

Coulomb's Law: stationary charge =

$$\vec{E} = -\frac{q}{4\pi\epsilon_0} \frac{\hat{a}_r}{r^2}$$

so actually



In 1880's; Maxwell came up with modifications:

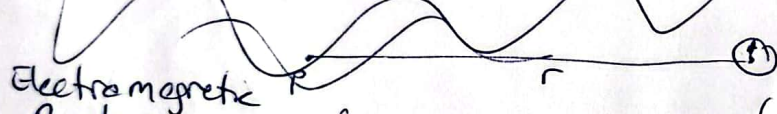
moving charge results in time dependence

$$\vec{E}(t) = \frac{-q}{4\pi\epsilon_0} \left[\frac{\hat{a}_r'}{r'^2} + \frac{r'}{c} \frac{d}{dt} \left(\frac{\hat{a}_r}{r^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \vec{a}_r' \right]$$

You have to look at the position of the charge at some earlier time to calculate the electric field at position 'P'. Since field takes time to reach Point 'P'. Remember it moves with light of speed 'c'. So $\vec{E}$ should depend on the position at an earlier time. This is called retarded position and time.

time takes to reach 'P' = $\frac{r}{c}$ so retarded time: $t - \frac{r}{c}$

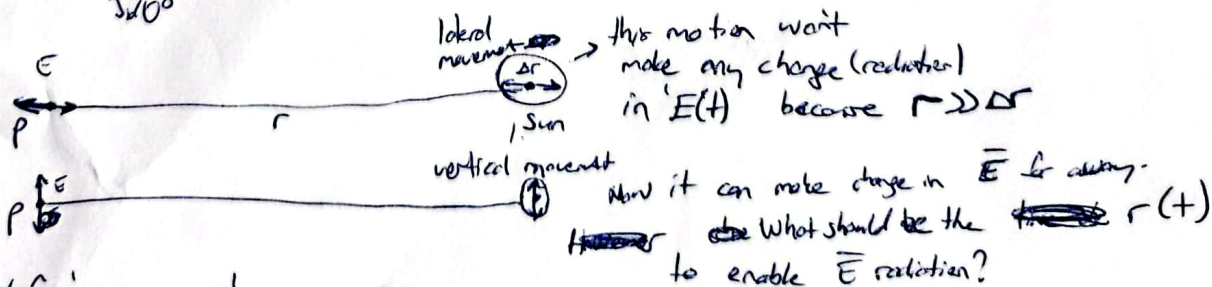
For example: think about a charge on sun moving around its surface



Electromagnetic Radiation: refers to the waves of the electromagnetic field, propagating (radiating) through space carrying electromagnetic radiant energy

Now check $E(t)$; which term can generate radiation?

Assume Sun: it is 1.5×10^{11} m far from us and speed of light is 3×10^8 m/s
means that time = $\frac{1.5 \times 10^{11}}{3 \times 10^8} = 500$ sec. $\Rightarrow$ retarded time: $t - 500$ s



$$\left(\frac{\hat{a}_r'}{r'^2} + \frac{r'}{c} \frac{d}{dt} \left(\frac{\hat{a}_r}{r^2} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \vec{a}_r' \right)$$

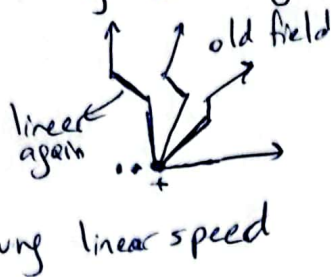
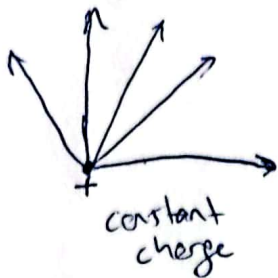
$\frac{1}{r^2} \rightarrow \frac{1}{r^2}$
same $\rightarrow$ stationary!

can have radiation
 $\rightarrow$ turns out that it can radiate at only $1/r$ dependence.

$\rightarrow$ need acceleration!

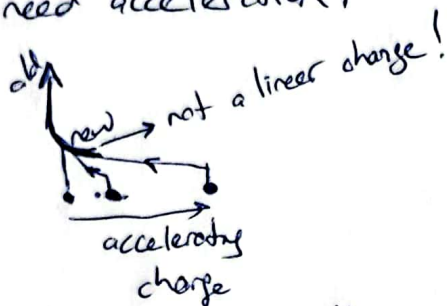
①

Water gate idea understanding Electromagnetic Radiation



⇒ similar to each other
no change with linear movement
still linear field profile!

we need acceleration;



now we understand that we need 2nd derivative wrt 't' should be non zero
and we know that e^{wt} has infinite non-zero derivative ability and $\cos wt$, or $\sin wt$

$\omega \rightarrow$ angular Hz radian/s.
radial velocity

draw $\cos(\omega t)$ versus ωt and $\cos(\omega t)$ versus 't'

assume $\omega = \frac{\pi}{4}$ radian/s

at $t=0$
 $t=1$ sec

$t=2$ s.

$t=3$ s

$t=4$ s

ωt	$\cos(\omega t)$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$
π	-1

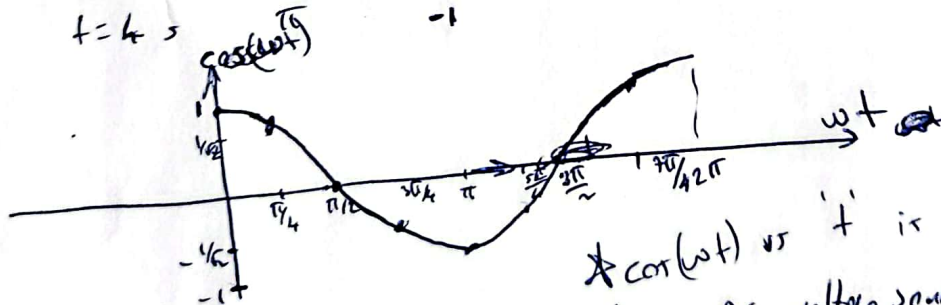
$t=5$ sec

$t=6$

$t=7$

$t=8$

ωt	$\cos \omega t$
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$
$\frac{3\pi}{2}$	0
$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}$
2π	1



for $\omega = \frac{\pi}{4}$

* $\cos(\omega t)$ vs 't' is similar just put 't' in x-axis.

Now if we have 2 wires and a AC voltage source in the following schematic.



for $V_{AC} = A \cos \omega t$
 $0 < \omega t < \frac{\pi}{2}$

+ charge for wire on top
- charge for wire on bottom



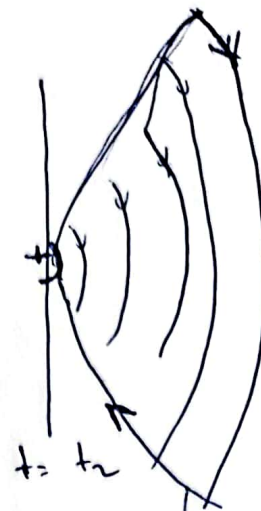
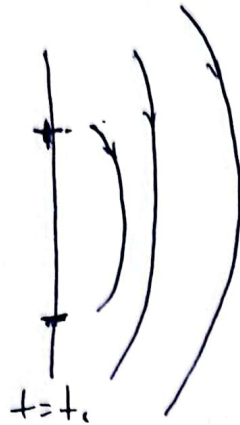
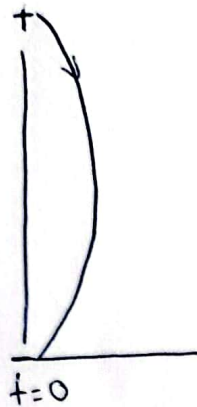
for $V_{AC} = A \sin \omega t$
 $\frac{\pi}{2} < \omega t < \pi$

increasing '-' charge on top wire
'+' charge on bottom wire

this is similar to:

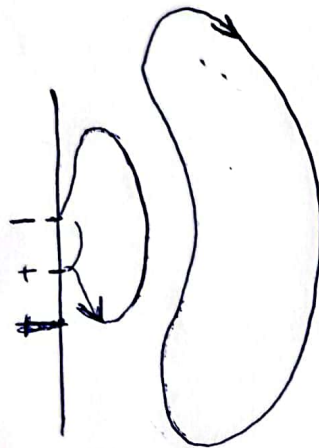


charges oscillating along a wire which produces radiating electric field.



draw the disturbance (radiation)

$t = t_3$



frequency of radiation is same as input voltage (V_{ac})

Watch youtube understandy electromagnetic radiation 0-5:40

watch: Lect 13 Electromagnetic Waves, solutions to Maxwell's 59:00-1:03

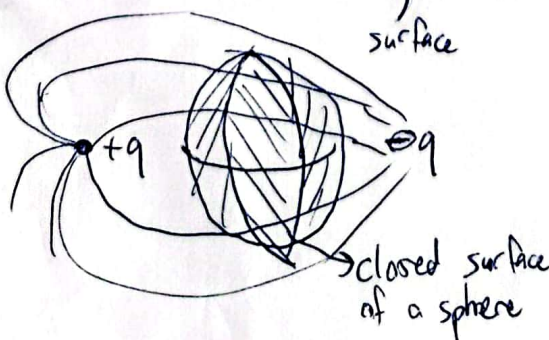
Maxwell's equations → ① Explain how magnetic field is generated

$\nabla \cdot \mathbf{D} = \rho_v$ → divergence of electric flux density ($\mathbf{D}$) is equal to volume charge density (ρ_v)

remember $\mathbf{D} = \epsilon \mathbf{E}$
↳ dielectric constant

so it means that $\oint \mathbf{D} \cdot d\mathbf{a} = \int \rho_v dV$
surface

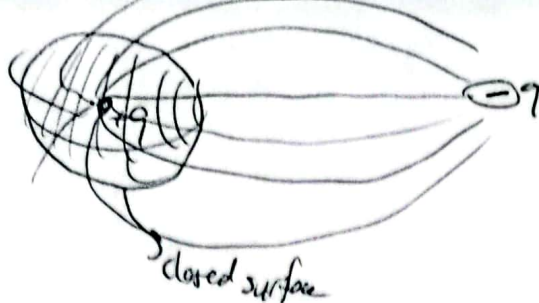
② Then simply draw $\mathbf{E}$, $\mathbf{H}$ field diagram on page 5.



$\vec{D}$ field going in and out is equal to each other since no charge is present inside the sphere.
 $\nabla \cdot \vec{D} = 0$

3

$$\nabla \cdot \mathbf{B} = 0$$



$$\oint \mathbf{D} \cdot d\mathbf{a} = \oint \mathbf{E} \cdot d\mathbf{a} = +q$$

$$\nabla \cdot \mathbf{B} = 0$$

Divergence of magnetic flux density ($\mathbf{B}$) is zero since there is no net magnetic charge. They always have N-S polarity independent of size. So net is always zero.

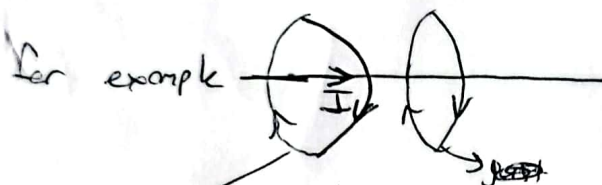
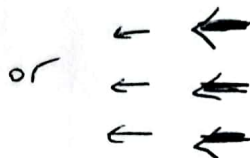
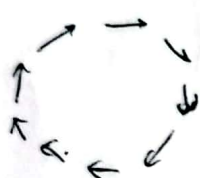
so magnetic field lines can't start or stop

$$\mathbf{B} = \mu \mathbf{H}$$

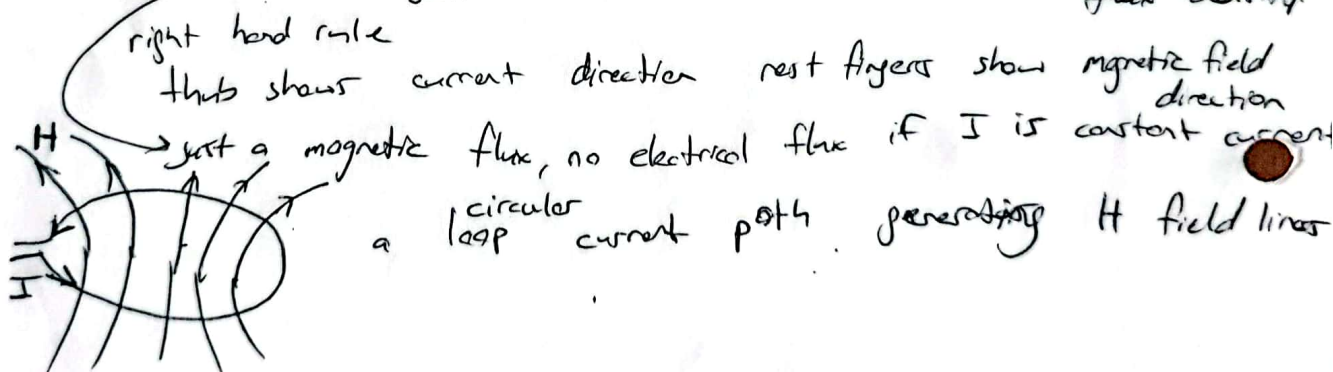
permeability

magnetic field lines can loop;

requires net + magnetic charge which doesn't exist.



a current carrying wire has rotating magnetic field and magnetic flux density.



$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

The curl of a Electric field is equal to - time derivative of magnetic flux density.

Curl: infinity small circulation of a vector field

assume

$$\mathbf{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$$

$$\nabla \times \mathbf{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) \hat{x} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) \hat{y} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \hat{z}$$

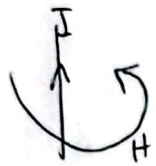
equates to $-\frac{\partial \mathbf{B}}{\partial t}$

$\mathbf{E}$ and $\mathbf{B}$ (thus $\mathbf{H}$) are coupled (broadly, indirectly)

(4)

$\nabla \times H = \frac{\partial D}{\partial t} + J$ infinitely small circulation of H is equal to current density if $\frac{\partial D}{\partial t} = 0$

density if $\frac{\partial D}{\partial t} = 0$

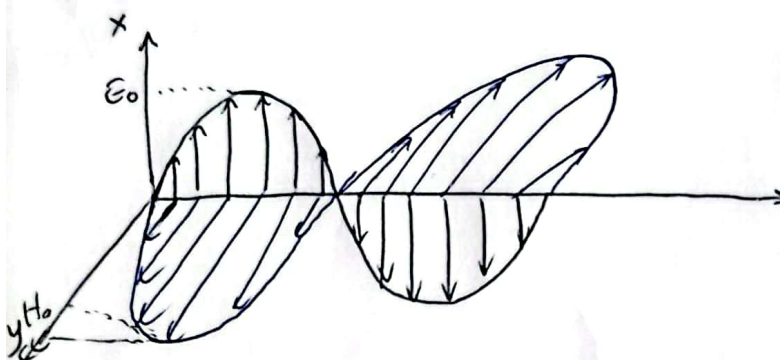


current and H are coupled.

if $\frac{\partial D}{\partial t}$ is not zero then

' H ' and ' D ' are coupled.

Overall it turns out that if $E(t)$ has non zero second time derivative, it is coupled with $H(t)$ meaning that they propagate together.



$$E(z,t) = E_0 \sin\left(2\pi\left(f + \frac{z}{\lambda}\right)\right)$$

$$H(z,t) = H_0 \sin\left(2\pi\left(f + \frac{z}{\lambda}\right)\right)$$

$$D = \epsilon_0 E$$

↓
permittivity

$$B = \mu_0 H$$

↓
permeability

Assume we have linearly polarized E and H field as depicted.

E field is a function of $(z \text{ and } t)$ and is along $\hat{x}$

$$\nabla \times E = - \frac{\partial E_x}{\partial z} \hat{y} = - \frac{2\pi}{\lambda} E_0 \cos\left(2\pi\left(f + \frac{z}{\lambda}\right)\right)$$

$$- \frac{\partial B}{\partial t} = - \frac{\partial}{\partial t} (H_0 \sin(2\pi(f + \frac{z}{\lambda}))) = -2\pi f B_0 \cos\left(2\pi\left(f + \frac{z}{\lambda}\right)\right)$$

$$\Rightarrow \frac{2\pi E_0}{\lambda} = 2\pi f B_0 \Rightarrow \frac{E_0}{B_0} = \lambda f = \text{speed} = c$$

$$\frac{E_0}{B_0} = \sqrt{\frac{1}{\mu_0 \epsilon_0}} \Rightarrow E_0 \cdot \sqrt{\epsilon_0} = \sqrt{\mu_0} H_0$$

[Movement of charges in a material induces thermal radiation which is also a kind of electromagnetic radiation.

Power of EMW = $S = E \times H = \frac{1}{2} E_0 \cdot H_0 = \frac{1}{2} E_0^2 \sqrt{\frac{\epsilon_0}{\mu_0}}$

density

→ comes due to nature of alternating signal
if it was constant then this would be '1'.

→ Photons; are elementary charges with zero rest mass and moving with 'c'. Energy of photon is linked to its frequency ν ;
 $E = h\nu$ where h is ~~planck~~ planck's constant ($6.62 \times 10^{-34} \text{ Js}$)

(5)

$$E = h \frac{c}{\lambda} = h \nu \quad c = \lambda \nu$$

converting wavelength to energy of a photon;

$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda(\text{nm})} \quad : \quad 6.62 \times 10^{-34}$$

Ex/ what is the energy of a blue (420 nm) photon?

$$E = \frac{1241}{420} \approx 3 \text{ eV}$$

(6)