Sekizinci baskıya yaklaşan ***Akışkanlar Mekaniği*** alanında, çok geniş bir şekilde adapte edilen ders kitabıdır. Yeni sekizinci baskı, problem çözme tekniğindeki kanıtlar, okuyucuların doğru sonucu bulmada ve sonuçların fiziksel anlamının ilişkisi açıcından, sistematik bir planın gelişmesine yardımcı olacaktır. Sekizinci baskı, bunun yanında enerji ve çevre ile birlikte akışkanlar mekaniğinin prensiplerini içeren alanları yeni videolarla görselleştirerek akışkanlar mekaniğinde ilginç yeni uygulamalar yardımıyla okuyucuyu motive etmektedir.

AKIŞKANLAR MEKANİĞİNİN BAŞLIKLARI, SI VERSİYON, SEKİZİNCİ BASKI

- ***Enerji ve Çevre üzerine vaka çalışması:*** Dünyada artan enerji ihtiyaçları doğrultusunda, akışkanlar mekaniği uygulamalarından yenilebilir kaynaklar üretmek.
- ***Tanıtım Videoları:*** Klasik NCFMF ve çeşitli kaynaklardan gelen yeni videolar, birçok kavramı görsel destekle kavramayı kolaylaştıran kaynak olarak kitabın web sayfasında öğrenci ve öğretim elemanı için kullanılabilir durumdadır.
- ***Bölüm 5:*** Hesaplamalı akışkanlar dinamiği üzerine, basit 1 ve 2 boyutlu akışların nümerik analizi için tablosal yaklaşım üzerine malzemeleri ve tartışmaları içermektedir.
- Kitabın tümündeki ölçüm birimleri tamamen SI sistemindedir.

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Akışkanlar Mekaniği

SEKİZİNCİ BASKIDAN ÇEVİRİ

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YAYINCILIK

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SI Baskısı

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PALME YAYINCILIK

Introduction to Fluid Mechanics

Chapter 1

Introduction

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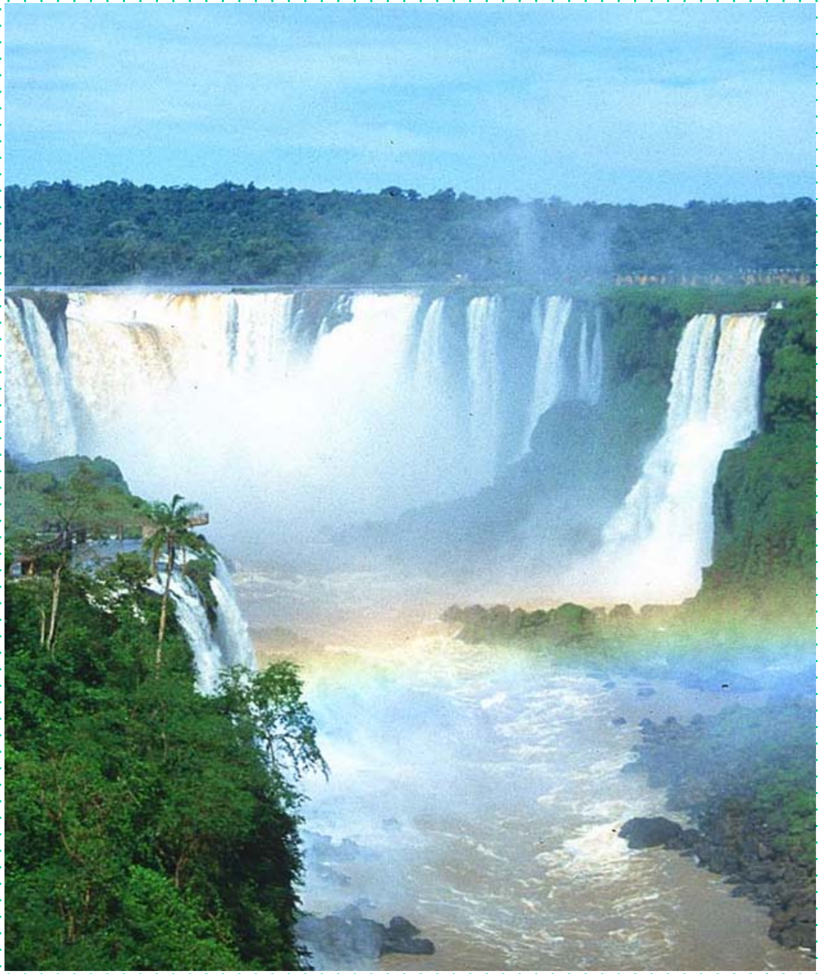
Yildiz Technical University
Mechanical Engineering Department
Besiktas, ISTANBUL

Main Topics

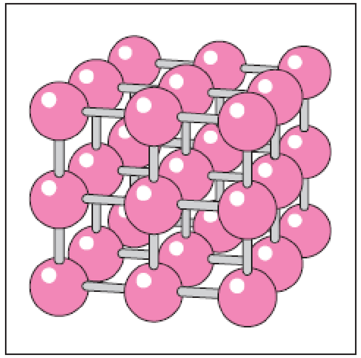
- ✓ Definition of a Fluid
- ✓ Basic Equations
- ✓ Methods of Analysis
- ✓ Dimensions and Units



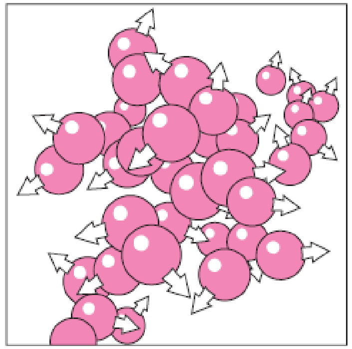
Fluid mechanics deals with liquids and gases in motion or at rest.

Intermolecular bonds are strongest in solids and weakest in gases.

Solid: The molecules in a solid are arranged in a pattern that is repeated throughout. Because of the small distances between molecules in a solid, the attractive forces of molecules on each other are large and keep the molecules at fixed positions.

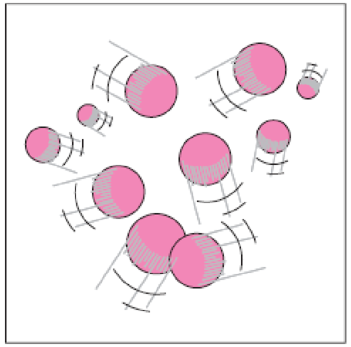


Liquid: In liquids molecules can rotate, move about each other and translate freely in the liquid phase.

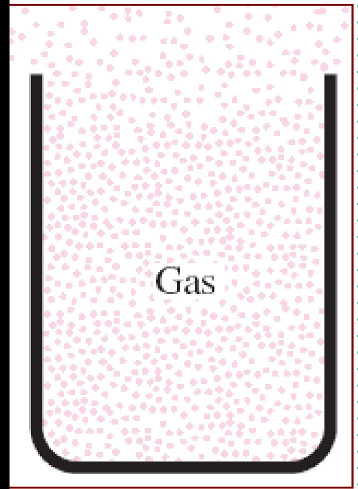


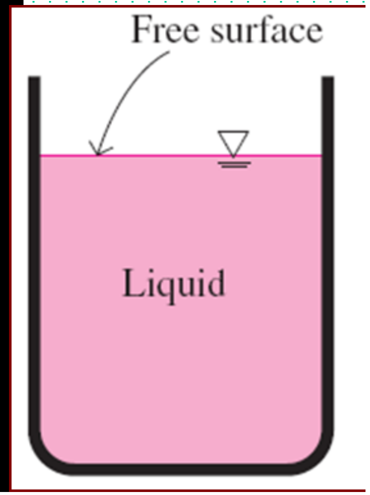
Gas: In the gas phase, the molecules are far apart from each other, and molecular ordering is nonexistent. Gas molecules move about at random, continually colliding with each other and the walls of the container in which they are contained.

Particularly at low densities, the intermolecular forces are very small, and collisions are the only mode of interaction between the molecules.

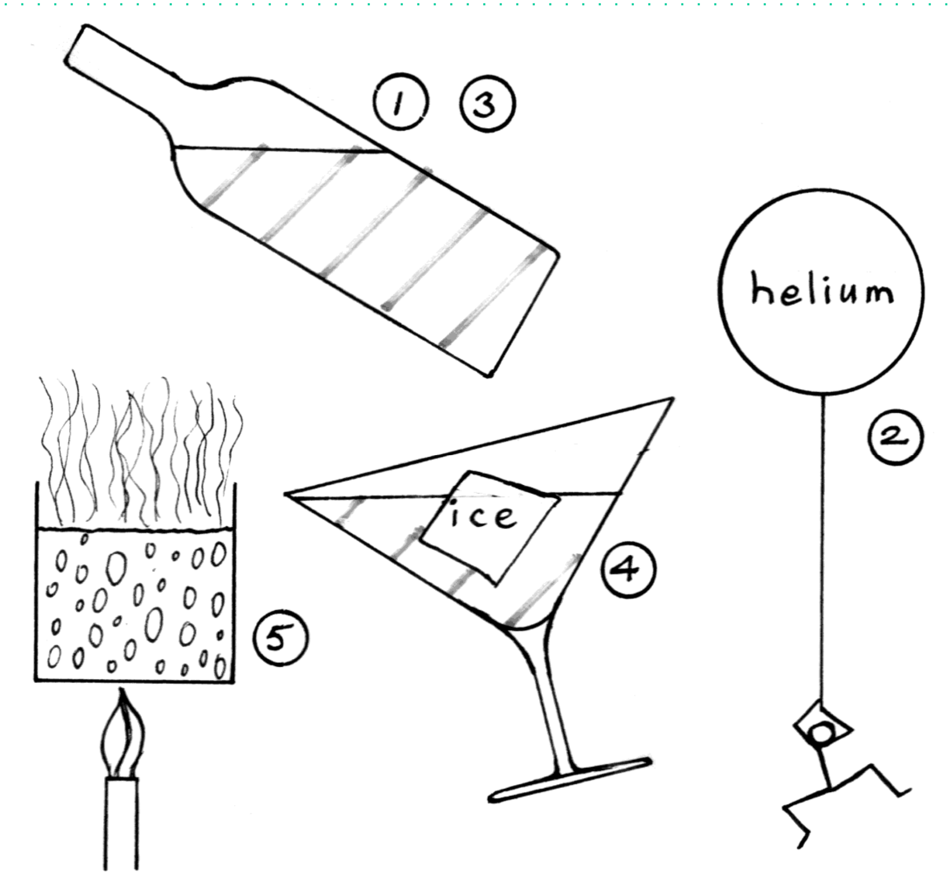


States of Matter





LIQUIDS AND GASES

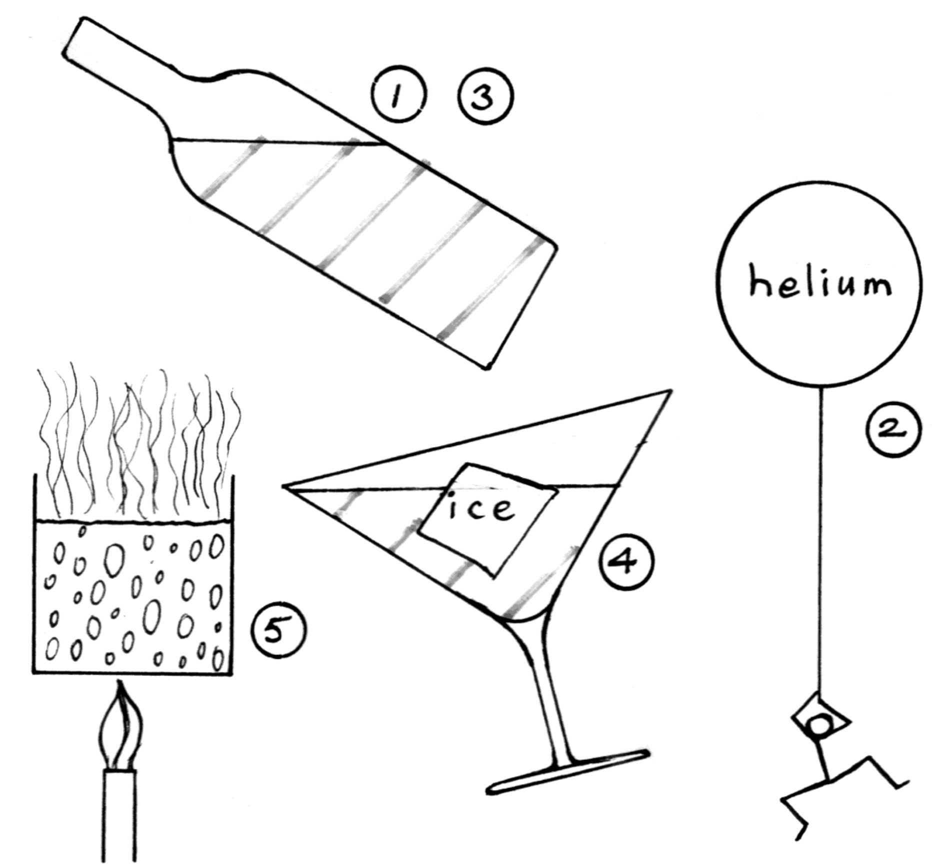


How many of the following statements are correct?

(a) all
(b) four
(c) three
(d) two
(e) one

(1) All fluids are liquids
(2) All gases are fluids
(3) All liquids are fluids
(4) If a solid melts it becomes liquid
(5) If a liquid evaporates it behaves like a gas

LIQUIDS AND GASES



How many of the following statements are correct?

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(d) two
(e) one

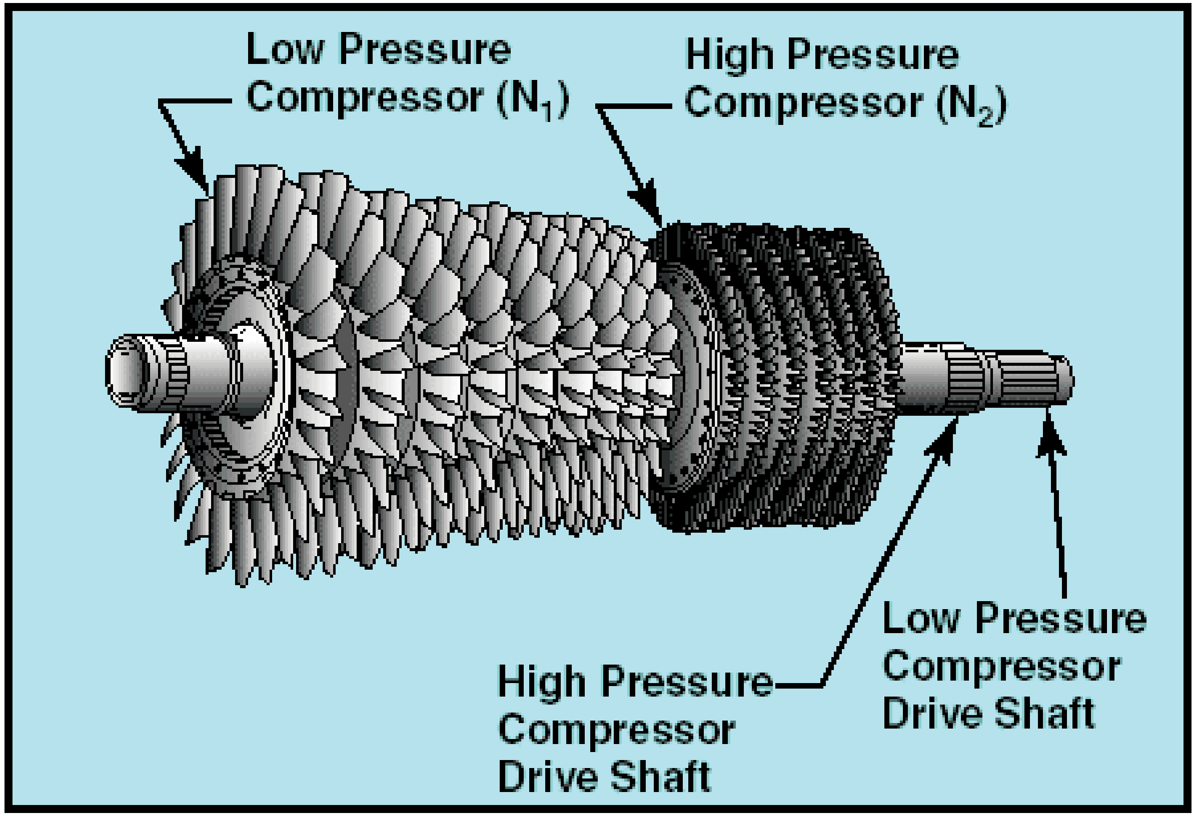
(1) All fluids are liquids
(2) All gases are fluids
(3) All liquids are fluids
(4) If a solid melts it becomes liquid
(5) If a liquid evaporates it behaves like a gas

1.2 Scope of Fluid Mechanics

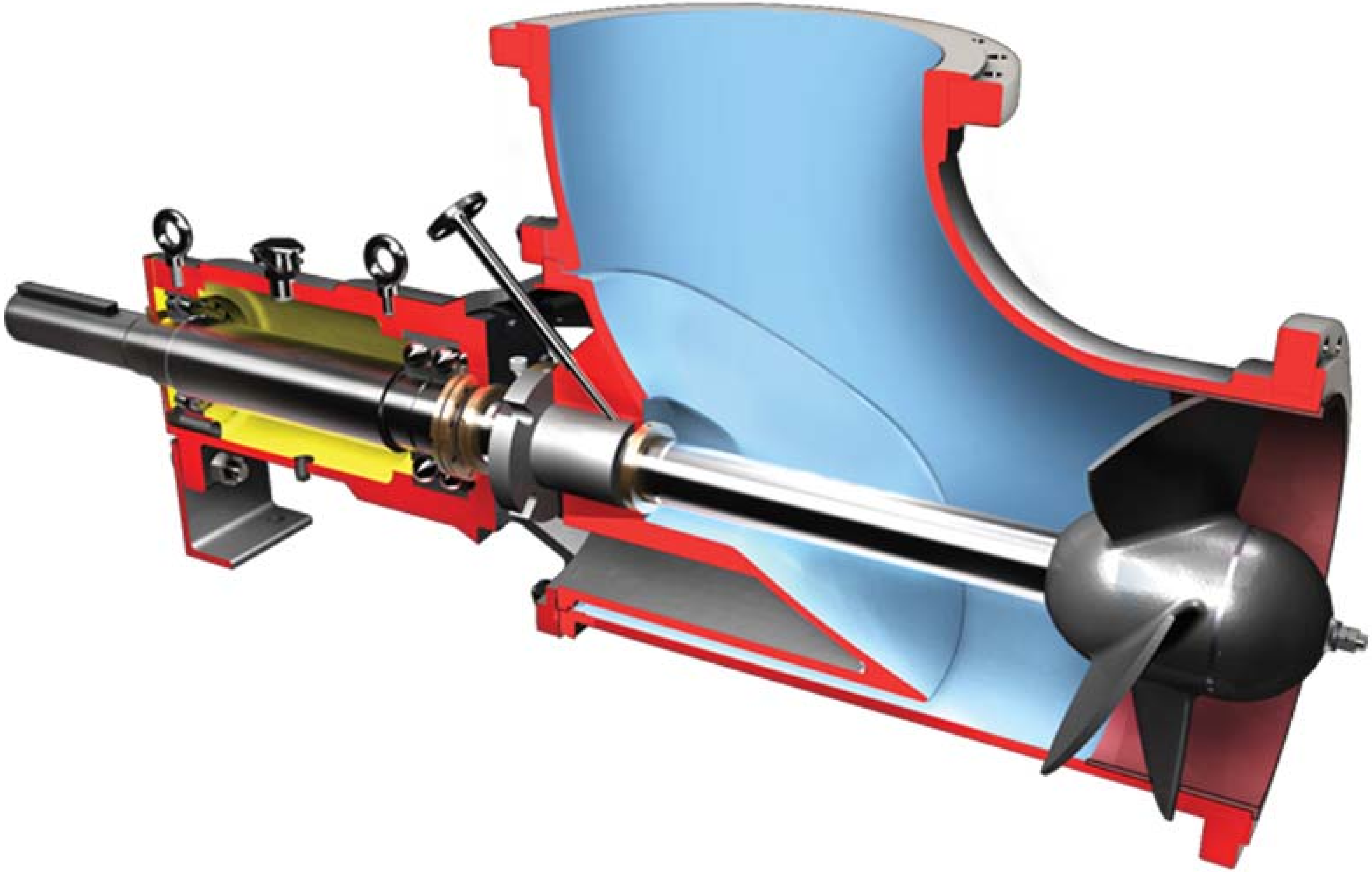
Fluid mechanics is the study of fluids at rest or in motion.
It has traditionally been applied in such areas as:

Application Areas of Fluid Mechanics

Compressors,



Axial Flow Elbow Pump

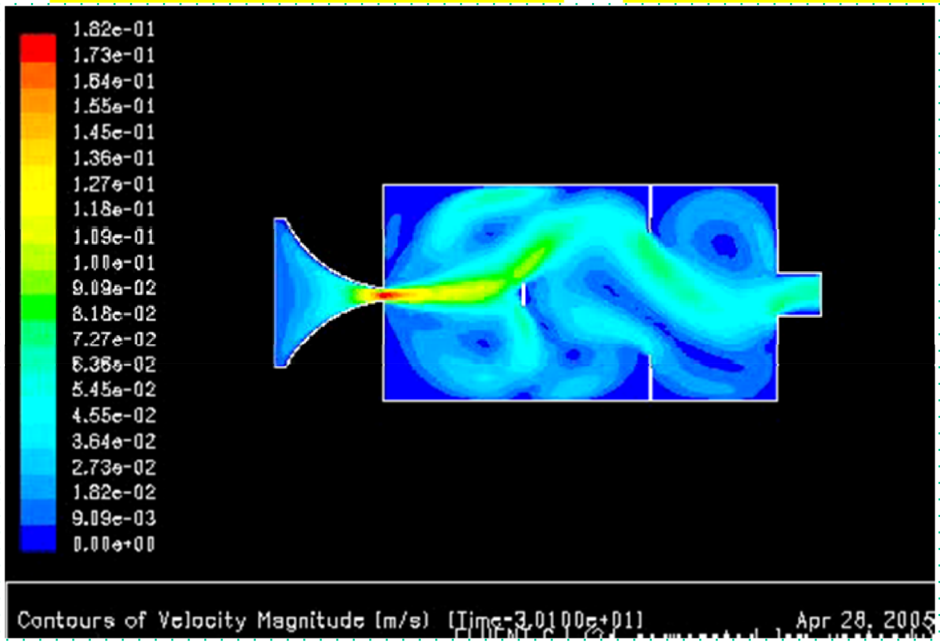


The piping systems needed in chemical plants; pump meters.

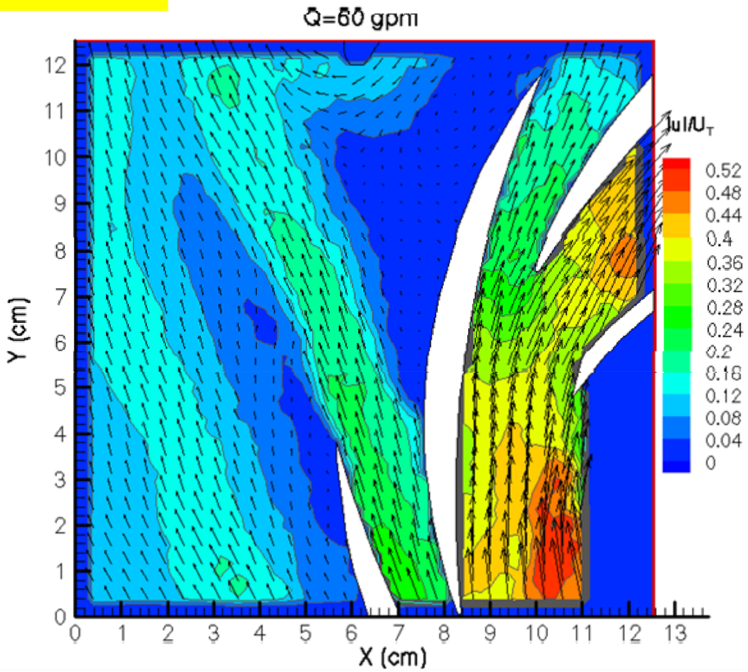
Wind Turbines

Piping and Plumbing Systems

Industrial applications



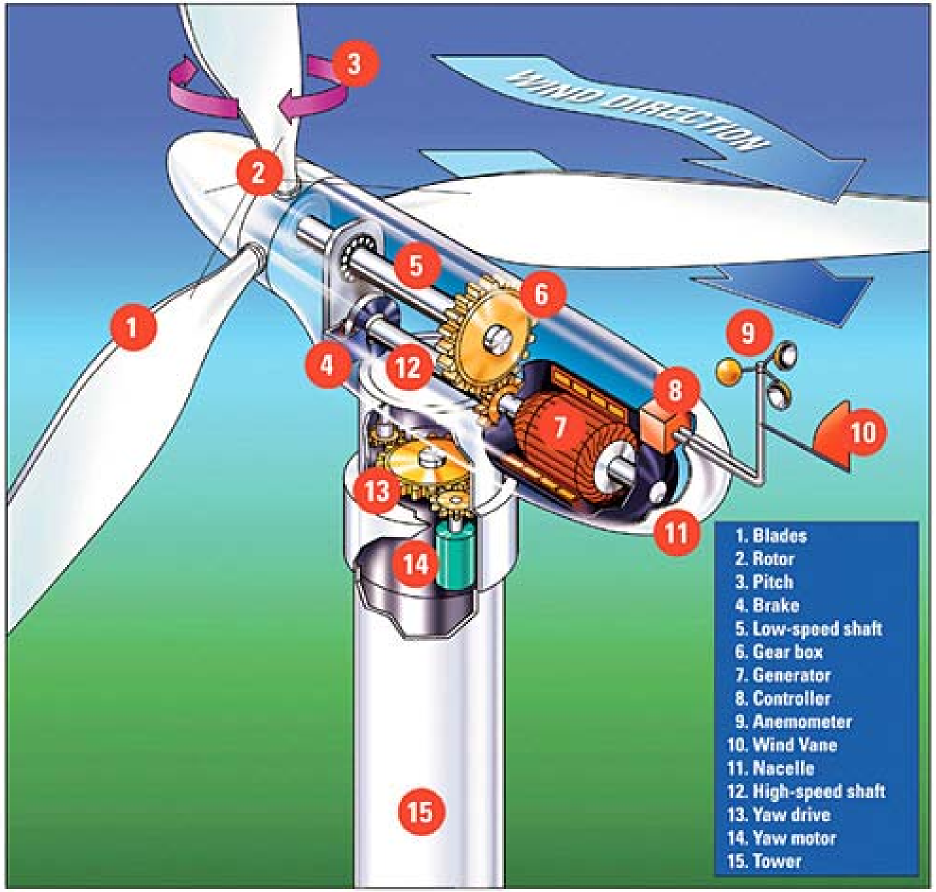
Target Flow meter



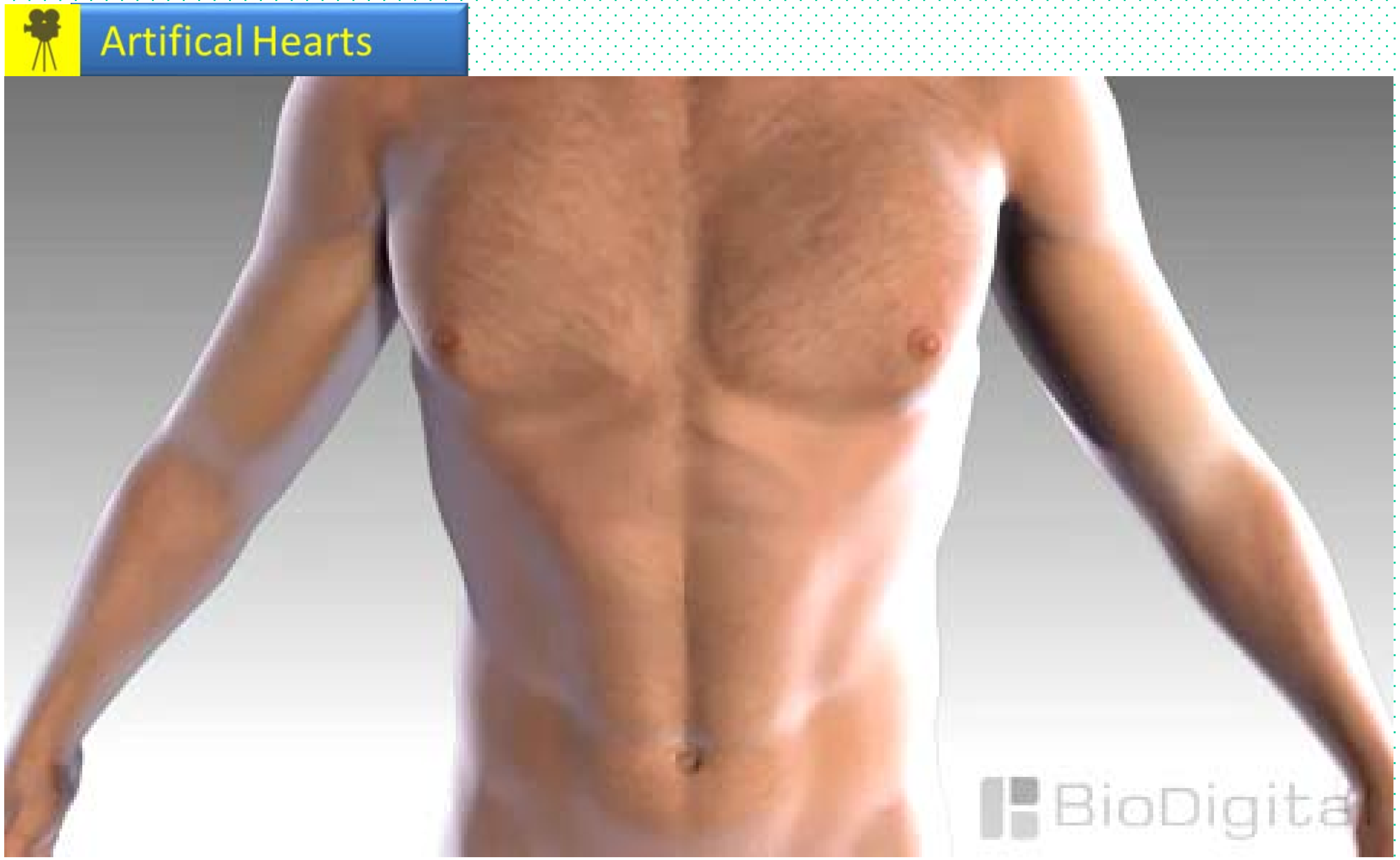
Centrifugal Pump flow in stall condition

Prof. Dr. Ali PINARBAŞI

Some examples include environmental and energy issues (e.g., containing oil slicks, large-scale wind turbines, energy generation from ocean waves, the aerodynamics of large buildings, and the fluid mechanics of the atmosphere and ocean and of phenomena such as tornadoes, hurricanes, and tsunamis);



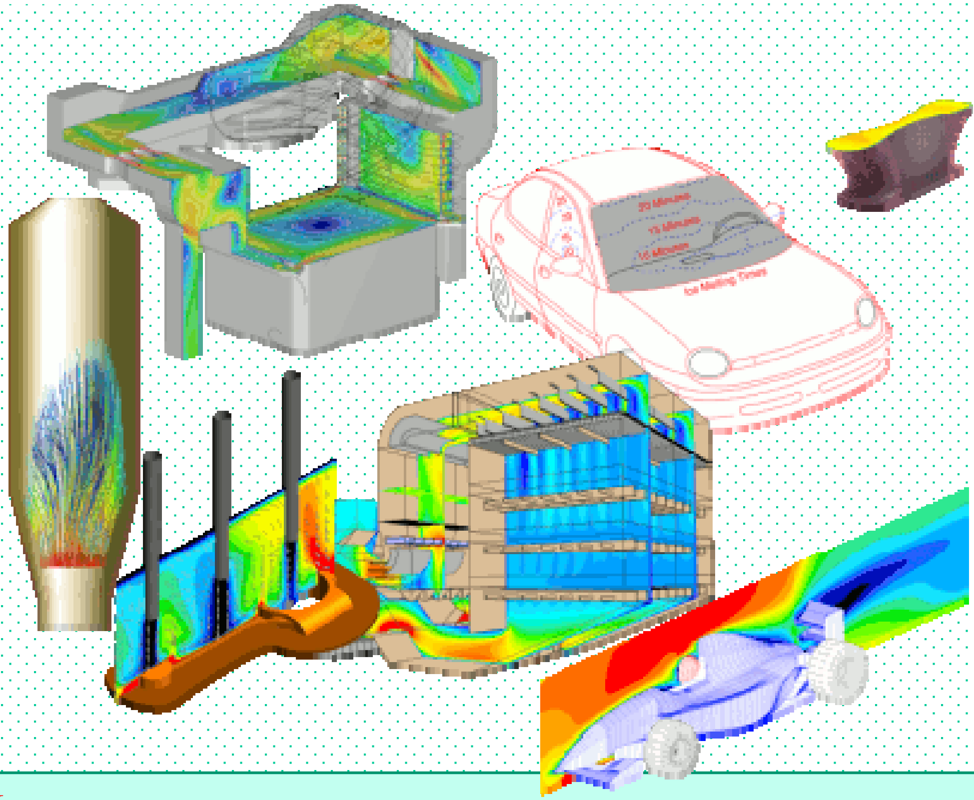
Biomechanics (e.g., artificial hearts and valves and other organs such as the liver; understanding of the fluid mechanics of blood, synovial fluid in the joints, the respiratory system, the circulatory system, and the urinary system);



Sport (design of bicycles and bicycle helmets, skis, and sprinting and swimming clothing, and the aerodynamics of the golf, tennis, and soccer ball);

Smart fluids” (e.g., in automobile suspension systems to optimize motion under all terrain conditions, military uniforms containing a fluid layer that is “thin” until combat, when it can be “stiffened” to give the soldier strength and protection, and fluid lenses with humanlike properties for use in cameras and cell phones);

CFD flow modeling
(Computational Fluid Dynamics)



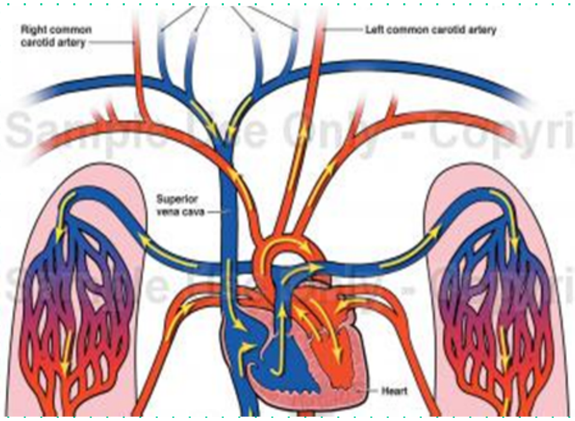
Microfluids (for extremely precise administration of medications)

Natural Flows and Weather

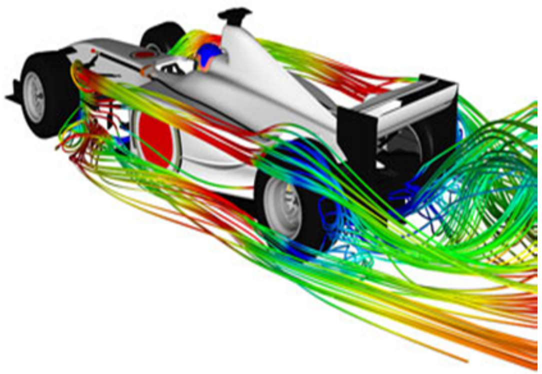
Boats

Aircraft and spacecraft

Power Plant



Human Body



Cars

Definition of a Fluid

When a shear stress is applied:

- ✓ **Fluids continuously deform**
  - under the application of a shear (tangential) stress
- ✓ **Solids deform or bend**
  - (e.g., when you type on a key- board, the springs under the keys compress)

Hence liquids and gases (or vapors) are the forms, or phases, that fluids can take.
We can see the difference between solid and fluid behavior.
If we place a specimen of either substance between two plates and then apply a shearing force F, each will initially deform; however, whereas a solid will then be at rest (assuming the force is not large enough to go beyond its elastic limit), a fluid will continue to deform as long as the force is applied.

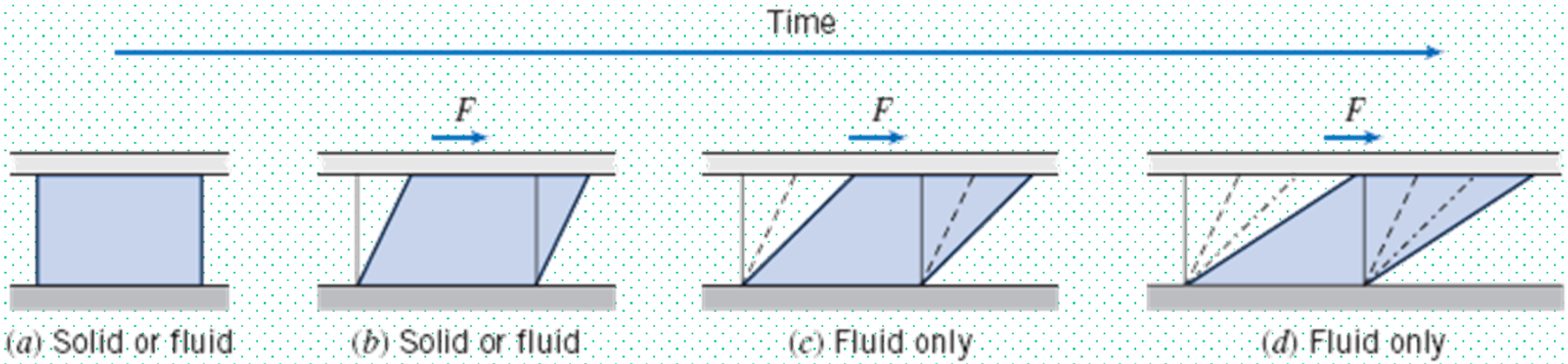
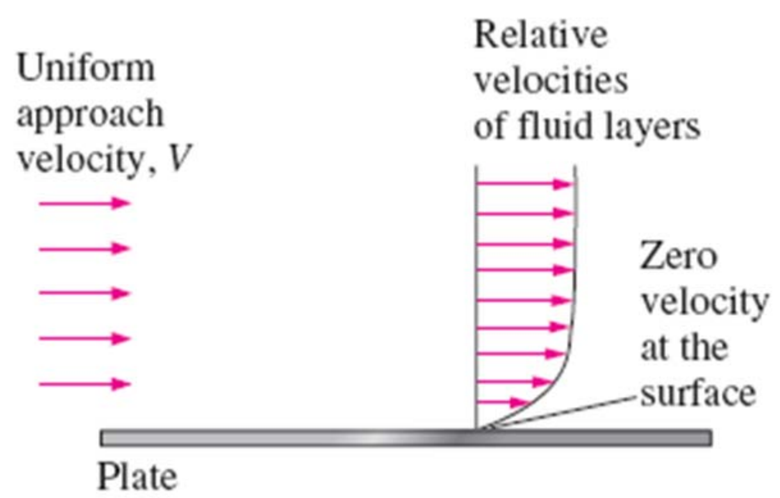


Fig. 1.1 Difference in behavior of a solid and a fluid due to a shear force.

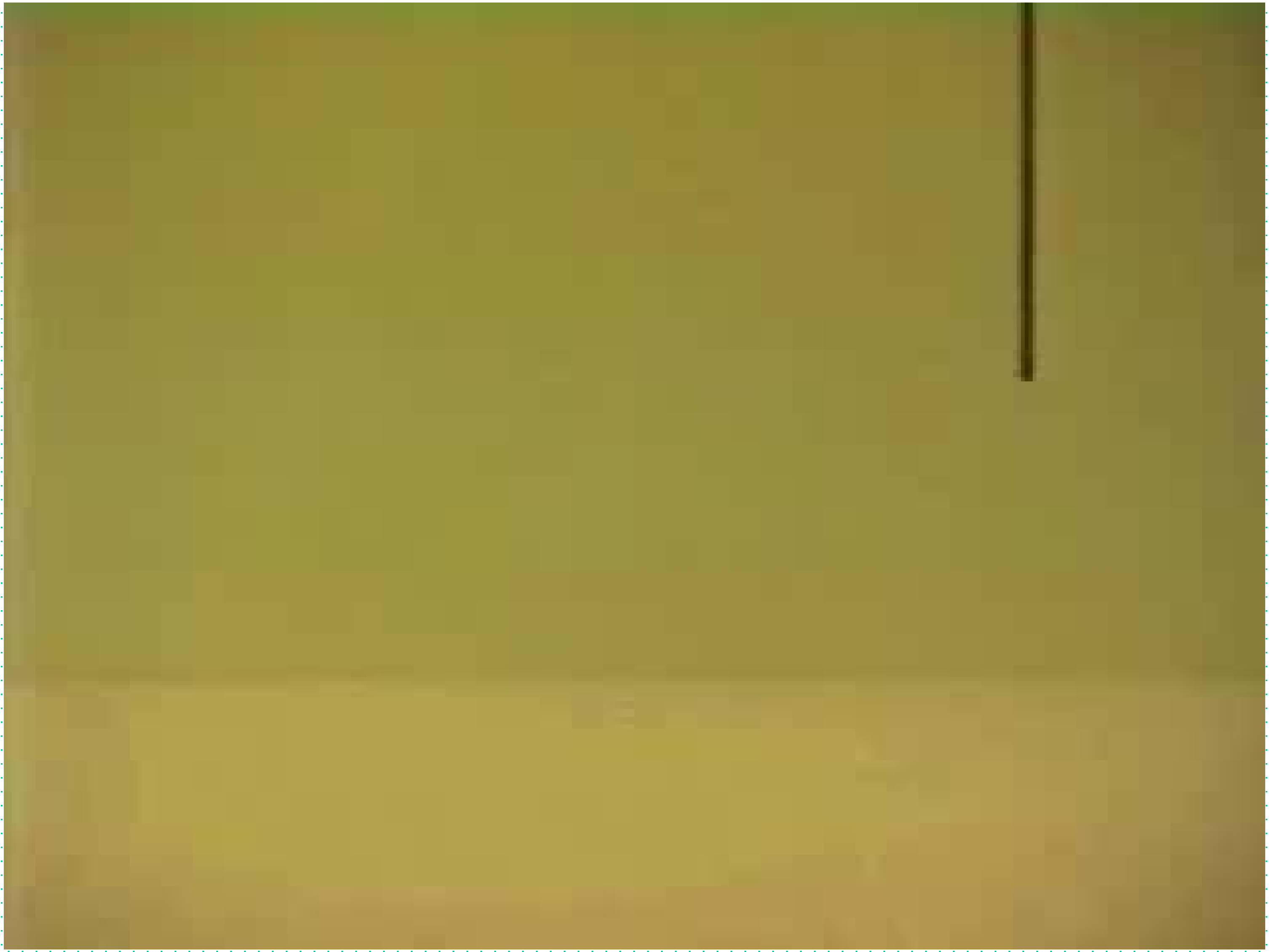
Note that a fluid in contact with a solid surface does not slip—it has the same velocity as that surface because of **the no- slip condition**, an experimental fact.



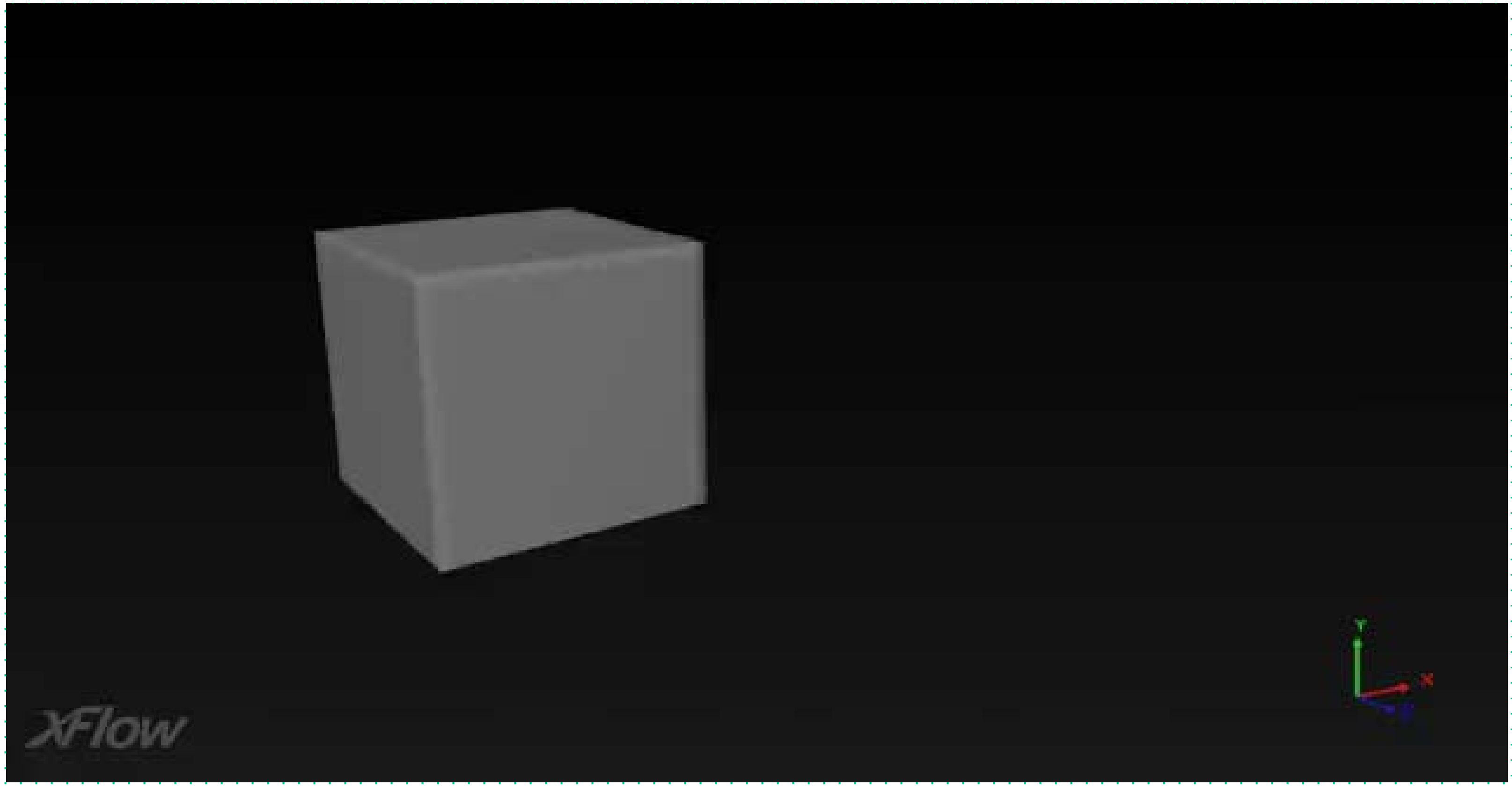
The amount of deformation of the solid depends on **the solid's modulus of rigidity G;**

But, the rate of deformation of the fluid depends on **the fluid's viscosity μ**.
We refer to solids as being elastic and fluids as being viscous.

No-Slip Condition

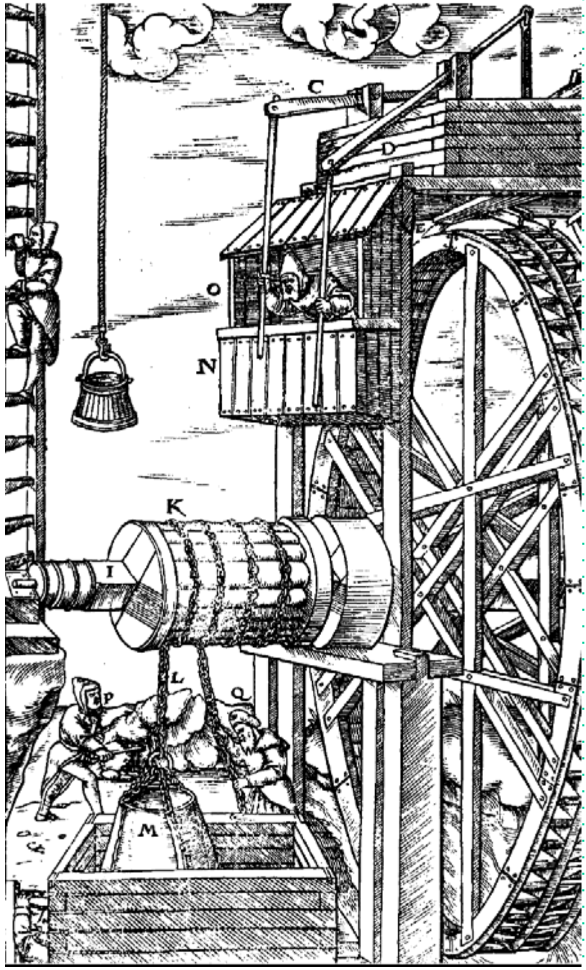


Flow Separation



1–3 A BRIEF HISTORY OF FLUID MECHANICS

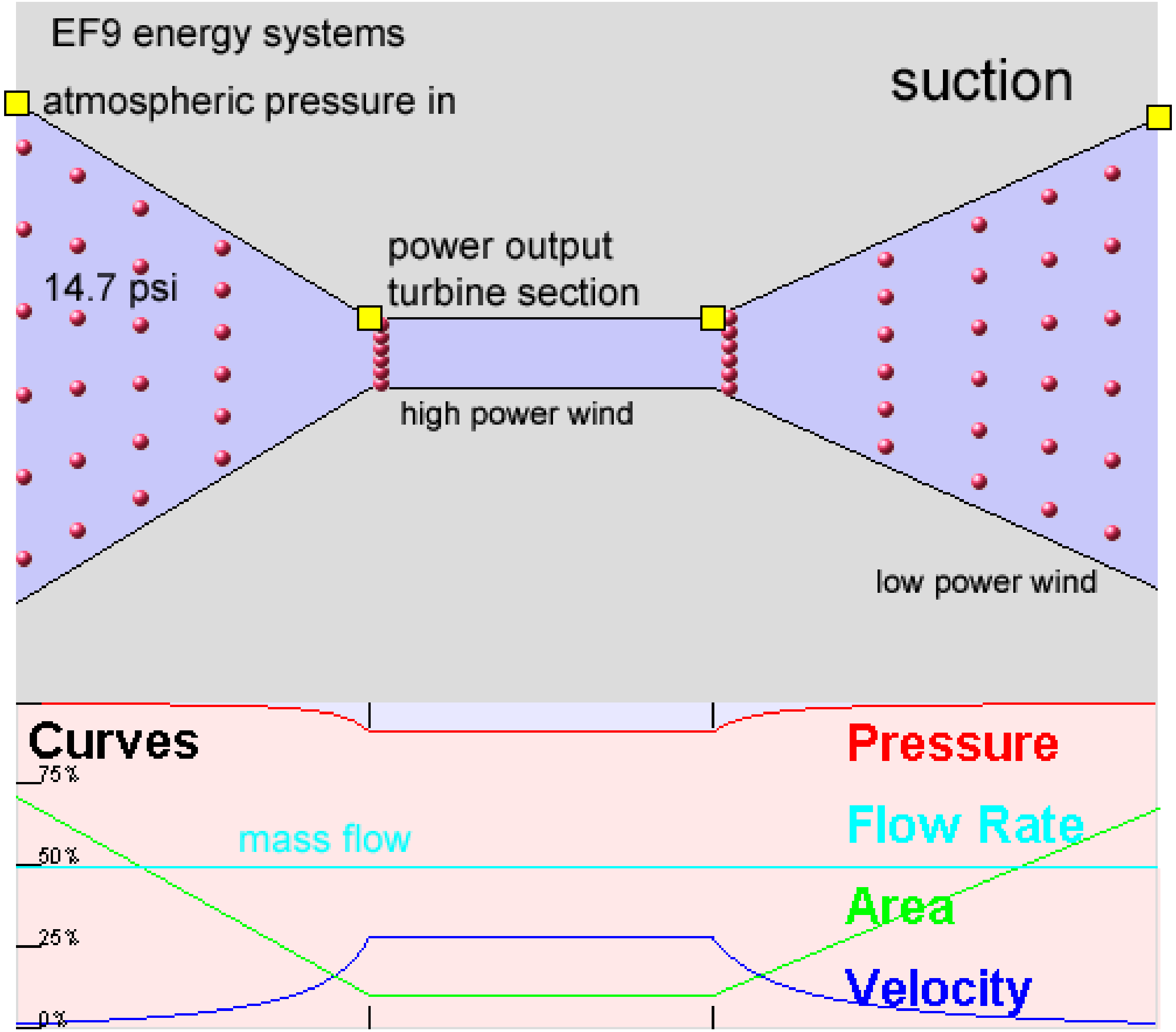
Segment of Pergamon pipeline. Each clay pipe section was 13 to 18 cm in diameter.



A mine hoist powered by a reversible water wheel.

450 × 286 - A **wind turbine** in northern Germany. It was the **world's largest turbine**.

Energy Line



EF9 energy systems
atmospheric pressure in
suction
14.7 psi
power output
turbine section
high power wind
low power wind
Curves
75 %
50 %
25 %
0 %
mass flow
Pressure
Flow Rate
Area
Velocity

Basic Equations

We need forms of the following

- ✓ Conservation of mass
- ✓ Newton's second law of motion
- ✓ The principle of angular momentum
- ✓ The first law of thermodynamics
- ✓ The second law of thermodynamics

Methods of Analysis

A **system** is defined as a fixed, identifiable quantity of mass; the system boundaries separate the system from the surroundings.

The boundaries of the system may be **fixed** or **movable**; however, no mass crosses the system boundaries.

✓ System (or "Closed System")

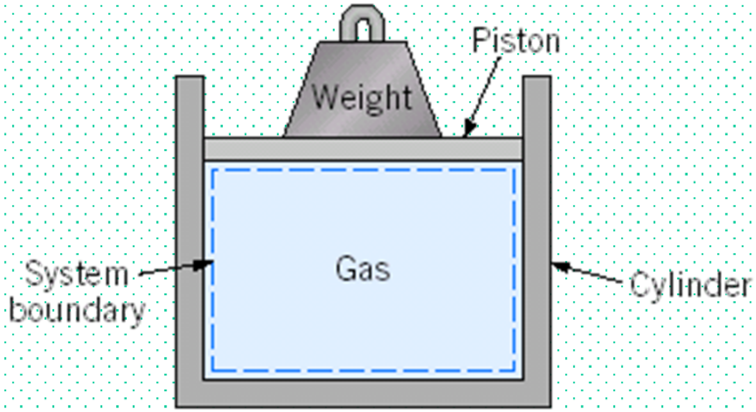


Fig. 1.2 Piston-cylinder assembly.

In the familiar piston-cylinder assembly from thermodynamics, the gas in the cylinder is the system.

If the gas is heated, the piston will lift the weight.

The system boundary Piston Cylinder boundary of the system thus moves. Heat and work may cross the boundaries of the system, but the quantity of matter within the system boundaries remains fixed. No mass crosses the system boundaries.

A control volume is an arbitrary volume in space through which fluid flows. The geometric boundary of the control volume is called **the control surface**.

The control surface may be real or imaginary; it may be at rest or in motion.

- ✓ **Control Volume (or "Open System")**

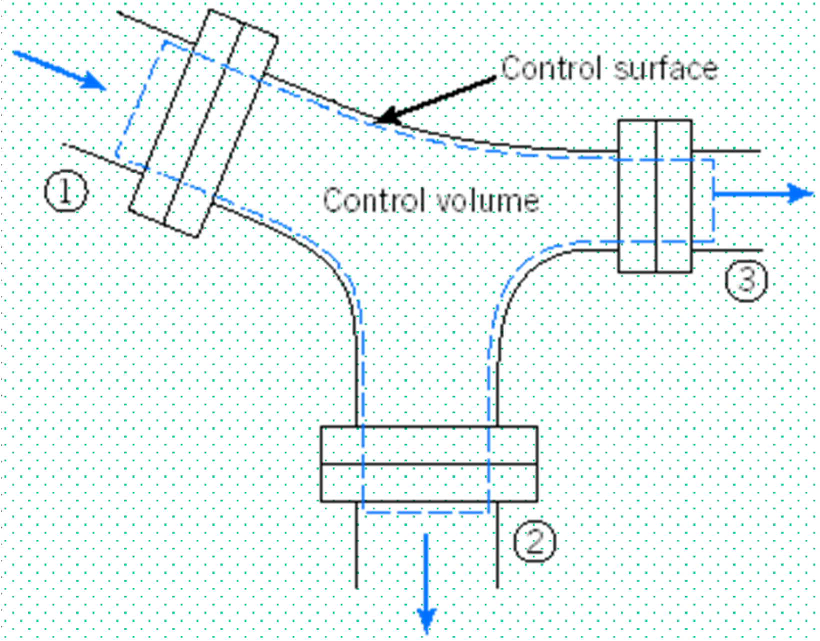
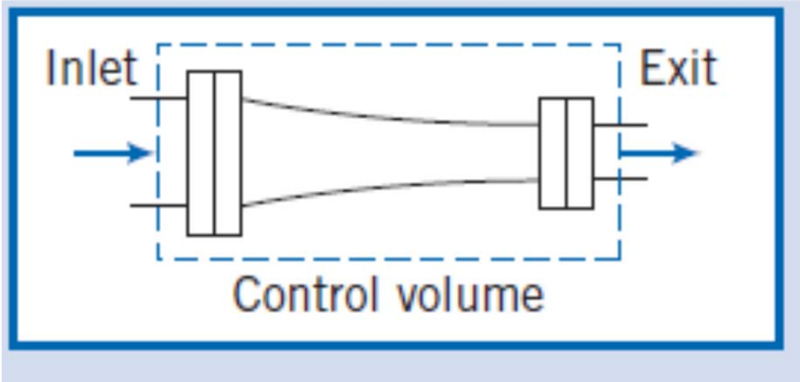


Fig. 1.3 Fluid flow through a pipe junction.

Example 1.2 Mass conservation applied to control volume

A reducing water pipe section has an inlet diameter of 50 mm and exit diameter of 30 mm. If the steady inlet speed (averaged across the inlet area) is 2.5 m/s, find the exit speed.



Given: Pipe, inlet $D_i = 50$ mm, exit $D_e = 30$ mm.
Inlet speed, $V_i = 2.5$ m/s.

Find: Exit speed, V_e.

Water is incompressible (density ρ=constant).

The mass flow is:

$$\dot{m} = \rho V A$$

$$\rho V_i A_i = \rho V_e A_e$$

Applying mass conservation,

$$V_e = V_i \frac{A_i}{A_e} = V_i \frac{\pi D_i^2/4}{\pi D_e^2/4} = V_i \left(\frac{D_i}{D_e}\right)^2$$

$$V_e = 2.7 \frac{\mathrm{m}}{\mathrm{s}} \left(\frac{50}{30}\right)^2 = 7.5 \frac{\mathrm{m}}{\mathrm{s}}$$

Dimensions and Units

Systems of Dimensions

- ✓ [M], [L], [t], and [T]
- ✓ [F], [L], [t], and [T]
- ✓ [F],[M], [L], [t], and [T]

Dimensions and Units

Systems of Units

- ✓ **MLtT**
  - SI (kg, m, s, K)
- ✓ **FLtT**
  - British Gravitational (lbf, ft, s, ^{o}R)
- ✓ **FMLtT**
  - English Engineering (lbf, lbm, ft, s, ^{o}R)

Dimensions and Units

Systems of Units

TABLE 1.1 Common Unit Systems

System of Dimensions	Unit System	Force F	Mass M	Length L	Time t	Temperature T
a. MLtT	Système International d'Unités (SI)	(N)	kg	m	s	K
b. FLtT	British Gravitational (BG)	lbf	(slug)	ft	s	°R
c. FMLtT	English Engineering (EE)	lbf	lbm	ft	s	°R

Dimensions and Units

Preferred Systems of Units

✓ **SI (kg, m, s, K)**

$$1\ \text{N} \equiv 1\ \text{kg} \cdot \text{m/s}^2$$

✓ **British Gravitational (lbf, ft, s, °R)**

$$1\ \text{slug} \equiv 1\ \text{lbf} \cdot \text{s}^2/\text{ft}$$

$$1\ \text{slug} \equiv 32.2\ \text{lbm}$$

Introduction to Fluid Mechanics

Chapter 2

Fundamental Concepts

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Yildiz Technical University
Mechanical Engineering Department
Besiktas, ISTANBUL

A drop forms when liquid is forced out of a small tube. The shape of the drop is determined by a balance of pressure, gravity, and surface tension forces.

Main Topics

- ✓ Fluid as a Continuum
- ✓ Velocity Field
- ✓ Stress Field
- ✓ Viscosity
- ✓ Surface Tension
- ✓ Description and Classification of Fluid Motions

Fluid as a Continuum

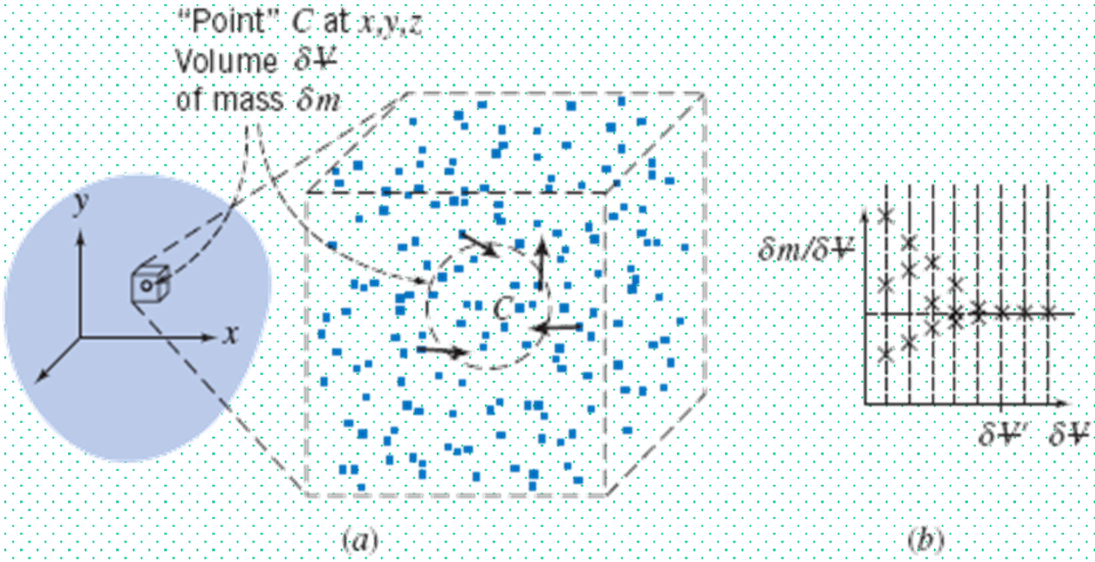


Fig. 2.1 Definition of density at a point.

$$\rho \equiv \lim_{\delta\forall \to \delta\forall'} \frac{\delta m}{\delta\forall}$$

$$\rho = \rho(x, y, z, t)$$

The most common being air and water as "smooth," i.e., being a continuous medium. Unless we use specialized equipment, we are **not aware of the underlying molecular nature of fluids**.

Velocity Field

Important property defined by a field is the velocity field

$$\vec{V} = \vec{V}(x,\ y,\ z,\ t)$$

The velocity vector, $\vec{V}$, can be written in terms of its three scalar components.

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

Velocity Field

Consider also

- ✓ Steady and Unsteady Flows
- ✓ 1D, 2D, and 3D Flows
- ✓ Timelines, Pathlines, and Streaklines

✓Steady and Unsteady Flows

If properties at every point in a flow field do not change with time, the flow is termed steady. Stated mathematically, the definition of steady flow is

$$\frac{\partial \eta}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = 0 \quad \text{or} \quad \rho = \rho(x, y, z)$$

$$\frac{\partial \vec{V}}{\partial t} = 0 \quad \text{or} \quad \vec{V} = \vec{V}(x, y, z)$$

In steady flow, any property may vary from point to point in the field, but all properties remain constant with time at every point.

✓1D, 2D, and 3D Flows

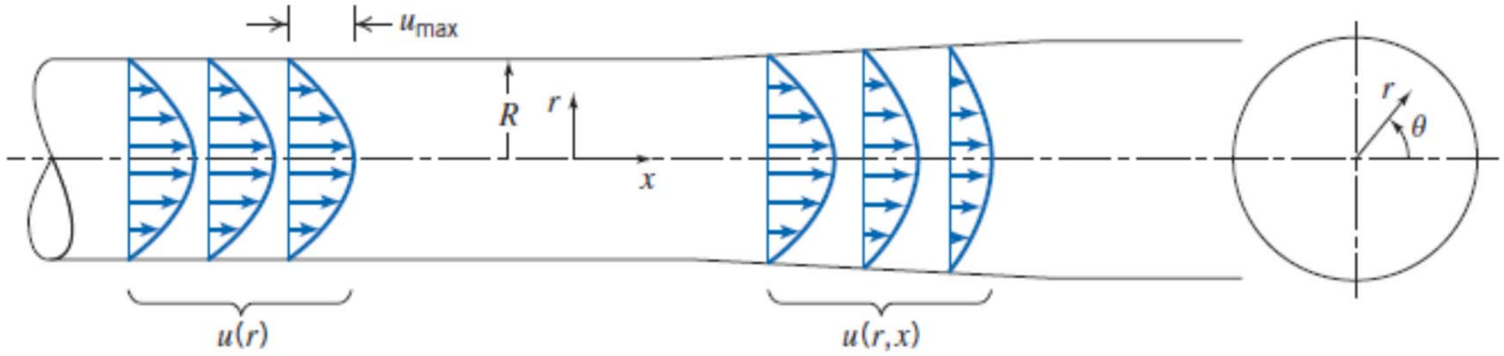


Fig. 2.2 Examples of one- and two-dimensional flows.

A flow is classified as one-, two-, or three-dimensional depending on the number of space coordinates required to specify the velocity field.

The velocity field may be a function of three coordinates and time. Such a flow field is termed **three-dimensional** (it is also **unsteady**).

✓1D, 2D, and 3D Flows

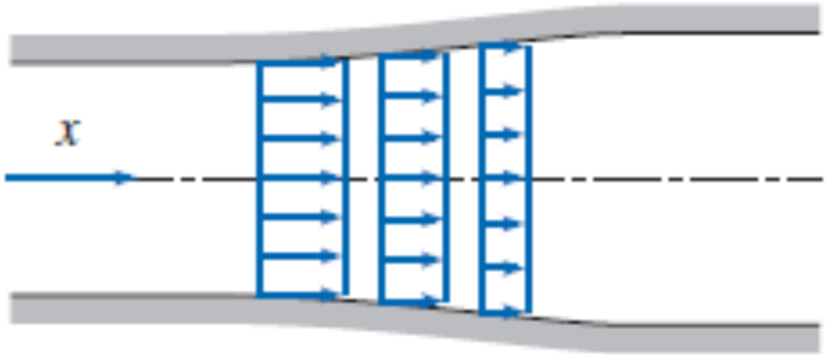


Fig. 2.3 Example of uniform flow at a section.

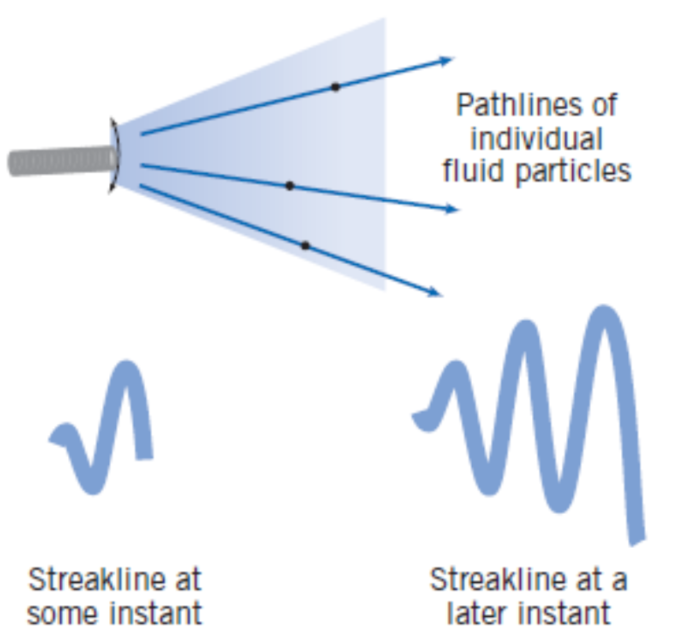
To simplify the analysis it is often convenient to use the notion of **uniform flow** at a given cross section.

In a flow that is uniform at a given cross section, the velocity is constant across any section normal to the flow.

✓Timelines, Pathlines, and Streaklines

If a number of adjacent fluid particles in a flow field are marked at a given instant, they form a line in the fluid at that instant; this line is called a **timeline**.

A pathline is the path or trajectory traced out by a moving fluid particle.



After a short period of time we would have a number of identifiable fluid particles in the flow, all of which had, at some time, passed through one fixed location in space.

The line joining these fluid particles is defined as a **streakline**.

Streaklines and Pathlines

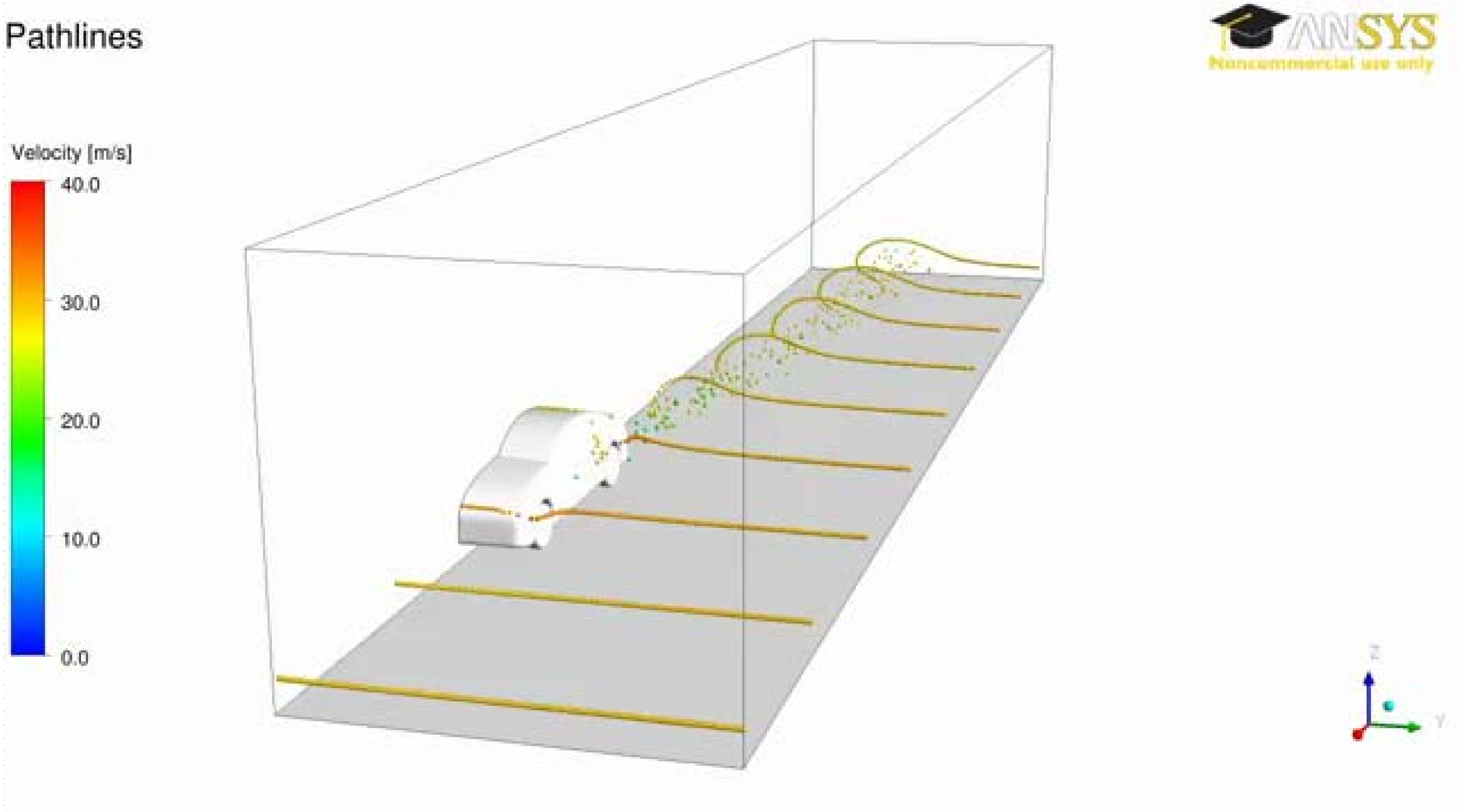
Streaklines vs. Pathlines

A visual explanation

Ramin Mehran

2010

Timelines around a car



✓Streamlines

Streamlines are lines drawn in the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field. **Since the streamlines are tangent to the velocity vector at every point** in the flow field, there can be no flow across a streamline.

$$\left.\frac{dy}{dx}\right)_{\text{streamline}} = \frac{v(x, y)}{u(x, y)}$$

Pathline

$$\left.\frac{dx}{dt}\right)_{\text{particle}} = u(x, y, t) \qquad \left.\frac{dy}{dt}\right)_{\text{particle}} = v(x, y, t)$$

Streakline

$$x_{\text{streakline}}(t_0) = x(t, x_0, y_0, t_0) \qquad y_{\text{streakline}}(t_0) = y(t, x_0, y_0, t_0)$$

Streamlines over a airfoil

Stress Field

Each fluid particle can experience: **surface forces** (pressure, friction) that are generated by contact with other particles or a solid surface; and **body forces**

Surface forces on a fluid particle lead to stresses.

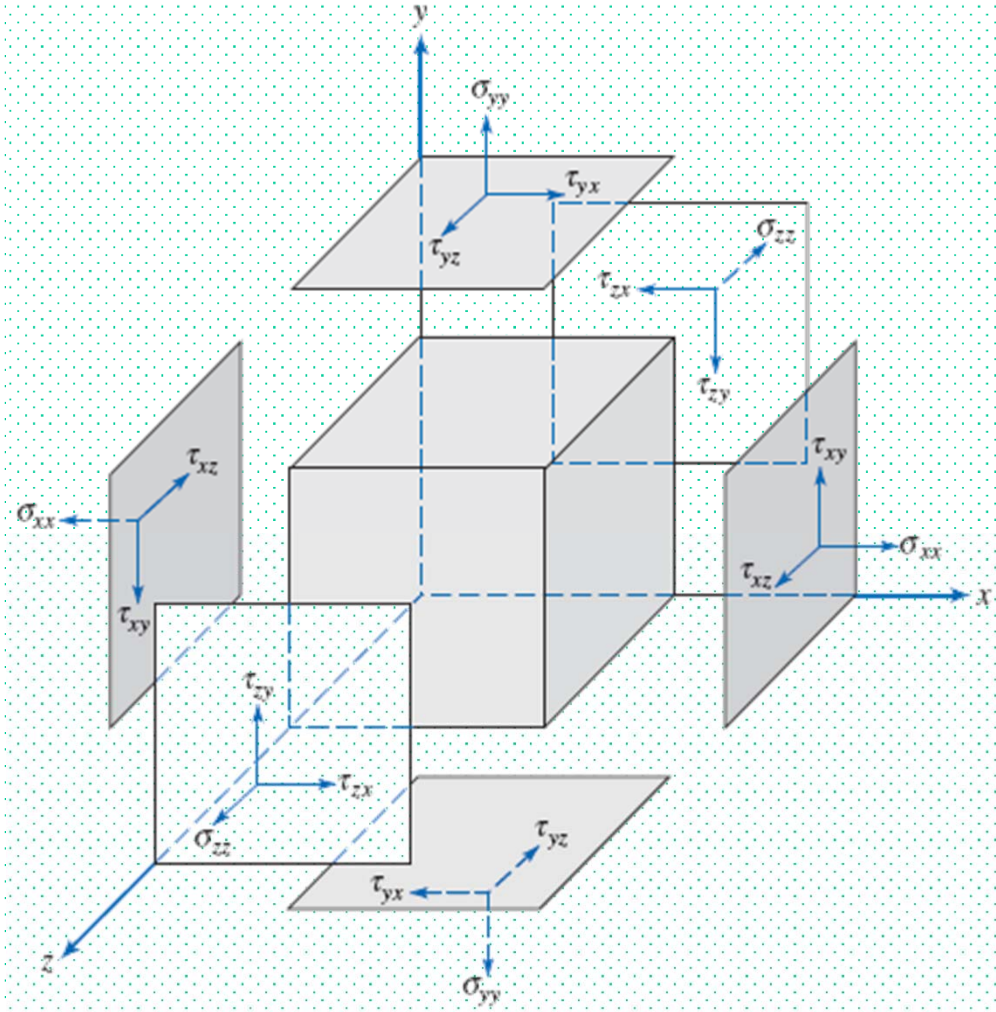


Fig. 2.8 Notation for stress.

$$\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n} \qquad \tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$$

The stress at a point is specified by the nine components

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

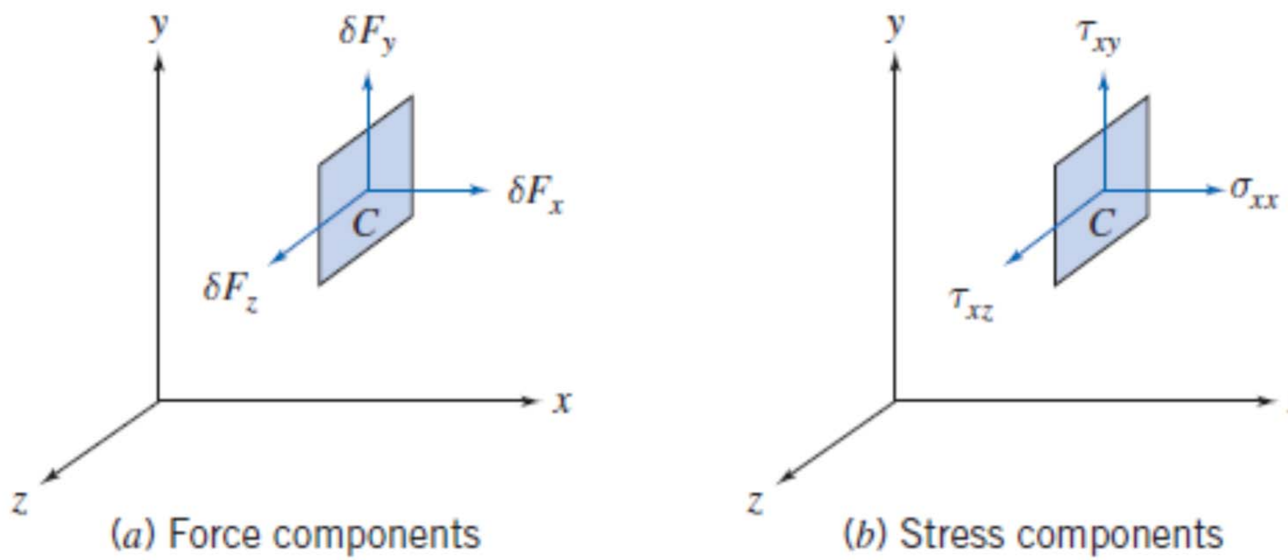


Fig. 2.7 Force and stress components on the element of area δA_x.

Viscosity

For a **solid**, stresses develop when the material is **elastically deformed or strained;** for a **fluid**, shear stresses arise due to **viscous flow**.

Hence we say solids are elastic, and fluids are viscous.

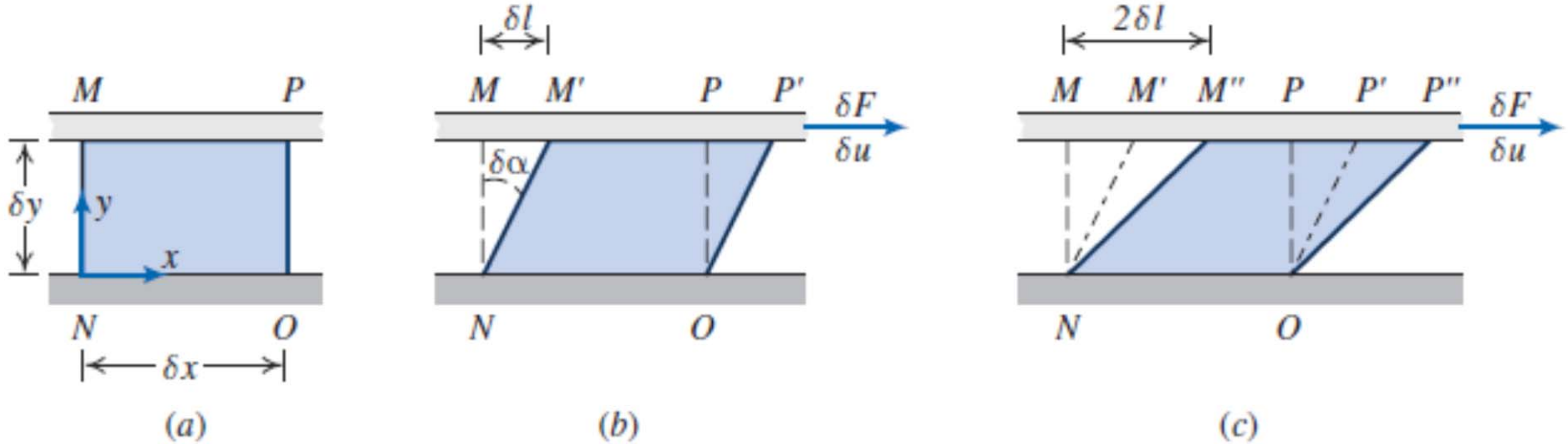


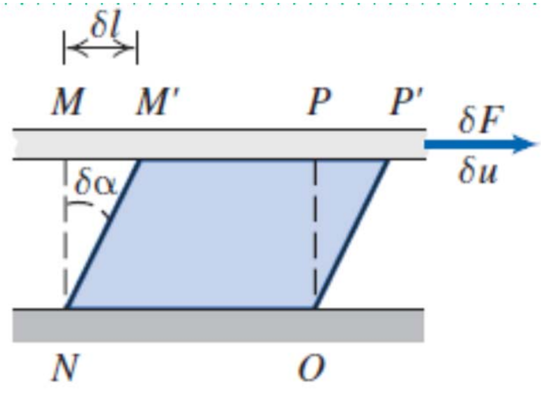
Fig. 2.9 (*a*) Fluid element at time *t*, (*b*) deformation of fluid element at time $t + \delta t$, and (*c*) deformation of fluid element at time $t + 2\delta t$.

Consider the behavior of a fluid element between the two infinite plates, initially at rest at time t and suppose a constant rightward force δFx is applied to the upper plate so that it is dragged across the fluid at constant velocity δu.

The relative shearing action of the infinite plates produces a shear stress, τyx, which acts on the fluid element and is given by

$$\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{dF_x}{dA_y}$$

$$\text{deformation rate} = \lim_{\delta t \to 0} \frac{\delta\alpha}{\delta t} = \frac{d\alpha}{dt}$$



$$\delta l = \delta u\, \delta t \qquad \delta l = \delta y\, \delta\alpha$$

$$\frac{\delta\alpha}{\delta t} = \frac{\delta u}{\delta y} \qquad \frac{d\alpha}{dt} = \frac{du}{dy}$$

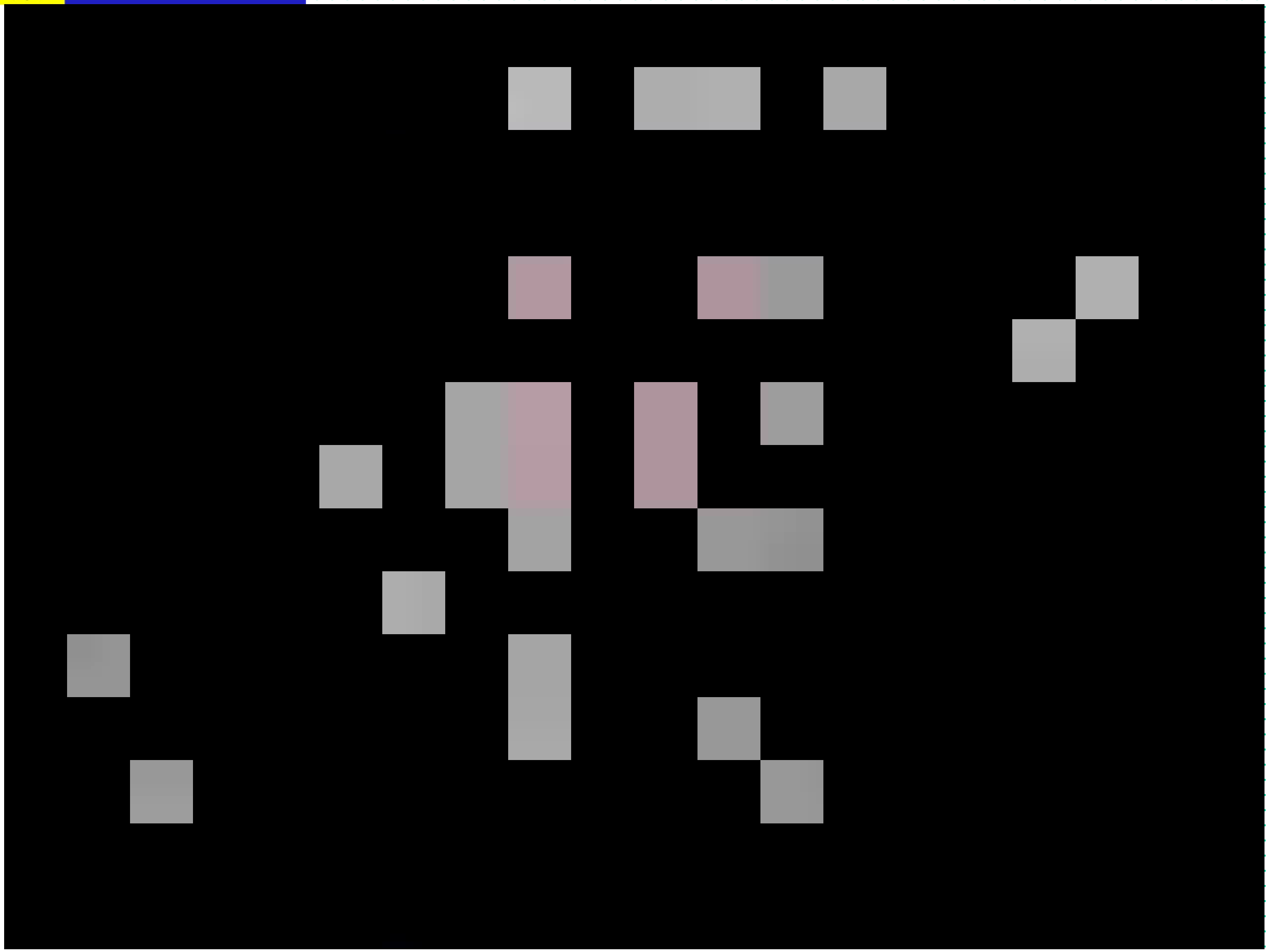
Shear stress τ_{yx}, experiences a rate of deformation (shear rate) given by du/dy.

What is the relation between shear stress and shear rate?

Fluids in which shear stress is directly proportional to rate of deformation are Newtonian fluids.

The term non-Newtonian is used to classify all fluids in which shear stress is not directly proportional to shear rate.

Viscosity



Viscosity

✓ **Newtonian Fluids**

- **Most of the common fluids (water, air, oil, etc.)**
- **“Linear” fluids**

$$\tau_{yx} \propto \frac{du}{dy}$$

$$\tau_{yx} = \mu \frac{du}{dy}$$

Example 2.2 VISCOSITY AND SHEAR STRESS IN NEWTONIAN FLUID

An infinite plate is moved over a second plate on a layer of liquid as shown. For small gap width, *d*, we assume a linear velocity distribution in the liquid. The liquid viscosity is 0.0065 g/cm.s and its specific gravity is 0.88.

Determine:

(a) The absolute viscosity of the liquid, in N s/m2.

(b) The kinematic viscosity of the liquid, in m2/s.

(c) The shear stress on the upper plate, in N/m2.

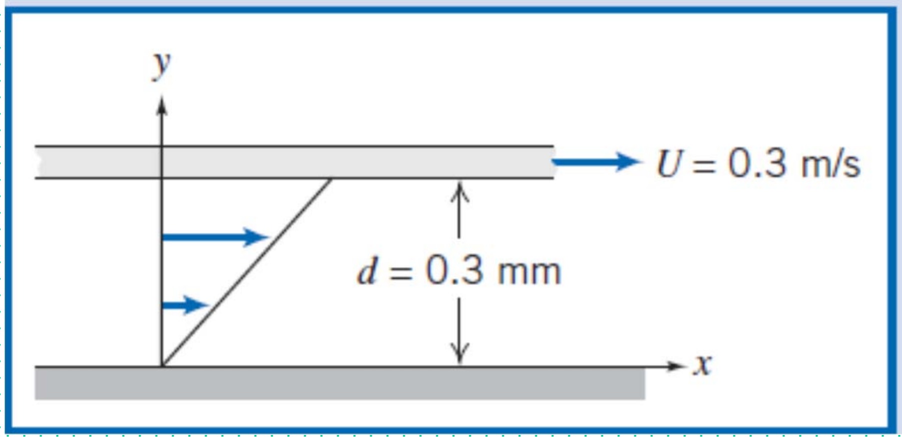
(d) The shear stress on the lower plate, in Pa.

(e) The direction of each shear stress calculated in parts (c) and (d).

$$\mu = 0.0065 \text{ g/cm} \cdot \text{s}$$
$$SG = 0.88$$

Assumptions:

(1) Linear velocity distribution
(2) Steady flow
(3) $\mu = \text{constant}$



(a) $\mu = 0.0065 \dfrac{\text{g}}{\text{cm}\cdot\text{s}} \times \dfrac{\text{kg}}{1000\ \text{g}} \times \dfrac{9.81\ \text{m}}{\text{s}^2} \times \dfrac{100\ \text{cm}}{\text{m}} \times \dfrac{\text{s}^2}{9.81\ \text{m}}$

$\mu = 6.5 \times 10^{-4}\ \text{N}\cdot\text{s/m}^2 \longleftarrow$

(b) $\nu = \dfrac{\mu}{\rho} = \dfrac{\mu}{\text{SG}\ \rho_{H_2O}}$

$$= 6.5 \times 10^{-4} \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times \frac{\text{m}^3}{(0.88)1000\ \text{kg}}$$

$$= 6.5 \times 10^{-4} \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \times \frac{\text{s}}{\text{m}^2} \times \frac{\text{m}^3}{(0.88)1000\ \text{kg}}$$

$\nu = 7.39 \times 10^{-7}\text{m}^2/\text{s} \longleftarrow$

(c) $\tau_{\text{upper}} = \tau_{yx,\,\text{upper}} = \mu \left.\dfrac{du}{dy}\right)_{y=d}$

Since u varies linearly with y,

$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{U-0}{d-0} = \frac{U}{d}$$

$$= 0.3 \frac{\text{m}}{\text{s}} \times \frac{1}{0.3\ \text{mm}} \times 1000 \frac{\text{mm}}{\text{m}} = 1000\ \text{s}^{-1}$$

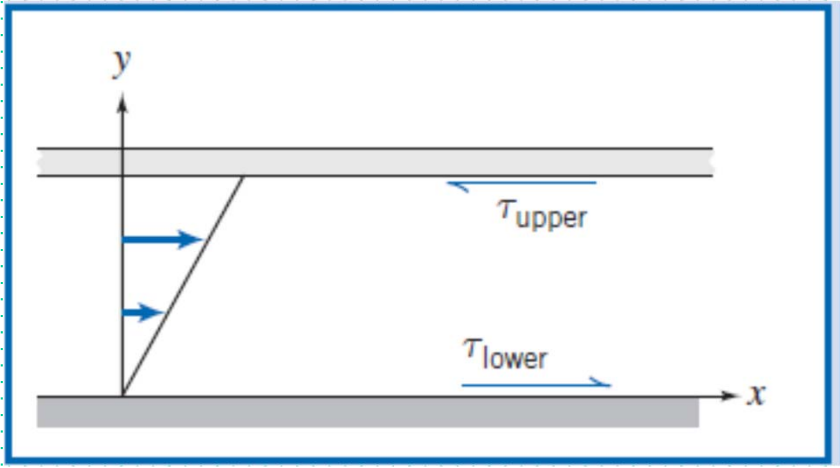
$\tau_{\text{upper}} = \mu \dfrac{U}{d} = 6.5 \times 10^{-4} \dfrac{\text{N}\cdot\text{s}}{\text{m}^2} \times \dfrac{1000}{\text{s}} = 0.65\ \text{N/m}^2 \longleftarrow$

(d) $\tau_{lower} = \mu \dfrac{U}{d} = 0.65\ \mathrm{N/m^2} \times \dfrac{\mathrm{Pa \cdot m^2}}{\mathrm{N}} = 0.65\ \mathrm{Pa}$

(e) Directions of shear stresses on upper and lower plates.

$\left\{\begin{array}{l}\text{The upper plate is a negative } y \text{ surface; so} \\ \text{positive } \tau_{yx} \text{ acts in the negative } x \text{ direction.}\end{array}\right\}$

$\left\{\begin{array}{l}\text{The lower plate is a positive } y \text{ surface; so} \\ \text{positive } \tau_{yx} \text{ acts in the positive } x \text{ direction.}\end{array}\right\}$



Viscosity

✓ **Non-Newtonian Fluids**

- **Special fluids (e.g., most biological fluids, toothpaste, some paints, etc.)**
- **"Non-linear" fluids**

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n$$

$$\eta = k|du/dy|^{n-1}$$

is referred to as the **apparent viscosity**.

$$\tau_{yx} = k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} = \eta\frac{du}{dy}$$

Viscosity

✓Non-Newtonian Fluids

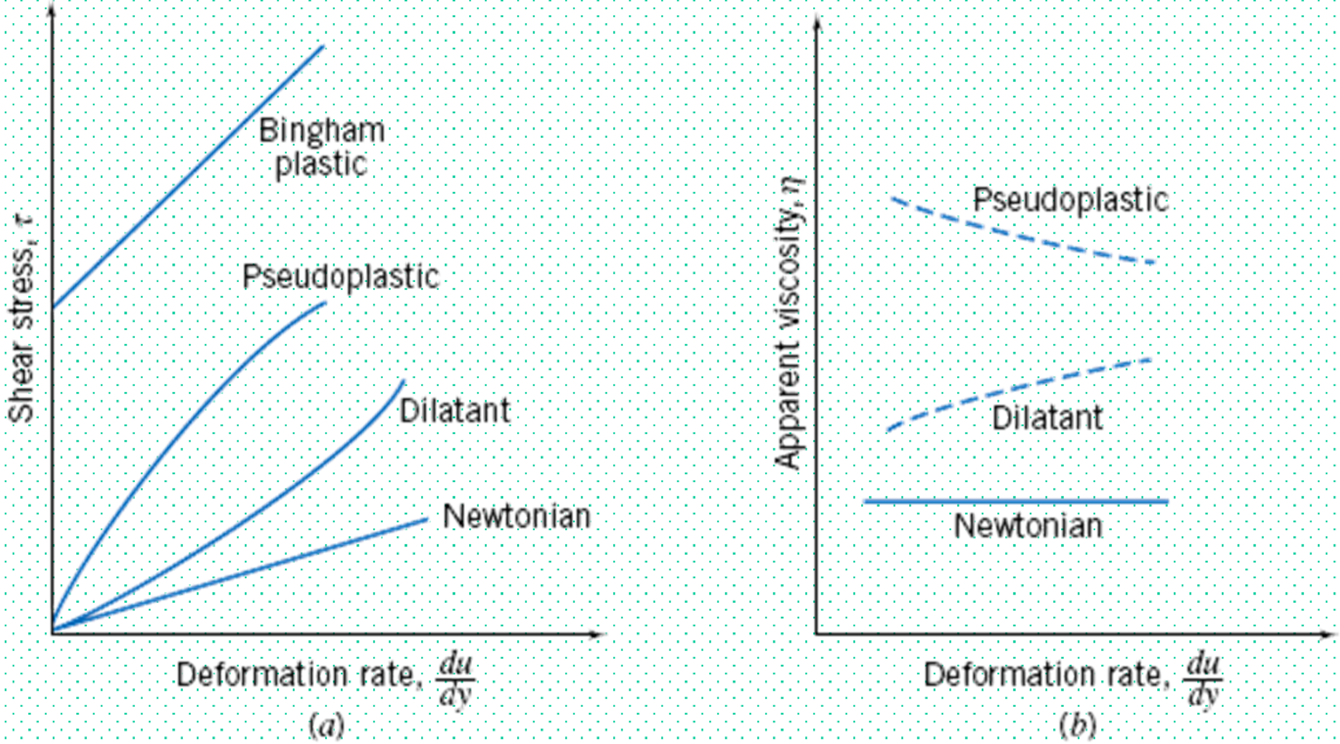


Fig. 2.10 (a) Shear stress, τ, and (b) apparent viscosity, η, as a function of deformation rate for one-dimensional flow of various non-Newtonian fluids.

Surface Tension

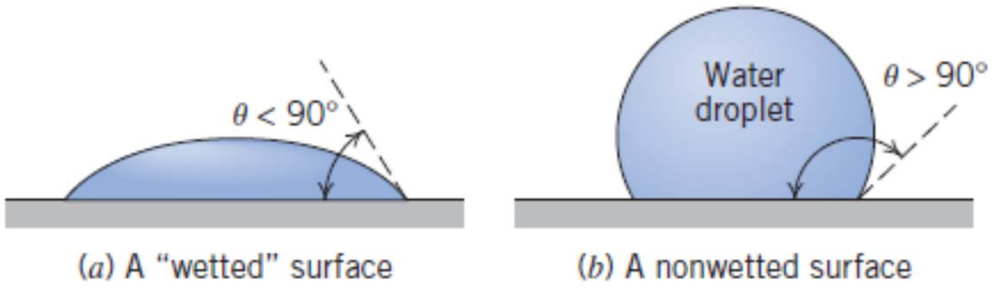


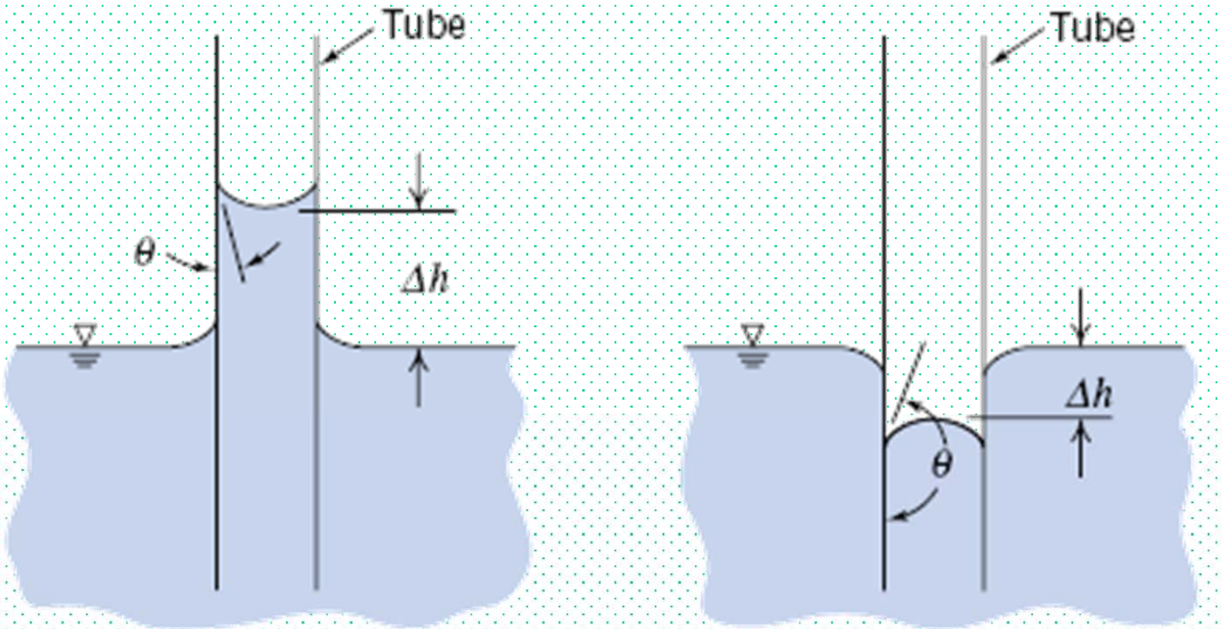
Fig. 2.11 Surface tension effects on water droplets.

When your car needs waxing: Water droplets tend to appear somewhat flattened out. After waxing, you get a nice "beading" effect.

There are two features to this membrane: **the contact an**gle, θ, and the magnitude of **the surface tension**, σ (N/m).
Both of these depend on the type of liquid and the type of solid surface with which it shares an interface.

A liquid as "wetting" a surface when the contact angle $\theta<90^o$.
The car's surface was wetted before waxing, and not wetted after.
This is an example of effects due to surface tension.

Surface Tension



(a) Capillary rise (θ < 90°) (b) Capillary depression (θ > 90°)

Fig. 2.12 Capillary rise and capillary depression inside and outside a circular tube.

the most important effect of surface tension is the creation of a curved meniscus that appears in manometers or barometers, leading to a (usually unwanted) capillary rise (or depression),

Description and Classification of Fluid Motions

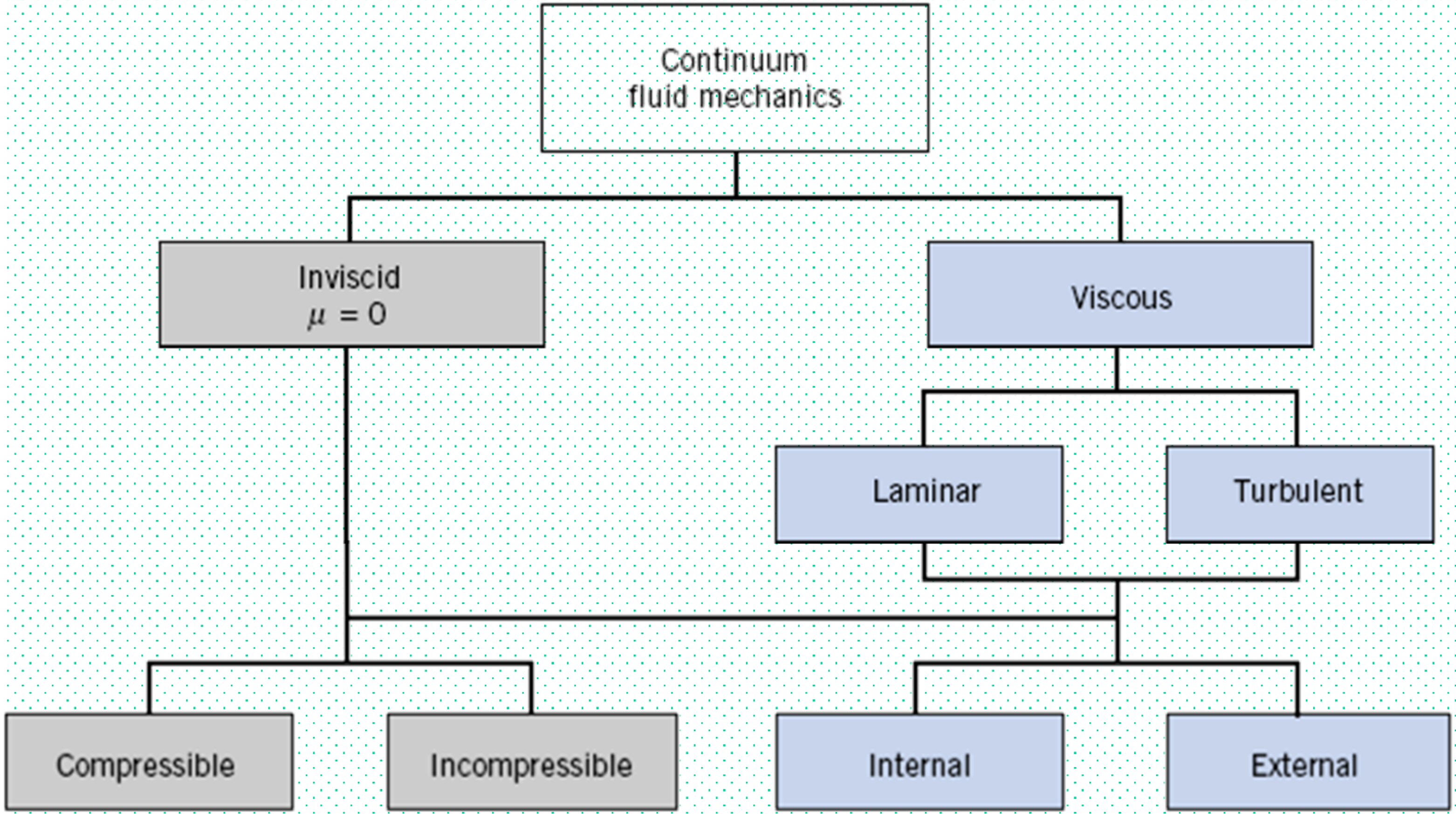
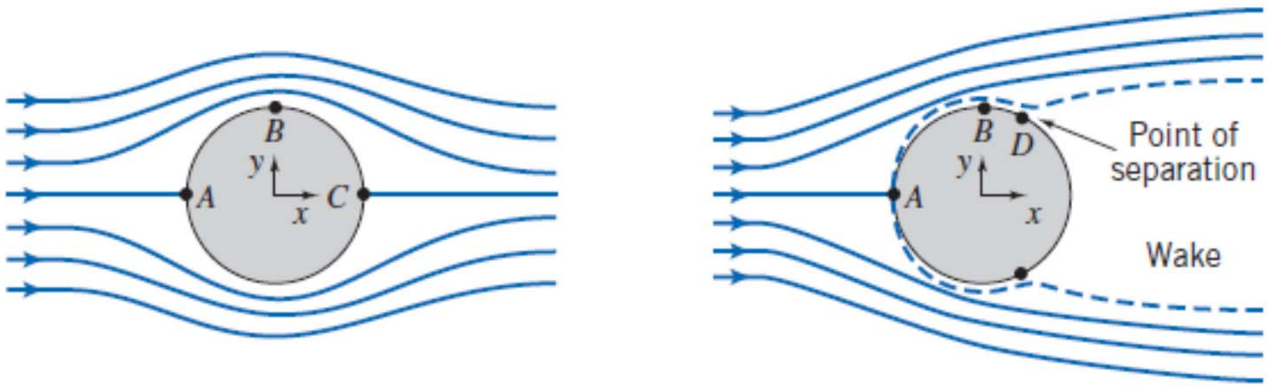


Fig. 2.13 Possible classification of continuum fluid mechanics.



(a) Inviscid flow (b) Viscous flow

Fig. 2.14 Qualitative picture of incompressible flow over a sphere.

the idealized notion of frictionless flow, called inviscid flow.

The streamlines are symmetric front-to-back. Because the mass flow between any two streamlines is constant, wherever streamlines open up, the velocity must decrease, and vice versa. Hence we can see that the velocity in the vicinity of points A and C must be relatively low; at point B it will be high. In fact, the air comes to rest at points A and C: They are stagnation points.

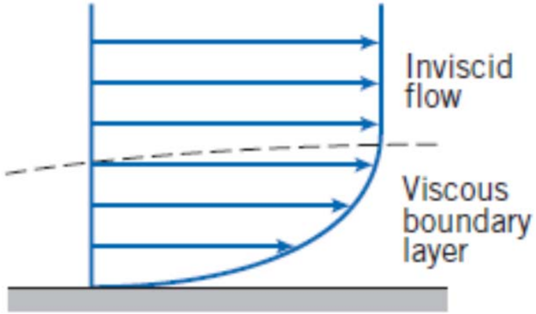


Fig. 2.15 Schematic of a boundary layer.

The no-slip condition requires that the velocity everywhere on the surface be zero, but inviscid theory states that even though friction is negligible in general for high-Reynolds number flows, there will always be a thin *boundary layer*, in which friction is significant and across the width of which the velocity increases rapidly from zero (at the surface) to the value inviscid flow theory predicts (on the outer edge of the boundary layer).

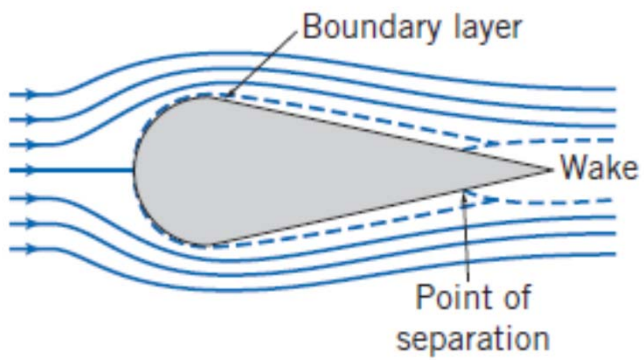


Fig. 2.16 Flow over a streamlined object.

The drag force in most aerodynamics is due to the low-pressure wake: If we can reduce or eliminate the wake, drag will be greatly reduced. If we consider once again why the separation occurred, we recall two features: *Boundary layer* friction slowed down the particles, but so did the adverse pressure gradient.
The streamlines open up gradually, and hence the pressure will increase slowly, to such an extent that fluid particles are not forced to separate from the object until they almost reach the end of the object.

Laminar and Turbulent Flows

A *laminar flow* is one in which the fluid particles move in smooth layers; a *turbulent flow* is one in which the fluid particles rapidly mix as they move along due to random 3D velocity fluctuations.

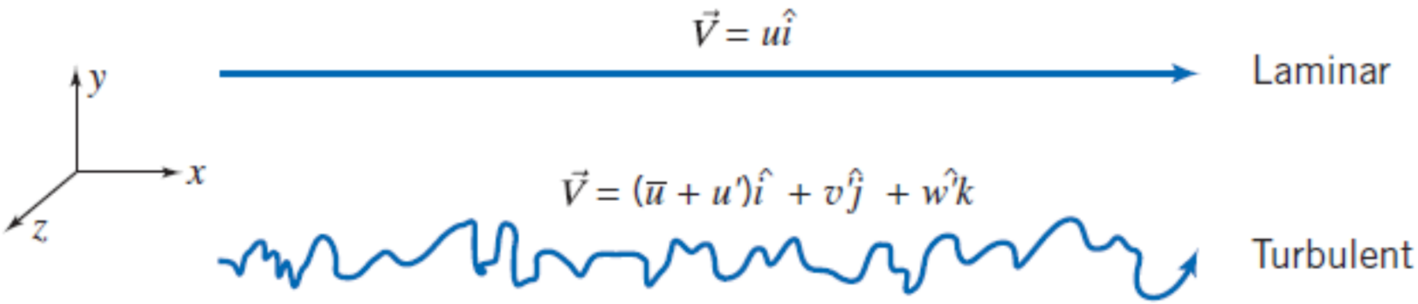


Fig. 2.17 Particle pathlines in one-dimensional laminar and turbulent flows.

Laminar versus Turbulent Flow

Compressible and Incompressible Flows

Flows in which variations in density are negligible are termed *incompressible*; when density variations within a flow are not negligible, the flow is called *compressible.*

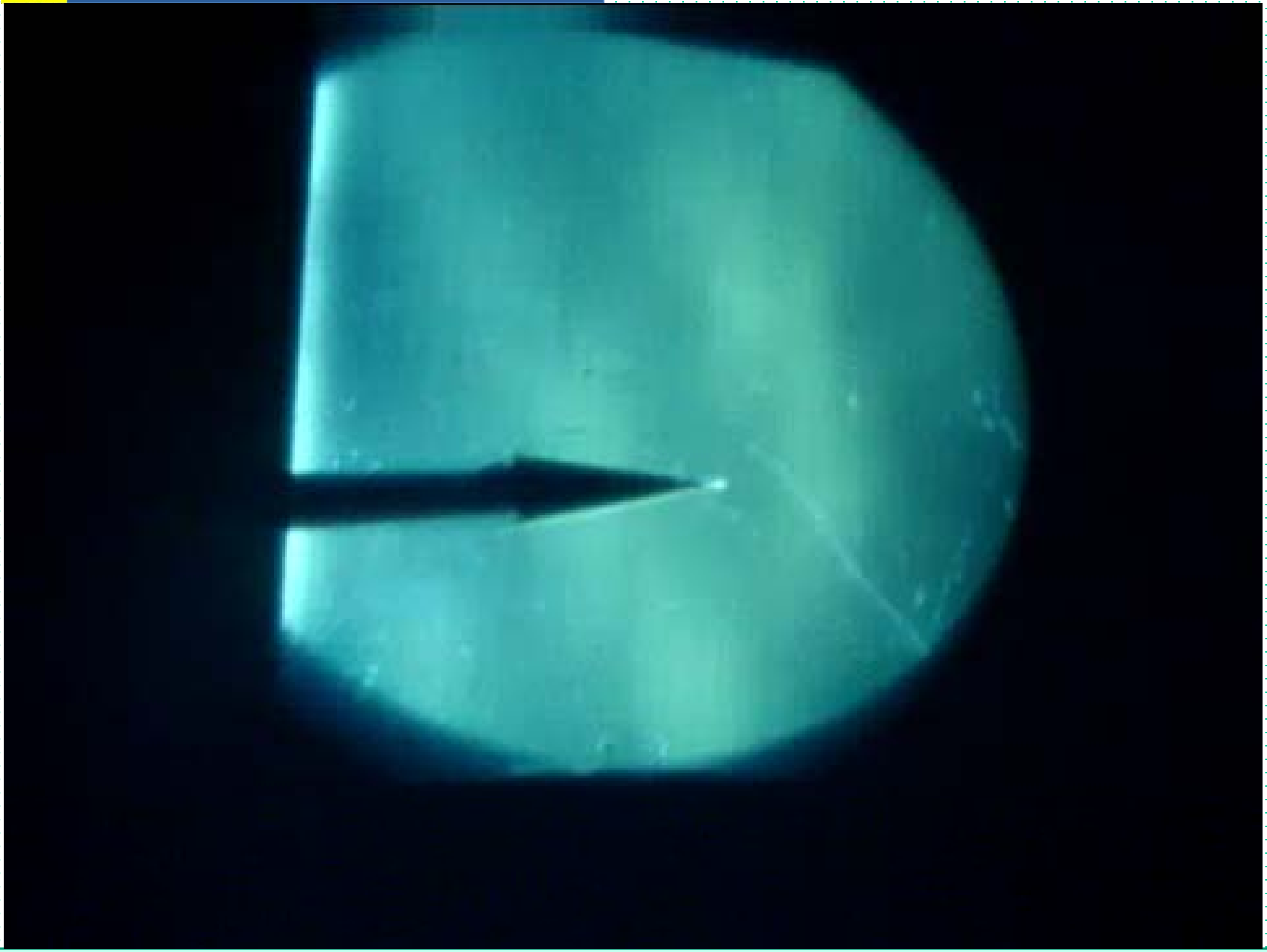
Pressure and density changes in liquids are related by the bulk compressibility modulus, or modulus of elasticity,

$$E_v \equiv \frac{dp}{(d\rho/\rho)}$$

Thus gas flows with M<0.3 can be treated as *incompressible*; a value of M=0.3 in air at standard conditions corresponds to a speed of approximately 100 m/s.

$$M \equiv \frac{V}{c}$$

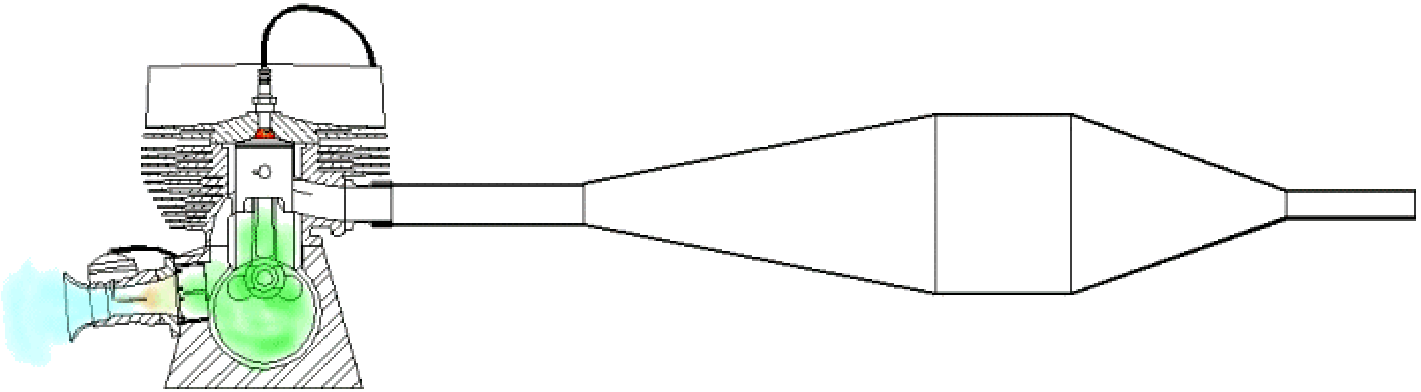
Supersonic Wind Tunnel



Internal and External Flows

Flows completely bounded by solid surfaces are called *internal or duct flows*. Flows over bodies immersed in an unbounded fluid are termed *external flows*. Both internal and external flows may be laminar or turbulent, compressible or incompressible.

External flow around a car



DENSITY

If the relative density of mercury is 13.6, the actual density of mercury is

(a) 13.6 kg/m^3
(b) 136 kg/m^3
(c) 1/13.6 kg/m^3
(d) 1360 kg/m^3
(e) 13600 kg/m^3

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(e) 13600 kg/m^3

VISCOSITY

The **dynamic** viscosity of pure glycerol is 1.5 Pa.s and its density 1262 kg/m^3.
The **kinematic** viscosity of pure glycerol is

(a) 1.5 m^2/s
(b) 1893 Pa.s
(c) 1.19 x 10^{-3} m^2/s
(d) 1.19 x 10^{-3} Pa.s

VISCOSITY

The **dynamic** viscosity of pure glycerol is 1.5 Pa.s and its density 1262 kg/m^3.
The **kinematic** viscosity of pure glycerol is

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(b) 1893 Pa.s
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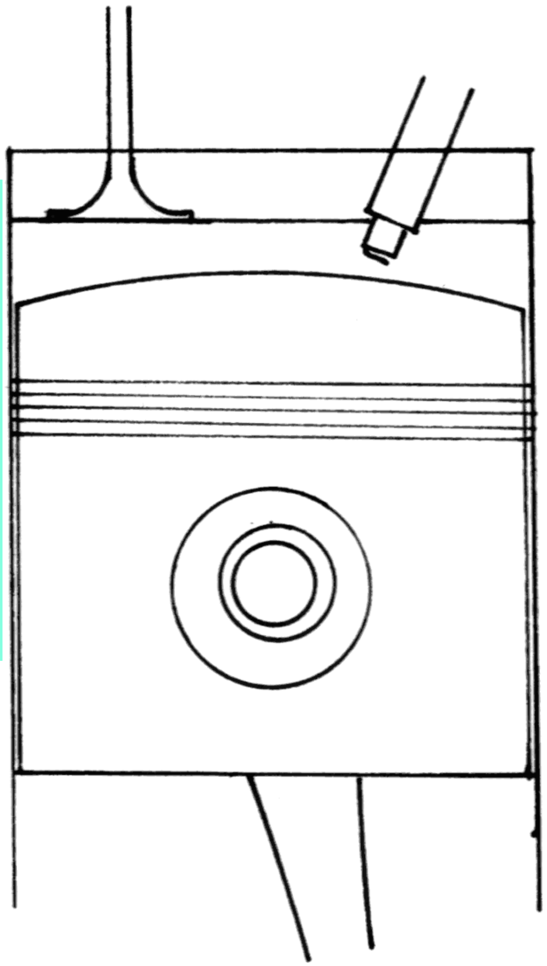
VISCOSITY

Shear stress $\tau = \mu$ x velocity gradient

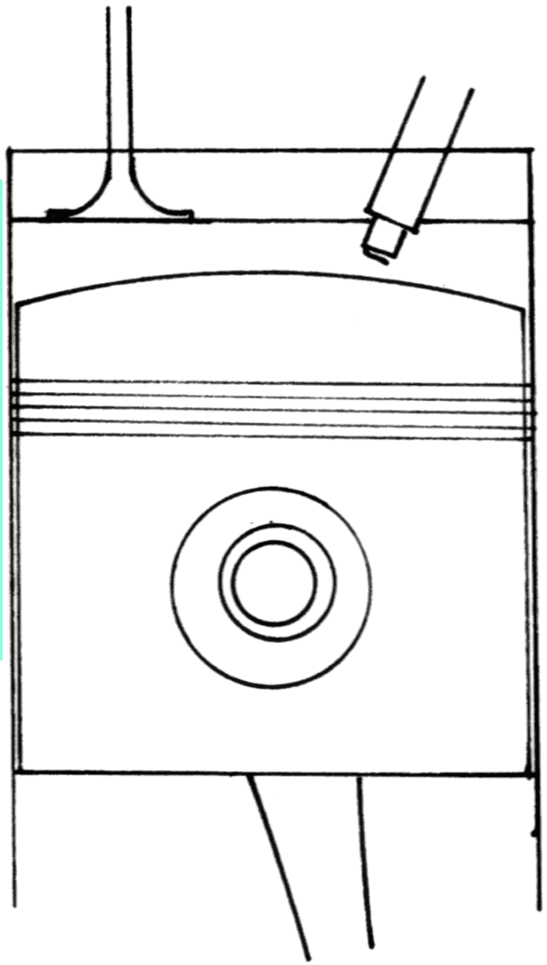
Peak rpm for a racing engine is 18000 rpm which corresponds to a maximum piston speed of 25 m/s. The viscosity of the lubricating oil in the gap between a piston and the cylinder wall is 0.016 Pa.s. If the gap width is 100 μm, is the shear stress acting on the piston at maximum speed

(a) 400 Pa
(b) 0.04 bar
(c) 0.4 bar



VISCOSITY

Shear stress $\tau = \mu$ x velocity gradient



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(a) 400 Pa
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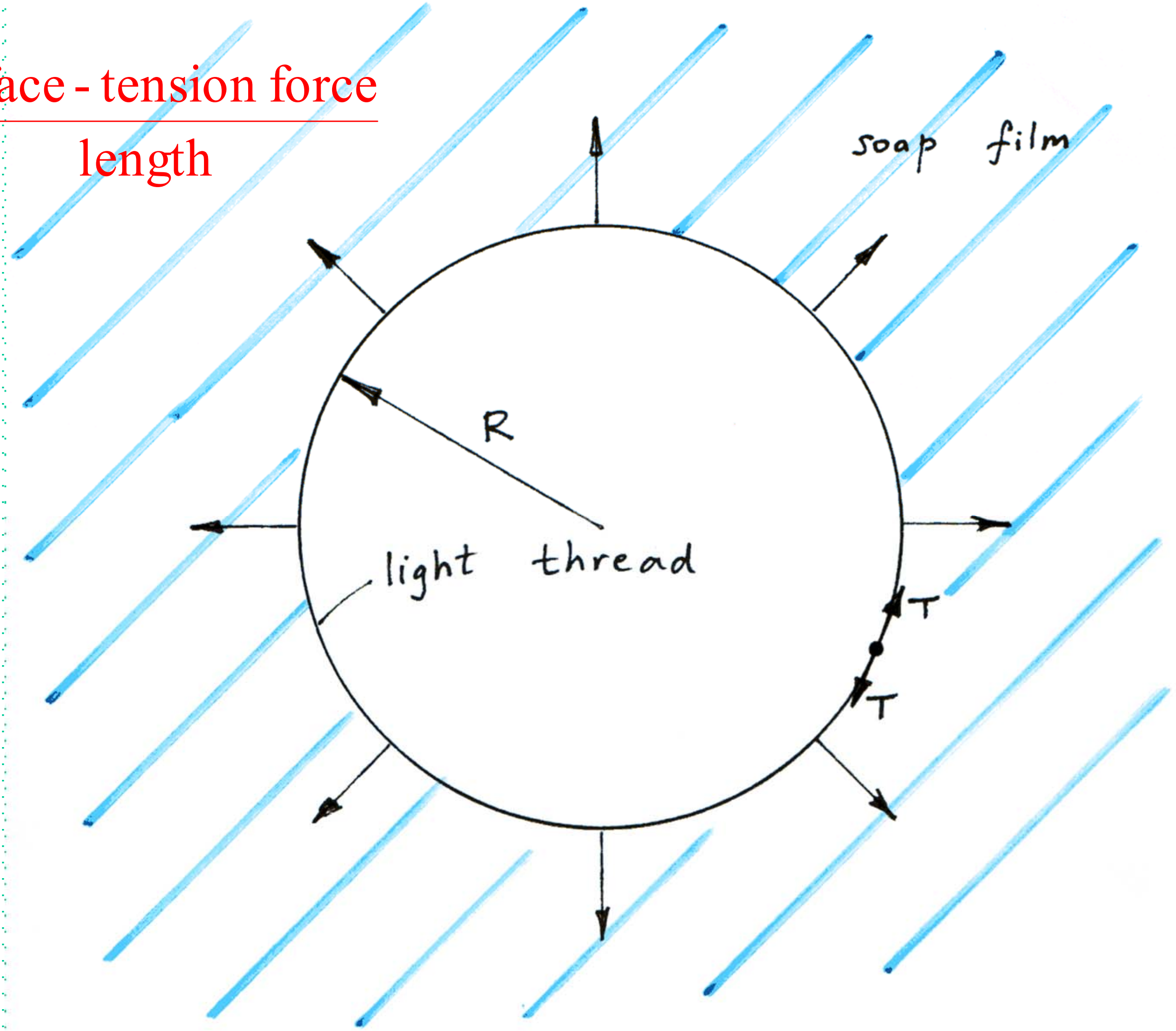
SURFACE TENSION

surface tension $\sigma \equiv \dfrac{\text{surface - tension force}}{\text{length}}$



The tension force T in the thread due to surface tension is

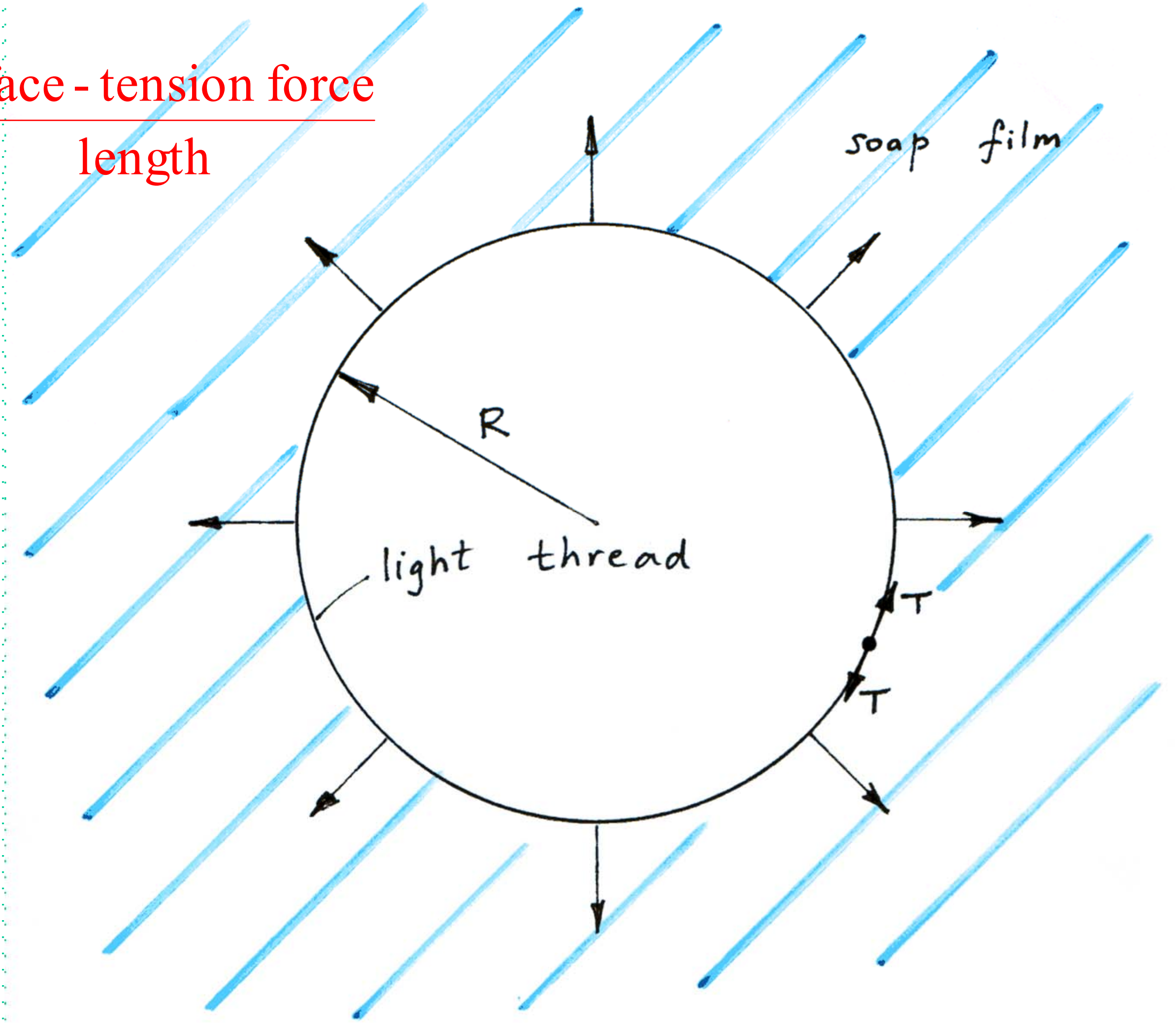
(a) $2\pi\, \sigma R$
(b) $\pi\, \sigma R$
(c) σR

SURFACE TENSION

surface tension $\sigma \equiv \dfrac{\text{surface - tension force}}{\text{length}}$

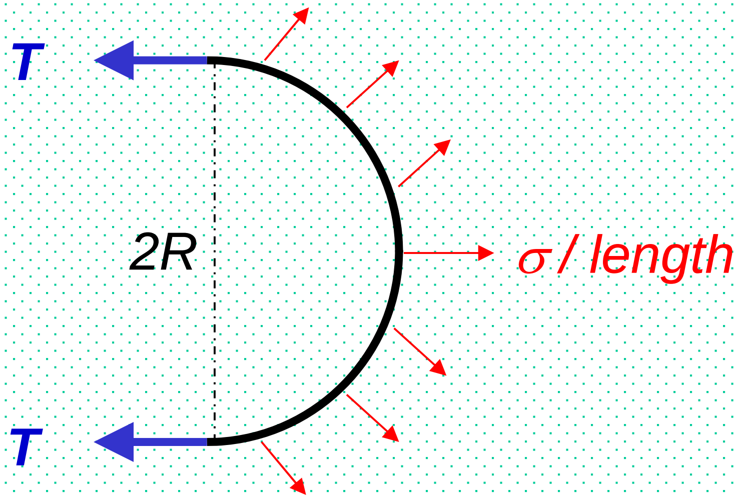
The tension force T in the thread due to surface tension is

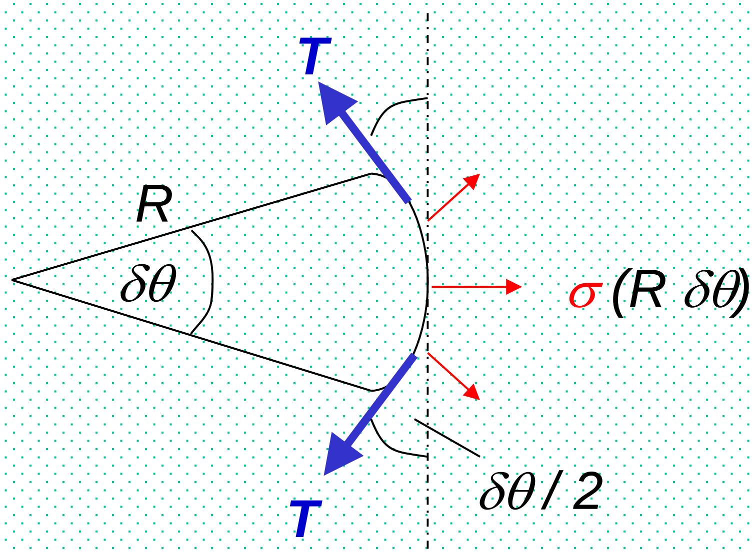
(a) $2\pi\,\sigma R$
(b) $\pi\,\sigma R$
(c) σR



SURFACE TENSION

$$\text{surface tension } \sigma \equiv \frac{\text{surface-tension force}}{\text{length}}$$





force balance in horizontal direction

$$2T = \sigma\, 2R$$

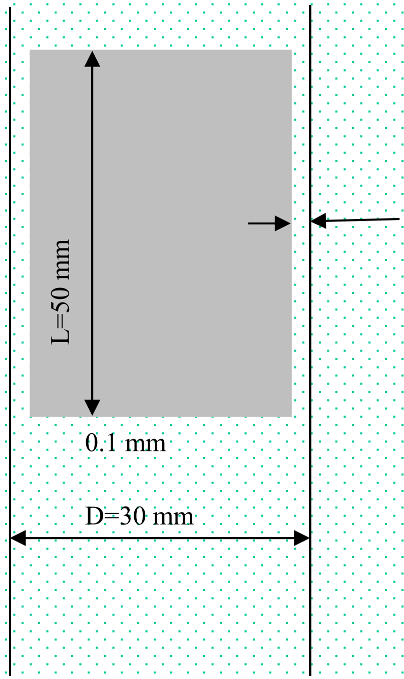
$$T = \sigma R$$

$$-2T \sin(\delta\theta / 2) + \sigma (R\, \delta\theta) = 0$$

$$T\, \delta\theta = \sigma R\, \delta\theta \qquad T = \sigma R$$

Example

A solid cylinder of weight W slides down in a vertical pipe which has a diameter of D=30 mm. The length of solid cylinder is L=50 mm, the clearance between the pipe and the block is 0.1 mm and filled with oil (ρ_{oil}=890 kg/m^3, μ_{oil}=0.40 kg/m s). Assuming linear velocity distribution in the film and neglecting the air drag, find the terminal velocity of the solid cylinder if the density of the solid cylinder is ρ_s=1300 kg/m^3.



$$\tau = \frac{F}{A} = \mu \frac{V}{l} \rightarrow V =$$

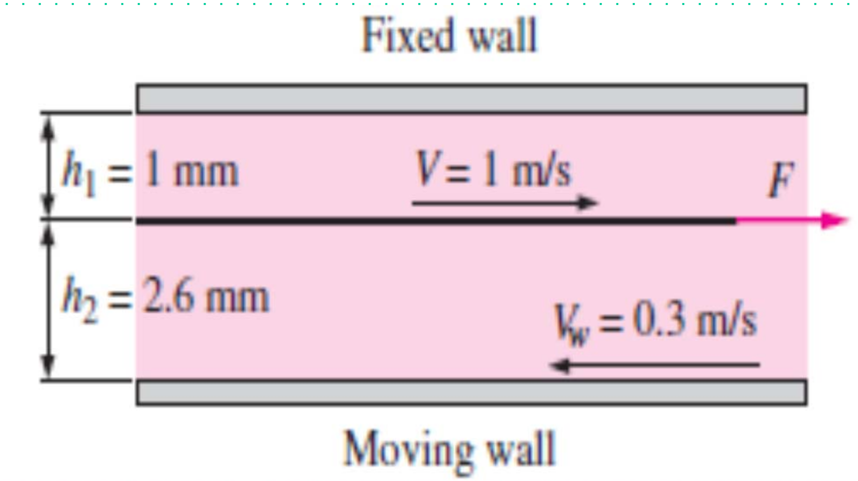
$$F = G = \rho g \frac{\pi d^2}{4} L \rightarrow G = 1300 * 9{,}81 * \frac{\pi * (0{,}03 - 0{,}0002)^2}{4} 0{,}05 = 0.44 \text{ N}$$

$$V = \frac{0.44 * 0{,}0001}{0{,}40 * 3.14(0{,}03 - 0{,}0002) * 0{,}05} \rightarrow V = 0.0234\, m/s$$

Example

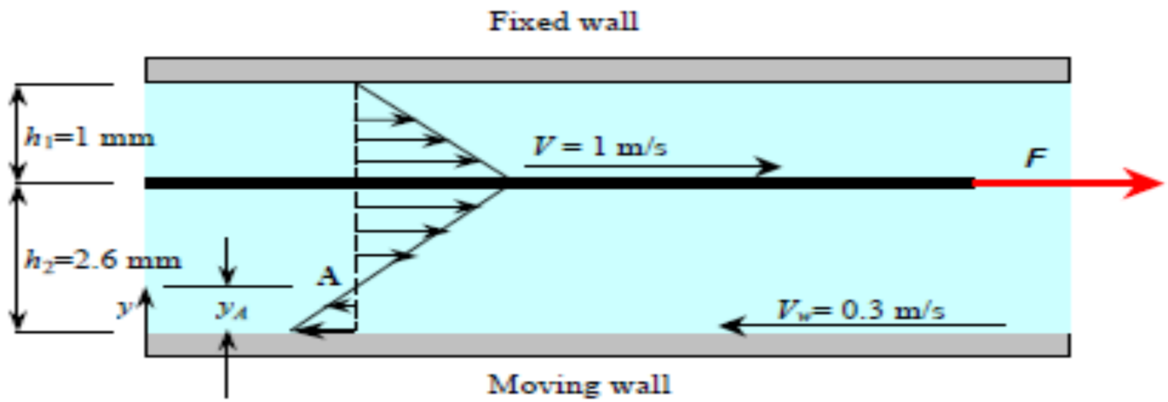
A thin 20-cm 20-cm flat plate is pulled at 1 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity of 0.3 m/s. The dynamic viscosity of oil is 0.027 Pa.s. Assuming the velocity in each oil layer to vary linearly,

(*a*) plot the velocity profile and find the location where the oil velocity is zero
(*b*) determine the force that needs to be applied on the plate to maintain this motion.



(*a*) plot the velocity profile and find the location where the oil velocity is zero

$$\frac{2.6 - y_A}{y_A} = \frac{1}{0.3} \quad \rightarrow \quad y_A = 0.60 \text{ mm}$$



(*b*) the force that needs to be applied on the plate to maintain this motion.

$$F_{shear,upper} = \tau_{w,up} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_1} = 0.027 \cdot (0.2 \cdot 0.2) \frac{1}{1 \cdot 10^{-3}} = 1.08\, N$$

$$F_{shear,lower} = \tau_{w,low} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V - V_w}{h_2} = 0.027 \cdot (0.2 \cdot 0.2) \frac{1-(-0.3)}{2.6 \cdot 10^{-3}}$$
$$= 0.54\, N$$

$$F = F_{shear,upper} + F_{shear,lower} = 1.08 + 0.54 = 1.62\, N$$

Introduction to Fluid Mechanics

Chapter 3

Fluid Statics

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Yildiz Technical University
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Besiktas, ISTANBUL

Main Topics

- ✓ The Basic Equations of Fluid Statics
- ✓ Pressure Variation in a Static Fluid
- ✓ Hydrostatic Force on Submerged Surfaces
- ✓ Buoyancy

The Basic Equations of Fluid Statics

✓ **Body Force**

$$d\vec{F}_B = \vec{g}\,dm = \vec{g}\,\rho\,d\forall$$

$$d\vec{F}_B = \rho\vec{g}\;dx\;dy\;dz$$

The Basic Equations of Fluid Statics

✓Surface Force

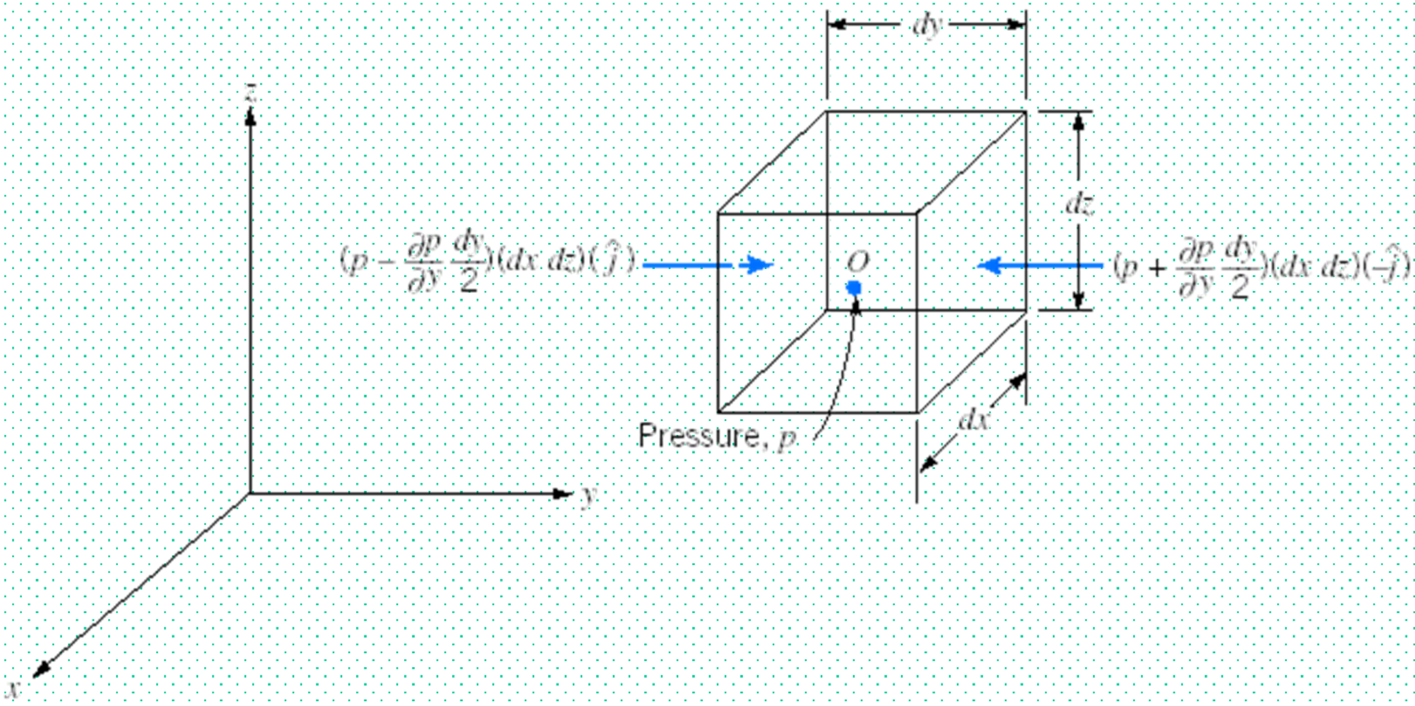


Fig. 3.1 Differential fluid element and pressure forces in the y direction.

The Basic Equations of Fluid Statics

✓ **Surface Force**

$$p_L = p + \frac{\partial p}{\partial y}(y_L - y) = p + \frac{\partial p}{\partial y}\left(-\frac{dy}{2}\right) = p - \frac{\partial p}{\partial y}\frac{dy}{2}$$

$$p_R = p + \frac{\partial p}{\partial y}(y_R - y) = p + \frac{\partial p}{\partial y}\frac{dy}{2}$$

The Basic Equations of Fluid Statics

✓ Surface Force

$$d\vec{F}_S = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)dx\,dy\,dz$$

$$d\vec{F}_S = -\text{grad}\,p\,(dx\,dy\,dz) = -\nabla p\,dx\,dy\,dz$$

The Basic Equations of Fluid Statics

✓Total Force

$$d\vec{F} = d\vec{F}_S + d\vec{F}_B = (-\nabla p + \rho\vec{g})\, d\forall$$

$$\frac{d\vec{F}}{d\forall} = -\nabla p + \rho\vec{g}$$

The Basic Equations of Fluid Statics

✓ Newton's Second Law

$$\frac{d\vec{F}}{d\forall} = \rho\vec{a} = 0 \qquad\qquad -\nabla p + \rho\vec{g} = 0$$

$$\begin{array}{cccc} -\nabla p & + & \rho\vec{g} & = 0 \\ \left\{\begin{array}{c}\text{net pressure force}\\ \text{per unit volume}\\ \text{at a point}\end{array}\right\} & + & \left\{\begin{array}{c}\text{body force per}\\ \text{unit volume}\\ \text{at a point}\end{array}\right\} & = 0 \end{array}$$

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} + \rho g_x &= 0 \quad x \text{ direction} \\ -\frac{\partial p}{\partial y} + \rho g_y &= 0 \quad y \text{ direction} \\ -\frac{\partial p}{\partial z} + \rho g_z &= 0 \quad z \text{ direction} \end{aligned}\right\}$$

$g_x = 0$, $g_y = 0$, and $g_z = -g$

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g$$

The Basic Equations of Fluid Statics

✓ Pressure-Height Relation

$$\frac{dp}{dz} = -\rho g \equiv -\gamma$$

Restrictions:	(1) Static fluid.
	(2) Gravity is the only body force.
	(3) The z axis is vertical and upward.

$$p_{gage} = p_{absolute} - p_{atmosphere}$$

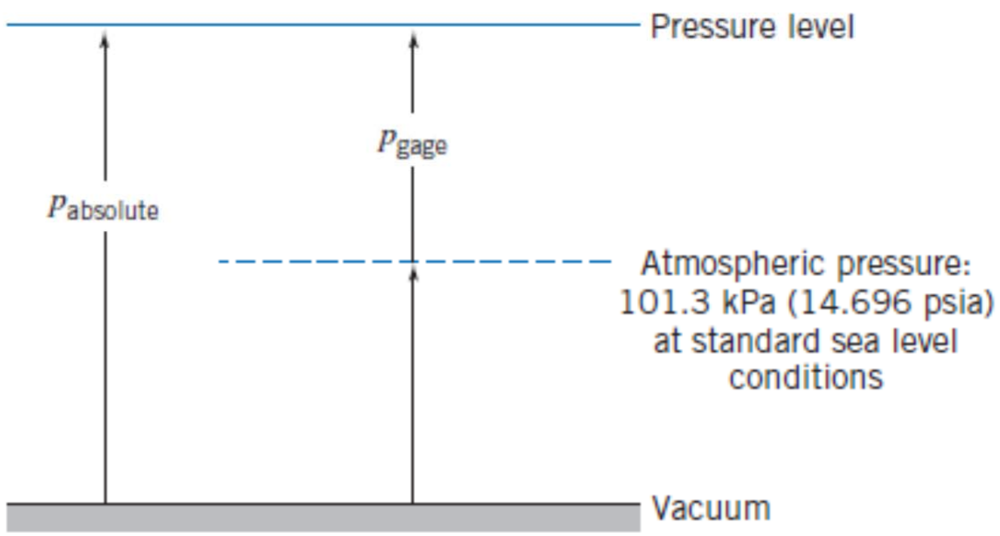


Fig. 3.2 Absolute and gage pressures, showing reference levels.

Pressure values must be stated with respect to a reference level. If the reference level is a vacuum, pressures are termed *absolute*. Most pressure gages indicate a pressure difference—the difference between the measured pressure and the ambient level.

Pressure levels measured with respect to atmospheric pressure are termed *gage pressures*.

$$p = p_0\left(1 - \frac{mz}{T_0}\right)^{g/mR} = p_0\left(\frac{T}{T_0}\right)^{g/mR}$$

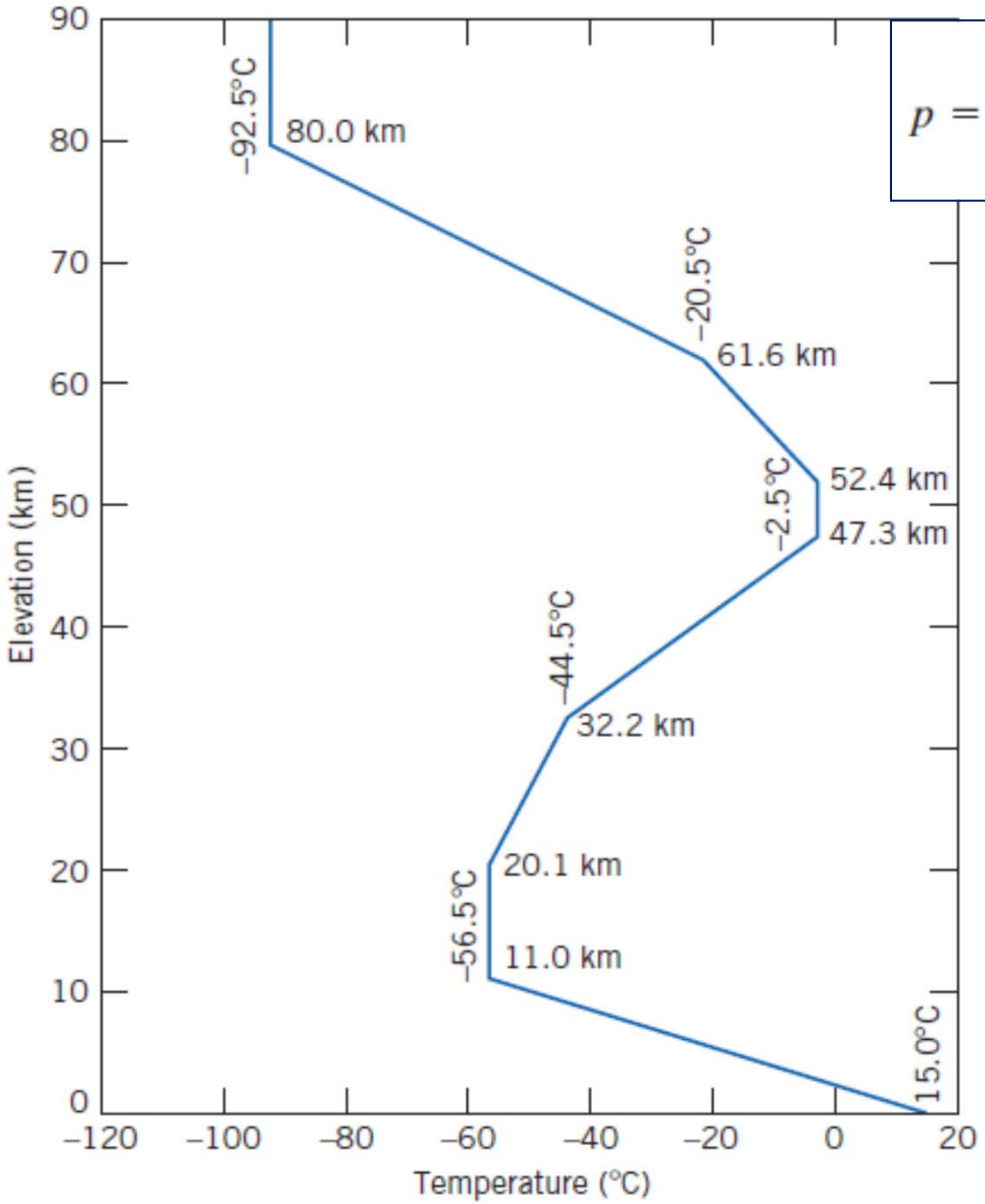


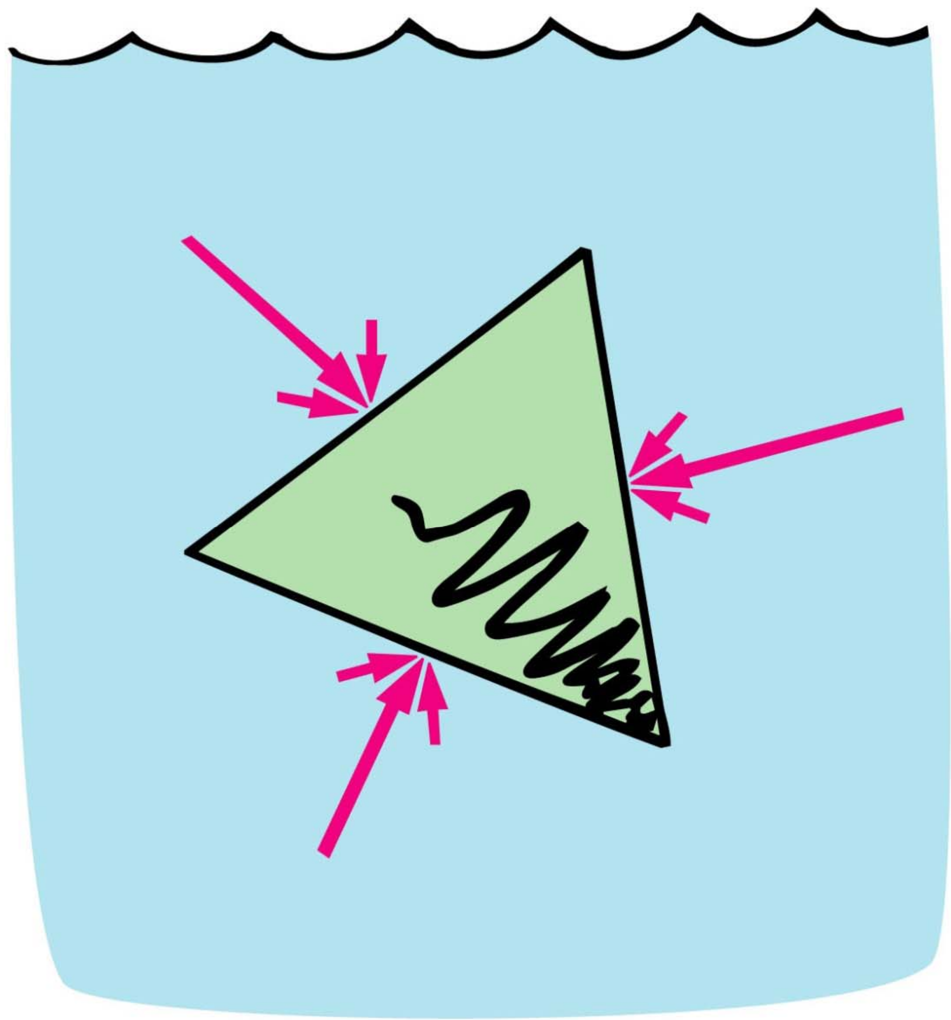
Fig. 3.3 Temperature variation with altitude in the U.S. Standard Atmosphere.

Table 3.1

Sea Level Conditions of the U.S. Standard Atmosphere

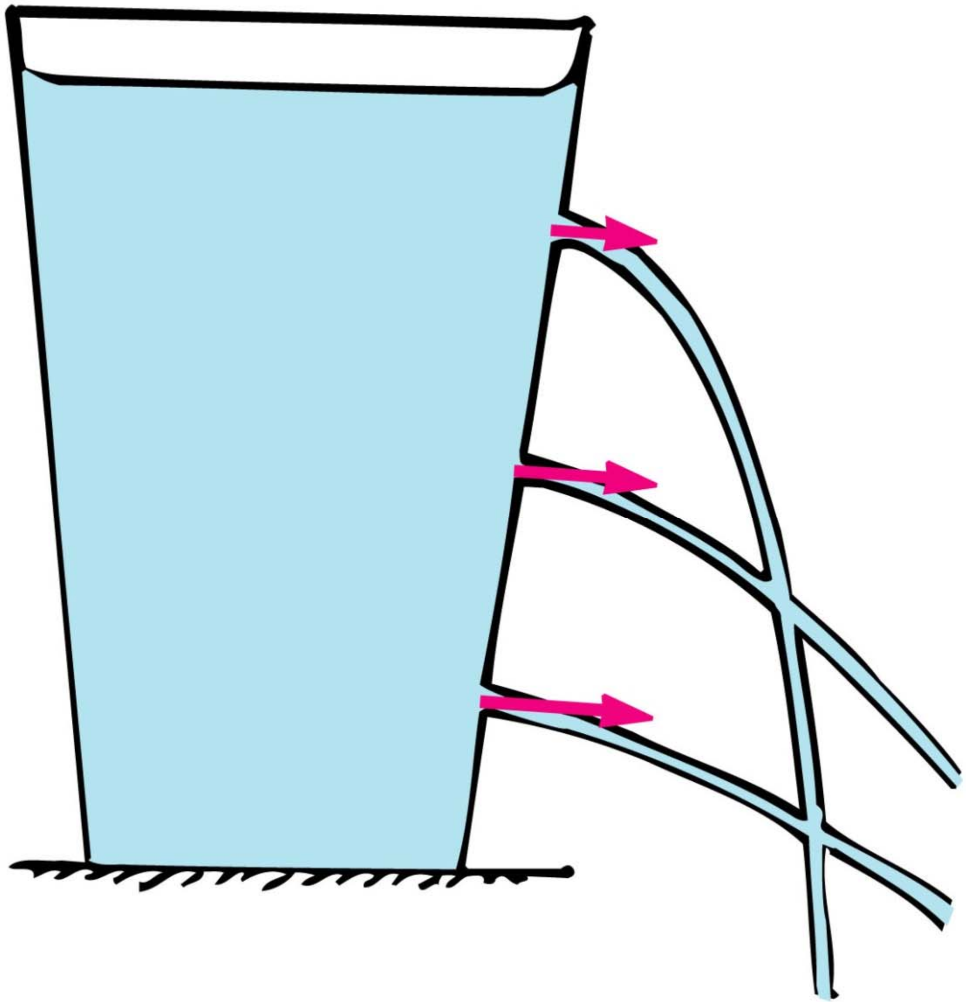
Property	Symbol	SI
Temperature	T	15°C
Pressure	p	101.3 kPa (abs)
Density	ρ	1.225 kg/m^3
Specific weight	γ	—
Viscosity	μ	1.789×10^{-5} kg/(m·s) (Pa·s)

Pressure in a Fluid



Pressure acts perpendicular to the surface and increases at greater depth.

Pressure in a Fluid



Pressure Variation in a Static Fluid

- ✓ Incompressible Fluid (density=constant)
- ✓ Manometers

$$\frac{dp}{dz} = -\rho g = \text{constant}$$

$$\int_{p_0}^{p} dp = -\int_{z_0}^{z} \rho g \, dz$$

$$p - p_0 = -\rho g(z - z_0) = \rho g(z_0 - z)$$

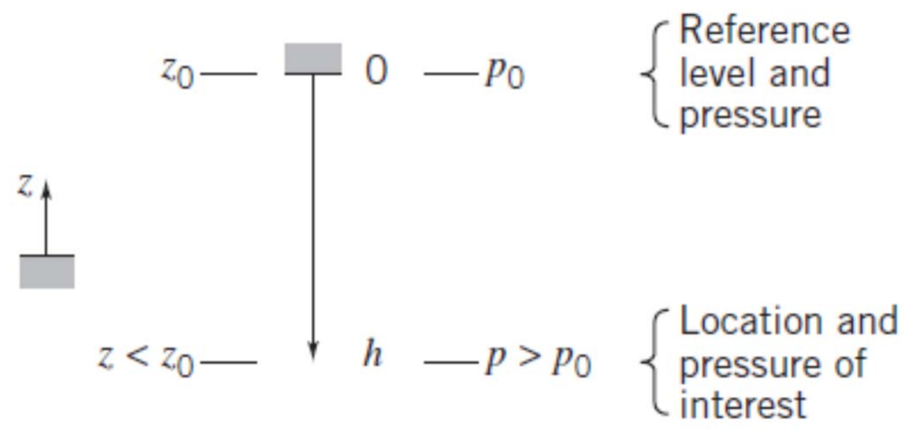


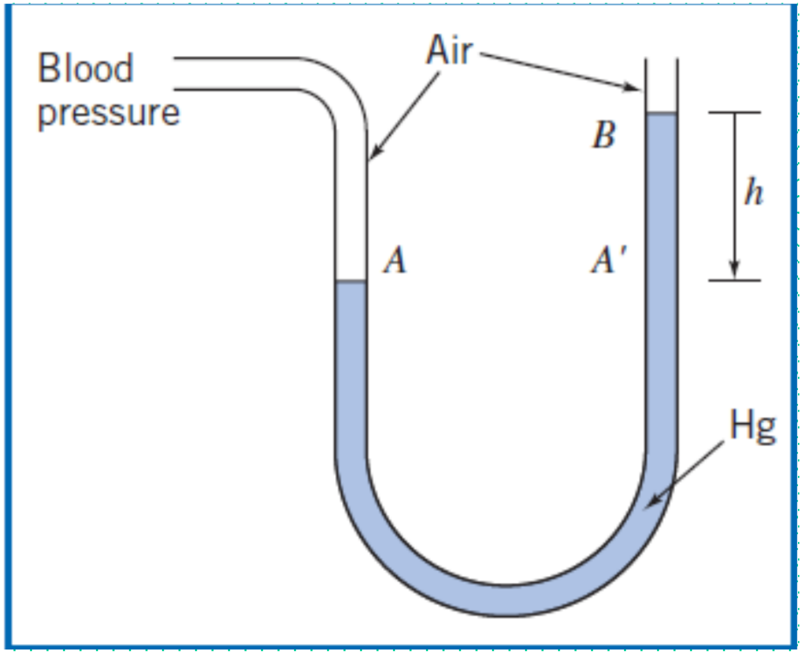
Fig. 3.4 Use of z and h coordinates.

$$p - p_0 = \rho g h \qquad \Delta p = g \sum_i \rho_i h_i$$

Hoover Dam.

Example 3.1 SYSTOLIC AND DIASTOLIC PRESSURE

Normal blood pressure for a human is 120/80 mm Hg. By modeling a sphygmomanometer pressure gage as a U-tube manometer, convert these pressures to kPa.



Assumptions: (1) Static fluid.
(2) Incompressible fluids.
(3) Neglect air density (<< Hg density).

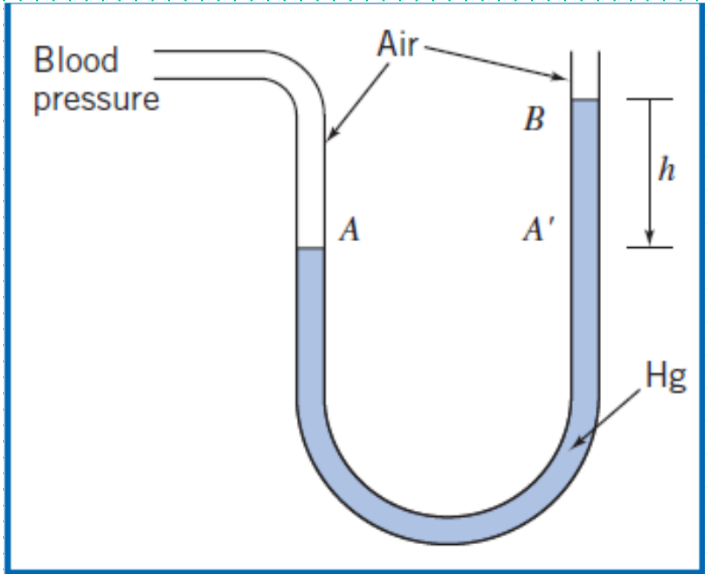
$$p_{A'} = p_B + \rho_{Hg} g h = SG_{Hg} \rho_{H_2O} g h$$

$$p_A = p_{A'} = SG_{Hg} \rho_{H_2O} g h$$

(h=120 mm Hg)

$$p_{systolic} = p_A = 13.6 \times 1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 120 \frac{m}{100} \times \frac{N \cdot s^2}{kg \cdot m}$$

$$= 16{,}000 \frac{N}{m^2} = 16\,kPa$$

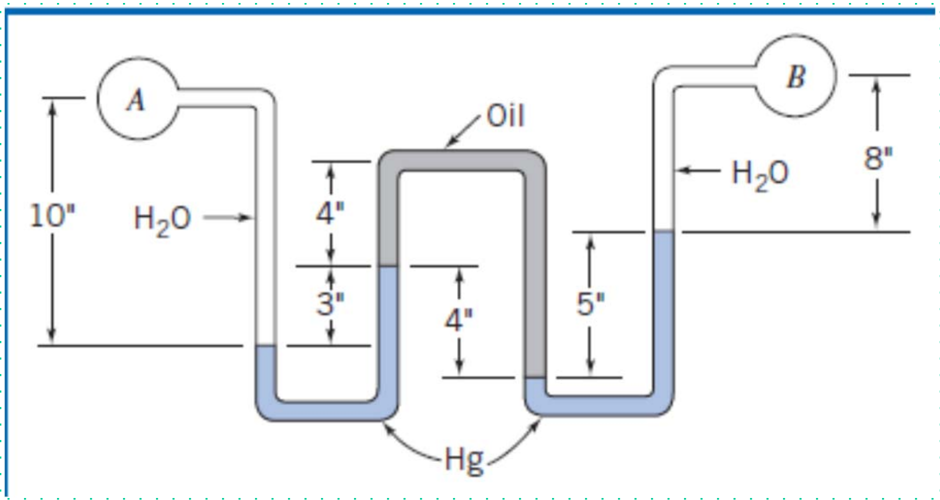


(h=80 mm Hg)

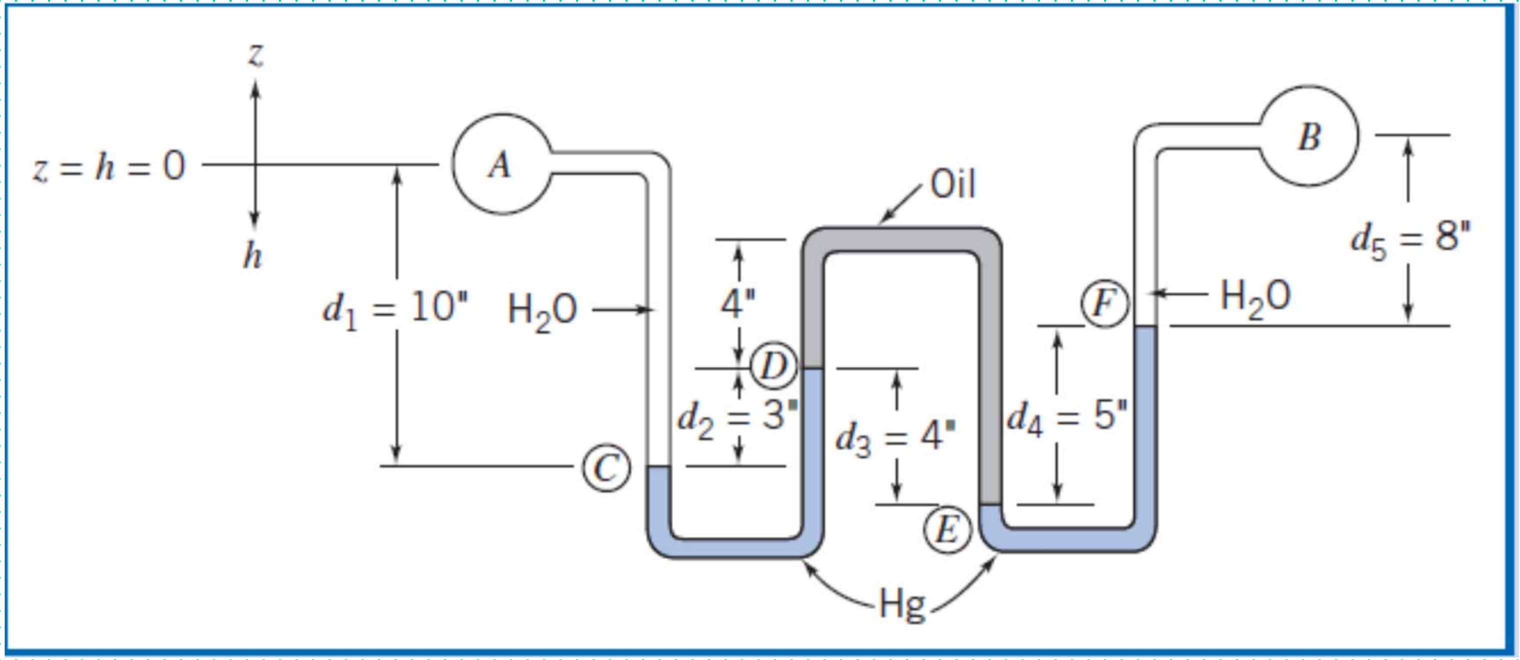
$$p_{diastolic} = 10.67\,kPa$$

Example 3.3 MULTIPLE-LIQUID MANOMETER

Water flows through pipes A and B. Lubricating oil is in the upper portion of the inverted U. Mercury is in the bottom of the manometer bends. Determine the pressure difference, $p_A - p_B$, in units of kPa.



Assumptions: (1) Static fluid.
(2) Incompressible fluid.



$$p_A - p_B = \Delta p = g(\rho_{H_2O} d_5 + \rho_{Hg} d_4 - \rho_{oil} d_3 + \rho_{Hg} d_2 - \rho_{H_2O} d_1)$$

$$p_2 - p_1 = \rho g(h_2 - h_1)$$

$$p_C - p_A = +\rho_{H_2O} g d_1$$

$$p_D - p_C = -\rho_{Hg} g d_2$$

$$p_E - p_D = +\rho_{oil} g d_3$$

$$p_F - p_E = -\rho_{Hg} g d_4$$

$$p_B - p_F = -\rho_{H_2O} g d_5$$

$$
\begin{aligned}
p_A - p_B &= (p_A - p_C) + (p_C - p_D) + (p_D - p_E) + (p_E - p_F) + (p_F - p_B) \\
&= -\rho_{H_2O} g d_1 + \rho_{Hg} g d_2 - \rho_{oil} g d_3 + \rho_{Hg} g d_4 + \rho_{H_2O} g d_5
\end{aligned}
$$

$$
\rho = SG \rho_{H_2O} \text{ with } SG_{Hg} = 13.6 \text{ and } SG_{oil} = 0.88
$$

$$
\begin{aligned}
p_A - p_B &= g(-\rho_{H_2O} d_1 + 13.6\rho_{H_2O} d_2 - 0.88\rho_{H_2O} d_3 + 13.6\rho_{H_2O} d_4 + \rho_{H_2O} d_5) \\
&= g\rho_{H_2O}(-d_1 + 13.6 d_2 - 0.88 d_3 + 13.6 d_4 + d_5) \\
p_A - p_B &= g\rho_{H_2O}(-250 + 1020 - 88 + 1700 + 200)\ \text{mm} \\
p_A - p_B &= g\rho_{H_2O} \times 2582\ \text{mm} \\
&= 9.81 \frac{\text{m}}{\text{s}^2} \times 1000 \frac{\text{kg}}{\text{m}^3} \times 2582 \frac{\text{m}}{1000} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \\
p_A - p_B &= 25.33\ \text{kPa}
\end{aligned}
$$

Thi
and
mat

Pressure Variation in a Static Fluid

✓ Compressible Fluid: Ideal Gas

$$dp = -\rho g \, dz \qquad\qquad p = \rho R T$$

Need additional information, e.g., $T(z)$ for atmosphere

$$\boxed{p = p_0 \left(1 - \frac{mz}{T_0}\right)^{g/mR} = p_0 \left(\frac{T}{T_0}\right)^{g/mR}}$$

Hydrostatic Force on Submerged Surfaces

✓ Plane Submerged Surface

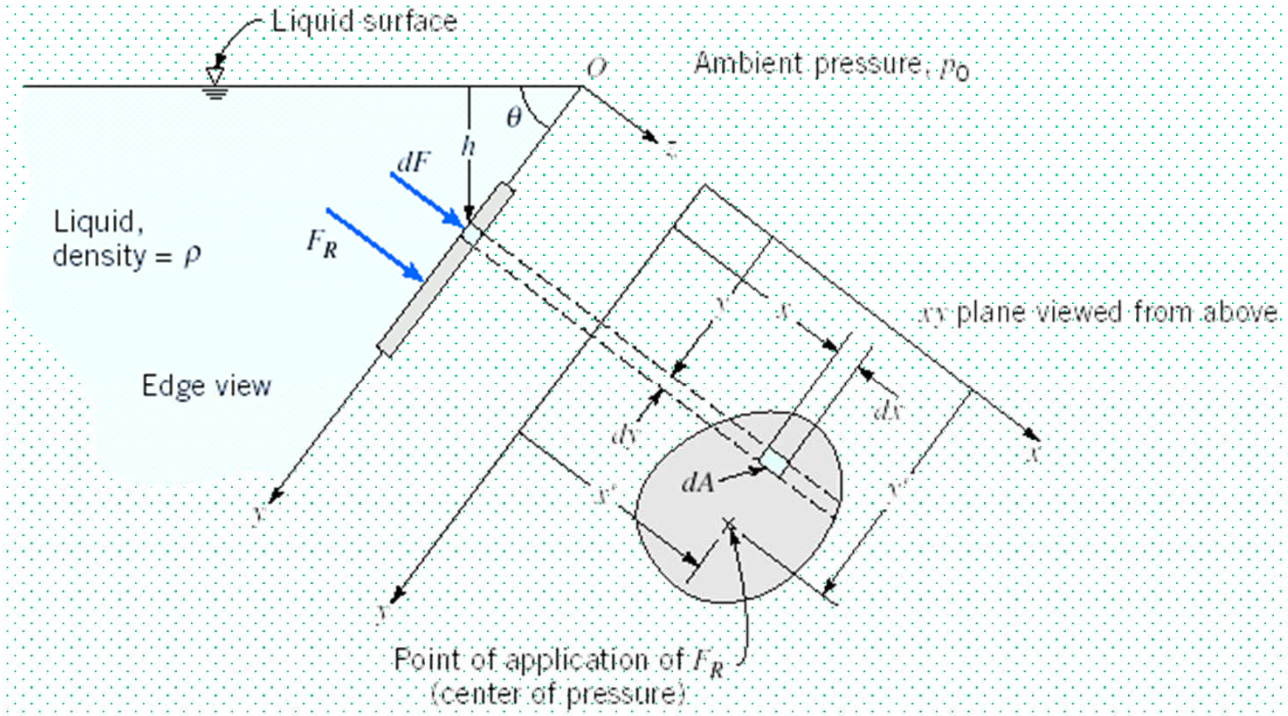


Fig. 3.5 Plane submerged surface.

Hydrostatic Force on Submerged Surfaces

✓ Plane Submerged Surface

$$F_R = \int_A p\, dA$$

$$y' F_R = \int_A y p\, dA \qquad\qquad x' F_R = \int_A x p\, dA$$

We can find F_R, and y' and x', by integrating, or …

Hydrostatic Force on Submerged Surfaces

- ✓ **Plane Submerged Surface**
  - **Algebraic Equations – Total Pressure Force**

$$F_R = p_c A$$

$$y' = y_c + \frac{\rho g \sin\theta \, I_{\hat{x}\hat{x}}}{F_R} \qquad x' = x_c + \frac{\rho g \sin\theta \, I_{\hat{x}\hat{y}}}{F_R}$$

Hydrostatic Force on Submerged Surfaces

✓ **Plane Submerged Surface**

- **Algebraic Equations – Net Pressure Force**

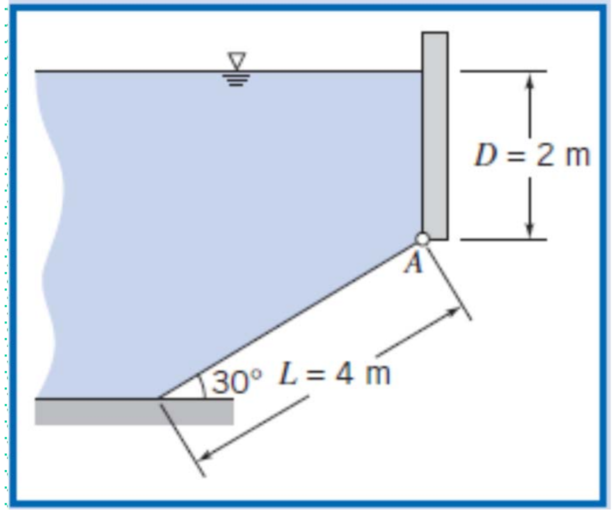
$$F_R = p_{c_{\text{gage}}} A$$

$$\boxed{y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c}}$$

$$\boxed{x' = x_c + \frac{I_{\hat{x}\hat{y}}}{A y_c}}$$

Example 3.5 RESULTANT FORCE ON INCLINED PLANE SUBMERGED SURFACE

The inclined surface shown, hinged along edge A, is 5 m wide. Determine the resultant force, F_R, of the water and the air on the inclined surface.



Solution:

In order to completely determine F_R, we need to find

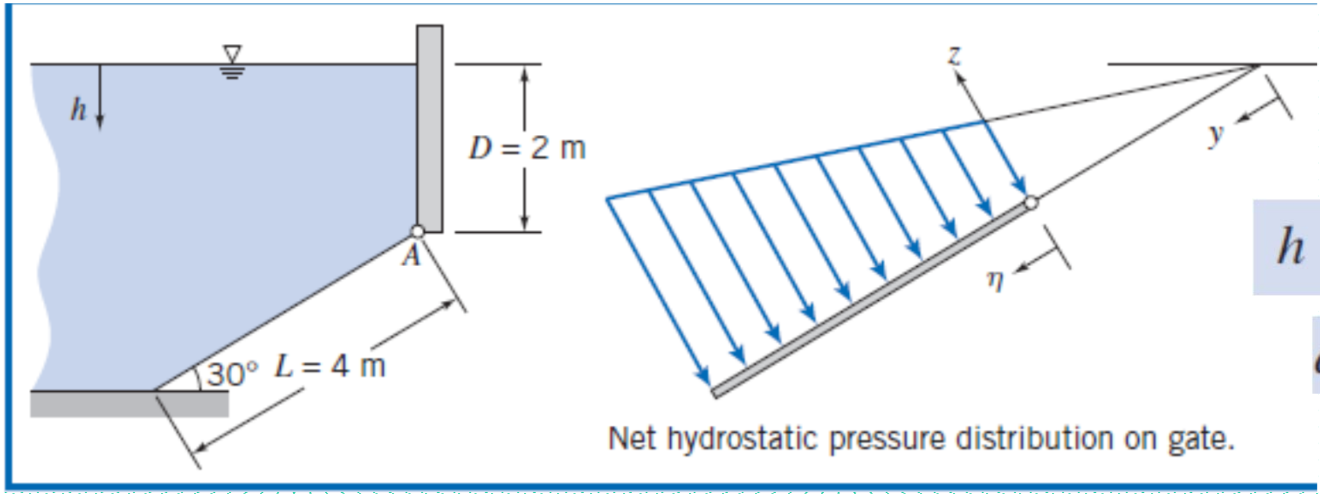
(a) the magnitude and

(b) the line of action of the force (the direction of the force is perpendicular to the surface).

We will solve this problem by using

(i) direct integration and

(ii) the algebraic equations.



Net hydrostatic pressure distribution on gate.

$$h = D + \eta \sin 30^\circ$$

$$dA = w\, d\eta$$

$$p = p_0 + \rho g h \quad F_R = \int_A p\, dA \quad \eta' F_R = \int_A \eta p\, dA \quad x' F_R = \int_A x p\, dA$$

$$F_R = \int_A p dA = \int_0^L \rho g (D + \eta \sin 30^\circ) w\, d\eta$$

$$= \rho g w \left[D\eta + \frac{\eta^2}{2} \sin 30^\circ \right]_0^L = \rho g w \left[DL + \frac{L^2}{2} \sin 30^\circ \right]$$

$$= 999 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 5\text{m} \left[2\,\text{m} \times 4\,\text{m} + \frac{16\,\text{m}^2}{2} \times \frac{1}{2} \right] \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$$

$$F_R = 588\,\text{kN}$$

$$\eta' F_R = \int_A \eta p \, dA$$

$$\eta' = \frac{1}{F_R}\int_A \eta p\,dA = \frac{1}{F_R}\int_0^L \eta p w \, d\eta = \frac{\rho g w}{F_R}\int_0^L \eta(D + \eta \sin 30^\circ)\, d\eta$$

$$= \frac{\rho g w}{F_R}\left[\frac{D\eta^2}{2} + \frac{\eta^3}{3}\sin 30^\circ\right]_0^L = \frac{\rho g w}{F_R}\left[\frac{DL^2}{2} + \frac{L^3}{3}\sin 30^\circ\right]$$

$$= 999\frac{\text{kg}}{\text{m}^3} \times 9.8\frac{\text{m}}{\text{s}^2} \times \frac{5\,\text{m}}{5.88\times 10^5\,\text{N}}\left[\frac{2\,\text{m}\times 16\,\text{m}^2}{2} + \frac{64\,\text{m}^3}{3}\times\frac{1}{2}\right]\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$\eta' = 2.22\,\text{m} \quad \text{and} \quad y' = \frac{D}{\sin 30^\circ} + \eta' = \frac{2\,\text{m}}{\sin 30^\circ} + 2.22\,\text{m} = 6.22\,\text{m} \longleftarrow$$

$$x' = \frac{1}{F_R}\int_A xp\, dA$$

$$x' = \frac{1}{F_R}\int_A \frac{w}{2} p\, dA = \frac{w}{2F_R}\int_A p\, dA = \frac{w}{2} = 2.5\,\text{m} \longleftarrow$$

Algebraic Equations

$$F_R = p_c A = \rho g h_c A = \rho g \left(D + \frac{L}{2}\sin 30^\circ \right) Lw$$

$$F_R = \rho g w \left[DL + \frac{L^2}{2}\sin 30^\circ \right]$$

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c}$$

$$x' = x_c + \frac{I_{\hat{x}\hat{y}}}{A y_c}$$

$$y_c = \frac{D}{\sin 30^\circ} + \frac{L}{2} = \frac{2\,\text{m}}{\sin 30^\circ} + \frac{4\,\text{m}}{2} = 6\,\text{m}$$

$$A = Lw = 4\,\text{m} \times 5\,\text{m} = 20\,\text{m}^2$$

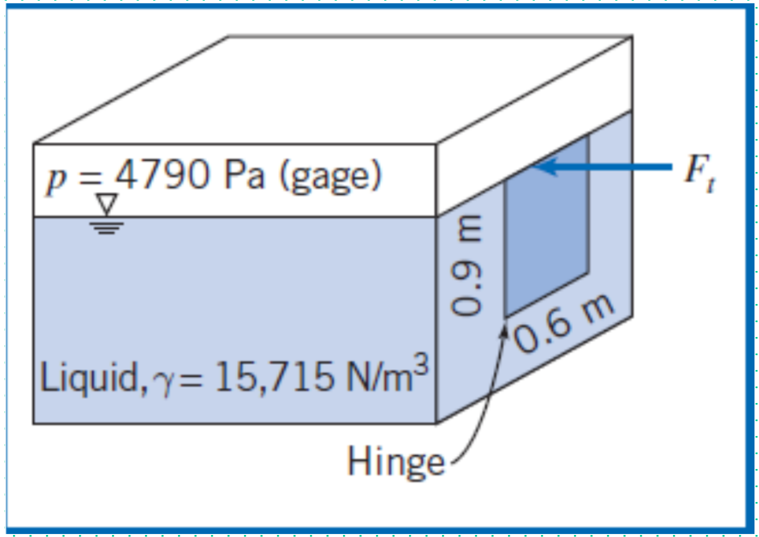
$$I_{\hat{x}\hat{x}} = \frac{1}{12} w L^3 = \frac{1}{12} \times 5\,\text{m} \times (4\,\text{m})^3 = 26.7\,\text{m}^2$$

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c} = 6\,\text{m} + 26.7\,\text{m}^4 \times \frac{1}{20\,\text{m}^2} \times \frac{1}{6\,\text{m}} = 6.22\,\text{m}$$

$$I_{\hat{x}\hat{y}} = 0 \text{ and } x' = x_c = 2.5\,\text{m}.$$

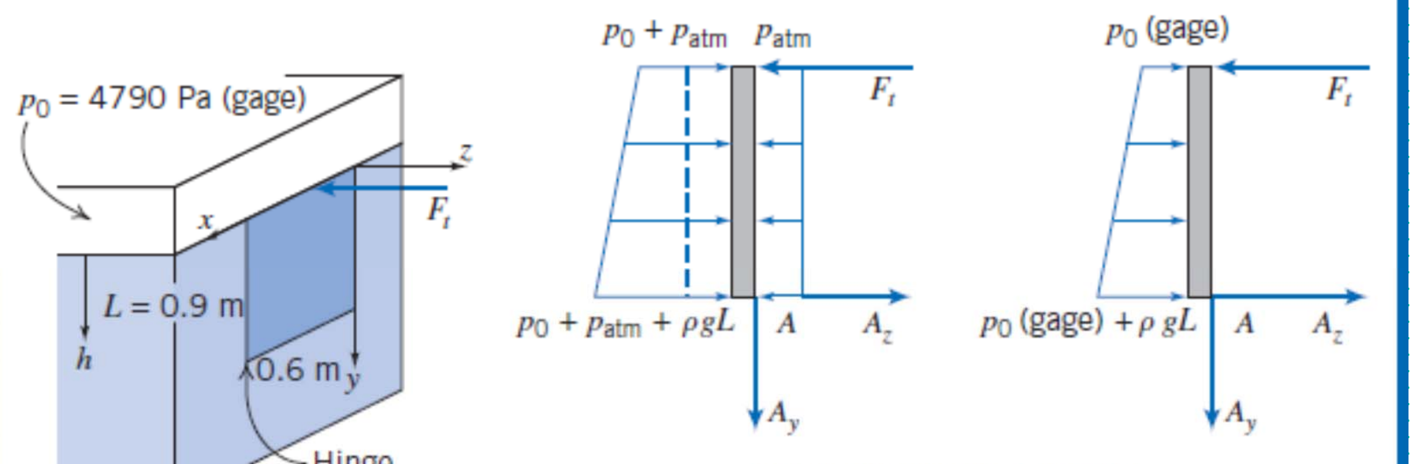
Example 3.6 FORCE ON VERTICAL PLANE SUBMERGED SURFACE WITH NONZERO GAGE PRESSURE AT FREE SURFACE

The door shown in the side of the tank is hinged along its bottom edge. A pressure of 4790 Pa (gage) is applied to the liquid free surface. Find the force, F_t, required to keep the door closed.

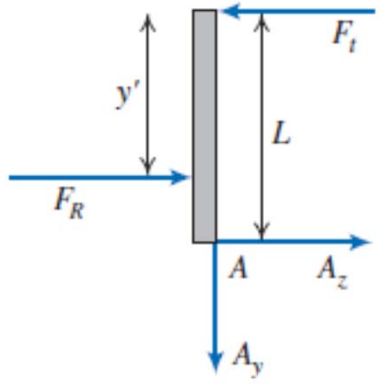


Solution:

This problem requires a free-body diagram (FBD) of the door. The pressure distributions on the inside and outside of the door will lead to a net force (and its location) that will be included in the FBD.



Free-body diagrams of door



Force free-body diagram

$$F_R = (p_0 + \rho g h_c)A = \left(p_0 + \gamma \frac{L}{2}\right) bL$$

$$y' = y_c + \frac{\rho g \sin 90^\circ \, I_{\hat{x}\hat{x}}}{F_R} = \frac{L}{2} + \frac{\gamma b L^3/12}{\left(p_0 + \gamma \frac{L}{2}\right) bL} = \frac{L}{2} + \frac{\gamma L^2/12}{\left(p_0 + \gamma \frac{L}{2}\right)}$$

$$\sum M_A = F_t L - F_R(L - y') = 0 \qquad \text{or} \qquad F_t = F_R\left(1 - \frac{y'}{L}\right)$$

$$F_t = \left(p_0 + \gamma \frac{L}{2}\right) bL \left[1 - \frac{1}{2} - \frac{\gamma L^2/12}{\left(p_0 + \gamma \frac{L}{2}\right)}\right]$$

$$F_t = \left(p_0 + \gamma \frac{L}{2}\right) \frac{bL}{2} + \gamma \frac{bL^2}{12} = \frac{p_0 bL}{2} + \frac{\gamma bL^2}{6}$$

$$= 4790 \frac{\text{N}}{\text{m}^2} \times 0.6\,\text{m} \times 0.9\,\text{m} \times \frac{1}{2} + 15,715 \frac{\text{N}}{\text{m}^3} \times 0.6\,\text{m} \times 0.81\,\text{m}^2 \times \frac{1}{6}$$

$$F_t = 2566\,\text{N} \longleftarrow$$

Hydrostatic Force on Submerged Surfaces

✓Curved Submerged Surface

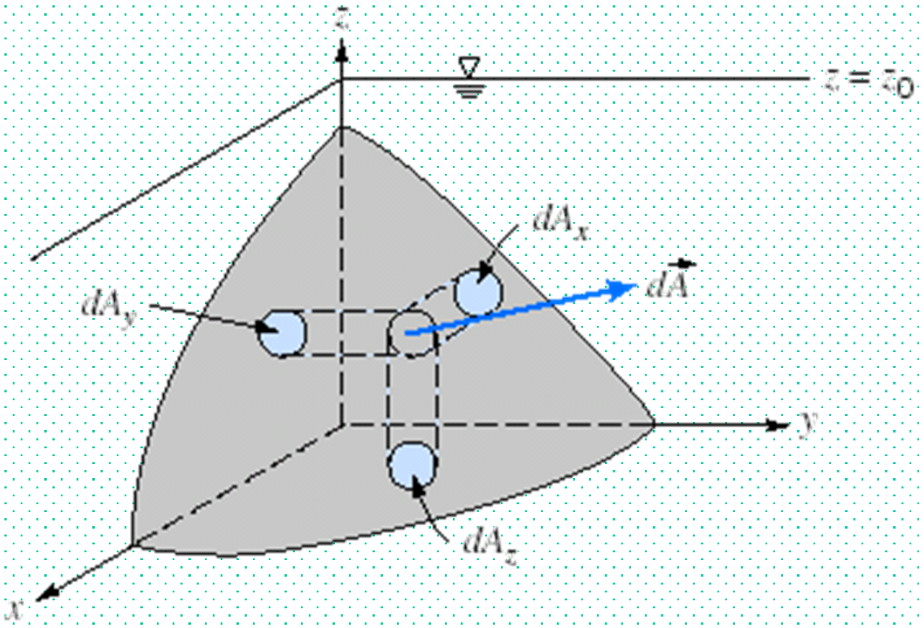


Fig. 3.7 Curved submerged surface.

Hydrostatic Force on Submerged Surfaces

✓ Curved Submerged Surface

- Horizontal Force = Equivalent Vertical Plane Force
- Vertical Force = Weight of Fluid Directly Above (+ Free Surface Pressure Force)

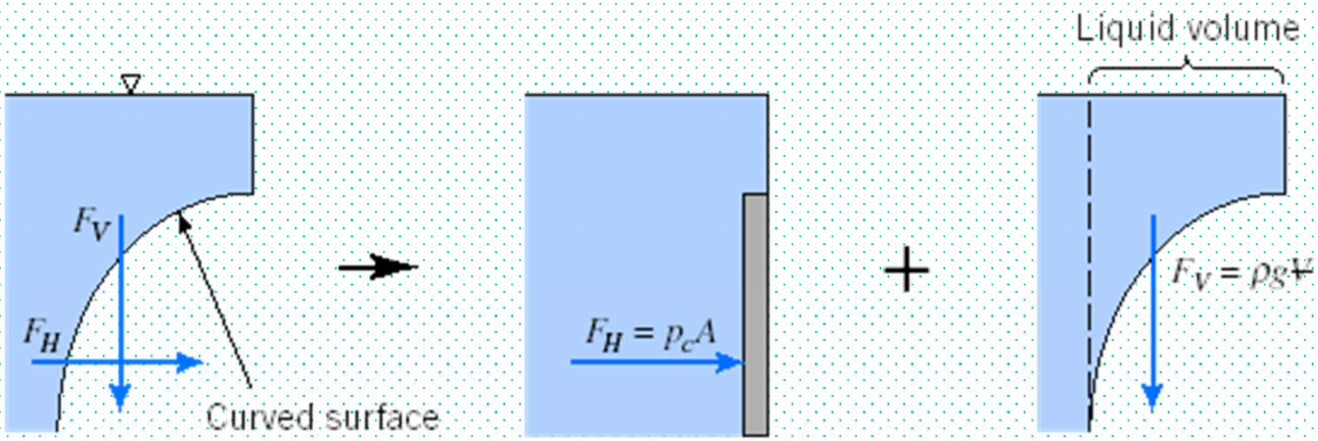
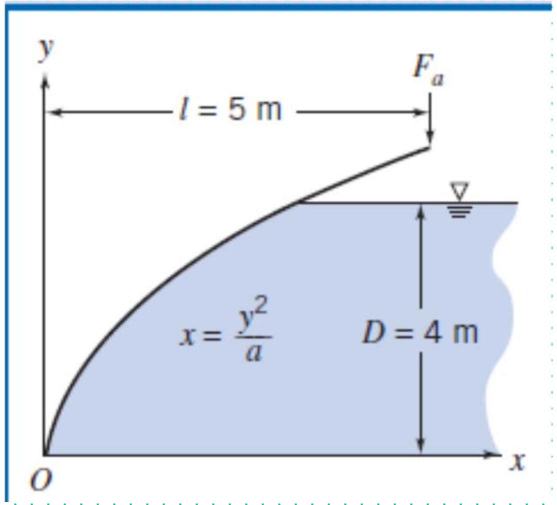


Fig. 3.8 Forces on curved submerged surface.

$$F_H = p_c A \quad \text{and} \quad F_V = \rho g V\!\!\!\!-$$

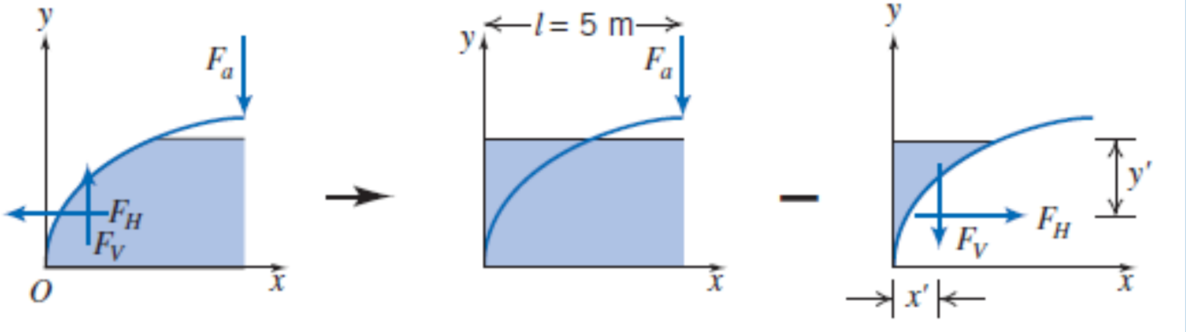
Example 3.7 FORCE COMPONENTS ON A CURVED SUBMERGED SURFACE

The gate shown is hinged at O and has constant width, w=5 m. The equation of the surface is $x=y^2/a$, where a=4 m. The depth of water to the right of the gate is D=4 m. Find the magnitude of the force, F_a, applied as shown, required to maintain the gate in equilibrium if the weight of the gate is neglected.



Solution:

We will take moments about point O after finding the magnitudes and locations of the horizontal and vertical forces due to the water. The free body diagram (FBD) of the system is shown above in part (a).



(a) System FBD (b) Null fluid forces (c) Fluid forces

$$F_H = p_c A \qquad y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c} \qquad F_V = \rho g \forall \qquad x' = \text{water center of gravity}$$

For F_H, the centroid, area, and second moment of the equivalent vertical flat plate

$$y_c = h_c = D/2, \qquad A = Dw, \text{ and } I_{\hat{x}\hat{x}} = wD^3/12.$$

$$F_H = p_c A = \rho g h_c A$$

$$= \rho g \frac{D}{2} D w = \rho g \frac{D^2}{2} w = 999 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times \frac{(4\,\text{m}^2)}{2} \times 5\,\text{m} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$$

$$F_H = 392\,\text{kN}$$

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{Ay_c}$$

$$= \frac{D}{2} + \frac{wD^3/12}{wDD/2} = \frac{D}{2} + \frac{D}{6}$$

$$y' = \frac{2}{3}D = \frac{2}{3} \times 4\,\text{m} = 2.67\,\text{m}$$

For F_V, we need to compute the weight of water "above" the gate. To do this we define a differential column of volume *(D-y)w dx* and integrate

$$F_V = \rho g \forall = \rho g \int_0^{D^{2/a}} (D-y)w\,dx = \rho g w \int_0^{D^{2/a}} (D - \sqrt{a}x^{1/2})dx$$

$$= \rho g w \left[Dx - \frac{2}{3}\sqrt{a}x^{3/2} \right]_0^{D^{3/a}} = \rho g w \left[\frac{D^3}{a} - \frac{2}{3}\sqrt{a}\frac{D^3}{a^{3/2}} \right] = \frac{\rho g w D^3}{3a}$$

$$F_V = 999\,\frac{\text{kg}}{\text{m}^3} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 5\,\text{m} \times \frac{(4)^3\text{m}^3}{3} \times \frac{1}{4\,\text{m}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 261\,\text{kN}$$

$$x'F_V = \rho g \int_0^{D^{2/a}} x(D-y)w\,dx = \rho g w \int_0^{D^{2/a}} (D - \sqrt{a}x^{3/2})dx$$

$$x'F_V = \rho g w \left[\frac{D}{2}x^2 - \frac{2}{5}\sqrt{a}x^{5/2}\right]_0^{D^{2/a}} = \rho g w \left[\frac{D^5}{2a^2} - \frac{2}{5}\sqrt{a}\frac{D^5}{a^{5/2}}\right] = \frac{\rho g w D^5}{10a^2}$$

$$x' = \frac{\rho g w D^5}{10a^2 F_V} = \frac{3D^2}{10a} = \frac{3}{10} \times \frac{(4)^2\,\mathrm{m}^2}{4\,\mathrm{m}} = 1.2\,\mathrm{m}$$

Determine the fluid forces, take moments about O,

$$\sum M_O = -lF_a + x'F_V + (D-y')F_H = 0$$

$$F_a = \frac{1}{l}[x'F_V + (D-y')F_H]$$

$$= \frac{1}{5\,\mathrm{m}}[1.2\,\mathrm{m} \times 261\,\mathrm{kN} + (4-2.67)\,\mathrm{m} \times 392\,\mathrm{kN}]$$

$$F_a = 167\,\mathrm{kN} \qquad \xleftarrow{\qquad\qquad F_a \qquad\qquad}$$

Buoyancy

If an object is immersed in a liquid, or floating on its surface, the net vertical force acting on it due to liquid pressure is termed ***buoyancy***.

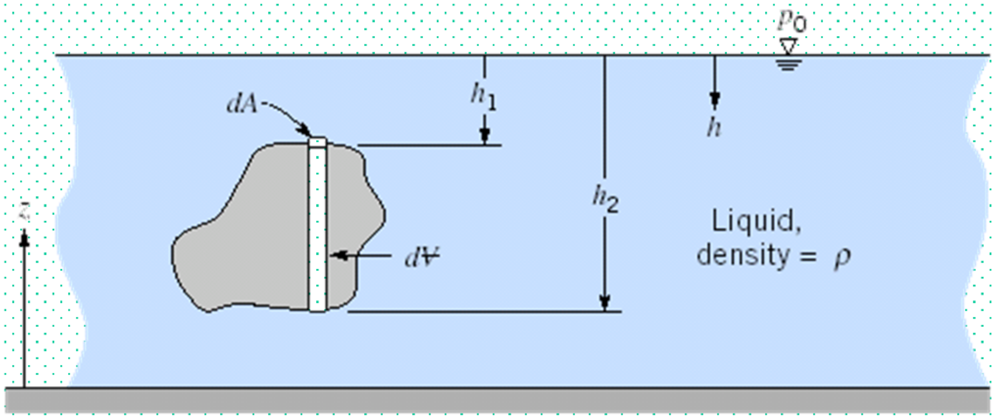


Fig. 3.9 Immersed body in static liquid.

$$dF_z = (p_0 + \rho g h_2)\, dA - (p_0 + \rho g h_1)\, dA$$

$$dF_z = \rho g (h_2 - h_1)\, dA$$

Buoyancy

$$F_{\text{buoyancy}} = \rho g \forall$$

This relation reportedly was used by Archimedes in 220 B.C. to determine the gold content in the crown of King Hiero II. Consequently, it is often called "Archimedes' Principle."

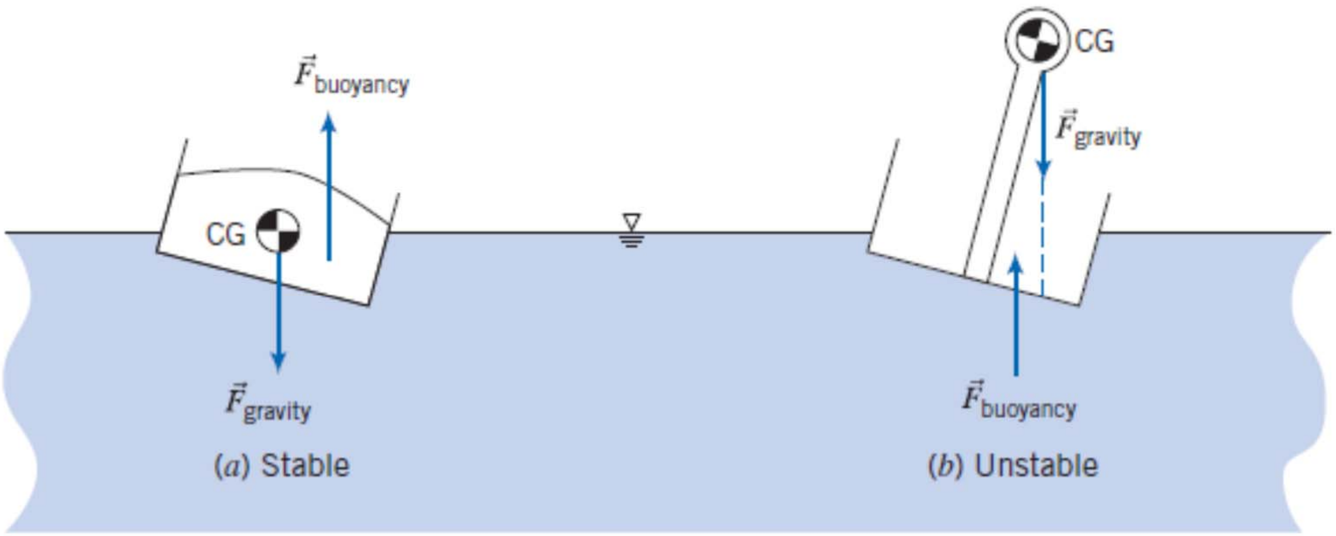
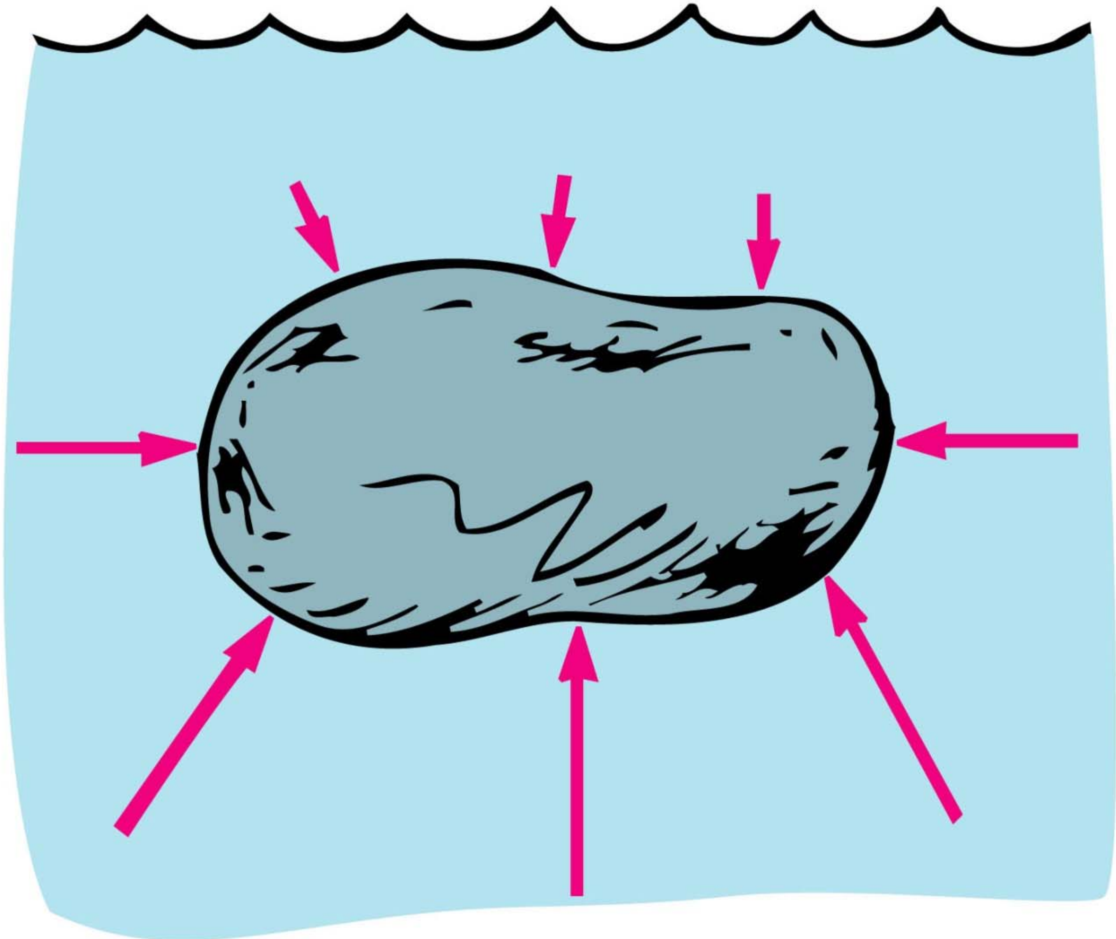


Fig. 3.10 Stability of floating bodies.

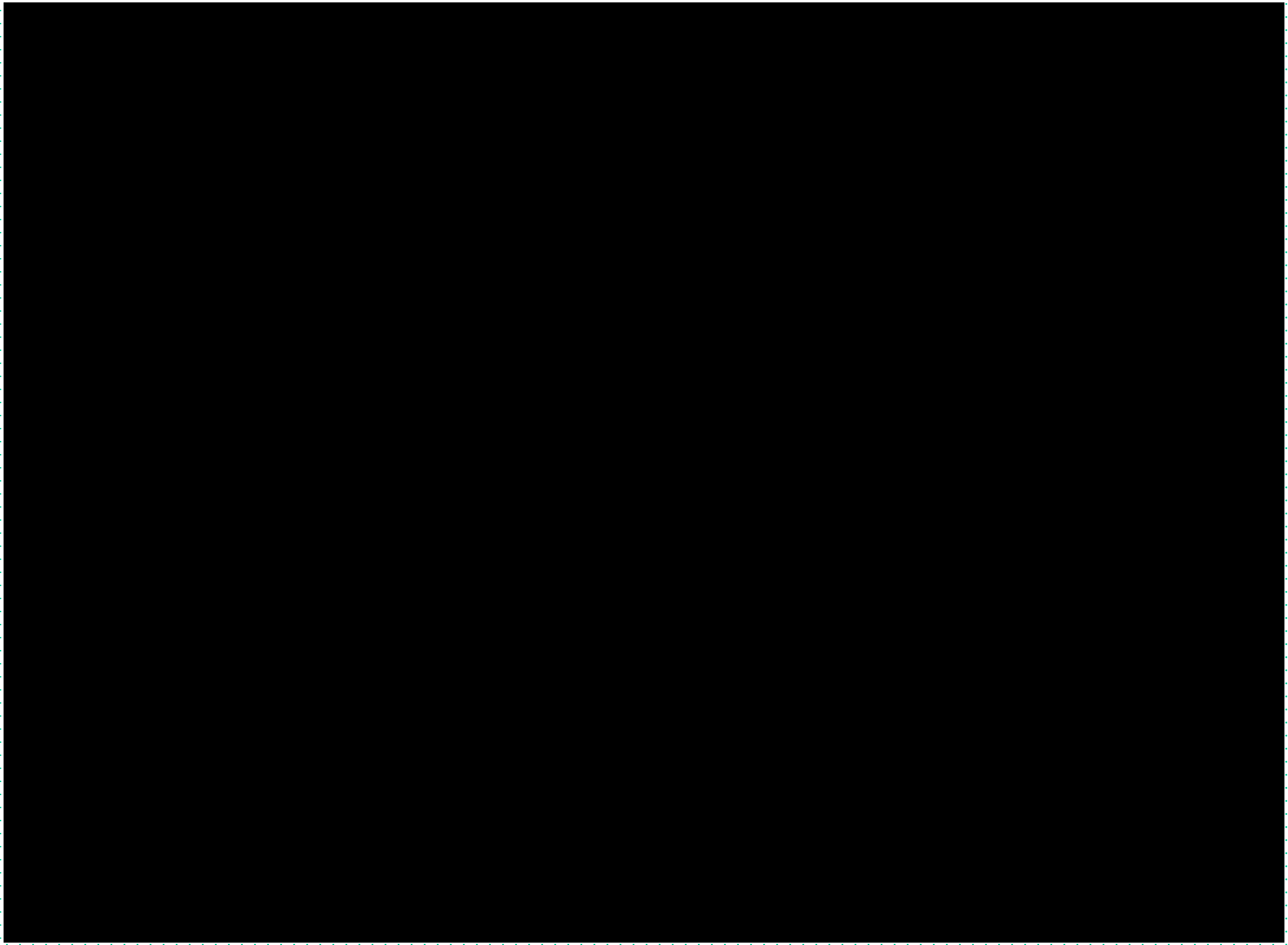
Buoyancy



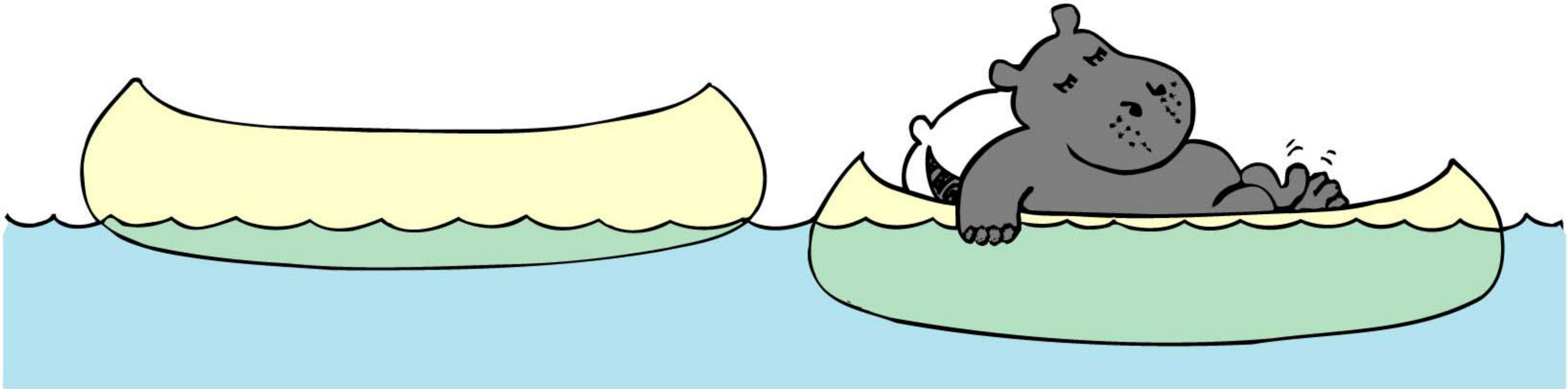
Net upward force is called the buoyant force!!!

Easier to lift a rock in water!!

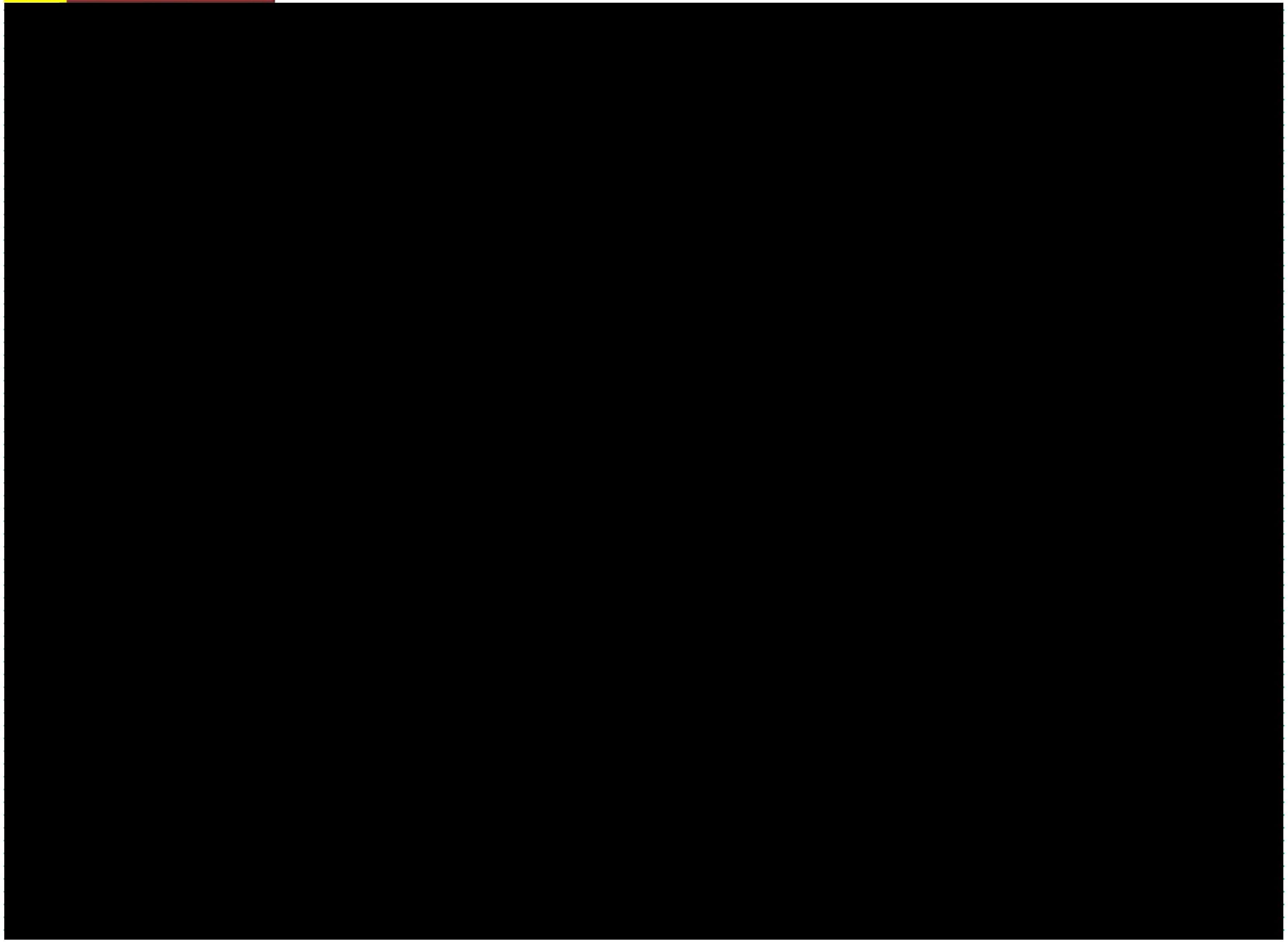
Buoyancy



Flotation

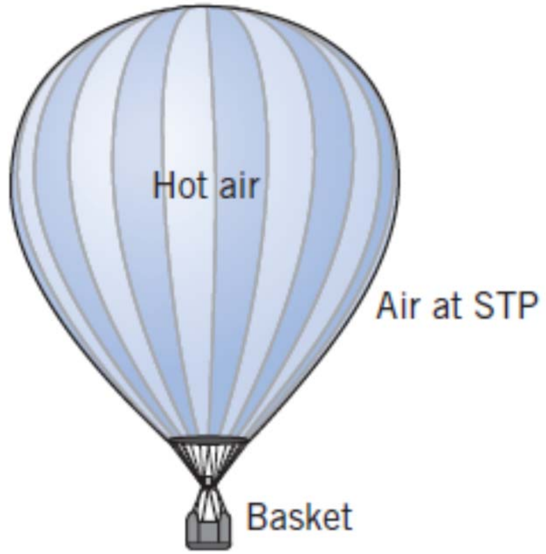


Stability



Example 3.8 BUOYANCY FORCE IN A HOT AIR BALLOON

A hot air balloon (approximated as a sphere of diameter 15 m) is to lift a basket load of 2670 N. To what temperature must the air be heated in order to achieve liftoff?



Assumptions: (1) Ideal gas.
(2) Atmospheric pressure throughout

$$F_{\text{buoyancy}} = \rho g \forall \qquad \sum F_y = 0 \qquad p = \rho R T$$

Summing vertical forces

$$\sum F_y = F_{\text{buoyancy}} - W_{\text{hot air}} - W_{\text{load}} = \rho_{\text{atm}} g \forall - \rho_{\text{hot air}}\, g \forall - W_{\text{load}} = 0$$

$$\rho_{\text{hot air}} = \rho_{\text{atm}} - \frac{W_{\text{load}}}{g\forall} = \rho_{\text{atm}} - \frac{6W_{\text{load}}}{\pi d^3 g}$$

$$= 1.227\frac{\text{kg}}{\text{m}^3} - 6 \times \frac{2670\,\text{N}}{\pi(15)^3\,\text{m}^3} \times \frac{\text{s}^2}{81\,\text{m}} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}$$

$$\rho_{\text{hot air}} = (1.227 - 0.154)\frac{\text{kg}}{\text{m}^3} = 1.073\frac{\text{kg}}{\text{m}^3}$$

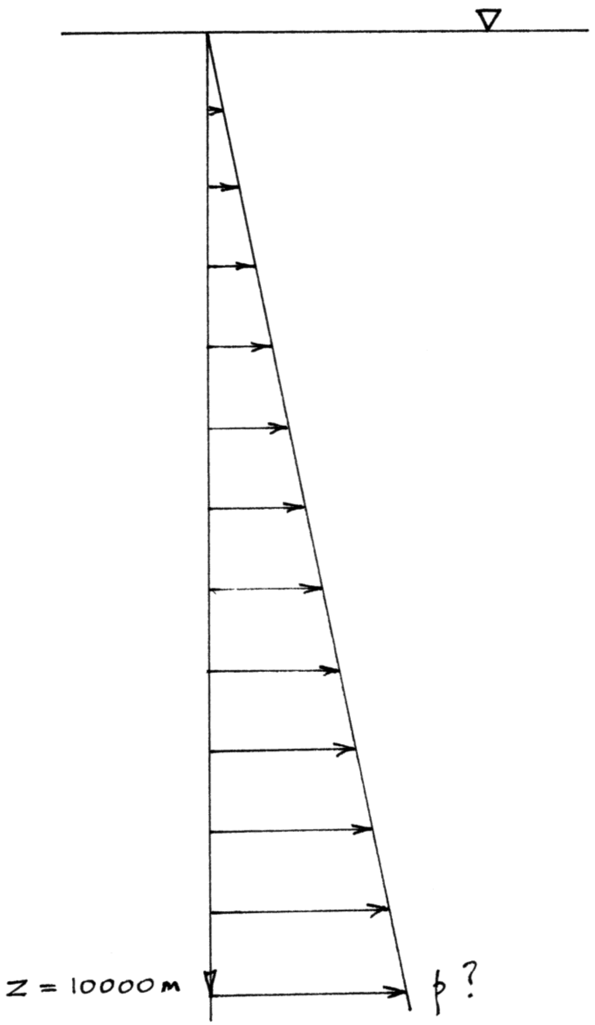
to obtain the temperature of this hot air, use the ideal gas equation;

$$\frac{p_{\text{hot air}}}{\rho_{\text{hot air}} R T_{\text{hot air}}} = \frac{p_{\text{atm}}}{\rho_{\text{atm}} R T_{\text{atm}}}$$

$$T_{\text{hot air}} = T_{\text{atm}}\frac{\rho_{\text{atm}}}{\rho_{\text{hot air}}} = (273 + 15)^\circ\text{K} \times \frac{1.227}{1.073} = 329^\circ\text{K}$$

$$T_{\text{hot air}} = 56^\circ\text{C} \longleftarrow$$

Hydrostatics – Interactive question



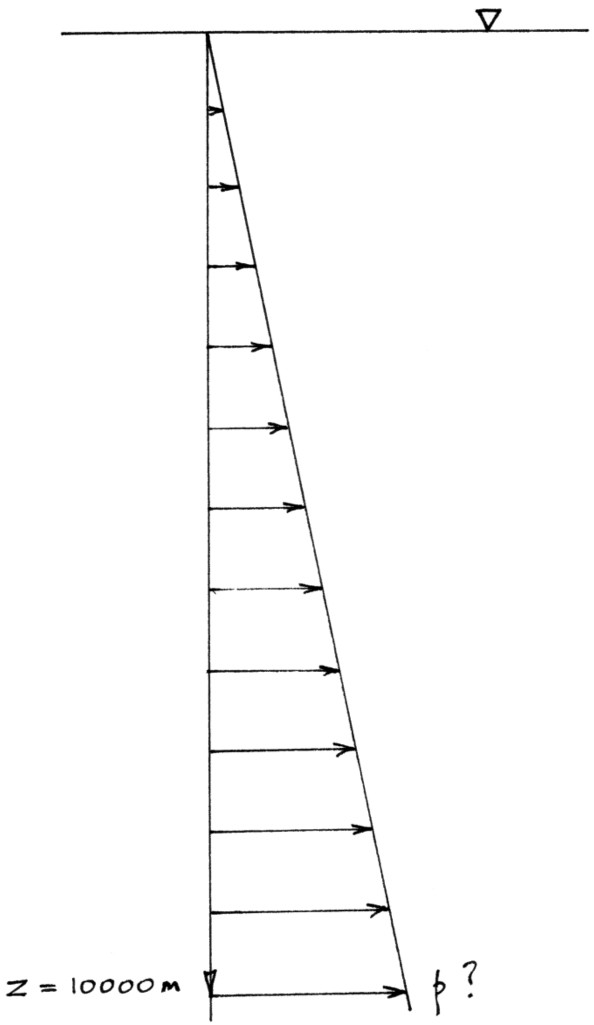
The density of seawater is 1030 kg/m^3. Which of the following values is correct for the hydrostatic pressure at a depth of 10 000 m?

(a) 101 bar

(b) 1010 bar

(c) 10101 bar

Hydrostatics – Interactive question



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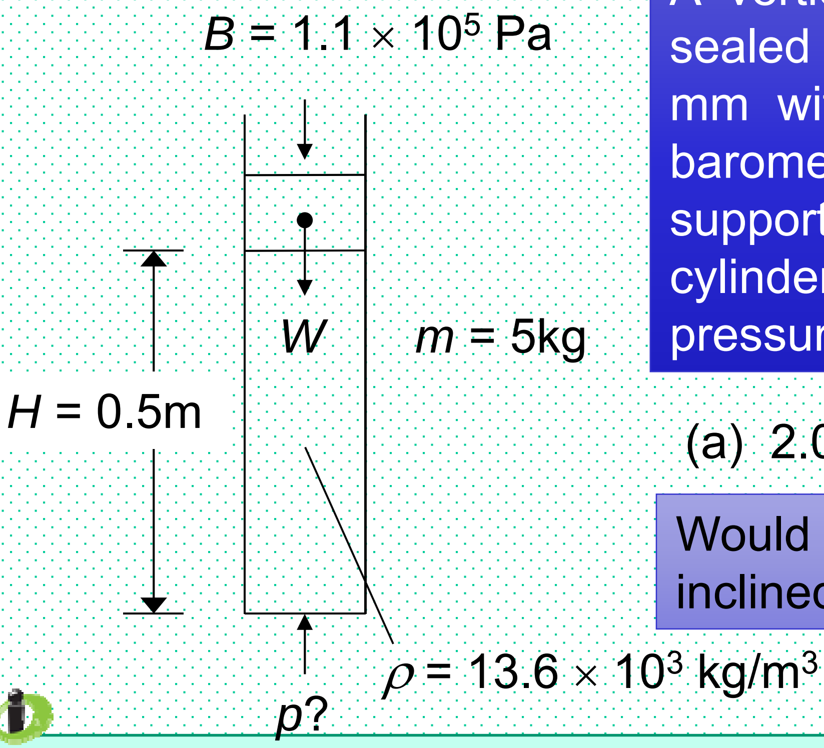
Hydrostatics – Interactive question

Principle: $\Delta p = \rho g \Delta z$

Application: Calculation of pressure due to a column of liquid supporting a weight and subject to external pressure.



A vertical cylinder of inside diameter 50 mm is sealed at the bottom and filled to a depth of 500 mm with mercury of relative density 13.6. The barometric pressure is 1.1 bar. If the mercury supports a piston of mass 5 kg (a perfect fit in the cylinder with no leakage or friction), what is the pressure at the bottom of the cylinder?

(a) 2.02 bar (b) 1.32 bar (c) 0.2 bar

Would the answer be different if the cylinder were inclined to the vertical?

(a) yes (b) no

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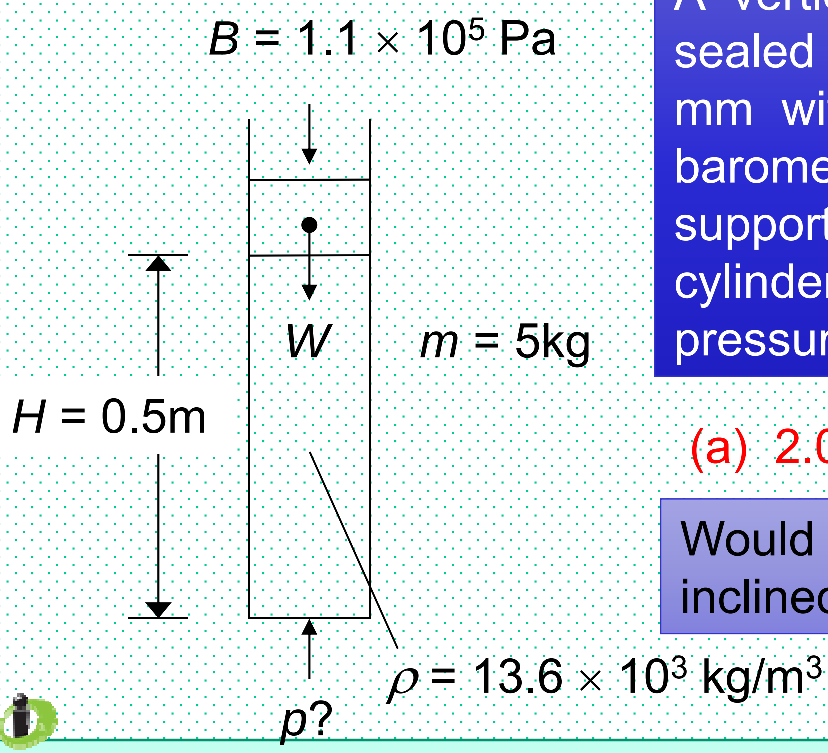

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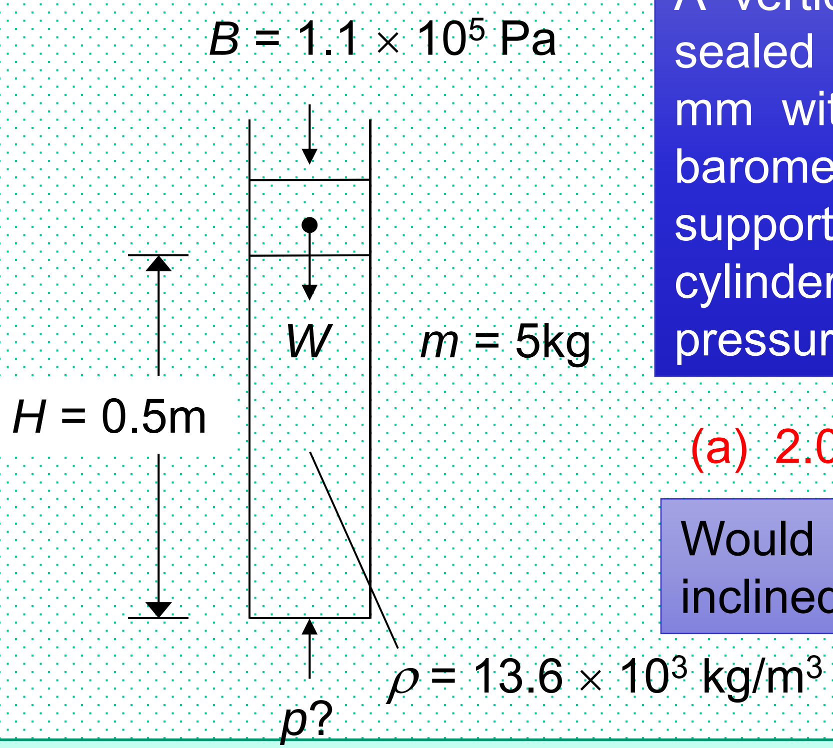

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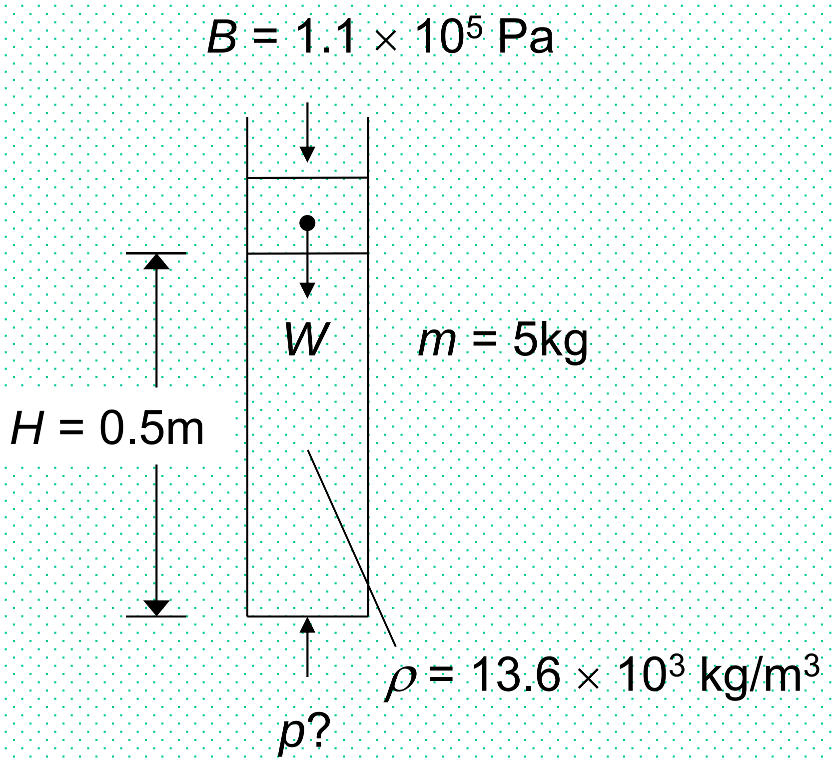


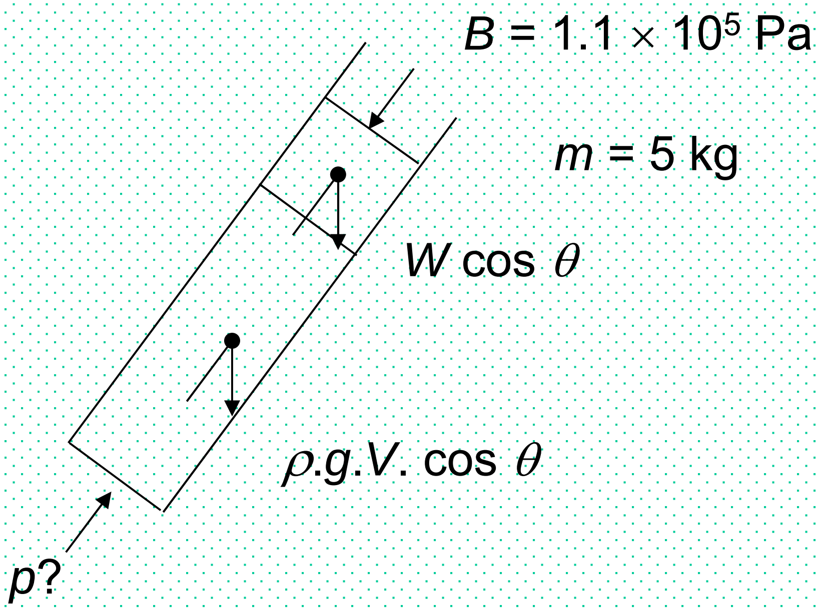
$$p\left(\frac{\pi D^2}{4}\right) - B\left(\frac{\pi D^2}{4}\right) - W - \rho g\left(\frac{\pi D^2}{4}\right)H = 0$$

$$p = B + W\left(\frac{4}{\pi D^2}\right) + \rho g H$$

$$p = 1.1x10^5 + 5\left(\frac{4}{\pi .0.05^2}\right) + 13.6x10^3 x9.81x0.5$$

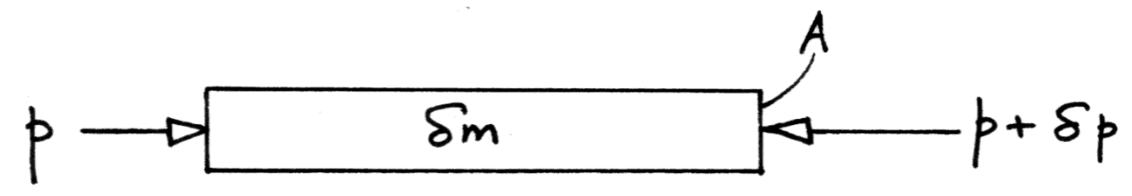
$p = 2.02$ bar

Hydrostatics – Interactive question



$$p = B + W\left(\frac{4}{\pi D^2}\right)\cos\theta + \rho g H \cos\theta$$

Manometry (pressure measurement)



$$p.A \;-\; (p + \delta p).A = \delta m.a = 0 \qquad \Delta\delta p = 0$$

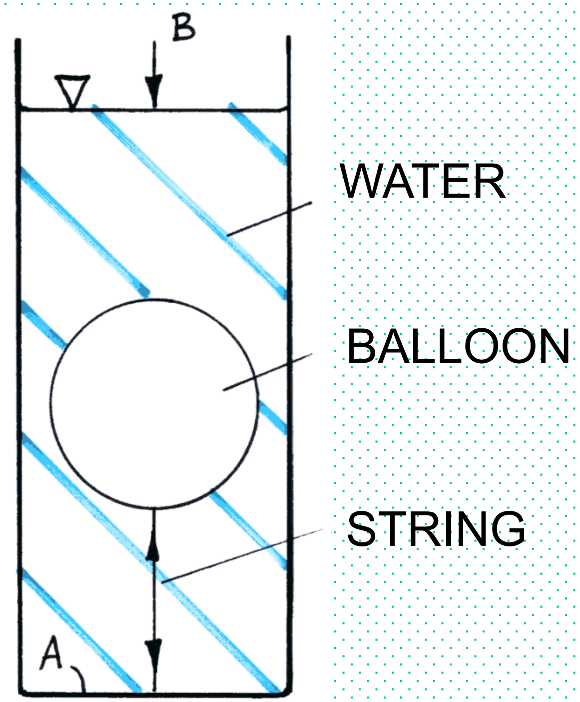
FLUID AT REST – ISOBARS HORIZONTAL

(EQUAL PRESSURES AT SAME LEVEL IN SINGLE FLUID)

Hydrostatics – Interactive question

Principle: Hydrostatic pressure increases in proportion to depth in a uniform density fluid at rest

Application: Pressure variation in a vessel containing an object submerged within a liquid.



With the balloon held under water, is the pressure at A

(a) Higher than without the balloon

(b) Unchanged

(c) Lower than without the balloon?

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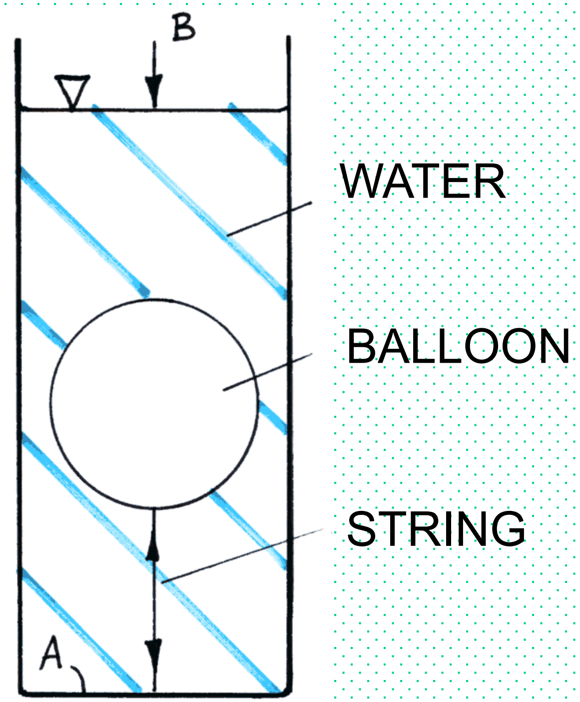

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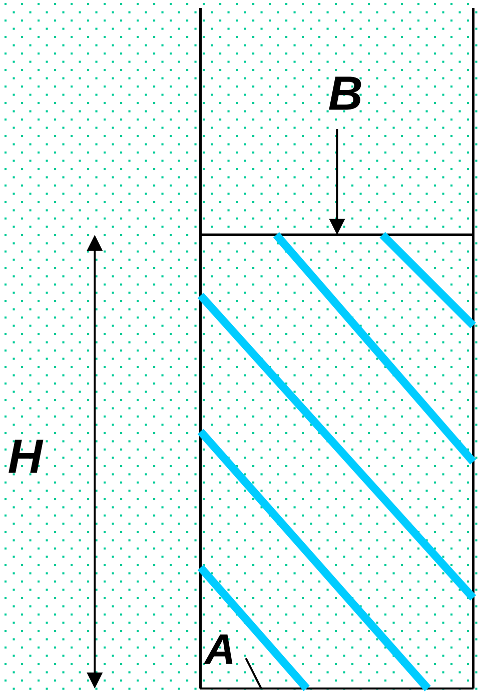
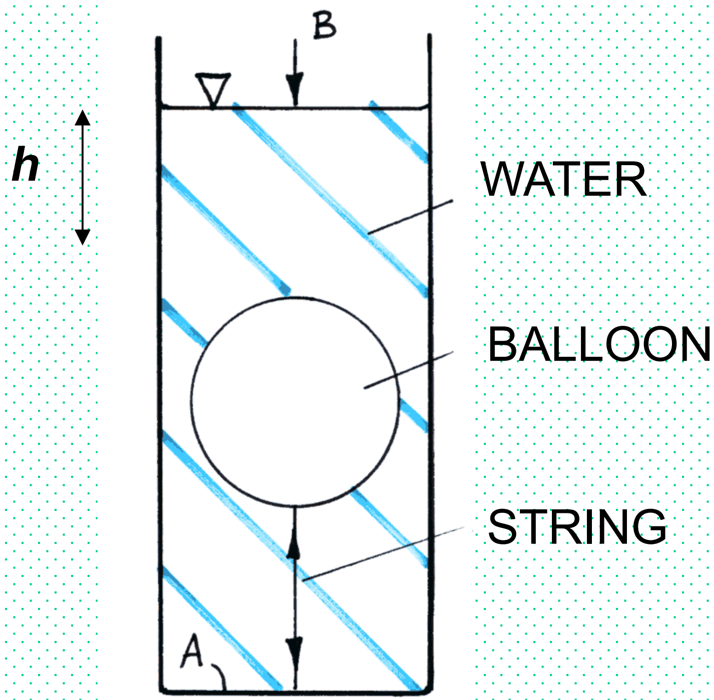




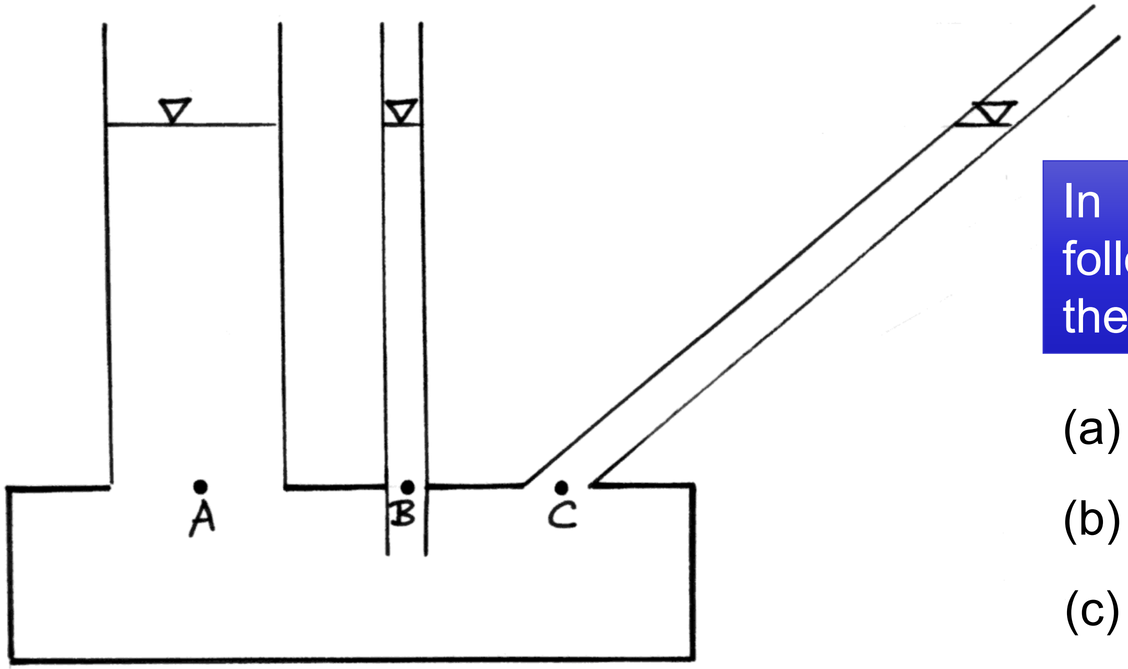
$p_A = \rho\, g\, H$

$p_A = \rho\, g\, (H + h)$

Hydrostatics – Interactive question

Principle: Lines of constant pressure (isobars) in a continuous volume of a constant-density fluid at rest are horizontal

Application: Pressure variation in vertical and inclined tubes



In the diagram, which of the following conditions applies to the pressure at A, B, and C?

(a) $p_A > p_B$ $p_B < p_C$

(b) $p_A = p_B$ $p_B < p_C$

(c) $p_A = p_B = p_C$

(d) $p_A > p_B$ $p_B > p_C$

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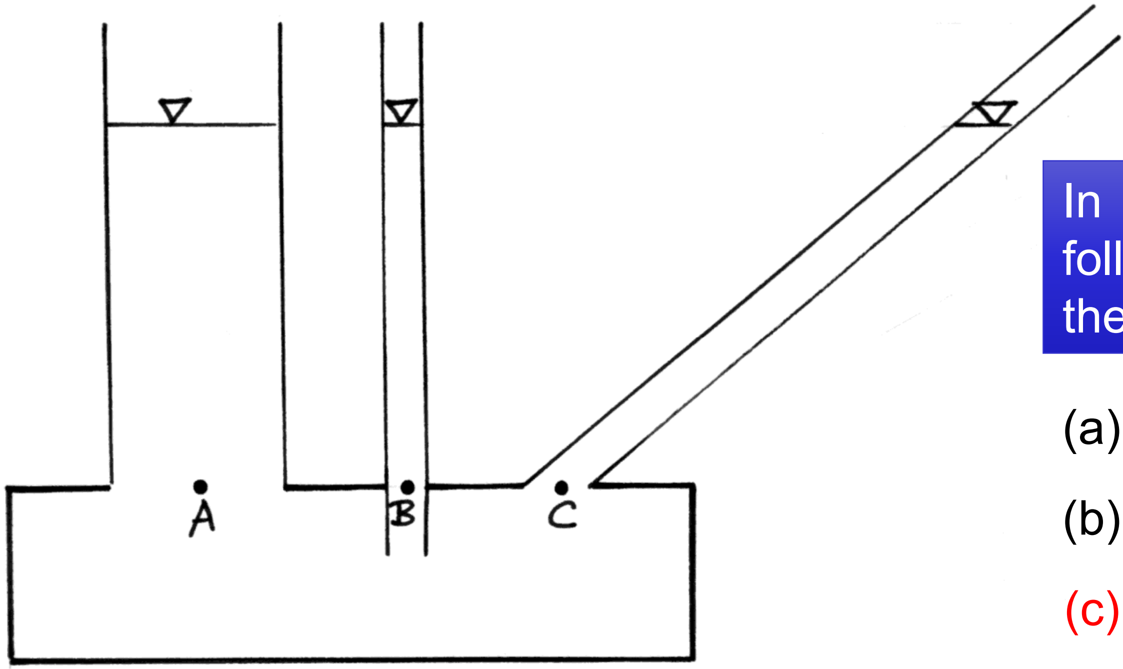

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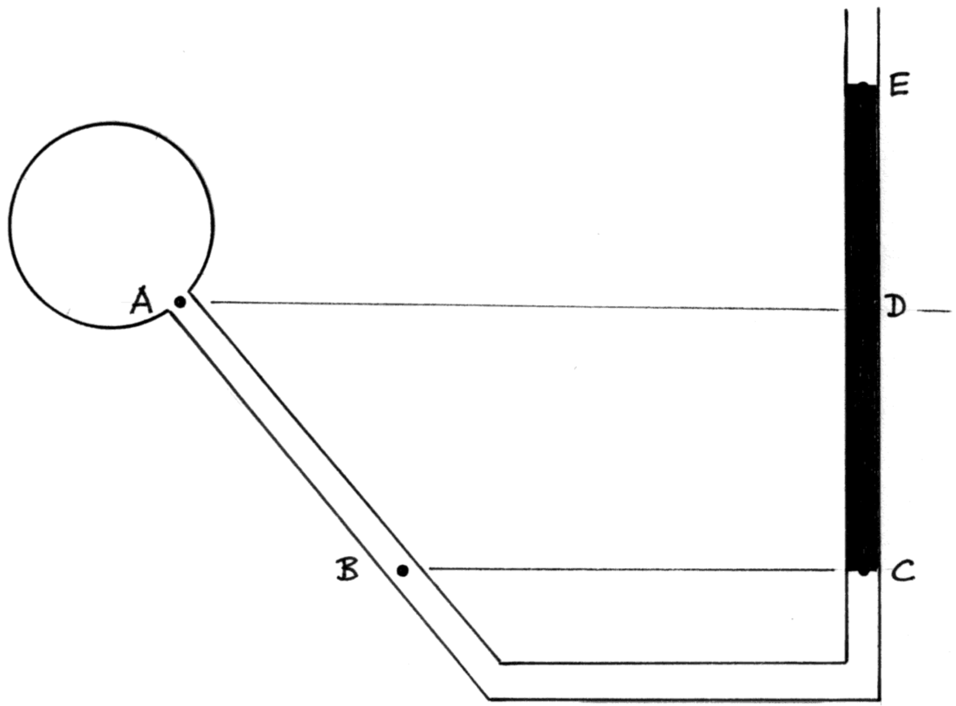
Application: Pressure variation in a manometer



In the manometer system, the points of equal pressure are

(a) A and D

(b) A and C

(c) B and C

(d) B and D

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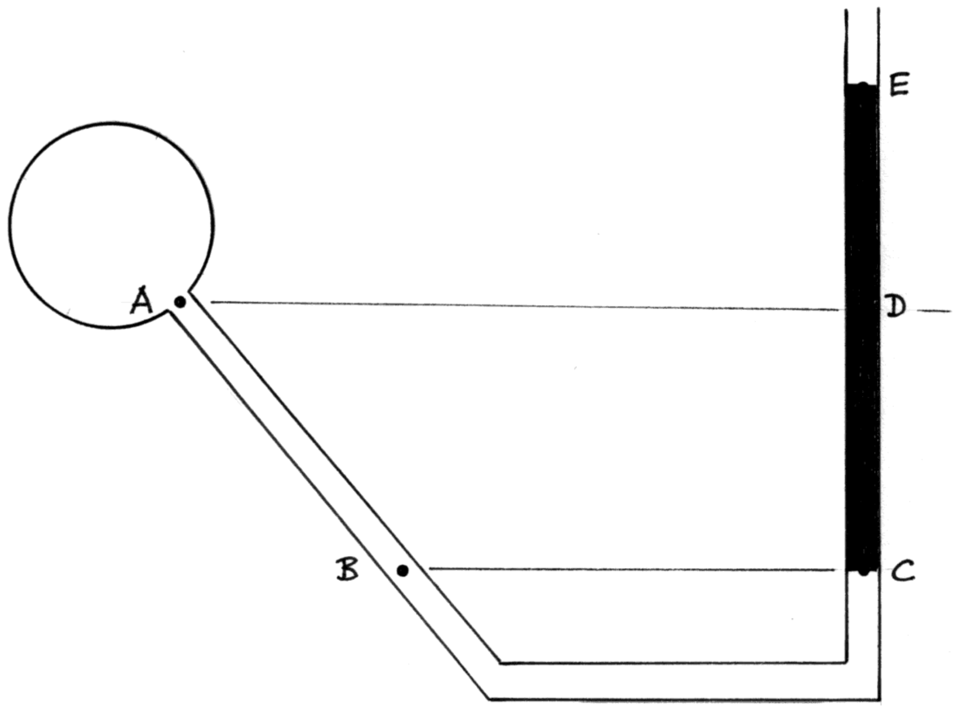

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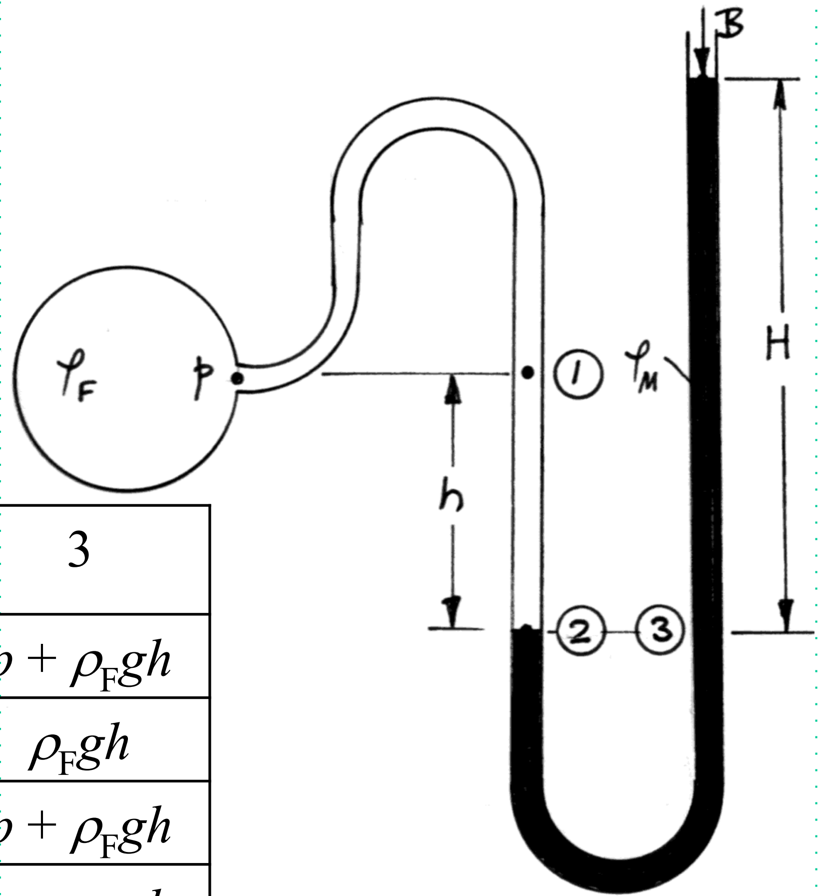
Hydrostatics – Interactive question

Principle: In a fluid at rest, isobars (i.e. lines or surfaces of constant pressure are horizontal)

Application: U - tube manometer



The following table shows the pressures at locations 1, 2 and 3. Which row of the table is correct?

	1	2	3	3
(a)	P	$p - \rho_F gh$	$B - \rho_M gH$	$p + \rho_F gh$
(b)	0	$\rho_M gh$	$\rho_M gH$	$\rho_F gh$
(c)	P	$p + \rho_F gh$	$B + \rho_M gH$	$p + \rho_F gh$
(d)	p	$p + \rho_F gh$	$B - \rho_M gH$	$p - \rho_F gh$

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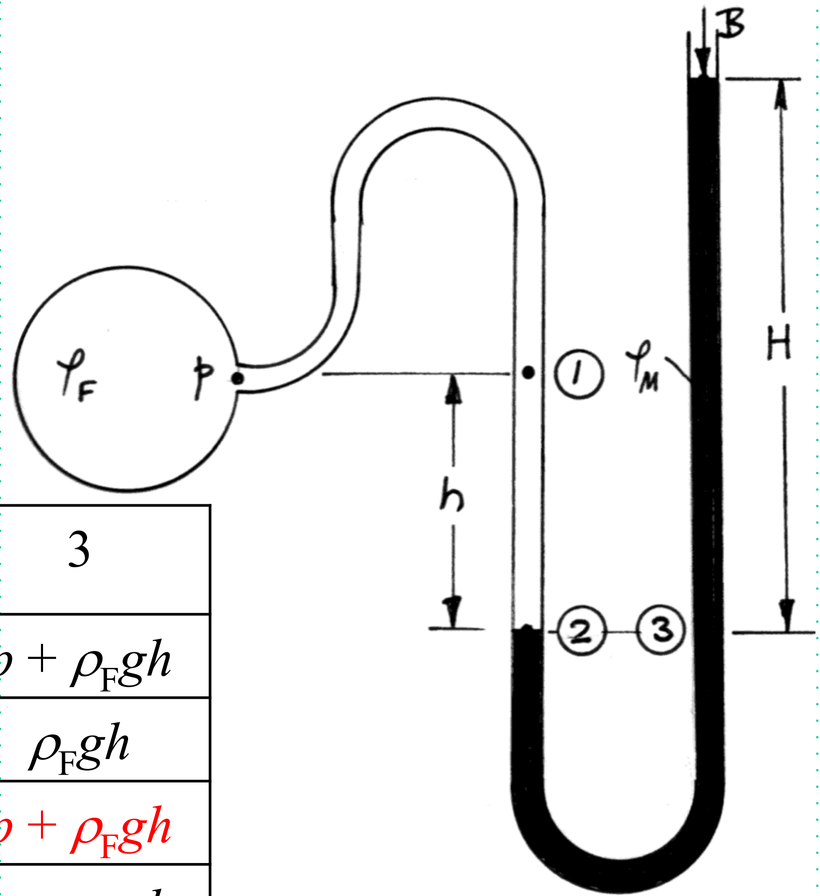

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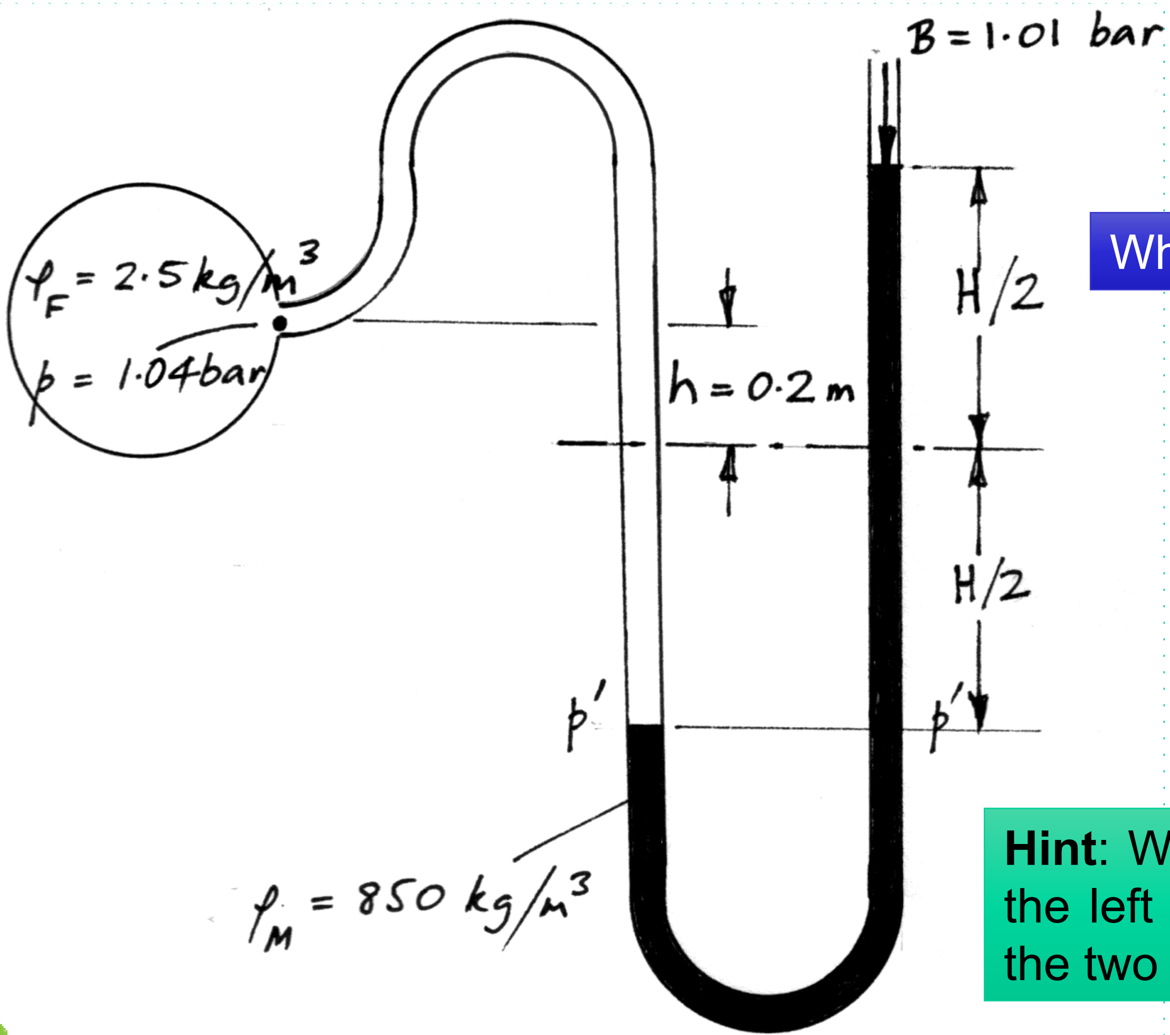
(c)	P	$p + \rho_F gh$	$B + \rho_M gH$	$p + \rho_F gh$
(d)	p	$p + \rho_F gh$	$B - \rho_M gH$	$p - \rho_F gh$

Hydrostatics – Interactive question



Which is the correct value for *H*?

(a) 0.361 m

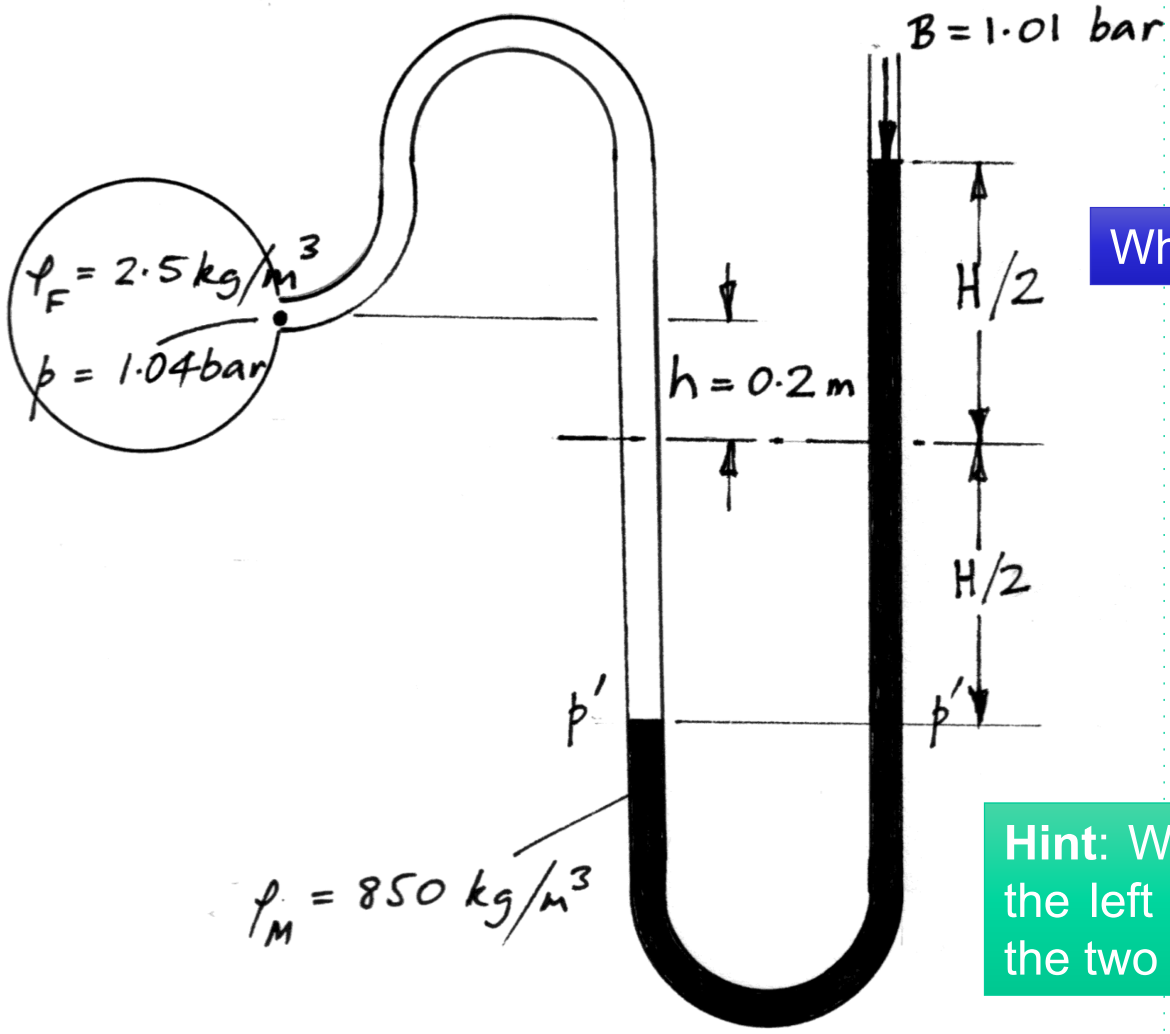
(b) 0.365 m

(c) 0.371 m

(d) 0.376 m

Hint: Write down equations for *p′* on the left and right-hand sides, equate the two and solve for *H*.

Hydrostatics – Interactive question



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Hydrostatics – Interactive question

LHS $p' = p + \rho_F g\left(h + \frac{H}{2}\right)$ RHS $p' = B + \rho_M gH$

$$p = B + \rho_M gH + \rho_F g\left(h + \frac{H}{2}\right)$$

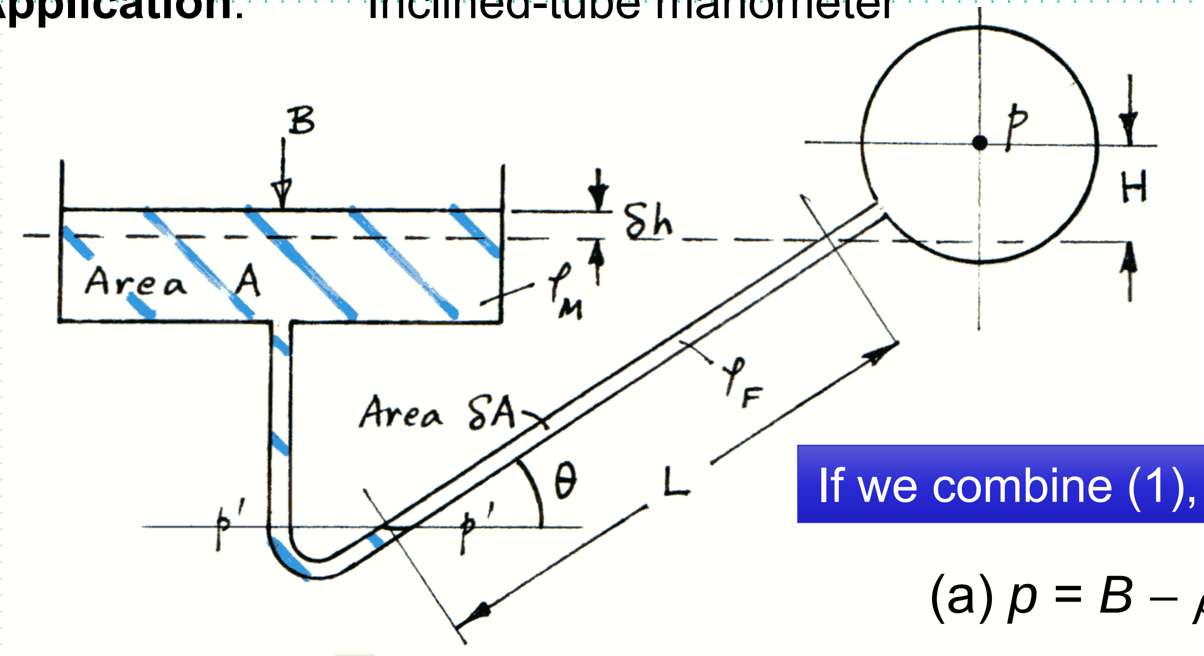
$$H = \frac{(1.04 - 1.01)10^5 + (2.5x9.81x0.2)}{\left(850 - \frac{2.5}{2}\right)x9.81}$$

$$H = 0.361\ m$$

Hydrostatics – Interactive question

Principles: Equal volumes and hydrostatic pressure variation

Application: Inclined-tube manometer



If we combine (1), (2) and (3), which is correct?

$L.\delta A = \delta h.A$ (1)

LHS $p' = B + \rho_M g\,(\delta h + L\sin\theta)$ (2)

RHS $p' = p + \rho_F g\,(H + L\sin\theta)$ (3)

(a) $p = B - \rho_F gH + \rho_M gL\left(\sin\theta + \dfrac{\delta A}{A}\right)$

(b) $p = B + \rho_M gL\sin\theta$

(c) $p = B + \rho_F gH + \rho_M g\ L\dfrac{\delta A}{A}$

(d) $p = B - \rho_F gH + \rho_M g\ L\dfrac{\delta A}{A} + (\rho_M - \rho_F)gL\sin\theta$

Hydrostatics – Interactive question

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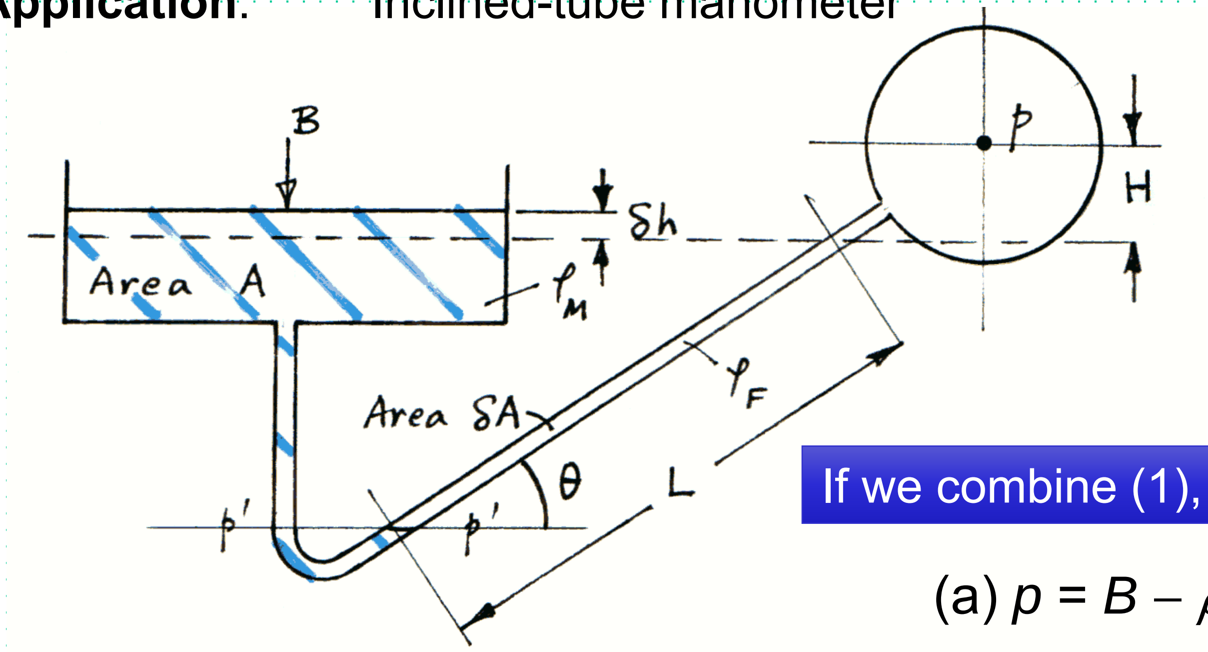

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(c) $p = B + \rho_F gH + \rho_M g\, L\dfrac{\delta A}{A}$

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$$L.\delta A = \delta h.A \qquad (1)$$

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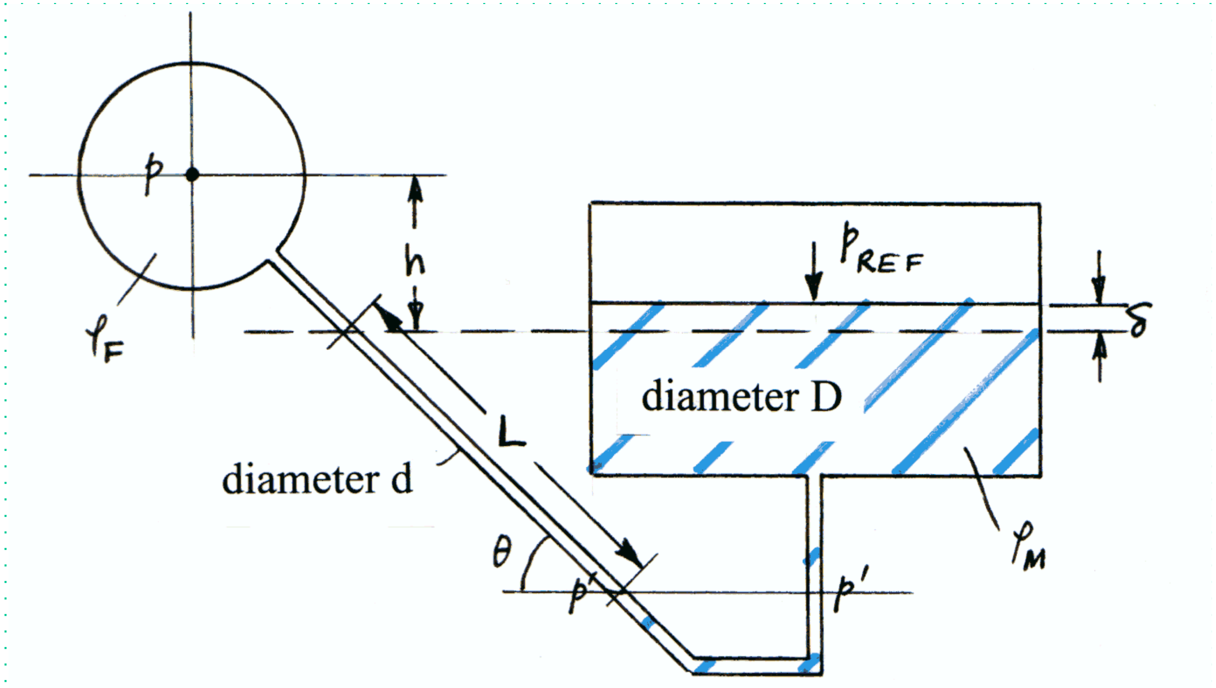
Hydrostatics – Interactive question

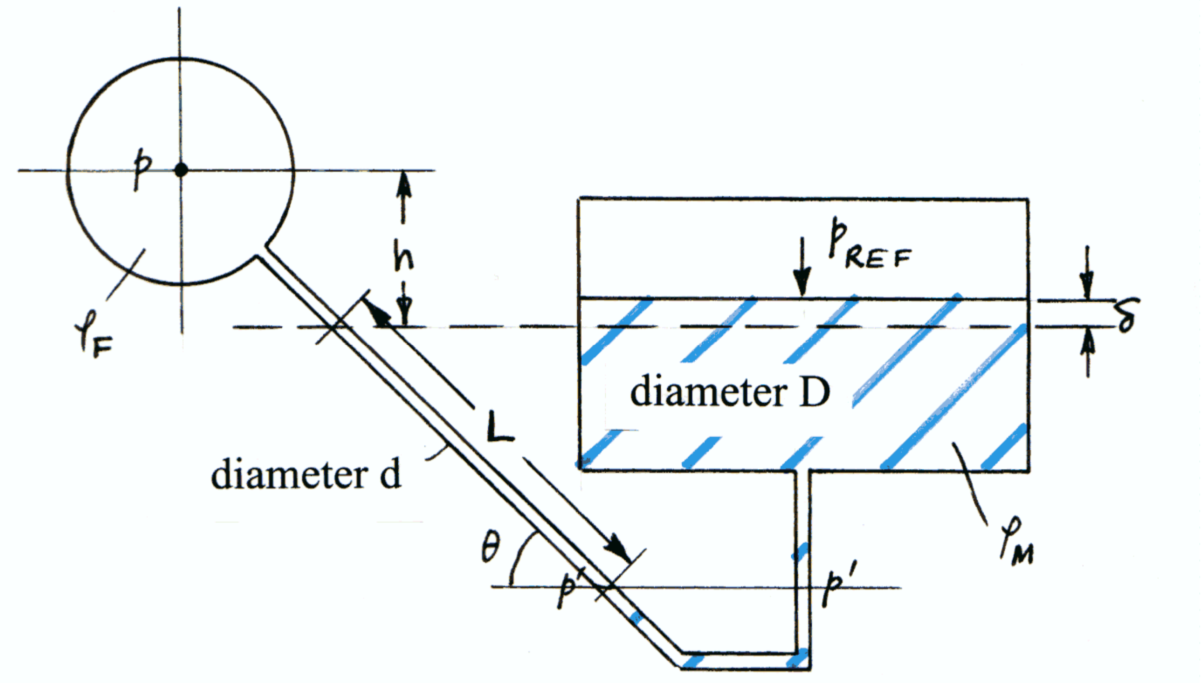


Principle: Equal volumes

If L = 400 mm, d = 5 mm and D = 100 mm, which is the correct value for δ?

(a) 20 mm (b) 1 mm (c) 0.2 mm

Hydrostatics – Interactive question



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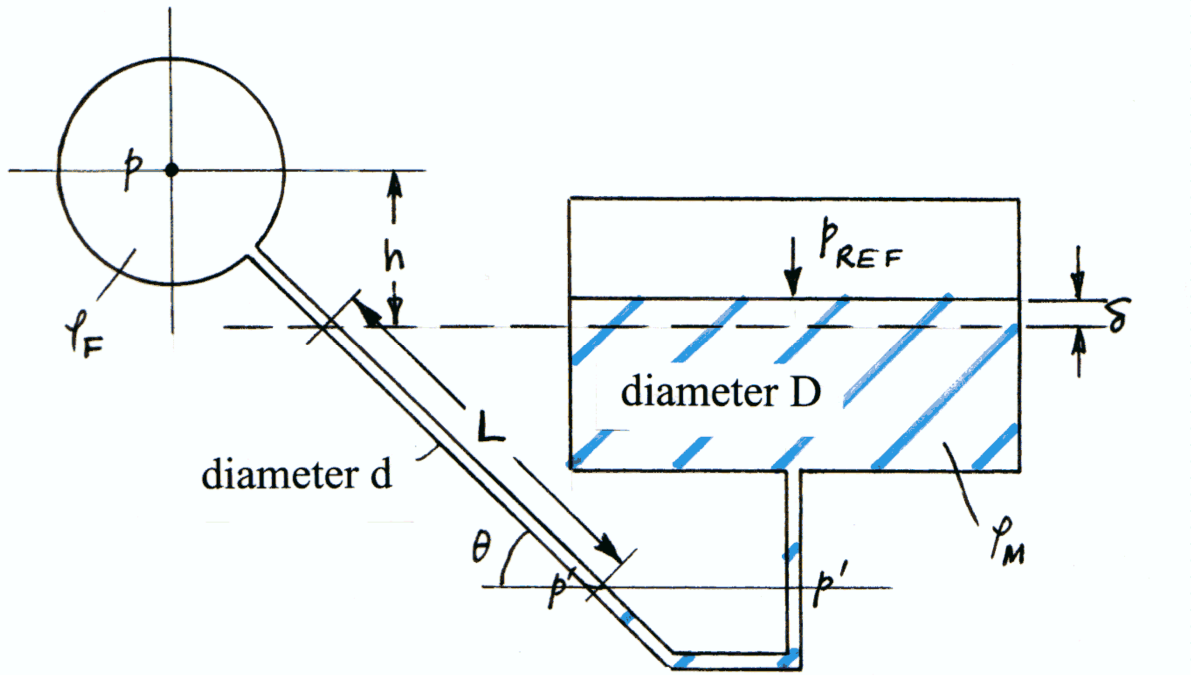
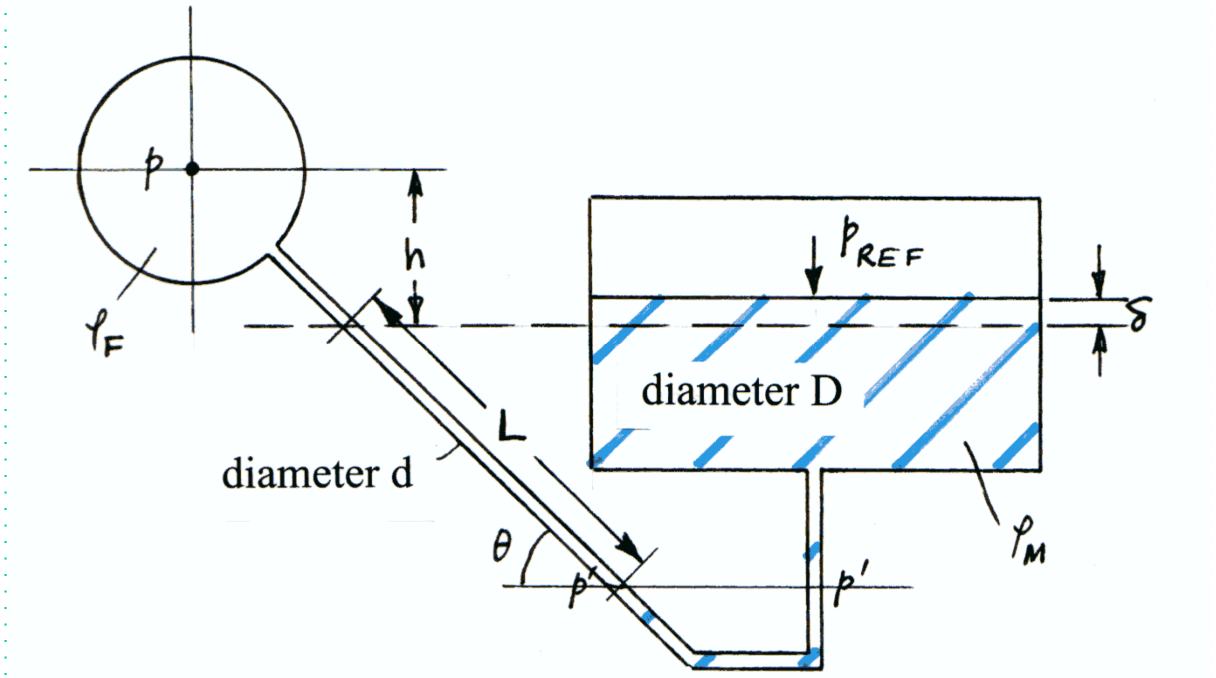


Principle: Hydrostatic pressure variation

If p_{REF} = 0.015 bar, θ = 30°, ρ_M = 850 kg/m^3, ρ_F = 2 kg/m^3 and h = 0.2 m, which is the correct value for p?

(a) 0.3 bar (b) 713 Pa (c) 3168 Pa (d) 1.7 bar

Hydrostatics – Interactive question



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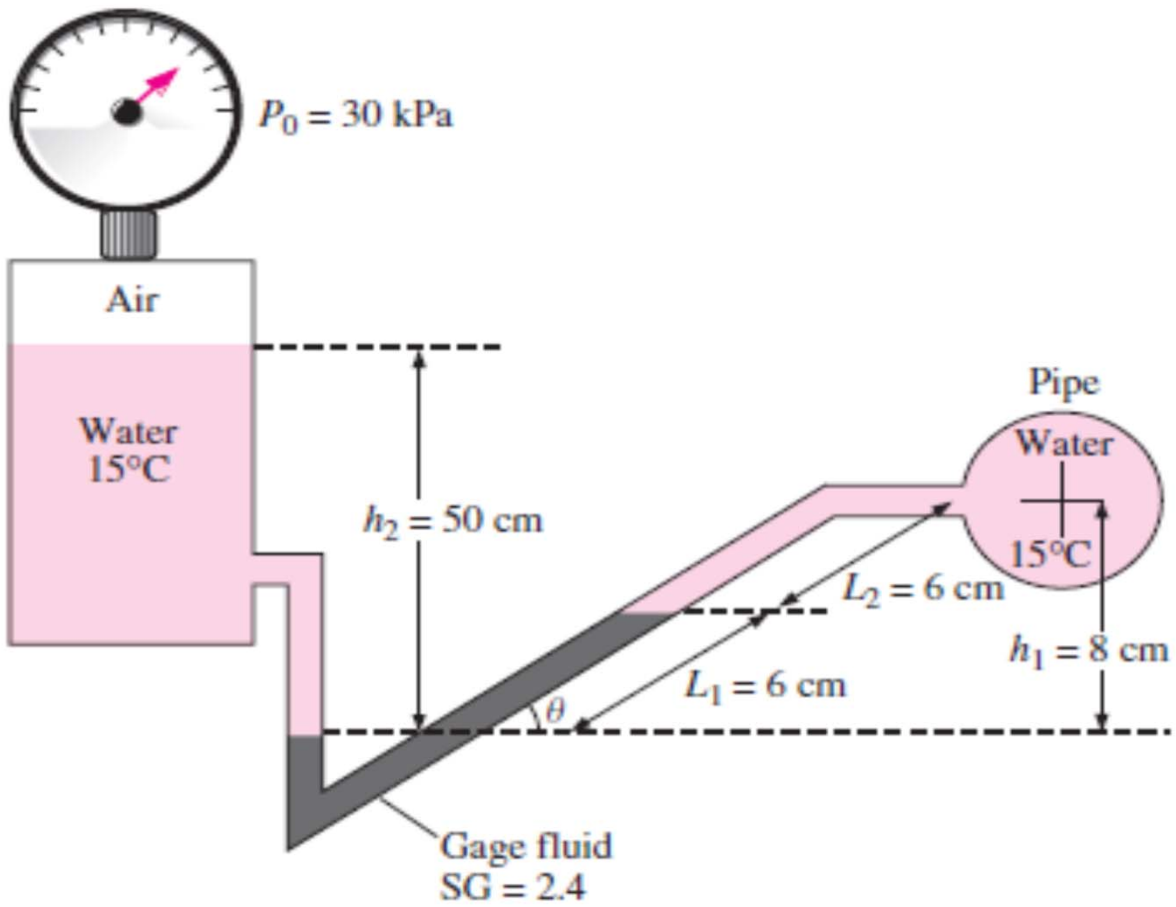
Hint: calculate *p′* for the right-and left-hand side, then find p from the equation

The pressure of water flowing through a pipe is measured by the arrangement shown in Figure. For the values given, calculate the pressure in the pipe.



$$P_{gage} + P_w g h_{w1} - \rho_{gage} g h_{gage} - \rho_w g h_{w2} = P_{water}$$

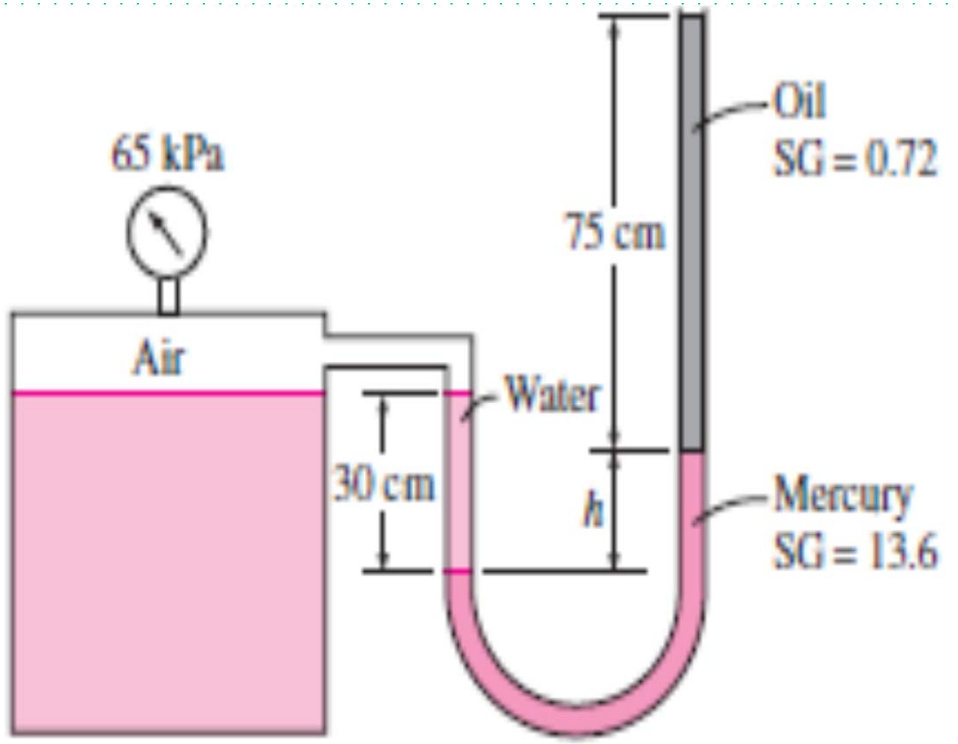
$$P_{water} = P_{gage} + \rho_w g\left(h_{w1} - SG_{gage} h_{gage} - h_{w2}\right)$$

$$= P_{gage} + \rho_w g\left(h_2 - SG_{gage} L_1 Sin\theta - L_2 Sin\theta\right)$$

θ =8/12= 0.6667

$$P_{water} = 30 + 1000 \cdot 9.81 \cdot (0.50 - 2.4 \cdot 0.06 \cdot 0.6667 - 0.06 \cdot 0.6667)/1000$$

The gage pressure of the air in the tank shown in Figure is measured to be 65 kPa. Determine the differential height *h* of the mercury column.



$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

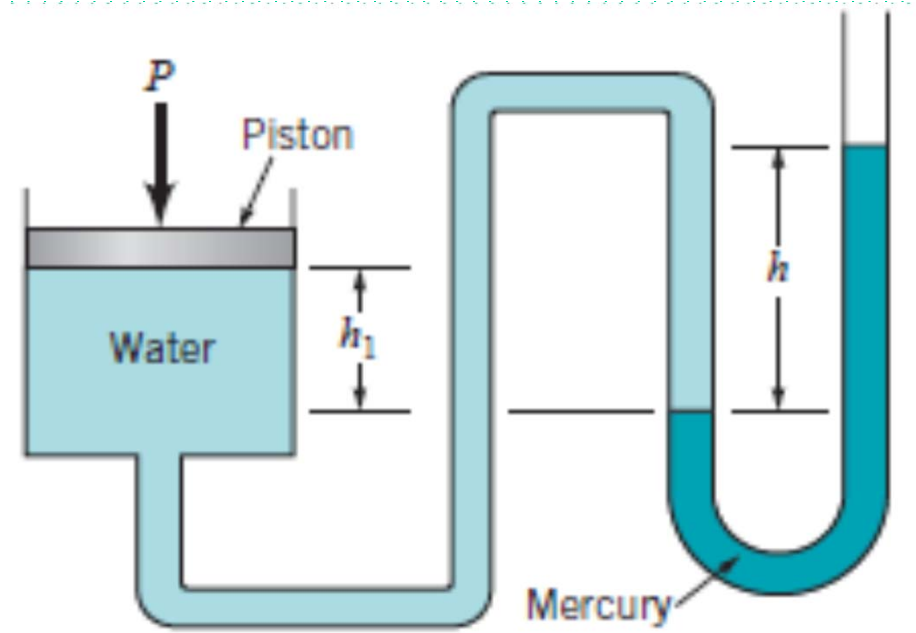
$$P_1 - P_{atm} = \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} - \rho_w g h_w$$

$$\frac{P_{1,gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{s,Hg} h_{Hg} - h_w$$

$$\left(\frac{65}{1000 \cdot 9.81}\right) \cdot 1000 = 0.72 \cdot 0.75 + 13.6 \cdot h_{Hg} - 0.3$$

$$\Rightarrow \; h_{Hg} = 0.47\ m$$

A piston having a cross-sectional area of 0.07 m^2 is located in a cylinder containing water as shown in Figure. An open U-tube manometer is connected to the cylinder as shown. For h_1=60 mm and h_2=100 mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.

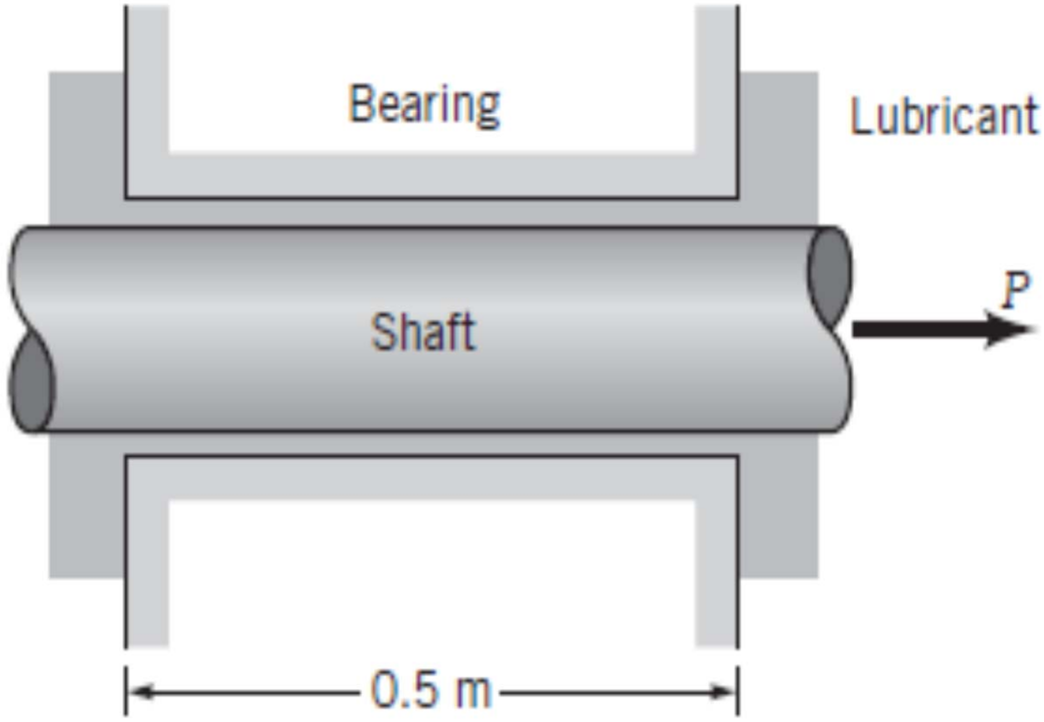


$$F = PA \quad P + \gamma_{water} h_1 - \gamma_{Hg} h = 0$$

$$P = \gamma_{Hg} h - \gamma_{water} h_1 = 133 * 0.1 - 9.90 * 0.06 = 1.27\ kN/m^2$$

$$F = (1.27x10^3)(0.07) = 889\ N$$

A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Figure. The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of 8.0 10^{-4} m^2/s and a specific gravity of 0.91. Determine the force *P* required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.



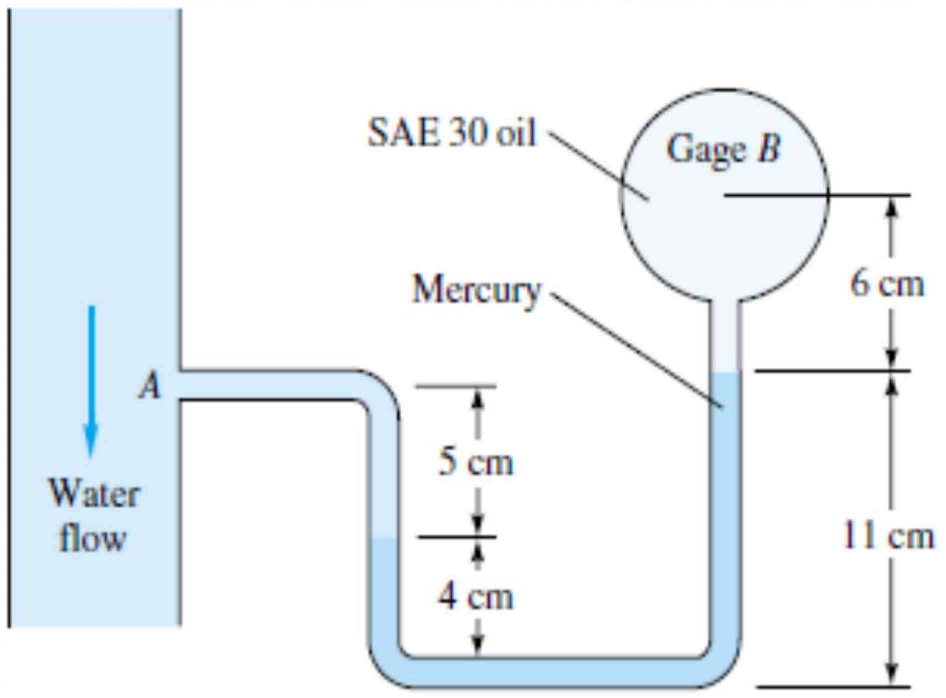
$$\sum F_x = 0 \qquad F = \tau A \qquad A = \pi D L$$

$$\tau = \mu \frac{V}{b}, \qquad so\ that, \quad F = \mu \frac{V}{b}\,\pi D L \qquad \mu = \nu\rho$$

$$F = \frac{(8.10^{-4}) * 0.91x\,10^{3} * 3\pi * 0.025 * 0.5}{0.0003} = 285.74\ N$$

Pressure gage *B* is to measure the pressure at point *A* in a water flow. If the pressure at *B* is 87 kPa, estimate the pressure at *A*, in kPa. Assume all fluids are at 20°C.

$$(\gamma_w = 9790\,\frac{N}{m^3}, \gamma_{mercury} = 133\,100\,\frac{N}{m^3}, \gamma_{oil} = 8720\,\frac{N}{m^3})$$



$$p_A - \gamma_W(\Delta z)_W - \gamma_M(\Delta z)_M - \gamma_O(\Delta z)_O = p_B$$

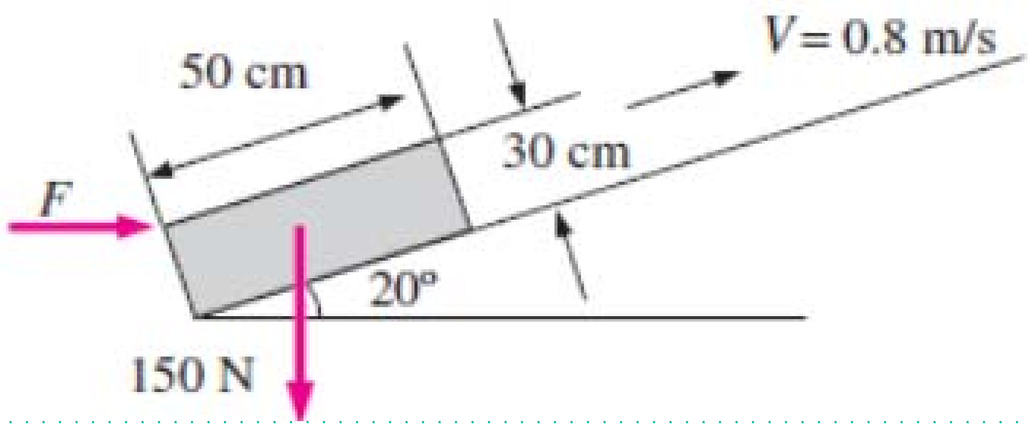
$$p_A - (9790 \text{ N/m}^3)(-0.05 \text{ m}) - (133{,}100 \text{ N/m}^3)(0.07 \text{ m}) - (8720 \text{ N/m}^3)(0.06 \text{ m})$$

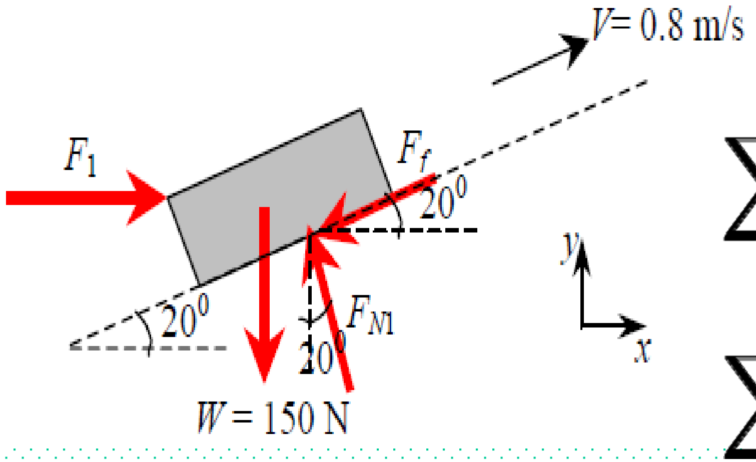
$$= p_A + 489.5 \text{ Pa} - 9317 \text{ Pa} - 523.2 \text{ Pa} = p_B = 87{,}000 \text{ Pa}$$

$$p_A = 96{,}351 \text{ Pa} = 96.4 \text{ kPa}$$

A 50x30x20-cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27.

(a) Determine the force *F* that needs to be applied in the horizontal direction.
(b) If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa.s is applied between the block and inclined surface, determine the percent reduction in the required force.





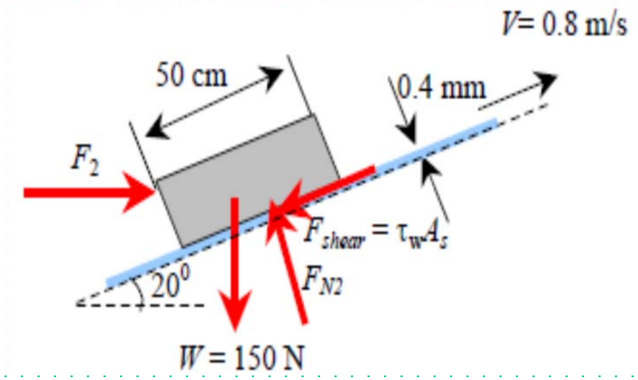
$$\sum F_x = 0: \qquad F_1 - F_{shear}\cos 20 - F_{N1}\sin 20 =$$

$$\sum F_y = 0: \qquad F_{N1}\cos 20 - F_{shear}\sin 20 - W =$$

Friction force: $F_f = fF_{N1}$

$$F_{N1} = \frac{W}{\cos 20 - f\,Sin\,20} = \frac{150}{\cos 20 - 0.27\,Sin\,20} = 177.0\,N$$

$$F_1 = F_f\cos 20 + F_{N1}\sin 20 = (0.27x177)\cos 20 + 177\sin 20 = 105.5\,N$$



$$F_{shear} = \tau_w A_s = \mu A_s \frac{V}{h} = 0.012 \cdot \frac{(0.5x0.2)0.8}{4x10^{-4}} = 2.4\ N$$

$$\sum F_x = 0: \qquad F_2 - F_{shear}\cos 20 - F_{N2}\sin 20 = 0$$

$$\sum F_y = 0: \qquad F_{N2}\cos 20 - F_{shear}\sin 20 - W = 0$$

$$F_{N2} = \frac{F_{shear}\sin 20 + W}{\cos 20} = \frac{(2.4\sin 20 + 150)}{\cos 20} = 160.5\ N$$

$$F_2 = F_{shear}\cos 20 + F_{N2}\sin 20 = 2.4\cos 20 + 160.5\sin 20 = 57.2\ N$$

Percentage reduction in the required force: $\frac{F_1 - F_2}{F_1} 100 = \frac{105.5 - 57.2}{105.5} 100 = 45.8\ \%$