HOMEWORK 3

1 The data $\{x[0], x[1], \ldots, x[N-1]\}$ are observed where the $x[n]$'s are independent and identically distributed (IID) as $\mathcal{N}(0, \sigma^2)$. We wish to estimate the variance σ^2 as

$$\hat{\sigma^2} = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n].$$

Is this an unbiased estimator? Find the variance of $\hat{\sigma^2}$ and examine what happens as $N \to \infty$.

2 The heart rate h of a patient is automatically recorded by a computer every 100 ms. In 1 s the measurements $\{\hat{h}_1, \hat{h}_2, \ldots, \hat{h}_{10}\}$ are averaged to obtain $\hat{h}$. If $E(\hat{h}_i) = \alpha h$ for some constant α and $\text{var}(\hat{h}_i) = 1$ for each i, determine whether averaging improves the estimator if $\alpha = 1$ and $\alpha = 1/2$. Assume each measurement is uncorrelated.

3 Two samples $\{x[0], x[1]\}$ are independently observed from a $\mathcal{N}(0, \sigma^2)$ distribution. The estimator

$$\hat{\sigma^2} = \frac{1}{2}(x^2[0] + x^2[1])$$

is unbiased. Find the PDF of $\hat{\sigma^2}$ to determine if it is symmetric about σ^2.