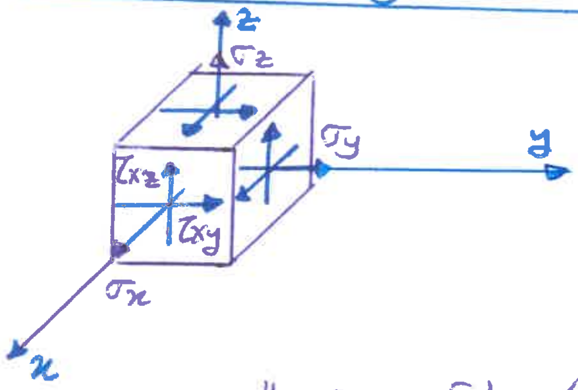
$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy} = \frac{\tau_{xy}}{G}$$

Matris formda yazılırsa;

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \frac{(1+\nu)}{E}\begin{bmatrix}1-\nu & -\nu & 0\\ -\nu & 1-\nu & 0\\ 0 & 0 & 2\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

bağıntıları elde edilir.

5.1.5) Üç Boyutlu Lineer Elastik İzotrop Durum



Gerilmeler

$$\sigma_x = \frac{E\nu}{(1+\nu)(1-2\nu)}\left(\varepsilon_x+\varepsilon_y+\varepsilon_z\right)+2G\varepsilon_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x+\nu(\varepsilon_y+\varepsilon_z)\right]$$

$$\sigma_y = \frac{E\nu}{(1+\nu)(1-2\nu)}\left(\varepsilon_x+\varepsilon_y+\varepsilon_z\right)+2G\varepsilon_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y+\nu(\varepsilon_x+\varepsilon_z)\right]$$

$$\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

$$\tau_{xz} = G\gamma_{xz} = \frac{E}{2(1+\nu)}\gamma_{xz}$$

$$\tau_{yz} = G\gamma_{yz} = \frac{E}{2(1+\nu)}\gamma_{yz}$$

Bağıntılar matris formunda yazılırsa; $\{\sigma\} = [D]\{\varepsilon\}$ olur

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}(1-\nu) & \nu & \nu & 0 & 0 & 0\\ \nu & (1-\nu) & \nu & 0 & 0 & 0\\ \nu & \nu & (1-\nu) & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}$$

$$\left.\begin{aligned}\lambda &= \frac{E\nu}{(1+\nu)(1-2\nu)}\\ \mu = G &= \frac{E}{2(1+\nu)}\end{aligned}\right\}$$ ile $[D]$ matrisi $\longrightarrow$ $$[D] = \begin{bmatrix}(\lambda+2\mu) & \lambda & \lambda & 0 & 0 & 0\\ \lambda & (\lambda+2\mu) & \lambda & 0 & 0 & 0\\ \lambda & \lambda & (\lambda+2\mu) & 0 & 0 & 0\\ 0 & 0 & 0 & \mu & 0 & 0\\ 0 & 0 & 0 & 0 & \mu & 0\\ 0 & 0 & 0 & 0 & 0 & \mu\end{bmatrix}$$

Şekil Değiştirmeler

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

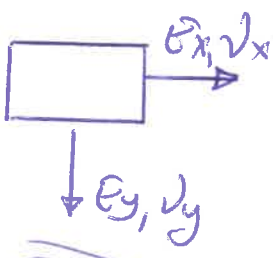
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad ; \quad \gamma_{xz} = \frac{\tau_{xz}}{G} \quad ; \quad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

5.2) ORTOTROP DURUM

Birbirine dik iki düzleme göre malzeme simetrisi olan cisimlerdir. Birbirine dik iki düzleme göre simetri olması durumunda bu iki düzleme dik üçüncü düzlemde simetri kendiliğinden sağlanır. Dolayısıyla birbirine dik 3 düzleme göre simetrisi olan malzemelerde bağımsız malzeme sabitlerinin sayısı 9'dur. Bu malzemelere ortogonallı anizotropik kelimelerinin kısaltılmışı olan ortotropik malzeme adı verilir. Örneğin; ahşap, lifli kompozit malzeme.

5.2.1) İki Boyutlu Lineer Elastik Ortotrop Durum (Düzlem Gerilme Hali)



$$\frac{\nu_x}{E_x} = \frac{\nu_y}{E_y}$$

Gerilmeler

$$\sigma_x = \frac{E_x}{1-\nu_x\nu_y}\left(\varepsilon_x + \nu_y\varepsilon_y\right)$$

$$\sigma_y = \frac{E_y}{1-\nu_x\nu_y}\left(\varepsilon_y + \nu_x\varepsilon_x\right)$$

$$\tau_{xy} = \frac{E_x E_y}{E_x(1+\nu_y) + E_y(1+\nu_x)}\gamma_{xy} = G\gamma_{xy}$$

Bu bağıntıda $G = \dfrac{E_x E_y}{E_x(1+\nu_y) + E_y(1+\nu_x)}$ şeklindedir.

matris formda yazılırsa;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\frac{1}{1-\nu_x\nu_y}\begin{bmatrix}E_x & E_x\nu_y & 0\\ E_y\nu_x & E_y & 0\\ 0 & 0 & G(1-\nu_x\nu_y)\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

şeklindedir.

Şekil Değiştirmeler

$$\varepsilon_x=\frac{\sigma_x}{E_x}-\nu_y\frac{\sigma_y}{E_y}$$

$$\varepsilon_y=-\nu_x\frac{\sigma_x}{E_x}+\frac{\sigma_y}{E_y}$$

$$\gamma_{xy}=\frac{[E_x(1+\nu_y)+E_y(1+\nu_x)]}{E_xE_y}\tau_{xy}=\frac{\tau_{xy}}{G}$$

5.22) İki Boyutlu Lineer Elastik Ortotrop Durum (Düzlem Şekil Değiştirme Hali)

Gerilmeler

$$\sigma_x=\frac{E_x}{(1-\nu_z\nu_y)(1-\nu_x\nu_z)-\nu_x\nu_y(1+\nu_z)^2}\left[(1-\nu_z\nu_y)\varepsilon_x+\nu_y(1+\nu_z)\varepsilon_y\right]$$

$$\sigma_y=\frac{E_y}{(1-\nu_z\nu_y)(1-\nu_x\nu_z)-\nu_x\nu_y(1+\nu_z)^2}\left[(1+\nu_z)\nu_x\varepsilon_x+(1-\nu_x\nu_z)\varepsilon_y\right]$$

$$\sigma_z=\frac{E_z}{E_x}\nu_x\sigma_x+\frac{E_z}{E_y}\nu_y\sigma_y$$

$$\tau_{xy}=\left[(1-\nu_z\nu_y)(1-\nu_x\nu_z)-\nu_x\nu_y(1+\nu_z)^2\right]G\cdot\gamma_{xy}$$

Yer Değiştirmeler

$\varepsilon_z\equiv 0$

$$\varepsilon_x=\frac{(1-\nu_x\nu_z)}{E_x}\sigma_x-\frac{\nu_y(1+\nu_z)}{E_y}\sigma_y$$

$$\varepsilon_y=-\frac{\nu_x(1+\nu_z)}{E_x}\sigma_x+\frac{(1-\nu_z\nu_y)}{E_y}\sigma_y$$

$$\gamma_{xy}=\frac{\tau_{xy}}{[(1-\nu_z\nu_y)(1-\nu_x\nu_z)-\nu_x\nu_y(1+\nu_z)^2]}\cdot\frac{1}{G}$$

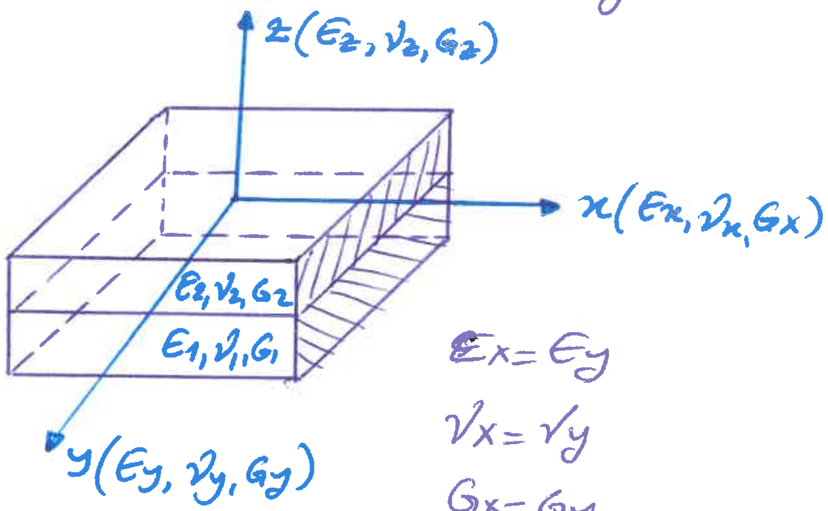
matris formda yazılırsa,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{1}{(1-\nu_z\nu_y)(1-\nu_x\nu_z)-\nu_x\nu_y(1+\nu_z)^2} \begin{bmatrix} E_x(1-\nu_z\nu_y) & E_x\nu_y(1+\nu_z) & 0 \\ E_y\nu_x(1+\nu_z) & E_y(1-\nu_x\nu_z) & 0 \\ 0 & 0 & AG \end{bmatrix}$$

$$A = (1-\nu_z\nu_y)(1-\nu_x\nu_z)-\nu_x\nu_y(1+\nu_z)^2$$

5.2.3) Üç Boyutlu Tetragonal Simetriye Sahip Ortotrop Malzeme Durumu

Ortotrop bir cisimde, malzemenin elastik özellikleri, uygun olarak seçilen bir eksen takımının iki eksenı doğrultusunda aynı, üçüncü eksen doğrultusunda farklıdır. Yani elastik özellikler birbirine dik üç düzleme göre ikişer ikişer simetriktir. Burada buna ek olarak birde iki eksenin üçüncü eksen etrafında dönmesi halinde elastik özelliklerin değişmemesi söz konusudur. Kısaca elastik özellikler eksenel simetriktir. Bu özel simetriden dolayı ortotrop malzemeye tetragonal simetrili denir. Bu durumda bağımsız sabitlerin sayısı 6 olur.



$E_x = E_y$

$\nu_x = \nu_y$

$G_x = G_y$

Gerilmeler

$$\sigma_x=\frac{E_z}{(1+\nu_x)(1-\nu_x-2\frac{E_x}{E_z}\nu_z^2}\left[(1-\nu_x^2)\varepsilon_x+\frac{E_x}{E_z}\nu_z(1+\nu_x)\varepsilon_y+\frac{E_x}{E_z}\nu_z(1+\nu_x)\varepsilon_z\right]$$

$$\sigma_y=\frac{E_z}{(1+\nu_x)(1-\nu_x-2\frac{E_x}{E_z}\nu_z^2)}\left[\frac{E_x}{E_z}\nu_z(1+\nu_x)\varepsilon_x+\frac{E_x}{E_z}\left(1-\frac{E_x}{E_z}\nu_z^2\right)\varepsilon_y+\frac{E_x}{E_z}\left(\nu_x+\frac{E_x}{E_z}\nu_z^2\right)\varepsilon_z\right]$$

$$\sigma_z=\frac{E_z}{(1+\nu_x)(1-\nu_x-2\frac{E_x}{E_z}\nu_z^2)}\left[\frac{E_x}{E_z}\nu_z(1+\nu_x)\varepsilon_x+\frac{E_x}{E_z}\left(\nu_x+\frac{E_x}{E_z}\nu_z^2\right)\varepsilon_y+\frac{E_x}{E_z}\left(1-\frac{E_x}{E_z}\nu_z^2\right)\varepsilon_z\right]$$

$$\tau_{xy}=G_x.\gamma_{xy}\;;\quad \tau_{xz}=G_z.\gamma_{xz}\;;\quad \tau_{yz}=G_z.\gamma_{yz}$$

matris formunda yazılırsa;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix}=\frac{E_z}{(1+\nu_x)(1-\nu_x-2\frac{E_x}{E_z}\nu_z^2)}\begin{bmatrix}(1-\nu_x^2) & \frac{E_x}{E_z}\nu_z(1+\nu_x) & \frac{E_x}{E_z}.\nu_z(1+\nu_x) & 0 & 0 & 0\\ \frac{E_x}{E_z}\nu_z(1+\nu_x) & \frac{E_x}{E_z}\left(1-\frac{E_x}{E_z}\nu_z^2\right) & \left(\nu_x+\frac{E_x}{E_z}\nu_z^2\right)\frac{E_x}{E_z} & 0 & 0 & 0\\ \frac{E_x}{E_z}\nu_z(1+\nu_x) & \frac{E_x}{E_z}\left(\nu_x+\frac{E_x}{E_z}\nu_z^2\right) & \frac{E_x}{E_z}\left(1-\frac{E_x}{E_z}\nu_z^2\right) & 0 & 0 & 0\\ 0 & 0 & 0 & AG_x & 0 & 0\\ 0 & 0 & 0 & 0 & AG_y & 0\\ 0 & 0 & 0 & 0 & 0 & AG_z\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}$$

olur. Burada verilen $A=\dfrac{(1-\nu_x)(1-\nu_x-2\frac{E_x}{E_z}\nu_z^2)}{E_z}$ 'dir.

Şekil Değiştirmeler

$$\varepsilon_x=\frac{\sigma_x}{E_x}-\nu_x\frac{\sigma_y}{E_x}-\nu_z\frac{\sigma_z}{E_z}$$

$$\varepsilon_y=-\frac{\nu_x}{E_x}\sigma_x+\frac{\sigma_y}{E_x}-\nu_z\frac{\sigma_z}{E_z}$$

$$\varepsilon_z=-\frac{\nu_x}{E_x}\sigma_x-\frac{\sigma_y}{E_x}+\frac{\sigma_z}{E_z}$$

$$\gamma_{xy}=\frac{\tau_{xy}}{G_x}\;;\quad \gamma_{xz}=\frac{\tau_{xz}}{G_z}\;;\quad \gamma_{yz}=\frac{\tau_{yz}}{G_z}$$ 'dir

5.3) Isı Değişimi Etkisi ve Elastisite Bağıntıları

Cisimlerde ısı değişimi etkisi ile yalnızca boy değişimleri (ε) oluşur. Kayma şekil değişimleri (γ) meydana gelmez. Burada α_t; ısı genleşme katsayısı, Δt ise ısı değişimini göstermek üzere elastisite bağıntıları aşağıda verilmiştir.

5.3.1) Isı Değişimi ve Düzlem Gerilme Hali

$$\left.\begin{aligned}\varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) + \alpha_t\cdot\Delta t = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \alpha_t\cdot\Delta t \\ \varepsilon_y &= \frac{1}{E}(-\nu\sigma_x + \sigma_y) + \alpha_t\cdot\Delta t = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} + \alpha_t\cdot\Delta t\end{aligned}\right\} \begin{array}{c}\text{Dış yük} \\ + \\ \text{ısı etkisi}\end{array}$$

Yalnız ısı etkisi durumunda;

$$\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\alpha_t\cdot\Delta t \\ \alpha_t\cdot\Delta t \\ 0\end{Bmatrix}$$

$$\{\sigma\} = [D]\{\varepsilon\} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix} \quad \text{idi.}$$

Isı etkisi durumunda;

$$\begin{Bmatrix}\sigma_x \\ \sigma_y \\ \tau_{xy}\end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x = \alpha_t\cdot\Delta t \\ \varepsilon_y = \alpha_t\cdot\Delta t \\ \gamma_{xy} = 0\end{Bmatrix}$$

Buradan;

$$\sigma_x = \frac{E}{1-\nu^2}\left[\alpha_t\cdot\Delta t + \nu\alpha_t\cdot\Delta t\right] = \frac{E}{1-\nu}\alpha_t\cdot\Delta t$$

$$\sigma_y = \frac{E}{1-\nu^2}\left[\nu\alpha_t\cdot\Delta t + \alpha_t\cdot\Delta t\right] = \frac{E}{1-\nu}\alpha_t\cdot\Delta t$$

$\tau_{xy} = 0$ elde edilir.

Sonuçta ısı değişiminden oluşan gerilmeler;

$$\sigma_x = \sigma_y = \frac{E}{1-\nu}\,\alpha_t \cdot \Delta t$$ dır. Buradan matris halde;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu}\,\alpha_t \cdot \Delta t \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$ yazılabilir. Burada

ısıya ait elastisite vektörü $\{D\} = \frac{E}{1-\nu}\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$ 'dır.

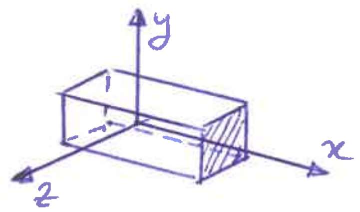
Dış yük etkisini de katarsak gerilme bağıntıları;

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\,\varepsilon_y\right) + \frac{E}{1-\nu}\,\alpha_t \cdot \Delta t$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\,\varepsilon_x\right) + \frac{E}{1-\nu}\,\alpha_t \cdot \Delta t$$

$$\tau_{xy} = G \cdot \gamma_{xy} + 0$$

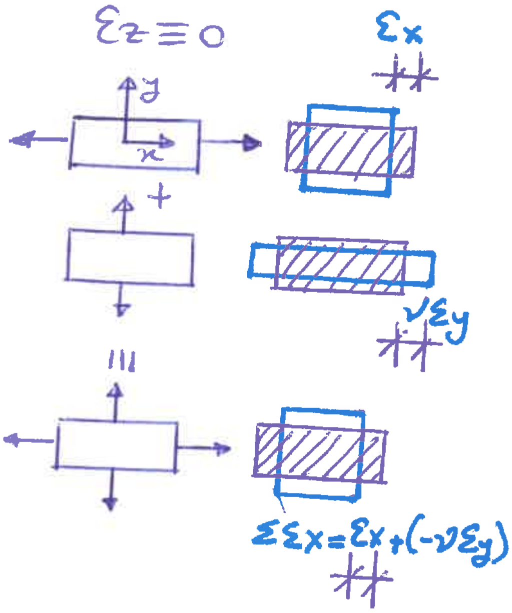
5.3.2) Isı Değişimi ve Düzlem Şekil Değiştirme Hali



$\sigma_z \neq 0$

$\varepsilon_z \equiv 0$

Düzlem şekil değiştirme halinde, birim şekil değiştirmeler; $\varepsilon_x, \varepsilon_y$ 'dir. $\varepsilon_z = 0$ 'dır.



$$\varepsilon_x = \alpha_t \cdot \Delta t \qquad \varepsilon_{x_t} = \varepsilon_x - \nu\,\varepsilon_y$$

$$\varepsilon_y = \alpha_t \cdot \Delta t \quad \text{veya} \quad \varepsilon_{y_t} = \varepsilon_y - \nu\,\varepsilon_x$$

Bu durumda;

$$\varepsilon_x = \alpha_t \Delta t - \nu\,\alpha_t \Delta t = (1-\nu)\,\alpha_t \Delta t$$

$$\varepsilon_y = -\nu\,\alpha_t \Delta t + \alpha_t \Delta t = (1-\nu)\,\alpha_t \Delta t$$

olur.

Gerilme bağıntıları;

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_x + (1-\nu)\varepsilon_y\right]$$

Düzlem Şekil Değiştirme Durumunda;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\varepsilon_x = (1-\nu)\alpha_t\Delta t$$

$$\varepsilon_y = (1-\nu)\alpha_t\Delta t$$

De bu durumda;

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)(1-\nu)\alpha_t\,\Delta_t + \nu(1-\nu)\alpha_t\Delta t\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu(1-\nu)\alpha_t\Delta t + (1-\nu)(1-\nu)\alpha_t\Delta t\right]$$

Buradan;

$$\sigma_x = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)}\alpha_t\Delta t$$

$$\sigma_y = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)}\alpha_t\Delta t$$

$$\tau_{xy} = 0$$

Bağıntıların matris formunda yazılımı;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{(1-2\nu)}\left(\frac{1-\nu}{1+\nu}\right)\alpha_t\,\Delta t\begin{Bmatrix}1\\ 1\\ 0\end{Bmatrix}$$

şeklinde dir. Isı elastisite sabitleri matrisi, düzlem şekil değiştirme halinde;

$$\{D\}_t = \frac{E}{(1-2\nu)}\left(\frac{1-\nu}{1+\nu}\right)\begin{Bmatrix}1\\ 1\\ 0\end{Bmatrix}$$

dir.

Sonuç olarak dış yüklerle ısı değişiminin birlikte dikkate alındığında iki düzlem şekil değiştirme hali için gerilme bağıntıları;

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)} \left\{ \left[(1-\nu)\varepsilon_x + \nu \varepsilon_y \right] + (1-\nu)\alpha_t \cdot \Delta t \right\}$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)} \left\{ \left[\nu \varepsilon_x + (1-\nu)\varepsilon_y \right] + (1-\nu)\alpha_t \Delta t \right\}$$

$$\tau_{xy} = G \gamma_{xy}$$

olarak elde edilir. Yine bu durumda şekil değiştirme bağıntıları;

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu \sigma_y) + (1-\nu)\alpha_t \Delta t$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu \sigma_x) + (1-\nu)\alpha_t \Delta t$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$ şeklinde bulunur.

5.3.3) Isı Değişimi ve Üç Boyutlu Durum

Isı Değişimi nedeniyle oluşan birim şekil değiştirmeler;

$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha_t \cdot \Delta t$ 'dir. Buradan gerilmeler;

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)\varepsilon_x - \nu(\varepsilon_y + \varepsilon_z) \right].$$

$$= \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)\alpha_t \cdot \Delta t + \nu(2\alpha_t \Delta t) \right]$$

$$= \frac{E}{(1-2\nu)} \alpha_t \Delta t$$

$$\sigma_y = \frac{E}{(1-2\nu)} \alpha_t \Delta t$$ $\tau_{xy} = \tau_{yz} = \tau_{xz} = 0$ olur.

$$\sigma_z = \frac{E}{(1-2\nu)} \alpha_t \cdot \Delta t$$

Isı elastik sabitler matrisi ise;

$$\{D\} = \frac{E}{(1-2\nu)} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$ şeklindedir.

Isı ve dış yükler dahil olması durumunda elastisite bağıntıları;

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha_t \cdot \Delta t$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha_t \cdot \Delta t$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha_t \cdot \Delta t$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \;;\quad \gamma_{xz} = \frac{\tau_{xz}}{G} \;;\quad \gamma_{yz} = \frac{\tau_{yz}}{G}$$ dır.

5.3.4) Isı Değişimi ve Ortotrop Durum

Ortotrop durumda şekil değiştirmeler; $\varepsilon_x = \alpha_x \cdot \Delta t$

$\varepsilon_y = \alpha_y \cdot \Delta t$

$\gamma_{xy} = 0$ 'dır.

$$\frac{\nu_x}{E_x} = \frac{\nu_y}{E_y}$$

Elastisite bağıntıları matris halde;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{1}{1-\nu_x\nu_y} \begin{Bmatrix} E_x(\alpha_x + \nu_y\alpha_y) \\ E_y(\nu_x\alpha_x + \alpha_y) \\ 0 \end{Bmatrix}$$

olarak bulunur.

5.4) LEVHA, PLAK VE KABUKLARIN ELASTİSİTE BAĞINTILARI

5.4.1) İzotrop Levhanın Bünye Denklemleri

Kesit tesirleri ile şekil değiştirmeler arasındaki elastisite bağıntıları;

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = C \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Buradaki katsayılar matrisi;

$$[D] = \begin{bmatrix} c & c\nu & 0 \\ c\nu & c & 0 \\ 0 & 0 & c\frac{1-\nu}{2} \end{bmatrix} \qquad c = \frac{Et}{1-\nu^2}$$

5.4.2) İzotrop Plağın Bünye Denklemleri

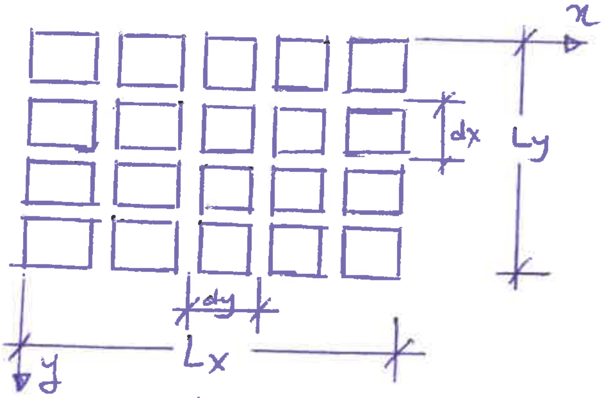
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = D \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

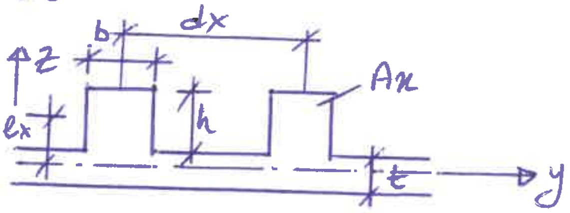
Buradaki katsayılar matrisi;

$$[D] = \begin{bmatrix} D & D\nu & 0 \\ D\nu & D & 0 \\ 0 & 0 & \frac{1-\nu}{2}D \end{bmatrix} \qquad D = \frac{Et^3}{12(1-\nu^2)} = \frac{EI}{(1-\nu^2)}$$

I: Kesit atalet momenti

5.4.3) Nervürlü Ortotrop Plağın Bünye Denklemleri





$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{54} & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

$$C_{11} = C + \frac{EA_x}{d_x} \qquad C_{44} = D + \frac{EI_x}{d_x} = D + \frac{E(I_x + A_x \cdot e_x^2)}{d_x}$$

$$C_{12} = \nu C = C_{21} \qquad C_{45} = \nu D$$

$$C_{22} = C + \frac{EA_y}{d_y} \qquad C_{55} = D + \frac{EI_y}{d_y} = D + \frac{E(I_y + A_y \cdot e_y^2)}{d_y}$$

$$C_{33} = \frac{1-\nu}{2}\, C \qquad C_{66} = (1-\nu)D + \frac{1}{2}\left[\frac{GJ_x}{d_x} + \frac{GJ_y}{d_y}\right]$$

Burada; $C = \frac{Et}{1-\nu^2}$, $D = \frac{Et^3}{12(1-\nu^2)}$, d_x ve d_y sırasıyla x ve y doğrultusundaki nervür aralığı, A_x ve A_y nervür alanları, I_x ve I_y ise x ve y yönlerindeki atalet momentleridir.

5.4.4) İzotrop Kabullenen Bünye Denklemleri

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{54} & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

Buradan İzotrop durum için;

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C & \nu C & 0 & 0 & 0 & 0 \\ \nu C & C & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2}C & 0 & 0 & 0 \\ 0 & 0 & 0 & D & \nu D & 0 \\ 0 & 0 & 0 & \nu D & D & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-\nu}{2}D \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

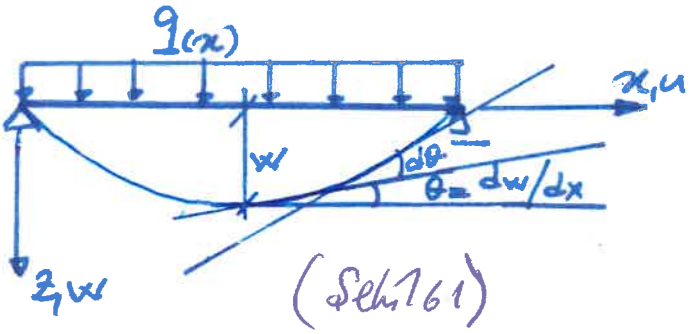
Burada, $C = \dfrac{Et}{1-\nu^2}$ $\quad D = \dfrac{Et^3}{12(1-\nu^2)}$ 'dir.

BÖLÜM II

ENERJİ YÖNTEMLERİ

6) TOPLAM POTANSİYEL ENERJİ

6.1) Toplam Potansiyel Enerjinin 1. Varyasyonu (Denge Denklemi)



(Şekil 6.1)

$w = (\text{çökme})$

$\dfrac{dw}{dx} = w' = \theta = (\text{dönme})$

$-\dfrac{d^2w}{dx^2} = -\dfrac{d\theta}{dx} = -\dfrac{1}{\rho} = -\dfrac{M}{EI}$

$-EIw'' = M$ (moment)

$-\dfrac{d^3w}{dx^3} = \dfrac{d^2\theta}{dx^2} = -\dfrac{1}{EI}\dfrac{dM}{dx} = -\dfrac{V}{EI}$

$-EIw''' = V$ (Kesme Kuvveti)

$\dfrac{d^4w}{dx^4} = \dfrac{d^3\theta}{dx^3} = \dfrac{1}{EI}\dfrac{d^2M}{dx^2} = \dfrac{1}{EI}\dfrac{dV}{dx} = \dfrac{1}{EI}q(x)$

$EIw^{IV} = q$ (Yayılı Yük)

çubuğun diferansiyel denklemi de;

$$EI\frac{d^4w}{dx^4} = q(x) \rightarrow EIw^4 = q(x)$$

olur.

Sınır koşulları

$x=0$'da $w=0$		
$x=L$'de $w=0$		Geometrik şartlar
$x=0$'da $w''=0$		
$x=L$'de $w''=0$		dinamik şartlar.

Bir çubuğun sınır koşullarını sağlayan pek çok yer değiştirme fonksiyonu $u=u(x)$ içinden yalnızca gerçek çözümü sağlayan biri $u=u_0(x)$ potansiyel enerjinin ifadesini (Π) minimum yapar. Başka bir deyişle toplam potansiyel enerji, sistemin gerçek denge konumunda bir ekstremden geçer. Böylece belirli bir integral ifadeyi ekstrem yapan konumun bulunmasına problemi indirgemiş oluruz. Bu ekstrem değer bir minimumdur. Bu hesaba da Varyasyon Hesabı denir.

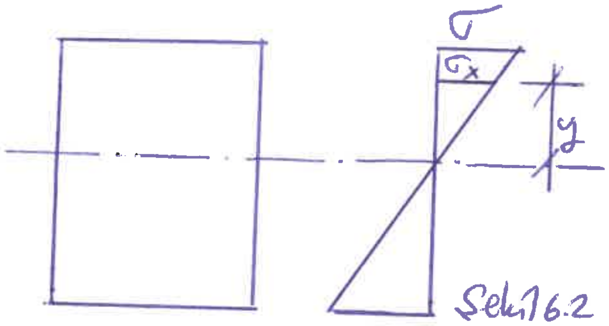
$$\Pi = U + W$$ (Toplam potansiyel enerji)

Burada; U: İç kuvvetlerin toplam potansiyel enerjisi

W: dış " " " "

dir.

6.1.1.) İç Kuvvetlerin Potansiyel Enerjisi



Şekil 6.2

$$\sigma_x = \frac{M}{I} \cdot y$$

$$\varepsilon_x = y \cdot \frac{d\theta}{dx} = y \cdot \frac{M}{EI}$$

$$U_1 = \int \frac{1}{2} \sigma_x \cdot \varepsilon_x \, (dA)\, dx = \int_V \frac{1}{2} \cdot \frac{M}{I} \cdot y \cdot \frac{My}{EI} \, dA\, dx = \frac{1}{2} \int \frac{M^2}{EI} dx$$

$$U_1 = \frac{1}{2} \int EI \left(\frac{d^2 w}{dx^2} \right)^2 dx \qquad \int y^2 dA = I$$

$$U = \int \frac{1}{2} \frac{M^2}{EI} dx + \int \frac{1}{2} \frac{N^2 L}{EA} dx + \int \frac{1}{2} \frac{V^2}{k' AG} dx$$

basit eğilme: $\frac{1}{2} \int EI (w'')^2 dx$

basit uzama: $\frac{1}{2} \int EA (u')^2 dx$

basit kayma: $\frac{1}{2} \int GA' (\gamma')^2 dx$

6.1.2) Dış Kuvvetlerin Potansiyel Enerjisi

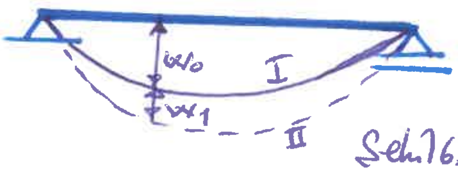
$$W = -\int q \cdot w \, dx$$

6.1.3) Toplam Potansiyel Enerji

$$\Pi = U + W = \int_0^\ell \frac{1}{2} \frac{M^2}{EI} dx - \int_0^\ell q \cdot w \, dx$$

$$= \int_0^\ell \left(\frac{EI}{2} (w'')^2 - q \cdot w \right) dx$$

6.1.4) Toplam Potansiyel Enerjinin Minimizasyonu



Şekil 6.3

Değişim (varyasyon): Çubuktaki yer değiştirme $w \to w_0 + w_1$ olsun. Skaler bir büyüklük olan Π toplam potansiyel enerji de $\Pi + \delta\Pi$ kadar artar.

I → Gerçek konum (denge durumu)
II → Komşu konum

$$\delta\Pi = \Pi_2 - \Pi_1$$

$$\delta\Pi = \Pi_2(w_0+w_1) - \Pi_1(w_0)$$

$$\delta\Pi = \left[\int_0^l \frac{EI}{2}(w_0'' + w_1'')^2 dx - \int_0^l q\,(w_0+w_1)\,dx\right] - \left[\int_0^l \frac{EI}{2}(w_0'')^2 dx - \int_0^l q\,w_0\,dx\right]$$

$$\delta\Pi = \int_0^l EI\,w_0''\,w_1''\,dx - \int_0^l q\,w_1\,dx + \int_0^l \frac{EI}{2}(w_1'')^2 dx$$

$$\int_0^l \frac{EI}{2}(w_1'')^2 dx = \alpha > 0$$

2. İkinci mertebe terim daima sıfırdan büyük ve pozitif

Bu denkleme kısmi integral uygularsak;

$$\int u\,dv = uv - \int v\,du$$

$$\int_0^l EI\,w_0''\,w_1''\,dx = \left|EI\,w_0''\,w_1'\right|_0^L - \left|EI\,w_0'''\,w_1\right|_0^L + \int_0^l EI\,w_0^{IV}\,w_1\,dx$$

Dinamik Şartlar $\begin{cases} x=0 \to w''=0 \\ x=L \to w''=0 \end{cases}$ $\quad$ $\left.\begin{matrix} x=0 \to w_1=0 \\ x=L \to w_1=0 \end{matrix}\right\}$ Sınır Şartları

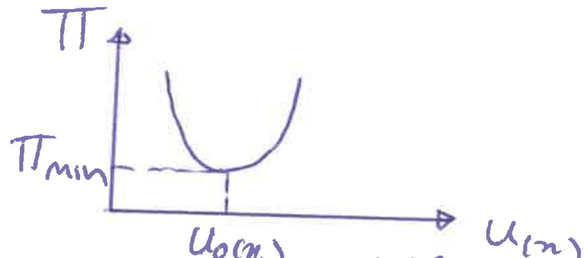
$$\delta\Pi = \int_0^L EI\,w_0^{IV}\,w_1\,dx - \int_0^l q\,w_1\,dx + \alpha = \left[\int_0^l (EI\,w_0^{IV} - q)\,dx\right] w_1 + \alpha$$

Hatırlatma $\longrightarrow EI\dfrac{d^4w}{dx^4} = q(x) \longrightarrow EI\,w_0^{IV} - q = 0$

o bloğundan parantez içindeki kısım sıfırdır.

Bu durumda $\delta\Pi = \alpha > 0$ kalır. Bunun anlamı denge konumuna komşu olan bütün konumlardaki toplam potansiyel enerji; denge konumundakinden daha fazladır. O halde "1" konumundaki (denge konumundaki) toplam potansiyel enerji minimumdur.

$$\Pi_1 = \Pi_{min}$$



Şekil 6.4

İkinci mertebeden bir büyüklük olan α birinci mertebeden büyüklüklerin yanında ihmal edilirse $\alpha = 0$ yazılır ve denge hali için durağanlık koşulu elde edilir.

$$\delta\Pi = 0 \quad (\text{durağanlık koşulu})$$

Ayrıca $\delta\Pi$'deki birinci dereceden ve ikinci dereceden terimlerin toplamına Π'nin birinci ve ikinci varyasyonu denir. Bunlar $\delta\Pi$ ve $\frac{\delta^2\Pi}{2}$ ile gösterilir.

$$\delta\Pi = \delta\Pi + \frac{1}{2!}\delta^2\Pi$$

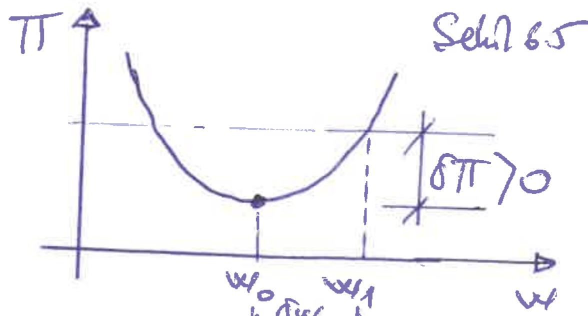
$$\delta\Pi = \int_0^l (EI\, w_0'' w_1'' - q . w_1)\, dx = 0 \quad \text{denge denklemi}$$

$$\frac{1}{2!}\delta^2\Pi = \int_0^l \frac{EI}{2}(w_1'')^2 dx = \alpha > 0$$

Bu durumda denge hali için;

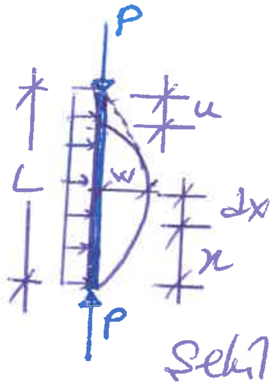
$$\left.\begin{array}{l} \delta\Pi = 0 \\ \frac{1}{2!}\delta^2\Pi > 0 \end{array}\right\} \text{ise } \Pi \text{ minimumdur.}$$

Şekil 6.5



Bu durumda elastik bir sistemde, geometrik sınır şartlarını sağlayan bütün yer değiştirme konumları arasında; gerçek konum toplam potansiyel enerjiyi minimum yapandır.

6.2) Toplam Potansiyel Enerjinin 2. Varyasyonu (Stabilite Denklemi)



Şekil 6.6

Eksenel yüklü bir elemanda boyuna yer değiştirmeye ek olarak enine yönde de yer değiştirmeler meydana gelirse bu elemanda burkulma problemi var demektir (Şekil 6.6). Çubuktaki toplam potansiyel enerji;

$\Pi = (U_m + U_b) + W$ 'dir. Burada;

U_m : Membran şekil değiştirme enerjisi
U_b : Eğilme " " "
W : Dış yük potansiyel enerjisi 'dir.

$$\Pi = \left[\frac{EA}{2} \int_0^L \varepsilon^2 dx + \frac{EI}{2} \int_0^L (w'')^2 dx \right] + P\left[u(L) - u(0)\right]$$

$$\Pi = \int_0^L \frac{EA}{2} \varepsilon^2 dx + \int_0^L \frac{EI}{2} (w'')^2 dx + \int_0^L P u' dx$$

Burada,

u' : eksenel kuvvete bağlı doğrusal şekil değiştirme
w'' : eğilmeye bağlı doğrusal şekil değiştirmedir.

$\varepsilon = u' + \frac{1}{2}(w')^2$ 'dir ve merkezi eksendeki uzama şekil değişimidir. Buradaki;

w' = büyük sapmaya bağlı doğrusal olmayan şekil değiştirmedir.

ε değerini denklemde yerine yazarsak;

$$\Pi=\int_0^L\left[\frac{EA}{2}\left(u'+\frac{1}{2}w'^2\right)^2+\frac{EI}{2}(w'')^2+Pu'\right]dx$$

u_0 ve w_0 denge konumunu, u_1 ve w_1 ise sonsuz küçük artımın olduğu ikinci konumu göstermek üzere ve buna göre;

$$\left.\begin{aligned} u &\rightarrow u_0+u_1 \\ w &\rightarrow w_0+w_1 \end{aligned}\right\}\text{artımlar}$$

$$u_0=-\frac{P}{EA}x \rightarrow u_0'=-\frac{P}{EA}\ ;\ w_0\cong 0$$

$$\left[\varepsilon=-\frac{P}{EA}\ ,\ \frac{du}{dx}=-\frac{P}{EA}\ ;\longrightarrow u=-\frac{P}{EA}x\right]$$ Kısalmadan dolayı $(-)$

Artımsal değerleri denklemde yerine yazarsak;

$$\Pi+\Delta\Pi=\int_0^L\left\{\frac{EA}{2}\left[(u_0'+u_1')+\frac{1}{2}(w_0'+w_1')^2\right]^2+\frac{EI}{2}(w_0''+w_1'')^2+\right.$$
$$\left.+P(u_0'+u_1')\right\}dx$$

$$\Pi=\int_0^L\left[\frac{EA}{2}\left(u_0'+\frac{1}{2}w_0'^2\right)^2+\frac{EI}{2}(w_0'')^2+Pu_0'\right]dx$$

$$\Delta\Pi=(\Pi+\Delta\Pi)-\Pi$$

Böylece potansiyel enerjideki değişimi seriye açarsak;

$$\Delta\Pi=\delta\Pi+\frac{1}{2!}\delta^2\Pi+\frac{1}{3!}\delta^3\Pi+---$$

$$\frac{1}{2!}\delta^2\Pi=\frac{1}{2}\int_0^L\left[EA(u_1')^2+EI(w_1'')^2-P(w_1')^2\right]dx$$

olur.

Yukarıda verilen $\frac{1}{2!}\delta^2\Pi$ bağıntısı ikinci dereceden terimler toplamını vermektedir. Üçüncü ve dördüncü dereceden oluşan terimler ihmal edilmiştir.

Burada kritik yük olarak tanımlanan P'nin küçük değerleri için; u_1 ve w_1'in bütün sıfır olmayan varyasyonları için $\delta^2\Pi > 0$ 'dır.

$$\delta(\delta^2\Pi)=0 \quad , \quad \delta\Pi=0$$

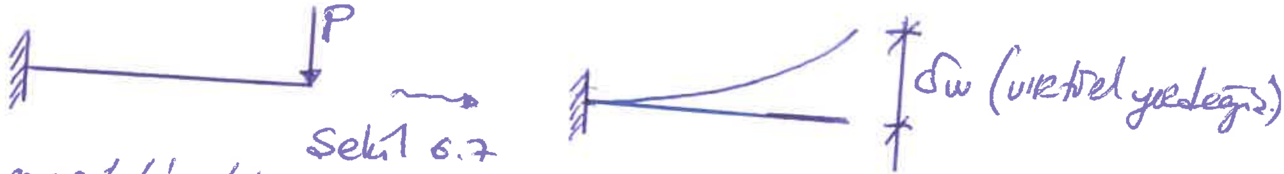
verilen bağıntıların u_1 ve w_1'e göre türevleri de sıfırdır.

$$\frac{\partial(\delta^2\Pi)}{\partial u_1}=0 \qquad \frac{\partial(\delta^2\Pi)}{\partial w_1}=0$$

6.3) VİRTÜL İŞ PRENSİBİ

Virtüel, gerçekte nasıl olduğunu bilmediğimiz bir fiziksel büyüklüğün mevcut sınır koşulları ile uyumlu olacak biçimde varsayacağımız oluşumunu ifade ederken kullandığımız bir terimdir. Virtüel kuvvetler ve virtüel deplasmanlar metodu olarak ikiye ayrılır.

Örneğin, aşağıdaki şekilde kiriş ucuna P yükü etkisin. Kiriş ucundaki yerdeğiştirmeyi virtüel olarak ele aldığımızda yer değiştirmeyi yukarıya veya aşağıya doğru alabiliriz. Ancak yer değiştirme eğrisi kirişin sınır koşullarını sağlamalıdır.



Şekil 6.7

Dengedeki bir cisim, sınır koşullarıyla uyumlu bir virtüel şekil değiştirme yaptığında, iç kuvvetlerin işi (δU), dış kuvvetlerin virtüel işine (δW) eşit olur.

$$\delta U = \delta W$$

u gerçek yerdeğiştirme, δu virtüel yerdeğiştirme, ε gerçek deformasyon, $\delta\varepsilon$ virtüel deformasyon ise;

$$U = \frac{1}{2}\int_V \sigma \cdot \varepsilon \cdot dV \quad \text{idi.} \qquad \{\sigma\} = [D]\{\varepsilon\}$$

$$\delta U = \int \{\delta\varepsilon\}^T \{\sigma\} dV = \int_V \{\delta\varepsilon\}^T [D] \{\varepsilon\} dV$$

X kütlesel kuvveti, p yüzeysel kuvveti, q çizgisel kuvveti, P münferit kuvveti gösterirse dış kuvvetlerin virtüel işi;

$$\delta W = \int_V \{\delta u\}^T \{X\} dV + \int_A \{\delta u\}^T \{p\} dA + \int_S \{\delta u\}^T \{q\} ds + P$$

$$\delta U = \delta W$$

$$\delta U = \int_V \{\delta \varepsilon\}^T [D] \{\varepsilon\} dv \quad \text{veya} \quad \delta U = \int_V \{\delta \varepsilon\}^T \{\sigma\} dv$$

$$\{\sigma\} = [D] \left(\{\varepsilon\} - \{\varepsilon_0\} \right) + \{\sigma_0\} - \alpha . \Delta_t \{D_T\}$$

Burada;

$[D]$ = malzemenin elastisite sabiti'

$\{\varepsilon_0\}$ = başlangıç şekil değiştirme

$\{\sigma_0\}$ = " gerilmesi

α = genleşme katsayısı

Δt = sıcaklık değişimi

$\{D_T\}$ = sıcaklık elastisite matrisi'dir.

$$\delta U = \int_V \{\delta\varepsilon\}_i^T [D] \{\varepsilon\} dV - \int_V \{\delta\varepsilon\}_i^T [D] \{\varepsilon_0\} dV + \int_V \{\delta\varepsilon\}_i^T \{\sigma_0\} dV - \int_V \{\delta\varepsilon\}_i^T . \alpha . \Delta_t \{D_T\} dV$$

$$\delta W = \int_V \{\delta u\}^T \{X\} dV + \int_A \{\delta u\}_i^T \{p\} dA + \int_S \{\delta u\}_i^T \{q\} ds + \{d\}^T P - \int_A c \{\delta w\}_i^T \{w\} dA$$

Burada;

u: yer değiştirme

X: kütlesel kuvvet

p: yüzeysel yük

q: çizgisel "

P: tekil yük

c: elastik yataklanma katsayısı'dır.

$$\delta U = \delta W$$

BÖLÜM III

7) SONLU ELEMANLAR YÖNTEMİ

7.1) GİRİŞ

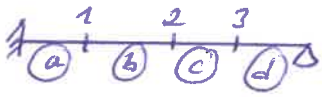
Sonlu Elemanlar Yöntemi; sürekli bir sistemi problemin karakterine uygun sonlu elemanlara ayırarak elde edilen elemanlar üzerinde iç ve dış kuvvetlerin enerjisinin minimizasyonu ve sonra bu elemanların birleştirilmesi tarzında bir uygulama getirir. Bunun sonucu olarak mesnet şartları, sisteme ait özellikler dış yüklerin sürekli ya da ani değişimleri kolayca gözönüne alınabilir. Dolayısıyla sonlu elemanlar yöntemi analitik metotlarla çözülemeyen karışık problemlere uygulanabilir. Yüzeysel sistemin tipik bölgelerinde eleman boyutları küçültülerek o bölgenin daha hassas incelenmesi mümkün olur. Diğer bir avantajı da, sınır şartlarının problemin çözüm sırasına göre en son adımda hesaplara dahil edilmesidir. Böylelikle çeşitli sınır şartlarını probleme uygularken baştaki yoğun hesaplara girilmez.

Sonlu elemanlar yönteminin kabuk sistemlere uygulanışına ait çalışmaların bir kısmında, sistem düzlemsel sonlu elemanlara ayrılmaktadır. Bu tip hesaplarda eleman özel eksenlerinin sistem eksenlerine dönüştürülmesinde, sistem rijitlik matrisinin hesabında bazı güçlüklerle karşılaşılmaktadır. Kabuk formuna uygun eğrisel sonlu eleman kullanılması, bu güçlükleri ortadan kaldırmaktadır. Yapının geometrisine bağlı olarak elemanın şekli bir, iki veya üç boyutlu olabilir. Eleman sınırları düz veya eğri çizgi olabilir.

Sonlu elemanlar yönteminde sistem sonlu sayıda elemana ayrılmaktadır. Eleman boyutları küçüldükçe problemin hata oranı azalmakta, fakat çözüm süreci uzamaktadır. Sistemi oluşturan elemanların her birine sonlu eleman denir ve birleştikleri köşe noktaları da düğüm noktaları olarak adlandırılır (Şekil 7.1).

1) Çözüm yapılacak eleman çubuk ise; (1 boyutlu ise)

sonlu elemanlara ayrılması;



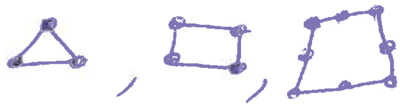
eleman seçimi;

2) Çözüm yapılacak eleman yüzey ise (2 boyutlu ise);

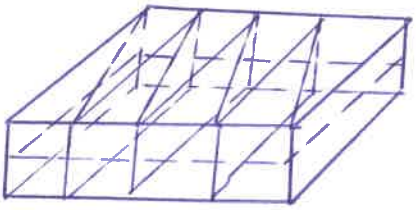
u (x,y)

v (x,y)

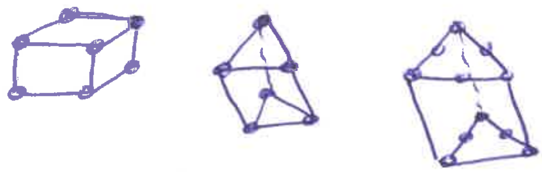
sonlu eleman seçimi;



3) Çözüm yapılacak eleman üç boyutlu ise;

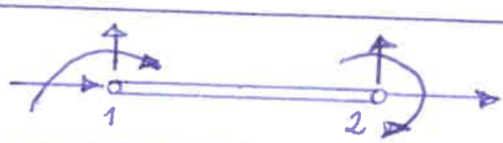
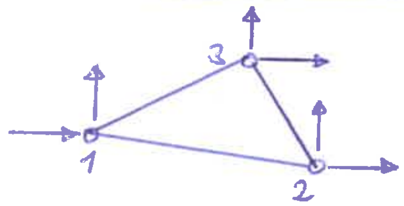
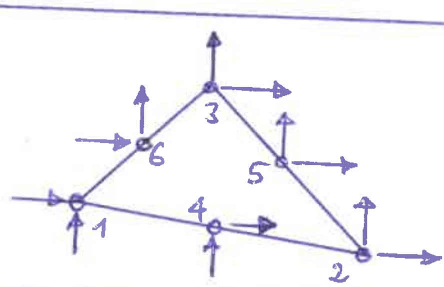
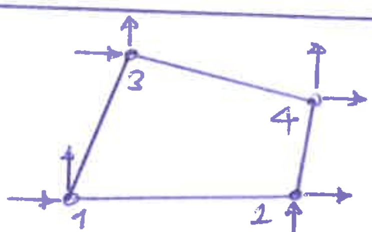
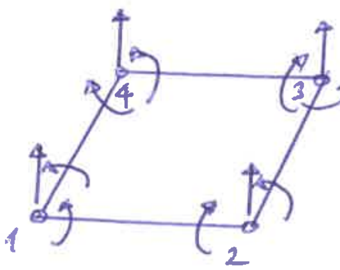


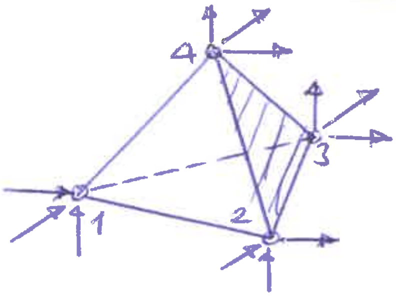
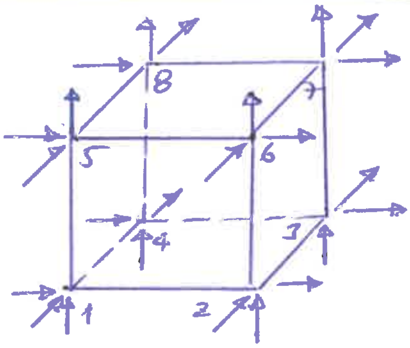
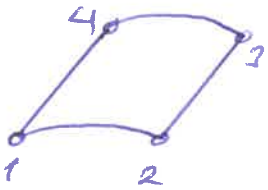
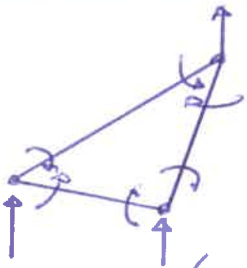
sonlu eleman seçimi;



Sonlu eleman yüzeyinin şekil değiştirmesi; düğüm noktalarının deplasman parametrelerine bağlı olarak ifade edilebilir. Deplasman parametreleri; deplasman bileşenleri, dönmeler ve burulma eğriliği gibi deplasman vektörlerini içermektedir. Eğilme hesaplarında düğüm noktalarının deplasman parametrelerin belirlenmesi, sistemin deplasman yüzeyinin ve her düğüm noktasındaki kesit tesirlerinin bulunması için yeterlidir. Stabilite hesabında ise, bu deplasman parametrelerine göre kurulan denklem takımının (Δ) katsayılar determinantını sıfır yapan yük, yani kritik yük (P_{kr}) belirlenir.

Bazı Sonlu Eleman Tipleri ve Serbestlik Dereceleri

Eleman	Şekil	Uygulama	Düğüm noktası	Düğüm noktası serbestlik derecesi
Kafes (Bar) Çubuğu		Kafes elemanı	2	1
Kiriş (Beam)		Çerçeve	2	3
Üçgen (Triangular)		Düzlem Gerilme	3	2
Üçgen (Triangular)		"	6	2
Dörtgen (Quadrilateral)		"	4	2
Dikdörtgen (Rectangular)	 (eğilme)	Plak	4	3

Eleman	Şekil	Uygulama	Düğüm noktası	Düğüm Noktası Serbestlik Derecesi
Dört üçgen yüzlü (Tetrahedron)		Üç eksenli gerilme	4	3
Prizma (Prism)		"	8	3
Dörtgen kabuk eleman (Rectangular shell element)		silindirik kabuk, tonoz	4	5-6
Üçgen eleman	 (eğilme)	plak	3	3

7.2) Deplasman Fonksiyonlarının Seçimi

Deplasman fonksiyonları; rijit cisim hareketi ve sabit deformasyon şartını sağlayacak şekilde seçilmelidir. Koordinat ekseni değişince çözüm farklı olmamalıdır. Bunun için deplasman fonksiyonları ya tam polinom veya tabii koordinatların fonksiyonu şeklinde olmalıdır. Elemanın içinde ve kenarında sürekli olmalıdır. Ayrıca iç ve dış kuvvetlerin içindeki türevlerde de sürekli olmalıdır.

Herhangi bir (e_i) elemanına ait ve bu elemanın içinde ya da sınırları üzerinde bir (i) noktasındaki deplasman (yer değiştirme) vektörü $\{u\}$ olsun;

$$\{u\}^e = \begin{Bmatrix} u(x,y) \\ v(x,y) \\ w(x,y) \end{Bmatrix} = [\phi(x,y)]\{a\}$$

Burada $[\phi(x,y)]$ seçilen deplasman fonksiyonudur. Hesap kolaylığı bakımından genellikle polinom seçilir. Pascal üçgeni polinomların seçilmesine yardımcı olur. Polinomlarda türev ve integral işlemleri daha kolaydır. Gerçek çözüme istenilen yaklaşımı sağlamak mümkündür. Pascal polinomları "şekil fonksiyonları" bölümünde anlatılmıştır. Seçilen deplasman fonksiyonları tam bir polinom ise geometrik izotropi sağlanır. Eğer polinom tam değil; fakat simetrisi varsa, yine geometrik izotropi vardır. Bağıntıda verilen $\{a\}$ bilinmeyen katsayılardır. Bu $\{a\}$ katsayılarının sayısı, bir elemandaki düğüm noktalarının deplasman parametrelerinin toplam sayısına eşit olmalıdır.

ÖRNEK : Yer değiştirme vektörü bir polinom olsun;

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

Bu durumda deplasman fonksiyonu;

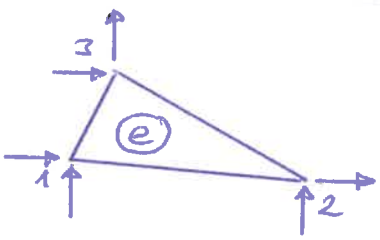
$$\{\phi_{(x,y)}\} = [1, x, x^2, x^3]$$

ve bilinmeyen katsayılar matrisi;

$$\{a\} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \quad \text{olacaktır.}$$

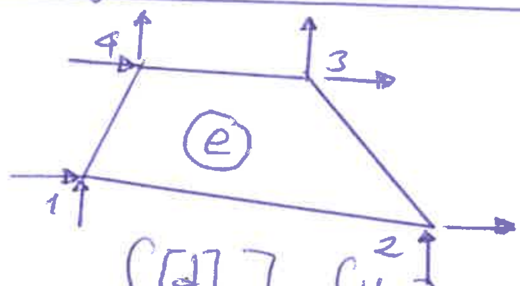
Seçilen elemanın düğüm noktası parametreleri ise;

Üç düğüm noktalı elemanda



$$\{d\} = \begin{Bmatrix} [d]_1 \\ [d]_2 \\ [d]_3 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

dört düğüm noktalı elemanda



$$\{d\} = \begin{Bmatrix} [d]_1 \\ [d]_2 \\ [d]_3 \\ [d]_4 \end{Bmatrix} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

şeklindedir. Elemanın herhangi bir (i) düğüm noktasında tarif edilen deplasman parametreleri en genel halde;

$$\{d\}_i = \{u, v, w, \theta_x, \theta_y, \theta_z, \tau, \ldots\}$$

$$i = 1, 2, 3, 4 \ldots$$

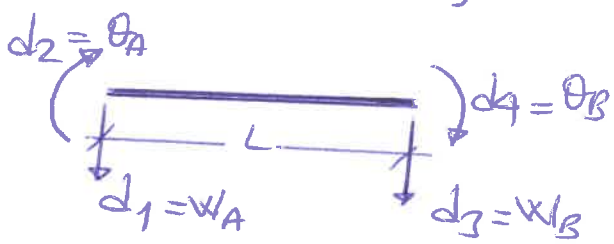
şeklindedir.

Eleman düğüm noktası deplasmanları $\{d\}$ ile, polinom sabitleri $\{a\}$ arasındaki bağı veren $[A]$ bağ matrisi ise, elemanın düğüm noktalarının deplasman parametrelerinin $\phi(x,y)$ ve $\phi(x,y)$'nin türevleri cinsinden yazılmış değerlerin düğüm noktası koordinatlarının yerlerine konulması ile bulunur.

Örneğin; Yer değiştirme vektörü,

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \quad \text{olsun.}$$

$$w = [\phi(x)]\{a\} = [1, x, x^2, x^3]\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$



A (0,0) B(L,0)

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$\theta = \frac{dw}{dx} = 0 + a_2 + 2a_3 x + 3a x^2$$

A(0,0) noktasında X yerine 0 yazılırsa; $w_A = a_1 + 0 + 0 + 0$

$\theta_A = 0 + a_2 + 0 + 0$

B(L,0) " x " L "

$$w_B = a_4 + a_2 L + a_3 L^2 + a_4 L^3$$

$\theta_B = 0 + a_2 + 2a_3 L + 3a L^2$ olur. Bu durumda $[A]$ matrisi,

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L & L^2 & L^3 \\ 0 & 1 & 2L & 3L^2 \end{bmatrix} \quad \text{elde edilir.}$$

Burada $\{d\}$ ile $\{a\}$ matrisleri bilinmeyenlerdir. Bağ matrisi vasıtasıyla bilinmeyen yalnız $\{d\}$ deplasman parametreleri kalır ve $\{a\}$ katsayıları $\{d\}$ deplasman parametreleri cinsinden hesaplanır.

$$\{d\} = [A]\{a\}$$

$$\{a\} = [A]^{-1}\{d\}$$

$[B] = [A]^{-1}$ dersek;

$\{a\} = [B]\{d\}$ olur. Bu bağıntıyı $\{u\} = \begin{Bmatrix} u(x,y) \\ v(x,y) \\ \phi(x,y) \end{Bmatrix} = [\phi(x,y)]\{a\}$ bağıntısında yerine koyarsak;

$$\{u\} = \begin{Bmatrix} u(x,y) \\ v(x,y) \\ w(x,y) \end{Bmatrix} = [\phi(x,y)][B]\{d\}$$

elde edilir ki; bu durumda elemanın deplasman vektörü, elemanın düğüm noktalarının deplasman parametreleri cinsinden belirlenmiş olur. Bağıntıdaki $[\phi(x,y)][B]$ ifadesine şekil fonksiyonu denir.

$$[\phi(x,y)][B] = [N]$$

Bağıntıdan deplasman parametresi kadar şekil fonksiyonu elde edilir.

$$\{u\} = [N]\{d\}$$

Şekil değiştirmeler ise;

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \underbrace{\begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix}}_{} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$[\Delta]$ = operatör matris

$$\{\varepsilon\} = [\Delta]\{u\}$$

$$\{u\} = [N]\{d\}$$

$$\{\varepsilon\} = \underbrace{[\Delta][N]}\{d\} = [G]\{d\}$$

$[G]$: şekil değiştirme matrisi

(1) düğüm noktasındaki d_1 deplasman parametresine ait N_1 şekil fonksiyonu öyle hesaplanır ki; (1) düğüm noktasının (x, y, z) koordinatları konduğu zaman,

$$N_1 = 1 \; ; \quad N_2 = N_3 = \dots N_n = 0$$

olur. Belirlenen bu şekil fonksiyonları, eleman şekli ve boyutu değişmediği sürece her elemanda aynıdır. Yalnız deplasman parametreleri elemandan elemana değişik. Problemde bu deplasman parametreleri bilinmeyen olarak hesaplanır.

7.3) Genel Dış Yük Halinde Sonlu Elemanlar Yöntemiyle Eğilme Hesabı

Dengede olan bir cismin, herhangi bir yer ve şekil değiştirmesinde iç kuvvetlerin işi, dış kuvvetlerin işine eşittir. İç kuvvetlerin virtüel işini δU, dış kuvvetlerin virtüel işini δW olarak gösterirsek;

$$\delta U = \delta W \quad \text{'dir.}$$

Tabii durum gerçek yükleme, i birim durumu ise viktüel yer değiştirme ve viktüel şekil değiştirme durumu olarak alınıp viktüel iç kuvvetlerle, viktüel dış kuvvetler işini yazarsak;

$$\delta U = \int_V \{\delta \varepsilon\}_i^T \sigma \, dV$$

$$\{\sigma\} = [D]\left(\{\varepsilon\} - \{\varepsilon_0\}\right) + \{\sigma_0\} - \alpha . \Delta t \{D_T\}$$

Burada;

$[D]$ = malzeme elastisite matrisini
$\{\varepsilon_0\}$: başlangıç şekil değiştirmesini
$\{\sigma_0\}$: başlangıç gerilmesini
α : genleşme katsayısını
Δt : sıcaklık değişimini
$\{D_T\}$: sıcaklık elastisite matrisini göstermektedir.

$\{\sigma\}$ ifadesini δU bağıntısında yerine yazarsak;

$$\delta U = \int_V \{\delta\varepsilon\}_i^T [D]\{\varepsilon\} dV - \int_V \{\delta\varepsilon\}_i^T [D]\{\varepsilon_0\} dV + \int_V \{\delta\varepsilon\}_i^T \{\sigma_0\} dV - \int_V \{\delta\varepsilon\}_i^T \alpha \Delta t \{D_T\} dV$$

$$\delta W = \int_V \{\delta u\}_i^T \{X\} dV + \int_A \{\delta u\}_i^T \{P\} dA + \int_S \{\delta u\}_i^T \{q\} ds + \{d\}^T P - \int_A c \{\delta w\}_i^T (w) dA$$

şeklinde bağıntılar elde edilir. Burada;

u: yer değiştirmeyi
X: kütle kuvvetini
p: yüzeysel yükü
q: çizgisel "
P: tekil "
c: elastik yataklanma katsayısını göstermektedir.

74) Sonlu Elemanlar Yardımı İle Stabilite Hesabı

Kesit kalınlıklarının sistem boyutlarına göre çok küçük olması halinde yapının narinliği söz konusu olur. Narinliğin artması büyük deplasmanlara sebep olacağından, birinci mertebe teorisi yerine, ikinci mertebe teorisiyle hesap gerekmektedir. Dolayısıyla sistemin stabilite hesabı önem kazanmaktadır. Stabilite hesabında denge konumu bellidir. Denge durumunun kararlı olup olmadığı araştırılır ve kararlı denge konumunu oluşturan şartlar incelenir.

Stabilite hesabında yapılan kabuller;

a) Denge denklemleri şekil değiştirmiş sistem üzerinde yazıldığı için elastik stabilite lineer olmayan bir teoridir. Yük-yer değiştirme arasında artık bir orantılılık bulunmaz. Bu nedenle etkiyen dış yüklerin aralarındaki oran sabit kalacak şekilde arttıkları kabul edilir.

b) Sistem lineer olmadığından süperpozisyon kuralı, Betti-Karşıtlık Teoremi geçerli değildir.

c) Membran kesit tesirlerinin büyük değerler aldığı göz önüne alınarak yalnızca bunların [No] ikinci mertebe etkileri hesaba katılarak, eğilme ve burulma momentlerinin ikinci mertebe etkileri ihmal edilecektir.

Toplam Potansiyel Enerji Denklemi;

$$\Pi = U + W$$

şekil değiştirme enerjisi U; eğilme ve membran şekil değiştirme enerjisi olarak dikkate alınırsa,

$$\Pi = U_m + U_e + W$$ olur.

$$U_M = \frac{C}{2} \iint \left(\varepsilon_x^2 + \varepsilon_y^2 + 2\nu \varepsilon_x \varepsilon_y + \frac{1-\nu}{2} \gamma_{xy}^2 \right) dx\, dy$$

$$U_b = \frac{D}{2} \iint \left(k_x^2 + k_y^2 + 2\nu k_x k_y + 2(1-\nu) k_{xy}^2 \right) dx\, dy$$

$$U = \frac{C}{2} \iint \left(\varepsilon_x^2 + \varepsilon_y^2 + 2\nu \varepsilon_x \varepsilon_y + \frac{1-\nu}{2} \gamma_{xy}^2 \right) dx\,dy + \frac{D}{2} \iint \left(k_x^2 + k_y^2 + 2\nu k_x k_y + 2(1-\nu) k_{xy}^2 \right) dx\,dy$$

Uzaysal bir sistemin yüzeyi üzerine etkiyen yüklerin x, y, z doğrultularındaki bileşenleri P_x, P_y, P_z ise ve elemanın ortalama yüzeyi üzerindeki bir noktanın yerdeğiştirme bileşenleri de u, v ve w ise tatbik edilen yüklerin potansiyel enerjisi için bağıntı;

$$W = -\iint \left(P_x \cdot u + P_y \cdot v + P_z \cdot w \right) dx\, dy \quad \text{olur}$$

Sistem konservatif ise dış kuvvetlerin potansiyel enerjisi negatiftir. Bu bağıntıda yerdeğiştirmeler ve kuvvetler aynı yöndedir. Yukarıda verilen membran ve eğilme şekil değiştirme enerji bağıntıları ince, elastik bir dairesel kabuğa ait olup ε_x, ε_y ve γ_{xy} kabuk orta yüzeyinin uzama ve kayma şekildeğiştirme bileşenleri; k_x, k_y ve k_{xy} ise ortalama yüzeyin eğrilik değişimi ve burulma eğriliğidir. Bunlara kabuğun deformasyon-deplasman bağıntıları veya kinematik münasebetleri veya uygunluk denklemleri denir. Ortalama yüzeyin u, v, w deplasman bileşenlerinin fonksiyonlarıdır.

Novozhilov teorisine göre, basık olmayan dairesel silindirlerin nonlineer deformasyon-deplasman bağıntıları ortalama yüzeyin u, v, w deplasman bileşenleri cinsinden ifadeleri aşağıdaki şekildedir. Burada r silindirin yarıçapını göstermektedir.

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\beta_x^2 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(-\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_y = \frac{\partial u}{\partial y} + \frac{w}{r} + \frac{1}{2}\beta_y^2 = \frac{\partial v}{\partial y} + \frac{w}{r} + \frac{1}{2}\left(\frac{v}{r} - \frac{\partial w}{\partial y}\right)^2$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \beta_x\beta_y = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\left(-\frac{\partial w}{\partial x}\right)\left(\frac{v}{r} - \frac{\partial w}{\partial y}\right)$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, \quad k_y = \frac{\partial v}{r\partial y} - \frac{\partial^2 w}{\partial y^2}, \quad 2k_{xy} = \frac{2\partial v}{r\partial x} - 2\frac{\partial^2 w}{\partial x \partial y}$$

$$\beta_x = -\frac{\partial w}{\partial x} \qquad \beta_y = \frac{v}{r} - \frac{\partial w}{\partial y}$$

7.4.1) Stabilitede Minimum Potansiyel Enerji Kriteri

Gerçek denge konumuna yakın ikinci bir konum ele alırsak;

$u \rightarrow u_0 + u_1$

$v \rightarrow v_0 + v_1$

$w \rightarrow w_0 + w_1$ olacaktır

Burada u_0, v_0 ve w_0 denge konumuna karşı gelen yerdeğiştirmelerdir. u_1, v_1 ve w_1 de ikinci konumu elde etmek için verilen sonsuz küçük miktarlardır. u, v ve w ise ikinci konuma karşı gelen yer değiştirmelerdir. Şimdi bükme şekil değiştirme bağıntıları;

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_x^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^4$$

Potansiyel enerjideki değişim ise,

$$\Delta U = \delta U + \frac{1}{2!}\delta^2 U + \frac{1}{3!}\delta^3 U \; --- \text{ şeklindedir.}$$

Burada $\frac{1}{2!}\delta^2 U$ değeri ; ($w_0 = 0$ için)

$$\frac{1}{2!}\delta^2(\varepsilon_x^2) = \left(\frac{\partial u}{\partial x}\right)^2 + \frac{\partial u_0}{\partial x}\left(\frac{\partial w_1}{\partial x}\right)^2 \text{ 'dir.}$$

$\varepsilon_y, \gamma_{xy}, k_x, k_y$ ve k_{xy} için benzer ifadeleri yazarsak membran ve eğilme şekil değiştirme enerjisinin ikinci varyasyonu aşağıdaki şekli alır;

$$\frac{1}{2!}\delta^2 U_m = \frac{C}{2}\iint\Big\{(\delta\varepsilon_x)^2+(\delta\varepsilon_y)^2+2\nu(\delta\varepsilon_x)(\delta\varepsilon_y)+\frac{1-\nu}{2}(\delta\gamma_{xy})^2$$

$$+\Big[(\varepsilon_{x_0}+\nu\varepsilon_{y_0})(\delta^2\varepsilon_x)+(\varepsilon_{y_0}+\nu\varepsilon_{x_0})(\delta^2\varepsilon_y)+\frac{1-\nu}{2}\gamma_{xy_0}(\delta^2\gamma_{xy})$$

$$\frac{1}{2!}\delta^2 U_b = \frac{D}{2}\iint\Big[(\delta k_x)^2+(\delta k_y)^2+2\nu(\delta k_x)(\delta k_y)+2(1-\nu)(\delta k_{xy})^2\Big]dx\,dy \quad dx\,dy$$

Bu durumda toplam enerjinin ikinci varyasyonu;

$$\frac{1}{2!}\delta^2 U = \frac{C}{2}\iint\Big(\varepsilon_{x_1}^2+\varepsilon_{y_1}^2+2\nu\,\varepsilon_{x_1}\varepsilon_{y_1}+\frac{1-\nu}{2}\gamma_{xy_1}^2\Big)dx\,dy$$

$$+\frac{1}{2}\iint\Big(N_{x_0}\beta_{x_1}^2+N_{y_0}\beta_{y_1}^2+2N_{xy_0}\beta_{x_1}\beta_{y_1}\Big)dx\,dy$$

$$+\frac{D}{2}\iint\Big[k_{x_1}^2+k_{y_1}^2+2\nu\,k_{x_1}k_{y_1}+2(1-\nu)k_{xy_1}^2\Big]dx\,dy$$

Bu bağıntıdaki deplasman-deformasyon ilişkisi ise;

$$\varepsilon_{x_1}=\frac{\partial u_1}{\partial x} \qquad \varepsilon_{y_1}=\frac{\partial v_1}{\partial y}+\frac{w_1}{r} \qquad \gamma_{xy_1}=\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial x}$$

$$k_{x_1}=-\frac{\partial^2 w_1}{\partial x^2} \qquad k_y=\frac{\partial v_1}{r\,\partial y}-\frac{\partial^2 w_1}{\partial y^2} \qquad 2k_{xy_1}=\frac{2\partial v_1}{r\,\partial x}-\frac{2\partial^2 w_1}{\partial x\partial y}$$

$$\beta_{x_1}=-\frac{\partial w_1}{\partial x} \qquad \beta_y=\frac{v_1}{r}-\frac{\partial w_1}{\partial y}$$

Buradaki N_{x_0}, N_{y_0} ve N_{xy_0} değerleri membran denge denklemlerinden elde edilen membran kuvvetleridir

$$N_{x_0}=C\,(\varepsilon_{x_0}+\nu\varepsilon_{y_0})$$

$$N_{y_0}=C\,(\varepsilon_{y_0}+\nu\varepsilon_{x_0})$$

$$N_{xy_0}=C\,\frac{1-\nu}{2}\gamma_{xy_0}$$

Dış yüklerin potansiyel enerjisini veren bağıntı;

$$W = -\iint (P_x \cdot u + P_y \cdot v + P_z \cdot w)\, dx\,dy$$ idi. Bu bağıntının

ikinci varyasyonunda ikinci dereceden terimler ihtiva etmediğinden $\delta^2 W = 0$ olur. Bu durumda;

$$\delta^2 \Pi = \delta^2 U_m + \delta^2 U_b$$ olur.

$\frac{1}{2!}\delta U$ bağıntısındaki (1) indisleri atılıp matris formunda yazılırsa;

$$\frac{1}{2!}\delta \Pi = \frac{1}{2}\iint \{\varepsilon\}^T [D] \{\varepsilon\}\, dx\,dy + \frac{1}{2}\iint \{\beta\}^T [N_0] \{\beta\}\, dx\,dy$$

Bağıntı açık şekilde yazılırsa;

$$\frac{1}{2!}\delta^2 \Pi = \frac{1}{2}\iint \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix}^T \begin{bmatrix} C\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} & [0] \\ [0] & D\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} dx\,dy + \frac{1}{2}\iint \begin{Bmatrix} -\frac{\partial w}{\partial x} \\ \frac{v}{r} - \frac{\partial w}{\partial y} \end{Bmatrix}^T \begin{bmatrix} N_{x0} & N_{xy0} \\ N_{xy0} & N_{y0} \end{bmatrix} \begin{Bmatrix} -\frac{\partial v}{\partial x} \\ \frac{v}{r} - \frac{\partial w}{\partial r} \end{Bmatrix} dx\,dy$$

$\{\varepsilon\} = [F]\{a\}$; $\{\beta\} = [G]\{a\}$ $[F];[G]$: Türev matrisleridir.

$$\{\varepsilon\} = [F][B]\{d\}$$

$$\{\beta\} = [G][B]\{d\}$$

Bu durumda;

$$\frac{1}{2!}\delta^2 \Pi = \frac{1}{2}\{d\}^T \Big(\iint [B]^T [F]^T [D] [F] [B]\, dx\,dy + \iint [B]^T [G]^T [N_0] [G] [B]\, dx\,dy \Big) \{d\}$$

olacaktır.

Bağıntının kapalı çözümü;

$$\delta^2\Pi = \{d\}^T\left([ke] + \lambda[ne]\right)\{d\}$$ olur.

Burada; $[ke]$ = eleman rijitlik matrisi

λ : yük parametresi

$[ne]$: eleman geometrik yük matrisidir.

$[ne]$ İkinci mertebe etkilerinden meydana gelen ilave yüklerle sisteme geçme çevirme matrisleri veya birleştirme metodu ile yapılarak $[K]$ sistem rijitlik matrisi ve $[N]$ sistem geometrik yük matrisi elde edilir. Elemanın deplasman parametreleri $\{d_{ei}\}$; sistemin deplasman parametreleri $\{D_s\}$ ise; bütün sistemin potansiyel enerjisinin minimizasyonunu yaparsak;

$$\frac{\partial(\delta^2\Pi)_{sistem}}{\partial D_s} = \sum \frac{\partial(\delta^2\Pi)_{eleman}}{\partial d_{ei}} = 0 \qquad (D_s = D_1, D_2, D_3, \ldots D_n)$$

elde edilir

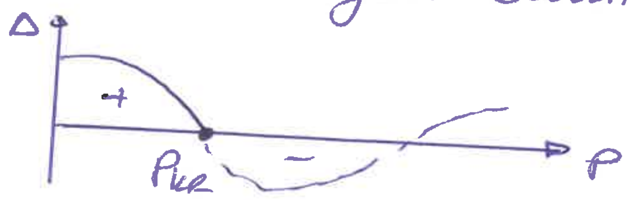
7.4.2) Stabilite Probleminin Çözümü

Burkulma yükü, sistemin ilk durumunu kararsız hale getiren en küçük yük parametresi olarak tanımlanabilir. Dış yüklerden oluşan membran kesit tesirleri bir yük parametresine bağlı olarak ifade edilebilir. İkinci mertebe hesaplarında süperpozisyon prensibi geçerli olmadığı için yükü belli bir değerin katları şeklinde gösterebiliriz. $[N_o] = \lambda[n_o]$ gibi. Burada λ yük parametresidir. Stabilite denklemi;

$$\left([K] + \lambda[N]\right)\{D_s\} = 0$$ şeklindedir.

Bu denklem takımına problemin sınır şartları uygulanır. Sistemin ilk konumunun kararsız hale gelmesi ve $\{\Delta_s\} = 0$'dan farklı koordinatlı başka bir denge konumunun bulunması için yukarıdaki denklem takımının katsayılar (Δ) determinantının sıfır olmasını gerektirir. (Δ) determinantını sıfır yapan yük iki şekilde bulunur:

1) Sıfırdan başlayarak P'ye artan değerler verilir ve (Δ) determinantı hesaplanır. Pozitiften negatife geçtiği bölgede bu aralıklar sıklaştırılarak (Δ) determinantını sıfır yapan P_{kr} değeri bulunmuş olur.



2) Ardışık yaklaşım yöntemi ile de P_{kr} yük hesaplanır.

7.5) Dinamik Dış Etkiler Halinde Sonlu Elemanlar Yöntemi

İnşaat mühendisliğinde deprem, darbe kuvvetleri, makine titreşim kuvvetleri ve ani patlama kuvvetleri zamanla değişen kuvvetlerdir. Dış etkilerin zamana bağlı olarak sonlu bir hızla değişmesi halinde atalet kuvvetlerinin de gözönüne alınması gerekmektedir. Sisteme etkiyen kuvvetler zamanın bir fonksiyonu ise, bu kuvvetlerin etkidiği yapıların tepkisi'de zamanın bir fonksiyonudur ve zamana bağlı olarak meydana gelecek yer değiştirmelerin ivmeleri, atalet kuvvetlerini meydana getirirler. Bu durumda sistem iki tip yükün etkisi altında düşünülebilir; harekete neden olan dış yük ve hareketin ivmelenmesine karşı duran atalet kuvvetleri. Yapı özellikleri ve etkiyen kuvvetlerden yola çıkarak, titreşim sistemine ait mekanik bir yay-kütle modeli oluşturulur ve kütlelere ait titreşim denklemleri kurularak çözüme ulaşılır.

Eleman üzerine yayılı kütleler düğüm noktalarına toplanabilir. Eleman boyutlarının büyük olması halinde gerçek sonuca iyi yaklaşmak için nokta kütle sayısını arttırmak ve mümkün olduğu kadar yayılı kütle haline yaklaşmak gerekir. Yüzeysel kütlelerden doğan atalet kuvvetlerinin gerçek yayılışları ile hesaba katılması uygundur. Yayılı kütlelerin ortalama yüzeye indirgendiği kabulü yapılacaktır. Zamana bağlı şekil değiştirme, ortalama yüzeyin yer değiştirmeleri cinsinden belirlendiğinden, yayılı atalet kuvvetleri yalnız yer değiştirme bileşenlerine bağlı olacaktır. Dönme bileşenlerine bağlı olarak ayrıca yüzeysel yayılı momentler meydana gelmeyecektir. Herhangi bir t anındaki yüzeysel elemanın hareket denklemi D'Alembert prensibine göre; sönme etkisi eden atalet kuvvetleri, sönüm etkileri ve yay kuvvetleriyle beraber dış kuvvetler denge halinde olmalıdır. Bilindiği gibi sönüm terimleri de, μ sürtünme katsayısı olmak üzere viskoz bir ortamda hızla orantılı bir sürtünme kuvveti gibi düşünülmektedir. Birim alana gelen kütle yoğunluğu ρ olduğuna göre; titreşime maruz kütlenin etkisi altında kaldığı atalet kuvveti, sönüm kuvveti ve yay kuvvetlerinin ilave dış yükler gibi düşünülmesiyle virtüel iş teoremi uygulanabilir.

δU iç kuvvetlerin, δW dış kuvvetlerin işi ise;

$$\delta U = \int_V \{\delta\varepsilon\}_i \sigma \, dV$$

$$\{\sigma\} = [D]\left(\{\varepsilon\} - \{\varepsilon_0\}\right) + \{\sigma_0\} - \alpha.\Delta t \{D_T\}$$

ε_0 başlangıç şekil değiştirmeleri ve σ_0 başlangıç gerilmeleri ihmal edersek;

$$\{\sigma\} = [D]\{\varepsilon\} - \alpha.\Delta t \{D_T\}$$ elde edilir.

$$\delta U = \int_V \{\delta\varepsilon\}_i^T [D]\{\varepsilon\} dV - \int_V \{\delta\varepsilon\}_i^T \alpha.\Delta t \{D_T\} dV$$

$$\delta W = -\rho \int_V \{\delta u\}_i^T \{\ddot{u}\} dV - \mu \int_V \{\delta u\}_i^T \{\dot{u}\} dV - c \int_A \{\delta u\}_i \{u\} dA$$
$$+ \int_A \{\delta u\}_i^T \{p\} dA$$

Tabii durum gerçek yükleme, i birim durumu ise virtüel deplasman ve virtüel şekil değiştirme durumu olarak alınıp; virtüel iş teoremi uyarınca dış kuvvetlerle, iç kuvvetlerin eşitliğini yazarak;

$\delta U = \delta W$ olur. Virtüel yer ve şekil değiştirmeleri de

$$\delta u = [N]\{\delta d\}$$

$$\delta \dot{u} = [N]\{\delta \dot{d}\}$$

$$\delta \ddot{u} = [N]\{\delta \ddot{d}\}$$

$\delta \varepsilon = [G]\{\delta d\} = [\Delta N]\{\delta d\}$ şeklindedir. Bu durumda;

$$\delta U = \delta W$$

$$\{\delta d\}^T\left[\int_V [\Delta N]^T[D][\Delta N]dV\right]\{d\} - \{\delta d\}^T[\alpha \Delta t]\int_V [\Delta N]^T\{D_T\}dV = $$
$$-\{\delta d\}^T\left[\rho\int_V [N]^T[N]dV\right]\{\ddot{d}\} - \{\delta d\}^T\left[\mu\int_V [N]^T[N]dV\right]\{\dot{d}\}$$
$$-\{\delta d\}^T\left[c\int_A [N_{(w)}]^T[N_{(w)}]dA\right]\{d\} + \{\delta d\}^T\int_A [N]^T p(t)dA$$

Buradan;

$[ke] = \int_V [\Delta N]^T[D][\Delta N]dV$: eleman rijitlik matrisi

$[Ce] = \mu\int_V [N]^T[N]dV$: " sönüm "

$[Me] = \rho\int_V [N]^T[N]dV$: " kütle "

$[Se] = c\int_A [N(w)]^T[N_{(w)}]dA$: " elastik yataklanma matrisi

$\{q_e(t)\}_T = \alpha \cdot \Delta t\int_V [\Delta N]^T\{D_T\}dV$: Elemanın zamana bağlı sıcaklık değişimi

$\{q_e(t)\}_o = \int_A [N]^T p(t)dA$: " " " dış yük

kısaltmaları yaparsak bağıntı;

$$[k_e]\{d(t)\} + [Ce]\{\dot{d}(t)\} + [Me]\{\ddot{d}(t)\} + [Se]\{d(t)\} = \{q_e(t)\}_o + \{q_e(t)\}_T$$

halini alır. Buradan sisteme geçiş;

$$[K]\{d(t)\}+[C]\{\dot{d}(t)\}+[M]\{\ddot{d}(t)\}+[S]\{d(t)\}=\{Q(t)\}_0+\{Q(t)\}$$

halini alır.

7.5.1) Sönümsüz Serbest Titreşim

Sönümsüz serbest titreşimde sönüm matrisi, zamana bağlı dış yük ve sıcaklık değişimi sıfır olacaktır. Elastik yataklanma etkileri de yoksa yukarıdaki bağıntı;

$$[K]\{d(t)\}+[M]\{\ddot{d}(t)\}=0$$ olur.

Titreşim basit harmonik hareket olduğundan bu homojen diferansiyel denklemin çözümü;

$$\{d\}=\{\bar{d}\}\sin(\omega t+\theta)$$ 'dır.

Burada $\{\bar{d}\}$ kütlelerin öteleme genlikleridir. Bu ifadenin türevi alınıp bağıntıdaki yerine yazılırsa titreşim bağıntısı;

$$([K]-\omega^2[M])\{\bar{d}\}=0$$ şeklini alır. $\{\bar{d}\}\neq 0$ çözümü için katsayılar determinantı sıfır olmalıdır.

$$\left|[K]-\omega^2[M]\right|=0$$

Bu determinanta frekans determinantı denir. Frekans determinantını sıfır yapan ω değerlerine özel frekans denir. $\lambda=\omega^2$ yazılır ve bağıntı $[M]^{-1}$ ile çarpılırsa;

$$\left|[K][M]^{-1}-\lambda[M][M]^{-1}\right|=0$$

$$[A]-\lambda[I]=0$$ olur.

Bu durumda problem özdeğer problemine dönüşür ve

n. mertebeden sabit katsayılı bir polinomun köklerinin bulunmasına indirgenmiş olur. n adet kökü olan bu denklemden n adet λ_i; dolayısıyla w_i özel açısal frekansları bulunur. Her w_i özel değerine karşılık $([K]-w^2[M])\{\bar{d}\}_i=0$ homojen denkleminden $\{\bar{d}\}_i$ değerleri bulunur. $\{\bar{d}\}_i$ vektörüne, w_i serbest titreşim frekansına karşı gelen serbest titreşim mod şekli denir. $\{\bar{d}\}_i$ vektörü yer değiştirmelere karşı gelmektedir. $\{\bar{d}\}_i$ kolon matrisleri yan yana yerleştirilerek kare $[\phi]$ modal matrisi elde edilir.

$$[\phi]=[\{\bar{d}\}_1,\{\bar{d}\}_2,----\{\bar{d}\}_n]$$

Matrisin $\{\bar{d}\}_{ij}$ elemanı; j. modda, i. serbestlik derecesi doğrultusundaki yer değiştirmeyi göstermektedir.

$$\{d\}=\{\bar{d}\}_1 D_1+\{\bar{d}\}_2 D_2+------+\{\bar{d}\}_n D_n \longrightarrow \{d\}=[\phi]\{D\}$$

Buradaki D_i değerlerine genelleştirilmiş koordinat denir. $\{\bar{d}\}_i$ ve $\{\bar{d}\}_j$ mod vektörleri hem kütle matrisine, hem de rijitlik matrisine göre ortogonaldir.

7.5.2) Sönümsüz Zorlanmış Titreşim

$$[K]\{d(t)\}+[M]\{\ddot{d}(t)\}=\{Q(t)\}.$$

7.5.3) Sönümlü Serbest Titreşim

$$[K]\{d(t)\}+[C]\{\dot{d}(t)\}+[M]\{\ddot{d}(t)\}=0$$

7.5.4) Sönümlü Zorlanmış Titreşim

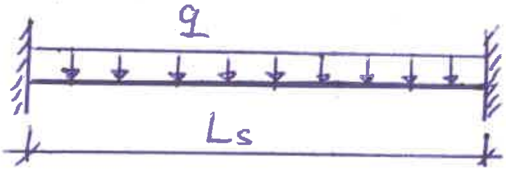
$$[K]\{d(t)\}+[C]\{\dot{d}(t)\}+[M]\{\ddot{d}(t)\}=\{Q(t)\}.$$

BÖLÜM V

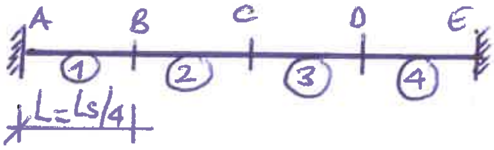
9) SONLU ELEMANLAR YÖNTEMİ İLE ÇUBUK HESABI

9.1) Çubuğun Eğilme Hesabı

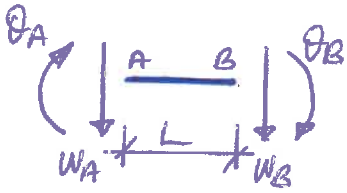
Çubuğun eğilme hesabı için L_s uzunluğunda, düzgün yayılı yükle yüklenmiş bir elemanı dikkate alalım ve bu elemanı 4 parçaya bölelim.

a) 

← Ele alınan çubuk elemanı

b) 

← Çubuğun sonlu elemanlara ayrılması

c) 

← Sonlu eleman parçası ve kabul edilen yer değiştirmeler

d) $A(0,0)\ ;\ B(L,0)$ ← Elemanın koordinatları

e) $$\{d\} = \left\{\begin{bmatrix} w_A \\ \theta_A \\ w_B \\ \theta_B \end{bmatrix}\right\} = \left\{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}\right\}$$ ← Elemanın düğüm noktası yerdeğiştirme parametreleri (matrisi)

f) 

g) $$\{D\} = \left\{\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix}\right\} = \left\{\begin{bmatrix} w_A \\ \theta_A \\ w_B \\ \theta_B \\ w_C \\ \theta_C \end{bmatrix}\right\}$$ ← Sistem yer değiştirme matrisi

9.1.1) Deplasman Fonksiyonlarının Seçimi

Deplasman fonksiyonları rijit cisim hareketi ve sabit deformasyon şartını sağlayacak şekilde seçilmelidir. Koordinat ekseni değişince çözüm farklı olmamalıdır. Bunun için deplasman fonksiyonu ya tam polinom veya tabii koordinatların fonksiyonu şeklinde olmalıdır. Ayrıca iç ve dış kuvvetlerin içindeki türevlerde de sürekli olmalıdır. Bu çalışmada deplasman fonksiyonu polinom olarak seçilmiştir. Elemanda bilinmeyen dört adet deplasman parametresi olduğu için, dört adet bilinmeyenli deplasman fonksiyonu seçilmiştir.

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$w = [\phi(x)]\{a\} = [1, x, x^2, x^3]\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$

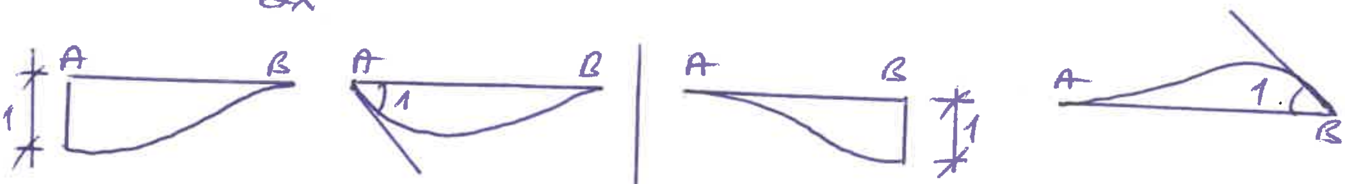
9.1.2) Şekil Fonksiyonlarının Tayini

Şekil fonksiyonlarını iki şekilde hesaplamak mümkündür. Birincisi çubuk uç deplasmanlarının hesabı ile, ikincisi de bağ matrisleri yardımıyladır.

a) Çubuk uç deplasmanları yardımı ile şekil fonksiyonlarının tayini:

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

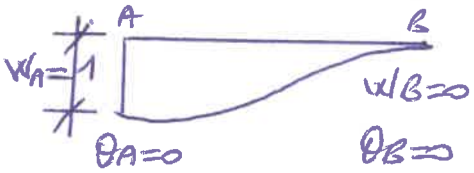
$$\theta = \frac{\partial w}{\partial x} = a_2 + 2a_3 x + 3a_4 x^2$$



A ucunun çökmesi ve dönmesi

B ucunun çökmesi ve dönmesi

– N_1 şekil fonksiyonunun tayini ($w_A=1$, $w_B=\theta_A=\theta_B=0$)



Koordinatlar $A(0,0)$; $B(L,0)$

$$w=a_1+a_2x+a_3x^2+a_4x^3$$

$$\theta=\frac{\partial w}{\partial x}=0+a_2+2a_3x+3a_4x^2$$

A ve B ucunun koordinat değerlerini w_A, θ_A, w_B ve θ_B bağıntılarında yerlerine yazarsak;

$$\left.\begin{aligned} w_A&=1=a_1+0+0+0\\ \theta_A&=0=0+a_2+0+0\\ w_B&=0=a_1+a_2L+a_3L^2+a_4L^3\\ \theta_B&=0=0+a_2+2a_3L+3a_4L^2\end{aligned}\right\}$$

Bu değerlere göre;

$a_1=1$; $a_3=-\frac{3}{L^2}$

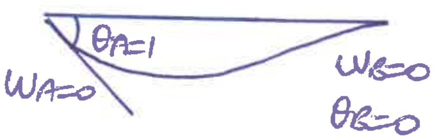
$a_2=0$; $a_4=\frac{2}{L^3}$

bulunur. Bu durumda da;

$N_1=1-\frac{3}{L^2}x^2+\frac{2}{L^3}x^3$ olarak N_1 şekil fonksiyonu bulunur.

$\xi=\frac{x}{L}$ denilirse $\longrightarrow$ $N_1=1-3\xi^2+2\xi^3$ olur.

– N_2 şekil fonksiyonunun tayini ($\theta_A=1$, $w_A=w_B=\theta_B=0$)



$$\left.\begin{aligned} w_A&=0=a_1+0+0+0\\ \theta_A&=1=0+a_2+0+0\\ w_B&=0=a_1+a_2L+a_3L^2+a_4L^3\\ \theta_B&=0=0+a_2+2a_3L+3a_4L^2\end{aligned}\right\}$$

Bu çözümden;

$a_1=0$; $a_2=1$

$a_3=-\frac{2}{L}$; $a_4=\frac{1}{L^2}$

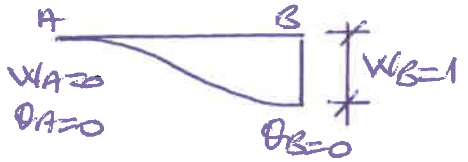
elde edilir.

Bu durumda N_2 şekil fonksiyonu;

$N_2=L\left(\frac{x}{L}-2\frac{x^2}{L^2}+\frac{x^3}{L^3}\right)$ olarak bulunur. $\xi=\frac{x}{L}$ ise

$N_2=L(\xi-2\xi^2+\xi^3)$ olur.

- N_3 şekil fonksiyonunun tayini ($w_B=1$, $w_A=\theta_A=\theta_B=0$)



$$\left.\begin{aligned} w_A &= 0 = a_1+0+0+0 \\ \theta_A &= 0 = 0+a_2+0+0 \\ w_B &= 1 = a_1+a_2L+a_3L^2+a_4L^3 \\ \theta_B &= 0 = 0+a_2+2a_3L+3a_4L^2 \end{aligned}\right\}$$

Bu çözümden;

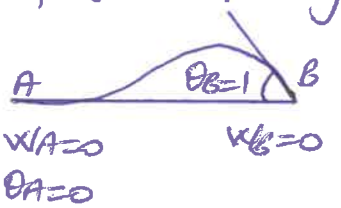
$a_1=0$; $a_2=0$

$a_3=\frac{3}{L^2}$; $a_4=-\frac{2}{L^3}$

$$N_3 = 3\frac{x^2}{L^2}-2\frac{x^3}{L^3}$$

$$N_3 = 3\xi^2-2\xi^3$$

- N_4 şekil fonksiyonunun tayini ($\theta_B=1$, $w_A=\theta_A=w_B=0$)



$$\left.\begin{aligned} w_A &= 0 = a_1+0+0+0 \\ \theta_A &= 0 = a_2+0+0 \\ w_B &= 0 = a_1+a_2L+a_3L^2+a_4L^3 \\ \theta_B &= 1 = a_2+2a_3L+3a_4L^2 \end{aligned}\right\}$$

$a_1=0$; $a_2=0$

$a_3=-\frac{1}{L}$; $a_4=\frac{1}{L^2}$

$$N_4 = L\left(-\frac{x^2}{L^2}+\frac{x^3}{L^3}\right)$$

$$N_4 = L\left(-\xi^2+\xi^3\right)$$

b) Bağ matrisi yardımıyla şekil fonksiyonlarının tayini

Eleman düğüm noktası deplasmanları $\{d\}$ ile polinom sabitleri $\{a\}$ arasındaki bağı veren $[A]$ bağ matrisi ise, elemanın düğüm noktası deplasman parametrelerinin $\phi(x)$ ve $\phi(x)$'in türevleri cinsinden yazılmış bileşenlerinde düğüm noktası koordinatlarının yerine konulmasıyla bulunur.

$$\{d\} = [A]\{a\}$$

$$\{a\} = [A]^{-1}\{d\}$$

$[A]^{-1} = [B]$ dersek;

$$\{a\} = [B]\{d\}$$

$w = [\phi(x)]\{a\} = [\phi(x)][B]\{d\}$ Bu ifadedeki $[\phi(x)][B]$'ye şekil fonksiyonu denir.

$$[N] = [\phi(x)][B]$$

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$\theta = \frac{\partial w}{\partial x} = a_2 + 2a_3 x + 3a_4 x^2$$

A(0,0) ve B(L,0) düğüm noktası koordinatları x ve bu koordinatlar ilgili bağıntılarda yerlerine yazılırsa;

$$\left.\begin{aligned} w_A &= a_1 + a_2 x + a_3 x^2 + a_4 x^3 \\ \theta_A &= a_2 + 2a_3 x + 3a_4 x^2 \end{aligned}\right|_{x=0} \qquad \left.\begin{aligned} w_B &= a_1 + a_2 x + a_3 x^2 + a_4 x^3 \\ \theta_B &= a_2 + 2a_3 x + 3a_4 x^2 \end{aligned}\right|_{x=L}$$

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L & L^2 & L^3 \\ 0 & 1 & 2L & 3L^2 \end{bmatrix}$$

matrisi elde edilir. Buradan $[A]^{-1} = [B]$ matrisi hesaplanır.

$$[B] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/L^2 & -2/L & 3/L^2 & -1/L \\ 2/L^3 & 1/L^2 & -2/L^3 & 1/L^2 \end{bmatrix}$$

$$w = [\phi(x,y)]\{a\} = [\phi(x,y)][B]\{d\}$$

$$[N_1, N_2, N_3, N_4] = \underbrace{[1, x, x^2, x^3][B]}_{[N]}$$

$[A]^{-1} = [B]$ matrisinin matlab programı ile hesabı

```
>> syms L
>> syms A
>> syms B
>> syms N1
>> syms N2
>> syms N3
>> syms N4
>> syms x
>> syms x1
>> [A]=[1 0 0 0;0 1 0 0;1 L L^2 L^3;0 1 2*L 3*L^2]

A =

[ 1, 0,    0,     0]
[ 0, 1,    0,     0]
[ 1, L, L^2,    L^3]
[ 0, 1, 2*L, 3*L^2]

>> [B]=[A]^-1

B =

[       1,      0,       0,      0]
[       0,      1,       0,      0]
[ -3/L^2,   -2/L,   3/L^2,   -1/L]
[  2/L^3,  1/L^2, -2/L^3,  1/L^2]

>> [x1]=[N1 N2 N3 N4]
```

$[x_1] = [N_1, N_2, N_3, N_4] * [B]$

```
x1 =

[ N1, N2, N3, N4]

>> [x1]=[1 x x^2 x^3]*[B]

x1 =

[ (2*x^3)/L^3 - (3*x^2)/L^2 + 1, x - (2*x^2)/L + x^3/L^2, (3*x^2)/L^2 - (2*x^3)/L^3,
x^3/L^2 - x^2/L]

>>
```

$$[N_1, N_2, N_3, N_4] = [1, x, x^2, x^3] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/L^2 & -2/L & 3/L^2 & -1/L \\ 2/L^3 & 1/L^2 & -2/L^3 & 1/L^2 \end{bmatrix}$$

$$N_1 = 1 - \frac{3}{L^2}x^2 + \frac{2}{L^3}x^3$$

$$N_2 = x - \frac{2}{L}x^2 + \frac{1}{L^2}x^3 = L\left(\frac{x}{L} - \frac{2}{L^2}x^2 + \frac{1}{L^3}x^3\right)$$

$$N_3 = \frac{3}{L^2}x^2 - \frac{2}{L^3}x^3$$

$$N_4 = -\frac{1}{L}x^2 + \frac{1}{L^2}x^3 = L\left(-\frac{x^2}{L^2} + \frac{x^3}{L^3}\right)$$

$\xi = \frac{x}{L}$ de ;

$$N_1 = 1 - 3\xi^2 + 2\xi^3$$

$$N_2 = L(\xi - 2\xi^2 + \xi^3)$$

$$N_3 = 3\xi^2 - 2\xi^3$$

$$N_4 = L(-\xi^2 + \xi^3)$$

9.1.3) Eleman Rijitlik Matrisinin Bulunması

Toplam potansiyel enerjinin birinci varyasyonu önceden de verildiği gibi;

$$\delta\Pi^e = \delta U^e + \delta W^e$$

$$\delta\Pi^e = \int_0^L \left[EI(w'')^2 - q.w\right] dx \quad \text{şeklindedir.}$$

Burada;

$$\delta U^e = \int_0^L [EI(w'')^2]\,dx = \int_0^L \{\varepsilon\}^T [D]\{\varepsilon\}\,dx = \{d\}^T [k_e]\{d\}$$

$$\delta W^e = -\int_0^L q\,w\,dx = -\int_0^L [N_1, N_2, N_3, N_4]^T q\,dx = -\{d\}^T [q_e]$$

$$\delta \Pi^e = \{d\}^T \left([k_e]\{d\} - [q_e] \right)$$

Burada $[k_e]$ eleman eşitlik matrisi, $[q_e]$ dış yük matrisidir.
Eleman eşitlik matrisi hesaplanırken 2 yöntem uygulanır:

a) İntegral matrisleri yardımıyla
b) Şekil fonksiyonları "

a) İntegral Matrisleriyle Eşitlik Matrisinin Hesabı

$$\varepsilon = -\frac{\partial^2 w}{\partial x^2} = -w''$$

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$w' = a_2 + 2a_3 x + 3a_4 x^2$$

$$-w'' = -2a_3 - 6a_4 x$$

$\{\varepsilon\} = [F]\{a\}$ Buradaki F matrisi türev matrisidir.

$$\{\varepsilon\} = [0, 0, -2, -6x] \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$

$$\{\varepsilon\} = [F][B]\{d\}$$

$$[F] = [0, 0, -2, -6x]$$

$$[k_e] = \int_0^L \{\varepsilon\}^T [D]\{\varepsilon\}\,dx$$

$$[k_e] = \int_0^L [B]^T [F]^T [EI][F][B]\,dx$$ olur.

$[H] = EI \int_0^L [F]^T [F]\,dx$ integral matrisidir.

$$[ke]=EI\int_0^L \begin{bmatrix} 1 & 0 & -3/L^2 & 2/L^3 \\ 0 & 1 & -2/L & 1/L^2 \\ 0 & 0 & 3/L^2 & -2/L^3 \\ 0 & 0 & -1/L & 1/L^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -2 \\ -6x \end{bmatrix} \begin{bmatrix} 0 & 0 & -2 & -6x \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/L^2 & -2/L & 3/L^2 & -1/L \\ 2/L^3 & 1/L^2 & -2/L^3 & 1/L^2 \end{bmatrix}$$

Hesaplamalardan sonra eleman rijitlik matrisi;

$$[ke]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$ şeklinde belirlenir.

b) Şekil Fonksiyonlarından Faydalanarak Eleman Rijitlik Matrisinin Hesabı

$$[ke]=\int_0^L \{\varepsilon\}^T [D] \{\varepsilon\}\, dx \;, \quad \{\varepsilon\}=-\frac{\partial^2 w}{\partial x^2} \;, \quad w=\underbrace{[\phi(x)][B]}_{[N]}\{d\}$$

$$\varepsilon=\frac{\partial^2 w}{\partial x^2}=-\frac{\partial^2 [N]}{\partial x^2}\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

$$[ke]=\int_0^L \left\{\left(-\frac{\partial^2 [N]}{\partial x^2}\right)^T [D] \left(-\frac{\partial^2 [N]}{\partial x^2}\right) dx\right.$$

$$[ke]=\int_0^L [-N_1'', -N_2'', -N_3'', -N_4'']^T [EI] [-N_1'', -N_2'', -N_3'', -N_4]\, dx$$

$$[ke]=EI\int_0^L \begin{bmatrix} N_1''N_1'' & N_1''N_2'' & N_1''N_3'' & N_1''N_4'' \\ N_2''N_1'' & N_2''N_2'' & N_2''N_3'' & N_2''N_4'' \\ N_3''N_1'' & N_3''N_2'' & N_3''N_3'' & N_3''N_4'' \\ N_4''N_1'' & N_4''N_2'' & N_4''N_3'' & N_4''N_4'' \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}$$

$$N_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \qquad N_2 = L\left(\frac{x}{L} - \frac{2x^2}{L^2} + \frac{x^3}{L^3}\right)$$

$$N_1' = -\frac{6x}{L^2} + \frac{6x^2}{L^3} \qquad N_2' = L\left(\frac{1}{L} - \frac{4x}{L^2} + \frac{3x^2}{L^3}\right)$$

$$N_1'' = -\frac{6}{L^2} + \frac{12x}{L^3} \qquad N_2'' = L\left(-\frac{4}{L^2} + \frac{6x}{L^3}\right)$$

Bu durumda;

$$k_{11} = EI\int_0^L N_1''N_1''\,dx = EI\int_0^L\left(-\frac{6}{L^2} + \frac{12x}{L^3}\right)\left(-\frac{6}{L^2} + \frac{12x}{L^3}\right)dx = \frac{12EI}{L^3}$$

$$k_{12} = EI\int_0^L N_1''N_2''\,dx = EI\int_0^L\left(-\frac{6}{L^2} + \frac{12x}{L^3}\right)\left(-\frac{4}{L^2} + \frac{6x}{L^3}\right)L\,dx = \frac{6EI}{L^2}$$

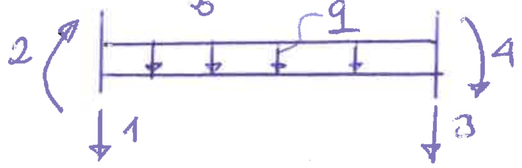
Bunun sonucunda;

$$[k_e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$ elde edilir.

9.1.4) Eleman Yük Matrisinin Hesabı

Elemana etkiyen yayılı yükler, eşdeğer kuvvete çevrilerek düğüm noktasına tekil yük gibi etki ettirilir. Bu durumda yük matrisi elemanda;

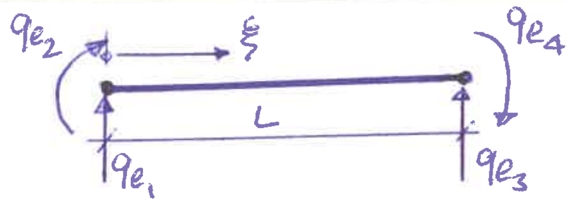
$$\{q_e\} = \int_0^L [N_1, N_2, N_3, N_4]^T q\,dx$$ şeklinde olacaktır.



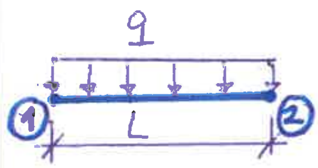
$$\{q_e\} = \int_0^L \left[\left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right)\left(L\left(\frac{x}{L} - \frac{2x^2}{L^2} + \frac{x^3}{L^3}\right)\right)\left(3\frac{x^2}{L^2} - 2\frac{x^3}{L^3}\right)\left(L\left(-\frac{x^2}{L^2} + \frac{x^3}{L^3}\right)\right)\right]^T q\,dx$$

$$\{q_e\} = q\left[\left(\frac{L}{2}\right), \left(\frac{L^2}{12}\right), \left(\frac{L}{2}\right), \left(-\frac{L^2}{12}\right)\right]^T$$

Çeşitli Dış Yüklerin Düğüm Noktalarına Aktarılması
Eşdeğer Kuvvet ve Moment Değerleri

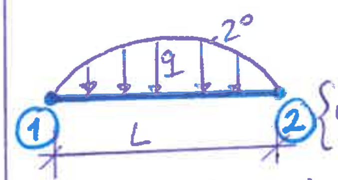


$$\{q_e\} = \begin{Bmatrix} q_{e1} \\ q_{e2} \\ q_{e3} \\ q_{e4} \end{Bmatrix} \qquad [N] = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{Bmatrix} = \begin{Bmatrix} 1-3\xi^2+2\xi^3 \\ L(-\xi+2\xi^2-\xi^3) \\ 3\xi^2-2\xi^3 \\ L(\xi^2-\xi^3) \end{Bmatrix}$$



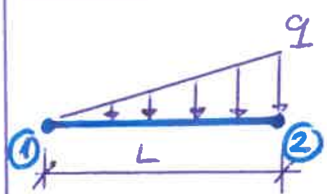
$q(\xi) = q$

$$\{q_e\} = q \begin{Bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{Bmatrix}$$



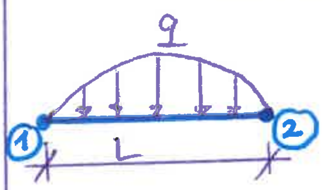
$q(\xi) = 4q(\xi-\xi^2)$

$$\{q_e\} = q \begin{Bmatrix} L/3 \\ L^2/15 \\ L/3 \\ -L^2/15 \end{Bmatrix}$$



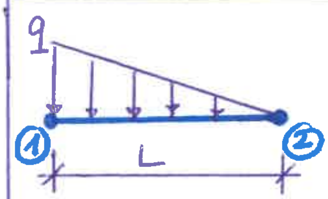
$q(\xi) = q\xi$

$$\{q_e\} = q \begin{Bmatrix} \frac{3}{20}L \\ L^2/30 \\ \frac{7}{20}L \\ -L^2/20 \end{Bmatrix}$$



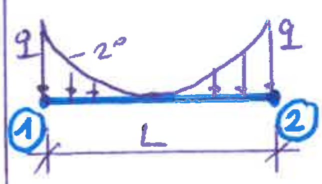
$q(\xi) = q\sin(\pi\xi)$

$$\{q_e\} = q \begin{Bmatrix} L/\pi \\ 2L^2/\pi^3 \\ L/\pi \\ -2L^2/\pi^3 \end{Bmatrix}$$



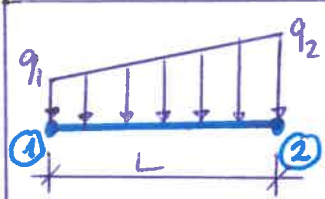
$q(\xi) = q(1-\xi)$

$$\{q_e\} = q \begin{Bmatrix} \frac{7}{20}L \\ L^2/20 \\ \frac{3}{20}L \\ -L^2/30 \end{Bmatrix}$$



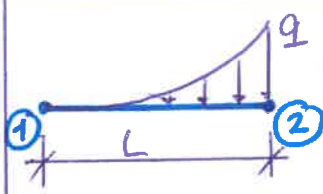
$q(\xi) = q-4q(\xi-\xi^2)$

$$\{q_e\} = q \begin{Bmatrix} L/6 \\ L^2/60 \\ L/6 \\ -L^2/60 \end{Bmatrix}$$



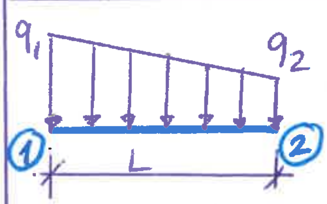
$q(\xi) = q_1+(q_2-q_1)\xi$

$$\{q_e\} = \frac{L}{60} \begin{Bmatrix} 21q_1+9q_2 \\ L(3q_1+2q_2) \\ 9q_1+21q_2 \\ -L(2q_1+3q_2) \end{Bmatrix}$$



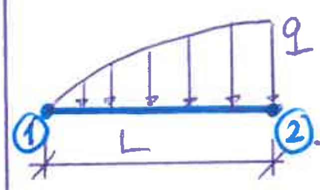
$q(\xi) = q\xi^2$

$$\{q_e\} = q \begin{Bmatrix} L/15 \\ L^2/60 \\ 4L/15 \\ -L^2/30 \end{Bmatrix}$$



$q(\xi) = q_1+(q_2-q_1)\xi$

$$\{q_e\} = \frac{L}{60} \begin{Bmatrix} 21q_1+9q_2 \\ L(3q_1+2q_2) \\ 9q_1+21q_2 \\ -L(2q_1+3q_2) \end{Bmatrix}$$



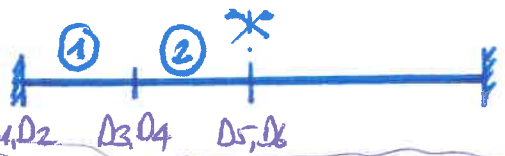
$q(\xi) = q\sqrt{\xi}$

$$\{q_e\} = q \begin{Bmatrix} \frac{80}{315}L \\ \frac{16}{315}L^2 \\ \frac{130}{315}L \\ -\frac{20}{315}L^2 \end{Bmatrix}$$

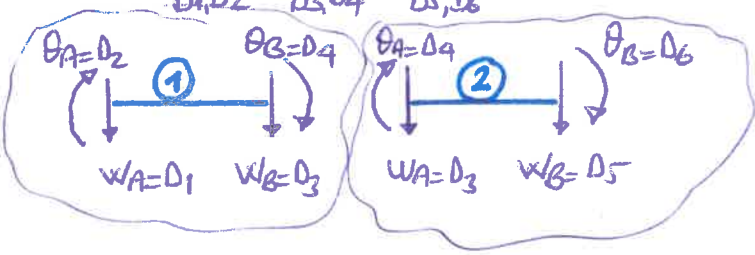
9.1.5) Sisteme Geçiş

Sisteme geçiş çevirme matrisleriyle ve birleştirme yöntemi ile yapılmaktadır.

a) Çevirme matrisleriyle sisteme geçiş



Genel sistemde yer değiştirmeler



Elemanlardaki yer değiştirmeler

1. eleman için çevirme matrisi:

$$\begin{bmatrix} w_A \\ \theta_A \\ w_B \\ \theta_B \end{bmatrix}_{1.eleman} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{C_{e_1}} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix}_{sistem}$$

* 1. elemandaki yer değiştirmeler ($w_A, \theta_A, w_B, \theta_B$), genel sistemdeki ($D_1, D_2, D_3, D_4$) yer değiştirmelerle çakıştırılmıştır.

2. eleman için çevirme matrisi:

$$\begin{bmatrix} w_A \\ \theta_A \\ w_B \\ \theta_B \end{bmatrix}_{2.eleman} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{C_{e_2}} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix}_{sistem}$$

* 2. elemandaki yer değiştirmeler ($w_A, \theta_A, w_B, \theta_B$), genel sistemdeki ($D_3, D_4, D_5, D_6$) yer değiştirmelerle çakıştırılmıştır.

Not: Bu sistemde kullanılan sonlu eleman şekli tek olduğundan 1 ve 2 nolu elemanlarda eleman yer değiştirmeleri aynı isimle anılmaktadır. Aksi durumda isimler değişecektir.

Genişleme matrislerinden faydalanarak sistem rijitlik matrisi;

$$[Ke_1] = [Ce_1]^T [ke][Ce_1] \longleftarrow \text{1. elemanın sisteme dönüşümü}$$

$$[Ke_2] = [Ce_2]^T [ke][Ce_2] \longleftarrow \text{2. " " "}$$

$$[Ke_1] = \frac{EI}{L^3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[Ke_1] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12 & -6L & 0 & 0 \\ 6L & 2L^2 & -6L & 4L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[Ke_2] = \frac{EI}{L^3} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[Ke_2] = \frac{EI}{L^3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 6L & -12 & 6L \\ 0 & 0 & 6L & 4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Toplam Sistem Rijitlik Matrisi;

$$[K]_s = [Ke_1] + [Ke_2]$$

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Sistem yük vektörünün hesabı;

$$\{Qe_1\} = [Ce_1]^T \{qe_1\}$$

$$\{Qe_2\} = [Ce_2]^T \{qe_2\}$$

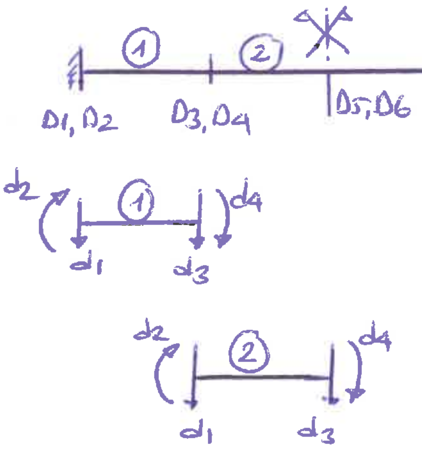
$$\{Qe_1\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} q \begin{bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{bmatrix} = q \begin{bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \\ 0 \\ 0 \end{bmatrix}$$

$$\{Qe_2\} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} q \begin{bmatrix} L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{bmatrix} = q \begin{bmatrix} 0 \\ 0 \\ L/2 \\ L^2/12 \\ L/2 \\ -L^2/12 \end{bmatrix}$$

$$\{Q\} = \{Qe_1\} + \{Qe_2\}$$

$$\{Q\} = q \begin{bmatrix} L/2 \\ L^2/12 \\ L \\ 0 \\ L/2 \\ -L^2/12 \end{bmatrix}$$

b) Biriktirme yöntemiyle sisteme geçiş



Eleman No	Eleman deplasman parametresi	Sistemde deplasman parametresi
1	$d_1=w_A$; $d_3=w_B$ $d_2=\theta_A$; $d_4=\theta_B$	D_1 ; D_3 D_2 ; D_4
2	$d_1=w_A$; $d_3=w_B$ $d_2=\theta_A$; $d_4=\theta_B$	D_3 ; D_5 D_4 ; D_6
Çubuk uç noktaları	↑ i ucu ; ↑ j ucu	↑ i ucu ; ↑ j ucu

Eleman Rijitlik Matrisi ; $[k_e]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}$

	D_1	D_2	D_3	D_4	D_5	D_6
D_1	$\overset{1}{k}_{11}$	$\overset{1}{k}_{12}$	$\overset{1}{k}_{13}$	$\overset{1}{k}_{14}$	0	0
D_2	$\overset{1}{k}_{21}$	$\overset{1}{k}_{22}$	$\overset{1}{k}_{23}$	$\overset{1}{k}_{24}$	0	0
D_3	$\overset{1}{k}_{31}$	$\overset{1}{k}_{32}$	$\overset{1}{k}_{33}+\overset{2}{k}_{11}$	$\overset{1}{k}_{34}+\overset{2}{k}_{12}$	$\overset{2}{k}_{13}$	$\overset{2}{k}_{14}$
D_4	$\overset{1}{k}_{41}$	$\overset{1}{k}_{42}$	$\overset{1}{k}_{43}+\overset{2}{k}_{21}$	$\overset{1}{k}_{44}+\overset{2}{k}_{22}$	$\overset{2}{k}_{23}$	$\overset{2}{k}_{24}$
D_5	0	0	$\overset{2}{k}_{31}$	$\overset{2}{k}_{32}$	$\overset{2}{k}_{33}$	$\overset{2}{k}_{34}$
D_6	0	0	$\overset{2}{k}_{41}$	$\overset{2}{k}_{42}$	$\overset{2}{k}_{43}$	$\overset{2}{k}_{44}$

k_{ij}
↙ yer ↘ sebep

Eleman rijitlik matrisi $[k_e]=\dfrac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$

$$K_{33}=\overset{1}{k}_{33}+\overset{2}{k}_{11}=\frac{EI}{L^3}(12+12)=\frac{24EI}{L^3}$$

$$K_{34}=\overset{1}{k}_{34}+\overset{2}{k}_{12}=\frac{EI}{L^3}(-6L+6L)=0$$

$$K_{43}=\overset{1}{k}_{43}+\overset{2}{k}_{21}=\frac{EI}{L^3}(-6L+6L)=0$$

$$K_{44}=\overset{1}{k}_{44}+\overset{2}{k}_{22}=EI/L^3\,(4L^2+4L^2)=\frac{EI}{L^3}(8L^2)$$

Sistem rijitlik matrisi ; $[K]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$

Sistem rijitlik matrisi $[K]$, yük matrisi $\{Q\}$ ve yer değiştirme matrisi $\{D_s\}$ ile ;

$$[K]\{D_s\}=\{Q\} \text{ olur.}$$

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = q\begin{bmatrix} L/2 \\ L^2/12 \\ L \\ 0 \\ L/2 \\ -L^2/12 \end{bmatrix}$$

9.1.6) Sınır Koşulları

Sonlu elemanlar yönteminin diğer yöntemlere göre avantajı sınır koşullarının probleme en sonunda uygulanmasıdır. Bu problemde simetriden faydalanıldığından simetri koşuluna göre $D_6=0$, mesnet koşullarına göre $D_1=D_2=0$ (Ankastre mesnette $v=v'=0$) olur. Bu durumda rijitlik, yer değiştirme ve yük matrisinin ilgili satır ve sütunları sıfırlanır.

$$\frac{EI}{L^3}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 & -12 & 0 \\ 0 & 0 & 0 & 8L^2 & -6L & 0 \\ 0 & 0 & -12 & -6L & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ D_3 \\ D_4 \\ D_5 \\ 0 \end{bmatrix} = q\begin{bmatrix} 0 \\ 0 \\ L \\ 0 \\ L/2 \\ 0 \end{bmatrix}$$

$$24D_3 - 12D_5 = qL\left(\frac{L^3}{EI}\right) \quad \text{---- } 1$$

$$8L^2D_4 - 6LD_5 = 0 \quad \text{---------- } 2$$

$$-12D_3 - 6LD_4 + 12D_5 = q\frac{L}{2}\,\frac{L^3}{EI} \quad \text{--- } 3$$

Denklemlerin çözümünden; $D_3 = \frac{3}{8}q\frac{L^4}{EI}$; $D_4 = \frac{1}{2}q\frac{L^3}{EI}$

$D_5 = \frac{2}{3}q\frac{L^4}{EI}$ bulunur.

9.1.7) Sistem momentlerinin hesabı

$$M_i = -EIw'' \longrightarrow M_1 = -EI\left[N_1'' \; N_2'' \; N_3'' \; N_4''\right]\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

1. elemanda (M_A ve M_B^1 hesabı)

$$M_1 = -\frac{EI}{L^2}\left[\left(-6+\frac{12x}{L}\right) \; L\left(-4+\frac{6x}{L}\right) \; \left(6-\frac{12x}{L}\right) \; L\left(-2+\frac{6x}{L}\right)\right]\begin{bmatrix} 0 \\ 0 \\ \frac{3}{8}q\frac{L^4}{EI} \\ \frac{1}{2}q\frac{L^3}{EI} \end{bmatrix}$$

$$M_1 = -\left(\frac{5}{4} - \frac{6x}{4L}\right)qL^2$$

$$x=0 \longrightarrow M_1 = -\frac{5}{4}qL^2 \longrightarrow L = \frac{L_s}{4} \longrightarrow M_A = -\frac{5}{64}qL_s^2 = -\frac{1}{12{,}8}qL_s^2$$

(Kesin çözüm $M_1 = -\frac{qL_s^2}{12}$)

$$x=L \longrightarrow M_B^1 = \frac{qL^2}{4}$$

2. elemandan (M_B^2 ve M_C hesabı)

$$M_2 = -\frac{EI}{L^2}\left[\left(-6+\frac{12x}{L}\right)\; L\left(-4+\frac{6x}{L}\right)\; \left(6-\frac{12x}{L}\right)\; L\left(-2+\frac{6x}{L}\right)\right] \begin{bmatrix} \frac{3}{8} q \frac{L^4}{EI} \\ \frac{1}{2} q \frac{L^3}{EI} \\ \frac{2}{3} q \frac{L^4}{EI} \\ 0 \end{bmatrix}$$

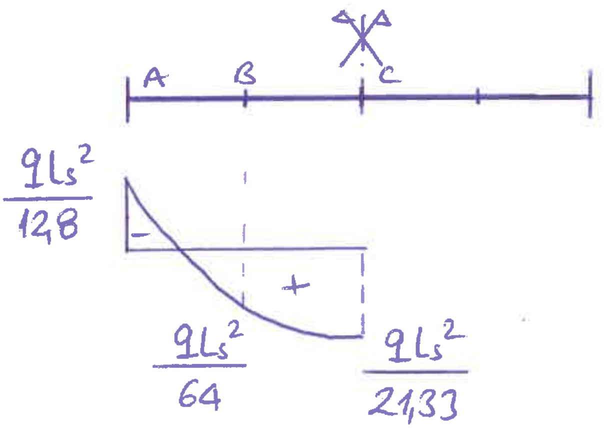
$$M_2 = \left(\frac{1}{4} + \frac{2x}{4L}\right) qL^2$$

$$x=0 \longrightarrow M_B^2 = \frac{qL^2}{4}$$

$$x=L \longrightarrow M_C = \frac{3}{4} qL^2 \longrightarrow L = \frac{L_s}{4} \longrightarrow M_C = \frac{3}{64} qL_s^2 = \frac{1}{21{,}33} qL_s^2$$

(Kesin çözüm $M_C = \frac{qL_s^2}{24}$)

$$M_B = (M_B^1 + M_B^2)/2 = \frac{qL^2}{4} \longrightarrow L = \frac{L_s}{4} \longrightarrow M_B = \frac{1}{64} qL_s^2$$



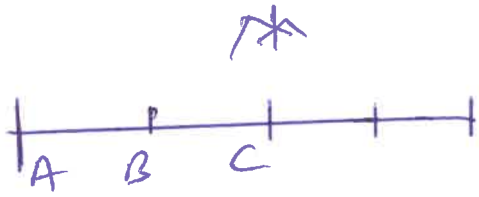
Rijitlik Matrisleri Yardımıyla Kesit Tesirlerinin Hesabı

$$\{F\}^e = [K]^e \cdot \{D\}^e - \{Q\}^e$$

1. nolu elemanın kesit tesirleri:

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}^1 = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ \frac{3}{8}q\frac{L^4}{EI} \\ \frac{1}{2}q\frac{L^3}{EI} \end{Bmatrix} - \begin{Bmatrix} \frac{qL}{2} \\ \frac{qL^2}{12} \\ \frac{qL}{2} \\ -\frac{qL^2}{12} \end{Bmatrix} = \begin{Bmatrix} -2qL \\ -\frac{4}{3}qL^2 \\ qL \\ -\frac{qL^2}{6} \end{Bmatrix}$$

$L = \frac{L_s}{4}$ olduğundan;



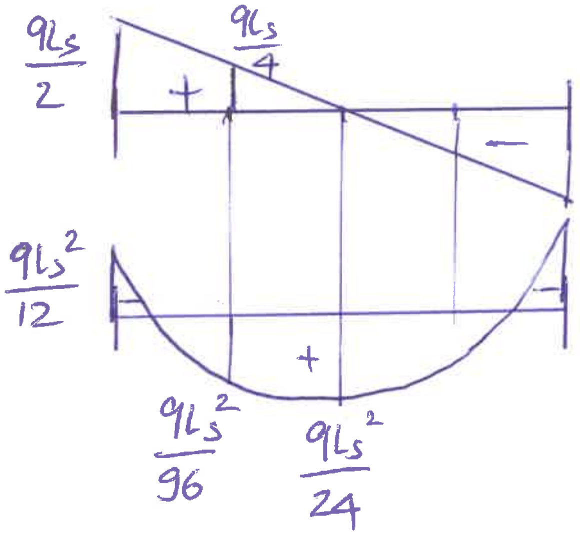
$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}^1 = \begin{Bmatrix} V_A \\ M_A \\ V_B \\ M_B \end{Bmatrix} = \begin{Bmatrix} -\frac{qL_s}{2} \\ -\frac{qL_s^2}{12} \\ \frac{qL_s}{4} \\ -qL_s^2/96 \end{Bmatrix}$$

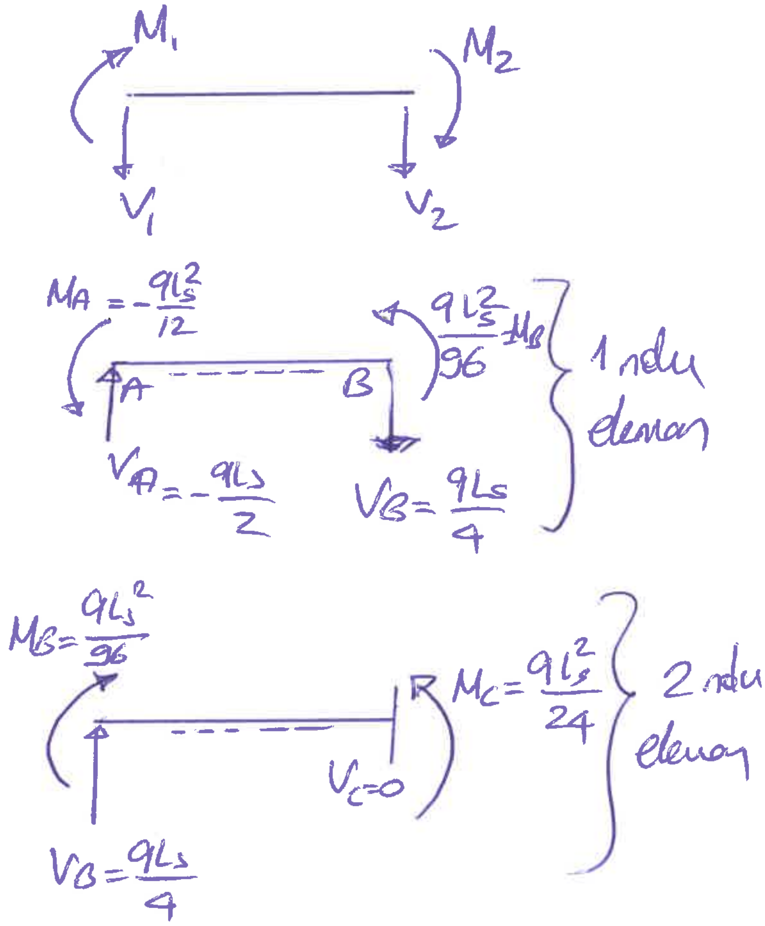
2. nolu elemanın kesit tesirleri

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}^2 = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \frac{3}{8}q\frac{L^4}{EI} \\ \frac{1}{2}q\frac{L^3}{EI} \\ \frac{2}{3}q\frac{L^4}{EI} \\ 0 \end{Bmatrix} - \begin{Bmatrix} \frac{qL}{2} \\ \frac{qL^2}{12} \\ \frac{qL}{2} \\ -\frac{qL^2}{12} \end{Bmatrix} = \begin{Bmatrix} -qL \\ qL^2/6 \\ 0 \\ -\frac{2}{3}qL^2 \end{Bmatrix}$$

$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}^2 = \begin{Bmatrix} V_B \\ M_B \\ V_C \\ M_C \end{Bmatrix} = \begin{Bmatrix} -qL_s/4 \\ qL_s^2/96 \\ 0 \\ -qL_s^2/24 \end{Bmatrix}$$

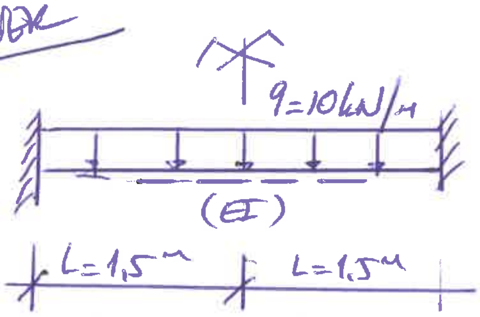
85a





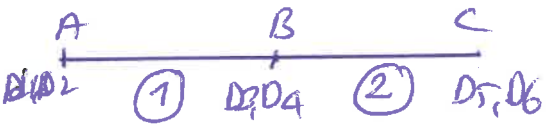
Yukarıdaki hesaplardan da anlaşılacağı gibi kesit tesirlerini varyasyon yöntemi ya da şekil fonksiyonları yardımıyla hesapladığımızda elde edilen sonuçlar yaklaşık olacaktır. Halbuki rijitlik matrisi yardımıyla bulunan kesit tesirleri kesin sonuçlardır.

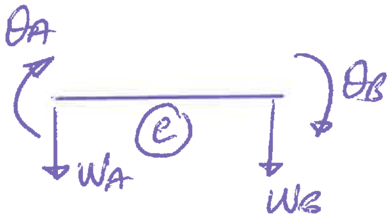
ÖRNEK



$EI = 3\times10^4\ kNm^2$

$q = 10\ kN/m$





$$[k_e] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\{Q\}^e = \begin{Bmatrix} qL/2 \\ qL^2/12 \\ qL/2 \\ -qL^2/12 \end{Bmatrix}$$

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad [C_2] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} [K_1]^e &= [C_1]^T[k_e][C_1] \\ [K_2]^e &= [C_2]^T[k_e][C_2] \end{aligned}\right\} [K]_s = [K_1]^e + [K_2]^e$$

$$\left.\begin{aligned} \{Q_1\}^e &= [C_1]^T\{Q\}^e \\ \{Q_2\}^e &= [C_2]^T\{Q\}^e \end{aligned}\right\} [Q_s] = \{Q_1\}^e + \{Q_2\}^e$$

85c

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{Bmatrix} = \begin{Bmatrix} qL/2 \\ qL^2/12 \\ qL \\ 0 \\ qL/2 \\ -qL^2/12 \end{Bmatrix}$$

$D_1 = D_2 = D_4 = D_5 = D_6 = 0$

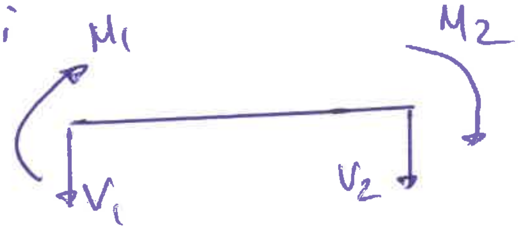
$$D_3 = w_B = \frac{qL^4}{24EI} = \frac{10 \cdot 1{,}5^4}{24 \cdot 3 \cdot 10^4} = 7{,}03 \times 10^{-5}\ \text{m.}$$

Eleman Kesit Tesirleri

1 nolu eleman

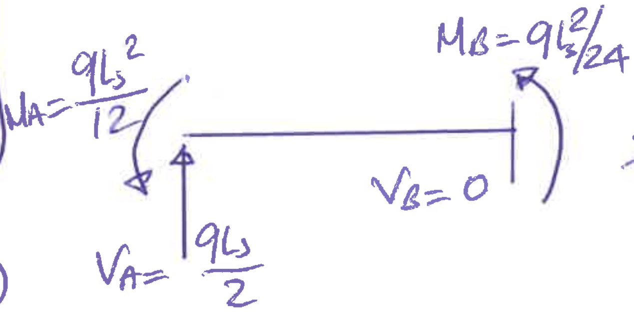
$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}^1 = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \frac{qL^4}{24EI} \\ 0 \end{Bmatrix} - \begin{Bmatrix} qL/2 \\ qL^2/12 \\ qL/2 \\ -qL^2/12 \end{Bmatrix} = \begin{Bmatrix} -qL \\ -qL^2/3 \\ 0 \\ -qL^2/6 \end{Bmatrix}$$

$L = \frac{L_s}{2}$ olduğundan;

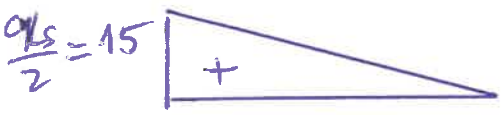


seçilen

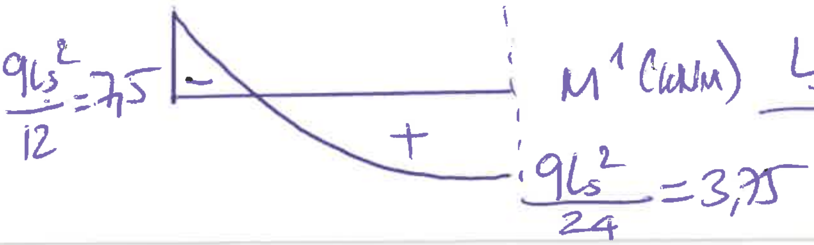
$$\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}^1 = \begin{Bmatrix} V_A \\ M_A \\ V_B \\ M_B \end{Bmatrix}^1 = \begin{Bmatrix} -\frac{qL_s}{2} \\ -\frac{qL_s^2}{12} \\ 0 \\ -\frac{qL_s^2}{24} \end{Bmatrix}$$



sonuç



V^1 (kN)



M^1 (kNm) $L_s = 3$ m

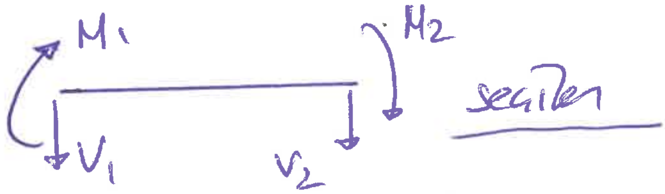
85 d

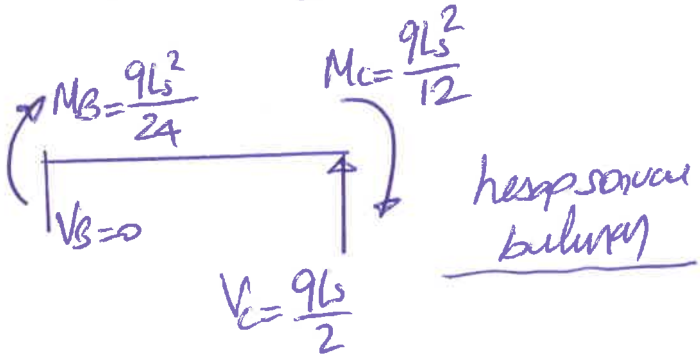
2 nolu eleman

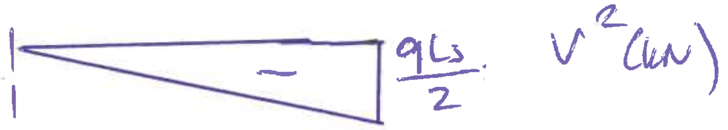
$$\begin{Bmatrix}V_1\\M_1\\V_2\\M_2\end{Bmatrix}^2 = \frac{EI}{L^3}\begin{bmatrix}12&6L&-12&6L\\6L&4L^2&-6L&2L^2\\-12&-6L&12&-6L\\6L&2L^2&-6L&4L^2\end{bmatrix}\begin{Bmatrix}qL^4/24EI\\0\\0\\0\end{Bmatrix} - \begin{Bmatrix}qL/2\\qL^2/12\\qL/2\\-qL^2/12\end{Bmatrix} = \begin{Bmatrix}0\\qL^2/6\\-qL\\qL^2/3\end{Bmatrix}$$

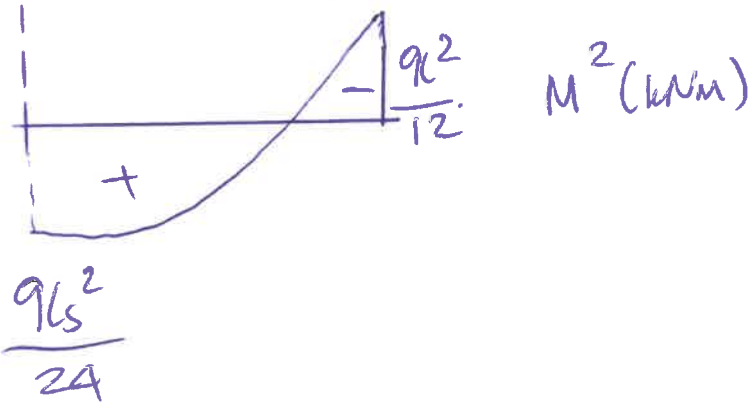
$L = \frac{L_s}{2}$ olduğundan;

$$\begin{Bmatrix}V_1\\M_1\\V_2\\M_2\end{Bmatrix} = \begin{Bmatrix}V_B\\M_B\\V_C\\M_C\end{Bmatrix} = \begin{Bmatrix}0\\qL_s^2/24\\-qL_s/2\\qL_s^2/12\end{Bmatrix}$$

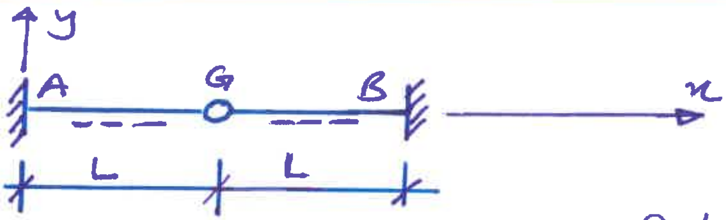


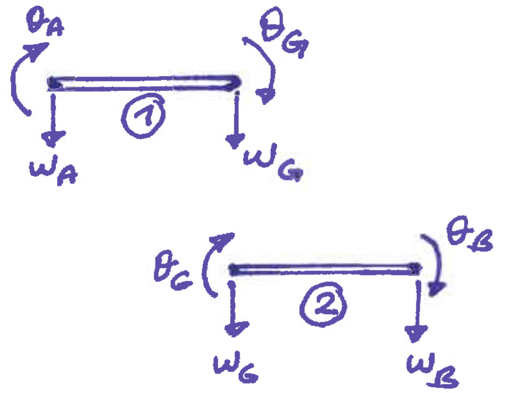






ÇUBUKTA MAFSAL OLMASI DURUMU

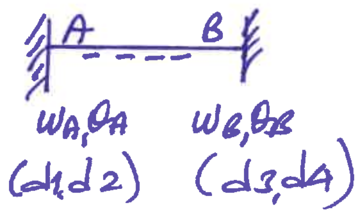




Şekilde görüldüğü gibi iki ucu rijit çubuğun orta noktasında moment mafsalı bulunmaktadır. Çubuktaki yerdeğiştirmeler şekilde verilmiştir.

a) İki ucu rijit çubuğun rijitlik matrisi

$$[k]=\frac{EI}{L^3}\begin{array}{c} \begin{array}{cccc} w_A & \theta_A & w_B & \theta_B \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} w_A \\ \theta_A \\ w_B \\ \theta_B \end{array}$$



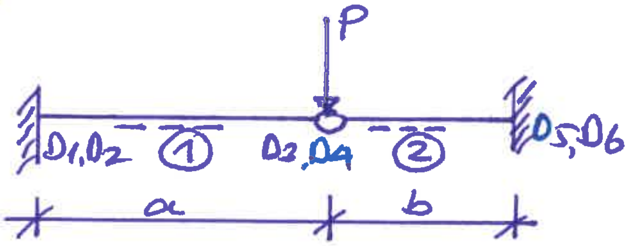
b) Sağ ucu mafsallı çubuğun rijitlik matrisi

$$[k]=\frac{EI}{L^3}\begin{array}{c} \begin{array}{cccc} w_A & \theta_A & w_B & \theta_B \end{array} \\ \left[\begin{array}{ccc|c} 3 & 3L & -3 & 0 \\ 3L & 3L^2 & -3L & 0 \\ -3 & -3L & 3 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} w_A \\ \theta_A \\ w_B \\ \theta_B \end{array}$$

c) Sol ucu mafsallı çubuğun rijitlik matrisi

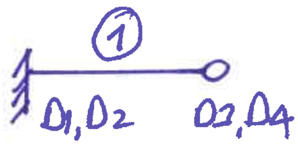
$$[k]=\frac{EI}{L^3}\begin{array}{c} \begin{array}{cccc} w_A & \theta_A & w_B & \theta_B \end{array} \\ \left[\begin{array}{c|c|cc} 3 & 0 & -3 & 3L \\ \hline 0 & 0 & 0 & 0 \\ \hline -3 & 0 & 3 & -3L \\ 3L & 0 & -3L & 3L^2 \end{array}\right] \end{array} \begin{array}{c} w_A \\ \theta_A \\ w_B \\ \theta_B \end{array}$$

4 ÖRNEK

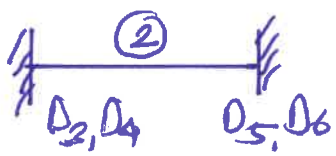


Mafsalı 1 nolu çubuğun sağında şeklinde hesaplayacağız.

$$[k_1]_e = \frac{EI}{a^3} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} 3 & 3a & -3 & 0 \\ 3a & 3a^2 & -3a & 0 \\ -3 & -3a & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array}$$



$$[k_2]_e = \frac{EI}{b^3} \begin{array}{c} \begin{array}{cccc} D_3 & D_4 & D_5 & D_6 \end{array} \\ \begin{bmatrix} 12 & 6b & -12 & 6b \\ 6b & 4b^2 & -6b & 2b^2 \\ -12 & -6b & 12 & -6b \\ 6b & 2b^2 & -6b & 4b^2 \end{bmatrix} \end{array} \begin{array}{c} \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{array}$$



Çevirme Matrisleri

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad [C_2] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K_1]^e = [C_1]^T [k_1]_e [C_1]$$

$$[K_2]^e = [C_2]^T [k_2]_e [C_2]$$

$$[K_s] = [K_1]^e + [K_2]^e$$

$$\{Q\} = \begin{Bmatrix} 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$D_1 = D_2 = D_5 = D_6 = 0$$

$$EI\begin{bmatrix} 3/a^3 & 3/a^2 & -3/a^3 & 0 & 0 & 0 \\ 3/a^2 & 3/a & -3/a^2 & 0 & 0 & 0 \\ -3/a^3 & -3/a^2 & 3/a^3+12/b^3 & 6/b^2 & -12/b^3 & 6/b^2 \\ 0 & 0 & 6/b^2 & 4/b & -6/b^2 & 2/b \\ 0 & 0 & -12/b^3 & -6/b^2 & 12/b^3 & -6/b^2 \\ 0 & 0 & 6/b^2 & 2/b & -6/b^2 & 4/b \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

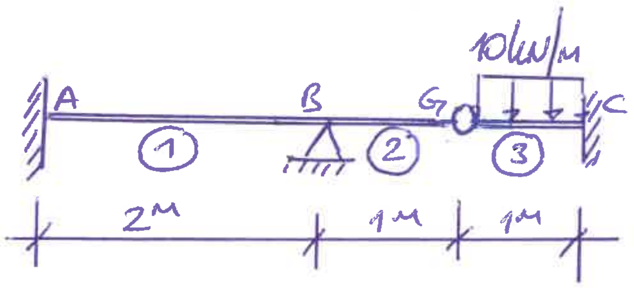
$$EI\begin{bmatrix} 3/a^3+12/b^3 & 6/b^2 \\ 6/b^2 & 4/b \end{bmatrix} \begin{Bmatrix} D_3 \\ D_4 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

$$D_3 = w_3 = \frac{-a^3 b^3 P}{3(b^3+a^3)EI} \qquad D_4 = \theta_3 = \frac{a^3 b^2 P}{2(b^3+a^3)EI}$$

Ödev: Çubuk kuvvetlerini (kesit tesirleri) bularak M, V diyagramlarını çiziniz.

ÚLOHA 4

ÚLOHA

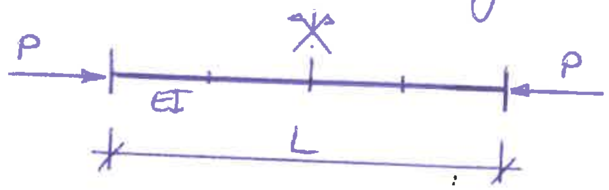


$E = 210\ GPa$, $I = 2 \cdot 10^{-4}\ m^4$

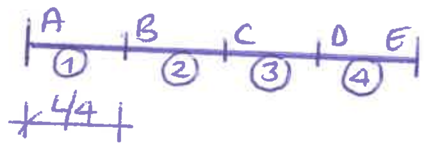
85j

9.2) Çubuğun Burkulma Hesabı

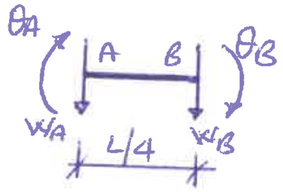
Eksenel P basınç kuvvetine maruz L uzunluğunda bir çubuğu ele alalım. Ve çubuğu 4 eşit parçaya bölelim;



← Ele alınan çubuk eleman



← Çubuğun sonlu elemanlara ayrılması



← Sonlu eleman parçası ve yer değiştirmeler

$$\{d\}^T = [w_A, \theta_A, w_B, \theta_B] = [d_1, d_2, d_3, d_4]$$ ← yer değiştirmeler

$A(0,0)$; $B(L,0)$ ← Eleman koordinatları

9.2.1) Deplasman Fonksiyonlarının Seçimi

Deplasman fonksiyonları rijit cisim hareketi ve sabit deformasyon şartını sağlayacak şekilde seçilmelidir. Koordinat ekseni değişince çözüm farklı olmamalıdır. Bunun için deplasman fonksiyonu ya tam polinom veya tabii koordinatların fonksiyonu şeklinde olmalıdır. Elemanın içinde ve kenarlarında sürekli olmalıdır. Ayrıca iç ve dış kuvvetlerin içindeki türevlerde de sürekli olmalıdır. Burada deplasman fonksiyonu polinom olarak seçilmiştir. Elemanda bilinmeyen dört deplasman parametresi olduğundan, dört bilinmeyenli deplasman fonksiyonu seçilmiştir.

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$w = [\phi(x)]\{a\} = [1, x, x^2, x^3]\{a\}$$

9.2.2) Şekil Fonksiyonlarının Tayini

Eleman düğüm noktası deplasmanları $\{d\}$ ile polinom sabitleri arasındaki bağı veren $[A]$ matrisi ise; elemanın düğüm noktası deplasman parametrelerinin $\phi(x)$ ve $\phi(x)$'in türevleri cinsinden yazılmış bileşenlerinde, düğüm noktası koordinatlarının yerlerine konulmasıyla bulunur.

$$\{d\} = [A]\{a\}$$

$$\{a\} = [A]^{-1}\{d\} \qquad [A]^{-1} = [B] \text{ ise;}$$

$$\{a\} = [B]\{d\}$$

$$w = \underbrace{[\phi(x)][B]}_{[N]}\{d\} = [N]\{d\}$$

$[N]$: şekil fonksiyonu

$$w = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$w' = \theta = a_2 + 2a_3 x + 3a_4 x^2$$

$$\{d\} = [A]\{a\}$$

$$d_1 = (w_A) = w(x=0) = a_1$$

$$d_2 = \theta_A = w'(x=0) = a_2$$

$$d_3 = w_B = w(x=L/4) = a_1 + a_2\frac{L}{4} + a_3\frac{L^2}{16} + a_4\frac{L^3}{64}$$

$$d_4 = \theta_B = w'(x=L/4) = a_2 + 2a_3\frac{L}{4} + 3a_4\frac{L^2}{16}$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & L/4 & L^2/16 & L^3/64 \\ 0 & 1 & L/2 & 3L^2/16 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$

$$[B] = [A]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix}$$

$$[N]=[\phi(x)][B]=[1, x, x^2, x^3][B]$$

$$[N_1, N_2, N_3, N_4]=[1, x, x^2, x^3]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix}$$

$$N_1=1-\frac{48x^2}{L^2}+\frac{128x^3}{L^3}$$

$$N_2=x-\frac{8x^2}{L}+\frac{16x^3}{L^2}$$

$$N_3=\frac{48x^2}{L^2}-\frac{128x^3}{L^3}$$

$$N_4=-\frac{4x^2}{L}+\frac{16}{L^2}x^3$$

9.2.3) Eleman Rijitlik Matrisinin Hesabı

Uçlarından eksenel P basınç yüküne maruz bir çubuğun toplam potansiyel enerjisinin 2'inci varyasyonunu yazarsak;

$$\delta^2\Pi^e=\int_0^{L/4}[EI(w'')^2-P(w')^2]dx$$

$$\delta^2\Pi^e=\int_0^{L/4}\{\varepsilon\}^T[D]\{\varepsilon\}dx+\int_0^{L/4}\{\beta\}^T[N_0]\{\beta\}dx$$

$$\delta^2\Pi^e=\{d\}^T[ke]\{d\}-P\{d\}^T[ne]\{d\}$$

$$\{d\}^T[ke]\{d\}=EI\int_0^{L/4}(w'')^2dx$$

$$\lambda\{d\}^T[ne]\{d\}=-P\int_0^{L/4}(w')^2dx$$

$$\delta^2\Pi^e=\{d\}^T([ke]+\lambda[ne])\{d\}$$

Burada $N_0 = -P$, $[ke]$ eleman rijitlik matrisi, λ yük parametresi, $[ne]$ geometrik yük matrisidir. Eleman rijitlik ve geometrik yük matrislerini integral matrisleri ile hesaplarsak;

$$w'' = -2a_3 - 6a_4 x$$

$$\{\varepsilon\} = [F]\{a\} \quad ; \quad [F] = [0, 0, -2, -6x]$$

$$\{\varepsilon\} = [F][B]\{d\}$$

$$[ke] = EI \int_0^{L/4} [B]^T [F]^T [F][B]\, dx$$

$$[ke] = EI \int_0^{L/4} \begin{bmatrix} 1 & 0 & -\frac{48}{L^2} & \frac{128}{L^3} \\ 0 & 1 & -\frac{8}{L} & \frac{16}{L^2} \\ 0 & 0 & \frac{48}{L^2} & -\frac{128}{L^3} \\ 0 & 0 & -\frac{4}{L} & \frac{16}{L^2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -2 \\ -6x \end{bmatrix} \begin{bmatrix} 0 & 0 & -2 & -6x \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix} dx$$

$$[ke] = EI \int_0^{L/4} \begin{bmatrix} 0 & 0 & -\frac{192}{L^2} + \frac{1536x}{L^3} & -\frac{576x}{L^2} + \frac{4608x^2}{L^3} \\ 0 & 0 & -\frac{32}{L} + \frac{192x}{L^2} & -\frac{96x}{L} + \frac{576x^2}{L^2} \\ 0 & 0 & \frac{192}{L^2} - \frac{1536x}{L^3} & \frac{576x^2}{L^2} - \frac{4608x^2}{L^3} \\ 0 & 0 & -\frac{16}{L} + \frac{192x}{L^2} & -\frac{48x}{L} + \frac{576x^2}{L^2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix} dx$$

$$[k_e]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}$$

$$k_{11}=EI\int_0^{L/4}\left[-\frac{48}{L^2}\left(-\frac{192}{L^2}+\frac{1536x}{L^3}\right)+\frac{128}{L^3}\left(-\frac{576x}{L^2}+\frac{4608x^2}{L^3}\right)\right]dx$$

$$=EI\left[-\frac{48}{L^2}\left|-\frac{192x}{L^2}+\frac{1536x^2}{2L^3}\right|_0^{L/4}+\frac{128}{L^3}\left|-\frac{576x^2}{2L^2}+\frac{4608x^3}{3L^3}\right|_0^{L/4}\right]$$

$$=\frac{768EI}{L^3}$$

$$[k_e]=\frac{EI}{L^3}\begin{bmatrix} 768 & 96L & -768 & 96L \\ 96L & 16L^2 & -96L & 8L^2 \\ -768 & -96L & 768 & -96L \\ 96L & 8L^2 & -96L & 16L^2 \end{bmatrix}$$

9.2.4) Eleman Geometrik Yük Matrisi

$w' = a_2+2a_3x+3a_4x^2$ ise ve;

$\{\beta\}=[G]\{a\} \longrightarrow [G]=[0, 1, 2x, 3x^2]$

$\{\beta\}=[G][B]\{d\}$

$[n_e]=\int_0^{L/4}[B]^T[G]^T[G][B]\,dx$ olur.

$$[n_e]=\int_0^{L/4}\begin{bmatrix}1 & 0 & -\frac{48}{L^2} & \frac{128}{L^3}\\ 0 & 1 & -\frac{8}{L} & \frac{16}{L^2}\\ 0 & 0 & \frac{48}{L^2} & -\frac{128}{L^3}\\ 0 & 0 & -\frac{4}{L} & \frac{16}{L^2}\end{bmatrix}\begin{bmatrix}0\\1\\2x\\3x^2\end{bmatrix}\begin{bmatrix}0 & 1 & 2x & 3x^2\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L}\\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2}\end{bmatrix}dx$$

$$[n_e]=\int_0^{L/4}\begin{bmatrix}0 & \left(-\frac{96x}{L^2}+\frac{384}{L^3}x^2\right) & \left(-\frac{192}{L^2}x^2+\frac{768}{L^3}x^3\right) & \left(-\frac{288}{L^2}x^3+\frac{1552}{L^3}x^4\right)\\ 0 & \left(1-\frac{16x}{L}+\frac{48}{L^2}x^2\right) & \left(2x-\frac{32}{L^2}x^2+\frac{96}{L^2}x^3\right) & \left(3x^2-\frac{48}{L}x^3+\frac{144x^4}{L^2}\right)\\ 0 & \left(\frac{96x}{L^2}-\frac{384}{L^3}x^2\right) & \left(\frac{192}{L^2}x^2-\frac{768}{L^3}x^3\right) & \left(\frac{288}{L^2}x^3-\frac{1552x^4}{L^3}\right)\\ 0 & \left(-\frac{8x}{L}+\frac{48x^2}{L^2}\right) & \left(-\frac{16x^2}{L}+\frac{96x^3}{L^2}\right) & \left(-\frac{24}{L}x^3+\frac{144x^4}{L^2}\right)\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L}\\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2}\end{bmatrix}dx$$

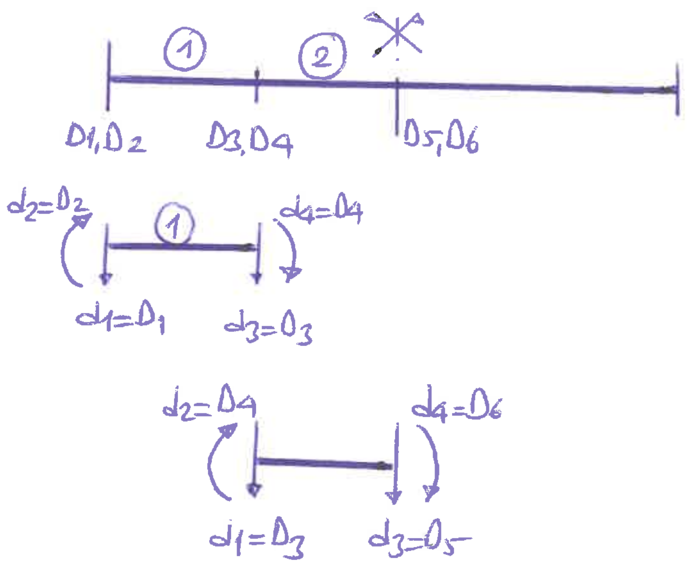
$$[n_e]=\begin{bmatrix}n_{11} & n_{12} & n_{13} & n_{14}\\ n_{21} & n_{22} & n_{23} & n_{24}\\ n_{31} & n_{32} & n_{33} & n_{34}\\ n_{41} & n_{42} & n_{43} & n_{44}\end{bmatrix}=\frac{1}{L}\begin{bmatrix}\frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10}\\ \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120}\\ -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10}\\ \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30}\end{bmatrix}$$

$$n_{11}=\int_0^{L/4}\left[-\frac{48}{L^2}\left(-\frac{192}{L^2}x^2+\frac{768}{L^3}x^3\right)+\frac{128}{L^3}\left(-\frac{288}{L^2}x^3+\frac{1552}{L^3}x^4\right)\right]dx$$

$$=\left[-\frac{48}{L^2}\left|-\frac{192}{L^2}\frac{x^3}{3}+\frac{768}{L^3}\frac{x^4}{4}\right|_0^{L/4}+\frac{128}{L^3}\left|-\frac{288}{L^2}\frac{x^4}{4}+\frac{1552}{L^3}\frac{x^5}{5}\right|_0^{L/4}\right]$$

$$=\frac{24}{5L}$$

9.2.5) Çevirme Matrisleri



Birinci eleman için çevirme matrisi;

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{[Ce_1]} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix}$$

İkinci eleman için çevirme matrisi;

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{[Ce_2]} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix}$$

9.2.6) Sistemin Rijitlik ve Geometrik Yük Matrislerinin Hesabı

$$[Ke_1] = [Ce_1]^T [ke] [Ce_1]$$

$$[Ke_1] = \frac{EI}{L^3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 768 & 96L & -768 & 96L \\ 96L & 16L^2 & -96L & 8L^2 \\ -768 & -96L & 768 & -96L \\ 96L & 8L^2 & -96L & 16L^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[Ke_1] = \frac{EI}{L^3} \begin{bmatrix} 768 & 96L & -768 & 96L & 0 & 0 \\ 96L & 16L^2 & -96L & 8L^2 & 0 & 0 \\ -768 & -96L & 768 & -96L & 0 & 0 \\ 96L & 8L^2 & -96L & 16L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[Ke_2] = \frac{EI}{L^3} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 768 & 96L & -768 & 96L \\ 96L & 16L^2 & -96L & 8L^2 \\ -768 & -96L & 768 & -96L \\ 96L & 8L^2 & -96L & 16L^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[Ne_1] = [Ce_1]^T [n_e] [Ce_1]$$

$$[Ne_1] = \frac{1}{L} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10} \\ \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} \\ -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10} \\ \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[Ne_2] = [Ce_2]^T [n_e] [Ce_2]$$

$$[Ne_2] = \frac{1}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10} \\ \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} \\ -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10} \\ \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[Ne_1] = \frac{1}{L} \begin{bmatrix} \frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10} & 0 & 0 \\ \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} & 0 & 0 \\ -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10} & 0 & 0 \\ \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad [Ne_2] = \frac{1}{L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10} \\ 0 & 0 & \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} \\ 0 & 0 & -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10} \\ 0 & 0 & \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30} \end{bmatrix}$$

Sistem rijitlik matrisi $[K]=[k_{e1}]+[k_{e2}]$

$$[K]=\frac{EI}{L^3}\begin{bmatrix} 768 & 96L & -768 & 96L & 0 & 0 \\ 96L & 16L^2 & -96L & 8L^2 & 0 & 0 \\ -768 & -96L & 768 & -96L & 0 & 0 \\ 96L & 8L^2 & -96L & 16L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{EI}{L^3}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 768 & 96L & -768 & 96L \\ 0 & 0 & 96L & 16L^2 & -96L & 8L^2 \\ 0 & 0 & -768 & -96L & 768 & -96L \\ 0 & 0 & 96L & 8L^2 & -96L & 16L^2 \end{bmatrix}$$

$$[K]=\frac{EI}{L^3}\begin{bmatrix} 768 & 96L & -768 & 96L & 0 & 0 \\ 96L & 16L^2 & -96L & 8L^2 & 0 & 0 \\ -768 & -96L & 1536 & 0 & -768 & 96L \\ 96L & 8L^2 & 0 & 32L^2 & -96L & 8L^2 \\ 0 & 0 & -768 & -96L & 768 & -96L \\ 0 & 0 & 96L & 8L^2 & -96L & 16L^2 \end{bmatrix}$$

Sistem geometrik yük matrisi; $[N]=[N_{e1}]+[N_{e2}]$

$$[N]=\frac{1}{L}\begin{bmatrix} \frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10} & 0 & 0 \\ \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} & 0 & 0 \\ -\frac{24}{5} & -\frac{L}{10} & \frac{48}{5} & 0 & -\frac{24}{5} & \frac{L}{10} \\ \frac{L}{10} & -\frac{L^2}{120} & 0 & \frac{L^2}{15} & -\frac{L}{10} & -\frac{L^2}{120} \\ 0 & 0 & -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10} \\ 0 & 0 & \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30} \end{bmatrix}$$

$$\frac{\partial(\delta^2\Pi)}{\partial D_s}=0 \longrightarrow \left([K]-P[N]\right)\{D_s\}=0$$

$$\frac{EI}{L^3}\begin{bmatrix} 768 & 96L & -768 & 96L & 0 & 0 \\ 96L & 16L^2 & -96L & 8L^2 & 0 & 0 \\ -768 & 96L & 1536 & 0 & -768 & 96L \\ 96L & 8L^2 & 0 & 32L^2 & -96L & 8L^2 \\ 0 & 0 & -768 & -96L & 768 & -96L \\ 0 & 0 & 96L & 8L^2 & -96L & 16L^2 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} - P\frac{1}{L}\begin{bmatrix} \frac{24}{5} & \frac{L}{10} & -\frac{24}{5} & \frac{L}{10} & 0 & 0 \\ \frac{L}{10} & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} & 0 & 0 \\ -\frac{24}{5} & -\frac{L}{10} & \frac{48}{5} & 0 & -\frac{24}{5} & \frac{L}{10} \\ \frac{L}{10} & -\frac{L^2}{120} & 0 & \frac{L^2}{15} & -\frac{L}{10} & -\frac{L^2}{120} \\ 0 & 0 & -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & -\frac{L}{10} \\ 0 & 0 & \frac{L}{10} & -\frac{L^2}{120} & -\frac{L}{10} & \frac{L^2}{30} \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} =$$

Sınır Şartları

$x=0 \longrightarrow \left.\begin{array}{l} w=0;\ D_1=0 \\ w'=0;\ D_2=0 \end{array}\right\}$ Ankastre mesnet

$x=\frac{L}{2} \longrightarrow w'=0;\ D_6=0 \}$ simetriden

$$\frac{EI}{L^3}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1536 & 0 & -768 & 0 \\ 0 & 0 & 0 & 32L^2 & -96L & 0 \\ 0 & 0 & -768 & -96L & 768 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ D_3 \\ D_4 \\ D_5 \\ 0 \end{bmatrix} - P\frac{1}{L}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{48}{5} & 0 & -\frac{24}{5} & 0 \\ 0 & 0 & 0 & \frac{L^2}{15} & -\frac{L}{10} & 0 \\ 0 & 0 & -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ D_3 \\ D_4 \\ D_5 \\ 0 \end{bmatrix} = 0$$

$$\frac{EI}{L^3}\left(1536D_3-768D_5\right)-P\frac{1}{L}\left(\frac{48}{5}D_3-\frac{24}{5}D_5\right)=0$$

$$\frac{EI}{L^3}\left(32L^2D_4-96L\,D_5\right)-P\frac{1}{L}\left(\frac{L^2}{15}D_4-\frac{L}{10}D_5\right)=0$$

$$\frac{EI}{L^3}\left(-768D_3-96L\,D_4+768D_5\right)-P\frac{1}{L}\left(-\frac{24}{5}D_3-\frac{L}{10}D_4+\frac{24}{5}D_5\right)=0$$

$$\left(\frac{EI}{L^3}1536-\frac{P}{L}\frac{48}{5}\right)D_3-\left(768\frac{EI}{L^3}-\frac{24}{5}\frac{P}{L}\right)D_5=0$$

$$\left(32L^2\frac{EI}{L^3}-\frac{P}{L}\frac{L^2}{15}\right)D_4-\left(96L\frac{EI}{L^3}-\frac{L}{10}\frac{P}{L}\right)D_5=0$$

$$-\left(768\frac{EI}{L^3}-\frac{P}{L}\frac{24}{5}\right)D_3-\left(96L\frac{EI}{L^3}-\frac{L}{10}\frac{P}{L}\right)D_4+\left(768\frac{EI}{L^3}-\frac{24}{5}\frac{P}{L}\right)D_5$$

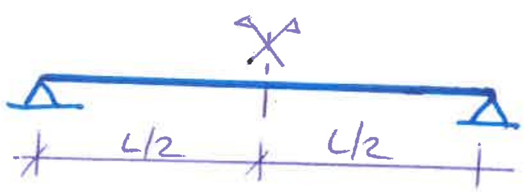
Sistemin ilk konumunun kararsız hale gelmesi ve $\{D_s\}=0$'dan farklı kararlı bir başka denge konumunun bulunması için yukarıdaki denklem takımının katsayılar determinantının sıfır olması gerekir. (Δ) determinantını sıfır yapan yük kritik yüktür.

$$\Delta=\begin{vmatrix}\left(\frac{EI}{L^2}1536-P\frac{48}{5}\right) & 0 & -\left(768\frac{EI}{L^2}-P\frac{24}{5}\right)\\ 0 & \left(32EI-\frac{PL^2}{15}\right) & -\left(96\frac{EI}{L}-P\frac{L}{10}\right)\\ -\left(768\frac{EI}{L^2}-P\frac{24}{5}\right) & -\left(96\frac{EI}{L}-P\frac{L}{10}\right) & \left(768\frac{EI}{L^2}-P\frac{24}{5}\right)\end{vmatrix}=0$$

$$0{,}3P^2L^2-166{,}4PEI+\frac{6144(EI)^2}{L^2}=0$$

$$\underset{\text{sonlu eleman}}{P_{kr}}=\frac{39{,}766EI}{L^2}\quad;\quad\underset{\text{kesin çözüm}}{P_{kr}}=\frac{4\pi^2EI}{L^2}\cong\frac{39{,}5EI}{L^2}$$

Sistemin 2 ucunun basit mesnetli olması halinde;



$x=0\rightarrow w=0$ $(D_1=0)$ $\quad x=\frac{L}{2}\rightarrow w'=0$ $(D_6=0)$

$$\frac{EI}{L^3}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16L^2 & -96L & 8L^2 & 0 & 0 \\ 0 & -96L & 1536 & 0 & -768 & 0 \\ 0 & 8L^2 & 0 & 32L^2 & -96L & 0 \\ 0 & 0 & -768 & -96L & 768 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \end{bmatrix} - \frac{P}{L}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{L^2}{30} & -\frac{L}{10} & -\frac{L^2}{120} & 0 & 0 \\ 0 & -\frac{L}{10} & \frac{48}{5} & 0 & -\frac{24}{5} & 0 \\ 0 & -\frac{L^2}{120} & 0 & \frac{L^2}{15} & -\frac{L}{10} & 0 \\ 0 & 0 & -\frac{24}{5} & -\frac{L}{10} & \frac{24}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ 0 \end{bmatrix} = 0$$

$$\left(16L^2\frac{EI}{L^2} - \frac{PL^2}{30}\right)D_2 - \left(96L\frac{EI}{L^2} - P\frac{L}{10}\right)D_3 + \left(8L^2\frac{EI}{L^2} + P\frac{L^2}{120}\right)D_4 = 0$$

$$-\left(96L\frac{EI}{L^2} - \frac{PL}{10}\right)D_2 + \left(1536\frac{EI}{L^2} - P\frac{48}{5}\right)D_3 - \left(768\frac{EI}{L^2} - P\frac{24}{5}\right)D_5 = 0$$

$$\left(8L^2\frac{EI}{L^2} + \frac{PL^2}{120}\right)D_2 + \left(32L^2\frac{EI}{L^2} - P\frac{L^2}{15}\right)D_4 - \left(96L\frac{EI}{L^2} - P\frac{L}{10}\right)D_5 = 0$$

$$-\left(768\frac{EI}{L^2} - \frac{P\,24}{5}\right)D_3 - \left(96L\frac{EI}{L^2} - P\frac{L}{10}\right)D_4 + \left(768\frac{EI}{L^2} - P\frac{24}{5}\right)D_5 = 0$$

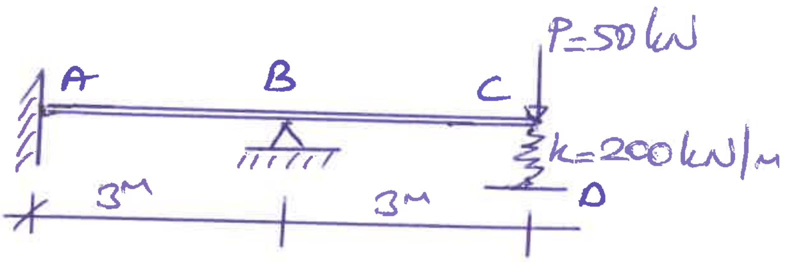
$$\begin{vmatrix} \left(16EI - \frac{PL^2}{30}\right) & -\left(96\frac{EI}{L} - P\frac{L}{10}\right) & \left(8EI + P\frac{L^2}{120}\right) & 0 \\ -\left(96\frac{EI}{L} - \frac{PL}{10}\right) & \left(1536\frac{EI}{L^2} - P\frac{48}{5}\right) & 0 & -\left(768\frac{EI}{L^2} - P\frac{24}{5}\right) \\ \left(8EI + \frac{PL^2}{120}\right) & 0 & \left(32EI - P\frac{L^2}{15}\right) & -\left(96\frac{EI}{L^2} - P\frac{L}{10}\right) \\ 0 & -\left(768\frac{EI}{L^2} - \frac{P24}{5}\right) & -\left(96\frac{EI}{L} - P\frac{L}{10}\right) & \left(768\frac{EI}{L^2} - P\frac{24}{5}\right) \end{vmatrix} = 0$$

$$\underset{\text{sonlu eleman}}{P_{kr}} = \frac{9{,}87\,EI}{L^2} \quad ; \quad \underset{\text{kesin çözüm}}{P_{kr}} = \frac{\pi^2 EI}{L^2} \cong \frac{9{,}86\,EI}{L^2}$$

ÇUBUKTA ELASTİK YAY OLMASI DURUMUNDA

Çubukta elastik yay olması durumu

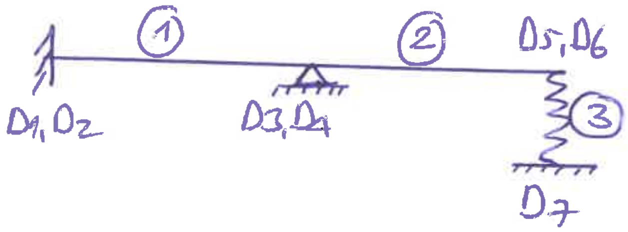
ÖRNEK



$E = 210 GPa = 21 \cdot 10^{7}\ kN/m^2$

$I = 2 \cdot 10^{-4}\ m^4$

$k = 200\ kN/m$



$$[k_1] = [k_2] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \qquad [k_3] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad [C_3] = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[C_2] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$[Ke_1] = [C_1]^T [k_1] [C_1]$$

$$[Ke_2] = [C_2]^T [k_2] [C_2]$$

$$[Ke_3] = [C_3]^T [k_3] [C_3]$$

$$[K]_s = [Ke_1] + [Ke_2] + [Ke_3]$$

E2

$$
\begin{array}{ccccccc}
D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 \\
(\upsilon_1) & (\varphi_1) & (\upsilon_2) & (\varphi_2) & (\upsilon_3) & (\varphi_3) & (\upsilon_4)
\end{array}
$$

$$
\begin{bmatrix}
\frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 & 0 \\
\frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & 0 & 0 \\
-\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{24EI}{L^3} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\
\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 \\
0 & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3}+k & -\frac{6EI}{L^2} & -k \\
0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\
0 & 0 & 0 & 0 & -k & 0 & k
\end{bmatrix}
\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -P \\ 0 \\ 0 \end{Bmatrix}
$$

$$
\begin{bmatrix}
\frac{8EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{6EI}{L^2} & \frac{12EI}{L^3}+k & -\frac{6EI}{L^2} \\
\frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\begin{Bmatrix} D_4\,(\varphi_2) \\ D_5\,(\upsilon_3) \\ D_6\,(\varphi_3) \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ -P \\ 0 \end{Bmatrix}
$$

$$
\frac{EI}{L^3}
\begin{bmatrix}
8L^2 & -6L & 2L^2 \\
-6L & 12+k' & -6L \\
2L^2 & -6L & 4L^2
\end{bmatrix}
\begin{Bmatrix} D_4 \\ D_5 \\ D_6 \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ -P \\ 0 \end{Bmatrix}
\qquad k' = \frac{kL^3}{EI}
$$

$D_4\,(\varphi_2) = -\dfrac{3PL^2}{EI}\left(\dfrac{1}{12+7k'}\right)$; $D_5\,(\upsilon_3) = -\dfrac{7PL^3}{EI}\left(\dfrac{1}{12+7k'}\right)$

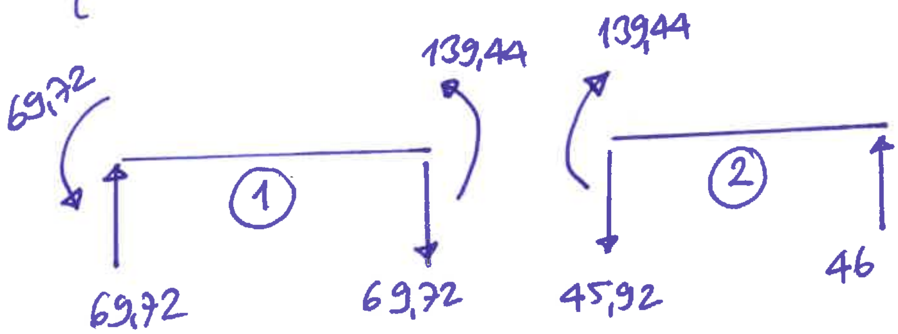
$D_6\,(\varphi_3) = -\dfrac{9PL^2}{EI}\left(\dfrac{1}{12+7k'}\right)$ $D_4 = -0{,}00249$ rad.; $D_5 = -0{,}0174$ m; $D_6 = -0{,}00247$ rad.

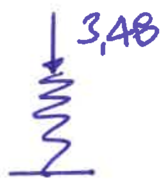
E3

$$\begin{Bmatrix} V_A \\ M_A \\ V_{B_1} \\ M_{B_1} \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0{,}00249 \end{Bmatrix} \cong \begin{Bmatrix} -69{,}72 \\ -69{,}72 \\ 69{,}72 \\ -139{,}44 \end{Bmatrix}$$

$$\begin{Bmatrix} V_{B2} \\ M_{B2} \\ V_c \\ M_c \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ -0{,}00249 \\ -0{,}0174 \\ -0{,}00747 \end{Bmatrix} \cong \begin{Bmatrix} 45{,}92 \\ 13944 \\ -46 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} V_{CD} \\ V_{DC} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} -0{,}0174 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -3{,}48 \\ 3{,}48 \end{Bmatrix}$$





EK:1A

KAFES SİSTEMLERİN SONLU ELEMANLAR YÖNTEMİ İLE ÇÖZÜMÜ

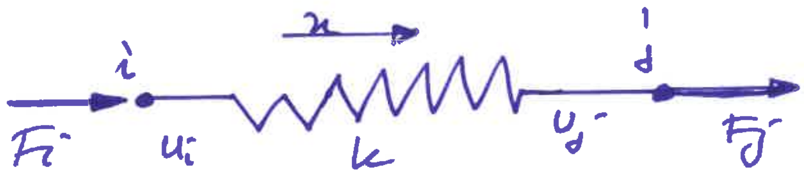
Kafes Sistemlerin Sonlu Elemanlar Yöntemi İle Hesabı

1- Giriş

Kafes sistemlerin çözümünde yay elemanlarından faydalanmak sonlu elemanlarda basitliği yönünden tercih edilmektedir. Eleman ve sistem rijitliğini tariflemek son derece kolay olmaktadır.

2- Yay Elemanı

Doğrusal elastik davranış gösteren yay elemanında yerdeğiştirmeler u, yay kuvveti F, yay rijitliği veya yay katsayısı k ise;



$$F = k(u_j - u_i) = k \cdot \Delta u$$

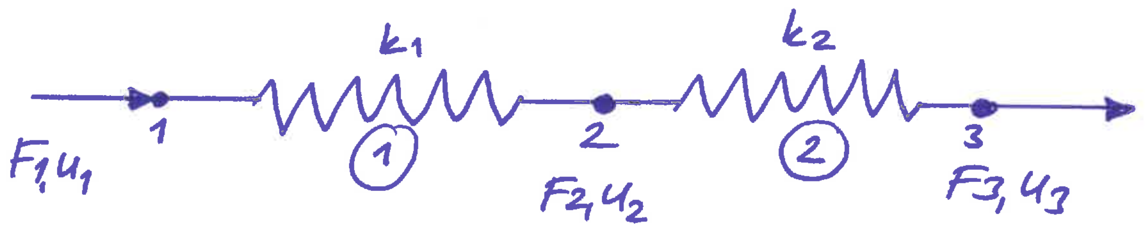
$$F_i + F_j = 0 \longrightarrow F_i = -F_j$$

$$\left.\begin{aligned} F_i + k(u_j - u_i) &= 0 \\ F_j - k(u_j - u_i) &= 0 \end{aligned}\right\} \quad \begin{aligned} F_i &= ku_i - ku_j \\ F_j &= -ku_i + ku_j \end{aligned}$$

veya $[k]\{u\} = \{F\}$ 'dir.

$$\begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$ olur.

İkili yay durumunda ise;



$\{u^1\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$ $\quad \{u^2\} = \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$ elemanda yerdeğiştirmeler

$\{F\}^1 = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$ $\quad \{F\}^2 = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$ Elemanda kuvvetler

$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} ; \quad \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

Genel matrisleri ise;

$$[Ce_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad Ce_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[ke]_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[ke]_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

$$[K]_s = [ke_1] + [ke_2] = \begin{bmatrix} k_1 & -k_1 & 0 \\ 0 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

$$\{u\}_s = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \qquad \{F\}_s = \begin{Bmatrix} F_1 \\ F_1+F_2 \\ F_2 \end{Bmatrix}$$

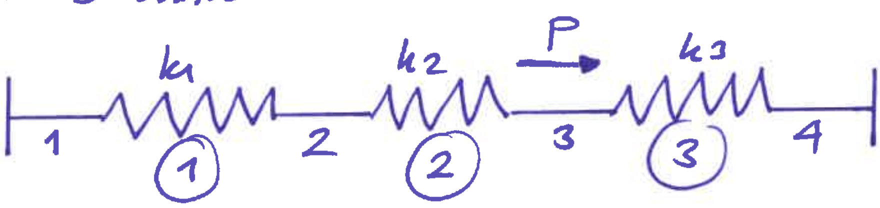
$$\{K\}_s \{u\}_s = \{F\}_s$$

ÖRNEK : 1

Şekilde verilen sistemin 2 ve 3 nolu düğüm noktalarının yerdeğiştirmelerini, mesnet tepkilerini ve yay kuvvetlerini bulunuz.

$k = 100$ kN/m

$P = 0{,}5$ kN



$$k_1 = k_3 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \qquad k_2 = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix}$$

$$[C_{e_1}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [C_{e_2}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [C_{e_3}] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[k]_s = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 100+200 & -200 & 0 \\ 0 & -200 & 200+100 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 300 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix}$$

$$\begin{bmatrix} +100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 300 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 0 \\ P \\ R_2 \end{Bmatrix} \qquad \begin{array}{l} u_1 = 0 \\ u_4 = 0 \end{array}$$

4/17

$$\begin{bmatrix} 300 & -200 \\ -200 & 300 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ P \end{Bmatrix} \longrightarrow \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0{,}002\,m \\ 0{,}003\,m \end{Bmatrix}$$

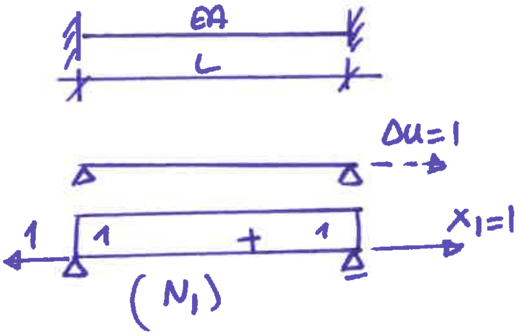
$$F_1 = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad F_2 = \begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} \quad F_3 = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$$

$$F_1 = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix}^1 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \begin{Bmatrix} 0 \\ 0{,}002 \end{Bmatrix} = \begin{Bmatrix} -0{,}2 \\ 0{,}2 \end{Bmatrix} kN$$

$$F_2 = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix}^2 = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} 0{,}002 \\ 0{,}003 \end{Bmatrix} = \begin{Bmatrix} -0{,}2 \\ 0{,}2 \end{Bmatrix} kN$$

$$F_3 = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix}^3 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \begin{Bmatrix} 0{,}003 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0{,}3 \\ -0{,}3 \end{Bmatrix} kN$$

Eksenel Kuvvet Elemanı



$$\delta_{1w} = -1.1 = -1$$

$$\delta_{11} = \frac{L.1.1}{EA}$$

$$\delta_{1w} + \delta_{11}.x_1 = 0$$

$$x_1 = \frac{EA}{L}$$

$$\boxed{F = \frac{EA}{L}}$$

$$\sigma = \frac{F}{A} \rightarrow F = \sigma.A = E.\frac{\Delta u}{L}.A = \frac{EA}{L}.\Delta u$$

$$\downarrow$$

$$\sigma = E.\varepsilon = E.\frac{\Delta u}{L}$$

şeklinde olacağı ve $k = \frac{EA}{L}$ değerinin çubuğun eksenel rijitliğini temsil ettiği dikkate alınırsa;

$$F_i = \frac{EA}{L}(u_i - u_j) = \overbrace{\frac{EA}{L}}^{k} u_i - \overbrace{\frac{EA}{L}}^{k} u_j$$

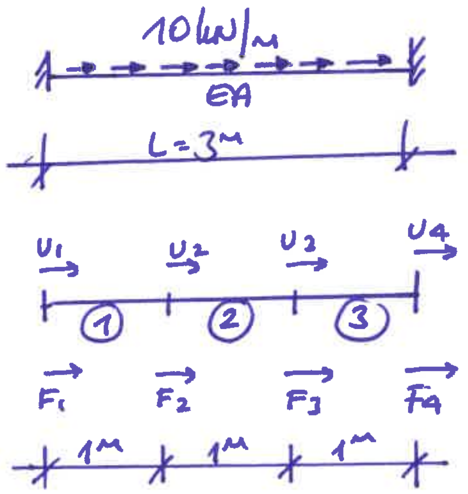
$$F_j = -\frac{EA}{L}(u_i - u_j) = -\frac{EA}{L} u_i + \frac{EA}{L} u_j$$

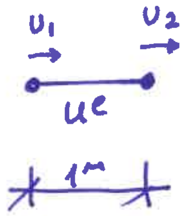
$$\begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

olur.

örnek

Şekilde görülen iki ucu ankastre normal kuvvet çubuğun mesnet tepkilerini ve çubuk kuvvetlerini hesaplayınız.





$$k_1 = k_2 = k_3 = EA\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[Ce_1] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad [Ce_2] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad [Ce_3] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K_1]^e = [Ce_1]^T [K_1][Ce_1] = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K_2]^e = [Ce_2]^T [K_2][Ce_2] = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K_3]^e = [Ce_3]^T [K_3][Ce_3] = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$[K]_s = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}$$

$u_1 = 0$

$u_4 = 0$

Yerdeğiştirmelerin Hesabı

$$\frac{EA}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$F_1 = 5\ kN$

$F_2 = 10\ kN$

$F_3 = 10\ kN$

$F_4 = 5\ kN$

$$\frac{EA}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 10 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 10/EA \\ 10/EA \end{Bmatrix}$$

Mesnet Tepkilerinin Hesabı

$$\frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}\begin{Bmatrix} 0 \\ 10/EA \\ 10/EA \\ 0 \end{Bmatrix} = \begin{Bmatrix} R_{Ax}+5 \\ 10 \\ 10 \\ R_{Bx}+5 \end{Bmatrix}$$

$$\frac{EA}{L}\left(-10/EA\right) = R_{Ax}+5$$

$$\underline{R_{Ax} = -15\ kN}$$

$$\frac{EA}{L}\left(-10/EA\right) = R_{Bx}+5$$

$$\underline{R_{Bx} = -15\ kN}$$

Çubuk kuvvetlerinin hesabı

$$F_1 = ku = F_e - F_q$$

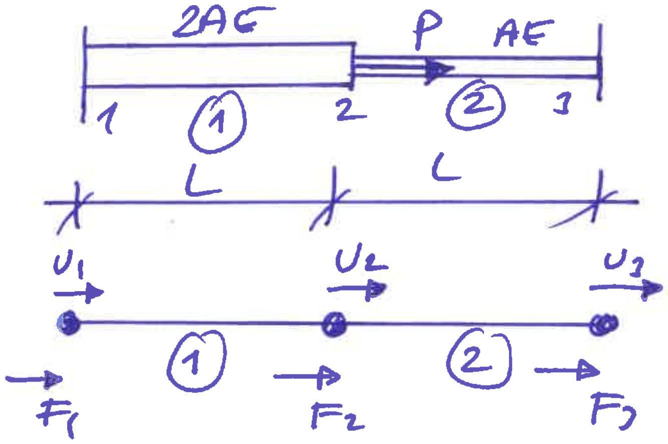
$$F_1^e = \begin{Bmatrix} N_i \\ N_j \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} - \begin{Bmatrix} 5 \\ 5 \end{Bmatrix}$$

$$= \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 10/EA \end{Bmatrix} - \begin{Bmatrix} 5 \\ 5 \end{Bmatrix} = \begin{Bmatrix} -15 \\ 5 \end{Bmatrix} kN$$

$$F_2^e = \begin{Bmatrix} N_i \\ N_j \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 10/EA \\ 10/EA \end{Bmatrix} - \begin{Bmatrix} 5 \\ 5 \end{Bmatrix} = \begin{Bmatrix} -5 \\ -5 \end{Bmatrix}$$

$$F_3^e = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 10/EA \\ 0 \end{Bmatrix} - \begin{Bmatrix} 5 \\ 5 \end{Bmatrix} = \begin{Bmatrix} 5 \\ -15 \end{Bmatrix}$$

ÖRNEK 2: Yerdeğiştirmeleri hesaplayınız.



$$k_1 = \frac{2EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[Ce_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad [Ce_2] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} [k_1]^e &= [Ce_1]^T [k_1][Ce_1] \\ [k_2]^e &= [Ce_2]^T [k_2][Ce_2] \end{aligned}\right\} [K]_s = [K_1]^e + [K_2]^e$$

$$[K_s] = \frac{EA}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

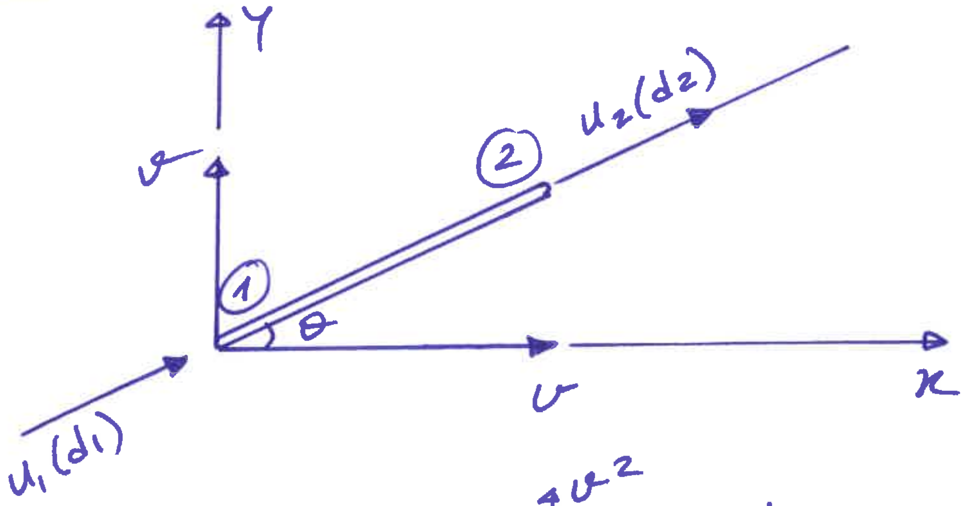
Yerdeğiştirmelerin Hesabı

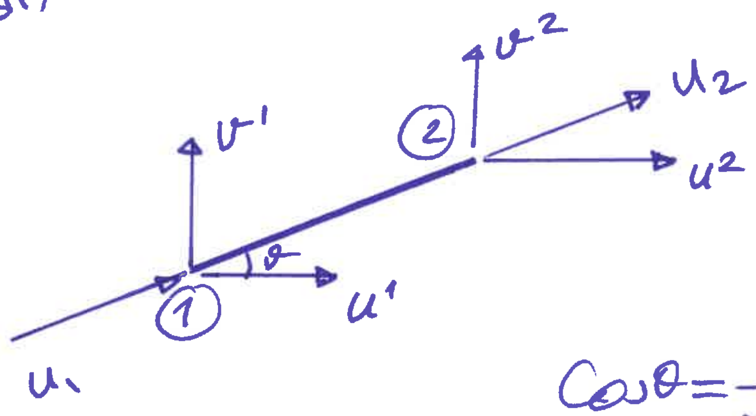
$$\frac{EA}{L}\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \qquad \begin{aligned} U_1 &= 0 \\ U_3 &= 0 \\ F_2 &= P \end{aligned}$$

$$\frac{3EA}{L} U_2 = P \rightarrow U_2 = \frac{PL}{3EA}$$

KAFES SİSTEMLER

Dönüşüm Matrisi





$$\cos\theta = \frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

$$u_1 = u^1 \cdot \cos\theta + v^1 \sin\theta$$

$$\cos\theta = \frac{x_2 - x_1}{L}$$

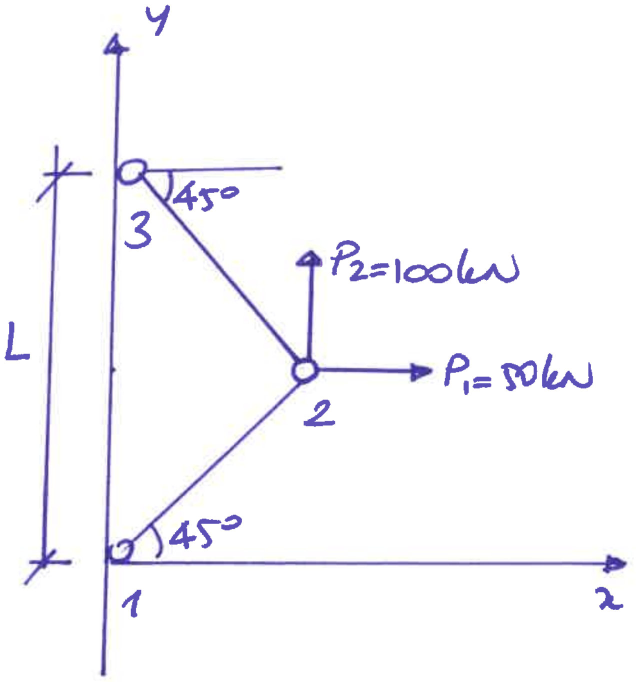
$$u_2 = u^2 \cdot \cos\theta + v^2 \sin\theta$$

$$\sin\theta = \frac{y_2 - y_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

$$[T] = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

$$\sin\theta = \frac{y_2 - y_1}{L}$$

ÖRNEK 1



$$[k_1] = [k_2] = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

DN	X	Y
1	0	0
2	L/2	L/2
3	0	L

EN	i	j	L	c	s
1	1	2	$\frac{L\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
2	2	3	$\frac{L\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

$$[T_1] = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$k_1 = [T_1]^T [k_1] [T_1] = \frac{EA}{2}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$k_2 = [T_2]^T [k_2] [T_2] = \frac{EA}{2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

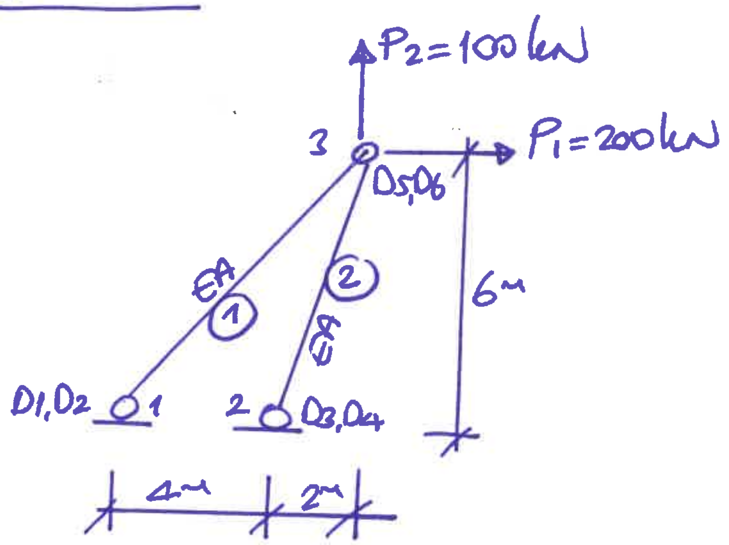
$$\frac{EA}{2L} \begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & -1 & 1 \\ -1 & -1 & 0 & 2 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix}$$

$u_1 = 0, \; v_1 = 0, \; u_3 = 0, \; v_3 = 0$

$$\frac{EA}{2L} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} F_{2x} \\ F_{2y} \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = \frac{L}{EA} \begin{Bmatrix} 500 \\ 100 \end{Bmatrix}$$

ÖRNEK : 3



$E = 200$ GPa

$A = 0{,}01\ m^2$

$L_1 = \sqrt{6^2 + 6^2} = 8{,}49\ m.$

$L_2 = \sqrt{(6-4)^2 + 6^2} = 6{,}32\ m.$

DN	X(m)	Y(m)
1	0	0
2	4	0
3	6	6

EN	i	j	L	c	s
1	1	3	8,49	0,707	0,707
2	2	3	6,32	0,316	0,949

$$c_1 = \cos\theta_1 = \frac{X_3 - X_1}{L_1} = \frac{6}{8{,}49} = 0{,}707$$

$$S_1 = \sin\theta_1 = \frac{y_3 - y_1}{L_1} = \frac{6}{8{,}49} = 0{,}707$$

$$c_2 = \cos\theta_2 = \frac{X_3 - X_2}{L_2} = \frac{2}{6{,}32} = 0{,}316$$

$$S_2 = \sin\theta_2 = \frac{y_3 - y_2}{L_2} = \frac{6}{6{,}32} = 0{,}949$$

$$[T_1] = \begin{bmatrix} 0{,}707 & 0{,}707 & 0 & 0 \\ 0 & 0 & 0{,}707 & 0{,}707 \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} 0{,}316 & 0{,}949 & 0 & 0 \\ 0 & 0 & 0{,}316 & 0{,}949 \end{bmatrix}$$

$$[Ce_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[Ce_2] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$k_1 = \frac{EA}{L_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$k_1 = \frac{(200.10^9)0{,}01}{8{,}49}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$k_1 = 2357.10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$k_2 = \frac{EA}{L_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= \frac{(200.10^9)0{,}01}{6{,}32}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$= 3162.10^5\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$[K_1] = [T_1]^T[k_1][T_1] = 2357.10^5\begin{vmatrix}0{,}50 & 0{,}50 & -0{,}50 & -0{,}50\\ 0{,}50 & 0{,}50 & -0{,}50 & -0{,}50\\ -0{,}50 & -0{,}50 & 0{,}50 & 0{,}50\\ -0{,}50 & -0{,}50 & 0{,}50 & 0{,}50\end{vmatrix}$$

$$[K_2] = [T_2]^T[k_2][T_2] = 3162.10^5\begin{vmatrix}0{,}10 & 0{,}30 & -0{,}10 & -0{,}30\\ 0{,}30 & 0{,}90 & -0{,}30 & -0{,}90\\ -0{,}10 & -0{,}30 & 0{,}10 & 0{,}30\\ -0{,}30 & -0{,}90 & 0{,}30 & 0{,}90\end{vmatrix}$$

$$[K_S]^1 = [C_{e_1}]^T[k_1][C_{e_1}] = 2357.10^5\begin{vmatrix}0{,}50 & 0{,}50 & 0 & 0 & -0{,}50 & -0{,}50\\ 0{,}50 & 0{,}50 & 0 & 0 & -0{,}50 & -0{,}50\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -0{,}50 & -0{,}50 & 0 & 0 & 0{,}50 & 0{,}50\\ -0{,}50 & -0{,}50 & 0 & 0 & 0{,}50 & 0{,}50\end{vmatrix}$$

$$[k_s]^2 = [C_{e2}]^T [k_2] [C_{e2}] = 3162 \cdot 10^5 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0{,}10 & 0{,}30 & -0{,}1 & -0{,}3 \\ 0 & 0 & 0{,}3 & 0{,}9 & -0{,}3 & -0{,}9 \\ 0 & 0 & -0{,}1 & -0{,}3 & 0{,}1 & 0{,}3 \\ 0 & 0 & -0{,}3 & -0{,}9 & 0{,}3 & 0{,}9 \end{bmatrix}$$

$$[k_s] = [k_1]^s + [k_2]^s = 10^5 \begin{bmatrix} 1178{,}5 & 1178{,}5 & 0 & 0 & -1178{,}5 & -1178{,}5 \\ 1178{,}5 & 1178{,}5 & 0 & 0 & -1178{,}5 & -1178{,}5 \\ 0 & 0 & 316{,}2 & 948{,}6 & -316{,}2 & -948{,}6 \\ 0 & 0 & 948{,}6 & 2845{,}8 & -948{,}6 & -2845{,}8 \\ -1178{,}5 & -1178{,}5 & -316{,}2 & -948{,}6 & 1494{,}7 & 2127{,}1 \\ -1178{,}5 & -1178{,}5 & -948{,}6 & -2845{,}8 & 2127{,}1 & 4024{,}3 \end{bmatrix}$$

$$[k_s]\{u\} = \{F\}$$

$$10^5 \begin{bmatrix} \ddots & & \\ & \ddots & \\ & & \ddots \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix}$$

$u_1 = 0$

$u_2 = 0$

$u_3 = 0$

$u_4 = 0$

$$10^5 \begin{bmatrix} 1494{,}7 & 2127{,}1 \\ 2127{,}1 & 4024{,}3 \end{bmatrix} \begin{Bmatrix} u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 200 \cdot 10^3 \\ 100 \cdot 10^3 \end{Bmatrix} \Rightarrow \begin{matrix} u_5 = 0{,}003573 \text{ m} \\ u_6 = -0{,}001857 \text{ m} \end{matrix}$$

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Mesnet Tepkilerinin Hesabı

$$10^5 \begin{bmatrix} \ddots & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0{,}003973 \\ -0{,}001857 \end{Bmatrix} = \begin{Bmatrix} R_{1x} \\ R_{1y} \\ R_{2x} \\ R_{2y} \\ 200 \cdot 10^3 \\ 100 \cdot 10^3 \end{Bmatrix}$$

$$10^{-3}\left[10^5(-1178{,}5 \times 0{,}003973) - 10^5(-1178{,}5 \times 0{,}001857)\right] = R_{1x}$$

$R_{1x} = -250\ \text{kN}$

$$10^{-3}\left[10^5(\text{''} \times \text{''}) - \text{''}(\text{''} \times \text{''})\right] = R_{1y}$$

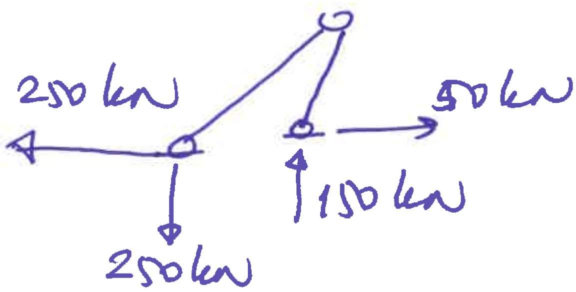
$R_{1y} = -250\ \text{kN}$

$$10^{-3}\left[10^5(-316{,}2 \times 0{,}003973) - 10^5(-948{,}6 \times 0{,}001857)\right] = R_{3x}$$

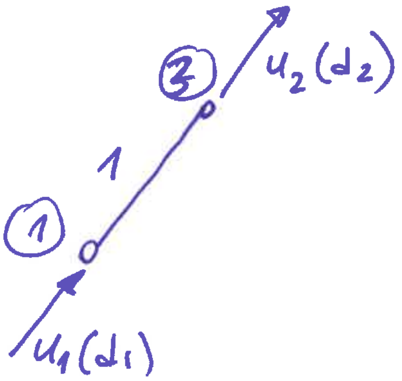
$R_{3x} = 50\ \text{kN}$

$$10^{-3}\left[10^5(-948{,}6 \times \text{''}) - 10^5(-2845{,}8 \times \text{''})\right] = R_{3y}$$

$R_{3y} = 150\ \text{kN}$



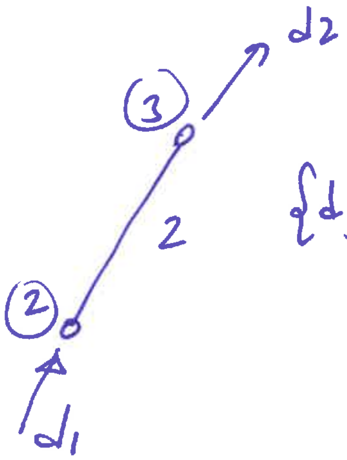
1 nolu elemanın lokal yerdeğiştirmeleri



$$\{d\}^1 = \begin{Bmatrix} d_1^1 \\ d_2^1 \end{Bmatrix} = [T_1]\{u^1\} = \begin{bmatrix} 0{,}707 & 0{,}707 & 0 & 0 \\ 0 & 0 & 0{,}707 & 0{,}707 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0{,}0035[illegible] \\ -0{,}001[illegible] \end{Bmatrix}$$

$$= \begin{Bmatrix} 0 \\ 0{,}0015\,m \end{Bmatrix}$$

2 nolu elemanın lokal yerdeğiştirmeleri



$$\{d\}^2 = \begin{Bmatrix} d_1^2 \\ d_2^2 \end{Bmatrix} = [T_2]\{u^2\} = \begin{bmatrix} 0{,}316 & 0{,}949 & 0 & 0 \\ 0 & 0 & 0{,}316 & 0{,}949 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0{,}00397[illegible] \\ -0{,}00167 \end{Bmatrix}$$

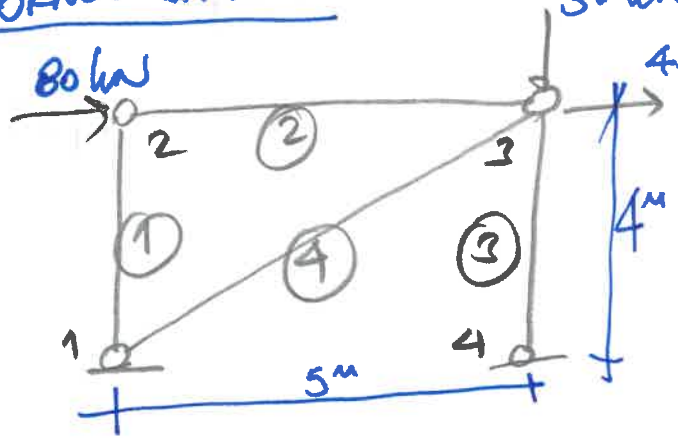
$$= \begin{Bmatrix} 0 \\ -0{,}0005\,m \end{Bmatrix}$$

Elemanlardaki Eksenel Kuvvetler

$$F^{(1)} = k_1 \cdot u^1 = 10^5 \begin{bmatrix} 2357 & -2357 \\ -2357 & 2357 \end{bmatrix} \begin{Bmatrix} 0 \\ 0{,}0015 \end{Bmatrix} = \begin{Bmatrix} -353553 \\ 353553 \end{Bmatrix} N$$

$$F^{(2)} = k_2 \cdot u^2 = 10^5 \begin{bmatrix} 3162 & -3162 \\ -3162 & 3162 \end{bmatrix} \begin{Bmatrix} 0 \\ -0{,}0005 \end{Bmatrix} = \begin{Bmatrix} 158114 \\ -158114 \end{Bmatrix} N$$

ÖRNEK KAFES



D.N	X	Y
1	0	0
2	0	4
3	5	4
4	5	0

E.N	i (L)	d (R)	L	C	S
1	1	2	4	0	1
2	2	3	5	1	0
3	3	4	4	0	-1
4	1	3	6,403	0,78	0,624

$$[k_1] = \begin{bmatrix} 2\times10^6 & -2\times10^6 \\ -2\times10^6 & 2\times10^6 \end{bmatrix} \qquad [k_2] = 10^5 \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \qquad [k_3] = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$$[k_4] = \begin{bmatrix} 624\,907 & -624\,907 \\ -\text{''} & +\text{''} \end{bmatrix}$$

$$[t_1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [t_2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad [t_3] = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$[t_4] = \begin{bmatrix} 0{,}78 & 0{,}624 & 0 & 0 \\ 0 & 0 & 0{,}78 & 0{,}624 \end{bmatrix}$$

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad [C_2] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$[C_3] = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad [C_4] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

K1/A

$$[K_s] = 10^6 \begin{bmatrix} 0{,}3809 & 0{,}3047 & 0 & 0 & -0{,}3809 & -0{,}3047 & 0 & 0 \\ 0{,}3047 & 2{,}2438 & 0 & -2 & -0{,}3047 & -0{,}2438 & 0 & 0 \\ 0 & 0 & 0{,}8 & 0 & -0{,}8 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 \\ -0{,}3809 & -0{,}3047 & -0{,}8 & 0 & 1{,}1809 & 0{,}3047 & 0 & 0 \\ -0{,}3047 & -0{,}2438 & 0 & 0 & 0{,}3047 & 2{,}2438 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{bmatrix} = \begin{bmatrix} R \\ 0 \\ 80 \\ 0 \\ 40 \\ -50 \\ 0 \\ 0 \end{bmatrix}$$

$D_3 = 0{,}4734 \times 10^{-3}$

$D_4 = 0$

$D_5 = 0{,}3734 \times 10^{-3}$

$D_6 = -0{,}0730 \cdot 10^{-3}$

K2/4

Lokal yerdeğiştirmeler

$$\{d\}^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0,4734 \cdot 10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\{d\}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 0,4734 \cdot 10^{-3} \\ 0 \\ 0,3734 \cdot 10^{-3} \\ -0,0730 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 0,4734 \cdot 10^{-3} \\ 0,3734 \cdot 10^{-3} \end{Bmatrix}$$

$$\{d\}^3 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} 0,3734 \cdot 10^{-3} \\ -0,0730 \cdot 10^{-3} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} +0,0730 \cdot 10^{-3} \\ 0 \end{Bmatrix}$$

$$\{d\}^4 = \begin{bmatrix} 0,7809 & 0,6247 & 0 & 0 \\ 0 & 0 & 0,7809 & 0,6247 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0,3734 \cdot 10^{-3} \\ -0,0730 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0,246 \cdot 10^{-3} \end{Bmatrix}$$

K3/4

Eleman kuvvetleri

$$F_1 = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix}$$

$$F_2 = 10^5 \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{Bmatrix} 0{,}4734 \cdot 10^{-3} \\ 0{,}3724 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 80 \\ -80 \end{Bmatrix}$$

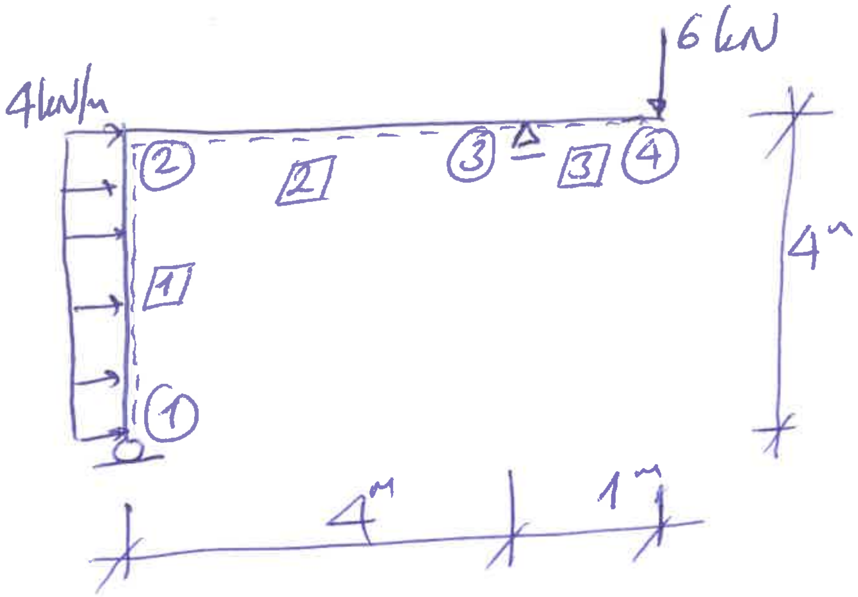
$$F_3 = 10^6 \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{Bmatrix} 0{,}0730 \cdot 10^{-3} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 146 \\ -146 \end{Bmatrix}$$

$$F_4 = \begin{bmatrix} 624707 & -624707 \\ -624707 & 624707 \end{bmatrix} \begin{Bmatrix} 0 \\ 0{,}246 \cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} -153{,}678 \\ +153{,}678 \end{Bmatrix}$$

K4/4

EK:1B

ÇERÇEVELERİN SONLU ELEMANLAR YÖNTEMİ İLE ÇÖZÜMÜ



2-4 αραm

$I = 16 \cdot 10^4\ cm^4$

$A = 120\ cm^2$

$E = 21 \cdot 10^6\ N/cm^2$

$EA_2 = EA_3 = 252 \cdot 10^{10}\ N$

$EI_2 = EI_3 = 3{,}36 \cdot 10^{12}\ Ncm$

1-2 αραm

$I = 72 \cdot 10^4\ cm^4$

$A = 2400\ cm^2$

$E = 21 \cdot 10^6\ N/cm^2$

$EA_1 = 5{,}04 \cdot 10^{10}\ N$

$EI_1 = 1{,}512 \cdot 10^{13}\ Ncm$

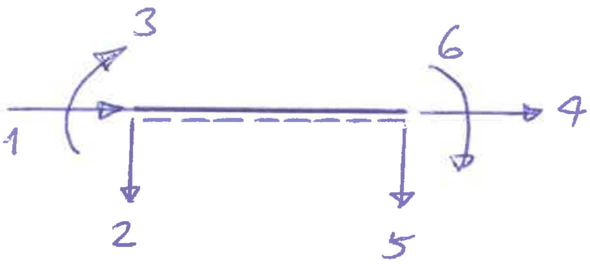
D.N	X (cm)	Y (cm)
1	0	0
2	0	400
3	400	400
4	500	400

E.N	l	r	Li (cm)	c	s
1	1	2	400	0	1
2	2	3	400	1	0
3	3	4	100	1	0

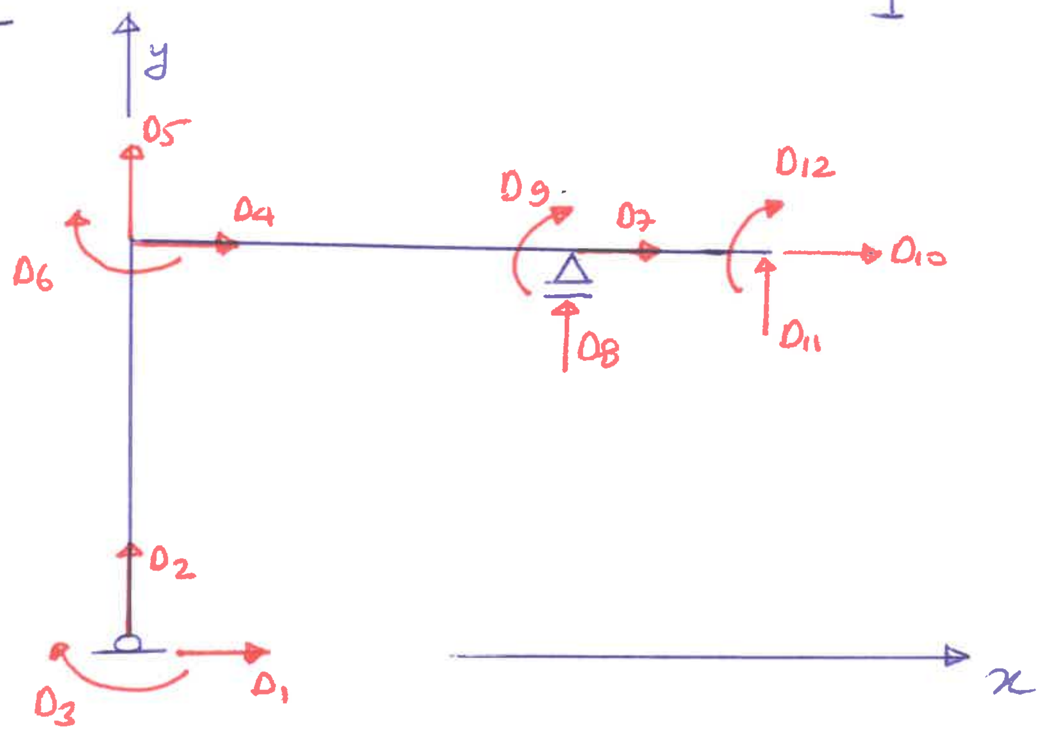
$c_1 = \cos\theta_1 = \frac{x_2 - x_1}{L_1} = \frac{0}{400} = 0 \quad ; \quad s_1 = \sin\theta_1 = \frac{y_2 - y_1}{L_1} = \frac{400}{400} = 1$

$c_2 = \cos\theta_2 = \frac{x_3 - x_2}{L_2} = \frac{400-0}{400} = 1 \quad ; \quad s_2 = \sin\theta_2 = \frac{y_3 - y_2}{L_2} = \frac{400-400}{400} = 0$

$c_3 = \cos\theta_3 = \frac{x_4 - x_3}{L_3} = \frac{500-400}{100} = 1 \quad ; \quad s_3 = \sin\theta_3 = \frac{y_4 - y_3}{L_3} = \frac{400-400}{100} = 0$



$$[k_e]=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



Sistem serbestlikleri

$$[k_1] = 10^6 \begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 2{,}835 & 567 & 0 & -2{,}835 & 567 \\ 0 & 567 & 151200 & 0 & -567 & 75600 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -2{,}835 & -567 & 0 & 2{,}835 & -567 \\ 0 & 567 & 75600 & 0 & -567 & 151200 \end{bmatrix}$$

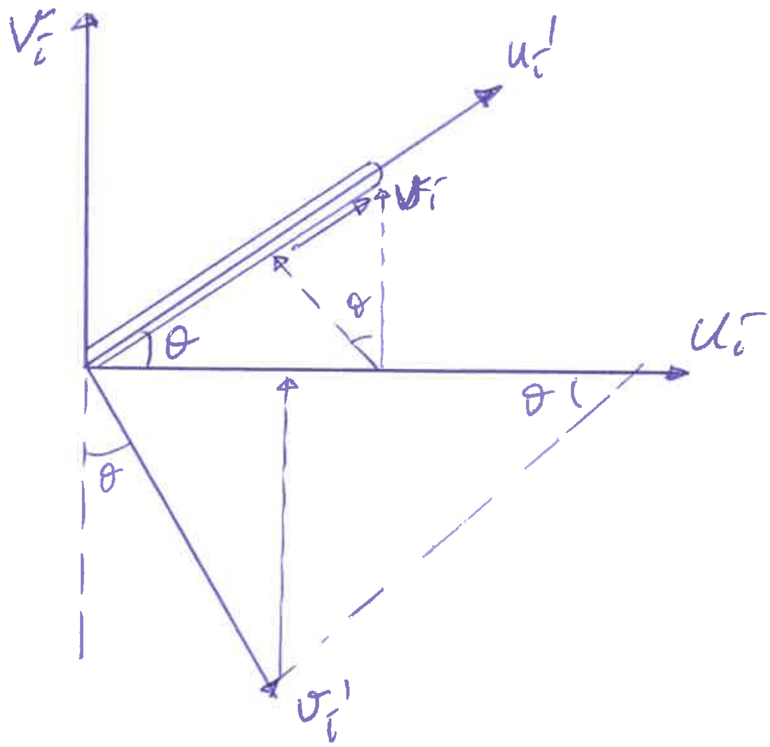
$$[k_2] = 10^6 \begin{bmatrix} 63 & 0 & 0 & -63 & 0 & 0 \\ 0 & 0{,}63 & 126 & 0 & -0{,}63 & 126 \\ 0 & 126 & 33600 & 0 & -126 & 16800 \\ -63 & 0 & 0 & 63 & 0 & 0 \\ 0 & -0{,}63 & -126 & 0 & 0{,}63 & -126 \\ 0 & 126 & 16800 & 0 & -126 & 33600 \end{bmatrix}$$

$$[k_3] = 10^6 \begin{bmatrix} 252 & 0 & 0 & -252 & 0 & 0 \\ 0 & 40{,}32 & 2016 & 0 & -40{,}32 & 2016 \\ 0 & 2016 & 134400 & 0 & -2016 & 67200 \\ -252 & 0 & 0 & 252 & 0 & 0 \\ 0 & -40{,}32 & -2016 & 0 & 40{,}32 & -2016 \\ 0 & 2016 & 67200 & 0 & -2016 & 134400 \end{bmatrix}$$

$$[T] = \begin{bmatrix} c & s & 0 & & & \\ s & -c & 0 & & \underline{0} & \\ 0 & 0 & 1 & 0 & 0 & 0 \\ & & 0 & c & s & 0 \\ & \underline{0} & 0 & s & -c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad [T_2] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[T_3] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



$$[ke_1] = [T_1]^T [k_1] [T_1]$$

$$[ke_2] = [T_2]^T [k_2] [T_2]$$

$$[ke_3] = [T_3]^T [k_3] [T_3]$$

$$u_i' = u_i \cdot \cos\theta + v_i \cdot \sin\theta$$

$$v_i' = u_i \sin\theta - v_i \cdot \cos\theta$$

$$[Ce_1]=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0\end{bmatrix}$$

$$[Ce_2]=\begin{bmatrix}0&0&0&1&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0\end{bmatrix}$$

$$[Ce_3]=\begin{bmatrix}0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&0&0&1\end{bmatrix}$$

$$[ke]^1=[Ce_1]^T[ke_1][Ce_1]$$

$$[ke]^2=[Ce_2]^T[ke_2][Ce_2]$$

$$[ke]^3=[Ce_3]^T[ke_3][Ce_3]$$

$$\{Q\}^T=\left\{\overset{D_1}{\frac{qL}{2}+R_{AH}},\ \overset{D_2}{0+R_{AV}},\ \overset{D_3}{+\frac{qL^2}{12}},\ \overset{D_4}{\frac{qL}{2}},\ \overset{D_5}{0},\ \overset{D_6}{-\frac{qL^2}{12}},\ \overset{D_7}{0},\ \overset{D_8}{R_{BV}},\ \overset{D_9}{0},\ \overset{D_{10}}{0},\ \overset{D_{11}}{-6},\ \overset{D_{12}}{0}\right\}$$

$$=\{8+R_{AH},\ R_{AV},\ +53{,}333,\ 8,\ 0,\ -53{,}33,\ 0,\ R_{BV},\ 0,\ 0,\ -6,\ 0\}$$

$$[ke_1] = 10^6 \begin{vmatrix} 2.835 & 0 & 567 & -2.835 & 0 & 567 \\ 0 & 126 & 0 & 0 & -126 & 0 \\ 567 & 0 & 151200 & -567 & 0 & 75600 \\ -2.835 & 0 & -567 & 2.835 & 0 & -567 \\ 0 & -126 & 0 & 0 & 126 & 0 \\ 567 & 0 & 75600 & -567 & 0 & 151200 \end{vmatrix}$$

$[ke_1]=[T_1]^T[k_1][T_1]$

$$[ke_2] = 10^6 \begin{vmatrix} 63 & 0 & 0 & -63 & 0 & 0 \\ 0 & 0.63 & -126 & 0 & -0.63 & -126 \\ 0 & -126 & 33600 & 0 & 126 & 16800 \\ -63 & 0 & 0 & 63 & 0 & 0 \\ 0 & -0.63 & 126 & 0 & 0.63 & 126 \\ 0 & -126 & 16800 & 0 & 126 & 33600 \end{vmatrix}$$

$[ke_2]=[T_2]^T[k_2][T_2]$

$$[ke_3] = 10^6 \begin{vmatrix} 252 & 0 & 0 & -252 & 0 & 0 \\ 0 & 40.32 & -2016 & 0 & -40.32 & -2016 \\ 0 & -2016 & 134400 & 0 & 2016 & 67200 \\ -252 & 0 & 0 & 252 & 0 & 0 \\ 0 & -40.32 & 2016 & 0 & 40.32 & 2016 \\ 0 & -2016 & 67200 & 0 & 2016 & 134400 \end{vmatrix}$$

$[ke_3]=[T_3]^T[k_3][T_3]$

$[ke]^1 = [ce_1]^T\,[ke_1][ce_1]$

Birimler Ncm, ve N' dur.

$[ke]^2 = [ce_2]^T\,[ke_2][ce_2]$

$[ke]^3 = [ce_3]^T\,[ke_3][ce_3]$

$$[Ksis] = [ke]^1 + [ke]^2 + [ke]^3$$

$$
is] = 10^6 \begin{bmatrix}
\cancel{2.8350} & \cancel{0} & \cancel{567} & \cancel{-2.835} & \cancel{0} & \cancel{567} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\
\cancel{0} & \cancel{126} & \cancel{0} & \cancel{0} & \cancel{-126} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\
567 & 0 & 151200 & -567 & 0 & 75600 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2.835 & 0 & -567 & 65.835 & 0 & -567 & -63 & 0 & 0 & 0 & 0 & 0 \\
0 & -126 & 0 & 0 & 126.63 & -126 & 0 & -0.63 & -126 & 0 & 0 & 0 \\
567 & 0 & 75600 & -567 & -126 & 184800 & 0 & 126 & 16800 & 0 & 0 & 0 \\
0 & 0 & 0 & -63 & 0 & 0 & 315 & 0 & 0 & -252 & 0 & 0 \\
\cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-0.63} & \cancel{126} & \cancel{0} & \cancel{40.95} & \cancel{1890} & \cancel{0} & \cancel{-40.32} & \cancel{2020} \\
0 & 0 & 0 & 0 & -126 & 16800 & 0 & -1890 & 168000 & 0 & 2016 & 67200 \\
0 & 0 & 0 & 0 & 0 & 0 & -252 & 0 & 0 & 252 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -40.32 & 2016 & 0 & 40.32 & 2016 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2016 & 67200 & 0 & 2016 & 134400
\end{bmatrix} \{D
$$

$$
\{Q\} = \begin{Bmatrix}
\cancel{\frac{qL}{2} + R_{AH}} \\
\cancel{0 + R_{AV}} \\
-\frac{qL^2}{12} \\
\frac{qL}{2} \\
0 \\
\frac{qL^2}{12} \\
0 \\
\cancel{R_{BV}} \\
0 \\
0 \\
-6 \\
0
\end{Bmatrix} = \begin{Bmatrix}
800 + R_{AH} \\
R_{AV} \\
533333.33 \\
80000 \\
0 \\
-533333.33 \\
0 \\
R_{BV} \\
0 \\
0 \\
-6000 \\
0
\end{Bmatrix}
$$

$[K]\{D\} = \{Q\}$ 'den; [indirgenmemiş matristen]

$(10^6)(567 \times 0,000171705) + (-2835 \times 0,060216491) + (567 \times 0,000115268) = 8000 + R_{AH}$

$R_{AH} = -16000$ N

$(10^6)(-126 \times 7,51968E-05) = R_{AV} \longrightarrow R_{AV} = -9500$ N

$(10^6)(-0,63 \times 7,53968E-05) + (126 \times 0,000115268) + (+1890 \times 3,9494E-05) - (40,32 \times 0,003354167)$

$+ 2020 \times 3,0565E-05 \cong 15500$ N $= R_{BV}$

Tüm sistemin indirgenmemiş rijitlik matrisi hesabından R_{AV}=-9500 N, R_{AH}=-16000 N, R_{BV}=15500 N bulunmuştur.

$$\{D\} veya\ \{U\} = [K_{sis}]^{-1} * \{Q\}$$

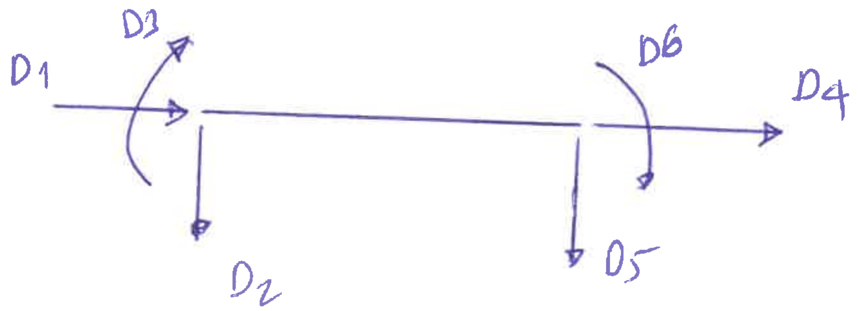
$$\{D\} veya\ \{U\} = \begin{Bmatrix} U1 \\ U2 \\ U3 \\ U4 \\ U5 \\ U6 \\ U7 \\ U8 \\ U9 \\ U10 \\ U11 \\ U12 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0.000171705 \\ 0.060216491 \\ 7.53968\mathrm{E}-05 \\ 0.000115268 \\ 0.060216491 \\ 0 \\ -3.9494\mathrm{E}-05 \\ 0.060216491 \\ 0.003354167 \\ -3.0565\mathrm{E}-05 \end{Bmatrix} \begin{Bmatrix} - \\ - \\ radyan \\ cm \\ cm \\ radyan \\ cm \\ - \\ radyan \\ cm \\ cm \\ radyan \end{Bmatrix}$$

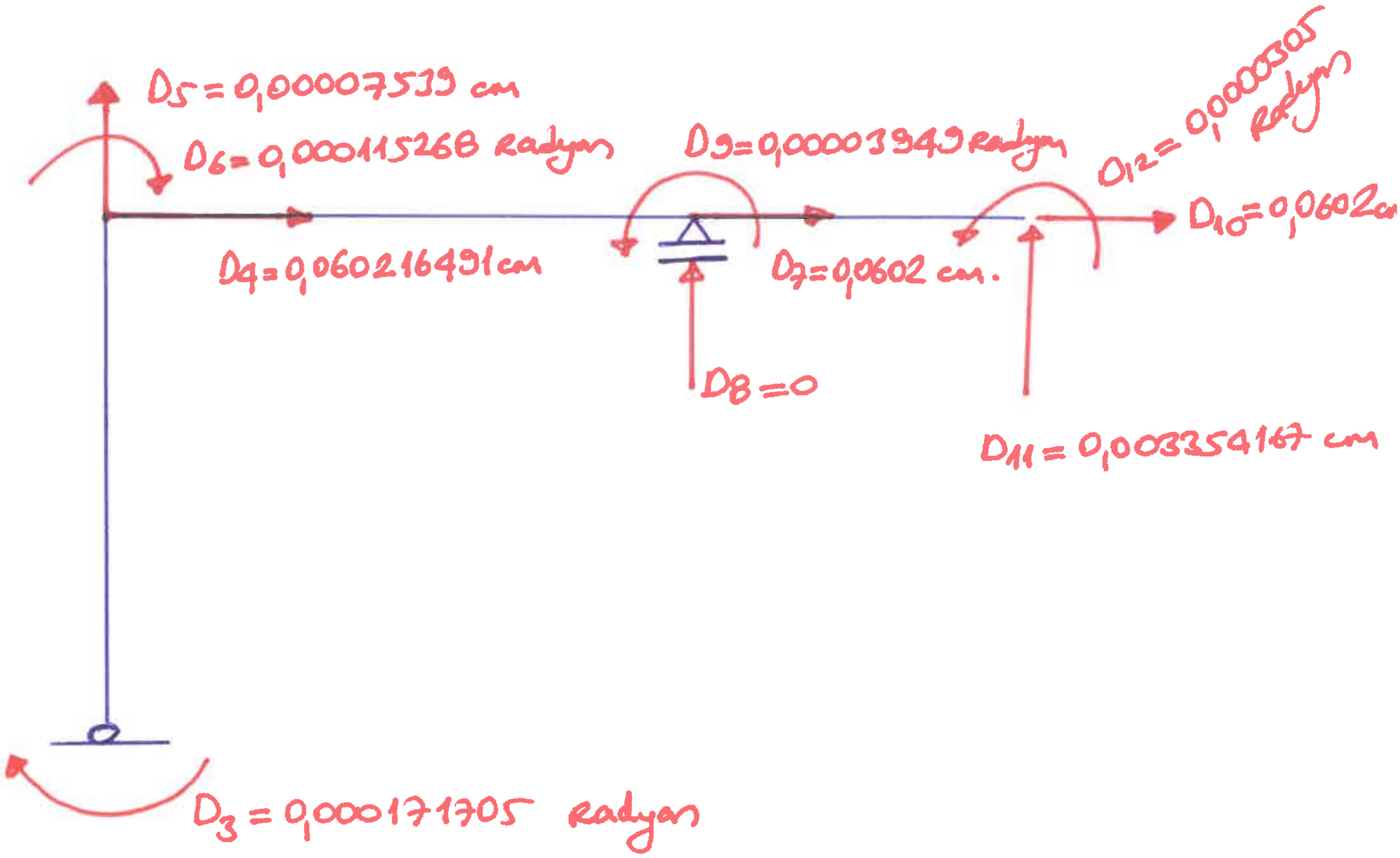
Yerel Koordinatlarda Yerdeğiştirmeler

$$U_1' = [T_1] * \{U\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.000171705 \\ 0.060216491 \\ 7.53968\mathrm{E}-05 \\ -0.000115268 \end{Bmatrix} = \begin{Bmatrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.000171705 \\ 7.53968\mathrm{E}-05 \\ 0.060216491 \\ 0.000115268 \end{Bmatrix}$$

$$U_2' = [T_2] * \{U\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0.060216491 \\ 7.53968\mathrm{E}-05 \\ -0.000115268 \\ 0.060216491 \\ 0 \\ 3.9494\mathrm{E}-05 \end{Bmatrix} = \begin{Bmatrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \end{Bmatrix} \begin{Bmatrix} 0.060216491 \\ -7.53968\mathrm{E}-05 \\ 0.000115268 \\ 0.060216491 \\ 0 \\ -0.39494\mathrm{E}-05 \end{Bmatrix}$$

$$U_3' = [T_3] * \{U\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0.060216491 \\ 0 \\ 3.9494\mathrm{E}-05 \\ 0.06021491 \\ 0.003354167 \\ -3.0565\mathrm{E}-05 \end{Bmatrix} = \begin{Bmatrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \end{Bmatrix} \begin{Bmatrix} 0.060216491 \\ 0 \\ -3.9494\mathrm{E}-05 \\ 0.060216491 \\ -0.003354167 \\ -3.05655\mathrm{E}-05 \end{Bmatrix}$$




D5 = 0,00007519 cm
D6 = 0,000115268 Radyan
D9 = 0,00003949 Radyan
D12 = 0,0000305 Radyan
D10 = 0,0602 cm
D4 = 0,060216491 cm
D7 = 0,0602 cm.
D8 = 0
D11 = 0,003354167 cm
D3 = 0,000171705 Radyan

Kesit Tesirleri

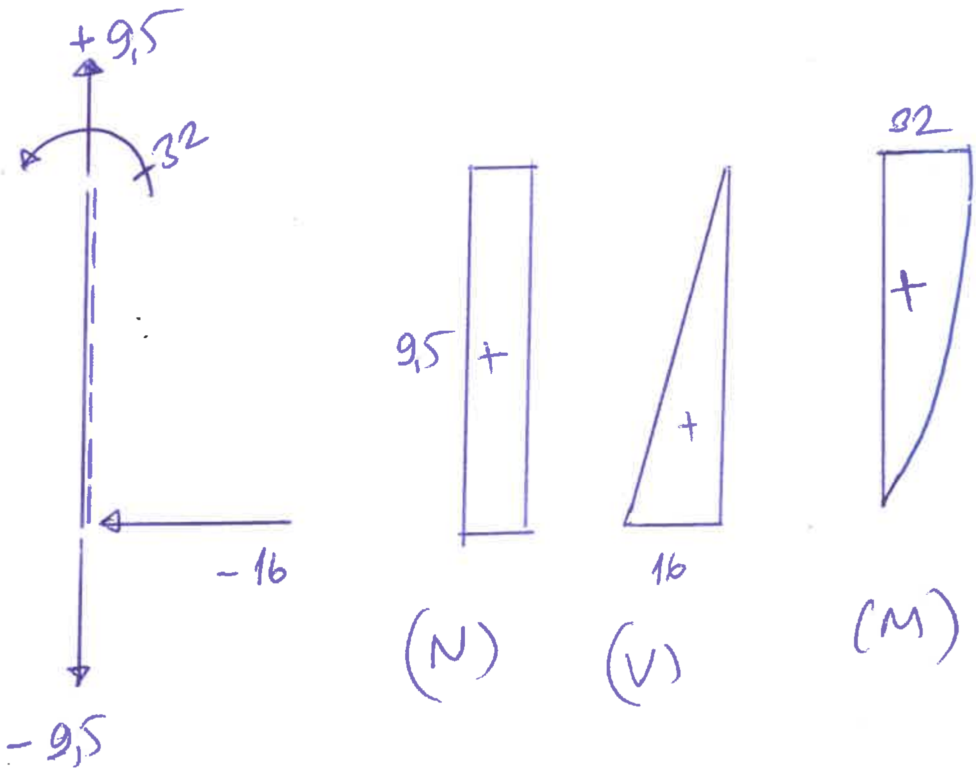
$$\{F\}_e = [k]_e\{D\}_e - \{Q\}_e$$

$$\{f_1\}+\{Q_1\}=\begin{Bmatrix} N_1^1 \\ V_1^1 \\ M_1^1 \\ N_2^1 \\ V_2^1 \\ M_2^1 \end{Bmatrix}+\begin{Bmatrix} 0 \\ 8000 \\ 533333.33 \\ 0 \\ 8000 \\ -533333.33 \end{Bmatrix}=[k_1]*\{U_1'\}$$

$$=10^6\begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 2.835 & 567 & 0 & -2.835 & 567 \\ 0 & 567 & 151200 & 0 & -567 & 75600 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -2.835 & -567 & 0 & 2.835 & -567 \\ 0 & 567 & 75600 & 0 & -567 & 151200 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 0.000171705 \\ 7.53968E-05 \\ 0.060216491 \\ 0.000115268 \end{Bmatrix}*10^{-3}+\begin{Bmatrix} 0 \\ -8000 \\ -533333.33 \\ 0 \\ -8000 \\ 533333.33 \end{Bmatrix}=$$

$$\begin{Bmatrix} -9.5 \\ -8 \\ 533.333 \\ 9.5 \\ 8 \\ -3733333.33 \end{Bmatrix}+\begin{Bmatrix} 0 \\ -8 \\ -533.333 \\ 0 \\ -8 \\ 533333.33 \end{Bmatrix}=\begin{Bmatrix} -9.5 \\ -16 \\ 0 \\ 9.5 \\ 0 \\ -32 \end{Bmatrix}\begin{matrix} kN \\ kN \\ kNm \\ kN \\ kN \\ kNm \end{matrix}\begin{Bmatrix} N1 \\ V1 \\ M1 \\ N2 \\ V2 \\ M2 \end{Bmatrix}$$

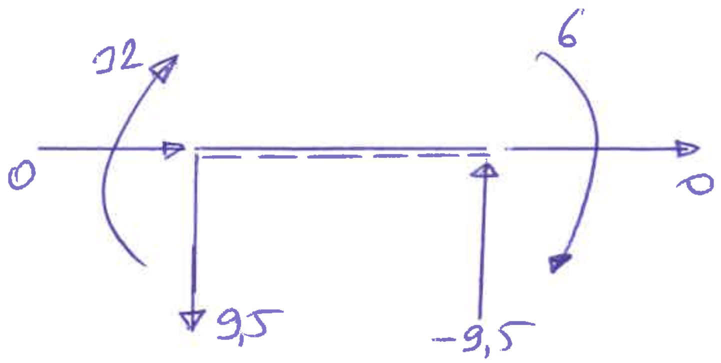
Ncm N

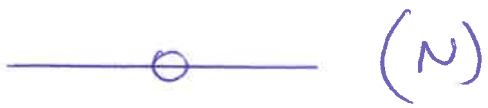


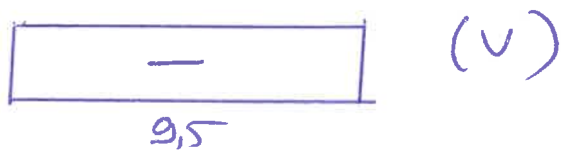
$$\{f_2\}+\{Q_2\}=\begin{Bmatrix}N_1^2\\V_1^2\\M_1^2\\N_2^2\\V_2^2\\M_2^2\end{Bmatrix}+\begin{Bmatrix}0\\0\\0\\0\\0\\0\end{Bmatrix}=[k_2]*\{U_2'\}$$

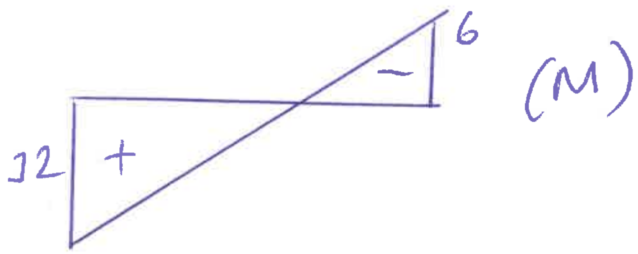
$$=10^6\begin{bmatrix}63&0&0&-63&0&0\\0&0.63&126&0&-0.63&126\\0&126&33600&0&-126&16800\\-63&0&0&63&0&0\\0&-0.63&-126&0&0.63&-126\\0&126&16800&0&-126&33600\end{bmatrix}\begin{Bmatrix}0.060216491\\-7.53968E-05\\0.000115268\\0.060216491\\0\\-3.9494E-05\end{Bmatrix}*10^{-3}-\begin{Bmatrix}0\\0\\0\\0\\0\\0\end{Bmatrix}=$$

$$=\begin{Bmatrix}0\\9.5\\32\\0\\-9.5\\6\end{Bmatrix}\begin{matrix}kN\\kN\\kNm\\kN\\kN\\kNm\end{matrix}\begin{Bmatrix}N1\\V1\\M1\\N2\\V2\\M2\end{Bmatrix}$$





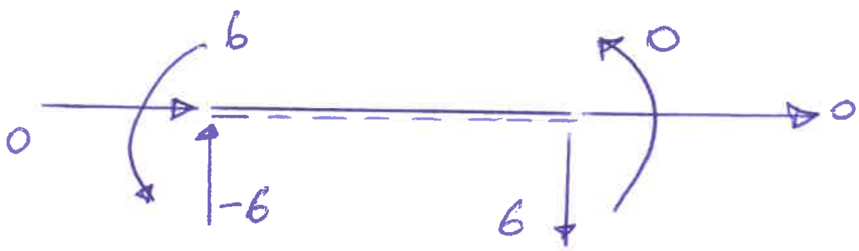


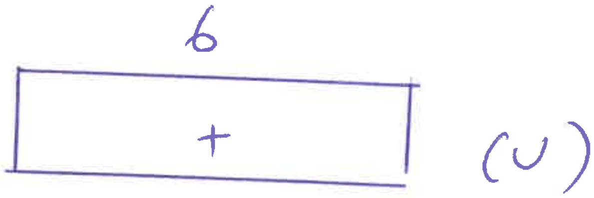


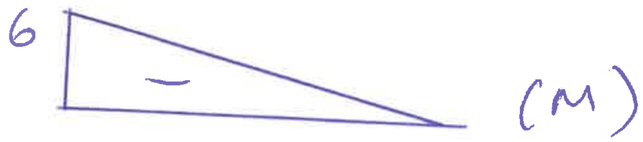
$$\{f_3\} + \{Q_3\} = \begin{Bmatrix} N_1^3 \\ V_1^3 \\ M_1^3 \\ N_2^3 \\ V_2^3 \\ M_2^3 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = [k_3] * \{U_3'\}$$

$$= 10^6 \begin{bmatrix} 252 & 0 & 0 & -252 & 0 & 0 \\ 0 & 40.32 & 2016 & 0 & -40.32 & 2016 \\ 0 & 2016 & 134400 & 0 & -2016 & 67200 \\ -252 & 0 & 0 & 252 & 0 & 0 \\ 0 & -40.32 & -2016 & 0 & 40.32 & -2016 \\ 0 & 2016 & 67200 & 0 & -2016 & 134400 \end{bmatrix} \begin{Bmatrix} 0.060216491 \\ 0 \\ -3.9494\mathrm{E}-05 \\ 0.060216491 \\ -0.003354167 \\ -3.05655\mathrm{E}-05 \end{Bmatrix} * 10^{-3} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

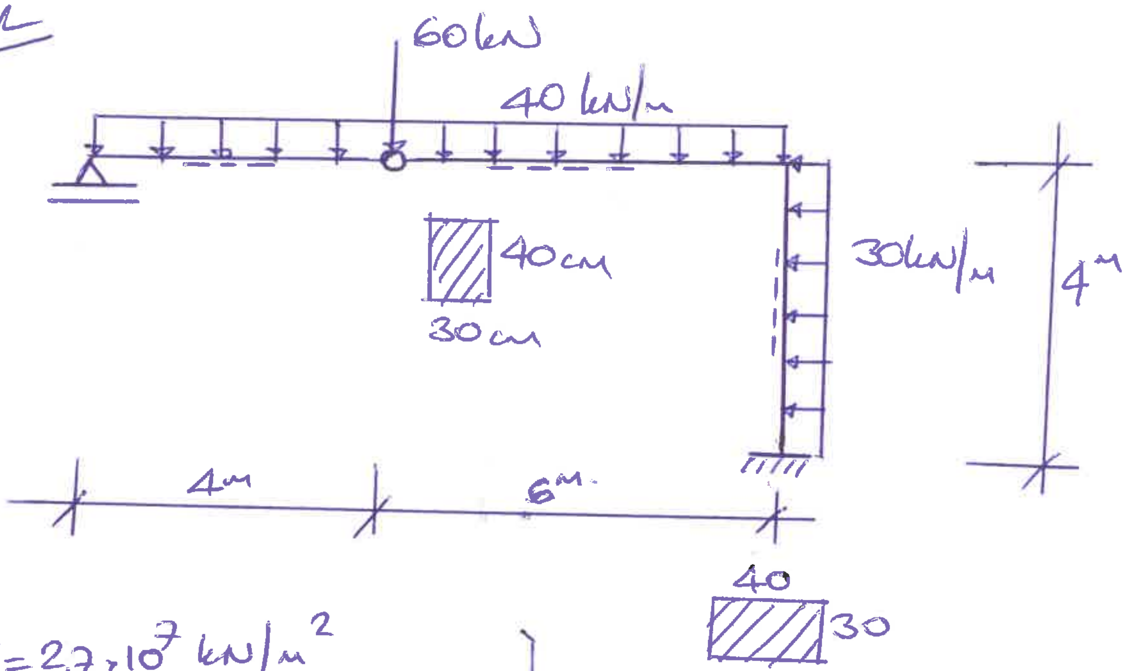
$$= \begin{Bmatrix} 0 \\ -6 \\ -6 \\ 0 \\ 6 \\ 0 \end{Bmatrix} \begin{matrix} kN \\ kN \\ kNm \\ kN \\ kN \\ kNm \end{matrix} \begin{Bmatrix} N1 \\ V1 \\ M1 \\ N2 \\ V2 \\ M2 \end{Bmatrix}$$







Örnek



$E = 2{,}7 \times 10^7 \ kN/m^2$

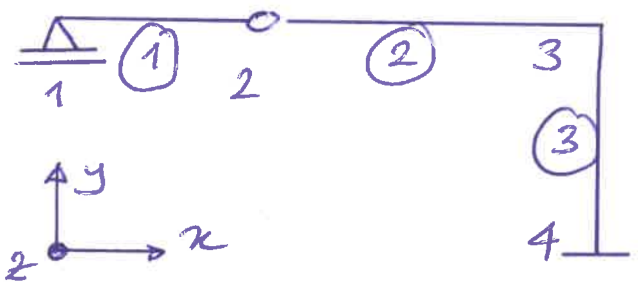
$I = \dfrac{0{,}30 \times 0{,}40^3}{12} = 1{,}6 \times 10^{-3} \ m^4$

$A = 0{,}12 \ m^2$

$EA = 324 \times 10^4 \ kN$

$EI = 4{,}32 \times 10^4 \ kNm^2$

Verilen sistemin yerdeğiştirmelerini ve kesit tesirlerini hesaplayalım.



D.N	X(m)	Y(m)
1	0	4
2	4	4
3	10	4
4	10	0

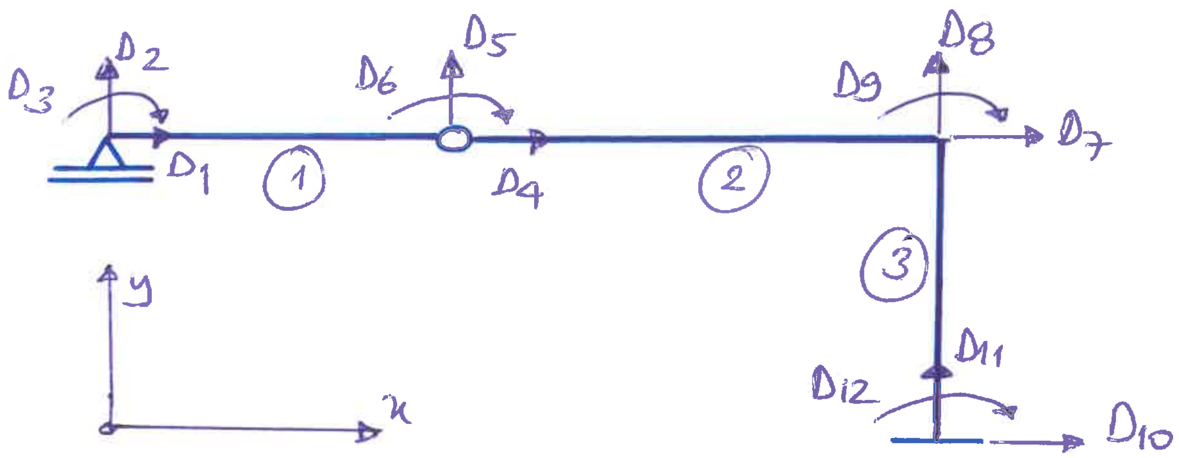
E.N	l	r	Li(m)	c	s
1	1	2	4	1	0
2	2	3	6	1	0
3	3	4	4	0	-1

$c = \dfrac{x_r - x_l}{L}$

$s = \dfrac{y_r - y_l}{L}$

$c_1 = \dfrac{x_2 - x_1}{L_1} = \dfrac{4-0}{4} = 1$ | $c_2 = \dfrac{x_3 - x_2}{L_2} = \dfrac{10-4}{6} = 1$ | $c_3 = \dfrac{x_4 - x_3}{L_3} = \dfrac{10-10}{4} = 0$

$s_1 = \dfrac{y_2 - y_1}{L_1} = \dfrac{4-4}{4} = 0$ | $s_2 = \dfrac{y_3 - y_2}{L_2} = \dfrac{4-4}{6} = 0$ | $s_3 = \dfrac{y_4 - y_3}{L_3} = \dfrac{0-4}{4} = -1$



Mafsal 1 nolu elemanda düşünülmüştür.

$$
[k_1] = \begin{array}{c}
\begin{array}{cccccc} u_1 & v_1 & \varphi_1 & u_2 & v_2 & \varphi_2 \end{array} \\
\left[\begin{array}{ccccc|c}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\
0 & \frac{3EI}{L^2} & \frac{3EI}{L} & 0 & -\frac{3EI}{L^2} & 0 \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\begin{array}{l} u_1 \\ v_1 \\ \varphi_1\,(\theta_1) \\ u_2 \\ v_2 \\ \varphi_2 \end{array}
$$

$$
[k_1] = 10^4 \begin{bmatrix}
81 & 0 & 0 & -81 & 0 & 0 \\
0 & 0{,}2025 & 0{,}81 & 0 & -0{,}2025 & 0 \\
0 & 0{,}81 & 3{,}24 & 0 & -0{,}81 & 0 \\
-81 & 0 & 0 & 81 & 0 & 0 \\
0 & -0{,}2025 & -0{,}81 & 0 & 0{,}2025 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
[k_2] = \begin{array}{c@{}c}
\begin{matrix} \quad u_1 & \quad v_1 & \quad \varphi_1 & \quad u_2 & \quad v_2 & \quad \varphi_2 \end{matrix} & \\
\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix} &
\begin{matrix} u_1 \\ v_1 \\ \varphi_1 \\ u_2 \\ v_2 \\ \varphi_2 \end{matrix}
\end{array}
$$

$$
[k_2] = 10^4 \begin{bmatrix}
54 & 0 & 0 & -54 & 0 & 0 \\
0 & 0{,}24 & 0{,}72 & 0 & -0{,}24 & 0{,}72 \\
0 & 0{,}72 & 2{,}88 & 0 & -0{,}72 & 1{,}44 \\
-54 & 0 & 0 & 54 & 0 & 0 \\
0 & -0{,}24 & -0{,}72 & 0 & 0{,}24 & -0{,}72 \\
0 & 0{,}72 & 1{,}44 & 0 & -0{,}72 & 2{,}88
\end{bmatrix}
$$

$$[k_3]=10^4\begin{bmatrix} 81 & 0 & 0 & -81 & 0 & 0 \\ 0 & 0{,}81 & 1{,}62 & 0 & -0{,}81 & 1{,}62 \\ 0 & 1{,}62 & 4{,}32 & 0 & -1{,}62 & 2{,}16 \\ -81 & 0 & 0 & 81 & 0 & 0 \\ 0 & -0{,}81 & -1{,}62 & 0 & 0{,}81 & -1{,}62 \\ 0 & 1{,}62 & 2{,}16 & 0 & -1{,}62 & 4{,}32 \end{bmatrix}$$

$$[T_1]=[T_2]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad [T_3]=\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[C_1]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_2]=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_3] = \begin{bmatrix} 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$\{Q\}_s = \left\{\begin{array}{c} 0 \\ -\frac{40\times4}{2} + R_{AV} \\ 0 \\ 0 \\ -\frac{40\times4}{2} - 60 - \frac{40\times6}{2} \\ \frac{40\times6^2}{12} \\ -\frac{30\times4}{2} \\ -\frac{40\times6}{2} \\ -\frac{40\times6^2}{12} + \frac{30\times4^2}{12} \\ -\frac{30\times4}{2} + R_{BH} \\ 0 + R_{BV} \\ -\frac{30\times4^2}{12} + M_B \end{array}\right\} \begin{array}{c} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{array} = \left\{\begin{array}{c} 0 \\ -80 + R_{AV} \\ 0 \\ 0 \\ -260 \\ 120 \\ -60 \\ -120 \\ -80 \\ -60 + R_{BH} \\ R_{BV} \\ -40 + M_B \end{array}\right\}$$

$$[k_{e_1}] = [T_1]^T [k_1] [T_1]$$

$$[k_{e_2}] = [T_2]^T [k_1] [T_2]$$

$$[k_{e_3}] = [T_3]^T [k_1] [T_3]$$

$$[K_e]^1 = [C_1]^T [k_{e_1}] [C_1]$$

$$[K_e]^2 = [C_2]^T [k_{e_2}] [C_2]$$

$$[K_e]^3 = [C_3]^T [k_{e_3}] [C_3]$$

$$[K]_s = [K_e]^1 + [K_e]^2 + [K_e]^3$$

$$
10^4 \begin{bmatrix}
81 & 0 & 0 & -81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0{,}2025 & -0{,}81 & 0 & -0{,}2025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0{,}81 & 3{,}24 & 0 & 0{,}81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-81 & 0 & 0 & 135 & 0 & 0 & -54 & 0 & 0 & 0 & 0 & 0 \\
0 & -0{,}2025 & 0{,}81 & 0 & 0{,}4425 & -0{,}72 & 0 & -0{,}24 & -0{,}72 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0{,}72 & 2{,}88 & 0 & 0{,}72 & 1{,}44 & 0 & 0 & 0 \\
0 & 0 & 0 & -54 & 0 & 0 & 54{,}81 & 0 & -1{,}62 & -0{,}81 & 0 & -1{,}62 \\
0 & 0 & 0 & 0 & -0{,}24 & 0{,}72 & 0 & 81{,}24 & 0{,}72 & 0 & -81 & 0 \\
0 & 0 & 0 & 0 & -0{,}72 & 1{,}44 & -1{,}62 & 0{,}72 & 7{,}20 & 1{,}62 & 0 & 2{,}16 \\
0 & 0 & 0 & 0 & 0 & 0 & -0{,}81 & 0 & 1{,}62 & 0{,}81 & 0 & 1{,}62 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -81 & 0 & 0 & 81 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1{,}62 & 0 & 2{,}16 & 1{,}62 & 0 & 4{,}32
\end{bmatrix}
\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ -80+R_{AV} \\ 0 \\ 0 \\ -260 \\ 120 \\ -60 \\ -120 \\ -80 \\ -60+R_{BH} \\ R_{BV} \\ -40+M_B \end{Bmatrix}
$$

$$D_2 = D_{10} = D_{11} = D_{12} = 0$$

$D_1 = -0{,}3111$ m. ; $D_3 = 0{,}3237$ rad. ; $D_4 = -0{,}3111$ m. ; $D_5 = -1{,}2949$ m ; $D_6 = -0{,}2435$ rad.

$D_7 = -0{,}3111$ m ; $D_8 = -0{,}000469$; $D_9 = -0{,}1519$ rad.

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$$10^4\begin{bmatrix} 81 & 0 & 0 & -81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0{,}2025 & -0{,}81 & 0 & -0{,}2025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0{,}81 & 3{,}24 & 0 & 0{,}81 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -81 & 0 & 0 & 135 & 0 & 0 & -54 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0{,}2025 & 0{,}81 & 0 & 0{,}4425 & -0{,}72 & 0 & -0{,}24 & -0{,}72 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0{,}72 & 2{,}88 & 0 & 0{,}72 & 1{,}44 & 0 & 0 & 0 \\ 0 & 0 & 0 & -54 & 0 & 0 & 54{,}81 & 0 & -1{,}62 & -0{,}81 & 0 & -1{,}62 \\ 0 & 0 & 0 & 0 & -0{,}24 & 0{,}72 & 0 & 81{,}24 & 0{,}72 & 0 & -81 & 0 \\ 0 & 0 & 0 & 0 & -0{,}72 & 1{,}44 & -1{,}62 & 0{,}72 & 7{,}20 & 1{,}62 & 0 & 2{,}16 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0{,}81 & 0 & 1{,}62 & 0{,}81 & 0 & 1{,}62 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -81 & 0 & 0 & 81 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1{,}62 & 0 & 2{,}16 & 1{,}62 & 0 & 4{,}32 \end{bmatrix} \begin{Bmatrix} D_1 = -0{,}3111 \\ D_2 = 0 \\ D_3 = 0{,}3237 \\ D_4 = -0{,}3111 \\ D_5 = -1{,}2949 \\ D_6 = -0{,}2485 \\ D_7 = -0{,}3111 \\ D_8 = -0{,}000469 \\ D_9 = -0{,}1519 \\ D_{10} = 0 \\ D_{11} = 0 \\ D_{12} = 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -80+R_{AV} \\ 0 \\ 0 \\ -260 \\ 120 \\ -60 \\ -120 \\ -80 \\ -60+R_{BH} \\ R_{BV} \\ -40+M_B \end{Bmatrix}$$

$[k]\{D\}-\{Q\}=\begin{Bmatrix} R_V \\ R_H \end{Bmatrix}$

2. satır dikkate alınırsa.

$10^4(-0{,}81 \cdot 0{,}3237 - 0{,}2025 \cdot (-1{,}2949)) = -80 + R_{AV} \longrightarrow R_{AV} \cong 80$ kN

10. satır dikkate alınırsa

$10^4(-0{,}81 \cdot (-0{,}3111) + 1{,}62(-0{,}1519)) = -60 + R_{BH} \longrightarrow R_{BH} \cong 120$

12. satır dikkate alınırsa.

$10^4(-1{,}62(-0{,}3111) + 2{,}16(-0{,}1519)) = -40 + M_B$

$M_B \cong 1800$ kNm

11. satır dikkate alınırsa.

$10^4(-81 \cdot (-0{,}000469)) = R_{BV} \longrightarrow R_{BV} \cong 380$ kN

Yerel Koordinatlarda Yerdeğiştirmeler

$$\{u_1\}' = [T_1]\{u\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -0{,}3111 \\ 0 \\ 0{,}3237 \\ -0{,}3111 \\ -1{,}2949 \\ -0{,}2435 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} = \begin{Bmatrix} -0{,}3111 \\ 0 \\ 0{,}3237 \\ -0{,}3111 \\ +1{,}2949 \\ -0{,}2435 \end{Bmatrix}$$

$$\{u_2\}' = [T_2]\{u\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -0{,}3111 \\ -1{,}2949 \\ -0{,}2435 \\ -0{,}3111 \\ -0{,}000469 \\ -0{,}1519 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} = \begin{Bmatrix} -0{,}3111 \\ +1{,}2949 \\ -0{,}2435 \\ -0{,}3111 \\ +0{,}000469 \\ -0{,}1519 \end{Bmatrix}$$

$$\{u_3\}' = [T_3]\{u\} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -0{,}3111^{*} \\ 0{,}000469 \\ -0{,}1519 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} = \begin{Bmatrix} 0{,}000469 \\ 0{,}3111 \\ -0{,}1519 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

* $u = -0{,}3111$ m. yerdeğiştirme genel sistemde D_7'ye denk düşmekte ve genel sistemde yatay yerdeğiştirmedir. (3) nolu çubuk içinse düşey yerdeğiştirmedir.

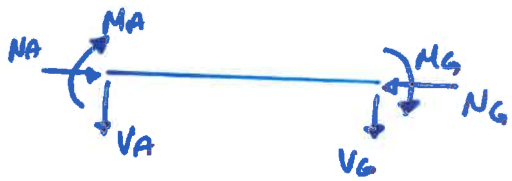
Çubuk kuvvetlerinin hesabı

1 nolu çubuk

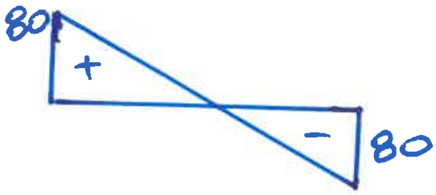
$$\{F\}^e = [K]^e \cdot \{D\}^e - \{Q\}^e$$

$$\{F\} = \begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = 10^4 * \begin{bmatrix} 81 & 0 & 0 & -81 & 0 & 0 \\ 0 & 0,2025 & 0,81 & 0 & -0,2025 & 0 \\ 0 & 0,81 & 3,24 & 0 & -0,81 & 0 \\ -81 & 0 & 0 & 81 & 0 & 0 \\ 0 & -0,2025 & -0,81 & 0 & 0,2025 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} -0,3111 \\ 0 \\ 0,3237 \\ -0,3111 \\ +1,2949 \\ -0,2435 \end{Bmatrix} - \begin{Bmatrix} 0 \\ +80 \\ 0 \\ 0 \\ +80 \\ 0 \end{Bmatrix}$$

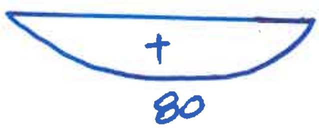
$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} N_A \\ V_A \\ M_A \\ N_G \\ V_G \\ M_G \end{Bmatrix} \cong \begin{Bmatrix} 0 \\ -80 \\ 0 \\ 0 \\ +80 \\ 0 \end{Bmatrix}$$



(N)



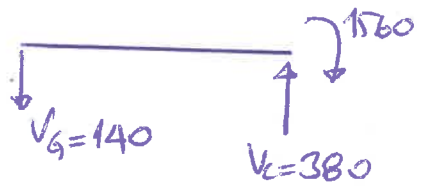
(V, kN)

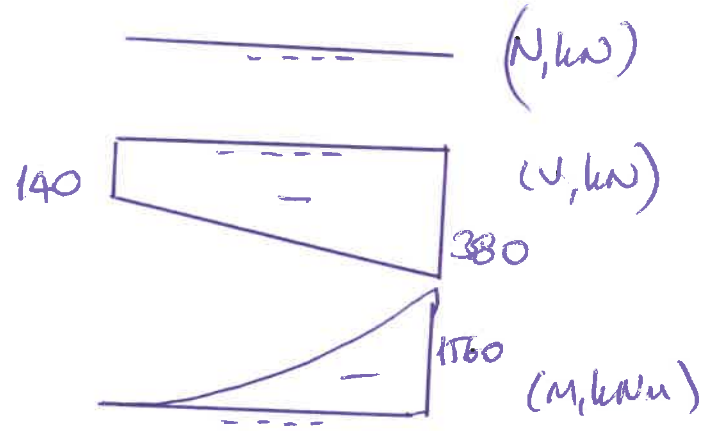


(M, kNm)

2 nolu çubuk

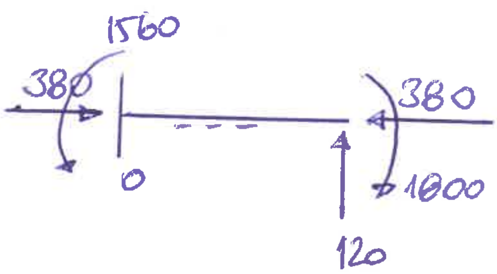
$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = 10^4 \times \begin{bmatrix} 54 & 0 & 0 & -54 & 0 & 0 \\ 0 & 0{,}24 & 0{,}72 & 0 & -0{,}24 & 0{,}72 \\ 0 & 0{,}72 & 2{,}88 & 0 & -0{,}72 & 1{,}44 \\ -54 & 0 & 0 & 54 & 0 & 0 \\ 0 & -0{,}24 & -0{,}72 & 0 & 0{,}24 & -0{,}72 \\ 0 & 0{,}72 & 1{,}44 & 0 & -0{,}72 & 2{,}88 \end{bmatrix} \begin{Bmatrix} -0{,}3111 \\ 1{,}2949 \\ -0{,}2435 \\ -0{,}3111 \\ 0{,}000469 \\ -0{,}1519 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 120 \\ 120 \\ 0 \\ 120 \\ -120 \end{Bmatrix} \stackrel{u}{=} \begin{Bmatrix} 0 \\ 140 \\ 0 \\ 0 \\ -380 \\ 1560 \end{Bmatrix}$$

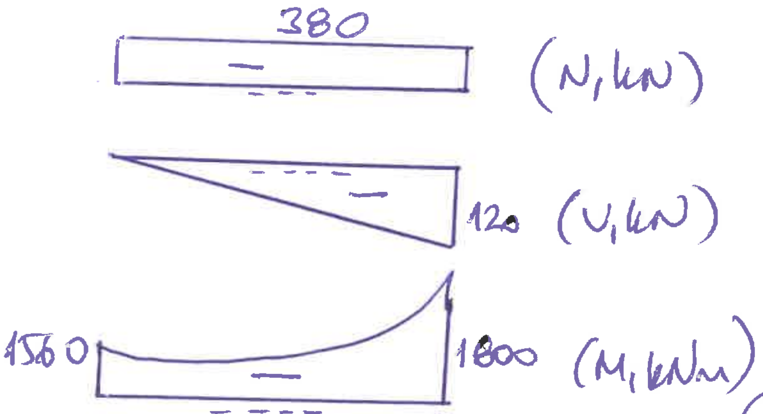




3 nolu çubuk

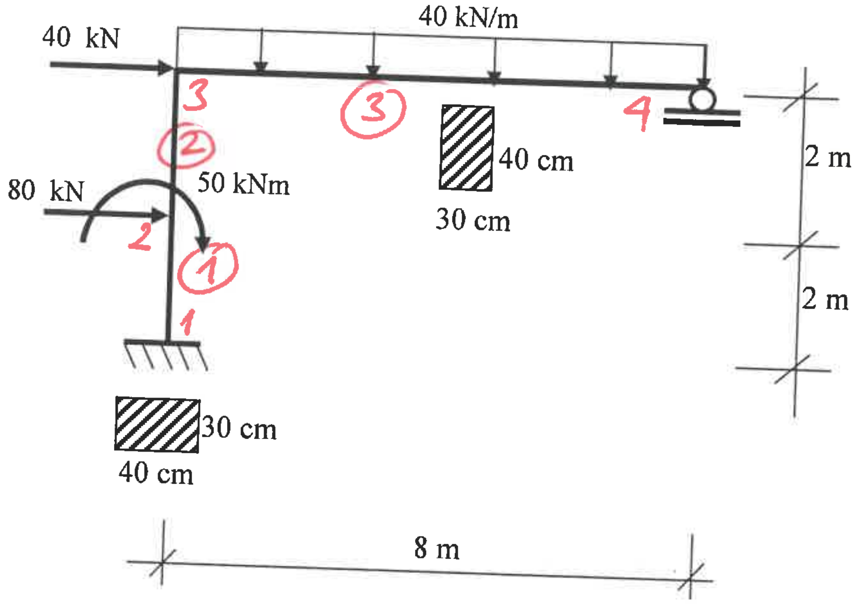
$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix} = 10^4 \times \begin{bmatrix} 81 & 0 & 0 & -81 & 0 & 0 \\ 0 & 0{,}81 & 1{,}62 & 0 & -0{,}81 & 1{,}62 \\ 0 & 1{,}62 & 4{,}32 & 0 & -1{,}62 & 2{,}16 \\ -81 & 0 & 0 & 81 & 0 & 0 \\ 0 & -0{,}81 & -1{,}62 & 0 & 0{,}81 & -1{,}62 \\ 0 & 1{,}62 & 2{,}16 & 0 & -1{,}62 & 4{,}32 \end{bmatrix} \begin{Bmatrix} 0{,}000469 \\ 0{,}3111 \\ -0{,}1519 \\ 0 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 60 \\ 40 \\ 0 \\ 60 \\ -40 \end{Bmatrix} \stackrel{u}{=} \begin{Bmatrix} 380 \\ 0 \\ -1560 \\ -380 \\ -120 \\ 1800 \end{Bmatrix}$$





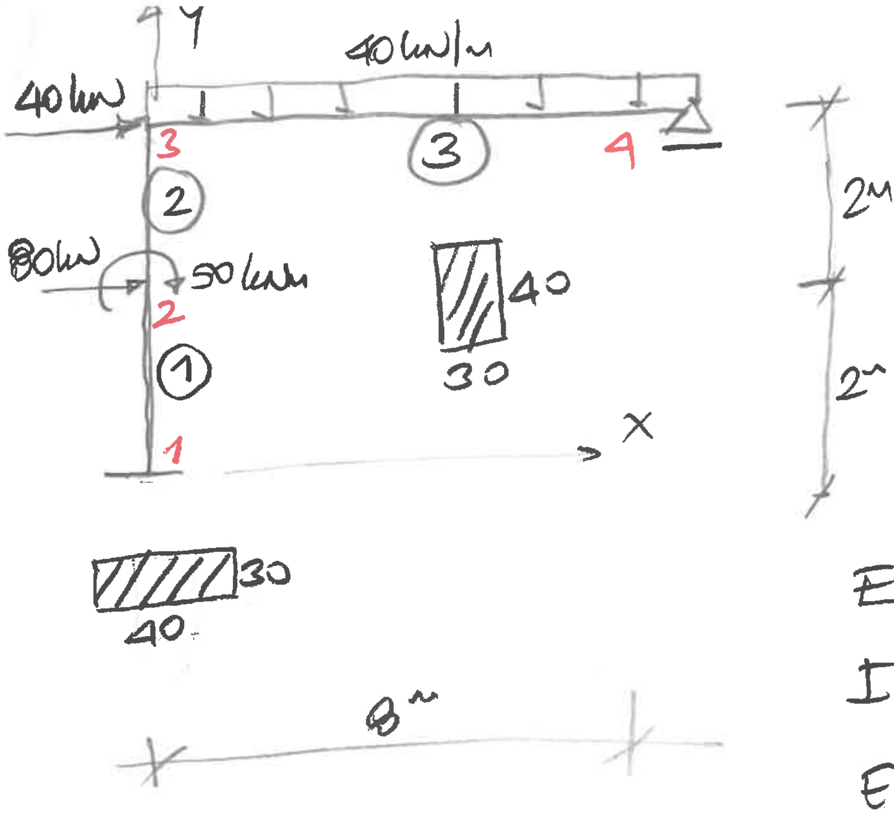
ÖRNEK: Şekilde verilen sistemin düğüm noktalarının yerdeğiştirmelerini ve çubuk kuvvetlerini sonlu elemanlar yöntemi ile bulunuz.

(E:21x10^6 kN/ m^2)



Not: Tekil yük bulunan çubuklarda, tekil yükün etkidiği noktaları düğüm noktası olarak ele almak işlem kolaylığı sağlayacaktır!

ÖRNEK
1-9 sayfa



DN	X	Y
1	0	0
2	0	2
3	0	4
4	8	4

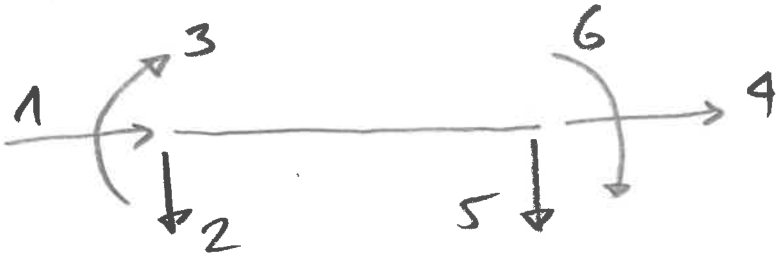
$E = 21 \times 10^6 \ kN/m^2$

$I = \dfrac{0{,}3 \times 0{,}4^3}{12} = 1{,}6 \times 10^{-3} \ m^4$

$EA = 252 \times 10^4 \ kN$

$EI = 3{,}36 \times 10^4 \ kNm^2$

E.N	$\bar{i}$	$\bar{j}$	L	c	s
1	1	2	2	0	1
2	2	3	2	0	1
3	3	4	8	1	0



Eleman yerdeğiştirme yönleri

$$[k_e] = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix}$$

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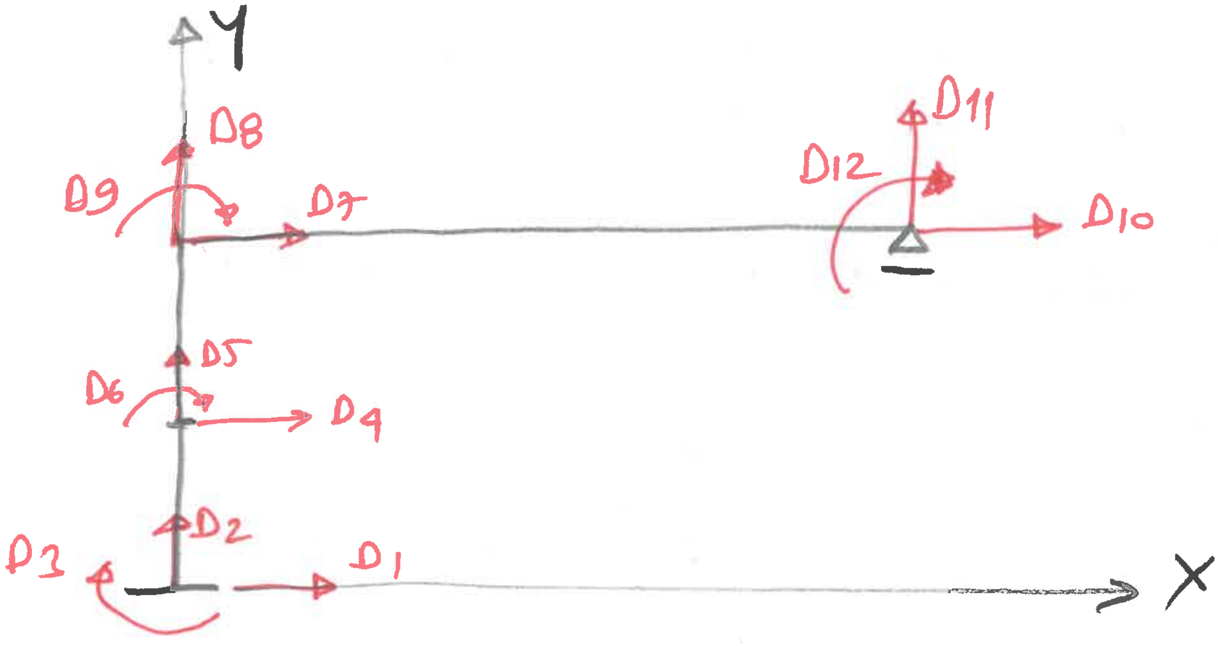


$$[k_1]=[k_2]=10^4\begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 5{,}04 & 5{,}04 & 0 & -5{,}04 & 5{,}04 \\ 0 & 5{,}04 & 6{,}72 & 0 & -5{,}04 & 3{,}36 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -5{,}04 & -5{,}04 & 0 & 5{,}04 & -5{,}04 \\ 0 & 5{,}04 & 3{,}36 & 0 & -5{,}04 & 6{,}72 \end{bmatrix}$$

$$[k_3]=10^4\begin{bmatrix} 31{,}5 & 0 & 0 & -31{,}5 & 0 & 0 \\ 0 & 0{,}07875 & 0{,}315 & 0 & -0{,}07875 & 0{,}315 \\ 0 & 0{,}315 & 1{,}68 & 0 & -0{,}315 & 0{,}84 \\ -31{,}5 & 0 & 0 & 31{,}5 & 0 & 0 \\ 0 & -0{,}07875 & -0{,}315 & 0 & 0{,}07875 & -0{,}315 \\ 0 & 0{,}315 & 0{,}84 & 0 & -0{,}315 & 1{,}68 \end{bmatrix}$$

č. 2/9

$$[T_1] = [T_2] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad [T_3] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[C_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_2] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$[C_3] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$[ke_1] = [T_1]^T [k_1] [T_1]$; $[K]_1 = [C_1]^T [ke_1] [C_1]$

$[ke_2] = [T_2]^T [k_2] [T_2]$; $[K]_2 = [C_2]^T [ke_2] [C_2]$

$[ke_3] = [T_3]^T [k_3] [T_3]$; $[K]_3 = [C_3]^T [ke_3] [C_3]$

$[K]_L = [K]_1 + [K]_2 + [K]_3$

δ.3/9

$$
10^4
\begin{bmatrix}
5{,}04 & 0 & 5{,}04 & -5{,}04 & 0 & 5{,}04 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 126 & 0 & 0 & -126 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5{,}04 & 0 & 6{,}72 & -5{,}04 & 0 & 3{,}36 & 0 & 0 & 0 & 0 & 0 & 0 \\
-5{,}04 & 0 & -5{,}04 & 10{,}08 & 0 & 0 & -5{,}04 & 0 & 5{,}04 & 0 & 0 & 0 \\
0 & -126 & 0 & 0 & 252 & 0 & 0 & -126 & 0 & 0 & 0 & 0 \\
5{,}04 & 0 & 3{,}36 & 0 & 0 & 13{,}44 & -5{,}04 & 0 & 3{,}36 & 0 & 0 & 0 \\
0 & 0 & 0 & -5{,}04 & 0 & -5{,}04 & 36{,}54 & 0 & -5{,}04 & -31{,}5 & 0 & 0 \\
0 & 0 & 0 & 0 & -126 & 0 & 0 & 126{,}07875 & -0{,}315 & 0 & -0{,}07875 & -0{,}315 \\
0 & 0 & 0 & 5{,}04 & 0 & 3{,}36 & -5{,}04 & -0{,}315 & 8{,}4 & 0 & 0{,}315 & 0{,}84 \\
0 & 0 & 0 & 0 & 0 & 0 & -31{,}5 & 0 & 0 & 31{,}5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0{,}07875 & 0{,}315 & 0 & 0{,}07875 & 0{,}315 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0{,}315 & 0{,}84 & 0 & 0{,}315 & 1{,}68
\end{bmatrix}
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 80 \\ 0 \\ 50 \\ 40 \\ -\frac{40\cdot 8}{2} \\ \frac{40\cdot 8^2}{12} \\ 0 \\ -\frac{40\cdot 8}{2} \\ -\frac{40\cdot 8^2}{12} \end{bmatrix}
$$

$D_4 = 0{,}0197$ m. | $D_7 = 0{,}0599$ m. | $D_0 = 0{,}0599$ m.

$D_5 = -0{,}000131$ m | $D_8 = -0{,}0003$ m.

$D_6 = 0{,}0173$ rad | $D_9 = 0{,}0221$ rad | $D_{12} = -0{,}0238$

4

0,4/9

$$10^4 \begin{bmatrix} 5{,}04 & 0 & 5{,}04 & -5{,}04 & 0 & 5{,}04 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 126 & 0 & 0 & -126 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5{,}04 & 0 & 6{,}72 & -5{,}04 & 0 & 3{,}36 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5{,}04 & 0 & -5{,}04 & 10{,}08 & 0 & 0 & -5{,}04 & 0 & 5{,}04 & 0 & 0 & 0 \\ 0 & -126 & 0 & 0 & 252 & 0 & 0 & -126 & 0 & 0 & 0 & 0 \\ 5{,}04 & 0 & 3{,}36 & 0 & 0 & 13{,}44 & -5{,}04 & 0 & 3{,}36 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5{,}04 & 0 & -5{,}04 & 36{,}54 & 0 & -5{,}04 & -31{,}5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -126 & 0 & 0 & 126{,}07875 & -0{,}315 & 0 & -0{,}07875 & -0{,}315 \\ 0 & 0 & 0 & 5{,}04 & 0 & 3{,}36 & -5{,}04 & -0{,}315 & 8{,}4 & 0 & 0{,}315 & 0{,}84 \\ 0 & 0 & 0 & 0 & 0 & 0 & 31{,}5 & 0 & 0 & 31{,}5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0{,}07875 & 0{,}315 & 0 & 0{,}07875 & 0{,}315 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0{,}315 & 0{,}84 & 0 & 0{,}315 & 1{,}68 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0{,}0197 \\ -0{,}000131 \\ 0{,}0173 \\ 0{,}0599 \\ -0{,}000262 \\ 0{,}0221 \\ 0{,}0599 \\ 0 \\ -0{,}022 \end{bmatrix} = \begin{bmatrix} R_{AH} \\ R_{AV} \\ M_A \\ 80 \\ 0 \\ 50 \\ 40 \\ -160 \\ 213{,}333 \\ 0 \\ -160+R_{BV} \\ -213{,}333 \end{bmatrix}$$

$R_{AH} \overset{\checkmark}{=} -120{,}00\ kN$

$R_{AV} =^{\checkmark}\ 165{,}06\ kN$

$M_A = -411{,}60\ kNm$

$-160 + R_{BV} = -5{,}11 \rightarrow R_{BV} =^{\checkmark} 154{,}90$

8.5/9

Yerel Koordinatlarda Yerdeğiştirmeler

$$\{u_1\}^1 = [T_1]\{u\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0{,}0197 \\ -0{,}000131 \\ 0{,}0173 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0{,}000131 \\ 0{,}0197 \\ 0{,}0173 \end{Bmatrix}^{1.e}$$

$$\{u_2\}^2 = [T_2]\{u\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0{,}0197 \\ -0{,}000131 \\ 0{,}0173 \\ 0{,}0599 \\ -0{,}000262 \\ 0{,}0221 \end{Bmatrix} = \begin{Bmatrix} -0{,}000131 \\ 0{,}0197 \\ 0{,}0173 \\ -0{,}000262 \\ 0{,}0599 \\ 0{,}0221 \end{Bmatrix}^{2.e}$$

$$\{u_3\}^3 = [T_3]\{u\} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0{,}0599 \\ -0{,}000262 \\ 0{,}0221 \\ 0{,}0599 \\ 0 \\ -0{,}0238 \end{Bmatrix} = \begin{Bmatrix} 0{,}0599 \\ 0{,}000262 \\ 0{,}0221 \\ 0{,}0599 \\ 0 \\ -0{,}0238 \end{Bmatrix}$$

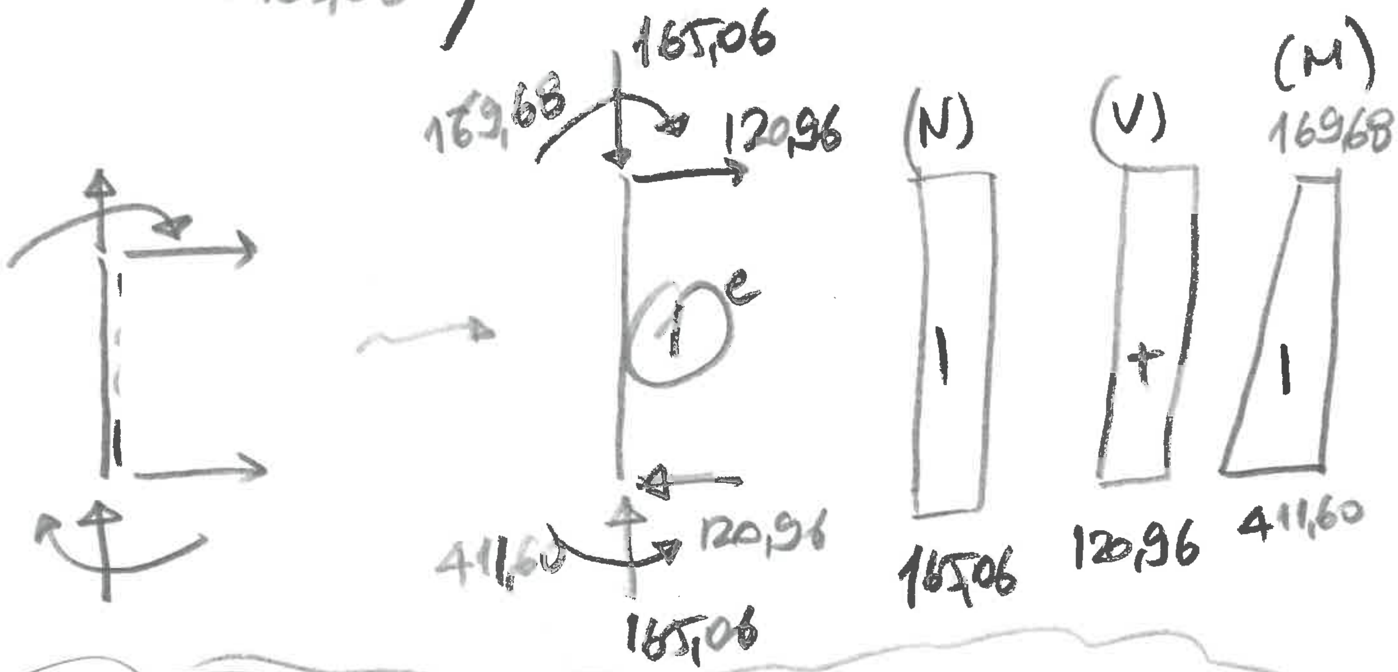
4
0.5/9

Çubuk Kuvvetlerinin Hesabı

1. eleman

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{1.e} = 10^4 \begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 5{,}04 & 5{,}04 & 0 & -5{,}04 & 5{,}04 \\ 0 & 5{,}04 & 6{,}72 & 0 & -5{,}04 & 3{,}36 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -5{,}04 & -5{,}04 & 0 & 5{,}04 & -5{,}04 \\ 0 & 5{,}04 & 3{,}36 & 0 & -5{,}04 & 6{,}72 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0{,}000131 \\ 0{,}0197 \\ 0{,}0123 \end{Bmatrix} -^{*} \begin{Bmatrix} 0 \end{Bmatrix}$$

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{1.e} = \begin{Bmatrix} 165{,}06 \\ -120{,}96 \\ -411{,}60 \\ -165{,}06 \\ 120{,}96 \\ 169{,}68 \end{Bmatrix}$$



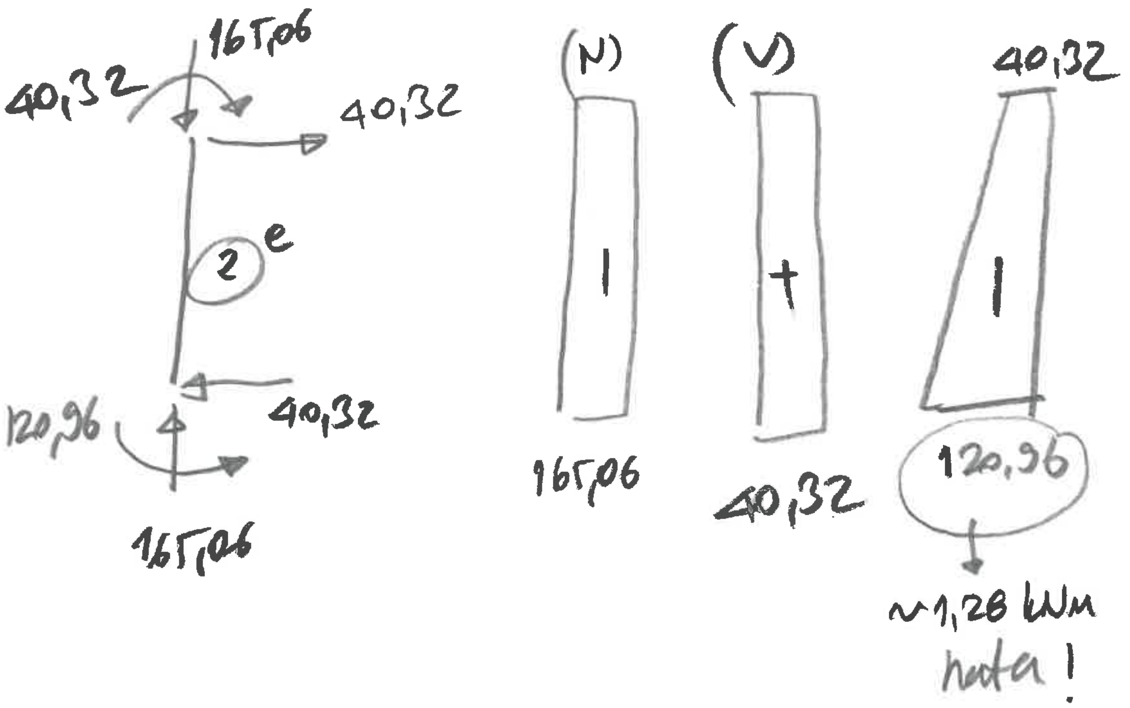
* Not: $D_5 = -0{,}000131$ değeri hesaplarda $-0{,}0001$ olarak görülebilir. Bu durumda N değeri 126 kN gibi ($N_{gerçek} \cong 165$ kN) farklı bir değer hesaplanabilmektedir.

0.7/9

2. eleman

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{2e} = 10^4 \begin{bmatrix} 126 & 0 & 0 & -126 & 0 & 0 \\ 0 & 5{,}04 & 5{,}04 & 0 & -5{,}04 & 5{,}04 \\ 0 & 5{,}04 & 6{,}72 & 0 & -5{,}04 & 3{,}36 \\ -126 & 0 & 0 & 126 & 0 & 0 \\ 0 & -5{,}04 & -5{,}04 & 0 & 5{,}04 & -5{,}04 \\ 0 & 5{,}04 & 3{,}36 & 0 & -5{,}04 & 6{,}72 \end{bmatrix} \begin{Bmatrix} -0{,}000131 \\ 0{,}0197 \\ 0{,}0173 \\ -0{,}000262 \\ 0{,}0599 \\ 0{,}0221 \end{Bmatrix} - \begin{Bmatrix} \underline{0} \end{Bmatrix}$$

$$\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{2e} = \begin{Bmatrix} 165{,}06 \\ -40{,}32 \\ -120{,}96 \\ -165{,}06 \\ 40{,}32 \\ 40{,}32 \end{Bmatrix}$$



ö.8/9

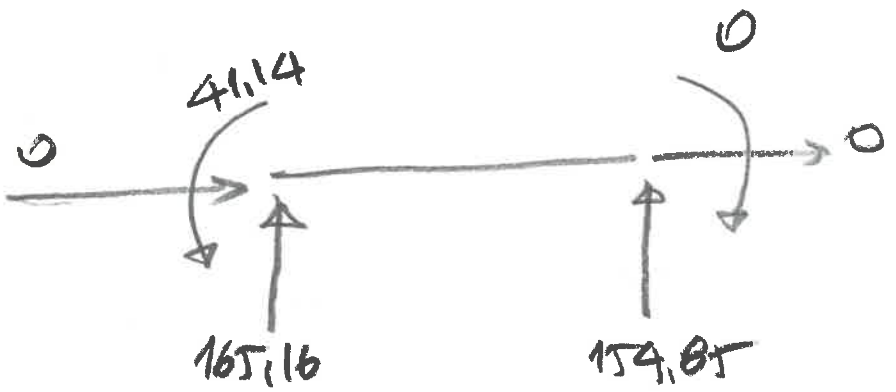
3.eleman)

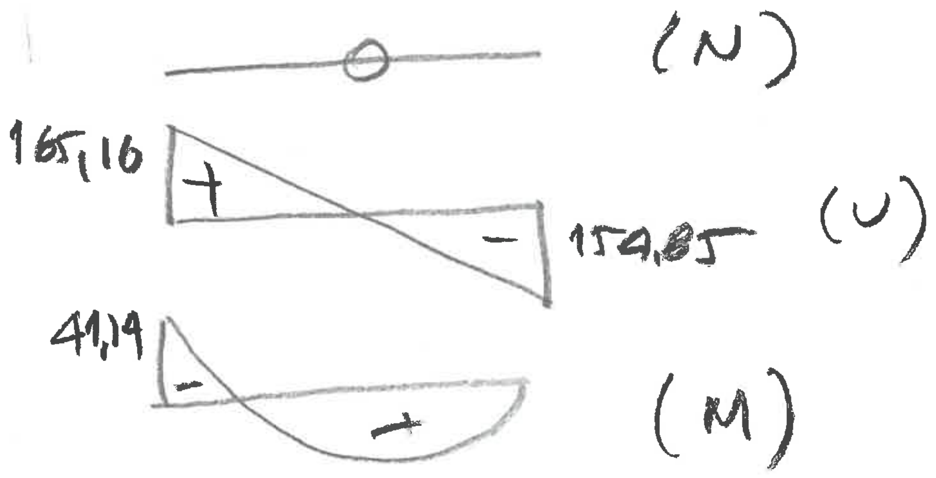
$$
\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{3.e} = 10^4 \begin{bmatrix} 31,5 & 0 & 0 & -31,5 & 0 & 0 \\ 0 & 0,07875 & 0,315 & 0 & -0,07875 & 0,315 \\ 0 & 0,315 & 1,68 & 0 & -0,315 & 0,84 \\ -31,5 & 0 & 0 & 31,5 & 0 & 0 \\ 0 & -0,07875 & -0,315 & 0 & 0,07875 & -0,315 \\ 0 & 0,315 & 0,84 & 0 & -0,315 & 1,68 \end{bmatrix} \begin{Bmatrix} 0,0599 \\ 0,000262 \\ 0,0221 \\ 0,0599 \\ 0 \\ -0,0238 \end{Bmatrix} - \begin{Bmatrix} 0 \\ +160 \\ 213,333 \\ 0 \\ +160 \\ -213,333 \end{Bmatrix}
$$

$$
\begin{Bmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \end{Bmatrix}^{3.e} = \begin{Bmatrix} 0 \\ -165,15 \\ -41,14 \\ 0 \\ -154,85 \\ 0 \end{Bmatrix}
$$

$-41,14 \rightarrow$ 0,62 kNm hata!

elemanda yönler aşağıya doğru pozitif alınmıştır



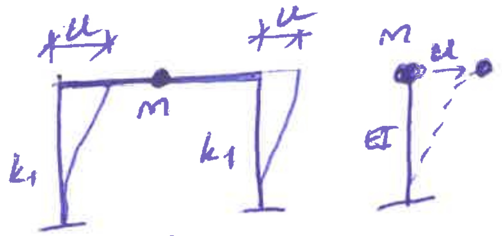


ö.9/9

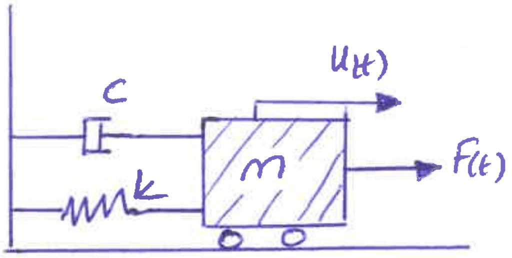
EK:2

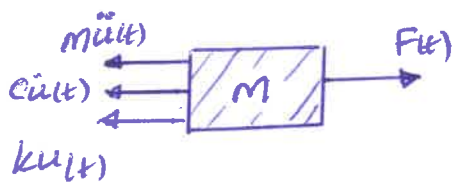
ÇERÇEVELERİN SONLU ELEMANLAR YÖNTEMİ İLE SERBEST TİTREŞİMİ

SERBEST TİTREŞİM



Şekil 1





$m\ddot{u}_{(t)} + c\dot{u}_{(t)} + ku_{(t)} = F_{(t)}$

(Hareket Denklemi)

m: kütle matrisi

$\ddot{u}_{(t)}$: ivme vektörü

c: sönüm matrisi

$\dot{u}_{(t)}$: hız vektörü

k: rijitlik matrisi

u: yerdeğiştirme vektörü

$F_{(t)}$: zamana bağlı dış kuvvet

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}$$

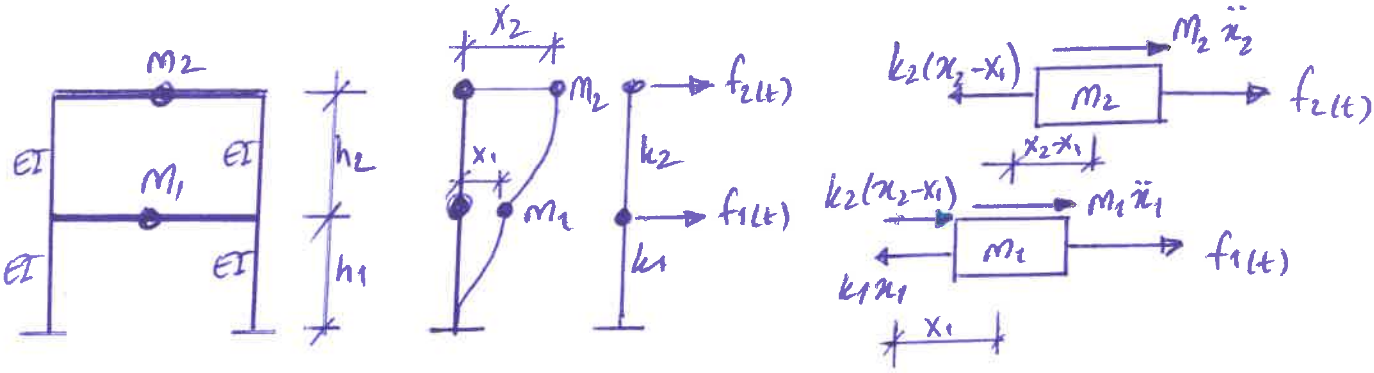
$([K] - \omega_i^2 [M])\{u_i\} = 0$ özdeğer problemi

ω_i = sistemin doğal açısal frekansı

$T = \frac{2\pi}{\omega_i}$ sistemin doğal titreşim periyodu

Şayet, Şekil 1'de verilen çerçeve kayma çerçevesi olarak dikkate alınıyorsa kolon rijitliği k_1,

$k_1 = \frac{12EI}{h^3}$ olarak hesaplanacaktır.



$$\Sigma F = m.a$$

$$f_1(t) - k_1.x_1 + k_2(x_2-x_1) = m_1.\ddot{x}_1 \rightsquigarrow -m_1\ddot{x}_1 - k_1.x_1 - k_2.x_1 + k_2.x_2 = -f_1(t)$$

$$f_2(t) - k_2(x_2-x_1) = m_2.\ddot{x}_2 \rightsquigarrow -m_2.\ddot{x}_2 - k_2.x_2 + k_2.x_1 = -f_{(2)}(t)$$

$$m_1.\ddot{x}_1 + (k_1+k_2)x_1 - k_2.x_2 = f_1(t) \quad ---(1)$$

$$m_2.\ddot{x}_2 - k_2.x_1 + k_2.x_2 = f_2(t) \quad ---(2)$$

(1) ve (2) bağıntıları matris formunda yazılırsa;

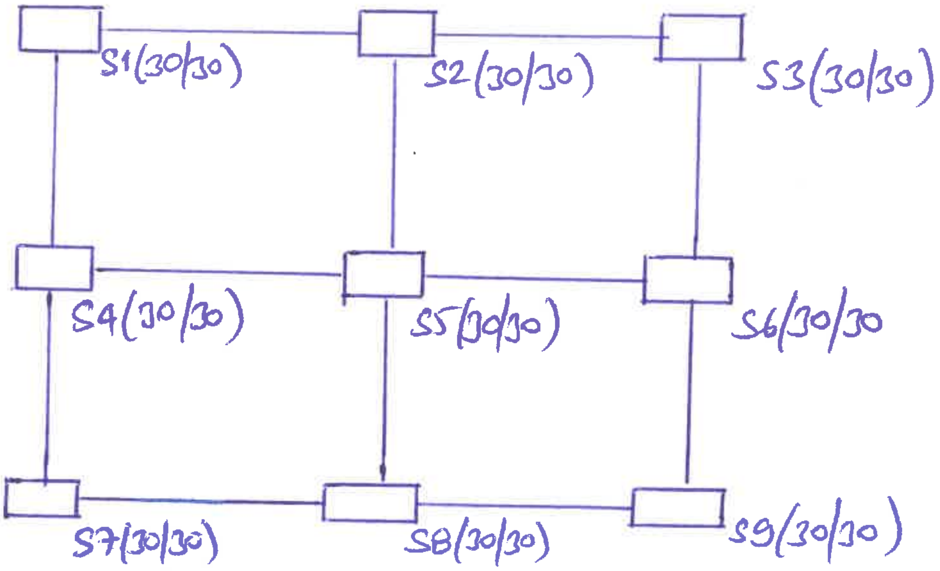
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & +k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

Bağıl serbest titreşim durumunda;

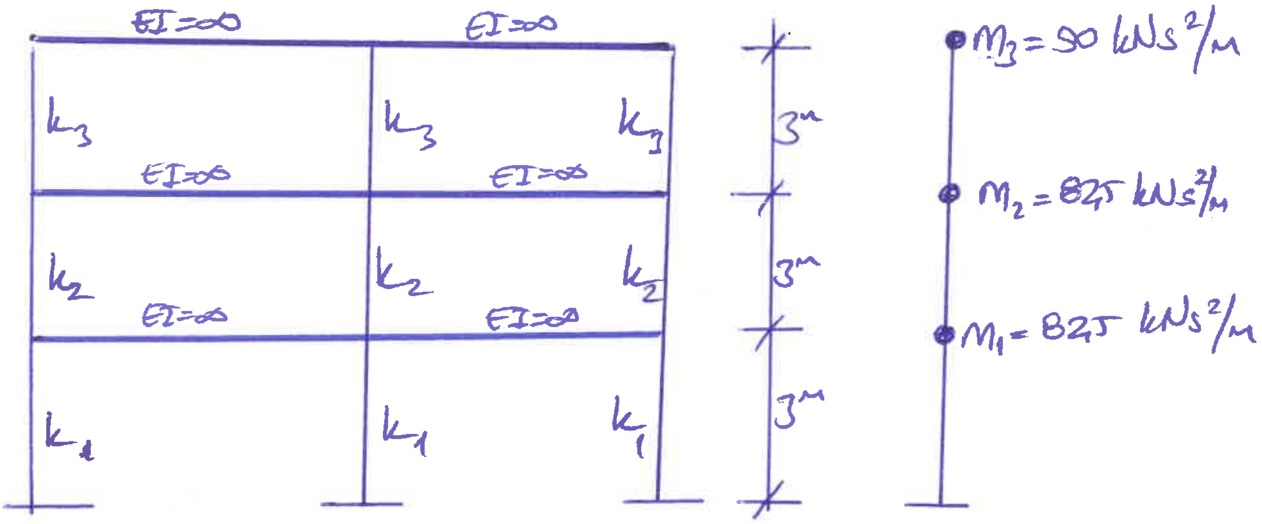
$$\left| \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & +k_2 \end{bmatrix} - \omega^2 \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \right| = 0$$

olarak ele alınacaktır. Buradan denklemin kökleri olan ω_i^2 sistemin doğal açısal frekansları hesaplanacaktır. Hesaplanan ω_i'ler her bir periyoda karşı geleceğinden $T_i = \frac{2\pi}{\omega_i}$ ile serbestlik derecesi kadar periyot bulunabilecektir.

ÖRNEK



$E_c = 2{,}8 \cdot 10^7 \ kN/m^2$



$$k = \frac{12EI}{h^3} = 12 \times 2{,}8 \times 10^7 \times \frac{0{,}30 \cdot 0{,}3^3}{12} \times \frac{1}{3^3} = 8400 \ kN/m$$

Kat rijitliği $\Sigma k_1 = \Sigma k_2 = \Sigma k_3 = 9 \cdot k = 75600 \ kN/m$

$$[K] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 151200 & -75600 & 0 \\ -75600 & 151200 & -75600 \\ 0 & -75600 & 75600 \end{bmatrix}.$$

$$[M] = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} = \begin{bmatrix} 825 & 0 & 0 \\ 0 & 825 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

$$\left| [K] - \omega^2 [M] \right| = 0$$ serbest titreşim

$$\left| \begin{bmatrix} 151200 & -75600 & 0 \\ -75600 & 151200 & -75600 \\ 0 & -75600 & 75600 \end{bmatrix} - \omega^2 \begin{bmatrix} 825 & 0 & 0 \\ 0 & 825 & 0 \\ 0 & 0 & 50 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 151200 - 825\omega^2 & -75600 & 0 \\ -75600 & 151200 - 825\omega^2 & -75600 \\ 0 & -75600 & 75600 - 50\omega^2 \end{vmatrix} = 0$$

$\omega^2 = \lambda$ ile ve determinant çözümü yapılırsa;

$$(151200 - 825\omega^2)\left[(151200 - 825\omega^2)(75600 - 50\omega^2) - (-75600)(-75600)\right]$$
$$-(-75600)\left[(-75600)(75600 - 50\omega^2) - (0)(-75600)\right] = 0$$

elde edilir. Bağıntı yeniden düzenlenirse;

$$3403125\,\lambda^3 - 1761952500\,\lambda^2 + 22718556 \cdot 10^5\,\lambda - 432081216 \cdot 10^6 = 0$$

Denklemin çözümünden;

$$\lambda_1 = \omega_1^2 = 0{,}2291 \cdot 10^3\ (rad/sec)^2 \longrightarrow \omega_1 = 15{,}1357\ rad/s$$

$$\lambda_2 = \omega_2^2 = 1{,}7130 \cdot 10^3\ (rad/sec)^2 \longrightarrow \omega_2 = 41{,}3883\ rad/s$$

$$\lambda_3 = \omega_3^2 = 3{,}2354 \cdot 10^3\ (rad/sec)^2 \longrightarrow \omega_3 = 56{,}8803\ rad/s$$

değerleri elde edilir. Buradan;

$$T_1 = \frac{2\pi}{\omega_1} = 0{,}42\ s.$$

$$T_2 = \frac{2\pi}{\omega_2} = 0{,}15\ s.$$

$$T_3 = \frac{2\pi}{\omega_3} = 0{,}11\ s.$$

olarak periyotlar belirlenir.

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Yukarıda verilen denklemde yığılı kütle olarak ele alınan kütle matrisi yayılı olarak dikkate alınacak ise;

$$[m^e] = \int_V \rho\, \phi^T \phi \, dV = \int_0^1 \phi^T \phi \, (L\,d\xi) \int_A \rho\, dA$$

bağıntısı kullanılacaktır. Bağıntıdaki;

$$\phi^T = \begin{Bmatrix} \phi_i \\ \varphi_i \\ \phi_j \\ \varphi_j \end{Bmatrix} = \begin{Bmatrix} 1-3\xi^2+2\xi^3 \\ L\xi-2L\xi^2+L\xi^3 \\ 3\xi^2-2\xi^3 \\ -L\xi^2+L\xi^3 \end{Bmatrix}$$

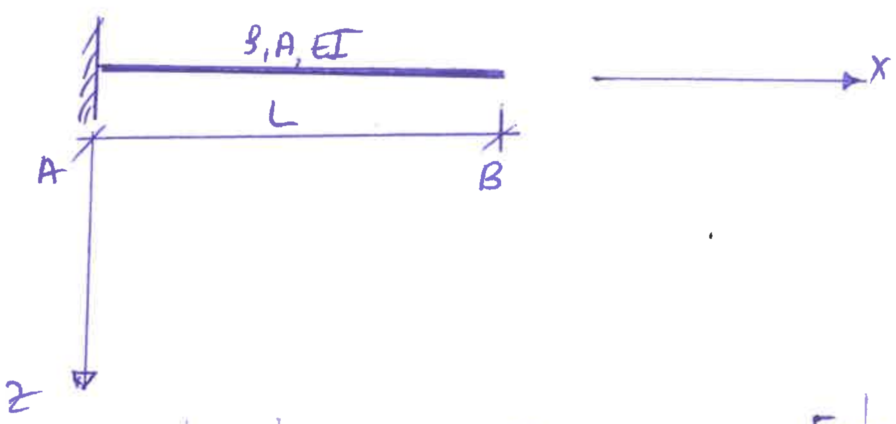
ρ = birim hacim kütledir.

Bu durumda kütle matrisi;

$$[m^e] = L\int_0^1 \begin{bmatrix} 1-3\xi^2+2\xi^3 \\ L(\xi-2\xi^2+\xi^3) \\ 3\xi^2-2\xi^3 \\ L(-\xi^2+\xi^3) \end{bmatrix} \begin{bmatrix} 1-3\xi^2+2\xi^3 & L(\xi-2\xi^2+\xi^3) & 3\xi^2-2\xi^3 & L(-\xi^2+\xi^3) \end{bmatrix} d\xi \int_A \rho\, dA$$

$$= \frac{\rho A L}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

ÖRNEK



$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & 6L & -12 & 6L\\ 6L & 4L^2 & -6L & 2L^2\\ -12 & -6L & 12 & -6L\\ 6L & 2L^2 & -6L & 4L^2\end{bmatrix} - \omega^2\frac{\rho A L}{420}\begin{bmatrix}156 & 22L & 54 & -13L\\ 22L & 4L^2 & 13L & -3L^2\\ 54 & 13L & 156 & -22L\\ -13L & -3L^2 & -22L & 4L^2\end{bmatrix}\right)\begin{Bmatrix}D_1\\ D_2\\ D_3\\ D_4\end{Bmatrix} =$$

$$\left(\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix} - \omega^2\frac{\rho A L}{420}\begin{bmatrix}156 & -22L\\ -22L & 4L^2\end{bmatrix}\right)\begin{Bmatrix}D_3\\ D_4\end{Bmatrix} = 0$$

$\lambda = \omega^2$ De,

$$\left|\begin{bmatrix}\frac{12EI}{L^3} & -\frac{6EI}{L^2}\\ -\frac{6EI}{L^2} & \frac{4EI}{L}\end{bmatrix} - \begin{bmatrix}\frac{156\rho A L}{420}\lambda & -\frac{22}{420}\rho A L^2\lambda\\ -\frac{22}{420}\rho A L^2\lambda & +\frac{4}{420}\rho A L^3\lambda\end{bmatrix}\right| = 0$$

$$\begin{vmatrix}\frac{12EI}{L^3} - \frac{156\rho A L}{420}\lambda & -\frac{6EI}{L^2} + \frac{22}{420}\rho A L^2\lambda\\ -\frac{6EI}{L^2} + \frac{22}{420}\rho A L^2\lambda & \frac{4EI}{L} - \frac{4}{420}\rho A L^3\lambda\end{vmatrix} = 0$$

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$$\frac{48(EI)^2}{L^4} - \frac{48}{420}\rho\frac{AL^3}{L^3}EI\lambda - \frac{624}{420}EI\rho A\lambda + \frac{624}{420^2}\rho^2A^2L^4\lambda^2$$

$$-\left[\frac{36(EI)^2}{L^4} - \frac{132}{420}\frac{EI}{L^2}\rho AL^2\lambda - \frac{132}{420}\frac{EI}{L^2}\rho AL^2\lambda + \frac{484}{420^2}\rho^2A^2L^4\lambda^2\right] = 0$$

$$\alpha = \omega^2\frac{\rho AL^4}{EI} \quad \text{De;}$$

$$\frac{12(EI)^2}{L^4} - \alpha\left(\frac{48}{420}\frac{(EI)^2}{L^4}\right) - \alpha\left(\frac{624}{420}\frac{(EI)^2}{L^4}\right) + \alpha^2\left(\frac{624}{420^2}\frac{(EI)^2}{L^4}\right)$$
$$+\alpha\left(\frac{132}{420}\frac{(EI)^2}{L^4}\right) + \alpha\left(\frac{132}{420}\frac{(EI)^2}{L^4}\right) - \alpha^2\left(\frac{484}{420^2}\frac{(EI)^2}{L^4}\right) = 0$$

$$\frac{12(EI)^2}{L^4} - \frac{408}{420}\frac{(EI)^2}{L^4}\alpha + \frac{1}{1260}\frac{(EI)^2}{L^4}\alpha^2 = 0$$

$$12 - 0{,}971\,\alpha + \frac{1}{1260}\alpha^2 = 0$$

$$\alpha_1 = 12{,}48 \qquad \alpha_2 = 1211{,}52$$

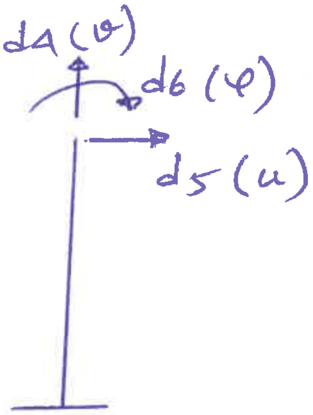
$$12{,}48 = \omega_1^2\frac{\rho AL^4}{EI} \longrightarrow \omega_1 = \sqrt{\frac{12{,}48\,EI}{\rho AL^4}}$$

$$1211{,}52 = \omega_2^2\frac{\rho AL^4}{EI} \longrightarrow \omega_2 = \sqrt{\frac{1211{,}52\,EI}{\rho AL^4}}$$

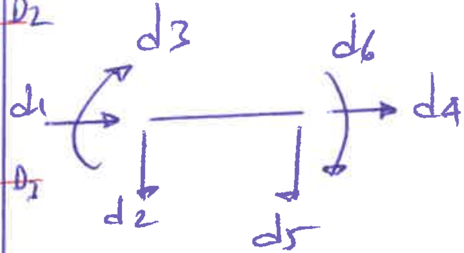
ÖRNEK

$M = 10\ kNs^2/m$

$EI = 20250\ kNm^2$



$$[K_e] = \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{cccccc|c} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & \\ \frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 & D_1 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & D_2 \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & D_3 \\ \frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 & D_4 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & D_5 \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & D_6 \end{array}$$



Boyuna yerdeğiştirme olmayacağından $D_4 = 0$

$$\left| [K] - \omega^2 [M] \right| = 0$$

$$\left| \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} - \omega^2 \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 9000 & -13500 \\ -13500 & 27000 \end{bmatrix} - \omega^2 \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 9000 - 10\omega^2 & -13500 \\ -13500 & 27000 - 10\omega^2 \end{vmatrix} = 0$$

$\omega^2 = \lambda$ ise

$$(9000 - 10\lambda)(27000 - 10\omega^2) - (-13500)(-13500) = 0$$

$$243000000 - 90000\lambda - 270000\lambda + 100\lambda^2 - 13500^2 = 0$$

$$60750000 - 360000\lambda + 100\lambda^2 = 0$$

$$607500 - 3600\lambda + \lambda^2 = 0$$

$$\lambda_1 = \omega_1^2 = 177{,}50 \longrightarrow \omega_1 = 13{,}32 \text{ rad/s}$$

$$\lambda_2 = \omega_2^2 = 3422{,}50 \longrightarrow \omega_2 = 58{,}50 \text{ rad/s}$$

$$T_1 = \frac{2\pi}{\omega_1} = 0{,}47 \text{ sn.}$$

$$T_2 = \frac{2\pi}{\omega_2} = 0{,}10 \text{ sn.}$$

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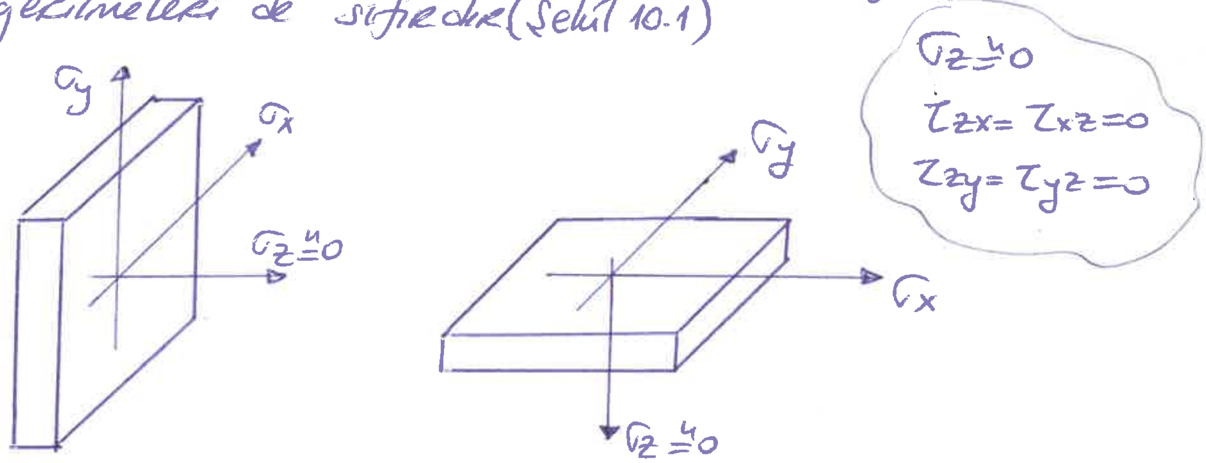
BÖLÜM VI

10) DÜZLEM GERİLME VE DÜZLEM ŞEKİL DEĞİŞTİRME HALİ

10.1) Düzlem Gerilme Hali

Bir elemanın ayrıma kesitindeki gerilme vektörlerinin doğrultuları aynı düzlem içinde ise buna iki eksenli gerilme hali veya düzlem gerilme hali denir. Levha veya plak gibi elemanlar bu gerilme hali ile ifade edilirler. Düzlem gerilme halinde eleman kalınlığı boyunca gerilmeler sıfırdır. Buna bağlı olarak da kayma gerilmeleri de sıfırdır (Şekil 10.1)



Şekil 10.1. Düzlem gerilme hali

Bu durumda gerilme tansörü üçü bağımsız olmak üzere 4 skaler büyüklükten oluşacaktır.

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} \qquad |\tau_{xy}| = |\tau_{yx}| \quad \text{------} (1)$$

Birim şekil değiştirmeler ise;

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix}$$

şeklindedir. ------ (2)

Gerilmeleri şekil değiştirmeler cinsinden yazarsak;

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D]\{\varepsilon\} = [D]\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{-----} (3)$$

İzotrop bir malzemede gerilmeler cinsinden elastisite bağıntıları;

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y) \quad \text{---------} (4)$$

$$\sigma_z = 0$$

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{-------} (5)$$

$$\{\sigma\} = [D]\{\varepsilon\} \quad \text{--------------} (6)$$

Birim şekil değiştirmeler cinsinden elastisite bağıntıları;

$$\left.\begin{aligned} \varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) \\ \varepsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) \end{aligned}\right\} \varepsilon_z = -\frac{\nu}{(1-\nu)}(\varepsilon_x + \varepsilon_y) \quad \text{----} (7)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

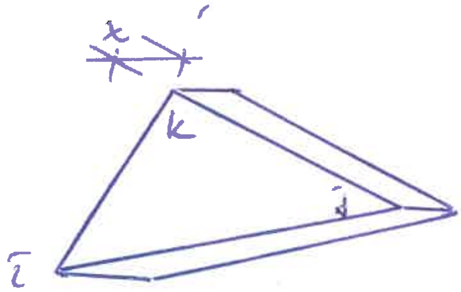
KO

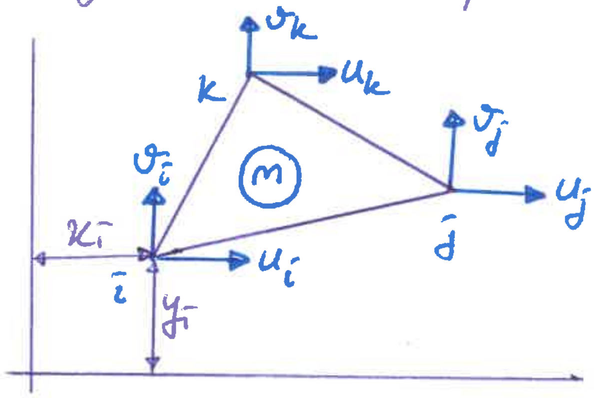
$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad \text{------} \quad (8)$$

$$\{\varepsilon\} = [D]^{-1}\{\sigma\} \quad \text{------} \quad (9)$$

10.2) İnce Levha

t kalınlığında üçgen bir levha alırsak ve bu elemanın her düğüm noktasında u ve v yer değiştirmelerine sahip olduğunu düşünürsek;





Buradaki yer değiştirme;

$$\{d_i\} = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \qquad \{d_j\} = \begin{Bmatrix} u_j \\ v_j \end{Bmatrix} \qquad \{d_k\} = \begin{Bmatrix} u_k \\ v_k \end{Bmatrix} \quad \text{----} \quad (10)$$

$$\{d\}_e = \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix}$$ şeklinde olacaktır. ------ (11)

Şekilden de görüleceği üzere üçgen levha için her düğüm noktasında x ve y yönündeki yer değiştirmelerden dolayı 2 serbestlik derecesi bulunmaktadır. Toplam 6 serbestlik derecesi bulunan elemanda düğüm noktalarındaki yer değiştirmelerin koordinatları lineer polinom olarak;

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y$$
$$v = \alpha_4 + \alpha_5 x + \alpha_6 y \qquad \text{--------- (12)}$$

şeklinde tanımlanabilir. Matris formunda;

$$u^m = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$
$$\text{--------- (13)}$$
$$v^m = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix}$$

şeklinde gösterilebilir. Bu durumda düğüm noktası koordinatlarına göre yer değiştirme fonksiyonları;

$$u_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i$$
$$v_i = \alpha_4 + \alpha_5 x_i + \alpha_6 y_i$$

$$u_j = \alpha_1 + \alpha_2 x_j + \alpha_3 y_j \qquad \text{--------- (14)}$$
$$v_j = \alpha_4 + \alpha_5 x_j + \alpha_6 y_j$$

$$u_k = \alpha_1 + \alpha_2 x_k + \alpha_3 y_k$$
$$v_k = \alpha_4 + \alpha_5 x_k + \alpha_6 y_k$$

şeklinde yazılabilir. Matris formunda yazılırsa;

$$\left\{u^m_{(x,y)}\right\} = \begin{bmatrix} u_i \\ \vartheta_i \\ u_j \\ \vartheta_j \\ u_k \\ \vartheta_k \end{bmatrix} = \begin{bmatrix} u^m_i \\ u^m_j \\ u^m_k \end{bmatrix} \quad \text{---------- (15)}$$

$$\left\{u^m_{(x,y)}\right\} = \begin{bmatrix} u_i \\ \vartheta_i \\ u_j \\ \vartheta_j \\ u_k \\ \vartheta_k \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i & y_i \\ 1 & x_j & y_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_j & y_j \\ 1 & x_k & y_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{bmatrix} \quad \text{------ (16)}$$

şeklinde elde edilir. Buradan kapalı ifade;

$\left\{u^m_{(x,y)}\right\} = \left[h_{(x,y)}\right]\left\{\alpha\right\}$ (17) şeklindedir. Buradaki $h(x,y)$ yaklaşım matrisi, ele alınan (m) elemanının i, j veya k gibi herhangi bir noktasının yer değiştirme vektörünü, düğüm noktalarının yer değiştirme vektörü ile tanımlar.

(14) bağıntısıyla ifade edilen yer değiştirme fonksiyonu α bilinmeyen katsayılarından ve $u_{(x,y)}$ düğüm noktası koordinatlarından oluşmaktadır. Matris formunda ifade edilen (17) bağıntısı düzenlenirse;

$$\left\{\alpha\right\} = \left[h_{(x,y)}\right]^{-1}\left\{u^m_{(x,y)}\right\} \quad \text{----- (18)}$$

olur.

$$\left[h_{(x,y)}\right]^{-1} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & h_{36} \\ h_{41} & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} \\ h_{51} & h_{52} & h_{53} & h_{54} & h_{55} & h_{56} \\ h_{61} & h_{62} & h_{63} & h_{64} & h_{65} & h_{66} \end{bmatrix} \text{------} (19)$$

$$h_{11} = \left[(x_j * y_k - x_k * y_j)/(x_i * y_j - x_j * y_i - x_i * y_k + x_k * y_i + x_j * y_k - x_k * y_j)\right]$$

$$h_{21} = \left[(y_j - y_k)/(x_i * y_j - x_j * y_i - x_i * y_k + x_k * y_i + x_j * y_k - x_k * y_j)\right]$$

$$h_{31} = \left[-(x_j - x_k)/(x_i * y_j - x_j * y_i - x_i * y_k + x_k * y_i + x_j * y_k - x_k * y_j)\right]$$

$$h_{41} = 0$$

$$h_{51} = 0$$

$$h_{61} = 0$$

$$h_{12} = 0$$

$$h_{22} = 0$$

$$h_{32} = 0$$

$$h_{42} = \left[(x_j * y_k - x_k * y_j)/(x_i * y_j - x_j * y_i - x_i * y_k + x_k * y_i + x_j * y_k - x_k * y_j)\right]$$

$$h_{52} = \left[(y_j - y_k)/(x_i * y_j - x_j * y_i - x_i * y_k + x_k * y_i + x_j * y_k - x_k * y_j)\right]$$

$$h_{62} = \left[-(x_j - x_k)/(x_i * y_j - x_j * y_i - x_i * y_k + x_k * y_i + x_j * y_k - x_k * y_j)\right]$$

$$h_{13}=\left[-(x_i^* y_k - x_k^* y_i)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{23}=\left[-(y_i - y_k)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{33}=\left[(x_i - x_k)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{43}=0$$

$$h_{53}=0$$

$$h_{63}=0$$

$$h_{14}=0$$

$$h_{24}=0$$

$$h_{34}=0$$

$$h_{44}=\left[-(x_i^* y_k - x_k^* y_i)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{54}=\left[-(y_i - y_k)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{64}=\left[(x_i - x_k)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_i)\right]$$

$$h_{15}=\left[(x_i^* y_j - x_j^* y_i)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_i)\right]$$

$$h_{25}=\left[(y_i - y_j)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y + x_j^* y_k - x_k^* y_i)\right]$$

$$h_{35}=\left[-(x_i - x_j)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_i)\right]$$

$$h_{45}=0$$

$$h_{55}=0$$

$$h_{65}=0$$

$h_{16}=0$

$h_{26}=0$

$h_{36}=0$

$$h_{46}=\left[(x_i^* y_j - x_j^* y_i)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{56}=\left[(y_i - y_j)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

$$h_{66}=\left[-(x_i - x_j)/(x_i^* y_j - x_j^* y_i - x_i^* y_k + x_k^* y_i + x_j^* y_k - x_k^* y_j)\right]$$

Buradan;

$$\{\alpha\} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & h_{16} \\ h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & h_{26} \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & h_{36} \\ h_{41} & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} \\ h_{51} & h_{52} & h_{53} & h_{54} & h_{55} & h_{56} \\ h_{61} & h_{62} & h_{63} & h_{64} & h_{65} & h_{66} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix}$$ xe;

Bunun sonucunda;

$$\alpha_1 = \frac{x_j y_k - x_k y_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)} u_i +$$

$$\frac{-x_i y_k + x_k y_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)} u_j +$$

$$\frac{x_i y_j - x_j y_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)} u_k$$

$$\alpha_2 = \frac{y_j - y_k}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, u_i$$

$$+ \frac{y_k - y_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, u_j$$

$$+ \frac{y_i - y_j}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, u_k$$

$$\alpha_3 = \frac{x_k - x_j}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, u_i$$

$$+ \frac{x_i - x_k}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, u_j$$

$$+ \frac{x_j - x_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, u_k$$

$$\alpha_4 = \frac{x_j y_k - x_k y_j}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, v_i$$

$$+ \frac{x_k y_i - x_i y_k}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, v_j$$

$$+ \frac{x_i y_j - x_j y_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, v_k$$

$$\alpha_5 = \frac{y_j - y_k}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, v_i$$

$$+ \frac{y_k - y_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, v_j$$

$$+ \frac{y_i - y_j}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)}\, v_k$$

$$\alpha_6 = \frac{x_k - x_j}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)} v_i$$

$$+ \frac{x_i - x_k}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)} v_j$$

$$+ \frac{x_j - x_i}{x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j)} v_k$$

$\alpha_1, \alpha_2\, \alpha_3, \alpha_4, \alpha_5$ ve α_6 bağıntılarının paydası determinantlar ve;

$$2\Delta = \det \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j) \quad \text{-----(20)}$$

şeklindedir. Bu durumda α'lar;

$$\alpha_1 = \frac{1}{2\Delta}\left[(x_j y_k - x_k y_j)u_i + (x_k y_i - x_i y_k)u_j + (x_i y_j - x_j y_i)u_k\right]$$

$$\alpha_2 = \frac{1}{2\Delta}\left[(y_j - y_k)u_i + (y_k - y_i)u_j + (y_i - y_j)u_k\right]$$

$$\alpha_3 = \frac{1}{2\Delta}\left[(x_k - x_j)u_i + (x_i - x_k)u_j + (x_j - x_i)u_k\right] \quad \text{------(21)}$$

$$\alpha_4 = \frac{1}{2\Delta}\left[(x_j y_k - x_k y_j)v_i + (x_k y_i - x_i y_k)v_j + (x_i y_j - x_j y_i)v_k\right]$$

$$\alpha_5 = \frac{1}{2\Delta}\left[(y_j - y_k)v_i + (y_k - y_i)v_j + (y_i - y_j)v_k\right]$$

$$\alpha_6 = \frac{1}{2\Delta}\left[(x_k - x_j)v_i + (x_i - x_k)v_j + (x_j - x_i)v_k\right]$$

şeklini alacaktır

(14) bağıntısını (12) bağıntısına adapte edersek;

$$u = (\alpha_1+\alpha_2 x_i+\alpha_3 y_i)u_i + (\alpha_1+\alpha_2 x_j+\alpha_3 y_j)u_j + (\alpha_1+\alpha_2 x_k+\alpha_3 y_k)u_k$$

$$v = (\alpha_4+\alpha_5 x_i+\alpha_6 y_i)v_i + (\alpha_4+\alpha_5 x_j+\alpha_6 y_j)v_j + (\alpha_4+\alpha_5 x_k+\alpha_6 y_k)v_k$$

şeklini alır. Buradaki α değerleri için aşağıdaki kısaltmaları yaparsak (21) bağıntısı;

$a_i = x_j y_k - x_k y_j$	$a_j = x_k y_i - x_i y_k$	$a_k = x_i y_j - x_j y_i$
$b_i = y_j - y_k$	$b_j = y_k - y_i$	$b_k = y_i - y_j$
$c_i = x_k - x_j$	$c_j = x_i - x_k$	$c_k = x_j - x_i$

$$\alpha_1 = \frac{1}{2\Delta}\left[a_i u_i + a_j u_j + a_k u_k\right]$$

$$\alpha_2 = \frac{1}{2\Delta}\left[b_i u_i + b_j u_j + b_k u_k\right]$$

$$\alpha_3 = \frac{1}{2\Delta}\left[c_i u_i + c_j u_j + c_k u_k\right] \quad \text{--------- (22)}$$

$$\alpha_4 = \frac{1}{2\Delta}\left[a_i v_i + a_j v_j + a_k v_k\right]$$

$$\alpha_5 = \frac{1}{2\Delta}\left[b_i v_i + b_j v_j + b_k v_k\right]$$

$$\alpha_6 = \frac{1}{2\Delta}\left[c_i v_i + c_j v_j + c_k v_k\right]$$

şeklini alır. u ve v bağıntıları ise;

$$u=\frac{1}{2\Delta}\left[(a_i+b_ix+c_iy)u_i+(a_j+b_jx+c_jy)u_j+(a_k+b_kx+c_ky)u_k\right]$$

$$v=\frac{1}{2\Delta}\left[(a_i+b_ix+c_iy)v_i+(a_j+b_jx+c_jy)v_j+(a_k+b_kx+c_ky)v_k\right]$$

------ (23)

(23) bağıntısındaki parantez içi terimler düğüm noktalarına ait şekil fonksiyonlarını ifade etmektedir.

$$N_i=\frac{1}{2\Delta}(a_i+b_ix+c_iy)$$

$$N_j=\frac{1}{2\Delta}(a_j+b_jx+c_jy) \qquad \text{---------- (24)}$$

$$N_k=\frac{1}{2\Delta}(a_k+b_kx+c_ky)$$

Matris formunda yazılırsa;

$$\{u\}_e=[N]\{d\}$$

$$\begin{Bmatrix}u\\v\end{Bmatrix}=\begin{bmatrix}N_i & 0 & N_j & 0 & N_k & 0\\ 0 & N_i & 0 & N_j & 0 & N_k\end{bmatrix}\begin{Bmatrix}u_i\\v_i\\u_j\\v_j\\u_k\\v_k\end{Bmatrix} \qquad \text{----- (25)}$$

olur. Birim şekil değiştirmeler;

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\underbrace{[\Delta][N]}_{[G]}\{d\}=[G]\{d\} \text{ 'dir.}$$

$$\{\varepsilon\}=\frac{1}{2\Delta}\begin{bmatrix}\frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix}\begin{bmatrix}N_i & 0 & N_j & 0 & N_k & 0\\ 0 & N_i & 0 & N_j & 0 & N_k\end{bmatrix}\{d\}$$

$$\{\varepsilon\} = \frac{1}{2\Delta}\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & \frac{\partial N_j}{\partial x} & 0 & \frac{\partial N_k}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 & \frac{\partial N_j}{\partial y} & 0 & \frac{\partial N_k}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & \frac{\partial N_j}{\partial y} & \frac{\partial N_j}{\partial x} & \frac{\partial N_k}{\partial y} & \frac{\partial N_k}{\partial x} \end{bmatrix}\{d\}$$

$$\{\varepsilon\} = \frac{1}{2\Delta}\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix}$$

İzotrop malzeme için gerilme bağıntıları;

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D]\{\varepsilon\} = [D][G]\{d\} \quad \text{------} (26)$$

$$\{\sigma\} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\frac{1}{2\Delta}\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix}$$

$$\{\sigma\} = \frac{E}{2\Delta(1-\nu^2)} \begin{bmatrix} b_i & \nu c_i & b_j & \nu c_j & b_k & \nu c_k \\ \nu b_i & c_i & \nu b_j & c_j & \nu b_k & c_k \\ \frac{1-\nu}{2} c_i & \frac{1-\nu}{2} b_i & \frac{1-\nu}{2} c_j & \frac{1-\nu}{2} b_j & \frac{1-\nu}{2} c_k & \frac{1-\nu}{2} b_k \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{Bmatrix}$$

$$\{\sigma\} = [D][G]\{d\} \text{ elde edilir.} \quad \text{------} (27)$$

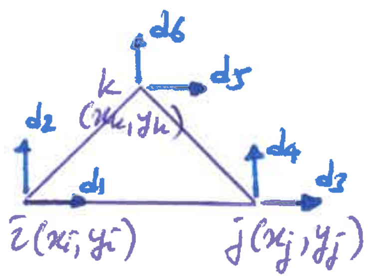
Eleman rijitlik matrisi ise;

$$[k_e] = \int_V [G]^T [D][G]\, dV \quad \text{----------} (28)$$

$$[k_e] = \frac{Et}{4\Delta^2(1-\nu^2)} \int \begin{bmatrix} b_i & 0 & c_i \\ 0 & c_i & b_i \\ b_j & 0 & c_j \\ 0 & c_j & b_j \\ b_k & 0 & c_k \\ 0 & c_k & b_k \end{bmatrix} \begin{bmatrix} b_i & \nu c_i & b_j & \nu c_j & b_k & \nu c_k \\ \nu b_i & c_i & \nu b_j & c_j & \nu b_k & c_k \\ \frac{1-\nu}{2} c_i & \frac{1-\nu}{2} b_i & \frac{1-\nu}{2} c_j & \frac{1-\nu}{2} b_j & \frac{1-\nu}{2} c_k & \frac{1-\nu}{2} b_k \end{bmatrix} dxdy$$

$$[k_e] = \frac{Et}{4\Delta^2(1-\nu^2)} \iint \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} dxdy \quad \text{---} (29)$$

$$[k_e] = \frac{Et}{4\Delta^2(1-\upsilon^2)} \iint \begin{bmatrix} b_i^2 + (\frac{1-\upsilon}{2})c_i^2 & \upsilon b_i c_i + (\frac{1-\upsilon}{2})b_i c_i & b_i b_j + (\frac{1-\upsilon}{2})c_i c_j & \upsilon b_i c_j + (\frac{1-\upsilon}{2})b_j c_i & b_i b_k + (\frac{1-\upsilon}{2})c_i c_k & \upsilon b_i c_k + (\frac{1-\upsilon}{2})b_k c_i \\ \upsilon b_i c_i + (\frac{1-\upsilon}{2})b_i c_i & \left(\frac{1-\upsilon}{2}\right) b_i^2 + c_i^2 & \upsilon b_j c_i + (\frac{1-\upsilon}{2})b_i c_j & c_i c_j + (\frac{1-\upsilon}{2})b_i b_j & \upsilon b_k c_i + (\frac{1-\upsilon}{2})b_i c_k & c_i c_k + (\frac{1-\upsilon}{2})b_i b_k \\ b_i b_j + (\frac{1-\upsilon}{2})c_i c_j & \upsilon b_j c_i + (\frac{1-\upsilon}{2})b_i c_j & b_j^2 + (\frac{1-\upsilon}{2})c_j^2 & \upsilon b_j c_j + (\frac{1-\upsilon}{2})b_j c_j & b_j b_k + (\frac{1-\upsilon}{2})c_j c_k & \upsilon b_j c_k + (\frac{1-\upsilon}{2})b_k c_j \\ \upsilon b_i c_j + (\frac{1-\upsilon}{2})b_j c_i & c_i c_j + (\frac{1-\upsilon}{2})b_i b_j & \upsilon b_j c_j + (\frac{1-\upsilon}{2})b_j c_j & (\frac{1-\upsilon}{2})b_j^2 + c_j^2 & \upsilon b_k c_j + (\frac{1-\upsilon}{2})b_j c_k & c_j c_k + (\frac{1-\upsilon}{2})b_j b_k \\ b_i b_k + (\frac{1-\upsilon}{2})c_i c_k & \upsilon b_k c_i + (\frac{1-\upsilon}{2})b_i c_k & b_j b_k + (\frac{1-\upsilon}{2})c_j c_k & \upsilon b_k c_j + (\frac{1-\upsilon}{2})b_j c_k & b_k^2 + (\frac{1-\upsilon}{2})c_k^2 & \upsilon b_k c_k + (\frac{1-\upsilon}{2})b_k c_k \\ \upsilon b_i c_k + (\frac{1-\upsilon}{2})b_k c_i & c_i c_k + (\frac{1-\upsilon}{2})b_i b_k & \upsilon b_j c_k + (\frac{1-\upsilon}{2})b_k c_j & c_j c_k + (\frac{1-\upsilon}{2})b_j b_k & \upsilon b_k c_k + (\frac{1-\upsilon}{2})b_k c_k & (\frac{1-\upsilon}{2})b_k^2 + c_k^2 \end{bmatrix} dx\,dy \quad \text{--(30)}$$



$a_i = x_j y_k - x_k y_j$

$b_i = y_j - y_k$

$c_i = x_k - x_j$

$a_j = x_k y_i - y_k x_i$

$b_j = y_k - y_i$

$c_j = x_i - x_k$

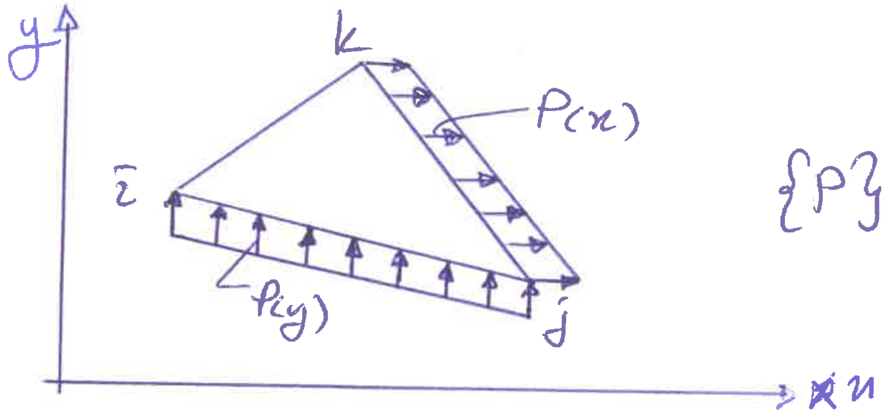
$a_k = x_i y_j - x_j y_i$

$b_k = y_i - y_j$

$c_k = x_j - x_i$

10.2.1) Yük Matrisi

a) Düzgün Yayılı Yük Durumu (Edge load)



$$\{P\} = \begin{Bmatrix} P_x \\ P_y \end{Bmatrix}$$

Sisteme etkiyen yayılı yük olması durumunda;

$$\{q\}_s = -\int [N]^T \{P\}\, ds$$

$$\{q_i\} = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_i\, dx\, dy$$

$$\{q_j\} = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_j\, dx\, dy \quad \text{-------------} (31)$$

$$\{q_k\} = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_k\, dx\, dy$$

N_i, N_j ve N_k şekil fonksiyonlarını (31) bağıntısındaki yerlerine yazarsak;

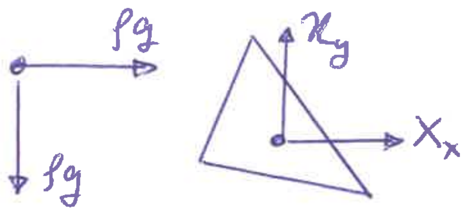
$$\{q_i\} = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int \frac{1}{2\Delta}(a_i + b_i x + c_i y)\, dx\, dy$$

$$= -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int \frac{1}{2\Delta} a_i\, dx\, dy = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \frac{a_i}{2} \quad \text{------} (32)$$

$$\{q_j\} = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int \frac{1}{2\Delta}(a_j + b_j x + c_j y)\, dx\, dy$$

$$\{q_k\} = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int \frac{1}{2\Delta}(a_k + b_k x + c_k y)\, dx\, dy$$

b) Kütlesel Yük (Body Forces)



$$\{X\} = \begin{Bmatrix} X_x(x,y) \\ X_y(x,y) \end{Bmatrix} = \begin{Bmatrix} \rho g \\ -\rho g \end{Bmatrix}$$

ρ = yoğunluk

W = ağırlık = $At\,(\rho g)$

$$\{X\} = \begin{Bmatrix} \rho g \\ -\rho g \end{Bmatrix}$$

$$\{q\}_X = -\int_V [N]^T \{X\}\, dV \quad \text{------} \quad (33)$$

c) Yayılı Kütle (Consistent mass)

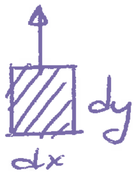
$$[M] = \int_V \rho [N]^T [N]\, dV \quad \text{------} \quad (34)$$

$$[M] = \begin{bmatrix} 1/6 & 1/12 & 1/12 & 0 & 0 & 0 \\ 1/12 & 1/6 & 1/12 & 0 & 0 & 0 \\ 1/12 & 1/12 & 1/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/6 & 1/12 & 1/12 \\ 0 & 0 & 0 & 1/12 & 1/6 & 1/12 \\ 0 & 0 & 0 & 1/12 & 1/12 & 1/6 \end{bmatrix} A \cdot \rho \cdot t \quad \text{------} \quad (35)$$

d) Isı Değişimi

$$\{q\}_t = -\alpha_t(\Delta t)\left(\int_v [G]^T dv\right)\{D_T\} \quad \text{------} \quad (36)$$

Isı değişiminde elastisite matrisi $\{D_T\}$;



$\rightarrow$

$$\varepsilon_x = \alpha_{tx}(\Delta t)$$
$$\varepsilon_y = \alpha_{ty}(\Delta t)$$
$$\gamma_{xy} = 0$$

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \alpha_{tx}(\Delta T) \\ \alpha_{ty}(\Delta T) \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \begin{Bmatrix} \alpha_{tx} \cdot \Delta T \\ \alpha_{ty} \cdot \Delta T \\ 0 \end{Bmatrix} = \begin{Bmatrix} \alpha_{tx} D_{11} + \alpha_{ty} D_{12} \\ \alpha_{tx} D_{21} + \alpha_{ty} D_{22} \\ 0 \end{Bmatrix} \Delta T$$

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$= \begin{Bmatrix} D_{11}+D_{12} \\ D_{21}+D_{22} \\ 0 \end{Bmatrix} \alpha \cdot \Delta T$$

malzeme ortotrop ise α_{tx}, α_{ty}'dir.

" izotrop ise $\alpha_{tx} = \alpha_{ty} = \alpha_t$

izotrop malzemede;

$$D_{11} + D_{12} = \frac{E}{1-\nu^2} + \frac{\nu E}{1-\nu^2} = \frac{E}{1-\nu}$$

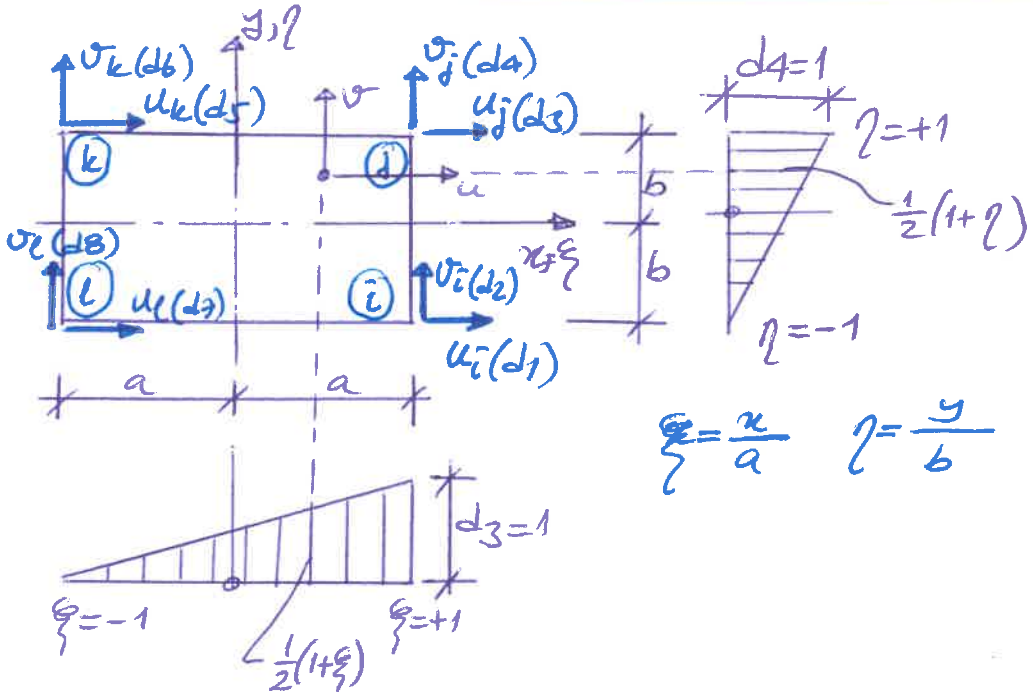
$$D_{21} + D_{22} = \frac{E}{1-\nu}$$

$$= \alpha \cdot \Delta T \cdot \frac{E}{1-\nu} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

elde edilir.

$$\{D\}_T = \frac{E}{1-\nu} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

10.3) İnce Levhaların Dörtgen Elemanlarla Hesabı



$$\xi=\frac{x}{a} \qquad \eta=\frac{y}{b}$$

Şekil fonksiyonu $\quad N_i=\frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)$ ------ (37)

$$\{u\}_e=[N]\{d\}$$

$$\begin{Bmatrix} u \\ v \end{Bmatrix}=\begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 & N_l & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k & 0 & N_l \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \\ u_l \\ v_l \end{Bmatrix} \quad \text{------} \quad (38)$$

$$\{d\}^T=[u_i, v_i;\ u_j, v_j;\ u_k, v_k;\ u_l, v_l] \quad \text{- - - - - - -} \quad (39)$$

Seçilen dörtgen elemanın her düğüm noktasında 2 yerdeğiştirme bileşeni bulunmaktadır. Elemanın şekil fonksiyonu (37) bağıntısı ile verilmiştir. Buna göre düğüm noktalarının şekil fonksiyonları;

$$
\begin{array}{l|ll}
N_i = \frac{1}{4}(1+\xi)(1+\eta) & \xi_i = +1 & \eta_i = +1 \\
N_j = \frac{1}{4}(1+\xi)(1-\eta) & \xi_i = +1 & \eta_i = -1 \\
N_k = \frac{1}{4}(1-\xi)(1+\eta) & \xi_i = -1 & \eta_i = +1 \\
N_l = \frac{1}{4}(1-\xi)(1-\eta) & \xi_i = -1 & \eta_i = -1
\end{array}
\quad \text{------} (40)
$$

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial u/\partial y + \partial v/\partial x \end{Bmatrix}
$$

$$
\{\varepsilon\} = \begin{bmatrix}
\frac{\partial N_i}{\partial x} & 0 & \frac{\partial N_j}{\partial x} & 0 & \frac{\partial N_k}{\partial x} & 0 & \frac{\partial N_l}{\partial x} & 0 \\
0 & \frac{\partial N_i}{\partial y} & 0 & \frac{\partial N_j}{\partial y} & 0 & \frac{\partial N_k}{\partial y} & 0 & \frac{\partial N_l}{\partial y} \\
\frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & \frac{\partial N_j}{\partial y} & \frac{\partial N_j}{\partial x} & \frac{\partial N_k}{\partial y} & \frac{\partial N_k}{\partial x} & \frac{\partial N_l}{\partial y} & \frac{\partial N_l}{\partial x}
\end{bmatrix} \{d\}
$$

$$
\frac{\partial N_i}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{4}(1+\xi+\eta+\xi\eta)\right] = \frac{1}{4a}(1+\eta)
$$

$$
\frac{\partial N_j}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{4}(1+\xi-\eta-\xi\eta)\right] = \frac{1}{4a}(1-\eta)
$$

$$
\frac{\partial N}{\partial x} = \frac{\partial N}{\partial \xi}\frac{\partial \xi}{\partial x} = \frac{\partial N}{\partial \xi}\cdot\frac{1}{a}
$$

$$
\frac{\partial N}{\partial y} = \frac{\partial N}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial N}{\partial \eta}\cdot\frac{1}{b}
$$

------ (41)

$$
\frac{\partial N_k}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{4}(1-\xi+\eta-\xi\eta)\right] = -\frac{1}{4a}(1+\eta)
$$

$$
\frac{\partial N_l}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{4}(1-\xi-\eta+\xi\eta)\right] = \frac{1}{4a}(-1+\eta)
$$

$$
= -\frac{1}{4a}(1-\eta)
$$

$$\frac{\partial N_i}{\partial y} = \frac{1}{4b}(1+\xi) \quad ; \quad \frac{\partial N_j}{\partial y} = -\frac{1}{4b}(1+\xi)$$

$$\frac{\partial N_k}{\partial y} = \frac{1}{4b}(1-\xi) \quad ; \quad \frac{\partial N_l}{\partial y} = -\frac{1}{4b}(1-\xi)$$

$$\{\varepsilon\} = \frac{1}{4ab}\begin{bmatrix} b(1+\eta) & 0 & b(1-\eta) & 0 & -b(1+\eta) & 0 & -b(1-\eta) & 0 \\ 0 & a(1+\xi) & 0 & -a(1+\xi) & 0 & a(1-\xi) & 0 & -a(1-\xi) \\ a(1+\xi) & b(1+\eta) & -a(1+\xi) & b(1-\eta) & a(1-\xi) & -b(1+\eta) & -a(1-\xi) & -b(1-\eta) \end{bmatrix}\{d\} \qquad (42)$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D]\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad [D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Izotropna ravninska tah

$$\{\varepsilon\} = \underbrace{[\Delta][N]}_{[G]}\{d\} \longrightarrow \{\sigma\} = [D][G]\{d\}$$

$$\{\sigma\} = \frac{1}{4ab}\underbrace{\frac{E}{(1-\nu^2)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}}_{[D]}\underbrace{\begin{bmatrix} b(1+\eta) & 0 & b(1-\eta) & 0 & -b(1+\eta) & 0 & -b(1-\eta) & 0 \\ 0 & a(1+\xi) & 0 & -a(1+\xi) & 0 & a(1-\xi) & 0 & -a(1-\xi) \\ a(1+\xi) & b(1+\eta) & -a(1+\xi) & b(1-\eta) & a(1-\xi) & -b(1+\eta) & -a(1-\xi) & -b(1-\eta) \end{bmatrix}}_{[G]}\{d\}$$

$$\{\sigma\} = \frac{E}{4ab(1-\upsilon^2)}\begin{bmatrix} b(1+\eta) & \upsilon a(1+\xi) & b(1-\eta) & -\upsilon a(1+\xi) & -b(1+\eta) & \upsilon a(1-\xi) & -b(1-\eta) & -\upsilon a(1-\xi) \\ \upsilon b(1+\eta) & a(1+\xi) & \upsilon b(1-\eta) & -a(1+\xi) & -\upsilon b(1+\eta) & a(1-\xi) & -\upsilon b(1-\eta) & -a(1-\xi) \\ a\left(\frac{1-\upsilon}{2}\right)(1+\xi) & b\left(\frac{1-\upsilon}{2}\right)(1+\eta) & -a\left(\frac{1-\upsilon}{2}\right)(1+\xi) & b\left(\frac{1-\upsilon}{2}\right)(1-\eta) & a\left(\frac{1-\upsilon}{2}\right)(1-\xi) & -b\left(\frac{1-\upsilon}{2}\right)(1+\eta) & -a\left(\frac{1-\upsilon}{2}\right)(1-\xi) & -b\left(\frac{1-\upsilon}{2}\right)(1-\eta) \end{bmatrix}\{d\}$$

$$\downarrow$$

$$[D][G]$$

Eleman Rijitlik Matrisi

$$[ke] = \int_V [G]^T [D][G]\, dV$$

$$[ke] = \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\int_A \begin{bmatrix} b(1+\eta) & 0 & a(1+\xi) \\ 0 & a(1+\xi) & b(1+\eta) \\ b(1-\eta) & 0 & -a(1+\xi) \\ 0 & -a(1+\xi) & b(1-\eta) \\ -b(1+\eta) & 0 & a(1-\xi) \\ 0 & a(1-\xi) & -b(1+\eta) \\ -b(1-\eta) & 0 & -a(1-\xi) \\ 0 & -a(1-\xi) & -b(1-\eta) \end{bmatrix}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{bmatrix} b(1+\eta) & 0 & b(1-\eta) & 0 & -b(1+\eta) & 0 & -b(1-\eta) & 0 \\ 0 & a(1+\xi) & 0 & -a(1+\xi) & 0 & a(1-\xi) & 0 & -a(1-\xi) \\ a(1+\xi) & b(1+\eta) & -a(1+\xi) & b(1-\eta) & a(1-\xi) & -b(1+\eta) & -a(1-\xi) & -b(1-\eta) \end{bmatrix} dxdy$$

$$[ke] = \frac{Et}{16a^2b^2(1-\nu^2)}\int_{-a}^{a}\int_{-b}^{b}\left[\quad\right] dx\,dy$$

$$\xi = \frac{x}{a} \qquad \eta = \frac{y}{b}$$

$$[ke]=\frac{Et}{16a^2b^2(1-\nu^2)}\int_A \begin{bmatrix} k11 & k12 & k13 & k14 & k15 & k16 & k17 & k18 \\ k21 & k22 & k23 & k24 & k25 & k26 & k27 & k28 \\ k31 & k32 & k33 & k34 & k35 & k36 & k37 & k38 \\ k41 & k42 & k43 & k44 & k45 & k46 & k47 & k48 \\ k51 & k52 & k53 & k54 & k55 & k56 & k57 & k58 \\ k61 & k62 & k63 & k64 & k65 & k66 & k67 & k68 \\ k71 & k72 & k73 & k74 & k75 & k76 & k77 & k78 \\ k81 & k82 & k83 & k84 & k85 & k86 & k87 & k88 \end{bmatrix} dxdy$$

$$k_{11} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[b^2(1+\eta)^2 + a^2\left(\frac{1-\nu}{2}\right)(1+\xi)^2 \right] dxdy$$

$$k_{12} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1+\eta)(1+\xi) + \left(\frac{1-\nu}{2}\right) ab(1+\eta)(1+\xi) \right] dxdy$$

$$k_{13} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[b^2(1+\eta)(1-\eta) - a^2\left(\frac{1-\nu}{2}\right)(1+\xi)^2 \right] dxdy$$

$$k_{14} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1+\xi)(1+\eta) + ab\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\eta) \right] dxdy$$

$$k_{15} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-b^2(1+\eta)^2 + a^2\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\xi) \right] dxdy$$

$$k_{16} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1+\eta)(1-\xi) - ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1+\xi) \right] dxdy$$

$$k_{17} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-b^2(1+\eta)(1-\eta) - a^2\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\xi) \right] dxdy$$

$$k_{18} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1+\eta)(1-\xi) - ab\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\eta) \right] dxdy$$

$$k_{21} = k_{12}$$

$$k_{22} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[a^2(1+\xi)^2 + b^2\left(\frac{1-\nu}{2}\right)(1+\eta)^2 \right] dA$$

$$k_{23} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1-\eta)(1+\xi) - ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1+\xi) \right] dxdy$$

$$k_{24} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-a^2(1+\xi)^2 + b^2\left(\frac{1-\nu}{2}\right)(1-\eta)(1+\eta) \right] dxdy$$

$$k_{25} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1+\eta)(1+\xi) + ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\xi) \right] dxdy$$

$$k_{26} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[a^2(1+\xi)(1-\xi) - b^2\left(\frac{1-\nu}{2}\right)(1+\eta)^2 \right] dxdy$$

$$k_{27} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1+\xi)(1-\eta) - ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\xi) \right] dxdy$$

$$k_{28} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-a^2(1+\xi)(1-\xi) - b^2\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\eta) \right] dxdy$$

$k31 = k13$

$k32 = k23$

$$k33 = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[b^2(1-\eta)^2 + a^2\left(\frac{1-\nu}{2}\right)(1+\xi)^2 \right] dxdy$$

$$k34 = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1-\eta)(1+\xi) - ab\left(\frac{1-\nu}{2}\right)(1-\eta)(1+\xi) \right] dxdy$$

$$k35 = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-b^2(1+\eta)(1-\eta) - a^2\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\xi) \right] dxdy$$

$$k36 = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1-\eta)(1-\xi) + ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1+\xi) \right] dxdy$$

$$k37 = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-b^2(1-\eta)^2 + a^2\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\xi) \right] dxdy$$

$$k38 = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1-\eta)(1-\xi) + ab\left(\frac{1-\nu}{2}\right)(1+\xi)(1-\eta) \right] dxdy$$

$$k_{41} = k_{14}$$

$$k_{42} = k_{24}$$

$$k_{43} = k_{34}$$

$$k_{44} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[a^2(1+\xi)^2 + b^2\left(\frac{1-\nu}{2}\right)(1-\eta)^2 \right]$$

$$k_{45} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1+\eta)(1+\xi) + ab\left(\frac{1-\nu}{2}\right)(1-\eta)(1-\xi) \right] dxdy$$

$$k_{46} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-a^2(1+\xi)(1-\xi) - b^2\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\eta) \right] dxdy$$

$$k_{47} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1-\eta)(1+\xi) - ab\left(\frac{1-\nu}{2}\right)(1-\eta)(1-\xi) \right] dxdy$$

$$k_{48} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[a^2(1+\xi)(1-\xi) - b^2\left(\frac{1-\nu}{2}\right)(1-\eta)^2 \right] dxdy$$

$k_{51} = k_{15}$

$k_{52} = k_{25}$

$k_{53} = k_{35}$

$k_{54} = k_{45}$

$$k_{55} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[b^2(1+\eta)^2 + a^2\left(\frac{1-\nu}{2}\right)(1-\xi)^2 \right] dxdy$$

$$k_{56} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1+\eta)(1-\xi) - ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\xi) \right] dxdy$$

$$k_{57} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[b^2(1+\eta)(1-\eta) - a^2\left(\frac{1-\nu}{2}\right)(1-\xi)^2 \right] dxdy$$

$$k_{58} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1+\eta)(1-\xi) - ab\left(\frac{1-\nu}{2}\right)(1-\eta)(1-\xi) \right] dxdy$$

$$k_{61} = k_{16}$$

$$k_{62} = k_{26}$$

$$k_{63} = k_{36}$$

$$k_{64} = k_{46}$$

$$k_{65} = k_{56}$$

$$k_{66} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[a^2(1-\xi)^2 + b^2\left(\frac{1-\nu}{2}\right)(1+\eta)^2 \right] dx\,dy$$

$$k_{67} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-\nu ab(1-\eta)(1-\xi) + ab\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\xi) \right] dx\,dy$$

$$k_{68} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[-a^2(1-\xi)^2 + b^2\left(\frac{1-\nu}{2}\right)(1+\eta)(1-\eta) \right] dx\,dy$$

$k_{71} = k_{17}$

$k_{72} = k_{27}$

$k_{73} = k_{37}$

$k_{74} = k_{47}$

$k_{75} = k_{57}$

$k_{76} = k_{67}$

$$k_{77} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[b^2(1-\eta)^2 + a^2\left(\frac{1-\nu}{2}\right)(1-\xi)^2 \right] dxdy$$

$$k_{78} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[\nu ab(1-\eta)(1-\xi) + ab\left(\frac{1-\nu}{2}\right)(1-\eta)(1-\xi) \right] dxdy$$

$k_{81} = k_{18}$ $\quad$ $k_{84} = k_{48}$

$k_{82} = k_{28}$ $\quad$ $k_{85} = k_{58}$

$k_{83} = k_{38}$ $\quad$ $k_{86} = k_{68}$

$k_{87} = k_{78}$

$$k_{88} = \frac{Et}{16a^2b^2(1-\nu^2)} \int_A \left[a^2(1-\xi)^2 + b^2\left(\frac{1-\nu}{2}\right)(1-\eta)^2 \right] dxdy$$

Rijitlik matrisindeki bazı terimlerin integrasyon değerlerini yazarak terimleri kısaltabiliriz.

$$\int_V \eta \, dV = t \int_{-a}^{a}\int_{-b}^{b} \frac{y}{b}\, dA = \frac{t}{b}\left[\frac{y^2}{2} \times \Big|_{-a}^{a} \Big|_{-b}^{b}\right] = \frac{t}{b}\left(\frac{b^2}{2} - \frac{b^2}{2}\right) \times$$

$$= 0$$

$$\int_V \xi \, dV = 0$$

$$\int_V dV = t \int_A dA = t x y \Big|_{-a}^{a} \Big|_{-b}^{b} = t(a+a)(b+b) = 4abt$$

$$\int_V \xi^2 dV = \frac{t}{a^2}\int_A x^2 dA = \frac{t}{a^2}\left[\frac{x^3}{3} \cdot y \Big|_{-a}^{a} \Big|_{-b}^{b}\right] = \frac{t}{a^2}\left[\frac{a^3}{3} + \frac{a^3}{3}\right]\left[b+b\right]$$

$$= \frac{t}{a^2}\left(2\,\frac{a^3}{3}\right)2b = \frac{4}{3}\, tab$$

$$\int_V \eta^2 dV = \frac{4}{3}\, tab$$

$$\int_V \xi \eta \, dV = \frac{t}{ab}\int_A xy\, dA = \frac{t}{ab}\left[\frac{x^2}{2} \cdot \frac{y^2}{2} \Big|_{-a}^{a} \Big|_{-b}^{b}\right] = 0$$

$$\mathfrak{z} = \frac{b}{a} \; ; \quad \frac{1}{\mathfrak{z}} = \frac{a}{b} \quad \text{ve} \quad D_{11} = \frac{E}{1-\nu^2}$$

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \longrightarrow [D] = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}$$

$$D_{12} = D_{21} = \frac{E}{1-\nu^2}\nu$$

$$D_{33} = \frac{E}{(1+\nu)2}$$

$$k_{11} = \frac{1}{16a^2b^2} \frac{E}{(1-\nu^2)} \left[b^2(4abt) + a^2\left(\frac{1-\nu}{2}\right)(4abt) \right]$$

$$= \frac{Et}{4(1-\nu^2)} \left(\frac{b}{a}\right) + \frac{Et}{4(1-\nu^2)} \left(\frac{1-\nu}{2}\right)\left(\frac{a}{b}\right)$$

$$= \frac{Et}{4(1-\nu^2)} \left(\frac{b}{a}\right) + \frac{Et}{4} \frac{1}{2(1+\nu)} \left(\frac{a}{b}\right)$$

$$= \frac{t}{4} D_{11} \cdot 3 + \frac{t}{4} D_{33} \cdot \frac{1}{3} = \frac{t}{4}\left(D_{11} \cdot 3 + D_{33} \cdot \frac{1}{3}\right)$$

$$k_{12} = \frac{t}{16a^2b^2} \frac{E}{(1-\nu^2)} \left[\nu ab\,(4ab) + \left(\frac{1-\nu}{2}\right) ab\,(4ab) \right]$$

$$= \frac{t}{4} \frac{E}{(1-\nu^2)} \nu + \frac{t}{4} \frac{E}{(1-\nu^2)} \left(\frac{1-\nu}{2}\right)$$

$$= \frac{t}{4} \frac{E}{(1-\nu^2)} \nu + \frac{t}{4} \frac{E}{(1+\nu)2}$$

$$= \frac{t}{4}\left(D_{12} + D_{33}\right)$$

$$k_{13} = \frac{t}{16a^2b^2} \frac{E}{(1-\nu^2)} \left[b^2\left(4ab - \frac{4}{3}ab\right) - a^2\left(\frac{1-\nu}{2}\right)\left(4ab + \frac{4}{3}ab\right) \right]$$

$$= \frac{t}{16a^2b^2} \frac{E}{(1-\nu^2)} \left[b^2\left(\frac{8}{3}ab\right) - a^2\left(\frac{1-\nu}{2}\right)\left(\frac{16}{3}ab\right) \right]$$

$$= \frac{t}{3} \frac{E}{(1-\nu^2)} \frac{b}{2a} - \frac{tE}{2(1+\nu)} \frac{1}{3} \frac{a}{b}$$

$$= \frac{t}{3}\left(D_{11} \cdot \frac{3}{2} - D_{33} \frac{1}{3}\right)$$

$$k_{14} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab\,(4ab) + ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= -\frac{t}{4}\,\frac{E}{(1-\nu^2)}\,\nu + \frac{t}{4}\,\frac{E}{(1+\nu)2}$$

$$= \frac{t}{4}\left(-D_{12} + D_{33}\right)$$

$$k_{15} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-b^2\left(4ab + \frac{4}{3}ab\right) + a^2\left(\frac{1-\nu}{2}\right)\left(4ab - \frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-b^2\left(\frac{16}{3}ab\right) + a^2\left(\frac{1-\nu}{2}\right)\left(\frac{8}{3}ab\right)\right]$$

$$= -\frac{t}{3}\,\frac{b}{a}\,\frac{E}{(1-\nu^2)} + \frac{t}{3}\,\frac{1}{2}\,\frac{a}{b}\,\frac{E}{(1+\nu)2}$$

$$= \frac{t}{3}\left(-D_{11}\cdot 3 + D_{33}\cdot\frac{1}{2\,3}\right)$$

$$k_{16} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\,\frac{E}{(1-\nu^2)}\,\nu - \frac{t}{4}\,\frac{E}{(1+\nu)2}$$

$$= \frac{t}{4}\left(D_{12} - D_{33}\right)$$

$$k_{17} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-b^2\left(4ab - \frac{4}{3}ab\right) - a^2\left(\frac{1-\nu}{2}\right)\left(4ab - \frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-b^2\left(\frac{8}{3}ab\right) - a^2\left(\frac{1-\nu}{2}\right)\left(\frac{8}{3}ab\right)\right]$$

$$= -\frac{t}{6}\,\frac{b}{a}\,\frac{E}{(1-\nu^2)} - \frac{t}{6}\,\frac{a}{b}\,\frac{E}{(1+\nu)2}$$

$$= -\frac{t}{6}\left(D_{11}\cdot 3 + D_{33}\cdot\frac{1}{3}\right)$$

$$k_{18} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= -\frac{t}{4}\,\frac{E}{(1-\nu^2)}\,\nu - \frac{t}{4}\,\frac{E}{(1+\nu)2}$$

$$= -\frac{t}{4}\left(D_{12} + D_{33}\right)$$

$$k_{22} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(4ab + \frac{4}{3}ab\right) + b^2\left(\frac{1-\nu}{2}\right)\left(4ab + \frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(\frac{16}{3}ab\right) + b^2\left(\frac{1-\nu}{2}\right)\left(\frac{16}{3}ab\right)\right]$$

$$= \frac{t}{3}\,\frac{a}{b}\,\frac{E}{(1-\nu^2)} + \frac{t}{3}\,\frac{b}{a}\,\frac{E}{(1+\nu)2}$$

$$= \frac{t}{3}\left(D_{22}\cdot\frac{1}{\beta} + D_{33}\cdot\beta\right)$$

$$k_{24} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-a^2\left(4ab + \frac{4}{3}ab\right) + b^2\left(\frac{1-\nu}{2}\right)\left(4ab - \frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-a^2\left(\frac{16}{3}ab\right) + b^2\left(\frac{1-\nu}{2}\right)\left(\frac{8}{3}ab\right)\right]$$

$$= -\frac{t}{3}\,\frac{a}{b}\,D_{22} + \frac{t}{3}\,\frac{b}{2a}\,\frac{E}{(1+\nu)2}$$

$$= \frac{t}{3}\left(-D_{22}\cdot\frac{1}{\beta} + D_{33}\cdot\frac{\beta}{2}\right)$$

$$k_{23} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\left(D_{12} - D_{33}\right)$$

$$k_{25} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab) + ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= -\frac{t}{4}\,\frac{E}{(1-\nu^2)}\,\nu + \frac{t}{4}\,\frac{E}{(1+\nu)2}$$

$$= \frac{t}{4}\left(-D_{21} + D_{33}\right)$$

$$k_{26} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(4ab - \frac{4}{3}ab\right) - b^2\left(\frac{1-\nu}{2}\right)\left(4ab + \frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(\frac{8}{3}ab\right) - b^2\left(\frac{1-\nu}{2}\right)\left(\frac{16}{3}ab\right)\right]$$

$$= \frac{t}{6}\,\frac{a}{b}\,D_{22} - \frac{t}{3}\,\frac{b}{a}\,D_{33}$$

$$= \frac{t}{3}\left(D_{22}\cdot\frac{1}{2z} - D_{33}\cdot z\right)$$

$$k_{27} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= -\frac{t}{4}D_{21} - \frac{t}{4}D_{33}$$

$$= -\frac{t}{4}\left(D_{21} + D_{33}\right)$$

$$k_{28} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-a^2\left(4ab - \frac{4}{3}ab\right) - b^2\left(\frac{1-\nu}{2}\right)\left(4ab - \frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-a^2\left(\frac{8}{3}ab\right) - b^2\left(\frac{1-\nu}{2}\right)\left(\frac{8}{3}ab\right)\right]$$

$$= -\left(\frac{t}{6}\,\frac{a}{b}\,D_{22} + \frac{t}{6}\,D_{33}\,\frac{b}{a}\right)$$

$$= -\frac{t}{6}\left(D_{22}\cdot\frac{1}{z} + D_{33}\cdot z\right)$$

$$k_{33} = \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\left[b^2\left(4ab+\frac{4}{3}ab\right)+a^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\left[b^2\left(\frac{16}{3}ab\right)+a^2\left(\frac{1-\nu}{2}\right)\left(\frac{16}{3}ab\right)\right]$$

$$= \frac{t}{3}\frac{b}{a}\frac{E}{(1-\nu^2)} + \frac{t}{3}\frac{a}{b}\frac{E}{(1+\nu)2}$$

$$= \frac{t}{3}\left(D_{11}\cdot 3 + D_{33}\cdot\frac{1}{3}\right)$$

$$k_{34} = \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= -\frac{t}{4}\frac{E}{(1-\nu^2)}\nu - \frac{t}{4}\frac{E}{(1+\nu)2}$$

$$= -\frac{t}{4}\left(D_{12} + D_{33}\right)$$

$$k_{35} = \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\left[-b^2\left(4ab-\frac{4}{3}ab\right)-a^2\left(\frac{1-\nu}{2}\right)\left(4ab-\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\left[-b^2\left(\frac{8}{3}ab\right)-a^2\left(\frac{1-\nu}{2}\right)\left(\frac{8}{3}ab\right)\right]$$

$$= -\frac{t}{6}\frac{b}{a}\frac{E}{(1-\nu^2)} - \frac{t}{6}\frac{a}{b}\frac{E}{(1+\nu)2}$$

$$= -\frac{t}{6}\left(D_{11}\cdot 3 + D_{33}\cdot\frac{1}{3}\right)$$

$$k_{36} = \frac{t}{16a^2b^2}\frac{E}{(1-\nu^2)}\left[\nu ab(4ab) + ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\frac{E}{(1-\nu^2)}\nu + \frac{t}{4}\frac{E}{(1+\nu)2}$$

$$= \frac{t}{4}\left(D_{12} + D_{33}\right)$$

$$k_{37} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-b^2\left(4ab+\frac{4}{3}ab\right)+a^2\left(\frac{1-\nu}{2}\right)\left(4ab-\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(-D_{11}\cdot\zeta + D_{33}\cdot\frac{1}{2\zeta}\right)$$

$$k_{38} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab)+ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\left(-D_{12}+D_{33}\right)$$

$$k_{44} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(4ab+\frac{4}{3}ab\right)+b^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{22}\,\frac{1}{\zeta} + D_{33}\cdot\zeta\right)$$

$$k_{45} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[\nu ab(4ab)+ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\left(D_{21}+D_{33}\right)$$

$$k_{46} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-a^2\left(4ab-\frac{4}{3}ab\right)-b^2\left(\frac{1-\nu}{2}\right)\left(4ab-\frac{4}{3}ab\right)\right]$$

$$= -\frac{t}{6}\left(D_{22}\,\frac{1}{\zeta} + D_{33}\cdot\zeta\right)$$

$$k_{47} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[\nu ab(4ab)-ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\left(D_{21}-D_{33}\right)$$

$$k_{48} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(4ab-\frac{4}{3}ab\right)-b^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{22}\,\frac{1}{2\zeta} - D_{33}\cdot\zeta\right)$$

$$k_{55} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[b^2\left(4ab+\frac{4}{3}ab\right)+a^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{11}\cdot 3 + D_{33}\cdot\frac{1}{3}\right)$$

$$k_{56} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= -\frac{t}{4}\left(D_{12} + D_{33}\right)$$

$$k_{57} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[b^2\left(4ab-\frac{4}{3}ab\right)-a^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{11}\frac{3}{2} - D_{33}\frac{1}{3}\right)$$

$$k_{58} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[\nu ab(4ab) - ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\left(D_{12} - D_{33}\right)$$

$$k_{66} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(4ab+\frac{4}{3}ab\right)+b^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{22}\frac{1}{3} + D_{33}\cdot 3\right)$$

$$k_{67} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-\nu ab(4ab) + ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

$$= \frac{t}{4}\left(-D_{21} + D_{33}\right)$$

$$k_{68} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[-a^2\left(4ab+\frac{4}{3}ab\right)+b^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(-D_{22}\,\frac{1}{3}+D_{33}\,\frac{3}{2}\right)$$

$$k_{77} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[b^2\left(4ab+\frac{4}{3}ab\right)+a^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{11}\cdot 3+D_{33}\cdot\frac{1}{3}\right)$$

$$k_{78} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[\nu ab(4ab)+ab\left(\frac{1-\nu}{2}\right)(4ab)\right]$$

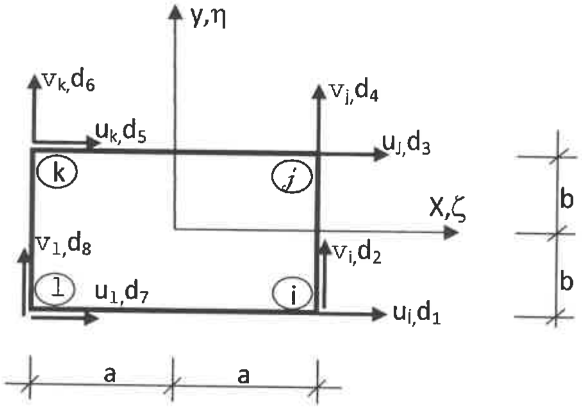
$$= \frac{t}{4}\left(D_{12}+D_{33}\right)$$

$$k_{88} = \frac{t}{16a^2b^2}\,\frac{E}{(1-\nu^2)}\left[a^2\left(4ab+\frac{4}{3}ab\right)+b^2\left(\frac{1-\nu}{2}\right)\left(4ab+\frac{4}{3}ab\right)\right]$$

$$= \frac{t}{3}\left(D_{22}\,\frac{1}{3}+D_{33}\cdot 3\right)$$

Yukarıda verilen sonuçlar aşağıda tabloda özetlenmiştir. Veya bir başka açıdan rijitlik matrisi yeniden oluşturulursa;

$$\left.\begin{aligned} k_{11} &= \frac{Et}{4(1-\nu^2)}\left(3+\frac{(1-\nu)}{2\,3}\right) \\ k_{12} &= \frac{Et}{4(1-\nu^2)}\left(\frac{\nu+1}{2}\right) \end{aligned}\right\}$$ şeklinde elde edilecektir



$$[D] = \frac{E}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \dfrac{1-\upsilon}{2} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}$$

$$\mathfrak{z} = {}^{b}/_{a}\,; \quad {}^{1}/_{\mathfrak{z}} = {}^{a}/_{b}$$

$$D_{22} = D_{11} = \frac{E}{1-\upsilon^2}$$

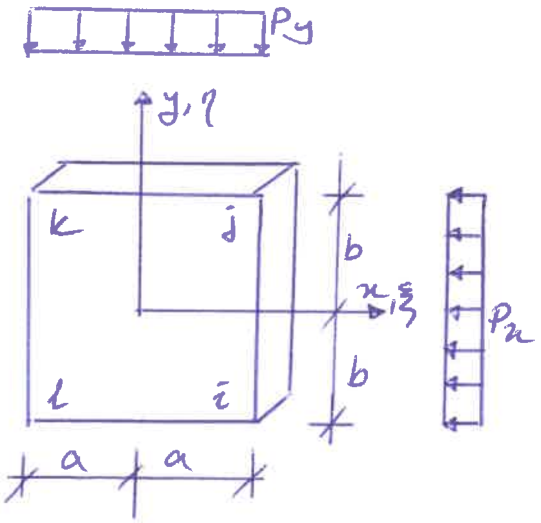
$$D_{12} = D_{21} = \frac{E}{1-\upsilon^2}\upsilon$$

$$D_{33} = \frac{E}{(1+\upsilon)2}$$

$$[k_e] = \left[\begin{array}{cccccccc}
\frac{t}{4}\left(D_{11}\cdot\mathfrak{z} + \frac{D_{33}}{\mathfrak{z}}\right) & & & & \textit{simetrik} & & & \\
\frac{t}{4}(D_{12}+D_{33}) & \frac{t}{3}\left(\frac{D_{22}}{\mathfrak{z}} + D_{33}\cdot\mathfrak{z}\right) & & & & & & \\
\frac{t}{4}\left(D_{11}\cdot\frac{\mathfrak{z}}{2} - \frac{D_{33}}{\mathfrak{z}}\right) & \frac{t}{4}(D_{12}-D_{33}) & \frac{t}{3}\left(D_{11}\cdot\mathfrak{z} + \frac{D_{33}}{\mathfrak{z}}\right) & & & & & \\
\frac{t}{4}(-D_{12}+D_{33}) & \frac{t}{3}\left(-\frac{D_{22}}{\mathfrak{z}} + D_{33}\cdot\frac{\mathfrak{z}}{2}\right) & \frac{t}{4}(-D_{12}-D_{33}) & \frac{t}{3}\left(\frac{D_{22}}{\mathfrak{z}} + D_{33}\cdot\mathfrak{z}\right) & & & & \\
\frac{t}{3}\left(-D_{11}\cdot\mathfrak{z} + \frac{D_{33}}{2\mathfrak{z}}\right) & \frac{t}{4}(-D_{21}+D_{33}) & \frac{t}{6}\left(-D_{11}\cdot\mathfrak{z} - \frac{D_{33}}{\mathfrak{z}}\right) & \frac{t}{4}(D_{21}+D_{33}) & \frac{t}{3}\left(D_{11}\cdot\mathfrak{z} + \frac{D_{33}}{\mathfrak{z}}\right) & & & \\
\frac{t}{4}(D_{12}-D_{33}) & \frac{t}{3}\left(\frac{D_{22}}{2\mathfrak{z}} - D_{33}\cdot\mathfrak{z}\right) & \frac{t}{4}(D_{12}+D_{33}) & \frac{t}{6}\left(-\frac{D_{22}}{\mathfrak{z}} - D_{33}\cdot\mathfrak{z}\right) & \frac{t}{4}(-D_{12}-D_{33}) & \frac{t}{3}\left(\frac{D_{22}}{\mathfrak{z}} + D_{33}\cdot\mathfrak{z}\right) & & \\
\frac{t}{6}\left(-D_{11}\cdot\mathfrak{z} - \frac{D_{33}}{\mathfrak{z}}\right) & \frac{t}{4}(-D_{21}-D_{33}) & \frac{t}{3}\left(-D_{11}\cdot\mathfrak{z} + \frac{D_{33}}{2\mathfrak{z}}\right) & \frac{t}{4}(D_{21}-D_{33}) & \frac{t}{3}\left(D_{11}\cdot\frac{\mathfrak{z}}{2} - \frac{D_{33}}{\mathfrak{z}}\right) & \frac{t}{4}(-D_{21}+D_{33}) & \frac{t}{3}\left(D_{11}\cdot\mathfrak{z} + \frac{D_{33}}{\mathfrak{z}}\right) & \\
\frac{t}{4}(-D_{12}-D_{33}) & \frac{t}{6}\left(-\frac{D_{22}}{\mathfrak{z}} - D_{33}\cdot\mathfrak{z}\right) & \frac{t}{4}(-D_{12}+D_{33}) & \frac{t}{3}\left(\frac{D_{22}}{2\mathfrak{z}} - D_{33}\cdot\mathfrak{z}\right) & \frac{t}{4}(D_{12}-D_{33}) & \frac{t}{3}\left(-\frac{D_{22}}{\mathfrak{z}} + D_{33}\cdot\frac{\mathfrak{z}}{2}\right) & \frac{t}{4}(D_{12}+D_{33}) & \frac{t}{3}\left(\frac{D_{22}}{\mathfrak{z}} + D_{33}\cdot\mathfrak{z}\right)
\end{array}\right]$$

10.3.1) Yük Matrisi

a) Düzgün Yayılı Yük Durumu (Edge Load)



$$\{q\}_s = -\int_s [N]^T \{P\} \, ds$$

$$\{q\}_i = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_i \, dx\,dy$$

$$\{q\}_j = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_j \, dx\,dy$$

$$\{q\}_k = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_k \, dx\,dy$$

$$\{q\}_l = -\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \int N_l \, dx\,dy$$

$$\{q\}_s = -\int \begin{bmatrix} N_i & 0 \\ 0 & N_i \\ N_j & 0 \\ 0 & N_j \\ N_k & 0 \\ 0 & N_k \\ N_l & 0 \\ 0 & N_l \end{bmatrix} dx\,dy = \begin{Bmatrix} -P_x . b \\ 0 \\ -P_x . b \\ -P_y . a \\ 0 \\ -P_y . a \\ 0 \\ 0 \end{Bmatrix}$$

$$\{q\}_i = -\int_s N_i \cdot P_x \, dy = -P_x \int_s \frac{1}{4}(1+\xi+\eta+\xi\eta) \, dy$$

$$\{q\}_i = -P_x . b$$

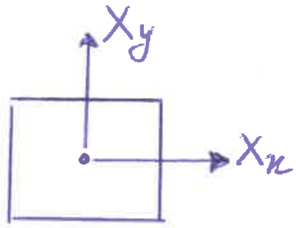
$$\int_{-b}^{b} dy = 2b \; ; \; \int_{-b}^{b} \xi \, dy = 2b \; ; \; \int_{-b}^{b} \eta \, dy = 0$$

$$\int_{-b}^{b} \xi\eta \, dy = 0$$

$$\{q\}_l = -P_x . b$$

$$\{q\}_j = \{q\}_k = -P_x . b$$

b) Kütlesel Yük (Body Forces)



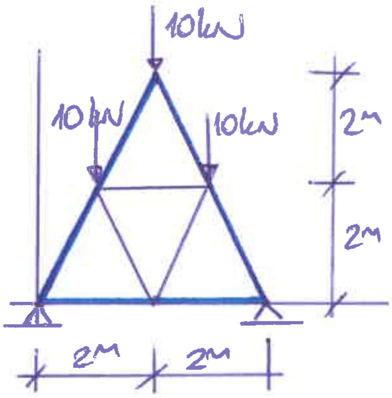
$$\{X\} = \begin{Bmatrix} X_x(x,y) \\ X_y(x,y) \end{Bmatrix}$$

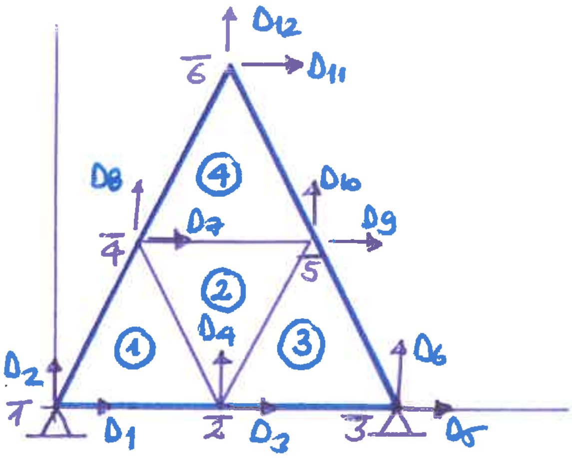
$$\{q\}_X = -\int_V [N]^T \{X\} \, dV$$

c) Yayılı yük (Consistent mass)

$$[M] = \rho \int [N][N] \, dV$$

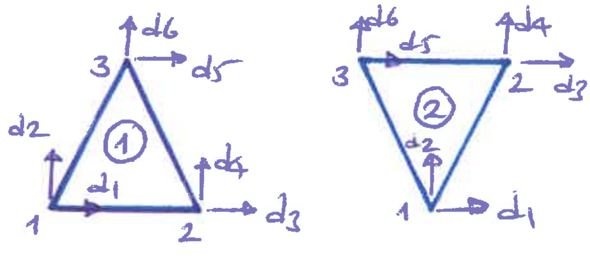
ÖRNEK 1) Şekilde boyutları ve tipi verilen üçgen levhanın yer-değiştirme ve gerilme hesaplarını yapınız. ($\nu=0{,}20$)





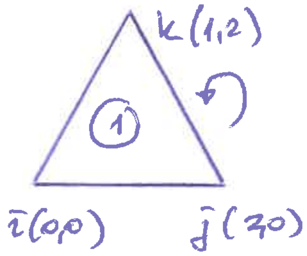
Düğüm noktaları koordinatları

Nokta No	x (m)	y (m)
1	0	0
2	2	0
3	4	0
4	1	2
5	3	2
6	2	4



Tip1 Tip2

1. eleman



$$\begin{cases} a_i = x_j y_k - x_k y_j = 2\times2 - 1\times0 = 4 \\ b_i = y_j - y_k = 0-2 = -2 \\ c_i = x_k - x_j = 1-2 = -1 \end{cases}$$

$$\begin{cases} a_j = x_k y_i - y_k x_i = 1\times0 - 0 = 0 \\ b_j = y_k - y_i = 2-0 = 2 \\ c_j = x_i - x_k = 0-1 = -1 \end{cases}$$

$$\begin{cases} a_k = x_i y_j - x_j y_i = 0 - 2\times0 = 0 \\ b_k = y_i - y_j = 0-0 = 0 \\ c_k = x_j - x_i = 2-0 = 2 \end{cases}$$

$b_i + b_j + b_k = 0$

$c_i + c_j + c_k = 0$

Alan $\Delta = \dfrac{2\times2}{2} = 2\ m^2$

$$\{\varepsilon\} = \frac{1}{2x2}\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}\{d\}$$

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$\{\sigma\} = \frac{E}{4(1-0{,}2^2)}\begin{bmatrix} 1 & 0{,}2 & 0 \\ 0{,}2 & 1 & 0 \\ 0 & 0 & \frac{1-0{,}2}{2} \end{bmatrix}\begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -2 & -1 & 2 & 2 & 0 \end{bmatrix}\{d\}$$

$$\{\sigma\} = \frac{E}{4(1-0{,}2^2)}\begin{bmatrix} -2 & -0{,}2 & 2 & -0{,}2 & 0 & 0{,}4 \\ -0{,}4 & -1 & 0{,}4 & -1 & 0 & 2 \\ -0{,}4 & -0{,}8 & -0{,}4 & 0{,}8 & 0{,}8 & 0 \end{bmatrix}\{d\}$$

1. elemanın rijitlik matrisi

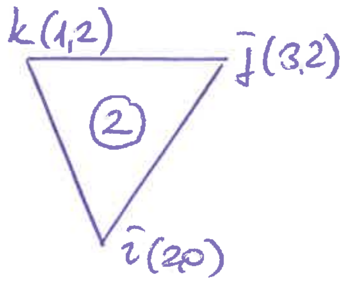
$$[ke] = \iint [G]^T [D][G]\, t\, dx\, dy$$

$$[ke_1] = \frac{Et}{16(1-0{,}2^2)}\iint \begin{bmatrix} -2 & 0 & -1 \\ 0 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} -2 & -0{,}2 & 2 & -0{,}2 & 0 & 0{,}4 \\ -0{,}4 & -1 & 0{,}4 & -1 & 0 & 2 \\ -0{,}4 & -0{,}8 & -0{,}4 & 0{,}8 & 0{,}8 & 0 \end{bmatrix} dx\,dy$$

$$\iint dx\,dy = 2$$

$$[ke_1] = \frac{Et}{8(1-0{,}2^2)}\begin{bmatrix} 4{,}4 & 1{,}2 & -3{,}6 & -0{,}4 & -0{,}8 & -0{,}8 \\ 1{,}2 & 2{,}6 & 0{,}4 & -0{,}6 & -1{,}6 & -2{,}0 \\ -3{,}6 & 0{,}4 & 4{,}4 & -1{,}2 & -0{,}8 & 0{,}8 \\ -0{,}4 & -0{,}6 & -1{,}2 & 2{,}6 & 1{,}6 & -2{,}0 \\ -0{,}8 & -1{,}6 & -0{,}8 & 1{,}6 & 1{,}6 & 0 \\ -0{,}8 & -2{,}0 & 0{,}8 & -2{,}0 & 0 & 4{,}0 \end{bmatrix}$$

2. eleman



$$\begin{cases} a_i = x_j y_k - x_k y_j = 3\times2 - 1\times2 = 4 \\ b_i = y_j - y_k = 2-2=0 \\ c_i = x_k - x_j = 1-3=-2 \end{cases}$$

$$\begin{cases} a_j = x_k y_i - y_k x_i = 1\times0 - 2\times2 = -4 \\ b_j = y_k - y_i = 2-0=2 \\ c_j = x_i - x_k = 2-1=1 \end{cases}$$

$$b_i + b_j + b_k = 0$$

$$c_i + c_j + c_k = 0$$

$$\text{Alan } \Delta = \frac{2\times2}{2} = 2\,m^2$$

$$\begin{cases} a_k = x_i y_j - x_j y_i = 2\times2 - 3\times0 = 4 \\ b_k = y_i - y_j = 0-2=-2 \\ c_k = x_j - x_i = 3-2=1 \end{cases}$$

$$\{\varepsilon\} = \frac{1}{2\times2}\begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -2 & 0 & 1 & 0 & 1 \\ -2 & 0 & 1 & 2 & 1 & -2 \end{bmatrix}\{d\}$$

$$\{\sigma\} = \frac{E}{4(1-0{,}2^2)}\begin{bmatrix} 1 & 0{,}2 & 0 \\ 0{,}2 & 1 & 0 \\ 0 & 0 & \frac{1-0{,}2}{2} \end{bmatrix}\begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -2 & 0 & 1 & 0 & 1 \\ -2 & 0 & 1 & 2 & 1 & -2 \end{bmatrix}\{d\}$$

$$\{\sigma\} = \frac{E}{4(1-0{,}2^2)}\begin{bmatrix} 0 & -0{,}4 & 2 & 0{,}2 & -2 & 0{,}2 \\ 0 & -2 & 0{,}4 & 1 & -0{,}4 & 1 \\ -0{,}8 & 0 & 0{,}4 & 0{,}8 & 0{,}4 & -0{,}8 \end{bmatrix}\{d\}$$

$$[ke_2] = \frac{Et}{16(1-0{,}2^2)}\iint \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \\ -2 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} 0 & -0{,}4 & 2 & 0{,}2 & -2 & 0{,}2 \\ 0 & -2 & 0{,}4 & 1 & -0{,}4 & 1 \\ -0{,}8 & 0 & 0{,}4 & 0{,}8 & 0{,}4 & -0{,}8 \end{bmatrix} dx\,dy$$

$$[k_{e2}] = \frac{Et}{8(1-0{,}2^2)} \begin{bmatrix} 1{,}6 & 0 & -0{,}8 & -1{,}6 & -0{,}8 & 1{,}6 \\ 0 & 4 & -0{,}8 & -2 & 0{,}8 & -2 \\ -0{,}8 & -0{,}8 & 4{,}4 & 1{,}2 & -3{,}6 & -0{,}4 \\ -1{,}6 & -2 & 1{,}2 & 2{,}6 & 0{,}4 & -0{,}6 \\ -0{,}8 & 0{,}8 & -3{,}6 & 0{,}4 & 4{,}4 & -1{,}2 \\ 1{,}6 & -2 & -0{,}4 & -0{,}6 & -1{,}2 & 2{,}6 \end{bmatrix}$$

Sistem rijitlik matrisi

a) Çevirme matrisleri ile;

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{bmatrix}$$

$$
\begin{bmatrix} d1 \\ d2 \\ d3 \\ d4 \\ d5 \\ d6 \end{bmatrix}_4 =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \end{bmatrix}
$$

Buradan;

$$
\left.
\begin{aligned}
[Ke_1] &= [Ce_1]^T [ke_1] [Ce_1] \\
[Ke_2] &= [Ce_2]^T [ke_2] [Ce_2] \\
[Ke_3] &= [Ce_3]^T [ke_3] [Ce_3] \\
[Ke_4] &= [Ce_4]^T [ke_4] [Ce_4]
\end{aligned}
\right\} [Ke] = [Ke_1] + [Ke_2] + [Ke_3] + [Ke_4]
$$

$$
[K] = \frac{Et}{8(1-\nu^2)}
\begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} & K_{17} & K_{18} & K_{19} & K_{1,10} & K_{1,11} \\
K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} & K_{27} & K_{28} & K_{29} & K_{2,10} & K_{2,11} \\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} & K_{37} & K_{38} & K_{39} & K_{3,10} & K_{3,11} \\
K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} & K_{47} & K_{48} & K_{49} & K_{4,10} & K_{4,11} \\
K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} & K_{57} & K_{58} & K_{59} & K_{5,10} & K_{5,11} \\
K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} & K_{67} & K_{68} & K_{69} & K_{6,10} & K_{6,11} \\
K_{71} & K_{72} & K_{73} & K_{74} & K_{75} & K_{76} & K_{77} & K_{78} & K_{79} & K_{7,10} & K_{7,11} \\
K_{8,1} & K_{8,2} & K_{8,3} & K_{8,4} & K_{8,5} & K_{8,6} & K_{8,7} & K_{8,8} & K_{8,9} & K_{8,10} & K_{8,11} \\
- & - & - & - & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - & - & - & -
\end{bmatrix}
$$

$$Ke1 = \begin{bmatrix}
4.4 & 1.2 & -3.6 & -0.4 & 0.0 & 0.0 & -0.8 & -0.8 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.2 & 2.6 & 0.4 & -0.6 & 0.0 & 0.0 & -1.6 & -2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-3.6 & 0.4 & 4.4 & -1.2 & 0.0 & 0.0 & -0.8 & 0.8 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.4 & -0.6 & -1.2 & 2.6 & 0.0 & 0.0 & 1.6 & -2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.8 & -1.6 & -0.8 & 1.6 & 0.0 & 0.0 & 1.6 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.8 & -2.0 & 0.8 & -2.0 & 0.0 & 0.0 & 0.0 & 4.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{bmatrix} \frac{Et}{8(1-0{,}2^2)}$$

$$Ke2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.6 & 0 & 0 & 0 & -0.8 & 1.6 & -0.8 & -1.6 & 0 & 0 \\
0 & 0 & 0 & 4 & 0 & 0 & 0.8 & -2 & -0.8 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.8 & 0.8 & 0 & 0 & 4.4 & -1.2 & -3.6 & 0.4 & 0 & 0 \\
0 & 0 & 1.6 & -2 & 0 & 0 & -1.2 & 2.6 & -0.4 & -0.6 & 0 & 0 \\
0 & 0 & -0.8 & -0.8 & 0 & 0 & -3.6 & -0.4 & 4.4 & 1.2 & 0 & 0 \\
0 & 0 & -1.6 & -2 & 0 & 0 & 0.4 & -0.6 & 1.2 & 2.6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \frac{Et}{8(1-0{,}2^2)}$$

$$Ke3 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4.4 & 1.2 & -3.6 & -0.4 & 0 & 0 & -0.8 & -0.8 & 0 & 0 \\
0 & 0 & 1.2 & 2.6 & 0.4 & -0.6 & 0 & 0 & -1.6 & -2 & 0 & 0 \\
0 & 0 & -3.6 & 0.4 & 4.4 & -1.2 & 0 & 0 & -0.8 & 0.8 & 0 & 0 \\
0 & 0 & -0.4 & -0.6 & -1.2 & 2.6 & 0 & 0 & 1.6 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.8 & -1.6 & -0.8 & 1.6 & 0 & 0 & 1.6 & 0 & 0 & 0 \\
0 & 0 & -0.8 & -2 & 0.8 & -2 & 0 & 0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \frac{Et}{8(1-0{,}2^2)}$$

$$Ke4 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4.4 & 1.2 & -3.6 & -0.4 & -0.8 & -0.8 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.2 & 2.6 & 0.4 & -0.6 & -1.6 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & -3.6 & 0.4 & 4.4 & -1.2 & -0.8 & 0.8 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.4 & -0.6 & -1.2 & 2.6 & 1.6 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.8 & -1.6 & -0.8 & 1.6 & 1.6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.8 & -2 & 0.8 & -2 & 0 & 4
\end{bmatrix} \frac{Et}{8(1-0{,}2^2)}$$

$$[K] = \frac{Et}{8(1-0.2^2)} \begin{bmatrix} 4.4 & 1.2 & -3.6 & -0.4 & 0.0 & 0.0 & -0.8 & -0.8 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.2 & 2.6 & 0.4 & -0.6 & 0.0 & 0.0 & -1.6 & -2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -3.6 & 0.4 & 10.4 & 0.0 & -3.6 & -0.4 & -1.6 & 2.4 & -1.6 & -2.4 & 0.0 & 0.0 \\ -0.4 & -0.6 & 0.0 & 9.2 & 0.4 & -0.6 & 2.4 & -4.0 & -2.4 & -4.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -3.6 & 0.4 & 4.4 & -1.2 & 0.0 & 0.0 & -0.8 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.4 & -0.6 & -1.2 & 2.6 & 0.0 & 0.0 & 1.6 & -2.0 & 0.0 & 0.0 \\ -0.8 & -1.6 & -1.6 & 2.4 & 0.0 & 0.0 & 10.4 & 0.0 & -7.2 & 0.0 & -0.8 & -0.8 \\ -0.8 & -2.0 & 2.4 & -4.0 & 0.0 & 0.0 & 0.0 & 9.2 & 0.0 & -1.2 & -1.6 & -2.0 \\ 0.0 & 0.0 & -1.6 & -2.4 & -0.8 & 1.6 & -7.2 & 0.0 & 10.4 & 0.0 & -0.8 & 0.8 \\ 0.0 & 0.0 & -2.4 & -4.0 & 0.8 & -2.0 & 0.0 & -1.2 & 0.0 & 9.2 & 1.6 & -2.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.8 & -1.6 & -0.8 & 1.6 & 1.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.8 & -2.0 & 0.8 & -2.0 & 0.0 & 4.0 \end{bmatrix}$$

b) Biriktirme Yöntemi İle;

Bu yöntemde düğüm noktalarında birleşen elemanların her düğüm noktasındaki ortak yer değiştirmeleri toplanır. Örneğin;

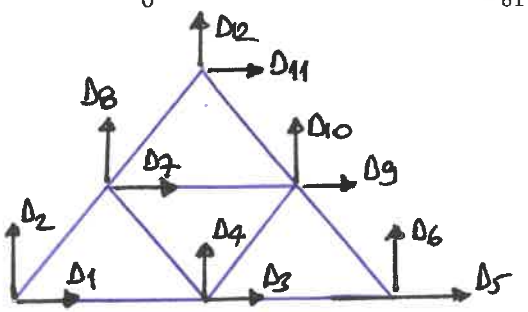
$K_{33} = k^1_{33} + k^2_{11} + k^1_{11}$ (Bu düğüm noktasında 1, 2 ve 3 elemanları birleşmektedir (1 nolu elemanın 2. ucu, 2 nolu elemanın 1. ucu ve 3 nolu elemanın 1. ucu)

1 nolu elemanın 2. ucu $\rightarrow k^1_{33}$
2 " " 1. " $\rightarrow k^2_{11}$
3 " " 1. " $\rightarrow k^1_{11}$

$K_{9,10} = k^2_{34} + k^1_{56} + k^1_{34}$ (Bu düğüm noktasında 2, 3 ve 4 elemanları birleşmektedir (2 nolu elemanın 2. ucu, 3 nolu elemanın 3. ucu ve 4 nolu elemanın 2. ucu)

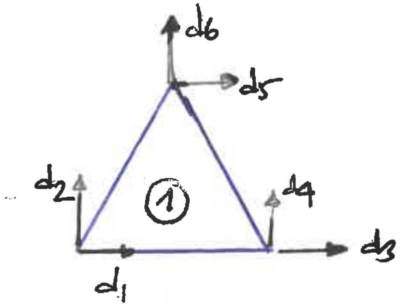
2 nolu elemanın 2. ucu $\rightarrow k^2_{34}$
3 " " 3. " $\rightarrow k^1_{56}$
4 " " 2. " $\rightarrow k^1_{34}$

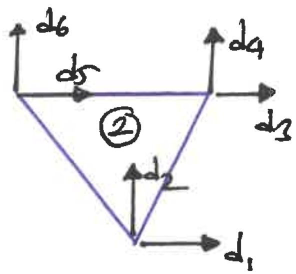
$$[K] = \frac{Et}{8(1-0{,}2^2)}\begin{bmatrix}
k_{11}^1 & k_{12}^1 & k_{13}^1 & k_{14}^1 & 0 & 0 & k_{15}^1 & k_{16}^1 & 0 & 0 & 0 & 0 \\
k_{21}^1 & k_{22}^1 & k_{23}^1 & k_{24}^1 & 0 & 0 & k_{25}^1 & k_{26}^1 & 0 & 0 & 0 & 0 \\
k_{31}^1 & k_{32}^1 & k_{33}^1+k_{11}^2+k_{11}^1 & k_{34}^1+k_{12}^2+k_{12}^1 & k_{13}^1 & k_{14}^1 & k_{35}^1+k_{15}^2 & k_{36}^1+k_{16}^2 & k_{15}^1+k_{13}^2 & k_{16}^1+k_{14}^2 & 0 & 0 \\
k_{41}^1 & k_{42}^1 & k_{43}^1+k_{21}^2+k_{21}^1 & k_{44}^1+k_{22}^2+k_{22}^1 & k_{23}^1 & k_{24}^1 & k_{45}^1+k_{25}^1 & k_{46}^1+k_{26}^2 & k_{25}^1+k_{23}^2 & k_{26}^1+k_{24}^2 & 0 & 0 \\
0 & 0 & k_{31}^1 & k_{32}^1 & k_{33}^1 & k_{34}^1 & 0 & 0 & k_{35}^1 & k_{36}^1 & 0 & 0 \\
0 & 0 & k_{41}^1 & k_{42}^1 & k_{43}^1 & k_{44}^1 & 0 & 0 & k_{45}^1 & k_{46}^1 & 0 & 0 \\
k_{51}^1 & k_{52}^1 & k_{53}^1+k_{51}^2 & k_{54}^1+k_{52}^2 & 0 & 0 & k_{55}^1+k_{55}^2+k_{11}^1 & k_{56}^1+k_{56}^2+k_{12}^1 & k_{13}^1+k_{53}^2 & k_{14}^1+k_{54}^2 & k_{15}^1 & k_{16}^1 \\
k_{61}^1 & k_{62}^1 & k_{63}^1+k_{61}^2 & k_{64}^1+k_{62}^2 & 0 & 0 & k_{65}^1+k_{65}^2+k_{21}^1 & k_{66}^1+k_{66}^2+k_{22}^1 & k_{23}^1+k_{63}^2 & k_{24}^1+k_{64}^2 & k_{25}^1 & k_{26}^1 \\
0 & 0 & k_{51}^1+k_{31}^2 & k_{52}^1+k_{32}^2 & k_{53}^1 & k_{54}^1 & k_{31}^1+k_{35}^2 & k_{32}^1+k_{36}^2 & k_{55}^1+k_{34}^2+k_{56}^1 & k_{34}^1+k_{33}^2+k_{33}^1 & k_{35}^1 & k_{36}^1 \\
0 & 0 & k_{61}^1+k_{41}^2 & k_{62}^1+k_{42}^2 & k_{63}^1 & k_{64}^1 & k_{41}^1+k_{45}^2 & k_{42}^1+k_{46}^2 & k_{43}^1+k_{43}^2+k_{65}^1 & k_{44}^1+k_{44}^2+k_{66}^1 & k_{45}^1 & k_{46}^1 \\
0 & 0 & 0 & 0 & 0 & 0 & k_{51}^1 & k_{52}^1 & k_{53}^1 & k_{54}^1 & k_{55}^1 & k_{56}^1 \\
0 & 0 & 0 & 0 & 0 & 0 & k_{61}^1 & k_{62}^1 & k_{63}^1 & k_{64}^1 & k_{65}^1 & k_{64}^1
\end{bmatrix}$$



$k_{33} = 4{,}4 + 1{,}6 + 4{,}4 = 10{,}4$ ✓

$k_{9,10} = 1{,}2 + 0 - 1{,}2 = 0$ ✓





Yük Matrisi

Sisteme etkiyen yükler tekil yüklerdir ve düğüm noktalarına etkimektedir. Bu durumda kuvvetler yer değiştirmelere karşı olacak şekilde yük matrisi düzenlenecektir

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{Bmatrix} \Rightarrow \{Q\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -10 \\ 0 \\ -10 \\ 0 \\ -10 \end{Bmatrix}$$

Yer Değiştirmelerin hesabı

Sistem rijitlik matrisi $[K]$, yük matrisi $\{Q\}$ ve yer değiştirme matrisi $\{D\}$ ise;

$[K][D] = \{Q\}$ şeklinde yazılacaktır

$$
\frac{Et}{8(1-0{,}2^2)}
\begin{bmatrix}
4{,}4 & 1{,}2 & -3{,}6 & -0{,}4 & 0 & 0 & -0{,}8 & -0{,}8 & 0 & 0 & 0 & 0 \\
1{,}2 & 2{,}6 & 0{,}4 & -0{,}6 & 0 & 0 & -1{,}6 & -2{,}0 & 0 & 0 & 0 & 0 \\
-3{,}6 & 0{,}4 & 10{,}4 & 0 & -3{,}6 & -0{,}4 & -1{,}6 & 2{,}4 & -1{,}6 & -2{,}4 & 0 & 0 \\
-0{,}4 & -0{,}6 & 0 & 9{,}2 & 0{,}4 & -0{,}6 & 2{,}4 & -4{,}0 & -2{,}4 & -4{,}0 & 0 & 0 \\
0 & 0 & -3{,}6 & 0{,}4 & 4{,}4 & -1{,}2 & 0 & 0 & -0{,}8 & 0{,}8 & 0 & 0 \\
0 & 0 & -0{,}4 & -0{,}6 & -1{,}2 & 2{,}6 & 0 & 0 & -1{,}6 & -2{,}0 & 0 & 0 \\
-0{,}8 & -1{,}6 & -1{,}6 & 2{,}4 & 0 & 0 & 10{,}4 & 0 & -7{,}2 & 0 & -0{,}8 & -0{,}8 \\
-0{,}8 & -2{,}0 & 2{,}4 & -4{,}0 & 0 & 0 & 0 & 9{,}2 & 0 & -1{,}2 & -1{,}6 & -2{,}0 \\
0 & 0 & -1{,}6 & -2{,}4 & -0{,}8 & 1{,}6 & -7{,}2 & 0 & 10{,}4 & 0 & -0{,}8 & 0{,}8 \\
0 & 0 & -2{,}4 & -4{,}0 & 0{,}8 & -2{,}0 & 0 & -1{,}2 & 0 & 9{,}2 & 1{,}6 & -2{,}0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0{,}8 & -1{,}6 & -0{,}8 & 1{,}6 & 1{,}6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0{,}8 & -2{,}0 & 0{,}8 & -2{,}0 & 0 & 4{,}0
\end{bmatrix}
\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -10 \\ 0 \\ -10 \\ 0 \\ -10 \end{Bmatrix}
$$

Sınır koşulları

Sabit mesnet koşulu $\longrightarrow D_1 = D_2 = D_5 = D_6 = 0$

Simetri koşulu $\longrightarrow D_3 = D_{11} = 0 \;;\; D_7 = -D_9 \;;\; D_8 = D_{10}$

Sistemin çözümünden;

$D_3 = 0$

$D_4 = -5{,}52 \dfrac{8(1-0{,}2^2)}{Et}$

$D_7 = 0{,}37 \dfrac{8(1-0{,}2^2)}{Et}$

$D_8 = -6{,}13 \dfrac{8(1-0{,}2^2)}{Et}$

$D_9 = -0{,}37 \dfrac{8(1-0{,}2^2)}{Et}$

$D_{10} = -6{,}13 \dfrac{8(1-0{,}2^2)}{Et}$

$D_{11} = 0$

$D_{12} = -8{,}49 \dfrac{8(1-0{,}2^2)}{Et}$

Gerilmelerin Hesabı

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_{1.eleman} = \frac{8(1-0{,}2^2)}{Et}\;\frac{E}{4(1-0{,}2^2)}\begin{bmatrix}-2 & -0{,}2 & 2 & -0{,}2 & 0 & 0{,}4\\ -0{,}4 & -1 & 0{,}4 & -1 & 0 & 2\\ -0{,}4 & -0{,}8 & -0{,}4 & 0{,}8 & 0{,}8 & 0\end{bmatrix}\begin{array}{c c}\begin{bmatrix}0\\ 0\\ 0\\ -5{,}52\\ 0{,}37\\ -6{,}13\end{bmatrix} & \begin{array}{c}D_1\\ D_2\\ D_3\\ D_4\\ D_7\\ D_8\end{array}\end{array}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_{1.eleman} = \frac{1}{t}\begin{bmatrix}-2{,}696\\ -13{,}480\\ -8{,}240\end{bmatrix}\; kN/m^2$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_{2.eleman} = \frac{2}{t}\begin{bmatrix}0 & -0{,}4 & 2 & 0{,}2 & -2 & 0{,}2\\ 0 & -2 & 0{,}4 & 1 & -0{,}4 & 1\\ -0{,}8 & 0 & 0{,}4 & 0{,}8 & 0{,}4 & -0{,}8\end{bmatrix}\begin{array}{c c}\begin{bmatrix}0\\ -5{,}52\\ -0{,}37\\ -6{,}13\\ 0{,}37\\ -6{,}13\end{bmatrix} & \begin{array}{c}D_3\\ D_4\\ D_9\\ D_{10}\\ D_7\\ D_8\end{array}\end{array}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_{2.eleman} = \frac{1}{t}\begin{bmatrix}-3{,}448\\ -3{,}032\\ 0\end{bmatrix}\; kN/m^2$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_{3.eleman} = \frac{2}{t}\begin{bmatrix}-2 & -0{,}2 & 2 & -0{,}2 & 0 & 0{,}4\\ -0{,}4 & -1 & 0{,}4 & -1 & 0 & 2\\ -0{,}4 & -0{,}8 & -0{,}4 & 0{,}8 & 0{,}8 & 0\end{bmatrix}\begin{array}{c c}\begin{bmatrix}0\\ -5{,}52\\ 0\\ 0\\ -0{,}37\\ -6{,}13\end{bmatrix} & \begin{array}{c}D_3\\ D_4\\ D_5\\ D_6\\ D_9\\ D_{10}\end{array}\end{array}$$

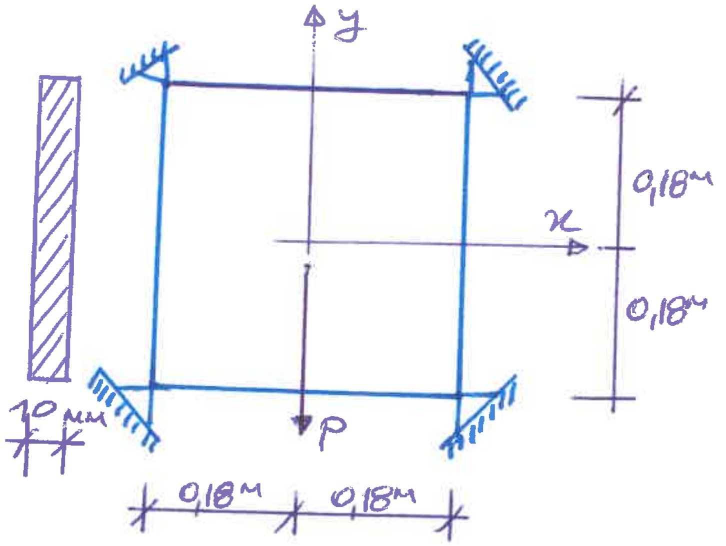
$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_{3.eleman} = \frac{1}{t}\begin{bmatrix}\quad\\ \quad\\ \quad\end{bmatrix}\; kN/m^2$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_{4\text{-eleman}} = \frac{2}{t} \begin{bmatrix} -2 & -0{,}2 & 2 & -0{,}2 & 0 & 0{,}4 \\ -0{,}4 & -1 & 0{,}4 & -1 & 0 & 2 \\ -0{,}4 & -0{,}8 & -0{,}4 & 0{,}8 & 0{,}8 & 0 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 0{,}37 \\ -0{,}13 \\ -0{,}37 \\ -0{,}13 \\ 0 \\ -8{,}49 \end{bmatrix} \end{array} \begin{array}{c} D_7 \\ D_8 \\ D_9 \\ D_{10} \\ D_{11} \\ D_{12} \end{array}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_{4\text{-eleman}} = \frac{1}{t} \begin{bmatrix} -4{,}848 \\ -10{,}032 \\ 0 \end{bmatrix} \quad kN/m^2$$

ÖRNEK 2)

Şekilde yükleme durumu ve boyutları verilen levhayı üçgen ve dörtgen sekil fonksiyonları ile çözerek gerilmeleri ve yer değiştirmeleri hesaplayınız.



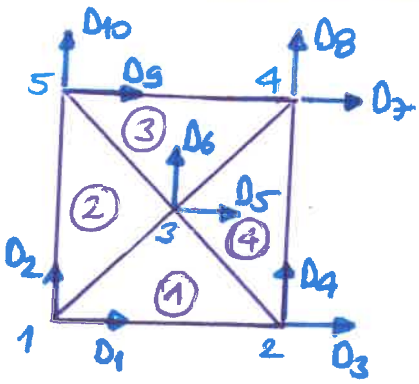
$E = 210000$ MPa

$\nu = 0{,}25$

$t = 10$ mm

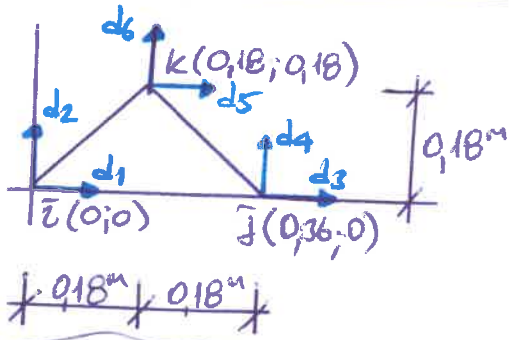
$P = 420$ kN

a) Üçgen elemanlarla çözüm



Koordinatlar	X(m)	y(m)
1	0	0
2	0,36	0
3	0,18	0,18
4	0,36	0,36
5	0	0,36

1 nolu eleman



$\Delta = \dfrac{0{,}36 \times 0{,}18}{2} = 0{,}0324$

$a_i = 0{,}36 \times 0{,}18 = 0{,}0648$

$b_i = -0{,}18$

$c_i = 0{,}18 - 0{,}36 = -0{,}18$

$a_j = 0{,}18 \times 0 - 0{,}18 \times 0 = 0$

$b_j = 0{,}18 - 0 = 0{,}18$

$c_j = -0{,}18$

$a_k = 0 - 0{,}36 \times 0 = 0$

$b_k = 0 - 0 = 0$

$c_k = 0{,}36 - 0 = 0{,}36$

$b_i + b_j + b_k = 0$

$c_i + c_j + c_k = 0$

$$\{\varepsilon\} = \frac{1}{2 \times 0{,}0324} \begin{bmatrix} -0{,}18 & 0 & 0{,}18 & 0 & 0 & 0 \\ 0 & -0{,}18 & 0 & -0{,}18 & 0 & 0{,}36 \\ -0{,}18 & -0{,}18 & -0{,}18 & 0{,}18 & 0{,}36 & 0 \end{bmatrix} \{d\}$$

$$\{\sigma\} = \frac{E}{2 \times 0{,}0324\,(1-0{,}25^2)} \begin{bmatrix} 1 & 0{,}25 & 0 \\ 0{,}25 & 1 & 0 \\ 0 & 0 & \frac{1-0{,}25}{2} \end{bmatrix} \begin{bmatrix} -0{,}18 & 0 & 0{,}18 & 0 & 0 & 0 \\ 0 & -0{,}18 & 0 & -0{,}18 & 0 & 0{,}36 \\ -0{,}18 & -0{,}18 & -0{,}18 & 0{,}18 & 0{,}36 & 0 \end{bmatrix} \{d\}$$

$$\{\sigma\} = \frac{E}{0{,}0648\,(1-0{,}25^2)} \begin{bmatrix} -0{,}18 & -0{,}045 & 0{,}18 & -0{,}045 & 0 & 0{,}09 \\ -0{,}045 & -0{,}18 & 0{,}045 & -0{,}18 & 0 & 0{,}36 \\ -0{,}0675 & -0{,}0675 & -0{,}0675 & 0{,}0675 & 0{,}135 & 0 \end{bmatrix} \{d\}$$

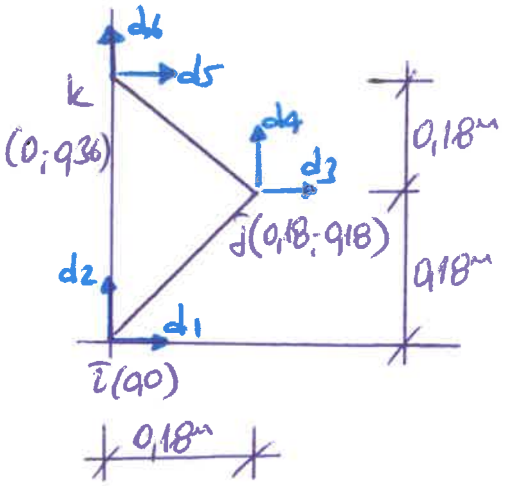
$$[k_{e1}] = \frac{Et}{4\Delta^2(1-\nu^2)} \iint \begin{bmatrix} -0{,}18 & 0 & -0{,}18 \\ 0 & -0{,}18 & -0{,}18 \\ 0{,}18 & 0 & -0{,}18 \\ 0 & -0{,}18 & 0{,}18 \\ 0 & 0 & 0{,}36 \\ 0 & 0{,}36 & 0 \end{bmatrix} \begin{bmatrix} -0{,}18 & -0{,}045 & 0{,}18 & -0{,}045 & 0 & 0{,}09 \\ -0{,}045 & -0{,}18 & 0{,}045 & -0{,}18 & 0 & 0{,}36 \\ -0{,}0675 & -0{,}0675 & -0{,}0675 & 0{,}0675 & 0{,}135 & 0 \end{bmatrix} dx\,dy$$

$t = 0{,}01\ m^2$, $\Delta = 0{,}0324\ m^2$

$\nu = 0{,}25$

$$[k_{e1}] = \frac{Et}{0{,}1296\,(1-0{,}25^2)} \begin{bmatrix} 0{,}04455 & 0{,}02025 & -0{,}02025 & -0{,}00405 & -0{,}0243 & -0{,}0162 \\ 0{,}02025 & 0{,}04455 & 0{,}00405 & 0{,}02025 & -0{,}0243 & -0{,}0648 \\ -0{,}02025 & 0{,}00405 & 0{,}04455 & -0{,}02025 & -0{,}0243 & 0{,}0162 \\ -0{,}00405 & 0{,}02025 & -0{,}02025 & 0{,}04455 & 0{,}0243 & -0{,}0648 \\ -0{,}0243 & -0{,}0243 & -0{,}0243 & 0{,}0243 & 0{,}0486 & 0 \\ -0{,}0162 & -0{,}0648 & 0{,}0162 & -0{,}0648 & 0 & 0{,}1296 \end{bmatrix}$$

2 ndu eleman



$a_i = 0{,}0648 \quad b_i = -0{,}18 \quad c_i = -0{,}18$

$a_j = 0 \quad b_j = 0{,}36 \quad c_j = 0$

$a_k = 0 \quad b_k = -0{,}18 \quad c_k = 0{,}18$

$$\iint dx\,dy = 0{,}0324$$

$$\{\varepsilon\} = \frac{1}{2 \times 0{,}0324} \begin{bmatrix} -0{,}18 & 0 & 0{,}36 & 0 & -0{,}18 & 0 \\ 0 & -0{,}18 & 0 & 0 & 0 & 0{,}18 \\ -0{,}18 & -0{,}18 & 0 & 0{,}36 & 0{,}18 & -0{,}18 \end{bmatrix} \{d\}$$

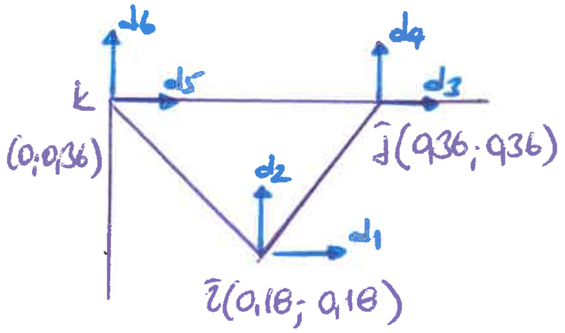
$$\{\sigma\} = \frac{E}{2 \times 0{,}0324(1-0{,}25^2)} \begin{bmatrix} 1 & 0{,}25 & 0 \\ 0{,}25 & 1 & 0 \\ 0 & 0 & 0{,}375 \end{bmatrix} \begin{bmatrix} -0{,}18 & 0 & 0{,}36 & 0 & -0{,}18 & 0 \\ 0 & -0{,}18 & 0 & 0 & 0 & 0{,}18 \\ -0{,}18 & -0{,}18 & 0 & 0{,}36 & 0{,}18 & -0{,}18 \end{bmatrix} \{d\}$$

$$\{\sigma\} = \frac{}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} -0{,}18 & -0{,}045 & 0{,}36 & 0 & -0{,}18 & 0{,}045 \\ -0{,}045 & -0{,}18 & 0{,}09 & 0 & -0{,}045 & 0{,}18 \\ -0{,}0675 & -0{,}0675 & 0 & 0{,}135 & 0{,}0675 & -0{,}0675 \end{bmatrix} \{d\}$$

$$[ke_2] = \frac{Et}{4\Delta^2(1-0{,}25^2)} \iint \begin{bmatrix} -0{,}18 & 0 & -0{,}18 \\ 0 & -0{,}18 & -0{,}18 \\ 0{,}36 & 0 & 0 \\ 0 & 0 & 0{,}36 \\ -0{,}18 & 0 & 0{,}18 \\ 0 & 0{,}18 & -0{,}18 \end{bmatrix} \begin{bmatrix} -0{,}18 & -0{,}045 & 0{,}36 & 0 & -0{,}18 & 0{,}045 \\ -0{,}045 & -0{,}18 & 0{,}09 & 0 & -0{,}045 & 0{,}18 \\ -0{,}0675 & -0{,}0675 & 0 & 0{,}135 & 0{,}0675 & -0{,}0675 \end{bmatrix} dx\,dy$$

$$[Ke_2] = \frac{Et}{0{,}1296(1-0{,}25^2)} \begin{bmatrix} 0{,}04455 & 0{,}02025 & -0{,}0648 & -0{,}0243 & 0{,}02025 & 0{,}00405 \\ 0{,}02025 & 0{,}04455 & -0{,}0162 & -0{,}0243 & -0{,}00405 & -0{,}02025 \\ -0{,}0648 & -0{,}0162 & 0{,}1296 & 0 & -0{,}0648 & 0{,}0162 \\ -0{,}0243 & -0{,}0243 & 0 & 0{,}0486 & 0{,}0243 & -0{,}0243 \\ 0{,}02025 & -0{,}00405 & -0{,}0648 & 0{,}0243 & 0{,}04455 & -0{,}02025 \\ 0{,}00405 & -0{,}02025 & 0{,}0162 & -0{,}0243 & -0{,}02025 & 0{,}04455 \end{bmatrix}$$

3 nolu eleman



$a_i = 0{,}1296$	$a_j = -0{,}0648$	$a_k = 0$
$b_i = 0$	$b_j = 0{,}18$	$b_k = -0{,}18$
$c_i = -0{,}36$	$c_j = 0{,}18$	$c_k = 0{,}18$

$$\{\varepsilon\} = \frac{1}{0{,}0648} \begin{bmatrix} 0 & 0 & 0{,}18 & 0 & -0{,}18 & 0 \\ 0 & -0{,}36 & 0 & 0{,}18 & 0 & 0{,}18 \\ -0{,}36 & 0 & 0{,}18 & 0{,}18 & 0{,}18 & -0{,}18 \end{bmatrix} \{d\}$$

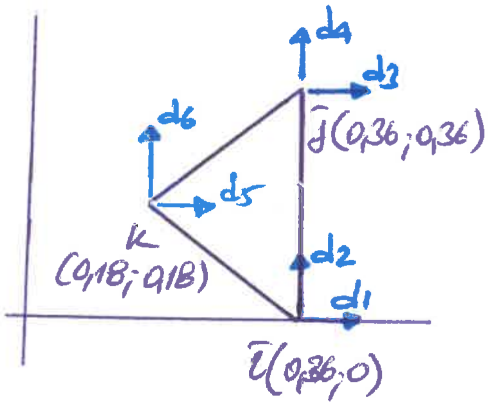
$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} 1 & 0{,}25 & 0 \\ 0{,}25 & 1 & 0 \\ 0 & 0 & 0{,}375 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0{,}18 & 0 & -0{,}18 & 0 \\ 0 & -0{,}36 & 0 & 0{,}18 & 0 & 0{,}18 \\ -0{,}36 & 0 & 0{,}18 & 0{,}18 & 0{,}18 & -0{,}18 \end{bmatrix} \{d\}$$

$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} 0 & -0{,}09 & 0{,}18 & 0{,}045 & -0{,}18 & 0{,}045 \\ 0 & -0{,}36 & 0{,}045 & 0{,}18 & -0{,}045 & 0{,}18 \\ -0{,}135 & 0 & 0{,}0675 & 0{,}0675 & 0{,}0675 & -0{,}0675 \end{bmatrix} \{d\}$$

$$[Ke_3] = \frac{Et}{4\Delta^2(1-0{,}25^2)} \iint \begin{bmatrix} 0 & 0 & -0{,}36 \\ 0 & -0{,}36 & 0 \\ 0{,}18 & 0 & 0{,}18 \\ 0 & 0{,}18 & 0{,}18 \\ -0{,}18 & 0 & 0{,}18 \\ 0 & 0{,}18 & -0{,}18 \end{bmatrix} \begin{bmatrix} 0 & -0{,}09 & 0{,}18 & 0{,}045 & -0{,}18 & 0{,}045 \\ 0 & -0{,}36 & 0{,}045 & 0{,}18 & -0{,}045 & 0{,}18 \\ -0{,}135 & 0 & 0{,}0675 & 0{,}0675 & 0{,}0675 & -0{,}0675 \end{bmatrix} d$$

$$[Ke_3] = \frac{Et}{0{,}1296(1-0{,}25^2)} \begin{bmatrix} 0{,}0486 & 0 & -0{,}0243 & -0{,}0243 & -0{,}0243 & 0{,}0243 \\ 0 & 0{,}1296 & -0{,}0162 & -0{,}0648 & 0{,}0162 & -0{,}0648 \\ -0{,}0243 & -0{,}0162 & 0{,}04455 & 0{,}02025 & -0{,}02025 & -0{,}00405 \\ -0{,}0243 & -0{,}0648 & 0{,}02025 & 0{,}04455 & 0{,}00405 & 0{,}02025 \\ -0{,}0243 & 0{,}0162 & -0{,}02025 & 0{,}00405 & 0{,}04455 & -0{,}02025 \\ 0{,}0243 & -0{,}0648 & -0{,}00405 & 0{,}02025 & -0{,}02025 & 0{,}04455 \end{bmatrix}$$

4 nolu eleman



$a_i = 0$ | $a_j = -0{,}0648$ | $a_k = 0{,}1296$

$b_i = 0{,}18$ | $b_j = 0{,}18$ | $b_k = -0{,}36$

$c_i = -0{,}18$ | $c_j = 0{,}18$ | $c_k = 0$

$$\{\varepsilon\} = \frac{1}{0{,}0648} \begin{bmatrix} 0{,}18 & 0 & 0{,}18 & 0 & -0{,}36 & 0 \\ 0 & -0{,}18 & 0 & 0{,}18 & 0 & 0 \\ -0{,}18 & 0{,}18 & 0{,}18 & 0{,}18 & 0 & -0{,}36 \end{bmatrix} \{d\}$$

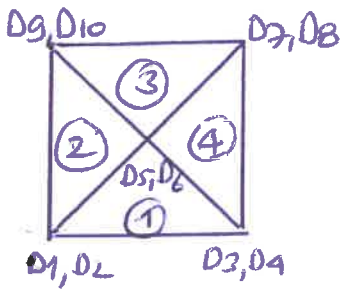
$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} 1 & 0{,}25 & 0 \\ 0{,}25 & 1 & 0 \\ 0 & 0 & 0{,}375 \end{bmatrix} \begin{bmatrix} 0{,}18 & 0 & 0{,}18 & 0 & -0{,}36 & 0 \\ 0 & -0{,}18 & 0 & 0{,}18 & 0 & 0 \\ -0{,}18 & 0{,}18 & 0{,}18 & 0{,}18 & 0 & -0{,}36 \end{bmatrix} \{d\}$$

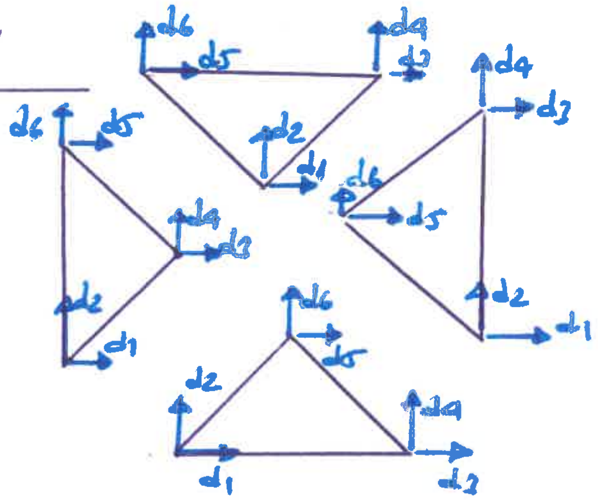
$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} 0{,}18 & -0{,}045 & 0{,}18 & 0{,}045 & -0{,}36 & 0 \\ 0{,}045 & -0{,}18 & 0{,}045 & 0{,}18 & -0{,}09 & 0 \\ -0{,}0675 & 0{,}0675 & 0{,}0675 & 0{,}0675 & 0 & -0{,}135 \end{bmatrix} \{d\}$$

$$[k_{e4}] = \frac{Et}{4\Delta^2(1-0{,}25^2)} \iint \begin{bmatrix} 0{,}18 & 0 & -0{,}18 \\ 0 & -0{,}18 & 0{,}18 \\ 0{,}18 & 0 & 0{,}18 \\ 0 & 0{,}18 & 0{,}18 \\ -0{,}36 & 0 & 0 \\ 0 & 0 & -0{,}36 \end{bmatrix} \begin{bmatrix} 0{,}18 & -0{,}045 & 0{,}18 & 0{,}045 & -0{,}36 & 0 \\ 0{,}045 & -0{,}18 & 0{,}045 & 0{,}18 & -0{,}09 & 0 \\ -0{,}0675 & 0{,}0675 & 0{,}0675 & 0{,}0675 & 0 & -0{,}135 \end{bmatrix} dx\,dy$$

$$[k_{e4}] = \frac{Et}{0{,}1296(1-0{,}25)^2} \begin{bmatrix} 0{,}04455 & -0{,}02025 & 0{,}02025 & -0{,}00405 & -0{,}0648 & 0{,}0243 \\ -0{,}02025 & 0{,}04455 & 0{,}00405 & -0{,}02025 & 0{,}0162 & -0{,}0243 \\ 0{,}02025 & 0{,}00405 & 0{,}04455 & 0{,}02025 & -0{,}0648 & -0{,}0243 \\ -0{,}00405 & -0{,}02025 & 0{,}02025 & 0{,}04455 & -0{,}0162 & -0{,}0243 \\ -0{,}0648 & 0{,}0162 & -0{,}0648 & -0{,}0162 & 0{,}1296 & 0 \\ 0{,}0243 & -0{,}0243 & -0{,}0243 & -0{,}0243 & 0 & 0{,}0486 \end{bmatrix}$$

Gerilim Matrisleri





$$[Ce_1]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[Ce_2]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[Ce_3]=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[Ce_4]=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Buradan eleman rijitlik matrislerini hesaplarsak;

$$[Ke_1]=[Ce_1]^T[ke_1][Ce_1]$$

$$[Ke_2]=[Ce_2]^T[ke_2][Ce_2]$$

$$[Ke_3]=[Ce_3]^T[ke_3][Ce_3]$$

$$[Ke_4]=[Ce_4]^T[Ke_4][Ce_4]$$

$$[Ke1]=\begin{bmatrix}
0.04455 & 0.02025 & -0.02025 & -0.00405 & -0.0243 & -0.0162 & 0 & 0 & 0 & 0 \\
0.02025 & 0.04455 & 0.00405 & 0.02025 & -0.0243 & -0.0648 & 0 & 0 & 0 & 0 \\
-0.02025 & 0.00405 & 0.04455 & -0.02025 & -0.0243 & 0.0162 & 0 & 0 & 0 & 0 \\
-0.00405 & 0.02025 & -0.02025 & 0.04455 & 0.0243 & -0.0648 & 0 & 0 & 0 & 0 \\
-0.0243 & -0.0243 & -0.0243 & 0.0243 & 0.0486 & 0 & 0 & 0 & 0 & 0 \\
-0.0162 & -0.0648 & 0.0162 & -0.0648 & 0 & 0.1296 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$[Ke2]=\begin{bmatrix}
0.04455 & 0.02025 & 0 & 0 & -0.0648 & -0.0243 & 0 & 0 & 0.02025 & 0.00405 \\
0.02025 & 0.04455 & 0 & 0 & -0.0162 & -0.0243 & 0 & 0 & -0.00405 & -0.02025 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0648 & -0.0162 & 0 & 0 & 0.1296 & 0 & 0 & 0 & -0.0648 & 0.0162 \\
-0.0243 & -0.0243 & 0 & 0 & 0 & 0.0486 & 0 & 0 & 0.0243 & -0.0243 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.02025 & -0.00405 & 0 & 0 & -0.0648 & 0.0243 & 0 & 0 & 0.04455 & -0.02025 \\
0.00405 & -0.02025 & 0 & 0 & 0.0162 & -0.0243 & 0 & 0 & -0.02025 & 0.04455
\end{bmatrix}$$

$$[Ke3]=\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0486 & 0 & -0.0243 & -0.0243 & -0.0243 & 0.0243 \\
0 & 0 & 0 & 0 & 0 & 0.1296 & -0.0162 & -0.0648 & 0.0162 & -0.0648 \\
0 & 0 & 0 & 0 & -0.0243 & -0.0162 & 0.04455 & 0.02025 & -0.02025 & -0.00405 \\
0 & 0 & 0 & 0 & -0.0243 & -0.0648 & 0.02025 & 0.04455 & 0.00405 & 0.02025 \\
0 & 0 & 0 & 0 & -0.0243 & 0.0162 & -0.02025 & 0.00405 & 0.04455 & -0.02025 \\
0 & 0 & 0 & 0 & 0.0243 & -0.0648 & -0.00405 & 0.02025 & -0.02025 & 0.04455
\end{bmatrix}$$

$$[Ke4]=\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.04455 & -0.02025 & -0.0648 & 0.0243 & 0.02025 & -0.00405 & 0 & 0 \\
0 & 0 & -0.02025 & 0.04455 & 0.0162 & -0.0243 & 0.00405 & -0.02025 & 0 & 0 \\
0 & 0 & -0.0648 & 0.0162 & 0.1296 & 0 & -0.0648 & -0.0162 & 0 & 0 \\
0 & 0 & 0.0243 & -0.0243 & 0 & 0.0486 & -0.0243 & -0.0243 & 0 & 0 \\
0 & 0 & 0.02025 & 0.00405 & -0.0648 & -0.0243 & 0.04455 & 0.02025 & 0 & 0 \\
0 & 0 & -0.00405 & -0.02025 & -0.0162 & -0.0243 & 0.02025 & 0.04455 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

matrisleri elde edilir. Toplam Rijitlik matrisi;

$[Ke] = [Ke_1] + [Ke_2] + [Ke_3] + [Ke_4]$ şeklindedir.

$$\frac{K}{0{,}1296/1-}\begin{bmatrix} 0.0891 & 0.0405 & -0.02025 & -0.00405 & -0.0891 & -0.0405 & 0 & 0 & 0.02025 & 0.00405 \\ 0.0405 & 0.0891 & 0.00405 & 0.02025 & -0.0405 & -0.0891 & 0 & 0 & -0.00405 & -0.02025 \\ -0.02025 & 0.00405 & 0.0891 & -0.0405 & -0.0891 & 0.0405 & 0.02025 & -0.00405 & 0 & 0 \\ -0.00405 & 0.02025 & -0.0405 & 0.0891 & 0.0405 & -0.0891 & 0.00405 & -0.02025 & 0 & 0 \\ -0.0891 & -0.0405 & -0.0891 & 0.0405 & 0.3564 & 0 & -0.0891 & -0.0405 & -0.0891 & 0.0405 \\ -0.0405 & -0.0891 & 0.0405 & -0.0891 & 0 & 0.3564 & -0.0405 & -0.0891 & 0.0405 & -0.0891 \\ 0 & 0 & 0.02025 & 0.00405 & -0.0891 & -0.0405 & 0.0891 & 0.0405 & -0.02025 & -0.00405 \\ 0 & 0 & -0.00405 & -0.02025 & -0.0405 & -0.0891 & 0.0405 & 0.0891 & 0.00405 & 0.02025 \\ 0.02025 & -0.00405 & 0 & 0 & -0.0891 & 0.0405 & -0.0203 & 0.00405 & 0.0891 & -0.0405 \\ 0.00405 & -0.02025 & 0 & 0 & 0.0405 & -0.0891 & -0.0041 & 0.02025 & -0.0405 & 0.0891 \end{bmatrix} \begin{bmatrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \\ D7 \\ D8 \\ D9 \\ D10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -420 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3564 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3564 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D5 \\ D6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -420 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Buradan $D5 = 0$

$$D6 = -\frac{420 \times 0{,}1296(1-0{,}25^2)}{21 \times 10^7 \times 0{,}01 \times 0{,}3564} = -6{,}82 \times 10^{-5}\ m.$$

Gerilmelerin hesabı;

1 nolu eleman

$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} -0{,}18 & -0{,}045 & 0{,}18 & -0{,}045 & 0 & 0{,}09 \\ -0{,}045 & -0{,}18 & 0{,}045 & -0{,}18 & 0 & 0{,}36 \\ -0{,}0675 & -0{,}0675 & -0{,}0675 & 0{,}0675 & 0{,}135 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -6{,}82\cdot10^{-5} \end{bmatrix}$$

$$\sigma_x^1 = \frac{E}{0{,}0648(1-0{,}25^2)} (0{,}09)(-6{,}82\cdot10^{-5})\cdot 10^{-3} = -21{,}22 \ N/mm^2$$

$$\sigma_y^1 = \frac{E}{0{,}0648(1-0{,}25^2)} (0{,}36)(-6{,}82\cdot10^{-5})\cdot 10^{-3} = -84{,}88 \ N/mm^2$$

$$\tau_{xy}^1 = 0$$

2 ve 4 nolu elemanlar

$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} -0{,}18 & -0{,}045 & 0{,}36 & 0 & -0{,}18 & 0{,}045 \\ -0{,}045 & -0{,}18 & 0{,}09 & 0 & -0{,}045 & 0{,}18 \\ -0{,}0675 & -0{,}0675 & 0 & 0{,}135 & 0{,}0675 & -0{,}0675 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -6{,}82\cdot10^{-5} \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma_x^{24} = \frac{E}{0{,}0648(1-0{,}25^2)} (0)(-6{,}82\cdot10^{-5})\cdot 10^{-3} = 0$$

$$\sigma_y^{24} = 0$$

$$\tau_{xy} = \frac{E}{0{,}0648(1-0{,}25^2)} (0{,}135)(-6{,}82\cdot10^{-5})\cdot 10^{-3} = -31{,}82 \ N/mm^2$$

3 nolu eleman

$$\{\sigma\} = \frac{E}{0{,}0648(1-0{,}25^2)} \begin{bmatrix} 0 & -0{,}09 & 0{,}18 & 0{,}045 & -0{,}18 & 0{,}045 \\ 0 & -0{,}36 & 0{,}045 & 0{,}18 & -0{,}045 & 0{,}18 \\ -0{,}135 & 0 & 0{,}0675 & 0{,}0675 & 0{,}0675 & -0{,}0675 \end{bmatrix} \begin{bmatrix} 0 \\ -6{,}82\cdot10^{-5} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma_x^3 = \frac{E}{0{,}0648(1-0{,}25^2)} (-0{,}09)(-6{,}82\cdot10^{-5})\cdot 10^{-3} = 21{,}22 \ N/mm^2$$

$$\sigma_y^3 = \quad \text{"} \quad (-0{,}36)(\ \text{"}\ )\ \text{"} = 84{,}88 \ \text{"} \qquad \tau_{xy}^3 = 0$$

Sistemi incelediğimizde yalnızca D_6 yer değiştirmesinin olduğu, diğer yerdeğiştirmelerin sıfır olduğu görülmektedir. Bu durumda 1 elemanında k_{66}, 2 elemanında k_{44}, 3 elemanında k_{22} ve 4 elemanında k_{66} rijitlik değerlerini alabiliriz. Üçgen elemanlar için hazırlanan (30) nolu tablodan;

$$k^3_{22}=\frac{Et}{4\Delta(1-\nu^2)}\left[\left(\frac{1-\nu}{2}\right)b_i^2+c_i^2\right]=\frac{Et}{4\Delta(1-\nu^2)}\,0{,}1296 \quad \left\| \begin{array}{l} b_i=0 \\ c_i=-0{,}36 \end{array}\right.$$

$$k^2_{44}=\frac{Et}{4\Delta(1-\nu^2)}\left[\left(\frac{1-\nu}{2}\right)b_j^2+c_j^2\right]=\frac{Et}{4\Delta(1-\nu^2)}\,0{,}0486 \quad \left\| \begin{array}{l} b_j=0{,}36 \\ c_j=0 \end{array}\right.$$

$$k^1_{66}=\frac{Et}{4\Delta(1-\nu^2)}\left[\left(\frac{1-\nu}{2}\right)b_k^2+c_k^2\right]=\frac{Et}{4\Delta(1-\nu^2)}\,0{,}1296 \quad \left\| \begin{array}{l} b_k=0 \\ c_k=0{,}36 \end{array}\right.$$

$$k^4_{66}=\frac{Et}{4\Delta(1-\nu^2)}\left[\left(\frac{1-\nu}{2}\right)b_k^2+c_k^2\right]=\frac{Et}{4\Delta(1-\nu^2)}\,0{,}0486 \quad \left\| \begin{array}{l} b_k=-0{,}36 \\ c_k=0 \end{array}\right.$$

$$K_{66}=\frac{Et}{4\Delta(1-\nu^2)}(0{,}1296\times2+0{,}0486\times2)=\frac{Et}{4\Delta(1-\nu^2)}\,0{,}3564$$

$$K_{66}\times D_6=-Q$$

$$\frac{Et}{4\Delta(1-\nu^2)}\,0{,}3564\,D_6=-420$$

$$D_6=-\frac{420\times0{,}1296\times(1-0{,}25^2)}{21\times10^7\times0{,}01\times0{,}3564}=-6{,}82\times10^{-5}\,m.$$

Benzer şekilde gerilmeler hesaplanırsa;

1 nolu eleman için;

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{0{,}0648(1-0{,}25^2)}\begin{bmatrix}-0{,}18 & -0{,}045 & 0{,}18 & -0{,}045 & 0 & 0{,}09\\ -0{,}045 & -0{,}18 & 0{,}045 & -0{,}18 & 0 & 0{,}36\\ -0{,}0675 & -0{,}0675 & -0{,}0675 & 0{,}0675 & 0{,}135 & 0\end{bmatrix}\begin{bmatrix}d_1\\ d_2\\ d_3\\ d_4\\ d_5\\ d_6\end{bmatrix}$$

*1 nolu eleman için O6'ya karşı gelen yerdeğiştirme d_6'dır. Dolayısıyla her satırda d_6'ya karşı gelen katsayılar ele alınacaktır.

$$\sigma_x^1 = \frac{E}{0{,}0648(1-0{,}25^2)}(0{,}09)(-6{,}82\times10^{-5})\times10^{-3} = -21{,}22 \ N/mm^2$$

$$\sigma_y^1 = \frac{E}{0{,}0648(1-0{,}25^2)}(0{,}36)(-6{,}82\times10^{-5})\times10^{-3} = -84{,}88 \ N/mm^2$$

$$\tau_{xy}^1 = 0$$

2 ve 4 nolu elemanlar için;

*2 ve 4 nolu elemanlarda O6'ya karşı gelen yer değiştirme d_4'tür

$$\sigma_x^{2,4} = \frac{E}{0{,}0648(1-0{,}25^2)}(0)(-6{,}82\times10^{-5})\times10^{-3} = 0$$

$$\sigma_y^{2,4} = 0$$

$$\tau_{xy}^{2,4} = \frac{E}{0{,}0648(1-0{,}25^2)}(0{,}135)(-6{,}82\times10^{-5})\times10^{-3} = -31{,}82 \ N/mm^2$$

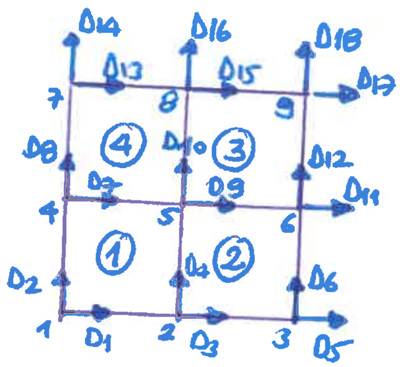
3 nolu eleman için;

$$\sigma_x^3 = \frac{E}{0{,}0648(1-0{,}25^2)}(-0{,}09)(-6{,}82\times10^{-5})\times10^{-3} = 21{,}22 \ N/mm^2$$

$$\sigma_y^3 = \frac{E}{0{,}0648(1-0{,}25^2)}(-0{,}36)(-6{,}82\times10^{-5})\times10^{-3} = 84{,}88 \ N/mm^2$$

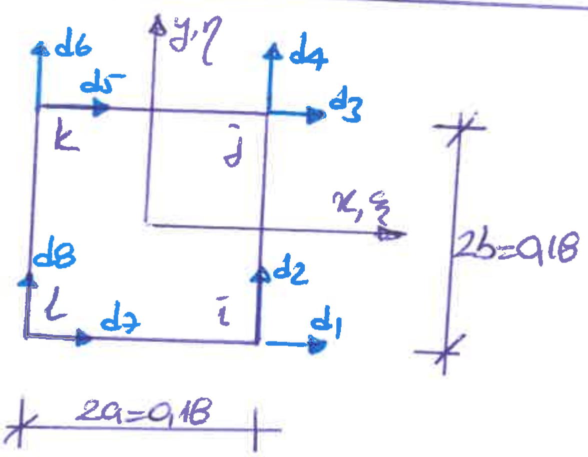
$$\tau_{xy}^3 = 0$$

b) Dörtgen elemanlarla çözüm



Koordinatlar	x(m)	y(m)
1	0	0
2	0,18	0
3	0,36	0
4	0	0,18
5	0,18	0,18
6	0,36	0,18
7	0	0,36
8	0,18	0,36
9	0,36	0,36

Eleman şekil fonksiyonun hesabı



$$\xi = \frac{x}{a} \qquad \eta = \frac{y}{b}$$

$$D = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$D_{11} = D_{22} = \frac{E}{1-\nu^2} = \frac{21\cdot 10^7}{1-0{,}25^2} = 22{,}4\cdot 10^7$$

$$D_{12} = D_{21} = \frac{E}{1-\nu^2}\nu = \frac{21\cdot 10^7}{1-0{,}25^2}0{,}25 = 5{,}6\cdot 10^7$$

$$D_{33} = \frac{E}{(1+\nu)2} = \frac{21\cdot 10^7}{(1+0{,}25)2} = 8{,}4\cdot 10^7$$

*

$$[ke] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} & k_{48} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} & k_{58} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{67} & k_{68} \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77} & k_{78} \\ k_{81} & k_{82} & k_{83} & k_{84} & k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix}$$

$$\beta = \frac{b}{a} = 1$$

$$k_{11} = \frac{0{,}01}{4}\left(22{,}4\times10^7 + 8{,}4\times10^7\right) = 77\times10^4$$

$$k_{21} = \frac{0{,}01}{4}\left(5{,}6\times10^7 + 8{,}4\times10^7\right) = 35\times10^4$$

$$k_{22} = \frac{0{,}01}{3}\left(22{,}4\times10^7 + 8{,}4\times10^7\right) = 102{,}67\times10^4$$

$$k_{31} = \frac{0{,}01}{4}\left(\frac{22{,}4}{2}\times10^7 - 8{,}4\times10^7\right) = 7\times10^4$$

$$k_{32} = \frac{0{,}01}{4}\left(5{,}6\times10^7 - 8{,}4\times10^7\right) = -7\times10^4$$

$$k_{33} = \frac{0{,}01}{3}\left(22{,}4\times10^7 + 8{,}4\times10^7\right) = 102{,}67\times10^4$$

$$k_{41} = \frac{0{,}01}{4}\left(-5{,}6\times10^7 + 8{,}4\times10^7\right) = 7\times10^4$$

$$k_{42} = \frac{0{,}01}{3}\left(-22{,}4\times10^7 + \frac{8{,}4}{2}\times10^7\right) = -60{,}67\times10^4$$

$$k_{43} = \frac{0{,}01}{4}\left(-5{,}6\times10^7 - 8{,}4\times10^7\right) = -35\times10^4$$

$$k_{44} = \frac{0{,}01}{3}\left(22{,}4\times10^7 + 8{,}4\times10^7\right) = 102{,}67\times10^4$$

$$k_{51} = \frac{0{,}01}{3}\left(-22{,}4\times10^7 + \frac{8{,}4}{2}\times10^7\right) = -60{,}67\times10^4$$

$$k_{52} = \frac{0{,}01}{4}\left(-5{,}6\times10^7 + 8{,}4\times10^7\right) = 7\times10^4$$

$$k_{53} = \frac{0{,}01}{6}\left(-22{,}4\times10^7 - 8{,}4\times10^7\right) = -51{,}33\times10^4$$

$$k_{54} = \frac{0{,}01}{4}\left(5{,}6\times10^7 + 8{,}4\times10^7\right) = 35\times10^4$$

$$k_{55} = \frac{0{,}01}{3}\left(22{,}4\times10^{7} + 8{,}4\times10^{7}\right) = 102{,}67\times10^{4}$$

$$k_{61} = \frac{0{,}01}{4}\left(5{,}6\times10^{7} - 8{,}4\times10^{7}\right) = -7\times10^{4}$$

$$k_{62} = \frac{0{,}01}{3}\left(\frac{22{,}4}{2}\times10^{7} - 8{,}4\times10^{7}\right) = 9{,}33\times10^{4}$$

$$k_{63} = \frac{0{,}01}{4}\left(5{,}6\times10^{7} + 8{,}4\times10^{7}\right) = 35\times10^{4}$$

$$k_{64} = \frac{0{,}01}{6}\left(-22{,}4\times10^{7} - 8{,}4\times10^{7}\right) = -51{,}33\times10^{4}$$

$$k_{65} = \frac{0{,}01}{4}\left(-5{,}6\times10^{7} - 8{,}4\times10^{7}\right) = -35\times10^{4}$$

$$k_{66} = \frac{0{,}01}{3}\left(22{,}4\times10^{7} + 8{,}4\times10^{7}\right) = 102{,}67\times10^{4}$$

$$k_{71} = \frac{0{,}01}{6}\left(-22{,}4\times10^{7} - 8{,}4\times10^{7}\right) = -51{,}33\times10^{4}$$

$$k_{72} = \frac{0{,}01}{4}\left(-5{,}6\times10^{7} - 8{,}4\times10^{7}\right) = -35\times10^{4}$$

$$k_{73} = \frac{0{,}01}{3}\left(-22{,}4\times10^{7} + \frac{8{,}4}{2}\times10^{7}\right) = -60{,}67\times10^{4}$$

$$k_{74} = \frac{0{,}01}{4}\left(5{,}6\times10^{7} - 8{,}4\times10^{7}\right) = -7\times10^{4}$$

$$k_{75} = \frac{0{,}01}{3}\left(\frac{22{,}4}{2}\times10^{7} - 8{,}4\times10^{7}\right) = 9{,}33\times10^{4}$$

$$k_{76} = \frac{0{,}01}{4}\left(-5{,}6\times10^{7} + 8{,}4\times10^{7}\right) = 7\times10^{4}$$

$$k_{77} = \frac{0{,}01}{3}\left(22{,}4\times10^{7} + 8{,}4\times10^{7}\right) = 102{,}67\times10^{4}$$

$$k_{81} = \frac{0{,}01}{4}\left(-5{,}6\times10^{7} - 8{,}4\times10^{7}\right) = -35\times10^{4}$$

$$k_{82} = \frac{0{,}01}{6}\left(-22{,}4\times10^{7} - 8{,}4\times10^{7}\right) = -51{,}33\times10^{4}$$

$$k_{83} = \frac{0{,}01}{4}\left(-5{,}6\times10^{7} + 8{,}4\times10^{7}\right) = 7\times10^{4}$$

$$k_{84} = \frac{0{,}01}{3}\left(\frac{22{,}4}{2}\times10^{7} - 8{,}4\times10^{7}\right) = 9{,}33\times10^{4}$$

$$k_{85} = \frac{0{,}01}{4}\left(5{,}6\times10^{7} - 8{,}4\times10^{7}\right) = -7\times10^{4}$$

$$k_{86} = \frac{0{,}01}{3}\left(-22{,}4\times10^{7} + \frac{8{,}4}{2}\times10^{7}\right) = -60{,}67\times10^{4}$$

$$k_{87} = \frac{0{,}01}{4}\left(5{,}6\times10^{7} + 8{,}4\times10^{7}\right) = 35\times10^{4}$$

$$k_{88} = \frac{0{,}01}{3}\left(22{,}4\times10^{7} + 8{,}4\times10^{7}\right) = 102{,}67\times10^{4}$$

	770000	350000	70000	70000	-606700	-70000	-513300	-350000
	350000	1026700	-70000	-606700	70000	93300	-350000	-513300
	70000	-70000	1026700	-350000	-513300	350000	-606700	70000
ke1	70000	-606700	-350000	1026700	350000	-513300	-70000	93300
	-606700	70000	-513300	350000	1026700	-350000	93300	-70000
	-70000	93300	350000	-513300	-350000	1026700	70000	-606700
	-513300	-350000	-606700	-70000	93300	70000	1026700	350000
	-350000	-513300	70000	93300	-70000	-606700	350000	1026700

$$
ce1\quad \begin{matrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
$$

$$
ce2\quad \begin{matrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
$$

$$
ce3\quad \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
$$

$$
ce4\quad \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
$$

$$ke_1 = [c_{e1}]^T[k_e][c_{e1}]$$

$$ke_2 = [c_{e2}]^T[k_e][c_{e2}]$$

$$ke_3 = [c_{e3}]^T[k_e][c_{e3}]$$

$$ke_4 = [c_{e4}]^T[k_e][c_{e4}]$$

	1026700	350000	-513300	-350000	0	0	93300	70000	-606700	-70000	0	0	0	0	0	0	0	0
	350000	1026700	-350000	-513300	0	0	-70000	-606700	70000	93300	0	0	0	0	0	0	0	0
	-513300	-350000	770000	350000	0	0	-606700	-70000	70000	70000	0	0	0	0	0	0	0	0
	-350000	-513300	350000	1026700	0	0	70000	93300	-70000	-606700	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	93300	-70000	-606700	70000	0	0	1026700	-350000	-513300	350000	0	0	0	0	0	0	0	0
	70000	-606700	-70000	93300	0	0	-350000	1026700	350000	-513300	0	0	0	0	0	0	0	0
ke1	-606700	70000	70000	-70000	0	0	-513300	350000	1026700	-350000	0	0	0	0	0	0	0	0
	-70000	93300	70000	-606700	0	0	350000	-513300	-350000	1026700	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

ke2

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1026700	350000	-513300	-350000	0	0	93300	70000	-606700	-70000	0	0	0	0	0	0
0	0	350000	1026700	-350000	-513300	0	0	-70000	-606700	70000	93300	0	0	0	0	0	0
0	0	-513300	-350000	770000	350000	0	0	-606700	-70000	70000	70000	0	0	0	0	0	0
0	0	-350000	-513300	350000	1026700	0	0	70000	93300	-70000	-606700	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	93300	-70000	-606700	70000	0	0	1026700	-350000	-513300	350000	0	0	0	0	0	0
0	0	70000	-606700	-70000	93300	0	0	-350000	1026700	350000	-513300	0	0	0	0	0	0
0	0	-606700	70000	70000	-70000	0	0	-513300	350000	1026700	-350000	0	0	0	0	0	0
0	0	-70000	93300	70000	-606700	0	0	350000	-513300	-350000	1026700	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ke3	0	0	0	0	0	0	0	0	1E+06	350000	-513300	-350000	0	0	93300	70000	-606700	-70000
	0	0	0	0	0	0	0	0	350000	1026700	-350000	-513300	0	0	-70000	-606700	70000	93300
	0	0	0	0	0	0	0	0	-5E+05	-350000	770000	350000	0	0	-606700	-70000	70000	70000
	0	0	0	0	0	0	0	0	-4E+05	-513300	350000	1026700	0	0	70000	93300	-70000	-606700
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	93300	-70000	-606700	70000	0	0	1026700	-350000	-513300	350000
	0	0	0	0	0	0	0	0	70000	-606700	-70000	93300	0	0	-350000	1026700	350000	-513300
	0	0	0	0	0	0	0	0	-6E+05	70000	70000	-70000	0	0	-513300	350000	1026700	-350000
	0	0	0	0	0	0	0	0	-70000	93300	70000	-606700	0	0	350000	-513300	-350000	1026700

Ke4

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1026700	350000	-513300	-350000	0	0	93300	70000	-606700	-70000	0	0
0	0	0	0	0	0	350000	1026700	-350000	-513300	0	0	-70000	-606700	70000	93300	0	0
0	0	0	0	0	0	-513300	-350000	770000	350000	0	0	-606700	-70000	70000	70000	0	0
0	0	0	0	0	0	-350000	-513300	350000	1026700	0	0	70000	93300	-70000	-606700	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	93300	-70000	-606700	70000	0	0	1026700	-350000	-513300	350000	0	0
0	0	0	0	0	0	70000	-606700	-70000	93300	0	0	-350000	1026700	350000	-513300	0	0
0	0	0	0	0	0	-606700	70000	70000	-70000	0	0	-513300	350000	1026700	-350000	0	0
0	0	0	0	0	0	-70000	93300	70000	-606700	0	0	350000	-513300	-350000	1026700	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Ke(sis)

1026700	4E+05	-5E+05	-350000	0	0	93300	70000	-606700	-70000	0	0	0	0	0	0	0	0
350000	1E+06	-4E+05	-513300	0	0	-70000	-606700	70000	93300	0	0	0	0	0	0	0	0
-513300	-4E+05	2E+06	700000	-513300	-350000	-606700	-70000	163300	140000	-606700	-70000	0	0	0	0	0	0
-350000	-5E+05	7E+05	2E+06	-350000	-513300	70000	93300	-140000	-1E+06	70000	93300	0	0	0	0	0	0
0	0	-5E+05	-350000	770000	350000	0	0	-606700	-70000	70000	70000	0	0	0	0	0	0
0	0	-4E+05	-513300	350000	1E+06	0	0	70000	93300	-70000	-606700	0	0	0	0	0	0
93300	-70000	-6E+05	70000	0	0	2E+06	0	-1E+06	0	0	0	93300	70000	-606700	-70000	0	0
70000	-6E+05	-70000	93300	0	0	0	2E+06	0	-1E+06	0	0	-70000	-606700	70000	93300	0	0
-606700	70000	2E+05	-140000	-606700	70000	-1E+06	0	4E+06	0	-1E+06	0	-606700	-70000	163300	140000	-606700	-70000
-70000	93300	1E+05	-1E+06	-70000	93300	0	-1E+06	0	4E+06	0	-1E+06	70000	93300	-140000	-1E+06	70000	93300
0	0	-6E+05	70000	70000	-70000	0	0	-1E+06	0	2E+06	0	0	0	-606700	-70000	70000	70000
0	0	-70000	93300	70000	-606700	0	0	0	-1E+06	0	2E+06	0	0	70000	93300	-70000	-606700
0	0	0	0	0	0	93300	-70000	-606700	70000	0	0	1E+06	-350000	-513300	350000	-70000	-606700
0	0	0	0	0	0	70000	-606700	-70000	93300	0	0	-350000	1E+06	350000	-513300	0	0
0	0	0	0	0	0	-606700	70000	163300	-140000	-606700	70000	-513300	350000	2E+06	-700000	-513300	350000
0	0	0	0	0	0	-70000	93300	140000	-1E+06	-70000	93300	350000	-513300	-700000	2E+06	350000	-513300
0	0	0	0	0	0	0	0	-606700	70000	70000	-70000	0	0	-513300	350000	1026700	-350000
0	0	0	0	0	0	0	0	-70000	93300	70000	-606700	0	0	350000	-513300	-350000	1026700

[K][D]={Q}

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		D1=0		0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		D2=0		0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		D3=0		0
2053400	0	0	0	93300	0	-1213400	0	93300	0	0	0	0	0	0		D4		0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0		D5=0		0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0		D6=0		0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0		D7=0		0
93300	0	0	0	2053400	0	-1026600	0	0	0	0	0	93300	0	0		D8		0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0		D9=0		0
-1213400	0	0	0	-1026600	0	4106800	0	-1026600	0	0	0	-1213400	0	0	*	D10	=	-420
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0		D11=0		0
93300	0	0	0	0	0	-1026600	0	2053400	0	0	0	93300	0	0		D12		0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0		D13=0		0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		D14=0		0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		D15=0		0
0	0	0	0	93300	0	-1213400	0	93300	0	0	0	2053400	0	0		D16		0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0		D17		0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1		D18		0

$$[D] = [K]^{-1}\{Q\}$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6.53E-07 & 0 & 0 & 0 & 1.12E-07 & 0 & 2.98E-07 & 0 & 1.12E-07 & 0 & 0 & 0 & 1.66E-07 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.12E-07 & 0 & 0 & 0 & 5.99E-07 & 0 & 2.43E-07 & 0 & 1.12E-07 & 0 & 0 & 0 & 1.12E-07 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2.98E-07 & 0 & 0 & 0 & 2.43E-07 & 0 & 5.41E-07 & 0 & 2.43E-07 & 0 & 0 & 0 & 2.98E-07 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.12E-07 & 0 & 0 & 0 & 1.12E-07 & 0 & 2.43E-07 & 0 & 5.99E-07 & 0 & 0 & 0 & 1.12E-07 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1.66E-07 & 0 & 0 & 0 & 1.12E-07 & 0 & 2.98E-07 & 0 & 1.12E-07 & 0 & 0 & 0 & 6.53E-07 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
*
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -420 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.00013 = D4 \\ 0 \\ 0 \\ 0 \\ -0.0001 = D8 \\ 0 \\ -0.00023 = D10 \\ 0 \\ -0.0001 = D12 \\ 0 \\ 0 \\ 0 \\ -0.00013 = D16 \\ 0 \\ 0 \end{bmatrix}$$

1 elemanı 1 noktası : $\xi=-1$ $\eta=-1$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0 & 0 & 0.18 & 0 & 0 & 0.045 & -0.18 & -0.045 \\ 0 & 0 & 0.045 & 0 & 0 & 0.18 & -0.045 & -0.18 \\ 0 & 0 & 0 & 0.0675 & 0.0675 & 0 & -0.0675 & -0.0675 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 0 \\ -0.00013 \\ 0 \\ -0.00023 \\ 0 \\ -0.0001 \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} D3 \\ D4 \\ D9 \\ D10 \\ D7 \\ D8 \\ D1 \\ D2 \end{array}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = 6913600 \begin{Bmatrix} -0.0000045 \\ -0.000018 \\ -0.000015525 \end{Bmatrix} = \begin{Bmatrix} -31.1112 \\ -12444 \\ -10733 \end{Bmatrix}$$

1 elemanı 2 noktası : $\xi=0$ $\eta=-1$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0 & 0.0225 & 0.18 & -0.0225 & 0 & 0.0225 & -0.18 & -0.0225 \\ 0 & 0.09 & 0.045 & -0.09 & 0 & 0.09 & -0.045 & -0.09 \\ 0.03375 & 0 & -0.03375 & 0.0675 & 0.03375 & 0 & -0.03375 & -0.0675 \end{bmatrix} \begin{bmatrix} 0 \\ -0.00013 \\ 0 \\ -0.00023 \\ 0 \\ -0.0001 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ -0.000015525 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -107.334 \end{Bmatrix}$$

1 Elemanı 5 noktası : $\xi=0 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0.0225 & 0.09 & -0.0225 & -0.09 & 0.0225 & -0.09 & -0.0225 \\ 0.0225 & 0.09 & 0.0225 & -0.09 & -0.0225 & 0.09 & -0.0225 & -0.09 \\ 0.03375 & 0.03375 & -0.03375 & 0.03375 & 0.03375 & -0.03375 & -0.03375 & -0.03375 \end{bmatrix} \begin{bmatrix} 0 \\ -0.00013 \\ 0 \\ -0.00023 \\ 0 \\ -0.0001 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ -0.000008775 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -60.667 \end{Bmatrix}$$

1 Elemanı 4 noktası : $\xi=-1 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0 & 0.09 & 0 & -0.09 & 0.045 & -0.09 & -0.045 \\ 0.0225 & 0 & 0.0225 & 0 & -0.0225 & 0.18 & -0.0225 & -0.18 \\ 0 & 0.03375 & 0 & 0.03375 & 0.0675 & -0.03375 & -0.0675 & -0.03375 \end{bmatrix} \begin{bmatrix} 0 \\ -0.00013 \\ 0 \\ -0.00023 \\ 0 \\ -0.0001 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} -0.0000045 \\ -0.000018 \\ -0.000008775 \end{Bmatrix} = \begin{Bmatrix} -31.1112 \\ -124.44 \\ -60.6668 \end{Bmatrix}$$

2 Elemanı 6 noktası : $\xi=+1 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0.045 & 0.09 & -0.045 & -0.09 & 0 & -0.09 & 0 \\ 0.0225 & 0.18 & 0.0225 & -0.18 & -0.0225 & 0 & -0.0225 & 0 \\ 0.0675 & 0.03375 & -0.0675 & 0.03375 & 0 & -0.03375 & 0 & -0.03375 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0001 \\ 0 \\ -0.00023 \\ 0 \\ -0.00013 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0.0000045 \\ 0.000018 \\ 0.000008775 \end{Bmatrix} = \begin{Bmatrix} 31.1112 \\ 124.4448 \\ 60.66684 \end{Bmatrix}$$

2 Elemanı 5 noktası : $\xi=0 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0.0225 & 0.09 & -0.0225 & -0.09 & 0.0225 & -0.09 & -0.0225 \\ 0.0225 & 0.09 & 0.0225 & -0.09 & -0.0225 & 0.09 & -0.0225 & -0.09 \\ 0.03375 & 0.03375 & -0.03375 & 0.03375 & 0.03375 & -0.03375 & -0.03375 & -0.03375 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0001 \\ 0 \\ -0.00023 \\ 0 \\ -0.00013 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ 0.000008775 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 60.66684 \end{Bmatrix}$$

2 Elemanı 2 noktası : $\xi=0 \quad \eta=-1$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0 & 0.0225 & 0.18 & -0.0225 & 0 & 0.0225 & -0.18 & -0.0225 \\ 0 & 0.09 & 0.045 & -0.09 & 0 & 0.09 & -0.045 & -0.09 \\ 0.03375 & 0 & -0.03375 & 0.0675 & 0.03375 & 0 & -0.03375 & -0.0675 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0001 \\ 0 \\ -0.00023 \\ 0 \\ -0.00013 \end{bmatrix} \end{array} \begin{array}{c} D5 \\ D6 \\ D11 \\ D12 \\ D9 \\ D10 \\ D3 \\ D4 \end{array}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ 0.000002025 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 14.00004 \end{Bmatrix}$$

2 Elemanı 3 noktası : $\xi=+1 \quad \eta=-1$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0 & 0.045 & 0.18 & -0.045 & 0 & 0 & -0.18 & 0 \\ 0 & 0.18 & 0.045 & -0.18 & 0 & 0 & -0.045 & 0 \\ 0.0675 & 0 & -0.0675 & 0.0675 & 0 & 0 & 0 & -0.0675 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.0001 \\ 0 \\ -0.00023 \\ 0 \\ -0.00013 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0.0000045 \\ 0.000018 \\ 0.000002025 \end{Bmatrix} = \begin{Bmatrix} 31.1112 \\ 124.4448 \\ 14.00004 \end{Bmatrix}$$

3 Elemanı 6 noktası : $\xi=+1 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0.045 & 0.09 & -0.045 & -0.09 & 0 & -0.09 & 0 \\ 0.0225 & 0.18 & 0.0225 & -0.18 & -0.0225 & 0 & -0.0225 & 0 \\ 0.0675 & 0.03375 & -0.0675 & 0.03375 & 0 & -0.03375 & 0 & -0.03375 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 0 \\ -0.0001 \\ 0 \\ 0 \\ 0 \\ -0.00013 \\ 0 \\ -0.00023 \end{bmatrix} \end{array} \begin{array}{c} D11 \\ D12 \\ D17 \\ D18 \\ D15 \\ D16 \\ D9 \\ D10 \end{array}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} -0.0000045 \\ -0.000018 \\ 0.000008775 \end{Bmatrix} = \begin{Bmatrix} -31.1112 \\ -124.445 \\ 60.66684 \end{Bmatrix}$$

3 Elemanı 9 noktası : $\xi=+1 \quad \eta=+1$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.18 & 0.045 & 0 & -0.045 & -0.18 & 0 & 0 & 0 \\ 0.045 & 0.18 & 0 & -0.18 & -0.045 & 0 & 0 & 0 \\ 0.0675 & 0.0675 & -0.0675 & 0 & 0 & -0.0675 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.0001 \\ 0 \\ 0 \\ 0 \\ -0.00013 \\ 0 \\ -0.00023 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} -0.0000045 \\ -0.000018 \\ 0.000002025 \end{Bmatrix} = \begin{Bmatrix} -31.1112 \\ -124.445 \\ 14.00004 \end{Bmatrix}$$

3 Elemanı 8 noktası : $\xi=0 \quad \eta=+1$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.18 & 0.0225 & 0 & -0.0225 & -0.18 & 0.0225 & 0 & -0.0225 \\ 0.045 & 0.09 & 0 & -0.09 & -0.045 & 0.09 & 0 & -0.09 \\ 0.03375 & 0.0675 & -0.03375 & 0 & 0.03375 & -0.0675 & -0.03375 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.0001 \\ 0 \\ 0 \\ 0 \\ -0.00013 \\ 0 \\ -0.00023 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ 0.000002025 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 14.00004 \end{Bmatrix}$$

3 Elemanı 5 noktası : $\xi=0 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0.0225 & 0.09 & -0.0225 & -0.09 & 0.0225 & -0.09 & -0.0225 \\ 0.0225 & 0.09 & 0.0225 & -0.09 & -0.0225 & 0.09 & -0.0225 & -0.09 \\ 0.03375 & 0.03375 & -0.03375 & 0.03375 & 0.03375 & -0.03375 & -0.03375 & -0.03375 \end{bmatrix} \begin{bmatrix} 0 \\ -0.0001 \\ 0 \\ 0 \\ 0 \\ -0.00013 \\ 0 \\ -0.00023 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ 0.000008775 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 60.66684 \end{Bmatrix}$$

4 Elemanı 5 noktası : $\xi=0 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0.0225 & 0.09 & -0.0225 & -0.09 & 0.0225 & -0.09 & -0.0225 \\ 0.0225 & 0.09 & 0.0225 & -0.09 & -0.0225 & 0.09 & -0.0225 & -0.09 \\ 0.03375 & 0.03375 & -0.03375 & 0.03375 & 0.03375 & -0.03375 & -0.03375 & -0.03375 \end{bmatrix} \begin{array}{c} \begin{bmatrix} 0 \\ -0.00023 \\ 0 \\ -0.00013 \\ 0 \\ 0 \\ 0 \\ -0.0001 \end{bmatrix} \end{array} \begin{array}{c} D9 \\ D10 \\ D15 \\ D16 \\ D13 \\ D14 \\ D7 \\ D8 \end{array}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ -0.000008775 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -60.66684 \end{Bmatrix}$$

4 elemanı 7 noktası : $\xi=-1 \quad \eta=+1$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.18 & 0 & 0 & 0 & -0.18 & 0.045 & 0 & -0.045 \\ 0.045 & 0 & 0 & 0 & -0.045 & 0.18 & 0 & -0.18 \\ 0 & 0.0675 & 0 & 0 & 0.0675 & -0.0675 & -0.0675 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.00023 \\ 0 \\ -0.00013 \\ 0 \\ 0 \\ 0 \\ -0.0001 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0.0000045 \\ 0.000018 \\ -0.000015525 \end{Bmatrix} = \begin{Bmatrix} 31.1112 \\ 124.4448 \\ -107.3336 \end{Bmatrix}$$

4 elemanı 8 noktası : $\xi=0 \quad \eta=+1$

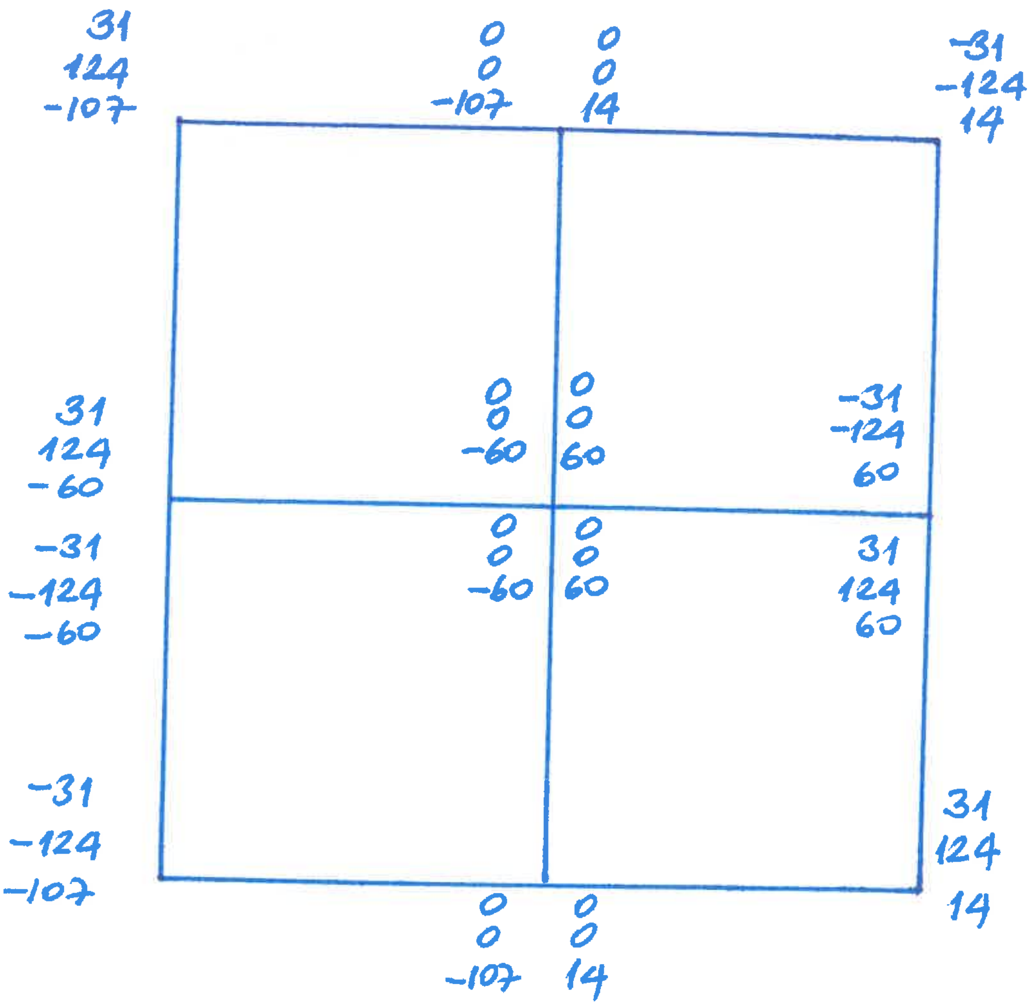
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.18 & 0.0225 & 0 & -0.0225 & -0.18 & 0.0225 & 0 & -0.0225 \\ 0.045 & 0.09 & 0 & -0.09 & -0.045 & 0.09 & 0 & -0.09 \\ 0.03375 & 0.0675 & -0.03375 & 0 & 0.03375 & -0.0675 & -0.03375 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.00023 \\ 0 \\ -0.00013 \\ 0 \\ 0 \\ 0 \\ -0.0001 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0 \\ 0 \\ -0.000015525 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -107.3336 \end{Bmatrix}$$

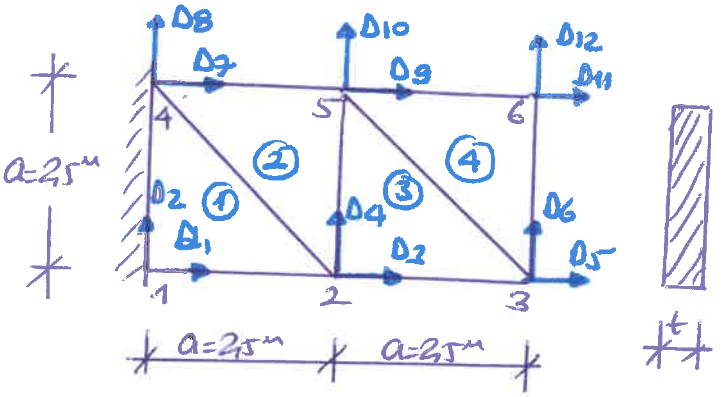
4 Elemanı 4 noktası : $\xi=-1 \quad \eta=0$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{4ab(1-\nu^2)} \begin{bmatrix} 0.09 & 0 & 0.09 & 0 & -0.09 & 0.045 & -0.09 & -0.045 \\ 0.0225 & 0 & 0.0225 & 0 & -0.0225 & 0.18 & -0.0225 & -0.18 \\ 0 & 0.03375 & 0 & 0.03375 & 0.0675 & -0.03375 & -0.0675 & -0.03375 \end{bmatrix} \begin{bmatrix} 0 \\ -0.00023 \\ 0 \\ -0.00013 \\ 0 \\ 0 \\ 0 \\ -0.0001 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 6913600 \begin{Bmatrix} 0.0000045 \\ 0.000018 \\ -0.000008775 \end{Bmatrix} = \begin{Bmatrix} 31.1112 \\ 124.44 \\ -60.66684 \end{Bmatrix}$$


31
124
-107
0
0
-107
0
0
14
-31
-124
14
31
124
-60
0
0
-60
0
0
60
-31
-124
60
-31
-124
-60
0
0
-60
0
0
60
31
124
60
-31
-124
-107
31
124
14
0
0
-107
0
0
14

ÖRNEK:3) Şekilde boyutları verilmiş olan levhanın 6 nolu düğüm noktası düşey yönde 3 cm çökme yapabilmesi durumunda taşıyabileceği düğüm noktası yüklerini bulunuz. 3 numaralı alanın $\Delta t = 30°C$ ısınması durumunda düğüm noktası yüklerini hesaplayınız.



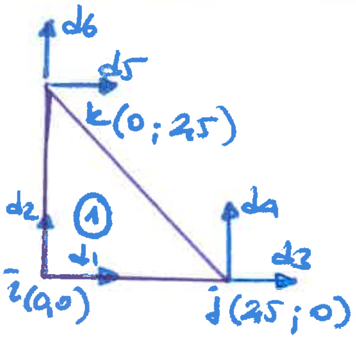
$E = 210000\ N/mm^2 = 21 \times 10^7\ kN/m^2$

$\nu = 0{,}30$, $t = 0{,}03\ m$

$\alpha_t = 1{,}2 \times 10^{-4}\ 1/°C$

$\delta_{6y} = 0{,}02\ m.$

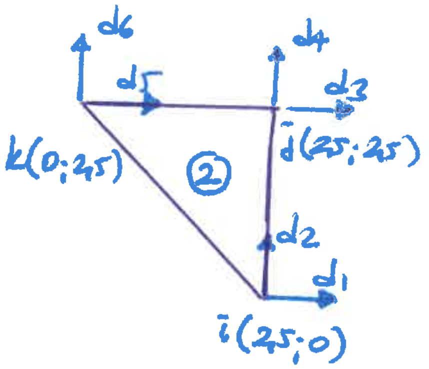
1 nolu eleman



$a_i = 6{,}25$	$a_j = 0$	$a_k = 0$
$b_i = -2{,}5$	$b_j = 2{,}5$	$b_k = 0$
$c_i = -2{,}5$	$c_j = 0$	$c_k = 2{,}5$

$$\Delta = \frac{2{,}5 \times 2{,}5}{2} = 3{,}125$$

2 nolu eleman



$a_i = 6{,}25$	$a_j = -6{,}25$	$a_k = 6{,}25$
$b_i = 0$	$b_j = +2{,}5$	$b_k = -2{,}5$
$c_i = -2{,}5$	$c_j = 2{,}5$	$c_k = 0$

$\Delta = 3{,}125$

1 nolu eleman

$$\{\varepsilon\} = \frac{1}{2\times 3{,}125}\begin{bmatrix} -2.5 & 0 & 2.5 & 0 & 0 & 0 \\ 0 & -2.5 & 0 & 0 & 0 & 2.5 \\ -2.5 & -2.5 & 0 & 2.5 & 2.5 & 0 \end{bmatrix}\{d\}$$

$$\{\sigma\} = \frac{E}{6{,}25(1-0{,}3^2)}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix} * \begin{bmatrix} -2.5 & 0 & 2.5 & 0 & 0 & 0 \\ 0 & -2.5 & 0 & 0 & 0 & 2.5 \\ -2.5 & -2.5 & 0 & 2.5 & 2.5 & 0 \end{bmatrix}\{d\}$$

$$\{\sigma\} = \frac{E}{6{,}25(1-0{,}3^2)}\begin{bmatrix} -2.5 & -0.75 & 2.5 & 0 & 0 & 0.75 \\ -0.75 & -2.5 & 0.75 & 0 & 0 & 2.5 \\ -0.875 & -0.875 & 0 & 0.875 & 0.875 & 0 \end{bmatrix}\{d\}$$

$$[k_{e1}] = \frac{Et}{4(\Delta^2)(1-0{,}3^2)}\iint \begin{bmatrix} -2.5 & 0 & -2.5 \\ 0 & -2.5 & -2.5 \\ 2.5 & 0 & 0 \\ 0 & 0 & 2.5 \\ 0 & 0 & 2.5 \\ 0 & 2.5 & 0 \end{bmatrix} * \begin{bmatrix} -2.5 & -0.75 & 2.5 & 0 & 0 & 0.75 \\ -0.75 & -2.5 & 0.75 & 0 & 0 & 2.5 \\ -0.875 & -0.875 & 0 & 0.875 & 0.875 & 0 \end{bmatrix} dx\,dy$$

$$\iint dx\,dy = A$$

$$[k_{e1}] = \frac{Et}{4\times 3{,}125(1-0{,}3^2)}\begin{bmatrix} 8.438 & 4.0625 & -6.25 & -2.1875 & -2.188 & -1.875 \\ 4.063 & 8.4375 & -1.875 & -2.1875 & -2.188 & -6.25 \\ -6.25 & -1.875 & 6.25 & 0 & 0 & 1.875 \\ -2.188 & -2.1875 & 0 & 2.1875 & 2.1875 & 0 \\ -2.188 & -2.1875 & 0 & 2.1875 & 2.1875 & 0 \\ -1.875 & -6.25 & 1.875 & 0 & 0 & 6.25 \end{bmatrix}$$

$$[ce1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[ce3] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

2 nolu eleman

$$\{\varepsilon\}=\frac{1}{2\times 3{,}125}\begin{bmatrix} 0 & 0 & 2.5 & 0 & -2.5 & 0 \\ 0 & -2.5 & 0 & 2.5 & 0 & 0 \\ -2.5 & 0 & 2.5 & 2.5 & 0 & -2.5 \end{bmatrix}\{d\}$$

$$\{\sigma\}=\frac{E}{6{,}25(1-0{,}3^2)}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix} * \begin{bmatrix} 0 & 0 & 2.5 & 0 & -2.5 & 0 \\ 0 & -2.5 & 0 & 2.5 & 0 & 0 \\ -2.5 & 0 & 2.5 & 2.5 & 0 & -2.5 \end{bmatrix}\{d\}$$

$$\{\sigma\}=\frac{E}{6{,}25(1-0{,}3^2)}\begin{bmatrix} 0 & -0.75 & 2.5 & 0.75 & -2.5 & 0 \\ 0 & -2.5 & 0.75 & 2.5 & -0.75 & 0 \\ -0.875 & 0 & 0.875 & 0.875 & 0 & -0.875 \end{bmatrix}\{d\}$$

$$[ke_2]=\frac{Et}{4(\Delta^2)(1-0{,}3^2)}\iint\begin{bmatrix} 0 & 0 & -2{,}5 \\ 0 & -2{,}5 & 0 \\ 2{,}5 & 0 & 2{,}5 \\ 0 & 2{,}5 & 2{,}5 \\ -2{,}5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & -0.75 & 2.5 & 0.75 & -2.5 & 0 \\ 0 & -2.5 & 0.75 & 2.5 & -0.75 & 0 \\ -0.875 & 0 & 0.875 & 0.875 & 0 & -0.875 \end{bmatrix} dx\,dy$$

$$[ke_2]=\frac{Et}{4\times 3{,}125(1-0{,}3^2)}\begin{bmatrix} 2.1875 & 0 & -2.188 & -2.188 & 0 & 2.1875 \\ 0 & 6.25 & -1.875 & -6.25 & 1.875 & 0 \\ -2.1875 & -1.875 & 8.438 & 4.0625 & -6.25 & -2.1875 \\ -2.1875 & -6.25 & 4.063 & 8.4375 & -1.875 & -2.1875 \\ 0 & 1.875 & -6.25 & -1.875 & 6.25 & 0 \\ 2.1875 & 0 & -2.188 & -2.188 & 0 & 2.1875 \end{bmatrix}$$

$$[\text{ce2}]=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[\text{ce4}]=\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\left[K_{e3}\right] = \frac{Et}{125(1-0.3^2)} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.4375 & 4.0625 & -6.25 & -2.1875 & 0 & 0 & -2.1875 & -1.875 & 0 & 0 \\ 0 & 0 & 4.0625 & 8.4375 & -1.875 & -2.1875 & 0 & 0 & -2.1875 & -6.25 & 0 & 0 \\ 0 & 0 & -6.25 & -1.875 & 6.25 & 0 & 0 & 0 & 0 & 1.875 & 0 & 0 \\ 0 & 0 & -2.1875 & -2.1875 & 0 & 2.1875 & 0 & 0 & 2.1875 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.1875 & -2.1875 & 0 & 2.1875 & 0 & 0 & 2.1875 & 0 & 0 & 0 \\ 0 & 0 & -1.875 & -6.25 & 1.875 & 0 & 0 & 0 & 0 & 6.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[K_{e4}\right] = \frac{Et}{125(1-0.3^2)} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.1875 & 0 & 0 & 0 & 0 & 2.1875 & -2.1875 & -2.1875 \\ 0 & 0 & 0 & 0 & 0 & 6.25 & 0 & 0 & 1.875 & 0 & -1.875 & -6.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.875 & 0 & 0 & 6.25 & 0 & -6.25 & -1.875 \\ 0 & 0 & 0 & 0 & 2.1875 & 0 & 0 & 0 & 0 & 2.1875 & -2.1875 & -2.1875 \\ 0 & 0 & 0 & 0 & -2.1875 & -1.875 & 0 & 0 & -6.25 & -2.1875 & 8.4375 & 4.0625 \\ 0 & 0 & 0 & 0 & -2.1875 & -6.25 & 0 & 0 & -1.875 & -2.1875 & 4.0625 & 8.4375 \end{bmatrix}$$

$$[Ke_1] = \frac{Et}{12{,}5(1-0{,}3^2)} \begin{bmatrix} 8.4375 & 4.0625 & -6.25 & -2.1875 & 0 & 0 & -2.1875 & -1.875 & 0 & 0 & 0 & 0 \\ 4.0625 & 8.4375 & -1.875 & -2.1875 & 0 & 0 & -2.1875 & -6.25 & 0 & 0 & 0 & 0 \\ -6.25 & -1.875 & 6.25 & 0 & 0 & 0 & 0 & 1.875 & 0 & 0 & 0 & 0 \\ -2.1875 & -2.1875 & 0 & 2.1875 & 0 & 0 & 2.1875 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.1875 & -2.1875 & 0 & 2.1875 & 0 & 0 & 2.1875 & 0 & 0 & 0 & 0 & 0 \\ -1.875 & -6.25 & 1.875 & 0 & 0 & 0 & 0 & 6.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[Ke_2] = \frac{Et}{12{,}5(1-0{,}3^2)} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.1875 & 0 & 0 & 0 & 0 & 2.1875 & -2.1875 & -2.1875 & 0 & 0 \\ 0 & 0 & 0 & 6.25 & 0 & 0 & 1.875 & 0 & -1.875 & -6.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.875 & 0 & 0 & 6.25 & 0 & -6.25 & -1.875 & 0 & 0 \\ 0 & 0 & 2.1875 & 0 & 0 & 0 & 0 & 2.1875 & -2.1875 & -2.1875 & 0 & 0 \\ 0 & 0 & -2.1875 & -1.875 & 0 & 0 & -6.25 & -2.1875 & 8.4375 & 4.0625 & 0 & 0 \\ 0 & 0 & -2.1875 & -6.25 & 0 & 0 & -1.875 & -2.1875 & 4.0625 & 8.4375 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K] = [Ke_1] + [Ke_2] + [Ke_3] + [Ke_4]$$

$$[K][\Delta] = [Q]$$

$$
\frac{21\times10^{7}\times0.03}{125(1-0.3^{3})}
\begin{bmatrix}
8.4375 & 4.0625 & -6.25 & -2.1875 & 0 & 0 & -2.1875 & -1.875 & 0 & 0 & 0 & 0 \\
4.0625 & 8.4375 & -1.875 & -2.1875 & 0 & 0 & -2.1875 & -6.25 & 0 & 0 & 0 & 0 \\
-6.25 & -1.875 & 16.875 & 4.0625 & -6.25 & -2.1875 & 0 & 4.0625 & -4.375 & -4.0625 & 0 & 0 \\
-2.1875 & -2.1875 & 4.0625 & 16.875 & -1.875 & -2.1875 & 4.0625 & 0 & -4.0625 & -12.5 & 0 & 0 \\
0 & 0 & -6.25 & -1.875 & 8.4375 & 0 & 0 & 0 & 0 & 4.0625 & -2.1875 & -2.1875 \\
0 & 0 & -2.1875 & -2.1875 & 0 & 8.4375 & 0 & 0 & 4.0625 & 0 & -1.875 & -6.25 \\
-2.1875 & -2.1875 & 0 & 4.0625 & 0 & 0 & 8.4375 & 0 & -6.25 & -1.875 & 0 & 0 \\
-1.875 & -6.25 & 4.0625 & 0 & 0 & 0 & 0 & 8.4375 & -2.1875 & -2.1875 & 0 & 0 \\
0 & 0 & -4.375 & -4.0625 & 0 & 4.0625 & -6.25 & -2.1875 & 16.875 & 4.0625 & -6.25 & -1.875 \\
0 & 0 & -4.0625 & -12.5 & 4.0625 & 0 & -1.875 & -2.1875 & 4.0625 & 16.875 & -2.1875 & -2.1875 \\
0 & 0 & 0 & 0 & -2.1875 & -1.875 & 0 & 0 & -6.25 & -2.1875 & 8.4375 & 4.0625 \\
0 & 0 & 0 & 0 & -2.1875 & -6.25 & 0 & 0 & -1.875 & -2.1875 & 4.0625 & 8.4375
\end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.02 \end{bmatrix}
=
\begin{array}{c}
\begin{bmatrix} \\ \\ \\ \\ -24230.8 \\ -69230.8 \\ 0 \\ 0 \\ -20769.2 \\ -24230.8 \\ 44999.99 \\ 93461.51 \end{bmatrix}
\end{array}
\begin{array}{l} Q1 \\ Q2 \\ Q3 \\ Q4 \\ Q5 \\ Q6\ (kN) \\ Q7 \\ Q8 \\ Q9 \\ Q10 \\ Q11 \\ Q12 \end{array}
$$

$Q_5 = 24230{,}8$ kN

$Q_6 = 69230{,}8$ kN

$Q_{11} = 45000$ kN

$Q_{12} = 93461{,}57$ kN

Isı Yükü Hesabı

$$\{q\}_T = -\alpha_t(\Delta t)\left(\int_V [G]^T dv\right)\{D\}_T$$

$$[G]^T = \frac{1}{2\Delta}\begin{bmatrix} b_i & 0 & c_i \\ 0 & c_i & b_i \\ b_j & 0 & c_j \\ 0 & c_j & b_j \\ b_k & 0 & c_k \\ 0 & c_k & b_k \end{bmatrix} \qquad \{D\}_T = \frac{E}{1-\nu}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$dV = t.\Delta \qquad \iint dx\,dy = \Delta$$

$$\{q\}_T = -\alpha_T(\Delta T)\frac{1}{2\Delta}\begin{bmatrix} b_i & 0 & c_i \\ 0 & c_i & b_i \\ b_j & 0 & c_j \\ 0 & c_j & b_j \\ b_k & 0 & c_k \\ 0 & c_k & b_k \end{bmatrix}\Delta t \cdot \frac{E}{1-\nu}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\{q\}_T = -\alpha_T(\Delta T)\frac{Et}{2(1-\nu)}\begin{bmatrix} b_i \\ c_i \\ b_j \\ c_j \\ b_k \\ c_k \end{bmatrix}$$

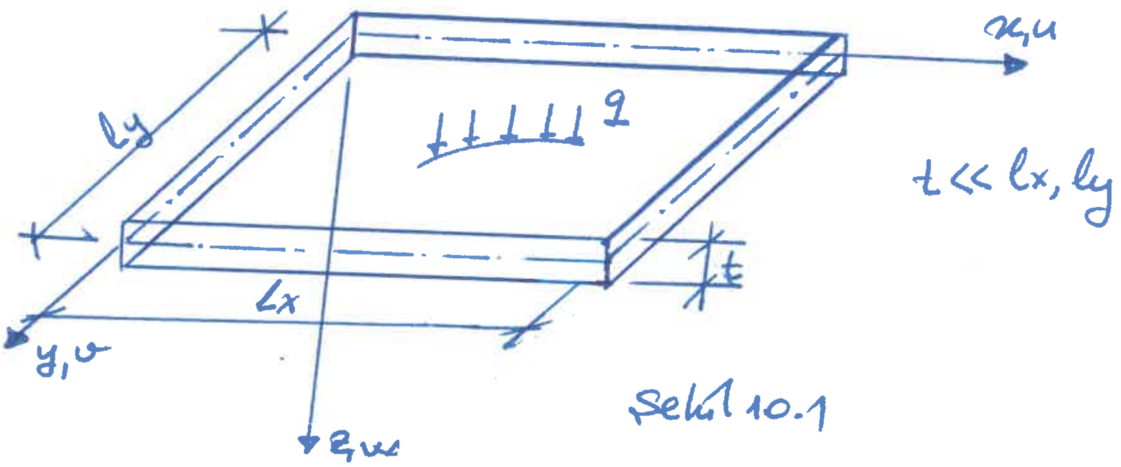
$$\{q\}_T = -0{,}00012\times(-30)\,\frac{21\times10^7\times0{,}03}{2(1-0{,}30)}\begin{bmatrix} -2{,}5 \\ -2{,}5 \\ 2{,}5 \\ 0 \\ 0 \\ 2{,}5 \end{bmatrix} = \overset{\{q\}_T}{\begin{bmatrix} -40500 \\ -40500 \\ 40500 \\ 0 \\ 0 \\ 40500 \end{bmatrix}}$$

$\{Q\} = -\Sigma q_T$

$Q_3 = 40500$
$Q_4 = 40500$
$Q_5 = -40500$
$Q_6 = 0$
$Q_9 = 0$
$Q_{10} = -40500$

10.4) İnce Plâklar

Her iki boyutu yanında kalınlığı çok küçük olan ve yükleri orta düzlemine dik olan taşıyıcı elemana plâk denir (Şekil 10.1)



Şekil 10.1

10.4.1) İnce plâklarda yapılan kabûller

a) Plâk geometrisi açısından yapılan kabûller;

1) Plâk kalınlığı diğer iki boyutu yanında oldukça küçüktür.

2) Plâğa etkiyen yükler plâk orta düzlemine diktir.

3) Yer değiştirmeler plâk kalınlığına oranla çok küçüktür.

4) Plâk kalınlığınca bütün orta noktaların geometrik yeri yine bir düzlemdir.

b) Malzeme açısından yapılan kabûller;

1) Malzeme homojendir.

2) Malzeme her doğrultuda aynı davranışı gösteren izotrop bir malzemedir.

3) Malzeme lineer elastiktir ve Hooke Kanunu geçerlidir.

c) Hesaplar açısından yapılan kabûller;

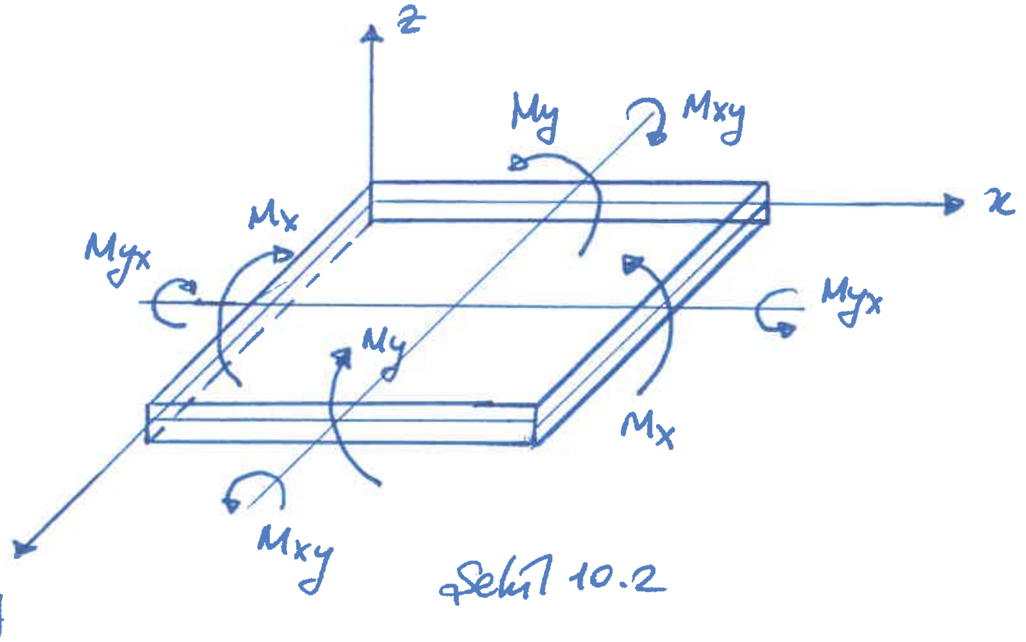
1) Plâk orta düzlemine dik doğrultudaki başka bir deyişle plâk kalınlığı boyunca meydana gelen gerilmeler (σ_z) çok küçük olduğundan ihmal edilebilir (Düzlem Gerilme Hali).

2) Plâk kalınlığı boyunca oluşan yer değiştirmelerinde plâk kalınlığında bir değişim olmaz.

3) Plâk kesitinin orta düzleminde şekil değişikliği meydana gelmeyecektir.

4) Bernoulli Hipotezi geçerlidir. Düzlem kesitler şekil değiştirdikten sonra da düzlem kalırlar ve eğilmiş çubuk eksenine diktirler.

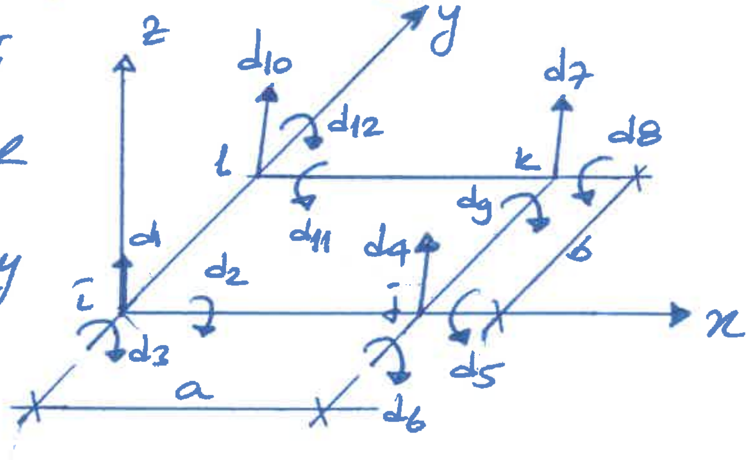
Şekil 10.2'de plâk elemanda meydana gelen kesit tesirleri verilmiştir.



Şekil 10.2

10.4.2) İnce Plâkların Dikdörtgen Şekil Fonksiyonları İle Hesabı

Şekil 10.3'de de verildiği gibi dikdörtgen plâk elemanın her düğüm noktasında bir düşey yer değiştirme ve her iki doğrultuda dönme olmak üzere toplam 12 serbestliği bulunmaktadır.



Şekil 10.3

Düğüm noktası serbestliklerine göre eleman deplasman parametreleri;

$$\{d\}^T = [w_i, \theta_{x_i}, \theta_{y_i}, w_j, \theta_{x_j}, \theta_{y_j}, w_k, \theta_{x_k}, \theta_{y_k}, w_l, \theta_{x_l}, \theta_{y_l}]$$

$$= [d_1, d_2, d_3 ; d_4, d_5, d_6 ; d_7, d_8, d_9 ; d_{10}, d_{11}, d_{12}]$$

Pascal polinomlarından faydalanarak deplasman fonksiyonu seçilirse;

$$w = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 x^2 y + a_9 x y^2$$
$$+ a_{10} y^3 + a_{11} x^3 y + a_{12} x y^3$$

olarak belirlenir. Deplasman fonksiyonundan faydalanarak x ve y doğrultularındaki dönmeler;

$$\theta_x = \frac{\partial w}{\partial y} = a_3 + a_5 x + 2a_6 y + a_8 x^2 + 2a_9 xy + 3a_{10} y^2 + a_{11} x^3 + 3a_{12} x y^2$$

$$\theta_y = -\frac{\partial w}{\partial x} = -a_2 - 2a_4 x - a_5 y - 3a_7 x^2 - 2a_8 xy - a_9 y^2 - 3a_{11} x^2 y - a_{12} y^3$$

şeklinde elde edilir.

10.4.3) Şekil Fonksiyonlarının Belirlenmesi

Deplasman fonksiyonu w, önceden de verildiği gibi

$$w = [\phi(x,y)]\{a\}$$ şeklinde yazılabilir.

$$w = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, xy^3] \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{Bmatrix}$$

$$[\phi(x,y)] = [1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3, x^3y, xy^3]$$

veya

$$w = [N]\{d\} = [N_1, N_2, N_3 \,\text{-------}\, N_9, N_{10}, N_{11}, N_{12}] \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{Bmatrix}$$

yazılabilir.

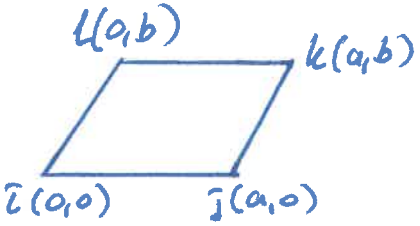
$w = [\phi(x,y)]\{a\}$ ile ve $\{a\} = [B]\{d\}$ olduğundan;

$$w = \underbrace{[\phi(x,y)]\,[B]}_{[N]}\{d\}$$

$[N] = [\phi(x,y)][B] \longrightarrow$ şekil fonksiyonunu verir.

[A] bağ matrisini ve dolayısıyla [B] matrisini elde etmek için düğüm noktası koordinatlarından;

$i(0,0)$, $j(a,0)$, $k(a,b)$, $l(0,b)$



$$w = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 + a_7 x^3 + a_8 x^2 y + a_9 xy^2 + a_{10} y^3 + a_{11} x^3 y + a_{12} xy^3$$

$$\theta_x = \frac{\partial w}{\partial y} = a_3 + a_5 x + 2a_6 y + a_8 x^2 + 2a_9 xy + 3a_{10} y^2 + a_{11} x^3 + 3a_{12} xy^2$$

$$\theta_y = -\frac{\partial w}{\partial x} = -a_2 - 2a_4 x - a_5 y - 3a_7 x^2 - 2a_8 xy - a_9 y^2 - 3a_{11} x^2 y - a_{12} y^3$$

$$[A] = \begin{array}{c} \left[\begin{array}{cccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & a & 0 & a^2 & 0 & 0 & a^3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & a & 0 & 0 & a^2 & 0 & 0 & a^3 & 0 \\
0 & -1 & 0 & -2a & 0 & 0 & -3a^2 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & a & b & a^2 & ab & b^2 & a^3 & a^2b & ab^2 & b^3 & a^3b & ab^3 \\
0 & 0 & 1 & 0 & a & 2b & 0 & a^2 & 2ab & 3b^2 & a^3 & 3ab^2 \\
0 & -1 & 0 & -2a & -b & 0 & -3a^2 & -2ab & -b^2 & 0 & -3a^2b & -b^3 \\
\hline
1 & 0 & b & 0 & 0 & b^2 & 0 & 0 & 0 & b^3 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 & 0 & 3b^2 & 0 & 0 \\
0 & -1 & 0 & 0 & -b & 0 & 0 & 0 & -b^2 & 0 & 0 & -b^3
\end{array}\right] \end{array}
\begin{array}{c} \\ i \\ \\ \\ j \\ \\ \\ k \\ \\ \\ l \\ \\ \end{array}$$

$$[B]=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-3/a^2 & 0 & 2/a & 3/a^2 & 0 & 1/a & 0 & 0 & 0 & 0 & 0 & 0 \\
-1/ab & -1/a & 1/b & 1/ab & 1/a & 0 & -1/ab & 0 & 0 & 1/ab & 0 & -1/b \\
-3/b^2 & -2/b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3/b^2 & -1/b & 0 \\
2/a^3 & 0 & -1/a^2 & -2/a^3 & 0 & -1/a^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
3/a^2b & 0 & -2/ab & -3/a^2b & 0 & -1/ab & 3/a^2b & 0 & 1/ab & -3/a^2b & 0 & 2/ab \\
3/ab^2 & 2/ab & 0 & -3/ab^2 & -2/ab & 0 & 3/ab^2 & -1/ab & 0 & -3/ab^2 & 1/ab & 0 \\
2/b^3 & 1/b^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2/b^3 & 1/b^2 & 0 \\
-2/a^3b & 0 & 1/a^2b & 2/a^3b & 0 & 1/a^2b & -2/a^3b & 0 & -1/a^2b & 2/a^3b & 0 & -1/a^2b \\
-2/ab^3 & -1/ab^2 & 0 & 2/ab^3 & 1/ab^2 & 0 & -2/ab^3 & 1/ab^2 & 0 & 2/ab^3 & -1/ab^2 & 0
\end{bmatrix}$$

$[N] = [\phi(x,y)][B]$ bağıntısından;

$$N_1 = \frac{2x^3}{a^3} - \frac{3x^2}{a^2} - \frac{3y^2}{b^2} + \frac{2y^3}{b^3} + \frac{3xy^2}{ab^2} + \frac{3x^2y}{a^2b} - \frac{2xy^3}{ab^3} - \frac{2x^3y}{a^3b} - \frac{xy}{ab} + 1$$

$$N_2 = y - (2y^2)/b + \frac{y^3}{b^2} - \frac{xy}{a} + \frac{2xy^2}{ab} - \frac{xy^3}{ab^2}$$

$$N_3 = \frac{2x^2}{a} - x - \frac{x^3}{a^2} + \frac{xy}{b} - \frac{2x^2y}{ab} + \frac{x^3y}{a^2b}$$

$$N_4 = \frac{3x^2}{a^2} - \frac{2x^3}{a^3} - \frac{3xy^2}{ab^2} - \frac{3x^2y}{a^2b} + \frac{2xy^3}{ab^3} + \frac{2x^3y}{a^3b} + \frac{xy}{ab}$$

$$N_5 = \frac{xy}{a} - \frac{2xy^2}{ab} + \frac{xy^3}{ab^2}$$

$$N_6 = \frac{x^2}{a} - \frac{x^3}{a^2} - \frac{x^2y}{ab} + \frac{x^3y}{a^2b}$$

$$N_7 = \frac{3xy^2}{ab^2} + \frac{3x^2y}{a^2b} - \frac{2xy^3}{ab^3} - \frac{2x^3y}{a^3b} - \frac{xy}{ab}$$

$$N_8 = \frac{xy^3}{ab^2} - \frac{xy^2}{ab}$$

$$N_9 = -\frac{x^3y}{a^2b} + \frac{x^2y}{ab}$$

$$N_{10} = \frac{3y^2}{b^2} - \frac{2y^3}{b^3} - \frac{3xy^2}{ab^2} - \frac{3x^2y}{a^2b} + \frac{2xy^3}{ab^3} + \frac{2x^3y}{a^3b} + \frac{xy}{ab}$$

$$N_{11} = \frac{y^3}{b^2} - \frac{y^2}{b} + \frac{xy^2}{ab} - \frac{xy^3}{ab^2}$$

$$N_{12} = -\frac{x^3y}{a^2b} + \frac{2x^2y}{ab} - \frac{xy}{b}$$

$\xi = \frac{x}{a}$; $\eta = \frac{y}{b}$ dersek ve gerekli düzenlemeleri yaparsak şekil fonksiyonları;

$$N_1 = 1 - \xi\eta - (3-2\xi)(1-\eta)\xi^2 - (1-\xi)(3-2\eta)\eta^2$$

$$N_2 = [(1-\xi)(1-\eta)^2\eta]b$$

$$N_3 = [-\xi(1-\xi)^2(1-\eta)]a$$

$$N_4 = (3-2\xi)\xi^2(1-\eta) + \xi\eta(1-\eta)(1-2\eta)$$

$$N_5 = [\xi\eta(1-\eta)^2]b$$

$$N_6 = [(1-\xi)\xi^2(1-\eta)]a$$

$$N_7 = (3-2\xi)\xi^2\eta - \xi\eta(1-\eta)(1-2\eta)$$

$$N_8 = [-\xi(1-\eta)\eta^2]b$$

$$N_9 = [(1-\xi)\xi^2\eta]a$$

$$N_{10} = (1-\xi)(3-2\eta)\eta^2 + \xi(1-\xi)(1-2\xi)\eta$$

$$N_{11} = [-(1-\xi)(1-\eta)\eta^2]b$$

$$N_{12} = [-\xi(1-\xi)^2\eta]a$$

şeklinde elde edilir.

Şekil değiştirme – Yer değiştirme Bağıntıları

İnce plaklarda şekil değiştirme - yer değiştirme bağıntısı;

$$\{\varepsilon\} = \begin{Bmatrix} -\dfrac{\partial^2 w}{\partial x^2} \\[2mm] -\dfrac{\partial^2 w}{\partial y^2} \\[2mm] -2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

şeklindedir. Diğer yandan $w = [N]\{d\}$ olduğundan bağıntı;

$$(1) \quad \{\varepsilon\} = \begin{Bmatrix} -\dfrac{\partial^2}{\partial x^2} \\[2mm] -\dfrac{\partial^2}{\partial y^2} \\[2mm] -2\dfrac{\partial^2}{\partial x \partial y} \end{Bmatrix} [N_i]\{d\} = [\Delta][N_i]\{d\}$$

$$\{\varepsilon\} = \underbrace{\begin{bmatrix} -N_{1,xx} & -N_{2,xx} & -N_{3,xx} & \cdots & -N_{12,xx} \\ -N_{1,yy} & -N_{2,yy} & -N_{3,yy} & \cdots & -N_{12,yy} \\ -2N_{1,xy} & -2N_{2,xy} & -2N_{3,xy} & \cdots & -2N_{12,xy} \end{bmatrix}}_{[\Delta][N_i]} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{12} \end{Bmatrix}$$

Diferansiyel Bağıntılar

$$\frac{\partial}{\partial x} = \frac{1}{a}\frac{\partial}{\partial \xi}$$

$$\frac{\partial}{\partial y} = \frac{1}{b}\frac{\partial}{\partial \eta}$$

$$\frac{\partial^2}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2}{\partial \xi^2}$$

$$\frac{\partial^2}{\partial y^2} = \frac{1}{b^2}\frac{\partial^2}{\partial \eta^2}$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{1}{ab}\frac{\partial^2}{\partial \xi \partial \eta}$$

$$-N_{1,xx} = -\frac{\partial^2 N_1}{\partial x^2} = -\frac{1}{a^2}\frac{\partial^2 N_1}{\partial \xi^2} = (1-2\xi)(1-\eta)\frac{6}{a^2}$$

$$-N_{1,yy} = -\frac{\partial^2 N_1}{\partial y^2} = -\frac{1}{b^2}\frac{\partial^2 N_1}{\partial \eta^2} = (1-\xi)(1-2\eta)\frac{6}{b^2}$$

$$-2N_{1,xy} = -2\frac{\partial^2 N_1}{\partial x \partial y} = -\frac{2}{ab}\frac{\partial^2 N_1}{\partial \xi \partial \eta}$$

$$= \frac{2}{ab}\left[1 - 6\xi(1-\xi) - 6\eta(1-\eta)\right]$$

veya bilindiği gibi $\{\varepsilon\}=[F][B]\{d\}$ idi. Bu bağıntıdan faydalanarak da $\{\varepsilon\}$ bulunabilir. Türev matrisi olan [F];

$$-\frac{\partial^2 w}{\partial x^2}=-2a_4-6a_7x-2a_8y-6a_{11}xy$$

$$-\frac{\partial^2 w}{\partial y^2}=-2a_6-2a_9x-6a_{10}y-6a_{12}xy$$

$$-2\frac{\partial^2 w}{\partial x\partial y}=-2a_5-4a_8x-4a_9y-6a_{11}x^2-6a_{12}y^2$$

$$[F]=\begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & -6x & -2y & 0 & 0 & -6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -2x & -6y & 0 & -6xy \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & -4x & -4y & 0 & -6x^2 & -6y^2 \end{bmatrix}$$

şeklinde elde edilir. Bu durumda;

$$\{\varepsilon\}=[F][B]\{d\} \quad (2)$$

bağıntısı ile hesap yapılırsa önceden bulunan $\{\varepsilon\}$ tablosu elde edilmiş olur.

Örneğin 1. sütun terimleri;

$$G_{11} = \frac{6}{a^2} - \frac{12x}{a^3} - \frac{6y}{a^2 b} + \frac{12xy}{a^3 b}$$

$$= \frac{6}{a^2}\left(1 - \frac{2x}{a} - \frac{y}{b} + \frac{2xy}{ab}\right)$$

$$= \frac{6}{a^2}\left(1 - 2\xi - \eta + 2\xi\eta\right)$$

$$= \frac{6}{a^2}(1-2\xi)(1-\eta)$$

$$G_{21} = \frac{6}{b^2} - \frac{12y}{b^3} - \frac{6x}{ab^2} + \frac{12xy}{ab^3}$$

$$= \frac{6}{b^2}\left(1 - \frac{2y}{b} - \frac{x}{a} + \frac{2xy}{ab}\right)$$

$$= \frac{6}{b^2}\left(1 - 2\eta - \xi + 2\xi\eta\right)$$

$$= \frac{6}{b^2}(1-\xi)(1-2\eta)$$

$$G_{31} = \frac{2}{ab} - \frac{12x}{a^2 b} - \frac{12y}{ab^2} + \frac{12x^2}{a^3 b} + \frac{12y^2}{ab^3}$$

$$= \frac{2}{ab}\left(1 - \frac{x}{a} + \frac{6y}{b} + \frac{6x^2}{a^2} + \frac{6y^2}{b^2}\right)$$

$$= \frac{2}{ab}\left(1 - \xi + 6\eta + 6\xi^2 + 6\eta^2\right)$$

$$= \frac{2}{ab}\left[1 - 6\xi(1-\xi) - 6\eta(1-\eta)\right]$$

$$([F] * [B])^T =$$

veya

$$([\Delta] * [N_i])^T = \begin{bmatrix} (1-2\xi)(1-\eta)\frac{6}{a^2} & (1-\xi)(1-2\eta)\frac{6}{b^2} & [1-6\xi(1-\xi)-6\eta(1-\eta)]\frac{2}{ab} \\ 0 & (1-\xi)(2-3\eta)\frac{2}{b} & (1-3\eta)(1-\eta)\frac{2}{a} \\ -(2-3\xi)(1-\eta)\frac{2}{a} & 0 & -(1-3\xi)(1-\xi)\frac{2}{b} \\ -(1-2\xi)(1-\eta)\frac{6}{a^2} & \xi(1-2\eta)\frac{6}{b^2} & [-1+6\xi(1-\xi)+6\eta(1-\eta)]\frac{2}{ab} \\ 0 & \xi(2-3\eta)\frac{2}{b} & (1-3\eta)(1-\eta)\frac{2}{a} \\ -(1-3\xi)(1-\eta)\frac{2}{a} & 0 & \xi(2-3\xi)\frac{2}{b} \\ -(1-2\xi)\eta\frac{6}{a^2} & -\xi(1-2\eta)\frac{6}{b^2} & [1-6\xi(1-\xi)-6\eta(1-\eta)]\frac{2}{ab} \\ 0 & \xi(1-3\eta)\frac{2}{b} & \eta(2-3\eta)\frac{2}{a} \\ -(1-3\xi)\eta\frac{2}{a} & 0 & -\xi(2-3\xi)\frac{2}{b} \\ (1-2\xi)\eta\frac{6}{a^2} & -(1-\xi)(1-2\eta)\frac{6}{b^2} & [-1+6\xi(1-\xi)+6\eta(1-\eta)]\frac{2}{ab} \\ 0 & (1-\xi)(1-3\eta)\frac{2}{b} & -\eta(2-3\eta)\frac{2}{a} \\ -(2-3\xi)\eta\frac{2}{a} & 0 & (1-3\xi)(1-\xi)\frac{2}{b} \end{bmatrix}$$

Gerilme Bağıntıları

Başlangıç şekil değiştirmelerle başlangıç gerilmeler dikkate alındığında gerilme bağıntısı;

$$\{\sigma\}=\{M\}=[D]\left(\{\varepsilon\}-\{\varepsilon_0\}\right)+\{\sigma_0\}$$

'dır. Başlangıç durumunun dikkate alınmadığı durumlarda bağıntı;

$$\{\sigma\}=\{M\}=[D]\{\varepsilon\}$$

olacaktır. Malzeme homojen ve izotrop ise;

$$[D]=\frac{Et^3}{12(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}$$

olacaktır. Bu durumda gerilme bağıntısı;

$$\{\sigma\}=\begin{Bmatrix}M_x\\ M_y\\ M_z\end{Bmatrix}=\frac{Et^3}{12(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}-\frac{\partial^2 w}{\partial x^2}\\ -\frac{\partial^2 w}{\partial y^2}\\ -2\frac{\partial^2 w}{\partial x \partial y}\end{Bmatrix}$$

halini alacaktır.

Gerilme bağıntısının çözümünü 2 şekilde yapabiliriz;

1- $\{\sigma\} = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = [D][\Delta][N_i]\{d\}$

$$= \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \underbrace{\begin{bmatrix} -N_{1,xx} & -N_{2,xx} & \cdots & -N_{12,xx} \\ -N_{1,yy} & -N_{2,yy} & \cdots & -N_{12,yy} \\ -2N_{1,xy} & -2N_{2,xy} & \cdots & -2N_{12,xy} \end{bmatrix}}_{[G]} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{12} \end{Bmatrix}$$

$$= [D][G]\{d\}$$

2- $\{\sigma\} = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = [D]\underbrace{[F][B]}_{[G]}\{d\}$

$$= [D][G]\{d\}$$

Bu durumda elde edilen $[D][G]$ matrisi aşağıda verilmiştir. $[D][G]$ matrisinden hemen sonra gerilme matrisi $\{\sigma\}$ yer almaktadır.

$$
\left([D][F][B]\right)^T = \quad \text{veya} \quad \left([D][\Delta][N]\right)^T =
\left|\begin{array}{ccc}
\frac{6}{a^2}(1-2\xi)(1-\eta)+\frac{6\nu}{b^2}(1-\xi)(1-2\eta) & \frac{6\nu}{a^2}(1-2\xi)(1-\eta)+\frac{6}{b^2}(1-\xi)(1-2\eta) & \frac{1-\nu}{ab}[1-6\xi(1-\xi)-6\eta(1-\eta)] \\
\frac{2\nu}{b}(1-\xi)(2-3\eta) & \frac{2}{b}(1-\xi)(2-3\eta) & \frac{1-\nu}{a}(1-3\eta)(1-\eta) \\
-\frac{2}{a}(2-3\xi)(1-\eta) & -\frac{2\nu}{a}(2-3\xi)(1-\eta) & -\frac{1-\nu}{b}(1-3\xi)(1-\xi) \\
-\frac{6}{a^2}(1-2\xi)(1-\eta)+\frac{6\nu}{b^2}(1-2\eta)\xi & -\frac{6\nu}{a^2}(1-2\xi)(1-\eta)+\frac{6}{b^2}(1-2\eta)\xi & \frac{1-\nu}{ab}[-1+6\xi(1-\xi)+6\eta(1-\eta)] \\
\frac{2\nu}{b}(2-3\eta)\xi & \frac{2}{b}(2-3\eta)\xi & -\frac{1-\nu}{a}(1-3\eta)(1-\eta) \\
-\frac{2}{a}(1-3\xi)(1-\eta) & -\frac{2\nu}{a}(1-3\xi)(1-\eta) & \frac{1-\nu}{b}(2-3\xi)\xi \\
-\frac{6}{a^2}(1-2\xi)\eta-\frac{6\nu}{b^2}(1-2\eta)\xi & -\frac{6\nu}{a^2}(1-2\xi)\eta-\frac{6}{b^2}(1-2\eta)\xi & \frac{1-\nu}{ab}[1-6\xi(1-\xi)-6\eta(1-\eta)] \\
\frac{2\nu}{b}(1-3\eta)\xi & \frac{2}{b}(1-3\eta)\xi & \frac{1-\nu}{a}(2-3\eta)\eta \\
-\frac{2}{a}(1-3\xi)\eta & -\frac{2\nu}{a}(1-3\xi)\eta & -\frac{1-\nu}{a}(2-3\xi)\xi \\
\frac{6}{a^2}(1-2\xi)\eta-\frac{6\nu}{b^2}(1-\xi)(1-2\eta) & \frac{6\nu}{a^2}(1-2\xi)\eta-\frac{6}{b^2}(1-\xi)(1-2\eta) & \frac{1-\nu}{ab}[-1+6\xi(1-\xi)+6\eta(1-\eta)] \\
\frac{2\nu}{b}(1-\xi)(1-3\eta) & \frac{2}{b}(1-\xi)(1-3\eta) & -\frac{1-\nu}{a}(2-3\eta)\eta \\
-\frac{2}{a}(2-3\xi)\eta & -\frac{2\nu}{a}(2-3\xi)\eta & \frac{1-\nu}{b}(1-3\xi)(1-\xi)
\end{array}\right|
$$

$$\{\sigma\}=\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}=\frac{Et^3}{12(1-\nu^2)}*\begin{bmatrix}
\frac{6}{a^2}(1-2\xi)(1-\eta)+\frac{6\nu}{b^2}(1-\xi)(1-2\eta) & \frac{6\nu}{a^2}(1-2\xi)(1-\eta)+\frac{6}{b^2}(1-\xi)(1-2\eta) & \frac{1-\nu}{ab}[1-6\xi(1-\xi)-6\eta(1-\eta)] \\
\frac{2\nu}{b}(1-\xi)(2-3\eta) & \frac{2}{b}(1-\xi)(2-3\eta) & \frac{1-\nu}{a}(1-3\eta)(1-\eta) \\
-\frac{2}{a}(2-3\xi)(1-\eta) & -\frac{2\nu}{a}(2-3\xi)(1-\eta) & -\frac{1-\nu}{b}(1-3\xi)(1-\xi) \\
-\frac{6}{a^2}(1-2\xi)(1-\eta)+\frac{6\nu}{b^2}(1-2\eta)\xi & -\frac{6\nu}{a^2}(1-2\xi)(1-\eta)+\frac{6}{b^2}(1-2\eta)\xi & \frac{1-\nu}{ab}[-1+6\xi(1-\xi)+6\eta(1-\eta)] \\
\frac{2\nu}{b}(2-3\eta)\xi & \frac{2}{b}(2-3\eta)\xi & -\frac{1-\nu}{a}(1-3\eta)(1-\eta) \\
-\frac{2}{a}(1-3\xi)(1-\eta) & -\frac{2\nu}{a}(1-3\xi)(1-\eta) & \frac{1-\nu}{b}(2-3\xi)\xi \\
-\frac{6}{a^2}(1-2\xi)\eta-\frac{6\nu}{b^2}(1-2\eta)\xi & -\frac{6\nu}{a^2}(1-2\xi)\eta-\frac{6}{b^2}(1-2\eta)\xi & \frac{1-\nu}{ab}[1-6\xi(1-\xi)-6\eta(1-\eta)] \\
\frac{2\nu}{b}(1-3\eta)\xi & \frac{2}{b}(1-3\eta)\xi & \frac{1-\nu}{a}(2-3\eta)\eta \\
-\frac{2}{a}(1-3\xi)\eta & -\frac{2\nu}{a}(1-3\xi)\eta & -\frac{1-\nu}{a}(2-3\xi)\xi \\
\frac{6}{a^2}(1-2\xi)\eta-\frac{6\nu}{b^2}(1-\xi)(1-2\eta) & \frac{6\nu}{a^2}(1-2\xi)\eta-\frac{6}{b^2}(1-\xi)(1-2\eta) & \frac{1-\nu}{ab}[-1+6\xi(1-\xi)+6\eta(1-\eta)] \\
\frac{2\nu}{b}(1-\xi)(1-3\eta) & \frac{2}{b}(1-\xi)(1-3\eta) & -\frac{1-\nu}{a}(2-3\eta)\eta \\
-\frac{2}{a}(2-3\xi)\eta & -\frac{2\nu}{a}(2-3\xi)\eta & \frac{1-\nu}{b}(1-3\xi)(1-\xi)
\end{bmatrix}^T$$

Eleman Rijitlik Matrisinin Hesabı

Seçilen dikdörtgen elemanın rijitlik matrisi de iki şekilde hesaplanabilmektedir. Bunlardan ilki;

(1) $$[k_e] = \int_0^a\int_0^b [B]^T [F]^T [D][F][B]\, dx\,dy$$

şeklindedir. Rijitlik matrisinin ikinci hesaplanış şekli

(2) $$[k_e] = \int_0^a\int_0^b [\Delta N]^T [D][\Delta N]\, dx\,dy$$ 'dir.

Burada $\int_0^a\int_0^b dx\,dy = ab\, d\xi\, d\eta$ 'dir. Bu durumda bağıntı;

$$[k_e] = ab\int_0^1\int_0^1 [\Delta N]^T [D][\Delta N]\, d\xi\, d\eta$$ olur.

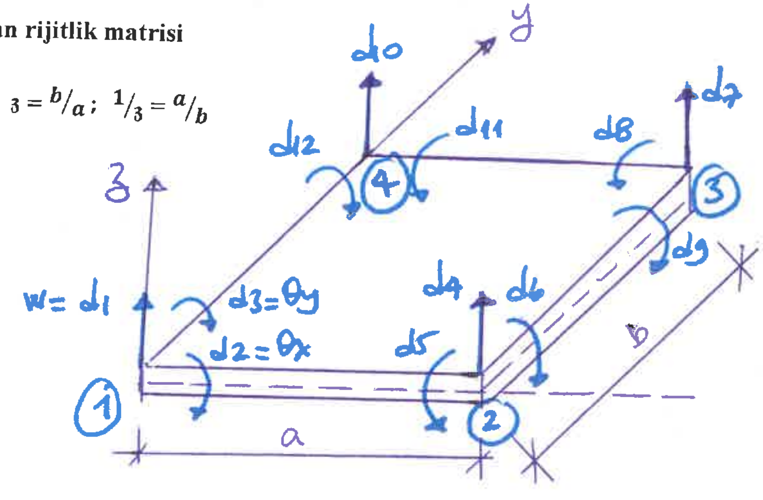
Bağıntıyı açık olarak yazarsak;

$$[k_e] = \frac{Et^3}{12(1-\nu^2)}\frac{1}{ab}\int_0^1\int_0^1 \begin{bmatrix} -N_{1,xx} & -N_{1,yy} \\ -N_{2,xx} & -N_{2,yy} \\ -N_{3,xx} & -N_{3,yy} \\ \vdots & \vdots \\ -N_{12,xx} & -N_{12,yy} \end{bmatrix} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} -N_{1,xx} & -N_{2,xx} & \cdots & -N_{12,xx} \\ -N_{1,yy} & -N_{2,yy} & \cdots & -N_{12,yy} \\ -2N_{1,xy} & -2N_{2,xy} & \cdots & -2N_{12,xy} \end{bmatrix} d\xi\, d\eta$$

Her iki bağıntıya göre hesaplanan eleman rijitlik matrisi aşağıda yer almaktadır.

Homojen ve izotrop plağın eleman rijitlik matrisi

$$\zeta = {}^{b}/_{a}\,;\quad {}^{1}/_{\zeta} = {}^{a}/_{b}$$



$$[k_e] = \frac{Et^3}{2(1-v^2)ab}
\begin{bmatrix}
4\left(\frac{1}{\zeta^2}+\zeta^2\right)+\frac{14-4v}{5} & & & & & & & & & & & \\
\left(\frac{2}{\zeta^2}+\frac{1+4v}{5}\right)b & \frac{4}{3}\left(\frac{1}{\zeta^2}+\frac{1-v}{5}\right)b^2 & & & & & & & & & & \\
-\left(2\zeta^2+\frac{1+4v}{5}\right)a & -vab & \frac{4}{3}\left(\zeta^2+\frac{1-v}{5}\right)a^2 & & & & & & & & & \\
2\left(\frac{1}{\zeta^2}-2\zeta^2\right)-\frac{14-4v}{5} & \left(\frac{1}{\zeta^2}-\frac{1+4v}{5}\right)b & \left(2\zeta^2+\frac{1-v}{5}\right)a & 4\left(\frac{1}{\zeta^2}+\zeta^2\right)+\frac{14-4v}{5} & & & & & & & & \\
\left(\frac{1}{\zeta^2}-\frac{1+4v}{5}\right)b & \frac{2}{3}\left(\frac{1}{\zeta^2}-\frac{2(1-v)}{5}\right)b^2 & 0 & \left(\frac{2}{\zeta^2}+\frac{1+4v}{5}\right)b & \frac{4}{3}\left(\frac{1}{\zeta^2}+\frac{1-v}{5}\right)b^2 & & & & & & & \\
-\left(2\zeta^2+\frac{1-v}{5}\right)a & 0 & \frac{2}{3}\left(\zeta^2-\frac{1-v}{10}\right)a^2 & \left(2\zeta^2+\frac{1+4v}{5}\right)a & vab & \frac{4}{3}\left(\zeta^2+\frac{1-v}{5}\right)a^2 & & & & & & \\
-2\left(\frac{1}{\zeta^2}+\zeta^2\right)+\frac{14-4v}{5} & -\left(\frac{1}{\zeta^2}-\frac{1-v}{5}\right)b & \left(\zeta^2-\frac{1-v}{5}\right)a & 2\left(\zeta^2-2\frac{1}{\zeta^2}\right)-\frac{14-4v}{5} & -\left(\frac{2}{\zeta^2}+\frac{1-v}{5}\right)b & \left(\zeta^2-\frac{1+4v}{5}\right)a & 4\left(\frac{1}{\zeta^2}+\zeta^2\right)+\frac{14-4v}{5} & & & & & \\
\left(\frac{1}{\zeta^2}-\frac{1-v}{5}\right)b & \frac{1}{3}\left(\frac{1}{\zeta^2}+\frac{1-v}{5}\right)b^2 & 0 & \left(\frac{2}{\zeta^2}+\frac{1-v}{5}\right)b & \frac{2}{3}\left(\frac{1}{\zeta^2}-\frac{1-v}{10}\right)b^2 & 0 & -\left(\frac{2}{\zeta^2}+\frac{1+4v}{5}\right)b & \frac{4}{3}\left(\frac{1}{\zeta^2}+\frac{1-v}{5}\right)b^2 & & & & \\
-\left(\zeta^2-\frac{1-v}{5}\right)a & 0 & \frac{1}{3}\left(\zeta^2+\frac{1-v}{5}\right)a^2 & \left(\zeta^2-\frac{1+4v}{5}\right)a & 0 & \frac{2}{3}\left(\zeta^2-\frac{2(1-v)}{5}\right)a^2 & \left(2\zeta^2+\frac{1+4v}{5}\right)a & -vab & \frac{4}{3}\left(\zeta^2+\frac{1-v}{5}\right)a^2 & & & \\
2\left(\zeta^2-\frac{2}{\zeta^2}\right)-\frac{14-4v}{5} & -\left(\frac{2}{\zeta^2}+\frac{1-v}{5}\right)b & -\left(\zeta^2-\frac{1+4v}{5}\right)a & -2\left(\frac{1}{\zeta^2}+\zeta^2\right)+\frac{14-4v}{5} & -\left(\frac{1}{\zeta^2}-\frac{1-v}{5}\right)b & -\left(\zeta^2-\frac{1-v}{5}\right)a & 2\left(\frac{1}{\zeta^2}-2\zeta^2\right)-\frac{14-4v}{5} & -\left(\frac{1}{\zeta^2}-\frac{1+4v}{5}\right)b & -\left(2\zeta^2+\frac{1-v}{5}\right)a & 4\left(\frac{1}{\zeta^2}+\zeta^2\right)+\frac{14-4v}{5} & & \\
\left(\frac{2}{\zeta^2}+\frac{1-v}{5}\right)b & \frac{2}{3}\left(\frac{1}{\zeta^2}-\frac{1-v}{10}\right)b^2 & 0 & \left(\frac{1}{\zeta^2}-\frac{1-v}{5}\right)b & \frac{1}{3}\left(\frac{1}{\zeta^2}+\frac{1-v}{5}\right)b^2 & 0 & -\left(\frac{1}{\zeta^2}-\frac{1+4v}{5}\right)b & \frac{2}{3}\left(\frac{1}{\zeta^2}-\frac{2(1-v)}{5}\right)b^2 & 0 & -\left(\frac{2}{\zeta^2}+\frac{1+4v}{5}\right)b & \frac{4}{3}\left(\frac{1}{\zeta^2}+\frac{1-v}{5}\right)b^2 & \\
-\left(\zeta^2-\frac{1+4v}{5}\right)a & 0 & \frac{2}{3}\left(\zeta^2-\frac{2(1-v)}{5}\right)a^2 & \left(\zeta^2-\frac{1-v}{5}\right)a & 0 & \frac{1}{3}\left(\zeta^2+\frac{1-v}{5}\right)a^2 & \left(2\zeta^2+\frac{1-v}{5}\right)a & 0 & \frac{2}{3}\left(\zeta^2-\frac{1-v}{10}\right)a^2 & -\left(2\zeta^2+\frac{1+4v}{5}\right)a & vab & \frac{4}{3}\left(\zeta^2+\frac{1-v}{5}\right)a^2
\end{bmatrix}$$

Eleman Yük Matrisi

Plağa etkiyen statik bir yük ise; bu durumda yük, tekil, yayılı veya çizgisel yük olabilir. Çizgisel yük çubuk konusunda verildiği gibi ele alınır ve yük hangi düğüm noktaları arasında ise tipki çubuk eleman gibi ele alınır ve yükün düğüm noktalarına etkimesi sağlanır. Yük tekil bir yük ise ilgili düğüm noktasında yükün yönüne göre dağılımı yapılır. Yük yayılı bir halde ise;

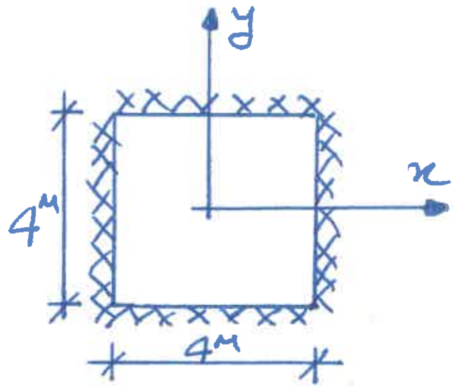
$$\{q_e\} = \int_0^a \int_0^b [N]^T \{P\}\, dx\, dy$$

$$\{q_e\} = ab \int_0^1 \int_0^1 [N_1\ N_2\ \cdots\ N_{12}]^T \{P\}\, d\xi\, d\eta$$

bağıntısı ile hesaplanabilir.

ÖRNEK

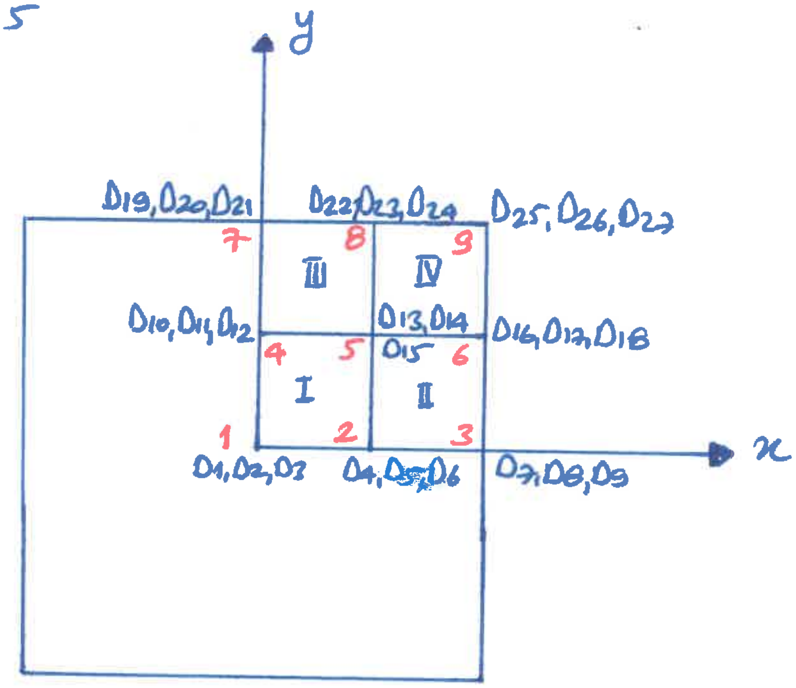
Şekilde verilen 4 kenarı ankastre plağın kesit tesirlerini hesaplayınız.

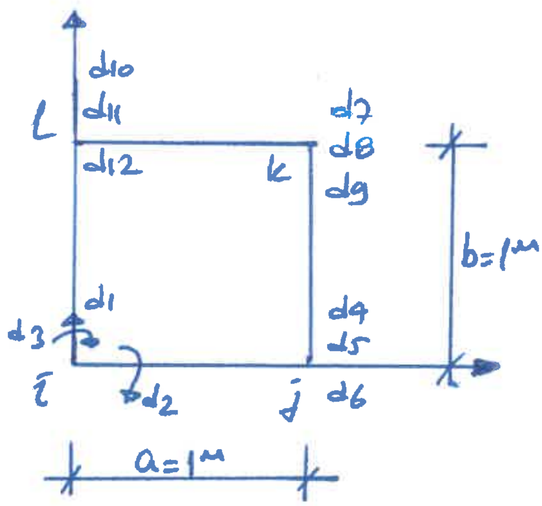


$d=(h_f)=15\ cm$

$E=21.10^7\ kN/m^2$

$\nu=0,15$





$\xi=\dfrac{x}{a}$

$\eta=\dfrac{y}{b}$

1) Eleman rijitlik matrisinin oluşturulması

$\lambda=\dfrac{b}{a}=1$; $\dfrac{1}{\lambda}=\dfrac{a}{b}=1$; $\nu=0,15$; $E=21.10^7\ kN/m^2$

$d=15\ cm$

2) Eleman Yük Matrisi

$$\{q_e\} = \iint [N]^T \{P\}\, dx\, dy$$

$$\{q_e\} = ab \int_0^1\int_0^1 [N_1, N_2, \text{----} N_{12}]^T \{P\}\, d\xi\, d\eta$$

$$\{q_e\} = pab \int_0^1\int_0^1 \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ N_{12} \end{bmatrix} d\xi\, d\eta = \begin{bmatrix} q_{e_1} \\ q_{e_2} \\ q_{e_3} \\ \vdots \\ q_{e_{12}} \end{bmatrix}$$

Bu integrasyondan;

$$\{q_e\}^T = \Big[pab\tfrac{1}{4};\ pab^2\tfrac{1}{24};\ -pa^2b\tfrac{1}{24};\ pab\tfrac{1}{4};\ +pab^2\tfrac{1}{24};$$

$$pa^2b\tfrac{1}{24};\ pab\tfrac{1}{4};\ -pab^2\tfrac{1}{24};\ pa^2b\tfrac{1}{24};\ pab\tfrac{1}{4};$$

$$-pab^2\tfrac{1}{24};\ -pa^2b\tfrac{1}{24}\Big]$$

3) Genişletme matrisleri

Yük Matrisinden hemen sonra genişleme matrisleri verilmiştir.

$$[k_e] = \frac{Et^3}{2(1-\nu^2)ab} * \begin{array}{cccccccccccc|c}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & \\
10.68 & 2.32 & -2.32 & -4.68 & 0.68 & -2.17 & -1.32 & 0.83 & -0.83 & -4.68 & 2.17 & -0.68 & 1 \\
2.32 & 1.56 & -0.15 & 0.68 & 0.44 & 0 & -0.83 & 0.39 & 0 & -2.17 & 0.61 & 0 & 2 \\
-2.32 & -0.15 & 1.56 & 2.17 & 0 & 0.61 & 0.83 & 0 & 0.39 & -0.68 & 0 & 0.44 & 3 \\
-4.68 & 0.68 & 2.17 & 10.68 & 2.32 & 2.32 & -4.68 & 2.17 & 0.68 & -1.32 & 0.83 & 0.83 & 4 \\
0.68 & 0.44 & 0 & 2.32 & 1.56 & 0.15 & -2.17 & 0.61 & 0 & -0.83 & 0.39 & 0 & 5 \\
-2.17 & 0 & 0.61 & 2.32 & 0.15 & 1.56 & 0.68 & 0 & 0.44 & -0.83 & 0 & 0.39 & 6 \\
-1.32 & -0.83 & 0.83 & -4.68 & -2.17 & 0.68 & 10.68 & -2.32 & 2.32 & -4.68 & -0.68 & 2.17 & 7 \\
0.83 & 0.39 & 0 & 2.17 & 0.61 & 0 & -2.32 & 1.56 & -0.15 & -0.68 & 0.44 & 0 & 8 \\
-0.83 & 0 & 0.39 & 0.68 & 0 & 0.44 & 2.32 & -0.15 & 1.56 & -2.17 & 0 & 0.61 & 9 \\
-4.68 & -2.17 & -0.68 & -1.32 & -0.83 & -0.83 & -4.68 & -0.68 & -2.17 & 10.68 & -2.32 & -2.32 & 10 \\
2.17 & 0.61 & 0 & 0.83 & 0.39 & 0 & -0.68 & 0.44 & 0 & -2.32 & 1.56 & 0.15 & 11 \\
-0.68 & 0 & 0.44 & 0.83 & 0 & 0.39 & 2.17 & 0 & 0.61 & -2.32 & 0.15 & 1.56 & 12
\end{array}$$

$$\{q_e\} = \begin{array}{|c|l} 3.75 & 1 \\ 0.625 & 2 \\ -0.625 & 3 \\ 3.75 & 4 \\ 0.625 & 5 \\ 0.625 & 6 \\ 3.75 & 7 \\ -0.625 & 8 \\ 0.625 & 9 \\ 3.75 & 10 \\ -0.625 & 11 \\ -0.625 & 12 \end{array}$$

Çevirme Matrisleri

$$
\{c_{e1}\} =
\begin{array}{|ccccccccccccccccccccccccccc}
\mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} & \mathbf{15} & \mathbf{16} & \mathbf{17} & \mathbf{18} & \mathbf{19} & \mathbf{20} & \mathbf{21} & \mathbf{22} & \mathbf{23} & \mathbf{24} & \mathbf{25} & \mathbf{26} & \mathbf{27} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Çevirme Matrisleri

$$
\{c_{e2}\} =
\begin{array}{ccccccccccccccccccccccccccc}
\mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} & \mathbf{15} & \mathbf{16} & \mathbf{17} & \mathbf{18} & \mathbf{19} & \mathbf{20} & \mathbf{21} & \mathbf{22} & \mathbf{23} & \mathbf{24} & \mathbf{25} & \mathbf{26} & \mathbf{27} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Çevirme Matrisleri

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$\{c_{e3}\} =$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

Çevirme Matrisleri

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	2
	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
$\{c_{e4}\} =$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0

Sistem rijitlik ve yük matrisi

[ke]=

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27		Q
1	10.68	2.32	-2.32	-4.68	0.68	-2.17	0	0	0	-4.68	2.17	-0.68	-1.32	0.83	-0.83	0	0	0	0	0	0	0	0	0	0	0	0	D1	3.75
2	2.32	1.56	-0.15	0.68	0.44	0	0	0	0	-2.17	0.61	0	-0.83	0.39	0	0	0	0	0	0	0	0	0	0	0	0	0	D2	0.625
3	-2.32	-0.15	1.56	2.17	0	0.61	0	0	0	-0.68	0	0.44	0.83	0	0.39	0	0	0	0	0	0	0	0	0	0	0	0	D3	-0.625
4	-4.68	0.68	2.17	21.36	4.64	0	-4.68	0.68	-2.17	-1.32	0.83	0.83	-9.36	4.34	0	-1.32	0.83	-0.83	0	0	0	0	0	0	0	0	0	D4	7.5
5	0.68	0.44	0	4.64	3.12	0	0.68	0.44	0	-0.83	0.39	0	-4.34	1.22	0	-0.83	0.39	0	0	0	0	0	0	0	0	0	0	D5	1.25
6	-2.17	0	0.61	0	0	3.12	2.17	0	0.61	-0.83	0	0.39	0	0	0.88	0.83	0	0.39	0	0	0	0	0	0	0	0	0	D6	0
7	0	0	0	-4.68	0.68	2.17	10.68	2.32	2.32	0	0	0	-1.32	0.83	0.83	-4.68	2.17	0.68	0	0	0	0	0	0	0	0	0	D7	3.75
8	0	0	0	0.68	0.44	0	2.32	1.56	0.15	0	0	0	-0.83	0.39	0	2.17	0.61	0	0	0	0	0	0	0	0	0	0	D8	0.625
9	0	0	0	-2.17	0	0.61	2.32	0.15	1.56	0	0	0	-0.83	0	0.39	0.68	0	0.44	0	0	0	0	0	0	0	0	0	D9	0.625
10	-4.68	-2.17	-0.68	-1.32	-0.83	-0.83	0	0	0	21.36	0	-4.64	-9.36	0	-4.34	0	0	0	-4.68	2.17	-0.68	-1.32	0.83	-0.83	0	0	0	D10	7.5
11	2.17	0.61	0	0.83	0.39	0	0	0	0	0	3.12	0	0	0.88	0	0	0	0	-2.17	0.61	0	-0.83	0.39	0	0	0	0	D11	0
12	-0.68	0	0.44	0.83	0	0.39	0	0	0	-4.64	0	3.12	4.34	0	1.22	0	0	0	-0.68	0	0.44	0.83	0	0.39	0	0	0	D12	-1.25
13	-1.32	-0.83	0.83	-9.36	-4.34	0	-1.32	-0.83	-0.83	-9.36	0	4.34	42.72	0	0	-9.36	0	-4.34	-1.32	0.83	0.83	-9.36	4.34	0	-1.32	0.83	-0.83	D13	15
14	0.83	0.39	0	4.34	1.22	0	0.83	0.39	0	0	0.88	0	0	6.24	0	0	0.88	0	-0.83	0.39	0	-4.34	1.22	0	-0.83	0.39	0	D14	0
15	-0.83	0	0.39	0	0	0.88	0.83	0	0.39	-4.34	0	1.22	0	0	6.24	4.34	0	1.22	-0.83	0	0.39	0	0	0.88	0.83	0	0.39	D15	0
16	0	0	0	-1.32	-0.83	0.83	-4.68	-2.17	0.68	0	0	0	-9.36	0	4.34	21.36	0	4.64	0	0	0	-1.32	0.83	0.83	-4.68	2.17	0.68	D16	7.5
17	0	0	0	0.83	0.39	0	2.17	0.61	0	0	0	0	0	0.88	0	0	3.12	0	0	0	0	-0.83	0.39	0	2.17	0.61	0	D17	0
18	0	0	0	-0.83	0	0.39	0.68	0	0.44	0	0	0	-4.34	0	1.22	4.64	0	3.12	0	0	0	-0.83	0	0.39	0.68	0	0.44	D18	1.25
19	0	0	0	0	0	0	0	0	0	-4.68	-2.17	-0.68	-1.32	-0.83	-0.83	0	0	0	0.68	-2.32	-2.32	-4.68	-0.68	-2.17	0	0	0	D19	3.75
20	0	0	0	0	0	0	0	0	0	2.17	0.61	0	0.83	0.39	0	0	0	0	-2.32	1.56	0.15	-0.68	0.44	0	0	0	0	D20	-0.625
21	0	0	0	0	0	0	0	0	0	-0.68	0	0.44	0.83	0	0.39	0	0	0	-2.32	0.15	-1.56	2.17	0	0.61	0	0	0	D21	-0.625
22	0	0	0	0	0	0	0	0	0	-1.32	-0.83	0.83	-9.36	-4.34	0	-1.32	-0.83	-0.83	-4.68	-0.68	2.17	21.36	-4.64	0	-4.68	-0.68	-2.17	D22	7.5
23	0	0	0	0	0	0	0	0	0	0.83	0.39	0	4.34	1.22	0	0.83	0.39	0	-0.68	0.44	0	-4.64	3.12	0	-0.68	0.44	0	D23	-1.25
24	0	0	0	0	0	0	0	0	0	-0.83	0	0.39	0	0	0.88	0.83	0	0.39	-2.17	0	0.61	0	0	3.12	2.17	0	0.61	D24	0
25	0	0	0	0	0	0	0	0	0	0	0	0	-1.32	-0.83	0.83	-4.68	2.17	0.68	0	0	0	-4.68	-0.68	2.17	10.68	-2.32	2.32	D25	3.75
26	0	0	0	0	0	0	0	0	0	0	0	0	0.83	0.39	0	2.17	0.61	0	0	0	0	-0.68	0.44	0	-2.32	1.56	-0.15	D26	-0.625
27	0	0	0	0	0	0	0	0	0	0	0	0	-0.83	0	0.39	0.68	0	0.44	0	0	0	-2.17	0	0.61	2.32	-0.15	1.56	D27	0.625

İndirgenmiş Sistem Rijitlik Matrisi

[ke]=

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27			Q
1	10.68	0	0	-4.68	0	-2.17	0	0	0	-4.68	2.17	0	-1.32	0.83	-0.83	0	0	0	0	0	0	0	0	0	0	0	0	D1	=	3.75
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
4	-4.68	0	0	21.36	0	0	0	0	0	-1.32	0.83	0	-9.36	4.34	0	0	0	0	0	0	0	0	0	0	0	0	0	D4		7.5
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
6	-2.17	0	0	0	0	3.12	0	0	0	-0.83	0	0	0	0	0.88	0	0	0	0	0	0	0	0	0	0	0	0	D6		0
7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
10	-4.68	0	0	-1.32	0	-0.83	0	0	0	21.36	0	0	-9.36	0	-4.34	0	0	0	0	0	0	0	0	0	0	0	0	D10		7.5
11	2.17	0	0	0.83	0	0	0	0	0	0	3.12	0	0	0.88	0	0	0	0	0	0	0	0	0	0	0	0	0	D11		0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0
13	-1.32	0	0	-9.36	0	0	0	0	0	-9.36	0	0	42.72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	D13		15
14	0.83	0	0	4.34	0	0	0	0	0	0	0.88	0	0	6.24	0	0	0	0	0	0	0	0	0	0	0	0	0	D14		0
15	-0.83	0	0	0	0	0.88	0	0	0	-4.34	0	0	0	0	6.24	0	0	0	0	0	0	0	0	0	0	0	0	D15		0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0		0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0		0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0		0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0		0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0		0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0		0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0		0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0		0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0		0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0		0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0		0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0		0

$$K = \frac{Et^3}{12(1-\upsilon^2)} *$$

	1	4	6	10	11	13	14	15		Q
1	10.68	-4.68	-2.17	-4.68	2.17	-1.32	0.83	-0.83	***D1***	3.75
4	-4.68	21.36	0	-1.32	0.83	-9.36	4.34	0	***D4***	7.5
6	-2.17	0	3.12	-0.83	0	0	0	0.88	***D6***	0
10	-4.68	-1.32	-0.83	21.36	0	-9.36	0	-4.34	***D10***	7.5
11	2.17	0.83	0	0	3.12	0	0.88	0	***D11***	0
13	-1.32	-9.36	0	-9.36	0	42.72	0	0	***D13***	15
14	0.83	4.34	0	0	0.88	0	6.24	0	***D14***	0
15	-0.83	0	0.88	-4.34	0	0	0	6.24	***D15***	0

D1=5.34/D

D4=3.18/D

D6=3.88/D

D10=3.18/D

D11=-3.88/D

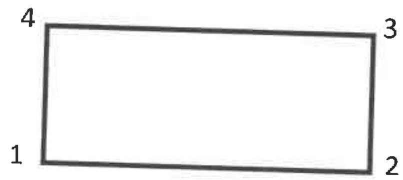
D13=1.92/D

D14=-2.38/D

D15=2.38/D

1 nolu eleman için gerilme hesabı;

1 nolu köşe için $\xi=x/a=0$; $\eta=y/b=0$



Eleman Düğüm Noktası $\longrightarrow$ 1 2 3 4

$$\begin{Bmatrix} M_{x1} \\ M_{y1} \\ M_{xy1} \end{Bmatrix} = \frac{Et^3}{12(1-v^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} 6.9 & 0.6 & -4 & -6 & 0 & -2 & 0 & 0 & 0 & -0.9 & 0.3 & 0 \\ 6.9 & 4 & -0.6 & -0.9 & 0 & -0.3 & 0 & 0 & 0 & -6 & 2 & 0 \\ 0.85 & 0.85 & -0.85 & -0.85 & -0.85 & 0 & 0.85 & 0 & 0 & -0.85 & 0 & 0.85 \end{array}\right] \begin{bmatrix} 5.34/D \\ 0 \\ 0 \\ \hline 3.18/D \\ 0 \\ 3.88/D \\ \hline 1.92/D \\ -2.38/D \\ 2.38/D \\ \hline 3.18/D \\ -3.88/D \\ 0 \end{bmatrix} \begin{matrix} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \\ D13 \\ D14 \\ D15 \\ D10 \\ D11 \\ D12 \end{matrix}$$

M_{x1}=5.98 kNm/m

M_{y1}=5.98 kNm/m

M_{xy1}=0.76 kNm/m

1 nolu eleman için gerilme hesabı;

2 nolu köşe için $\xi=x/a=1$; $\eta=y/b=0$

Eleman Düğüm Noktası $\longrightarrow$ 1 2 3 4

$$\begin{Bmatrix} M_{x2} \\ M_{y2} \\ M_{xy2} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \begin{array}{ccc:ccc:ccc:ccc} \multicolumn{3}{c}{1} & \multicolumn{3}{c}{2} & \multicolumn{3}{c}{3} & \multicolumn{3}{c}{4} \end{array}$$

$$\left[\begin{array}{ccc:ccc:ccc:ccc} -6 & 0 & 2 & 6.9 & 0.6 & 4 & -0.9 & 0.3 & 0 & 0 & 0 & 0 \\ -0.9 & 0 & 0.3 & 6.9 & 4 & 0.6 & -6 & 2 & 0 & 0 & 0 & 0 \\ 0.85 & 0.85 & 0 & -0.85 & -0.85 & -0.85 & 0.85 & 0 & 0.85 & -0.85 & 0 & 0 \end{array}\right] \begin{Bmatrix} 5.34/D \\ 0 \\ 0 \\ 3.18/D \\ 0 \\ 3.88/D \\ 1.92/D \\ -2.38/D \\ 2.38/D \\ 3.18/D \\ -3.88/D \\ 0 \end{Bmatrix} \begin{array}{c} D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \\ D13 \\ D14 \\ D15 \\ D10 \\ D11 \\ D12 \end{array}$$

M_{x2}=2.98 kNm/m

M_{y2}=3.18 kNm/m

M_{xy2}=-0.51 kNm/m

1 nolu eleman için gerilme hesabı;

3 nolu köşe için $\xi=x/a=1$; $\eta=y/b=1$

Eleman Düğüm Noktası ⟶ 1 2 3 4

$$\begin{Bmatrix} M_{x3} \\ M_{y3} \\ M_{xy3} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} 0 & 0 & 0 & -0.9 & -0.3 & 0 & 6.9 & -0.6 & 4 & -6 & 0 & 2 \\ 0 & 0 & 0 & -6 & -2 & 0 & 6.9 & -4 & 0.6 & -0.9 & 0 & 0.3 \\ 0.85 & 0 & 0 & -0.85 & 0 & -0.85 & 0.85 & -0.85 & 0.85 & -0.85 & 0.85 & 0 \end{array}\right] \begin{array}{|c|c} 5.34/D & D1 \\ 0 & D2 \\ 0 & D3 \\ \hline 3.18/D & D4 \\ 0 & D5 \\ 3.88/D & D6 \\ \hline 1.92/D & D13 \\ -2.38/D & D14 \\ 2.38/D & D15 \\ \hline 3.18/D & D10 \\ -3.88/D & D11 \\ 0 & D12 \end{array}$$

M_{x2}=2.25 kNm/m

M_{y2}=2.25 kNm/m

M_{xy2}=-1.79 kNm/m

1 nolu eleman için gerilme hesabı;

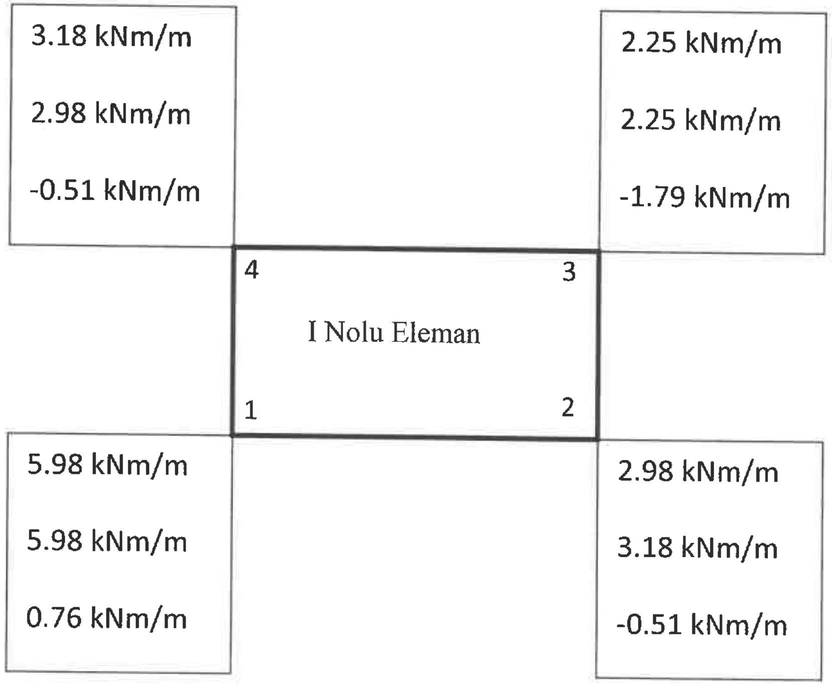
4 nolu köşe için $\xi = x/a = 0$; $\eta = y/b = 1$

$$
\begin{array}{c}
\text{Eleman Düğüm Noktası} \longrightarrow \\
\begin{Bmatrix} M_{x4} \\ M_{y4} \\ M_{xy4} \end{Bmatrix} = \dfrac{Et^3}{12(1-\upsilon^2)} *
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} & 1 & & & 2 & & & 3 & & & 4 & \end{array} \\
\left|\begin{array}{ccc:ccc:ccc:ccc}
-0.9 & -0.3 & 0 & 0 & 0 & 0 & -6 & 0 & -2 & 6.9 & -0.6 & -4 \\
-6 & -2 & 0 & 0 & 0 & 0 & -0.9 & 0 & -0.3 & 6.9 & -4 & -0.6 \\
0.85 & 0 & -0.85 & -0.85 & 0 & 0 & 0.85 & -0.85 & 0 & -0.85 & 0.85 & 0.85
\end{array}\right|
\end{array}
\left|\begin{array}{c}
5.34/D \\ 0 \\ 0 \\ \hdashline 3.18/D \\ 0 \\ 3.88/D \\ \hdashline 1.92/D \\ -2.38/D \\ 2.38/D \\ \hdashline 3.18/D \\ -3.88/D \\ 0
\end{array}\right|
\begin{array}{c}
D1 \\ D2 \\ D3 \\ D4 \\ D5 \\ D6 \\ D13 \\ D14 \\ D15 \\ D10 \\ D11 \\ D12
\end{array}
$$

M_{x2}=3.18 kNm/m

M_{y2}=2.98 kNm/m

M_{xy2}=-0.51 kNm/m



2 nolu eleman için gerilme hesabı;

1 nolu köşe için $\xi=x/a=0$; $\eta=y/b=0$

Eleman Düğüm Noktası ⟶ 1 2 3 4

$$\begin{Bmatrix} M_{x1} \\ M_{y1} \\ M_{xy1} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left|\begin{array}{ccc:ccc:ccc:ccc} 6.9 & 0.6 & -4 & -6 & 0 & -2 & 0 & 0 & 0 & -0.9 & 0.3 & 0 \\ 6.9 & 4 & -0.6 & -0.9 & 0 & -0.3 & 0 & 0 & 0 & -6 & 2 & 0 \\ 0.85 & 0.85 & -0.85 & -0.85 & -0.85 & 0 & 0.85 & 0 & 0 & -0.85 & 0 & 0.85 \end{array}\right| \begin{vmatrix} 3.18/D \\ 0 \\ 3.88/D \\ \hdashline 0 \\ 0 \\ 0 \\ \hdashline 0 \\ 0 \\ 0 \\ \hdashline 1.92/D \\ -2.38/D \\ 2.38/D \end{vmatrix} \begin{matrix} D4 \\ D5 \\ D6 \\ D7 \\ D8 \\ D9 \\ D16 \\ D17 \\ D18 \\ D13 \\ D14 \\ D15 \end{matrix}$$

M_{x1}=3.98 kNm/m

M_{y1}=3.33 kNm/m

M_{xy1}=-0.20 kNm/m

2 nolu eleman için gerilme hesabı;

2 nolu köşe için $\xi = x/a = 1$; $\eta = y/b = 0$

$$
\begin{array}{cccccccccccccc}
 & & \text{Eleman Düğüm Noktası} \longrightarrow 1 & & & 2 & & & 3 & & & 4 & & \\
\begin{Bmatrix} M_{x2} \\ M_{y2} \\ M_{xy2} \end{Bmatrix} & = \dfrac{Et^3}{12(1-\upsilon^2)} * & \left[\begin{array}{ccc|ccc|ccc|ccc} -6 & 0 & 2 & 6.9 & 0.6 & 4 & -0.9 & 0.3 & 0 & 0 & 0 & 0 \\ -0.9 & 0 & 0.3 & 6.9 & 4 & 0.6 & -6 & 2 & 0 & 0 & 0 & 0 \\ 0.85 & 0.85 & 0 & -0.85 & -0.85 & -0.85 & 0.85 & 0.85 & 0 & -0.85 & 0 & 0 \end{array}\right]
\end{array}
\begin{array}{c} \\ \end{array}
\left[\begin{array}{c} 3.18/D \\ 0 \\ 3.88/D \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 1.92/D \\ -2.38/D \\ 2.38/D \end{array}\right]
\begin{array}{c} D4 \\ D5 \\ D6 \\ D7 \\ D8 \\ D9 \\ D16 \\ D17 \\ D18 \\ D13 \\ D14 \\ D15 \end{array}
$$

M_{x2}=-11.32 kNm/m

M_{y2}=-1.7 kNm/m

M_{xy2}=1.07 kNm/m

2 nolu eleman için gerilme hesabı;

3 nolu köşe için $\xi=x/a=1$; $\eta=y/b=1$

$$\begin{array}{l} \text{Eleman Düğüm Noktası} \longrightarrow \\ \begin{Bmatrix} M_{x3} \\ M_{y3} \\ M_{xy3} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{ccc|ccc|ccc|ccc} 0 & 0 & 0 & -0.9 & -0.3 & 0 & 6.9 & -0.6 & 4 & -6 & 0 & 2 \\ 0 & 0 & 0 & -6 & -2 & 0 & 6.9 & -4 & 0.6 & -0.9 & 0 & 0.3 \\ 0.85 & 0 & 0 & -0.85 & 0 & -0.85 & 0.85 & -0.85 & 0.85 & -0.85 & 0.85 & 0 \end{array}\right] \end{array} \begin{array}{c} \left\{\begin{array}{c} 3.18/D \\ 0 \\ 3.88/D \\ \hdashline 0 \\ 0 \\ 0 \\ \hdashline 0 \\ 0 \\ 0 \\ \hdashline 1.92/D \\ -2.38/D \\ 2.38/D \end{array}\right\} \end{array} \begin{array}{c} D4 \\ D5 \\ D6 \\ D7 \\ D8 \\ D9 \\ D16 \\ D17 \\ D18 \\ D13 \\ D14 \\ D15 \end{array}$$

M_{x2}=-6.76 kNm/m

M_{y2}=-1.01 kNm/m

M_{xy2}=-0.95 kNm/m

2 nolu eleman için gerilme hesabı;

4 nolu köşe için $\xi = x/a = 0$; $\eta = y/b = 1$

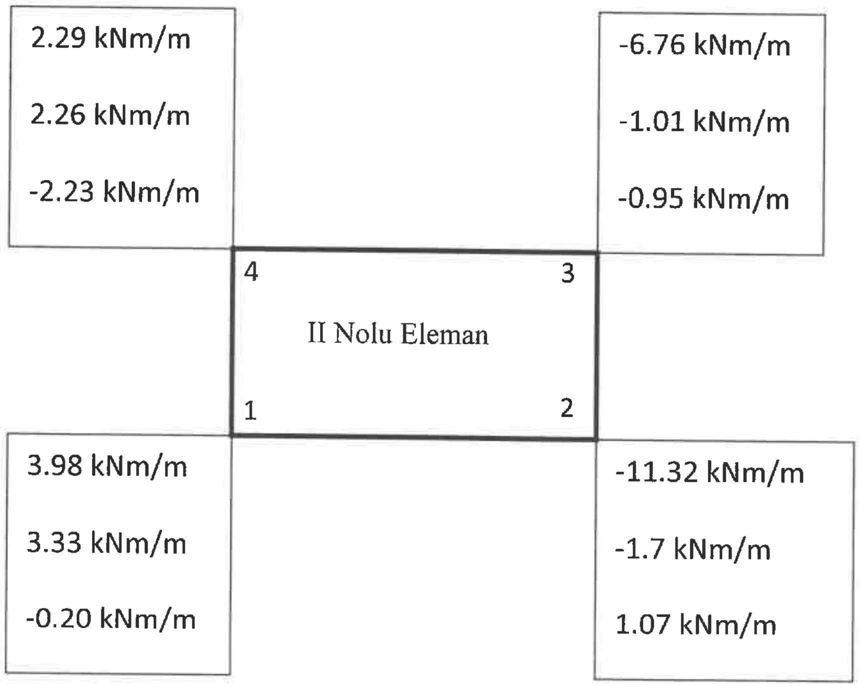
Eleman Düğüm Noktası ⟶ 1 2 3 4

$$\begin{Bmatrix} M_{x4} \\ M_{y4} \\ M_{xy4} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left| \begin{array}{ccc|ccc|ccc|ccc} -0.9 & -0.3 & 0 & 0 & 0 & 0 & -6 & 0 & -2 & 6.9 & -0.6 & -4 \\ -6 & -2 & 0 & 0 & 0 & 0 & -0.9 & 0 & -0.3 & 6.9 & -4 & -0.6 \\ 0.85 & 0 & -0.85 & -0.85 & 0 & 0 & 0.85 & -0.85 & 0 & -0.85 & 0.85 & 0.85 \end{array} \right| \left| \begin{array}{c} 3.18/D \\ 0 \\ 3.88/D \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 1.92/D \\ -2.38/D \\ 2.38/D \end{array} \right| \begin{array}{c} D4 \\ D5 \\ D6 \\ D7 \\ D8 \\ D9 \\ D16 \\ D17 \\ D18 \\ D13 \\ D14 \\ D15 \end{array}$$

M_{x2}=2.29 kNm/m

M_{y2}=2.26 kNm/m

M_{xy2}=-2.23 kNm/m



3 nolu eleman için gerilme hesabı;

1 nolu köşe için $\xi=x/a=0$; $\eta=y/b=0$

Eleman Düğüm Noktası $\longrightarrow$ 1 2 3 4

$$\begin{Bmatrix} M_{x1} \\ M_{y1} \\ M_{xy1} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} 6.9 & 0.6 & -4 & -6 & 0 & -2 & 0 & 0 & 0 & -0.9 & 0.3 & 0 \\ 6.9 & 4 & -0.6 & -0.9 & 0 & -0.3 & 0 & 0 & 0 & -6 & 2 & 0 \\ 0.85 & 0.85 & -0.85 & -0.85 & -0.85 & 0 & 0.85 & 0 & 0 & -0.85 & 0 & 0.85 \end{array}\right] \begin{array}{c|c} 3.18/D & D10 \\ -3.88/D & D11 \\ 0 & D12 \\ \hline 1.92/D & D13 \\ -2.38/D & D14 \\ 2.38/D & D15 \\ \hline 0 & D22 \\ 0 & D23 \\ 0 & D24 \\ \hline 0 & D19 \\ 0 & D20 \\ 0 & D21 \end{array}$$

M_{x1}=3.33 kNm/m

M_{y1}=3.98 kNm/m

M_{xy1}=-0.20 kNm/m

3 nolu eleman için gerilme hesabı;

2 nolu köşe için $\xi = x/a = 1$; $\eta = y/b = 0$

Eleman Düğüm Noktası ⟶ 1 2 3 4

$$\begin{Bmatrix} M_{x2} \\ M_{y2} \\ M_{xy2} \end{Bmatrix} = \frac{Et^3}{12(1-v^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} -6 & 0 & 2 & 6.9 & 0.6 & 4 & -0.9 & 0.3 & 0 & 0 & 0 & 0 \\ -0.9 & 0 & 0.3 & 6.9 & 4 & 0.6 & -6 & 2 & 0 & 0 & 0 & 0 \\ 0.85 & 0.85 & 0 & -0.85 & -0.85 & -0.85 & 0.85 & 0 & 0.85 & -0.85 & 0 & 0 \end{array}\right] \begin{Bmatrix} 3.18/D \\ -3.88/D \\ 0 \\ 1.92/D \\ -2.38/D \\ 2.38/D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} D10 \\ D11 \\ D12 \\ D13 \\ D14 \\ D15 \\ D22 \\ D23 \\ D24 \\ D19 \\ D20 \\ D21 \end{matrix}$$

M_{x2}=2.26 kNm/m

M_{y2}=2.29 kNm/m

M_{xy2}=-2.23 kNm/m

3 nolu eleman için gerilme hesabı;

3 nolu köşe için ξ=x/a=1 ; η=y/b=1

Eleman Düğüm Noktası ⟶ 1 2 3 4

$$\begin{Bmatrix} M_{x3} \\ M_{y3} \\ M_{xy3} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} 0 & 0 & 0 & -0.9 & -0.3 & 0 & 6.9 & -0.6 & 4 & -6 & 0 & 2 \\ 0 & 0 & 0 & -6 & -2 & 0 & 6.9 & -4 & 0.6 & -0.9 & 0 & 0.6 \\ 0.85 & 0 & 0 & -0.85 & 0 & -0.85 & 0.85 & -0.85 & 0.85 & -0.85 & 0.85 & 0 \end{array}\right] \begin{array}{|c|c} 3.18/D & D10 \\ -3.88/D & D11 \\ 0 & D12 \\ \hline 1.92/D & D13 \\ -2.38/D & D14 \\ 2.38/D & D15 \\ \hline 0 & D22 \\ 0 & D23 \\ 0 & D24 \\ \hline 0 & D19 \\ 0 & D20 \\ 0 & D21 \end{array}$$

M_{x2}=-1.01 kNm/m

M_{y2}=-6.76 kNm/m

M_{xy2}=-0.95 kNm/m

3 nolu eleman için gerilme hesabı;

4 nolu köşe için $\xi=x/a=0$; $\eta=y/b=1$

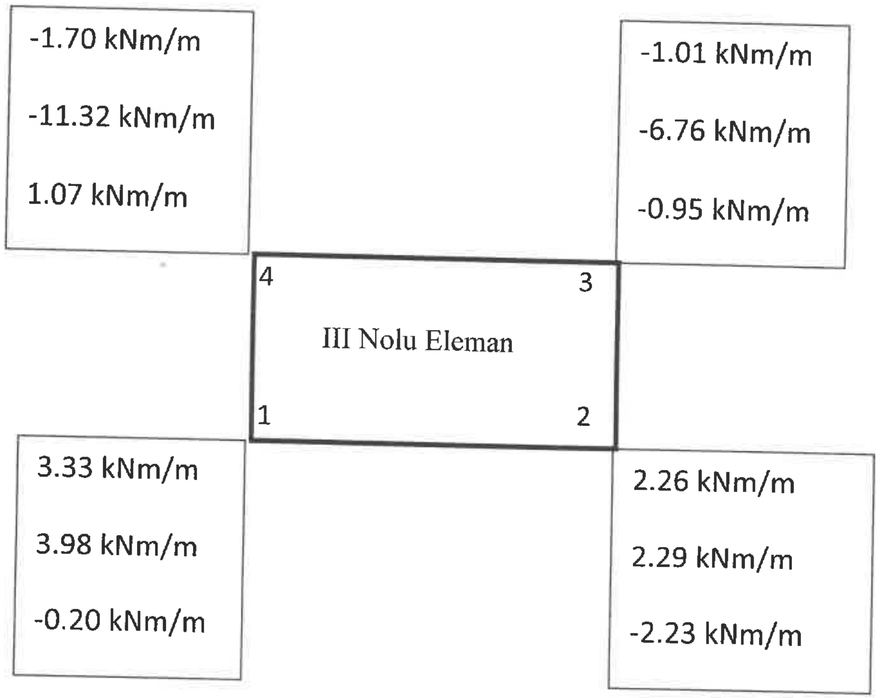
Eleman Düğüm Noktası $\longrightarrow$

$$
\begin{Bmatrix} M_{x4} \\ M_{y4} \\ M_{xy4} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \begin{array}{c}
\begin{array}{cccccccccccc} & 1 & & & 2 & & & 3 & & & 4 & \end{array} \\
\left[\begin{array}{ccc:ccc:ccc:ccc}
-0.9 & -0.3 & 0 & 0 & 0 & 0 & -6 & 0 & -2 & 6.9 & -0.6 & -4 \\
-6 & -2 & 0 & 0 & 0 & 0 & -0.9 & 0 & -0.3 & 6.9 & -4 & -0.6 \\
0.85 & 0 & -0.85 & -0.85 & 0 & 0 & 0.85 & -0.85 & 0 & -0.85 & 0.85 & 0.85
\end{array}\right]
\end{array}
\left\{\begin{array}{c}
3.18/D \\ -3.88/D \\ 0 \\ \hdashline 1.92/D \\ -2.38/D \\ 2.38/D \\ \hdashline 0 \\ 0 \\ 0 \\ \hdashline 0 \\ 0 \\ 0
\end{array}\right\}
\begin{array}{c}
D10 \\ D11 \\ D12 \\ D13 \\ D14 \\ D15 \\ D22 \\ D23 \\ D24 \\ D19 \\ D20 \\ D21
\end{array}
$$

M_{x2}=-1.70 kNm/m

M_{y2}=-11.32 kNm/m

M_{xy2}=1.07 kNm/m



4 nolu eleman için gerilme hesabı;

1 nolu köşe için $\xi=x/a=0$; $\eta=y/b=0$

Eleman Düğüm Noktası $\longrightarrow$ 1 2 3 4

$$\begin{Bmatrix} M_{x1} \\ M_{y1} \\ M_{xy1} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \begin{vmatrix} 6.9 & 0.6 & -4 & -6 & 0 & -2 & 0 & 0 & 0 & -0.9 & 0.3 & 0 \\ 6.9 & 4 & -0.6 & -0.9 & 0 & -0.3 & 0 & 0 & 0 & -6 & 2 & 0 \\ 0.85 & 0.85 & -0.85 & -0.85 & -0.85 & 0 & 0.85 & 0 & 0 & -0.85 & 0 & 0.85 \end{vmatrix} \begin{vmatrix} 1.92/D \\ -2.38/D \\ 2.38/D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} D13 \\ D14 \\ D15 \\ D16 \\ D17 \\ D18 \\ D25 \\ D26 \\ D27 \\ D22 \\ D23 \\ D24 \end{matrix}$$

M_{x1}=2.30 kNm/m

M_{y1}=2.30 kNm/m

M_{xy1}=-2.41 kNm/m

4 nolu eleman için gerilme hesabı;

2 nolu köşe için ξ=x/a=1 ; η=y/b=0

Eleman Düğüm Noktası ⟶ 1   2   3   4

$$\begin{Bmatrix} M_{x2} \\ M_{y2} \\ M_{xy2} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} -6 & 0 & 2 & 6.9 & 0.6 & 4 & -0.9 & 0.3 & 0 & 0 & 0 & 0 \\ -0.9 & 0 & 0.3 & 6.9 & 4 & 0.6 & -6 & 2 & 0 & 0 & 0 & 0 \\ 0.85 & 0.85 & 0 & -0.85 & -0.85 & -0.85 & 0.85 & 0.85 & 0 & -0.85 & 0 & 0 \end{array}\right] \left[\begin{array}{c} 1.92/D \\ -2.38/D \\ 2.38/D \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} D13 \\ D14 \\ D15 \\ D16 \\ D17 \\ D18 \\ D25 \\ D26 \\ D27 \\ D22 \\ D23 \\ D24 \end{array}$$

M_{x2}=-6.76 kNm/m

M_{y2}=-1.01 kNm/m

M_{xy2}=-0.39 kNm/m

4 nolu eleman için gerilme hesabı;

3 nolu köşe için $\xi = x/a = 1$; $\eta = y/b = 1$

Eleman Düğüm Noktası $\longrightarrow$ 1 2 3 4

$$\begin{Bmatrix} M_{x3} \\ M_{y3} \\ M_{xy3} \end{Bmatrix} = \frac{Et^3}{12(1-\upsilon^2)} * \left[\begin{array}{ccc|ccc|ccc|ccc} 0 & 0 & 0 & -0.9 & -0.3 & 0 & 6.9 & -0.6 & 4 & -6 & 0 & 2 \\ 0 & 0 & 0 & -6 & -2 & 0 & 6.9 & -4 & 0.6 & -0.9 & 0 & 0.6 \\ 0.85 & 0 & 0 & -0.85 & 0 & -0.85 & 0.85 & -0.85 & 0.85 & -0.85 & 0.85 & 0 \end{array} \right] \left\{ \begin{array}{c} 1.92/D \\ -2.38/D \\ 2.38/D \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{array} \right\} \begin{array}{c} D13 \\ D14 \\ D15 \\ D16 \\ D17 \\ D18 \\ D25 \\ D26 \\ D27 \\ D22 \\ D23 \\ D24 \end{array}$$

M_{x2}=0 kNm/m

M_{y2}=0 kNm/m

M_{xy2}=1.63 kNm/m

4 nolu eleman için gerilme hesabı;

4 nolu köşe için $\xi=x/a=0$; $\eta=y/b=1$

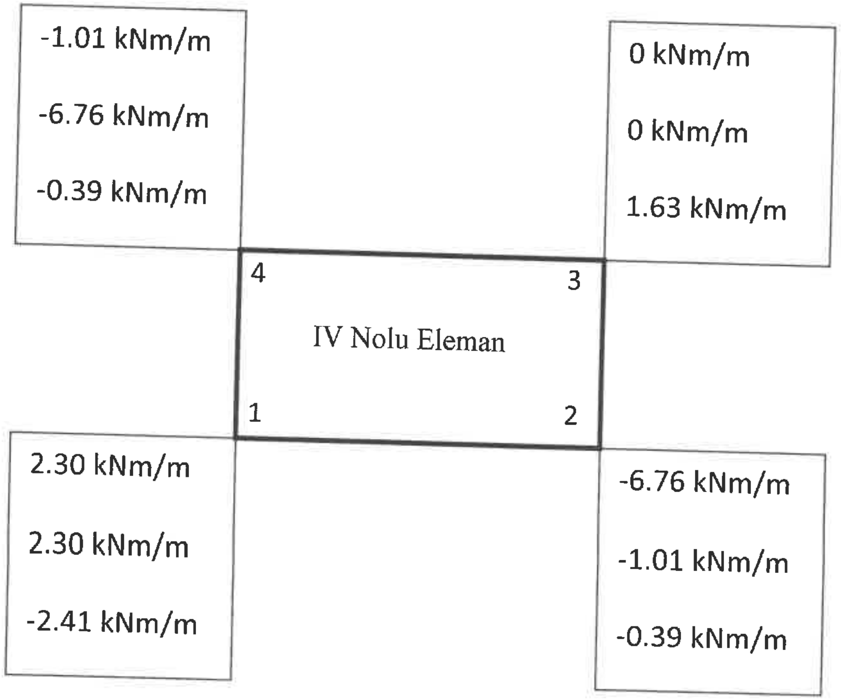
Eleman Düğüm Noktası ⟶ 1 2 3 4

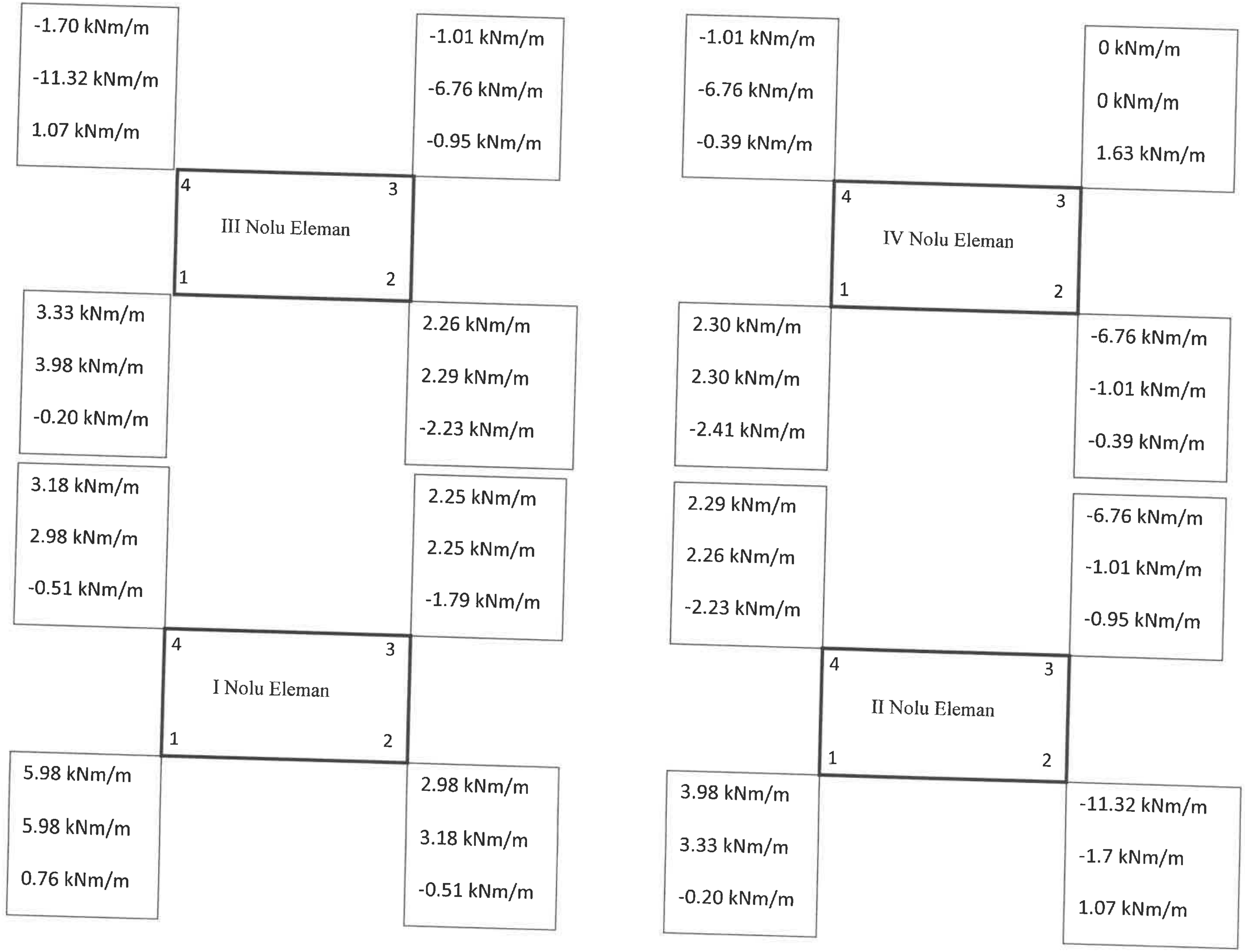
$$\begin{Bmatrix} M_{x4} \\ M_{y4} \\ M_{xy4} \end{Bmatrix} = \frac{Et^3}{12(1-\nu^2)} * \left|\begin{array}{ccc|ccc|ccc|ccc} -0.9 & -0.3 & 0 & 0 & 0 & 0 & -6 & 0 & -2 & 6.9 & -0.6 & -4 \\ -6 & -2 & 0 & 0 & 0 & 0 & -0.9 & 0 & -0.3 & 6.9 & -4 & -0.6 \\ 0.85 & 0 & -0.85 & -0.85 & 0 & 0 & 0.85 & -0.85 & 0 & -0.85 & 0.85 & 0.85 \end{array}\right| \left|\begin{array}{c} 1.92/D \\ -2.38/D \\ 2.38/D \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{array}\right| \begin{array}{c} D13 \\ D14 \\ D15 \\ D16 \\ D17 \\ D18 \\ D25 \\ D26 \\ D27 \\ D22 \\ D23 \\ D24 \end{array}$$

M_{x2}=-1.01 kNm/m

M_{y2}=-6.76 kNm/m

M_{xy2}=-0.39 kNm/m





-1.70 kNm/m
-11.32 kNm/m
1.07 kNm/m
-1.01 kNm/m
-6.76 kNm/m
-0.95 kNm/m
4
3
III Nolu Eleman
1
2
3.33 kNm/m
3.98 kNm/m
-0.20 kNm/m
2.26 kNm/m
2.29 kNm/m
-2.23 kNm/m
3.18 kNm/m
2.98 kNm/m
-0.51 kNm/m
2.25 kNm/m
2.25 kNm/m
-1.79 kNm/m
4
3
I Nolu Eleman
1
2
5.98 kNm/m
5.98 kNm/m
0.76 kNm/m
2.98 kNm/m
3.18 kNm/m
-0.51 kNm/m
-1.01 kNm/m
-6.76 kNm/m
-0.39 kNm/m
0 kNm/m
0 kNm/m
1.63 kNm/m
4
3
IV Nolu Eleman
1
2
2.30 kNm/m
2.30 kNm/m
-2.41 kNm/m
-6.76 kNm/m
-1.01 kNm/m
-0.39 kNm/m
2.29 kNm/m
2.26 kNm/m
-2.23 kNm/m
-6.76 kNm/m
-1.01 kNm/m
-0.95 kNm/m
4
3
II Nolu Eleman
1
2
3.98 kNm/m
3.33 kNm/m
-0.20 kNm/m
-11.32 kNm/m
-1.7 kNm/m
1.07 kNm/m

Tüm sistemdeki 2 nolu düğümün momentleri;

$$M_x = \frac{2{,}98+3{,}98}{2} = 3{,}48 \text{ kNm/m}$$

$$M_y = \frac{3{,}18+3{,}33}{2} = 3{,}26 \quad "$$

$$M_{xy} = \frac{-0{,}51-0{,}20}{2} = -0{,}36 \quad "$$

Tüm sistemdeki 5 nolu düğümün momentleri

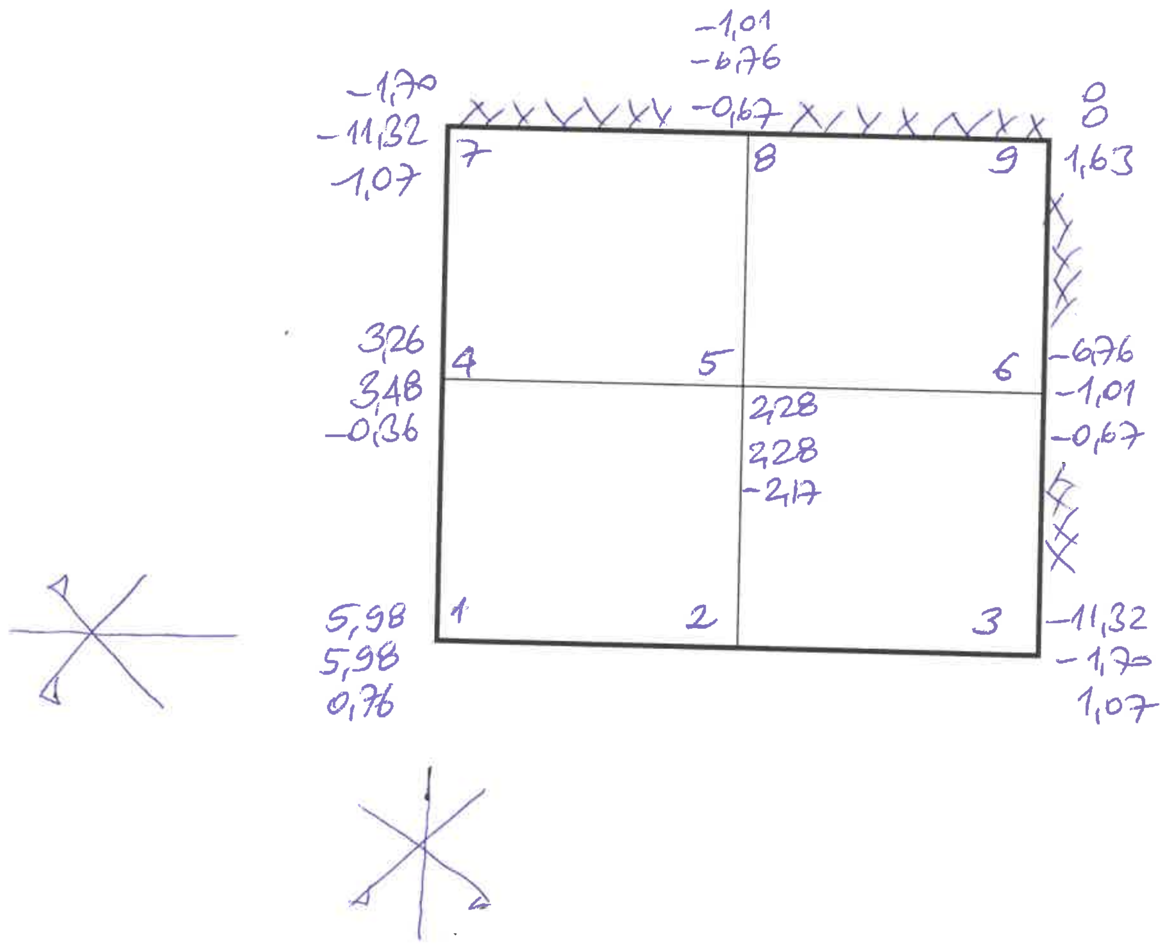
$$M_x = \frac{2{,}25+2{,}29+2{,}26+2{,}30}{4} = 2{,}28 \text{ kNm/m}$$

$$M_y = \frac{2{,}25+2{,}26+2{,}29+2{,}30}{4} = 2{,}28 \quad "$$

$$M_{xy} = \frac{-1{,}79-2{,}23-2{,}23-2{,}41}{4} = -2{,}17 \quad "$$

Diğer düğüm noktaları da benzer şekilde bulunur

Sonuçlar


-1,01
-6,76
-0,67
-1,70
-11,32
1,07
0
0
1,63
7
8
9
3,26
3,48
-0,36
4
5
6
-6,76
-1,01
-0,67
2,28
2,28
-2,17
5,98
5,98
0,76
1
2
3
-11,32
-1,70
1,07

ORTHOTROP PLÂKLARIN SONLU ELEMANLAR YÖNTEMİ İLE HESABI

Orthotrop plâklarda deplasman fonksiyonu, eğitlik matrisi ve yük matrisinin oluşturulması yönteminde izotrop plâklardan bir farkı yoktur. Ancak elastisite bağıntısı

$$[D]=\begin{bmatrix} C_{44} & C_{45} & 0 \\ C_{45} & C_{55} & 0 \\ 0 & 0 & C_{66} \end{bmatrix}$$

$$C_{44}=D+\frac{E(I_x+A_x.e_x^2)}{d_x}$$

$$C_{55}=D+\frac{E(I_y+A_y.e_y^2)}{d_y}$$

$$C_{66}=(1-\nu)D+\frac{1}{2}\left(\frac{GI_x}{d_x}+\frac{GI_y}{d_y}\right)$$

$$C_{45}=\nu D$$

şeklinde olacaktır.

Elastik Zemine Oturan İnce Plâkların Eğilmeli Hesabı

Plâk deplasman fonksiyonu ve eleman eğitlik matrisi plâk teorisinde verildiği gibidir.

$$w=a_1+a_2x+a_3y+a_4x^2+a_5xy+a_6y^2+a_7x^3+a_8x^2y+a_9xy^2+a_{10}y^3+$$
$$+a_{11}x^3y+a_{12}xy^3$$

$$[k_e]=\iint [B]^T[F]^T[D][F][B]\,dx\,dy \quad \text{veya}$$

$$[k_e]=ab\int_0^1\int_0^1 [\Delta N]^T[D][\Delta N]\,d\xi\,d\eta$$

Zemin Etki Matrisi

Zemin etki matrisine elastik yataklanma matrisi de denir. Elemanda;

$$[S_e] = c \iint [N]^T [N] \, dx\,dy$$

$$[N] = [\phi(x,y)][B]$$

$$[S_e] = c \iint [B]^T [\phi(x,y)]^T [\phi(x,y)][B] \, dx\,dy$$ 'dir. Burada

c elastik yataklanma katsayıdır.

Sisteme Geçiş

$\left([K]+[S]\right)\{D\} = \{Q\}$ bağıntısından $\{D\}$ yer-değiştirmeler bulunur. Buradan sistemde oluşan gerilmeler hesaplanır.

EK:3

MATLAB ÖRNEKLERİ

KOD1-İKİ MATRİS ÇARPIMI

$$[\boldsymbol{C1}] = [\boldsymbol{A1}] * [\boldsymbol{B1}]$$

$$[\boldsymbol{A1}] * [\boldsymbol{B1}] = \begin{bmatrix} 3 & 4 & 7 & -4 & 2 \\ 2 & -1 & 0 & -7 & 1 \\ 3 & 5 & 11 & 1 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 2 & -6 & 3 \\ 3 & 4 & 5 & 10 \\ -1 & 11 & 3 & -2 \\ 0 & 5 & 1 & 5 \\ 3 & 7 & 2 & 3 \end{bmatrix}$$

```
>> A1=[3 4 7 -4 2 ; 2 -1 0 -7 1; 3 5 11 1 2];
```

$$A1 = \begin{bmatrix} 3 & 4 & 7 & -4 & 2 \\ 2 & -1 & 0 & -7 & 1 \\ 3 & 5 & 11 & 1 & 2 \end{bmatrix}$$

```
>> B1=[1 2 -6 3; 3 4 5 10; -1 11 3 -2; 0 5 1 5; 3 7 2 3];
```

$$B1 = \begin{bmatrix} 1 & 2 & -6 & 3 \\ 3 & 4 & 5 & 10 \\ -1 & 11 & 3 & -2 \\ 0 & 5 & 1 & 5 \\ 3 & 7 & 2 & 3 \end{bmatrix}$$

```
>> C1=A1*B1
```

$$C1 = \begin{bmatrix} 14 & 93 & 23 & 21 \\ 2 & -28 & -22 & -36 \\ 13 & 166 & 45 & 48 \end{bmatrix}$$

KOD2-ÜÇ MATRİSİN ÇARPIMI

$$[\boldsymbol{D2}] = [\boldsymbol{A2}] * [\boldsymbol{B2}] * [\boldsymbol{C2}]$$

$$[A2] * [B2] * [C2] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

clear, clc

>>syms L

>>syms x

>> A2=[1 0 0 0 ; 0 1 0 0 ; 0 0 1 0; 0 0 0 1; 0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0]

$$A2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

>> B2=[12 6 -12 6; 6 4 -6 2; -12 -6 12 -6; 6 2 -6 4]

$$B2 = \begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix}$$

```
>> C2=[ 1 0 0 0 0 0 0 0 ; 0 1 0 0 0 0 0 0 ; 0 0 1 0 0 0 0 0 ; 0 0 0 1 0 0 0 0 ]
```

$$C2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
>> D2=A2*B2*C2
```

$$D2 = \begin{bmatrix} 12 & 6 & -12 & 6 & 0 & 0 & 0 & 0 \\ 6 & 4 & -6 & 2 & 0 & 0 & 0 & 0 \\ -12 & -6 & 12 & -6 & 0 & 0 & 0 & 0 \\ 6 & 2 & -6 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

KOD3-DÖRT MATRİSİN ÇARPIMI

$$Carp\_3 = [A3] * [B3] * [C3] * [D3]$$

$$[A3]*[B3]*[C3]*[D3] = \begin{bmatrix} 1 & 0 & -\frac{3}{L^2} & \frac{2}{L^3} \\ 0 & 1 & -\frac{2}{L} & \frac{1}{L^2} \\ 0 & 0 & \frac{3}{L^2} & -\frac{2}{L^3} \\ 0 & 0 & -\frac{1}{L} & \frac{1}{L^2} \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ -2 \\ -6x \end{bmatrix} * [0 \quad 0 \quad -2 \quad -6x] * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{2}{L} & \frac{3}{L^2} & -\frac{1}{L} \\ \frac{2}{L^3} & \frac{1}{L^2} & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix}$$

```
clear, clc
syms L
syms x
```

```
>> A3=[ 1 0 -3/(L^2) 2/(L^3) ; 0 1 -2/L 1/(L^2); 0 0 3/(L^2) -2/(L^3); 0 0 -1/L 1/(L^2)]
```

$$A3 = \begin{bmatrix} 1 & 0 & -\frac{3}{L^2} & \frac{2}{L^3} \\ 0 & 1 & -\frac{2}{L} & \frac{1}{L^2} \\ 0 & 0 & \frac{3}{L^2} & -\frac{2}{L^3} \\ 0 & 0 & -\frac{1}{L} & \frac{1}{L^2} \end{bmatrix}$$

```
>> B3=[ 0; 0; -2; -6*x]
```

$$B3 = \begin{bmatrix} 0 \\ 0 \\ -2 \\ -6x \end{bmatrix}$$

```
>> C3 =[ 0, 0, -2, -6*x]
```

$C3 = [0 \quad 0 \quad -2 \quad -6x]$

```
>> D3=[ 1 0 0 0 ; 0 1 0 0 ; -3/(L^2) -2/L 3/(L^2) -1/L; 2/(L^3) 1/(L^2) -2/(L^3) 1/(L^2)];
```

$$D3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{2}{L} & \frac{3}{L^2} & -\frac{1}{L} \\ \frac{2}{L^3} & \frac{1}{L^2} & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix}$$

>> Carp_3= A3*B3*C3*D3

$$Carp\_3 = \begin{bmatrix} 12x\frac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^3}-3\frac{\frac{24x}{L^3}-\frac{12}{L^2}}{L^2} & 6x\frac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^2}-2\frac{\frac{24x}{L^3}-\frac{12}{L^2}}{L} & 3\frac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L^2}-12x\frac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^3} & 6x\frac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^2}-\frac{\frac{24x}{L^3}-\frac{12}{L^2}}{L} \\ 12x\frac{\left(\frac{6x}{L^2}-\frac{4}{L}\right)}{L^3}-3\frac{\frac{12x}{L^2}-\frac{8}{L}}{L^2} & 6x\frac{\left(\frac{6x}{L^2}-\frac{4}{L}\right)}{L^2}-2\frac{\frac{12x}{L^2}-\frac{8}{L}}{L} & 3\frac{\left(\frac{12x}{L^2}-\frac{8}{L}\right)}{L^2}-12x\frac{\frac{6x}{L^2}-\frac{4}{L}}{L^3} & 6x\frac{\left(\frac{6x}{L^2}-\frac{4}{L}\right)}{L^2}-\frac{\frac{12x}{L^2}-\frac{8}{L}}{L} \\ 3\frac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L^2}-12x\frac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^3} & 2\frac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L}-6x\frac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^2} & 12x\frac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^3}-3\frac{\frac{24x}{L^3}-\frac{12}{L^2}}{L^2} & \frac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L}-6x\frac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^2} \\ 12x\frac{\left(\frac{6x}{L^2}-\frac{2}{L}\right)}{L^3}-3\frac{\frac{12x}{L^2}-\frac{4}{L}}{L^2} & 6x\frac{\left(\frac{6x}{L^2}-\frac{2}{L}\right)}{L^2}-2\frac{\frac{12x}{L^2}-\frac{4}{L}}{L} & 3\frac{\left(\frac{12x}{L^2}-\frac{4}{L}\right)}{L^2}-12x\frac{\frac{6x}{L^2}-\frac{2}{L}}{L^3} & 6x\frac{\left(\frac{6x}{L^2}-\frac{2}{L}\right)}{L^2}-\frac{\frac{12x}{L^2}-\frac{4}{L}}{L} \end{bmatrix}$$

KOD4-DÖRT MATRİSİN ÇARPIMININ İNTEGRALİ

Çubuk eleman rijitlik matrisinin hesabı [ke]

$$[\boldsymbol{Carp\_3\_intg}] = \int_0^L [\boldsymbol{A3}] * [\boldsymbol{B3}] * [\boldsymbol{C3}] * [\boldsymbol{D3}]$$

```
>> InT_D3=int(Carp_3, x, 0, L)
```

$$Carp\_3 = \int_0^L \begin{bmatrix} 12x\dfrac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^3} - 3\dfrac{\frac{24x}{L^3}-\frac{12}{L^2}}{L^2} & 6x\dfrac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^2} - 2\dfrac{\frac{24x}{L^3}-\frac{12}{L^2}}{L} & 3\dfrac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L^2} - 12x\dfrac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^3} & 6x\dfrac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^2} - \dfrac{\frac{24x}{L^3}-\frac{12}{L^2}}{L} \\ 12x\dfrac{\left(\frac{6x}{L^2}-\frac{4}{L}\right)}{L^3} - 3\dfrac{\frac{12x}{L^2}-\frac{8}{L}}{L^2} & 6x\dfrac{\left(\frac{6x}{L^2}-\frac{4}{L}\right)}{L^2} - 2\dfrac{\frac{12x}{L^2}-\frac{8}{L}}{L} & 3\dfrac{\left(\frac{12x}{L^2}-\frac{8}{L}\right)}{L^2} - 12x\dfrac{\frac{6x}{L^2}-\frac{4}{L}}{L^3} & 6x\dfrac{\left(\frac{6x}{L^2}-\frac{4}{L}\right)}{L^2} - \dfrac{\frac{12x}{L^2}-\frac{8}{L}}{L} \\ 3\dfrac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L^2} - 12x\dfrac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^3} & 2\dfrac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L} - 6x\dfrac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^2} & 12x\dfrac{\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)}{L^3} - 3\dfrac{\frac{24x}{L^3}-\frac{12}{L^2}}{L^2} & \dfrac{\left(\frac{24x}{L^3}-\frac{12}{L^2}\right)}{L} - 6x\dfrac{\frac{12x}{L^3}-\frac{6}{L^2}}{L^2} \\ 12x\dfrac{\left(\frac{6x}{L^2}-\frac{2}{L}\right)}{L^3} - 3\dfrac{\frac{12x}{L^2}-\frac{4}{L}}{L^2} & 6x\dfrac{\left(\frac{6x}{L^2}-\frac{2}{L}\right)}{L^2} - 2\dfrac{\frac{12x}{L^2}-\frac{4}{L}}{L} & 3\dfrac{\left(\frac{12x}{L^2}-\frac{4}{L}\right)}{L^2} - 12x\dfrac{\frac{6x}{L^2}-\frac{2}{L}}{L^3} & 6x\dfrac{\left(\frac{6x}{L^2}-\frac{2}{L}\right)}{L^2} - \dfrac{\frac{12x}{L^2}-\frac{4}{L}}{L} \end{bmatrix}$$

$$InT_{D3} = \begin{bmatrix} \dfrac{12}{L^3} & \dfrac{6}{L^2} & -\dfrac{12}{L^3} & \dfrac{6}{L^2} \\ \dfrac{6}{L^2} & \dfrac{4}{L} & -\dfrac{6}{L^2} & \dfrac{2}{L} \\ -\dfrac{12}{L^3} & -\dfrac{6}{L^2} & \dfrac{12}{L^3} & -\dfrac{6}{L^2} \\ \dfrac{6}{L^2} & \dfrac{2}{L} & -\dfrac{6}{L^2} & \dfrac{4}{L} \end{bmatrix}$$

KOD5-ŞEKİL FONKSİYONUNDAN FAYDALANARAK ELEMAN RİJİTLİK MATRİSİNİN HESABI

$$[k_e] = \int_0^L \begin{bmatrix} -N_1'' \\ -N_2'' \\ -N_3'' \\ -N_4'' \end{bmatrix} [-N_1'' \quad -N_2'' \quad -N_3'' \quad -N_4''] \, \mathrm{dx}$$

clear, clc

>>syms L

>>syms x

>> N1=1-3*(x^2)/(L^2)+2*(x^3)/(L^3)

$$N1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}$$

>> N2=L*(x/L-2*(x^2)/(L^2)+(x/L)^3)

$$N2 = L(\frac{x}{L} - \frac{2x^2}{L^2} + \frac{x^3}{L^3})$$

>> N3=3*x/(L^2)-2*((x/L)^3)

$$N3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}$$

>> N4=-1*(x^2)/L+(x^3)/(L^2)

$$N4 = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

```
>> N1_dev2=diff(diff(N1))
>> N2_dev2=diff(diff(N2))
>> N3_dev2=diff(diff(N3))
>> N4_dev2=diff(diff(N4))
```

$$N1\_dev2 = -\frac{6}{L^2} + \frac{12x}{L^3}$$

$$N2\_dev2 = L(-4/L^\wedge 2 \ + 6x/L^\wedge 3\ )$$

$$N3\_dev2 = \frac{6}{L^2} - \frac{12x}{L^3}$$

$$N4\_dev2 = -\frac{2}{L} + \frac{6x}{L^2}$$

>> NN_VEC1=-[N1_dev2; N2_dev2; N3_dev2; N4_dev2]

$$NN\_VEC1 = \begin{bmatrix} \frac{6}{L^2} - \frac{12x}{L^3} \\ -L(-4/L\text{^}2\ + 6x/L\text{^}3\,) \\ -\frac{6}{L^2} + \frac{12x}{L^3} \\ \frac{2}{L} - \frac{6x}{L^2} \end{bmatrix}$$

>> NN_VEC2=-[N1_dev2 N2_dev2 N3_dev2 N4_dev2]

$$NN\_VEC2 = \begin{bmatrix} \frac{6}{L^2} - \frac{12x}{L^3} & -L(-4/L\text{^}2\ + 6x/L\text{^}3\,) & -\frac{6}{L^2} + \frac{12x}{L^3} & \frac{2}{L} - \frac{6x}{L^2} \end{bmatrix}$$

>> kL=NN_VEC1*NN_VEC2;

$$kL = \begin{bmatrix} \left(\frac{12x}{L^3} - \frac{6}{L^2}\right)^2 & L\left(\left(\frac{6x}{L^3} - \frac{4}{L^2}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right)\right) & -\left(\frac{12x}{L^3} - \frac{6}{L^2}\right)^2 & \left(\frac{6x}{L^2} - \frac{2}{L}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right) \\ L\left(\left(\frac{6x}{L^3} - \frac{4}{L^2}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right)\right) & L^2\left(\frac{6x}{L^3} - \frac{4}{L^2}\right)^2 & L\left(\left(\frac{6x}{L^3} - \frac{4}{L^2}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right)\right) & L\left(\left(\frac{6x}{L^2} - \frac{2}{L}\right) * \left(\frac{6x}{L^3} - \frac{4}{L^2}\right)\right) \\ -\left(\frac{12x}{L^3} - \frac{6}{L^2}\right)^2 & -L\left(\left(\frac{6x}{L^3} - \frac{4}{L^2}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right)\right) & \left(\frac{12x}{L^3} - \frac{6}{L^2}\right)^2 & -\left(\frac{6x}{L^2} - \frac{2}{L}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right) \\ \left(\frac{6x}{L^2} - \frac{2}{L}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right) & L\left(\left(\frac{6x}{L^2} - \frac{2}{L}\right) * \left(\frac{6x}{L^3} - \frac{4}{L^2}\right)\right) & \left(\frac{6x}{L^2} - \frac{2}{L}\right) * \left(\frac{12x}{L^3} - \frac{6}{L^2}\right) & \left(\frac{6x}{L^2} - \frac{2}{L}\right)^2 \end{bmatrix}$$

```
>> InT_NN_VEC=int(kL, x, 0, L)
```

$$kL = \int_0^L \begin{bmatrix} \left(\frac{12x}{L^3}-\frac{6}{L^2}\right)^2 & L\left(\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)\right) & -\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)^2 & \left(\frac{6x}{L^2}-\frac{2}{L}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right) \\ L\left(\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)\right) & L^2\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)^2 & L\left(\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)\right) & L\left(\left(\frac{6x}{L^2}-\frac{2}{L}\right)*\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)\right) \\ -\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)^2 & -L\left(\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right)\right) & \left(\frac{12x}{L^3}-\frac{6}{L^2}\right)^2 & -\left(\frac{6x}{L^2}-\frac{2}{L}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right) \\ \left(\frac{6x}{L^2}-\frac{2}{L}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right) & L\left(\left(\frac{6x}{L^2}-\frac{2}{L}\right)*\left(\frac{6x}{L^3}-\frac{4}{L^2}\right)\right) & \left(\frac{6x}{L^2}-\frac{2}{L}\right)*\left(\frac{12x}{L^3}-\frac{6}{L^2}\right) & \left(\frac{6x}{L^2}-\frac{2}{L}\right)^2 \end{bmatrix}$$

$$InT\_NN\_VEC = \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix}$$

KOD6-KİRİŞ MOMENTLERİNİN HESABI

$$M_1 = -\frac{EI}{L^2}\begin{bmatrix} -N_1^{"} & -N_2^{"} & -N_3^{"} & -N_4^{"} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \frac{3}{8}q\frac{L^4}{EI} \\ \frac{1}{2}q\frac{L^3}{EI} \end{bmatrix}$$

```
clear, clc
syms L
syms x
syms q
syms E
syms I
>> N1=1-3*(x^2)/(L^2)+2*(x^3)/(L^3)
```

$$N1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}$$

```
>> N2=L*(x/L-2*(x^2)/(L^2)+(x/L)^3)
```

$$N2 = L(\frac{x}{L} - \frac{2x^2}{L^2} + \frac{x^3}{L^3})$$

```
>> N3=3*x/(L^2)-2*((x/L)^3)
```

$$N3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3}$$

```
>> N4=-1*(x^2)/L+(x^3)/(L^2)
```

$$N4 = -\frac{x^2}{L} + \frac{x^3}{L^2}$$

```
>> N1_dev2=diff(diff(N1))
>> N2_dev2=diff(diff(N2))
>> N3_dev2=diff(diff(N3))
>> N4_dev2=diff(diff(N4))
```

$$N1\_dev2 = -\frac{6}{L^2} + \frac{12x}{L^3}$$

$$N2\_dev2 = L(-4/L^2 + 6x/L^3)$$

$$N3\_dev2 = \frac{6}{L^2} - \frac{12x}{L^3}$$

$$N4\_dev2 = -\frac{2}{L} + \frac{6x}{L^2}$$

>> NN_VEC2=-[N1_dev2 N2_dev2 N3_dev2 N4_dev2]

$$NN\_VEC2 = \left[\frac{6}{L^2} - \frac{12x}{L^3} \quad -L(-4/L^\wedge 2\ + 6x/L^\wedge 3\ ) \quad -\frac{6}{L^2} + \frac{12x}{L^3} \quad \frac{2}{L} - \frac{6x}{L^2}\right]$$

>> M1=-(E*I/(L^2))*(-NN_VEC2)*[0 ; 0 ; 3*q*(L^4)/(8*E*I); (1/2)*q*(L^3)/(E*I)]

$$M1 = \frac{3qL^2}{8}\left(\frac{12x}{L^3} - \frac{6}{L^2}\right) - \frac{qL}{2}\left(\frac{6x}{L^2} - \frac{2}{L}\right)$$

>> M1_x_0=subs(M1,x,0)

M1_x_0 =

-(5*q)/4

>> M1_x_L=subs(M1,x,L)

M1_x_L =

q/4

>> M2=-(E*I/(L^2))*(-NN_VEC2)*[3*q*(L^4)/(8*E*I) ; (1/2)*q*(L^3)/(E*I) ; ((2/3)*q*(L^4))/(E*I); 0]

$$M2 = \frac{7qL^\wedge 2}{24}\left(\frac{12x}{L^3} - \frac{6}{L^2}\right) - \frac{qL^2}{2}\left(\frac{6x}{L^3} - \frac{4}{L^2}\right)$$

>> M2_x_0=subs(M2,x,0)

M2_x_0 =

q/4

>> M2_x_L=subs(M2,x,L)

M2_x_L =

(3*q)/4

KOD7-ÇUBUK ELEMAN RJİTLİK MATRİSİNİN HESABI

$$[k_e] = EI\int_0^{L/4} \begin{bmatrix} 1 & 0 & -\frac{48}{L^2} & \frac{128}{L^3} \\ 0 & 1 & -\frac{8}{L} & \frac{16}{L^2} \\ 0 & 0 & \frac{48}{L^2} & -\frac{128}{L^3} \\ 0 & 0 & -\frac{4}{L} & \frac{16}{L^2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -2 \\ -6x \end{bmatrix} [0 \quad 0 \quad -2 \quad -6x] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix}$$

```
clear, clc
syms L
syms x
syms q
syms E
syms I
```

```
>> A4=[1 0 -48/L^2 128/L^3;0 1 -8/L 16/L^2;0 0 48/L^2 -128/L^3;0 0 -4/L 16/L^2]
```

$$A4 = \begin{bmatrix} 1 & 0 & -\frac{48}{L^2} & \frac{128}{L^3} \\ 0 & 1 & -\frac{8}{L} & \frac{16}{L^2} \\ 0 & 0 & \frac{48}{L^2} & -\frac{128}{L^3} \\ 0 & 0 & -\frac{4}{L} & \frac{16}{L^2} \end{bmatrix}$$

```
>> B4=[0;0;-2;-6*x]
```

$$B4 = \begin{bmatrix} 0 \\ 0 \\ -2 \\ -6x \end{bmatrix}$$

```
>> C4=[0 0 -2 -6*x]
```

$$C4 = \begin{bmatrix} 0 & 0 & -2 & -6x \end{bmatrix}$$

```
>> D4=A4.'
```

$$D4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix}$$

>> kL_2=A4*B4*C4*D4

$$\mathrm{kL\_2} = \begin{bmatrix} 768x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^3} - 48\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L^2} & 96x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^2} - 8\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L} & 48\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L^2} - 768x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^3} & 96x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^2} - 4\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L} \\ 768x\frac{\left(\frac{96x}{L^2}-\frac{16}{L}\right)}{L^3} - 48\frac{\frac{192x}{L^2}-\frac{32}{L}}{L^2} & 96x\frac{\left(\frac{96x}{L^2}-\frac{16}{L}\right)}{L^2} - 8\frac{\frac{192x}{L^2}-\frac{32}{L}}{L} & 48\frac{\left(\frac{192x}{L^2}-\frac{32}{L}\right)}{L^2} - 768x\frac{\frac{96x}{L^2}-\frac{16}{L}}{L^3} & 96x\frac{\left(\frac{96x}{L^2}-\frac{16}{L}\right)}{L^2} - 4\frac{\frac{192x}{L^2}-\frac{32}{L}}{L} \\ 48\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L^2} - 768x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^3} & 8\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L} - 96x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^2} & 768x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^3} - 48\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L^2} & 4\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L} - 96x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^2} \\ 768x\frac{\left(\frac{96x}{L^2}-\frac{8}{L}\right)}{L^3} - 48\frac{\frac{192x}{L^2}-\frac{16}{L}}{L^2} & 96x\frac{\left(\frac{96x}{L^2}-\frac{8}{L}\right)}{L^2} - 8\frac{\frac{192x}{L^2}-\frac{16}{L}}{L} & 48\frac{\left(\frac{192x}{L^2}-\frac{16}{L}\right)}{L^2} - 768x\frac{\frac{96x}{L^2}-\frac{8}{L}}{L^3} & 96x\frac{\left(\frac{96x}{L^2}-\frac{8}{L}\right)}{L^2} - 4\frac{\frac{192x}{L^2}-\frac{16}{L}}{L} \end{bmatrix}$$

>> INT_kL_2=E*I*int(kL_2,0,L/4)

$$\mathrm{kL\_2} = \int_0^{L/4} \begin{bmatrix} 768x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^3} - 48\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L^2} & 96x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^2} - 8\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L} & 48\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L^2} - 768x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^3} & 96x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^2} - 4\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L} \\ 768x\frac{\left(\frac{96x}{L^2}-\frac{16}{L}\right)}{L^3} - 48\frac{\frac{192x}{L^2}-\frac{32}{L}}{L^2} & 96x\frac{\left(\frac{96x}{L^2}-\frac{16}{L}\right)}{L^2} - 8\frac{\frac{192x}{L^2}-\frac{32}{L}}{L} & 48\frac{\left(\frac{192x}{L^2}-\frac{32}{L}\right)}{L^2} - 768x\frac{\frac{96x}{L^2}-\frac{16}{L}}{L^3} & 96x\frac{\left(\frac{96x}{L^2}-\frac{16}{L}\right)}{L^2} - 4\frac{\frac{192x}{L^2}-\frac{32}{L}}{L} \\ 48\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L^2} - 768x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^3} & 8\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L} - 96x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^2} & 768x\frac{\left(\frac{768x}{L^3}-\frac{96}{L^2}\right)}{L^3} - 48\frac{\frac{1536x}{L^3}-\frac{192}{L^2}}{L^2} & 4\frac{\left(\frac{1536x}{L^3}-\frac{192}{L^2}\right)}{L} - 96x\frac{\frac{768x}{L^3}-\frac{96}{L^2}}{L^2} \\ 768x\frac{\left(\frac{96x}{L^2}-\frac{8}{L}\right)}{L^3} - 48\frac{\frac{192x}{L^2}-\frac{16}{L}}{L^2} & 96x\frac{\left(\frac{96x}{L^2}-\frac{8}{L}\right)}{L^2} - 8\frac{\frac{192x}{L^2}-\frac{16}{L}}{L} & 48\frac{\left(\frac{192x}{L^2}-\frac{16}{L}\right)}{L^2} - 768x\frac{\frac{96x}{L^2}-\frac{8}{L}}{L^3} & 96x\frac{\left(\frac{96x}{L^2}-\frac{8}{L}\right)}{L^2} - 4\frac{\frac{192x}{L^2}-\frac{16}{L}}{L} \end{bmatrix}$$

$$\mathrm{INT\_kL\_2} = \begin{bmatrix} \frac{768}{L^3} & \frac{96}{L^2} & -\frac{768}{L^3} & \frac{96}{L^2} \\ \frac{96}{L^2} & \frac{16}{L} & -\frac{96}{L^2} & \frac{8}{L} \\ -\frac{768}{L^3} & -\frac{96}{L^2} & \frac{768}{L^3} & -\frac{96}{L^2} \\ \frac{96}{L^2} & \frac{8}{L} & -\frac{96}{L^2} & \frac{16}{L} \end{bmatrix}$$

KOD8-GEOMETRİK YÜK MATRİSİNİN HESABI

ÇUBUK ELEMANI Ls=L/4 için

$$[n_e] = EI\int_0^{L/4}\begin{bmatrix} 1 & 0 & -\frac{48}{L^2} & \frac{128}{L^3} \\ 0 & 1 & -\frac{8}{L} & \frac{16}{L^2} \\ 0 & 0 & \frac{48}{L^2} & -\frac{128}{L^3} \\ 0 & 0 & -\frac{4}{L} & \frac{16}{L^2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2x \\ 3x^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2x & 3x^2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix}$$

```
clear, clc
syms L
syms x
```

```
>> A5= [1 0 -48/L^2 128/L^3;0 1 -8/L 16/L^2;0 0 48/L^2 -128/L^3;0 0 -4/L 16/L^2]
```

$$A5 = \begin{bmatrix} 1 & 0 & -\frac{48}{L^2} & \frac{128}{L^3} \\ 0 & 1 & -\frac{8}{L} & \frac{16}{L^2} \\ 0 & 0 & \frac{48}{L^2} & -\frac{128}{L^3} \\ 0 & 0 & -\frac{4}{L} & \frac{16}{L^2} \end{bmatrix}$$

```
>>B5=[0;1;2*x;3*x^2]
```

$$B5 = \begin{bmatrix} 0 \\ 1 \\ 2x \\ 3x^2 \end{bmatrix}$$

```
>> C5=[0 1 2*x 3*x^2]
```

$$C5 = \begin{bmatrix} 0 & 1 & 2x & 3x^2 \end{bmatrix}$$

>> D5=[1 0 -48/L^2 128/L^3;0 1 -8/L 16/L^2;0 0 48/L^2 -128/L^3;0 0 -4/L 16/L^2]'

$$D5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{48}{L^2} & -\frac{8}{L} & \frac{48}{L^2} & -\frac{4}{L} \\ \frac{128}{L^3} & \frac{16}{L^2} & -\frac{128}{L^3} & \frac{16}{L^2} \end{bmatrix}$$

>> nL=A5*B5*C5*D5

$$\mathrm{nL} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{41} & k_{43} & k_{44} \end{bmatrix}$$

$$k_{11} = \frac{96x\left(\frac{96x}{L^2} - \frac{9384x^2}{L^3}\right)}{L^2} - \frac{384x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^3}$$

$$k_{12} = \frac{384x^2}{L^3} - \frac{96x}{L^2} + \frac{16x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L} - \frac{48x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2}$$

$$k_{13} = \frac{384x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2} - \frac{96x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2}$$

$$k_{14} = \frac{8x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L} - \frac{48x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2}$$

$$k_{21} = \frac{384x^2\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L^3} - \frac{96x\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L^2}$$

$$k_{22} = \frac{48x^2}{L^2} - \frac{16x}{L} - \frac{16x\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L} + \frac{48x^2\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L^2} + 1$$

$$k_{23} = \frac{96x\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L^2} - \frac{384x^2\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L^3}$$

$$k_{24} = \frac{48x^2\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L^2} - \frac{8x\left(\frac{48x^2}{L^2} - \frac{16x}{L} + 1\right)}{L}$$

$$k_{31} = \frac{384x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^3} - \frac{96x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2}$$

$$k_{32} = \frac{96x}{L^2} - \frac{384x^2}{L^3} - \frac{16x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L} + \frac{48x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2}$$

$$k_{33} = \frac{96x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2} - \frac{384x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^3}$$

$$k_{34} = \frac{48x^2\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L^2} - \frac{8x\left(\frac{96x}{L^2} - \frac{384x^2}{L^3}\right)}{L}$$

$$k_{41} = \frac{96x\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L^2} - \frac{384x^2\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L^3}$$

$$k_{42} = \frac{48x^2}{L^2} - \frac{8x}{L} + \frac{16x\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L} - \frac{48x^2\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L^2}$$

$$k_{43} = \frac{384x^2\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L^3} - \frac{96x\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L^2}$$

$$k_{44} = \frac{8x\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L} - \frac{48x^2\left(\frac{8x}{L} - \frac{48x^2}{L^2}\right)}{L^2}$$

```
>> INT_nL=int(nL,0,L/4)
```

$$\mathrm{INT\_nL} = \int_0^{L/4} \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{41} & k_{43} & k_{44} \end{bmatrix}$$

$$\mathrm{INT\_nL} = \begin{bmatrix} \frac{24}{5L} & \frac{1}{10} & -\frac{24}{5L} & \frac{1}{10} \\ \frac{1}{10} & \frac{L}{30} & -\frac{1}{10} & -\frac{L}{120} \\ -\frac{24}{5L} & -\frac{1}{10} & \frac{24}{5L} & -\frac{1}{10} \\ \frac{1}{10} & -\frac{L}{120} & -\frac{1}{10} & \frac{L}{30} \end{bmatrix}$$

KOD9 -GEOMETRİK YÜK MATRİSİNİN HESABI

ÇUBUK ELEMANI Ls=1 m. için

$$[n_e] = EI\int_0^1 \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2x \\ 3x^2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2x & 3x^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix}$$

```
clear, clc
syms L
syms x
```

```
>> A6=[1 0 0 0;0 1 0 0;-3 -2 3 -1;2 1 -2 1].'
```

$$A6 = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

```
>>B6=[0;1;2*x;3*x^2]
```

$$B6 = \begin{bmatrix} 0 \\ 1 \\ 2x \\ 3x^2 \end{bmatrix}$$

```
>> C6=[0 1 2*x 3*x^2]
```

$$C6 = \begin{bmatrix} 0 & 1 & 2x & 3x^2 \end{bmatrix}$$

```
>> D6=[1 0 0 0;0 1 0 0;-3 -2 3 -1;2 1 -2 1].'
```

$$D6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix}$$

```
>> nL_2=A6*B6*C6*D6
```

$$nL\_2 = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{41} & k_{43} & k_{44} \end{bmatrix}$$

$k_{11} = 0$

$k_{12} = 6x^2 - 6x$

$k_{13} = 12x - 6x(-6x^2 + 6x) + 3x^2(-6x^2 + 6x) - 12x^2$

$k_{14} = 4x(-6x^2 + 6x) - 6x - 3x^2(-6x^2 + 6x) + 6x^2$

$k_{21} = 0$

$k_{22} = 3x^2 - 4x + 1$

$k_{23} = 8x + 6x(3x^2 - 4x + 1) - 3x^2(3x^2 - 4x + 1) - 6x^2 - 2$

$k_{24} = 3x^2(3x^2 - 4x + 1) - 4x(3x^2 - 4x + 1) - 4x + 3x^2 + 1$

$k_{31} = 0$

$k_{32} = -6x^2 + 6x$

$k_{33} = 6x(-6x^2 + 6x) - 12x - 3x^2(-6x^2 + 6x) + 12x^2$

$k_{34} = 6x - 4x(-6x^2 + 6x) + 3x^2(-6x^2 + 6x) - 6x^2$

$k_{41} = 0$

$k_{42} = 3x^2 - 2x$

$k_{43} = 4x - 6x(-3x^2 + 2x) + 3x^2(-3x^2 + 2x) - 6x^2$

$k_{44} = 4x(-3x^2 + 2x) - 2x - 3x^2(-3x^2 + 2x) + 3x^2$

```
>> INT_nL_2=int(nL_2,0,1)
```

$$INT\_nL\_2 = \int_0^1 \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{41} & k_{43} & k_{44} \end{bmatrix}$$

$$INT\_nL\_2 = \begin{bmatrix} 0 & -1 & -\frac{1}{10} & \frac{1}{10} \\ 0 & 0 & -\frac{3}{10} & \frac{2}{15} \\ 0 & 1 & \frac{1}{10} & -\frac{1}{10} \\ 0 & 0 & \frac{1}{5} & -\frac{1}{30} \end{bmatrix}$$